YuMi Deadly Maths
North Lakes Project

Maths Fiesta

Lesson Plans and Activities
Years 7–9

North Lakes State College
10 and 17 February 2014

Prepared by the YuMi Deadly Centre
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ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at QUT which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

The YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

Contents

1 Years 7–9 Lesson Plans

1.1 Year 7/8/9: Perspective – it all depends on the way you look at it!

Learning goal: Students will view and record objects from different perspectives.

ACMMG161 Draw different views of prisms formed by combinations of prisms. Using aerial views of buildings and other 3D structures to visualise the structure of the building or prism.

ACMMG218 Solve problems involving the surface area and volume of right prisms.

Big ideas: Mental rotation, perspective and projections.

Resources: “3 views” – houses and aeroplanes; picture board(s); original perspective puzzle; chart to record the competition outcome; spaceship refuelling module; Multilink blocks; squared paper, dotty (hexagonal) paper.

Reality

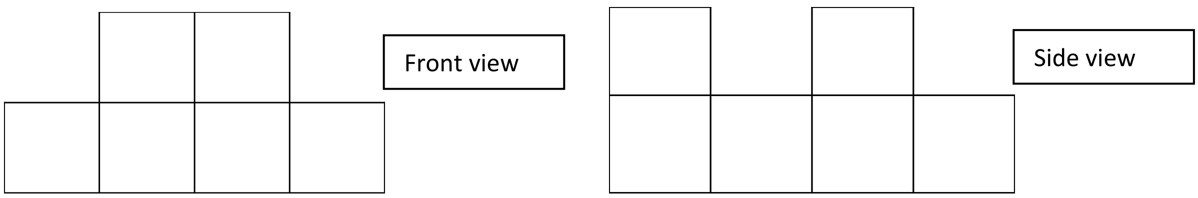
Prior experience Does your road/house look different when you approach from a different direction?

Looking at common objects in the school from different perspectives.

Local knowledge Templates including workshop signboards, maps and plans (such as on smart phones, iPads, etc.)

Kinaesthetic Pick up familiar objects and view different sides, e.g. “3 views” of house plans or aeroplane plans. Move around objects and view them from several different angles.

Perspective puzzle:



Using blocks, can you build a shape with these two views? Find the most and the least number of blocks possible.

Abstraction

Body Walk around to view objects from different perspectives. Draw things from different directions and perspectives

Hand Create with blocks the object represented on the picture board. Use as many blocks as possible. Record your counts. Then use the fewest possible. Justify and record.

Represent these on hexagonal dotty paper or squared paper.

Mind View an object from one direction and imagine what it looks like from other positions.

Creativity Piet Hein (Danish philosopher), in a boring lecture, created in his mind the 6 different (unique) shapes made from 4 cube pieces each (and one three-block piece) which fitted together to form a 3 × 3 cube. Google: Soma cube, Piet Hein.

Mathematics

Language Maximum, minimum, volume, surface area, perspective, net. Front, back, side, above views.

Practice	Record on hexagonal paper (3D) and on squared paper (a plan view) the resulting shapes. Choose other shapes to draw.
Connections	Geometry (flip, slide, turn, or reflection, rotation, translation), surface area, volume, pentominoes, Soma cube – Calculate the volume and surface area of the shapes that are seen from unusual directions.

Reflection

Validation Get students to go back into their world and to draw things in their world from different directions/perspectives.

Application Problems and applications that have to do with mental rotation and perspective. Templates including workshop signboards, maps; architecture, art, design, town planning. For example, how did the ancient Athenians construct a building whose sections were perfect squares from the bottom of the hill?

Extension *Flexibility*. Think of more than one way that a shape could look like for different directions or perspectives – what would a person on an elephant wearing a sombrero look like from a helicopter – what about other perspectives?

Reversing. We have gone from perspectives to shape, now go from shape to perspectives (e.g. construct something from cubes and others draw front, side and back views).

Generalising. Many shapes and objects have different views from different perspectives. Can we form a taxonomy of differences – e.g. 3D shapes in perspective, 3D shapes from front, side, etc. What things can't we know? e.g. the 2D shapes in a net of a shape give clues to view from different points of view.

Changing parameters. What if the shapes change in some way (like doubling height but tripling sideways growth), how will the different views (side, back, front, etc.) change?

Note: Can extend tessellations to 3D shapes – Escher-type drawings from perspectives (e.g. curved perspective lines) – also useful to look at NAPLAN questions.

1.2 Year 7/8/9: Percent – fractions, decimals and change

Learning goal: Develop understanding of percent and solve real-life problems.

Big ideas: Concept of fraction as part of a whole; concept of fraction as an operator; concept of decimal as a part of a whole (decimals as tenths, hundredths and thousandths); flexibility of decimal point (ones ←→ hundredths); relationship vs transformation (P = % × A relates to $A \xrightarrow{\times\%} P$); Triadic principles (A, % and P can all be unknowns).

Resources: Masking tape, coloured paper (various sizes), grid paper, place-value chart, post-it notes.

Reality

Prior experience Percent – students complete a brainstorming activity using post-it notes. On the post-it notes they identify where they have seen the word percent and what this term means to them.

Also fraction understandings – part→whole and whole→part; common fraction ←→ decimal fraction relationships; relationships as change and reverse change (inverse), percentage as an operator (percentage = percent × amount); three types of percentage problems (triadic principle).

Abstraction

Body Have students identify the relationship between percentages and decimal numbers. Use a dice for students to choose numbers and show on the PV mat (looking at normal decimals changing to % as a movement of 2 places to left as hundredths become the unit) and then on the large grid (looking at hundredths as a percent and a decimal).

Break students into small groups and give them several large piece of butchers' paper to estimate certain percentages (considering the butchers' paper being one whole and the percentage being a part of it).

Hand Have students use dice, place-value chart and grids to show the relationship between percentages and decimal numbers by rolling their dice, writing it as a percent and a decimal number, shading in the grid to show.

Give students several different-sized pieces of coloured paper to estimate percentages. Collect paper and have a group discussion on how they decided on their choices.

Mind Complete visualisation activities based on the body and hand activities.

Creativity Discuss if had a different total than 100 to build percent around – make up your own per "something"?

Mathematics

Language Percent, hundredth, profit, loss, discount, and so on.

Practice Have students look at application of percent (e.g. 27% of $65) and see it as an operator P (percentage) = % (percent) × A (amount). Have students translate this into change and use the following model to help identify how to solve problems that require a direct percent, a profit and a loss. They will need to link their learning from converting percentages into decimal numbers – this is because the multiplier can be a fraction (27% = 27/100) or a decimal (0.27) and decimal is easier to work with, particularly with calculators.

$$\text{START} \xrightarrow{\text{X multiplier}} \text{FINISH}$$

$$A \xrightarrow{\times\%} P$$

$$\$65 \xrightarrow{\times 0.27} \$17.55$$

Connections Connect percentage problems to fraction and decimal change problems.

Reflection

Validation	Have students go back to their world – look at situations using percent – banks, interest, loans, discount, profit, loss, and so on.
Application	Have students choose a reflection question and answer it in their books. Share the reflections and relate back to their reality.
Extension	*Flexibility*. Relate percent to fractions, decimals, and to ratio – try to think of everything same as 25% (e.g. 15 minutes, 90 degrees, 250 mm, and so on). *Reversing*. Do all three things in the triad – given A and % → find P, given A and P → find %, given % and P → find A. *Generalising*. Say to student that they have an amount (A) and a percent (%) and we are not giving the numbers – how do they go about finding the percentage (P); what do they say the process generally involves – what do you do first, second, and so on – repeat this for when A and P are given and P and % are given. *Changing parameters*. Set problems that go from one percent to another (e.g. I sold a house a year later for 35% profit for $420 000, how much interest did I have to pay the bank for the loan to buy it at 9.5% interest per year?).

1.3 Year 8/9: Problem solving – acting it out

Learning goal:	Students will understand the strategy of solving problems by "Acting it out" using Polya's four stages.
Big idea:	Acting out strategy; Polya's stages.
Resources:	Poster with problems; digit cards; money.

Reality

Prior experience	Problems that students have solved. Polya's four stages (at least informally).
Local knowledge	**See** the problem; understand what the problem involves. Discuss whether they have solved a problem by doing it – by acting out what to do. Try to find some situation where students solve problems by acting them out. Sport? Construction?
Kinaesthetic	When you find this problem – get the students to act it out. Highlight for the students the organisation of this and the kinaesthetic parts: What is the problem? What are we asked to do? What do we have to find? What are we given? What do we want? What is the important data? What can be ignored? When we have done the problem, what have we learnt?

Abstraction

Body	Act the problem out, accounting for all possibilities.
Hand	Model the problem, selecting appropriate notation to picture it, e.g. diagram, pattern, table, list, or graph.
Mind	Visualise the problem.
Creativity	Design a **plan** that will help solve the problem.

Mathematics

Language	Polya's four stages: See, Plan, Do, Check.
Practice	Do the maths to solve the problem. Carry out the plan. Revise if necessary. If it doesn't work, go back to SEE and PLAN again.
Connections	Make the connections to the Acting (body), Modelling (hand) and Visualising (mind) in solving problems.

Reflection

Check	Compare answer to the problem task. Did you find what was asked? Does the solution meet all the conditions? Does it make sense? Is it reasonable? If the answer is not exact, is it close enough? Is it the only solution?
Application	Apply the "Act it out" strategy to other problems using Polya's four stages.
Extension	*Flexibility.* See if students can solve another way or get another solution (e.g. in handshake problem – what if wives shake with husbands?). *Reversing*. Give an answer and ask students to make up a problem. *Generalising*. Give a problem and ask students for steps he/she would use. See if students can solve an acting-out problem by visualising. *Changing parameters.* Develop other strategies to solve problems, e.g. write a number sentence, identify sub-tasks, work backwards, check for hidden assumptions, trial and error (guess and check).

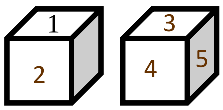
Act it out

1. Six married couples meet at the start of a tour. If they shake hands as introductions are made, how many different handshakes take place?

2. A used car salesman buys a car for $6 000 and sells it for $8 000. He then buys it back for $7 500 and finally sells it for $10 000. What profit does he make?

3. Three men pay $10 each (total $30) for a hotel room. The manager says the cost was really $25 and returns $5 with the bell boy. The bell boy pockets $2 and returns $3 to the men. The men, therefore, paid $9 each, total $27. The bell boy's $2 makes $29. What happened to the other $1?

4. A desk calendar is composed of 2 cubes that can be lifted up and positioned together with any side facing the front e.g. 24 below. What are the remaining numbers on the 2 cubes?



Remember all dates from 01 to 31 must be possible.

5. The number 2938 has these properties: thousands digit is even, hundreds digit is odd and all four digits are different.

How many four digit numbers have these properties?

6. On a trip, John wishes to average 80 km per hour. At halfway, he finds he is averaging 40 km per hour. What average speed must John travel in the second half to achieve his goal of 80 km per hour overall?

1.4 Year 7/8/9: Algebra – visualising solutions to linear equations

Learning goal: Visualising solving linear equations using balance rule and backtracking.

Big ideas: Balance rule, backtracking, body → hand → mind learning sequence.

Resources: Paper and pen, posters with equations, change-change "function machine", students' walking and acting out a mass balance.

Reality

Prior experience Understanding arithmetic as change and transformation.
Understanding inverse change – backtracking.
Can act out a problem along a line (e.g. *Uncle gives us money, we spend $7, Auntie gives us $10 we spend $16, we have $12 left, how much did we get from Uncle?*) and backtrack for answer.
Understanding arithmetic equations and equations with unknowns – equals, balance rule.
Able to translate real-world problems into arithmetic equations and equations with unknowns.

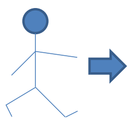
Local knowledge Find something of interest to students that involves an unknown component, e.g. problems **FROM** *Jack, Sue and Jess each won a 4th division prize, they put their money together with the $17 that Joy had and paid $109 for a table of 4 at the banquet, how much was the 4th division prize?* **TO** *The Jones and Smiths went to Splash-World where there was a special children's rate of $5 per child, the Jones had three adults and two children (who cost $10 to enter), while the Smiths had two adults and one child (who cost $5), the Smiths spent $25 on snacks but the Jones did not spend any extra on snacks, both families spent the same altogether, what was the adult price?* **TO** *Tom's quarter share in the money, when added to $5 in his pocket, was equal to a half share less $26 that was spent on a meal, how much was the total amount of money?*

Kinaesthetic Act out these problems, with students playing the parts and play money, and translate the actions into equations.

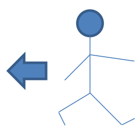
Abstraction

Body (1) **Change and backtracking**

- Give students a linear problem that involves one use of variable (e.g. *the fishers each caught 7 fish, they gave 5 to a friend, they had 23 left, how many fishers?*). Discuss what is unknown (e.g. the number of fishers). Discuss how we could represent this (as ? or as a letter f). Write the problem as an equation with the unknown (e.g. $7f-5=23$ or $?\times7-5=23$).
- Work with students to act out this problem as a change. Look at LHS of equation (the expressions, $7f-5=23$). Discuss how it could be acted out. Admit it could be forward along a line for multiplication and then backward for subtraction. However, say we are looking for a more abstract change – can we think of the action as steps forward, one step for each operation? Let students develop their own ideas.

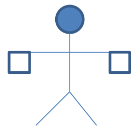


- Encourage students to see there is more than one way to act out the change: (a) starting from zero, it is $+7f$ and -5; (b) starting from f, it is $\times7$ and -5. Spend time looking at the difference between (a) and (b).
- Work with students to act out backtracking – actions that undo what has been changed. Get students to walk the forward steps, and then walk **backwards** those steps undoing each operation as they go. Encourage students to see and walk the backtracks for the two changes: (a) backtracking to zero, it is $+5$ and $-7f$; and (b) backtracking to f, it is -5 and $\div7$. [*Note*: Backtracking to zero is very useful for solving quadratics.]



- Practise the following for a variety of linear equations with one use of variable on one side only – do all the equation types (e.g. $3y-2=16$, $y/4+3=5$, $5(y+3)=35$, where y is the unknown):
(a) one use of variable problems → expressions → changes → backtracks, **and**
(b) backtracks → changes →expressions → one use of variable problems.

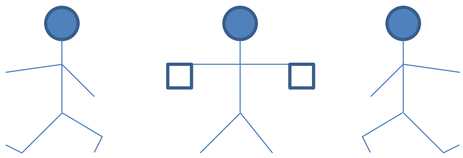
(2) **Balance and balance rule**



- Use students to act out balance – both hands out and horizontal. Act out LHS > RHS and LHS < RHS. Act out LHS = RHS → LHS > RHS → LHS = RHS.
- Discuss how the dot point above could happen and lead to the **balance rule**, that is, "to keep in balance and equal, whatever is done to one side, the same has to be done to the other side". Discuss how this holds for more than physical balance situation – encourage students to think of the balance as mathematical – all operations can be done to both sides.

(3) **Solving one-use-of-unknown linear equations using balance and backtracking together**

- Say to students that we are going to act out balance and change together for problems leading to linear equations where the variable is on one side and on both sides. Discuss that, as we are now using both sides, the idea is to make changes so that we have **unknown alone on one side** and numbers on the other (discuss that a=b is the same as b=a so does not matter which side the unknown is on). Say we begin with simpler one-use-of-variable on one side problems, such as 7*f*–5=23.



- Set up the action – organise the class into groups of three students – middle ones to act out balance, LHS ones to act out LHS of equation, and RHS ones to act out RHS of equation (these roles should continually change so all students get experience). Middle students hold a paper on each side with 7*f*–5 on LHS and 23 on RHS.
- Restate we have to get unknown (e.g. *f*) alone on one side and numbers on the other, and discuss which side can change to get *f* (e.g. LHS). LHS students walk 7*f*–5 and backtrack to get *f*, and, in doing this, identify changes +5 and ÷7 as the way to get back to *f*.
- Organise the students to make these changes on the balance – RHS students begin at +5 and then move onto ÷7; middle students ensure this happens to both sides; and LHS students change the 23. If needed LHS and RHS students can write the changes they make on the paper that is in middle students' hands. Action keeps going until have *f* = 4. The forward changes and the backtracking are shown below in <u>arrowmath</u> notation, e.g.

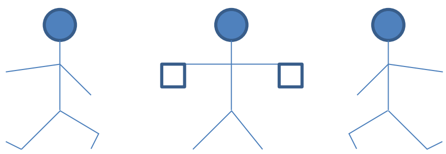
$$f \xrightarrow{\times 7} \quad \xrightarrow{-5} 23 \qquad 4 \xleftarrow{\div 7} 28 \xleftarrow{+5} 23$$

- Repeat this for other problems and equations which have one use of unknown.

[*Note*: Year 7 students could end at this point.]

(4) **Solving general linear equations using balance and backtracking together**

- Move on to using students to act out balance for problems and equations with unknowns on both sides (e.g. *5 fishers in two boats all caught the same number of fish, the 2 in the first boat received 4 more fish from a friend, the 3 in the second boat gave 6 away, at the end each boat had the same number of fish, how many fish did each fisher catch?*). Write down the operations for each boat (e.g. 1^{st} boat: ×2+4; 2^{nd} boat: ×3−6).
- Discuss what we do not know (e.g. number of fish each caught). Introduce this as an unknown. Discuss what kind of symbol (e.g. *f*). Write down the equation (e.g. 2*f*+4 = 3*f*–6).



- Again use a combination of acting out balance and change to solve this – three students as before. LHS student backtracks 2*f*+4; RHS backtracks 3*f*–6. Encourage students to see that backtracking 3*f*–6 to get a single *f* can be +6 ÷3 or +6 −2*f*, and that backtracking 2*f*+4 to get a single *f* can be −4 ÷2 or −4 −*f*; while backtracking 3*f*–6 to get a number is −3*f* and backtracking 2*f*+4 to get a number is −2*f*. Need to look at both sides to find the backtrack of which expression is best at getting a single *f* on one side and only numbers on the other side. Encourage students to look for the most efficient backtrack. Interestingly, inefficient backtrack will still get the right answer, but they will take longer, e.g.

Following RHS backtrack $+6\ -2f$ is efficient

	$2f+4 = 3f-6$
$+6$	$2f+10 = 3f$
$-2f$	$10 = f$

Note: This backtrack is efficient because it gets rid of *f*s on the LHS as it reduces the *f*s on the RHS to one.

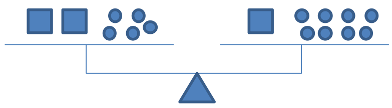
Following the RHS backtrack $+6\ \div 3$ is not efficient

	$2f+4 = 3f-6$
$+6$	$2f+10 = 3f$
$\div 3$	$^{2}/_{3}f+10/3 = f$
$-^{2}/_{3}f$	$10/3 = {}^{1}/_{3}f$
$\times 3$	$10 = f$

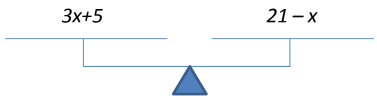
Note the need for extra backtracking makes this backtrack inefficient.

- Repeat this process of balance and backtracking for other problems and equations which have unknowns on both sides – act out getting the answer. Use a variety of problem and equation types (e.g. 3Y + 2 = 26 – Y; 5 – 4Y = 23 – 2Y; Y/3 – 1 = 2Y – 36; and so on).
- Reverse the above and go from an answer (e.g. y=6) to an equation to a problem. Repeat this for more examples.

Hand Repeat the above process, short circuiting it where possible, for (a) drawings of balances on paper, (b) boxes for variables and counters for ones (initially), and (c) expressions on each side of the drawing (just before going to equations), as on right.



3x+5 = x+8



3x+5 = 21–x

Act out doing the same to other side as doing to one side. Act on variables to ensure that all variables end up on one side, then do the same for numbers but ending up on other side. Remember backtrack to get the operation that is done to both sides (remember +5 means −5 to remove it from a side and −5 has to be also done to other side).

Mind Look at equations, shut eyes and imagine the balance and backtracking on either side that get equation into a variable = number form.

Creativity If you can, encourage students to work out their own actions for backtracking and balancing.

Mathematics

Language Ensure students understand: equation, expression, equals, backtracking, and balance rule; and know that expression = expression gives equation.

Practice Give out problems and ensure that students practise the following:

- Problem → equation → solution for unknown
- Solution for unknown → equation → problem

Connections Undertake lessons that look at similarities and differences between different equation types: (a) 2Y+3 = 17, one use of variable on one side (tend to be of ×3 −6 type backtracks); (b) 3Y−7 = 25−Y, variables on both sides but not too complex (tend to + and – variables to remove from one side); and (c) Y/3+2= 48−2Y, division of variables (act to get all variables in (b) form and then solve).

Undertake lessons that connect equations/solutions with line graphs/intersection points.

Reflection

Validation Get students to find things in their life that require the new knowledge.

Application Give problems and applications, and rich tasks, that require solutions to linear equations.

Extension *Flexibility* – try to think of all linear equation types. *Reversing* – go from solution to problem via equation. *Generalising* – try to see patterns that give quick answers to equation types, and be able to run through the types. *Changing parameters* – try to apply methods to quadratics (where does it work?).

Worksheet

1. One-use-of-unknown equations

   (a) $3x + 4 = 13$ [3]

   (b) $2x - 3 = 23$ [13]

   (c) $31 - 2x = 17$ [7]

   (d) $26 - 5x = 11$ [3]

   (e) $\frac{x}{3} + 2 = 5$ [9]

   (f) $11 - \frac{x}{4} = 27$ [–64]

2. Unknowns-on-both-sides equations

   (a) $3x - 2 = x + 4$ [3]

   (b) $4x + 6 = 2x + 10$ [2]

   (c) $3 - 2x = x - 12$ [5]

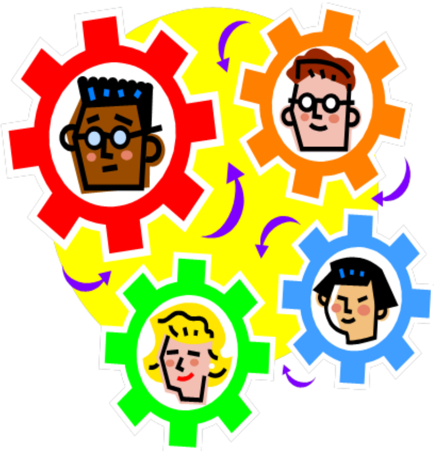
   (d) $17 - 3x = 11 - x$ [3]

   (e) $\frac{x}{2} + 7 = 2x - 5$ [8]

   (f) $15 - \frac{3x}{2} = 16 - 2x$ [2]

   (g) $9 - \frac{5x}{4} = 2 - \frac{2x}{3}$ [12]

2.1 Some Strange Stories



Keeping the cogs, the little grey cells, turning.

A MYSTERIOUS STORY

Peter, Mary, Bill and Sally live in the same house. One night, Peter and Mary went out to the movies. When they got back home, they found Sally beaten up and dead on the floor. Bill was not arrested. He was not questioned for any crime. Why not?

A COLOURFUL STORY

Jane listened entranced. “The bear walked 2 kilometres south, 2 kilometres east,” said her teacher, “then the same distance north and ended up where he started from. How can this be?” asked her teacher. “I don’t know!” exclaimed Jane, “but I know the colour of the bear!” Why was Jane able to do this?

A WET STORY

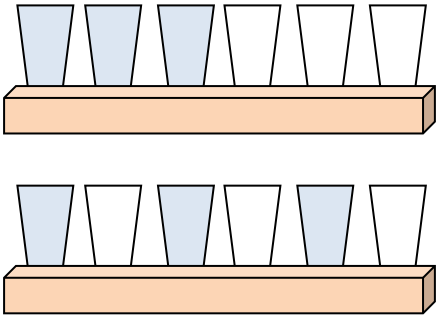
A man went for a walk. It started to rain. He did not have a hat. He was not carrying an umbrella. He kept walking. His clothes got wet. His shoes got wet. Still his hair did not get wet. How come?

A RUNNING STORY

A man was running home. Near home, he met a masked man. He stopped. Then he turned around and ran back to where he had started. Why?

A DEDUCTIVE STORY

Sherlock Holmes was passing a house at night. A man screamed from inside, “Don’t Bill!” There was a shot. Sherlock Holmes rushed inside the house to find a dead man and a smoking gun being watched by a lawyer, a priest and a doctor. Sherlock Holmes arrested the priest immediately and took him to the police. The priest confessed to the murder. How did Sherlock Holmes know the priest was guilty when he did not see the murder?

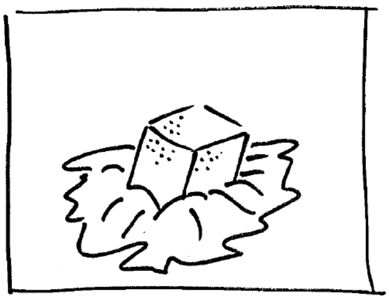


A MOVING STORY

John was perplexed. When he first entered the room, the six glasses were placed as in the top picture. All the glasses containing water were on the left. When he entered the room the second time, the glasses containing water were alternating with the empty glasses as in the bottom picture. Forensic evidence said only one glass had been moved. How could that be?

A DRY STORY

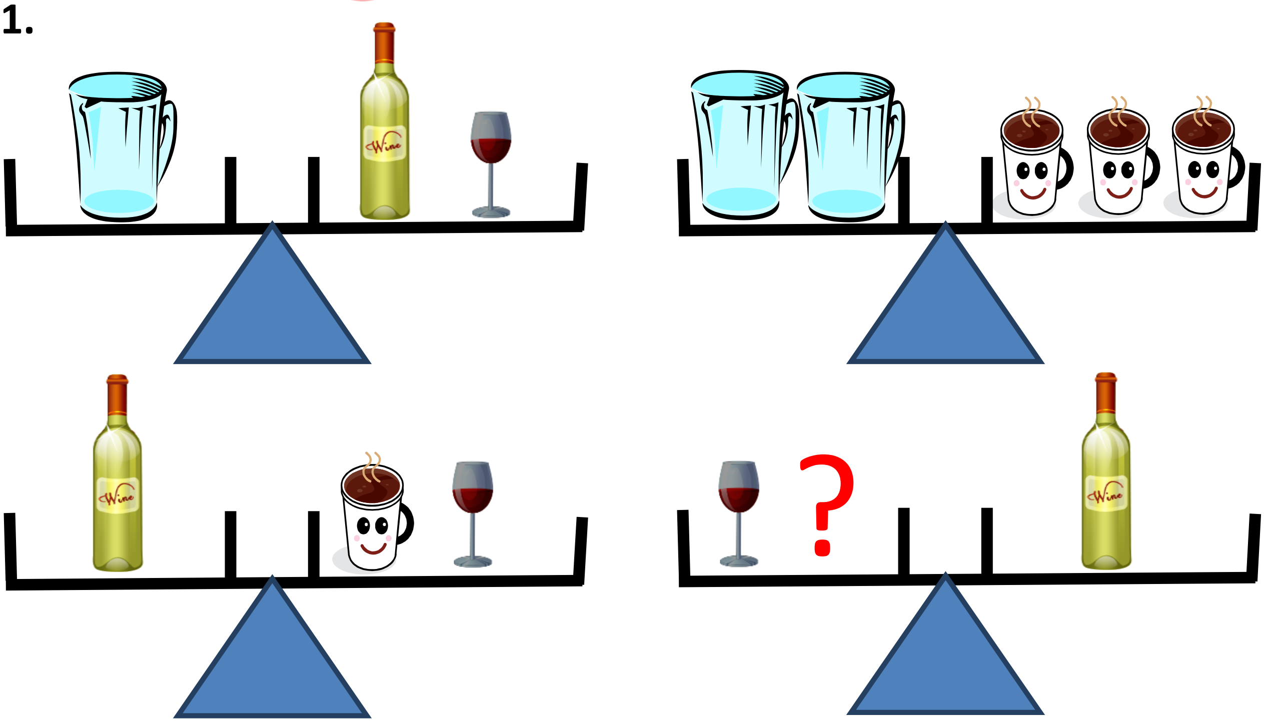
A woman unwrapped a lump of sugar. She put it into her coffee. The sugar did not get wet. How can this be?



2.2 Algebra Antics

Algebra Antics

1.



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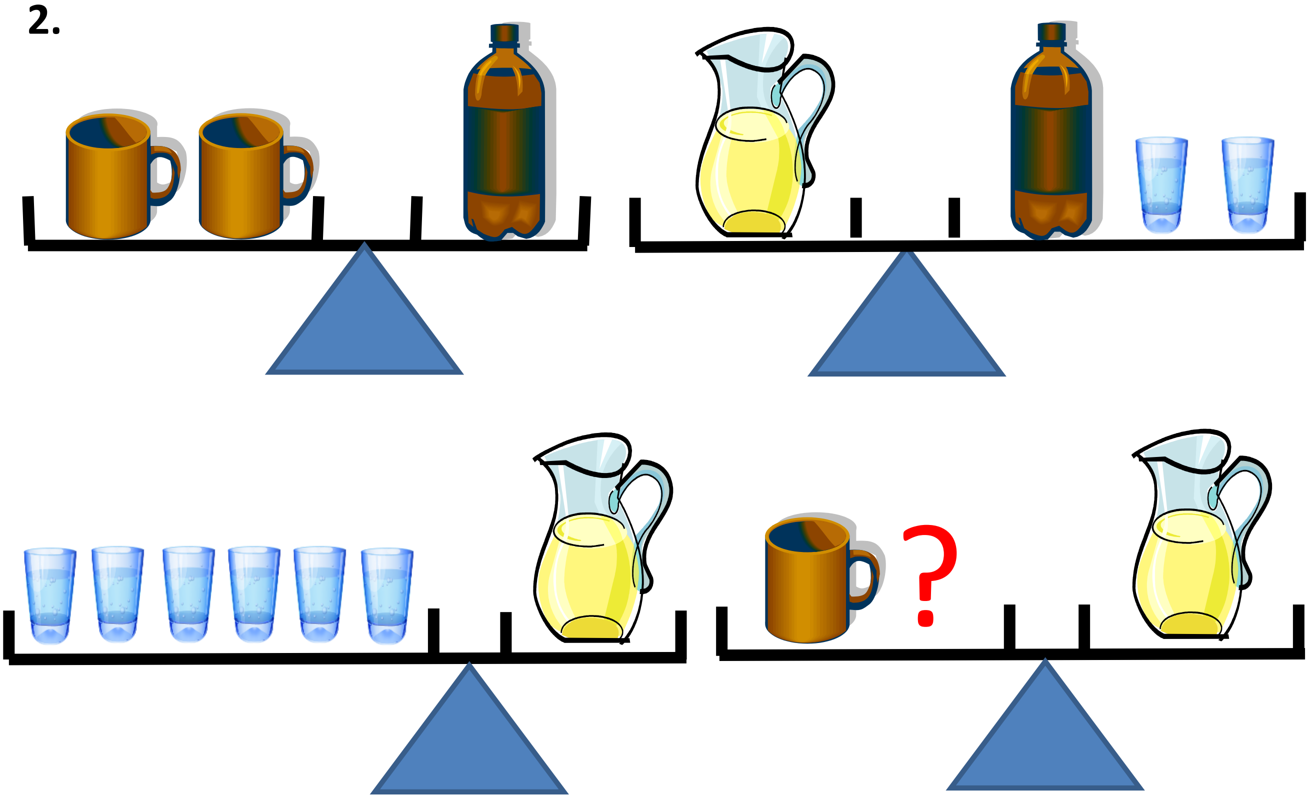

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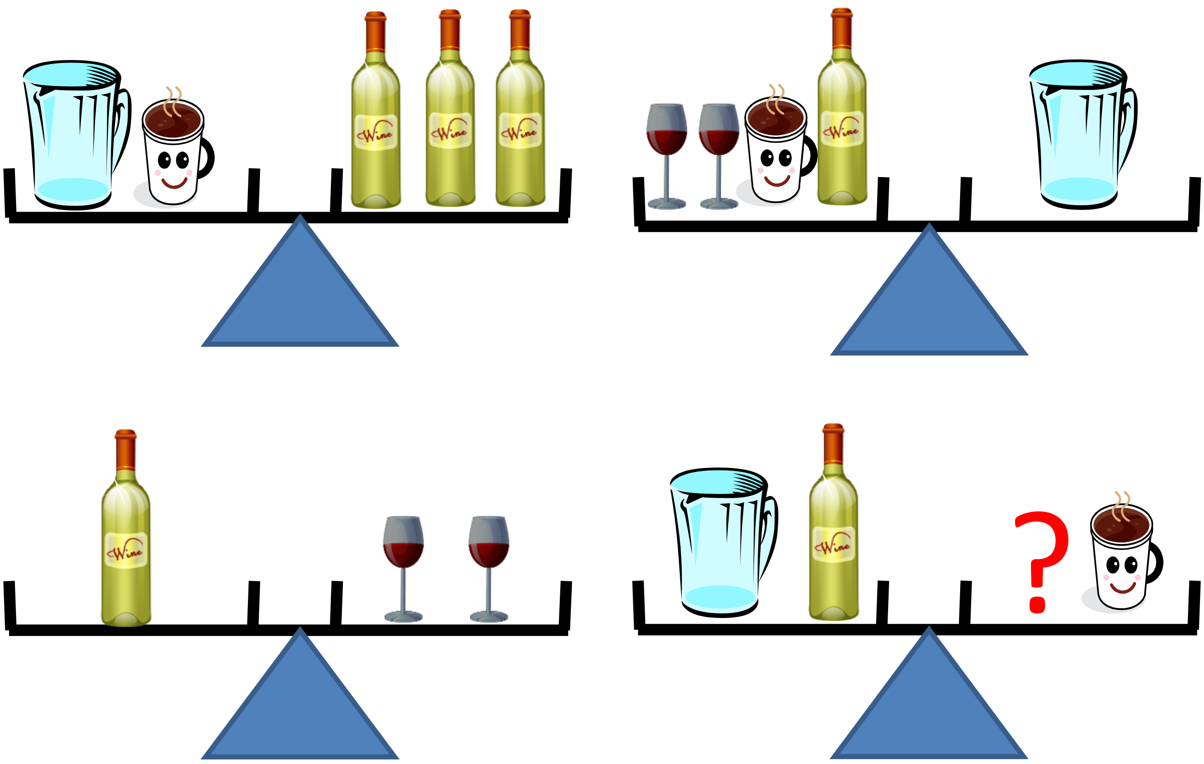

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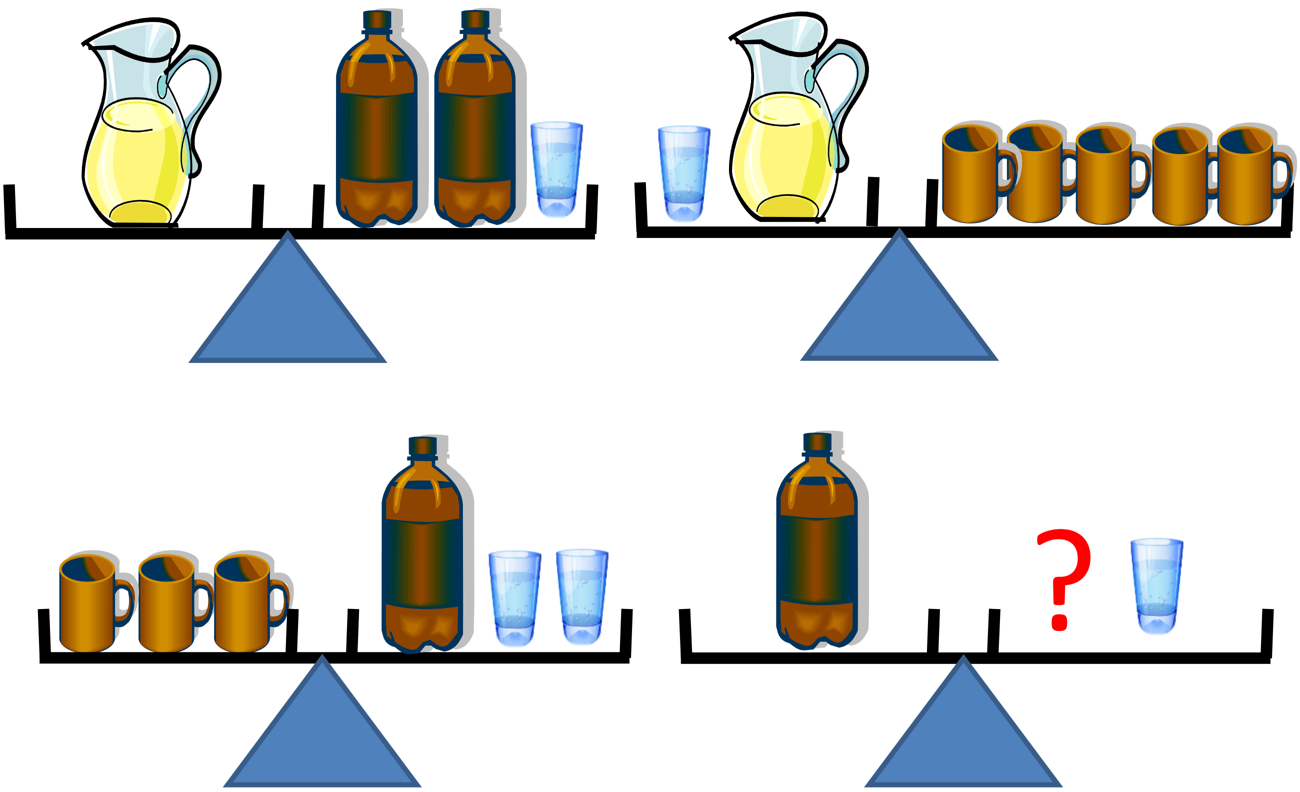


Algebra Antics

Numbers represent the sum of the objects in each row or column.

				**45**
				**38**
				**36**
23	**21**	**41**	**34**	

 ?

 ?

 ?

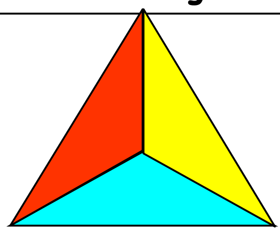
2.3 Dynamic Designs

McMahon's 4-colour triangles, Tessellations and Escher-type designs

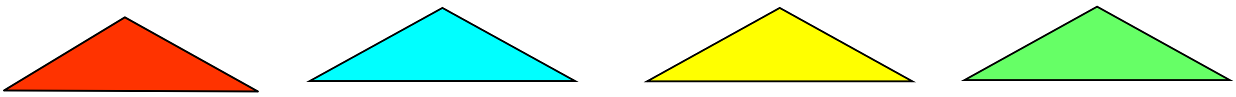
Dynamic Designs

McMahon's 4-COLOUR triangles – Wksheet 1

This triangle …



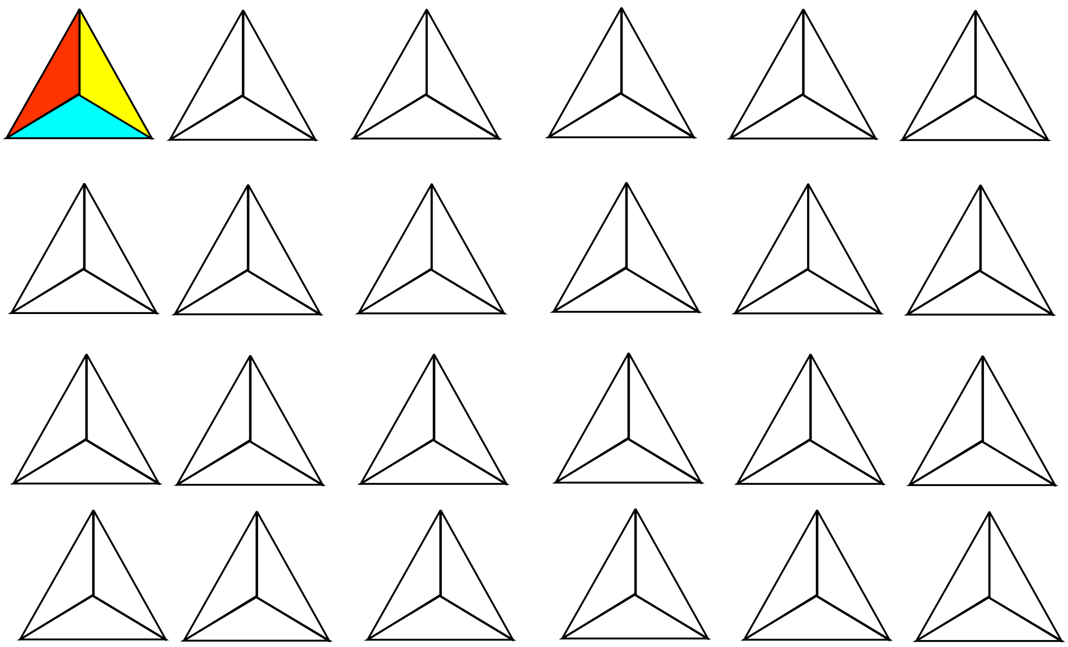
… was made from 3 of these coloured triangles.



Make another 23 triangles that are all coloured differently.

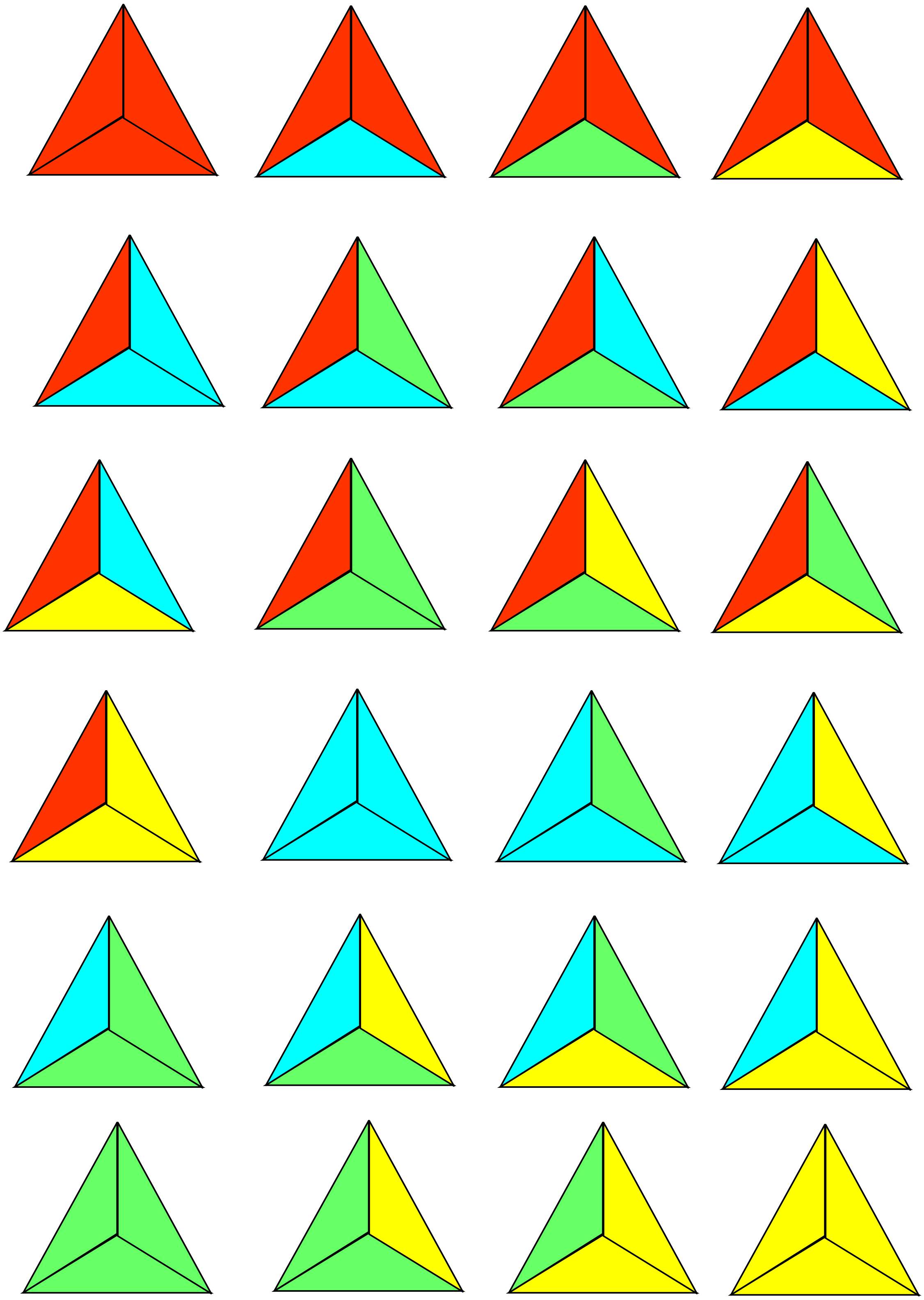
However, if you can rotate one of the triangles so that it is the same as another triangle, then it is NOT a different triangle.

GOOD LUCK!



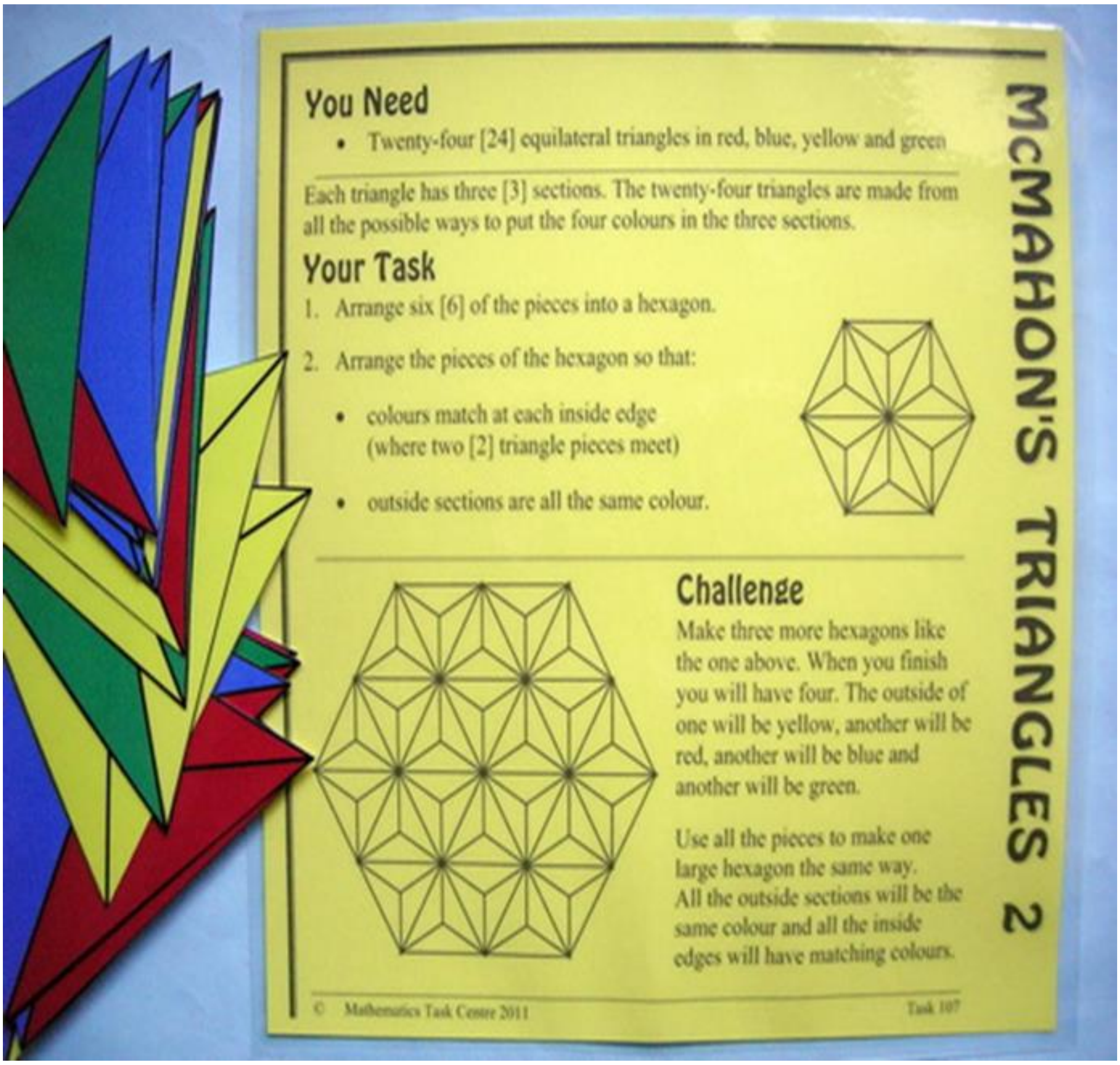
McMahon's 4-colour triangles – Wksht 2

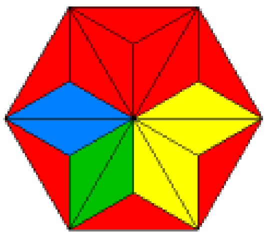
Dynamic Designs



McMahon's 4-colour triangles – Wksht 3

Dynamic Designs





The hexagon with the red border and all inside edges the same colour is done for you. Copy the model using six pieces from the bag.

Challenge: Work with the triangles to make hexagons with:

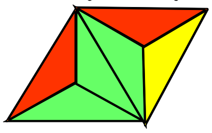
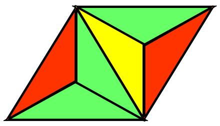
a) a yellow border b) a blue border c) a green border

McMahon's 4-colour triangles – Wksht 4

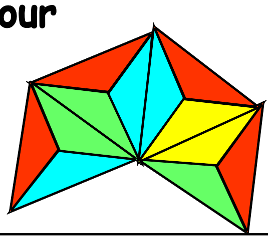
Dynamic Designs

McMahon triangles are used as follows:

- **triangles can only be placed together if colour is common**

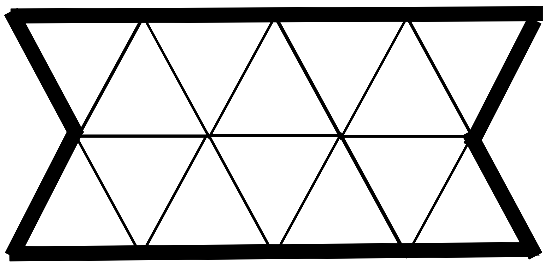
  - allowed  **- not allowed** 

- **edge of the shape must have a common colour**

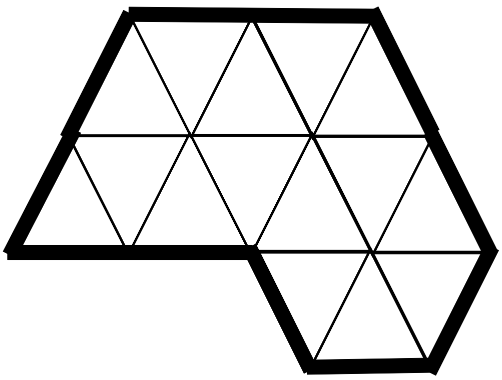
  - as in this partially complete example 

USE TRIANGLES TO MAKE THE FOLLOWING

(1)



(2)



(3) A large hexagon from all 24 triangles

(4) A large rhombus from all 24 triangles

]

Tessellation and Escher Art

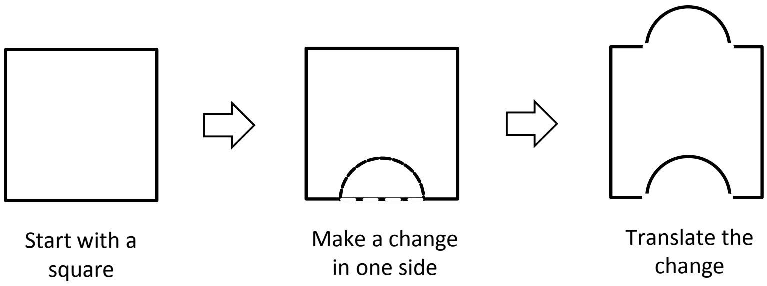
Dynamic Designs

Tessellating shapes are the bases of two art forms – Escher-type art and fabric design.

Escher-type art

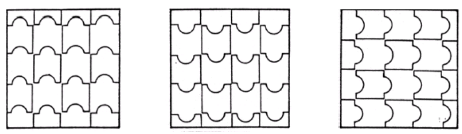
This is based on doing something to one side of a tessellating shape and undoing it to the other.

1. Start with a tessellating shape e.g. square, triangle, rectangle.
2. Cut out any shape and sticky tape it to the opposite side.
3. Repeat this process 3 or 4 times.
4. Copy the final shape onto 2 different colours of paper.
5. Tessellate the shapes to create a pattern.
6. Note the use of flipping, sliding and turning.

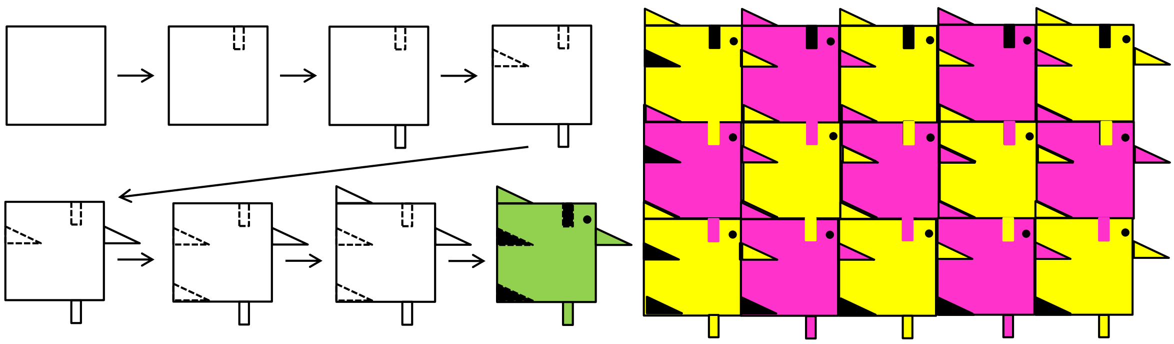


The finished shape has the same area as the original shape. Why?

7. Tessellate using two different colours.



Be an Escher artist or a fabric designer. Create your own design.



Cut out copies on different coloured paper and make an artistic tessellating Escher-type design.

Display your art in the classroom.

2.4 Matchbox Madness

Matchbox Madness

An Edward de Bono Puzzle

Each person takes six matchboxes.

Using the six matchboxes, place them side by side so that **every** matchbox is touching:

(a) the side of TWO other matchboxes

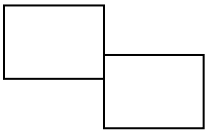
(b) the side of THREE other matchboxes

(c) the side of FOUR other matchboxes

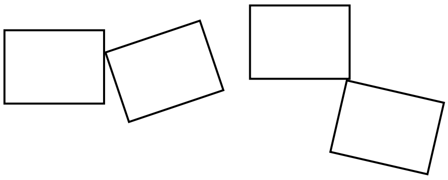
(d) the side of FIVE other matchboxes

Note: A match box is touching if part of a surface touches part of a surface

Looking down: Allowed



Not allowed



The same for looking sideways

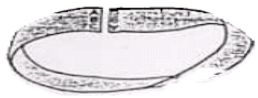
2.5 Mobius Mobility

Mobius Mobility

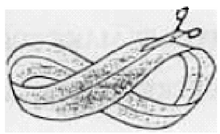
Mobius strip

In all the activities below, always do the activity with a normal cylinder/ring with no twists and compare so you can see how the Mobius strip is different. Cut out a flat strip of paper about 50 cm long. Roll into a circle and glue one end on to the other end.

1. Make a Mobius strip. Cut out a flat strip of paper about 50 cm long. Twist the paper (once) and then join the two ends to make a closed ring.



2. Try to colour one side of the strip red and the other side green (use any two different colours). What do you notice?
3. Try to draw a line along the centre of the strip continuing until you come back to the same point from which you started. Are you convinced that this strip has only one side?
4. Predict what you think will happen if you cut along the middle of the strip as shown. Validate by cutting. (Cut along the line you have already drawn.) Were you surprised by what happened?
5. Make another Mobius strip. This time draw a line that is 1 third of the width. Continue to draw the line the same distance from the edge until you come back to the same point from which you started. Cut along the line you have just drawn. Do you have a chain of Mobius strips?



6. Make a Mobius strip where one end is twisted twice before it is glued to the other end. Repeat activities above. Make Mobius strips that have 3 or 4 twists and repeat activities above again. Can you discover a pattern emerging?

The Mobius strip is a thing
Which somewhat resembles a ring
But given the strength
To travel its length
You still haven’t done anything.

A mathematician named Klein
Thought the Mobius strip was divine
Said he, “If you glue
The edges of two
You’ll get a weird bottle like mine.”

2.6 Geo Genius – Measurement and Shape

Measurement

Geo Genius

Using the geoboard or dotted paper, make as many shapes as you can with an area of 36 cm². Find the shape with the largest perimeter. Why is this so?

Using the geoboard, make as many shapes as you can that have a perimeter of 36 cm. Find the shape with the largest area. Can you find applications for this in the real world?

When a square piece of paper is folded in half, vertically, the resulting rectangle has a perimeter of 39 cm. In square centimetres, find the area of the original square sheet of paper.

Jeff is tiling a mosaic with fewer than 100 tiles. When he made five equal rows, he had one left over; with four equal rows, he had one left over; and with nine equal rows, he had none left over. How many tiles did Jeff have?

Challenge: Show on your geoboard how an L-shaped shape can be cut into 4 smaller same size and same shape L-shapes.

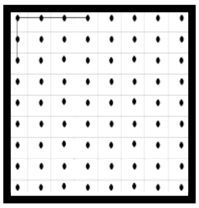
Geo Genius

Paths and regions

Choose 1 nail on the geoboard. Choose another nail as far away as possible. Make a path of line segments from one nail to the other. How many times did your lines cross? How many angles did you make?

Paths and regions

If the geoboard has 25 nails and there are 2 nails on the inside of a region and 3 nails on the outside, where are the other 20? Make such a region on your board.



Shapes

Make the following:

(i) a square inside a square;

(ii) a triangle inside a rectangle;

(iii) a square inside a triangle inside a square;

(iv) a parallelogram overlapping a trapezium.

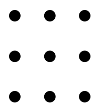
Shapes

Try to make a star from several triangles.

Puzzle

Select a 3 × 3 set of nails (as shown).

Construct 5 isosceles triangles that have different sizes.



2.7 Winning Ways – Games

Nim, Noughts and Crosses, Kaooa, French Military game, SOS and Slither

Winning Ways

The focus of these task is not to play the games to beat the other player, but to work together as a group to figure out strategies to win.

NIM

Rules

There are many games of Nim, which is a war game. We will play a simple game for two players which starts with 14 counters in a line. Each player in turn, takes one or two from either end of the line. The winner is the player who does not take the last counter. (*Note:* The game can be played by drawing 14 circles in a row and crossing them out.) The game should always be won by the first player as there is a strategy that always wins.

Instructions

1. Students play each game in turn, trying to work out the winning strategy.
2. If you are having difficulty with Nim, ask the following questions:
    - What number of counters would you like to be left with? What number would you not like to be left with?
    - How can you ensure you leave your opponent with the number of counters that means they lose?
    - If there were 15 counters, the second player would always win. Why?

Modifications that change strategies

3. What if the game of Nim was changed so that you could remove 1, 2 or 3 counters from either end of the line of counters? If the start was still 14 counters in a line, how would this change the strategy? Would the first player still win? With what strategy?
4. If the 14 counters of Nim were put in a circle and you were allowed to remove 1 counter or 2 counters that were adjacent (a counter had not been removed from between them), would this change the strategy? Would the first player still win? With what strategy? Or would this give the game to the second player (and with what strategy)? Remember, the player who takes the last counter loses.

What strategies can you find to solve this?
Five consecutive numbers added together give a total of 150. What are the numbers?

Winning Ways

NOUGHTS AND CROSSES

Rules: Noughts and Crosses is a simple alignment game. It is played on a simple board which has nine cells and can be quickly drawn on any piece of paper (see on right). It is played by two players who take turns. To begin, one player takes the O and the other the X. The rules are that players in turn place their O or X in one of the nine cells that is vacant. The winner is the first player to get three of their O's or X's in a row, column or diagonal. Otherwise the game is a draw.

Instructions

1. Play the game, trying to work together to work out winning moves. Students need to work out which strategies provide the best chance of winning and of achieving a draw.
2. Students share strategies. If not much is being achieved, ask these questions:
   - What is the most important square or cell to fill?
   - If you fill this cell, what mistake by your opponent will ensure you win? What can you do to maximise your chances of this error?
   - If your opponent fills this cell, what do you have to do to ensure you draw?
   - Is there a way to win without filling this most important cell first? Why does it work?
3. Try these "end games" – it will help you to develop strategies.
   - X plays next and can win no matter what O does! How?

	X	O

   - O plays next and if places O wrongly, X wins no matter what. Why is this cell wrong for O?

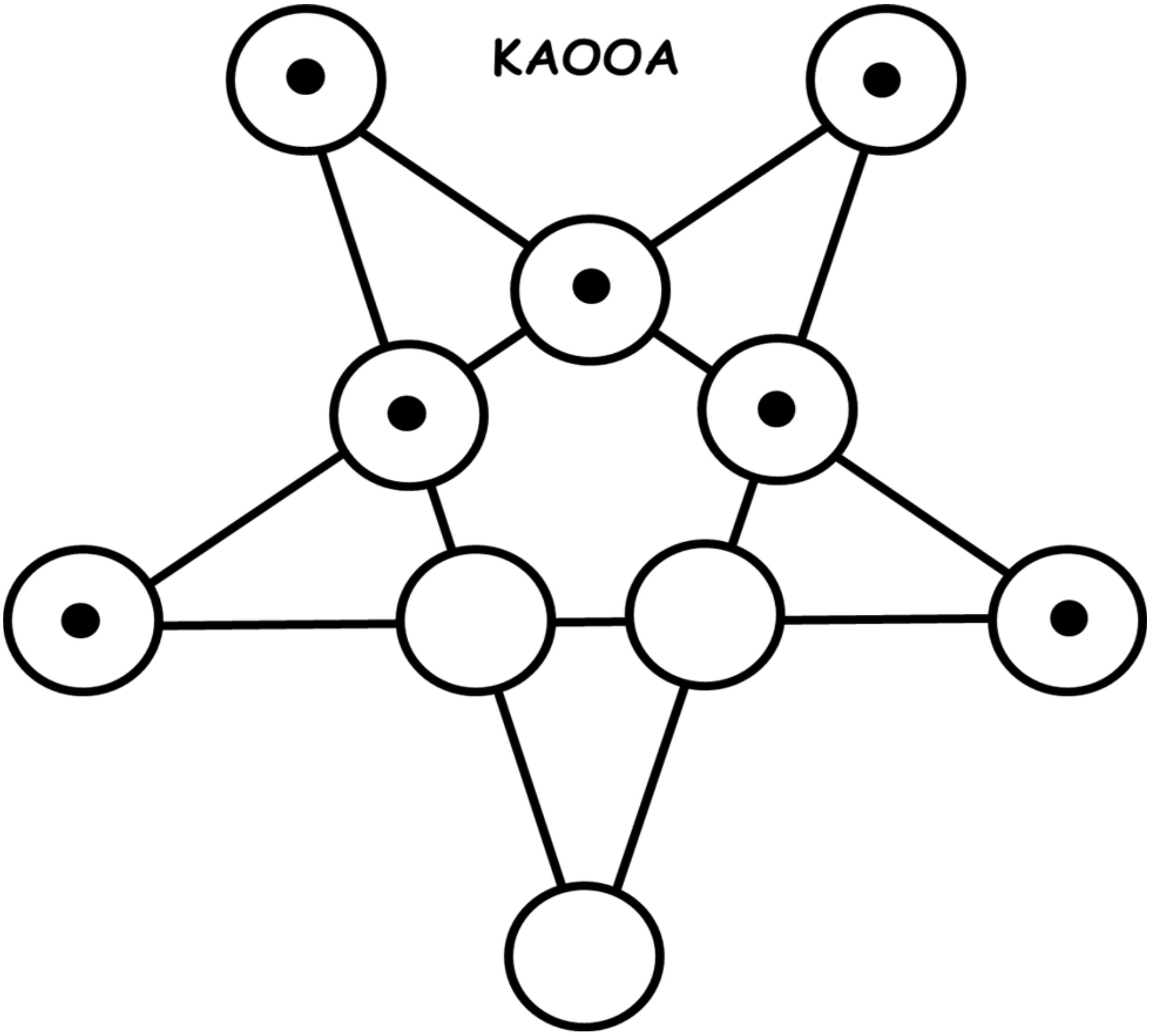
		O
	X	
X		

   - O plays next and if places O wrongly, X wins no matter what. Why is this cell wrong for O?

		X
	O	
X		

4. Write down your strategies:
   - Strategy for how to win starting from middle cell.
   - Strategy for how not to lose when opposition starts from middle cell.
   - Strategy for how to win from a corner cell.
   - Strategy for how not to lose when opposition starts from a corner cell.
5. Why does a corner cell player often win if the other player is a regular middle cell player?
6. What if in the game of Noughts and Crosses, you could put either a nought or a cross when it was your turn to play? What is the strategy now?
7. What if three in a row was a loss?

Winning Ways



Kaooa Rules

Kaooa is a hunt game played on a board of 10 circles presented in the diagram above. It is a game for two players – one is the seven Kaooas and the other is the tiger they hunt.

The seven Kaooas are placed on the circles that have the little circles inside, the tiger is then placed anywhere on the other three circles. One Kaooa moves first, then the tiger, and so on. Kaooas can move along any line to a vacant circle. The tiger moves in the same way but can eat (jump) a Kaooa by going over one along a line to a vacant circle.

Kaooas win if they trap the tiger (there is no legitimate move for the tiger) while the tiger wins if it can eat one Kaooa. The game should be won by the Kaooas.

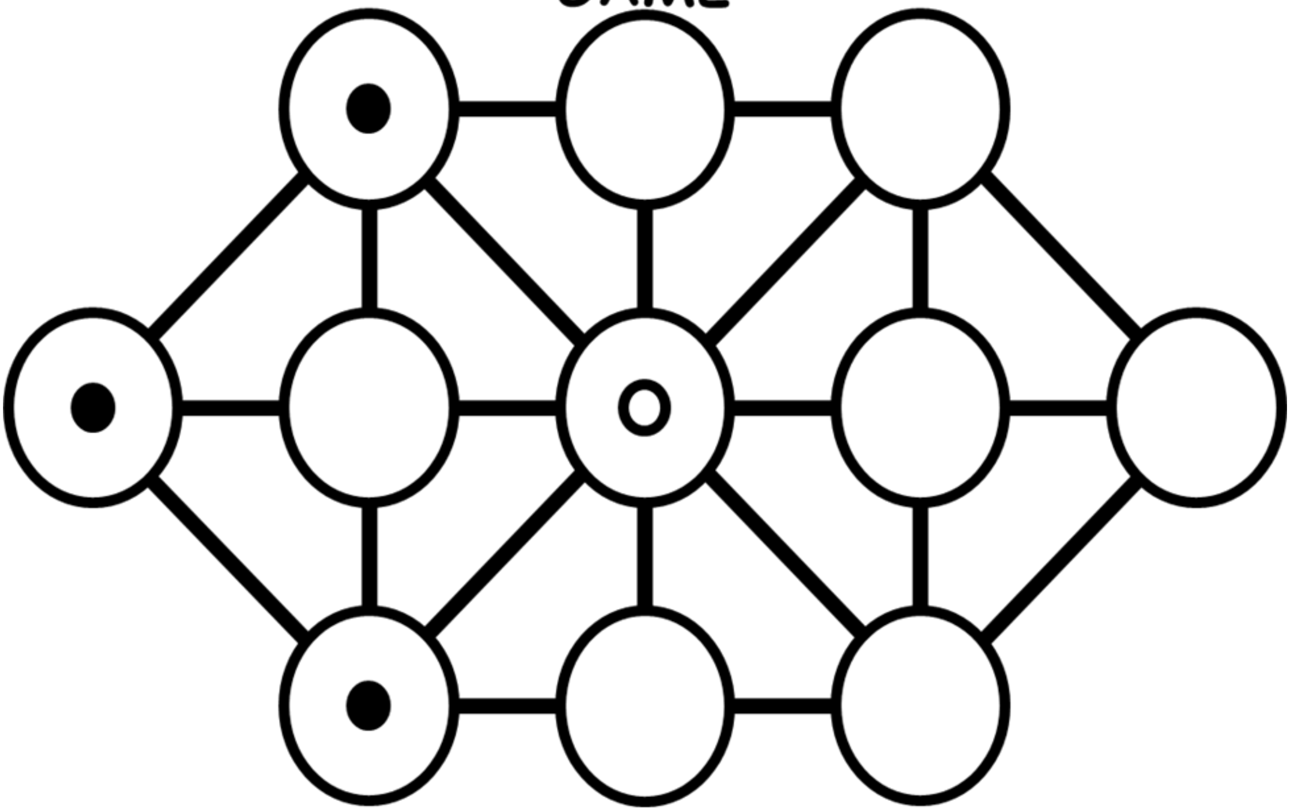
(a) How can you stop being eaten (where can you not leave a cell open)?

(b) What are the important cells to fill?

(c) Can your strategy work with the tiger starting in any of the three cells available?

Winning Ways

FRENCH MILITARY GAME

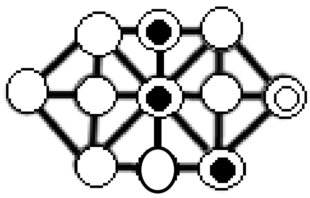


Rules

This game is a hunt game which uses a board as above. The board has 11 cells – the three with small black circles are where the three English soldiers are placed, the one with the small white circle is where Napoleon is placed. One soldier moves first – soldier can move one cell forward/sideways along any line to a vacant cell but not backwards (even diagonally). Napoleon can move one cell in any direction along a line to a vacant cell. The English soldiers win if they can pin Napoleon (so that he has no legitimate move). All other results, continuous moving between spaces ("stalemate") or Napoleon getting behind the English soldiers to where they cannot go back to get him, are a win for Napoleon.

Instructions

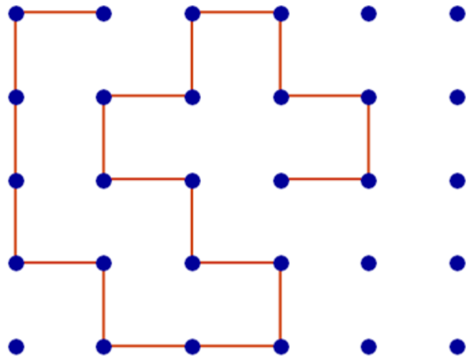
1. Analyse the game. Who wins and how? What are good strategies – for Napoleon and for the soldiers? Who should always win? (Remember – this is a French game!)
2. On the right is an end game (English to move). How can the English win? How can Napoleon get away?



Remember: The focus of this task is not to play the games to beat the other player, but to work together as a group to figure out strategies to win.

Winning Ways

SLITHER



Number of players: 2

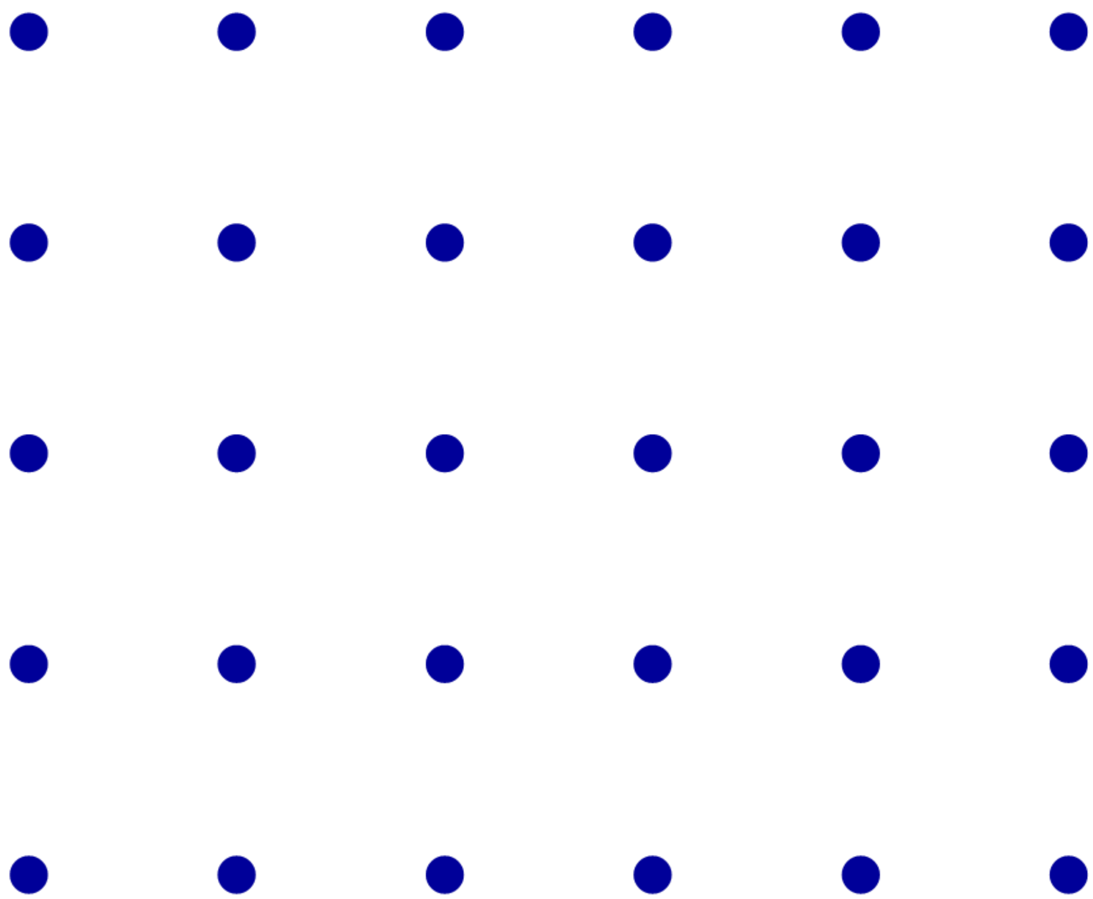
Materials: Textas, dotted paper

Object of game:

- To force your opponent to close the path.

Rules:

- Players take turns to draw a line joining pairs of dots either horizontally or vertically.
- The line must be drawn on either end of the line already formed.
- No dot may have more than two lines going to it.



Winning Ways

SOS

Number of players: 2

Materials: Textas, grid paper

Object of game:

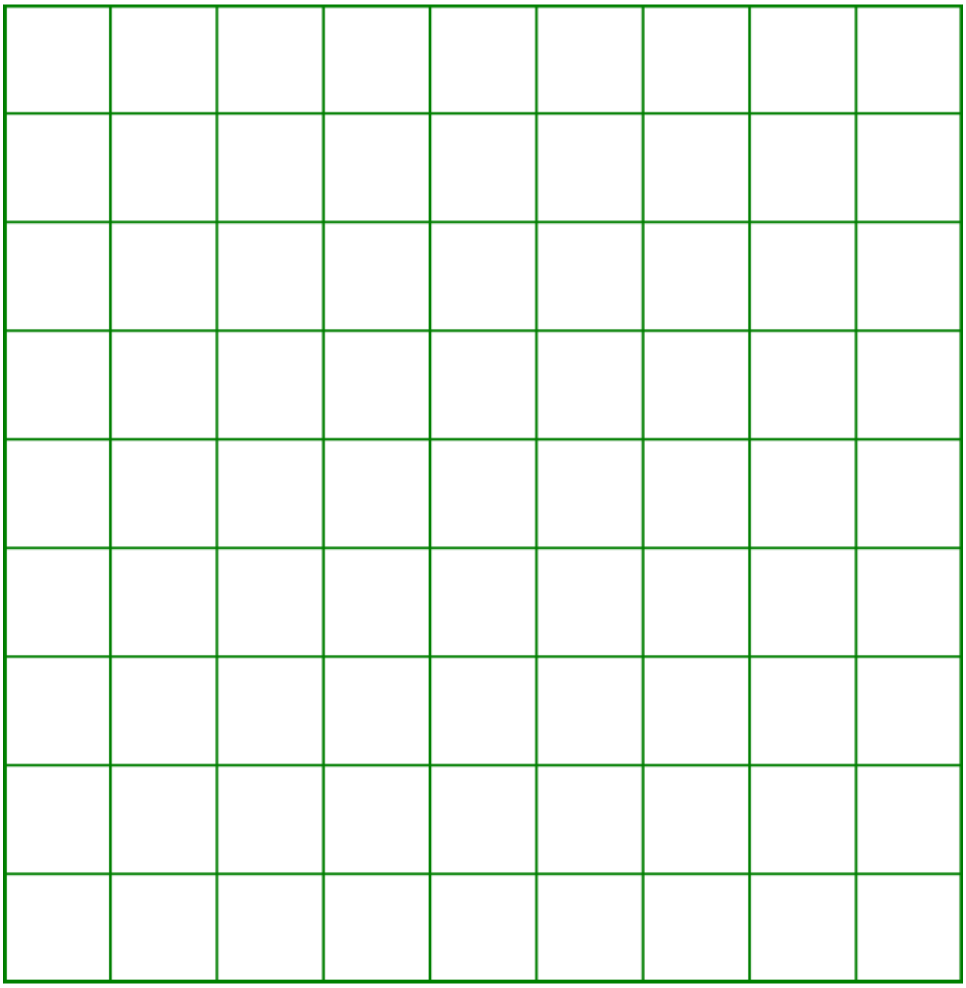
- To create the highest number of S-O-S sequences on the board.

Rules:

- Players take turns to write either an S or an O in one of the squares of the grid. Each player can choose either letter on each turn.
- If a player succeeds in creating a straight S-O-S sequence (either vertically, horizontally or diagonally), they claim that sequence and have another turn.
- S-O-S sequences can be claimed by one player by circling and by the other by drawing a line through it.

Variations:

- The same game can be played on a different-sized grid.



2.8 Mathaerobics

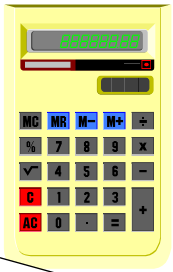
Mathaerobics

All in a Minute

How many sit-ups can you do in a minute?
How many push-ups can you do in a minute?
How many star jumps?
How many times can you bend and touch your toes?

Compare your rates with a friend.

2.9 Crazy Calculators



Lucky 7 puzzle

You are allowed to press the following buttons only: 4, +, −, ×, ÷.

How many presses to make 7 on the display? (The person who does it with the fewest moves is the winner.) Who can get it within 7 presses?

The "Big One"

At no time during the game should either player press C (Clear)

Two players – **one** calculator. Player 1 enters a number between 0 and 99 (without showing Player 2), then presses ÷, the number again and =. Player 2 takes the calculator (which is showing 1 on the display), then tries to guess Player 1's entry number by pressing a guess number (e.g., 43), followed by =. The number on the screen may be correct but, if not, it should help Player 2 make a better 2nd guess, For example, if the Player 2's number is <1, then s/he must enter a larger number; if >1, Player 2 must press a smaller number.

The game ends when Player 2 gets a 1 on the screen. Player 2's score is the number of guesses s/he made before getting 1 to appear on the screen. Keep a tally of the entries made to get back to 1. Change roles; play 5 games each; the player with the lowest total score wins.

CRAZY CALCULATORS

The Talking Calculator

On an upside down calculator, the word hOLES can be made by the number 53704 – **you try it**.

53704 – 29861 = 23843, so 23843 + 29861 = 53704.

We can make a talking calculator by writing the sum and a clue e.g. 23843 + 29861 = ________ What do worn out socks have in common with a coalmine?

Make up your own talking calculator problems!

Calculator NIM (or Force Out)

Two players – one calculator. The purpose of this game is to be the person who "forces" the number to 0.

Player 1 chooses a number between 1 and 5.

Moves: Subtract 0.1, 0.2, 0.3 in any one turn.

Estimate Factors and/or Products

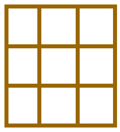
Two players – 1 calculator each

Player A and Player B both enter 56 × . Player A then enters another factor (e.g., 27) without letting Player B see it. Player A shows Player B the product (1512). Player B then needs to estimate what the 2nd factor is. S/He inserts the 2nd factor to see how close s/he is; S/he then tries another factor (without pressing Clear).Player B records each answer (product s/he gets and the factor inserted). How many trials did it take Player B to work out the missing factor. Swap roles – Player A becomes Player B and B becomes A.

2.10 Matchstick Magic and Rome Rules

Matchstick Magic

1.



Use your matchsticks to create the above pattern of 9 squares.

Remove **4** sticks so that:

a) 5 squares remain

b) 6 squares remain

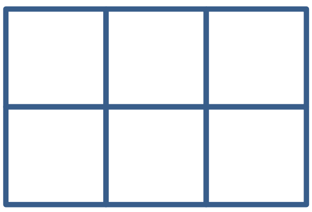
c) 7 squares remain

Keep a record of your moves.

2.

Arrange 17 sticks as below.

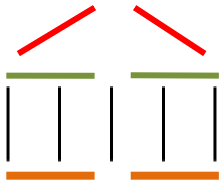
1. Remove 4 sticks leaving 3 squares
2. Remove 5 sticks leaving 3 squares



3.

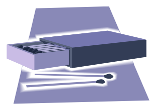
An 11-stick façade of a house is shown below.

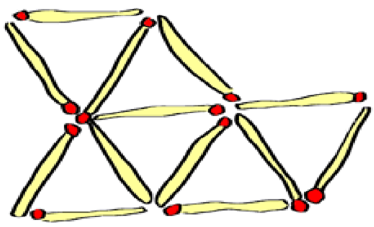
1. Move 2 sticks and get 11 squares.



2. Move 4 sticks and get 15 squares.

4.





Remove only 3 matches and leave 3 triangles.

I I I I = I + I I + I I I   Move one stick to make this true.

Challenges!! Rome Rules

Roman Numerals

1 = I	6= VI
2 = II	7 = VII
3 = III	8 = VIII
4 = IV	9 = IX
5 = V	10 = X

50 = L

2.

The statements below are made with matchsticks but, in each case, one stick has been misplaced. Can you move **one and only one stick** so that the statement will be correct?

a) IV – II = VII
b) VI – X = IV
c) IX – XI = III
d) VI = II
e) L V – II = LV

1.

1. Use 4 matchsticks to show that half of ELEVEN is six.

2. Use 6 matchsticks to form this fraction : $\frac{\text{I}}{\text{VII}}$

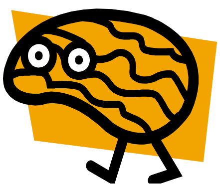
   By moving just one stick, change the fraction so that its value is:

   a) one third; b) eight thousand

3.

Create your own puzzle using Roman Numerals.

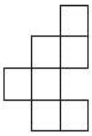
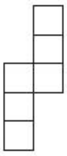
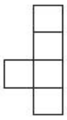
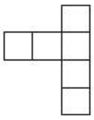
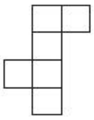
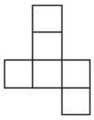
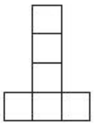
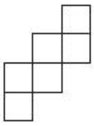
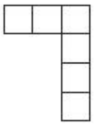
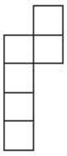
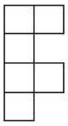
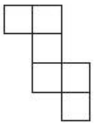
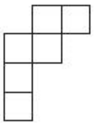
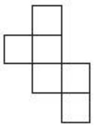
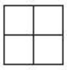
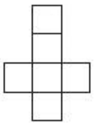
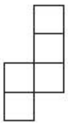
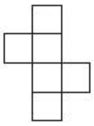
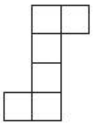
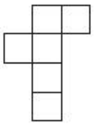
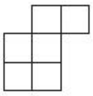
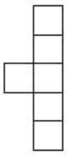
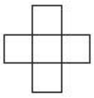
Fool a friend.



2.11 Clever Cubes and Who's a Star

Clever Cubes

Which of the nets below can be folded into a cube? If in doubt, use the plastic square polydrons to make the nets and then clip together to construct a cube. Keep a record of the numbers that make a cube. We've ticked one for you.

 1.	 2.	 3.	 4.	 5.	 6.
 7.	 8.	 9.	 10.	 11.	 12.
 13.	 14.	 15.	 16. ✓	 17.	 18.
 19.	 20.	 21.	 22.	 23.	 24.

Who's a Star?

1. Clever Cube Construction

a) Examine the images on the faces of the displayed cube.
Transcribe them on to the net of a cube.
Fold the net to make the solid.
Compare your cube with the original.
Are they the same?

b) Design your own colourful cube.

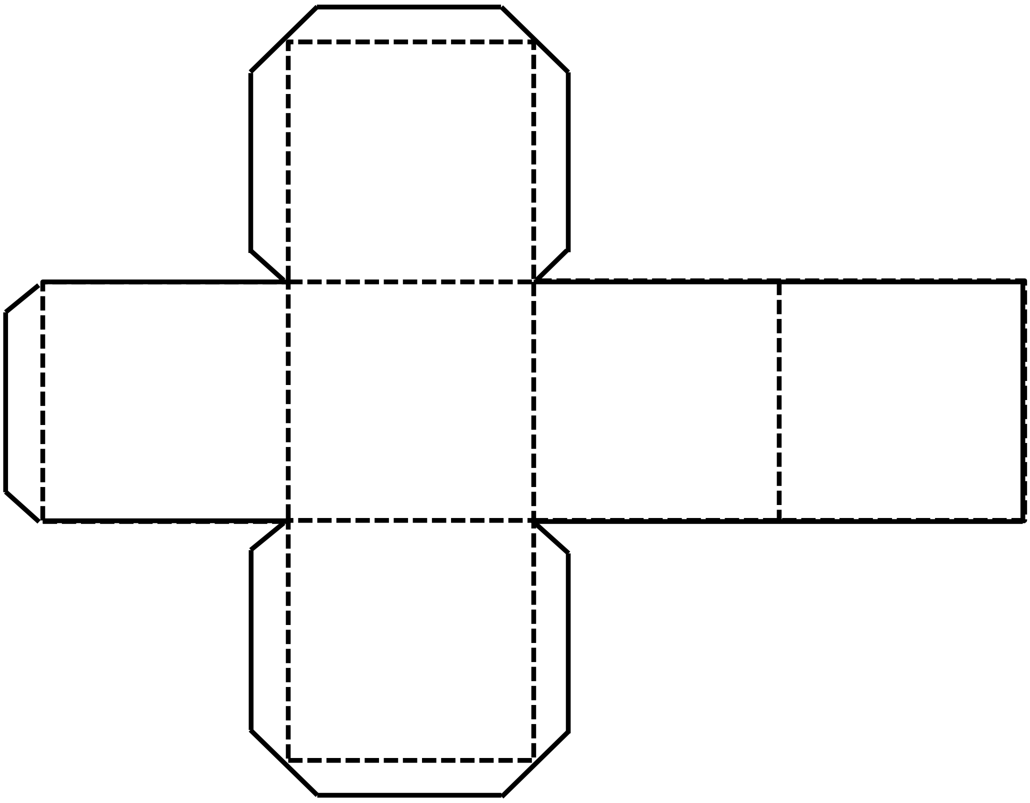
2. Powerful Pentominoes

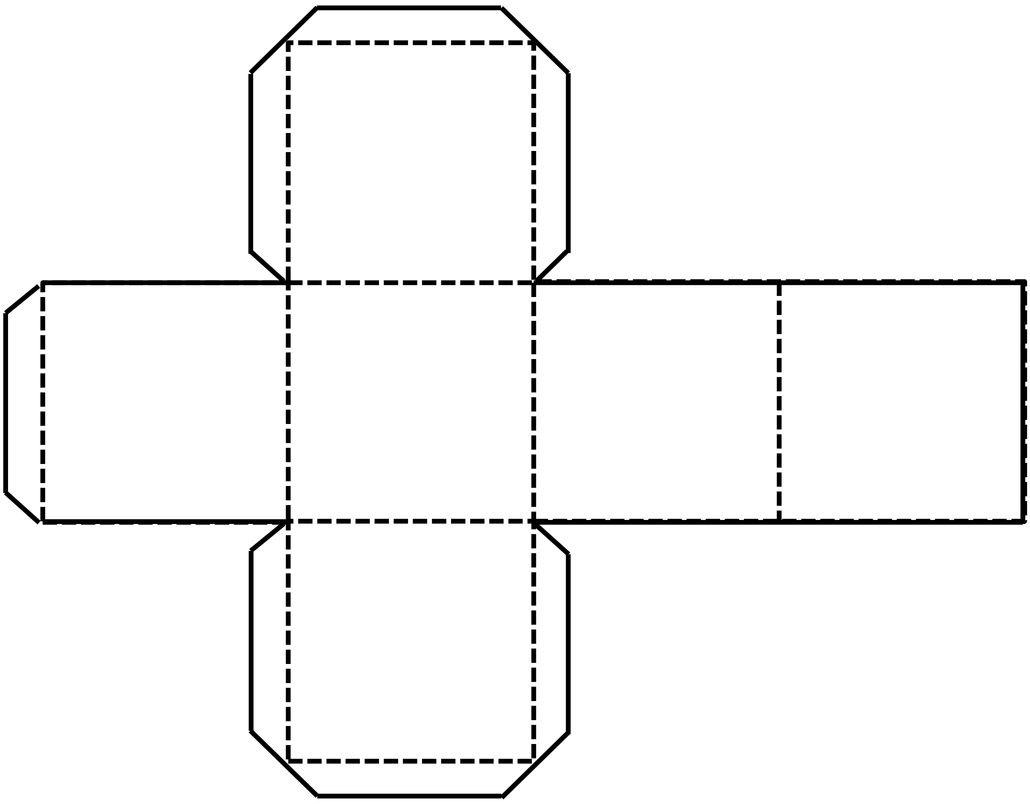
a) Nets of cubes use 6 square polydrons.
Nets of pentominoes are made with 5 square polydrons.
Can you make all twelve pentomino nets?

b) Create an animal using all twelve pentominoes

c) Which of the pentomino nets can be folded to make a cube without a top?

Cube nets





2.12 Ps and Qs: Prickly Problems and Quirky Questions

PRICKLY PROBLEMS

1. Six married couples meet at the start of the tour.

If they shake hands as the introductions are made, how many different handshakes take place?

2. A used car salesman buys a car for $6 000 and sells it for $8 000.

He then buys it back for $7 500 and finally sells it for $10 000.

What profit does he make?

3. 59 tennis players enter a knockout competition.

How many games must be played to determine a winner?

4. The number 2938 has these properties: thousands digit is even, hundreds digit is odd, and all four digits are different.

How many four-digit numbers have these properties?

5. On a trip, John wishes to average 80 km per hour.

At halfway, he finds he is averaging 40 km per hour.

What average speed must John travel in the second half to achieve his goal of 80 km per hour overall?

Quirky Questions

Three men pay $10 each (total $30) for a hotel room. The manager says the cost was really $25 and returns $5 with the bell boy. The bell boy pockets $2 and returns $3 to the men. The men therefore paid $9 each, total $27. The bellboy's $2 makes $29. What happened to the other $1?

You want to know whether the way to a town is on your left or right. You meet two people. One always lies, the other always tells the truth but you don't know who is who. What question can you ask that ensures you will get the correct direction from either person?

Four famous sports people entered a television studio. One was a tennis player, one was a swimmer, one was a golfer, and the other was a chess player. Use the clues to find out who played what sport:

* Mr Cook is not good at chess.

* Both Mr Scott's and Ms Pluck's sports involve a ball.

* Ms Hunt can't swim at all.

* Neither Ms Pluck nor Ms Hunt play tennis.

Tommy invited some friends from school and several cousins to watch a DVD for his 9th birthday party. Each school friend is 9 years old and each cousin is 11 years old. None of his cousins attend his school. If the sum of the ages of Tommy, his friends, and his cousins is 122, how many of Tommy's school friends and how many cousins attended the party?

The distance from Cool Hill to Daisy Valley is 24 kilometres. The distance from Benjy Bay to Cool Hill is 2/3 of the distance from Cool Hill to Daisy Valley. The distance from Apple Valley to Benjy Bay is 3/8 of the distance from Benjy Bay to Cool Hill. What is the distance from Apple Valley to Benjy Bay?

★★★★★

3 Appendices

3.1 Activities and resources: Years 7–9

The following table summarises the 12 activities, and the total resources required for each activity.

	Activities	Resources (to be divided into three sets)
1	Some Strange Stories	• 12 books; no extra resources
2	Algebra Antics	• Activity cards (two), pencils and paper
3	Dynamic Designs – McMahon's Triangles – Escher Art	• Four activity cards for McMahon's Triangles, one activity card for Tessellations and Escher Art • A4 paper – two different colours (1/2 ream of each) • 12 sticky tapes/glues
4	Matchbox Madness	• Activity card, 144 matchboxes (3 containers of 48)
5	Mobius Mobility	• Activity card, 18 rolls of narrow strips of paper • 48 pencils of 2 different colours (red and green) • 18 glue sticks (or sticky tapes)
6	Geo Genius – Measurement – Shape	• Activity cards (one each for Measurement and Shape) • 30 dotted paper sheets • 18 notebooks, 18 pencils • 30 geoboards, 3 bags rubber bands (50-60 per bag)
7	Winning ways – Games	• Activity cards for 6 games • 12 back-to-back useable game sheets, 18 pencils
8	Mathaerobics	• Activity card • 18 one-minute egg timers • Posters and pens
9	Crazy Calculators	• Activity card • 24 calculators • Scrap paper, pencils
10	Matchstick Magic and Rome Rules	• Activity cards (one for each activity) • 24 matchsticks in a zip lock plastic bag, 10 per table = 30 bags of 24 matchsticks
11	Clever Cubes and Who's a Star?	• Activity cards (one for each activity) • 72 copies of nets of cubes (2 to a page) – cut in half • 30 bags of 6 square polydrons (click together 2D shapes) • 48 pencils of 2 different colours (red and black) • 18 glue sticks, 24 scissors
12	Ps and Qs – Prickly Problems – Quirky Questions	• Activity cards (one for each activity) • 72 notebooks, 18 pencils

3.2 Student passport: Years 7–9

Geo Genius Draw the star you made with several triangles. **Challenge - Geo Measurement** a) 10 posts = ____ posts b) 100 posts = ____ posts c) 1000 posts = ____ posts	**Winning Ways** What game did you play? ____________________________ What was your winning strategy? ____________________________ ____________________________ ____________________________ ____________________________ ____________________________	**Some Strange Stories** **Story:** ____________________ ____________________________ **Answer:** ____________________________ ____________________________ ____________________________ ____________________________ ____________________________
Crazy Calculators What was your favourite calculator game? ____________________________ ____________________________ Why?_________________________ ____________________________ ____________________________	**Matchbox Madness** Draw how you arranged the matchboxes so the sides touched 3 other matchboxes.	**Matchstick Magic** Circle which matchstick puzzle you found the easiest. Put a cross through the hardest one. 1a 1b 1c 2a 2b 3a 3b 4 **Challenge - Rome Rules**
Dynamic Designs **McMahon's Triangles** **Escher Art**	**Prickly Problems** 1._____ 2._____ 3._____ 4._____ 5._____ **Quirky Questions**	**Mobius Mobility**
Clever Cubes How many of the nets made a cube? _____ **Challenge – Who's a star?** **Powerful Pentominoes** a) How many pentomino nets did you make? _____ b) What animal did you make? ____________________________	**Mathaerobics** How many in one minute? **You** **Friend** **Sit-ups** ☐ ☐ **Push ups** ☐ ☐ **Star jumps** ☐ ☐ **Touch toes** ☐ ☐	**Algebra Antics**

3.3 Timetable 10 February: Years 7–9 Hall Activities

From 10:20am, class rotations will be conducted in Sessions Two, Three and Four. The hall will be divided into three areas with twelve tables in each area. **YDC staff will be present in each of the areas.**

Timetable

Session Two

Time	Area One	Area Two	Area Three
10:20 –11:30		Year 9: A, B, C, D, E, Immersion & Honours	

Session Three

Time	Area One	Area Two	Area Three
12:00 –1:10	Year 7: A, B & C	Year 9: F, G, H & I	

Session Four

Time	Area One	Area Two	Area Three
1:50 –3:00	Year 7: D, E & F	Year 8: A, B, C, D, E & Honours	

Twelve year-level appropriate activities will be available in each session.

3.4 Timetable 17 February: Years 7–9 Lessons

Session One

Time	YDC Teacher 1	YDC Teacher 2	YDC Teacher 3	YDC Teacher 4
8:55 – 9:25	Year 8: F	Year 8: G	Year 7: A	Year 7: B
9:30 – 10:00	Year 8: H	Year 8: I	Year 7: C	Year 7: D

Session Two

Time	Teacher 1	Teacher 2	Teacher 3	Teacher 4
10:25 – 10:55	Year 9: A	Year 9: B	Year 9: C	Year 9: D
11:00 – 11:30	Year 9: E	Year 9: Immersion	Year 9: Honours	Year 7: E

Session Three

Time	Teacher 1	Teacher 2	Teacher 3	Teacher 4
12:05 – 12:35	Year 9: F	Year 9: G	Year 9: H	Year 9: I
12:40 – 1:10				Year 7: F

Session Four

Time	Teacher 1	Teacher 2	Teacher 3	Teacher 4
1:55 – 2:25	Year 8: A	Year 8: B	Year 8: Honours	Year 8: C
2:30 – 3:00	Year 8: D	Year 8: E		

Four teachers will demonstrate a part of the RAMR cycle for these classes. The same year-level lesson will be modelled to the next class in the rotation. The venue: classroom, undercover area or oval will depend on the lesson content and availability of outside space.

YuMiDeadly

Growing community through education

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