YuMi Deadly Maths

Year 9 Teacher Resource:

MG – How much does it hold?

Prepared by the YuMi Deadly Centre

Faculty of Education, QUT

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QUT

YuMiDeadly

Growing community through education

ACKNOWLEDGEMENT

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research.qut.edu.au/ydc/

Year 9 Measurement and Geometry

How much does it hold?

Learning goal Students will calculate the surface area and volume of right prisms and cylinders.

Content description Measurement and Geometry – Using units of measurement

- Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)
- Solve problems involving the surface area and volume of right prisms (ACMMG218)

Big idea Measurement – interpretation vs construction

Resources Different types/sizes of empty tin cans and right prisms, rolls of paper, Maths Mat, elastics, large congruent paper circles, 30 cm rulers, metre rulers, paper strips

Reality

Local knowledge Discuss where cylinders and right prisms are found in the local environment (e.g. electricity poles, cans of food, boxes, buildings).

Prior experience Check students' knowledge of cylinders and prisms:

- *What are the properties of a cylinder?* [Bases are two congruent circles and vertical face is a rectangle with one side equal to the perimeter of its circular base.]
- *What comprises the surface area of a cylinder?* [the two base circles and the vertical rectangle]
- *What is a right prism?* [It is a 3D shape with bases of two congruent polygons and all vertical faces are rectangles that are perpendicular to the bases.]
- *Why are they called "right" prisms?* [They are called right prisms because the angles between the base and sides are right angles.]
- *What comprises the surface area of right prisms?* [the two congruent base polygons and the number of rectangular faces equal to the sides of the base of the polygon]
- *What two measures are needed to calculate the volume of any regular 3D shape?* [the area of the base and the perpendicular height]

Kinaesthetic Ask students to bring an empty can and a right prism. Have examples on a table, e.g. 825 g can of tomatoes, can of soft drink, Toblerone box, cube, cereal box.

Distribute rolls of paper and have students investigate the surface area of the shapes they have brought. For the cylinder (can), students trace and cut out the two circles that form the base and cut the paper to the height of the can then wrap the roll around the can and mark where the two ends meet. Cut the paper to that length.

Ask students:

- *What measurement of the can has the paper been wrapped around?* [perimeter/ circumference]
- *So what is the length of the paper equal to?* [perimeter/circumference of the circle]
- *Form the net of the cylinder. What shapes do you have?* [two equal circles joined at points, top and bottom, of the length of the vertical face of the cylinder]
- *What comprises the surface area of the cylinder?* [the two circle bases and the one rectangular vertical face]
- *What measurements do you need to calculate the surface area of your cylinder?* [radius of the base circle, the circumference of the base circle that is the length of the vertical face, and the perpendicular height of the can]

- *How will you calculate the total surface area?* [Find the area of the base circle, multiply it by 2 (for the two bases) and add on the area of the rectangular vertical face.]
- *What formulas are needed?* [Area of circle × 2 + Area of rectangle]
- *What is the formula for area of circle/rectangle?* [πr^2 / L × prism height]
- *What is the length equal to?* [circumference of circle base]
- *So what will the formula be for the surface area of the cylinder after this substitution?*
- **Surface area of cylinder** = $2\pi r^2 + \pi D$ × prism ht *or* $2\pi r^2 + 2\pi r$ × prism ht

Follow the same process to cut and lay out the nets for triangular prisms, cubes, square prisms, rectangular prisms. Establish the formulas:

- **Surface area of triangular prism** = base × perpendicular height [of triangular base] + L [equal to perimeter of triangular base] × prism height, that is:

  SA = base × perp ht + perim of base × prism ht

(*Note*: Area of triangle = ½ base × perp ht and as there are 2 bases, 2 × ½ base × perp ht = base × perp ht.)

- **Surface area of square prism** = $s^2 + 4s$ × prism ht
- **Surface area of rectangular prism** = 2 L × B [of bases] + L [perim of base] × prism ht

From this, have students generalise the formula for surface area of any cylinder or right prism:

- *What are the common elements in each of the formulas above?* [The surface area depends on the **area of the two bases** and the **area of the vertical rectangular faces** whose total length is equal to the perimeter of the base and whose perpendicular height is equal to the prism height.]
- *So what is the formula for surface area of any cylinder or right prism?*

Surface area right prism = 2 × area of prism base + perimeter of base × prism height

Let surface area = SA; area of prism base = B; perimeter of base = P; height = h, then:

$$\boldsymbol{SA = 2B + Ph}$$

Reverse:

- *If you knew the surface area and the dimensions of the base, would you be able to calculate the height of the prism?* [yes – use above formula to isolate height]

$$\text{prism height} = \frac{\text{surface area} - (2 \times \text{area of prism base})}{\text{perimeter of base}}$$

$$\boldsymbol{h = \frac{SA - 2B}{P}}$$

Abstraction

Body **Maths Mat and elastics – square prism**

Have students make a square on the mat. Have four students (approximately the same height) stand within the square each holding an elastic from the corners of the square base up to the top of their heads:

- *What measurement is being demonstrated?* [perpendicular height of the square prism]

Take another elastic and have the four students hold part of this elastic so that it mirrors the square base on the mat:

- *What shape can be seen at any cross-section of the square prism?* [a square]
- *What factor, then, is being multiplied all throughout the square prism?* [the area of the square base]

Have students crouch down still holding all elastics but adjusting the prism height:

- *What factor stays the same?* [the area of the base]
- *What factor changes?* [the prism height]
- *What is being measured by multiplying the area of the base by the prism height?* [the volume inside the square prism]
- *What general formula can be applied to finding the volume of a square prism?* [volume = area of base × prism height or, in the case of a square prism, $s^2 \times h$]

Follow the same process to establish formulas for triangular prisms and rectangular prisms.

Triangular prism: V = area of prism base × prism ht → V = ½ prism base × perp ht × prism ht

Rectangular prism: V = L × W × prism ht

- *What is the only formula we need to find the volume of any right prism?*

Volume of right prism = area of base × prism height

Let volume = V; area of prism base = B; height = h, then:

$$\boldsymbol{V = B \times h}$$

Investigate for cylinders

Have students place a large paper circle on the mat. Have a number of students take 30 cm rulers and stand them upright around the circumference of the circle. Ask another student to place a second paper circle on top of the rulers so that the base and the top are parallel.

- *What shape, or factor, is being repeated throughout the prism height?* [the area of the circle base]

Have students bring the metre rulers for students to hold against the 30 cm rulers. Other students place a third paper circle on top of the metre rulers.

- *What shape or factor is being repeated throughout the prism height?* [area of circle base]
- *What factor is changing in the cylinder?* [the cylinder height]
- *What formula can be developed to find the volume of a cylinder?* [volume of cylinder = area of circle base × prism height]
- *Compare the formulas established for the 3D shapes above. What elements are the same in finding the volume?* [the area of the prism base × prism height]
- *What, then, is the general formula for the volume of all right prisms?* [volume of right prism = area of prism base × prism height]

Reverse:

- *If you are given the volume of the right prism and any two of the other dimensions, how do you find the missing dimension?* [use the formula to isolate the missing dimension and substitute the known dimensions into the formula]

e.g. Width of rectangular prism (W) = $\frac{\text{Volume}}{\text{L} \times \text{H}}$

Hand

Have students swap their can with a partner whose can size is different. Using strips of paper, students cut lengths equal to the circumference, diameter/radius and cylinder height. Use the general formula for volume then substitute the appropriate measures for a cylinder to calculate the volume using calculators.

For example, the volume of an 825 g can of tomatoes – dimensions from paper strips are: diameter = 9.8 cm, radius = 4.9 cm, height = 11 cm

Volume = Area of base × cylinder height

$$\begin{aligned} V &= A \times H \\ &= \pi r^2 \times H \\ &= 3.14 \times 4.9 \times 4.9 \times 11 \\ &= 829.31\ \text{cm}^3 \end{aligned}$$

Considering that 1 cm³ of volume = 1 g of mass = 1 mL capacity of water, the formula used to find volume of any right prism has been validated, as 829.31 cm³ is very close to 825 g, allowing that the contents may not have quite reached the top of the can or the can may be concave.

Students then fill the can to its capacity to verify the water capacity of the can of tomatoes. *What do you predict the capacity to be?* [around 825 to 830 mL of water].

Mind

Ask students to close their eyes and visualise a small cylindrical test tube with dimensions of 1 cm radius and height of 11 cm. *What volume of liquid would it hold if it were filled 1 cm below the top?* [approx. 31.4 cm^3] *What is the height of a tin can whose volume is 375 cm³ and the area of its base is 125 cm²? What could this can hold?*

Creativity

Students create measurements for their own right prisms, identify what the contents could be and calculate the surface area and volume.

Mathematics

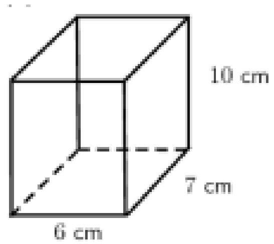
Language/ symbols

length, width, radius, perpendicular height, face, surface area, volume, right prism, cylinder, square prism, triangular prism, rectangular prism

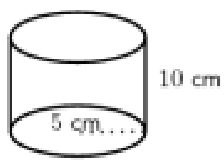
Practice

1. Draw nets of triangular, square, rectangular prisms and a cylinder, including dimensions, to demonstrate that the surface area equals twice area of base plus perimeter of base × prism height. Use calculators to find the surface area in each case using the dimensions you have chosen.
2. If a litre of paint covers an area of 2 m^2, how much paint does a painter need to cover:
   (a) a rectangular swimming pool with dimensions 4 m × 3 m × 2.5 m (the inside walls and floor only)?
   (b) the inside walls and floor of a circular reservoir with diameter 4 m and height 2.5 m?
3. Use the general formula for volume to calculate the volume of the prisms below:

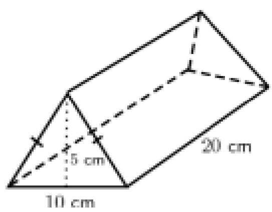
(a)



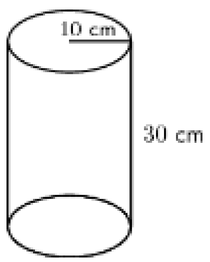
(b)



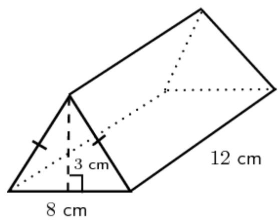
(c)



(d)



(e)



4. A rectangular prism of volume 3200 mm³ has a rectangular base of length 10 mm and width 8 mm. Find the height h of the prism.

5. The area of one square face of a cube is equal to 64 cm^2. Find the volume of the cube.
6. A 4.9 m long water trough has a semicircular base with a diameter of 140 cm. Find the capacity of water it can hold in litres.

Connections Relate to perimeter, area, metric tables and their conversion, capacity and mass.

Reflection

Validation Students discuss situations where surface area and volume would require calculation in the real world, e.g. painting, tiling, covering boxes (surface area) or filling swimming pools, containers, excavations (volume). Students validate their partner's creative work.

Application/ problems Provide applications and problems for students to apply to different real-world contexts independently; e.g. *A landscape architect is designing raised gardens of different shapes for a local park. He wants to compare how much soil will be required for each type: a square garden with a side of 3 m, a rectangular garden with L = 4 m and W = 2 m, and a circular garden with a diameter of 3 m. All are the same height, 0.4 m. What is the total amount of soil required if one of each type was built?*

Extension **Flexibility**. Students are able to substitute a specific formula for different shapes into the general formulas for calculating both surface area and volume of 3D shapes.

Reversing. Students are able to move between starting with dimensions of 3D shape ↔ calculating surface area or volume ↔ finding a dimension when given surface area or volume and two dimensions, starting from and moving between any given point.

Generalising. *A right prism has a polygon as its base and vertical faces perpendicular to the base. The base and top surface are congruent (i.e. the same shape and size) and parallel. If any cross-section of a prism is taken parallel to those bases by making a cut through the prism parallel to the bases, the cross-section will be congruent to the bases. It is a "right" prism because the angles between the base and sides are right angles.*

The general formulas for surface area and volume of right prisms are all that students need to understand and know, as the specific formula for surface area or volume of each different type of right prism may then be inserted into the general formula. This lessens the student's cognitive load.

Changing parameters. Students explore the surface area and volume of combined geometrical solids.

Teacher's notes

- Ensure that students have a sound understanding of the properties of 3D shapes and conversion of metric measurements before proceeding to introduce surface area and volume.
- Converting volume into capacity (mL = millilitres, kL = kilolitres, ML = megalitres):

  1 mL = 1 cm^3 1 kL = 1000 L = 1 m^3

  1 L = 1000 mL = 1000 cm^3 1 ML = 1000 kL = 1000 m^3
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a can, students look at it, remove the can, students then close their eyes and see the can in their mind, then make a mental picture of a different type of can.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html

- Explicit teaching that aligns with students’ understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

