

YuMi Deadly Maths

Overview

Philosophy, Pedagogy,
Change and Culture

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

Prep to Year 9: Introductory Resource – Overview





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The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

DEVELOPMENT OF THIS BOOK

This version of the YuMi Deadly Maths Overview book is a modification and extension of a book developed as part of the Teaching Indigenous Mathematics Education (TIME) project funded by the Queensland Department of Education and Training from 2010–12. The YuMi Deadly Centre acknowledges the Department’s role in the development of YuMi Deadly Maths and in funding the first version of this book.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at QUT which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

The YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

The YuMi Deadly Centre can be contacted at ydc@qut.edu.au. Its website is <http://ydc.qut.edu.au>.

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ABOUT YUMI DEADLY MATHS

From 2000–09, researchers who are now part of the YuMi Deadly Centre (YDC) collaborated with principals and teachers predominantly from Aboriginal and Torres Strait Islander schools and occasionally from low socio-economic status (SES) schools in a series of small projects to enhance student learning of mathematics. These projects tended to focus on a particular mathematics strand (e.g. whole-number numeration, operations, algebra, measurement) or on a particular part of schooling (e.g. middle school teachers, teacher aides, parents). They resulted in the development of specialist materials but not a complete mathematics program (these specialist materials can be accessed via the YDC website, <http://ydc.qut.edu.au>).

In October 2009, YDC received funding from the Queensland Department of Education and Training through the Indigenous Schooling Support Unit, Central-Southern Queensland, to develop a train-the-trainer project, called the **Teaching Indigenous Mathematics Education** or **TIME** project. The aim of the project was to enhance the capacity of schools in Central and Southern Queensland Indigenous and low SES communities to teach mathematics effectively to their students. The project focused on Years P to 3 in 2010, Years 4 to 7 in 2011 and Years 7 to 9 in 2012, covering all mathematics strands in the Australian Curriculum: Number and Algebra, Measurement and Geometry, and Probability and Statistics. The work of the TIME project across these three years enabled YDC to develop a cohesive mathematics pedagogical framework, **YuMi Deadly Maths**, that covers all strands of the *Australian Curriculum: Mathematics* and now underpins all YDC projects.

YuMi Deadly Maths (YDM) is designed to enhance mathematics learning outcomes, improve participation in higher mathematics subjects and tertiary courses, and improve employment and life chances. YDM is unique in its focus on creativity, structure and culture with regard to mathematics and on whole-of-school change with regard to implementation. It aims for the highest level of mathematics understanding and deep learning, through activity that engages students and involves teachers, parents and community. With a focus on big ideas, an emphasis on connecting mathematics topics, and a pedagogy that starts and finishes with students' reality, it is effective for all students. It works successfully in different schools/communities as it is not a scripted program and encourages teachers to take account of the particular needs of their students. Being a train-the-trainer model, it can also offer long-term sustainability for schools.

YDC believes that changing mathematics pedagogy will not improve mathematics learning unless accompanied by a whole-of-school program to challenge attendance and behaviour, encourage pride and self-belief, instil high expectations, and build local leadership and community involvement. YDC has been strongly influenced by the philosophy of the Stronger Smarter Institute (C. Sarra, 2003) which states that any school has the potential to rise to the challenge of successfully teaching their students. YDM is applicable to all schools and has extensive application to classrooms with high numbers of at-risk students. This is because the mathematics teaching and learning, school change and leadership, and contextualisation and cultural empowerment ideas that are advocated by YDC represent the best practice for **all** students.

YDM is now available direct to schools face-to-face and online. Individual schools can fund YDM in their own classrooms (contact ydc@qut.edu.au or 07 3138 0035). This Overview resource is part of the provision of YDM direct to schools and is the first in a series of resources that fully describe the YDM approach and pedagogical framework for Prep to Year 9. It overviews (a) the beliefs that underpin YDM; (b) the YDM philosophy and pedagogy; (c) cultural implications; (d) relationships to whole-school change and leadership; and (e) implementation of YDM in schools (along with integration with existing programs). Because YDM is largely implemented within an action-research model, the resources undergo amendment and refinement as a result of school-based training and trialling. The ideas in this resource will be refined into the future.

YDM underlies three projects available to schools: YDM Teacher Development Training (TDT) in the YDM pedagogy; YDM AIM training in remedial pedagogy to accelerate learning; and YDM MITI training in enrichment and extension pedagogy to build deep learning of powerful maths and increase participation in Years 11 and 12 advanced maths subjects and tertiary entrance.

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List of Abbreviations

ABS	Australian Bureau of Statistics
AIM	Accelerated Inclusive Mathematics
DET	Department of Education and Training
MITI	Mathematicians in Training Initiative
NAPLAN	National Assessment Program – Literacy and Numeracy
QCAR	Queensland Curriculum, Assessment and Reporting
RAMR	Reality–Abstraction–Mathematics–Reflection
SES	socio-economic status
TIME	Teaching Indigenous Mathematics Education
YDC	YuMi Deadly Centre
YDM	YuMi Deadly Maths

1 *Beliefs and Imperatives*

Australia is currently at a crisis point with very low numbers of students wishing to undertake careers in mathematics, in the mathematical sciences and other disciplines based on mathematics, or in the teaching and learning of mathematics. At the same time, there has been a rise in employment opportunities in areas where mathematics is required, notably engineering and trade occupations such as electrician. This has led to a skills shortage that is threatening the viability of the industries that underpin Australia's economic wellbeing, and has resulted in strong government initiatives in skills and training.

Aboriginal, Torres Strait Islander, and low socio-economic status (SES) students are underperforming in mathematics, exacerbating the crisis in mathematics and also negatively affecting their futures as their mathematics is often too low for any traineeship, apprenticeship or employment opportunity. This leads to welfare dependence, all the difficulties with respect to health, substance abuse, violence and crime that can go with this, and low employment and life chances. If these students' mathematics learning can be improved, it results in a win-win situation with people moving from welfare to productive employment, reducing the skills shortage and building better futures, a convergence of economic and social benefit.

The mission of the YuMi Deadly Centre (YDC) is to work with these mathematically underperforming Aboriginal, Torres Strait Islander, and low SES students and collaborate with their teachers, schools and communities in projects to enhance their mathematics learning and improve their employment and life chances. As part of its work in these projects, YDC has developed a mathematics pedagogical framework, called YuMi Deadly Maths (YDM), which is designed to efficiently improve mathematics outcomes for Aboriginal, Torres Strait Islander, and low SES students.

This chapter outlines the beliefs and imperatives that underlie the design of YDM, and the rationale for the components of YDM that are described in this book.

1.1 Context and imperatives

This section looks at YDM in terms of present-day context and provides some rationale for how YDM has been positioned, leading to the YDM imperatives.

1.1.1 Context and rationale

From a mathematics/mathematics-education perspective, there is a shortage of students wishing to undertake careers in mathematics, in the mathematical sciences and other disciplines based on mathematics, or in the teaching and learning of mathematics (Thomas, 2009). This has resulted in mathematics departments within universities closing down or being subsumed into other (emerging) discipline areas such as complex systems. As a consequence, there is a shortage of mathematics teachers to the point that nearly half the teachers of mathematics in secondary schools have no pre-service training in mathematics or mathematics education.

From a student perspective, Aboriginal and Torres Strait Islander students are generally two years behind their non-Indigenous counterparts and low SES students are similarly behind when compared against students from high socio-economic families (Marginson, 2008). A major factor in Indigenous and low SES disengagement and underachievement is that students are confronted with a Eurocentric mathematics curriculum. From this perspective, mathematics is presented in a purely abstract form with very little connection to Indigenous and low SES people, culture and world view. This disassociation with the subject usually manifests itself in statements from students like "Why do I need to learn this?" and "What will I use this for?".

Thus, to enhance engagement and achievement, YDC has asked fundamental questions such as “What is mathematics?” and “How does culture relate to mathematics?”. The exploration of answers to these questions has resulted in the development of a philosophy and pedagogy for YDM (see Chapter 2) that aims to allow Indigenous and low SES students to explore mathematics on their own terms, through their world view, and that values and utilises the cultural and social capital that these students bring to the classroom (Bourdieu, 1973). Given that the emphasis is on allowing students to be creative and self-expressive, we would argue that the philosophy will not only allow Indigenous and low SES students to connect with mathematics but also allow these students to excel in learning mathematics.

Cultural and social capital (Bourdieu, 1973)

Australian education is frequently described and perceived as providing opportunity in return for hard work and attention (meritocracy). However, schools often contribute more to social reproduction than social mobility. One means of analysing different school outcomes for students is through Bourdieu’s (1973) theories of habitus and cultural capital. By definition, habitus relates to acquired ways of thinking, seeing, acting, speaking, walking and eating (Wadham, Pudsey, & Boyd, 2007). Habitus as a word looks and sounds much like habits which we understand as patterns of behaviour or ways of acting that are performed automatically without conscious thought.

An individual’s culture represents their community’s shared understanding of symbols and signs, language, values and meanings, beliefs, norms, rituals and material objects used to interpret the world around them. An individual’s cultural capital consists of their learned ability to interpret socially constructed messages and practices they engage with and value. While all individuals possess cultural capital that continues to build throughout their lifetime, initial cultural capital formation varies according to the social practices and beliefs of the individual’s family and local community and may or may not match the cultural capital valued or legitimised within the dominant culture.

Social capital represents the ability of a student’s parents to utilise the school system to ensure success through enacting choice and voice (Ball & Vincent, 2001) in their child’s education. Parents with high social capital to impart to their children make use of networks of friends and acquaintances to assist their child with their education, and their own understanding of the education system acquired from their own academic qualifications, professional reputations and status. As a result, these parents are quick to volunteer and position themselves as partners and managers in the education of their children (Meadmore, 2004).

Traditional Eurocentric education practices in Australia legitimise the values, beliefs and practices of the middle and upper class. As a result children from middle and upper class families experience a natural fit with the cultural capital of the school whereas Indigenous and low SES students need to learn this as well as the curriculum. As Bourdieu (1973) states:

The culture’s educational system demands of everyone alike that they have what it does not give. This consists mainly of linguistic and cultural competence and that relationship of familiarity with culture which can only be produced by family upbringing when it transmits the dominant culture.

1.1.2 Imperatives

Mathematics programs to close the gap between Aboriginal, Torres Strait Islander, and low SES and mainstream students can vary in many ways.

1. In terms of the direction and freedom given to teachers, some programs supply scripts of what to say from which teachers cannot deviate; others provide sequences of lesson plans, back-up resources and materials from which teachers have some freedom to choose and in terms of which teachers have some freedom to tailor to their students’ needs; and some provide knowledge and exemplar lessons in order for teachers to plan their own sequences and fashion their own script.

2. In terms of the focus of their instruction, some programs see their role in meeting the functional needs of their students, so that they can recall, and use in standard situations, the content and procedures that have been specified for certain vocations; other programs believe that it is important to understand the mathematics behind the specific content and procedures so that this knowledge is portable and can be translated to new situations.
3. In terms of the scope of the program, some programs restrict themselves to only what happens in the mathematics classroom; others provide support for some out of classroom activities such as homework, excursions and clubs; while others see that their role extends to the school as a whole and the school's relationships with parents/carers and community.

YDM is designed to adopt a distinct position within the options available for a mathematics program. This position is described in sections 1.2 and 1.3.

Over the last 15 years, YDC staff have been trialling teaching ideas with Indigenous and low SES students. Looking back, these ideas have been based on beliefs and imperatives with respect to what students are capable of learning, what teachers are capable of teaching and how these relate to school and community. In general, YDC staff have found that success is the greatest when teaching encompasses all the knowledge required to understand as well as to do.

These experiences are distilled into the following **six imperatives** on which YDM is based:

1. that all people deserve the deepest mathematics teaching and learning that empowers them to understand their world mathematically and to solve their problems in their reality;
2. that all people can be empowered in their lives by mathematics if they understand it as a conceptual structure, life-describing language, and problem-solving tool;
3. that all people can excel in mathematics and remain strong and proud in their culture and heritage if taught actively, contextually, with respect and high expectations and in a culturally safe manner;
4. that all teachers can be empowered to teach mathematics with the outcomes above if they have the support of their school and system and the knowledge, resources and expectations to deliver effective pedagogy;
5. that all communities can benefit from the mathematics teaching and learning practices above if school and community are connected through high expectations in an education program of which mathematics is a part; and
6. that a strong empowering mathematics program can profoundly and positively affect students' future employment and life chances, and have a positive influence on school and community.

These imperatives are based on particular views of the **nature** of four components of YDM:

- mathematics
- mathematics learning
- mathematics teaching
- school–community connections.

The positioning of YDM is now described, and illustrated by diagrams, in terms of these four components. Each diagram shows two two-way dichotomies that are **social** (vertical) and **cognitive** (horizontal) and place YDM into the quadrant that it represents.

1.2 Mathematics and mathematics learning

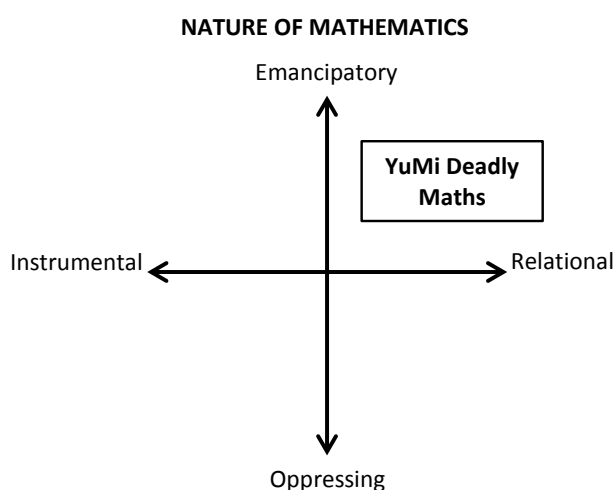
We begin by looking at YDM with respect to the inputs for the program: YDM's position regarding mathematics and mathematics learning.

1.2.1 Mathematics

For mathematics, the two dichotomies are **instrumental–relational** and **oppressing–emancipatory**. From a cognitive perspective, the dichotomy first described by Skemp (1976), instrumental vs relational, reflects two ends of perceptions of the nature of mathematics. Mathematics is seen from an instrumental perspective as a collection of definitions, rules and procedures that find answers in particular situations; while it is seen from a relational perspective as a structure of concepts, strategies and principles that provide meaning and underpin application and problem solving. For example, if the problem was conversion between millimetres (mm) and metres (m), instrumental mathematics would consist of the rules: number of m = number of mm \div 1000, and number of mm = number of m \times 1000 and the procedure of moving the decimal point to achieve this (e.g. 1564mm = 1.564 – move decimal point 3 places to left; 2.87m = 2870mm – move decimal point 3 places to right). Against this, relational mathematics would connect metrics to whole number and decimal numeration and the flexible role of the unit in representing these numbers with symbols (e.g. m to thousands place and mm to ones place and m to ones place and mm to thousandths place depending on the unit); relate metric conversion to multiplicative structure in terms of how place-value positions relate by multiplying and dividing by powers of 10; and show that metric conversion in general, and the mm and m relationship in particular, is an **extension** of knowledge of whole and decimal numbers. In this way, the student gains length metric conversion **plus** an understanding of number that enables all other metric conversions to be understood and lays the foundation of higher mathematics.

From a social perspective, mathematics can be seen as emancipatory or oppressing. Emancipatory mathematics contains the ideas that enable students to understand their position in the world and to analyse, and take control of, the factors that determine this role; while oppressing mathematics contains only the ideas that enable students to fit in. For example, if the problem is getting a job, oppressing mathematics would simply be the content for the entry level of a particular job; while emancipatory mathematics would be this content plus extra content to understand how to progress in the job or to use the job for own progression, and how the job operates, or could operate in terms of the general good. A particular example is that Indigenous and low SES students are trained for construction jobs in some programs by experiencing only limited mathematics with respect to the procedures and formulae related to the job. They do not have the underlying mathematics (e.g. projective geometry and algebra of compensation) that would enable them to design constructions and prepare quotes which are the basis of promotion and becoming a builder or contractor.

As the diagram on the right shows, YDM is in the **emancipatory/relational quadrant**. Its aim is to reveal mathematics as a connected structure that provides students with the knowledge to take control of their lives and become what they wish. YDM contains activities that effectively build this form of mathematics.



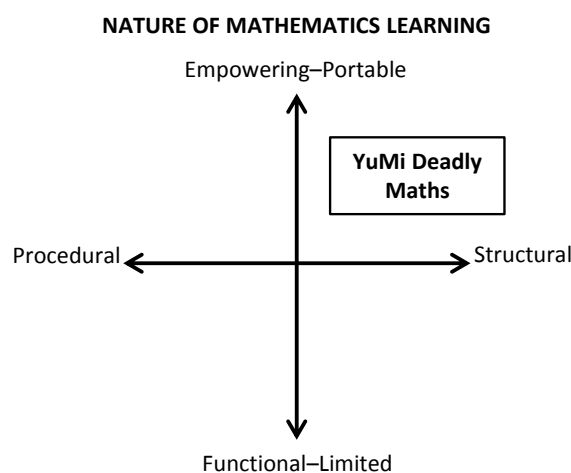
1.2.2 Mathematics learning

For mathematics learning, the two dichotomies are **procedural–structural** and **functional–empowering**. From a cognitive perspective, procedural learning is focusing on the rote learning of instrumental mathematics content where the student learns disparate facts, rules and procedures as pieces of knowledge to be recalled, while structural learning is focusing on acquiring relational mathematics content as rich schema (where mathematically equivalent understandings are connected and form integrated networks of information). From a social perspective, learning can focus on functional outcomes, that is, mathematics content **limited** to the immediate need for it, or it can focus on empowering outcomes, that is, mathematics content that is **portable** and can be translated or transferred to a wide variety of situations. The power of mathematics lies in its portability and portability depends on structural understanding (see section 2.3). Rich schema enables knowledge to be applied in all the components that are connected. It facilitates recall because knowledge is stored as whole structures not individual components, and it enhances problem solving as the connections enable other knowledge to be considered as well as the knowledge that is the focus of the problem.

An example for both dichotomies is that of volume formulae, say for the trades of welding or boilermaking. A cylindrical tank has to be built that is 4 m in diameter and of a height that it will hold 40,000 litres or 40 kilolitres or cubic metres of grain. The boilermaker has to calculate the height. The mathematics to solve this problem can be taught functionally by providing the boilermaker with the formula for the volume (i.e. $V = \pi R^2 H$), the formula for height (i.e. $H = V/\pi R^2$), and the value of π , and the boilermaker can work out the height with a calculator for this particular context. The mathematics can also be taught in an empowering manner as follows.

1. The formula for a cylinder can be generated from experiencing area of a rectangle and circumference of a circle in real situations through making a circle and rectangle πR by R and through area of prism/cylinder in area of base \times height (see YDM Measurement book).
2. The algebra of changing the subject of a formula can be generated by experiencing the balance rule in many equivalence situations, that is, by students inquiring rather than just listening to the teacher.
3. The process of translating real-world problems to mathematics symbols and translating them back into reality can be experienced throughout (1) and (2).
4. This will enable the problem to be solved in that situation but also provides understanding that can be portably transferred to any volume and area situation, any balance algebra situation (such as budgets for quotes), and to many problem-solving situations – it empowers the student.

As the diagram on the right shows, YDM is in the **empowering–portable/structural** quadrant. Its aim is to reveal mathematics as a connected structure that provides students with portable knowledge that is theirs and not reliant on memory of a rule. YDM can provide many activities that effectively enable this type of mathematics learning.



1.3 Mathematics teaching and school–community connections

We continue looking at YDM with respect to the outputs for the program: YDM’s position regarding mathematics teaching and school–community connections.

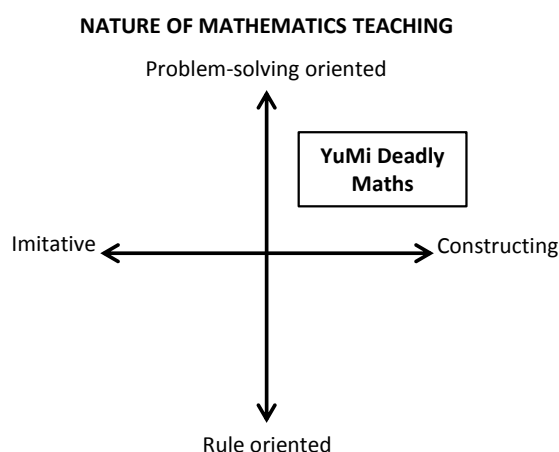
1.3.1 Mathematics teaching

For mathematics teaching, the two dichotomies are **imitative–constructing** and **rule–problem**. From a cognitive perspective, imitative teaching is the traditional textbook exposition teaching where a worksheet or a textbook page of exercises is provided, the teacher shows how the first example is completed, then the teacher works through one or two more with the students, and finally the rest are given to the students to be done by **imitating** the teacher’s process (the simplistic “I do, we do, you do” exposition approach) while the teacher wanders, checks and helps. On the opposite end, constructing teaching focuses on providing experiences from which the students can construct their own knowledge in a context where discussion with teachers and peers leads to development of language and symbols (that is, social constructivism). Imitative teaching leads to only being able to reproduce procedures when specific examples are provided; students are often confused by small changes in the form of presentation of examples. Constructing requires new knowledge to be accommodated, and partly generated, within students’ existing knowledge and leads to ownership, flexibility and meaning.

From a social perspective, rule-oriented teaching emphasises recall of ideas as definitions, and learning of procedures as rules; it provides collections of rules and procedures to be learnt by repetition. Against this, problem-oriented teaching or teaching by inquiry starts from real problems (from the perspective of the student), acts out and models these problems looking at a variety of models and strategies, develops the mathematical language and symbolic activities that will solve the problem, connects this new mathematics knowledge to appropriate existing knowledge, finally translating the solution back into the problem situation from whence the teaching process started.

An example for both dichotomies is the addition algorithm for two digits (e.g. $48 + 24$). Imitative and rule-oriented teaching begins by taking one process (usually the most efficient) and describing and showing how it works; for example, separate ones and tens, align ones and tens, add ones, rename any sets of ten ones as one ten, add tens plus any extras due to renaming, and represent final number of tens and ones as a numeral. Then it encourages the students to practise the procedure until they can do it accurately. Problem-based and constructing teaching begins by finding a problem context (e.g. money – spending \$48 and \$24, length – travelling 48 km and then 24 km) and acting out/modelling the problem often in more than one way or with more than one strategy. The following three are all useful: (a) *separation* (e.g. adding \$1 coins and \$10 notes and changing ten \$1 coins to one \$10 note if needed); (b) *sequencing* (e.g. moving along number line from 0 to 48, moving a further 20 to 68 and a further 4 to 72); and (c) *compensation* (e.g. add 2 to 48 to make 50 and compensate by subtracting 2 from 24 to make 22, then $48 + 24 = 50 + 22 = 72$). Modelling these strategies would be followed by activities to: (a) construct a way of both interpreting problems and undertaking one or more approaches/strategies to get the answer; (b) practise what has been constructed; (c) connect the whole-number form of the strategy to other uses of it; (d) apply the strategy back to the starting problems; and (e) see if it can be extended to more advanced mathematics (e.g. $348 + 424$).

The interesting point to note in this example is that the models and strategies are more important than the calculations. These three approaches/strategies are all useful as big ideas that recur across the years of mathematics and are used in many contexts: (a) separation can be applied to algebra (e.g. $23 + 41$, which can be separated into $20 + 40 = 60$ and



$3 + 1 = 4$, means that $2x + 3y + 4x + y = 2x + 4x + 3y + y = 6x + 4y$; (b) number-line modelling in sequencing can be applied to equations and leads to a powerful way to solve algebraic equations; and (c) compensation leads into equivalence of expressions (e.g. $a + b = a + k + b - k$).

As the diagram on the previous page shows, YDM is in the **problem-oriented/constructing** quadrant. Its aim is to teach students to solve problems through revealing the structure of mathematics. YDM can provide many activities that effectively enable this type of mathematics learning. Such revelation is best done in an inquiry mode.

1.3.2 School–community connections

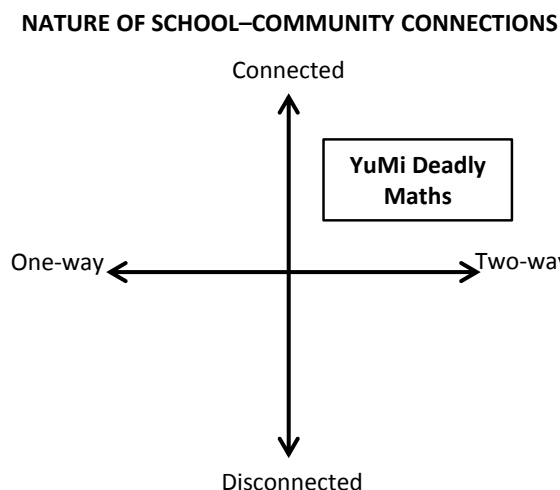
For school–community connections, the dichotomies are **one-way–two-way** and **disconnected–connected**. From a cognitive perspective, schools and their mathematics programs can relate to the community as if they are the experts, always operating the knowledge flow from school to community; or they can see themselves in a collaboration or co-construction where there is mutual learning and a two-way flow of knowledge. From a social perspective, schools can be disconnected from their community, operating in their own world and their own way within the school fence and seeing themselves as independent from the community (e.g. a world within a world); or they can be connected, with strong relationships between school, students, parents/carers and community members, and seeing themselves as part of the community, allowing the community access and some control over the schools' facilities.

An example for both dichotomies is with respect to the library. Schools can see the library as theirs and to be used by students in class time only, remaining neat, tidy and unused outside of school hours and disconnected from the community; or they can see the library as a community resource, to be organised so that it is used by all members of the community, inside and outside of school hours, and is set up as a place for community members to gather and meet (e.g. Cherbourg State School's Community Centre). This uses the library to connect school and community. However, even more than this is possible, as the above is still one-way. What if the library started to gather the stories of community members, to become a place in which the community stored and shared **its** knowledge? Students could gather examples of community mathematics knowledge among other stories, and these could be added to the collection. In this way, the library is **two-way strong**.

This involvement with, or connection to, community has to also be part of school education with respect to mathematics. For example, looking at community methods for measurement, or bringing in a tradesperson to show how they do mathematics in his/her trade, or looking at activities (e.g. fixing the road, building a park) or events (e.g. catering for a celebration) in the local community. In the past, this was particularly difficult for teachers in Indigenous communities. In a project run by YDC staff in the early 2000s, teachers were requested to develop a mathematics unit of work in which they worked with community members (in this project, Elders of Indigenous communities) to plan and teach the unit. Very few teachers (less than 25%) were willing and able to attempt the task, even with support of YDC staff.

The problem was that they believed Indigenous culture had very little mathematics that could be used. However, Indigenous communities are strong in mathematics in terms of pattern and relationship.

As the diagram on the right shows, YDM is in the **connected/two-way** quadrant. Its aim is be part of a connected school which values community knowledge and welcomes community members into school to share their knowledge. YDM can provide examples of how this is one of the most effective ways for Indigenous and low SES students to learn, including for mathematics.



1.4 Overview of this book

This book now describes and explores YDM in terms of the perspectives above: mathematics, mathematics learning, mathematics teaching, and school–community connections. It also looks at how YDM can be implemented in schools and as such explores aspects of professional learning, classroom trials, teacher change and action research.

As we will see throughout this series of YDM books, when things are mathematically similar they are also able to be taught with similar techniques and materials (they are educationally similar). This makes learning easier for students because they are working with and reinforcing familiar patterns, as well as providing deeper insights into connected mathematical structures. For example, whole numbers and decimals have similar patterns and reasoning and can be taught with similar materials; rate, ratio and percent are also similar concepts and can all be taught and represented with double number lines.

Thus, in this book, mathematics and mathematics learning are joined into one chapter called philosophy, learning and culture; mathematics teaching is subsumed under pedagogical framework; and school–community connections is looked at in terms of school change and leadership. The chapters of this book are:

Chapter 1: Beliefs and imperatives

Chapter 2: Philosophy, learning and culture

Chapter 3: Pedagogical framework

Chapter 4: School change and leadership

Chapter 5: Implementing YDM

This book is designed to introduce YDM, which is used in many YDC projects. Most people will receive this book because they are in a YDC project. However, because of its generic nature, this book will not give details of particular projects. Thus, this book may be accompanied by a small handout which describes how YDM is being used in the particular project in which you are involved. The handout gives information on how YDM is being implemented, gives dates of professional learning inputs (if appropriate), and describes the online support.

2 *Philosophy, Learning and Culture*

This section looks at how mathematics and learning is treated within YDM. It begins by describing the ontological and epistemological framework on which YDM is based, and then looks at the three features of this framework, creativity, symbols, and cultural bias, from a learning context.

2.1 Philosophical framework

YDM's philosophy was developed by exploring the connection between culture and mathematics for two reasons: (1) to value the cultural capital students bring to the classroom; and (2) to challenge the Eurocentric nature of Australian school mathematics. To achieve this, we had to ask the fundamental question: What is mathematics?

The framework in Figure 2.1 (adapted from Matthews, 2006) encapsulates YDM's view on this fundamental question of the ontology and epistemology of mathematics whereby mathematics starts from observations in a perceived reality. The observer chooses an aspect of a real-life situation (represented by a grey circle in Figure 2.1), and then creates an *abstract* representation of the real-life situation using a range of mathematical symbols. The observer uses the mathematics in its abstract form to explore particular attributes and behaviours of the real-life situation and to communicate these ideas to others. To validate, extend and apply this mathematics, the observer *critically reflects* on their mathematical representation to ensure that it fits with their observations of reality, to see if extensions and modifications can be made to further generalise the mathematics and to transfer the mathematics to solve other similar life problems.

Although the above description is complete in terms of the creation of mathematics, it is missing one crucial aspect in its relationships, and this is the interplay or dichotomy between reality and mathematics. Our perceptions of reality are inexact unlike the mathematics that emerges from abstraction from that reality. Thus, reality and mathematics are two ends of a spectrum – inexact to exact and back again. The crucial point here is that mathematics topics must be seen in both worlds. It is important that mathematics be understood in reality, and seen in terms of error, uncertainty and diversity, as well as a logical structure. This is obvious for probability, statistics and measurement, but it also has to apply to arithmetic, algebra and geometry. Yes, mathematics is a pure construction of the mind but it is also of useful application in the world.

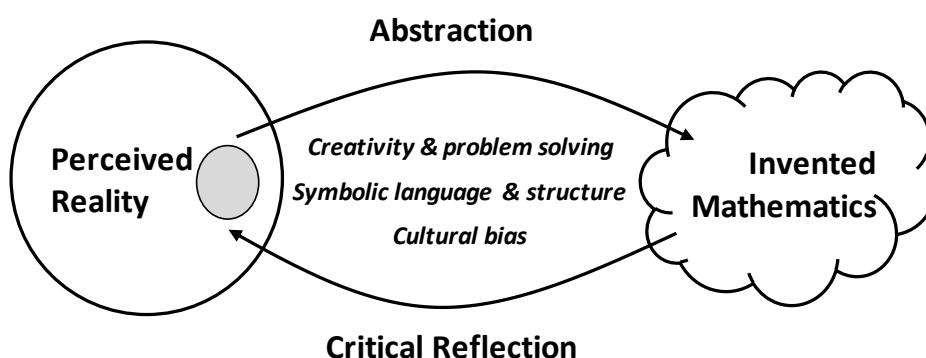


Figure 2.1 Relationship between perceived reality and invented mathematics (adapted from Matthews, 2006)

As shown in Figure 2.1, there are three other features of the model that are a consequence of the cycle from reality to mathematics and return. Both abstraction and reflection are *creative and problem-solving* acts; mathematics as a language and structure is built around *symbols* which carry concepts, strategies and relationships from reality to the abstract and back to reality; and the mathematics and how it is used in reality is framed by the *cultural bias* of the person creating the abstraction and reflection.

2.1.1 The creation of mathematics

The abstraction and critical reflection processes form an important cycle (thesis–antithesis–synthesis) with perceived reality and invented mathematics. It is through this cycle that mathematics knowledge is *created*, *developed* and *refined* and it is through this cycle that mathematics is applied and used to *solve problems in reality*. Mathematical knowledge is created (the thesis) by abstraction from perceived reality. This knowledge is trialled within itself for consistency (proof) and against reality for effectiveness (application). Problems that emerge in proof or application (the antithesis) are used to amend the mathematics (the synthesis) and the cycle continues. However, equally important with consistency and proof is effectiveness and application. We learn mathematics for the beauty of its patterns and relationships and to understand our world.

The act of abstraction requires the learner to generalise a mathematical idea from examples in the world to symbols in the invented world of mathematics. It means that the learner has to move from reality to symbols, for example, connecting the real-life situation of three children joining two children to make five children with the symbols $2 + 3 = 5$. The recommended way to do this is to move through a sequence of representations of the mathematical idea from reality to abstract (as in Figure 2.2). The representations can be external (real-world activities, materials, images, pictures, language and symbols,) or internal (mental images of external representations), with learning occurring when structural connections are made between the two (Halford, 1993). The external representations facilitate the internal representations while accompanying language and actions become increasingly abstract (as in Figure 2.2).

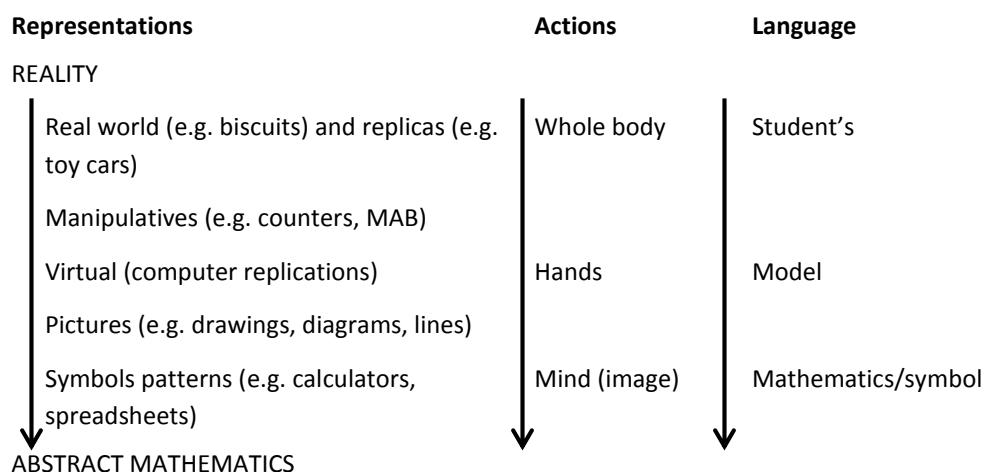


Figure 2.2 Abstraction sequence from reality to mathematics

Reflection is similarly more powerful than it seems at first glance. It requires the learner to validate their mathematics learning against their everyday life, thus generating ownership of the knowledge. However, it is also a method of extending learning as the reflection acts on the abstracted mathematics in relation to reality. For example, students can reflect on $3 + 4 = 7$ and see that if one addend, say the 3, was reduced by 2, then the sum, 7, has to be reduced by 2 to keep the equation equal, the beginning of the balance rule. The extension of knowledge through critical reflection can be assisted by the use of four generic strategies: flexibility, generalising, reversing and changing parameters.

Four generic strategies (adapted from Baturo, Cooper, Doyle, & Grant, 2007)

1. **Flexibility.** This strategy considers other ways to reach the same idea, different ways to think about what has been learnt, and unusual applications of the idea (e.g. **non-prototypic** examples). For example, 61 is 6 tens and 1 one but it can also be thought of as 7 tens less 9, half a hundred plus 11, one hour and one minute, and so on.

2. **Reversing.** This strategy takes the process by which the idea was learnt and reverses it. For example, the idea of “two-thirds” is built by taking a thing as a whole, partitioning it into three equal parts and considering two of the parts; reversing is to take a thing as two-thirds and to construct a whole.
3. **Generalising.** This strategy looks for ways to make the idea more general. For example, if 4 tens and 6 ones is “forty-six” (fourty-six), then 1 ten and 6 ones should be “onety-six” rather than “sixteen” and fred tens and henry ones should be “fredty-henry”.
4. **Changing parameters.** This strategy assists generalisation and considers what happens if something in the idea is changed. For example, removing 2 from the left hand side of $3 + 4 = 7$ and considering how to keep the equation true leads to balance and compensation principles (see *Mathematics as Story Telling* in subsection 2.2.2).

YDM is designed for students to experience both abstraction and reflection. In traditional classrooms, many students only experience mathematics in its abstract form (i.e. they start and end within the mathematics cloud of Figure 2.1). Other students only experience the top half of the philosophical framework and do not reflect back to reality. All these students do not experience nor obtain an appreciation for the cycle of abstraction and reflection. Pedagogy that is centred on the cycle should lead to an authentic and deep mathematical literacy and allow students to achieve at a high standard in relation to their local culture and life.

2.1.2 Creativity, symbols and cultural bias

Creativity and problem solving, symbols as language and structure, and cultural bias are features of the model in Figure 2.1 that are a consequence of this figure’s Reality–Abstraction–Mathematics–Reflection cycle.

The first feature, **creativity and problem solving**, is particularly evident in the abstraction and critical reflection cycle. In particular, it is important to note that **creativity** makes this cycle similar to other artistic pursuits such as dance, music, painting and language as a different form of abstraction. Therefore, mathematics can be considered as another art form and, in theory, to be related to these other forms of abstractions. In essence, it is possible to develop empowering pedagogy that allows students to be creative and express themselves in the mathematics classroom. This would allow students to learn mathematics from their current knowledge (i.e. from the students’ social and cultural background), thereby providing agency through creativity and ownership over their learning.

Problem solving, on the other hand, represents the more directed and controlled aspect of creativity. Nevertheless, its role is important because it leads to an important way of perceiving mathematics, that is, as a tool for problem solving. This has become central to YDM, that is, to argue that the goal of mathematic learning is to empower students to use mathematics to solve problems in their lives.

The second feature, **symbols as a language and structure** are a product of the abstraction process. Symbols and their meanings are important features of the model since they connect the abstract representation with perceived reality. However, it is common that students do not make these connections easily and view mathematics as just sums (symbolic exercises) with no real meaning. This is further exacerbated for students when they first learn algebra, and letters are suddenly introduced into mathematics without any obvious reason except that we are now learning algebra.

Interestingly, focusing on creativity within mathematics provides the opportunity for students to generate their own symbols to represent their understanding of the mathematical process. These symbol systems can then be compared to and assist in understanding the meanings of standard symbols, symbolic **language** and their connection to reality. In addition, this can also lead to the teaching and learning of the **underlying structure** of mathematics, providing students with a holistic view of mathematics.

The third feature, **cultural bias**, exists in all aspects of the abstraction and critical reflection cycle. The observer expresses their cultural bias in the way they perceive reality and decide on which aspect of reality they wish to

focus. In the abstraction process, the form a symbol takes and the meanings that are attached to this symbol or group of symbols is biased by a cultural perspective. Finally, the critical reflection processes are underpinned by the cultural bias within the abstraction process and the observer's perception of reality. If we have an understanding and appreciation of the cultural bias within mathematics, new innovative pedagogy can be developed that moves beyond some cultural biases so that students can relate to mathematics but also gain a deep understanding for the current form of mathematics and how mathematics is used.

2.2 Creativity and problem solving

This section takes the first feature of the philosophical framework, creativity and problem solving, and looks at its role in learning within YDM. Therefore, to start we look at a set of learner-centred principles of mathematics learning.

2.2.1 Learner-centred principles

These principles (see Table 2.1) were developed from a review of the literature (Alexander & Murphy, 1998) concerning principles that research indicated enabled excellence in instruction. According to Alexander and Murphy, these learning principles act as invaluable tools for those who seek to understand and facilitate the learning of others. These principles are one of the bases of YDM with Aboriginal, Torres Strait Islander and low SES schools in the last 15 years.

Table 2.1 *Learner-centred principles (Alexander & Murphy, 1998)*

Dimensions of learning	Learner-centred principles
Knowledge base	One's existing knowledge serves as the foundation of all future learning by guiding organisation and representation, by serving as a basis of association with new information, and by colouring and filtering all new experiences.
Situation/context	Learning is as much a socially-shared knowledge as it is an individually-constructed enterprise.
Development and Individual differences	Learning, while ultimately a unique adventure for all, progresses through various common stages of development influenced by both inherited and experiential and environmental factors.
Strategic processing	The ability to reflect upon and regulate one's thoughts and behaviour is essential to learning and development.
Motivation and affect	Motivational or affective factors, such as intrinsic motivation, attributions for learning, and personal goals, along with motivational characteristics of learning tasks, play a significant role in the learning process.

These principles indicate some areas in which YDM's philosophical framework may need attention. The principle "knowledge base" is covered by the Reality component of the framework which ensures that the teaching of the new mathematical idea begins from the existing knowledge of the student, both local knowledge and mathematics knowledge. "Situation/context" is not explicitly covered by the framework but should be part of the Abstraction and Reflection processes. "Development and Individual differences" is taken into account by the framework in terms of the activities that are the basis of each of the components. "Strategic processing" is another that is implicit in the framework, particularly in the reflective generic strategies – see section 2.1.1. "Motivation and affect" is a very important part of YDM as is evident in many sections of this book.

2.2.2 Experiencing creativity and problem solving


Experiencing creativity

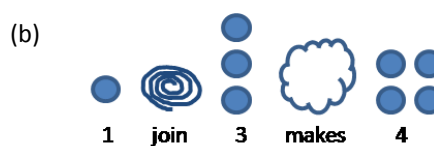
Analyses of the activity of mathematicians has continually equated them to poets, that is, the act of creating mathematics can be regarded as similar to the act of writing poetry. This can be seen in the words of mathematicians who refer to the best arguments not as correct or excellent but as “elegant” (which means correct and beautiful). In modern learning of mathematics, **creativity** has been largely lost.

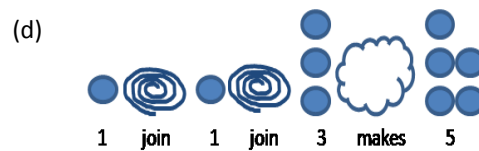
YDM would like to bring creativity back into mathematics and sees its role within abstraction and reflection. As shown in 2.1.1, the **four generic strategies** used to extend understanding are all creative. And if we give students licence to invent, the act of abstraction can be creatively simulated in the classroom – see Mathematics as Story Telling below.

Mathematics as Story Telling (Matthews, 2006)

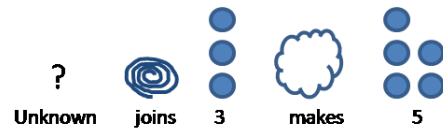
The basis of the Mathematics as Story Telling (MAST) activities is the need to teach students the role of mathematics symbols in telling stories and the creativity that brought about these symbols. MAST activities that develop understanding of symbols through allowing students to creatively invent their own symbols and use these to introduce big ideas tend to have seven steps like the example below.

1. **Step 1: Symbols in general.** Students explore how symbols can be assembled to tell a story, first in Indigenous situations (e.g. Indigenous art) and then creating and interpreting symbols for simple actions (e.g. walking and sitting at a desk).
2. **Step 2: Exploring the mathematical story** (*here simple addition*). Students act out a story (e.g. 2 students join 3 students to make 5 people). Discussion identifies story elements – objects (the 2, 3 and 5 people) and actions (joining, making).
3. **Step 3: Creating own symbols for the story.** Students create their own symbols to tell the story. They first do this free style (a drawing that represents the “joining” and the “making”) and discuss results (e.g. are they linear – showing the action left to right, or are they more holistic)? Secondly, the students create symbols in a more structured and linear setup (students use objects for students and drawing for “join” and “make” or “same as”), as in the example on right. Here the “join” picture is a vortex that picks up the 2 + 3 and the “making” picture is a cloud that brings them down together as rain.
4. **Step 4: Sharing symbols.** Students share symbols and explain their symbols’ meanings. Students then use other students’ symbol systems to represent other stories (e.g. 4 people join 7 people), and make up stories where other students’ symbols are used to represent addition.
5. **Step 5: Story modification** (*here introduces balance and compensation and adds further creativity*). Teacher removes one counter from the left hand side two counters and asks if story still true. Most students will say no. Teacher then asks how it can be made true again. The normal answers are (as below): (a) to put the counter back, (b) to remove a counter from right hand side (the balance principle for equations), (c) to add a counter to the three counters (the compensation principle); and (d) draw another vortex on the left hand side and put a counter in front of it.





6. **Step 6: Extension** (*here to unknown*). Teacher sets up story: “unknown number of people join 3 to make 5”. Students create their own symbol for unknown as on right. Students use balance principle to find unknown.
7. **Step 7: Introducing the formal symbols** (*here + and =*). Teacher introduces students to common formal symbols for join (+), results (=) and unknown (x) e.g. $2 + 3 = 5$ and $x + 3 = 5$. Begin to relate these symbols to everyday situation and to solve for unknown.



Inventing symbols (e.g. students could be asked to invent their own way of identifying the ones in decimal place values – e.g. circle it), inventing language (e.g. students could be asked to invent their own words for halves, thirds, quarters, fifths and so on), and inventing systems (e.g. students could be asked to make a number system for 12 fingered “Anastasians”, or invent a way of counting on one hand) are all ways students can be creative. The generic strategies of reversing (e.g. for a symmetry to shape activity, to follow the normal shape to symmetry activity, the students could be asked to construct a complex shape/design that has five lines of symmetry – i.e. not a regular pentagon) and flexibility (students could be asked to construct an uncommon shape and divide it into 100 equal parts that would be difficult for students to shade 25%) are also effective.

Experiencing problem solving

When it began in Greece, mathematics was the word for “the thinking that solves problems” and referred to a much wider set of experiences than it does today. When the first university insisted on mathematics training as a prerequisite, it did not refer to the calculations of which most of modern school mathematics is composed, it meant ability to solve problems. As argued by Wilson (1978) (cited in Ashlock, Johnson, Wilson, & Jones, 1983), mathematics started as problem solving but was then colonised by the procedures useful in problem solving until these procedures replaced problem solving as what mathematics was. YDM wishes to bring problem solving back as the start and the end of mathematics – to the point that being able to use mathematics in everyday life to solve problems is YDM’s aim for mathematics.

This means that it is crucial that mathematics be understood in terms of everyday life. Since the abstraction act removes context from everyday situations and reduces description to brief and concise symbolic patterns and relationships, this means time ensuring that students can go both ways – life to symbols and symbols to life. Thus, YDM **starts with real problems and ends with real problems** and, in between, ensures that there is a lot of problem solving at real and abstract levels.

Problem solving is “what you do when you do not know what to do”. Thus, what problems are depends on the problem solver (e.g. “24 heads and 34 feet, how many chickens and how many rabbits?” is a problem for many but not for those who know simultaneous equations where it is an exercise).

Problems are of two types: (a) those that are based in a domain of mathematics – **routine** problems (e.g. word problems or algebra problems); or (b) those that have no domain of knowledge – **creative** or non-routine problems (e.g. matchstick puzzles). Routine problems are solved most effectively by rich schematic knowledge of the domain of the problem; creative problems are solved most effectively by thinking skills and strategies.

As well, no-one solves problems without some failures, so positive affective traits are necessary. On top of this, problems are exercises if they can be solved by algorithmic procedures – problem solvers need strategies or general “rules-of-thumb” that point towards a solution (e.g. act it out) to help them work out what to do. Overall, problem solving requires so much more knowledge and affects than exercises.

To prepare students for both types of problems, and those in between, it is necessary for students to learn:

- (a) **mathematics content** as rich schema with important parts automated (automated knowledge can be used with no increase in cognitive load);
- (b) **positive affective traits** (e.g. motivation, perseverance, “have-a-go” attitude, strong self-concepts to handle failure);
- (c) a strong repertoire of **problem-solving strategies** (covering visual, language, checking, patterning and restructuring);
- (d) a strong set of **thinking skills** (visual, logical, flexible, creative, and patterning); and
- (e) **metacognition** (the ability to be aware and control own thinking; to plan, monitor, evaluate [includes checking and validating] and make decisions) – see subsection 2.2.3.

Note: A **Problem-Solving resource** based on the above has been developed for YDM and is available via the YDM Blackboard site that can be accessed by schools involved in YDM training.

2.2.3 Inquiry approach, investigation, and multi-representations

There are learning approaches that act to enhance creativity and problem solving. YDM advocates three of these in particular – inquiry, investigation, and multi-representations. The inquiry approach is related to two other approaches, complicity and metacognitive metaphors.

Inquiry, complicity, and metacognitive metaphors

YDM emphasises legitimising local knowledge, creating representations, building toolkits and reflecting on knowledge. **One of the best ways to do this is to build a community of learning in the classroom using the inquiry approach.** At its best, the inquiry approach eschews the teacher as “sage on the stage” and, to a lesser extent, as “guide on the side” and advocates a new role of “meddler in the middle” (McWilliam, 2005). It involves co-opting the students to be complicit in their own learning, a major indicator of success found by YDC staff in projects with underachieving Aboriginal and Torres Strait Islander students.

Thus the **inquiry approach** is advocated as a way of implementing mathematics learning based on YDM’s philosophical framework when it: (a) co-opts the students as **co-constructors** of their knowledge (Claxton, 1999); (b) involves students in discussion and debate without the necessity of common conclusions; (c) ensures all teaching and discussion is culturally safe; and (d) assumes the highest expectations of teachers that the students will handle the work. The most recommended teaching approach for mathematics teaching is *social constructivism* with students constructing their own knowledge in relation to discussion with peers and teachers (Cobb, 1994; English & Halford, 1995). The inquiry approach facilitates social constructivism, making learning in the framework social sharing and meeting the “situation/context” principle of Alexander and Murphy (1998).

Two examples of **complicity** are worth considering. In the first, from three years ago, the teacher gave a pre-test and shared the results with the class. As they studied the unit of work, he gave weekly small tests so the students could see and how much they were improving from the pre-test. He placed the weekly result as a bar graph at the front of the room. In the end, the students were working with the teacher to get the bar graph up to the highest level possible. At the end of the unit, the class performance was well above previous years. In the second example, happening now, all the teachers of a school have taught their students the pedagogical framework from Chapter 3, a framework that is based on Figure 2.1. The students have been taught that they will start from their reality, abstract the ideas to mathematics and reflect back to reality. The idea is that the students will be able to follow and undertake the pedagogy themselves. This should be powerful for the students because it makes them aware of their own learning – see subsection 2.2.4).

The last example of complicity is also an example of the approach of using **metacognitive metaphors**. In order to co-opt students as co-constructors of knowledge, YDM has found that it is necessary to provide teaching that enables students to learn how to understand themselves as learners, and to involve the students in the project as **co-researchers**. This is facilitated by instruction that provides models of learning as acquisition of knowledge (psychology), social negotiation (sociology) and data gathering (research). The models of learning taught to students are called metacognitive metaphors; they enable students to monitor and measure their own learning and to regulate their behaviour and learning actions (see subsection 2.2.4). Instruction that involves metacognitive metaphors allows learning to meet the “Strategic processing” principle of Alexander and Murphy (1998).

Open investigations

The pedagogy suggested by the YDM philosophical framework lends itself to an open investigative approach to mathematics where students explore an idea using creativity and problem solving to learn as much as they can. Investigations in mathematics can make lessons engaging, use relevant and meaningful contexts, use technology, develop essential skills, make it concrete and support the students to think, use the benefits of group work and those notions of multiple intelligences using kinaesthetic and visual thinking, and cater for the huge diversity of student interest and abilities while developing genuine understandings (not just provide rules without meaning).

Traditional mathematics has often involved routine “closed questions” such as $5 + 6 = \square$. An “open question” such as $\square + \square = 11$ leads to more possibilities and is often non-routine in nature. An investigation for example would go one step further and ask, “How many possibilities are there and how do you know when you have them all?” As Afzal Ahmed, one time professor of mathematics at Chichester, UK, once quipped: *If teachers of mathematics had to teach football, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

The use of open-ended investigations serves a dual purpose in that they can develop elements of the Australian Curriculum Proficiency strands such as understanding, fluency, problem solving, and reasoning as well as skills and content – both at the same time. Openness implies multiple entry and exit points allowing for mixed ability and students at varying levels to bring their own mathematics to the investigation. They also illustrate more closely the “true” nature of mathematics.

The philosophical framework of this section and the pedagogical framework of the next section are a lens that allows teachers to create investigations and enrich and adjust mathematical tasks by asking a series of questions such as:

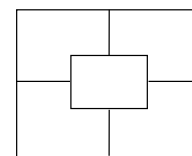
- Can I link the task to **reality**?
- Have I got an **interesting and meaningful context**?
- Can I make it **personal** for the students?
- Could I make it a **cooperative group challenge**?
- Can I provide choice and allow **student self-responsibility** and **ownership**?
- Can I embed the learning within a **story**?
- Can I use the **outdoors**?
- Can I involve students **physically** (kinaesthetic learning)?
- Could I use a **simulation role play**?
- Can I make it **concrete**?
- Can I exploit the **use of technology**?
- Can I use the **visual** aspects?
- Can I make the task **open-ended and investigative**?
- Can I make it a genuine **problem solving challenge**?
- Can I allow for **multiple entry and exit points (differentiation)**?
- Have I got **something for everyone**?
- Do the **symbols** the students are required to use have meaning in reality?

As teachers gain confidence, the progression to include and develop investigative type activities will become part of their everyday repertoire.

Multi-representations

This approach involves planning lessons with many representations that involve continuously linking representations. Representations are ways of considering mathematics ideas and include language, symbols, graphs, drawings, virtual materials, physical materials and acting out the idea in the real world, that is, all the components of Figure 2.2.

For example, students can count verbally “twenty-two, twenty-three, ...” and so on while, at the same time, adding bundling sticks onto a Tens/Ones chart, recording the number on paper, and adding one on a calculator. Duval (1999) argues that mathematics comprehension results from the coordination of at least two representational forms or registers; the multifunctional registers of natural language, and figures/diagrams, and the mono-functional registers of notation systems (symbols) and graphs. He further contends that learning is deepest when students can integrate registers or integrate representations. YDM uses a material called a **thinkboard** which is an A3 sheet divided into five regions – one for symbols, one for language, one for drawing, one for materials and one for a real-world story (as on right).



2.2.4 Metacognition

There is one extra attribute of creativity and problem solving – it lies in what is needed to relate mathematical ideas to applications. This relation requires students to know **when** to use an idea as well as what idea to use and how to use the idea. The basis of this “when” is **metacognition**, that is, awareness and control of own thinking particularly with regard to the ability to plan, monitor, evaluate and make decisions with regard to progress on an application of a mathematical idea.

Metacognition for application of knowledge can be enabled with two teaching strategies, namely, match-mismatch (Charles & Lester, 1982) and metacognitive training (Brown, 1995). **Match–mismatch** involves three explicit teaching steps: (1) teach the knowledge; (2) teach when to use the knowledge (i.e. for what problem types); and (3) provide experiences that match and mismatch situations (first providing tasks on all of which the idea can be used, and then tasks on only some of which the idea can be used, to give the students experience of “when”). This means that the three steps involve teaching the mathematics idea, applying the mathematics, and then determining when to apply the idea (as the idea is useful on only some of the activities).

Metacognitive training involves directly teaching students models of how learners think and to use these models to keep a record of their thinking (with the view of enabling them to be aware of and control their thinking). The training can be supported by activities, namely “random reporting”, “think–pair–share”, social interaction roles, and reciprocal teaching.

Metacognitive training activities

Four important activities that can enable students to improve their metacognition are as follows.

1. **Random reporting.** Problems are given to groups to solve. The group is told that, when they are finished, any member of the group could be chosen to report, so they have to ensure all members understand the problem and solution.
2. **Think–pair–share.** Give students a problem which has more than one possible way to solve it and ask them to work alone. After a time, put into pairs and ask them to listen to each other’s answers and then come to a common conclusion. A little further along, put students into groups of four and ask each group to come up with a common best answer. Students’ arguments about who are right make students aware of their thinking.

3. **Social interaction roles.** Place students in groups and assign a role to each member. One recommended way is groups of three with one student designated as leader (they make the decision if there is dissension), one as recorder (they write down and report the group's findings – only they can use a pen); and one as checker (they check the others' thinking). Change the roles for different problems. Ensure students stay in roles. The delineation of the roles causes all to consider the others' and their own thinking.
4. **Reciprocal teaching.** Students take turns to be the teacher of their group. The teachers are taken out and given the task they have to teach along with teaching hints. Students have to prepare a plan to ensure all students in their group learn. The role of teacher requires students to consider their own and other students' thinking.

2.3 Symbolic language and structure

The second feature of invented mathematics is that the abstracted form of the mathematics is written in symbols which are both a language and a structure. This section will look at this in terms of learning. It will cover what it means to be a language and a structure and then look at important components of this structure, big ideas, sequencing and connections

2.3.1 Symbols, language and rich schema

This subsection looks at symbols as language and introduces the notion of accessible knowledge and its relation to rich schema (or structured/relational knowledge).

Symbols as language

It is very important to understand that symbols are a shorthand language to tell stories about the world. For example, experiencing two objects and three objects joining to make five objects for many different types of objects leads to seeing that twoness plus threeness leads to fiveness and to the symbols $2 + 3 = 5$. These symbols, $2 + 3 = 5$, is the abstraction of the activity with the objects. It has gained something through the abstraction – it is much more powerful in that $2 + 3 = 5$ works for any objects of any measures, it is not restricted to the objects from which it was abstracted. However, it has lost something – meaning that it is not related to the life of the learner. Thus, this has to be given back so that the students can experience both the power of the symbols and the meaning associated with the symbols telling stories.

For algebra, it is even more extreme because, for arithmetic, numbers are abstracted from objects and then letter as variables are abstracted from numbers – a double abstraction. So this leads to many activities of students translating real-world situations to symbols and then translating symbols to real-world situations. For example, three boats going fishing and each catching five fish but leaving two behind when they gathered at the shore is $3x - 2$; and $3x - 2$ in shopping is buying three boxes of chocolates and getting a discount of \$2 for buying three boxes.

Seeing symbols as a language really helps understanding of mathematics in its symbol activities as well as in its applications. For example, solving $3x - 2 = 13$ is easier if you think of it as a story. For example, three car loads of people came to pick fruit and two people left; this left 13 people picking fruit; how many in each car? One can backtrack the story – before the two left, there had to be 15 people which is five per car.

At all levels of the journey from concrete objects to abstract generalities, students need the big ideas to be connected to reality (real-life contexts) so that their mathematics learning: (a) retains meaning and relevance for them; and (b) remains accessible and useful as a problem-solving tool for everyday activities and applications. Decontextualised abilities to manipulate algebraic entities and equations are meaningless if a student cannot see the real-life application of generalised mathematics.

Accessible knowledge

There are two types of mathematics knowledge – the first is **available** knowledge and this is what students use when they are asked to recall their knowledge; the second is **accessible** knowledge and this is what students use when they successfully apply their knowledge in a different context to where they learnt it. Obviously, accessible knowledge is the focus of YDM.

Teaching for accessible knowledge requires more than teaching for available knowledge. Accessing mathematics knowledge means knowing it on its own merits and knowing **when, where and how** it can be applied to task situations. Thus, accessible knowledge is based on mathematical ideas being held in **rich schemas**.

Rich schema

Rich schema is the name for knowledge being held in networks of nodes with connections between nodes that show similarities and differences. Thus, rich schemas are knowledge held as relational knowledge. Such schemas have four characteristics:


- (a) they *define* – completely describe all the ways the mathematical idea can be thought of;
- (b) they *connect* – store knowledge so that all relationships are evident and all related knowledge is connected;
- (c) they *apply* – contain all the different ways the mathematical idea can be used in reality; and
- (d) they *remember experiences* – organise and store all experiences students have had with the mathematical idea.

YDM has been designed to be effective in developing rich schema and, therefore, accessible knowledge. It involves abstraction and reflection and begins and ends with reality thus ensuring mathematical ideas are well defined, connected, applied and experienced. It identifies with techniques particularly useful for accessibility such as explicitly relating the mathematical ideas to as wide a collection of out-of-school experiences as possible and teaching that mathematics and its symbols form a highly connected structure, a language to describe the world and a set of tools for solving the world's problems.

In particular, the **generic strategies** (see subsection 2.1.1) are useful; *flexibility* gives the students a rich set of applications (particularly non-prototypic activities), *generalising* integrates knowledge which means there are less things to apply, *reversing* often means that real-world instances are constructed (the best way to teach interpreting the world), and *changing parameters* builds connections. Overall, the secret to accessible knowledge is to do **both sides** of the philosophical framework – to *abstract* the mathematical idea and *practise* the idea but not to stop there, to draw a breath and move on to *connections and reflections* (see Section 3).

2.3.2 Big ideas

Big ideas are mathematical ideas that apply across many levels of mathematics (i.e. across many years of school mathematics) and many topics of mathematics. For example, knowing that $2 + 3 = 3 + 2$ is an idea that just grows with the mathematics, showing how the big idea called the commutative principle recurs across many levels and types of mathematics:


$$\begin{array}{l} 2 + 3 = 3 + 2 \\ 367 + 2012 = 2012 + 367 \\ 4.65 + 23.8 = 23.8 + 4.65 \\ 34.2 \text{ m} + 27.9 \text{ m} = 27.9 \text{ m} + 34.2 \text{ m} \\ 2\frac{3}{7} + 4\frac{1}{6} = 4\frac{1}{6} + 2\frac{3}{7} \\ 5\text{h } 34\text{m} + 4\text{h } 56\text{m} = 4\text{h } 56\text{m} + 5\text{h } 34\text{m} \\ 2a + b = b + 2a \\ f(x) + g(x) = g(x) + f(x) \end{array}$$

Teaching based on big ideas requires two elements: (a) knowing what are the important big ideas; and (b) developing teaching and learning programs that build these ideas across Prep to Year 9. Big ideas (or mathematics principles) are relationships where the meaning is encapsulated in the way components are related, not the particular content that the components represent. For example, $2 + 3 = 5$ is a content relationship – 2 objects joined with 3 objects gives 5 objects. However, 2 tens + 3 tens are 5 tens is the beginning of the big idea that A of something plus B of something is A + B of the same thing. This idea grows across Prep to Year 9 as follows:

$$\begin{array}{l}
 2 \text{ tens} + 3 \text{ tens} = 5 \text{ tens} \\
 \frac{4}{7} + \frac{2}{7} = \frac{6}{7} \\
 3 \times 8 + 6 \times 8 = 9 \times 8 \\
 2 \text{ of any number} + 5 \text{ of any number} = 7 \text{ of any number} \\
 4x + 5x = 9x \\
 \downarrow \\
 ax + bx = (a + b)x
 \end{array}$$

There are a variety of big ideas. They can be understood in terms of five headings as follows.

1. **Global.** These are big ideas that hold for nearly all mathematics knowledge. For example, “Relationship vs Transformation” is the name of a global big idea that states that mathematics has three components: objects (such as people, chairs, numbers, equations, and so on), relationships between objects (e.g. 2, 3 and 5 related by addition $2 + 3 = 5$) and transformations or changes from one object to another (e.g. 2 changes to 5 by +3), and that every mathematics idea can be perceived as a relationship or as a transformation.
2. **Concepts.** These are big ideas around meaning. They include concept of number, concept of place value, concept of addition, concept of fraction and so on. Interestingly, many of these are composed of sub-concepts. For example, the concept of fraction has five sub-concepts – fraction as part of a whole ($\frac{3}{4}$ is a whole partitioned into 4 parts and taking 3 of these), fraction as part of a set ($\frac{3}{4}$ is considering a set of objects is one whole partitioned into 4 parts and taking 3 of these), fraction as a single point on a number line ($\frac{3}{4}$ is a point on the line between 0 and 1 that is 3 parts out of 4 along the line), fraction as division ($\frac{3}{4}$ is $3 \div 4$), and fraction as multiplier or operator ($\frac{3}{4}$ is multiplying by 3 and dividing by 4). Concepts tend to be related to particular topics (e.g. number, fractions) will be described in the modules where appropriate.
3. **Principles.** These are big ideas around properties or laws. They are relationships or changes that recur across the years of mathematics. For example, inverse shows how two things are opposite to, or undo, each other such as: forward–backward, right–left, clockwise–anticlockwise, expansion–contraction, addition–subtraction, multiplication–division, square–square root, $x^n - x^{-n}$ (multiplication), $x^n - x^{1/n}$ (power), differentiation–integration, and so on. Thus, principles are predominantly independent of context or topic and thus the present list (still being upgraded) will be described here. (*Note:* Some principles are repeated in different forms for different strands – e.g. inverse relation applies to subtraction, division, fractions and measurement).
4. **Strategies.** These are big ideas that show the way towards answers. They include the ways in which problems and examples are solved. They consist of generic strategies such “make a drawing” or “break into parts”; but they also cover strategies related to strands such as the three for addition which are “separate into parts, do parts separately, and combine”, “keep one section as is, and combine with parts for other section in sequence” and “do something simpler and then compensate”. Strategies are both generic and related to topics and will be described in the modules.
5. **Teaching.** These are different to the above; they are big ideas for teaching mathematics. The most obvious are related to the reality–abstraction–mathematics–reflection cycle. For example, reversing is a teaching big idea because it applies to nearly all situations.

In the books on how to teach mathematics that follow this book, big ideas are the starting points for instruction. If, in the early years, one can understand, even informally, the big ideas that overview a topic, then

learning of this topic becomes much easier as students move up the levels; that is, big ideas enable **acceleration**.

Note: A **Big Ideas resource** based on the above has been developed for YDM and is available via the YDM Blackboard site that can be accessed by schools involved in YDM training.

2.3.3 Connections and sequences

The basis of YDM is that mathematics should be taught so that it is accessible as well as available; that is, learnt as a rich schema containing knowledge of when and why as well as how. Rich schema has knowledge as connected nodes, which facilitates (a) recall, because it is easier to remember a structure than a collection of individual pieces of information; and (b) problem solving, because content knowledge that solves problems is usually peripheral, along a connection from the content on which the problem is based. This connectedness is enhanced if the sequences in mathematics teaching move along lines of connectedness. This means that, for example, division leads to fractions which lead to percent and ratio. In particular, the Mathematics component of the Reality–Abstraction–Mathematics–Reflection (RAMR) cycle is built around sequencing and connections.

Connecting and sequencing within and across topics

YDM is based on connections across and within mathematics domains. Knowing how these domains are connected enables teachers to draw on similar representations when sequencing work and to help students connect their new learning in the sequence to prior learning. For example, knowing that fractions and probability are structurally related through the part/whole concept helps teachers make better choices about sequencing probability materials and relating language from fractions to probability. Some important connections and sequences are below.

1. **Whole numbers, decimal numbers, integers, money and measurement.** This is because of the decimal relationship that adjacent place-value positions are related by $\times 10$ to the left and $\div 10$ to the right (e.g. 7 tens $\times 10 = 7$ hundreds; 70 cents $\times 10 = \$7$), and adjacent groups of three are related by $\times 1000$ to the left and $\div 1000$ to the right (e.g. 5 ones $\times 1000 = 5$ thousands; 23 mm $\times 1000 = 23$ m).
2. **Common fractions, percent, ratio and probability.** This is because they can all be considered in terms of parts and wholes, that is, common fractions are equal parts out of a whole (e.g. fraction $\frac{2}{3}$ is 2 equal parts out of 3 equal parts; probability $\frac{1}{6}$ is one chance out of 6 equal chances), percent is 100 equal parts out of a whole (27% is 27 equal parts out of 100 equal parts), and ratio is equal parts but part to part (e.g. ratio of cement to sand of 2:5 is 2 equal parts cements and 5 equal parts sand that makes up 7 equal parts of the whole).
3. **Addition, subtraction, multiplication and division.** This is because addition as joining and subtraction as separation are inverses, multiplication as joining of equal groups is repeated addition and division as separation into equal groups is the inverse of multiplication.
4. **Division, measurement with units and common fractions.** This is because all three separate into equal parts which means that they all have the inverse relation property (e.g. the bigger the divisor, the smaller the dividend; the larger the unit, the smaller the number of units; and the bigger the denominator, the smaller the fraction).
5. **Place value, measures and mixed numbers.** This is because they consist of different components where smaller components form larger components when there is sufficient of them (e.g. whole numbers have tens and ones with 10 ones equalling one ten; measures have, for example, hours and minutes with 60 minutes equalling one hour; mixed numbers have, for example, wholes and sixths with 6 sixths equalling one whole).

Connecting and sequencing across strands

Mathematics grew out of two views of reality: the first was number – the amount of discrete objects present; and the second was the world around us – the shapes and the spaces we live in. The basis of number was the unit, the one. Large numbers were formed by grouping these ones and small numbers (e.g. fractions) by partitioning these ones into equal parts. The operations of addition and multiplication, and the inverse operations of subtraction and division, were actions on these ones which joined and separated sets of numbers. Algebra was constructed by generalising number and arithmetic, and representing general results with letters. Figure 2.3 illustrates this connected sequence or relationship diagrammatically.

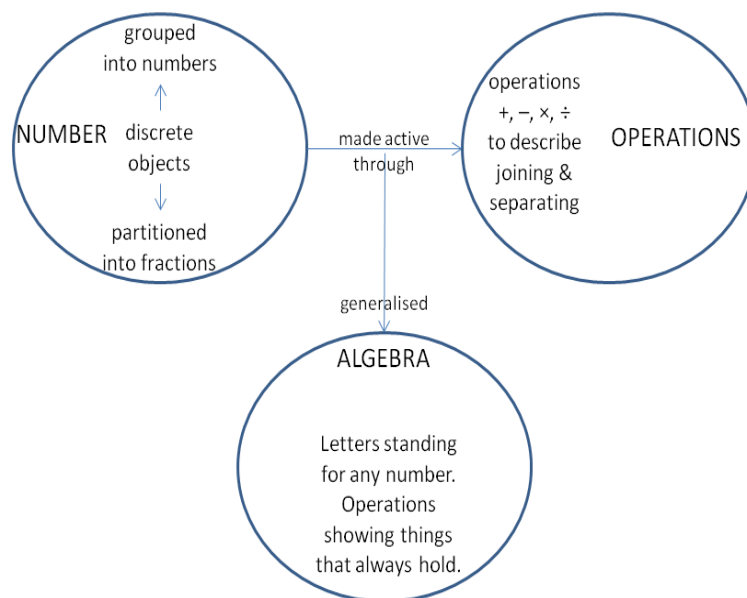


Figure 2.3 Connections and sequencing Number–Operations–Algebra

Number, operations and algebra, with input from geometry, gave rise to applications within measurement and statistics and probability. This relationship is diagrammatically represented by Figure 2.4.

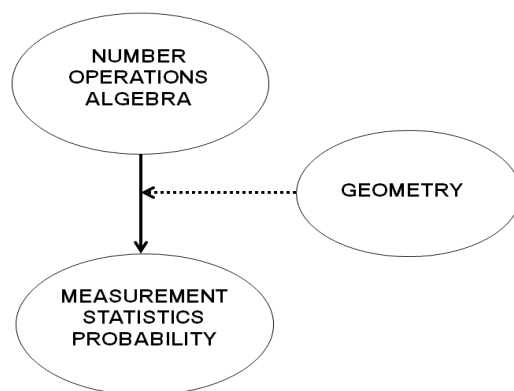


Figure 2.4 Connections and sequencing between strands

This gives a framework for Prep to Year 9 mathematics that enables mathematics as a whole to be considered as a connected sequence. It provides an overview and sequence for the connections upon which teaching should be built (e.g. number and geometry before measurement; relationship between probability and fractions).

Implications for teaching

YDM is based on the argument that knowledge of the structure of mathematics, particularly of connections and sequences (and big ideas), can assist teachers to be effective and efficient in teaching mathematics. This is because it enables teachers to:

- (a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;
- (b) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to and sequenced from other mathematics or based on the one big idea is easier to recall and provides options in problem solving;
- (c) *choose effective instructional materials, models and strategies* – mathematics that is connected to other mathematics or based around a big idea commonly can be sequenced so that it is taught with similar materials, models and strategies; and
- (d) *teach mathematics in a manner that makes it easier for later teachers to teach more advanced mathematics* – by preparing the linkages to other ideas and the foundations for the big ideas the later teacher will use.

Thus it is essential that teachers know the mathematics that precedes and follows what they are teaching, because they are then able to build on the past and prepare for the future.

2.4 Cultural bias

YDC's mathematics projects are and have been predominantly with Aboriginal and Torres Strait Islander and low SES students; this section looks at these two groups of students. The focus on these students enabled YDC to develop a pedagogical approach to mathematics that meets the needs of learners who have traditionally underachieved and who do not reflect the dominant culture of schools and Australian society.

As well, the belief of YDC is that such an approach, if remaining focused on mathematical structure, and not low-level functional numeracy, will also be applicable to students from other minority groups and also the dominant culture.

2.4.1 Aboriginal and Torres Strait Islander students

For Aboriginal and Torres Strait Islander students, the disassociation with mathematics is made worse by their social political situation within Australia, as dispossessed peoples, as well as the cultural divide between Indigenous and non-Indigenous people within Australia. As argued by Matthews (2006), Terra Nullius, the notion by which this country was claimed by Europeans, was more than just simply ignoring the existence of the Indigenous people of Australia. Under this notion the colonisers considered Indigenous people and their cultures as primitive with no value for a modern society. In fact, the colonisers considered Indigenous people as a dying race; a relic of the past. The story of David Unaipon is a living example of this.

The story of David Unaipon (Matthews, 2003)

David Unaipon was born in 1873 on the banks of the lower Murray River, a member of the Ngarrinjeri people. He has been recognised as Australia's first Aboriginal author and inventor, and was described by one newspaper as Australia's Leonardo da Vinci (Simons, 1994). In 1907, David Unaipon invented the modern shears that revolutionised the Australian shearing industry. From reading Newton's Laws of Motion, Unaipon converted curvilinear motion into linear motion to invent the hinge that drives the modern shear, a more efficient design. Even though he was commissioned by an Adelaide firm to examine the design of the shears, Unaipon could not secure the funds to keep the patent for the device. Subsequently, the idea was stolen from him and he never received any economic benefit or recognition for his invention (Simons, 1994; Shoemaker, 1989).

In 1914, David Unaipon also described the principle of the helicopter, based on the aerodynamics of the boomerang, long before the aircraft was invented. Similarly, in other writings, he predicted the development of lasers (Simons, 1994; Shoemaker, 1989). According to Jones (1990), Unaipon took out several other patents, which included a centrifugal motor, a multi-radial wheel and a mechanical propulsion device, all of which eventually lapsed (Jones, 1990). It is interesting to note that in one of Unaipon's writings on Aboriginal culture, Unaipon speculated about the inherent mathematical ability of his people with respect to their highly skilled fishing.

But let us review them as they stand there in their canoes. With a spear seven or eight feet long with prongs made of wood bound on to the end of a spear, the depth of the water four to six feet deep with weeds and reeds as described in the earlier part of this article. The water is of a chalky colour or of milk colour and the fisherman is not able to see an object more than four inches below the surface ... The fisherman knows that it is a Tcheri and he takes up his spear, knowing the depth and size of the fish. All this is done in a moment, ten yards away the fish is racing for life, and the speed at which it is travelling and the depth at which it is swimming has to be allowed for ... and the spear has to be thrown unerringly to strike the fish. An expert will probably make six correct catches and miss two (cited in Alexander, 1997).

As an author, Unaipon's written work on Aboriginal cultural bore a similar fate to his scientific endeavours when a British anthropologist, William Ramsay Smith, plagiarised his stories. Smith changed the tone of the text to that of an objective scientific observer and, in 1930, published the works in his name without acknowledging David Unaipon. Interestingly, Smith took out the reference to Indigenous mathematical abilities from Unaipon's original text on fishing (Alexander, 1997). David Unaipon died in 1969, at the age of 96, a bitter man with no recognition for his literature and inventions (Alexander, 1997). It is only after his death that Australia has honoured him on the fifty-dollar note. Even today most Australians would not have heard of David Unaipon; first Aboriginal writer and an accomplished scientist (Simons, 1994).

The devaluing of Indigenous cultures particularly within the sciences continues today. We are taught through our education system that "humankind" has evolved from hunter-gatherers to technologically advanced societies. This notion perpetuated by the education system does not provide a sense of pride for Indigenous students about their culture (at the bottom of the evolutionary ladder) and does not provide a place for Indigenous people and their knowledge within the sciences. It also ignores the reality that Aboriginal and Torres Strait Islander people have contributed to the scientific and economic advancement of Australia through bio-prospecting and have powerful forms of mathematics. For example, *kinship systems are highly abstract multiplicative frameworks*, the most advanced form of mathematical thinking, and a better basis for advanced mathematics than the additive relationships emphasised in Australian classrooms. Aboriginal and Torres Strait Islander people have also developed highly sophisticated ways of caring for country (land and sea) which rely heavily on "reading the signs", that is, having a form of mathematical knowledge that enables the complexity evident in total ecosystems to be understood.

Bio-prospecting (Matthews 2003)

On a larger scale, the exploitation of Indigenous knowledge is prevalent within the bio-prospecting field, where Indigenous botanical and medicinal knowledge is used to create pharmaceuticals for commercial purposes. Once the knowledge is taken, it is recast into Western scientific models and claimed as their own knowledge or “discovery”. To justify taking the knowledge and validate their discovery, researchers have devalued the Indigenous culture claiming they did not have a full scientific understanding (Foley, 2000, cited in Matthews, 2003; Smith, 1999). As an example, on 9 May 2001, ABC radio aired a report on the World Today about Harry Boeck, a New South Wales chemist who had claimed to discover a cure for golden staph. Golden staph are bacteria that are resistant to antibiotics and have caused many deaths from post-operative infections. The “new” drug was reported as being a “concoction of eucalypt and tea tree oils, plus six other chemicals”. In the report, Harry Boeck states that

I suppose my interest in natural remedies is – comes from reading and becoming intrigued with how people treated their conditions years ago. And Australian aboriginal people had remedies that they’ve used. Other cultures had remedies that they’ve used effectively, although they didn’t know why they were effective, but in many cases they were (Nolan, 2000, cited in Matthews, 2003).

Harry Boeck’s statement that Indigenous people or other cultures did not understand “why” presupposes that he had an intimate knowledge of these cultures and an understanding of the cultures’ ontologies, their picture of the world, that supports their “way of knowing” (Christie, 1991, 1993, cited in Matthews, 2003). Without this in-depth understanding, it appears that the Western world prejudices Indigenous people’s culture and knowledge as primitive or simplistic and lacking the capacity to understand “why”. The Western world and their “ways of knowing” still take the superior position, particularly with regards to science.

Disengagement is also a product of the mismatch between Indigenous cultures and the Eurocentric, standard-English culture of Australian schools. Aboriginal and Torres Strait Islander students predominantly come to school with a home language which is not standard English and with knowledge, skills, and patterns of interaction that are not appreciated by schools and do not match what facilitates success in school.

This mismatch is particularly evident in the way mathematics is taught in schools. Aboriginal and Torres Strait Islander students tend to be active holistic learners, appreciating overviews of subjects and conscious linking of ideas (Grant, 1998). In fact, Indigenous people have been characterised as belonging to “high-context culture groups” using a holistic (top-down) approach to information processing in which meaning is “extracted” from the environment and the situation. In contrast, mainstream Australian culture is characterised as a “low-context culture” and uses a linear, sequential building block (bottom-up) approach to information processing in which meaning is constructed (Ezeife, 2002).

School mathematics is traditionally presented in a compartmentalised form where the focus is on the details of the individual parts rather than the whole and relationships within the whole, a form of presentation that disadvantages Aboriginal and Torres Strait Islander students.

2.4.2 Low socio-economic status (SES) students

Historically, educational institutions have favoured higher to middle-class backgrounds, beliefs and practices. This is due to a number of factors including the history of the purpose of schooling across its development and the socio-economic backgrounds of the majority of teachers and curriculum developers (Meadmore, 1999). As such, there are pre-existing patterns of communication and interactions (or discourses) endemic to education in Australia which are not favourable to lower SES students (Meadmore, 1999). Thus, the middle-class Eurocentric culture of Australian schools and the implicitly understood patterns of communication and interactions serve to further marginalise students from low socio-economic backgrounds from school mathematics. The nature of discourses within school practices do not always successfully link to nor validate mathematical practices that may be part of low SES students’ out-of-school experiences (Baker, Street, &

Tomlin, 2006) leading to insufficient links being made between students' existing mathematical knowledge and practices and school mathematics. In these cases, students may disassociate from school mathematics and feel they cannot succeed, particularly if their home skills and knowledge are not valued nor actively sought (Thomson, 2002).

Expectations may also pose difficulties for low SES students as for Indigenous students. Low SES parents may perceive mathematics as alienating and unnecessary or too difficult for their children to learn; this can lead to students not expecting to succeed in mathematics, having low expectations of themselves and their future roles in society, and thus not participating in mathematics classrooms. Teachers may also have low expectations of low SES students and often believe that lower level or *life* numeracy is all that is needed for these students (Baker et al., 2006). The resulting emphasis of mathematics for these students becomes utilitarian, rote and procedural mathematics tasks that are not explicitly related to overarching mathematical structures.

2.4.3 Strengths and weaknesses of Eurocentric mathematics

Similar to other cultures, European societies developed their own mathematics to help them explain their world and solve their problems, particularly space, time, and eventually, number. When trading became a way of life, a need developed to be more precise in representing and quantifying value to have a shared agreement of how values could be compared ("is mine worth more than yours"), a more sophisticated process than quantification as it involves rate (e.g. 3 cows = 1 boat). Thus, Western mathematics developed a system of keeping track of "how many", "how much", and "what is it worth".

Over time the European mathematics' quantification and comparison system grew to encompass a variety of numbers (common and decimal fractions, percentages, rates and ratios), measures of time and shape (length, area, volume, mass and angle), and two operations (addition and multiplication) and their inverses (subtraction and division). The system was also generalised to findings that hold for all numbers and measures, and the resulting mathematics area of algebra has grown in importance as science and technology has expanded.

The strength and weakness of European mathematics lies in the success of its quantifying and comparing systems that underpinned the growth of science and technology. Advances in science and technology have transformed the way we live.

A common benefit usually espoused is that we live longer and healthier lives and Western society now has the tools to control our environment and ensure better living conditions; we can cool the hot, warm the cold, clear the land, bring in new plants and animals, and clothe, shelter, and feed large populations.

However, this triumph has affected European culture and society. Success has come to mean increasing numbers and continued growth; smaller numbers and negative growth are to be avoided (and are given names that signify failure, such as "recession"). The culture appears to have little ability to comprehend harmony and live sustainably. Instead, the people attempt to dominate the land, the sea, the weather, and the animals, birds and fish, with little understanding of the implications their actions will have on human life, which are ultimately poverty, hunger, war, pestilence and global warming.

Mathematics can be developed in a way that would reinforce planetary equity and sustainability (see Figure 2.5), however, such mathematics requires less dominance by number, less need for growth, and an emphasis on living in harmony with land and sea; a non-Eurocentric form of mathematics.

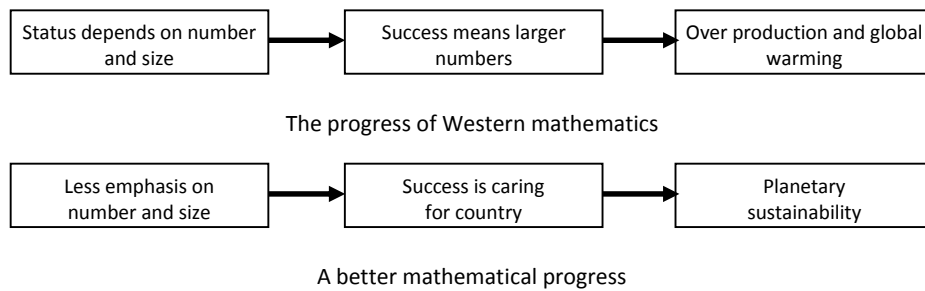


Figure 2.5 Two perspectives of progress

2.4.4 Implications for teaching

Traditional teaching of mathematics can confront Aboriginal and Torres Strait Islander and low SES students' cultures and perpetuate the belief that success in mathematics classrooms requires the rejection of culture (i.e. that one has to become "white" or middle class). The focus of previous mathematics-education projects by YDC staff with Aboriginal and Torres Strait Islander and low SES students has been to enhance mathematics outcomes but retain pride in culture and heritage. The result of this experience, in line with the analysis of this section, has been to **identify approaches to teaching mathematics that legitimise and include students' cultural capital and out-of-school experiences of mathematics.**

The first of these approaches is to confront the Eurocentric nature of traditional school mathematics by making students aware of the cultural implications of mathematics teaching, and draw attention to which mathematics ideas change perceptions of reality (e.g. dividing land into hectares builds perception that land can be treated as isolated pieces not as an interacting whole). In particular, it appears important to discuss the role of mathematics in European culture and to draw attention to the culture's strengths and weaknesses. It is important not to make mathematics a celebration of the school culture, but make it something that is available to all students.

The second of these approaches is to legitimise local Indigenous knowledge and integrate students' culture and mathematics instruction, to match the mathematics classroom to the cultural capital brought by the students (Bourdieu, 1973). This means contextualising mathematics into the life and culture of students, using examples, models and activities from the everyday lives of the students (e.g. balance can be understood in terms of equal weights of people being on each side of small planes). This requires the teacher to have knowledge of the out-of-school life of the students. It is particularly important when English is not the first language of the students or there are different meanings for common ideas (e.g. in the Torres Strait, fair shares does not mean equal shares). If teachers understand and accept the cultural and social capital students bring to the classroom as a starting point for instruction, the philosophical framework of Figure 2.1 can be used by teachers to help students: (1) create their own mathematical world; (2) understand the mathematical worlds of others; and (3) understand how these worlds relate to their everyday life and the everyday lives of others. The philosophical framework of YDM provides a platform from which teachers can assist students to effectively construct learnings of mathematics that relate to the students' reality (social constructivism) (Cobb, 1994). Such mathematics learnings facilitate a mathematics environment which offers students choices, has cultural pluralism, and supports rich home mathematics experiences.

An important legitimisation is to use a local cultural framework for learning. Local teacher aides, Elders, non-school-based personnel that the students respect, such as sport coaches or trade instructors and key community members, may be able to provide direction. In Queensland, there are two widely followed frameworks as follows.

Two Indigenous frameworks

Two frameworks that are used widely in Queensland Indigenous schools are the Aboriginal holistic planning and teaching framework (Grant, 1998) and the Torres Strait Islander educational framework (Foster, 2009). Both frameworks are designed to provide teachers with perspectives in which to place and discuss mathematical ideas.

The Aboriginal holistic planning and teaching framework focuses on six perspectives in relation to a mathematical idea. These perspectives are land, language, culture, time, place, and relationships. Their relationship to a topic is illustrated in Figure 2.6 below.

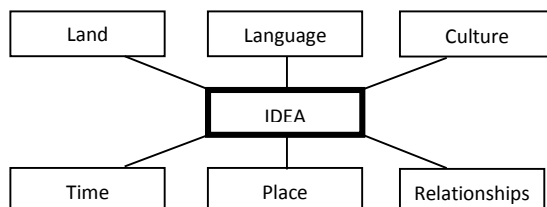


Figure 2.6 Aboriginal holistic planning and teaching framework (Grant, 1998)

Land, language and culture are straightforward in that they direct teachers to undertake activities in the context of the local environment, the local language and the local culture. Time means history and directs teachers to place the mathematics in the pre-colonial and colonial history of the local Aboriginal people. Relationships is also straightforward but place means to focus on interactions that come from a holistic view of place. Thus, place and relationship require the mathematics idea to be presented as part of a structure.

The Torres Strait Islander framework is based on the four directions of navigation as Islanders are seafaring people. It has four perspectives as in Figure 2.7 below (Geography, History, Culture and deep structure). All perspectives are straightforward and have strong similarities to the Grant framework.

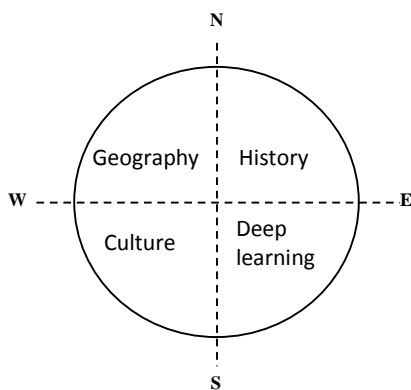


Figure 2.7 Torres Strait Islander educational framework

Source: Permission to use diagram from Torres Strait Islands Cultural Induction Program was granted by Mr Steve Foster (2009), Associate Principal, Primary, Tagai State College.

The third approach has already been covered; it is to present mathematics as a holistic structure of ideas that can empower the learner by focusing on underlying big ideas and using instructional strategies that relate acting, creating, modelling and imagining.

Finally, approaches to improve mathematics learning cannot stand alone. They need to be allied with whole-school activities to relate to local community, challenge attendance and behaviour, and have high expectations. This necessary whole-school change is discussed in Chapter 4, School Change and Leadership.

3 Pedagogical Framework

In the development of YDM, it was decided to use the philosophical framework of Matthews as the basis of the pedagogy. This section looks at mathematics teaching and describes the cycle that underlies the pedagogical framework, discusses the cycle's role in planning, and provides brief descriptions of the other mathematics teaching pedagogies that influenced the final framework.

3.1 RAMR cycle

To change the philosophical framework of Figure 2.1 to a pedagogical framework required two steps. The **first step** was to deconstruct the philosophical framework into components that can become pedagogical steps. To do this, the contexts of reality and mathematics and the processes of abstraction and reflection became four different types of instructional episodes. These were then linked into a cycle of instructional episodes that starts and ends with reality – the **Reality–Abstraction–Mathematics–Reflection** or **RAMR cycle** – see Figure 3.1 below.

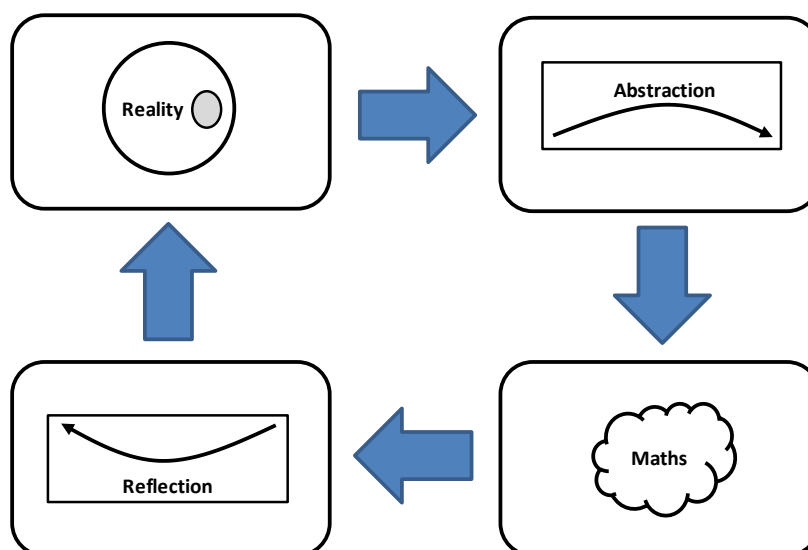


Figure 3.1 The components of the RAMR cycle

The **second step** was to flesh out the cycle by bringing in other pedagogies that had been useful in teaching mathematics to Indigenous and low SES students and to integrate them into the four components of the cycle. This formed the full framework that is described in subsection 3.2.3.

The RAMR cycle will be the basis of the teaching resources for the seven sub-strands of mathematics, namely, Number, Operations, Algebra, Geometry, Measurement, Statistics and Probability. These components are now described with an example before the full cycle is introduced in planning.

3.1.1 Components

The basis of YDM's pedagogical framework is the RAMR cycle, deconstructed as in Figure 3.1. Its four components are reality, abstraction, mathematics and reflection (as follows), which build a pedagogical framework for the development of a new mathematical idea.

Reality

The reality component of the cycle is where students: (a) access knowledge of their environment and culture; (b) utilise existing mathematics knowledge prerequisite to the new mathematical idea; and (c) experience real-world activities that act out the idea.

The focus in this component is to connect the new idea to existing ideas and everyday experiences. Among the kinaesthetic, physical and visualisation activities that predominate in this component, it is vital that students be provided with opportunities to generate their own experiences and verbalise their actions. This generation and verbalisation provides the students with ownership over their understanding of the mathematical idea.

Abstraction

The abstraction process is where students experience a variety of representations, actions and language that enable meaning to be developed in a way that carries mathematical ideas from reality to abstraction. We focus on activities with students' bodies, then activities with the hand and finally activities to ensure visual images are developed (i.e. body → hand → mind). Representations, actions and language will predominantly be as in Figure 2.2 in subsection 2.1.1.

However students should also be provided with opportunities to create their own representations, including language and symbols of the mathematical idea that has been initially experienced through physical activity (see subsection 2.2.2). This allows students to have a creative experience that will firstly, develop meaning and secondly, attach it to language and symbols. The sharing of other students' representations provides students with alternative views of the same idea attached to varied symbolic representations. Discussions on the use of different symbols enables students to: (a) critically reflect on their journey (enabling them to justify and "prove" their ideas); (b) understand the role of symbols in mathematics (enabling them to understand the relation between symbol, meaning and reality); and (c) be ready to appropriate (Ernest, 2005) the commonly accepted symbols of Eurocentric mathematics.

Mathematics

The mathematics component of the cycle is where students: (a) appropriate the formal language and symbols of Eurocentric mathematics; (b) reinforce the knowledge they have gained during the abstraction; and (c) build connections with other related mathematical ideas.

The focus is to assist students to construct their own set of tools (filling their "mathematical toolbox") that will enable them to recognise and recall mathematical ideas from the language and symbols associated with the ideas, thus adding to their bank of accessible knowledge. The connections between new and existing ideas enable better recall of mathematical ideas and improve problem solving. It is easier to remember ideas in terms of how they are related to each other (structural understanding) as opposed to many disconnected pieces of information. The ideas that help in problem solving are often connected peripherally to the central idea to which the problem refers.

Reflection

The critical reflection process is where the new mathematical ideas are: (a) considered in relation to reality in order to validate/justify understandings; (b) applied back to reality in order to solve everyday life problems; and (c) extended to new and deeper mathematical ideas through the use of generic or reflective strategies, namely, flexibility, reversing, generalising and changing parameters.

As well as reflecting on the mathematics they have learnt in relation to the world they live in, this process involves students' consideration of the journey from reality to mathematics via abstraction that they took in developing the mathematical ideas. It requires reflection on what they learnt, how they learnt it, and why they learnt it. It also requires them to justify their outcome.

Note: Along with continual **diagnosis** concerning whether students have learnt what is being taught, the two generic or reflective strategies of **flexibility** and **reversing** can be used in all four RAMR stages.

3.1.2 Application to lessons

Using the YDM pedagogical framework to plan and implement mathematics lessons requires a deconstruction of the mathematical ideas into the RAMR cycle, namely: reality, abstraction, mathematics and reflection, to be taught in sequence as a unit of work. However, it should be noted that some ideas may need more than one cycle to be completed, or that some ideas require parts of the cycle to be repeated before moving on to the next part of the cycle.

An example of such a deconstruction is as follows. It shows how the framework can be used to develop a unit of work to **teach comparison of 2-digit numbers**. This mathematical idea is one that encompasses all aspects of the RAMR cycle.

Reality

Look at *real-world* instances of comparison of 2-digit numbers (e.g. height, mass, street numbers), ensure students understand place value of 2-digit numbers, and set up real-world activities (kinaesthetic, physical and visual).

Use examples that incorporate both the set (for example, quantities in groups) and number-line models (for example, street numbers, temperatures). For example, take two 2-digit numbers, say 34 and 42, and compare by (a) placing side by side and counting (which comes first); (b) placing on a number line (which is further from the 0); (c) looking at \$34 and \$42 (which will buy more); and (d) cutting 34 cm and 42 cm lengths and placing side by side (which is longer).

Abstraction

Set up a sequence of representations (real world, physical, virtual, picture, patterns), along with actions and language. Follow the sequence **body → hand → mind**. Use a variety of representations and models. For example, construct own measurer out of 10 unit lengths of cardboard (stuck end to end) or 10 unit long straws (alternating colours threaded on string). Use this to measure things in classroom (estimate first). Identify how many tens and guess the ones. Draw different objects being measured, write in lengths and compare with other items.

Always *relate* language and symbols for the number to the tens and ones, the drawing and the reality. (*Note*: the drawing for 34 should be 3 hops of 10 and 4 hops of one – this should be easily seen as less than 4 hops of ten and 2 hops of one.) Prepare a virtual measure with tens marked and use to measure and compare virtual objects. Draw 0–100 on number lines, identify numbers under consideration, and compare. *Reverse* the process – start with numbers, draw measurer showing these amounts, and compare lengths.

Discuss what is important, the number of ones or the number of tens? (*Note*: this is crucial – the important learning is that the largest place value is the important one in comparison and order.) *Create* symbols for larger than and smaller than.

Mathematics

Identify the mathematics that the tens determine the larger number (the ones are only important when the tens are the same) and *introduce* formal language (e.g. larger, longer, etc.) and symbols (e.g. >, <).

Reinforce this with the game “chance order” (*board*: _ _ less than _ _; *rules*: draw four cards – 1–9 only – and place digits as drawn in the four places so that left hand side number is less than the right hand side number; *winner*: the player with the highest left hand side tens digit).

Connect the comparison on a number line with comparison in a set model (money, MAB, bundling sticks). Reinforce learning by counting sticks and showing that 42 sticks have more tens than 34 sticks. *Discuss* other situations which are connected to order.

Reflection

Validate the mathematical idea of comparison by setting problems that *apply* comparison to reality (e.g. who has more fish? what is the most expensive toy? who drives further?). Use the *four reflective strategies*:

- flexibility* – try to think up all the different ways we compare in the world (e.g. rain gauges, cars on a production line, mass, angle) – try to think of non-prototypic situations (e.g. thickness of paper);
- reversing* – in most activities teachers give students two numbers and ask for the larger, what if we give one number and ask the students to find a larger or to find a number that is between two numbers (even as difficult as what number is between 34 and 35); and
- generalising* – through discussion students see that the hundreds are most important in comparison of 3-digit numbers (and that the highest place-value position is most important in any comparison – that determining place value is done from R \rightarrow L, reading numbers and comparison is L \rightarrow R);
- changing parameters* – what if the numbers were negative?

3.1.3 Detailed RAMR cycle for planning lessons

To use the YDM pedagogical framework to plan instruction for a mathematical idea, it is deconstructed into components that are applied to produce a structured, instructional sequence for teaching the idea. Figure 3.2 briefly outlines how this can be done, **providing, in one diagram, all the steps of the RAMR cycle**. It should be noted that this cycle need not be rigidly followed. It is a checklist of what should be covered in a lesson or unit of work, but it sometimes does this best if it is flexibly used.

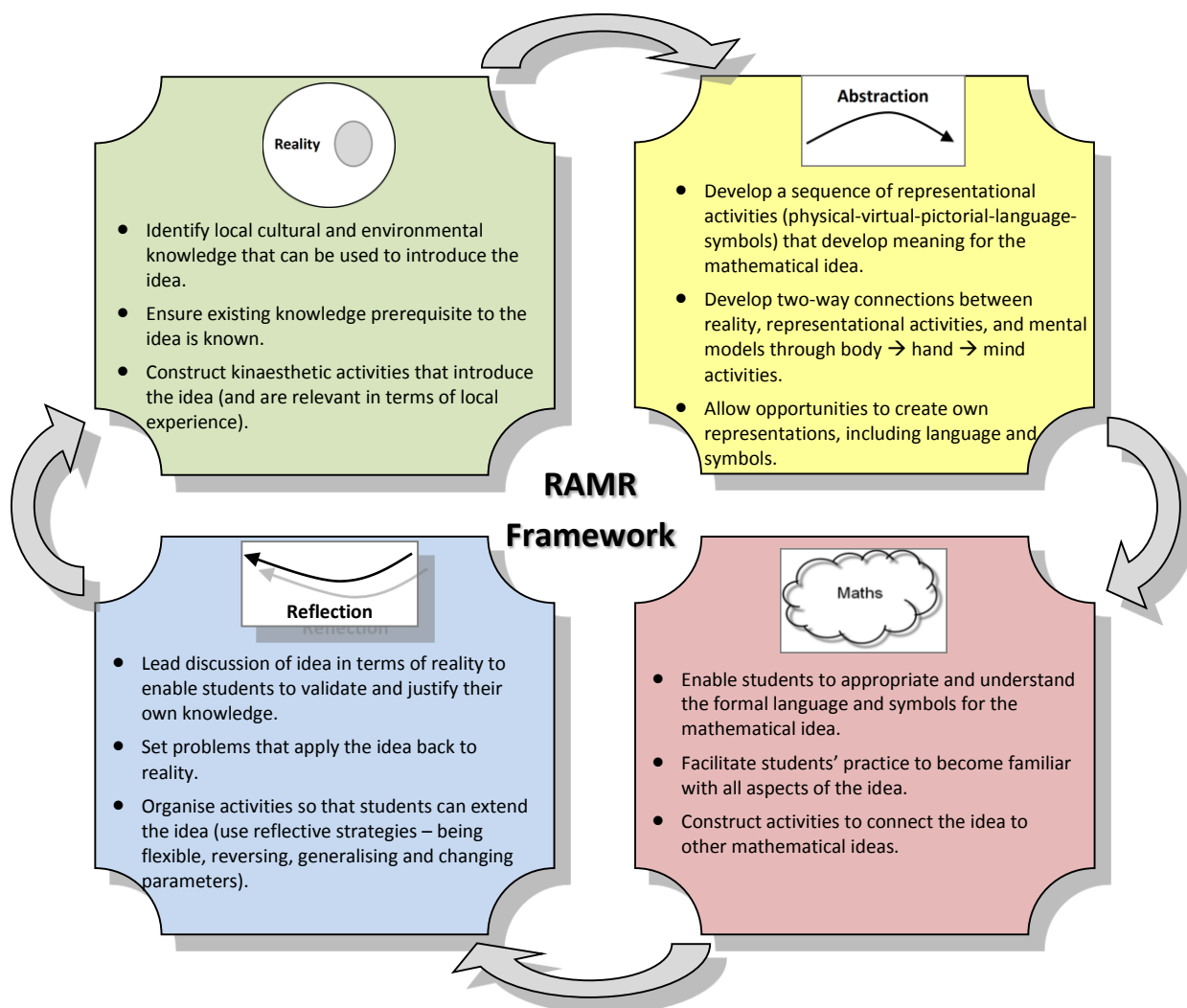


Figure 3.2 Using the RAMR cycle to plan mathematics instruction for a mathematical idea

Checking that prerequisite ideas are known is part of the Reality component of the cycle. Connected mathematical ideas are used in the Mathematics component of the cycle. Follow up ideas (or extensions of the ideas) are focused on during Reflection.

Flexibility and reversing can be used in all stages of the cycle. However, along with generalising and changing parameters, they have a powerful role in extending ideas at the end of the cycle.

3.2 Overall framework

The RAMR cycle is only part, albeit a very important part, of the YDM pedagogical framework. There are two other components:

- (a) **Developing a plan** for teaching across a period of time requires identification, diagnosis (or diagnostic assessment), analysis, planning, teaching, management and reflection. Of these seven activities, the RAMR framework applies to just two – the diagnosis and the teaching – as this section describes.
- (b) **Identifying the central mathematics** to be taught and the ideas to which it should be connected requires an understanding of mathematics structure, most importantly big ideas, connections and sequencing – as was discussed in section 2.3.

This section covers (a) above, that is, the Planning–Teaching cycle, discusses levels of instruction within a lesson, and provides the overall framework for how the RAMR and planning cycles come together. Identifying the central mathematics is left to the six books covering the *Australian Curriculum: Mathematics* strands.

3.2.1 YDM Planning–Teaching cycle

In 2011, YDC undertook a project to support teachers developing and evaluating their own teaching plans. Pre-post testing showed that YDM was successful in empowering the teachers to be able to implement a teaching plan for improving mathematics learning if the teachers acquired the knowledge and skills to have the following capacities, and to cycle through them as Figure 3.2 on the right shows.

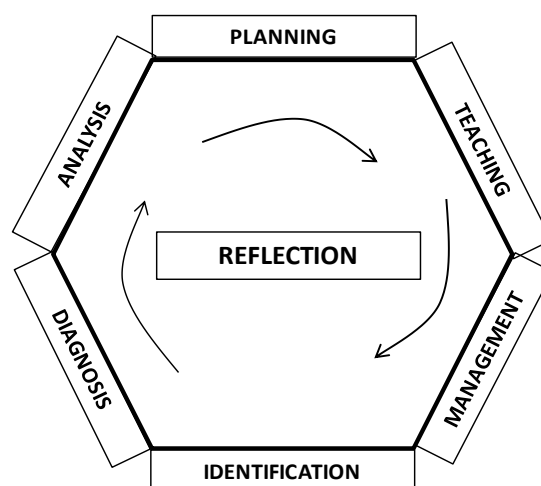


Figure 3.3 YDM Planning–Teaching cycle

1. **Identification.** Determining the **focus of the instruction** for the period being planned. This is initially determined by the curriculum but modified by knowledge of students' mathematics progress up to the start of the lessons.
2. **Assessment.** Identifying and administering **effective diagnostic assessment** tools. The RAMR cycle will assist here by showing what is needed for the starting step of the plan. Most diagnostic tests need modification to add in reality, kinaesthetic, connection and reflection items (particularly with respect to the four generic/reflection strategies).
3. **Analysis.** Using a spreadsheet (e.g. Excel) to **analyse the students' responses** to the assessment in terms of class needs and individual needs.
4. **Planning.** Determining **sequences of instruction** for the students so they can progress from where they are to where they should be.
5. **Teaching.** Having the mathematics and mathematics-education knowledge to **effectively teach** the plan. This is where the **RAMR cycle will be used** – it will help **develop the lessons** for each step in the teaching plan.

6. **Management.** Having the **general lesson knowledge to manage learning**, in particular to know after instruction which students know and do not know what has been taught, and what to do about the two groups.
7. **Reflection.** Having the ability (and knowledge) **to reflect on each of 1 to 6** and to modify instruction to **maintain effective learning**.

The Planning, Teaching and Management components of this YDM Planning–Teaching cycle are based on Shulman’s work on teacher knowledge to be able to teach in mathematics (and other disciplines). Shulman argued that there were three types of teacher knowledge – mathematics content knowledge, mathematics pedagogy knowledge, and general lesson planning knowledge.

This YDM Planning–Teaching cycle is very empowering for teachers – along with the RAMR cycle, it forms a framework which enables teachers to develop their own plans and sequences, their own scripts and lessons, for the specific needs of their students.

Note: YDM is developing its own series of diagnostic tests for major mathematics topics. The first of these (for whole and decimal numbers, fractions and probability) was published by state and federal Departments of Education and given free to every Queensland school. It is called *Developing Mathematics Understanding through Cognitive Diagnostic Assessment Tasks* (CDAT). As other diagnostic tests become available, we will make them available through the YDC Blackboard site.

3.2.2 Three levels of instruction

Within the Teaching and Management components of the Planning–Teaching cycle, it is important to ensure that lessons run well. To do this, it is important that teachers are familiar with instruction at three levels – technical, domain and generic (Baturo et al., 2007).

The RAMR cycle is very active, requiring teachers to provide rich experiences with materials. Students learn from thinking about their experiences with materials (through questions, discussion and debate); however, this is not possible without the teacher being able to manage the activity to start with, and knowing which questions should be asked. The three levels are as follows.

1. **Technical.** The technical level means becoming familiar and proficient with the use of materials (e.g. knowing the folds that enable $1/6$ to be constructed from a circle of paper). All materials have weaknesses; they have aspects that reinforce what you are teaching but have other aspects that distract or lead to error. Teachers need to know how to overcome these distractions (e.g. used incorrectly MAB can lead students to think that three longs and two units are represented by symbols 302).
2. **Domain.** The domain level means knowing what materials and what activities will provide experiences effective for learning. This will be the focus of the six resources on the mathematics sub-strands.
3. **Generic.** The generic level means knowing strategies that apply to any mathematics topic that will extend knowledge of the topic. Four of these have been discussed within the Reflection component and are flexibility, reversing, generalising, and changing parameters. The abiding issue with them, and the reflection process, is that teaching must not stop when a mathematical idea is acquired; this is only the beginning. Reflection in terms of the generic strategies can take the knowledge to a macro level where it provides a **holistic structure into which later ideas can fit or build upon**.

3.2.3 Overall framework

The power of mathematics lies in its generalised state, as a structure and language and a portable set of tools for solving many problems. Along with big ideas, connections and sequencing, the RAMR cycle and the Planning–Teaching cycle provide an overall YDM pedagogical framework for teaching and learning mathematics that empowers teachers to teach powerful mathematics effectively without scripts or detailed lesson plans provided by an external source. In this way, teachers can make mathematics learning real to their students.

Thus, the framework reminds us that **deep understanding** not shallow procedures must be the end point of mathematics instruction; and that this is the basis of YDM mathematics – improving teachers’ abilities to teach. Along with mathematics structure, the overall YDM pedagogical framework is an integration of the Planning–Teaching cycle and the Reality–Abstraction–Mathematics–Reflection cycle. It consists of the Planning–Teaching cycle with the RAMR cycle connected to it, as shown in Figure 3.4 below.

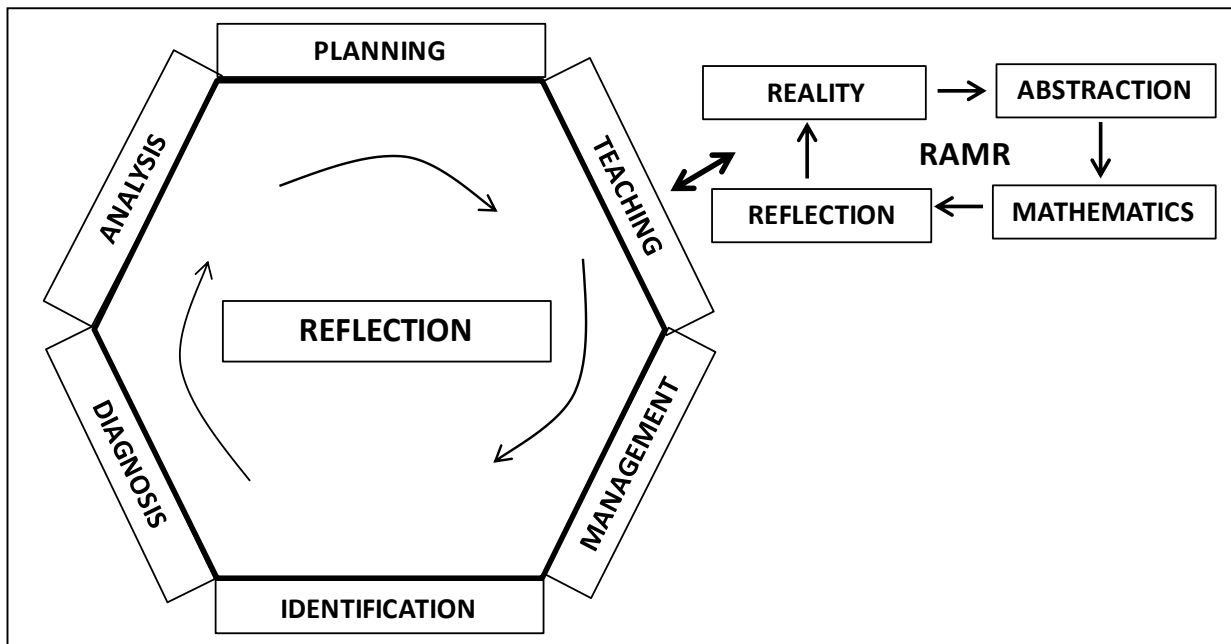


Figure 3.4 The YDM pedagogical framework

3.3 Relation to other instructional strategies

It is illuminating to compare the YDM pedagogical framework with other well-known instructional models and principles which have shown their effectiveness across many years, namely (1) Payne and Rathmell’s (1977) triangle model, (2) Wilson’s (1978) cycle (adapted by Ashlock et al., 1983), (3) Alexander and Murphy’s (1998) learner-centred principles, and (4) Baturó’s (1998) knowledge types adapted from Leinhardt (1990). It is interesting to see that a pedagogy that has emerged from an analysis of the nature of mathematics encompasses these models/principles.

3.3.1 Payne and Rathmell triangle

This framework for learning mathematics (Payne & Rathmell, 1977) relates real-world situations, representations, language and symbols (see Figure 3.5). It has been used by YDC staff for many years with success. It fits well with the YDM pedagogical framework, particularly the reality → abstracting → mathematics components. It starts from reality (real-world problems), connects to symbols (mathematics) and identifies representations as supporting the process of abstraction to language and symbols. It is a powerful way to think of the abstraction process.

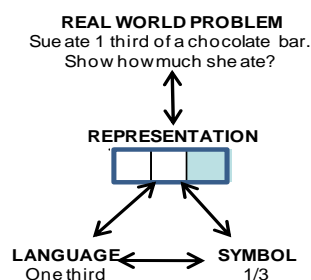


Figure 3.5 Payne and Rathmell triangle framework for teaching fraction $\frac{1}{3}$ (Payne & Rathmell, 1977)

In YDM's use of the triangle model, real-world situations are identified and modelled with body, hand and mind. This links to the use of physical, pictorial and virtual materials which are then connected to introduction of language and symbols, studied and reinforced as two-way connections. The triangle model can be seen as having three options, cultural, informal and formal: connections are made culturally (e.g. language and context), then informally (e.g. everyday situations and language), and finally in formal mathematical terms. The triangle model also has one other important feature. It gives credence to representation, particularly models such as arrays and number lines. It is not only the abstract symbols that provide the foundation for future mathematics; sometimes representations are equally if not more important (e.g. number tracks learnt with early counting can be extended to lines that enable advanced topics such as ratio and algebra to be easily comprehended).

3.3.2 Wilson cycle

The Wilson cycle is similar to the RAMR cycle in that it builds from reality back to reality. In mathematics, ideas are built on top of each other and, notwithstanding the power of gestalt leaps of understanding, it is often necessary to build large ideas from smaller steps. A sequence that has been used by YDC staff for many years with success, the Wilson cycle (adapted by Ashlock et al., 1983), specifies six steps as in Figure 3.6: (1) teach the idea informally (real-world situations, representations and informal language); (2) introduce the formal language and symbols; (3) undertake activities specifically to connect the new knowledge to existing knowledge; (4) practice (games, practice activities and worksheets); (5) apply knowledge to solve problems; and (6) see if students can undertake activities that can extend knowledge to new knowledge without having to go through all the steps. The cycle also advocates continuous *checking and diagnosis* of students' understandings to ensure no errors become habituated.

The similarity of the Wilson cycle to the RAMR cycle can be seen in the way the RAMR cycle's four components relate to the Wilson cycle's six components; Reality, Abstraction and the start of Mathematics cover "Introduce", the remainder of Mathematics covers "Consolidate", and Reflection back to Reality covers "Apply".

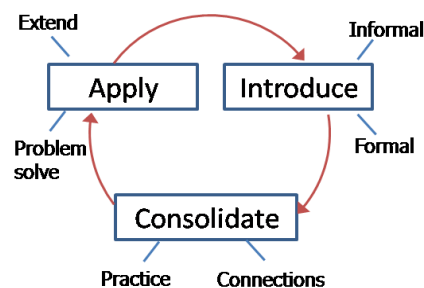


Figure 3.6 The Wilson cycle for teaching mathematics (Ashlock et al., 1983)

3.3.3 Baturó's knowledge types and Bruner's three stages

Baturó (1998) modified Leinhardt (1990) to produce a model of four knowledge types that have been used with success by YDC staff. This model argued that instruction in mathematics should develop four knowledge types: (1) entry (knowledge of mathematical ideas before instruction); (2) representational (knowledge of physical materials and pictures used to develop the ideas); (3) procedural (knowledge of definitions, rules and algorithms); and (4) structural (knowledge of relationships and concepts). The model argued that structural knowledge, the most powerful form of mathematical knowledge, is based on representations and connections, acquired by moving from surface features to underlying structure to abstraction (Sfard, 1991), and based on recognising structural connections between external (e.g. materials, pictures, word and symbols) and internal (e.g. mental models) representations (Halford, 1993). These four knowledge types are evident in YDM's pedagogical framework with entry at "Reality", representational at "Abstraction", and procedural and structural at "Mathematics". In particular, structural knowledge with its schematic connections is the major aim of the YDM approach. Bruner (1967) argued that instruction should move through the following three stages: enactive → iconic → symbolic. This is the basis of the RAMR cycle's three stages **body → hand → mind**.

4 School Change and Leadership

The experience of YDC staff in previous projects has shown that the **provision of new mathematics teaching ideas is, on its own, often insufficient for sustainable improvement** in Aboriginal and Torres Strait Islander and low SES students' learning of mathematics. The new ideas have struggled to have positive effects when low attendance and negative behaviour are endemic across a school, when school practices and learning spaces disengage students, when positive partnerships are not formed between teachers and their students and/or Indigenous teacher aides, when classrooms do not involve community leaders/members or acknowledge local knowledge, and when teachers do not believe that the students are capable of the work (e.g. Cooper, Batur, Warren, & Doig, 2004). The **ideas have been successful when they have been integrated into whole-school changes** that challenge attendance and behaviour, integrate and legitimise local community knowledge, build in practices that acknowledge and embrace the culture of the students, and change teacher attitudes towards a high-expectation relationship with the students (C. Sarra, 2005a).

Emancipatory, empowering and connected learning and teaching environments coupled with positive teacher and student beliefs and perceptions about learning **are necessary components** of school programs to improve students' life chances through education. In addition to paying attention to students' learning in terms of specific content and skills, positive teacher perceptions and community-school partnerships that aim to develop positive student identities and future aspirations are vital components of whole-school programs to ensure the best possible learning outcomes for all students.

With respect to Indigenous students, YDM supports the Stronger Smarter Institute's ideas with respect to school change and leadership. It shares the Institute's belief that it is possible to deliver high quality education right across Australia; and that this requires promoting that as Aboriginal and Torres Strait Islander people we should be strong in our hearts, proud of our identity, solid in our community – and smart in the way we do things, focused on high achievement, determined to succeed. YDM also believes that these ideas have wider applicability to any cultural group, particularly minority, refugee, and low SES groups.

This section discusses education experiences for Aboriginal, Torres Strait Islander and low SES students in the context of whole-school programs designed to enhance community-school partnerships and provide school environments which lead to high expectations for teachers and students and positive student identities.

4.1 Situating the context

An emancipatory environment involves “collaborative activity, engaged in by people who experience oppression and by those in solidarity with them, that is expressive of their effective agency and that aims both to alleviate the sufferings being experienced and to create conditions for the effective agency (or freedom) of everyone” (Lacey, 2002, p. 10).

Aboriginal, Torres Strait Islander and low SES students are often referred to as victims of their own circumstances, consequently reproducing their isolation and exclusion from education (Connell, Ashenden, Kessler, & Dowsett, 1982; Meadmore, 2004; Sanderson & Thomson, 2003). The focus of this section is to move away from this pervading view and examine some of the interrelated issues, namely, school change and leadership, and how these issues work in schools to create emancipatory environments that alter power in classrooms, and in doing so, provide Aboriginal, Torres Strait Islander and low SES students with access to an effective education rather than one that is isolating and exclusionary.

4.1.1 Exclusion and positive discrimination

The exclusion of Aboriginal, Torres Strait Islander and low SES students from education is evident in NAPLAN testing responses, student retention rates, and school completions (Australian Bureau of Statistics data, 2009; MCEECDYA, 2008). Early exit from school leads to long-term unemployment, crime and poor health (Sanderson & Thomson, 2003) which in turn unfairly places the education focus on the student and not on the education system. This is particularly evident for remote Indigenous students (as “Indigenous exclusion from education” below shows – but the arguments are just as effective for low SES students). Educators are less likely to examine causes of Aboriginal, Torres Strait Islander and low SES students’ underachievement than for other students and more likely to blame the students and their families and communities. Some of these factors include schools’ inability to establish a strong connection to Aboriginal and Torres Strait Islander families and the importance of valuing and recognising the culture and history of these families (Garcia & Guerra, 2004; Purdie & Buckley, 2010). Educators need to take responsibility for the performance of Aboriginal, Torres Strait Islander and low SES students and reflect on the effect of their own beliefs and assumptions (C. Sarra, 2009).

Indigenous exclusion from education

This exclusion was exemplified in the Australian Bureau of Statistics (ABS) data (2009) which reported that Indigenous people living in rural and remote areas of Australia were identified as less likely than those in urban areas to have completed Year 12 (ABS, 2009). Although improvements in school completions of Indigenous students have been recorded, “Indigenous people aged 15 years and over were still half as likely as non-Indigenous Australians to have completed school to Year 12 in 2006 (23% compared with 49%)” (ABS, 2009). These relative differences have remained unchanged since 2001. Further, Indigenous students continue to be the most educationally-disadvantaged group particularly within the area of mathematics, performing on national testing two years below their non-Indigenous counterparts (MCEECDYA, 2008). When these results are viewed together with student retention rates to Year 12 (see Lamb, 2009), it is not surprising that the lowest levels of post-compulsory school enrolments are recorded by young Indigenous people.

These statistics are disturbing particularly when early exit from school has been found to be associated with long-term unemployment, “early involvement in the juvenile justice system, and very poor health” (Sanderson & Thomson, 2003, p. 96). These social issues are often blamed on the numerous problems that have manifested within many Indigenous communities throughout Australia, which unfairly places the focus on the communities and their people, when it should be on ways to improve the effectiveness of the education that students receive (Aboriginal and Torres Strait Islander Social Justice Commissioner, 2008; Behrendt, 2008; C. Sarra, 2009).

Garcia and Guerra (2004) argue that students from low SES and racially or ethnically diverse backgrounds often experienced failure in schools because educators were unwilling or less likely to examine the causes of underachievement and failure among these students. Rather, and as noted previously, there is a tendency to blame the students, families and communities on poor academic results. They argue that educators must be willing to assume some responsibility for the failure of Indigenous students critically reflecting on their own beliefs and assumptions about Indigenous students and their families as responsible for such poor academic results (Garcia & Guerra, 2004; G. Sarra, 2007a). Sarra argues that most educators would have difficulty perceiving themselves as reinforcing the problem, and therefore would be less willing to actively reflect on their practice in ways that bring about change in their pedagogical practice and change within the education system.

Attempts to improve the educational situation for Aboriginal, Torres Strait Islander and low SES students have often involved positive discrimination (e.g. special entry programs, special scholarships, CSIRO STEM project in collaboration with BHP Billiton, and even the DET TIME project which funded the initial development of this resource). Positive discrimination in the interests of a particular group does, on occasion, generate resentment among those who do not benefit from such programs and, to some extent, embarrassment among those who do (Fraser, as cited in C. Sarra, 2005a). However equity demands that there be positive discrimination when different groups start from different situations regarding access to opportunities, and resentment and embarrassment are unwarranted.

4.1.2 Bucket analogy

Sarra (2005a) uses a bucket analogy to provide a powerful argument for positive discrimination. He relates opportunity to the amount of water in a bucket and the flow of access of opportunity to running taps. It is written from an Aboriginal and Torres Strait Islander perspective but applies to any group which is performing at a lower level than mainstream.

Bucket analogy (Sarra, 2005a)

Projects developed under the Aboriginal Employment Development Policy (AEDP) and the National Aboriginal and Torres Strait Islander Education Policy (NAEP) have positively discriminated in the interests of better outcomes for Indigenous people and as a means to address the discrepancies presented earlier. Fraser (as cited in C. Sarra, 2005a) is correct to argue that the notion of positively discriminating in the interests of a particular group does, on occasion, generate resentment among those who do not benefit from such programs and, to some extent, embarrassment among those who do. Consequently there is a necessity to generate better understanding about the need for such programs, and to encourage Aboriginal and Torres Strait Islander people to make use of such programs without feeling guilty.

In an effort to address this degree of misunderstanding, the following analogy presented is useful. Imagine two buckets into which flow opportunities (as in Figure A). One bucket represents opportunities for white Australia while the other is for Aboriginal and Torres Strait Islander Australia. In this context, “opportunities” mean opportunities in terms of access to education, enterprise, employment, health and housing. In addition to this is access to human rights such as equal wages, rightful access to wages earned, and the ability to do things such as own property, or move around freely within society without restriction.

Historically then it is clear that in “the lucky country” where we say “fair go for all”, white Australia has enjoyed a much greater flow of opportunity than Aboriginal and Torres Strait Islander Australia. Given this flow of access to opportunity in comparison to very restricted Aboriginal and Torres Strait Islander access to the same, the two buckets are comparatively at quite disparate levels.

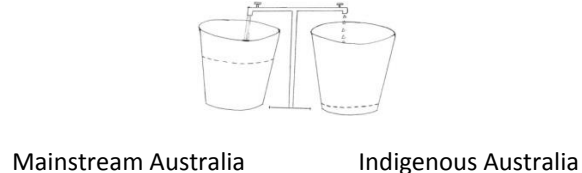


Figure A. Historical buckets of opportunities

Reflection on the discrepancies provided by this diagram simply provides a different way of observing those discrepancies in education, employment, health and criminal justice statistics presented earlier in this chapter. If as many mainstream Australians say, they want “all Australians to be equal”, then clearly both buckets must be at the same level (as in Figure B).

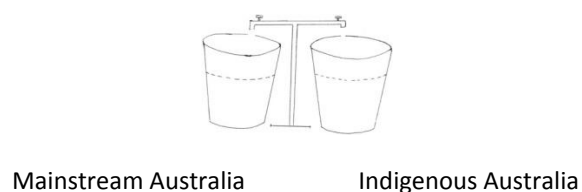


Figure B. Buckets of equal opportunities

For this level of equity to realistically occur, and both buckets eventually enjoy the same levels, one of four things would need to occur.

First, one could take opportunities away from the Mainstream Australia bucket to the extent that it is aligned with Aboriginal and Torres Strait Islander Australia. We could just tip some out while maintaining the flow into the other. As we extrapolate from the analogy this means taking opportunities away from white Australia so that their unemployment rate is 23%, the same as in Aboriginal Australia, and their school completion rate is brought back from 73% to 32%. Mainstream Australia's infant mortality rates would have to increase 3-5 times. Realistically this could never occur. It is interesting to note that while Mainstream Australia harbours the perception that Aboriginal people are privileged, I could never see them line up to be "this privileged". It is also worth noting that many Aboriginal and Torres Strait Islander Australians, having experienced the frustration of access to opportunity, would not want other Australians to endure the same demoralising experience.

Second, one could tip some out of Mainstream Australia's bucket, and increase the flow into the other until they become even. To extrapolate again this would mean restricting access to, or taking opportunities away from Mainstream Australia while increasing access for Aboriginal and Torres Strait Islander Australia. Again this is not realistic, and clearly, it is inhumane to see human beings denied access to such opportunity; even if white Australia did this to Aboriginal Australians for many years.

Third, one could stop the flow into Mainstream Australia's bucket and retain or increase the flow into Aboriginal and Torres Strait Islander Australia's bucket until the levels are aligned. Again, any effort to stop a human being's access to education, enterprise, employment, health and housing is inhumane. It is inhumane to stop access to human rights such as equal wages, rightful access to wages earned, and the ability to do things such as own property, or move around freely within society without restriction.

Fourth, one could maintain the existing flow of opportunity for Mainstream Australia, but then increase, above Mainstream Australia, the flow of opportunity to Aboriginal and Torres Strait Islander Australia, until both levels are the same. Once both levels are the same, we then ensure the flow to both is equal. If we do not increase the flow to Aboriginal Australia, it will never catch up to Mainstream Australia. Maintaining the existing flow to both buckets will see the discrepancies perpetuated. As we extrapolate this option we see that Mainstream Australia is not being denied any access to human rights and opportunity. We are not turning their tap down or off.

4.1.3 Increasing the flow of opportunity

In the interests of achieving equity, it is necessary to increase the flow of opportunity to Aboriginal, Torres Strait Islander and low SES Australia until all people have equal access. In real terms this means things like financially supporting families so their children can participate fully in school, providing special places in university courses for students and providing specific employment and enterprise programs that target the Aboriginal, Torres Strait Islander and low SES people. The programs in question do not represent a crutch for Aboriginal, Torres Strait Islander and low SES people to lean on, but rather they are a means to emancipation – insufficient perhaps, but still necessary.

School change and strong leadership can successfully address competencies in complex situations to create a stimulating learning environment where Aboriginal, Torres Strait Islander and low SES students can excel in their learning. Eurocentric and middle-class approaches to the teaching and learning of mathematics continue to exclude and marginalise these students from the very opportunities that education is purported to provide for their future (Rothbaum, Weisz, Pott, Miyake, & Morelli, 2000; De Plevitz, 2007). Because mathematics continues to be taught in ways that appear foreign to these students, that is, decontextualised and removed from students' cultures and communities, students are unable to gain successful entry points into learning about the subject, particularly if they are Indigenous (Matthews, Cooper, & Baturo, 2007). As a consequence, students are often blamed for their own failures rather than how the mathematics has been taught. The COAG *National Indigenous Reform Agreement* (2008) is a Federal Government policy which has identified six targets to address the disadvantage in Indigenous education, health and employment outcomes. Two of these targets

are (a) halving the gap between Indigenous and non-Indigenous students in reading, writing and numeracy by 2018; and (b) halving the gap for Indigenous students in Year 12 or equivalent attainment rates by 2020.

4.2 School change and leadership cycle

YuMi Deadly Maths has been influenced by the philosophy and success of the Stronger Smarter Institute and examples of leadership that have successfully changed learning in Indigenous schools (Cooper, Baturo, Warren, & Grant, 2006; Douglas, 2009; C. Sarra, 2005b, 2007). In particular, YDM supports the Stronger Smarter Institute's belief that change comes when students are strong in their hearts, proud of their identity, solid in their community, and smart in the way they do things – focused on high achievement and determined to succeed.

The YDM position on school change and leadership cycles through four imperatives (see Figure 4.1): **community–school partnerships, local leadership, positive student identity, and high expectations**. It aims to develop not only new capabilities but also shifts in thinking individually and collectively. The four imperatives are particularly important for Aboriginal, Torres Strait Islander and minority groups so as to increase community capacity within schools; that is, to empower Aboriginal and Torres Strait Islander people and minority groups to have a voice from the local community. The diagram below shows the cycle for Aboriginal and Torres Strait Islander people.

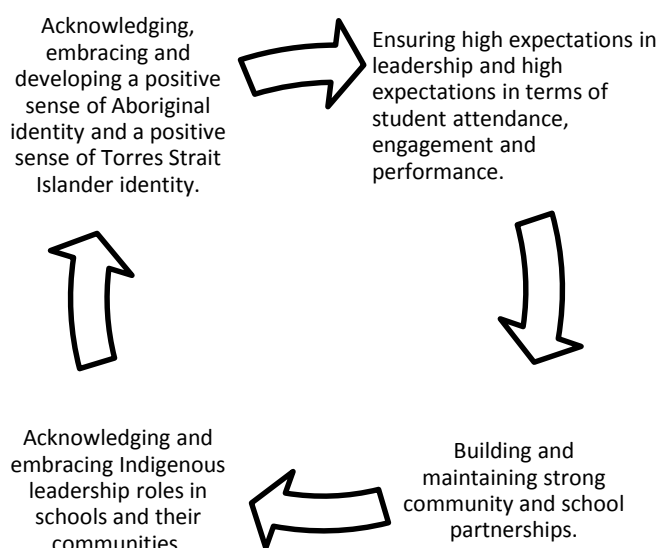



Figure 4.1 Cycle of school change and leadership

 In the following pages, the symbol on the left will be used to show that a school community and leadership imperative is being discussed.

The purpose of this section of the resource is to describe how to achieve these four imperatives and to keep the cycle moving, ensuring sustained growth towards enhanced learning. It aims to provide information crucial to supporting schools and their extended communities in creating and sustaining emancipatory environments that enhance the opportunities of the students who attend, and to challenge some of the mechanisms and processes that continue to reproduce disengagement among Aboriginal, Torres Strait Islander and low SES students within the schooling system. The development of an open, trusting and reciprocal relationship between the school and the Aboriginal and Torres Strait Islander communities can lead to innovative and dynamic opportunities to incorporate new knowledge into the learning and culture of the school.

4.2.1 Community–school partnerships


The first imperative of YDM's school change and leadership focus is building and sustaining strong school and community partnerships. The successes of changes to improve learning in Aboriginal and Torres Strait Islander and low SES schools in Queensland and other states have been highly related to the quality of the school–community partnerships. In this, it is important to note that the partnerships were different in each case.

A strengths-based approach is ideal when using YDM as this approach values the importance of working with our Aboriginal and Torres Strait Islander communities and utilising their strengths and aspirations to help facilitate productive change. McCashen and Anglicare Australia (2004, p. 11) stated:

The term “strengths approach” is used because it is an approach that is dependent primarily on positive attitudes about people and their potential for change, growth and learning.

This type of approach privileges the voices of Aboriginal and Torres Strait Islander people and their ways of knowing and being, which shares the power structure that has often been exploited by dominant cultures due to their ethnocentric views of superiority. In a school this genuine and authentic form of collaboration empowers individuals and communities to be part of the decision making and change processes required to enhance the learning of students.

Successful school change programs take account of the particular characteristics of both school and community (including abilities and needs of staff, students, parents and community members and the relative location and geography of the school and community). Successful programs in other schools can provide direction but no two programs can be completely alike and examples need to be adapted not copied.

 The development of good community–school partnerships results in close collaborative relationships and provides more efficient scope to improve outcomes for Aboriginal, Torres Strait Islander and low SES students; effective case studies are provided by Douglas (2009) and C. Sarra (2005a, 2005b) (see **Appendix A**: Two effective school–community partnerships). Issues relating to truancy, health, behaviour and juvenile justice are matters that should be resolved through the school and community working together in collaboration rather than the school acting in isolation. To develop such a partnership, schools need to work with their students, the parents and carers of their students and the community. This can be difficult if the community is not coherent or identifiable, is hostile, or believes that education is not their business. A list of ideas to get started follows.

1. **Have the right attitude and beliefs.** It is important to believe in partnerships, not be arrogant about qualifications and appearances, and not take a superior position. For Indigenous people, it is important to follow Smith's (1999) set of guidelines, as follows, for the protection of the rights, interests and sensitivities of people being studied:

- Respect for people
- Present yourself to people face-to-face
- Look, listen ... speak
- Share and host people, be generous
- Be cautious
- Do not trample over the mana (spirit and cultural heritage) of people
- Do not flaunt your knowledge – be humble

These appear to be also applicable in low SES schools and communities, and in any school with a recognisable cultural group.

2. **Find key stakeholders.** For Aboriginal and Torres Strait Islander schools, these are most likely to be Elders, traditional owners, Indigenous teacher aides and/or Community Education Counsellors, the community council, community organisations or relevant Indigenous school support centres located within the various jurisdictions of education departments. For low SES schools, look within education bodies similar to the above, community organisations, sporting clubs or any group that is successful (even the P & C). In all

cases, it is important that you try to use local people in the school to get started, but take care that you do not contact only one faction, and remember that gatekeepers may not be people in official positions.


3. **Start from the community's position.** Find out community members' strengths, expectations and difficulties with the school. Use this as a starting point to work out a position acceptable to both you and the community. Set up a committee to follow through on breaking down barriers to reduce problems and increase expectations. This requires many face-to-face visits and a willingness to leave the school. It also requires setting up a code of practice for involvement of school in community and community in school.
4. **Involve the community in the school and the school in the community.** Encourage community members to visit the school and be involved in the learning, and organise for the school to take part in community celebrations. Organise teachers to visit homes and to welcome locals to their classrooms. If possible, set aside school buildings/grounds for community use (e.g. coffee lounge, Elders' room) and ask for community buildings/grounds (e.g. local gymnasium) for school use. Organise the use of school resources for community use out of school hours (e.g. oval, computers, library) and vice versa (e.g. after-school programs). If possible, involve community in school decision making (including budgets).
5. **Use locals for all school jobs and make local community knowledge a legitimate part of the school curriculum.** Employ locals, set up a local knowledge/culture subject, and encourage teachers to use local knowledge – see 4.2.2 below.
6. **Make the wider society aware of your plans.** Work closely with the Department of Education and Training (DET) hierarchy (involve regional directors, specialist officers). Liaise regularly with police, hospitals, justice groups, churches, etc. Apply for extra funding from everywhere.

The benefits of school community partnerships are significant on three levels as follows.

- (a) the students have role models in school who are local and who are gaining or have gained educational qualifications;
- (b) the school has staff who know the students and their culture and can partner with teachers from outside; and
- (c) students are less likely to vandalise what locals have built or to confront behaviourally a local person.

4.2.2 Local leadership

The second imperative of YDM's school change and leadership focus is acknowledging and embracing local leadership roles in schools and school communities. This ensures that students are always in situations where there are Aboriginal, Torres Strait Islander and low SES models of leadership, and where there are people who can make decisions from a situation where they understand the reasons that students may have for not attending school or engaging in the classroom. It can be crucial for developing students' pride in their background and culture, and students' beliefs in the efficacy of school learning in assisting them to achieve their dreams. However, more than this, embracing local leadership enables transformation processes that enhance learning to emerge in schools and their communities.

 Local leadership is a contentious issue in that it is more than just employing locals, it is giving them power over non-locals in at least some situations. This can cause dissension in schools as power is often associated with educational qualifications, and local staff do not often have the qualifications that teachers would see as legitimising their power. It requires special management of teacher aides who provide classroom support to teachers (see **Appendix B**: Application at Cherbourg of Drucker's (1996) elements for teacher aides). However, where local leadership has been embraced, there have been spectacular improvements in attendance and learning outcomes; and where it has not, many destructive errors are evident. A list of ideas to get started (possibly in order of difficulty) is as follows.

1. **Have a clear mission for employing local staff and implement this.** The school and principal have to articulate local leadership as a clear goal so that everyone knows what the school stands for and what it


intends to achieve (the first element of Drucker, 1996, see **Appendix B**). It would be advantageous for this and other imperatives if this mission is developed collaboratively between the staff and community. Once the mission is developed, it should be implemented as soon as possible.

2. **Use the school–community partnership to implement local leadership roles.** Employ locals as general staff (e.g. reception, administration), grounds people (e.g. gardeners, cleaners), and teacher aides. Employ locals for all the maintenance jobs (e.g. painting, carpentry, windows) and tuckshop/catering. Set up a local knowledge/culture subject and employ local staff/teacher aides to plan and teach it. Let students see many locals having authority in the school.
3. **Encourage staff to accept local leadership.** Use the aim to enhance students' learning outcomes to argue for the strong role that local leadership plays in motivating students and for ensuring that there is cultural empowerment. In particular, raise the issue that there are situations (e.g. cultural and behavioural) where it is important that local teacher aides make decisions.
4. **Treat local positions as real positions.** The local positions have to be seen as real positions; they must bear the same responsibilities for results as other staff (e.g. confronting negative behaviour, high expectations). They must also be subject to appraisal and review; goals should be set with consequences if not met. The principal should have the same high expectations of local staff as of students. (This is the second element of Drucker, 1996, as discussed in **Appendix B**.)
5. **Set up training programs for local staff.** It is important to offer local staff opportunities for continuous learning, particularly for teacher aides. Local staff bring local knowledge into the school but they need to be provided with opportunities to expand it so they can take better paid roles; they should be encouraged to attend workshops, seminars and conferences like regular staff and to enrol in courses to gain higher qualifications. (This is the third element of Drucker, 1996, as in **Appendix B**.)
6. **Require teaching staff to form real partnerships with local teacher aides in which power is shared.** Present schools are failing Aboriginal, Torres Strait Islander and low SES students. Respect for local input represents the change needed to remediate this situation, and includes power sharing between non-local teachers, particularly with respect to culture and behaviour.
7. **Become a community-based school.** This is setting up procedures where principal and staff share power and decision making with the community with respect to how the school operates.

In practice, local leadership has been difficult for many schools, even those who have had success in their endeavours to improve learning, but represents one of the most influential changes on the overall ethos of the school.

4.2.3 Positive student identity

The third imperative of YDM's school change and leadership focus is acknowledging, embracing and developing a positive sense of Aboriginal, Torres Strait Islander or low SES identity (and other minority cultural identities) depending on the student. This is obviously vital to gaining a sense of individual success; students who have no pride in themselves nor belief that they can succeed are beaten before they start. However it is a contentious imperative; it can be seen as reinforcing the ideology of social mobility and meritocratic practices whereby anyone can become anything they want to be if they study and work hard within the existing Eurocentric school system. This ideology provides an illusion that traditional Eurocentric schooling is equitable and that failure is the fault of the student; in fact there is little mobility, dispossessed people's children predominantly stay dispossessed and the fault is with the system.

 For YDM, positive identity is seen in terms of the student, to improve the student's choices, and may require the school to change as much as the student.

This may also be one area where there are differences between Aboriginal and Torres Strait Islander students and low SES students that require different school actions. The former come from proud cultures which conflict

with the middle-class culture of schools, and positive identities can be built around those cultures. The latter have been judged inferior within their culture so positive identities have to be built by showing that their situation is not their or their families' fault. However, low SES students would benefit from realising there are strong cultures with the same issues as they have and which have been oppressed by their culture, and Indigenous students would also benefit from insight into systemic inequity and the unfairness of blaming the victim.

Activities that can build positive student identity are at one end straightforward whole-school processes such as having a school song that resonates with the students, and at the other complex pedagogical strategies requiring identification of characteristics of local culture (see Indigenous studies for all schools below).

Indigenous studies for all schools (G. Sarra, 2007b)

Embedding Indigenous perspectives in schools inclusively or as a discrete unit of work is needed in ALL schools for ALL students. It is crucial to stress here that this requires not an optional “boutique” course that will provide nothing more than a romantic and palatable tour through the land of the exotic *other*. Rather, what is required is nothing less than the teaching of Indigenous studies to ALL students in ALL schools, because all Australian students need to learn the truth about Australia's past so that, in turn, they can develop a foundation for a better understanding of the present situation of Aboriginal and Torres Strait Islander people – in the hope of making a better future for all of us (G. Sarra, 2007b).

It is also vital to note in this context that the development and delivery of Indigenous studies programs in schools is not primarily an attempt to lay blame for past atrocities; at the same time we should not seek to avoid the actuality of such events. Nor is an Indigenous studies program an attempt to promote any particular political stance; it is rather, in its essence, a purposeful and instructive structure, with the goal at its core of promoting social justice through education.

The incorporation of an Indigenous studies program into the syllabus recognises how Aboriginal people and Torres Strait Islander people and their cultures influence contemporary Australian society. An Indigenous studies program has scope to identify and acknowledge aspects of the cultures of Indigenous Australians and to provide views and space for debate around current Indigenous issues in Australia. It will enable students to learn about the ways of life of Indigenous peoples in Australia's past and present, how these have changed over time, their contribution to Australia today and their potential contribution in the future.

A list of ideas to get started is presented under the two headings as follows.

Whole-school processes

1. **Set up processes for building pride in self and school/community.** Initial processes are built around visible school-wide parade activities such as slogans (e.g. “Strong and Smart”, “Proud and Deadly”, “Bold and Brilliant”), school songs, and uniforms developed by the school community. These need to be created to relate to strengths of the school and community, and related to a system of school-wide rewards and incentives. However, they must also be related to ongoing work within classrooms challenging the students to live up to the slogan and using role models to show why one should be proud to be from that community.
2. **Set up whole-school processes for challenging behaviour.** Schools need a common behavioural management program used consistently in each classroom. Without this, unacceptable behaviour can prevent the best mathematics instruction activities achieving their goals. The most effective behaviour management programs focus on rewarding acceptable behaviour and building the ability of students to resist unacceptable activities of other students as well as deterring unacceptable behaviour. **Appendix C, section C1**, Activities to enhance behaviour, describes six behaviour management ideas.
3. **Set up whole-school processes across the school for supporting attendance.** Once again these are initially visible school-wide parade activities to reward attendance (e.g. by visits to popular fast food

establishments). However, they require ongoing commitment by teachers and aides to monitor students, and changes in school and classroom processes to attract students to the classroom (e.g. **Appendix C, section C1**, activities that support behaviour also support attendance).

4. **Ensure all classrooms are culturally and socially safe and empowering.** It is important to ensure productive teaching and learning pedagogy is meaningful to the social and cultural contexts of the local learner, particularly for Aboriginal and Torres Strait Islander students. One way to do this is to ensure that all teaching is culturally safe; check that the curriculum does not focus on issues that affront, in particular, Aboriginal and Torres Strait Islander students. A second way is to incorporate local cultural programs taught by Elders, local teacher aides and other community members into the curriculum. Such cultural studies programs are an important adjunct to programs to build positive identity. Interestingly, experiences in Cherbourg showed that such studies needed to go beyond boomerangs and bush tucker to discussions of the issues that affect present day communities, land rights, unemployment, crime, drunkenness, and abuse (G. Sarra, 2007b).

Pedagogical strategies

1. **Ensure all classrooms provide deep support for the school processes.** The school processes run at parade set the scene for programs to enhance learning in Aboriginal, Torres Strait Islander and low SES schools; programs in individual classrooms find it difficult to be successful if they are in isolation. However, the reverse is also true, central programs also have a low chance of success unless they are supported at the classroom level. In the most successful programs (C. Sarra, 2005a), this classroom support is provided at a deep level, through the class discussions that follow up the slogans and the rewards. This is where students discuss the questions about why it is important to be strong and smart and what has to be done to achieve this. Effective programs at the school level seem to be ones in which the school management gives precedence to the program and to leading the program and particularly takes this leadership role in the visible aspects of the program, the building of pride and the challenging of behaviour, but that this is consistently followed up with discussion at the classroom level and with the support of all teachers and aides. There are classroom activities that help support behaviour and attendance. One of these is in **Appendix C, section C2**: Brain Gym which focuses on activities to improve attention.
2. **Develop a strategy for encouraging an identity positive to the students.** The major focus of this section is to find a way to encourage a positive identity. This includes changing students' beliefs about themselves and their culture and background; they have to associate themselves and their culture with intelligence. The students need to come to believe three aspects, that: (a) they have the ability, with hard work, to understand and succeed at school (including in mathematics) to the same level as mainstream students; (b) succeeding at school, and working hard to do it, is important and a vital part of their future; and (3) this success does not have to be achieved by the student rejecting their background and culture. To do this requires identifying characteristics of the students, and their background and culture that can be used as a catalyst for this change in identity. In Cherbourg school, the "Strong and Smart" slogan was used; the principal redefined it to mean that students should work hard at school, and not fulfil the perceptions of most non-Indigenous people that Indigenous students fail. In Dajarra school, the upper primary teacher built a program around "Aim high, Beat yourself" (see Cooper et al., 2006) where he redefined for his students what being a success was (it was not beating others but trying as hard as you could and doing better than you did yesterday). He found that the students were happier trying hard to beat themselves than they were trying to beat others. It also meant that every student could be a success regardless as long as their performance improved.
3. **Set up culturally empowering learning spaces.** Schools need to be spaces where (regardless of their home context) students can experience a positive insight and understanding of what being themselves is all about. Schools need to be aware that, although Aboriginal and Torres Strait Islander and low SES students' home lives may include domestic violence and alcoholism, their role is to create learning experiences which enable students to understand that these unpalatable dynamics are the legacy of other historical

and sociological processes, and NOT the legacy of being an Aboriginal person, Torres Strait Islander or low SES person.

4. **Make learning intrinsically interesting.** Many Aboriginal, Torres Strait Islander and low SES students no longer believe in school-generated social mobility (that hard work in school will enable them to get a good job) although many still believe that sport may enable this. The situation can, of course, be tackled by changing students' opinions of school and showing them how success at school can open up life and employment opportunities. However, to achieve change in students' opinions, they have to be at school. Although, increased attendance can be achieved through various governmental interventions, making school intrinsically attractive so that students attend because of the interesting nature of school activities may be a more productive move. This requires the school to cater to the needs of the students and appeal to their desire to know. Flexibility in school hours and pedagogy could also be considered.

4.2.4 High expectations

The fourth imperative of YDM's school change and leadership focus is ensuring high expectations in leadership and high expectations in classrooms in terms of attendance, engagement and performance. Teachers' beliefs of their principal's perceptions of their capabilities as teachers and students' beliefs about their teacher's perceptions of their abilities can become self-fulfilling prophecies. Here, teachers and students may have a tendency to meet their supervisors' expectations. If the expectations are low, then performance is likewise (Wong & Wong, 2004).

It is almost impossible to hide low expectations; they emerge in small things such as lack of criticism. Underachieving schools tend to have both leadership and classroom expectations as low. To turn achievement around, these expectations have to become high which means that both principals and teachers need to change their beliefs. However, beliefs are difficult to change and many successful examples of school improvement only happened when low-expectation principals and teachers left and were replaced by high-expectation staff. Sadly, the converse is also true; high-performing schools have declined when high-expectation staff left.

High expectations in leadership

"High expectations in classrooms" is a common mantra for emancipatory school change; however, YDM's high-expectation imperative argues that it must be integrated with high expectation in leadership. This means that management (particularly the principal) should have high expectations of staff in the same way as a teacher has high expectations. This can be challenging for management as, like teachers with students, they have to appraise and criticise and not accept low performance. A list of ideas to get started is as follows. Note that most of these ideas integrate with the ideas for getting started with the other imperatives in the cycle.

1. **Ensure high expectations are part of the mission for the school and a whole-school process.** All staff need to be appraised and reviewed re expectations, including the leadership group. Staff and students need to see that the leadership of a school supports the high-expectation program before they will give the program the credit that will enable the school to regenerate academically. Thus, high expectations in classrooms need to be supported by the leadership group, particularly as it can lead to initial difficulties with the local community. The "Aim high, Beat yourself" program (Cooper et al., 2006) was criticised by local parents and carers until they saw it begin to work in positive ways for their children.
2. **Build a culture and ethos of high expectations.** As described in **Appendix C, section C3**, Organisation culture and leadership, a culture of high expectations can be built within schools, and a culture of low expectations can be destroyed (Schein, 2004). This may require the leaders to step outside the culture that made them.
3. **Organise professional development and visits to successful schools.** The move to high expectations can be a large step for some teachers and aides; professional development, discussion groups and visits to

schools successfully using it really help. The Woorabinda success was built on teachers and aides visiting Cherbourg.

4. **Encourage teachers to set up classroom plans.** A powerful way to assist teachers and aides to have high expectations is to encourage them to set up “high expectation” plans to achieve improvements in student performance and to keep records of this improvement using an action-research approach (Douglas, 2009).
5. **Be prepared to transfer staff who cannot meet high expectations.** Sometimes staff cannot fit into a culture; the only option is that they be moved on. In this, it is useful that the Queensland Department of Education and Training hierarchy is aware of the mission of the school.

High expectations in classrooms

Maintaining high expectations of students for student success is having a belief that students have the ability and can learn. As Wong and Wong (2004, p. 10) stated:

The belief in positive expectations is based on the research that whatever the teacher expects from the learner is what the learner will produce. If you believe that a student is a low-level, below average, slow learner, the student will perform as such because these are the beliefs you transmit to the student. If you believe that a student is a high ability, above average, capable learner, the student will perform as such because these are the expectations you transmit to the student.

The expectations that teachers display and continually reinforce to students influence the achievement of those students in their classrooms. If teachers communicate high expectations to the students then the students will try to succeed to meet those standards. This is the key to the pedagogical philosophy of high expectations. As C. Sarra (2005b) stated with regard to his use of high expectations in the Aboriginal school at Cherbourg:

Having low expectations of Aboriginal children in schools and tolerating things like absenteeism and bad student behaviour contributes significantly to the process of engineering a negative Aboriginal identity. Many schools, and many teachers within schools, without realising, are guilty of playing a part in such a process, and such processes will not be reversed until mainstream Australians in our schools and communities understand, develop and respond to the Aboriginal perceptions of being Aboriginal.

The process of challenging students to reflect on their identity and what it truly means to them, is indeed quite challenging in itself, and requires individuals to look deeply inside themselves. Here, the process can be greatly enhanced by effective teaching and learning practices and by activities that build positive identities such as Human Values Education subjects (G. Sarra, 2007c) and teaching techniques (see **Appendix C, section C4** for more explicit detail).

Human Values in Education is a program that has been implemented in schools with predominate numbers of Indigenous students however it is a program that can be used in all schools regardless of ethnicity and culture. It presents a background to human values, recognising the changing role of family in structuring values within our society. The program is most successful when implemented throughout the whole school. It involves using seven teaching techniques to teach a particular over a fortnight: mind map, quotation, story, music, group activity, silent sitting and creative visualisations (G. Sarra, 2007c).

4.3 Sustaining the change

Strong leadership that openly recognises the identities of Aboriginal and Torres Strait Islander students, instils a sense of pride in a student's heritage, and advocates high expectations of students, creates a highly functional and organised school culture (C. Sarra, 2003; Schein, 2004).

Improvements in learning by Aboriginal and Torres Strait Islander students in successful school–community partnerships and leadership projects tend not to be sustained when the staff that are responsible for the improvements leave the school (e.g. Douglas, 2009). The situation would be similar in low SES schools. To be sustainable, emancipatory change must continue after staff leave. How to do this is difficult because it requires the change in the first place, which is difficult enough, and then the special setting up of this change so that it continues beyond the school participation by existing staff and students who made the change.

4.3.1 Principles of effective learning and teaching

One positive for sustainable improvement in underachieving Aboriginal, Torres Strait Islander and low SES schools is that the Principles of Effective Learning and Teaching developed by Education Queensland (1994) advocate fairness and equity and acknowledge the rights and responsibilities of individuals to maximise access, participation and educational learning outcomes for all students. These principles include the following defining characteristics of effective learning and teaching:

- is founded on an understanding of the learner;
- requires active construction of meaning;
- enhances and is enhanced by a supportive and challenging environment;
- is enhanced through worthwhile learning partnerships; and
- shapes and responds to social and cultural contexts.

These principles support YDM’s school–community and leadership imperatives and cycle. They suggest quite strongly that if a teacher knows more about the social and cultural context of the learner, then there is much greater scope for effective learning and teaching to occur. These principles or others that might bear similar meaning should be constantly articulated in school–community and leadership programs, stressing the point that educators have a very important relationship with students and their parents and community. Accordingly there is incumbency upon educators, who stand by such rhetoric, to “shape and respond” to the social and cultural context of the students rather than blaming it for the high degree of failure. The principles must anchor good classroom management, behaviour management and a learning environment with high expectations (G. Sarra, 2007a).

Thus, if these principles are accepted by administrators and teachers, it means that new staff in a school with success in improving Aboriginal, Torres Strait Islander and low SES learning should continue the positive work by the previous staff. However, there are many ways to build successful school improvement projects and most will have local characteristics, so it is still important for sustainability that successful school projects have characteristics that enable new staff to become acquainted with them and come to own them.

4.3.2 Facilitating sustainable change

It is not easy to bring about emancipatory change and even harder for this change to last after the original staff leave. Some pointers on how to do this are below.

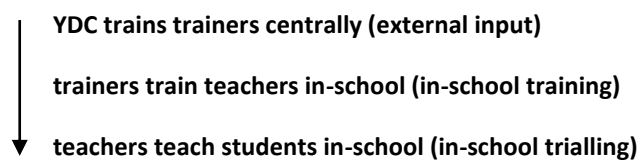
1. **Departing staff have to give up personal ownership characteristics if project is to be sustainable.** A project that is going to be taken over by others must be capable of operating without the specialist skills of one person. Therefore, to ensure sustainability, projects must be modified so that individual characteristics of staff are not important. This also applies to marketing the project so that it is identified with one person.
2. **Sustainable projects should have as many generic components as possible.** Staff coming into a project will not have particular knowledge of the community but will have generic knowledge of the type of project; they can more easily take over activities that are generic.
3. **Sustainable projects should be capable of component change.** Staff coming into a project might feel that certain components should be replaced (e.g. the behaviour management system). This should be possible without harming the effectiveness of the project.

4. **Sustainable projects should include documentation, professional learning and change-over.** People go but documentation remains and good documentation (similar to a textbook) can provide sustainability on its own. Professional learning brings new members up to speed. Change-over periods allow new staff to be inducted. Staggered change-over also helps.
5. **Sustainable projects should vest ownership in community and local staff.** The community and local staff tend to remain. They can provide the project commonality from one group of staff to the next. Professional learning should be vested with them.
6. **Sustainable projects should build in change.** Staff change but so do the community and the context of the school. Sustainable projects not only can handle change, they have change processes built in.
7. **Sustainable projects should build a school culture.** As described in **Appendix C, section C3** (Organisational culture and leadership), a change project can build a culture that remains after staff leave. This is common for elite schools where complete change in staff would not affect what the school stands for and aims to do. Similar can be achieved in underachieving Aboriginal, Torres Strait Islander and low SES schools. Students choose the school for its outcomes and staff volunteer to go to the school because of its mission. For example, a ghetto school in Los Angeles built a reputation of strong scores in algebra and had maintained this over many years.
8. **Sustainable projects need the support of the state Department of Education and Training (DET) hierarchy.** School staff change but the DET structure remains. Thus, DET can provide the base for ongoing activities to make sure that change in a particular school is sustainable.

5 Implementing YDM

Implementing YDM is a collaboration between YDC and school staff. It involves resources, training, trialling, reflecting and support. YDM expects that teachers will become familiar with mathematics as a language, structure and problem-solving tool, and will be able to use the YDM pedagogical framework (Planning–Teaching and RAMR cycles). If this is done, then student outcomes are excellent, but it often requires a willingness to **learn and change** from teachers.

As well, the schools collaborating with YDC often have attributes and programs in place that YDM needs to fit into and its activities tailored around. Most commonly, funding is not available for YDC to directly train all teaching staff. This means a **train-the-trainer approach** is used where:



This three-step process leaves two steps as in-school activities where, depending on funding, YDC staff may not be able to be present. This leaves a **major part of implementation** to the school.

In this section, we look generically at the process used by YDC to work with schools to implement YDM. We discuss the framework for implementation, the action-research processes and the roles of participants, the development of school plans, and how YDM can support testing regimes such as NAPLAN, lesson plans such as C2C, and students performing well below their age level.

5.1 Framework for implementation

This first section describes the framework by which YDM is usually implemented – looking at the model of professional learning (PL) and teacher change followed, the activities provided, and the steps for implementation.

5.1.1 Change model

The implementation of YDM is based on PL and teacher change being a cycle of affective readiness, pertinent external input, effective classroom trials, positive student responses, and supportive reflective sharing (Baturu, Warren, & Cooper, 2004; Clarke & Peter, 1993) that leads to further readiness and so on (see Figure 5.1).

The change model recognises that positive student responses along with initial readiness are crucial to successful change and that these are facilitated by:

- pertinent, relevant and innovative ideas and materials at input (the YDM resources, the PL activities and the website);
- just-in-time support before and during in-school training and classroom trials (on planning and modelling training and instruction);
- support of community, system, principal and other administration staff (e.g. Assistant/Deputy Principals, Heads of Curriculum, Heads of Department); and
- responding to feedback to data gathering in an action-research process during in-school training and classroom trials.

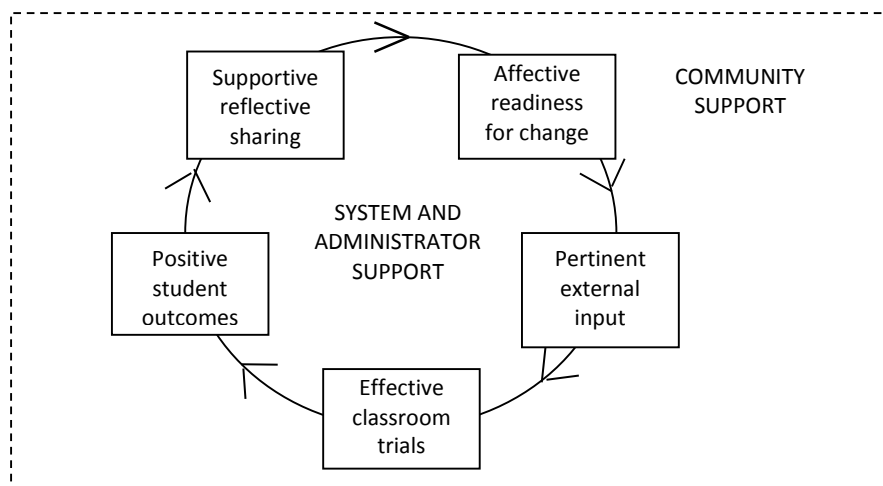


Figure 5.1 The YDC effective PL cycle (adapted from Clarke & Peters, 1993)

In particular, the YDM resources and PL workshops have been designed so that:

- their focus encompasses school change and leadership (principals, community, system and administration support) as well as mathematics and its learning and teaching (teachers);
- they provide examples of classroom activities designed to maximise mathematics learning outcomes by valuing local culture and knowledge, engaging student interest, building high teacher expectations and enabling positive student identity;
- they provide a framework for principals and teacher-trainers to work together to set up a supportive in-school training and trialling process;
- they set up contact between school (principal and trainers) and YDC staff to provide online support for in-school training and trialling; and
- they provide information so that each school runs an action-research process that provides feedback to amend resources and processes.

Thus, the YDM model relies on principals and key teachers participating in workshops to be:

TRAINERS

CHANGE AGENTS and

RESEARCHERS

This is a lot to ask of busy teaching professionals so, to assist, the YDC researchers provide **face-to-face and online training and resources**, including books and website resources, covering:

- the YDM pedagogy and philosophy and beliefs regarding school change and leadership;
- YDM strategies for teaching Number, Operations, Algebra, Geometry, Measurement, Statistics and Probability (including representations, problem solving and connections);
- PD workshop materials to support in-school training and trialling, and preparation for professional sharing; and
- materials to assist action-research data gathering.

As well, YDC staff will be assigned to each project to keep in regular contact. YDC pledges to **never give up on any teacher and school that requires help and is willing to work hard to improve the mathematics learning** and, therefore, the employment and life chances of Aboriginal, Torres Strait Islander, and low SES students.

5.1.2 Activities

YDM is normally implemented in a school as a **two-year train-the-trainer project** to build the capacity of the schools in relation to teaching mathematics to their students. The normal focus of the training is Indigenous and low SES students, but YDM works for all students. The train-the-trainer project is normally two stages across two years and focuses on the following each year:

First year: YDM's aims and activities, philosophy and pedagogy, school change and leadership, the *Australian Curriculum: Mathematics* strands of Number and Algebra, and action research and contact with YDC.

Second year: Review and reflection on the implementation in the first year, continued review of YDM, action research and YDC contact, the *Australian Curriculum: Mathematics* strands of Measurement and Geometry, and Statistics and Probability, and setting up sustainable YDM pedagogy in the school or region.

However, to take account of remote schools, particularly schools with a high turnover of staff, the above two stages may be replaced with the following three-stage process:

Stage 1 Preparation: Introduction of YDM's aims and activities, philosophy and pedagogy, school change and leadership, action research and contact with YDC, and collaboration with school to develop a project that meets the school's needs (usually one to two terms).

Stage 2 Training: Train-the-trainer activity with respect to Overview and *Australian Curriculum: Mathematics* strands of Number and Algebra, Measurement and Geometry, and Statistics and Probability (one year).

Stage 3 Sustainability: Assisting school with training new staff, and setting up sustainable YDM pedagogy in the school or region.

Across the two years, the implementation consists of the following four components.

1. **Professional learning (PL).** Across the two years, YDC provides: (a) **special PL** for Principals, Assistant/Deputy Principals, Heads of Curriculum (HoCs), Heads of Department (HoDs) and other administrators, to build awareness of YDM as a mathematics program and to develop a school plan for the implementation of YDM in school; (b) **intensive PL** for key teachers on YDM and how to train other teachers in its use (teachers who receive this training will be called **trainers** to distinguish them from teachers who are not intensively trained); (c) **specific topic-focused PL** for any interested teacher from the school to support the administration and trainers in implementing YDM; and (d) **sharing summits** for schools to share and celebrate their successes (usually at end of each year). This PL is provided **face-to-face**, **online** or as a **blended mixture**, depending on requirements of the school and funding.
2. **Resources.** YDC provides each trainer who receives PL with seven books providing information on the YDM approach to teaching mathematics and resources on how to implement this approach: *Overview*, *Number*, *Operations*, *Algebra*, *Geometry*, *Measurement*, and *Statistics and Probability*. These materials are also available on the YDM Blackboard site for all staff in the school to access.
3. **Online support.** YDC provides all project schools with support for the in-school training and trialling, and tailoring the program, through a blend of face-to-face and online methods depending on requirements of the school and funding. The online methods consist of: (a) **regular electronic communication** (e.g. school staff can email to have questions answered at any time); (b) access to the **YDM Blackboard site** containing project materials, discussion forum, lesson ideas, and extra resources that are specific to the school, school cluster, or project; and (c) an **underlying course** running on the YDM Blackboard site which provides YDM PL training, including videos, so that schools can use this with their new teachers.

4. **Action-research training.** YDC provides trainers with training in taking an action-research approach to their in-school work. Trainers are encouraged to regularly send to YDC a journal as evidence of their progress. YDC uses these journals to monitor classroom staff knowledge and practices and student numeracy outcomes, and construct theory that explains and predicts the relationships observed, particularly interactions between YDM training, resources and support \leftrightarrow classroom staff knowledge and practices \leftrightarrow student numeracy outcomes (cognitive and affective). YDC will also support any principal, assistant principal, trainer or teacher to undertake a higher degree (Masters of Education, Masters of Research, Professional Doctorate or Doctor of Philosophy) at QUT using their experience in the project as the basis of their study and research. YDC is developing a cohort of such people from all its projects and can provide cohort members with YDC/QUT Higher Degree Research support to guide them through their study.

Note: Specific extra training or school visits to work in teachers classrooms or with individual school planning can also be provided. YDC has many years of experience of working in classrooms and with **education assistants** (teacher aides) and **parents and community**.

5.1.3 Framework for implementation

Process

The PL and support activities will be organised across the two years so that there are opportunities for **in-school planning, training and trialling** between each YDC organised PL or support input. This means that the YDC input will be broken into separate components and spread across the two years. It is anticipated that the separate components will be as follows (unless school requirements combine some components):

- PL for principals/administrators and trainers (including sharing summits at end of each year);
- books and resources including an online training program;
- access to collections of lesson plans, videos, and resource books for all teaching staff; and
- online support and access to a YDC staff member as a phone and online contact.

In between professional development inputs and the summit, it is anticipated that schools will do what is needed to make the project a success:

- (a) prepare a plan indicating school direction for implementation with time for in-school training and trialling – between principal workshop and first intensive trainer PL input;
- (b) train all school staff in YDM, support trainers to trial ideas in classrooms and train and support staff to trial ideas in their classrooms, between the intensive PL inputs for trainers and between the last intensive PL input and the sharing summits;
- (c) share planned learning experiences using YDM with YDC staff and on the website between intensive trainer PLs and prepare a presentation for the Sharing Summit (to share success with other teachers and schools within the project) between last intensive trainer PL for the year and the summit; and
- (d) gather data on the effectiveness of their in-school training and trialling in relation to the teaching and learning of students and communicate this regularly with YDC staff across the year;
- (e) report on implementation of YDM in their school at the Sharing Summit.

The implementation of YDM is a combination of: (a) centrally organised PL inputs and school organised in-school activities, and of formal pre-planned training-support activities; and (b) informal ad hoc contact, and training-support-research activities. Of these activities, what is central, formal and planned is controllable by YDC but what is school organised, informal and ad hoc is not. Yet, it is these latter activities that are the most powerful and effective.

The **informal in-school processes** provide a unity to the PL inputs and school staff's actions in spite of their apparent separateness as individual activities; and, together with the formal inputs, enable opportunities for change, in spite of their separateness. Such informal processes include the following:

- schools meeting before any YDC input to discuss their involvement and give teachers ownership of the involvement;
- schools having meetings before the intensive trainer PLs to hear what was said at the administrator PL, look over this book and discuss the extent of the project, direction for implementation, and what they would like to achieve;
- the formation of a group of teachers to support the trainers and the setting up of a process by which all staff can receive some training and support, and setting up in-school processes for supporting and sharing in-school trials of YDM ideas (on top of what is available from YDC); and
- setting up in-school processes for gathering data, keeping contact with YDC, preparing feedback reports and sending/discussing these feedback reports to/with YDC, and working towards the Sharing Summit as a school project.

Training input and in-school activity

The central inputs and their relationship with in-school processes across the two years of YDM implementation are shown in Figure 5.2. PL inputs can be face-to-face, online or a blended mixture of both.

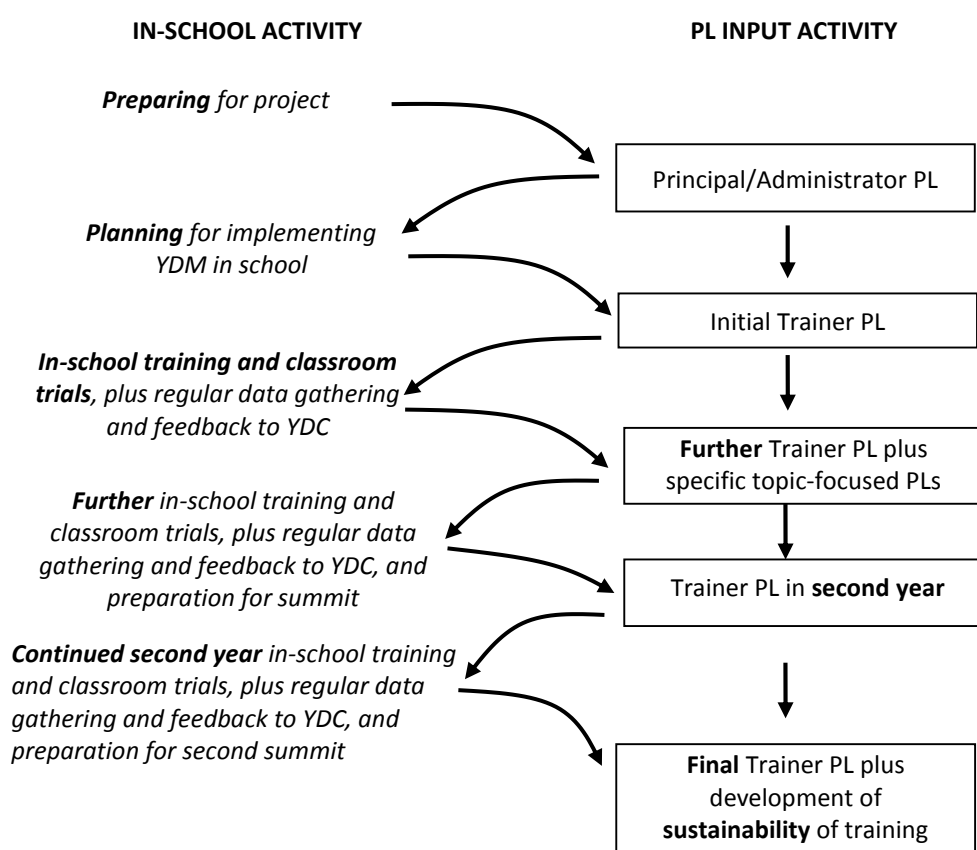


Figure 5.2 Relationship between PL inputs and in-school processes

Thus, the implementation of the YDM project is an interaction of PL input and in-school activity. This interaction can have many and few iterations. Present projects have been successful with only two trainer PLs before the summit. Others are being planned with many small trainer PLs mixed with specific PLs and even allowing schools to choose the PLs and the order in which they do them.

5.2 Research activities

This section looks at the research activities which are part of the YDM implementation. Research enables YDC to gather feedback on its YDM projects and to improve YDM's effectiveness for teaching mathematics. It also enables the teacher to adopt a powerful role in the classroom which is effective in enhancing student learning – “teacher as researcher”. This section looks at research by looking at theory and practice of action research and the roles and responsibilities of participants.

5.2.1 Action research

Change in practice does not occur simply by relaying information about teaching and learning; the power and potential of action research is needed to positively affect practice. When teachers adopt the role of teacher as researcher, their taken-for-granted assumptions about teaching and learning are challenged (Stringer, 2004). Such assumptions establish and maintain particular positions about the nature and practice of mathematics education as commonsensical (Hall, 1982).

Beliefs about teaching are largely tacit (Wertsch, 1994). That is, there is a tendency to operate from a sense of what is going on without actively reflecting on what our intentions might be and what is being said to students. Beliefs about learning and teaching can only be uncovered by engaging in systematic self-critical analysis of current instructional practices (Carr & Kemmis, 1986; Johnson, 2005; Stringer, 2004). Teaching and learning remains hidden unless we have some reason for making it explicit. Herein lies the rationale for participation in YDM.

Action-research cycle

YDC believes that the successful implementation of YDM in schools requires that school staff be involved in a school-wide action-research project. In this way, in-school training and classroom trialling of ideas can be monitored to improve student outcomes in terms of teacher capacity to teach and student learning outcomes. Considering action research as a “plan–act–observe–reflect” research cycle (Stringer 2004, p. 45) is an effective framework (see Figure 5.3) for designing action research and gathering evidence-based data related to training and teaching. An action research may be started at any point in the cycle. For example, a teacher develops a plan for teaching (Plan), enacts their plan (Act), observes student responses and learning within their class (Observe), reflects on this observation (Reflect), and then begins the cycle again by devising a new plan.

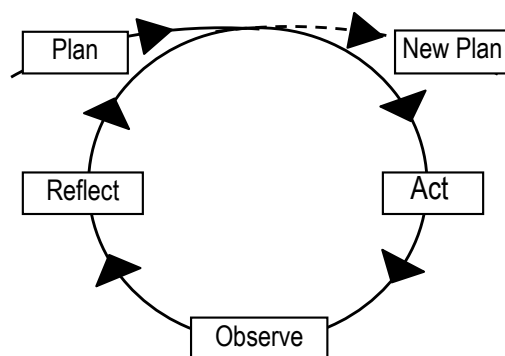


Figure 5.3 The plan-act-observe-reflect research cycle

An example of action research

Below is an example of action research in the classroom. **It shows that action-research practices fit well with the YDM pedagogical framework.**

Plan. Analysis of student work indicated weakness in place-value understanding and a plan is made to teach this area. A pre-test is recommended to provide useful evidence of this.

Act. Teaching is undertaken. For example, a RAMR cycle of activities is devised with abstracting activities encompassing body–hand–mind aspects. (The look–think–act process may be enacted in several loops within this overall cycle as individual activities are implemented to further student understanding).

Observe. Observations of classroom activities, short tests and worksheets indicate understanding of grouping tens and ones. Students’ place-value understanding improved. At this point analysis may show aspects that need further reinforcement or indicate that students are ready to continue on to further connected understandings. A post-test is a useful tool to provide evidence for reflection on effectiveness of planning and actions.

Reflect. Thinking about teaching and student learning leads to development of a new plan that may lead to a different way of teaching or new content.

While teachers express concern at the time this process takes to complete, it must be remembered that a concept taught to deep understanding with strong mental models will result in less teaching time required to extend this concept. Similarly, instances of mindful and focused implementation of YDM pedagogy supported by pre- and post-test results with examples of activities undertaken provide strong reflective evidence of the working of YDM within schools as well as a wealth of examples for teacher-trainers to draw from when creating presentations for the Sharing Summit.

Reflection and critique

Effective action research involves reflection on the observed results of activity to inform further planning and implementation of activity. It is important to see interactions within and between each tier of the project from an action-research viewpoint. As such the project team operates iterations of workshops and resources as a cycle where reflection by the team on activities, input from workshop participants through evaluation and feedback, and results of trials are important elements of planning. Similarly, interactions at each tier form their own action-research cycles.

The use of evaluation and feedback by the project team within the action-research cycle demonstrates the importance of working within a group when conducting action research. Group reflections on the outcomes of the action research means that each step of the action-research cycle is open to critique at all tiers of the project.

When conducting self-critique and reflection, an individual may make errors related to:

- (a) participants’ interpretations of the situation;
- (b) participants’ decisions about the question/s at issue, and what is and is not relevant; and
- (c) participants’ anticipations of initial and ongoing professional development.

Through involvement in action research, participants submit not only others’ account of activities to critique but also their own. This allows for shared understandings to develop between participants, more effectively illuminating positive practices and possibilities for change. Participants at each tier of interaction are a part of the situation undergoing change. Here each tier wants change, because they want to enhance learning. The viewpoints that YDM wants to support are those which are newly emerging in the course of the training itself; transcending those perceptions with which each participant started.

A genuine cycle of learning through participation in the project is operating when we can do things we could not do before. Evidence of new skills and capabilities deepens our confidence; real learning is occurring. Through participation in the YDM project activities, assumptions are made explicit, and ideas are communicated, challenged and supported through continual professional engagement in practice.

The culmination is the sharing of ideas, thus evaluating participation in the project, that is: What happened? What was your thinking, feeling at the time? What can you learn from this? What does this mean for the next time you teach?

5.2.2 Research activities

The **research activities** which are guided by an action-research model (Carr, 1986; Johnson, 2005) drive the project's practices and therefore comprise the following key data collection techniques:

- questionnaires and surveys – administered at the start and end of PL sessions and project implementation cycles; and
- ongoing reflective portfolios containing pre- and post-test results, teacher unit/lesson plan ideas and teacher analysis of results from teaching.

The research activities, along with the informal school processes, add a dimension to the implementation of YDM that gives it the capacity to overcome the lack of support at school level for training and trialling. They do this by:

- co-opting the principals, the teacher-trainers and, through them, the classroom teachers involved in trials as supporters of each other;
- involving principals, teacher-trainers and classroom teachers in gathering data on their own practices (both in-school training and classroom trials); and
- setting up ongoing contact between schools and YDC staff (the project team) that can provide some online support for in-school activities.

One way to integrate activity to ensure there is mutual support, comprehensive data gathering and effective implementation of YDM is for the school to see its involvement as an opportunity to undertake school-wide action research. However, the important point is that **the YDM project, and the activity in each school, must be a collaboration between schools and YDC**. The expectation is that everyone will learn from the experience in a way that facilitates the development of better resources throughout the implementation which in turn enhances student success in mathematics in future years.

The interactions between schools and YDC represent a synergy of research and practice. This relationship between theory and practice is a defining characteristic of design research (Cobb, 2003) and therefore drives the project's practices. While theory and practice are two different entities, they are interdependent and complementary phases of the change process.

The purpose of implementing YDM in a school project **is to improve practice**, not to simply affirm that the YDM approach succeeds. Therefore, its development is guided by ongoing formal evaluations at each tier so that key elements of the project are systematically addressed. It is also guided by informal evidence regularly gathered from principals, support consultants, teachers and students.

5.2.3 Roles and responsibilities

YDC believes that the success of change through the implementation of YDM necessitates a sustained and engaged commitment from all participants (Crevola & Hill, 2005). As Barth (1990) argues, those who have the greatest impact and influence on student achievement, progress, self-confidence, and behaviour are classroom teachers. To ensure this impact is positive requires transformational and emancipatory actions from schools and teachers.

The YDM train-the-trainer project means that interactions within schools involving teachers who are not teacher-trainers will occur alongside of formal central activities. The system and project team activity will be with principals and teacher-trainers in the PD workshops and through YDM resources. To enable the impact of the project to reach classroom teachers and students necessitates roles and responsibilities for the other four tiers.

YDC also recognises that for change to be sustainable and long lasting, it is necessary to challenge and extend participants' beliefs, understandings and practices (Crevola, 2000; see also DET, 2014). Thus, teachers' practices provide the foundation for reflecting and refining, based on students' learning needs. For this to occur requires

ongoing support to enable teachers to learn new and more complex teaching strategies and to successfully integrate and implement these into their repertoire. Therefore, maximising the impact of YDM requires the transformation of schools into organisations for lifelong learning with environments that explicitly encourage and enhance the capacity of teachers and students to be creative (Senge, 1994).

To maximise the chance to achieve the above, **all the participants** in the implementation are required to take on the roles and responsibilities designated.

1. **The system.** Education systems need to develop capacity and to do so may need the support of external systems. The initial decision of the Department of Education and Training in Queensland to implement the YDC TIME project was an indication of this support. Thus, it has acknowledged the philosophy and pedagogy that underpins YDM and reflects the view that it has the capacity to improve educational outcomes for Aboriginal, Torres Strait Islander and low SES students through demonstrating high standards and expectations of such students.
2. **YDC staff.** Implementation of YDM is based on the premise that improvements with the teaching and learning of mathematics require schools to function as learning organisations (Senge, 1994). Thus, processes for ongoing improvement such as data collection and reflection on in-school training and trialling of ideas are designed and written into the YDM project's framework of activities. To support this, YDC staff should maintain regular contact with principals and teacher-trainers as well as developing YDM resources and providing on campus (QUT) training. Other processes critical to the role of the project team include providing ongoing support for schools and managing the collection, collation and analysis of data to inform the progress of the project.
3. **Administrators.** Change that is transformational and emancipatory requires school leaders to be in agreement with and fully supportive of the YDM project. In their key position as leader, they play a critical role in initiating, implementing, sustaining and maintaining YDM. They also play a critical role in knowing and understanding the process of change and the changes themselves, here the pedagogy and philosophy of YDM. School leaders are required to engage in reflective conversations with their teachers, professional learning teams and students to debrief on the project's progress. Professional reflective journals play a key role, with school leaders recording their insights, challenges and successes. School leaders will be required to: (a) reflect on their students' test results prior to commencing YDM; (b) participate in administrators' PL input and develop school directions for implementation that provide all staff with incentives, time to PL and trial ideas in the classroom; (c) establish and make explicit in the school directions goals for elements they are wanting to change in their schools, and assist teacher-trainers to undertake in-school training and trialling; and (d) implement and reflect on their school directions for effective teaching for learning, gather data on what is happening in the school, keep a reflective journal of school YDM implementation progress, and assist in preparing reports and presentations.
4. **Trainers.** As internal change agents, teacher-trainers seek to provide the bridge between the teachers and the project team. The teacher-trainers address how YDM is contextualised (Bernstein, 1990) from the primary level of the university to implementation into the classroom. Here, the YDM pedagogy and philosophy framework become the launching pad for engagement in professional discussion and action with principals and classroom teachers. It is expected that teachers will be introduced to new teaching practices through their active participation in the project and, therefore, will learn more about these practices as they trial, practise, reflect and refine with the teacher-trainers and other colleagues in school (Crevola & Hill, 2005). To facilitate change in classrooms, teacher-trainers are required to: (a) attend YDM PL inputs; (b) set up and run the program of in-school training and classroom trialling support of other teachers and teacher aides as per their school directions; (c) sustain the momentum from participation in initial workshops to support the ongoing development and implementation of YDM; (d) gather data on in-school activities, keep a reflective portfolio and maintain contact with YDC; and (e) assist in preparing reports and presentations.

5. **Teachers who are not trainers.** Improvements in student mathematics learning have been shown to be the consequence of the teacher's participation in learning to improve teaching effectiveness. Through active engagement and participation in learning, teachers increase their knowledge and learn how to be more effective in their teaching. The success of any change and/or reform is highly dependent on which changes are supported by ongoing, well-structured professional development. To bring about change in their classrooms; teachers are required to: (a) reflect on their students' NAPLAN and QCAR results prior to commencing in-school training in YDM; (b) attend all in-school training and trial ideas in their own classroom; (c) establish and make explicit school directions and goals for what they are wanting to change (recorded in a professional reflective journal); and (d) implement and reflect on the principal's and professional learning team's school directions for implementation and their own plans for effective teaching for learning.
6. **Students.** However school change through the trial and implementation of YDM impacts on student outcomes, the improvement process must be measured by the extent to which such change impacts on student learning, and more specifically their mathematics learning. Thus, the effectiveness of change can be monitored by teachers who measure what is important and how it relates to student achievement and success. In YDM projects reflective portfolios from teachers provide valuable information that enables YDC staff to assess the performance of each participating school especially when accompanied by clear learning goals for students and pre- and post-test results. Comparison of school NAPLAN results that are publicly available on the web may also provide further indications of student learning improvements in mathematics.

5.3 Developing school directions

The success of YDM requires the development of school directions for implementation through a process of collaboration. Collaboration is intended to mean that everyone's viewpoints are taken as contributions to the school directions for understanding the situation. Such school directions are required to be developed by the principal and teachers to overtly and explicitly address what change is to occur within the school and classroom context.

A simple three-question framework provides a useful basis for systemic school planning as follows:

- Where are we now?
- Where to next?
- How will we get there?

Each school's responses to these questions will comprise the school directions for implementation of YDM. It is expected that each school will provide the YDC team with their initial school directions for implementation of YDM as well as revisions made in response to action-research processes undertaken by the professional learning team and teachers.

The YDM project provides the administrators and trainers with PL inputs to be disseminated to the classroom teachers in their schools, supported by YDM resources and online and telephone communication with YDC staff. Thus, to be effective, school directions require consideration of the following components:

- allocation of time for the principal to present the facets of the YDM approach (the framework of activities, the mathematics philosophy and pedagogy, the ideas on school change and leadership, and the proposed school directions);
- allocation of time for the teacher-trainer to train classroom teachers – the YDM teacher-trainer PL workshops involve six days of training, five days of which are on mathematics teaching and learning, so close to this amount of time may be needed for in-school training across the year;
- a regular program of school meetings (informal and formal) for teachers to share ideas and support each other;

- implementation of school change components other than mathematical focus (e.g. behaviour management, high expectations, community participation, local leadership) where applicable;
- a program of data gathering and contact with YDC staff including a process for regular communication with YDC staff; and
- a process for making YDM an important part of the year's work.

5.4 Integration with existing programs

This final section looks at how YDM can fit in with existing programs. Three situations are explored: (a) how YDM supports teaching for a **testing regime** that relies on tasks; (b) how YDM integrates with an inflexible, and even scripted, **textbook or lesson system** which provides information and directions for lessons for each day; and (c) how YDM can **accelerate mathematics learning where students are performing below their year level**.

5.4.1 Testing regimes, lesson systems and integration with other pedagogies

Testing

The major outcome of implementation of YDM in schools is to enhance mathematics learning performance. A result of this is improvement of test outcomes. One of the major test outcomes in Australia is NAPLAN. This testing regime is based on answering tasks which apply mathematical ideas. The question is whether YDM can improve test results in instruments like NAPLAN?

In testing regimes based on tasks, students have to **use accessible rather than available** mathematics knowledge (see section 2.3). The test items are such that there are two ways to make errors: (1) not having the knowledge available (i.e. not having learnt the knowledge); and (2) having the knowledge available but not accessing it (i.e. having learnt the knowledge but not realising that it can be applied in the task situation).

The design of YDM's pedagogical framework, particularly the RAMR cycle, is such that it **improves accessibility**. The Reality component places the mathematics in everyday tasks; the Abstraction component builds deep understanding, the Mathematics component ensures that knowledge is connected to other similar topics and thus what mathematical ideas are legitimate for, and the Reflection component, particularly through flexibility, builds knowledge of where all mathematics ideas can be applied. The RAMR cycle is therefore excellent in building available and accessible knowledge – it should build mathematics knowledge that works in NAPLAN.

Lesson systems

YDM provides teachers with a good pedagogical framework for **planning and sequencing effective mathematics instruction** for students that leads to robust understanding of mathematics and strong mental models. Curriculum documents and guidelines for implementing the intended curriculum (such as C2C units in Queensland or textbook systems) provide teachers with information about the deemed important content to teach and when it should be taught to meet nationally targeted mathematical development of understanding. The question is, how can YDM integrate with these?

YDM enriches these guidelines by providing teachers with a pedagogical framework and cycle within which to plan and teach connected learning experiences that assist students to progress in their mathematical thinking from concrete and tangible real-life experiences to representing their world using the abstract symbolisation of mathematics. The aim is for students' understanding of mathematics to be strong enough for them to also be able to understand all mathematical representations and recognise the real-world applications of these.

Most lesson systems (like C2C in Queensland) provide the "what" and the "when"; YDM provides the "**how**". Thus, YDM can **support and enrich** most textbook and lesson systems. YDM can be used as a basis for critiquing lessons. The **RAMR framework** shows all the steps necessary for teaching an idea. Thus, we can ask: does the lesson plan start with reality, does it have body/kinaesthetic activities, does it show all connections to other

similar mathematics, and does the lesson finish with reflection and extension? As well, the **Planning–Teaching cycle** can determine if the lesson is appropriate. For example, it is inappropriate for 4-digit numbers to be taught to students who only know 2-digit numbers even if it is in the year level program of lesson plans.

Integration with other pedagogies

Some schools with whom YDC collaborates follow common pedagogies across different subjects, for example, explicit instruction. YDC staff can work with, and integrate with, many of these pedagogies. This can be the initial point of collaboration where YDC and the school come to an acceptable way for YDM to fit in with the school's generic pedagogy. For example, explicit instruction is often built around three steps, "I do", "we do" and "you do" followed by reflection. The four stages of the RAMR cycle can fit in with these three steps if they are considered as major steps and not simply a way of teaching a worksheet of examples – Reality is "I do", Abstraction is "we do", Mathematics is "you do", and Reflection is reflection. However, this does extend what happens in each step.

5.4.2 Accelerating underperforming students' mathematics learning

YDM is particularly useful for students who perform mathematically below their school year level. YDC has a project that is using YDM to accelerate learning so these students catch up to their year level. This acceleration is based on a theory (Cooper & Warren, 2011; Warren & Cooper, 2009) for building big ideas that sees mathematics knowledge growing through **structured sequences** of instructional activities that go **across models/representations** not single or sets of lessons within a model/representation.

The theory argues that these structured sequences have the properties: (a) effective models and representations have strong isomorphism to desired internal mental models, few distracters, and many options for extension; (b) sequences of models/representations develop so there is increased flexibility, decreased overt structure, increased coverage and continuous connectedness to reality; (c) ideas behind consecutive steps are nested wherever possible (later thinking is a subset of earlier); (d) complexity can be facilitated by integrating models but may require the development of superstructures if complexity leads to compound difficulties; and (e) abstraction is facilitated by comparison of models/representations to show commonalities that represent the kernel of desired internal mental model.

This implies that mathematics learning of underperforming students can be accelerated if: (a) instruction, models, representations and language follow a nested line of abstraction from ability level to age level; and (b) instruction has its basis in carefully-built contextually and culturally appropriate foundational ideas enabling rapid and gestalt advances across higher level and more generalised topics (Holmes & Tait-McCutcheon, 2009; Thomas & Tagg, 2007).

The implication of this theory is that acceleration is enhanced if instruction is divided into vertical units of work (the project refers to these vertical units as **modules**) that structurally sequence mathematical ideas from students' performance level to their age level. In this way, the modules can build big ideas as well as concepts, strategies and processes. Early trials of this in YDC projects has shown that modules' first stages (that cover the performance level at which the student is have to be completed slowly and carefully to build the connections that frame out the big mathematics ideas, but then the later stages (that cover the later mathematical ideas) can be covered quickly in gestalt-like leaps of understanding.

The way to use YDM for underperforming students, and to enable speedy growth of knowledge (i.e. acceleration), is to: (a) find the students' present performance level; (b) plan a sequence of units to take the student from performance to age level and use the RAMR model to build big ideas and connections at each unit; and (c) spend time getting students to deeply understand the mathematics at the starting or performance level unit and then increase the speed of learning as go up the levels.

YDC has developed such a series of modules to assist schools to accelerate the knowledge of very underperforming junior secondary students. It was initially funded for Indigenous students (2010–13) and called

Accelerated Indigenous Mathematics (AIM). The idea was then used with large low SES secondary schools, funded by an Australian Research Council (ARC) Linkage grant and called XLR8 Maths project. It is now available to schools under the name of the Accelerated Inclusive Mathematics (AIM) project.

5.4.3 Extending and enriching students' mathematics understanding

YDM is also useful for strongly performing students and students who are gifted and talented with respect to mathematics. Many of these students can develop high abilities through rote learning rules but this is sometimes insufficient to enable them to succeed at a good level in mathematics-based subjects in university. This is because powerful mathematics is built on understanding mathematics as a deep structure.

Deep structure has two components. First, it is built on **rich schema**, meaning students having knowledge that: (a) completely defines the mathematical idea; (b) provides all applications of the knowledge; (c) has all connections identified between the knowledge and other mathematics knowledge; and (d) contains critiqued and recoverable experiences of that knowledge. Second, it is based on **big ideas**, that students' knowledge: (a) covers all the different concepts that make up a mathematics big idea; (b) encompasses a strong understanding of mathematics language, particularly as a shorthand concise symbolic structure that tells stories and describes life; and (c) has a rich repertoire and understanding of models, representations, and strategies.

Deep structure requires the following: (a) strong diagnosis to detect weaknesses; (b) rebuilding of knowledge from first principles where there is little understanding; (c) smooth seamless progressions of ideas, ensuring all meanings are included; (d) strong repertoires of models and strategies; and (e) not accepting correct answers from rote-learned processes without understanding of underlying knowledge. This means the implementation of student-centred practices, an acceptance that students have to construct their own knowledge by looking for similarities and differences in activity, and the adoption of high expectations towards quality of knowledge.

One way to do this is to build teaching around rich tasks that are: (a) multi-representational in terms of integrating models and representation but also multi-topic in terms of bringing many ideas together to reach solution; (b) challenging to the point of failure for many students but enabling progress for all students; and (c) bridging affect, skills and reflection, particularly in terms of technical, domain and generic knowledges.

To also add the interest, motivation and engagement that is required to maintain student interest in mathematics, rich tasks should: (a) be based on an **idea that is interesting to the students** (appropriate for the age level and interests of the students); (b) be **challenging but not too challenging** (a jump too far can affect confidence); and (c) **fit in and connect with previous knowledge** to be able to build superstructures that enable big ideas. If the content is too far from the students, there will be no construction of strong new knowledge. Without fit, there can be no deepening.

YDC has developed a project for enriching and extending students' mathematics knowledge to increase participation in mathematics and attendance at university. It is called Mathematicians in Training Initiative or MITI Maths and is now available to schools. It provides a pedagogy for deepening mathematics understanding, techniques to identify, modify and construct rich tasks, and a set of rich tasks covering the Australian Curriculum.

5.5 Sustainability

The last aspect of implementation of YDM in schools is to do so in a way that it becomes sustainable and YDC is no longer required to provide continual support. To this end, YDC is attempting to complete each project with training on sustainability in terms of the school no longer requiring YDC support.

This sustainability appears to have two components:

- (a) the school understands and implements YDM to a successful level in terms of their students' learning;
- and

- (b) a process is set up in the school for training of new teachers in the school's YDM approach so that it remains in the school.

What has to be sustained is more than the general YDM professional learning and includes the testing, lessons and pedagogies described in subsection 5.4.1, the acceleration described in subsection 5.4.2 and the extension and enrichment described in subsection 5.4.3. Thus, the implementation of sustainability is as shown in Figure 5.4.

There are difficulties in achieving this when the teachers do not stay in the school for long. A high turnover of teachers requires yearly training and a strong process of always having someone staying on who can do the training. Conversely, there can also be a problem with a lack of movement of teachers in that YDM might ossify for lack of new input.

YDM's answer to this is to: (a) encourage a staff member from each school or group of schools to take an interest in becoming a trainer and allow this person to shadow YDC PL staff; (b) prepare a special training component for a trainer and build this into the YDM training projects; and (c) develop an online course that schools who participate in, or complete, a training project can access. This online training course is intended to: (a) assist with training new staff; (b) keep the school up to date with new versions of YDM resources; and (c) have structures to allow school staff to maintain contact with staff in other schools in their position.

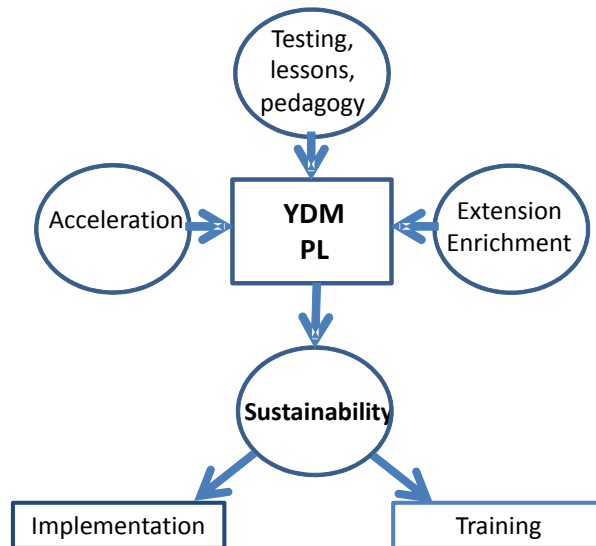


Figure 5.4 Implementing sustainability

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Appendix A:

Two effective school–community partnerships

Cherbourg State School

In most Indigenous community schools the community council is the leadership and consultation group. This council is elected by the local community like any other council. Of importance is that schools acknowledge and respect the role that the council plays in any community. When these processes are made explicit, there is the potential to pursue the best possible outcomes for the children of the community. Real partnerships mean that all parties benefit. An example of this partnership existed between Cherbourg State School and the local community council.

The partnership organised for Cherbourg people to work in the school, in particular as general office staff, grounds people, and teacher aides and to provide maintenance. The employment of local people as teacher aides was regarded as a vital component within the school management strategy because the aides were used in the classroom to assist teachers with the students to help improve their learning outcomes, particularly in literacy and numeracy. The employment of local people for maintenance recognised and employed a strength of the community. The Cherbourg Community Council were receiving national awards for the houses they were building in Cherbourg. Thus, it made much better sense for the council to provide school site maintenance, rather than seeing money go to agencies in Murgon. Also, this meant the students would obviously have more respect for their school when they saw their own people playing a significant role in maintaining it. As well, the school also took on local people as teacher aides.

This change in process established a sense of appreciation and good will with the council as a means to continue a productive partnership. Another simple strategy to ensure a productive and meaningful partnership was simply by placing a signature block for the elected council chairperson on all of the school's operational and strategic documents. Having this signature ensured the council had direct input into the directions of the school and the performance targets it set. For the school this provided some leverage to negotiate some shared resources to assist the school.

As a result of the partnership, the school was able to take people who were on a work for the dole program, and provide them with employment that engaged them in full-time work with an educational career path that could eventually see them as registered classroom teachers. Apart from the obvious benefits of having additional adults in each classroom, there was the highly significant benefit of having local Aboriginal community people in the classroom that were from precisely the same community context as the students who were sitting in the classroom.

These benefits are significant on two levels. First, the students have local community role models on a day-to-day basis, who are also engaged in a form of study, and provide an explicit example of someone who is from the same community context and is modelling daily a sense of being strong and smart. If many people from Cherbourg are working in the school and they are working to exceedingly high expectations, then there is no reason why we cannot expect the students to work to exceedingly high expectations (C. Sarra, 2007). Second, having more local Aboriginal community workers in the classroom provides a useful medium through which teachers can develop a much deeper understanding of the students' community context. Further, they provide the school with an immediate link to the community if problems arise with students or particular issues at school. This link is beneficial in enhancing the relationship between the school and the community.

Low SES state school in South Australia

In the 1970s, Tom Cooper was a teacher at a low SES state high school in Adelaide with the special duties to work with the community (he ran an after-school centre for youth and assisted the local community education committee to set up Australia's first community school). The school was in a suburb that had been built by the South Australian Housing Trust down-wind from the "sewerage farm" (where raw sewerage was treated in open ponds) to be low rental accommodation for welfare families. It was composed of identical maisonettes (double houses) and, as a suburb, had the lowest reputation in surveys as a place to live. There was considerable juvenile delinquency, substance abuse, crime, broken families and family violence, and many "latch key" children (children with a house key on a string around their neck because their parents did not return home until late). The reputation of the high school was so low in terms of student violence, engagement in classrooms and academic performance that teachers could not be sent there by allocation, teachers had to volunteer to be there. It was chosen from all low SES schools across Australia to be the first community school. It was allocated special funding for new buildings which would incorporate a health clinic, a community library, a youth centre, and special community sports facilities.

The reputation of the primary school beside the high school was not the same. Although having the same clientele, it was seen as a school that was making significant positive changes to students' engagement and performance. Tom approached the principal to find out how this was being done, so the approaches could be applied to the new community school. The approaches were remarkably similar to Cherbourg.

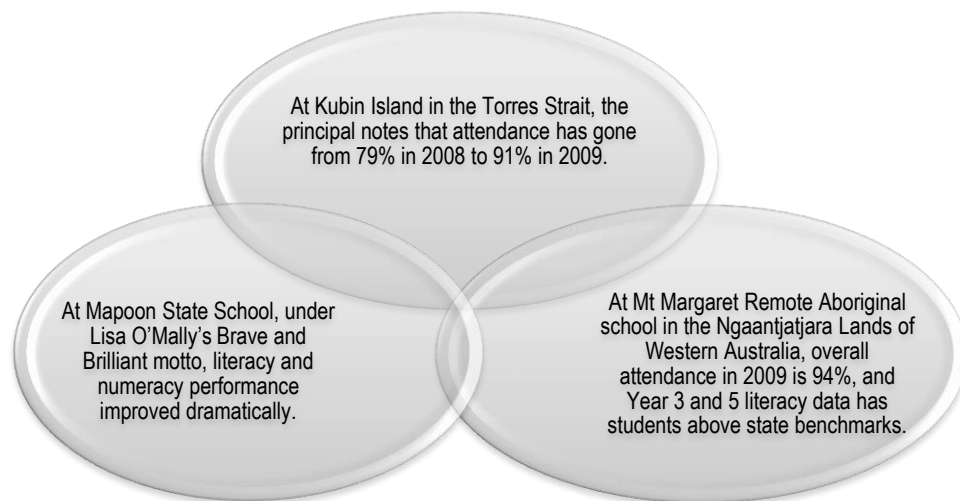
The principal made a decision that she was going to involve the community in her school. She obtained a double demountable and set it up as a drop in coffee lounge for the many unemployed and single parents – it had armchairs, tea and coffee making facilities and magazines (the high school had few facilities). She obtained permission to require teachers who came to her school to work into the evenings. Each teacher was required to visit each of their students' parents in the parents' home once per term and to be available to talk to parents at other times in the evenings. This was used as a starting point for a massive program of involving parents in classes – teachers had to strongly encourage parents to join in their classrooms. Finally, the principal set up a community committee to help run the school – she gave this committee control over the expenditure of school funds – she gave them real power.

This was a tremendous act of trust and a very courageous move in an education system which, though more flexible than other states, tended to operate in a risk aversion way. The strategy worked – students responded by seeing the school as part of their community and behaviour and performance improved.

Appendix B:

Application of Drucker's (1996) elements for teacher aides

Transformation processes emerge in Indigenous schools and their communities where school leadership acknowledges and embraces Indigenous leadership. That is, when school principals and other school leaders variously described as Indigenous Community Leaders (ICL), Indigenous Education Workers, knowledge workers, Indigenous Teacher Aides, but referred to here as ICL workers, work respectfully together in the interests of stronger smarter futures for their children, revolution occurs. For example, the Indigenous Education Leadership Institute (2008) reports that:



ICL workers are often referred to as the most valuable asset in schools, working tirelessly to support students. They field the hurt, confusion, anger and frustration of children from communities trying to cope with day-to-day life and then trying to learn. It is vital that school leaders actively manage and invest in them so they can continue to support and inspire Indigenous children. Such an investment would alleviate these workers taking on the extra burden of wondering about their employment conditions, how they will be adequately remunerated for the important role they fulfil and how they will be credentialed. G. Sarra (2007a) cites the work of Peter Drucker (1996) who describes the relationship between management and ICL workers:

*We know that knowledge people have to be managed as if they were volunteers. They have expectations, self-confidence, and, above all, a network. And that gives them mobility, which is probably the greatest change in the human condition. A very short time ago, if you were the son of a peasant, you were going to be a peasant. Even in this country, social mobility was almost unknown. Now, every one of the young people I know has his or her resume in the bottom drawer, which no blue collar worker ever did. So accept the fact that we have to treat almost anybody as a volunteer. They carry their tools in their heads and can go anywhere. And we know what attracts and holds volunteers. The first thing is a **clear mission**. People need to know what their organization stands for and is trying to accomplish. The second thing is **responsibility for results**, which means appraisal and review. And the third thing is **continuous learning**" (Drucker, 1996, cited in G. Sarra 2007a).*

Drucker (1996) emphasises that ICL workers who are mobile, have to be treated like volunteers. That is, they are mobile because they are not tied down in terms of locality. Here, their very mobility value adds to schools, because they are not locked into one school community but possibly several. In contrast, a teacher aide is much less mobile but this does not reduce the value they add to their respective school communities. We would argue that the teacher aide is valued because of the nature of the knowledge he/she possesses. This more than compensates for their lack of mobility. Indeed, within Indigenous education the teacher aide is often the only source of continuity between school and community, in contrast to non-Indigenous teachers who

come and go with great regularity and often do not know the community, the children, nor the culture. Providing stability in a school is crucial to any transformation and emancipation.

Drucker (1996) cites three basic elements for attracting and investing in ICL workers. The first of these is a **clear mission** statement. The principal of the school has to articulate a clear goal. The staff have to sign and agree to the achievement of such goals. They also need to know how to achieve the goals and receive feedback on how they are progressing. Drucker points out that at Pepsi everyone knows that the mission is to beat Coca Cola. If you do not sign on to that then you have no future. Moreover it is easy for the staff to tell if they are winning the sales war with Coca Cola (Drucker, cited in G. Sarra 2007a). A Cherbourg State School narrative elaborates this point further:

Cherbourg State School is approximately 300 kilometres northwest of Brisbane. It is situated in an Aboriginal community at Cherbourg with approximately 250 students. At Cherbourg State School the aim was to generate good academic outcomes for all students in the sense of achieving results comparable with any other school in the state from Kindergarten to year 7 and to nurture a strong and positive sense of what it means to be Aboriginal in today's society. This goal in turn was broken down into smaller goals. Here the goal of improving attendance at Cherbourg fitted into this paradigm very neatly. All staff at the school understood the importance of attendance. Having a school with irregular attendance was very demoralising for both staff and students and it meant a perpetuation of Cherbourg State School's negative exceptionality.

Taking measures such as isolating the phenomenon of unexplained absences first and dealing with that gave the staff a means of achieving the goal of improving attendances. A system of rewards was set in place. What was crucial here was not the significance or importance of the prize to be gained, but the processes of expectation that the pupils would, and recognition when, they had changed their behaviour (G. Sarra, 2007a).

Drucker's (1996) second element, **responsibility for results**, entails encouraging the teachers to see themselves as capable of contributing towards achieving the overall goal.

At Cherbourg State School the teachers were encouraged to reflect on what they might be doing or not doing in the classrooms that resulted in poor attendance. Was their teaching relevant, innovative, entertaining or were they going through the motions? With the teachers' aides he took the step of including them in this process. What were they doing to help the teacher understand and relate to the kids? What information or guidance were they giving which could help make the relationship between the teacher and the pupil more productive. Were they too encouraging the students to meet higher expectations or were they part of the process where the school was stuck in the slough of negative exceptionality?

The third element in Drucker's (1996) paradigm, **opportunities for continuous learning**, was very important in the management of the teacher aides.

They brought knowledge to the school. This in many ways constituted their value to the school. But they needed to be provided with the opportunity to expand their knowledge base for this is what made them valuable not least in market terms. Accordingly the teacher aides were enrolled in teacher education programs and were encouraged to attend workshops, seminars and conferences like regular knowledge workers (Drucker cited in G. Sarra 2007a).

Of significant importance to the success and transformation of a school are the relationships that are created and sustained over time. Such relationships include those between the school leaders, teachers, and ICL workers. It is through the maintenance of respectful and trusting relationships and reflection on practice and achievements that transformation and emancipation occurs.

Appendix C:

Effective activities in working in Indigenous schools

C1 Activities to enhance behaviour (G. Sarra, 2007a)

Below are six activities that have been successful in promoting positive behaviours that support learning for all students in the class.

C1.1 Routine

Setting and following the same routine every morning for the first half hour of each day provides students with an opportunity to settle into the day with ease. If students are late to school in the morning, they know exactly what is expected of them and what to do without any disruptions to others. It also removes any inhibitions of embarrassment or uneasiness they might experience when entering the classroom. As a teacher stated:

In my classroom each morning, children would enter the room, get their water bottles, sharpen their pencils, blow their noses if needed and order their tuckshop. Students would then sit on the carpet for roll call, morning talks and to visit the key ethic or value for the fortnight before commencing brain gym exercises followed by handwriting. The rest of the morning session was allocated to literacy until morning tea. There were two uninterrupted hours devoted to literacy each day.

A set routine each day in the classroom minimises behavioural problems and is a great classroom management strategy. Predictable and consistent classroom practices assist in producing a well-managed classroom.

C1.2 Consistency

No student wants disorganisation, constant change or chaos. Consistency is an important factor in a classroom that is supported by a safe, caring and predictable environment. Consistency from day one of the school year will play a vital role in the classroom management procedures. As a teacher, the first two to three weeks is usually spent teaching students the rules and routines of the classroom.

Here teacher educators, Wong and Wong (2004, p. 3) argue that “your success during the school year will be determined by what you do on the first day of school”:

Establishing and maintaining good control of the class in and out of the classroom and modelling to the students that you know what you are doing will assist in developing a well managed classroom. There is little consistency for students when they see a teacher who is disorganised with lack of control.

C1.3 Classroom management

The structures and procedures a teacher uses to maintain a positive and productive learning environment is essential for student learning to occur. In the first few weeks of school, setting up particular points in the classroom that are used when rotating students from one activity to the next is vital. This teaching strategy when applied assists in delivering a productive and cooperative working environment and minimises disruptive behaviour from students.

When students are in their working stations in my classroom there is no walking around the room at all without permission and noise is kept to a minimal level. It is worth stressing that this strategy is only effective if the rules and routines that have been established are consistent and are used daily so students become familiar with the process.

It takes just as much energy to achieve positive results as it does to achieve negative results. So why waste your energy on failing when that same energy can help you and your student succeed (Wong & Wong, 2004, p. 37).

The ideal here is that the procedures and routines that are used organise the classroom, so that the learning that takes place can run smoothly and stress-free so that every student is involved in a co-operative learning environment (Wong & Wong, 2004, p. 85).

C1.4 Points reward system

Students are seated in groups. Each group chooses a name for their group that relates to the unit being studied for the term. Points are given to individuals in the group or the whole group for a variety of reasons. Some of these include: completing activities on time, concentrating in class, working cooperatively in their groups, being a good friend, preparing and tidying up tasks, displaying kindness, compassion and other values concepts that are promoted within the school. At the end of the week the points are added up and the winning group with the most points receives a small toy.

C1.5 Class money

Students receive play money for attendance and good behaviour and performance and fines for poor behaviour. Once a week, students can spend their money to buy toys, classroom materials, time on the computer, etc. A class in an Indigenous school built a cardboard shopping centre with a bank and shops. The students kept a bank account with their money. On the purchasing day, shops were opened and students sold and gave change. The teacher also got the students to pay rent and power and to budget. Special presents (e.g. footballs) that required months to save enough class money were available in an attempt to encourage students to save.

C1.6 Invisible chair

At the commencement of the school year, day one, the invisible chair is introduced to the class. If students are disruptive, not working or exhibiting behaviour different from what is expected, then they may be sent to the invisible chair. A chair and desk is placed at the back of the classroom next to the teacher's desk and behind the rest of the class. However, the desk and chair is situated so that the student is still capable of completing all work tasks. Within the whole class, teachers and students see the student in the invisible chair as non-existent in the classroom and no assistance is given to him/her. When the student is ready to cooperate and do the right thing, assistance is given.

C2 Brain gym

Each morning prior to the students commencing class, each student's water bottle is filled with water that they use before and after their brain gym exercises and throughout the day. Brain gym exercises involve fun and enjoyable movement exercises each morning prior to commencing any learning activities in the morning and immediately after morning tea. The exercises are used to enhance student experiences of whole-brain learning. They stimulate the right and left brain to make learning easier, assist students to focus and concentrate and are effective with academic skills. Brain gym has been a successful strategy with behaviour management and classroom management. The exercises vary from activating the brain to improve academic skills to improving behavioural and postural correlates. Dennison and Dennison (1994, p. 1) explain:

Focusing is the ability to cross the participation midline, which separates the back and front of the body as well as the back (occipital) and frontal lobes. Incompletion of developmental reflexes results in the inability to express oneself with ease and to participate actively in the learning process. Students who are under focused are often labelled as, "inattentive", "unable to comprehend", "language delayed", or "hyperactive". Some children are over focused and try too hard. These movements which help to unblock focus are designated as back/front integration activities.

Brain gym is integrated each morning into the half an hour of routine activities and is designed to assist students to become focused and attentive and ready to work. Some of the exercises that students perform include the following.

1. **Cross-Crawl.** This exercise is done daily because it accesses both brain hemispheres simultaneously. The students bend the elbow and alternatively move one arm and touch its opposite leg and the other arm and touch its opposite leg. Cross-crawl improves left, right coordination, greater spatial awareness and enhances hearing and vision.
2. **The Elephant.** This exercise involves a movement in which the head, torso and pointing arm and hand function as one. Students pretend their arm and body are the elephant's trunk and write the number 8 slowly while focusing on their finger. Their whole body moves as one without any separate movements. Students pretend to glue their head to their shoulder so there is only one movement during this exercise. The Elephant activates the brain for: (a) crossing the auditory midline (including skills of attention, recognition, perception and memory); (b) short- and long-term memory; (c) thinking ability (Dennison & Dennison, 2004, p. 8).
3. **The Energiser.** This exercise is used to enhance students' concentration and attention. Students are seated at their desks. They position their hands, flat in front of their shoulders with their fingers pointing slightly inwards. The student has his/her forehead on the desk and takes a deep breath in, lifting first her forehead, then her neck and finally her upper back. As the student is lifting her forehead and neck she slowly exhales out.

C3 Organisational culture and leadership

The organisational culture and leadership in a school begins with the principals who have the potential to impose their own values and assumptions on a group. To illustrate, if the school demonstrates success and achievement and the underlying assumptions of members of the school community come to be taken for granted, we then have a culture that will define for later generations of members what types of leadership are acceptable. In this example, the culture defines the leadership in such a way that it is emancipatory and works to bring about success. Conversely, this process can work the other way. If the school runs into adaptive difficulties, that is, as its environment changes to the point where some of its driving assumptions are no longer well founded, the influence of leadership is once again evident but in ways that may be less likely to bring about success and achievement. Schein (2004, p. 11) elaborates this aspect further

Culture and leadership are two sides of the same coin; neither can really be understood by itself. On the one hand, cultural norms define how a given nation or organization will define leadership – who will get promoted, who will get the attention of followers. On the other hand, it can be argued that the only thing of real importance that leaders do is to create and manage culture; that the unique talent of leaders is their ability to understand and work with culture; and that it is an ultimate act of leadership to destroy culture when it is viewed as dysfunctional.

Strong leadership requires people in such positions to step outside the culture that created the leader and to start evolutionary change processes that are more adaptive and emancipatory. What is crucial for school leaders is the ability to perceive the limitations of one's own culture and to evolve the culture adaptively as Schein (2004, p. 17) explains

the culture of a group can now be defined as a pattern of shared basic assumptions that was learned by a group as it solved its problems of external adaptation and internal integration, that has worked well enough to be considered valid and, therefore, to be taught to new members as the correct way to perceive, think, and feel in relation to those problems.

If the culture of a school displays shared beliefs and assumptions that are anchored in high expectations, productive and meaningful teaching and learning practices, openly and actively embraces and acknowledges Aboriginal and Torres Strait Islander identity, builds and maintains strong community relationships and

acknowledges and embraces Indigenous leadership in schools, then the view of deficit thinking will be replaced with proactive and emancipatory thinking that provides real and meaningful change and the opportunity to see transformative change in classroom practices and the schools. This form of leadership underpins and indeed strongly supports the cyclic nature of the YDM philosophy.

C4 Human Values Education (G. Sarra, 2007c)

Table C1 is an example of a values framework. Each fortnight a sub-value is chosen by the staff in which all teaching members would use the seven teaching techniques. However one sub-value is chosen from each of the core values. The sub-values are selected at the beginning of each term in accordance with the perceived needs of students at that time.

Table C1 *A values framework*

RIGHT-CONDUCT	PEACE	TRUTH	LOVE	NON-VIOLENCE
Manners	Patience	Truthfulness	Kindness	Consideration
Living skills	Concentration	Optimism	Friendship	Cooperation
Helpfulness	Self-acceptance	Honesty	Forgiveness	Global awareness
Responsibility	Self-discipline	Determination	Generosity	Loyalty
Independence	Happiness	Fairness	Compassion	Citizenship
Perseverance	Thankfulness	Trust	Tolerance	Justice
Courage	Contentment	Reflection	Service	Respect

Seven teaching techniques

1. **Mind map.** A mind map is a teaching tool that can be used to ask students certain key questions relating to a sub-value. These key questions are used to help the teacher develop a clear understanding in their own mind about the meaning of the sub-value. Key questions for the mind map are:

- What is it?
- How do we feel when we practise it?
- How do others feel when we practise it?
- What does it look like?
- What are the benefits?
- What are the challenges?
- What are the limits?
- What is the opposite?
- What are some related values?

These questions are also used with the students to introduce the sub-value. The remaining techniques are intended to reinforce the concept and heighten the levels of each student's awareness.

2. **Quotation.** The quotations are positive statements that focus attention on the sub-value. They are designed to increase knowledge and reinforce the value for the fortnight. Daily repetition of the quotation should influence students' thoughts and focus the attention of the student on a significant aspect of the sub-value and its importance in life. These statements help the teacher heighten the levels of awareness for each student of the particular sub-value. The process of the teaching of the quotation could include discussion of how one might resolve a dilemma relating to the sub-value. The hope here is that in turn this might influence the behaviour of how a student might react in an actual situation.

Most of the quotations that are used are at a level from which the students can make meaningful interpretations of the message in the quotation. Each year level chooses a quotation that is appropriate for

their class. In the lower years a positive affirmation for manners might be, “**Good manners cost nothing – but are worth a lot.**” The intention here is that daily repetition of the quotation creates a positive picture of the sub-value in the mind of the student, expanding their capabilities for positive action.

3. **Silent sitting.** Silent sitting is a technique designed to calm the mind. The intention here is that in the classroom a calm mind is one clear of distractions and consequently able to learn more effectively.

The following is a detailed example of using the breath for quietening the mind:

Lie on the carpet on your back or your stomach and keep your legs straight with your arms beside your body. Gently close your eyes and take a deep breath in through your nose and out through your mouth. Concentrating on your breathing take a deep breath in and let it out slowly. Again take a deep breath in and let it out slowly. I want you to tense the muscles in your toes and relax (twice). Tense the muscles in your legs and relax (twice). Tense the muscles in your bottom and relax (twice). Tense the muscles in your stomach and relax (twice), Push your shoulders into the carpet and relax (twice). Tense the muscles in your fingers. Make a fist and squeeze as tight as you can – relax (twice). Tense the muscles in your face. Screw your face up as tight as you can – relax (twice). Tense the muscles in your whole body and relax. You are now feeling peaceful and relaxed. Stay with these feelings for a moment. Now quietly open your eyes and sit up.

4. **Creative visualisation.** The technique of visualisation is simply one of creating mental pictures of having or doing what it is you desire. Visualisation is an excellent technique for children because it utilises their already active imaginations, giving them an outlet to use their imagination in a positive way (Kehoe, 2002:54 in G. Sarra 2007). The creative visualisation aspect of the human values program enables students to portray themselves positively in a number of ways. The teacher reads a creative visualisation that has been written by the staff themselves or has been found in some literature. The visualisation is a sequence of descriptive pictures that express various circumstances or dilemmas that take the students on a journey to resolve the situation. The creative visualisation develops a deeper understanding of the sub-value and generates a positive feeling of peace and self-worth.
5. **Story telling.** Story telling allows students the opportunity to relate to a particular value by painting them a mental picture using characters, situations, moral dilemmas and resolutions. Students are able to relate to the context of a value if the story outlines the expectations and acceptable attitudes and behaviours that reflect the particular value. Careful consideration of stories is necessary so the correct message is reflected and positive role models are provided who display appropriate good behaviours and attitudes reflecting a strong sense of optimism, trust, honesty etc. Stories can also be created by the teacher about a specific student’s life, with a moral dilemma and a resolution reflecting a value.
6. **Group activities.** Group activities are lessons that are structured to reinforce knowledge and understanding of a value. The presentation of the activities will be a follow-on to the mind map, the quotation and the story. A group activity explores an aspect of the value allowing the students to cooperate in groups, encourages team spirit and social interaction and reinforces certain behaviour. Through role-playing students can re-enact life situations or stories that relate to a value and these simulated experiences allow students to learn and understand the meaning of a value through experiential learning. These activities can improve students’ self-confidence and give students some knowledge about how they might meet challenges in their lives. There are numerous ways a teacher can present activities to give a particular message to students.
7. **Music.** Music introduces students to the lyrics of a song to teach a particular value. Through singing, students have the opportunity to feel the rhythm of a song that reinforces the meaning of a value.

A list of some ideas to get staff started with regard to high expectations in classrooms is as follows. Again, most of these ideas integrate with the ideas for getting started with the other imperatives in the cycle.

1. **Reward high performance.** Students that meet performance levels need rewards. This can be at two levels – extrinsic (providing individuals with explicit, obvious and public rewards for attendance, good behaviour

and working hard) and intrinsic (providing classroom activity that is interesting and related to the world of the students).

2. **Have special programs.** There may also have to be special programs for students who need time out to be able to fit in. The area of behaviour is one in which tolerance needs to be discussed – common simple programs with consequences and which have behaviours for which there is zero tolerance within the mainstream program have been found to be effective.
3. **Use the other whole-school processes.** Equate high expectations to, for example, meeting the slogan of the school.
4. **Use school–community partnerships.** If the community can join with the school on a joint program to emphasise high expectations, then the work of staff can be reinforced at home and in the school with the local staff.



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