YuMi Deadly Maths

Literacy in Mathematics

Prep to Year 12

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

Prep to Year 12: Supplementary Resource 3 – **Literacy in Mathematics**

QUT

YuMiDeadly
Growing community through education

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YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at QUT dedicated to enhancing the learning of all students to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates YDC's vision: *Growing community through education*.

The YuMi Deadly Centre can be contacted at ydc@qut.edu.au. Our website is research.qut.edu.au/ydc.

ABOUT YUMI DEADLY MATHS AND THIS RESOURCE

From 2000–09, researchers who are now part of the YuMi Deadly Centre (**YDC**) collaborated with principals and teachers predominantly from Aboriginal and Torres Strait Islander schools and occasionally from low socio-economic status (SES) schools in a series of small projects to enhance student learning of mathematics. These projects tended to focus on a particular mathematics strand (e.g. whole-number numeration, operations, algebra, measurement) or on a particular part of schooling (e.g. middle school teachers, teacher aides, parents). They resulted in the development of specialist materials but not a complete mathematics program (these specialist materials can be accessed via the YDC website, research.qut.edu.au/ydc).

In October 2009, YDC received funding from the Queensland Department of Education and Training through the Indigenous Schooling Support Unit, Central-Southern Queensland, to develop a train-the-trainer project, called the **Teaching Indigenous Mathematics Education** or **TIME** project. The aim of the project was to enhance the capacity of schools in Central and Southern Queensland Indigenous and low SES communities to teach mathematics effectively to their students. The project focused on Years P to 3 in 2010, Years 4 to 7 in 2011 and Years 7 to 9 in 2012, covering all mathematics strands in the Australian Curriculum: Number and Algebra, Measurement and Geometry, and Probability and Statistics. The work of the TIME project across these three years enabled YDC to develop a cohesive mathematics pedagogical framework, **YuMi Deadly Maths**, that covers all strands of the *Australian Curriculum: Mathematics* and now underpins all YDC projects.

YuMi Deadly Maths (**YDM**) is designed to enhance mathematics learning outcomes, improve participation in higher mathematics subjects and tertiary courses, and improve employment and life chances. YDM is unique in its focus on creativity, structure and culture with regard to mathematics and on whole-of-school change with regard to implementation. It aims for the highest level of mathematics understanding and deep learning, through activity that engages students and involves teachers, parents and community. With a focus on big ideas, an emphasis on connecting mathematics topics, and a pedagogy that starts and finishes with students' reality, it is effective for all students. It works successfully in different schools/communities as it is not a scripted program and encourages teachers to take account of the particular needs of their students. Being a train-the-trainer model, it can also offer long-term sustainability for schools.

YDC believes that changing mathematics pedagogy will not improve mathematics learning unless accompanied by a whole-of-school program to challenge attendance and behaviour, encourage pride and self-belief, instil high expectations, and build local leadership and community involvement. YDC has been strongly influenced by the philosophy of the Stronger Smarter Institute (C. Sarra, 2003) which states that any school has the potential to rise to the challenge of successfully teaching their students. YDM is applicable to all schools and has extensive application to classrooms with high numbers of at-risk students. This is because the mathematics teaching and learning, school change and leadership, and contextualisation and cultural empowerment ideas that are advocated by YDC represent the best practice for **all** students.

This resource supplements the series of books that describe the YDM approach and pedagogical framework for Prep (Foundation) to Year 9. It focuses on an aspect of mathematics teaching and learning that is part of all topics, namely **literacy**. It discusses how literacy is different in mathematics, identifies six components of literacy in mathematics, and describes these components and how they relate to each other. It proposes a scope and sequence to teach the components across Foundation to Year 12. *Literacy in Mathematics* is the third supplementary YDM resource book. The others are *Big Ideas of Mathematics* and *Problem Solving*. Because YDM is largely implemented within an action-research model, the resources undergo amendment and refinement as a result of school-based training and trialling. The ideas in this resource will be refined into the future.

YDM underlies three types of projects available to schools: (a) general training in the YDM pedagogy (through a variety of projects titled with YDM acronym); (b) Accelerated Inclusive Mathematics (AIM) training in remedial pedagogy to accelerate learning (through AIM projects, XLR8 projects and AIM Early Understandings projects); and (c) Mathematicians in Training Initiative (MITI) training in enrichment/extension pedagogy to build deep learning of powerful maths and increase participation in advanced Years 11–12 and tertiary maths subjects.

Contents

Abbreviations

ACARA	Australian Curriculum, Assessment and Reporting Authority
NAPLAN	National Assessment Program – Literacy and Numeracy
QCAA	Queensland Curriculum and Assessment Authority
RAMR	Reality–Abstraction–Mathematics–Reflection

Figures and Tables

1 Summary and Overview

This chapter introduces this resource dealing with literacy in mathematics and summarises its major ideas. Some terms used throughout the book require explanation before proceeding further.

While *mathematical literacy* is used by some as a synonym for numeracy (particularly in the USA), in this resource it is used to describe the development of language and literacy in mathematics through the application of skills in speaking, listening, vocabulary, reading and writing.

A *text* is defined to be any form of communication, both spoken and written, and can include words, symbols, tables and visual images.

From a mathematics perspective, a *visual image* is a print or electronic picture or representation of something or someone, across a broad range from tables, graphs and diagrams that present information, through to illustrations intended to enhance engagement.

1.1 Overview of resource

This resource explains the unique features of literacy in mathematics. It deals with the three major components of literacy in mathematics: vocabulary, decoding (e.g. listening and reading) and encoding (e.g. speaking and writing). It provides a bridge between general literacy development and mathematics content and pedagogy. The main ideas underpinning this resource are summarised in this section and provide some strategies for developing the necessary skills.

The development of language in the context of mathematics is a progression, as follows:

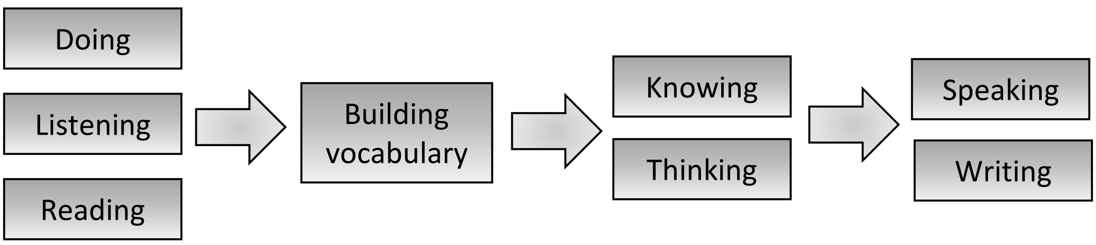


Figure 1.1 The process of developing language and literacy in mathematics

From a literacy perspective, listening and reading are the ways that we receive information from others, including building the mathematical vocabulary that we need to think and know. Using that vocabulary when speaking and writing are the ways that we transmit information to others. However, doing, knowing and thinking, more commonly thought of as aspects of pedagogy, are also essential parts of the literacy process. This means that pedagogy and literacy are inextricably entwined. This resource attempts to build on the mathematics pedagogy developed in the other YuMi Deadly Maths publications. In that sense it is a supplementary resource; however, there are inevitable overlaps between pedagogy and literacy.

Children first learn to communicate mathematically through speech and listening. As their mathematical ideas develop in both complexity and abstraction, written forms of communication (such as prose, symbols and visuals) come to the fore. The development of formal mathematical writing starts to become important in the upper primary years.

Literacy is a General Capability of the Australian Curriculum, requiring that students develop the knowledge, skills and confidence to interpret and use language effectively. As Figure 1.1 demonstrates, it is an important aspect of mathematics, as students are required to interpret and create a range of texts (spoken and written) typical of mathematics, including:

- posing and answering questions;
- discussing, producing and explaining solutions;
- understanding instructions and word problems; and
- engaging in mathematical problem solving.

In this resource, mathematical literacy is discussed in four contexts: listening and speaking, vocabulary, decoding and encoding. As symbols and visual images are important components of decoding and encoding, each is given a separate chapter. The book concludes by providing a program for teaching literacy to students at all levels in the context of mathematics to meet the demands of the Australian Curriculum.

Listening and speaking in the mathematics classroom serves two purposes: as a medium for students to develop their mathematical thinking; and as a strategy for teachers to engage students in mathematics as they assess and plan. Teachers need to use conversation to engage students in mathematics and continue to enable them to converse throughout the stages of the mathematics lesson. Establishing a culture of shared ideas and guiding students as they discuss their mathematical understanding enriches mathematical thinking. Conversation needs to be modelled, taught and revisited many times within the mathematics context/classroom.

The **mathematical vocabulary** consists of words used in everyday English and words with specific mathematical meanings. Mastery of these words is vital for effective communication. This resource proposes strategies for developing the mathematical vocabulary, and provides word lists for use in vocabulary development and spelling activities, classified by Australian Curriculum content strand and stage of schooling.

Children start **decoding** mathematical texts through listening. As they develop as mathematicians they encounter written mathematical texts in teacher presentations, written notes and instructions, and worded problems. The nature of mathematical texts makes them harder to decode than most other text types. It is critical that students know and can use the task words to tell them what to do in mathematics. Decoding the text of worded problems is particularly challenging, with studies reporting that up to 64% of errors in mathematical problem solving can be attributed to errors in the interpretation of the problem text. Mathematical problem texts should be read at least three times: a **quick scan** to gain an overall feel for the problem; a **close reading** to gather all the required details from the text; and after a solution has been found, a **final reading** to ensure that the proposed solution fully answers the question. In addition to decoding, students must develop the ability to **critically analyse** informative mathematics texts.

Creating a text is the process of **speaking and writing** to create a text. In the case of mathematics, the texts are informative. The process of writing is complex. There are two important aspects of writing in mathematics: the mathematics must be accurate, and the writing must be grammatically correct. It is easy for experienced writers such as teachers to underestimate the challenges for beginning writers. Therefore, the process of writing should be broken down into manageable steps for students, starting with speaking. An understanding of the symbols used in mathematics is essential. Some forms of mathematics writing such as reports, mathematical arguments, and definitions are unique to mathematics.

This resource has been developed for two audiences with differing needs. Teachers of mathematics in the primary years are usually skilled in teaching literacy across all curriculum areas. However, they may lack knowledge of how literacy is different in mathematics. For those teachers, this resource explains some of the features of mathematical literacy that differ from literacy in other curriculum areas. In contrast, teachers of secondary mathematics are usually experts in talking about mathematical content and writing mathematically, but may have little training or experience in the teaching of vocabulary and literacy. For secondary teachers the resource explains some of the basics of teaching literacy.

1.2 What is literacy in mathematics?

Literacy is a General Capability of the Australian Curriculum. According to the Australian Curriculum, Assessment and Reporting Authority (ACARA):

> Students become literate as they develop the knowledge, skills and dispositions to interpret and use language confidently for learning and communicating in and out of school and for participating effectively in society. Literacy involves students in listening to, reading, viewing, speaking, writing and creating oral, print, visual and digital texts, and using and modifying language for different purposes in a range of contexts. (ACARA, 2017, Home/F-10 Curriculum/General capabilities/ Literacy page)

In particular, in the context of mathematics, ACARA says that:

> In the Australian Curriculum: Mathematics, students learn the vocabulary associated with number, space, measurement and mathematical concepts and processes. This vocabulary includes synonyms, technical terminology, passive voice and common words with specific meanings in a mathematical context. Students develop the ability to create and interpret a range of texts typical of mathematics ranging from calendars and maps to complex data displays. Students use literacy to understand and interpret word problems and instructions that contain the particular language features of mathematics. They use literacy to pose and answer questions, engage in mathematical problem-solving, and to discuss, produce and explain solutions. (ACARA, 2017, Home/F-10 Curriculum/General capabilities/Literacy page)

The foundations of literacy are developed for all learning areas by the teachers of English. However, to progress mathematically, it is important for teachers to recognise that literacy in mathematics differs from literacy in other learning areas. It follows that it must be taught in the context of mathematics. While mathematics teachers draw on the foundations developed by teachers of English, it cannot be assumed that students automatically transfer the literacy skills taught in other learning areas to the mathematics context. For example, according to the Queensland Curriculum and Assessment Authority (QCAA): “The responsibility for developing and monitoring students’ abilities to use effectively the forms of language demanded [in mathematics] rests with the teachers of mathematics.” (QCAA, 2014, p. 37).

1.3 Cultural considerations

In the case of students who are learners of English as an additional language there are the language issues that arise in any learning area, including general comprehension and vocabulary development.

While some argue that mathematics transcends geographic boundaries and that the language of mathematics is universal, cultural factors are important in mathematics. For example:

- Indigenous cultures have a world view that emphasises the whole rather than the parts, whereas Eurocentric mathematics promotes the division of the whole into parts;
- currencies, and the symbols associated with money, vary from country to country;
- although the use of the imperial measurement system continues in only three countries (USA, Myanmar and Liberia), the cultural and trading influence of the USA and the partial use of imperial measurements in some other counties that have formally adopted the metric system (for example, the UK) means that students will encounter aspects of the imperial measurement system;
- the meaning of other symbols can vary, for example the decimal point;
- some cultures value computational fluency, and emphasise rote learning of number facts; and
- different computational methods and approaches to mathematical thinking are used in different parts of the world.

These cultural factors can impact on the use of mathematical language, even in the case of first-language English speakers.

1.4 Overview of resource

This resource comprises the following chapters:

Chapter 1: Summary and overview – considering the meaning of literacy in mathematics, and teaching and cultural implications;

Chapter 2: Listening and speaking – focusing on the early development of mathematical literacy through meaningful classroom conversations and discussions about mathematics generally, and specifically about computations and problem solving;

Chapter 3: The mathematical vocabulary – focusing on the lexicon of mathematics, particularly the nature of the vocabulary, sources of confusion, and spelling;

Chapter 4: Making meaning of mathematics texts (decoding) – dealing with reading and comprehension of mathematical instructions and problems, and diagnosing student decoding problems;

Chapter 5: Creating mathematics texts (encoding) – focusing on the writing process, the unique nature of mathematical writing, and the development of texts in the genres commonly used in mathematics;

Chapter 6: Symbols – explaining how symbols tell stories and examining the lexicon and syntax of mathematical symbols;

Chapter 7: Visual images – focusing on tables, visual images of all types (informative and narrative), the associated vocabulary and pedagogical strategies; and

Chapter 8: Literacy program – providing a program for teaching mathematical literacy from Foundation to Year 12.

Additionally there are six appendices that provide useful resources for teaching mathematical literacy.

2 Listening and Speaking

This chapter describes the role of listening and speaking in the mathematics classroom and its importance in the development of mathematical language. It is an important part of learning foundational mathematics concepts; however, the use of meaningful mathematics conversations amongst students is an often-underrated pedagogical tool in the secondary years. This chapter proposes some strategies for encouraging listening and speaking in the classroom.

Language is a social concept that is developed through social interactions for communication purposes. According to Lev Vygotsky (1978, 1986), an early 20th century Russian psychologist, language acquisition involves not only a child's exposure to words but also an interdependent process of growth between thought and language. He contended that children learn through social experiences in their communities. Through these interactions, more knowledgeable others (usually teachers, but can also be other students) impart skills, values and knowledge to learners. Vygotsky viewed language as humans' greatest tool. When directed to others, it is a means for communicating with the outside world, and when used internally as private speech, it is a medium for thinking.

Vygotsky hypothesised that thought and language are initially separate systems in the child, merging at around three years of age. Speech and thought then become interdependent, with thought becoming verbal and speech becoming representational. While Vygotsky's own research was not based on an analysis of classrooms activities, it has provided the stimulus for much research into the process of teaching and learning in schools. The application of Vygotsky's theories to education is commonly called a *sociocultural* educational perspective.

Vygotsky argued that "children solve practical problems with the help of their speech, as well as with their eyes and hands" (Vygotksy, 1978, p 26). It follows that conversation (listening and speaking) or discourse in the mathematics classroom is vital and encourages the dual purposes of language: as a medium for students to develop their mathematical thinking; and as a strategy for teachers to engage students in mathematics as they assess and plan. Teachers need to use conversation to engage students in mathematics and to continue to enable them to converse throughout the stages of the mathematics lesson. Establishing a culture of shared ideas and guiding students as they talk about their mathematical understanding enriches mathematical thinking. Conversation needs to be modelled, taught and revisited many times within the mathematics context/classroom.

Children first learn to communicate mathematically by listening to speech. They then learn to communicate mathematical ideas through speech, for example when counting and identifying fundamental concepts such as *more* and *less*. It is the means of developing the lexicon that is the foundation of mathematical communication. The development of the mathematical lexicon in the early years focuses on everyday English words and simple mathematical concepts such as number and shape, so it can be integrated with general vocabulary development. Although written forms of mathematical communication begin to be developed as students progress in their schooling, speech remains the most important form of mathematical communication.

2.1 Creating classroom conversations in mathematics

None of us can learn without the opportunity to practice. In classrooms where teachers talk most of the time, outcomes are lowered (Hattie, 2012). Teachers need to share the opportunity to talk with students. Switching from traditional teacher-centred classroom to a conversation-rich one is challenging for teachers and students. It requires a major shift in teaching, and students need guidance to learn to share their strategies and ideas effectively and confidently.

Strategies to build student mathematical conversations and confidence include the following:

1. Create ideas for dialogue with and amongst students, encouraging them to:
   (a) talk about their thinking (using age-appropriate language such as, or similar to: conclude, deduce, discover, elaborate, estimate, happen, hypothesise, imagine, infer, guess, organise, plan, predict, reason, remember, recognise, suggest);
   (b) tell and act out mathematical stories and situations; and
   (c) look, see and visualise – using their "maths eyes".
2. Give students opportunities to talk about their thinking and reasoning by describing, discussing, elaborating, explaining, justifying, and so on, supported by manipulatives and games.
3. Ask questions that give students opportunities to use mathematical and everyday vocabulary to verbalise their thinking.
4. Encourage active listening to each other and the teacher, through questioning, summarising and paraphrasing.
5. Develop the mathematical and everyday language needed to describe:
   (a) mathematical concepts – verbalising stages and/or parts; and
   (b) their own reasoning.

2.2 Conversations about problem solving

Discussing problems from the students' perspective provides opportunities to develop real mathematical conversations. These discussions can involve the whole class, small groups, or two people (either two students or student and teacher). They enable students to recognise (or invent) a problem situation and have a go at resolving it by making links to prior knowledge. Using familiar contexts and having a meaningful purpose will ensure an animated discussion.

Initially, conversations about problem solving give students the chance to pose questions and to develop plans to solve problems. By discussing strategies students discover that there are often several approaches that can be used to successfully solve a problem. It is important to understand that we don't all do the same thing and/or think the same way mathematically. Acknowledging others' point of view is a powerful conversation strategy in mathematics.

Digging deeper, conversations occur when students display their work and present their strategies, ideas and solutions. As students explain their thinking others can see connections and the usefulness of different methods. It is important that we, as effective teachers, emphasise that it is not just important to solve the problem but also to learn the strengths and weaknesses of different strategies. Discussions can then focus on which is the best strategy in terms of rigour, effort, and simplicity. Mathematicians often refer to the best solution as the "most elegant".

Using frameworks fosters conversation. Students need encouragement to engage in discussion in mathematics lessons (after all, there are very limited opportunities to practise mathematical conversations outside of the mathematics classroom) and using a simple concept map can enable introductory sharing of ideas between pairs of students. Using the concept map without the sharing would help students to record their own ideas but does not help them expand their thinking through conversation with another. It is a very effective way to practise using the language of mathematics in a meaningful way.

2.3 Conversations about computation

Telling problem stories, using language of reasoning and thinking and developing the ability to ask questions are vital for students to develop an understanding of computation. Students must see computation as a tool for solving real-world problems, not an end in itself. Undue emphasis on decontextualised computations leads to misconceptions about the nature of mathematics.

Encouraging and enabling students to discuss computation helps to develop efficient algorithms. It puts the focus on mental strategies. By comparing computational strategies, students learn that the optimum computational strategy varies according to the situation (e.g. 99 + 23 and 56 + 21 should require different methods).

Sharing emerging or unconventional methods for computation stretches everyone's mind. It illustrates that there are often many ways to do things and is useful to build connections. Trying different ways to do things can enrich thinking and understanding and ultimately help students determine the most efficient strategy.

2.4 Building classroom conversations and vocabulary

Maths vocabulary is often introduced as labels, lists of words or posters with definitions. However, students do not really understand what they mean until they have had first-hand experience with what the word applies to. Understanding develops as students use and hear words in context in the real world while attempting to solve problems.

Using classroom conversation and exploration of the world around them helps students to develop the concepts and ideas of mathematics. Allow students to converse using their own words to explain their understanding of the new concepts and gradually introduce the relevant mathematical terms to build students' vocabulary. The Abstraction phase of the Reality–Abstraction–Mathematics–Reflection (RAMR) cycle is rich in this process.

In mathematics small words make big differences. Many small words such as pronouns, prepositions and conjunctions make a big difference in the understanding of a mathematics problem. For example,

- *of* and *off* cause a lot of confusion in solving percentage problems, as the *percent of* something is quite distinct from the *percent off* something; and
- the word *a* can mean *any* in mathematics – when asking students to *show that a number divisible by 6 is even*, we aren't asking for a specific example, but for the students to show that all numbers divisible by 6 have to be even.

Teachers also need to be aware of the importance of enunciating the small but significant words precisely, not only to enhance computational skills but also to help students answer open-response questions more accurately.

The role of listening and speaking in developing vocabulary is explored in more detail in section 3.2.1. **Appendix A** provides a list of words used in mathematics that commonly confuse students.

2.5 Using mathematical language meaningfully

Talking gets students involved. The mathematics classroom should be an exciting place when we establish a culture of shared ideas and guide students as they discuss their mathematical understanding. Using the language of mathematics in a meaningful way is an everyday necessity. As with any other language, it should be practised regularly.

3 The Mathematical Vocabulary

This chapter describes the unique features of the mathematical vocabulary and some strategies for developing that vocabulary in students.

3.1 The nature of mathematical vocabulary

The mathematical lexicon (vocabulary) consists of three classes of words: those used in **everyday English**; words with different ordinary English and mathematical meanings (called **sub-technical** words); and words that have meaning only in the context of mathematics (called **technical** words) (Pierce & Fontaine, 2009).

Mathematics has many words with irregular plural forms, such as *die/dice*, or no plural form, such as *dozen*. Often, they originate from Latin or Greek: *axis/axes, radius/radii*. In some cases, there are two acceptable plurals, for example *formula/formulae/formulas*. In others, the different plural forms have subtle differences in meaning; for example, *indices* is the plural of *index* when used mathematically, but *indexes* is commonly used to describe the content lists at the back of books.

Appendix B provides a list of words in the mathematical vocabulary, based on the language demands of the Australian Curriculum, classified by content strand and stage of schooling. They can be used in the vocabulary and spelling activities described in the following subsections. However, students often encounter the words before the associated concept appears in the curriculum since, as we have seen, the language comes before the learning.

3.1.1 Everyday English words

Some particular types of everyday English words appear frequently in mathematical texts, and are crucial to their understanding. They include:

- prepositions such as *to*, *from*, *under*, *over*, *along*, *of*;
- adjectives such as *plus*, *minus*, *approximately*;
- adjectives with comparative and superlative forms such as *long/longer/longest*, *far/further/furthest*, *less/lesser/least*;
- quantifiers (another form of adjective) including numbers, both cardinal (*one*, *two*, *three*) and ordinal (*first*, *second*, *third*) and other words that show quantity such as *dozen*;
- logical operators such as *and*, *or*, *not*, *if … then*;
- words that are easily confused such as *of/off*, *compliment/complement* (see **Appendix A**);
- words derived from other parts of speech; for example, the adjectives *different*, *differing*, *differentiable* and *indifferent*, the verb *differentiate*, the adverb *differently*, nouns *difference*, *differentiation* and *differential* are all derived from the noun *differ* (Nippold & Sun, 2008);
- words that appear to describe the same concept, but have slight variations in meaning, such as *identical*, *same*, *equal*, *equivalent*, *congruent*;
- statements that are understood differently in mathematics; for example, *add* in everyday English implies an increased result, whereas in mathematics the result may be increased, reduced or the same, depending on the value to be added; and
- complex and sometimes contradictory strings of words and phrases, such as *lowest common denominator*, *mutually exclusive*, *increasing at a decreasing rate*, *if and only if* (Gough, 2007; Halliday, 1990; Spanos, Rhodes, Dale, & Crandall, 1988).

3.1.2 Sub-technical words

Some words have a different meaning in ordinary English than their meaning in mathematics, for example, *root, linear, variable, square, prime, mean, mode, product, operation, expression, dividend, difference, power, rational, function, a*. In many cases the mathematical meaning of these words is related to, but more precise than, the ordinary English meaning, for instance, *equality, similar*. They are called sub-technical words. Sub-technical words are often a source of confusion for beginning mathematicians. A list of those that can cause difficulties is included in **Appendix A**.

3.1.3 Technical words

Other words have meaning only in mathematics. Examples include *rectangle, coefficient, percent, median, hectares, binomial, denominator,* and *vinculum*. They may be used in everyday contexts, but always retain their mathematical meaning. These are called technical words.

3.2 Stages in the development of vocabulary

Young students and English language learners commence the development of their mathematical vocabulary initially through conversation. As their reading proficiency develops, written texts also become sources of vocabulary development. Figure 3.1 illustrates these stages in the development of mathematical vocabulary.

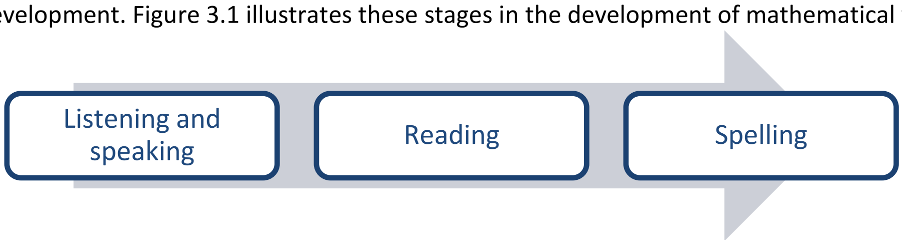


Figure 3.1 Stages in the development of mathematical vocabulary

3.2.1 Listening and speaking

Listening and speaking as aspects of literacy have been addressed in Chapter 2. This section considers the role of listening and speaking in developing vocabulary. Listening and speaking are critical to the development of a mathematical vocabulary. Children learn by example, and listening to mathematical talk is the first stage in the process of developing their own mathematical vocabulary. They then start to employ that vocabulary in their own speech. It is only after they become adept in the use of mathematical words in speech that they are ready to represent them in written forms using letters, numbers, other symbols and diagrams. We know that learning is reinforced when several senses are engaged. Thus, engaging several senses (sight, hearing and speech), is good pedagogical practice.

Foundational mathematical concepts are learnt by observing and discussing the attributes of objects (i.e. the properties, characteristics or features of an object). Attributes that young children notice are colour, size, mass, shape, sound, taste, texture and function or use. As they identify attributes they learn words that are relevant to each attribute (*thick, thin, large, small,* etc.). They progress to activities that require sorting, matching, comparing and ordering, and the associated language. In particular, comparing and ordering requires new language, particularly adjectives (such as *long/short*); comparative and superlative adjectives (such as *long/longer/longest*); comparing words (such as *more, less, the same*); words that describe location (such as *beginning, up, under*); and words that describe sequence (such as *first, second, next, last*). Counting leads to the language of number, both cardinal and ordinal.

Numeration leads to discussions about, and the development of vocabulary for, concepts associated with computation, equality, measurement, and statistics. Consideration of time, particularly the future, leads to the language of prediction and probability. Observation of the properties of objects and visualisation of those objects require the development of the language of geometry.

Figure 3.2 shows the five stages in coming to know a word and the concept(s) represented by that word (Beck, McKeown, & Kucan, 2001; Hipwell, 2015):

Stage 1 Does not know the word → **Stage 2** Has seen or heard the word → **Stage 3** Knows the spelling and meaning, but cannot explain it to others → **Stage 4** Knows the word well, can explain and use it in some contexts → **Stage 5** Fully understands the concept, can define and use it in all relevant contexts

Figure 3.2 Five stages in coming to know and understand a word

The strategies used for teaching mathematical vocabulary are influenced by the complexity of the meaning of the word. Many words have relatively straightforward meanings, for example *triangle*, and do not require a lot of teaching effort. In such cases, the required understanding is achieved when students have reached the fourth stage. However, some words have complex, varied and/or conceptual meanings that must be developed carefully. Examples include words with several meanings and words that embody complex ideas, such as *point,* having no dimension; *independent,* both variables and events; and *proof*. In such cases teachers should aim for the deep understanding of the type described in the fifth stage.

Pedagogical strategies that can be used to develop or deepen an understanding of a word are summarised in **Appendix C**. While space does not permit full explanations of each of these activities here, further information can be obtained by an internet search or by contacting the YuMi Deadly Centre. However, it is important to note that pedagogical strategies such as writing a word on the board, explaining its meaning, and/or providing a glossary, will not move students beyond the early stages of knowing a word.

There are often several ways to verbalise the same mathematical idea and students eventually need to be familiar with them all. The way a mathematical idea is verbalised is not necessary obvious from the way it is written. For example:

All two-digit numbers are written in the same way, with tens in the left column and ones in the right column, for example, 25. From 20 onwards, they are also verbalised that way: tens first and then the ones, for example, *twenty-five*. However, the numbers between 13 and 19 are verbalised in the opposite manner: ones first, then tens, for example, *fourteen*. Eleven and twelve are similar to the single-digit numbers with a name that has no logical connection to the quantity. Most other languages do not have this complication with the numbers between 11 and 19, with these numbers verbalised as *ten-one, ten-two, ten-three*, and so on. This applies to all Asian languages and is thought to be one reason that Asian students tend to do well at mathematics.

The verbalisation of fractions is even worse than numbers. A fraction is usually verbalised by saying the numerator first, as a cardinal value, and then the denominator, as an ordinal value, for example, *two fifths*. However, to say *two over five* is equally acceptable. A further complication is that if the denominator is two, we say *half* instead of *second*, and if it is four, we can choose between *quarter* and *fourth*. With these multiple methods of verbalising fractions, it is not surprising that students find them confusing.

In some cases, the written form of a mathematical idea does not give any information as to how it is verbalised. For example, students must learn that a ratio is verbalised as *three is to four.* There is nothing in the appearance of 3:4 to indicate this.

There are many homophones that can also lead to confusion in spoken mathematics, for example, *two/too/to, four/for/fore, sum/some, pi/pie, eight/ate, cent/sent/scent.* For other homophones, see the list of words that are commonly confused in **Appendix A**. Those with hearing difficulties may also confuse *five* and *nine*.

3.2.2 Reading

As students become proficient in reading it becomes another tool in the development of the mathematical vocabulary. The description of mathematical ideas in prose can introduce new vocabulary. The way that words are used to describe mathematical concepts can lead to productive classroom discussions about the mathematical and everyday meanings of words and the difference in interpretation of everyday and mathematical expressions.

Reading is a key part of the process of decoding a text and is discussed more extensively in Chapter 4.

3.2.3 Spelling

Spelling is an important part of knowing a word. However, it should become the focus of classroom activities only after the word is embedded in the students' vocabulary. Therefore, the explicit teaching of spelling is usually developed in the upper primary years.

Minor variations in spelling can change the meaning of a word: *minutes/minus, complement/compliment, principle/principal*. Homophones (see section 3.2.1 and Appendix A) can be a source of spelling confusion for some students (Durkin & Shire, 1991). In the upper primary years, mathematical words should be included as part of normal spelling activities. In secondary classrooms, mathematics teachers should periodically test students' spelling of mathematical words (possibly as a lesson starter, using NAPLAN-style questions). Incorrectly spelled words (whether mathematical or not), wherever they occur, must be corrected by the teacher.

Some effective strategies for teaching spelling include:

- Multiple choice questions where students have to select the option with the correct spelling (particularly good for homophones and words that are commonly confused).
- Find the little words inside a big word, without changing the order of the letters (for example, *repeatable* includes the words *pea, tab, pea, rep, peat, able, table, repeat*).
- Take a base word and add prefixes and suffixes to build up word families (for example *ratio* leads to *ratios, ration, rations, rationed, rationing, rational, rationals, rationale, rationalise, rationalised, rationalising, rationally, rationalist, rationalism, rationality, irrational, irrationals, irrationally, irrationality*).
- Break a word into syllables (by clapping or using chin drops); this is a strategy commonly used by good spellers.
- Focus on the Greek, Latin or French root of the word; for example, *circumference* is derived from the Latin words *circum* (around, about) and *ferre* (to carry or bear). Find other words that share the root.
- Focus on the 'tricky bit' in a word, for example, *independent* (the –ent at the end)*, parallelogram* (the location of the double and single letter "L").
- Focus on the pronunciation; for example, *simultaneous* is easy to spell if it is pronounced correctly. (Hipwell, 2013).
- Make crosswords involving mathematical words (students could make them for each other).
- Teach the spelling of the word and its plural if the plural is not formed by adding *s* or *es*.

Less effective strategies include:

- Spelling rules (such as *I before E except after C*) – many argue that they have so many exceptions that they can mislead students.
- Memorising word lists – words learnt in lists often stay in lists, that is, spelling learnt this way may not be transferred to other contexts such as sentences.

3.3 The teacher's dilemma

Mathematics teachers often confront a dilemma between, on the one hand, using vocabulary that students can understand easily and, on the other hand, using the appropriate mathematical terms. The QCAA advises that:

> When writing, reading, questioning, listening and talking about mathematics, teachers and students should use the specialised vocabulary related to the subject. Students should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions. (QCAA, 2014, p. 37)

Abedi (2009) found that it is important for students to be continually exposed to the value and richness of the mathematical lexicon (whilst Abedi's conclusions were based on studies of students without an English-speaking background, they apply to all students). However, there is a fine line between vocabulary that is an essential part of the mathematical content and vocabulary that adds unnecessary complexity. If students are unlikely to understand the essential mathematical vocabulary, teachers have the option of (a) pre-teaching the relevant vocabulary, or (b) providing explanations when the unfamiliar word(s) arise. Regardless of the strategies used to teach vocabulary, time invested in developing students' understanding of a rich mathematical lexicon will yield dividends when the literacy focus is extended to reading and writing.

4 Making Meaning of Mathematics Texts (Decoding)

While speech is always an important form of informal communication among mathematicians, as mathematical ideas develop in both complexity and abstraction, written forms of communication become more important. Symbolic mathematical forms are so efficient at communicating mathematical ideas (in addition to being a universal language used by mathematicians from all cultural backgrounds) that they become the dominant form of mathematical communication. Written English is used to support the symbolic expression of mathematics. It is at this stage that the literacy of mathematics starts to diverge from the literacies of other learning areas.

This chapter explains some of the challenges of interpreting and understanding (decoding) mathematics texts, and provides some pedagogical strategies to support learners in this area.

4.1 The nature of mathematical texts

As explained in Chapter 1, in this resource a *text* is any form of communication, both spoken and written. Texts are not a random accumulation of information, but are presented in a considered way to achieve a particular purpose. The word *text* comes from the Latin *texere*, meaning to weave. Thus a text is a linguistic structure purposefully woven out of words and/or signs. Mathematical texts use words, but can also include symbols, tables, and visual images.

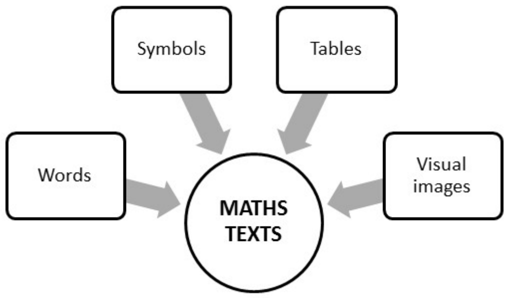


Figure 4.1 Forms of mathematical texts

Mathematical texts are harder to decode than most other texts. First, the reader must be able to interpret the words. This requires an understanding of the mathematical vocabulary, but also the ability to negotiate the complex syntactical (grammatical) structures often used in mathematical texts, including abstract and impersonal forms (Abedi, 2009; Abedi & Lord, 2001). Second, mathematical texts contain symbols and visual images that add to the complexity of decoding. The reader must be familiar with any symbols used, the conventions of presenting information in tables, and the wide variety of visual images used to present quantitative information (Newman, 1983). Because of their complexity, symbols and visual images are discussed separately in Chapters 6 and 7, respectively.

The concise nature of mathematical texts also adds to the problem of decoding. A lot of information is often packed into few words (called lexical density). Mathematic texts contain more concepts per sentence (and paragraph) than any other type of text. Unlike other texts, mathematics texts are concise, with little textual redundancy. Textual redundancy occurs when the same information is presented in several ways so that the meaning of an unfamiliar word can often be deduced from the surrounding text. In many mathematical texts, the inability to understand a key word can affect the understanding of the entire text. As Orton (2004) stated, "mathematics texts generally cannot be read quickly, for every word and every symbol is essential to the extraction of meaning" (p. 161). Because of lexical density, conciseness, and complexity in words and expressions, many mathematics texts about relatively simple concepts demand decoding skills that may be well above the grade level for which they are intended (Zevenbergen, 2001).

A further complexity in mathematical texts is that the page layout may require the eye to travel in a different way from the traditional left to right and top to bottom.

School students generally encounter written mathematical texts in four situations: (a) teacher presentations, usually accompanied by verbal explanation; (b) written notes, often in a textbook or handout; (c) written instructions; and (d) worded problems. The latter two provide the greatest challenge for students as they can be required to interpret them without assistance.

4.2 Reading and comprehension

Reading in the mathematics classroom serves two purposes: as a means of engaging students in mathematics; and to develop students' mathematical thinking. Teachers should guide students' reading to refine their thinking by asking them to re-read and consult various text forms throughout the lesson.

Reading aloud is often an underused strategy in the mathematics classroom. Teachers can read to students to build comprehension and understanding. Reading to students gives them the opportunity to experience, think about and consider the application of mathematics in real life. Strategic pauses during the reading can give students time to think and also demonstrate how to break up a mathematical text into its component parts. Asking students to read aloud is one of the most powerful strategies for helping students to understand the text. It can also assist teachers in diagnosing comprehension problems. Inconsistent pace, mispronunciations, omitting or inserting words all indicate different reading problems that, once identified, can be addressed. Junior secondary teachers should consider using this pedagogical strategy more often.

Asking students to read and comprehend a range of literature in which mathematics is represented has many benefits:

- they can read and understand how mathematics is a natural part of their physical and social worlds;
- they learn through books how mathematical ideas can be represented in different ways;
- they can focus on the patterns of number and colour to predict how stories and articles are constructed;
- they can use the natural context of stories and articles to discuss and reason about mathematical ideas;
- magazines, papers, and media, both print and digital, show how mathematics ideas can be represented in different contexts and ways; and
- they can explore what and how visual images are used to represent mathematical data and ideas in different texts and contexts.

It also has the incidental benefit of preparing students for NAPLAN reading tests.

4.2.1 Basic reading strategies

As well as using and modelling the usual basic reading strategies to make sense of a mathematics text, teachers can assist students by asking:

- *What is the main idea?*
- *How does this idea connect to what you know?*
- *Can you use the visual images to predict or anticipate the content?*

Teachers should look for key concepts or specialised vocabulary that need to be introduced before reading (because students may not obtain this meaning from the text) and consider whether other materials are needed to support the reading. Graphic organisers are particularly useful in building understandings in the classroom, for example, quadrant models, analysis grids, webs, and step processes.

By using guided reading in small groups, students can analyse using tools such as "what I know" and "what I predict" organisers. The teacher can ask students questions such as:

- *What would you be doing in this situation?*
- *What does the picture/chart/graph tell you?*
- *How does the title connect to what you are reading?*
- *Why are these words in capital letters?*
- *Why is there extra space here?*
- *What does that word mean in this context?*

As students develop, they can be encouraged to pose these, and other, questions for themselves.

4.2.2 Four ways of reading texts

There are four ways of reading texts (Hipwell, Carter, & Barton, 2017), as shown in Figure 4.2:

Skimming	Scanning	Continuous reading	Close reading
• involves glancing or browsing the text in a similar way to how a reader might read a magazine • the reader wants to get a general idea of what is in the text • not everything is read	• involves searching for something specific • the text is read with the purpose of locating key ideas or information • scanning is more purposeful and deliberate than skimming	• involves reading (usually words) in a sustained way • similar to the way that novels are read	• involves reading something in detail • the reader looks closely at every feature of the text • may involve re-reading the important and/or complex parts

Figure 4.2 Four ways of reading texts (Hipwell, Carter, & Barton, 2017)

These methods are used in different circumstances, depending on the purpose for reading. Students need to determine when each method is appropriate.

4.3 Interpreting written instructions

Written instructions generally use *task words* (listed in **Appendix D**) to tell students what to do (Carter & Hipwell, 2013b). Some task words are synonyms, for example, *show* and *demonstrate*. Others may appear to be similar, but have important differences, for example, *approximate* and *estimate*. Some task words are used frequently in mathematics, for example, *calculate* and *simplify*, and some have different everyday and mathematical meanings, for example, *evaluate*. It is critical that students know and can use the task words used regularly at their level of mathematics. This is a process that starts early with words such as *add* and *write* and continues throughout schooling leading to the higher order skills such as *analyse*, *assess* and *generalise*.

In many cases, students are asked to employ the skill embodied in a task word without that task word being used. For example, *why* or *why not* might be used in place of *explain*; *give reasons* means *justify*; *what are the similarities and differences* is another way of saying *compare*. Some mathematical instructions should not be interpreted literally:

- *find* x means *determine the value represented by the letter* x, not *locate the letter* x;
- *how can you get from A to B?* may refer to the path to be taken on a map, not the mode of travel;
- *describe the graph* asks for an interpretation of the graph, not references to lines, shading, words and orientation.

Students must be able to recognise the various ways in which instructions can be expressed.

4.4 Interpreting worded problems

Decoding the text of worded problems is challenging. Information is presented in words, but there may also be symbols, tables and visual images. Studies have found that up to 64% of errors in mathematical problem solving can be attributed to errors in the interpretation of the problem text (Cummins, Kintsch, Reusser, & Weimer, 1988; De Corte & Verschaffel, 1985). As the importance of teaching students how to interpret a problem text cannot be overstated, the ideas in this section are also presented in Supplementary Resource 2 on Problem Solving.

Mathematics problem texts are structured differently from other texts. A traditional reading paragraph includes an introductory topic sentence, intermediate sentences that expand on and support the main idea with details, and a summarising or linking sentence at the end. In mathematics problem texts, the key idea often comes at the end of the paragraph, in the form of a question or instruction. Students must learn to read the problem text differently.

Mathematical problem texts should be read at least three times: twice at the beginning and once at the end, described below.

4.4.1 Reading the text of a problem

First reading of the text of a problem

When first encountering a problem text, it should be **scanned** (read quickly) to gain an overall feel for the problem. The reader should form some idea of the context of the problem and what is required. However, when scanning, it is easy to overlook or misread a word and obtain the wrong impression, as in the following example:

Anne left home with $100 to go shopping. She spent all but $1. How much did she have left?

In the scanning stage, students might overlook the crucial word “but” and believe that Anne spent $1. In this case “but” is crucial to the meaning of the problem. Once the text is accurately decoded, the solution is easy. So, scanning must be followed by close reading.

Second reading of the text of a problem

Close reading is the second stage of reading a problem text, where the details are gathered from the text. It examines each word and phrase to extract all relevant information. There are four ways of extracting information from a problem text (Raphael, 1982, 1986):

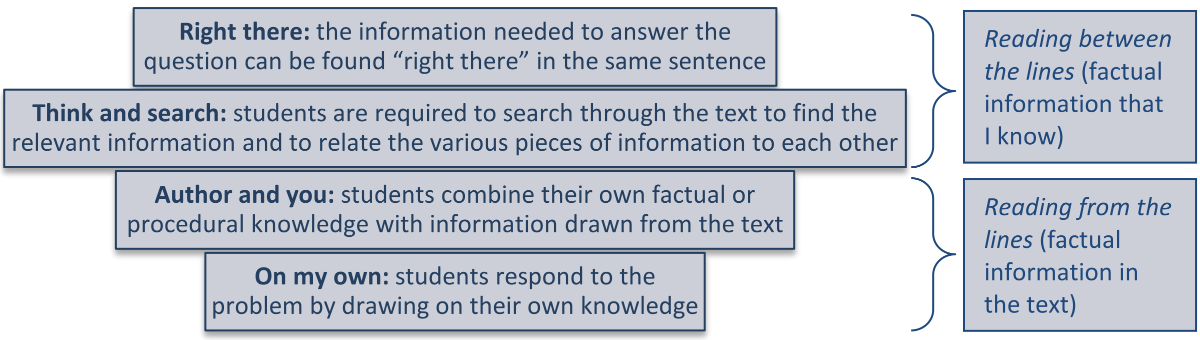


Figure 4.3 Four ways of extracting information from a problem text (Raphael, 1982, 1986)

Questions that students should ask of themselves as they read a worded problem text include:

- *What facts am I given in the text?* [often easily identified as numbers]
- *What am I asked to do?* [it can help to look for the question mark]
- *What additional facts do I know that are relevant?* [“author and you” approach]
- *Can I summarise the information in a table or diagram?*
- *What processes can I use?*

Consider the following examples:

> *There was some money on the table. John took away $8. This left $14. How much was on the table?*
>
> Some words can confuse students as this example shows. Focusing on “took away” and “left” may result in thinking the problem is solved by subtraction when in fact it is solved by addition.

> *The deacon said, “The trip to the Gold Coast takes me one hour and twenty minutes”. “That’s strange,” said the minister, “It only takes me 80 minutes”. How much faster is the minister?*
>
> Close reading is all that is needed here (and knowing that 1 hour and 20 minutes is 80 minutes). However, some students may not know what a deacon or a minister is and may be distracted by this irrelevant information. Research shows that students from lower socio-economic backgrounds are more likely to be confused by irrelevant contextual information (Cooper & Dunne, 2000).

If the problem involves multiple steps, it can be difficult to process all the information at once. Students may have to read the text more slowly, or several times, highlighting or underlining important sections. It can help to break the text up into meaningful chunks and phrases. The burden on short-term memory may result in earlier information being forgotten by the time the student reaches the end of the problem (Barton & Heidema, 2002). In such cases a framework for getting started with maths problems (**Appendix F**) may assist students in sorting through the information in the text.

Final reading of the text of a problem

Once a solution to the problem has been found (i.e. at the end), the text should be reread to check that:

- the proposed solution fully answers the question, that is, that all steps have been completed (it is easy to leave out a final step when the problem requires many steps);
- the answer is reasonable in the context of the problem (e.g. if the problem asked about the age of a person, an answer of 563 would clearly be incorrect);
- an appropriate degree of accuracy has been provided (e.g. money should be rounded to the nearest dollar [whole number] or cent [hundredth]);
- the answer uses the correct units (e.g. if the question asked for the number of metres, providing an answer in centimetres would be incorrect);
- the answer is presented in the required manner (e.g. if the question asks that the response be presented in a table).

4.4.2 Classifying information

The information in a problem text can be classified as:

- **given:** the information we work on and start from;
- **wanted:** the required answer(s);
- **needed:** anything we have to work out on the way to the answer; and
- **not needed:** any unnecessary/irrelevant/redundant information (“red herrings”).

Formally identifying information in this way can be useful exercise as the following example shows.

> *There were 900 cases of fruit at the depot. Three large trucks each removed 126 cases. They were followed by five small trucks which each removed 57 cases. How many cases were left at the depot after the large trucks had been loaded?*

The **given** information is the number of cases at the depot (900), the number of trucks (3 large and 5 small), and the capacity of each truck (large 126 cases and small 57 cases). The **wanted** information is the amount left after the large trucks have been loaded. The **needed** information is the total number of cases on the large trucks (3 × 126 = 378 cases). The **not needed** (irrelevant) information is the total number of cases on the small trucks (5 × 57 = 285 cases). Thus the number of cases left was 900 – 378 = 522 cases.

To assist in identifying the different types of information, students can annotate the text. For example, students could:

circle the given information

~~**cross out**~~ the not needed (irrelevant) information

underline the words that say what is wanted, and

write down what is needed.

If the problem is posed verbally, students should be encouraged to make notes, classifying and summarising the given, needed and wanted information.

In the example above, the given information is entirely numeric, and the question mark indicated the wanted information. The strategy of locating the given information by looking for the numbers and the wanted information by looking for the question mark works in this and many other mathematics problems. However, it has little to do with logic. More advanced students should be exposed to problems that do not contain numbers or question marks.

This example is also unusual because it includes unnecessary information. Many students consider that if they have not used all the information provided, they must have made an error. For this reason, students should frequently be exposed to questions that include irrelevant information. Real-life applications of mathematics are full of irrelevancies.

Students should also be encouraged to see that contextual information is often irrelevant to the calculation process. In this example, it did not matter that the trucks were in a depot, what was being removed (cases of fruit) by the trucks or the size of the trucks, beyond establishing that there were two different types of truck. What mattered were the mathematical relationships between the facts. Asking students to write their own problem using the same mathematical relationships in a different context, and comparing with their classmates' problems can help to illustrate this idea. However, the contextual information is important when it comes to interpreting the answer and writing it in a sentence.

If students struggle to develop a plan for solving a problem, it could be that they did not identify all the required information while reading the problem. Remember that worded problems in mathematics are often lexically dense (packing a lot of information into few words) making the decoding task challenging. For example, students might:

- overlook important words, for example, prepositions such as *to, from, above, below*, or words that indicate sequencing such as *initially, then, after*;
- dismiss some information as being irrelevant; for example, information such as *the older child, named Tom, likes ice-cream* may convey important details about number (older means exactly two), gender (male), or a comparison of ages (not twins), despite the object of the sentence (ice-cream) being irrelevant to the solution of the problem;
- fail to notice that they need to supply information not provided in the problem text, for example, the number of days in March, or the number of metres in a kilometre.

To understand what a problem text is asking students to do, they must know the meaning of key task words such as *identify, demonstrate, explain, predict, simplify* (see **Appendix D** for a list of these).

4.4.3 Conventions of mathematics problems

Some conventions about mathematics problem solving are summarised in the four so-called rules (or assumptions) of worded mathematical problems (Kintsch & Greeno, 1985):

1. Only the relationship between, and changes to, mathematical sets are important. Other information (such as names and context) are mathematically irrelevant.
2. All values are assumed to be exact, unless modified by words such as *approximately, at least, more than*.
3. All relevant information is assumed to be in the text of problem, so the reader does not have to consider whether extra information is needed.
4. The problem must be solved within the bounds of the question, no matter how unrealistic.

The first of these so-called rules says that contextual information is irrelevant. That is only partially true. Contextual information may be irrelevant to the calculation process, but it is used in a mathematical problem to give it authenticity. It also provides the clues as to the mathematical processes that are required. Additionally, while names and contexts may be unimportant during the calculation stage, the answer must be interpreted and presented in the context of the original problem.

As discussed earlier, in the artificial environment of the mathematics classroom, the data in a problem is usually limited to the information relevant to the solution. This can provide students with valuable (albeit unrealistic) clues as to how to solve the problem (if you have not used all the information given, then you must have made a mistake). Students should occasionally be exposed to problem texts that include irrelevant information, including illustrations.

Rules 3 and 4 represent a cynical view of problem solving in mathematics classes. However, they apply more often than we may care to admit, especially in examinations. We might expect students to introduce some memorised factual knowledge (for example, the number of days in a week), but they are not expected to collect additional data to solve a problem, unless explicitly told to (nor are they able to do so in a closed examination). More importantly, in the case of the fourth rule, we do not expect students to introduce additional information, even if it is informed by their life experience. For example, when calculating the number of trips required to take 33 people up four levels in an elevator if the maximum load is eight people, students are not permitted to assume that at least one person would choose to use the stairs rather than wait for the fifth trip of the elevator, even though it is a likely outcome (Cooper & Dunne, 2000). These assumptions should be discussed with students in the context of interpreting mathematical problem texts.

4.4.4 Deciding which operation to use

The translation of a real-world story into a mathematical process usually involves arithmetic operations. There are usually clues in the problem text as to the operations that are needed.

Clues from the type of mathematical objects (numbers)

One way of determining the arithmetic operation is to consider the numbers, or mathematical objects, that are involved. This is best done in three steps.

Step 1: Deciding if the operation is addition/subtraction or multiplication/division

Arithmetic operations such as addition, subtraction, multiplication and division are *binary* operations. That means that two inputs form one output. In some cases, there appear to be more than two inputs, for example, $2 + 3 + 4 = 9$, but this is really a case of repeated addition: $2 + 3 = 5$ and then $5 + 4 = 9$. Thus, binary operations have three parts, two of which are usually known (inputs) and one unknown (output). The relationship between the three numbers can help to identify which type of operation applies.

If all three numbers refer to the objects being considered, for example, 2 fish, 3 fish and 5 fish, then the operation is addition/subtraction. If one of the numbers describes how many groups, repeats or lots there are, while the other two numbers refer to objects, it is multiplication/division, for example, 20 fish, 5 fish and 4 groups. Multiplication/division also has equal-sized groups while addition/subtraction need not.

Step 2: Deciding if the operation is addition or subtraction

If the operation is addition/subtraction, look at the problem in terms of part-part-total (P-P-T). If the parts are known and the total is wanted, the operation is addition; if the total and one part are known and the other part is wanted, it is subtraction.

As shown in Figure 4.4, **addition** problems arise in four general formats: joining (action), finding the total (inaction); changing or comparing; and the inverse of subtraction.

Joining (action)	Finding total (inaction)	Compare/change	Inverse of subtraction
• Three students, two more joined them, how many altogether?	• $15 in notes, $6 in coins, how much money altogether?	• Started with four books, borrowed three more, how many books altogether?	• Gave away five fish, had six left, how many fish did I catch?

Figure 4.4 Four types of worded addition problems

Similarly, **subtraction** problems can occur as: separating (action); finding the part (inaction); changing or comparing; and the inverse of addition, as shown in Figure 4.5.

Separating (action)	Finding part (inaction)	Compare/change	Inverse of addition
• Seven students, three went home, how many remaining?	• $21 cash, $6 in coins, how much in notes?	• Grace has three books more than Alex, together they have nine books, how many does Alex have?	• Caught four fish in the morning and some more in the afternoon, had twelve in total, how many were caught in the afternoon?

Figure 4.5 Four types of worded subtraction problems

Step 3: Deciding if the operation is multiplication or division

If the problem is multiplication/division, look at the problem in terms of factor-factor-product (F-F-P). If the factors are known and the product is wanted, the operation is multiplication; if the product and one factor are known and the other factor is wanted, it is division.

As shown in Figure 4.6, multiplication can be presented as one of four models: combining equal groups or sets; total in an array; jumps along a number line; or the inverse of division.

Combining sets	Array	Number line	Inverse of division
• Bought four bags of six apples, how many apples altogether?	• Four rows of five tiles, how many tiles altogether?	• Five jumps of three along a number line, how far altogether?	• Divided fish equally between three families, each got seven fish, how many fish altogether?

Figure 4.6 Four types of worded multiplication problems

Similarly, division can be either partitioning into equal groups or sets; rows in an array; individual jumps along a number line; or the inverse of multiplication, as shown in Figure 4.7.

Partitioning sets	Array	Number line	Inverse of multiplication
• 24 apples shared among four students, how many each?	• 20 students lined up in four equal rows, how many students in each row?	• A relay race of 400 m, four runners, what distance was run by each runner?	• Seven went fishing, all caught the same number of fish, total of 21 fish, how many each?

Figure 4.7 Four types of worded division problems

Clues from the words

Often the words used in a problem text provide clues as to the operation needed by implying particular arithmetic operations. For example, *together* usually implies addition, *reduction* implies subtraction, *times* suggests multiplication and *shared* implies division. A list of such textual clues is in **Appendix E**.

However, **this method needs to be used with caution. Some words are not what they seem. There are no hard and fast rules.** See the guidelines and examples in the box below.

When the words *how many* or *how much* are followed by a comparative form of an adjective, such as *how much **further*** or *how many **more***, they imply the inverse operation. This is demonstrated in the following examples.

> Generally, the word *together* suggests addition, as in this simple problem:
>
> *Peter has $2 and Mary has $3. **How much** do they have together?*
>
> However, by modifying the question so that the comparative form of the adjective *more* is used after the words *how much*, subtraction is required:
>
> *Peter has $2 and Mary has $3. **How much more** than Peter does Mary have?*

In the next example the word *times* indicates multiplication:

> *The house is five **times** the length of the bedroom. If the bedroom is three metres long, how long is the house?*
>
> However, using the comparative adjective *longer* after the words *how many times* changes the operation to one of division:
>
> *The house is five times the length of the bedroom and ten times that of the bathroom. **How many times longer** is the bedroom than the bathroom?*

In other words, the use of a comparative form of adjective after the words *how much* or *how many* results in the inverse operation.

Clues from the units of measurement

Sometimes the units of measurement for the given, wanted and needed data can suggest the operation(s) to be used. For example:

> *Peter travels 9 km in 3 hours. What is his average speed?*

In this problem, speed would be measured in kilometres per hour (where *per* implies division). Since the number of hours and distance travelled in kilometres are given in the problem, then to produce an answer in kilometres per hour, the units needed for the answer suggest that the distance must be divided by the time.

In another example, area is measured in square units (cm^2, m^2, etc.). As multiplication is used to square a number, to find the area, it is necessary to multiply (not add) the length by the width. This can be a useful tip for those students who regularly confuse area and perimeter.

4.4.5 Determining the type of number required

The problem text can provide clues as to the type of number required in the answer. If the answer is a whole number (discrete data), words such as *how many* (suggesting counting), *fewer* or *fewest* are used. On the other hand, words that imply that the answer could be a fraction (continuous data), include words such *as how much, less* or *least*.

As mentioned earlier, it is difficult to present textual clues as hard and fast rules, as **there can be many exceptions**. However, the question *Are there any words in this problem that give you a clue about what to do?* can be a useful prompt for use with students struggling to make a start on a problem.

4.4.6 Graphic organiser to decode a problem text

The graphic organiser in **Appendix F** for interpreting a worded problem can assist students to deconstruct the text of a mathematics problem. The purpose of a graphic organiser is to structure a student's thinking. By providing each section for students to complete in turn, it gives students a starting point, allows them to think about one process at a time, suggests the sequence in which they should think about those processes, and provides an opportunity to record their ideas. In this case the graphic organiser has five sections:

1. **Reading from the lines**: the relevant data (factual information) *given* in the problem text.
2. **Reading between the lines**: the additional information from the students' own knowledge that is relevant to the problem.
3. **What I have to find out**: the *wanted* information.
4. **My ideas**: a place to record the student's thinking, including whether there is any *needed* information (that is, intermediate steps), what operations might be prompted by the text, an estimate of the answer (or the range of values in which the answer might lie).
5. **Putting it all together**: how the problem can be represented, for example, a diagram, a table, an equation.

If students are able to complete the graphic organiser they will have decoded and understood the problem text and should be heading in the right direction to solve the problem. Further, if the problem is used for assessment of the student, completion of the graphic organiser would give the teacher the opportunity to award partial credit for the problem, even if the answer has not been reached.

Graphic organisers do not work for every student. Some do not need the structured assistance. Others can dispense with the organiser once they are familiar with the structured approach that it represents.

4.5 Diagnosing student decoding problems

Where students are unable to correctly respond to a worded mathematics problem, Newman (1983) proposed five questions that could be used to diagnose the nature of the error.

QUESTION	DIAGNOSIS
Read the question aloud to me.	Difficulty with basic reading.
What does the question ask you to do?	Unable to understand what has been read.
How are you going to find the answer?	Unable to determine the process(es) needed to solve the problem.
Show me how to get the answer. As you do it, talk aloud about what you are doing.	Unable to apply the process(es) needed to solve the problem.
What does your answer mean?	Unable to interpret the answer in the context of the problem.

The first two questions diagnose decoding issues. Asking these five questions, in order, every time a student needs assistance ensures that the assistance is targeted appropriately. Students should be helped to overcome only the diagnosed weakness and then be allowed to continue with the solution independently.

4.6 Critically analysing mathematics texts

In the case of informative mathematics texts, once students can decode the text, the next step is to critically evaluate it. The reader should be able to analyse the text, taking a critical and questioning approach, to determine whether any findings and conclusions are reasonable and valid. In doing so, the reader can analyse the text using five broad criteria: (a) assessment of the text; (b) assessment of the creators and publishers of the text; (c) intentions of the author; (d) techniques used by the author to influence the reader; and (e) reader reaction. Table 4.1 lists some questions that students could ask under each of these headings, with examples of how they could apply to mathematics texts.

Table 4.1 *Critically analysing mathematics texts*

QUESTION	EXAMPLES IN THE CONTEXT OF MATHEMATICS OR STATISTICS
Assessment of the text	
Who is the target audience? How do you know?	Is the text written for students, other mathematicians or the general public?
When and where was the text created? How do you know?	How recent are any statistics quoted in the text?
What was happening in the world at the time the text was created?	Were there economic, social or political issues at the time the text was written that might have influenced the content?
Have there been any relevant events/changes since the text was created?	Have circumstances changed since the data was collected? Would the data, if collected today, be similar?
What is in the text? What is missing?	Is there an explanation of how the data was collected?
Is this text presenting a balanced view of the issue?	Are there questions that should have been asked that were not included? Are all survey questions framed in an unbiased manner?

QUESTION	EXAMPLES IN THE CONTEXT OF MATHEMATICS OR STATISTICS
Is the text fair? Does it treat the subject matter/sides/parties fairly?	Was the sample selected in a valid manner? Is the sample size appropriate? Are both sides of the issue discussed in an impartial manner?
Is all the relevant information provided?	Has all the relevant data been presented and/or discussed? Is a copy of the questionnaire available to the reader?
Is the mathematics correct?	Have the appropriate mathematical and/or statistical techniques been used? Have they been applied correctly?
What is real/true/verifiable in the text? What is not real/true/verifiable?	Are the calculations and/or data available to readers so they can conduct their own analysis?
Are the conclusions based on the mathematics valid?	Are the conclusions supported by the mathematics? Are there any unsubstantiated claims? Is there a discussion of the error/accuracy of the statistics or predictions presented?
What has been left out of this text that you would like to have seen included?	Have all possible explanations been included? What else would you like to know?
How might different people interpret the message of the text?	Do you need a degree in higher mathematics to make sense of the text?
Do you need to consult another source of information?	Do you need to see more data to make an informed judgement on the issue?
What would an alternative text say?	Are there any other explanations for the outcome of the analysis?
Assessment of the text creator/publisher	
Who created the text? What do you know about the author?	Has the author been paid or sponsored to conduct this work? By whom? Could any payments/sponsorship affect the conclusions?
Who published the text? What do you know about the publisher?	Is the text published by a reputable publisher? Who controls what is published? Has the text been reviewed by other experts?
Knowing what you know about who created/and or published this text, how do you expect them to treat the subject matter?	Is the author or publisher known for having or supporting particular views?
How might the creator of this text view the world? Why do you think that?	Is the author trying to change or defend the current situation (status quo)?
Who is making money from the text (what are the commercial implications)?	Does the text advertise a particular product or service? Does it promote (directly or indirectly) a sponsor's business interests?
Who benefits from this text? Who does not?	What is the effect of any recommendations?
Author intention	
What is the author's intent? What does the author want you to know/think?	What is the purpose of the text: to inform, argue, persuade?

QUESTION	EXAMPLES IN THE CONTEXT OF MATHEMATICS OR STATISTICS
How do you think the author sees the world? Who or what may have influenced the author's world view?	Does the author agree/disagree with the current situation (status quo)?
What is the author really trying to say? What are the author's values, attitudes and beliefs?	Are the author's values, attitudes or beliefs evident in the text? (They should not be obvious in most kinds of mathematics texts.)
Why does the author want you to have this information/think this way?	This question should be asked if the author's values, attitudes or beliefs are evident in a mathematics text.
What techniques did the author use to influence my thinking?	
How does the author achieve his/her intention?	What methods does the author use to persuade you to accept his/her conclusions?
What kind of language is used in this text? What is its influence on the message?	Does the text include unnecessary adjectives or emotive language?
What do the images suggest? What do the words suggest?	Are the data displays (graphs) reasonable?
How do the features of text influence the message?	Does the text seek to argue (presenting both sides of the case) or persuade (presenting only one side of the case)?
How does the medium influence the message of the text?	Would the text be more/less effective if it was presented in another medium (e.g. book, online, advertisement)?
How would this text be different if …	How would this text be different if it considered the alternative arguments or was written in an unbiased manner?
Reader reaction	
Does this information make sense to me? Do I agree with this text?	Does the text convince you?
Does this information agree with what I already know? Is the text changing the way I think?	Is your knowledge of the world/situation different because of this text?
What does this text mean to me?	Are the findings important? Do they matter? So what?
What did I learn about myself as a reader, writer, learner?	Do you need to do further research/learn more about the topic? Did you understand the mathematics?
What action do I need to take?	What will you do as a result of reading this text?

5 Creating Mathematics Texts (Encoding)

Writing in the mathematics classroom serves two purposes: as a medium for developing and recording students' mathematical thinking; and as a means for teachers to assess students' understanding (both formatively and summatively). Teachers should give students the opportunity to write in various forms throughout the lesson.

Mathematical texts contain symbols and visual images that add to their complexity. Symbols can be confusing, and students must know how to use them correctly. Graphs and tables must be accurate, not misleading, and presented according to the conventions of mathematics. Because of their complexity, symbols and visual images are discussed separately in Chapters 6 and 7, respectively.

This chapter explains the unique nature of writing in mathematics and how mathematics texts vary from other types of texts. It proposes some pedagogical strategies to support learners in this area.

5.1 Writing as a pedagogical tool

Writing in mathematics supports the learning of mathematics.

5.1.1 Recording mathematical thinking

Students should learn to record their mathematical thinking for several reasons:

- to make notes to assist in coping with complex situations (the human short-term memory can typically cope with only seven concepts at a time);
- to provide a holding place for their mathematical thinking that can be accessed later;
- to provide evidence of their thinking process, particularly in assessment contexts;
- to justify strategies and conclusions; and
- to share their mathematical ideas with others.

Additionally, the process of writing can slow down the pace of thinking, giving time to develop ideas not previously considered.

The process of recording mathematical thinking on paper can begin early – even before students are able to write in words and sentences. As shown in Figure 5.1, 'on paper' records can include:

- photographs of students' acting out or use of tools (such as manipulatives and grids) that represent their thinking;
- drawings, with or without labels, supporting the idea that mathematics tells stories;
- completed graphic organisers (see **Appendix F**);
- summaries of information provided;
- notes that record and display individual and group thinking;
- sentences and paragraphs describing their own thinking; and
- symbolic records of calculations.

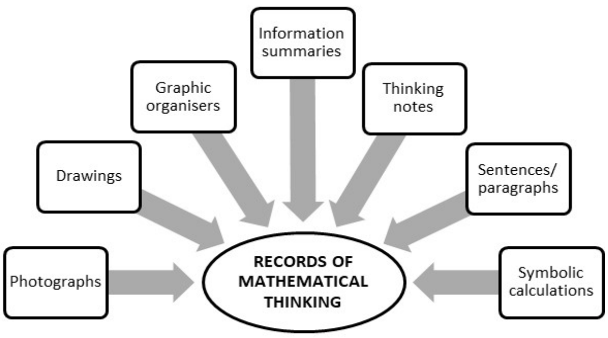


Figure 5.1 Methods of recording mathematical thinking

When solving problems, writing can help to clarify what it is the problem is asking and what information is available (see the use of a graphic organiser to deconstruct the text of a mathematics problem described in subsection 4.4.6). The process of writing about a mathematics problem will itself often lead to a solution. Written explanations of solutions to mathematics problem focus on what was done and why it works.

Written responses to mathematics problems can include:

- **Mathematics diaries and/or learning logs**: Students write comments and questions about what they are doing and reading; this provides opportunities to interact with what they have seen, done, listened to, and read.
- **Shared writing with a group**: Written responses are valuable for students to refer to when participating in small group discussion; the page becomes a holding place for their thoughts until they can build on them.
- **The ideas of others**: Students can share their ideas and strategies in small groups and consider others' views for including in or enriching their own explanations/solutions.

More generally, by writing, students are forced to think through, and find the meaning in, different mathematical situations. It helps them to think and ponder. The type of thinking and writing involved in justifying a strategy or explaining an answer is quite different from that needed to merely solve an equation.

To be effective, planning for mathematics writing involves some hard work on the part of the teacher to ensure the following:

- the problem must be **appropriate** for the students who are going to be writing about it;
- the students must know how to use **tools** such as manipulatives, diagrams, pictures /grids to work out their solutions before writing about them;
- the students must have **confidence** in their ability to respond to the problem as individuals, that is, 'work like a mathematician' or think of themselves as successful mathematics students;
- the students must feel **comfortable sharing their answers** without fear of being ridiculed – teachers and students have to accept all responses as worthy of discussion; and
- the problem must be **discussed with the whole group** or class and all strategies reported; students absorb mathematics from their peers when they share responses.

5.1.2 Writing strategies that support learning in mathematics

The following writing strategies help students to learn, both the mathematics and how to write about it.

1. **Rewriting** gives students the opportunity to develop their written responses more fully using cues from classmates or the teacher. This process gives students more time and thus a deeper connection with the subject.
2. **Creating (writing) new problems** gives students the opportunity to create contexts and mathematical problems to solve that are significant in their world (reality); for example, *Write as many problems as you can that use 2 + 3*. This is a powerful learning strategy and also builds familiarity with the language of problem texts.
3. **Recording what they have learnt** gives students more time to reflect on the topic, what they are learning, and identify their own mathematical thinking. They learn by explaining the answer and justifying their thinking.
4. **Keeping track** enables students to organise whatever it is they are learning and planning to write about. There are many graphic organisers to choose from to introduce and use here (see **Appendix F**).

5. **Illustrating** can enable students least likely to engage in mathematical writing, who often have well-developed visual-spatial skills and problem-solving skills. With careful encouragement and guidance these students can build the skills needed for writing and then move on to writing mathematical explanations.

5.1.3 Writing and student assessment

The object of any maths lesson is for students to be able to apply some aspect of mathematical thinking. Writing enables them to record not just the answer but the strategy they used to arrive at the answer. Partially correct solutions indicate to the teacher where they need to go next with the learning, that is, what must the student understand before we can progress?

Asking and answering open-ended questions reveals whether the students have learnt the subject procedurally or conceptually. Students are required to apply their skills, prove their solutions, draw generalisations, and make connections using words as well as diagrams – all categories of thinking that are classified at the higher end of commonly held educational objectives. This type of standards-based assessment helps students develop confidence in their mathematical thinking and provides more information for the assessor than would a single answer on a multiple-choice test.

5.1.4 Writing and speaking

Immature writers write with themselves as the audience, that is, they write as they would speak. However, writing is different from speech. Speaking is a relatively informal process. The text producer (speaker) and the audience (listener) are not usually separated in time and place, so that the listener can seek clarification if the level of explanation is insufficient. Adjectives such as *here*, *there*, *this*, *that* make sense when talking but may not when writing. Consequently, "speech written down" results in texts that do not meet the required levels of formality, detail, and precision. Students must understand that the way they write is different from the way they think and speak. The development of this important idea should start early.

5.2 The writing process

In addition to using writing as a tool for learning, the aim of teaching writing in the context of mathematics is to develop the ability to create texts that conform to the standards and conventions of mathematical writing. It has already been established that decoding mathematical texts is challenging. It follows that it is also difficult to create them. Creation of mathematical texts is a skill that develops over many years, drawing on general writing skills.

Creating a text is the process of composing or writing a text. In the case of mathematics, the texts are informative. There are two important aspects of writing in mathematics: the mathematics must be accurate and the writing must be grammatically correct (Carter & Hipwell, 2013b).

The process of writing is complex. It is easy for experienced writers to underestimate the challenges for beginning writers. They are required to control eight processes, often simultaneously:

- identifying rhetorical positions, such as purpose, audience, context, and stance;
- marshalling the required ideas and knowledge;
- planning what and how the ideas and knowledge will be presented;
- using the codes and conventions of written language;
- balancing processes strategically;
- monitoring the writing process;
- reading to review the text; and
- editing the text by correcting and revising.

Students can become overwhelmed if they attempt all eight processes simultaneously, especially in extended writing tasks. Teachers can assist developing writers by separating the processes into steps that can be dealt with sequentially. This section proposes some strategies for doing this, summarised in Figure 5.2 and described in the following subsections.

Identifying purpose and audience

↓

Planning, including use of graphic organisers

↓

Drafting the text, following conventions for mathematical writing

↓

Reviewing and editing

Figure 5.2 Steps involved in mathematical writing

5.2.1 Purpose and audience

Writers must have a clear idea of why they are writing and who they are writing for. Decisions about the purpose and audience of the writing determine the nature of the finished product. Mathematical writing usually has a specific purpose, such as:

- **informing**, for example, by describing, comparing and contrasting, explaining, classifying, defining and reporting;
- **arguing**, including debating, persuading and recommending;
- **justifying** and providing evidence; and
- **making decisions** by analysing, evaluating, generalising, or inferring.

In mathematical writing it is common to assume that the audience is educated and has a knowledge of mathematics. However, in some cases the audience may be younger and/or unfamiliar with mathematics. The nature of the audience determines the vocabulary (technical terms), the formality and complexity of the language, and the level of explanations and descriptions.

Teachers should encourage students to explicitly consider purpose and audience at the beginning of the writing process. In extended assessment tasks, part of the work to be submitted could include planning documents that describe the purpose and audience.

5.2.2 Planning

Planning of writing requires the generation of ideas, followed by the selection and sequencing of those ideas. In informative writing, the generation of ideas usually requires background knowledge of the topic, often obtained through research. However, in mathematics, the ideas can also be obtained from mathematical processes such as collecting and analysing data, calculating, modelling and simulating. Initially, all ideas should be recorded, possibly through a brainstorming process. After the ideas have been captured, students can then make judgements about their merit, the connections between them (assisting in paragraphing), and their sequencing in the final product.

Writing guides such as graphic organisers can assist students in planning their writing. The purpose of a graphic organiser is to structure students' thinking, by allowing them to:

- record their ideas (brainstorm);
- consider the merits of each idea individually;
- consider whether they have insufficient, sufficient or too much information (it is easier to add or delete information at this stage, rather than when the writing is complete);
- plan the sequence in which the ideas can be presented; and
- assemble the ideas in a form that suits the structure of the text type.

As the graphic organiser should reflect the thinking process associated with the relevant type of text, the selection of a graphic organiser should be a teacher decision, particularly for immature writers. Graphic organisers for the text types commonly used in mathematics are provided in **Appendix F** (Carter & Hipwell, 2013a).

It is recommended that, where a graphic organiser is provided as part of an activity or assessment item, students should be required to submit the completed graphic organiser as part of their planning documents. Students' ideas can be assessed from the completed graphic organiser, providing an opportunity for them to receive credit for their ideas even if their written expression is poor. With experience, students will become familiar with the thinking process behind the various text types and be able to plan their writing without the assistance of graphic organisers.

It is during planning that teachers should check that students have adequate content. At this stage the content can easily be added or removed to meet the required document length. Attempts to add content after drafting is complete can affect the structure and cohesiveness of the entire document. Removing excess content after it has been drafted to fit within a word limit represents wasted effort.

5.2.3 Construction of the text (drafting)

On completion of the planning process, including the completion of graphic organisers, the written text is drafted, following the codes and conventions appropriate to the purpose and audience of the text. Mathematical writing is different from writing in most other learning areas (although there are strong parallels with scientific writing). Students who write well in other learning areas may not be successful mathematical writers. It is the responsibility of mathematics teachers to guide students in the codes and conventions of mathematical writing. They include:

- appropriate technical vocabulary;
- spare, minimal forms of expression (concise);
- avoidance of deixis (words and phrases that cannot be understood without additional contextual information, for example, *here, there, this one*);
- seamless interweaving of the words, symbols, tables and visual images;
- standard methods of computation, transformation or proof (Ernest, 1998);
- standard forms of mathematical notation and abbreviations;
- an unbiased academic tone;
- impersonal presentations, including passive voice and third person; and
- formal language, avoiding colloquialisms, clichés, and contractions (Carter & Hipwell, 2013b).

In constructing the text, students should use the language appropriate to the text type. Beginning writers could be provided with a series of questions to guide the thinking and writing process. Writers with slightly more experience may not need prompting questions but may make use of a glossary of the relevant technical vocabulary, sentence starters appropriate to the text type, connecting words, and other useful mathematical language (Carter & Hipwell, 2013b). More mature writers should be encouraged to consider ways of avoiding repetition and achieving variety in sentence openings.

Mathematical reports, including some common mathematical text types, are discussed in the next subsection.

5.2.4 Mathematical reports

A mathematical report is used to document a research project, an investigation and/or the collection and analysis of data. A report makes extensive use of section headings: Introduction, Procedure (Method), Results, Conclusion and, optionally, Research, Discussion, Recommendations, Bibliography and Appendices. An abstract may be provided in the case of a longer report.

Mathematical reports include several text types:

SECTION	TEXT TYPE
Abstract	summarising
Introduction	introducing
Research	describing
Method	sequencing
Results	analysing/describing
Discussion	analysing
Conclusion	concluding
Recommendation	recommending
Appendices	various

A graphic organiser for planning a report is included in **Appendix F**. The white sections are mandatory, and the grey sections are optional.

Students can find it difficult to write introductions and conclusions, often uncertain as to what goes in these sections. The following suggestions may assist them:

INTRODUCTION	CONCLUSION
• An opening with impact, for example a quote, vignette, or incident. • Set the stage – what is the document about? • Aims – what are we trying to achieve? • Importance – why do we care? • Define – what are the important issues? • Foreshadow – what is to come?	• The main ideas – a summary of what you have already said. • Aims – were they met? • Evaluate – was the process worthwhile? • Decision – what did you decide and why? • Changes – what would you do differently? • Personal reflection – what did you learn (be specific)? • Recommendations – what should we do now? • Revisit opening – refer back to any opening quote, vignette, incident.

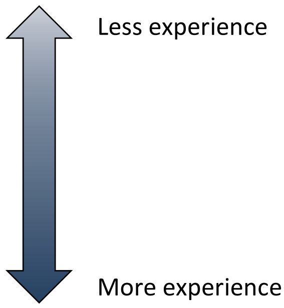
5.2.5 Editing

When drafting is complete, students should check that the finished product is acceptable. Any task descriptions and assessment criteria should be re-read to ensure that the draft meets those requirements. The document length should be compared to any word limit. The draft must be proofread to ensure that the document is clear and concise. Grammar, punctuation and spelling should be checked. Reading aloud (or mouthing the words) helps to identify repetitious text. Proofreading is most effective if it is done at least 24 hours after drafting is finished.

5.3 Pedagogical strategies for teaching writing

As we have seen in section 5.2, writing is a complex process. Effective writers take years to develop. Initially, considerable scaffolding may be required, then gradually withdrawn as the student gains experience. Writing tasks should initially be structured and brief, becoming longer and more open ended over time, for example:

- Fill in the missing information in the sentence(s) (cloze).
- Complete the sentence.
- Rephrase this sentence to make the answer.
- Write a sentence about ...
- Write a paragraph about ...
- Write several paragraphs about …



Teachers should require students to present their ideas regularly in written form, using sentences and paragraphs, when appropriate, and use these opportunities to explicitly teach the skills of mathematical writing.

YuMi Deadly Maths does not often encourage imitative forms of teaching and learning. However, they are useful when initiating students into a community of practice such as the conventions of writing mathematically. Fisher and Frey (2008) proposed a Gradual Release of Responsibility Model (Figure 5.3) that reflects the ideas about writing discussed in this resource:

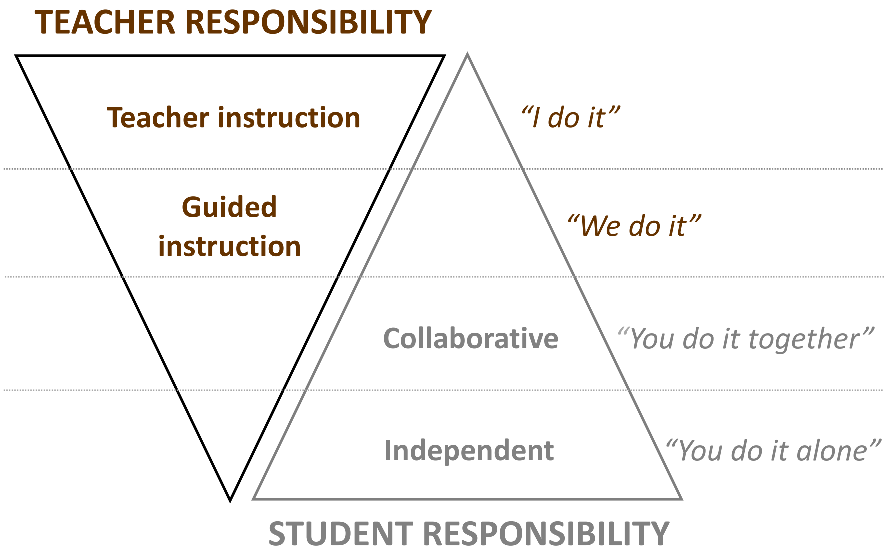


Figure 5.3 Gradual Release of Responsibility Model (Fisher & Frey, 2008)

While the use of triangles pointing up and down suggests a directionality, Fisher and Frey do not see the processes of modelled, guided, collaborative and independent learning as being strictly sequential. Instead teachers should move from one form of learning to another, as required by the lesson content and the needs of students.

6 Symbols

The language of mathematics includes a unique array of specialised symbols. This chapter looks at the use of symbols as a form of mathematical communication and their use in the construction of mathematical arguments and in mathematical definitions.

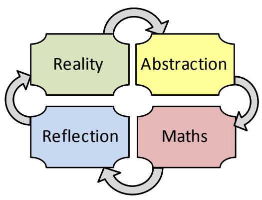
6.1 Symbols tell stories

The symbols of mathematics enable the world to be described succinctly and in a generalised way (e.g. 2 + 3 = 5 means caught 2 fish and then caught another 3 fish, or bought $2 chocolate and $3 drink, or joined a 2 m length of wood to a 3 m length ... and so on). This means that mathematical teaching and learning involves continual interchange between symbols and stories, between formal mathematics and the world.

6.1.1 The RAMR cycle

In the YuMi Deadly Maths RAMR cycle, symbols are introduced in the **mathematics** phase. However, as the sample RAMR below shows, students can start the **abstraction** process by acting out the story (*body*) and then pictorially representing the story (*hand*). The students' first attempt to represent the story will probably look like a storyboard – a series of illustrations of key parts of the action. However, if they are encouraged to simplify their illustrations (for succinctness) and standardise them (so that the same process is always represented by the same illustration), students will start to develop their own symbolic language. They will have developed an effective form of shorthand. If students are now introduced to the idea of language as a form of communication, they can investigate the utility of their individual symbolic languages as a means of communication (*mind*). They will discover the need for a shared lexicon (the various symbols) and syntax (conventions about how the symbols are used). This provides the opportunity to introduce the mathematical symbolic language in terms of lexicon and syntax. Students can **reflect** back to the original story to represent it using standard mathematical symbols. They can consider the potential for extending the use of mathematical symbols to simplify and generalise other mathematical stories.

6.1.2 Introducing symbols



Mathematics is a succinct symbolic language that describes everyday life. Students can be introduced to the idea of a mathematical symbolic language through the following RAMR activities.

Reality

Gather existing knowledge. *Find a story based in the students' reality that incorporates mathematical concepts. It could be a mathematical problem that has developed out of other classroom activities (e.g. how much floor space is needed to neatly stack all the chairs in the classroom?) or in the context of the students' lives (e.g. how much will it cost to take the family to the movies?). The wordier the story is, the better, as we want to illustrate how we can tell this story more concisely using symbols.*

Abstraction

Body. *Act out the story.*

Hand. *Illustrate the story. The students' first attempt to represent the story will probably look like a storyboard – a series of illustrations of key parts of the action. However, if they are encouraged to simplify their illustrations* *(for succinctness) and standardise them (so that the same process is always represented by the same illustration), students will start to develop their own symbolic language.*

Do not introduce mathematical symbols to the students yet, although they may think of this themselves.

Students should be able to develop an effective form of shorthand, which could be used for other, different stories. This is one of the purposes of symbols.

***Mind**. Introduce students to the idea of language as a form of communication. Investigate the utility of their individual symbolic languages as a means of communication by sharing their representation of the story with a friend. They will discover that they may not understand each other's symbols and there is a need for a shared lexicon (the various symbols) and syntax (conventions about how the symbols are used). Communication is another purpose for symbols.*

Where else are symbols used (e.g. Indigenous art, road signs, toilet and bathroom doors, the alphabet – upper and lower case, punctuation, maps, weather forecasts)? Do these 'languages' have a lexicon and syntax?

Mathematics

Introduce relevant parts of the mathematical symbolic language (this might have come up as one of the examples in the previous section) and ask students to rewrite the story using those mathematical symbols. Share this with a friend. Is it easier to understand each other's representation of the story when they use the same symbols? Do they need any rules (conventions) about how the symbols are used? Can they simplify their representations by removing any unnecessary symbols (e.g. do they need to write 4 × A for adult movie tickets, or is 4A sufficient? would this convention also work for four lots of eight?).

Reflection

***Reversing**. Compare their symbolic representation of the story with the original story. Can they reconstruct the story from the symbols?*

***Extend** the use of mathematical symbols to represent a variation of the original story, and then to a different story.*

***Generalising**. What is gained when we use mathematical symbols (simplicity? generality? conciseness?) and what is lost (detail? understanding?).*

When are mathematical symbols useful? When do they make things harder?

Symbols are the way that we generalise in mathematics.

***Advanced and cross-curricular ideas**. Do symbols have to be written? Can we symbolise ideas with actions? For example, how would you show someone to walk in a particular direction, or give commands to a trained dog? What symbols are used in dance? Can we symbolise ideas with images? For example, what might a sunset, a fadeout, or a section in black and white indicate in a movie? How does culture affect symbols?*

Indigenous cultures make extensive use of symbols, for example in dance, storytelling and art. Thus, Indigenous students, with suitable encouragement, can adapt those experiences to the learning of a mathematical symbolic language.

6.2 Conventional mathematical symbols

Mathematical symbols and notation form a language that is unique to the study of mathematics (although they are occasionally borrowed by other disciplines). This makes the study of mathematics different from most other disciplines. It also endows mathematics with a language that is understood by every other mathematician. It can be argued that the language of mathematics, as represented by mathematical numerals and notation, is the most widely used language on the planet. This is because, when compared to most other languages, the mathematical symbolic language has a small lexicon (symbols) and simple syntax (rules or conventions for using those symbols).

To be useful, symbols must be standardised, that is, interpreted in the same way by all mathematicians. A symbol is a representation that can stand for (symbolise, depict, encode or represent) something other than itself. This is particularly true of mathematical symbols. For example, the numeral 6 is simply a curly line on a piece of paper. There is nothing in the visual appearance of the numeral 6 that suggests that it represents the concept of six rather than, say, eight. However, its importance lies in what it represents. The numeral 6 can:

- be determined by counting;
- represent a collection of six objects;
- be the outcome of a measurement;
- be a location on a map; or
- be an equivalence class of sets.

It follows that the concept being represented by the numeral 6 can vary according to the context of the representation.

The symbols that we use to represent mathematical concepts do not stand alone. The symbol 6 only has meaning when it forms part of the larger set of digits 0, 1, 2, 3, ... and is combined with other mathematical symbols such as those for arithmetic operations, equality, and so on. In other words, the numeral 6 is meaningless apart from the system to which it belongs. Such systems of representation are structured by the conventions that underpin them.

The use of symbols and abbreviations when writing mathematical notation leads to conciseness, a characteristic that is valued in mathematical writing. When many mathematical symbols are used in combination with each other, it is often called *mathematical notation*.

Symbols, although easy to write, can be more difficult to interpret and use than words, for several reasons.

- Unlike words, symbols cannot be interpreted using a phoneme-grapheme relation (i.e. by sounding them out, or by analysing the spelling). The link between the written and spoken form of the symbol cannot usually be deduced and must be learnt.

- There may be different directionality when following symbolic instructions; for example, grouping symbols require the reader to work from the inside to the outside; the rule for order of operations requires that multiplication and division are undertaken ahead of addition and subtraction, regardless of the order in which they are written. Further, the symbols can often be verbalised in a variety of directions (for example, $8^2 + 6$ can be read as *eight squared plus six*; *the square of eight plus six*; *six is added to eight squared* or *six is added to the square of eight*).

- Expressions with similar appearances can have different meanings because of the spatial relationship of the elements, for example, 32 and 23; $-(3)^2$ and $(-3)^2$ or $2\pi r$ and πr^2.

- A meaning can be given to symbols that are omitted (e.g. $2x$ actually means $2 \times x$). There are further examples of omitted symbols in sections 6.2.1 and 6.2.3.

- The syntax of the mathematical symbolic language can be confusing; for example, it accepts $2 < x < 5$, but not $2 > x > 5$ or $2 < x > 5$.

- Mathematical symbols may be used differently in different parts of the world. In Latin American and European countries, the use of commas and stops are exchanged; for example, 4,325.64 in Australia, the UK or USA would be written as 4.325,64 in many other countries.

The complexities of interpreting symbolic mathematical language often cause students to skip over sections of text involving the extensive use of symbols.

The symbols used in mathematics can be classified into seven categories, as shown in Figure 6.1. This section deals with each of them in turn, and concludes with the conventions for their use.

Numerals
1, 2, 3, etc.

Pronumerals
a, k, x, etc.

Symbols for operations
+, ×, ÷, etc.

Grouping symbols
[ ], ( ), { }, etc.

Symbols for comparing values
=, ≈, ≥, etc.

Abbreviations (maths and measurement)
∴, ∞, cm, etc.

Geometric symbols
∠, Δ, ⊥, etc.

Figure 6.1 Types of symbols used in mathematics

6.2.1 Numerals

The Hindu-Arabic numerals are the fundamentals of our system of mathematics. Children learn the nine digits 1, 2, 3, ..., 9 from an early age. The concept of zero is more abstract and this symbol is generally introduced later. As they progress through the early years of schooling students learn to combine the ten digits to form larger whole numbers. This requires an understanding of place value.

The words *number*, *numeral* and *digit* are often confused. A number is a specific value. It may be written in words or numerals. A digit is one of the numerals from 0 to 9, which may be used singly or collectively to form a number. There are many different systems of numerals, including Hindu-Arabic, Roman, Egyptian, and so on.

According to the Australian Curriculum, students are introduced to the concept of fractions, both common and decimal, in the later years of primary schooling. By the end of middle school, students should have mastered both types of fractions and directed number.

The introduction of place value and fractions demonstrates that the meaning of a numeral depends on its location and proximity to other symbols. For example, consider the difference in meaning of the numeral 5 in the expressions 50, 525, $\frac{1}{5}$, $\sqrt[5]{2}$, 4^5, $5x$, $x + 5$, x^5, x_5.

Numerals can sometimes be omitted, with the value of the omitted numeral depending on the context:

What we write	What we really mean	Value of the omitted numeral(s)
$3x$	$3x + 0$	0
x	$1 \times x$	1
x	x^1	1
$\sqrt{x}$	$\sqrt[2]{x}$	2
$\log x$	$\log_{10} x$	10

6.2.2 Pronumerals

Numbers and numerals represent particular and unique values. When a value is unknown or may vary, numerals can no longer be used. In such cases the symbols for numerals are replaced by symbols such as letters from the English or Greek alphabets. However, other symbols such as ?, ☐, ★, __, can be used (and, in the early stages of teaching algebra, often are).

Pronumerals arise in three situations:

- When the value(s) are not known, for example in an equation such as $3x + 2 = 14$. In these cases the pronumeral is called an *unknown*. The purpose for using the unknown in this context is usually to solve the equation to find the value(s) that satisfy it.
- To explore what happens as value(s) are allowed to change, for example in a function or a relation. In this case, the pronumeral is called a *variable*. At least two variables are required: the *dependent* variable and one or more *independent* variables. The purpose of using variables could be to explore how changes to the value of the independent variable(s) affect the value of the dependent variable. Variables are often represented using letters at the end of the alphabet (x, y, z).
- When a generalised expression or relation is required, for example in the y-intercept form of a linear function $y = mx + c$. In this example the pronumerals m and c are called *parameters*. If the values of the parameters are subsequently determined, then they are usually substituted into the expression/ relation, for example, $y = 2x + 3$. Parameters are often represented using letters at the beginning of the alphabet (a, b, c).

The use of pronumerals, when combined with numerals and mathematical operators, is generally referred to as *algebra*.

When using several pronumerals, two systems can be used:

- a different letter for each different pronumeral (e.g. a, b, c, …); or
- the same letter with different subscripts (e.g. x_1, x_2, x_3, …).

The subscript system has the advantage of showing a common feature of the pronumerals (in the above example, the notation may be suggesting that all the pronumerals are x-values). However, it is essential that students understand that a different subscript makes the pronumerals as different from each other as they would be had different letters been used.

6.2.3 Operations

A binary operation is one in which two inputs are converted in a specified manner into a single output. The most common examples of binary operations are the four arithmetic operations. These are represented by the symbols +, –, × and ÷. However, there are alternative representations of these operations:

Operation	Possible representations	
Addition	3 + 4 + 6	$\begin{array}{r}3\\4\\\underline{6}\end{array}$
Subtraction	4 – 3	4 + (–3)
Multiplication	3 × 4 × 6	3·4·6
Division	3 ÷ 4	$\frac{3}{4}$

The concise nature of mathematical writing means that some symbols are assumed to be there, even if not actually written down. For example, if an operation symbol is not attached to a vertical list of numbers, it is assumed that the operation is addition (+), and $2x$ actually means $2 \times x$.

For many students, a source of difficulty is the large number of synonyms for each of the symbols representing the four arithmetic operations (see **Appendix E**).

Another binary operation that is represented by a symbol is the nth root of a number. This can be written using a radical sign: $\sqrt[n]{x}$ or $^n\surd x$, meaning the nth root of a number, x. If there is no superscript in front of the radical sign, for example $\surd x$, it is assumed to be a square root.

Some binary operations are represented using notation rather than a dedicated symbol. They include:

Fractional notation	$\frac{a}{b}$
Ratio notation	$a : b$
Index notation	a^b
Scientific notation	$a \times 10^b$

Each of these operations are binary because they has two inputs (a and b) and an output that is a single value.

6.2.4 Grouping symbols

Grouping symbols change the usual order in which arithmetic operations are performed. They require that operations within the grouping symbols are undertaken before operations outside the grouping symbols. If there are several grouping symbols, then:

- in the case of nested grouping symbols, operations are undertaken working from the inside (first) to the outside (last);
- in the case of a series of grouping symbols, operations within the grouping symbols are undertaken working from the left to the right.

Most students understand that the grouping symbols include *brackets* [ ], *parentheses* ( ) and *braces* { }, even if they name them incorrectly. However, they are less likely to understand that a line above or below an operation requires that the operation is undertaken first. For example:

- $\sqrt{3+2}$ requires that the addition is undertaken before the square root is taken;
- $\frac{3+2}{5+4}$ requires that both additions are undertaken before the division; and
- $\overline{3+2} \times 7$ requires that the addition is undertaken before the multiplication.

When there are multiple grouping symbols in a single expression, the preferred way to write them is {[( )]} with the parentheses on the inside, the square-shaped brackets used next, and braces used outermost.

6.2.5 Comparing values

A variety of symbols are used to compare two or more values:

Equality

=	Equal to
≡	Equivalent to, congruent
≈	Approximately equal to

Inequality

<, >	Less than, greater than
≤, ≥	Less than or equal to, greater than or equal to
≪, ≫	Much less than, much greater than
≠	Not equal to

Other

∝	Proportional to

By the end of middle schooling, students would be expected to understand all of these symbols. However, many students misunderstand the nature of the equals (=) symbol, believing that it means *I am now about to write the answer*. As explained in the YuMi Deadly Maths book on Operations, it is important that students understand that equals means *same value as*. Failure to grasp this concept from the beginning will cause problems when algebraic equations are introduced later.

Another source of confusion for some students is the fact that the word *is* can be used with each of the symbols that are used to compare values. For example:

ten *is* the same as five times two	10 = 5 × 2 (also ten *is* five times two)
ten *is* more than five	10 > 5
price *is* proportional to mass	$\mathrm{P} \propto \mathrm{m}$

Students must learn to look at the words that follow *is*, in order to make sense of the statement. However, in the case of equality, the word *is* may or may not include the words *the same as*.

6.2.6 Abbreviations

Mathematical abbreviations

A number of standard abbreviations are used in mathematics that students should be taught and encouraged to use, including:

∴	therefore
Q	because
∃	there is, there are
∀	for each, for every
∈	is an element of
∞	infinity
$f(x)$	the function, f, of x
%	percent (or per cent)
...	and so on
∑	the sum of
μ, $\overline{x}$	mean (μ is the population mean and $\overline{x}$ is a sample mean)

Mathematical abbreviations are only used in mathematical notation or informal (personal) note taking. They should not be used in formal situations in sentences (see section 6.2.8).

Abbreviation of units of measurement

Units of measurement are critical in mathematics. In most parts of the world the metric system is used. However, the imperial system is still used in three countries: USA, Liberia and Myanmar. In the UK the metric system has been officially adopted, but some aspects of the imperial system are still used alongside the metric system. This means that students may encounter imperial measurements and their abbreviations.

Early systems of measurement were based on easily observed phenomena such as the length of body parts, the distance that could be walked in a certain time, and astrological events (days, months, seasons, years). These systems involved non-standard units (for example, the length of a hand span may vary from person to person). Many traditional cultures, including Indigenous communities, still rely on such systems. Measurement was a practical, rather than an abstract, activity, so traditional units of measurement were developed only if there was a need for them in that culture. Students brought up in traditional communities may need additional support to ensure that they have mastered the metric system of measurement.

Middle school mathematics students are not generally expected to work with units that are not standard for their country. However, as non-standard units can appear in some documents and books (particularly if published some time ago) and on television, students occasionally encounter them. It is recommended that they have a basic familiarity with these units.

Almost all units of measurement have standard abbreviations. These are commonly used whenever symbols are used. However, in prose, the unabbreviated form should be used.

There are some alternative, but non-standard, abbreviations in use (for example, *cc* for cubic centimetres; *sec* for second; *hr* for hour). Students must be able to recognise them, but should be discouraged from using them in their own writing.

6.2.7 Geometric symbols

Students should be taught and encouraged to use the following symbols that are commonly used as abbreviations in geometry:

||	is parallel to
⊥	is perpendicular to
|||	is similar to
≡	is congruent to
∠	angle
Δ	triangle
○	circle
□	square
▯	rectangle

6.2.8 Combining mathematical symbols with other symbols

Combining with words

Care is needed when combining words and symbols, particularly in formal mathematical writing. Whole numbers less than ten should not be written as numerals in sentences (prose); instead, the number should be written out in full, for example, three cats instead of 3 cats. Beyond ten, numbers may be written in numerals, although exceptions can be made in the case of hundred, thousand, million, and so on (e.g. three million instead of 3 000 000). Large numbers with fewer than five digits are usually written with no spaces, for example 2018. If there are five or more digits, for ease of reading, a space (not comma) should be inserted between the groups of three, starting from the right, for example, 1 573 853. In the past, commas were used to separate groups of three digits, for example 1,573,853, but this should be discouraged to avoid confusion with the European representation of the decimal point.

Units of measurement, when used in a sentence, should be written out in full. Symbols should never be used in a sentence as an abbreviation; thus the symbol $\therefore$ should not be used as a substitute for the word *therefore*. It is appropriate to include expressions or equations using mathematical symbols in prose, but all pronumerals should be written in italic script form (Cambria Math and Times New Roman italic font work well). If the expression or equation is extensive, it may be centred on its own line.

Combining with computer notation

Some students use computer notation as a substitute for mathematical symbols, for example 3*2 instead of 3×2, 3/2 instead of $\frac{3}{2}$, SQRT(3) instead of $\sqrt{3}$. While easier to word-process, they are not standard mathematical notation. Students wishing to word-process mathematical notation should be shown how to use software such as Microsoft Equation or MathType. Alternatively, they should leave space in the document and neatly handwrite the notation after printing.

6.3 Mathematical arguments

Mathematical texts frequently include mathematical arguments presented in symbols (often called *mathematical notation*). It is beyond the scope of this resource to deal with the conventions of mathematical notation, although it is explained in detail in Carter and Hipwell (2013b). Students should be familiar with, and encouraged to use, the words commonly used to link the different parts of mathematical arguments, such as *if, so, where, while, let, since, because, therefore, for each, for every, if … then* (or the mathematical symbols used to represent some of these words). Effective presentations of extended mathematical arguments include some explanations in sentences. Finally, the conclusion of a mathematical argument usually requires a sentence to present the result in context.

6.4 Mathematical definitions

There is a convention for writing mathematical definitions. Brevity is essential. This is often achieved by the use of symbols, but even where students are unfamiliar with the symbolic representations, they can write a concise definition using words. For example, a square is commonly defined as a plane figure with four equal straight sides and four right angles. However, this is a list of properties rather than a definition – it includes too much detail. This list can be pared down further. A square can be defined as a plane figure with four equal straight sides and one right angle (since one right angle, when combined with four equal straight sides, must result in right angles at the other three vertices). It is an interesting challenge to ask students to write the most concise definitions possible for other geometric figures.

6.5 Teaching strategies

Just as the teaching of a language is about more than learning the vocabulary, the teaching of the mathematical symbolic language is about more than students learning the meaning of the symbols. As we have seen, it has two elements: symbols (lexicon) and conventions for their use (syntax). It follows that teaching strategies must focus on both elements.

As already mentioned in section 5.3, YuMi Deadly Maths does not often encourage imitative forms of teaching and learning. However, they can be useful when initiating students into a community of practice. Students should be encouraged to memorise the symbols of mathematics by regular exposure through the use of word walls, charts, flashcards and placemats. Making their own resources of this type can enhance student understanding and recall. The conventions associated with the use of mathematical symbols must be explicitly identified and explained. They are learnt by regular exposure through teacher modelling and student practice.

7 Visual Images

Mathematics is not only composed of words and symbols, it is also a pictorial language that uses visual models to communicate. From a mathematics perspective, a *visual image* is a print or electronic (digital) picture or representation of something or someone. It covers a broad range from tables, graphs and diagrams that present information, through to illustrations intended to enhance engagement. The term *visual image* is used in this resource in preference to terms used by other writers such as *graphics, information graphics* or *graphical images* to avoid the suggestion that visual images are limited to graphs or confusion with the subject known in many schools as graphics (Hipwell et al., 2017).

Visual images are an important form of mathematical communication, in school and extending into post-school education, training and employment. Perusal of most secondary mathematics textbooks reveals that visuals appear on almost every page. Assessment items such as NAPLAN tests also make extensive use of visuals. The expectation that students of mathematics should be able to encode and decode visuals that provide both quantitative and qualitative information requires a broad approach to developing strategies for using these visuals. Creating visual images in mathematics requires students to think visually, that is, to visualise. They need to explore what visual model a word conjures up for them and teachers should provide frequent opportunities to visualise and construct. This chapter looks at the use of visual images as a form of mathematical communication.

Students are likely to encounter a wide variety of visual images in mathematics. The extent of this variety makes it difficult for students to learn about and practise every possibility. It follows that it is neither practical nor likely to be successful for teachers to try to teach every variation or type of visual image. The traditional approach to teaching visuals is by purpose, with little transfer of knowledge between contexts. For example, number lines, linear measuring scales, measuring gauges, and timelines all rely on the interpretation of a scale in a single dimension, but they are often taught separately in mathematics and also in other learning areas, implying that they differ from each other. However, it is the characteristics (properties) of a visual (e.g. scale, direction, shape, colour) that primarily determine how it is decoded. A teaching approach that focuses on the *properties* of visual images is more likely to assist students in making meaning and in transferring knowledge between visual images with similar properties (Hipwell et al., 2017).

The nature of a visual image determines how it is used and how we write about it. As shown in Figure 7.1, mathematical visual images can be classified into three broad groups (Hipwell et al., 2017):

Tables	Informative visual images	Narrative visual images
• provide information in compact, summarised form	• show factual information • include graphs of all types, number lines and Cartesian planes, timelines, all types of maps, plans and blueprints, scale drawings, geometric diagrams, Venn diagrams, flowcharts, tree diagrams, hierarchies and networks	• used in many situations to add interest and tell a story • include illustrations, photographs and sketches

Figure 7.1 Types of mathematical visual images

The purpose and properties of a visual image also determine how it is used. Section 4.2.2 outlined four ways of reading texts: skimming, scanning, continuous reading and close reading. When reading visual images, scanning (looking for specifics) and close reading (paying attention to all the details) are more important reading practices than skimming and continuous reading.

7.1 Types of representations

Mathematical ideas can be represented in many forms, including stories, verbal and written descriptions, actions, manipulatives, diagrams, tables, ordered pairs/coordinates, maps, graphs (including number lines), simulations, algebraic equations and models. One of the aims of teaching mathematics is to develop an understanding of all representational forms and to be able to translate mathematical ideas from one representational form to another.

Many of these representational forms are visual images. It follows that students must be able to interpret (decode) and create (encode) visual images, and to provide spoken and written explanations of their meaning. These skills are developed in increasingly complex situations throughout schooling.

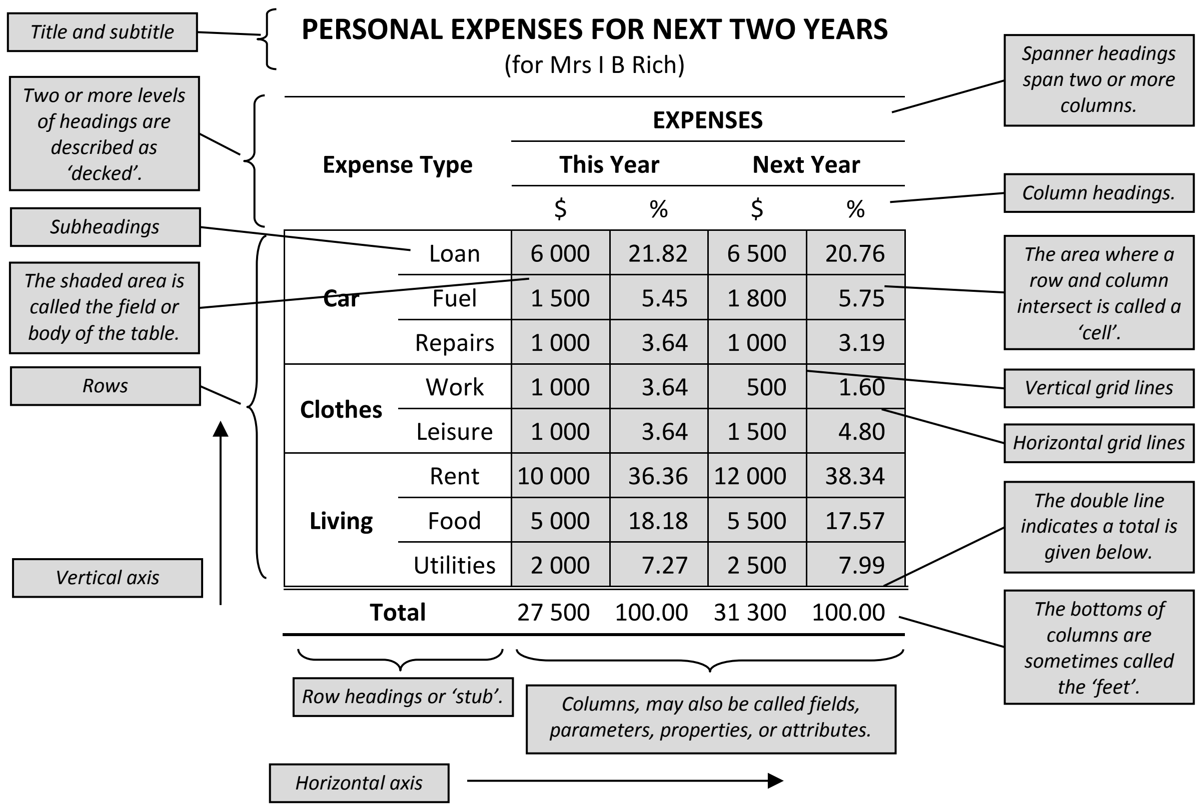
Mathematical visual images are used across the curriculum. However, their interpretation, use and preparation is initially developed in mathematics lessons.

7.2 Tables

Tables present data in a more compact form than sentences. They are one of the best ways to convey numerical information and to store data for rapid reference. It follows that tables are an important tool for solving mathematical problems. However, tables are difficult for inexperienced students to read, as they are read in unusual directions unlike the left-to-right and top-to-bottom approach used with prose. Students are required to track information horizontally and vertically and possibly from one table to another.

7.2.1 Parts of a table

The names of the different parts of a table are shown in Figure 7.2.



PERSONAL EXPENSES FOR NEXT TWO YEARS
(for Mrs I B Rich)

Expense Type		**EXPENSES**			
		This Year		**Next Year**	
		$	%	$	%
Car	Loan	6 000	21.82	6 500	20.76
	Fuel	1 500	5.45	1 800	5.75
	Repairs	1 000	3.64	1 000	3.19
Clothes	Work	1 000	3.64	500	1.60
	Leisure	1 000	3.64	1 500	4.80
Living	Rent	10 000	36.36	12 000	38.34
	Food	5 000	18.18	5 500	17.57
	Utilities	2 000	7.27	2 500	7.99
Total		27 500	100.00	31 300	100.00

Figure 7.2 The parts of a table

At their simplest, tables can be a way of presenting information in a list. However, even simple tables need careful introduction and analysis.

Tables can be compiled manually or digitally, often in spreadsheets. There are several conventions about how every table (print or digital) should be presented. They should all have a title so that the reader knows what the data is about. Depending on the length of the document, the tables may be numbered, for example *Table 3*. The data is arranged in rows and columns. Those rows and/or columns require headings and labels that explain the meanings of the cells. Row and/or column labels are often arranged in a particular order to make the relationships, trends, comparisons, distributions and/or anomalies easier to see. The data in the body of a table can be numeric or words. In the case of numeric data, the last row and far right-hand column may be used to display column and row totals. A grand total may be given in the bottom right-hand cell of the table.

7.2.2 Types of tables

One-way tables show information (attributes) about one type of variable along the same axis (usually horizontal). In a one-way table, each row may be called a record, and may not have a heading. The columns, which may also be called fields or attributes, show the different characteristics of each record, and should include a heading. An example of a one-way table is a statistical frequency distribution table.

In *two-way tables* the information in each row is combined with the information in each column to create the value shown in the cell formed by the intersection of that row and column. There are several types of two-way tables, including the following:

- Calendars, timetables and schedules.
- Grids that show the result when the row elements combine with the column elements in a defined manner, for example, a stem and leaf plot. Often the row and column headings are the same, resulting in a square table. When a square table is reflected in the diagonal (top left to bottom right), equivalent values are often found in the corresponding cells. Examples of this type of table are addition and multiplication tables and sporting results.
- Two-way *frequency* tables that show the number of occurrences in each category of two variables: one variable is shown in the rows and the second variable is given in the columns. An example of this type of table is one that shows the height and weight of a group of people.

If the order in which the data is presented matters, then the row elements are usually read before the column elements (for example, if tossing a coin twice then the data in rows would show the outcomes from the first toss of the coin, while the data in the columns would show the outcomes of the second toss).

Tables can be used for six purposes, as shown in Figure 7.3.

Summarising (e.g. a frequency distribution table showing various scores and their frequencies)	Comparing (e.g. a table that shows trends over time)	Recording and organising data (e.g. a table that records the responses to questions in a survey)
Showing relationships (e.g. a table of Cartesian coordinates)	Processing steps in an iterative process (e.g. a table used to calculate interest earned in a year and the value of the investment at year end, over several years)	Manipulating data (e.g. calculating percentages, often in a spreadsheet)

Figure 7.3 Six purposes of tables in mathematics

7.2.3 Interpretation of a table

The interpretation of a table requires explicit teaching. Unlike prose, tables are not read from left to right and top to bottom. Initially the table should be **scanned**, making note of the title and row and column headings to form an idea of what the table is about. Then the data in the individual cells may be **examined closely**, looking for values of interest, trends in the data and anomalies (unusual values). It is not always necessary to read the value of every cell to obtain an understanding of the data being presented in the table.

Examining the data in cells requires students to track information both horizontally and vertically. This may be challenging for younger students.

7.2.4 Creating tables

Creating a table is a complex skill, indicated by the number of issues that students need to consider when designing a table. Students must start by examining the data to go in the table and then consider a variety of issues, including:

- **title**: the subject matter of the table, with a date of preparation/collection of the data, wherever possible;
- **type of table**: one-way or two-way;
- **type of data to be shown**: discrete and nominal data will usually require the data to be presented as individual values, whereas continuous data (or discrete and nominal data with many values) will require that the data is arranged in groups (also called classes or categories);
- **space on the page**: this may affect the layout of the table as rows and columns may be transposed (swapped) to make it fit more comfortably on the page;
- **variables to be shown**: what type and how many; as we cannot show three-dimensional tables, more than two variables will complicate the table, often requiring more than one the table;
- **arrangement of headings**: typical arrangements include alphabetical, ascending or descending order; chronological; geographical; qualitative criteria (e.g. importance or preference); and relationship to other columns or rows (e.g. percentages may be adjacent to the corresponding numbers); subheadings may also be required;
- **contents of the cells**: words, numbers or symbols; single or multiple values; the cell contents will often determine column width and row height;
- **requirement for totals** (numeric data only);
- **requirement for grid lines, shading and/or colour**: if there is colour or shading, a key may be needed to explain the interpretation of the shading and colour; and
- **alignment of the data**: sentences or phrases words are usually left aligned; single words may be left aligned or centred; numbers must be aligned under the decimal points, or to the right (if all numbers contain the same number of decimal places).

This list shows that the creation of a table starts with consideration of the cell contents (note that this differs from the starting point when interpreting a table). Headings, titles and other embellishments can be added later.

7.2.5 Spreadsheets

Spreadsheet software makes it very easy to prepare, manipulate and present tables and to perform calculations on the data contained therein. Students should be taught to use spreadsheets as an extension of (but not an alternative to) their work with tables. They should be encouraged to use them as more than a word-processor, and to take advantage of the various calculations and functions available within the spreadsheet. Teachers should be willing to accept (and even encourage) student work submitted in spreadsheet form. Students can show their working by submitting the spreadsheet printout in both *Data View* (showing the results) and *Formula View* (showing the calculations and formulas used to generate the results). The preparation of a spreadsheet requires as much understanding of the relevant concepts as the preparation of a table manually.

Often the compilation of data in a table is a preliminary step for the preparation of a graph. It follows that there is a very close relationship between tables and graphs.

7.3 Informative visual images

Informative visual images show factual information. They include graphs of all types, number lines and Cartesian planes, timelines, all types of maps and scale drawings, geometric diagrams, Venn diagrams, flowcharts, tree diagrams, hierarchies, and networks. Understanding visual images is fundamental to success in mathematics. Several studies have highlighted the important role that the teacher plays in developing students' access to visual images.

Visual images present information in visual-spatial, rather than linguistic or symbolic formats. This draws on spatial ability, which is a composite of other abilities such as visual perception, visual processing and mental rotation. Students with difficulties in these areas are likely to experience difficulty with decoding visual images. Further, there can be gender differences in spatial ability, with girls generally requiring more support in this area than boys.

The interpretation of visual images also requires an understanding of the relevant mathematical content and the context of the problem. These issues are dealt with in the topic-specific YuMi Deadly Maths books. However, just as the comprehension of prose requires higher order thinking that goes beyond a grasp of spelling, grammar and punctuation, so the interpretation of visual images requires more than just teaching the underlying skills such as scaling, drawing axes and locating points.

There are strategies that can assist in interpreting visual images. As noted earlier, the usual approach to teaching visuals in mathematics is by content area (topic), with little attempt to transfer the knowledge of visuals between different contexts. For example, number lines, measuring scales (of all types), protractors and box and whisker plots are often taught separately, implying that they differ from each other. However, it is the *characteristics* (properties) of a visual (e.g. scale, direction, shape, colour) that primarily determine how it is interpreted. An approach that considers visuals according to their properties rather than their content is more compatible with the "Big Ideas" philosophy that underpins YuMi Deadly Maths.

Strategies for interpreting visual images depend on the nature of the image. However, all informative visual images share several characteristics. They should all have an explanatory **title**. Depending on the length of the document in which they are embedded, the visual images may be **numbered**, for example, *Figure 6*. **Labels** are used to convey additional information, for example:

- axes and scales in graphs;
- keys in maps and some graphs (such as pie charts); and
- annotations in diagrams.

Similar decoding strategies can be applied to visual images with similar properties. For example, the skill of interpreting scale can be applied to all visual images with one axis (such as number lines) or two axes (such as statistical graphs), and maps. Similarly, a knowledge of angles can be used to interpret maps, pie charts, geometric diagrams and Cartesian planes.

The first step in interpreting a visual image is to establish what type of visual image it is. When interpreting unfamiliar images, students should be encouraged to consider if the image is similar to anything they have seen before. Once a connection to a known visual image has been made, students' existing skills and knowledge can be applied to the unfamiliar situation. For example the visual image shown in Figure 7.4, resembling cigarettes in an ashtray, may be unlike anything students have seen before. However, the length of each cigarette indicates the percentage of smokers in that country, just like the columns of a column graph. Once the similarity between Figure 7.4 and column graphs is detected, the interpretation of the visual image is much easier.

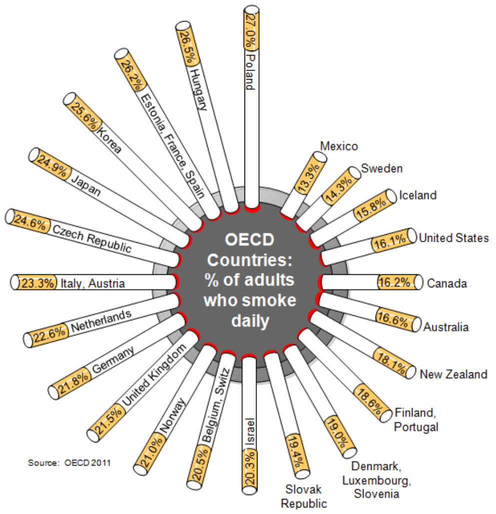


Figure 7.4 Example of an unfamiliar visual image

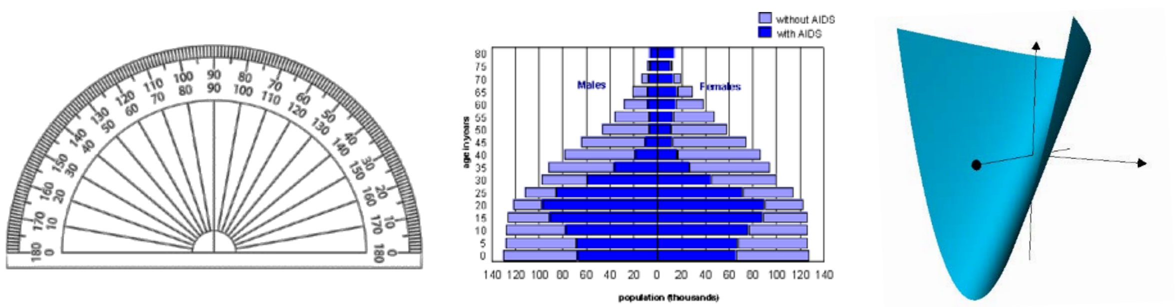
Research shows that students will not transfer existing skills and knowledge to an unfamiliar situation without practice.

7.3.1 Scaled visual images

Graphs and number lines

Many informative visual images convey information by the use of a scale on one, two or three axes. As shown in Figure 7.5, the axes may be in any orientation. A single axis may be curved (e.g. in an analogue clock face or protractor). If there are several axes, they are usually, but not always, orthogonal (meet at right angles). Examples include:

- **one axis**: number line, time line, thermometer, ruler, tape measure, measuring gauge, protractor, speedometer, analogue clock, divided bar graph, box and whisker plot;
- **two axes**: pictograph, column graph, bar graph, line graph, Cartesian plane, Argand diagram; and
- **three axes**: 3D Cartesian graph.



one axis two axes three axes

Figure 7.5 Examples of visual images with one, two and three scaled axes

In its simplest form, the scale is explicitly marked as a series of consecutive numbers and marks, usually starting at zero. However, in this form students often understand the axis to be a discrete scale, that is, a means of counting rather like beads on a number frame. This is often inadvertently reinforced by classroom activities where students use number lines as a concrete aid when adding and subtracting whole numbers. Students must understand that a scale is continuous, that is, measured, before they can progress to the higher order skills of interpreting and reasoning with scales (Lowrie & Diezmann, 2005).

This becomes apparent when some numbers are removed from the scale. For example, the scale may be marked in lots of five or ten, or even at irregular intervals. To make sense of the axis, students must be able to identify values even if they are not labelled. In these cases, counting will not help in finding the missing value(s). To make sense of the missing numbers, students must be able to deduce whether the scale is ascending or descending by comparison with two or more known quantities on the axis, and use proportion to estimate the value being shown. In other words, students are now using distance, proximity and reference points to interpret values on the axis. These are the skills of measurement, not counting.

If the scaled axis on a visual image does not show any values at all, the scale must be inferred. In these cases, students must know and apply the convention that movements up the page or to the right suggest increases, while movements down the page or to the left represent decreases. The order of marks matters, and marks that are closer together in distance are also closer together in value, and vice versa. It is still possible to make comparisons in these circumstances.

The first quadrant of a Cartesian plane has many similarities to column and line graphs. However, when negative values are included the graph is extended from one quadrant to four. The numbering of these quadrants can be a source of confusion as they start in the top right and move in an anticlockwise direction.

Once students have mastered the interpretation of scaled axes they are able to interpret a wide variety of graphs, both familiar and unfamiliar.

Maps and scale drawings

The common feature of maps and scale drawings, such as those shown in Figure 7.6, is the use of scale (explicit or implicit) to create a representation of a (usually real-world) situation. To interpret a map or scaled drawing, students need an understanding of distance, direction and symbols.

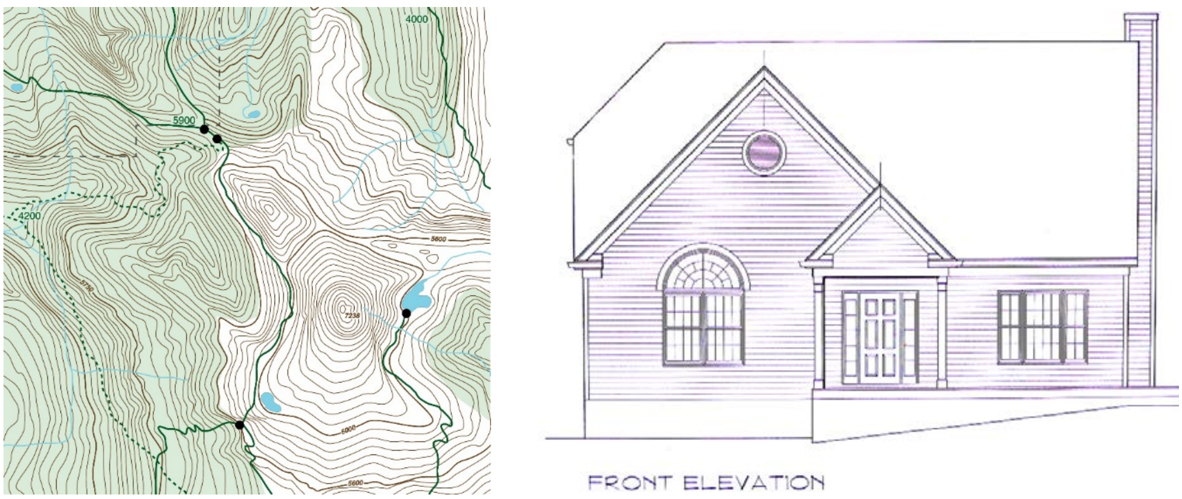


Figure 7.6 Examples of visual images that require an understanding of distance, direction and symbols

Interpretation of distance requires an understanding of scale. Scale can be represented in three ways: a measured line interval, equivalent measurements (for example, 1 cm ≡ 2 km) or a ratio (for example, 1:2 000 000), and some maps show all three forms. While the first two forms are also commonly used in graphs, the ratio method is generally only used with maps. Teaching of scale should connect these various representations.

Interpretation of direction needs an understanding of angles and the words used to describe direction. Students should understand both reference-based descriptors of direction (e.g. *left, right, up down*) and absolute descriptors of direction (e.g. *north, south,* compass bearings). They should understand the relationship between direction and angle. However, confusion can arise when considering angles in maps compared to the Cartesian plane: in maps, angles are measured in a clockwise direction starting at north (which usually points to the top of the page), whereas in the Cartesian plane, angles are measured in an anticlockwise direction, starting at the positive direction of the x-axis (which usually points to the right of the page).

Maps can make use of grids and coordinates such as latitude and longitude. This has obvious connections to the Cartesian plane. However, directionality is important. In a map, latitude (how far north or south, represented vertically) is given before the longitude (how far east or west, represented horizontally). In contrast, in the Cartesian plane, the x-coordinate (represented horizontally) is given before the y-coordinate (represented vertically).

Maps may use symbols, lines, colour and shading to show features. These are usually explained in a key. Often, the additional information provided on a map is very detailed and may even be on a different page (in the case of street directories and atlases), and as a result, be confusing. Students need practice in interpreting symbols, lines, colour and shading on a map.

To interpret explanations or instructions relating to maps and scaled drawings, students need an understanding of the relevant spatial language (for example, *between, through, from, to*). They should also practise more complex descriptions (such as *second on the left*).

7.3.2 Unscaled visual images – from picture to diagram

Younger students are more comfortable showing their understanding by drawing pictures. These form a basis for learning about more stylised representations. Moving from a detailed picture to a simplified shape is a step toward mathematical abstraction.

There is often confusion between the terms *diagram*, *picture* and *drawing*. In mathematics, a diagram is a representation used to show the relevant parts of a mathematical problem and how they fit together. It differs from a drawing or picture by excluding irrelevant (and often distracting) detail. Once this detail is removed, the diagram becomes an abstraction. In Figure 7.7, the left-hand image is a photograph of a ladder placed against a wall. There is a lot of extraneous detail (as often occurs in real-life contexts). The illustration in the centre is the same context, but much of the extraneous detail has been removed. However, the details of the ladder and the wall are still evident. In the right-hand case, all irrelevant detail has been removed to focus only on the mathematical relationships the triangular shape formed by the ladder and wall. It is no longer possible, nor is it necessary, to see that the sides of the triangle are a ladder and a wall.

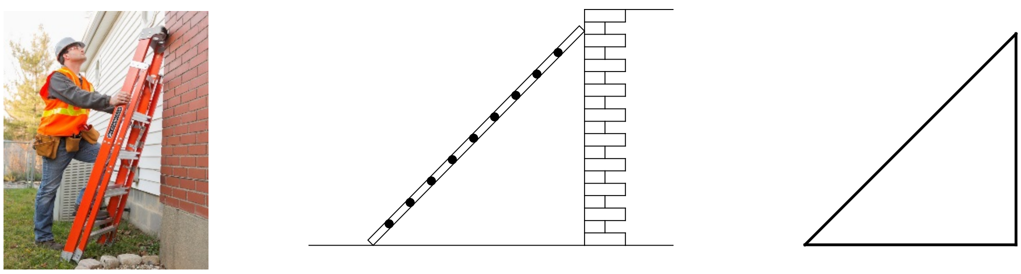


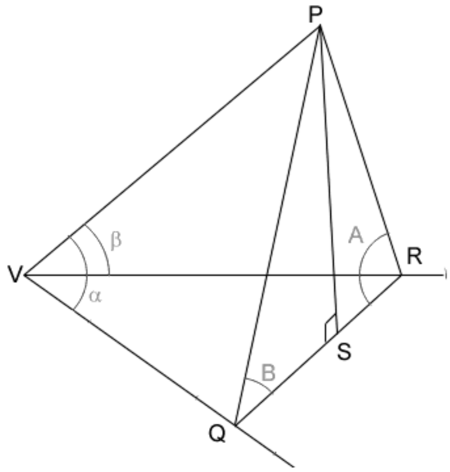
Figure 7.7 Depictions of a ladder against a wall

To successfully interpret the mathematical diagram, students need to recognise that the three images depict the same situation. In other words, their attention needs to be diverted from the surface detail to make it easier to focus on the mathematical relationships of the situation.

Most diagrams are not drawn to scale, nor are angles drawn exactly. If distance or angle are important they are annotated on the diagram. Students must understand that they cannot interpret an unscaled diagram by measuring. On the other hand, when preparing a diagram, it can assist understanding if the angles are drawn so that they are approximate representations of reality.

Geometric diagrams

A geometric diagram (figure) consists of points and lines (including curved lines, arcs and circles). Where lines intersect, angles are formed. One or more lines may form a closed shape (e.g. circle, ellipse, polygon). Three-dimensional geometric diagrams (figures) comprise points, lines, and surfaces (planes and curved surfaces, including spheres). Where lines and/or surfaces intersect, angles are formed. One or more surfaces may form a closed solid (for example, sphere, ellipsoid, polyhedron). Figure 7.8 shows two examples of geometric diagrams.



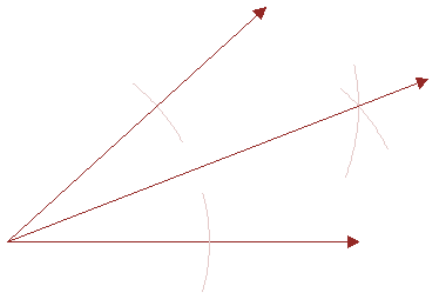


Figure 7.8 Examples of geometric diagrams

In geometric diagrams information is primarily encoded by the use of area, lines, points and angles. This results in an emphasis on shape.

To create (and interpret) geometric diagrams, students must understand these conventions.

- Lines, plane shapes and solids are defined by the points that lie in critical places on the figure (a line is identified by two points, a triangle by three points, a quadrilateral by four points, and so on). Since triangles, angles and arcs can all be labelled using three points, an additional symbol may be required to indicate what type of shape is involved.
- Length and angle size are indicated by annotations on the diagram. As geometric diagrams are not drawn to scale, they cannot be interpreted by measuring.
- It is important to be able to label points, lines (including curved lines) and shapes and to indicate lengths and size of angles. Upper case letters usually identify points, and lower-case letters are used to indicate values (such as measurements of length); Greek letters may be used to label the size of angles.
- The letter *O* is used as a label only for the centre of a circle or the origin of a Cartesian plane and the use of the letter *l* is avoided. This is to avoid confusion with the numerals 0 and 1. Where a figure is labelled with two or more letters, consecutive letters of the alphabet are often used.

A diagram of a single angle has two possible interpretations: the acute/right/obtuse angle and the reflex angle (except in the case of straight angles, where both angles are the same). The convention is that if the angle is unmarked, the smaller value of the angle is assumed. It follows that to indicate a reflex angle, it must be marked, usually with a small arc drawn near the vertex of the angle. Unlike maps and angles in the Cartesian plane, the direction of rotation of an angle is usually unimportant in a geometric diagram. Where it matters, the angle may be marked with an arc with an arrowhead to indicate the direction of rotation.

Other diagrams based on shape

There are a number of diagrams that, like geometric diagrams, are based on area, lines, points and angles, that is, shape (see Figure 7.9). They include Venn diagrams, pie charts (sector graphs), two-dimensional images of three-dimensional models, and tessellations.

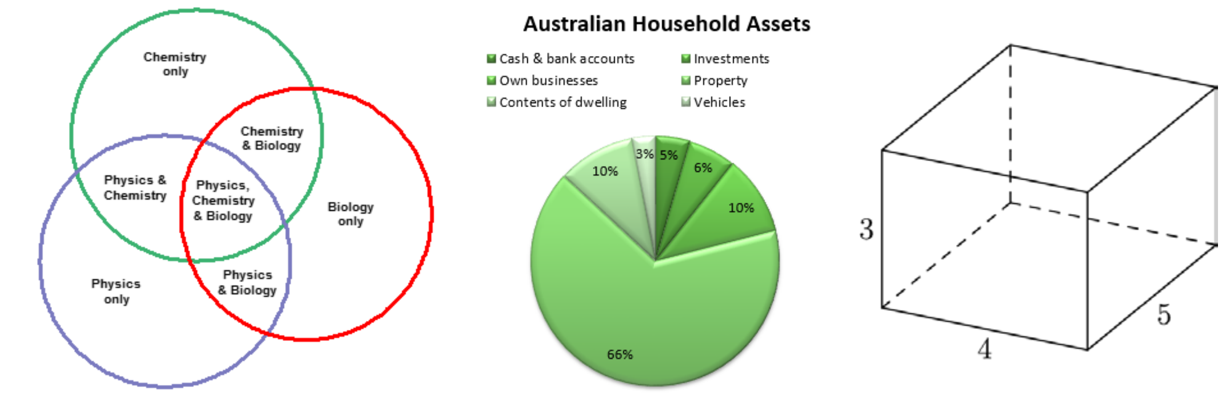


Figure 7.9 Examples of visual images that rely on shape

Diagrams that show connections

Some diagrams show connections (such as relationships or pathways) between various nodes (such as objects, people, data, variables). Some examples are shown in Figure 7.10. Labels, points or geometric shapes are typically used to represent the nodes, with lines (often with arrowheads) representing the connections. Where quantity is relevant it is usually shown by numbers written above the relevant symbol. These visual images may use symbols, depending on the purpose of the graphic, which are usually explained in a key.

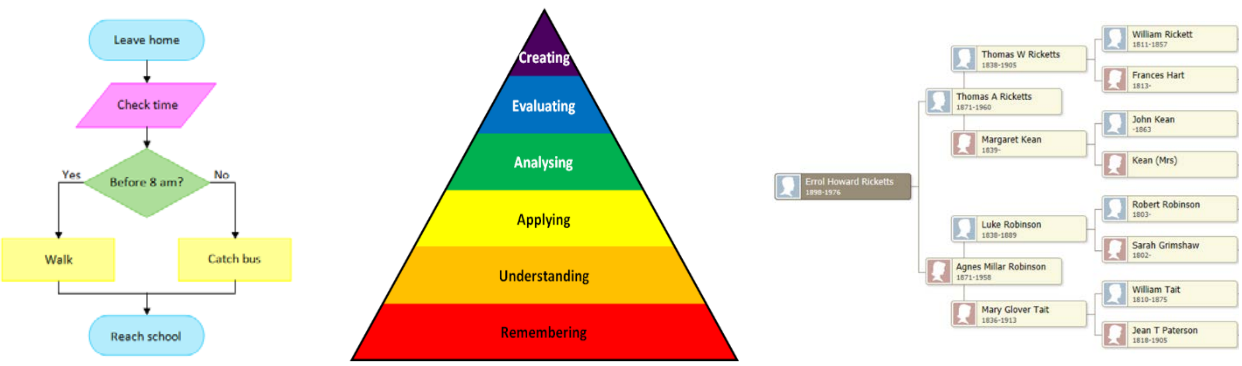


Figure 7.10 Examples of visual images that show connections and/or hierarchies

Students will often be familiar with some types of connection diagrams (e.g. simple tree diagrams in probability, mind maps, and simple networks such as “maps” of train and bus routes). They will also be familiar with equipment or machinery that relies on networks (e.g. a car GPS navigator that finds the best route between two locations).

However, they may not have explicitly considered them as a group that shares common features. In consequence, they may not readily transfer the skills and knowledge of one type of connection diagram to another.

Like other diagrams, connection diagrams only show relevant information. The stripping away of irrelevant information often results in very abstract representations of the real world (e.g. “maps” of rail networks, flow charts that detail the steps needed to undertake a task). Students may need assistance in understanding the diagram as a representation of reality. The interpretation and use of connection diagrams requires elements of logical deduction and use of systematic approaches.

7.3.3 Patterns

The interpretation of many informative visual images relies on recognising patterns. Recognising patterns may require elements of:

- counting or calculating to identify visual patterns based on number patterns;
- observing the repeated use of shapes, colour, shades, and so on;
- understanding symmetry (both line and point); and
- recognising transformations (such as flips, slides and turns).

Recognising visual patterns requires visual reasoning which differs from sequential reasoning such as that used in logical deduction. In some cases, the pattern may involve several components (e.g. more than one shape). Where this occurs, students should be encouraged to analyse the individual components of the pattern before putting them together to see the overall pattern.

7.3.4 Critical literacy

Words often accompany visual images, for example graphs that form part of a report, and diagrams in an explanation. A common statement is that a picture is worth a thousand words. This is often true, and explains why visual images are commonly used in mathematics where concise forms of communication are valued. The relationship between the visual image and the written text is one of the following:

The visual image is **vital** to the interpretation of the written text, for example there is information in the visual image that is not provided in the written text.	The visual image is **useful**, but the information can be found in other ways, for example if the information in the visual image is also provided in the written text.
The visual image is **irrelevant** to the interpretation of the written text, for example if nothing is lost if the visual image was omitted.	The visual image is **confusing, misleading or contradictory**, for example if the information in the visual image does not match the information in the written text.

Figure 7.11 Relationship between visual image and written text

Students should be exposed to all four situations and must know how to evaluate every visual image to ensure that they do not overlook important information.

In the case of informative visual images, once students can decode the image, the next step is to critically evaluate it. The reader should be able to analyse the image, taking a critical and questioning approach, to determine whether any findings and conclusions are reasonable and valid. Hipwell et al. (2017) list 33 questions that can be asked to assist a reader to critically analyse a visual image.

Misleading visual images

Is every picture worth a thousand words? Unfortunately, pictures can lie. Graphs are not always what they seem. A misleading graph does not necessarily mean that it is incorrect. But, unless they are examined carefully, they could give the wrong impression. Misleading visual images may be used when the creator does not know any better, or the creator of the image actually wants to mislead, hoping that the reader will not take the time to study the image in detail. Figure 7.12 shows two examples of misleading visual images.

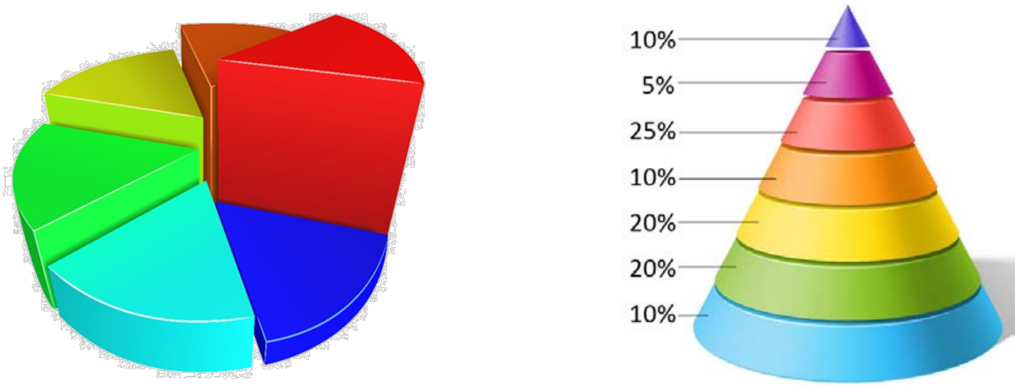


Figure 7.12 Examples of misleading visual images

Sometimes a graph can be very cluttered or contain a lot of empty space. When this happens, it may be necessary to alter the way the graph is presented. However, every attempt must be made to avoid misleading the reader. The creator of the graph should have a good reason for what has been done. Making the graph more attractive is not usually a good enough reason.

If a graph is found to be misleading, it calls into question anything else written by that author. Accordingly, students must take care to ensure that their graphs are not (intentionally or unintentionally) misleading.

7.4 Narrative visual images

Narrative visual images are generally used to appeal to the emotions. They rely on the perceptual properties of marks in the image (colour, shape, size, tone, texture, orientation) to convey information. They may be used in mathematics contexts to add interest and to assist in telling a story, for example, illustrations, photographs, and sketches. Narrative visual images may involve recognition of patterns or perception and association to understand the story they are telling.

While it is tempting to dismiss these visual images as being of less importance in mathematics, they are used very commonly, mainly to engage students. They should be drawn to students' attention when they occur, with discussions about the relevance to the remainder of the text (see Figure 7.11).

When narrative visual images appear in worded problem texts they are often intended to assist with interpreting the text, for example, by including a picture of the context. This is particularly common in standardised tests such as NAPLAN, where the test-writers cannot assume that all students are familiar with the context. In many cases they do not add to the information already provided by the words. However, on occasion they may supply important information not mentioned anywhere else (for example, a photograph of an advertisement might mention the price or a minimum purchase quantity that is not given anywhere else). Students must look carefully at every picture that they see.

7.5 The vocabulary of visual images

A further challenge in the teaching of visual literacy is the extensive vocabulary used to describe visual images, including: *artwork, audiovisual, caricature, cartoon, chart, depiction, design, diagram, display, drawing, figure, graph, graphic, illustration, image, map, picture, photo, photograph, plan, plot, sketch, visual aid* and *visual representation* for static images; and *animation, dynamic image, motion picture, movie, simulation* and *video* for moving images. Standard dictionary definitions of these words show that they all have slightly different meanings. Individual words can have multiple and even conflicting meanings.

For example, students seeking the definition of a *chart* from online sources may find that a chart is variously defined to be:

- a graph, diagram or table displaying detailed information (Cambridge dictionaries online, Wikipedia);
- a sheet presenting information in the form of graphs, diagrams or tables (Oxford dictionaries online, Merriam-Webster online dictionary, American heritage dictionary of the English language);
- a map used for navigation by sea or air (Cambridge dictionaries online, Oxford dictionaries online, Merriam-Webster online dictionary, American heritage dictionary of the English language, Wikipedia);
- a ranked listing of sales, for example of best-selling records (Cambridge dictionaries online, Oxford dictionaries online, Merriam-Webster online dictionary, American heritage dictionary of the English language, Wikipedia).

In contrast, the highly-regarded American Psychological Association manual for writing defines a *chart* to be a display of non-quantitative information.

Inconsistencies can also occur in the use of some words between learning areas, for example, the use of the word *histogram* in geography to refer to any type of column graph, whereas the same word is used in mathematics to refer to a very specific type of statistical graph. Teachers need to be aware of the difficulties caused by these vocabulary issues and ensure that various shades of meaning are discussed with students (Carter, Hipwell, & Quinnell, 2012).

7.6 Teaching strategies

Unlike in some other sections of this resource, explicit teaching is **not** a recommended method of learning about visual images. The extensive variety of visual images makes it difficult for students to learn every possibility. It follows that it is neither practical nor likely to be successful for teachers to try to teach every variation or type of visual image. As explained throughout this chapter, a teaching approach that focuses on the properties of visual images is more likely to assist students in making meaning.

8 Literacy Program

This section provides a suggested program for teaching literacy in the context of mathematics.

Primary teachers should ensure that their general literacy lessons also take account of the differences in literacy in the mathematics context (and in other learning areas). Secondary mathematics teachers should include literacy experiences in their pedagogy and cannot assume that students have learnt the required skills in other learning areas. If students are required to read or write mathematical texts in assessment, then these skills must be developed in mathematics lessons before the assessment occurs.

Inspection of the word lists in **Appendix B** shows that in the early years of schooling the emphasis is on developing a rich vocabulary of everyday English words that students can use in spoken and written forms to describe their mathematical ideas. This is best done in the context of cross-curricular language development. However, as students continue through their schooling, they are expected to develop a command of the technical and sub-technical language of mathematics. These words are best introduced in the context of mathematics, but can then be included in the more general classroom language and literacy activities.

By the time students enter secondary schooling, the literacy demands of mathematics start to diverge from those of other subjects. Where literacy in mathematics differs from literacy in other learning areas, it is logical that it is taught in the context of mathematics. The concise and efficient nature of mathematics writing can only be taught in the context of mathematics. According to the QCAA: “The responsibility for developing and monitoring students’ abilities to use effectively the forms of language demanded [in mathematics] rests with the teachers of mathematics.” (QCAA, 2014, p. 37). Thus, while mathematics teachers draw on the foundations developed by teachers of English, it cannot be assumed that literacy skills taught in other learning areas are automatically transferred to the mathematics context.

It is good literacy practice to have an approach to teaching that allows the mathematical literacies to be developed alongside the content. In other words, the teaching of mathematical ideas should include communication of those ideas. Teachers should seize all opportunities that arise in class to support the development of mathematical literacy.

Taking all the above into account, Table 8.1 below provides an effective program for development of mathematical literacy skills. It distributes across the clusters of year levels the six components: vocabulary development, spelling, speech, decoding, encoding, and the use of symbols.

8.1 Program for teaching literacy in mathematics

Table 8.1 *Program for teaching literacy in mathematics*

COMPONENT	LOWER PRIMARY YEARS	UPPER PRIMARY YEARS	JUNIOR SECONDARY YEARS	SENIOR SECONDARY YEARS
Listening	<u>Major focus</u> Listening to mathematical ideas. Following simple spoken instructions.	Listening to mathematical ideas and following instructions expressed using the appropriate mathematical terms.	Listening to mathematical ideas and following instructions expressed in mathematical terms.	

COMPONENT	LOWER PRIMARY YEARS	UPPER PRIMARY YEARS	JUNIOR SECONDARY YEARS	SENIOR SECONDARY YEARS
Speaking	Major focus Encouragement to verbalise mathematical ideas.	Encouragement to verbalise mathematical ideas using the appropriate mathematical terms.	Recognition that spoken mathematical language differs from written language.	
Building vocabulary	Major focus Everyday English words used to describe mathematical ideas.	Exposure to the technical words of mathematics, as they arise in the mathematics curriculum and associated classroom activities. Exposure to words with the same stem.	Introduction of the sub-technical words of mathematics, and continued development of technical words, as they occur in the mathematics curriculum and associated classroom activities.	Development and use of a rich mathematical vocabulary.
Reading and decoding	Understanding that mathematics tells stories. Reading activities to include mathematics contexts (e.g. Oxford Readers).	Major focus Decoding of simple worded problems and task words.	Major focus Decoding of complex worded problems and task words. Critical review of advertisements and graphs.	Decoding and critical review of selected mathematical reports and studies.
Spelling		Major focus General spelling activities to include mathematical words.	Mathematics teachers to ensure that students are able to spell technical, sub-technical and relevant everyday English words.	
Creating	Representing stories mathematically.	Understanding that speech and writing involve different language choices. Construction of simple mathematical sentences: describing, sequencing, summarising.	Understanding that speech and writing involve different language choices. Construction of one or more paragraphs: explaining, reflecting, comparing, analysing.	Major focus Construction of extended mathematical reports.
Symbols	Symbols that show number, addition, subtraction, equality.	Symbols for operations, fractions, comparisons, abbreviations for units of measurement.	Major focus Grouping symbols, geometric symbols, development of simple symbolic mathematical arguments	Major focus Development of extended symbolic mathematical arguments (higher level courses only).

COMPONENT	LOWER PRIMARY YEARS	UPPER PRIMARY YEARS	JUNIOR SECONDARY YEARS	SENIOR SECONDARY YEARS
Visuals/ displays	Exploring the use of simple displays including one- and two-way tables, number lines, graphs, maps and diagrams, generally involving small, positive whole numbers. Understanding that illustrations may not always convey useful mathematical information.	Interpretation and creation of more complex tables, statistical and Cartesian graphs, maps and diagrams, which display and compare quantities involving positive whole numbers and fractions. The relationship between a visual image and any associated words.	<u>Major focus</u> Interpretation and creation of more complex visual images. Preparation of complex tables, statistical and algebraic graphs, and geometric diagrams. Use of spreadsheet software to prepare tables and graphs.	<u>Major focus</u> Critical interpretation and evaluation of any type of table and visual image. Use of spreadsheet software, including formulas and functions.

8.2 Conclusion

This resource has provided an overview of the issues involved in teaching literacy in the mathematics learning area. Mathematics teachers should include literacy experiences in their pedagogy and cannot assume that students have learnt the required skills in other learning areas. If students are required to read or write mathematical texts in assessment, then these skills must be developed in mathematics lessons before the assessment occurs.

The mathematical vocabulary and symbols support a highly specialised and concise method of communication. In particular, students' understanding of the vocabulary of mathematics is essential to the decoding of mathematics texts, especially worded problems.

Teachers should not overlook the value of using writing as a learning tool. For example, confusion about area and perimeter could be resolved by asking students to complete a graphic organiser that compares the two and then writing a paragraph about the similarities and differences between them. Understandings of the strengths and limitations of the three common averages (mean, median and mode) could be developed by asking students to prepare a written explanation (what, how and why) about the circumstances in which each average is most effective.

Writing can also be used to support the reflection stage of the RAMR cycle. Teachers can use writing tasks to assess student understanding of important concepts, their proficiency in explaining and using those concepts, and their attitudes toward learning mathematics. Thus, writing in mathematics is beneficial for both teacher and student. Although it may take time and effort to introduce writing into the mathematics classroom, it is worth the effort.

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Appendix A: Words that are commonly confused

adjacent side/opposite side

affect/effect

amount/number

and/or

answer/solution

answer/total

approximate/estimate

arc/arch/ark

average/mean

bar graph/column graph/divided bar graph/histogram

base (of solid shape)/base (of a power or logarithm)

between/from...to

biannual/biennial

bisect/dissect

borrow/lend/loan/owe

brackets/braces/parentheses

capacity/volume

cent/sent/scent

circumference/perimeter

coefficient/constant

column/row

commission/commission rate

complement/compliment

complementary/supplementary

concave/convex

congruent/equal/equivalent/same

congruent/similar

converge/diverge

credit card/debit card/EFTPOS

deduction/induction

demonstration/proof

dependent variable/independent variable

descent/decent/dissent

discount/discount rate

discrete/continuous

discrete/discreet

distance/displacement

divide by/divide into

divisor/deviser

edge/side

eight/ate

ellipse/eclipse

ellipse/oval

ellipses/ellipsis

equal/equivalent

equation/expression/term

evaluate (find a value for)/evaluate (assess)

experiment/trial/event/outcome

explicit/implicit

extrapolate/interpolate

favourable (probability)/favourable (preferred)

four/for/fore

fourth/forth

horizontal/vertical

hour/our

hyperbola/hyperbole

income: gross/net/taxable

indices/indexes

infinite/undefined

intercept/intersect

interest/interest rate

latitude/longitude

left (direction)/left (remainder)/left (leave)

mass/weight
mean/median/mode
metre/meter
mode/mowed
nautical mile/statute mile
necessary/sufficient
not divisible/indivisible
not equal/unequal
number/digit/numeral
numerator/denominator
parentheses/parenthesis
pi/pie
plane/plain
population/sample
possible/probable
price: cost/buying/wholesale
price: selling/retail/marked/discounted
principal/principle
profit/profit margin
pronumeral/variable
qualitative/quantitative
remainder (division)/remainder (subtraction)
revolution/rotation
right angle (90°)/right angle (the angle to the right)
sector/segment
sequence/series
sign/sine
simplified form/simplification
speed/velocity
square metres/metres square
straight/strait
subscript/superscript
sum/operation
sum/some
survey/census
survey/questionnaire
theorem/theory
tonne/ton
trapezium/trapezoid
two/too/to
week/weak
whole/hole

Appendix B: Mathematical vocabulary

The lists below can be used as the basis for vocabulary and spelling activities to complement the study of particular topics in mathematics. The words have been mapped against the content descriptions for the various year levels of the Australian Curriculum.

Some words occur in many forms, e.g. *add*, *addition*, *adding*, *added*. In most of these cases, the list contains the stem word only.

In some cases where an unusual plural is given (e.g. formulae), a plural ending in *-s* (e.g. formulas) is also acceptable. If in doubt, consult the Macquarie Dictionary.

Number and Algebra

	Early years (Years F to 3)	**Upper primary (Years 4 to 6)**	**Lower secondary (Years 7 to 9)**	**Upper secondary (Years 10 to 12)**
Processes	about almost answer cover count on count explain false find join group guess how how many how much nearly problem question roughly sort think true what which who why	arithmetic calculate change (vary) compute convert estimate guess mathematics mental arithmetic number line number sentence puzzle repeat result round (off) rounded show working	affect approximate approximation arithmetic expression ascertain brackets demonstrate determine effect error evaluate exact value grouping symbols: brackets, braces, parentheses, vinculum imply infer law lower bound method nearest order convention/order of operation proof/prove response significant figures solution strategy truncate upper bound	analyse because conjecture exemplify hence hypothesis (pl. hypotheses) if imply increment infer interpret reasonable/reasonableness/ reasoning since synthesise therefore valid/validate verify when while

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Types of Numbers and Matrices	backwards close to dozen (no plural form) even number expanded form expanded notation first, second, third, ... etc. forwards just over just under last naught nil none number odd number one, two, three, ... etc. pair position zero	binary cardinal number composite number consecutive digit every other exactly gross (no plural form) hundred last but one, two, etc. numeral ordinal number palindromic number place value prime number score (no plural form) square number thousand triangular number	billion counting number directed number Fibonacci finite googol googolplex Hindu-Arabic numerals infinite/infinity integer irrational number magnitude million natural number negative number number system perfect square plus or minus positive number radical sign rational number real number Roman numerals sign square root surd transcendental number trillion whole number	abelian group absolute value altitude argument associative azimuth closure commutative complex number field group identity imaginary number inverse magnitude matrix (pl. matrices) modulo realise scalar set undefined vector
Comparing Numbers	alike altogether all attribute belongs between different enough equal from ... to greater than group high least less than like list low many match more than most none not equal order position right same size wrong	ascending order classify compare decreasing descending order few greatest identical increasing least majority maximum (pl. maxima) minimum (pl. minima) minority number line quantity set value	inequality nearest equivalent greater than or equal to less than or equal to	ambiguous insignificant negligible

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Operations	add all told altogether another another way as well collection count count on cover decrease deduct divide difference double, triple, quadruple each extra fit group increase join less minus more multiply of plus put reduce regroup remaining remove share sort subtract sum take (away) the same as times together total trade	diminish divisor factor highest common factor lowest common multiple multiple part portion product quotient remainder split	algorithm alternative method augment distribute/distributive dividend divisibility divisible/not divisible operation	binary unary unitary
Fractions, Decimals, Percentages, Ratio	part whole	cancel common denominator common fraction decimal fraction decimal place decimal point denominator equivalent fractions fraction half, third, quarter, fifth, etc. improper fraction invert mixed number numerator per cent/percent percentage proper fraction reciprocal reduced to simplest form simplified simplify vinculum vulgar fraction	decimal notation direct proportion equivalence equivalent ratio fractional notation golden ratio inverse inverse proportion per proportion proportional rate ratio recurring decimal repeating decimal terminating decimal unitary method variation	

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Index Notation			base cubed cubic number exponent exponential index notation index (plural indices) mantissa order of magnitude power scientific notation squared standard form	index form radical form
Finance	amount amount tendered bill budget buy cent change (money) coin cost deposit dollar exchange fee gain money note owe pay price purchase save sell spend swap trade withdraw	bank bank account bargain borrow budget business cheap currency dear expensive household interest value	accrue annual ATM (automatic teller machine) balance best buy closing balance cheque credit credit card credit charges credit limit daily debit debit card discount discount rate EFTPOS (electronic funds transfer at point of sale) fixed interest flat-rate interest gross income grow instalment interest rate invest lend loan lose loss minimum monthly payment money order monthly principal quarterly simple interest spreadsheet transaction weekly yearly	account amortise annuity appreciate available credit balance sheet biannually break even buying price commission commission rate compound interest compounding cost price costs depreciate exchange rate expenditure expenses GST (goods and services tax) income invoice mark up net income opening balance operating costs past value present value profit margin reducible reducing interest repayment rest period retail price retainer return revenue selling price semi-annually taxation wholesale price
Patterns	counting on missing order pattern position sort	arrangement compare equivalent number pattern predict repeating	sequence term	abstract abstraction generalise generalisation relationship

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Algebra	count on fit fit together join		algebra algebraic expression algebraic identity algebraic term backtracking balance binomial coefficient constant difference of two squares distributive law eliminate equation evaluate expand expression factorise formula (pl. formulae or formulas) function input inverse operation left-hand side like output perfect square pronumeral quadratic relation represent right-hand side rule satisfy simplify solution solve substitute symbol trinomial unlike	algebraic equation context dependent variable derive independent variable inequality model root of an equation simultaneous equations simultaneous solution transpose unique solution variable zero of an equation
Graphing	horizontal vertical	grid number line	axis (pl. axes) abscissa (pl. abscissae) Cartesian coordinates Cartesian plane coordinates curve domain gradient graphical method incline intercept linear number plane non-linear ordered pair ordinate origin quadrant range slope table of values	Argand diagram asymptote conic section cubic directrix (pl. directrices) ellipse focus (pl. foci) hyperbola (pl. hyperbolae) locus (pl. loci) parabola (pl. parabola) polar quartic secant tangent

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Rates and Calculus	fast quick slow		acceleration consumption rate density flow knot rapid rate speed travel graph velocity	area under a curve calculus derivative differentiation from first principles differentiation/differential integration/integral monotonic decreasing monotonic increasing optimum (pl. optima) rate of change

Measurement and Geometry

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Measurement	measure measurement	conversion convert metric system standard unit unit	formula mensuration rule	
Length	around backwards broad centimetre close deep distance far forwards height high long metre narrow near reverse ruler shallow short tall thick thin wide	boundary kilometre millimetre pace perimeter tape measure trundle wheel	girth inch mile odometer pi yard	angstrom displacement light year micrometre micron nanometre nautical mile statute mile
Area	inside narrow outside wide	area breadth cover depth width	acre altitude composite hectare net perpendicular height square units surface surface area	
Volume and Capacity	cup deep empty full jug shallow	capacity cubic litre millilitre volume	cross-section cubic units kilolitre megalitre pint	
Mass	balance heavy light mass scales weigh weight	gram kilogram milligram tonne	gross net ounce pound stone tare ton	gravity pulley

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Time	after afternoon age always analog/analogue arrive before begin calendar clock date day daytime depart digital duration early end finish fortnight hour January, February, etc. last last year late leap year leave midday midnight minute Monday, Tuesday, etc. month morning never new next next year night night-time noon now o'clock old season sometimes soon spring, summer, etc. start this year time today tomorrow usually watch week weekend when year yesterday young	24 hour time am (ante meridiem) century decade lunar month millennium pm (post meridiem) program second stopwatch timetable working day working week	AD (Anno Domini) BC (before Christ) BCE (before common era) CE (common era) CDT (Central Daylight Time) CST (Central Standard Time) daylight saving EDT (Eastern Daylight Time) EST (Eastern Standard Time) GMT (Greenwich Mean Time) international date line ME (modern era) quarter time zone WST (Western Standard Time)	instantaneous
Temp-erature	cold cool hot warm	degrees thermometer	boiling point Celsius centigrade freezing point	absolute zero Fahrenheit Kelvin

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Angles, Pythagoras & Trigonometry		acute degrees obtuse protractor reflex revolution right angle straight angle	adjacent clinometer cosine hypotenuse incline opposite Pythagoras sine tangent theorem triad trigonometry	depression elevation gradians minutes radians seconds
Geometry	arrow label ruler	classify compasses concave congruent convex diagram figure geometry object property protractor set square	adjacent bisect bisector construct extend orthogonal perpendicular bisector similar spatial	axiom convention corollary deduce demonstrate dividers network proof prove theorem
Location and Transformation	above after alongside around before behind below beside bottom down flip here in in front of inside left map next to off on on top out outside over pathway position right side by side slide symmetry there top turn under underneath up	east enlarge grid legend line symmetry north reduce reflection rotation rotational symmetry scale south tessellation translation west		

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Shape	circle corner curved edge face flat rectangle round side square straight surface triangle	arc centre chord closed cone cube cylinder decahedron diameter dodecahedron equilateral icosahedron irregular isosceles kite octahedron open parallelogram pentagon/hexagon/ heptagon, etc. polygon polyhedron prism pyramid radius (pl. radii) rectangular prism regular rhombus (pl. rhombi) scalene tangent tetrahedron trapezium/trapezoid triangular prism	pi	

Statistics and Probability

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Probability	card certain chance coin dice die doubt equally likely fair heads impossible improbable likely may (not) happen outcome possible predict probability probable spinner tails uncertain unfair unlikely very likely very unlikely will (not) happen	Ace, King, Queen, etc. pack/deck of cards Spades, Hearts, etc. standard pack/deck of cards	complement desired event expected experiment experimental probability favourable relative frequency repetition sample space theoretical probability tree diagram trial unfavourable	independent mutually exclusive probability tree diagram replacement simulate
Statistical Graphs	chart column graph graph picture graph	axis (pl. axes) bar graph divided bar graph key label line graph line plot pictogram pictograph pie chart plot scatter graph scattergram sector graph shade title	frequency histogram frequency polygon misleading skew	cumulative frequency histogram cumulative frequency polygon ogive

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Statistical Analysis	column count grid observe row sort swap table tally tally chart	data datum frequency information most common random	average bimodal categorical data centrality cumulative cumulative frequency dispersion extreme values frequency frequency distribution table mean median mode outlier qualitative data quantitative data random sample range sample score spread spreadsheet survey transpose typical variation	bias census class interval continuous data decile deviation discrete data distribution hypothesis line of best fit nominal data numerical data ordinal data parameter pentile percentile population quartile questionnaire regression line reliable sample statistic stem and leaf plot trend trend line two-way table unreliable Venn diagram

Technology

	Early years (Years F to 3)	Upper primary (Years 4 to 6)	Lower secondary (Years 7 to 9)	Upper secondary (Years 10 to 12)
Technology		calculator clear computer display enter execute key screen	alphanumeric CAS (computer algebra systems) clear all clear entry dynamic graphing software equation editing software format formula graphics calculator last answer memory memory operation key program reset scientific calculator scroll scroll key second function key shift key sign change key software package spreadsheet word processor	binary database hexadecimal matrix (pl. matrices) octal

Appendix C: Development of mathematical vocabulary

Knowledge of the target word	**Strategies and activities to develop students beyond this level**	**What these strategies will achieve**
Fully understands the concept, can define and use it in all relevant contexts.	Word usage (either directly or indirectly) shows that students have a deep understanding of the term/concept.	Deep understanding of the concept, illustrated by the ability to use and define the word.
	Students write their own precise and concise mathematical definition.	
	Students use the word appropriately in a sentence or authentic context.	
	Students write their own definition using the Frayer Model (Frayer, Fredrick, & Klausmeier, 1969).	
	Students complete the sentence "I will remember this word by …".	
	Students list other words that they might use with the target word.	
Knows the word well, can explain and use it in context.	Students generate their own vocabulary map.	
	Students list the properties (essential and non-essential) of the concept represented by the target word.	
	Students list synonyms and antonyms of the target word.	
	Teacher generates the vocabulary map.	Increased familiarity and a deeper understanding.
	Teacher gives examples, symbolic and/or visual representations of the target word.	
	Teacher pre-teaches the word and its meaning in context.	
	Students create sentences using the word appropriately.	
Knows the spelling and meaning, but cannot explain it clearly to others.	Mix and match activities (e.g. concentration, dominoes).	
	Teacher defines the word using expressions such as "something that" or "the process of".	
	Co-construction of lists of related words.	Superficial understanding of the word by engaging with a definition created by others.
	Teacher explains words and concepts that are commonly confused with the target word. Misconceptions are clarified. Mathematical meanings of the word are emphasised.	
	Teacher explains the etymology (e.g. Greek/Latin roots, base word, prefixes, suffixes, compound words, plural forms).	
	Teacher identifies the part of speech of the target word, and appropriate and inappropriate usage of the target word.	
Has seen or heard the word.	Teacher explains and students learn spelling, using various strategies.	
	Pronunciation.	
Does not know the word.	Glossary with definitions provided to the student (teacher, textbook, dictionary syllabus, handouts).	Increased familiarity with the target word and related words.
	Word lists (glossary, keywords, word walls) without definitions.	

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Appendix D: Task words

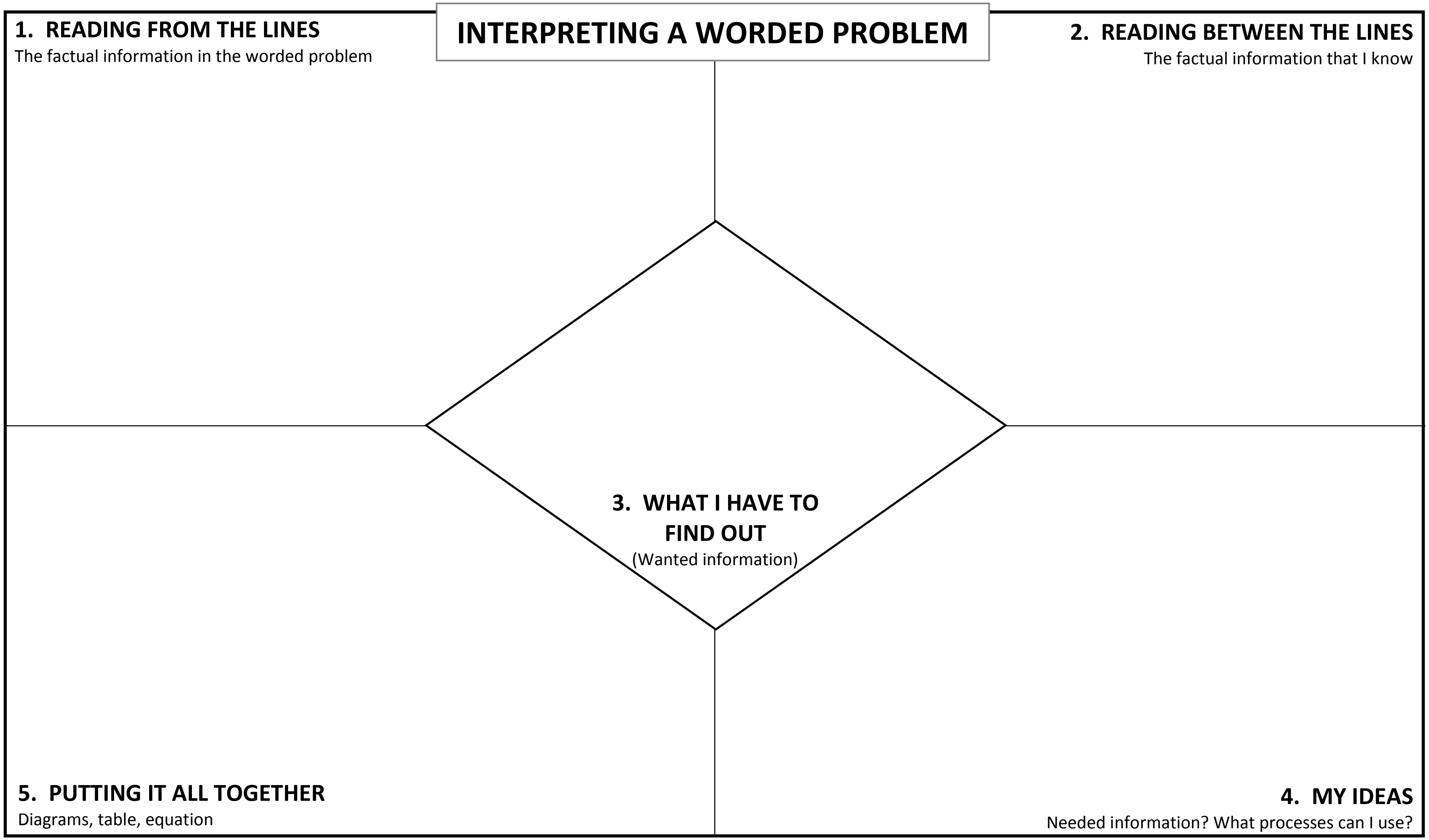
Abstract
Account for (give reasons)
Account for (try all possibilities)
Add
Analyse
Apply
Appraise
Appreciate
Approximate
Argue
Arrange (in order)
Assess
Assume
Break down
Calculate
Categorise
Check
Clarify
Classify
Combine
Comment on
Compare
Conclude
Consider
Construct
Contrast
Convert
Count
Create
Criticise
Debate
Decide
Decrease
Deduce
Defend
Define
Demonstrate
Derive
Derive (calculus)
Describe
Design
Determine
Develop
Devise
Differentiate (show differences)
Differentiate (calculus)
Discriminate
Discuss
Distinguish
Divide
Draw
Elaborate
Eliminate
Enumerate
Estimate
Evaluate (assess)
Evaluate (find the value of)
Examine
Exemplify
Expand
Experiment
Explain
Expound
Extend
Extract
Extrapolate (data)
Extrapolate (generalise)
Factorise
Find (locate)
Find (the answer to)
Formulate
Generalise
Give
Graph
Identify
Illustrate (provide a visual image)
Illustrate (provide an example)
Increase
Indicate
Infer
Integrate (calculus)
Interpolate
Interpret
Investigate
Judge
Justify
List
Locate
Measure
Memorise
Model
Multiply
Order
Organise
Outline
Paraphrase
Persuade
Plot
Predict
Prepare
Present
Propose
Prove
Quote
Rank
Recall
Recite
Recommend
Record
Recount
Reduce
Refer
Reflect (an image)
Reflect (on a process)
Relate
Review
Rotate
Select
Sequence
Show (that)
Simplify
Simulate
Sketch
Solve
State
Substitute
Subtract
Suggest
Sum
Summarise
Synthesise
Tabulate
Test
Trace
Transcribe
Transform
Translate (an image)
Translate (from one form to another)
Use
Validate
Value
Verify
Visualise

Appendix E: Words that suggest arithmetic operations

Addition	Subtraction	Multiplication	Division
accumulate add altogether all told and another augment bigger (than) credit deposit extra faster (than) forward further (than) gain greater (than) grows heavier (than) higher (than) increase longer (than) more (than) older (than) positive plus rise sum taller (than) thicker (than) together total up wider (than) with	backwards debit debt decrease deduct difference diminish discount down exceed fall fewer (than) from gone leave left (over) less (than) lighter (than) lose lower (than) minus narrower (than) nearer (than) negative net (e.g. income) off reduce remaining remove reverse shorter (than) slower (than) subtract take (away) thinner (than) withdraw younger (than)	array by commission double, triple, etc. factor groups of lots of magnify multiple multiply of product repeated square, cube, etc. taxation times times more (than) twice, thrice, etc. twofold, threefold, etc.	distribute divide divisible divisor factor fraction groups half, third, quarter, etc. halve left over out of parts per per cent or percent portion quotient rate reciprocal remainder share split

Note: Use these lists as a guide only; remember that these words could be used in a worded problem for any operation. See *Clues from the words* in section 4.4.4.

Appendix F: Graphic organisers



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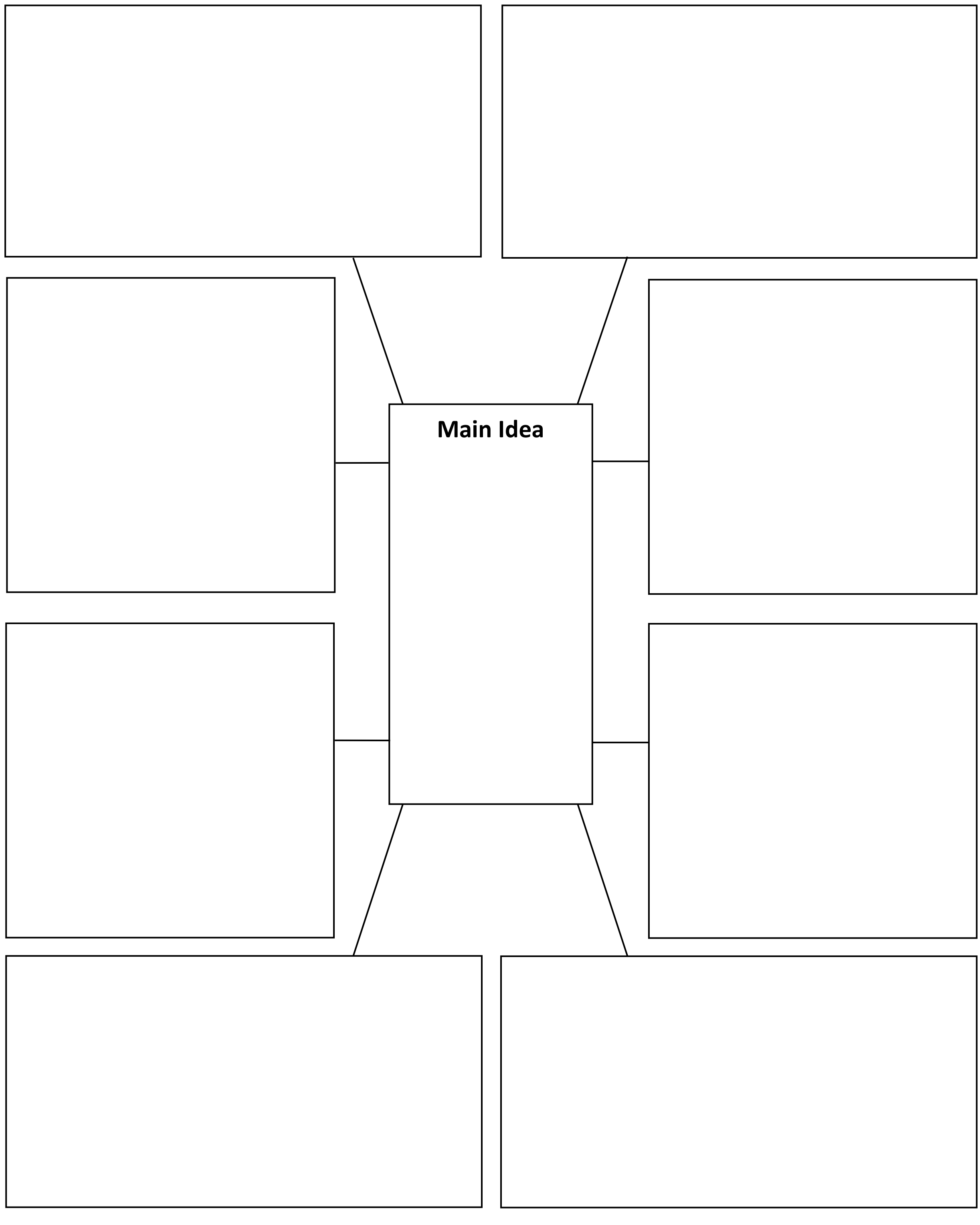
GRAPHIC ORGANISER FOR DESCRIPTION

Topic:



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GRAPHIC ORGANISER FOR COMPARISON

Topic:			
Issue	**Situation 1 Differences**	**Similarities**	**Situation 2 Differences**
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes
	Notes	Notes	Notes

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GRAPHIC ORGANISER FOR PROCEDURES

Topic:

INTRODUCTION
(Task summary, purpose, objective, desired outcome)

Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram
Step ….	**Insert here:** ☐ table ☐ list ☐ diagram

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GRAPHIC ORGANISER FOR JUSTIFYING

DECISION:

JUSTIFICATION:

because

because

because

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GRAPHIC ORGANISER FOR INFERRING

What I see

What I know

I say/think/hypothesise that

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GRAPHIC ORGANISER FOR EXPLAINING

WHAT	HOW	WHY

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PMI GRAPHIC ORGANISER FOR ANALYSING

PLUS	MINUS	INTERESTING

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SWOT GRAPHIC ORGANISER FOR ANALYSING

	STRENGTHS	WEAKNESSES
Analyse the idea		
	OPPORTUNITIES	**THREATS**
Look beyond the idea		

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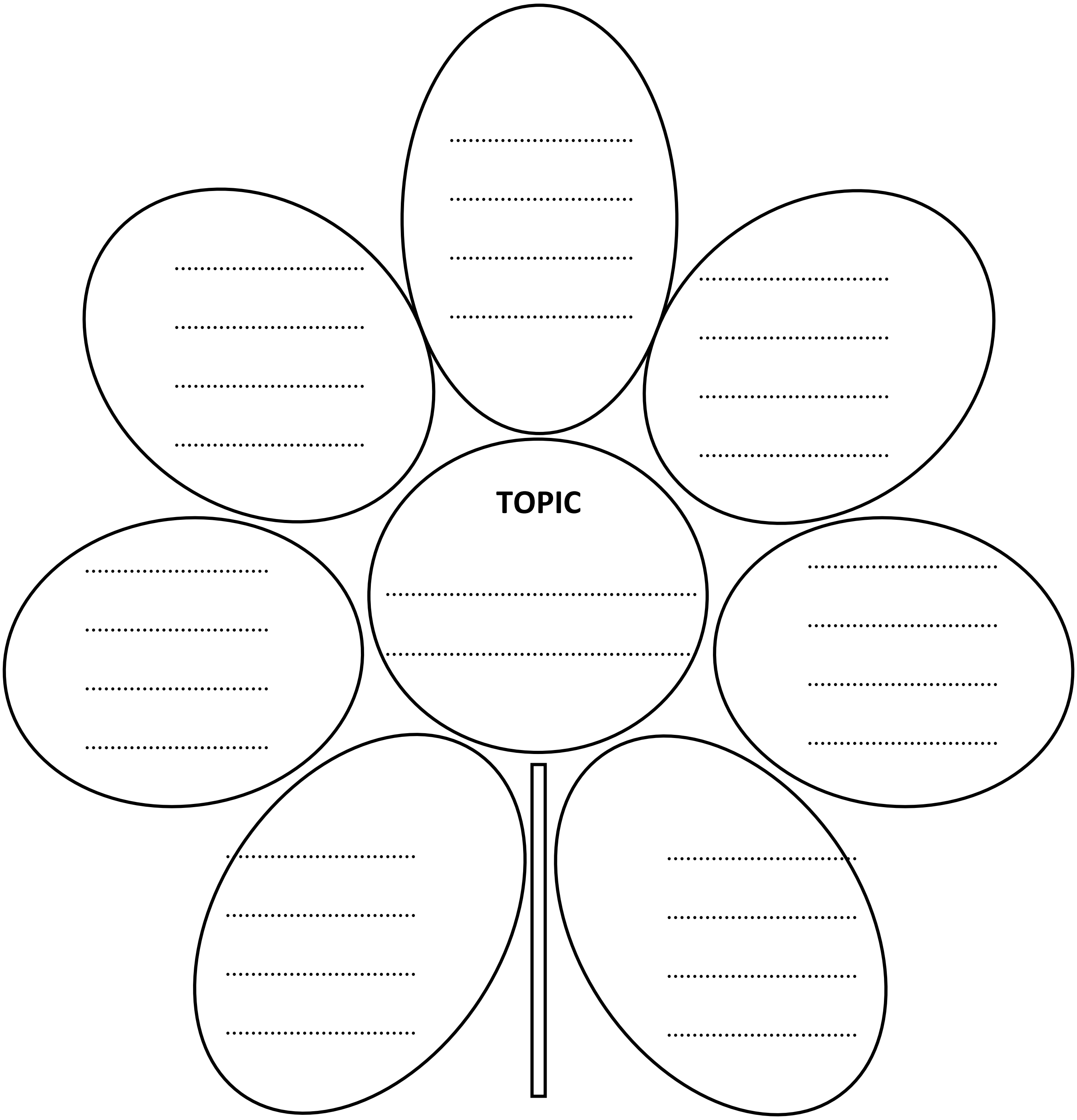
FLOWER GRAPHIC ORGANISER FOR ANALYSING



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GRAPHIC ORGANISER FOR PLANNING A MATHEMATICAL REPORT

Title: What is the name of the report?

Research: What information is needed?

Collect information for the bibliography during the research.

Procedure: List the steps followed (including collecting data).

- *Consider numbering these steps.*
- *Where/when was the data collected?*
- *Put copies of survey forms in an appendix.*

Results:

- *Put raw data in an appendix.*
- *Use tables and graphs to summarise the results.*
- *Discuss assumptions and their effects on the results.*
- *Comment on the precision and accuracy of the data collected.*

Conclusions: Findings and discussion.

Recommendation(s): What action are you proposing should be taken as a result of your report?

- *Consider numbering the recommendations.*

Appendices: For detailed information.

As the report is prepared, make notes of the details to be attached. Use these notes as a checklist for collating the final report.

Introduction: Why have you done this work?

If appropriate, explain the theory behind the work that you are doing.
*Write this section **after** the conclusion and any recommendations, but put it **before** the procedure.*

Abstract:

*Write this section **last**, but put it **before** the introduction.*

Other details:

- ✓ Does the report contain your name, class, teacher's name?
- ✓ Are any of the following needed: task sheet (put this on top), title page, table of contents?
- ✓ Has the report been proofread?
- ✓ Are all the pages numbered and collated in the correct order? Are they all stapled together?

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Prep to Year 12: Supplementary Resource 3 – **Literacy in Mathematics**

YuMiDeadly
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through the YuMi Deadly Centre
Faculty of Education
School of Teacher Education and Leadership
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059
CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
Email: ydc@qut.edu.au
Website: http://research.qut.edu.au/ydc