XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

XLR8 Unit 13

# Modelling linear relationships

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**ACKNOWLEDGEMENTS**

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.


**YUMI DEADLY CENTRE**

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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# *XLR8 Program: Scope and Sequence*

| | 2 year program | 3 year program |
|---|---|---|
| **Unit 01: Comparing, counting and representing quantity**<br>Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers. | 1 | 1 |
| **Unit 02: Additive change of quantities**<br>Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown). | 1 | 1 |
| **Unit 03: Multiplicative change of quantities**<br>Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers. | 1 | 1 |
| **Unit 04: Investigating, measuring and changing shapes**<br>Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs. | 1 | 1 |
| **Unit 05: Dealing with remainders**<br>Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning. | 1 | 1 |
| **Unit 06: Operations with fractions and decimals**<br>Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals. | 1 | 2 |
| **Unit 07: Percentages**<br>Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages. | 1 | 2 |

| | 2 year program | 3 year program |
|---|---|---|
| **Unit 08: Calculating coverage**<br>Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers. | 2 | 2 |
| **Unit 09: Measuring and maintaining ratios of quantities**<br>Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts. | 2 | 2 |
| **Unit 10: Summarising data with statistics**<br>Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics. | 2 | 2 |
| **Unit 11: Describing location and movement**<br>Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled. | 2 | 3 |
| **Unit 12: Enlarging maps and plans**<br>Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures. | 2 | 3 |
| **Unit 13: Modelling with linear relationships**<br>Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient. | 2 | 3 |
| **Unit 14: Volume of 3D objects**<br>Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects. | 2 | 3 |
| **Unit 15: Extended probability**<br>Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events. | 2 | 3 |

# Overview

## Context

In this unit, students will explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships. The investigation of rate and ratio is also examined using equations and algebraic calculations for finding gradient and distances between points on a line.

## Scope

This unit is based upon **proportional relationships** between **discrete items** and **measures** described using **rate notation**. **Proportional relationships** using **rate** describe **multiplicative relationships** between quantities that cannot be combined. Often, the use of the word 'per' in a rate describes how the **change in one quantity results in a proportional change in another**.

**Unit rate** may be used to describe the **relationship between different attributes** where the **second attribute** is reduced to a **single unit**. In linear relationships, rate describes a constant multiplicative relationship **between** measured attributes. Also, in simple linear relationships there is a common and constant multiplicative relationship **within** the measured attributes. Linear relationships may be represented using **tables** of data, straight-line **graphs** on the Cartesian plane and **linear equations**.

Rate may be used to describe **gradient** which defines the **change in the y-coordinate per the change in the x-coordinate** on Cartesian planes. The constant gradient of a straight line, when combined with Pythagoras' theorem, allows for the calculation of the **distance to the mid-point** between two locations on the Cartesian plane.

Common strategies to scaffold **proportional thinking** and rate-related problems include using **proportion tables**, **within and between strategies**, and **dual-scale number lines**. Ultimately, these strategies help students to identify the proportional relationships and in turn choose appropriate operations to **solve algebraically for unknown variables** in proportional relationships.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are applied within this area are highlighted in black.

## Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P13: YouTube Investigation* includes the use of proportion tables and line graphs to interpret and determine solutions to rate and direct proportion problems.

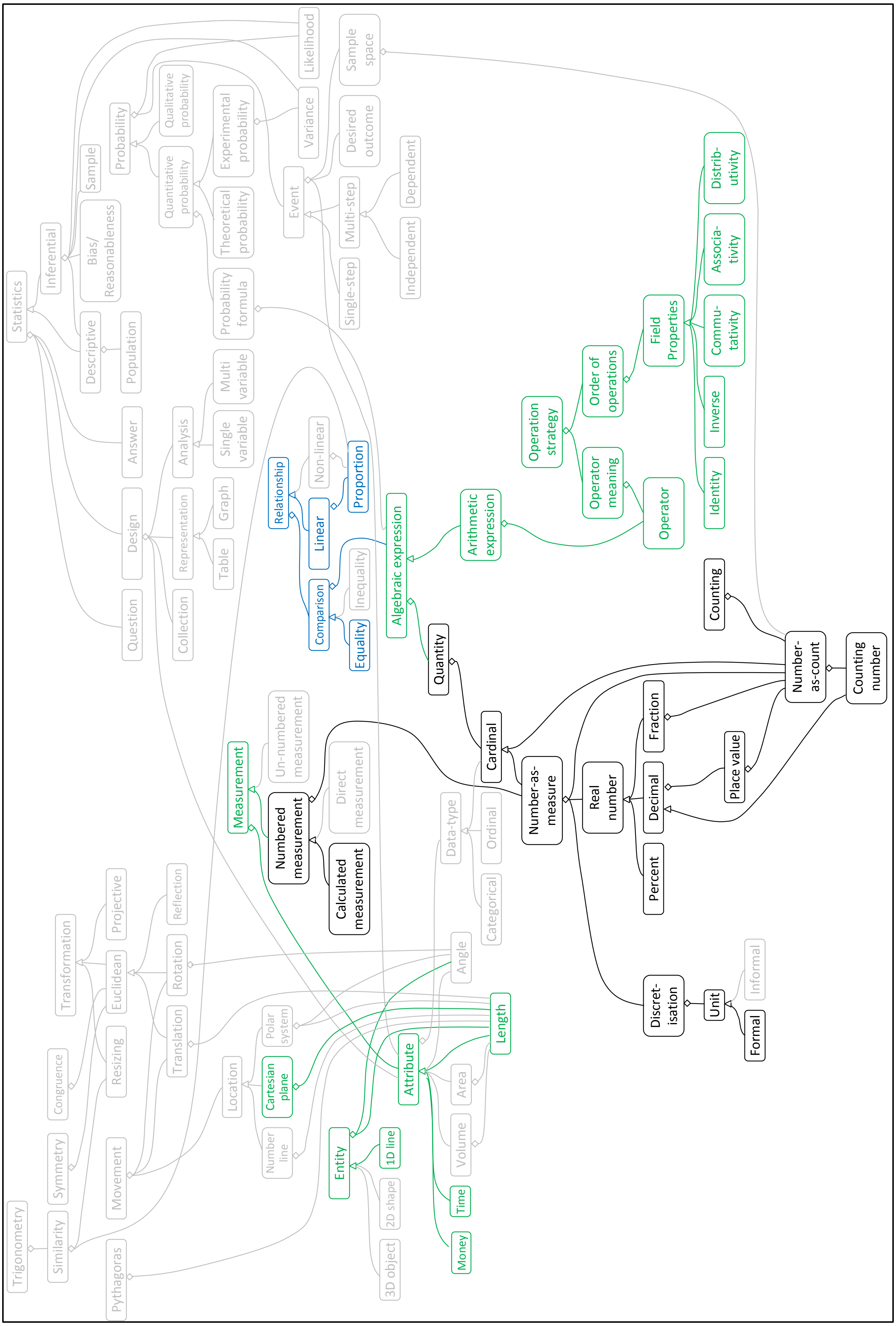



*Figure 1.* Scope of Unit 13

# Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

**Cycle 1: Relationships in tables and graphs**

In this cycle the concept of a mathematical relationship is introduced. In this case, that the value of one variable is dependent upon that of another. The cycle considers a variety of datasets presented as tables and graphs and uses mainly graphical techniques to interpret the data (that is, to solve for one variable when the other is set to a known value). The general idea of a relationship presented in this cycle will be narrowed down to linear relationships, straight-line graphs and equations in the following cycles.

**Cycle 2: Simple linear relationships**

Simple linear relationships between two variables can be represented in proportion tables or double number lines (as for ratio), on Cartesian planes as straight line graphs, and as formalised algebraic equations ($y = mx$). Before exploring unit rate, explore situations that occur with a second value that is not a unit so as to highlight the multiplicative relationship between the values. Rate is frequently expressed as unit rate (quantity of first attribute per one unit of second attribute, e.g., 60 km/hr is understood as 60 km per 1 hour of travel). All situations that can be described in terms of *per*, *for each* or *for every,* are rates. As a result, rate can be introduced early because it is used throughout early mathematics without being recognised. For example, *3 bags of lollies with 4 lollies in each bag* is really 3 × 4 lollies/bag. In fact, some educators argue that all simple multiplications are number × rate (e.g. 3 bags × 4 lollies/bag = 12 lollies). In this cycle, the value of rate will be limited to positive values.

**Cycle 3: Comparing simple linear relationships**

This cycle explores the effect caused by changing the coefficient in an equation on the gradient of a line. The effect on a linear relationship caused by the existence and changing of a constant value is also explored. Many of the contexts from the previous cycle can be re-used. For example, the cost of purchasing goods by weight which includes the purchase of a shopping bag.

**Cycle 4: Simultaneous solutions**

There are a range of methods for solving linear relationships simultaneously. These can include using algebraic tools and understandings like backtracking, balance, simultaneous equations and generating graphs for equations (identifying where the graphs cross).

**Cycle 5: Modelling real data with linear relationships**

Identifying linear relationships from real data requires some interpretation and often allowance for possible outliers in the data due to variance. Correlation and lines of best fit enable “messy” real-world correlation to be considered and predicted mathematically. Real data modelled using linear relationships can also be used to solve real-world problems to predict points of time and locations where collisions may occur.

## *Notes on Cycle Sequence:*

The proposed cycle sequence outlined may be completed sequentially as it stands.

## Literacy Development

Core to the development of number and algebra concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between them.

**Cycle 1: Relationships in tables and graphs**

Cartesian plane, x-coordinate, y-coordinate, variables, relationship between variables

**Cycle 2: Simple linear relationships**

Rate, per, for each, for every, km/hr, $/L, $/kg, comparison, variable, equation, unknowns, Cartesian planes, unknowns, gradient, slope, $y = mx$, linear relationship

**Cycle 3: Comparing simple linear relationships**

Gradient, slope, growing part constant, fixed part, multiplier, y-intercept, $y = mx + c$

**Cycle 4: Simultaneous solutions**

Variable, equation, balance, backtracking, Cartesian planes, simultaneous equations, solutions, unknowns, gradient, slope, constant, fixed part

**Cycle 5: Modelling real data with linear relationships**

Correlation, line of best fit, average, intersection, intercepts

Name:______________________________________ Date:________

# *Can you do this? #1*

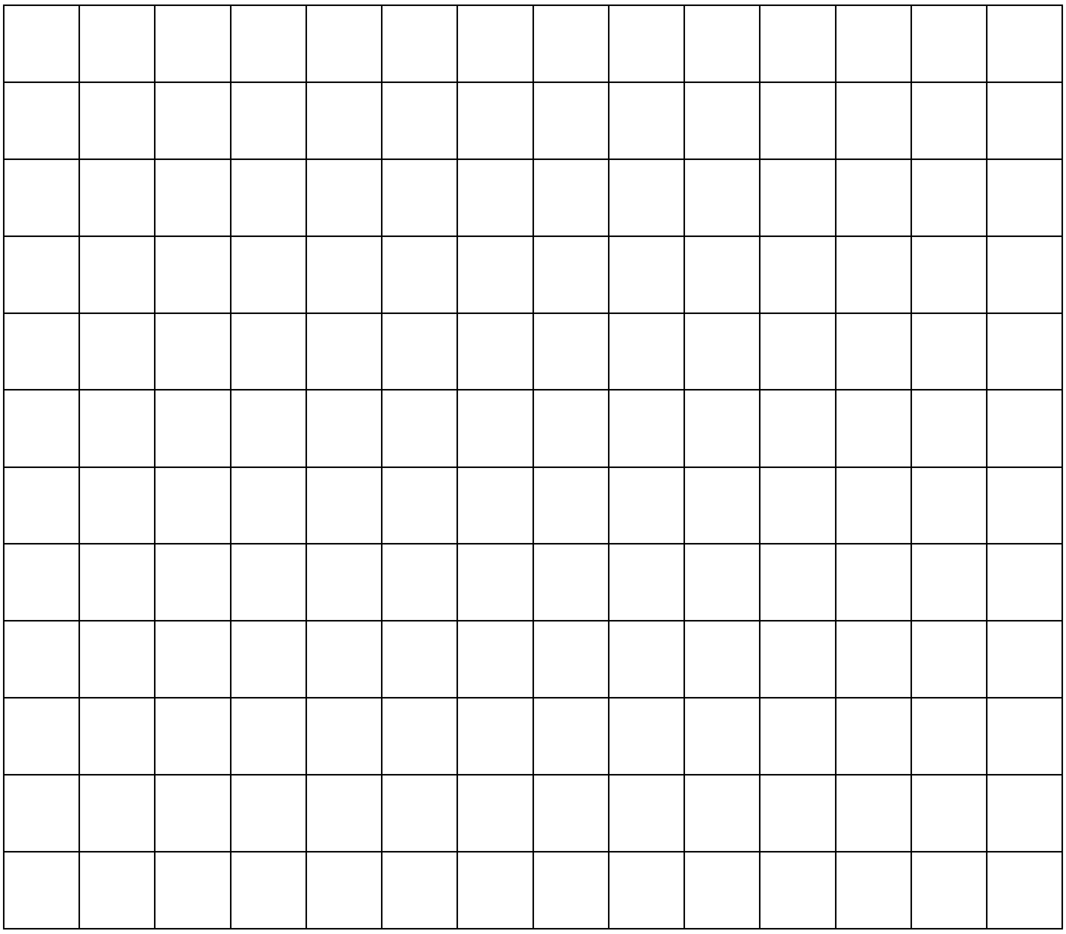
1. The following table of data represents the temperature at various times of the day.

| ***Time*** | 6:00 am | 9:00 am | 12:00 pm | 3:00 pm | 6:00 pm |
|---|---|---|---|---|---|
| ***Temperature*** | 12°C | 17°C | 21°C | 19°C | 14°C |

(a) Label the axes and scales on the Cartesian plane.

(b) Plot each point.

(c) Draw a line through the plotted points.



(d) Mark on the graph at approximately what time the in the morning the temperature was 19°C. ______________________________

(e) What was the approximate temperature at 5:00pm? _________

Obj. 13.1.1
a)i ☐
ii ☐
iii ☐
iv ☐

b)i ☐
ii ☐
iii ☐
iv ☐
v ☐
c) ☐

Obj. 13.1.2
d) ☐
e)i ☐
e)ii ☐

# Cycle 1: Relationships in Tables and Graphs

## Overview

### Big Idea

In this cycle the concept of a mathematical relationship is introduced. In this case, that the value of one variable is dependent upon that of another. The cycle considers a variety of datasets presented as tables and graphs and uses mainly graphical techniques to interpret the data (that is, to solve for one variable when the other is set to a known value). The general idea of a relationship presented in this cycle will be narrowed down to linear relationships, straight-line graphs and equations in the following cycles.

### Objectives

By the end of this cycle, students should be able to:

13.1.1 Create displays of relationships between variables using line graphs. [7NA178]

13.1.2 Investigate, interpret and analyse graphs of authentic data. [7NA180]

### Conceptual Links

Locating a point along a line requires students to know number names and symbols in order and the ability to place these along a number line with reference to zero. The number zero must be recognised as a starting point or origin rather than simply as a word meaning ‘nothing’. Students require knowledge of numerical sequences, number names and symbols in order to locate points along a number line from a reference point of zero. Students should be able to use a ruler to draw a line and partition it into same-sized segments. As a critical component of this cycle, learner understanding of additive and multiplicative number facts should be reinforced.

This cycle provides the basis for introducing proportional relationships and their graphical representation on the Cartesian plane.

### Materials

For Cycle 1 you may need:

- maths mat or large grid
- 1 cm grid paper
- string, rope or elastic
- paper

### Key Language

Cartesian plane, x-coordinate, y-coordinate, variables, relationship between variables

### Definitions

*Variables:* related attributes to be represented on a graph.

## *Assessment*

### *Anecdotal Evidence*

Some possible prompting questions:

- What are the variables you are graphing?
- Which axis is which variable?
- Can you create a line graph from your data pairs?
- How are the variables related?
- What are the units of measure for each variable?
- Is there a consistent relationship between the variables (does the graph have a straight line or close to a straight line)?
- If you look at the slope of the line, what does it tell you about the relationship between the variables? (For example, in a distance vs time running graph, a steeper slope suggests that students were running faster (more metres/second) and a straight horizontal line indicates no movement.)

### *Portfolio Task*

The student portfolio task *P13: YouTube Investigation* engages students in creating and interpreting displays of relationships between variables using line graphs.

# RAMR Cycle

## Reality

Remind students of previous graphing experiences. So far activities have explored creating bar graphs and circle graphs from counts of discrete items. Extend this idea to the generation of line graphs where the line represents movement between points (as for Unit 11).

Use numbered cards to mark an axis along two adjoining walls of the classroom. Start a student at location (0,0) (e.g., back left corner of the room). Plot the student's movement from the origin to the opposite corner of the room in vertical and horizontal movements ensuring that all movement is forward only (no backtracking). Graph the locations the student stops at on their path across the room. Discuss the graph in terms of change in y location and change in x location.

## Abstraction

The abstraction sequence for this cycle starts from students' previous experiences of graphing counted items to graphing relationships between values on the axes as line graphs. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Use a trundle wheel and stop watch to walk from one school fence to the other. Record how many seconds it takes to walk 10 m, 20 m, 30 m and so on to the other fence. Record the time elapsed and the change in distance in a table. It is not necessary to insist students walk at the same rate for each section they measure. A crooked or inconsistent line is more useful to explore.

***Resource*** Resource 13.1.1 Recording table

2. *Model/represent with materials.* Use large floor grids or maths mat to act out the line graph. Discuss with students which attributes they measured as they walked (change in time and change in location). Discuss which attributes to assign to each axis. Use students as markers to plot the time elapsed and the distance travelled from the table of collected data onto the grid. Use a length of string or elastic for students to hold to see the line of the graph. Ask students to place markers at their feet and lay the string/elastic along the grid so that they can step away and see the graph.
3. Discuss with students what information they can read from the graph. *Is the graph a straight line or a crooked line? What does this mean?* (If the graph is a straight line then students walked at a consistent pace, if it is crooked they walked faster or slower in some sections.) If the graph is a crooked line, try to determine from the graph which sections students walked faster or slower.
4. *Represent/model as graphs.* Use the table of values from the school ground walk to generate graphs on grid paper. Compare this graph with the larger graph created.

***Resource*** Resource 13.1.2 Grid papers

5. *Connect to mathematics.* If students have generated a crooked line (where they did not walk at a consistent rate), discuss how they could take an average value for walking across the school ground by using starting and finishing times and locations only. Ask students to use the graph to determine their position 10 seconds after they left their starting point at the fence. Explore some other examples to reinforce using a graph to find values.
6. *Connect to language.* Introduce language of per to describe the walking (e.g., 10 m per 3 seconds, 20 m per 6 seconds).

## Mathematics

### *Language/symbols and practice*

Discuss other line graphs where there is a relationship between the values represented on the axes. For example, graphs of relationships between change in temperature or change in location and time. Look at examples of line graphs and discuss the relationships shown in the graphs.

***Resource*** Resource 13.1.3 Examples of line graphs

Explore other instances of data that may be graphed using line graphs. For example:

- Change in temperature across a day (measure temperature at hourly intervals).
- Time to run 100 m from start to finish mapped against distance. Station students with stopwatches every 10 m along a hundred metre track and 10 m beyond, all students click start on their watch as the runner starts and click stop as the runner passes them. This data can be recorded in a table and graphed. It may show periods of acceleration and deceleration.
- If you have students who like to run, and enough students to record, try 200 m.
- Other interesting combinations could be change in cost compared with change in mass or capacity/volume (link to unit pricing).

***Resource*** Resource 13.1.4 Recording table
Resource 13.1.5 Temperature observations for 24 hours

## Reflection

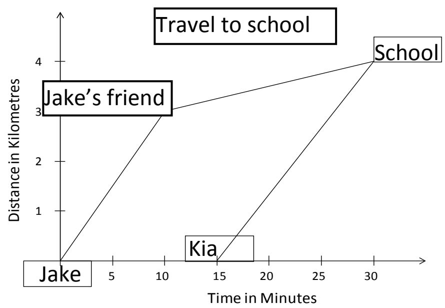
### *Apply the idea*

**Heart rates**. Take pulse before starting. Do step-up exercises for two minutes then retake pulse. Record pulse every two minutes until the heart rate has fallen back to normal. Record on a chart and plot the graph. Discuss the slow-down rate. (*Note*: Athletes drop back to normal very quickly.)

***Resource*** Resource 13.1.6 Heart rates

### *Extend the idea*

Extend students' understanding of relationships between variables. Describe what might be happening. For example, Jake used his scooter to travel to school. He met his friend on the way and they walked together. His sister, Kia, left after him and went to school on her bike. They all arrived together. *Why is there a bend in Jake's graph? How long did it take for Jake to get to school? How long did it take for Kia to get to school?*




Encourage students to create their own travel graphs including different types of travel (e.g., walking and cycling; walking, bus travel, then walking again). Discuss what the slopes on the graphs indicate about the relationship between time and distance travelled.

***Resource*** Resource 13.1.7 Extension line graphs

# *Teacher Reflective Notes*

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________________ Date:________

# *Can you do this? #2*

1. Bottles of soft drink contain 1.25 L. A bottle of soft drink costs $1.

   (a) Write this relationship as a rate comparing price per litre ($/L).

   ______________________________________________

   (b) Use a calculator to find the unit price per litre.

   ______________________________________________

2. Apple and pear juice is blended in the ratio of 3 parts apple juice to 2 parts pear juice (3:2).

   (a) Write this as a rate of parts apple juice per parts pear juice.

   ______________________________________________

   (b) Write this relationship as a unit rate of apple juice/pear juice.

   ______________________________________________

3. A factory packs apples in bags. There are 7 apples per bag.

   (a) How many apples in 12 bags? ______________________

   (b) How many bags are needed to pack 119 apples? __________

4. Phone calls to mobile phones are charged at a rate of $1.70 per minute.

   (a) Complete the proportion table for call costs.

| Cost | $1.70 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Minutes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

   (b) What is the cost of a call which takes 9 minutes? __________

   (c) If a phone call cost $17.00, how many minutes did the call last for?

   ______________________________________________

Obj. 13.2.1
a)i ☐
ii ☐

Obj. 13.2.4
b) ☐

Obj. 13.2.2
a)i ☐
ii ☐

Obj. 13.2.4
b) ☐

Obj. 13.2.3
a)i ☐
ii ☐
b)i ☐
ii ☐

Obj. 13.2.5
a)i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐
b) ☐
c) ☐

5. The following proportion table represents money earned at the rate of \$8 per hour.

| ***Time (hours)*** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ***Earnings (\$)*** | 8 | 16 | 24 | 32 | 40 |

(a) Label the axes and scales on the Cartesian plane.

(b) Plot each point.

(c) Draw a line through the plotted points.

(d) Extend the graph to 10 hours worked.

(e) Mark 8 hours of work on the graph. Write down how much had been earned at 8 hours worked? ______________________________

(f) Choose two coordinate pairs from the graph and write them down. i.__________________________ ii.____________________________

(g) Calculate the change in the x-axis between the two coordinate pairs. ______________________________________________

(h) Calculate the change in the y-axis between the two coordinate pairs. ______________________________________________

(i) Calculate the gradient of the line using the change in the y-axis and the change in the x-axis. _____________________________

(j) Write an equation for this line. ______________________________

6. Create a table of values for the equation *y = 3x*.

| ***x*** | | | | | |
|---|---|---|---|---|---|
| ***y*** | | | | | |

Obj. 13.2.6
a)i ☐
ii ☐
iii ☐
iv ☐
b)i ☐
ii ☐
iii ☐
iv ☐
v ☐
c) ☐

Obj. 13.2.7
d) ☐
e)i ☐
ii ☐

Obj. 13.2.10
f)i ☐ ☐
ii ☐ ☐

Obj. 13.2.8
g)i ☐
ii ☐

Obj. 13.2.9
h)i ☐
ii ☐

Obj. 13.2.10
i)i ☐
ii ☐

Obj. 13.2.11
j)i ☐
ii ☐

Obj. 13.2.12
i ☐ ☐
ii ☐ ☐
iii ☐ ☐
iv ☐ ☐
v ☐ ☐

# Cycle 2: Simple Linear Relationships

## Overview

### Big Idea

Simple linear relationships between two variables can be represented in proportion tables or double number lines (as for ratio), on Cartesian planes as straight line graphs, and as formalised algebraic equations ($y = mx$). Before exploring unit rate, explore situations that occur with a second value that is not a unit so as to highlight the multiplicative relationship between the values. Rate is frequently expressed as unit rate (quantity of first attribute per one unit of second attribute, e.g., 60 km/hr is understood as 60 km per 1 hour of travel). All situations that can be described in terms of *per*, *for each* or *for every,* are rates. As a result, rate can be introduced early because it is used throughout early mathematics without being recognised. For example, *3 bags of lollies with 4 lollies in each bag* is really 3 × 4 lollies/bag. In fact, some educators argue that all simple multiplications are number × rate (e.g. 3 bags × 4 lollies/bag = 12 lollies). In this cycle, rates will be limited to positive values.

### Objectives

By the end of this cycle, students should be able to:

13.2.1 Represents and interprets 'best buys' as rates. [7NA174]

13.2.2 Connects rate and ratio representations. [8NA188]

13.2.3 Solve a range of problems involving rates. [8NA188]

13.2.4 Calculates unit rate (e.g., 120 km in 2 hrs = average speed of 60 km/hr). [8NA188]

13.2.5 Generate a proportion table to solve rate problems involving direct proportion. [9NA208]

13.2.6 Plot points from a proportion table onto a Cartesian plane to generate a line graph. [8NA193]

13.2.7 Solve rate problems by extrapolation or interpolation of the line graph. [8NA194]

13.2.8 Identify change in the x-axis on the Cartesian plane. [9NA294]

13.2.9 Identify change in the y-axis on the Cartesian plane. [9NA294]

13.2.10 Find gradient of a line segment (interval) on the Cartesian plane. [9NA294]

13.2.11 Generate an equation for a line using gradient. [8NA194]

13.2.12 Solve linear equations for a range of values to create a table of values. [7NA179]

### Conceptual Links

Rate is a multiplicative relationship between dependent variables. The way of modelling rates in proportion tables or double number lines is the same as for ratio and proportion. Multiplication and division facts are prerequisite skills for rate. Students may also be given the opportunity to practice conversion of metric units through rate activities.

This cycle introduces the idea of rate which extends on previous proportional thinking ideas with ratio. Rate ideas are important for any future problem solving situations which require comparisons and proportional mixtures that involve different types of unit. These may be represented on Cartesian planes and as equations.

## Materials

For Cycle 2 you may need:

- calculators
- Cartesian planes (grid paper)
- dual scale number lines
- maths mat or large grid on floor

## Key Language

Rate, per, for each, for every, km/hr, $/L, $/kg, comparison, variable, equation, unknowns, Cartesian planes, unknowns, gradient, slope, $y = mx$, linear relationship

## Definitions

*Gradient:* slope of a line. Calculated using the formula: $\frac{\Delta y}{\Delta x}$ otherwise known as $\frac{\text{change in } y}{\text{change in } x}$.

## Assessment

### Anecdotal Evidence

Some possible prompting questions:

- What is the base amount?
- What amounts could you multiply the base amount by?
- Can you organise some values in a table?
- Can these values be represented on a Cartesian plane?
- If you extend the graph, what other values might you get?
- Can you find values on the graph that are between values in the table?
- Can you work out a value for the slope of the graph?
- Is your graph a straight line?
- Can you find an equation for the linear relationship?

### Portfolio Task

The student portfolio task *P13: YouTube Investigation* engages students with determining and interpreting rates including 'best buys'.

# *RAMR Cycle*

## Reality

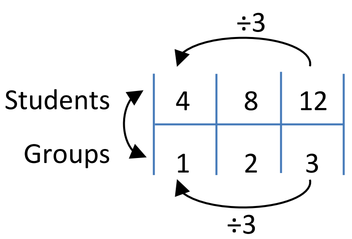
Rate problems can be addressed very effectively with real situations familiar to students. Simply filling pencil cases with twelve pencils per case is an example of early rate. Reinforce the language of *per* and connect clearly to twelve pencils for every pencil case. Students may also act out 3 groups with 4 students per group, 4 teams with 5 people per team and so on. Create other examples using rate language to extend the idea of rate. For example, speed is kilometres per hour (km/hr), cost of petrol is dollars per litre ($/L), unit prices in the shopping centre are dollars or cents per kg. Discuss local situations which are linear relationships (can be represented by line graphs). Look at different patterns and familiar situations that involve variable quantities. For example:

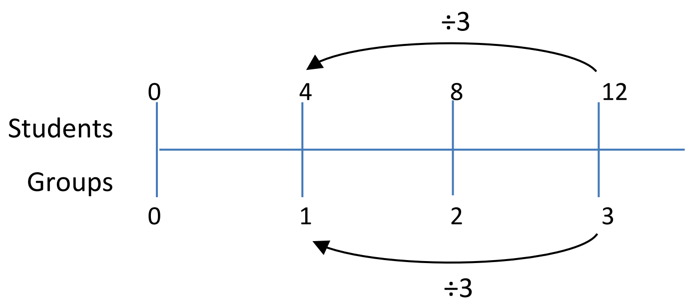
- A can of fuel will allow 2 hours of travel. Depending upon how many cans of fuel I take, how long will I be able to travel?
- I bought a $3 ice cream for everyone. How much was spent for 1 person, 2, 3, and so on?
- A candle is initially 30 cm tall. Every hour it burns it gets 2 cm shorter. How tall will the candle be after 6 hours?

## Abstraction

The abstraction sequence for this cycle begins with students' experience of concrete, simple rate problems. These experiences can be extended to explore solution strategies for less tangible and more complex rates and representations including proportion tables, graphs and equations. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Start with simple examples that can be acted out as in the Reality phase. For example, 12 students arranged in 3 groups.
2. *Represent/model with materials.* Use unifix cubes or counters to model arranging 12 into 3 groups. Identify how many counters per group. Identify the operation used to partition the 12 into 3 groups (division).
3. *Represent/model as drawings.* Rate problems can be modelled in proportion tables and on dual scale number lines. Highlight the multiplicative relationships within and between measures.




4. *Connect to language and symbols.* Write the relationship between students and groups as 12 students per 3 groups. Connect this to the operation to reduce to a unit rate of 4 students per group.
5. *Reverse the problem*. Change the problem into "total = number × rate" where the number relates to groups, the rate relates to students in each group, and the total to the total number of students. Explicitly connect to the operation used to find the total.

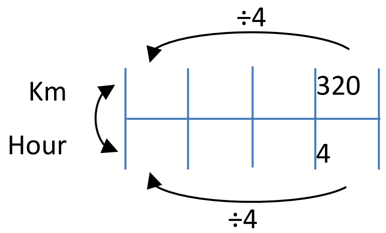
6. *Model/represent graphically*. Explore the construction of a graph to illustrate the relationship between students and groups. Position students on large floor grids or maths mat. Each student represents a different numbers to plot the values from the proportion tables. Ask students to hold a length of cord or elastic to see values that might land between their values. Put the cord down on the mat across the coordinates so that students can step away and see the graph.
7. *Represent/model as drawings.* Recreate the large graph as a line graph on squared paper using the proportion table. Compare this graph with the larger maths mat graph to see if they match.
8. *Connect to language and symbols.* Explore values along the graph and test the equation constructed in step 5 to see that the equation matches the line for any number of groups.
9. *Connect all representations.* Ensure that students can recognise the connection between the representation of 12 students/3 groups, 4 students/group, the proportion table, the graphical representation, and the equation for any number of groups as "total = number × rate".

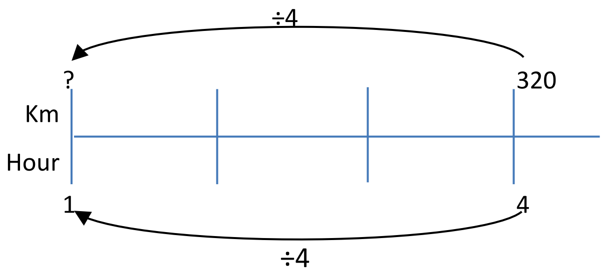
## Mathematics

In the Mathematics phase, students are expected to learn and practise four features of the concept of rate. These are:

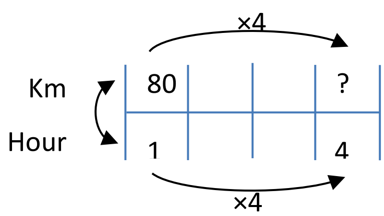
1. **Rate as comparison**. Rate is a comparison between two attributes that are usually of different types – like speed (km/hour) or cost ($/kg or $/L).
2. **Unit rate compares with one**. Rate is also a comparison where the second attribute is expressed as a unit value. For example, speed of 68 km across an hour is represented as 68 km/hr.

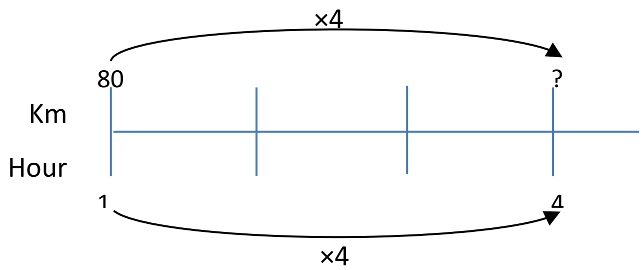
   **Rate is related to division.** For example, if I drove 320 km in 4 hours, to find the distance driven in 1 hour I need to divide by 4, then 320 ÷ 4 = 80 km, so the unit rate is 80 km/hr. If we know total distance and total time, then the unit rate can be found using division. This can be modelled in a proportion table or on a dual scale number line.




3. **Rate is the basis of multiplication**. When we apply multiplication, we multiply a number by a rate. For example, 3 bags with 4 lollies in each bag is 3 bags × 4 lollies/bag = 12 lollies. Similarly, 5 rows of 7 objects is 5 rows × 7 objects/row. If we know the rate and have a multiplier for the second attribute in the rate, we can use that multiplier to find the total amount of the first attribute in the rate. This can be modelled in a proportion table or using a dual scale number line as a means of determining the necessary operator. For example, if we are travelling at 80km/hr, how far we will travel in 4 hours can be modelled as follows:



### *Language/symbols and practice*

**Practise plotting linear relationships on Cartesian planes**

Provide students with opportunities to practice manipulating and solving for unknown variables (both independent and dependent) as well as determining rate. Use tables of data to generate graphs of simple linear relationships and encourage students to move flexibly between these various representations.

***Resource*** Resource 13.2.1 Rate problems
Resource 13.2.2 Grid paper

Real world activities may be used to generate data sets that students may be able to graph and determine relationships between variables. These include:

- Walking across the school ground at a steady pace.
- Adding marbles (one at a time) to an aluminium tray in water and recording the height of the tray above water after each marble is added.
- Increase in height of a chair stack as chairs are added (or increase in height of foam cup stack). Note that the height of one chair is the baseline. The measurement is the increase in height as chairs or cups are added.
- Rate of change of water level in a bucket with a slow leak.

**Finding unknown variables**

Provide students with opportunities to practice manipulating and solving for unknown variables (both independent and dependent) as well as determining rate. Encourage students to find, pose and graph their own problems involving rate. This may also be an opportunity to reinforce best buys problems, recognise the rate component of these and represent them graphically.

## Reflection

### *Check the idea*

This phase is an extension of the previous phase. Provide students with real problems involving rate that may be of interest to them. *Resource 13.2.3: Real rates* has some examples of real world examples of rates.

***Resource*** Resource 13.2.3 Real rates

### *Apply the idea*

Application of rate to real problems also involves interpretation of the results. It is important to engage students in further thinking and communication to ensure that they can interpret the answers they find from undertaking rate calculations. *Resource 13.2.4: Rates of water use* and *Resource 13.2.5: Rates of exchange* provide the basis for real investigations using rates that require students to interpret their findings and communicate results.

***Resource*** Resource 13.2.4 Rates of water use
Resource 13.2.5 Rates of exchange

## *Extend the idea*

**Generalise rules**

Encourage students to look for their own rules to assist them with generalisation of rate processes. Some of the following may arise:

- Rates are given as "number" "attribute"/"second attribute" (e.g., 24 km/hr and 6 litres/km). Rate problems, therefore, deal with these attributes. Looking at a lot of problems will show that attributes "cancel" as well as numbers (e.g., 3 hr @ 40 km/hr = 120 km – the hr/hr seems to become 1).
- This means that:
    - if rate and the first attribute is given, multiplication can be used to find totals; and
    - if rate and second attribute is given, the operation is usually divide.

Ensure students can generalise from growing pattern rules to equations, equations to Cartesian planes and from Cartesian planes to equations.

# *Teacher Reflective Notes*

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________________ Date:________

# *Can you do this? #3*

1. You have \$5 saved and earn \$8 per hour, how many hours will it take to earn enough to buy a \$69 phone?

   (a) Complete the table of values.

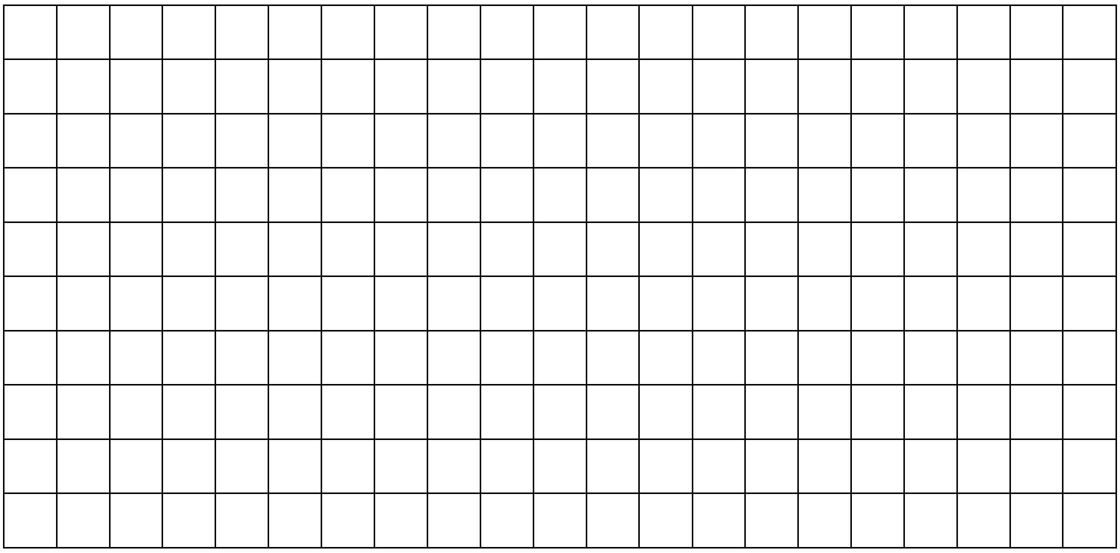
| | | | | | | |
|---|---|---|---|---|---|---|
| Total (\$) | | | | | | |
| Time (h) | | | | | | |

   (b) Write down the rule for calculating each total (\$). ____________

   (c) Calculate how many hours it will take to earn \$69? ___________

   (d) Graph this relationship on the Cartesian plane.

   (e) Mark on the Cartesian plane where you have earned \$69.



   (f) Your cousin also has \$5 saved but only earns \$7 per hour. Write down the rule for calculating total earnings after each hour for your cousin. ________________________________________

   (g) Graph this relationship on the Cartesian plane.

   (h) Mark on the Cartesian plane where your cousin has earned \$69.

Obj. 13.3.1
a)i □ □
ii □ □
iii □ □
iv □ □
v □ □
vi □ □

Obj. 13.3.3
b)i □
ii □

Obj. 13.2.3
c) i □
ii □

Obj. 13.2.6
d)i □ □
ii □ □
iii □ □
iv □ □
v □ □

Obj. 13.3.4
e) □

Obj. 13.3.3
f)i □
ii □

Obj. 13.2.6
g)i □ □
ii □ □
iii □

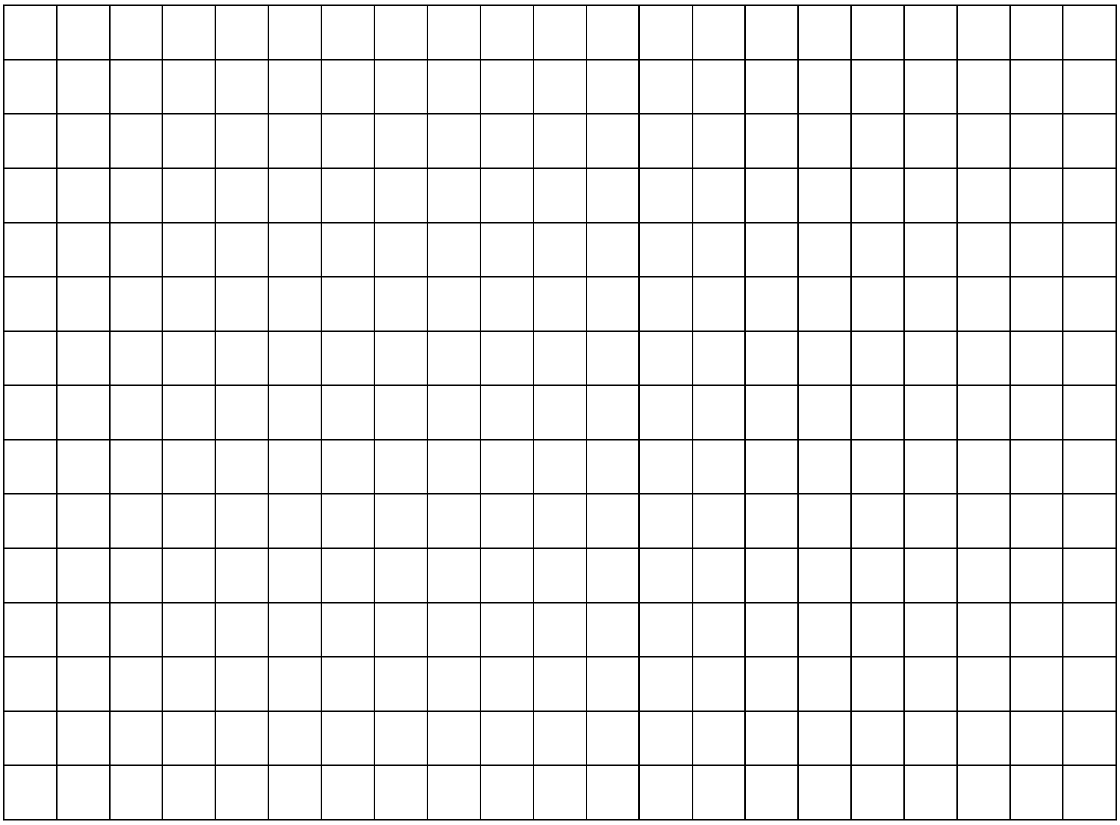
Obj. 13.3.4
h) □

2. The points on a line are recorded in a table.

   (a) Plot each point on the Cartesian plane.

   (b) Draw a line through the plotted points.

| | | | | |
|---|---|---|---|---|
| *y* | 1 | 3 | 5 | 7 |
| *x* | 0 | 1 | 2 | 3 |



   (c) What is the gradient? ________________________________

   (d) What is the y-intercept? ________________________________

   (e) Write the equation for this line. ________________________

   (f) Extend the graph to find the value of y if x = 6. ____________

Obj. 13.2.6
a)i □
ii □
iii □ □
iv □ □
v □ □
vi □ □
b) □

Obj. 13.3.2
c)i □
ii □

Obj. 13.3.2
d) □ □

Obj. 13.3.3
e)i □
ii □

Obj. 13.3.4
f) □

# Cycle 3: Comparing Simple Linear Relationships

## Overview

### Big Idea

This cycle explores the effect caused by changing the coefficient in an equation on the gradient of a line. The effect on a linear relationship caused by the existence and changing of a constant value is also explored. Many of the contexts from the previous cycle can be re-used. For example, the cost of purchasing goods by weight which includes the purchase of a shopping bag.

### Objectives

By the end of this cycle, students should be able to:

13.3.1 Generate a proportion table to solve rate problems involving direct proportion and a constant. [8NA193]

13.3.2 Find y-intercept of a line on the Cartesian plane. [8NA194]

13.3.3 Generate an equation for a line using gradient and y-intercept. [8NA194]

13.3.4 Use a graph of a simple linear relationship to solve a rate problem. [8NA194]

### Conceptual Links

Mapping rate relationships graphically on Cartesian planes requires students to be able to generate proportion tables for rate relationships. Students must also have the ability to apply multiplication and division number facts and generate Cartesian planes with appropriate use of scale.

This cycle extends the modelling of rate relationships to include Cartesian planes. Rate ideas are important for any future problem solving situations which require comparisons and proportional mixtures that involve different types of unit. Graphical representations can be an effective means of modelling and extrapolating values for rate relationships.

### Materials

For Cycle 3 you may need:

- calculators
- maths mat or large floor grid
- elastics or string
- dual scale number lines
- Cartesian planes

### Key Language

Rate, per, for each, for every, km/hr, $/L, $/kg, comparison, variable, equation, Cartesian planes, unknowns, gradient, slope, coefficient

### Definitions

*Coefficient:* multiplier of a variable. In simple linear equations, the coefficient ($m$) of $x$ is the gradient of the line $y = mx + c$.

## Assessment

### *Anecdotal Evidence*

Some possible prompting questions:

- What do you notice about the gradient (slope) of the line on the graph when you change the multiplier (coefficient) in the equation? What changed and what stayed the same?
- What do you notice about the gradient (slope) of the line on the graph when the multiplier (coefficient) in the equation is a positive number?
- What do you notice about the gradient (slope) of the line on the graph when the multiplier (coefficient) in the equation is a negative number?
- What do you notice about the position of the line on the graph when you have a constant value included in the equation? If the constant value changes, how does the line change or stay the same?

### *Portfolio Task*

The student portfolio task *P13: YouTube Investigation* engages students with solving rate problems using a line graph.

# RAMR Cycle

## Reality

As in the previous cycle, rate problems should be connected to situations familiar to students. Continuing the connected the use of proportion tables, line graphs and equations to model possible solutions to rate problems, this cycle focuses on comparing linear relationships. In particular, this cycle explores the effect on the gradient of a line as a result of changes to the coefficient, and the effect on the position of a line of change as a result of changes to the constant. Find a variety of these to share with students and discuss the reading of these representations.

Previously used linear relationships can be explored using "what if" questions. For example:

- A can of fuel will allow 2 hours of travel. Depending upon how many cans of fuel I take, how long will I be able to travel? What if the cans of fuel are smaller and will only allow 1.5 hours of travel? What if the fuel economy of the boat is improved and a can of fuel will allow 2.5 hours of travel?
- I bought a $3 ice cream for everyone. How much was spent for 1 person, 2, 3, and so on? What if ice creams were only $2.50? What if the price of ice creams has increased to $3.5?
- A candle is initially 30 cm tall. Every hour it burns it gets 2 cm shorter. How tall will the candle be after 6 hours? What if the candle is 40 cm tall to start with? What if new candles are bought that are only 20 cm tall?

Explore these problems intuitively and through language to ensure that students are familiar with the net effect of these changes. For example, if fuel cans are smaller and allow less travel, more will need to be taken for the same trip or less distance is to be travelled.

## Abstraction

The abstraction sequence for this cycle begins with students' experience of concrete, simple rate problems. These experiences can be extended to explore solution strategies for less tangible and more complex rates and representations including proportion tables, graphs and equations. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Generate an input-output table of values for a function machine by acting out a simple linear proportional relationship. Ask students to generate a human graph on a maths mat or large grid on the floor (students stand at the coordinates and hold a length of string or elastic to create the line). Place this line down on the grid.

   Change the multiplier in the function machine to generate a new set of coordinates. Create a human graph on the maths mat or large grid. Place the new line on the floor and ask students to step away. Discuss how the line is the same as and different from the original line. Repeat with another changed multiplier.

   Ask students to predict what will happen with the line if the multiplier is changed again.

2. *Represent/model as drawings.* Recreate the first two line graphs on paper. Engage students with generating an input-output table for the third multiplier and graphing this line to test their theory. Ensure that students can connect the values from the table with the equation and the equation with the drawn line on the graph. Reinforce the connection between the consistent multiplicative change in the input-output table, the gradient of the line and the coefficient ($m$) of $x$ in the equation in the form of $y = mx + c$.

**Resource**

Resource 13.3.1 Plotting input-output values on a Cartesian plane
Resource 13.3.2 Grid paper

## Mathematics

### *Language/symbols and practice*

**Exploring gradients**

Extend students by practising problems that explore a variety of linear relationships. Begin with a word problem that requires students to graph an equation on a Cartesian plane. Include questions that provide the reverse information – a graph from which students must derive the correct equation. Practice should include relationships that have positive or negative gradients and include both whole number and fractional gradients.

**Exploring y-intercept**

Once students are confident with identifying the gradient of a line, consider changing the problem to include a constant value other than 0. For example:

- There is enough fuel in my boat to travel for 1.5 hours. After this period, I will refill with cans of fuel. A can of fuel will allow 2 hours of travel. Depending upon how many cans of fuel I take, how long will I be able to travel? (gradient = 2, constant = 1.5)
- I bought a $3 ice cream for everyone plus a $5 chocolate. How much for 1 person, 2, 3, and so on? (gradient = 3, constant = 5)
- I had $13 and I paid $2 per hour to be at Timezone, how much money did I have left or how much do I have to go into debt? (gradient = $^{-}2$, constant = 13)
- A candle is initially 30 cm tall. Every hour it burns it gets 2 cm shorter. How tall will the candle be after 6 hours? (gradient = $^{-}2$, constant = 30)
- In a rhythm pattern using actions of claps and clicks, each student adds an additional click as the pattern moves from one student to the next (e.g., clap clap click, clap clap click click, clap clap click click click, …). How many clicks will a student perform depending on their number in the sequence? (gradient = 1, constant = 2)

Generate tables of values for these problems and graph the relationships. It may assist students to recognise the consistent change if these are acted out with function machines and an additional row is added to the proportion table. For example:

I bought a $3 ice cream for everyone plus a $5 chocolate. How much for 1 person, 2, 3, and so on? (gradient (m) = 3, constant (c) = 5)

| | $3 \times 0 + 5$ | $3 \times 1 + 5$ | $3 \times 2 + 5$ | $3 \times 3 + 5$ | $3 \times 4 + 5$ | $3 \times 5 + 5$ | $3 \times 6 + 5$ | $3 \times x + 5$ |
|---|---|---|---|---|---|---|---|---|
| Total cost (y) | 5 | 8 | 11 | 14 | 17 | 20 | 23 | |
| Number of people (x) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |

Identify where the graph crosses the y-axis (where the value of x is 0).

Explore examples with the same gradient and different constants (using the same Cartesian plane). Discuss what students notice about these graphs. Identify the relationship between the y-intercept and the constant value for the problem. Explore changing the constant value and discuss the resulting change in the position of the line on the graph.

***Resource*** Resource 13.3.3 Growing patterns to Cartesian planes
Resource 13.3.2 Grid paper

### Linear relationships with positive gradients and changing constants

Explore sets of examples with the same gradient and different constants (using the same Cartesian plane). Discuss what students notice about these graphs. Ensure that students are able to generate graphs from equations as well as equations from graphs.

***Resource*** Resource 13.3.4 Positive gradient with a constant practice

### Linear relationships with negative slopes and changing constants

Explore further graphs that slope down from left to right or have a negative slope or gradient.

Example situations might include starting with $10 and spending $2 per go on a game machine [$-2x + 10$ *or* $10 - 2x$].

Explore sets of examples with same gradient, changing constant and changing gradient, same constant as for positive graphs. Ensure that students are able to generate graphs from equations as well as equations from graphs.

***Resource*** Resource 13.3.5 Negative gradient with a constant practice

### Linear relationships with fractional or decimal slopes and constants

Explore graphs that have a fractional or decimal slope or gradient. Example situations might include starting with $10 and spending $0.50 per go on a game machine [$-0.5x + 10$ *or* $10 - 0.5x$].

Explore sets of examples with same gradient, changing constant and changing gradient, same constant as for positive graphs. Ensure that students are able to generate graphs from equations as well as equations from graphs.

***Resource*** Resource 13.3.6 Fractional gradient practice

## Reflection

### *Apply the idea*

Ensure that students can see the relationships in Mathematics above. Undertake applications as follows.

**Spaghetti Bridge**. Materials: spaghetti, foam cup, paper clip for hook. Use spaghetti, one strand, between two desks. Suspend the cup with an opened-out paper clip hooked over the spaghetti. Add nails (or similar) one at a time to see when it collapses. Repeat for two strands of spaghetti, then three, and so on. *What will they hold? How many strands would you need to support a mobile phone (or similar)?* Create a chart to record the results and graph the outcomes. (Note: Although not very heavy, cup & clip become a constant in this equation.)

***Resource*** Resource 13.3.7 Spaghetti bridge

# *Teacher Reflective Notes*

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________________ Date:________

# *Can you do this? #4*

1. A shop sells two different types of marble in bags: red and blue. Bag A containing three red marbles and two blue marbles weighs 66 g. Bag B containing one red marble and four blue marbles weighs 72 g.

   (a) Form an equation that describes Bag A. ____________________

   (b) Complete the table of values for Bag A.

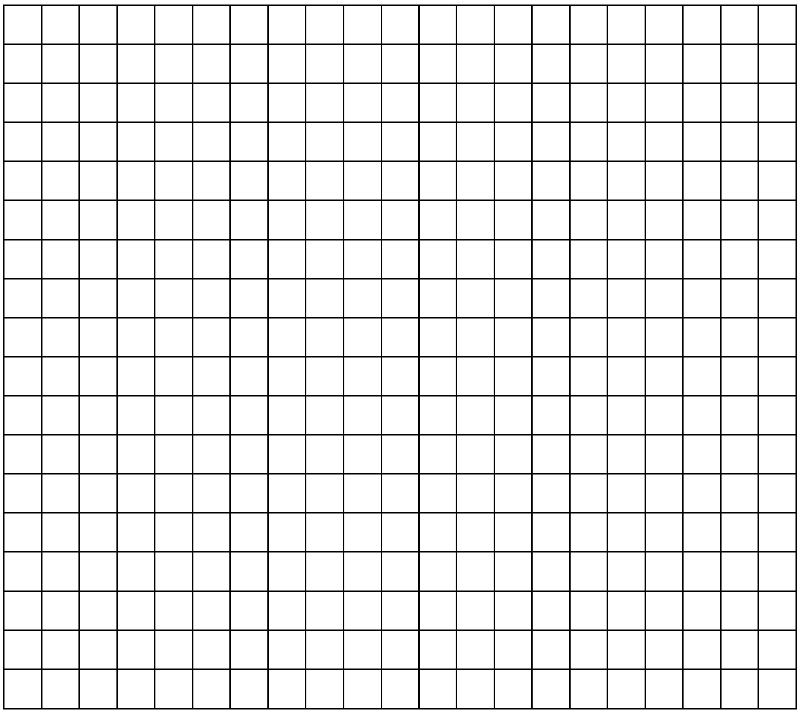
| *Red (r)* | 10 | 16 |
|---|---|---|
| *Blue (b)* | | |

   (c) Form an equation that describes Bag B. ____________________

   (d) Complete the table of values for Bag B.

| *Red (r)* | 10 | 16 |
|---|---|---|
| *Blue (b)* | | |

   (e) Plot these lines on the Cartesian plane.

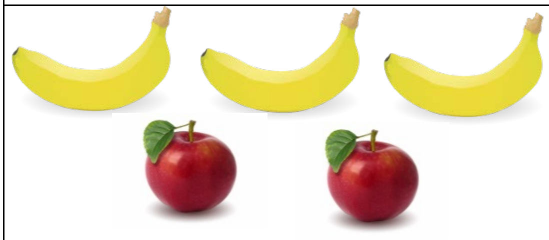
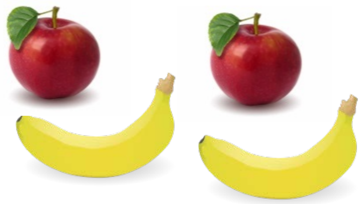



   (f) What are the weights of the marbles where the lines cross?

   Red ____________________ Blue ____________________

Obj. 13.4.1
a) i ☐
ii ☐
iii ☐
b) i ☐
ii ☐
c) i ☐
ii ☐
iii ☐
d) i ☐
ii ☐
e) i ☐
ii ☐
iii ☐
iv ☐
f) i ☐
ii ☐

2. Two people bought fruit at the shop.

| **Person A** | **Person B** |
|---|---|
|  |  |
| 122 cents | 92 cents |

(a) Form an equation for Person A.

(b) Form an equation for Person B.

(c) Solve simultaneously to find the cost of a banana.

(d) Calculate the cost of an apple.

Obj. 13.4.2
a)i ☐
ii ☐
iii ☐
b)i ☐
ii ☐
iii ☐
c)i ☐
ii ☐
iii ☐
d)i ☐
ii ☐
iii ☐

# Cycle 4: Simultaneous Solutions

## Overview

### Big Idea

There are a range of methods for solving linear relationships simultaneously. These can include using algebraic tools and understandings like backtracking, balance, simultaneous equations and generating graphs for equations (identifying where the graphs cross).

### Objectives

By the end of this cycle, students should be able to:

13.4.1 Solve simultaneous equations graphically. [8NA194]

13.4.2 Solve simultaneous equations algebraically. [8NA194]

### Conceptual Links

This cycle links directly to previous ideas of rate, proportion tables and graphical representations of linear relationships. Reading coordinates from a Cartesian plane and division are important prerequisite ideas for this cycle.

This cycle extends students' understandings of proportional relationships to include solving for values of x and y that will satisfy more than one equation.

### Materials

For Cycle 4 you may need:

- maths mat or large grid
- 1 cm grid paper
- rulers
- string, rope or elastic
- paper
- different colour pens or pencils

### Key Language

Variable, equation, balance, backtracking, Cartesian planes, simultaneous solutions, unknowns, gradient, slope, constant, fixed part

### Definitions

*Simultaneous solutions:* values for variables that satisfy more than one linear equation at the same time.

## *Assessment*

### *Anecdotal Evidence*

Some possible prompting questions:

- Can you create an equation for the question?
- Do the equations have the same variables?
- Can you graph each equation?
- Will a table of values or a proportion table help you?
- Is there a solution that works for both equations at the same time?
- If you add or subtract your equations, will you eliminate a variable?
- Can you substitute that value back into one of the equations to find the other variable?

### *Portfolio Task*

The student portfolio task *P13: YouTube Investigation* engages students in solving direct proportion problems using proportion tables.

# RAMR Cycle

## Reality

Start with problems that relate to students' lives like NRL or netball. For example, The "space shots" ball (a game where players line up and shoot three points as fast as they can), Team A was losing 8 to 13 to Team B at half time, but scored at 3 points per minute in the second half while Team B only scored at 2 points per minute. Answer these questions:

a) At what time in the second half did Team A pass a score of 26?
b) At what time in the second half did Team A pass Team B?
c) If halves are 10 minutes, what was the final score?

Act out the first few minutes of the second half in a table of values.

| Minutes | 0 | 1 | 2 | 3 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Team A | 8 | 11 | 14 | 17 | | | | | | | |
| Team B | 13 | 15 | 17 | 19 | | | | | | | |

Ask students to predict when they think Team A will pass a score of 26, whether they think Team A will pass Team B and what the final scores may be. Discuss strategies that could be used to answer these questions.

## Abstraction

The abstraction sequence for this cycle starts from students' previous experience of linear relationships plotted on Cartesian planes and extends these to find solutions for real problems. A suggested sequence of activities is as follows:

1. *Kinaesthetic*. Continue acting out the space shots problem from the reality for another minute.
2. *Record mathematically.* Suggest to students that a greater range of problems could be solved more quickly if another method were used. Relate the scores of the teams to equations that include the score at half-time (time = 0 for the second half), and the shooting rates of each team. Team A's score in the second half is $3m + 8$ and Team B's is $2m + 13$ where m is number of minutes.
3. *Apply graphing skills.* Graph each of the equations on the same Cartesian plane. Label each equation clearly (possibly even use different colours for each line to tell them apart).
4. Use the graph to find where Team A passes a score of 26, when Team A passes Team B, and what the scores are at 10 minutes.

   **Note:** (a) is answered by seeing when $3m + 8 = 26$; (b) is answered by seeing when $3m + 8 = 2m + 13$; and (c) is answered by the scores at 10 minutes.

| Minutes | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Team A | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 32 | 35 | 38 |
| Team B | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 |

## Mathematics

### *Language/symbols and practice*

***Simple examples***

Practise using graphs to solve other problems involving rates. For example,

> *You have \$5 and you earn \$8 per hour, how many hours will it take to earn enough to buy a \$69 phone?*

*Determine equations.* Start by creating a table of hours (e.g., 0, 1, 2, 3 …) versus money (e.g., \$5, \$13, \$21, \$29 …). Students should identify the differences in the table to obtain the growing part (which is 8) and fixed part (which is 5), and use this information to generate the expression $8n + 5$ in patterns or the equation $y = 8x + 5$ in graphs. The other side is the target and this means getting to \$69. Students need to show that such targets are a horizontal line, in this case at 69 or $y = 69$.

*Use line graphs to solve the problem.* Construct appropriate axes ($y$ axis 0 to 70 and $x$ axis 0 to 10). Draw the graphs. See where they cross. Then check to see that this is the correct time by calculating when you get to \$69 from the table.

***More complex examples***

Practise more examples of these types of problems. More complex examples may be generated by extending the simple problems. For example,

> *You have \$5 and you earn \$8 per hour. Bob has \$20 but only earns \$5 per hour. When do you have the same money as Bob? How many hours more does Bob have to work for his \$69 phone than you?*

*Determine equations*. Repeat the process of Part One creating tables for both you and Bob. The equations are, for you $y = 8x + 5$, for Bob $y = 5x + 20$, and the target for the phone $y = 69$.

*Use line graphs to solve the problem.* Draw axes, draw graphs, and see where they cross. Check results. See when Bob gets to or over \$69 by seeing the point on the $x$ axis where his graph cuts the line for $y = 69$.

### *Resource*

Resource 13.4.1 Solving for variable problems

### *Connections*

Also make connections between solving equations using graphical methods, algebraic skills of backtracking and balance to solve for unknowns. For example, the problems above could be solved by finding when $y = 8x + 5 = 69$ and $y = 5x + 20 = 69$. Similarly, students should engage with determining instances where $y = 8x + 5$ and $y = 5x + 20$ result in the same values for x and y (where the graphed functions intersect).

***Algebraic solutions***

Considering the previous example,

> *You have \$5 and you earn \$8 per hour. Bob has \$20 but only earns \$5 per hour. How many hours more does Bob have to work for his \$69 phone than you?*

Ensure students can recognise that they are finding values for x for each equation that result in a value of 69 for y. These problem types will be asking for when the same value is reached under differing conditions. In this example, the solutions for x will lie along a horizontal line where y = 69 and will be the x values where the linear functions intersect with this line. Students should find solutions for x for each equation where y = 69 by substituting this value for y in the equations and finding the corresponding x value. For example,

$$
\begin{aligned}
y = 8x + 5 &= 69 \\
8x &= 69 - 5 \\
8x &= 64 \\
x &= 64 \div 8 \\
x &= 8
\end{aligned}
\qquad\qquad
\begin{aligned}
y = 5x + 20 &= 69 \\
5x &= 69 - 20 \\
5x &= 49 \\
x &= 49 \div 5 \\
x &= 9.8
\end{aligned}
$$

So, you need to work 8 hours before you can buy the mobile phone. To have enough money, Bob needs to work for 9.8 hours. To answer the initial question, Bob needs to work 1.8 hours longer than you before he can buy the phone.

Given the base of the problem example, the following question is also possible.

> *You have $5 and you earn $8 per hour. Bob has $20 but only earns $5 per hour. When do you have the same money as Bob?*

This question is not asking for when both Bob and you have a set amount of money, but rather, at what point will you have worked the same number of hours and have the same amount of money. Students should recognise that this occurred previously where the two graphs of Bob's and your earnings intersected. Similarly, students need to also recognise that they are looking for instances where $y = 8x + 5 = 5x + 20$.

This can be explored using balance ideas as follows:

$$
\begin{aligned}
8x + 5 &= 5x + 20 \\
8x - 5x &= 20 - 5 \\
3x &= 15 \\
x &= 5
\end{aligned}
$$

Substituting x = 5 into either of the equations should give a value for y.

$$
\begin{aligned}
y &= 8x + 5 \\
y &= 8 \times 5 + 5 \\
y &= 45
\end{aligned}
$$

Thus Bob and you have the same amount of money after you have both worked for 5 hours. That amount of money is $45.

The same outcome may be reached by treating the equations as entities that may be operated on and solved simultaneously.

$$
\begin{aligned}
y &= 8x + 5 \\
\underline{-(\,y} &\underline{= 5x + 20)} \\
0 &= 3x - 15 \\
5 &= 3x \\
5 &= x
\end{aligned}
$$

Then solve for y as before.

***Resource*** Resource 13.4.2 More practice problems

## Reflection

### *Check the idea*

If possible, devise a problem relevant to students' interests that requires use of simultaneous solutions. The following worded problems may provide adaptable structures.

> ***Trading cards and collectibles:***
>
> *Paul collects baseball cards and football cards. The number of baseball cards is 10 more than twice the number of football cards. In total, Paul has 70 cards. How many of each type card does Paul have?*
>
> ***Tickets for events:***
>
> *Mayfield High School sold tickets for a school musical. Seats in the auditorium cost $6 each and balcony seats cost $4. A total of 200 tickets was sold and $960 was collected. How many of each type of ticket was sold.*
>
> ***Spending limits:***
>
> *Margie is responsible for buying a week's supply of food and medication for the dogs and cats at a local shelter. The food and medication for each dog costs twice as much as those supplies for a cat. She needs to feed 164 cats and 24 dogs. Her budget is $4240. How much can Margie spend on each dog for food and medication?*

### *Apply the idea*

Consider budgeting decisions that may be relevant for students. For example, comparing mobile phone plans with initial call + cost per minutes for calls or texts, possible amounts that may be spent on entertainment or snacks, combinations of items within collections.

The following problem structure may be useful:

> *A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totalled $487. The second order was for 6 bushes and 2 trees, and totalled $232. The bills do not list the per-item price. What were the costs of one bush and of one tree?*

### *Extend the idea*

Describe the path taken by two moving objects using linear equations (e.g., two boats sailing in the harbour, vehicles travelling along roads which will meet, people moving from location to location in different directions on paths that will cross). Note that these problems involving graphing distance or speed over time to determine the location through which both objects will pass. Follow up questions may determine whether both objects will pass through the location at the same time and consequently collide.

# *Teacher Reflective Notes*

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________________ Date:________

# *Can you do this? #5*

1. Jodie and Pedro are going to race each other. Jodie can run 5 metres per second, whereas Pedro can run 6 metres per second. Because she is slower, Pedro gives Jodie a head start of 13 m.

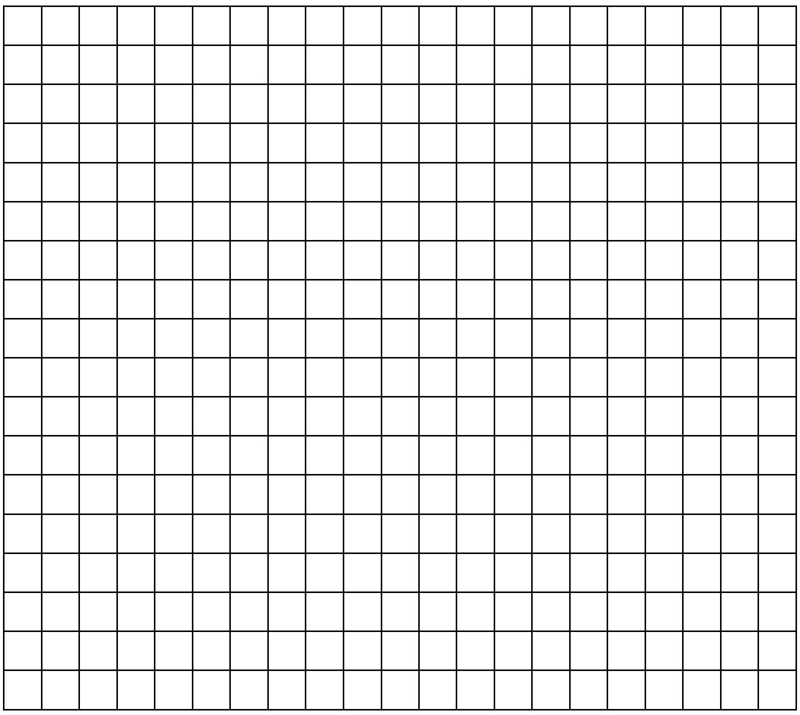
   (a) Complete the table of values for how far from the start and how long Jodie runs.

| *Time (s)* | 0 | 1 | 5 | 10 |
|---|---|---|---|---|
| *Distance (m)* | | | | |

   (b) Complete the table of values for how far from the start and how long Pedro runs.

| *Time (s)* | 0 | 1 | 5 | 10 |
|---|---|---|---|---|
| *Distance (m)* | | | | |

   (c) Graph the data for Jodie and Pedro on a Cartesian plane.




   (d) How far will Pedro have to run before he catches up to Jodie?

   ______________________________________________

   (e) How many seconds will it take Pedro to catch up to Jodie? _____

Obj. 13.5.1

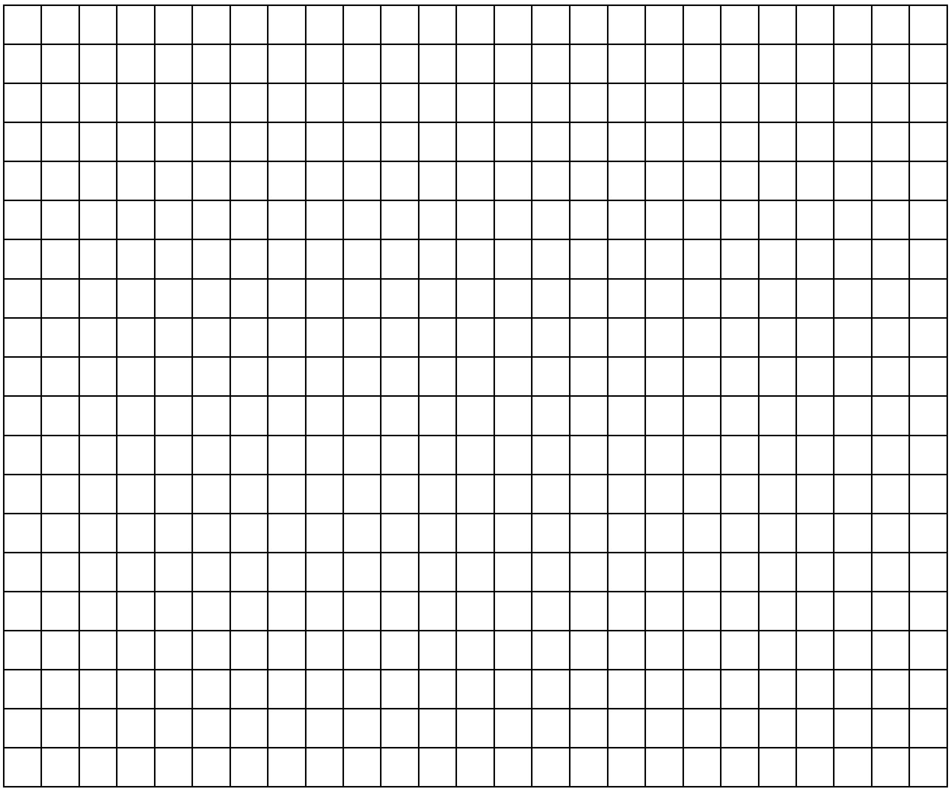
a) i ☐
ii ☐
iii ☐
iv ☐
b) i ☐
ii ☐
iii ☐
iv ☐
c) i ☐
ii ☐
iii ☐
iv ☐
d) ☐
e) ☐

2. The following table gives the weight (1000kg) and fuel efficiency (km/L) for a sample of ten cars.

| **Car** | **Weight (1000 kg)** | **Fuel efficiency (km/L)** |
|---|---|---|
| Chevrolet Camaro | 1.6 | 13 |
| Honda Accord | 1.5 | 13 |
| Lincoln Continental | 1.8 | 10 |
| Pontiac Grand Am | 1.4 | 13 |
| BMW 3-Series | 1.5 | 12 |
| Honda Civic | 1.1 | 16 |
| Toyota Camry | 1.5 | 14 |
| Hyundai Accent | 1.0 | 16 |
| Mazda Protege | 1.1 | 15 |
| Cadillac DeVille | 1.8 | 11 |

(a) Graph the weight of the cars against the fuel efficiency.

Weight
(1000 kg)



Fuel efficiency (km/L)

(b) Does there seem to be a relationship between the weight of a car and its fuel efficiency? ___________________________________

(c) Draw a line of best fit to describe this relationship.

(d) From your graph, predict the fuel efficiency of a car that weighs 2000 kg. _______________________________________________

Obj.
13.5.2
a) i □
ii □
iii □□□
□□□
□□□
□
b) □
c) □
d) □

# Cycle 5: Modelling Real Data with Linear Relationships

## Overview

### Big Idea

Identifying linear relationships from real data requires some interpretation and often allowance for possible outliers in the data due to variance. Correlation and lines of best fit enable “messy” real-world correlation to be considered and predicted mathematically. Real data modelled using linear relationships can also be used to solve real-world problems to predict points of time and locations where collisions may occur.

### Objectives

By the end of this cycle, students should be able to:

13.5.1 Use graphical methods to model real world problems. [8NA194]

13.5.2 Analyse and identify correlation between variables in simple linear relationships. [8NA194]

### Conceptual Links

Determining correlation between variables requires students to be able to identify variables and generate dot plots of data sets on graph paper. In order to determine a possible equation for the relationship between the variables, students will need to apply their understanding of gradient and y-intercept.

This cycle extends students’ understandings of dot plots to recognition of when a general linear trend is evident and the resulting line of best fit that may be applied in order to make predictions from the correlation indicated by the data.

### Materials

For Cycle 5 you may need:

- maths mat or large grid
- 1 cm grid paper
- string, rope or elastic
- grid paper

### Key Language

Correlation, line of best fit, average, intersection, intercepts

### Definitions

*Correlation:* identified relationship between variables.

*Line of best fit:* a line on a graph showing the general direction that a group of points seem to be heading.

## Assessment

### *Anecdotal Evidence*

Some possible prompting questions:

- What are the variables you are graphing?
- Which axis is which variable?
- How are the variables related?
- What are the units of measure for each variable?
- Is there a consistent relationship between the variables (does the graph have a straight line or close to a straight line)?
- If you look at the slope of the line, what does it tell you about the relationship between the variables? (For example, in a distance vs time running graph, a steeper slope suggests that students were running faster (more metres/second) and a straight horizontal line indicates no movement.)

### *Portfolio Task*

This cycle extends students' available skill set towards completion of the student portfolio task *P13: YouTube Investigation*.

# RAMR Cycle

## Reality

Explore instances where it may be feasible to test for correlation or a linear relationship. Examples may include height and arm span or arm length, height and foot length. Discuss other relationships that may or may not be correlated. For example, heights and eye colour should show no correlation as these are not related whereas heights and other body measures may be. Consider the variables list from *Resource 13.5.1: Census at School sample*. Ensure that students recognise which variables may be related and which variables may not be related.

## Abstraction

The abstraction sequence for this cycle starts from students' previous experiences of graphical representations of linear relationships with unknown amounts, and extends their thinking to estimation of linear correlation between variables using real data. A suggested sequence of activities is as follows:

1. *Kinaesthetic*. Select a pair of variables from *Resource 13.5.1: Census at School sample* to explore with students. For example, height and length of right foot. Ask students to generate a Cartesian plane on the floor or Maths Mat with axes, labels and scale. Determine heights of students and their foot lengths and record these as coordinates on a sticky note with their name to place on the graph. Ask students to stand on the graph if possible or place their sticky note coordinates on the graph in place. Discuss the trend in the data points. Highlight the trend using a length of string or elastic on the large graph. Discuss with students the best placement of the line to best capture the trend in the data. Connect to the language of "line of best fit".
2. *Record mathematically.* Ask each student to generate a paper copy of the graph on graph paper.
3. *Extrapolate from the data.* Use the graph to analyse the data. For example, find what the foot length of a person taller than the taller person might be, predict what the foot length of a person 133 cm tall might be.
4. *Generate an equation for the data.* Ask students to determine the gradient and y-intercept for the data set. Substitute possible heights of students into the equation to determine their possible foot lengths.
5. *Repeat with variables that have no correlation.* Repeat the activity steps using data that should not show a linear relationship. For example, arm span and eye colour. Highlight the difference between correlated data that shows a trend and data where there is no relationship.

***Resource*** Resource 13.5.1 Census at School sample
Resource 13.5.2 Grid paper

## Mathematics

### *Language/symbols and practice*

***Practice with further data sets***

Use the data set in *Resource 13.5.1: Census at School sample* to repeat the activity from the Abstraction phase. Compare the lines of best fit of both graphs and the equations for each. If the lines are different, contrast the equations and discuss what the differing gradients and y-intercepts mean for each sample.

Select other data sets that can be graphed and analysed. For example, temperature and rainfall for a summer month, temperature and rainfall for a winter month, boys' heights and foot lengths, girls' heights and foot lengths, boys' heights and arm spans, girls' heights and arm spans, football players years of competition and numbers of goals scored, basketball players years of competition and baskets scored, or any ideas that will suit the interests of your students. These examples may or may not be correlated.

***Practice with location and movement***

Return to location and movement examples as explored in Cycle 1. There is a range of problem types that may be considered including:

- The point of intersection of two paths with different starting locations and different directions of movement.
- The point of intersection of two paths with same starting locations but different times of start and different speeds of movement.
- The point of intersection of two paths with same start time, different starting locations, different directions of movement and same speed.
- The point of intersection of two paths with same start time, different starting locations, different directions of movement and different speed.

These examples can all be modelled using a Cartesian plane and the intersection of graphs located or equations for graphs determined and solved simultaneously.

## Reflection

### *Apply the idea*

***Explore fill rates and containers***

Explore examples with water flow rates and same or different size buckets. Model when the buckets will hold the same amount of water and/or which will overflow first.

Alternatively, explore examples of different sized water tanks leaking at the same rate. At which point will both water tanks hold the same fraction of their capacity.

### *Extend the idea*

Return to some of the more complex graphs from the Reflection phase of Cycle 1. Explore the creation of equations for each section of a complex line graph. Use the example to extrapolate sections of the graph to predict rates or locations if the change point did not happen. For example, the gradient of the heart rate graph should be steady, increase, and then decrease at a steady rate. Explore what the heart rate would have been had it continued to increase at the same rate instead of slowing with the cessation of exercise. Predict how long it might take to return to equilibrium from this increased state. (Arguably the continued increase in heart rate may not be physically possible but it is useful for theoretical consideration and the extrapolation of data.)

## *Teacher Reflective Notes*

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

# *Unit 13 Portfolio Task – Teacher Guide*

## *YouTube Investigation*

**Content Strand/s:** Number and algebra

**Resources Supplied:**
- Task sheet
- Teacher guide

**Other Resources Needed:**
- None

**Summary:**
Students work in the context of videos on YouTube and calculate which video was on average most popular. Students then calculate the cost and profit of uploading videos to YouTube with a sponsor and if this is a viable financial enterprise.

**Variations:**
- Find current popular YouTube videos and estimate the view rate across 3 months.

**ACARA Proficiencies Addressed:**
Understanding
Fluency
Problem Solving
Reasoning

**Content Strands:**

*Number and algebra*

13.1.1 Create displays of relationships between variables using line graphs. [7NA178]

13.1.2 Investigate, interpret and analyse graphs of authentic data. [7NA180]

13.2.1 Represents and interprets 'best buys' as rates. [7NA174]

13.2.4 Calculates unit rate (e.g., 120km in 2 hrs = average speed of 60km/hr). [8NA188]

13.2.7 Solve rate problems by extrapolation or interpolation of a line graph. [8NA194]

13.3.1 Generate a proportion table to solve rate problems involving direct proportion and a constant. [8NA193]

# *YouTube Investigation*

| | |
|---|---|
| **Name** | |
| **Teacher** | |
| **Class** | |

**Your Task:** Investigate how money is earned from YouTube

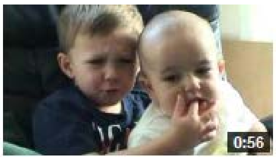
Within Portfolio Task 13, your work demonstrated the following characteristics:

| | | | A | B | C | D | E |
|---|---|---|---|---|---|---|---|
| Understanding and Fluency | Mathematical language and symbols | 13.2.7 Solve rate problems by extrapolation or interpolation of a line graph. | Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols | Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols | Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols | Use of aspects of mathematical terminology, diagrams and symbols | Use of everyday language |
| Problem Solving and Reasoning | Problem solving approaches | 13.2.1 Represents and interprets 'best buys' as rates. 13.2.4 Calculates unit rate. | Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar | Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations | Application of problem-solving approaches to investigate simple familiar situations | Some selection and application of problem-solving approaches in simple familiar situations. | Partial selection of problem-solving approaches |
| | Mathematical modelling | 13.1.1 Create displays of relationships between variables using line graphs. 13.3.1 Generate a proportion table to solve rate problems involving direct proportion and a constant. | Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar | Development of mathematical models and representations in complex familiar or simple unfamiliar situations | Development of mathematical models and representations in simple familiar situations | Statements about simple mathematical models and representations | Isolated statements about given mathematical models and representations |
| | Reasoning and justification | 13.1.2 Investigate, interpret and analyse relationships from graphs of authentic data. | Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached | Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached | Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached | Statements about choices made, strategies used and conclusions reached | Isolated statements about given strategies or conclusions |

Comments:

1. Below are four videos from YouTube that have become popular. Calculate the views <u>per **month**</u> for each of the videos below and make a decision about which video is the closest to being a YouTube sensation.

(a)




**Charlie bit my finger - again !**
by HDCYT • 7 years ago • 769,884,123 views
Charlie bit my finger - again ! Subscribe here: http://bit.ly/1oHVuUM Even had I thought of trying to get my boys to do this I probably ...

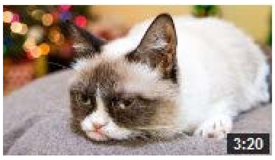
(b)


**Psycho Dad Shreds Video Games**
by McJuggerNuggets • 1 month ago • 11,633,071 views
An angry father runs over his son's video game collection with a lawn mower.
HD

(c)


**Mutant Giant Spider Dog (SA Wardega)**
by SA Wardega • 1 week ago • 87,629,139 views
FACEBOOK: http://www.facebook.com/sawardega SUBSCRIBE: http://www.youtube.com/user/wardegasa?sub_confirmation=1.
OFFICIAL HD

(d)




**Grumpy Cat Stars in "Hard To Be a Cat at Christmas" Music Video**
by Purina Friskies • 9 months ago • 3,028,495 views
Grumpy Cat, Colonel Meow, Oskar the Blind Cat, Nala Cat, and Hamilton the Hipster Cat have come together in an ...
HD

2. Graph the views of these videos assuming that the same view rate per month was maintained. Extend the graph to 6 months of time.

3. How many views might the 'Charlie bit my finger-again' video have at 5 months?
______________________________________________________________

4. How does this compare to the cat video? ______________________________

5. Generate an equation for the viewing rate for each graph using the gradients and y-intercepts.

a) ______________________________________________

b) ______________________________________________

c) ______________________________________________

d) ______________________________________________

YouTubepreneurs, the people who make money from creating videos for YouTube, are paid based on the number of views they receive. You've recently developed a small group of fans on YouTube and have a sponsor who is paying you at a rate of $1.50 per 1120 views.

6. Your video gets 64,953 views in one month, how much income does that generate?

   *(Round your answer to two decimal places*.)

If you uploaded 5 videos in one month each 0.75GB in size.

7. What is the total amount of data you used (in GB)

8. You have a data plan that costs $7.30 per Gigabyte, with no data limit, using this device how much did it cost to upload the five videos?

9. The monthly expenses that you have include:

   Movie-making software - $15 per month

   Internet – cost calculated in question 8

   Based on these expenses, and the income you calculated in Question 6 for each video, how much profit or loss did you make for this month?

10. a) If you made and uploaded 8 movies per month, how much income would you make for the month?

    b) If you made and uploaded 8 movies per month, what would your expenses be for the month?

    c) Based on these expenses and income, how much profit or loss would you make for the month? ____________________

11. Would you continue to make movies for YouTube to earn money? Why/Why not?

    .____________________

    .____________________

Name:________________________________________ Date:________

# *Can you do this now? Unit 13*

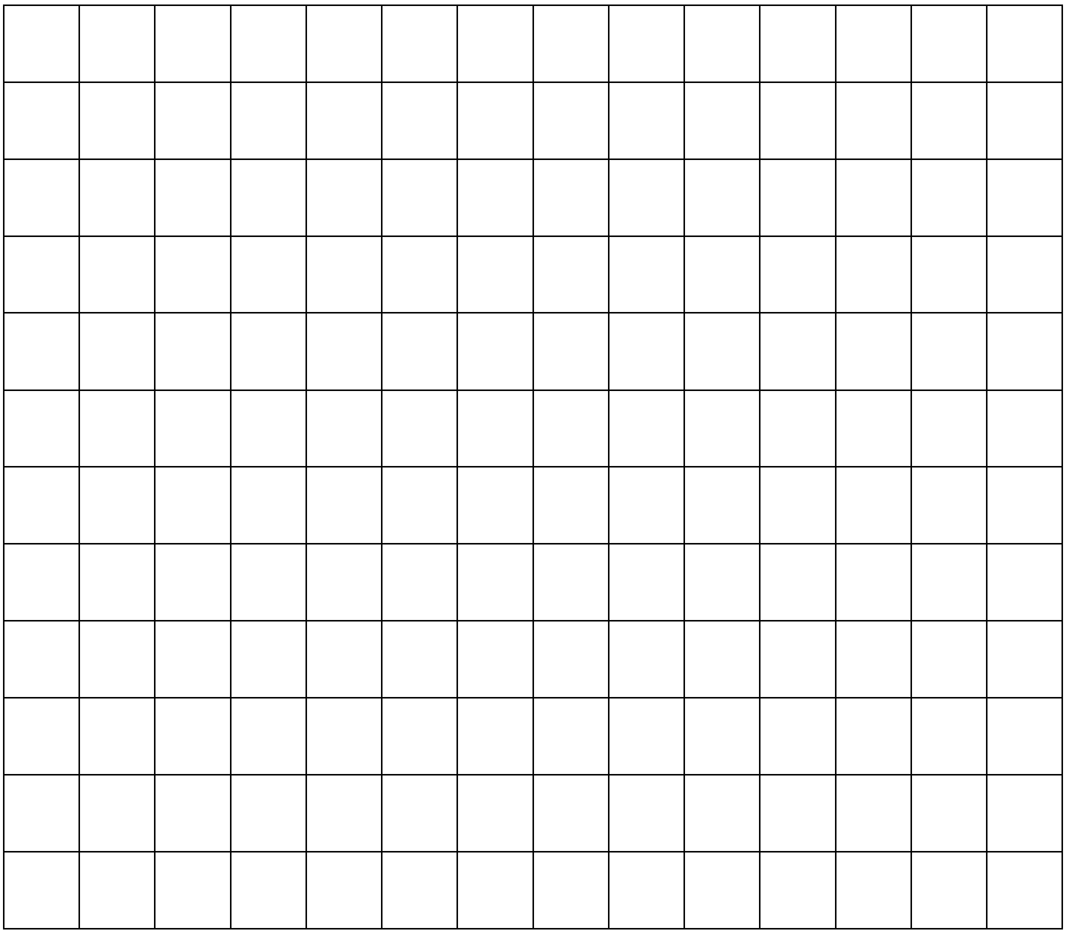
1. The following table of data represents the temperature at various times of the day.

| ***Time*** | 6:00 am | 9:00 am | 12:00 pm | 3:00 pm | 6:00 pm |
|---|---|---|---|---|---|
| ***Temperature*** | 12°C | 17°C | 21°C | 19°C | 14°C |

(a) Label the axes and scales on the Cartesian plane.

(b) Plot each point.

(c) Draw a line through the plotted points.



(d) Mark on the graph at approximately what time the in the morning the temperature was 19°C. ______________________________

(e) What was the approximate temperature at 5:00pm? _________

Obj.
13.1.1
a)i ☐
ii ☐
iii ☐
iv ☐

b)i ☐
ii ☐
iii ☐
iv ☐
v ☐
c) ☐

Obj.
13.1.2
d) ☐
e)i ☐
e)ii ☐

2. Bottles of soft drink contain 1.25 L. A bottle of soft drink costs $1.05.

   (a) Write this relationship as a rate comparing price per litre ($/L).

   ______________________________________________

   (b) Use a calculator to find the unit price per litre.

   ______________________________________________

Obj. 13.2.1
a)i ☐
ii ☐

Obj. 13.2.4
b) ☐

3. Apple and pear juice is blended in the ratio of 4 parts apple juice to 3 parts pear juice (4:3).

   (a) Write this as a rate of parts apple juice per parts pear juice.

   ______________________________________________

   (b) Write this relationship as a unit rate of apple juice/pear juice.

   ______________________________________________

Obj. 13.2.2
a)i ☐
ii ☐

Obj. 13.2.4
b) ☐

4. A factory packs apples in bags. There are 6 apples per bag.

   (a) How many apples in 12 bags? ______________________

   (b) How many bags are needed to pack 114 apples? __________

Obj. 13.2.3
a)i ☐
ii ☐
b)i ☐
ii ☐

5. Phone calls to mobile phones are charged at a rate of $1.72 per minute.

   (a) Complete the proportion table for call costs.

| Cost | $1.72 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Minutes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

   (b) What is the cost of a call which takes 9 minutes? __________

   (c) If a phone call cost $17.20, how many minutes did the call last for?

   ______________________________________________

Obj. 13.2.5
a)i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐
b) ☐
c) ☐

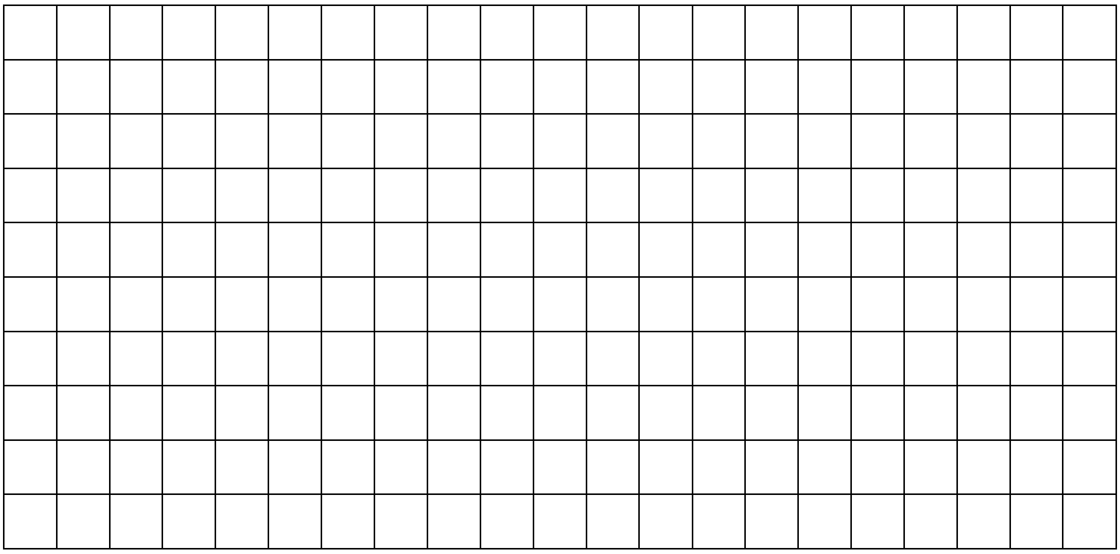
6. The following proportion table represents money earned at the rate of $7.50 per hour.

| ***Time (hours)*** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ***Earnings ($)*** | 7.50 | 15 | 22.50 | 30 | 37.50 |

(a) Label the axes and scales on the Cartesian plane.

(b) Plot each point.

(c) Draw a line through the plotted points.



(d) Extend the graph to 10 hours worked.

(e) Mark 7 hours of work on the graph. Write down how much had been earned at 7 hours worked? ______________________

(f) Choose two coordinate pairs from the graph and write them down. i. ____________________ ii. ____________________

(g) Calculate the change in the x-axis between the two coordinate pairs. ______________________

(h) Calculate the change in the y-axis between the two coordinate pairs. ______________________

(i) Calculate the gradient of the line using the change in the y-axis and the change in the x-axis. ______________________

(j) Write an equation for this line. ______________________

Obj. 13.2.6
a)i □
ii □
iii □
iv □
b)i □
ii □
iii □
iv □
v □
c) □

Obj. 13.2.7
d) □
e)i □
ii □

Obj. 13.2.10
f)i □ □
ii □ □

Obj. 13.2.8
g)i □
ii □

Obj. 13.2.9
h)i □
ii □

Obj. 13.2.10
i)i □
ii □

Obj. 13.2.11
j)i □
ii □

7. Create a table of values for the equation $y = 5x$.

| | | | | | |
|---|---|---|---|---|---|
| ***x*** | | | | | |
| ***y*** | | | | | |

8. You have \$9 saved and earn \$7 per hour, how many hours will it take to earn enough to buy a \$72 phone?

   (a) Complete the table of values.

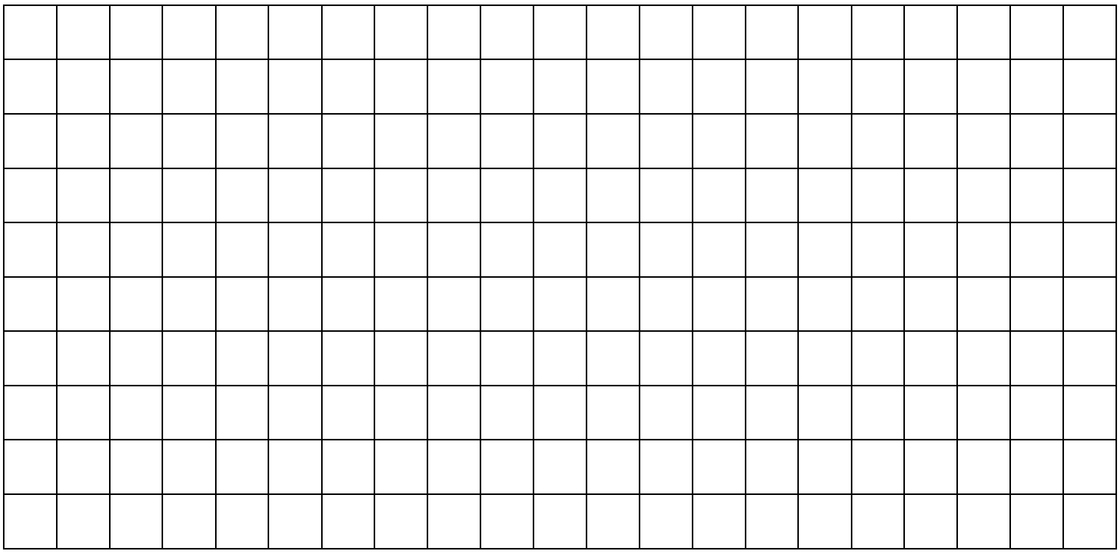
| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| Total (\$) | | | | | | |
| Time (h) | | | | | | |

   (b) Write down the rule for calculating each total (\$). ____________

   (c) Calculate how many hours it will take to earn \$72? ___________

   (d) Graph this relationship on the Cartesian plane.

   (e) Mark on the Cartesian plane where you have earned \$72.



   (f) Your cousin also has \$9 saved but only earns \$6 per hour. Write down the rule for calculating total earnings after each hour for your cousin. ____________________________________________

   (g) Graph this relationship on the Cartesian plane.

   (h) Mark on the Cartesian plane where your cousin has earned \$72.

Obj. 13.2.12
i □ □
ii □ □
iii □ □
iv □ □
v □ □

Obj. 13.3.1
a)i □ □
ii □ □
iii □ □
iv □ □
v □ □
vi □ □

Obj. 13.3.3
b)i □
ii □

Obj. 13.2.3
c) i □
ii □

Obj. 13.2.6
d)i □ □
ii □ □
iii □ □
iv □ □
v □ □

Obj. 13.3.4
e) □

Obj. 13.3.3
f)i □
ii □

Obj. 13.2.6
g)i □ □
ii □ □
iii □

Obj. 13.3.4
h) □

9. The points on a line are recorded in a table.

(a) Plot each point on the Cartesian plane.

(b) Draw a line through the plotted points.

| | | | | |
|---|---|---|---|---|
| *y* | 3 | 5 | 7 | 9 |
| *x* | 0 | 1 | 2 | 3 |

(c) What is the gradient? ______________________

(d) What is the y-intercept? ______________________

(e) Write the equation for this line. ______________________

(f) Extend the graph to find the value of y if x = 6. ______________

Obj. 13.2.6
a)i □
ii □
iii □ □
iv □ □
v □ □
vi □ □
b) □

Obj. 13.3.2
c)i □
ii □

Obj. 13.3.2
d) □ □

Obj. 13.3.3
e)i □
ii □

Obj. 13.3.4
f) □

10. A shop sells two different types of marble in bags: red and blue. Bag A containing two red marbles and three blue marbles weighs 65 g. Bag B containing four red marbles and one blue marble weighs 75 g.

(a) Form an equation that describes Bag A. ____________________

(b) Complete the table of values for Bag A.

| | | |
|---|---|---|
| *Red (r)* | 10 | 16 |
| *Blue (b)* | | |

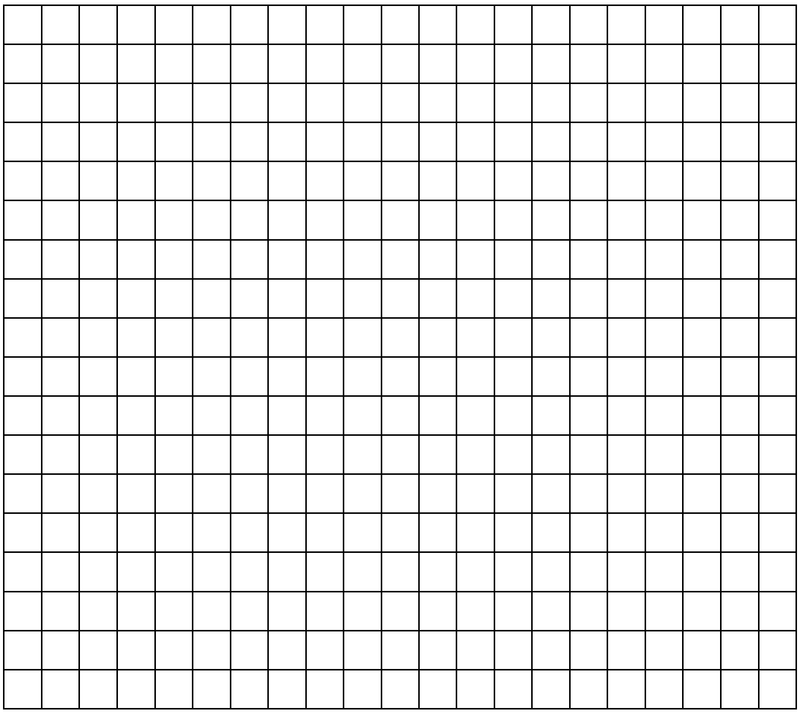
(c) Form an equation that describes Bag B. ____________________

(d) Complete the table of values for Bag B.

| | | |
|---|---|---|
| *Red (r)* | 10 | 16 |
| *Blue (b)* | | |

(e) Plot these lines on the Cartesian plane.

red



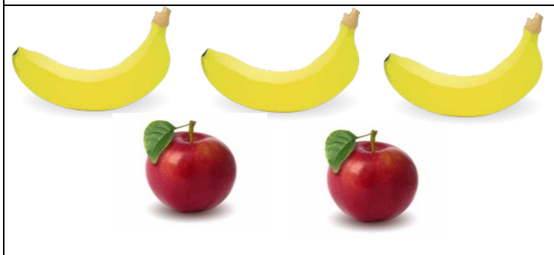
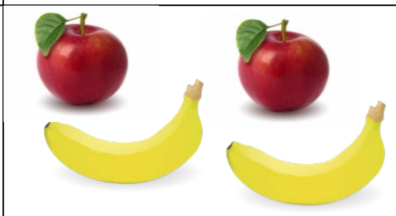
blue

(f) What are the weights of the marbles where the lines cross?

Red ____________________ Blue ____________________

Obj.
13.4.1
a) i □
ii □
iii □
b) i □
ii □
c) i □
ii □
iii □
d) i □
ii □
e) i □
ii □
iii □
iv □
f) i □
ii □

11.Two people bought fruit at the shop.

| Person A | Person B |
| --- | --- |
|  |  |
| 128 cents | 96 cents |

(a) Form an equation for Person A. ______________________

(b) Form an equation for Person B. ______________________

(c) Solve simultaneously to find the cost of a banana.

______________________

______________________

______________________

______________________

(d) Calculate the cost of an apple.

______________________

______________________

______________________

______________________

______________________

______________________

Obj. 13.4.2
a)i ☐
ii ☐
iii ☐
b)i ☐
ii ☐
iii ☐
c)i ☐
ii ☐
iii ☐
d)i ☐
ii ☐
iii ☐

12. Jodie and Pedro are going to race each other. Jodie can run 4 metres per second, whereas Pedro can run 6 metres per second. Because she is slower, Pedro gives Jodie a head start of 14 m.

(a) Complete the table of values for how far from the start and how long Jodie runs.

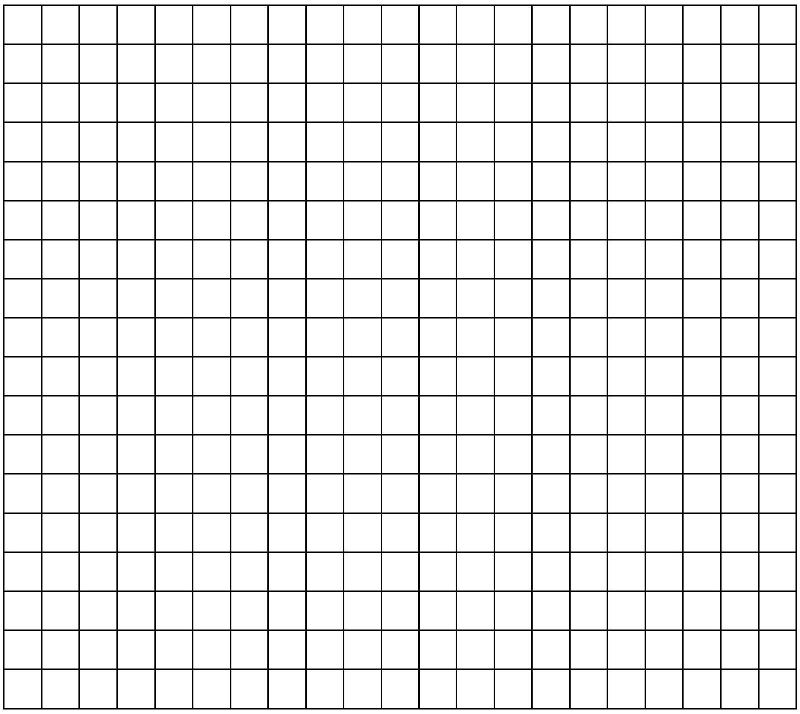
| *Time (s)* | 0 | 1 | 5 | 10 |
|---|---|---|---|---|
| *Distance (m)* | | | | |

(b) Complete the table of values for how far from the start and how long Pedro runs.

| *Time (s)* | 0 | 1 | 5 | 10 |
|---|---|---|---|---|
| *Distance (m)* | | | | |

(c) Graph the data for Jodie and Pedro on a Cartesian plane.

Distance (m)



Time (s)

(d) How far will Pedro have to run before he catches up to Jodie?

________________________________________

(e) How many seconds will it take Pedro to catch up to Jodie? _____

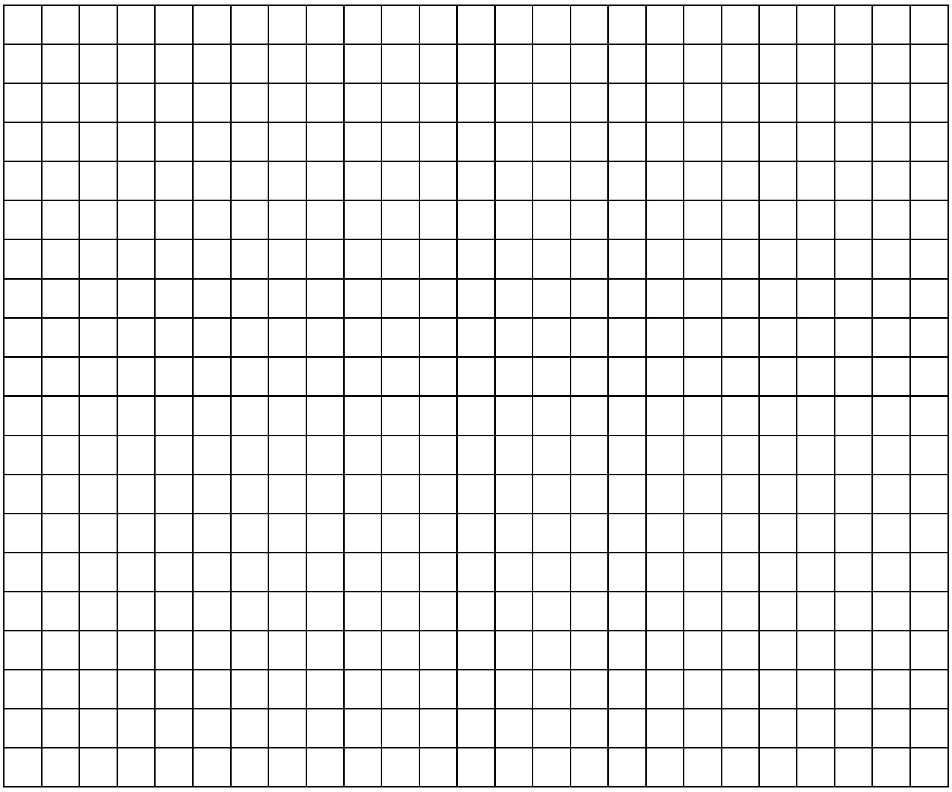
Obj. 13.5.1
a) i □
ii □
iii □
iv □
b) i □
ii □
iii □
iv □
c) i □
ii □
iii □
iv □
d) □
e) □

13. The following table gives the weight (1000kg) and fuel efficiency (km/L) for a sample of ten cars.

| **Car** | **Weight (1000 kg)** | **Fuel efficiency (km/L)** |
|---|---|---|
| Chevrolet Camaro | 1.6 | 13 |
| Honda Accord | 1.5 | 13 |
| Lincoln Continental | 1.8 | 10 |
| Pontiac Grand Am | 1.4 | 13 |
| BMW 3-Series | 1.5 | 12 |
| Honda Civic | 1.1 | 16 |
| Toyota Camry | 1.5 | 14 |
| Hyundai Accent | 1.0 | 16 |
| Mazda Protege | 1.1 | 15 |
| Cadillac DeVille | 1.8 | 11 |

(a) Graph the weight of the cars against the fuel efficiency.

Weight
(1000 kg)



Fuel efficiency (km/L)

(b) Does there seem to be a relationship between the weight of a car and its fuel efficiency? ____________________________________

(c) Draw a line of best fit to describe this relationship.

(d) From your graph, predict the fuel efficiency of a car that weighs 2000 kg. ______________________________________________

Obj.
13.5.2
a) i □
ii □
iii □□□
□□□
□□□
□
b) □
c) □
d) □

XLR8 – Accelerating Mathematics Learning

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