XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

XLR8 Unit 15

Extended probability

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.

Scope

This unit is based upon the **likelihood** of **chance events**. The **language of probability** and the **calculation of likelihood** as a number of chances out of a total **sample space** were explored in Unit 1 with a focus on identifying the possible outcomes of chance events and comparing the likelihood of **impossible**, **likely** and **certain** events.

The **outcomes** of a **chance event** and **subsequent chance events** may be **independent** or **dependent**. Any outcome has a **complementary outcome**. The **probability** of an **outcome** and its **complement** add to **1** or **100%.**

Notions of the **likelihood** of **dependent events** can result in many misconceptions including the lottery ticket syndrome and informal ideas that if a tossed coin has come up heads many times in a row then the next toss should be more likely to be tails when in fact these are **independent events** and the likelihood remains the same.

Determining when **events** are **independent**, **dependent** or **complementary** by considering whether or not the **sample space** has actually changed for subsequent events is an important skill. This results in the need to explore contexts where the sample space remains consistent (**with replacement** if drawing items from a deck, bag or hat, dice, coins) as well as contexts where the sample space changes (**without replacement** if drawing items from a deck, bag or hat).

Once students are able to determine the **likelihood** of a **single event** or **consecutive multiple events**, it is important to extend to the use of **likelihood** to make informed decisions or **inferences** from the **likelihood of an event** to justify their choices.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts connected to and reinforced are highlighted in green and number and algebra concepts and processes applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.

- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.

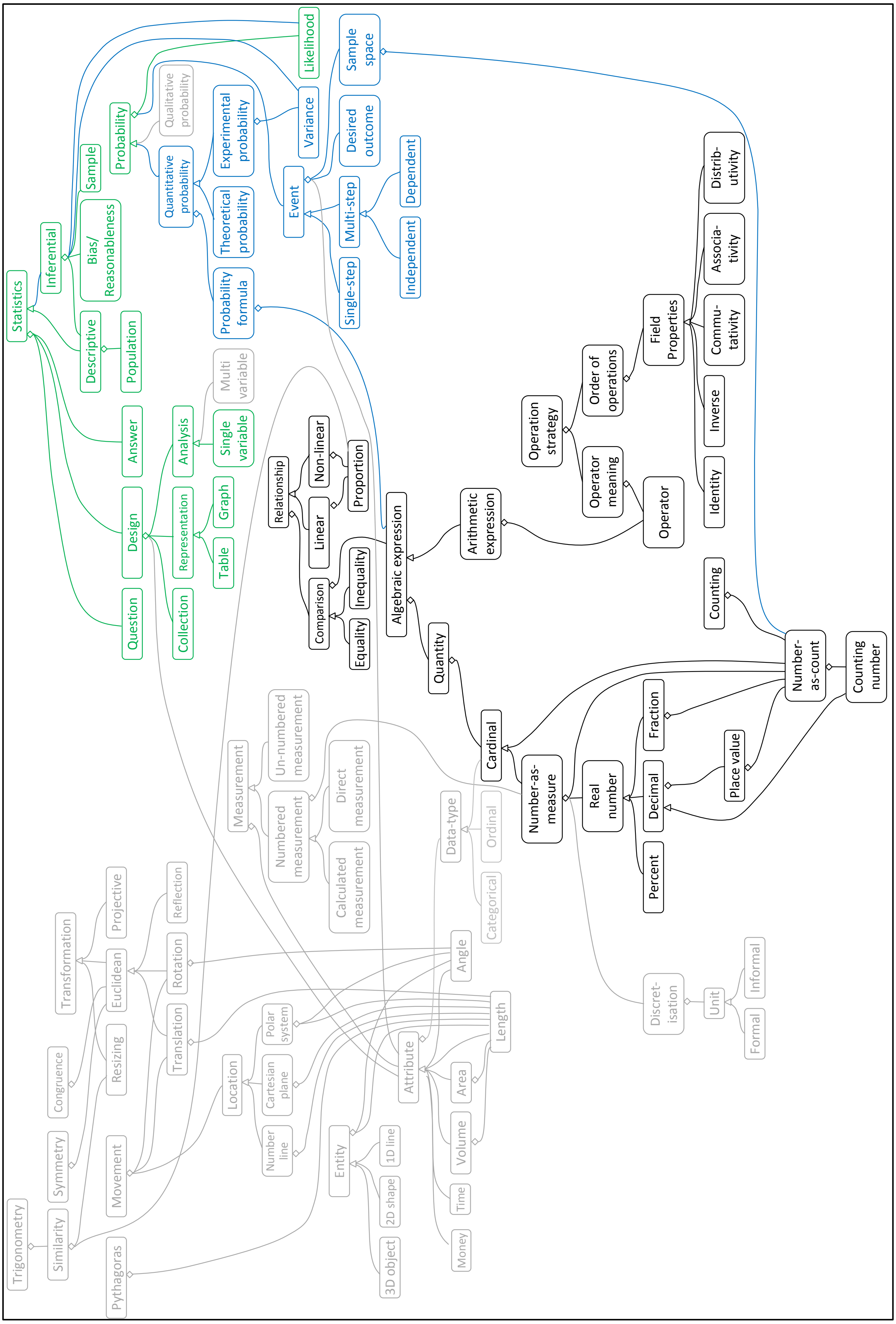


Figure 1. Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Single step probability

This cycle extends upon the notion of likelihood of chance events, informal and mathematical language to describe chance and sample space, equally likely outcomes and single-step experiments developed in Unit 1. Students are given the opportunity to consolidate understanding of representation of probability as a percentage or a fraction or decimal between 0 and 1 where 0 represents no chance, impossible and never and 1 represents absolute certainty. Students will explore theoretical and experimental probability with a view to recognising that many trials of an experiment are necessary before theoretical probability may be evident. Internet-based random event generators are ideal for this purpose.

Cycle 2: Multiple-step probability

This cycle extends from likelihood of a single chance event to consider likelihood of successive or combined chance events. Students will also explore theoretical and experimental probability of multiple-step chance events both with and without replacement. Students will explore probability with activities such as drawing cards from a deck, tossing multiple coins, and lottery games.

Cycle 3: A and B, A or B, A not B investigations in likelihood

This cycle extends from likelihood of single or multiple-step chance events to consider more closely those events that may be unrelated, complementary, mutually inclusive or mutually exclusive. Students should first be able to identify where one event cannot happen if the other happens, and events where the chance of one will not be affected by the occurrence of the other. Students will also use Venn diagrams as a tool for identifying events that may be inclusive or exclusive.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially as it stands.

Literacy Development

Core to the development of mathematical concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between terms.

Cycle 1: Single-step probability

Experiment, event, outcome, sample space, fraction, decimal, percentage, likelihood, chance, equally likely, range, probability, spinner

Cycle 2: Multiple-step probability

Experiment, event, outcome, sample space, multiple-step probability, subsequent events, successive events, with replacement, without replacement, tree diagrams, two-way tables, combinations

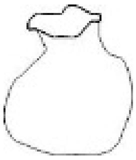
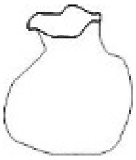
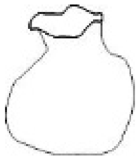
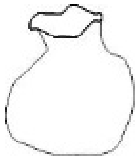
Cycle 3: A and B, A or B, A not B investigations in likelihood

Independent, dependent, complementary, inclusive, exclusive, A or B, A and B, A not B, universal set, null set, Venn diagram

Name:______________________________ Date:________

Can you do this? #1

1. Draw marbles in the bags so that the probability of getting a red marble is:

(a) $\frac{2}{3}$ 	(b) 1 	(c) 0 	(d) More than $\frac{1}{2}$ 

Obj. 15.1.1
1a) □
b) □
c) □
d) □

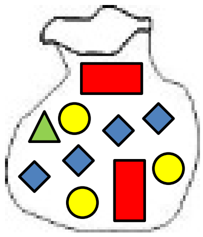
2. A bag has 5 green counters, 3 red counters and 2 blue counters. What is the chance of selecting a green counter?

   Circle the correct answer.

(a) $\frac{2}{3}$ (b) $\frac{3}{10}$ (c) $\frac{1}{2}$ (d) $\frac{6}{10}$

Obj. 15.1.1
2a) □
b) □
c) □
d) □

3. Write the probability as a decimal of selecting each shape block from the bag.

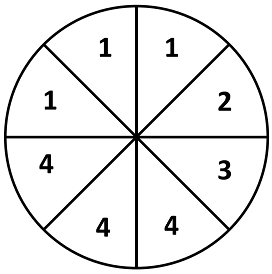


(a) Square _________ (b) Circle _________

(c) Triangle _________ (d) Rectangle _________

Obj. 15.1.2
3a) □
b) □
c) □
d) □

4. Show the percentage chance of spinning each number on the spinner.



(a) 1: ______________ (b) 2: ________________

(c) 3: ______________ (d) 4: ________________

Obj. 15.1.3
4a) □
b) □
c) □
d) □

5. Write how many heads and how many tails you would be likely to get if you flip a coin 100 times?

(a) Heads: ____________________ (b) Tails: _________________

(c) Why: ____

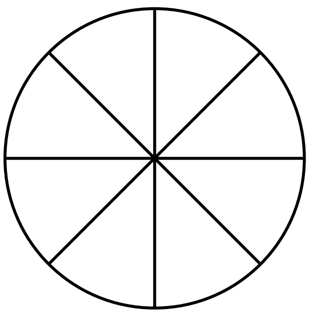
Obj. 15.1.4
5a) □
b) □
c) □

6. Write the numbers 1, 3, 6 and 9 onto the spinner so that you would be:

(a) most likely to get a 9

(b) just as likely to get a 3 as a 6

(c) least likely to get a 1.



Obj. 15.1.4
6a) □
b) □
c) □

7. If you draw one card out of a 52 card deck, replace it and shuffle the deck.

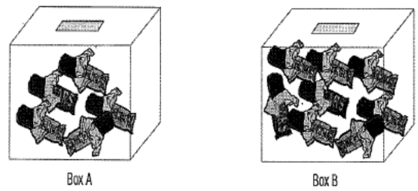
(a) What are the chances of drawing that same card out again? ______

(b) Is it a fair draw if you do not shuffle the cards? _________________

(c) Why/why not? ____

Obj. 15.1.4
a) □
b) □
c) □

8. Box A has 1 Mars Bar and 4 Snickers;
Box B has 2 Mars Bars and 6 Snickers.
(You cannot see inside the boxes).



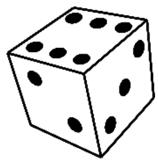
(a) If you want to get a Mars Bars when you take one out, circle the box you would choose.

(b) Why did you choose that box? ______________________________

____

Obj. 15.1.4
a) □
b) □

9. On a fair 6-sided die, what is the probability (from 0-1) of rolling each number?



(a) 1: ________ (b) 2: ________ (c) 3: ________

(d) 4: ________ (e) 5: ________ (f) 6: ________

(g) What probability do you have of rolling a number less than 5? ____

(h) What is the fraction chance of rolling a 6 or a 1? ________________

(i) What is the percentage chance of rolling the odd numbers? ______

(j) If you rolled the die 96 times, how many times should each number come up? ____

Obj. 15.1.4
a) □
b) □
c) □
d) □
e) □
f) □
g) □
h) □
i) □
j) □

10. When you roll the die 96 times, you record the following frequencies:

Observed Frequency	1: 14	2: 22	3: 13	4: 10	5: 16	6: 21
Experimental Probability						

a) Based on this data, write the experimental probability for each number in the table.

b) How does the experimental probability compare to the theoretical probability? ______________________________________

Obj. 15.1.5
a)□□□
□□□
b)□□□

Unit 1: Single-Step Probability

Overview

Big Idea

This cycle extends upon the notion of likelihood of chance events, informal and mathematical language to describe chance and sample space, equally likely outcomes and single-step experiments developed in Unit 01. Students are given the opportunity to consolidate understanding of representation of probability as a percentage or a fraction or decimal between 0 and 1 where 0 represents no chance, impossible and never and 1 represents absolute certainty. Students will also explore theoretical and experimental probability with a view to recognising that many trials of an experiment are necessary before theoretical probability may be evident. Internet-based random event generators are ideal for this purpose.

Within this cycle, students are exposed to a variety of common chance situations which may frequently take the form of games. Random event generators include tosses of coins, rolls of dice, drawing items from a deck, bag or box (including lottery or bingo), and spinners. Ensure when using games that the focus of activities is analysis of the points scoring and the likelihood of outcomes that makes the game fair or biased. Similarly, draw attention to common misconceptions that occur when combined with the rules of the game. For example, dice games where it is necessary to roll a specific number to start can result in the misplaced idea that it is harder to roll a 6.

Objectives

By the end of this cycle, students should be able to:

15.1.1Describe theoretical probabilities using fractions. [6SP144]

15.1.2Describe theoretical probabilities using decimals. [6SP144]

15.1.3Describe theoretical probabilities using percentages. [6SP144]

15.1.4Determine probabilities for the outcome of single step events. [7SP168]

15.1.5Compare observed frequencies across experiments with expected frequencies. [6SP146]

Conceptual Links

Probability requires students to be able to recognise attributes, count, and represent likelihood as a number of parts of a collection. Likelihood is represented as a fraction or decimal between 0 or 1 or percentages between 0% and 100%. Work within this cycle will rely on and reinforce students' understanding of division and division facts and fraction, decimal and percentage notations.

Materials

For Cycle 1 you may need:

- random event generators: dice, spinners, cards, coins, bingo counters/balls
- counters and opaque bags or envelopes
- Internet-based random event generators

Key Language

Experiment, event, outcome, sample space, fraction, decimal, percentage, likelihood, chance, equally likely, range, probability, spinner, independent events

Definitions

Experiment: a single trial of a chance event. For example, one roll of a die would be an experiment.

Experimental/frequentist probability: the likelihood of a chance event based on data from a collection of experiments. The greater the number of trials, the nearer to the theoretical probability the experimental probability should be.

Independent events: events that are not affected by the outcome of previous or subsequent events. For example, flipping a coin, each roll of a die, each run of a spinner, pulling counters from a bag with replacement, and pulling a card from a deck with replacement, are all independent events.

Sample space: the number of possible outcomes for a chance event.

Theoretical probability: the frequency of an outcome divided by the sample space. For example, a 6-sided die numbered 1 to 6, has a theoretical probability for each number of one sixth, a fair coin has a theoretical probability of one half.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many different outcomes are there for the event? What is the sample space you have?
- Are all outcomes equally likely?
- Is each event independent?
- What is the theoretical probability?
- What was the frequency of each outcome from your experiments?
- What is the experimental/frequentist probability?
- What do you notice when you compare the theoretical probability with the experimental probability? Did an outcome happen more or less than the expected number of times? Did an outcome happen the same as the expected number of times?

RAMR Cycle

Reality

Discuss chance events with students. Some examples may be, likelihood of rain on a sunny day or on a cloudy day, sunrise/sunset, seeing the moon during the day, seeing the moon at a specific time of the day, walking home, or being allowed to stay up past midnight. Revisit the informal language of probability discussed in Unit 01. Ask the students to order these words on a continuum and to suggest other terms for likelihood. Draw students' attention to descriptions of likelihood that involve number such as, 100%, 50%, 1 chance in 2, 50-50, 25-75 and so on.

Resource Resource 15.1.1 Continuum of probability language

Revise by identifying possible outcomes and defining sample space using the toss of a coin. Make sure that students can list all the possibilities (heads or tails) and the chance of tossing a head (1 chance out of 2, ½, 0.5, 50%, 50-50, even chance). Discuss the likelihood of tossing a head or a tail (2 chances out of 2, 2/2, 1, 100%, certain) and the likelihood of tossing something other than a head or a tail (0 chances out of 2, 0/2, 0, 0%, impossible).

Abstraction

The abstraction sequence for this cycle starts from students' understanding of the likelihood of chance events and familiar everyday language to describe likelihood. The ability to identify all possible outcomes of a chance event and assign a probability within the range of 0 to 1 is extended upon within this cycle to comparison of theoretical and experimental probability. A suggested sequence of activities is:

1. *Identify all possible outcomes of an experiment.* Examine a fair six-sided die to list all possible outcomes. Discuss how many of each outcome are on the die.

Introduce language. Ensure students understand that rolling a single die several times results in independent events. One roll of the die does not affect the outcome of subsequent rolls.

Assign theoretical probabilities to each outcome. Discuss the likelihood of each outcome as one chance out of six. Ensure that students understand that this represents the theoretical probability, discuss how this might be different if dice were actually rolled.

Represent symbolically. Discuss how to represent this as a fraction. Use calculators to convert to decimal. Convert decimal to percentage to represent as chance out of 100%.

Test experimental probability for each outcome. Use racetrack game to see which car finishes first. Record each roll of the dice in a tally table. At the end of the game, students should identify how many rolls total to finish the game and the experimental probability of each number rolled for their game. Generate a class total for each number by collating all tallies into one larger dataset. Recalculate the experimental probability. Explore how close to theoretical probability the class set of rolls reaches.

Generate a very large dataset for trials. Use an Internet-based dice roller to generate 1000 rolls of a dice and compare the experimental probability with theoretical probability.

Resource Resource 15.1.2 Racetrack game

Mathematics

Language/symbols and practice

Explore other random event generators that maintain a constant sample space such as spinners, drawing counters from bags, cards from decks (with replacement and shuffling between draws), use more than one die to generate larger totals. These provide practice for students in identifying possible outcomes, assigning theoretical probabilities and comparing with experimental results. Teachers can use the activities to address possible student misconceptions about probability.

Beetle game

A common misconception that students develop with dice is that it is harder to roll a six than any other number (this stems from the number of games that require a 6 to be rolled to start). This misconception may be addressed through The Beetle Game which assigns a dice number to each part of the beetle to be drawn. As the game progresses, the focus number that students need to draw their beetle changes, so most numbers are difficult to roll at some point. If students persist in their misconception, it is possible to reassign dice values to the parts to be drawn to make sure the teacher has not rigged the game. Students should also be encouraged to work out the likelihood of each number on a fair die occurring and the actual theoretical probability of each number appearing.

Resource Resource 15.1.3 The Beetle Game

Horse race game

To ensure that calculation of fraction probabilities includes all possible outcomes, it is important to build **sample spaces** of all possible outcomes. For example, if two dice are thrown and added, the sample space is as shown below. It means, for example, that the chance of getting a 7 is the number of outcomes giving 7 divided by the total number of outcomes = 6/36. Initially, to maintain the transparency of the sample space, it is important to not reduce fractions to their lowest form.

Two-dice sample space:

2	1,1					
3	1,2	2,1				
4	1,3	2,2	3,1			
5	1,4	2,3	3,2	4,1		
6	1,5	2,4	3,3	4,2	5,1	
7	1,6	2,5	3,4	4,3	5,2	6,1
8	2,6	3,5	4,4	5,3	6,2	
9	3,6	4,5	5,4	6,3		
10	4,6	5,5	6,4			
11	5,6	6,5				
12	6,6					

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Students should be encouraged to choose a horse to back in the first run through of The Horse Race game, determine which horse wins, then explore the actual likelihood of each horse winning. The sample space can be determined using either of the methods shown above. The game engages students with experimental probability versus theoretical probability. While horse 7 should win, it is still possible that horses 2 or 12 may come home first. Ensure students understand why horse 1 or horse 13 will never leave the starting gates.

Resource Resource 15.1.4 The Horse Race Game

Feud

The game of Feud also uses two dice with a point score assigned to specific outcomes.

Materials: 2 dice, 2 players.

Rules:

- Players take turns throwing dice and add the numbers.
- If the point sum is 2, 3, 4, 10, 11 or 12, player 1 receives one point. If the sum is 5, 6, 7, 8 or 9, player 2 receives one point.
- The first player to gain 10 points wins.

Questions (after many games):

- Is the game fair?
- Does it matter whether you are player 1 or 2?
- What extra numbers could we give player 1 to even the contest?

Making Fair with Spinners

Spinners may be difficult for students if they are relying solely on count of sectors. Spinners work on the amount of area for each sector. It is important to have students use tracing paper to validate that sectors on a spinner are all equally likely because the areas are the same before they look at the number of sectors that are coloured. Spinners may be coloured or numbered or, in the case of The Beetle Game or Twister they may have pictures of body parts on them. Any game that uses dice can be converted to a spinner game.

Games so far have engaged students with known or calculable theoretical probabilities which they can then compare with experimental probabilities. It is important to also provide opportunities to use the frequency or experiment probability to predict what items may be in the sample space.

Mixed Bags I

Provide students with opaque bags or envelopes containing ten coloured counters (three different colours). Engage students in guessing the contents of each bag without looking in the bag. To do this, students should make ten draws from the bag and record the colour of the counter they draw out. Each time the counter should be replaced and the bag shaken before redrawing. Once students have drawn from the bag ten times, they should consider the frequency of each coloured counter they have drawn to suggest how many of each colour is in the bag.

Students can use a deck of cards to provide a similar experience. Shuffle a selection of ten cards. Turn over the first card and record its value. Replace and reshuffle the cards. Students should consider the frequency of each card and suggest how many of each card is present.

Mixed Bags II

This activity may be extended by providing students with a set of probabilities for drawing coloured counters from a bag or cards from a deck. Ask students to construct a suitable set of counters or cards to match the probabilities. Initially, these probabilities should be in fraction form with the same denominators to indicate the sample space. As students develop competence, include examples where some probabilities have been reduced to lowest form or represented with decimals or percentages. For example, The Horse Race game generates probabilities of 1/36, 2/36, 3/36, 4/36, 5/36 and 6/36 and so on. These may also be represented as 1/18, 1/12, 1/9, 1/6. To solve these problems, students need to first generate a selection of probabilities with matching denominators to indicate the possible sample space of outcomes.

Reflection

Check the idea

Use *Resource 15.1.5: Tossing coin experiment* to further explore theoretical and experimental probability arising from tossing a fair coin and represent the outcomes of heads and tails cumulatively as percentages.

Resource Resource 15.1.5 Tossing coin experiment

Apply the idea

Explore *Resource 15.1.6: Cylinder experiment* to apply cumulative percentages of probability to a context other than tossing coins.

Resource Resource 15.1.6 Cylinder experiment

Extend the idea

Ask students to work in pairs to construct a chance game using one of the random event generators explored. Encourage students to set the scoring of outcomes for the game in such a way that the outcomes of the game are "rigged" or weighted unfairly towards one outcome. This should be done as subtly as possible. Engage students with playing games designed by other groups with the added task of identifying what outcome has been weighted to win and how this has been accomplished. This strategy extends students' understanding of the operation of chance events. It also engages students with extended reasoning about likelihood of chance events. When a loss occurs, opportunities for discussion are generated because the weighted outcome does not produce the expected result. This activity further highlights the potential misuse of probability in real world contexts.

The *Biased Die Game* may help to provide students with some ideas as to how this may be achieved.

Biased Die

Materials: Cubes with sides as follows: 1, 1, 2, 3, 3, 3.

Directions:

Create a biased die with numbers e.g. 1, 1, 2, 3, 3, 3. Roll the die and call the outcome for students to record on a table (this means they do not have the chance to study the die). Keep rolling until students think they know the arrangement of numbers on the die.

- A pattern may appear after 10 rolls, where students can make a hypothesis as to what numbers are featured on the die.
- After 20 rolls students may be in a better position to make a hypothesis regarding the number arrangement.
- Clear frequencies will be seen with over 50 rolls of tally data.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________ Date:________

Can you do this? #2

1. A coin is flipped and a die is rolled.

(a) How many different combinations could there be?
(Show your working.)

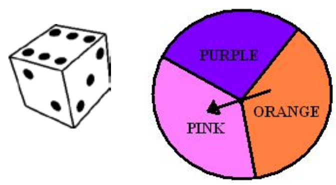
(b) What is the probability of tossing a head and rolling an even number?

____

(Show your working.)

(c) What combination has the probability outcome of 0.33? _____
(Show your working.)

2. A board game rolls a die and spins a spinner to determine the types of moves players can make. Use a tree diagram to show all possible outcomes of the die and spinner.



Obj.
15.2.1
a)□□□
□□□
□□□
□□□

b) □

c) □

Obj.
15.2.2
□□□
□□□
□□□
□□□
□□□
□□□
□□□

3. You are playing a game with two dice.

(a) Write down the possible combinations of dice to give each total.

Total	Combinations		Total	Combinations	
1			7		
2			8		
3			9		
4			10		
5			11		
6			12		

(b) How many combinations are there in total (include turnarounds).

____

(c) In the table, write the probability of each outcome as a fraction.

(d) Would you be more likely to roll a 7 or an 11?______________

(e) Why do you think this is? ____________________________

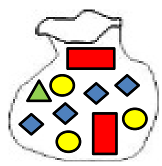
____

Obj. 15.2.1
a) □□□ □□□ □□□ □□□ □□□ □□□ □□□ □□□ □□□ □□□ □□□ □□□
b) □
c) □□□ □□□ □□□ □□□
d) □
e) □

4. When playing Monopoly, what are the chances of getting out of jail by rolling doubles? ______________________________

Obj. 15.2.1 □

5. A rectangle is drawn out of the bag and then put back in. What is the probability of a rectangle being drawn out of the bag on the next turn? ______________________



Obj. 15.2.4 □

6. These three markers are left to draw out of a bag. Markers drawn out of the bag are not put back.

12 2 13

Draw tree diagrams of the possible combinations of markers that might come out of the bag in the next two consecutive draws.

Obj. 15.2.3 □□□ □□ □□ □□

7. Two cards are drawn out of a deck of 52 cards one after the other.

(a) What is the chance of the first card being red? ______________

(b) What is the chance of the second card being a spade? __________

Obj. 15.2.5
a) □
b) □

Unit 2: Multiple-Step Probability

Overview

Big Idea

This cycle examines the likelihood of a single chance event before further exploring the possibility of successive or dependent chance events. Students investigate theoretical and experimental probability of multiple-step chance events both with and without replacement, for example, drawing cards from a deck, tossing multiple coins, and lottery games.

Objectives

By the end of this cycle, students should be able to:

15.2.1 Represent events and solve problems using two-way tables. [8SP292]

15.2.2 List all outcomes for two-step chance experiments with replacement using tree diagrams. [9SP225]

15.2.3 List all outcomes for two-step chance experiments without replacement using tree diagrams. [9SP225]

15.2.4 Determine probabilities for events with replacement. [9SP225]

15.2.5 Determine probabilities for events without replacement. [9SP225]

Conceptual Links

Probability requires students to be able to determine likelihood of events that may occur with or without replacement as well as determining the likelihood of events that involve multiple chance events. Work within this cycle will rely on and reinforce students' understanding of single-step probability, division and division facts and fraction, decimal and percentage notations. All probability activities should include consideration of the validity of results and what an answer means when used to make and justify decisions.

Materials

For Cycle 2 you may need:

- random event generators such as dice, coins, spinners, Internet based tools
- game boards
- Internet based random event generators
- 1 cm grid paper for quick generation of 2 way tables

Key Language

Experiment, event, outcome, sample space, multiple-step probability, subsequent events, successive events, with replacement, without replacement, tree diagrams, two-way tables, combinations

Definitions

With replacement: any instance of a chance experiment where the sample space is unchanged for subsequent draws. For example, flipping a coin, rolling a die, spinning a spinner, drawing a card from a deck and shuffling it back in before another card is drawn.

Without replacement: any instance of a chance experiment where the sample space changes after each experiment. For example, lottery or bingo draws where numbers are not replaced, card games where consecutive cards are drawn from a deck.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Is this a single chance event or a sequence of chance events?
- Does one chance event affect the next chance event in your trials?
- Are you replacing or not replacing the elements after each trial?
- Will the sample space be the same or different?
- How can you represent all the possible outcomes of each event?
- Do you have equal likelihood of each event occurring?

RAMR Cycle

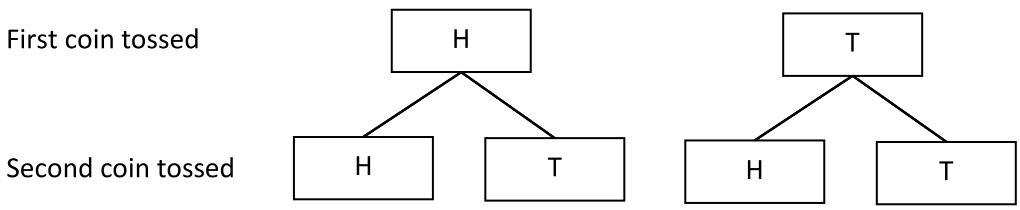
Reality

Discuss chance games with students that use combinations of random events generators. The simplest example is the very old Australian game of Two Up. A wide variety of games exist that rely on successive random events including any card game and commonly advertised lottery games. Discuss the sample space of successive events with students. For example, in Two Up the flip of one coin does not influence the outcome of the flip of the other coin. In card games, each card dealt that is not replaced in the deck and shuffled does influence the outcome of the next card. For example, in a deck of cards, if the Ace of Hearts is in view on the table and another appears in a subsequent draw, people tend to call “foul play”. See if students can identify other chance events where subsequent outcomes are/are not influenced by previous events.

Abstraction

The abstraction sequence for this cycle starts from students’ understanding of single-step probability. The ability to identify all possible outcomes of a chance event and assign a probability within the range of 0 to 1 is extended upon within this cycle to consider multiple-step probability with and without replacement. A suggested sequence of activities is:

1. *Identify all possible outcomes of an experiment.* Using the toss of two fair coins as an example, explore the sample space. Ask students to identify the possible outcomes of tossing two fair coins and predict the likelihood of each outcome. It is highly likely that students will identify three possible outcomes of both heads, both tails or one head and one tail.
2. Discuss the likelihood of each outcome with students. Ask them to predict the probability. It is highly likely that they will identify the probability as one chance in three for each outcome. Playing the game for a number of flips may start to indicate a higher prevalence of one head and one tail as an outcome (hopefully).
3. *Represent symbolically.* Discuss how to represent the possible outcomes of tossing two coins in a tree diagram with each coin toss represented as a sequence. This has been shown vertically here but could also be represented horizontally. Ensure that students understand that the outcome of one coin toss is still independent of the previous coin toss.



4. *Assign theoretical probabilities to each outcome.* Reexamine with students the likelihood of each outcome. It may help students to list the possible outcomes or ways to reach an outcome if the tree diagram does not help. For example, possible combinations of coins could be HH, HT, TH, TT. They should see that there is one way to have two heads, one way to have two tails and two possible ways to reach a head and a tail. This means that the probability is not out of three possible outcomes, but out of four possible combinations of outcomes. Highlight with students that the probability of each toss of the coin is ½, but as combinations of outcomes, the probability is ½ for the first coin, and ½ for the second coin. Connect to the mathematical process of ½ × ½ = ¼ to determine the likelihood of each outcome.

Mathematics

Language/symbols and practice

Continue students' exploration of multiple-step probability with further coins added to the toss. Use listing of possible combinations and tree diagrams to scaffold students' thinking to determine how many possible combinations of Heads and Tails arise when using three coins and the subsequent probability of each. For example, HHH, HTH, HTT … and so on. Count the outcomes that result in all heads, all tails, two heads and one tail, one head and two tails.

Planetfall Game

The Planetfall Game uses multiple-step probability equivalent to four successive tosses of a fair coin. Play the game first so that students can explore playing the game and scoring. Extend students' thinking by examining the likelihood of landing on each planet. Discuss whether the scoring used is enough to balance out the likelihood of landing on each planet. Ask students to create a game that uses the same system to go two levels further (six successive tosses of a fair coin). Instruct them to use the theoretical probability of landing on each outcome as a base for determining the score for each so that it is as close to an even chance as possible. **Note**: Students may choose a context for their game (e.g., countries, sporting teams, racing events, anything that captures their interest).

Resource Resource 15.2.1 Planetfall Game

Explore the use of two-way tables to record combinations of chance events that only combine two events, for example, choosing a mode of transport followed by a destination (modelled by tossing a coin and rolling a die). Assume that each choice is equally likely. **Toss of coin:** Heads to fly, Tails to drive. **Roll of die:** 1) Brisbane; 2) Adelaide; 3) Darwin; 4) Sydney; 5) Melbourne; 6) Perth

Destination Transport	Brisbane (1)	Adelaide (2)	Darwin (3)	Sydney (4)	Melbourne (5)	Perth (6)
Fly (Heads)						
Drive (Tails)						

Discuss the theoretical likelihood of each travel outcome. Toss a coin and roll a die ten times to determine which travel combination occurs most often. Consider ways to increase the number of possible destinations and/or modes of transport that can still be represented in a two-way table (for example, two dice or two coins). Determine whether each outcome is still equally likely.

Sandwich Bar Options

Complete *Resource 15.2.2 Sandwich bar options*. Once students have completed the exercise consider, if it is known that a number of the team have a greater preference for chicken over tuna, how should this change the initial decisions about the number of each type of lunch to make? Repeat the changing of parameters for other elements of the combinations. Discuss with students that these are dependent events. The outcome of the first lunch bag changes the possible options for subsequent choices.

Consider where there are even numbers of each option, what happens when the second person chooses lunch – determine how their options are different. This provides an opportunity to discuss probability of combinations of events because subsequent choices may change without replacement.

Resource Resource 15.2.2 Sandwich Bar Options

Multiple-step Probability with/without Replacement

Card games, bingo and lottery all provide opportunities to further explore multiple-step probability with/without replacement. Explore scenarios like drawing a card from a deck, replacing, shuffling and drawing another card. The likelihood of a card being drawn in this scenario does not change – it is possible to draw out the same card twice. Explore a number of cases of a specific card (7 hearts, probability of 1 in 52), and less specific cards (a 7, probability of 1 in 4; a heart, probability of 1 in 13; a picture card (K, Q, J, A), probability 16 in 52; a card with a value under 5, probability 16 in 52).

Follow this up by drawing a card from a deck and then determining the probability of the next card. For example, if a 7 hearts is drawn first up, what is the likelihood of following cards being …? (try a few examples here as above). Ensure students consider the changed sample space and the changed number of options. For example, there are only 51 cards remaining to choose from. So, there is 0 probability of drawing out another 7 hearts; 1 in 3 probability of drawing another 7; 1 in 12 probability of drawing another heart; 16 in 51 chance of a picture card; 16 in 51 chance of a card with a value under 5. Repeat this for a further card which will drop the possible cards to choose to 50 and so on.

Extend students further to consider the likelihood of each number from a lotto draw of 45 numbers. Traditionally numbers are not replaced when drawn. Consider the value of Systems games and the accompanying price changes. Are these better value for money in terms of cost per game?

Reflection

Check/Apply the idea

Explore chance outcomes with students that require them to use multiple-step probability with/without replacement.

Monopoly

Explore the likelihood of possible outcomes in Monopoly. For example, is it possible for anyone to land on Old Kent Road on the first turn? What other spaces are impossible? What spaces are possible? How likely are you to land on Community Chest on the first go? What is the most likely space to reach? How likely are you to land on Community Chest on the first go, and then Chance on the second go? How likely are you to go to jail for rolling three doubles in a row? Is it possible to land on The Angel Islington, Euston Road and Pentonville Road in that order in the first round? Is it possible to collect all the railway stations on the first go around the board? What numbers would you need to roll? What is the likelihood of that happening?

Resource Resource 15.2.3 Monopoly Board

Yahtzee

Consider the Bonus Points section of a Yahtzee score card. Compare the points with the likelihood of each occurrence. Are these fair? Are these events listed in order of likelihood? To really extend students, consider the likelihood of rolling certain combinations of dice (use calculators).

Resource Resource 15.2.4 Yahtzee Variations

Bingo

You are playing Bingo and need number 6 to win. If there are a hundred numbers and forty-five numbers have already been drawn, how likely is the next number to be 6?

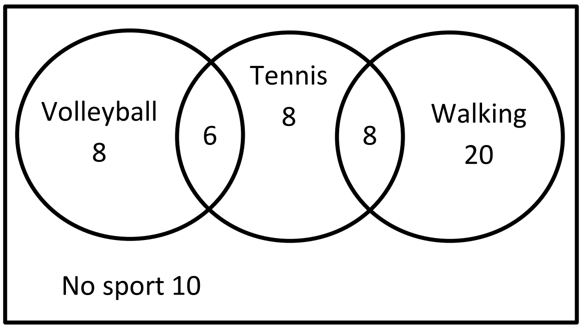
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________ Date:________

Can you do this? #3

1. This Venn diagram shows members of a sports club and the sports they play.



Write down the complementary event to people who play at least one sport. ____

Obj. 15.3.1

1 □

2. In the Venn diagram, people play volleyball, tennis, walking or no sport.

(a) How many members play at least two sports? _____________

(b) How many people play volleyball or tennis but not both? _________

(c) You are planning a barbecue for all sports club members who play volleyball and tennis. How many do you need to cater for? ___

Obj. 15.3.3

2a) □

b) □

c) □

3. A school survey found that 50% of students are likely to represent the school in sports. 25% of students are likely to choose football, 12% are likely to choose netball, and the remaining students are likely to play hockey.

   (a) What percentage of students are likely to play hockey? _______

   (b) In a class of 24 students, how many students in the class are likely to not play sport? ______________________________________

   (c) In a class of 24 students, how many are likely to play football?

Obj. 15.3.2

3a) □

b) □

c) □

4. A tyre manufacturer surveyed 2200 customers about their tyre preferences. 894 customers liked Tyre A only, 588 liked Tyre B only, and 496 liked both tyres equally. Draw a Venn diagram to represent the tyre manufacturer's survey results.

Obj. 15.3.4

4i □

ii □

iii □

iv □

5. (a) How many customers liked neither Tyre A nor Tyre B? ________

(b) How many customers liked Tyre A or Tyre B but not both? _____

(c) If you were the tyre manufacturer, based on the results of this survey, would you make more or less or the same number of each type of tyre? ____

Obj. 15.3.5

5a) □

b) □

c) □

6. (a) Based on the survey data, if a new customer comes into the store, what is the probability that they will like only Tyre A better? ______ (Show your working.)

(b) If a similar sample were surveyed with 4400 people, how many of them are likely to like only Tyre A better? ______________________

Obj. 15.3.6

6a) □

b) □

c) □

d) □

(c) Based on the survey data, what is the probability that a new customer will like either Tyre A or Tyre B the same? _____________

(d) If a similar sample were surveyed with 4400 people, how many of them are likely to like either Tyre A or Tyre B the same? _________

Unit 3: A and B, A or B, A not B Investigations in Likelihood

Overview

Big Idea

This cycle begins by exploring the likelihood of single or multiple-step chance events before more closely considering unrelated, complementary, mutually inclusive or mutually exclusive events. Students should first be able to identify where one event cannot happen if the other happens. They must also recognise when the chance of one event does not affect the occurrence of another. Students will use Venn diagrams as a tool for graphically representing events.

Objectives

By the end of this cycle, students should be able to:

15.3.1Identify complementary events. [8SP204]

15.3.2Use the sum of probabilities to solve problems. [8SP204]

15.3.3Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'. [8SP205]

15.3.4Represent events in Venn diagrams. [8SP292]

15.3.5Solve problems using Venn diagrams. [8SP292]

15.3.6Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'. [9SP226]

Conceptual Links

Probability requires students to be able to determine likelihood of events that may occur with or without replacement as well as determining the likelihood of events that involve multiple chance events. Work within this cycle will rely on and reinforce students' understanding of single-step probability, division and division facts and fraction, decimal and percentage notations. All probability activities should include consideration of the validity of results and what an answer means when used to make and justify decisions.

Materials

For Cycle 3 you may need:

- hoops
- length of rope or tape
- collection of pictures or items that may be sorted

Key Language

Venn diagram, independent events, complementary events, mutually inclusive, mutually exclusive, including, excluding, universal set, null set,

Definitions

Venn diagram: diagram to visually represent mathematical or logical sets, usually drawn as circles. Common elements of sets are represented by intersections of the circles.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What are all the events in the set you are looking at?
- Are there any overlaps between the events?
- Are there any gaps between the total number and the number accounted for in the events?
- How can you represent this visually?
- If there are some overlaps, how will you spread the totals for each event out so that the total in each circle is correct?
- How are the variables related?
- Are there any complementary events?
- Are there mutually exclusive events?
- Can you identify examples where the sets overlap?

RAMR Cycle

Reality

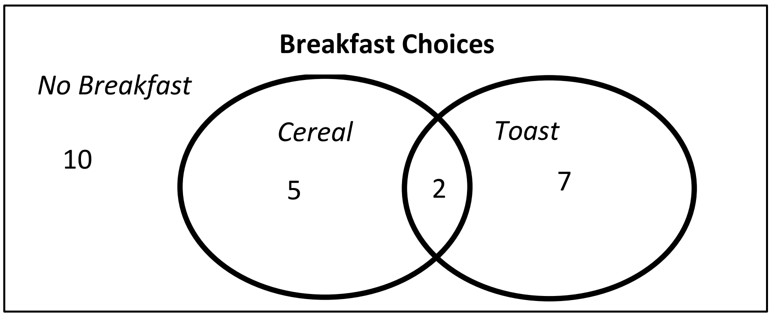
Remind students of the data activities from *P9: Are we really healthy?* (accompanying Unit 10: Summarising Data with Statistics). Quickly generate bar graphs to represent Breakfast Choices (no breakfast, bread, cereal) and Breakfast Choices (juice, milk, tea/coffee). Discuss what the graphical representation shows well and what is more difficult to determine. For example, it is easy to see how many students have no breakfast, bread or cereal. However, it is difficult to determine (unless the total number of students is known), how many students selected more than one option. In particular, two students selected both bread and cereal. Discuss other ways to represent the data to provide more detail.

Resource Resource 15.3.1 Breakfast choices sample data

Abstraction

The abstraction sequence for this cycle starts from students' understanding of graphical representations and extends to representation of data using Venn diagrams. Information represented on the Venn diagram can then be analysed in terms of likelihood of single events, combined events and complementary events. A suggested sequence of activities is:

1. *Kinaesthetic activity.* Use skipping ropes to generate large interlocking circles to stand in. Have students write their name on a piece of paper or sticky note and consider their typical breakfast choice – no breakfast, toast and/or cereal. Students who have no breakfast stand outside the circles. Students with toast or cereal stand in toast or cereal circles. Students who have both toast and cereal stand in the intersection of the circles. Alternatively, use sticky notes or names on paper with hoops on the floor or large circles drawn on the board to generate the Venn diagram.
2. *Represent with drawing/symbols*. Ask students to draw circles in their books and write students' names or use a numeric value to show how many students in each group.



3. *Connect to language*. Discuss the parts of the Venn diagram and what it represents with students. Consider the following questions: How many students in total? 5 + 7 + 2 + 10 = 24;
How many students have cereal? 5 + 2 = 7; How many students have toast? 7 + 2 = 9; How many students have toast and cereal? 2; How many students have no breakfast? 10. Introduce the language of the intersection between the sets (in this case both cereal and toast), and the union of the sets (cereal or toast or both). No breakfast or the complement of cereal or toast or both should also be made explicit. Mathematical notation for these may also be introduced as students develop competence.

4. *Connect to probability*. Use the values from the sample to make predictions about breakfast choices. For example, from this sample, how likely is it to choose a person who has both cereal and toast for breakfast. Ensure students can identify the sample space (all students), and the intersection of the cereal and toast sets (2). This will give them a likelihood of 2 out of 24.
5. Generate a Venn diagram to represent the sample data explored in the reality phase. Compare the likelihood of selecting a person who has both cereal and toast for breakfast with the class sample. Discuss how similar/different the values are, which sample may be representative, how good a predictor of fourteen year old students' breakfast choices the samples may be. Consider what occupations or community sectors might find this information useful and how they might gather or find this data. Compare and contrast likelihoods of other events represented on the Venn diagram.
6. *Extend thinking.* Consider further surveys of samples that are used to generate data to make predictions. For example, polls before elections, television viewing polls, favourite actor surveys, voting on The Voice, Australia's Got Talent, Survivor, Big Brother, and any other ideas that may occur to students. Discuss with students how many people they think are surveyed to generate these values. Ask students what they think is an appropriate sample size to work from for predicting trends from gathered data.

Mathematics

Language/symbols and practice

Provide students with further practice gathering and representing data on Venn diagrams. The Breakfast choices data includes fruit juice/milk/tea or coffee as options. This extends students to representing more than two options in a Venn diagram. Ensure students are able to count from the data or use tally marks to determine how many students to place in each section on the Venn diagram. Engage students in similar thinking comparing and contrasting the likelihood of different combinations of fruit juice, milk and/or tea/coffee.

The data set in *Resource 15.3.2: Student Work Sample Data* engages students with more than two choices to represent in circles as they consider students involved in paid work, community work and/or housework outside of school. Engage students in similar thinking comparing and contrasting the likelihood of different combinations of paid work, housework and/or community work within the class group. Compare this data with the sample of data extracted from the Australian Bureau of Statistics.

Resource Resource 15.3.2 Student work sample data

Additional worksheets to practice Venn diagrams and the interpretation of data may be found at the following sites:

Math Worksheets Land: http://www.mathworksheetsland.com/topics/5packs/logic-venn-5pack.pdf

Primary Resources.co.uk: http://www.primaryresources.co.uk/maths/mathsF1b.htm#carroll

Education World: http://www.educationworld.com/tools_templates/venn_diagram_templates.shtml

Math-Aids Worksheets: http://www.math-aids.com/Venn_Diagram/

Reflection

Check the idea

Students can practice their skills in working on Venn diagrams by accessing word problems at the following links.

Math Worksheets 4 Kids: http://www.mathworksheets4kids.com/venn-diagram.html

Common Core Sheets: http://www.commoncoresheets.com/Venn.php

Apply the idea

Engage students in choosing a suitable topic to survey. Students will be asked to gather, represent and analyse data using Venn diagrams. Ensure students are able to interpret the data in terms of what it might mean for a larger population.

Extend the idea

Provide students with a set of probabilities generated from a sample. Represent these probabilities on a Venn diagram to reconstruct a possible sample space and group profile.

Venn Diagram Worksheet: Mr Santowski's Math Page:
http://mrsantowski.tripod.com/2011IntegratedMath2/LessonNotes/IM10L47Venn_Diagrams_and_Probability.pdf

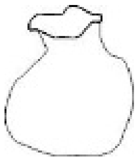
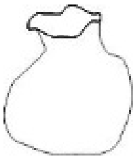
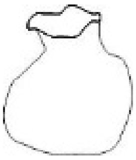
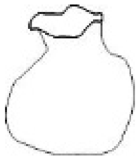
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________ Date:________

Can you do this now? Unit 15

1. Draw marbles in the bags so that the probability of getting a red marble is:

(a) $\frac{2}{5}$	(b) 1	(c) 0	(d) More than $\frac{3}{6}$
			

Obj. 15.1.1
1a) □
b) □
c) □
d) □

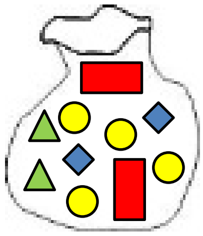
2. A bag has 5 green counters, 3 red counters and 2 blue counters. What is the chance of selecting a green counter?

   Circle the correct answer.

(a) $\frac{2}{3}$ (b) $\frac{3}{10}$ (c) $\frac{1}{2}$ (d) $\frac{6}{10}$

Obj. 15.1.1
2a) □
b) □
c) □
d) □

3. Write the probability as a decimal of selecting each shape block from the bag.

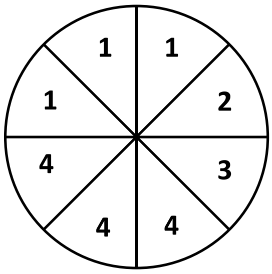


(a) Square ________ (b) Circle ________

(c) Triangle ________ (d) Rectangle ________

Obj. 15.1.2
3a) □
b) □
c) □
d) □

4. Show the percentage chance of spinning each number on the spinner.



(a) 1: ______________ (b) 2: ________________

(d) 3: ______________ (d) 4: ________________

Obj. 15.1.3
4a) □
b) □
c) □
d) □

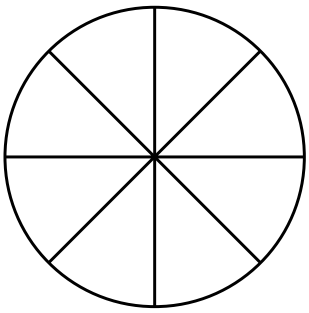
5. Write how many heads and how many tails you would be likely to get if you flip a coin 100 times?

(a) Heads: ____________________ (b) Tails: _________________

(c) Why: ____

Obj. 15.1.4
5a) □
b) □
c) □

6. Write the numbers 1, 3, 6 and 9 onto the spinner so that you would be:



(a) most likely to get a 1

(b) just as likely to get a 3 as a 9

(c) least likely to get a 6

Obj. 15.1.4
6a) □
b) □
c) □

7. If you draw one card out of a 52 card deck, replace it and shuffle the deck.

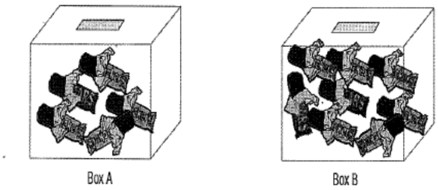
(a) What are the chances of drawing that same card out again? ______

(b) Is it a fair draw if you do not shuffle the cards? ________________

(c) Why/why not? ____

Obj. 15.1.4
7a) □
b) □
c) □

8. Box A has 1 Mars Bar and 4 Snickers; Box B has 2 Mars Bars and 6 Snickers. (You cannot see inside the boxes).



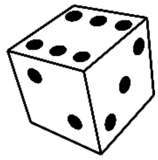
(a) If you want to get a Mars Bar when you take one out, circle the box you would choose.

(b) Why did you choose that box? ____________________________

____

Obj. 15.1.4
8a) □
b) □

9. On a fair 6-sided die, what is the probability (from 0-1) of rolling each number?



(a) 1: ________ (b) 2: ________ (c) 3: ________

(d) 4: ________ (e) 5: ________ (f) 6: ________

(g) What probability do you have of rolling a number less than 3? ____

(h) What is the fraction chance of rolling a 3 or a 4? _______________

(i) What is the percentage chance of rolling the even numbers? _____

(j) If you rolled the die 96 times, how many times should each number come up? ____

Obj. 15.1.4
9a) □
b) □
c) □
d) □
e) □
f) □
g) □
h) □
i) □
j) □

10. When you roll the die 96 times, you record the following frequencies:

Observed Frequency	1: 12	2: 25	3: 10	4: 8	5: 16	6: 25
Experimental Probability						

a) Based on this data, write the experimental probability in the table for each number.

b) How does the experimental probability compare to the theoretical probability? ____

Obj. 15.1.5
a)□□□
□□□
b)□□□

11. A coin is flipped and a die is rolled.

(a) How many different combinations could there be?
(Show your working.)

(b) What is the probability of tossing a head and rolling an odd number?

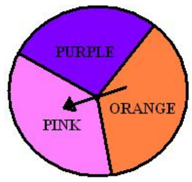
____

(Show your working.)

(c) What combination has the probability outcome of 0.33? _________
(Show your working.)

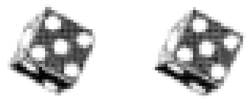
Obj. 15.2.1
a)□□□
□□□
□□□
□□□
b) □
c) □

12. A board game rolls a die and spins a spinner to determine the types of moves players can make. Use a tree diagram to show all possible outcomes of the die and spinner.



Obj. 15.2.2
□□□
□□□
□□□
□□□
□□□
□□□
□□□

13. You are playing a game with two dice.



(a) Write down the possible combinations of dice to give each total.

Total	Combinations		Total	Combinations	
1			7		
2			8		
3			9		
4			10		
5			11		
6			12		

(b) How many combinations are there in total (include turnarounds).

____

(c) In the table, write the probability of each outcome as a fraction.

(d) Would you be more likely to roll a 7 or an 11? ________________

(e) Why do you think this is? ______________________________

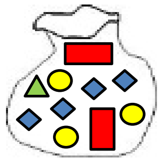
____

Obj. 15.2.1
a)□□□
□□□
□□□
□□□
□□□
□□□
□□□
□□□
□□□
□□□
□□□
□□□
b) □
c)□□□
□□□
□□□
□□□
d) □
e) □

14. When playing Monopoly, what are the chances of getting out of jail by rolling doubles? ______________________________________

Obj. 15.2.1
14 □

15. A rectangle is drawn out of the bag and then put back in. What is the probability of a rectangle being drawn out of the bag on the next turn? ___________________________



Obj. 15.2.4
15 □

16. These three markers are left to draw out of a bag. Markers drawn out of the bag are not put back.

12 2 13

Draw tree diagrams of the possible combinations of markers that might come out of the bag in the next two consecutive draws.

Obj. 15.2.3
□□□
□□
□□
□□

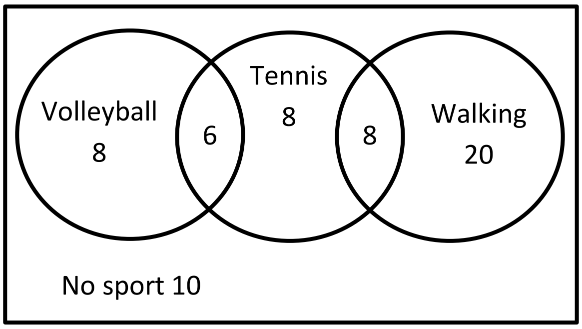
17. Two cards are drawn out of a deck of 52 cards one after the other.

(a) What is the chance of the first card being red? __________________

(b) What is the chance of the second card being a spade? ____________

Obj. 15.2.5
17a) □
b) □

18. This Venn diagram shows members of a sports club and the sports they play.



Write down the complementary event to people who play at least one sport. ____

Obj. 15.3.1
18 □

19. In the Venn diagram, people play volleyball, tennis, walking or no sport.

(a) How many members play at least two sports? __________________

(b) How many people play volleyball or tennis but not both? _________

(c) You are planning a barbecue for all sports club members who play volleyball and tennis. How many do you need to cater for? _______

Obj. 15.3.3
19a) □
b) □
c) □

20. A school survey found that 50% of students are likely to represent the school in sports. 25% of students are likely to choose football, 12% are likely to choose netball, and the remaining students are likely to play hockey.

(a) What percentage of students are likely to play hockey? _______

(b) In a class of 24 students, how many students in the class are likely to not play sport? ___________________________________

(c) In a class of 24 students, how many are likely to play football? _

____

Obj. 15.3.2
20a) □
b) □
c) □

21. A tyre manufacturer surveyed 2200 customers about their tyre preferences. 894 customers liked Tyre A only, 588 liked Tyre B only, and 496 liked both tyres equally. Draw a Venn diagram to represent the tyre manufacturer's survey results.

Obj. 15.3.4
21i □
ii □
iii □
iv □

22. (a) How many customers liked neither Tyre A nor Tyre B? ________

(b) How many customers liked Tyre A or Tyre B but not both? _____

(c) If you were the tyre manufacturer, based on the results of this survey, would you make more or less or the same number of each type of tyre? ____

Obj. 15.3.5
22a) □
b) □
c) □

23. (a) Based on the survey data, if a new customer comes into the store, what is the probability that they will like only Tyre A better? _____
(Show your working.)

Obj. 15.3.6
23a) □
b) □
c) □
d) □

(b) If a similar sample were surveyed with 4400 people, how many of them are likely to like only Tyre A better? ___________________

(c) Based on the survey data, what is the probability that a new customer will like either Tyre A or Tyre B the same? ____________

(d) If a similar sample were surveyed with 4400 people, how many of them are likely to like either Tyre A or Tyre B the same? _________

XLR8 – Accelerating Mathematics Learning

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