XLR8 – Accelerating Mathematics Learning

XLR8 Unit 14

Volume of 3D objects

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module’s development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras’ theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.

Scope

The relationships between **measures of three dimensional objects** are combined to **calculate solid volume** for **rectangular prisms**, **other prisms** and **cylinders**. These ideas are further connected to **mass** and **capacity**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts connected to and reinforced are highlighted in green and number and algebra concepts and processes applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

Diagnostic Worksheets: The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.

Anecdotal Evidence: Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.

Summative Worksheet: The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.

Portfolio task: The portfolio task *P14: Which Tank?* accompanying Unit 14 engages students with investigations into the dimensions and resulting volumes of various tanks.

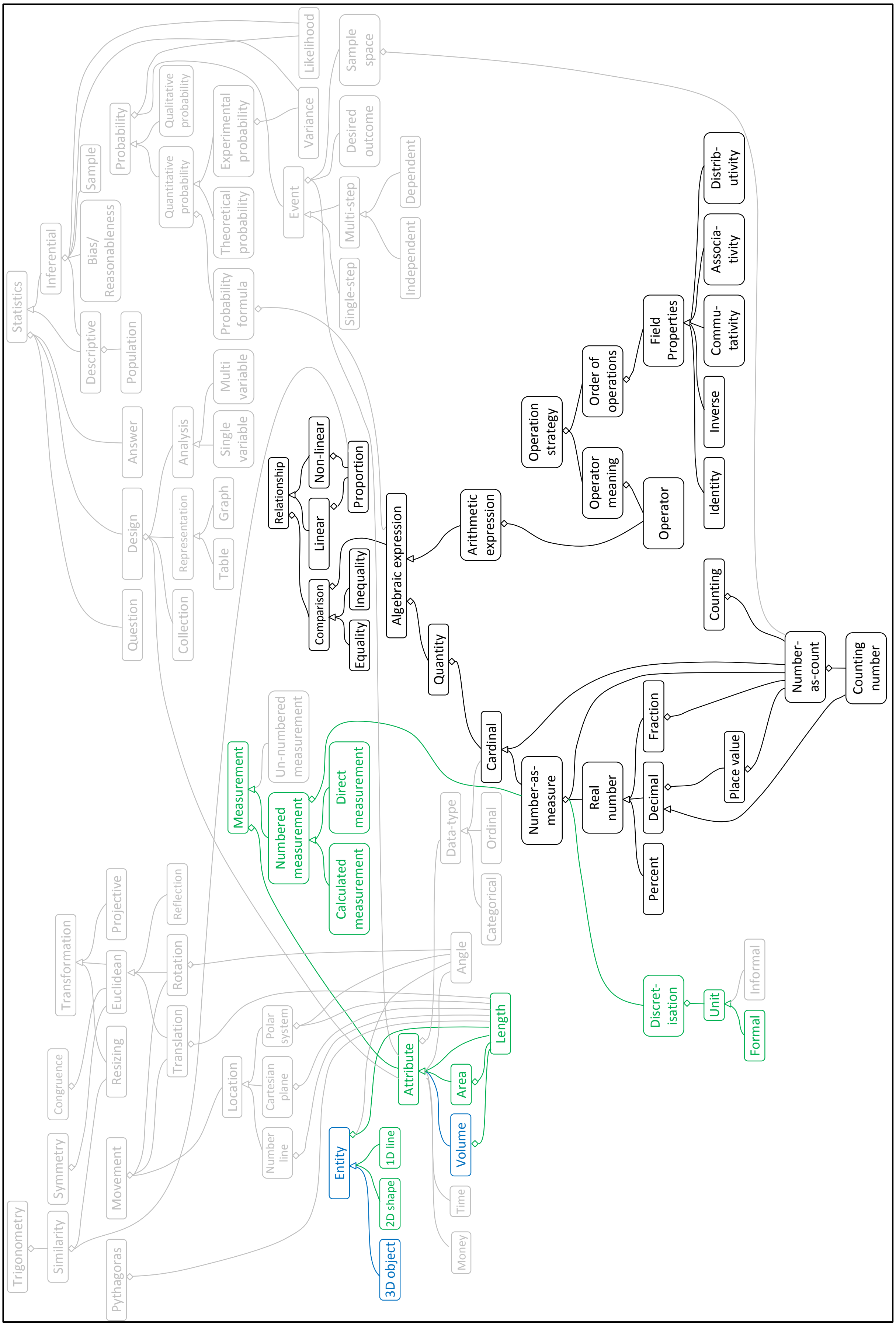


Figure 1. Scope of Unit 14

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Volume of prisms

In this cycle the calculation of volume of prisms is extended from students' understanding of calculated 2D area. The cycle also draws from previous student experience exploring the construction of 3D objects from 2D nets. This exploration should link to the concept of capacity. These ideas are further connected to calculations for volumes of triangular based prisms.

Cycle 2: Volume of cylinders

In this cycle the calculation of volume of cylinders is extended from students' understanding of calculated volume of rectangular and triangular based prisms. This exploration should link to the concept of capacity.

Cycle 3: Volume of pyramids and cones

Earlier cycles develop volume and connect capacity. When working with volume of pyramids and cones, however, it is more useful to use the concept of capacity to assist with the development of formulae. In this cycle students will explore the relationships between capacity of a cube and a square-based pyramid, and the capacity of a cylinder and a cone with the same size base. These relationships are then generalised to the formulae for volume of pyramids and cones.

Cycle 4: Relationship between surface area and volume

The surface area to volume relationship is similar to the perimeter to area relationship. For a fixed volume, surface area can vary widely, for a fixed surface area, spheres and cubes give the best volume. However, the volume to surface-area ratio increases as things grow in size. These ideas are useful for consolidating students' understanding of the difference between measures of volume and surface area. The ideas also provide additional practice in identifying the use of these calculations in real-world contexts.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially as it stands. If desired it may be possible to explore volume of pyramids and cones as extensions of their prism and cylinder counterparts. If stretched for time, exploration of the volume of pyramids and cones may be left as these are essentially beyond the Year 9 outcomes. However, inclusion of these objects will increase the range of real-world applications that students are able to engage with when considering the volume of composite objects in Cycle 5.

Literacy Development

The application of language consistent with the organisation of mathematical concepts is essential to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic). In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Volume of prisms

Box, rectangular prism, triangular based prism, capacity, mass, volume, cubic centimetre (cm^3), cc, area, base, height, three dimensional, length, width, rectangle, triangle, cubic millimetre (mm^3), cubic metre (m^3), cubic kilometre (km^3)

Cycle 2: Volume of cylinders

Cylinder, radius, diameter, circumference, cubic millimetre (mm^3), cubic centimetre (cm^3), cc, cubic metre (m^3), cubic kilometre (km^3)

Cycle 3: Volume of pyramids and cones

Pyramid, cone, capacity, volume, cubic centimetre (cm^3), cc, cubic millimetre (mm^3), cubic metre (m^3), cubic kilometre (km^3

Cycle 4: Relationship between surface area and volume

Surface area, square kilometres (km^2), square metres (m^2), square centimetres (cm^2), square millimetres (mm^2), volume, cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3), cubic millimetres (mm^3), 3D objects, 2D faces, formulae

Name:____ Date:_________

Can you do this? #1

1. Look at the things in your classroom.

   (a) Name two things that have volume.

   i. ______________________ ii. ______________________

   (b) Name one thing that does not have volume. ______________

   (c) Explain the difference. ______________________________

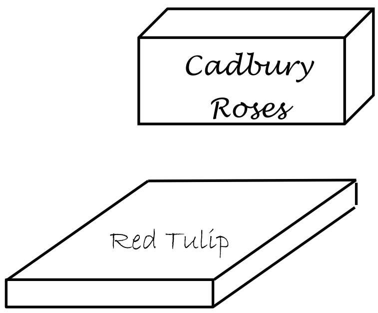
   ____

   (d) Can an object have volume but not capacity? _____________

   Why/why not? ____________________________________

Obj. 14.1.1
1.a)i □
ii □
b) □
c) □
d) □

2. How could you find out (without a ruler) which of the two boxes of chocolates has the greater capacity (holds more)?



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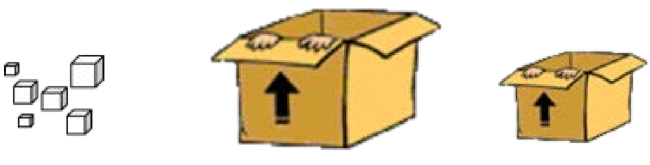
Obj. 14.1.1
2. □

3. Lots of small cubes (shown below) were used to measure the volume of Box A and Box B. When they were counted, Box B had more cubes than Box A.



Measuring cubes Box A Box B

   (a) Does this mean that Box B has a larger volume than Box A? ____

   (b) Why or why not? ________________________________

   ____

Obj. 14.1.1
3.a) □
b) □

4. A container has a capacity of 10 mL. What would be the volume of an ice cube made in this container? ____________________

Obj. 14.1.2
4. □

5. Write down something in the classroom, school or local area that you could measure in:

(a) km^3 ______________________________

(b) m^3 ______________________________

(c) cm^3 ______________________________

6. Convert the following volume measures:

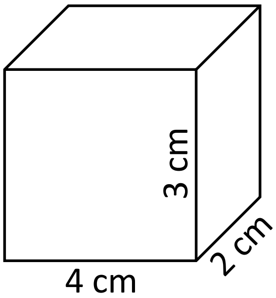
(a) $1\ 000\ 000\ cm^3$ = ______________ m^3

(b) $10\ cm^3$ = ______________ mm^3

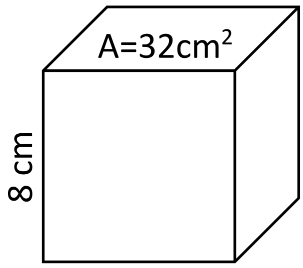
7. Find the volume of the following boxes:

(a) A box with height 3 cm, length 4 cm and width 2 cm.

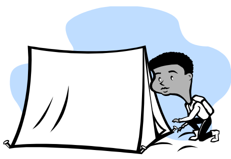
V = Area base × h = l × w × h



(b) A box with height 8 cm. The area of the top of the box is 32 cm^2.



8. Describe how to calculate the volume of this tent.



____

____

9. a) Here is a hexagonal based prism. Write a formula to calculate its volume. ______________

b) If its base has an area of 25 cm^2 and its height is 40cm, what is its volume? ______________

Obj. 14.1.3
5.a) □
b) □
c) □

Obj. 14.1.4
6.a) □
b) □

Obj. 14.1.5
7.a)i □
ii □
iii □

b)i □
ii □
iii □

Obj. 14.1.6
8.i □
ii □

Obj. 14.1.7
9.a) □

Obj. 14.1.8
9.b)i □
ii □
iii □

Cycle 1: Volume of Prisms

Overview

Big Idea

Volume is a measure of the amount of space enclosed or taken up by a three-dimensional (3D) object. It is measured by filling the interior of 3D objects by units. Volume refers to solid volume (e.g., cubic metres) and is different from capacity (e.g., litres). Solid volume is determined by the amount of space enclosed or displaced, that is, displacement as well as filling. It is calculated using formulae and is in cubic units (e.g., mm^3, cm^3, m^3, and km^3) because it relates to length in three directions.

In this cycle the calculation of volume of prisms is extended from students' understanding of calculated 2D area and their previous experience exploring the construction of 3D objects from 2D nets. This exploration should also be linked to the concept of capacity. These ideas are further connected to calculations for volumes of triangular based prisms.

Objectives

By the end of this cycle, students should be able to:

14.1.1 Connects volume and capacity as concepts. [6MG138]
14.1.2 Connects units of measure for volume and capacity. [6MG138]
14.1.3 Choose appropriate units of measurement for volume and capacity. [8MG195]
14.1.4 Convert between units of measure for volume. [8MG195]
14.1.5 Develop formula for and calculate the volume of rectangular prisms. [8MG198]
14.1.6 Develop formula for and calculate the volume of triangular prisms. [8MG198]
14.1.7 Develop formula for and calculate the volume of prisms in general. [8MG198]
14.1.8 Solve problems involving the volume of right prisms. [9MG218]

Conceptual Links

Calculation of solid volume relies on students' ability to identify the faces of a 3D object, identify a suitable base and height, measure dimensions and calculate area of their chosen base. Students also need to be aware of the need for common units for calculations. Solid volume calculation provides opportunities to reinforce number, place value, operation concepts and processes (especially multiplication and division).

This cycle reinforces students' knowledge of calculating area and extends this to volume. These ideas will be useful when exploring real world volume applications like volume of water tanks, petrol tanks or determining possible dimensions of tanks to give a particular volume in design problems.

Materials

For Cycle 1 you may need:

- boxes
- small cubes to count capacity
- selection of same size boxes filled with different items (sultana boxes are ideal filled with hobbyfill, fish tank gravel, small beads, tooth picks, rice, empty box
- rulers
- large jug or large beaker and water
- alternative to sultana boxes – raid the science materials for a set of same size items that have different density (usually 2 cm cubes of aluminium, tin, steel, lead …)

Key Language

Box, rectangular prism, triangular based prism, capacity, mass, volume, cubic centimetre (cm^3), cc, area, base, height, three dimensional, length, width, rectangle, triangle, cubic millimetre (mm^3), cubic metre (m^3), cubic kilometre (km^3), polyhedron, polyhedra, non-polyhedron, non-polyhedra

Definitions

Capacity: amount of liquid a 3D object can hold if hollow.

Non-polyhedra/Non-polyhedron: 3D objects with curved surfaces

Polyhedra/Polyhedron: 3D objects with flat surfaces

Prism: 3D object with flat surfaces and two congruent faces opposite each other joined by parallelograms. Prisms are named for the shape of their bases.

Volume: displacement of a solid object. Sometimes used interchangeably with capacity.

Assessment

Anecdotal Evidence

Some possible prompting questions:

What faces can you see on the 3D object? Does the object have flat or curved surfaces?

Which faces are the same (congruent)?

How can you work out volume? What measures do you have/know?

Are the measures all in the same unit?

What will happen to the volume of the shape if one of the dimensions is changed?

Portfolio Task

Skills developed throughout this cycle will contribute to the student portfolio task *P14: Which tank?*

RAMR Cycle

Reality

Students frequently confuse the attributes of mass, capacity and volume. Volume is an attribute of a 3D object. For liquids, it is called capacity or how much an object can hold; mass is heft, how hard the object pushes down on the hand; and volume is the amount of space occupied by the object. Experience small but heavy and large but light objects so that students see that mass (heaviness) is not necessarily directly related to volume (size). This can also be done using same size packets with different masses (e.g., sultana boxes filled with beads, rice, toothpicks, hobbyfill, fish tank gravel), or using mass/density sets from the Science Department. Relate understanding of volume back to everyday life – what things in the world have volume, what do not, what has large/small volume?

Abstraction

The abstraction sequence for this cycle begins with students' understanding of the attribute of volume and extends exploration of the attribute to strategies for calculating volume of rectangular prisms. These strategies are extended within the Mathematics phase to include other prisms. A suggested sequence of activities is:

1. *Kinaesthetic activity*. Start with re-experiencing length and area – how tall people are, the size of the area of their hands. Now look at how big/small people are. Compare a big person with a small person and discuss the different amount of space they take up curled into a ball under a desk. A bigger person should take up more space under a desk.
2. *Construct items with volume.* Allow students to make different-sized constructions. These could include building with blocks and other material or making things with plasticine or play dough. Make these larger in volume by adding more, or smaller in volume by taking material away.
3. *Directly compare*. Compare objects and discuss which have greater or smaller volume.
4. *Indirectly compare.* Explore volume as the amount of space an object takes up by immersing objects in water. Mark the initial water level, the increase for one object, and then see if the other makes the water go higher or lower. Alternatively, fill a bucket with the first object in it to the brim, remove this object and place in the second. Observe the changing water level.
5. *Non-standard units*. Pack small containers with same objects (e.g., marbles, MAB cubes or unifix cubes). Measure the volume of a tissue box by stacking unifix cubes inside. Discuss how to ensure there are no gaps or overlaps. Find the volume of objects that are rectangular prisms. Ask students to use cubes to "make" the object that is the same length, width and height and then count the cubes. Use MAB cubes and unifix cubes to measure the volume of the same item. Ensure that students understand that if they use smaller units they will have a more accurate measure with a greater number of units than if they use larger units (e.g., it takes more MAB cubes to build a tissue box sized object than it takes unifix cubes).
6. *Repeat the process.* Construct the object in layers. For example, the first layer may have 16 cubes. Discuss how many layers of cubes are needed to make the object.
7. *Develop the formula.* Students should be able to identify that the number of cubes they counted relates to the number of cubes in the base layer by the number of layers high. Generalise this to find the formula for volume as the area of the base by the height.
8. *Reversing*. Up to now the students have gone from container to volume. Reverse this to go from volume as an experience to constructing a suitable container. Thus, do activities where students construct containers for particular purposes or build things to certain sizes.

Mathematics

Connections

Identity and Inverse

Volume calculations for prisms and cylinders use multiplication of the 2D area of the base by the height. Students should recognise that if the height is a single unit, the volume will be the area of the base by one unit layer. Students also need to recognise that if they have the volume and two dimensions, they can use division to find the missing dimension. Ensure that students can see this connection when solving problems and provide practice with these types of problems.

Language/symbols and practice

Practice finding solid volume using non-standard units

Find volume non-standard units in local community (e.g., measuring boots of cars in terms of travel cases or bags of cement/mulch). Try to encourage students to think of ways non-standard volume units are used in the world, local and otherwise. Use history and look at other units used in the past (e.g., the gallon which was the amount of wheat in a standard barrel).

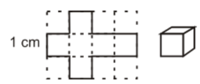
Common unit

After the need for a standard has been developed, time can be spent measuring volume with a class chosen unit. Discuss the most efficient shape for measuring volume. The cube can be argued to be the best because it packs well, is based on square area units, and leads to multiplication. Ensure students understand that they must use a consistent unit when counting volume.

Standard units and metric calculations

Find things in everyday life that are 1 mm^3, 1 cm^3, 1 m^3, and 1 km^3 to which students can refer back to. For example: (a) obtain a collection of small rectangular prisms (e.g., matchboxes or sultana boxes) and pack these with MAB units; (b) work out how many cubic metres of air in the classroom; and (c) find out how many cubic km of water in a nearby dam or soil and rock dug out of a nearby quarry.

Construct or experience mm^3, cm^3, m^3, and km^3. For 1 cm^3, use 1 cm grid paper, draw and cut out a net for a cube of side 1 cm, fold and tape to make the cube, and compare with a centicube or a MAB unit. Determine the volume of this in cubic millimetres ($10 \times 10 \times 10 = 1000$ mm^3).



For 1 m^3, make a cubic metre out of 12 metre rulers (or diagonally rolled newspaper sheets). Explore how many large MAB blocks will fit in a cubic metre. Explore the relationship: 1 000 000 cm^3 = 1 m^3.

For km^3, look on the Internet for something that is as large as this (e.g., the capacity of a very large dam, lake or bay/inlet).

Resource Resource 14.1.1 Estimation and calculation of volume

Consistent units of measure for calculation

When exploring non-standard units of measure for volume, students should have established that units used to determine volume need to be consistent. Ensure that students are able to extend this thinking to recognise that if a problem has dimensions in different units, that it is necessary for them to convert measures so that all dimensions are measured in the same unit before solving problems.

Metric conversions

Place value relationships and conversions can be explored using metric volume place value charts, number expanders and slide rules.

Resource Resource 14.1.2 Metric conversions

Volume of any prism

Repeat Abstraction steps 6, 7, 8 for other shaped-base prisms by creating an area of base units that can be layered to create the height. For example, Toblerone boxes (triangular prisms), The Pentagon (pentagonal-based prism), Quality Street chocolate tins (octagonal-based prism). Use thick examples of a given base shape and stack them. Discuss the pattern. Students should be able to discover that **Volume of any prism = area of base × H.**

Resource Resource 14.1.3 Volume practice problems
Resource 14.1.4 Missing dimension volume practice problems

Connections

Volume, capacity and mass

There is a particular relationship between the attributes of volume, capacity and mass. This relationship is unique to metric measures. A cube that is 1 cm × 1 cm × 1 cm has a volume of 1 cm^3. If this cube was filled with water (at 4° C) it would weigh 1 gram (g) and the capacity would be 1 millilitre (mL). So the relationship is 1 cm^3 = 1 mL = 1 g.

This relationship is seen in car engine sizes where the volume of the cylinders in the engine is sometimes described as cc (cubic centimetres) or as litres (L). A vehicle could have a 4000 cc motor which could also be described as a 4 L engine.

The relationships between units are:

1 cubic cm (cm^3) = 1 cm × 1 cm × 1 cm = 1 millilitre (mL) = 1 gram (g)

1 cubic decimetre (dm^3) = 10 cm × 10 cm × 10 cm = 1 litre (L) = 1000 mL = 1 kilogram (kg) = 1000 g

1 cubic metre (m^3) = 100 cm × 100 cm × 100 cm = 1 kilolitre (kL) = 1000 L = 1 tonne (t) = 1000 kg.

Reflection

Check the idea

Ensure students can calculate the volume of prisms and cylinders and determine missing dimensions. Encourage students to use a rough sketch to help them organise their thinking.

Apply the idea

Explore real world problems that require calculation of volume. For example, possible dimensions of tanks to fit within spaces (water tanks under eaves, petrol tanks for long, low spaces under cars or upright in the side of the car, box sizes for storage of multiple items).

Resource Resource 14.1.5 Real world volume practice problems

Extend the idea

Volume and proportion

As for area, it is important for students to explore what happens to volume when all dimensions of a prism are doubled. Here is an activity to show that doubling each dimension will increase the volume by 8 times.

Construct a rectangular prism from cubes. Make a second prism double the first in length, width and height. Discuss what happens to the volume. This can also be demonstrated by stacking sultana packets. If the length is doubled there will be two packets end to end. Doubling the width as well will add two more packets, and doubling the height will add another layer of four packets. Students can then count to see that doubling each dimension results in eight times as many packets. Test this idea out with tripling length, width and height. Does volume increase 9 times? (Challenge: Explore what happens when length and width change and height stays the same.)

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #2

1. Describe how you would find the volume of a cylinder.

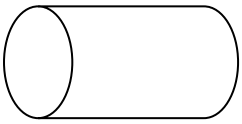
____

____

Obj. 14.2.1
1.i ☐
ii ☐

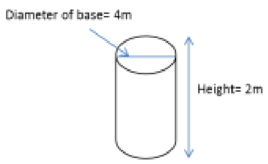
2. Find the volume of a can with length 10 cm and diameter 6cm.

Area of a circle = πr^2



Obj. 14.2.2
2.i ☐
ii ☐
iii ☐

3. Find the volume of this water tank. The height of the tank is 2 m and the diameter of the base is 4 m.



3.i ☐
ii ☐
iii ☐

Cycle 2: Volume of Cylinders

Overview

Big Idea

In this cycle the calculation of volume of cylinders is extended from students' understanding of calculated volume of rectangular and triangular based prisms. This exploration should also be linked to the concept of capacity.

Objectives

By the end of this cycle, students should be able to:

14.2.1 Develop formula for and calculate the volume of cylinders. [9MG217]
14.2.2 Solve problems involving the volume of cylinders. [9MG217]

Conceptual Links

Calculation of solid volume relies on students' ability to identify the faces of a 3D object, identify a suitable base and height, measure single dimensions and calculate area of their chosen base. In the case of cylinders, the base is circular and relies on students' ability to calculate area of a circle. Students also need to be aware of the need for common units for calculations. Solid volume calculation provides opportunities to reinforce number, place value and operation concepts and processes (especially multiplication and division).

This cycle reinforces students' knowledge of calculating area and extends this to volume. These ideas will be useful when exploring real world volume applications like volume of water tanks, petrol tanks or determining possible dimensions of tanks to give a particular volume in design problems.

Materials

For Cycle 2 you may need:

cylindrical containers

- rulers
- large jug or large beaker and water

Key Language

Can, cylinder, capacity, mass, volume, cubic centimetre (cm^3), cc, area, base, height, three dimensional, length, width, cylinder, radius, diameter, circumference, cubic millimetre (mm^3), cubic metre (m^3), cubic kilometre (km^3)

Definitions

Cylinder: two congruent circles joined by a rectangle.

Assessment

Anecdotal Evidence

Some possible prompting questions:

What faces can you see on the 3D object? Does the object have flat or curved surfaces?

Which faces are the same (congruent)?

How can you work out volume? What measures do you have/know?

How do you work out the area of a circle?

Are the measures all in the same unit?

What will happen to the volume of the shape if one of the dimensions is changed?

Portfolio Task

Skills developed throughout this cycle will contribute to the student portfolio task *P14: Which tank?*

RAMR Cycle

Reality

Volume of a cylinder can be related most easily to volume of a prism. The same process of finding the area of the base by the perpendicular height will work as long as the circular face is used as the base. Relate understanding of volume back to everyday life, for example, water bottles, Pringles boxes, tinned food, poly pipe, steel pipe, water tanks. Discuss properties of cylinders that make them more effective for these applications. For example, cylinders are more comfortable to hold than rectangles; tins are easier to roll or extrude from metals than fold; and, consider the relative strengths of shapes.

Abstraction

The abstraction sequence for this cycle starts from students' understanding of strategies for calculating volume of rectangular prisms and extends these to calculating volume of a cylinder. A suggested sequence of activities is:

1. *Kinaesthetic activity*. Start with re-experiencing volume of prisms. Explore creating a representative student from a school jumper and jeans (this could fit within the context of creating a school garden scarecrow). Discuss ways to find the volume of material needed to fill out the arms, legs and body of the student.
2. *Construct items with volume.* Allow students to make different-sized constructions that are cylindrical. These could include rolling light card and other material or making things with plasticine or play dough. Make these larger in volume by adding more, or smaller in volume by taking material away. Mark the height of the object in layers of equal thickness. Discuss how many layers are needed to make the object.
3. *Develop the formula.* Students should be able to identify that the volume relates to the area of the base layer by the number of layers high. Generalise this to find the formula for volume as the area of the base by the height (Volume of a cylinder = area of circular base × H = $\pi \times r2 \times H$).
4. *Reversing*. Up to now the students have gone from container to volume. Reverse this to go from volume as an experience to constructing a suitable container. Thus, do activities where students construct containers for particular purposes or build things to certain sizes.

Mathematics

Connections

Inverse

Volume calculations for prisms and cylinders use multiplication of the 2D area of the base by the height. Students also need to recognise that if they have the volume and two dimensions, they can use division to find the missing dimension. Ensure that students can see this connection when solving problems and provide practice with these types of problems.

Consistent units of measure for calculation

When exploring non-standard units of measure for volume, students should establish that there must be consistency in the units used for measuring the volume. Ensure that students are able to extend this thinking to recognise that if a problem has dimensions in different units, that it is necessary for them to convert measures so that all dimensions are measured in the same unit before solving problems.

Volume, capacity and mass

Use cylindrical objects to explore and reinforce the relationships between the attributes of volume, capacity and mass identified in Cycle 1.

Language/symbols and practice

Standard units and metric calculations

Find cylindrical things in everyday life that are 1 mm^3, 1 cm^3, 1 m^3, and 1 km^3 to which students can refer back. For example: (a) obtain a collection of small cylinders (e.g., M & M tubes, test tubes, straws) and find the volumes of these; (b) work out how many cubic metres of water in the school water tanks; and (c) find out how many cubic km of water in a nearby dam or soil and rock dug out of a nearby quarry.

Resource Resource 14.2.1 Volume practice problems (cylinders)
Resource 14.2.2 Missing dimension volume practice problems (cylinders)

Reflection

Check the idea

Ensure students can calculate the volume of cylinders and determine missing dimensions. Encourage students to use a rough sketch to help them organise their thinking.

Apply the idea

Explore real world problems that require calculation of volume. For example, possible dimensions of tanks to fit within spaces (water tanks under eaves, petrol tanks for long, low spaces under cars or upright in the side of the car, box sizes for storage of multiple items).

Resource Resource 14.2.3 Real world volume practice problems (cylinders)

Extend the idea

Volume and proportion

As for area, it is important for students to explore what happens to volume when all dimensions of a prism or cylinder are doubled. The following activity shows that doubling each dimension will increase the volume by 8 times.

1. Construct a cylinder from paper or light card.
2. Make a second cylinder double the first in length, width and height.
3. Discuss what happens to the volume: *This can also be demonstrated by stacking cylinders. If the length is doubled there will be two cylinders end to end. Doubling the width as well will add two more cylinders, and doubling the height will add another layer of four cylinders.*
4. Ensure students can then count to see that doubling each dimension results in eight times as many cylinders: *This can be tricky to visualise as the cylinders will have gaps between that may cause confusion but the basic idea can be tested this way. Test this idea out with tripling length, width and height. Does volume increase 9 times?*
5. Challenge: Explore what happens when radius changes and height stays the same.

Teacher Reflective Notes

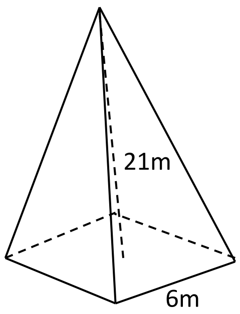
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #3

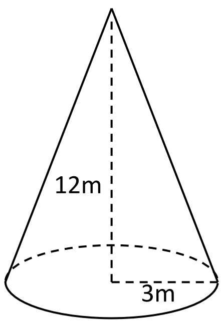
1. Calculate the volume of this square- based pyramid.

   ($V_{pyramid} = \frac{1}{3} \times$ volume of cube)



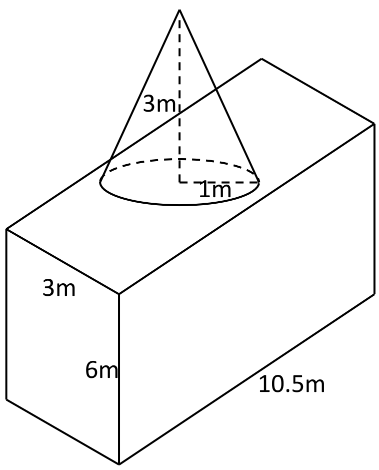
Obj.
14.3.1
1.i ☐
ii ☐
iii ☐

2. Calculate the volume of this cone. ($V_{cone} = \frac{1}{3} \times$ Volume of cylinder)



Obj.
14.3.2
2.i ☐
ii ☐
iii ☐

3. Calculate the volume of this composite shape.



Obj.
14.3.2
3.i ☐
ii ☐
iii ☐
iv ☐
vi ☐
vi ☐
vii ☐
viii ☐
ix ☐

Cycle 3: Volume of Pyramids and Cones

Overview

Big Idea

While earlier cycles develop volume and connect to capacity, with volume of pyramids and cones it is more useful to use the concept of capacity to assist with the development of formulae. In this cycle students will explore the relationships between capacity of a cube and the capacity of a square based pyramid. They will also learn about the capacity of a cylinder and the capacity of a cone with the same size base. These relationships are then generalised to the formulae for volume of pyramids and cones.

Objectives

By the end of this cycle, students should be able to:

14.3.1 Develop formula for and calculate the volume of pyramids to solve related problems. [10MG271]

14.3.2 Develop formula for and calculate the volume of cones to solve related problems. [10MG271]

Conceptual Links

Calculation of solid volume relies on students' ability to identify the faces of a 3D object, identify a suitable base and height, measure single dimensions and calculate area of their chosen base. In the case of pyramids and cones, the volume is a consistent fraction of the matching prisms and cylinders. These calculations provide students with the additional reinforcement of finding a third of a value or dividing by three. Students should be aware of the need for common units for calculations. Solid volume calculation provides opportunities to reinforce number, place value and operation concepts and processes (especially multiplication and division).

This cycle reinforces students' knowledge of calculating area and extends this to volume. These ideas will be useful when exploring real world volume applications like volume of water tanks, petrol tanks or determining possible dimensions of tanks to give a particular volume in design problems.

Materials

For Cycle 3 you may need:

nets for cubes
nets for square based pyramids to match cubes
rice, popping corn or sand
scoops or spoons
play dough
nets for cylinders
nets for cones to match cylinders
acrylic 3D solid sets (fillable)
large trays to contain rice, popcorn or sand when pouring
dental floss

Key Language

Capacity, mass, volume, cubic centimetre (cm^3), cc, area, base, height, three dimensional, length, width, cylinder, radius, diameter, circumference, rectangle, triangle, cubic millimetre (mm^3), cubic metre (m^3), cubic kilometre (km^3), pyramid, cone

Definitions

Cone: non-polyhedron with a flat base (frequently circular) that tapers smoothly to a point.

Pyramid: polyhedron with a base joined to an apex by triangles. Pyramids named by the 2D shape of their base.

Assessment

Anecdotal Evidence

Some possible prompting questions:

What faces can you see on the 3D object? Does the object have flat or curved surfaces?

Does it come to a point?

Which faces are the same (congruent)?

How can you work out volume? What measures do you have/know?

How do you work out the area of a circle?

Are the measures all in the same unit?

What will happen to the volume of the shape if one of the dimensions is changed?

Portfolio Task

Skills developed throughout this cycle will contribute to the student portfolio task *P14: Which tank?*

RAMR Cycle

Reality

Volume of pyramids and cones can be related most easily to volume of prisms and cylinders by connecting first to capacity. Relate understanding of volume back to capacity of objects in everyday life, for example, actual pyramids, air space inside roofs, top sections of bottles and containers that narrow to a cap. Discuss pyramids and cones as parts of wholes. This may be demonstrated easily by cutting a bottle where it changes from straight sided cylinder to pyramid (milk bottle) or cone (soft drink bottle). Invert the cone or pyramid inside the cylinder or box that is left so that students can see the pyramid or cone and the remaining empty space in the matching cube or cylinder. This can also be demonstrated using commercial acrylic 3D fillable shapes.

Abstraction

The abstraction sequence for this cycle starts from students' understanding of strategies for calculating volume of rectangular prisms and extends these ideas by exploring the relationship between the volume of a cube and its matching pyramid. This idea can be further extended to cones by repeating the steps for pyramids. A suggested sequence of activities is:

1. *Kinaesthetic activity*. Start with re-experiencing volume of prisms. Create rectangular prisms from play dough and calculate the volume. Discuss with students whether a pyramid the same height and base would have more, the same or less volume. Use dental floss as a cutter to cut away opposite sides to create a triangular based prism. Discuss with students what fraction of the object is left (should be about half if done accurately). Consider what else needs to be removed to create the pyramid (removing opposite sides to create a point at the top). Ask students to estimate or predict what volume of play dough is used in the pyramid compared to what has been cut away (precise fractions are not needed here as the idea is to generally agree that the pyramid has less volume than the rectangular prism and is going to be less than a half).

2. *Construct items with volume.* Create cubes from light card leaving one base open so they can be filled with rice or popping corn. Create matching pyramids from light card again leaving the base open so they can be filled with rice or popping corn.

1. Ask students to predict how many times they can fill the pyramid and pour it into the cube. Encourage students to carefully fill the pyramid and pour this into their cube. Adjust prediction. Repeat until the cube is full. Students should find that they filled the pyramid three times to fill the cube.

2. *Develop the formula.* Students should be able to identify the formula for the volume of the cube. Connect students' exploration of the capacity of pyramids and cubes to the formula and fraction ideas. They should be able to generalise that if a pyramid fills a cube three times, then the pyramid is one third of the volume of the cube. Generalise this to find the formula for volume of a pyramid as the volume of a cube the same base and height divided by three (Volume of a pyramid = $\frac{1}{3}$ × Volume of cube = $\frac{1}{3}$ × Base area × H).

3. Repeat these steps using rectangular prisms and rectangular based pyramids as well as cylinders and cones to reinforce the ideas and extend to these formulae.

Resource Resource 14.3.1 Nets

Mathematics

Connections

Inverse

Volume calculations use multiplication of the 2D area of the base by the height. Students also need to recognise that if they have the volume and two dimensions, they can use division to find the missing dimension. Ensure that students can see this connection when solving problems and provide practice with these types of problems.

Consistent units of measure for calculation

When exploring non-standard units of measure for volume, students should recognise that there must be consistency in the units used to determine volume. Ensure that students are able to extend this thinking to recognise that, before solving problems, it is necessary to convert measures so all dimensions are in the same unit.

Language/symbols and practice

Standard units and metric calculations

Find pyramids and cones in everyday life that are 1 mm^3, 1 cm^3, 1 m^3, and 1 km^3 to which students can refer back. For example: (a) obtain a collection of pyramids and cones (e.g., ice cream cones, tops of bottles, decorative chocolate boxes) and find the volumes of these; (b) work out volumes of pyramids or cones used with funnels or petrol tanks; and (c) find out how many cubic km of water in a nearby dam or soil and rock dug out of a nearby quarry.

Resource Resource 14.3.2 Volume practice problems (cones/pyramids)

Reflection

Check the idea

Ensure students can calculate the volume of pyramids and cones and determine missing dimensions. Encourage students to use a rough sketch to help them organise their thinking.

Apply the idea

Explore real world problems that require calculation of volume. For example, possible dimensions of tanks to fit within spaces (water tanks under eaves, petrol tanks for long, low spaces under cars or upright in the side of the car, box sizes for storage of multiple items).

Extend the idea

Explore finding the volumes of composite objects by separating the object into components, finding volumes and combining these for a total volume.

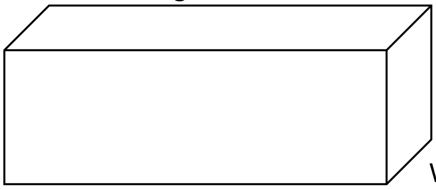
Resource Resource 14.3.3 Composite objects practice problems

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

Can you do this? #4



1. Calculate the surface area of this box (draw a net to help you).

2. The length of this box is doubled. Calculate the new surface area.

3. If this box was created as a cylinder with height 15 cm and radius 3 cm, what would its surface area be? (Draw a net to help you)

Obj.
14.4.1
1)i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐

2)i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐

Obj.
14.4.2
3)i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐

Cycle 4: Relationship Between Surface Area and Volume

Overview

Big Idea

The surface area to volume relationship is similar to the perimeter to area relationship. The surface area for a fixed volume can vary widely. However, the volume to surface-area ratio increases as things grow in size. When considering an object with a fixed surface area, spheres and cubes provide the greatest volume. These ideas are useful to explore to consolidate students' understanding of the difference between the measures of volume and surface area. They also provide additional practice in identifying the use of these calculations in real-world contexts.

Objectives

By the end of this cycle, students should be able to:

14.4.1 Calculate the surface area of polyhedra to solve related problems. [9MG218]
14.4.2 Calculate the surface area of cylinders to solve related problems. [9MG217]

Conceptual Links

Students have previously explored volume of solid objects in this Unit and surface area in *Unit 8: Cycle 7.* Calculation of surface area relies on students being able to identify the 2D faces of 3D objects and calculate their respective areas. Calculation of solid volume relies on students' ability to identify the faces of a 3D object, identify a suitable base and height, measure dimensions and calculate area of their chosen base. Students must be aware of the need for common units for calculations. Solid volume calculation provides opportunities to reinforce number, place value, operation concepts and processes (especially multiplication and division).

This cycle reinforces and connects students' knowledge of calculating surface area and calculation of solid volume. These ideas will be useful when exploring real world volume applications like volume of water tanks, petrol tanks or determining possible dimensions of tanks to give a particular volume in design problems. The ability to calculate and compare surface area and volume combined with ratio may be useful in real-world investigations involving heat transfer and optimisation of capacity.

Materials

For Cycle 4 you may need:

- A4 paper or light card
- large tray to contain spillage
- rice, popping corn or sand
- scoops or spoons

Key Language

Surface area, square kilometres (km^2), square metres (m^2), square centimetres (cm^2), square millimetres (mm^2), volume, cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3), cubic millimetres (mm^3), 3D objects, 2D faces, formulae

Definitions

Composite object: an irregular 3D object that is made up of simpler 3D objects.

Assessment

Anecdotal Evidence

Some possible prompting questions:

What objects can you see that the 3D object could be split into?

What faces can you see on the 3D objects? Do the objects have flat or curved surfaces?

Which faces are the same (congruent)?

How can you work out volume? What measures do you have/know?

How do you work out the area of a circle?

Are the measures all in the same unit?

What will happen to the volume of the shape if one of the dimensions is changed?

Portfolio Task

Skills developed throughout this cycle will contribute to the student portfolio task *P14: Which tank?*

RAMR Cycle

Reality

Discuss surface area and volume of objects with students. Ensure they can identify the differences between these two attributes. Consider the implications for each measure. For example, increasing volumes of cereal require larger packets, resulting in increased packaging costs. To minimise packaging costs, the lowest surface area for the volume is needed. Consider other applications of this idea.

Abstraction

1. *Kinaesthetic activity.* Revisit the scarecrow from Cycle 2 and the volume of stuffing material needed. Consider the surface area of cloth needed to create a different look for the scarecrow. Discuss what might be the outcome of making a shorter and stockier scarecrow. What would this mean for the amount of filling and cloth needed. Compare surface area of possible t-shirts.
2. *Represent with drawings.* Sketch the cylinders needed to create new arms for the scarecrow (these can be constructed from paper and tested by filling if necessary to scaffold student thinking).
3. Use the sketches or cylinders to identify the surfaces of the objects. Calculate the surface areas and volumes for each pair of scarecrow arms.
4. Compare the volume to surface area ratios to draw conclusions about the scarecrows' arms.

Resource Resource 14.4.1 Surface Area to Volume tables

Mathematics

Language/symbols and practice

Practice finding surface area and volumes of common packaged items to identify the optimal shapes and sizes for packaging of cereal, liquids, and other items. Explore the nets for each shape. Discuss some of the other constraints that may contribute to decisions when the "best" size and shape is not used. Some examples are, common widths of door shelves in fridges, shelf heights in pantries, sizes of card stock used for packaging.

Reflection

Apply the idea

Look after the baby.

In this activity, students explore the properties of volume and surface area in the real-world context of heat loss from solids depending on the amount of surface area that is available for cooling. The example used leads to consideration of heat retention/loss for babies.

Note: The following initial activity reinforces relationships developed in Unit 8: Cycle 7.

Start with surface area and perimeter comparisons:

Make a shape from 5 square counters. Note the area and perimeter. Record this information in a table for comparison and for noting patterns.

Make the shape again but double original scale, and then again, but triple. Note the areas and perimeters in the table for comparison and for noting patterns.

Identify the pattern in the relationship between area and perimeter as the size increases. Repeat the above for a 9-counter square, doubling and tripling the length and width. What pattern is emerging in the perimeter to area relationship?

Extend to surface area and volume comparisons:

Start with 3 cubes and make any 3D shape. Double and triple this in scale. Calculate volume and surface area. Record the information in a table for comparison and for noting patterns.

Identify the effect on surface area of doubling and tripling lengths of sides. What is the effect on volume?

Make cubes of side 1, 2, 3, 4, 5 and 6. Calculate area of base, volume and surface area for these cubes. Record the information in a table. What pattern emerges? What happens to the surface-area to volume ratio?

Consider the important consequence does this has for babies. Think about their ability or inability to stay cool or warm.

Resource Resource 14.4.2 Look after the baby tables

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 14 Portfolio Task – Teacher Guide

Which Tank?

Content Strand/s: Measurement and geometry

Resources Supplied:
- Task sheet
- Teacher guide

Other Resources Needed:
- None

Summary:

Using the context of buying a tank for the school, students identify which tank or tanks are appropriate for the school to purchase. Students justify their choice.

Variations:
- Repeat a similar problem within the context of grain silos on a farm.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Measurement and geometry

14.2.1 Develop formula for and calculate the volume of cylinders. [9MG217]

14.2.2 Solve problems involving the volume of cylinders. [9MG217]

14.3.1 Develop formula for and calculate the volume of pyramids to solve related problems. [10MG271]

14.3.2 Develop formula for and calculate the volume of cones to solve related problems. [10MG271]

Which Tank?

Name	
Teacher	
Class	

Your Task:

It is your task to investigate the volume of various tanks using their dimensions to identify which is the most economical.

Within Portfolio Task 14, your work demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	**Procedural fluency**	14.2.1 Develop formula for and calculate the volume of cylinders. 14.3.1 Develop formula for and calculate the volume of pyramids to solve related problems.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	14.2.2 Solve problems involving the volume of cylinders. 14.3.2 Develop formula for and calculate the volume of cones to solve related problems.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language

Comments:

Your class has been asked to select a new tank/s for your school. The tank/s need to hold 50000 L of water. Remember 1 m^3 holds 1 000 L of water

1. Determine the capacity of each tank. Use the space below to show your working.

Tank	Shape	Height	Base dimensions	Capacity	Cost
A	Cylinder	6 m	Radius = 2 m		$8500
B	Triangular Prism	7 m	Base = 5 m Height = 5 m		$7000
C	Square prism	3 m	Length = 3 m		$2000
D	Square prism	4 m	Length = 2 m		$2300
E	Cylinder	5 m	Diameter = 5 m		$6750

2. Use the information and the capacity that you have calculated to determine which tank is the best buy.
   You are able to get more than one tank.

   Tank: ____________________________

3. Justify your choice:

   ____

   ____

   ____

Name:______________________________ Date:________

Can you do this now? Unit 14

1. Look at the things in your classroom.

   (a) Name two things that have volume.

   i. ____________________ ii. ____________________

   (b) Name one thing that does not have volume. ______________

   (c) Explain the difference. ______________________________

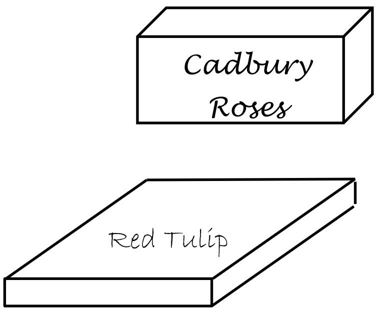
   ____

   (d) Can an object have volume but not capacity? ____________

   Why/why not? ____________________________________

Obj.
14.1.1
1.a)i ☐
ii ☐
b) ☐
c) ☐
d) ☐

2. How could you find out (without a ruler) which of the two boxes of chocolates has the greater capacity (holds more)?



   ____

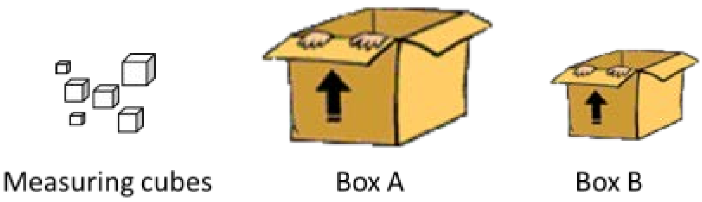
   ____

   ____

   ____

Obj.
14.1.1
2. ☐

3. Lots of small cubes (shown below) were used to measure the volume of Box A and Box B. When they were counted, Box B had more cubes than Box A.



   (a) Does this mean that Box A has a smaller volume than Box B? __

   (b) Why or why not? _______________________________

   ____

Obj.
14.1.1
3.a) ☐
b) ☐

4. A container has a capacity of 100 mL. What would be the volume of an ice cube made in this container? ______________________________

Obj. 14.1.2
4. ☐

5. Write down something in the classroom, school or local area that you could measure in:

(a) km^3 ____

(b) m^3 ____

(c) cm^3 ____

Obj. 14.1.3
5.a) ☐
b) ☐
c) ☐

6. Convert the following volume measures:

(a) $1\ m^3$ = ________________ cm^3

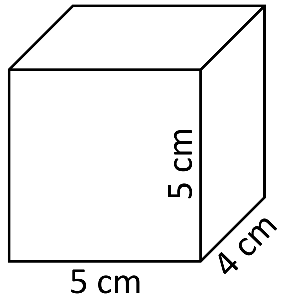
(b) $1000\ mm^3$ = ________________ cm^3

Obj. 14.1.4
6.a) ☐
6.b) ☐

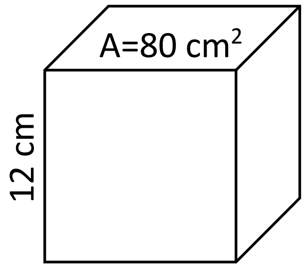
7. Find the volume of the following boxes:

(a) A box with height 5 cm, length 5 cm and width 4 cm.

V = Area base × h = l × w × h



(b) A box with height 12 cm. The area of the top of the box is 80 cm^2.



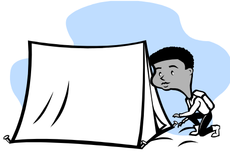
Obj. 14.1.5
7.a)i ☐
ii ☐
iii ☐

7.b)i ☐
ii ☐
iii ☐

8. Describe how to calculate the volume of this tent.

____

____



Obj. 14.1.6
8.a)i ☐
ii ☐

9. a) Here is an octagonal-based prism. Write a formula to calculate its volume. ______________________________

b) If its base has an area of 390 cm^2 and its height is 8 cm, what is its volume? ________________

Obj. 14.1.7
9.a) ☐
Obj. 14.1.8
b)i ☐
ii ☐
iii ☐

10. Describe how you would find the volume of a cylinder.

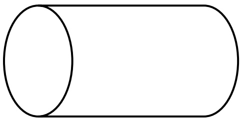
____

____

Obj. 14.2.1
10.i ☐
ii ☐

11. Find the volume of a can with length 8 cm and diameter 5 cm.

Area of a circle = πr^2



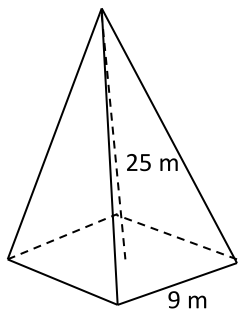
Obj. 14.2.2
11.i ☐
ii ☐
iii ☐

12. Find the volume of this water tank. The height of the tank is 2.4 m and the diameter of the base is 3.5 m.

12.i ☐
ii ☐
iii ☐

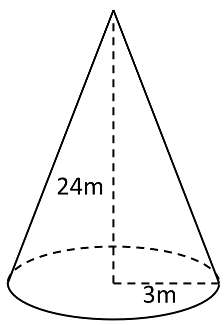
13. Calculate the volume of this square-based pyramid.

($V_{pyramid} = \frac{1}{3} \times$ Volume of cube)



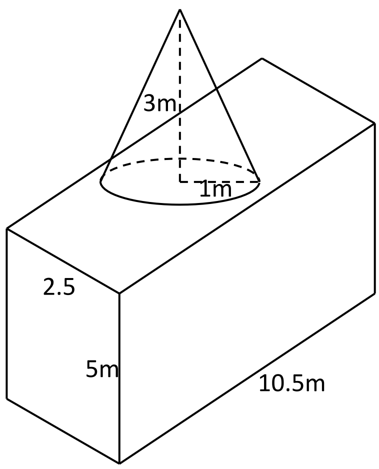
Obj.
14.3.1
13.i ☐
ii ☐
iii ☐

14. Calculate the volume of this cone. ($V_{cone} = \frac{1}{3} \times$ Volume of cylinder)



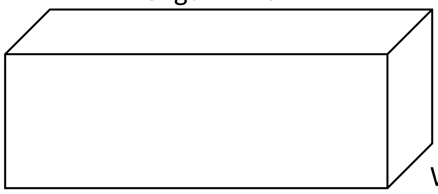
Obj.
14.3.2
14.i ☐
ii ☐
iii ☐

15. Calculate the volume of this composite shape.



Obj.
14.3.2
15.i ☐
ii ☐
iii ☐
iv ☐
vi ☐
vi ☐
vii ☐
viii ☐
ix ☐

16. Calculate the surface area of this box (draw a net to help you).



17. The length of this box is doubled. Calculate the new surface area.

18. If this box was created as a cylinder with height 12 cm and radius 3 cm, what would its surface area be? (Draw a net to help you)

Obj.
14.4.1
16.i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐
17.i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
Obj.
14.4.2
18.i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐

XLR8 – Accelerating Mathematics Learning

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