



## **XLR8 Unit 12**

# **Enlarging maps and plans**

**2016**

## ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <http://ydc.qut.edu.au> and staff can be contacted at [ydc@qut.edu.au](mailto:ydc@qut.edu.au).

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# XLR8 Program: Scope and Sequence

	2 year program	3 year program
<b>Unit 01: Comparing, counting and representing quantity</b> Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
<b>Unit 02: Additive change of quantities</b> Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
<b>Unit 03: Multiplicative change of quantities</b> Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
<b>Unit 04: Investigating, measuring and changing shapes</b> Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
<b>Unit 05: Dealing with remainders</b> Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
<b>Unit 06: Operations with fractions and decimals</b> Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
<b>Unit 07: Percentages</b> Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
<b>Unit 08: Calculating coverage</b> Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
<b>Unit 09: Measuring and maintaining ratios of quantities</b> Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
<b>Unit 10: Summarising data with statistics</b> Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
<b>Unit 11: Describing location and movement</b> Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
<b>Unit 12: Enlarging maps and plans</b> Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
<b>Unit 13: Modelling with linear relationships</b> Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
<b>Unit 14: Volume of 3D objects</b> Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
<b>Unit 15: Extended probability</b> Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3



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# Overview

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## Context

In this unit, students will develop their ability to describe proportional relationships between quantities of discrete items or measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.

## Scope

This unit is based upon **proportional relationships** between **measures** described using **ratio notation**. **Proportional relationships** using **ratios** describe **part to part relationships** (fractions describe part to whole relationships).

Ratio in measurement contexts may be used to create **similar figures, scale drawings, maps and plans**. Again, the **relationships between the parts within shapes** are maintained while **scaling** shapes up or down. **Similar figures**, there is a consistent relationship between the lengths of corresponding sides of the shape. **Similar shapes** have the **same sized angles** while **lengths of sides enlarge or reduce** by a **scale factor**. **Ratios** between **similar triangles** can be extended to exploring **ratios within right angle triangles** to develop the **trigonometric relationships** of **sine, cosine and tangent**.

Common strategies to scaffold **proportional thinking** include **proportion tables, within and between strategies**, and **dual-scale number lines**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts **developed** in this unit is highlighted in **blue**, concepts that may be **connected to and reinforced** are highlighted in **green** and number and algebra concepts and processes applied within this area are highlighted in black.

## Assessment

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed after teaching the unit. A comparison of the outcomes from the summative and diagnostic worksheets provides a measure of student improvement.
- *Portfolio task:* The portfolio task *P12: Measuring Shapes* accompanying Unit 12 engages students using scale drawings and ratio relationships to determine the real dimensions of aircraft, compare lengths within triangles and investigate image projection.

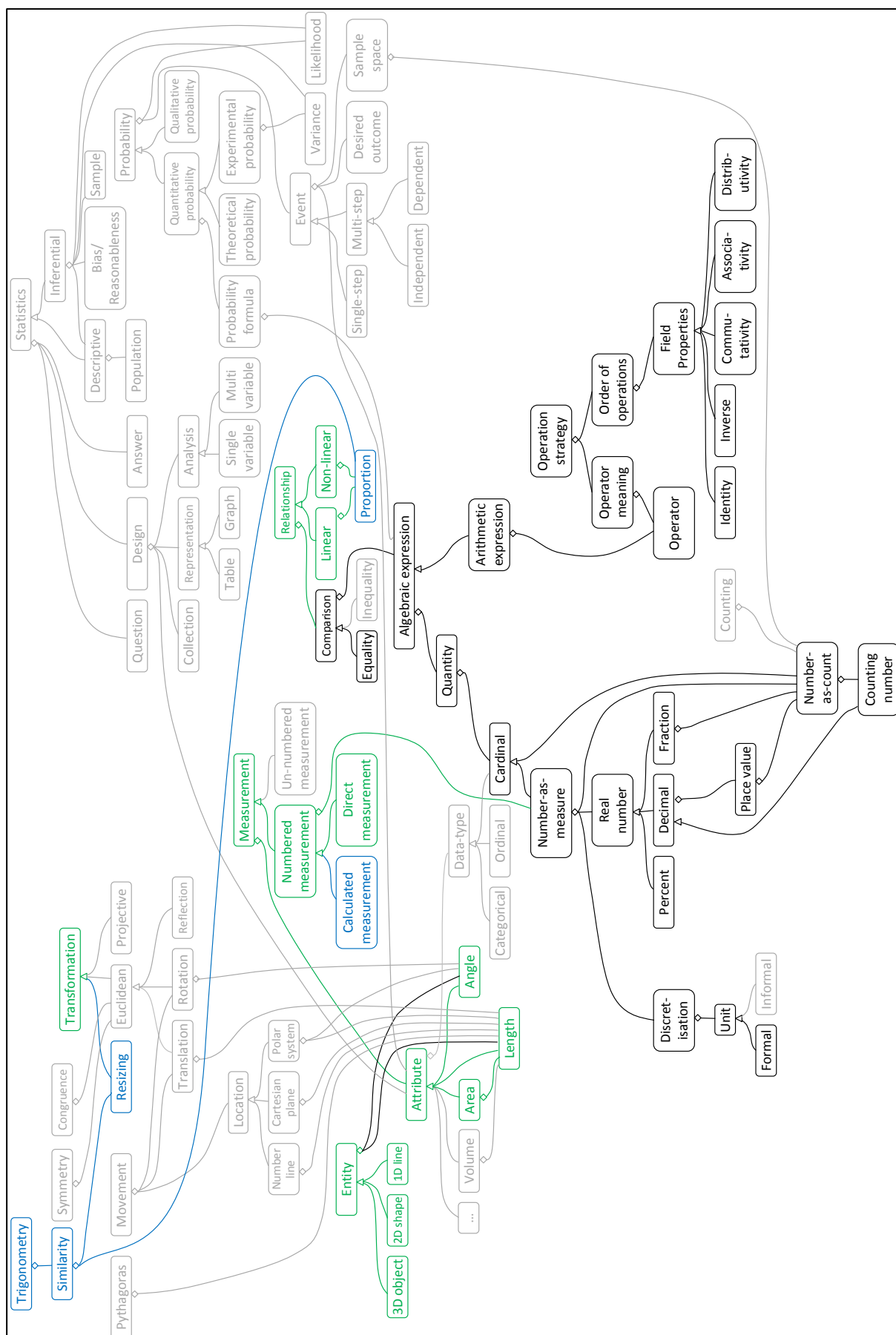


Figure 1. Scope of Unit 12



## Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

### Cycle 1: Scales and scale factors

Ratio notation is useful to describe the scale used to reduce full size constructions and objects to plan drawings and maps that provide a smaller overview of objects and their dimensions. These plans and maps serve to facilitate the planning process. Checking details on a small scale model assists planners to better prepare for the challenges and expenses of a full-scale project. Alternatively, large scales serve to elucidate items that are too small for the human eye to perceive. Unit ratio is frequently used to describe the scales of maps and plans. Scale and ratio can also be used for the comparison of dimensions in graphs of physical objects.

### Cycle 2: Ratio and angle relationships between and within similar shapes

Enlargement and reduction of shapes to create similar figures requires all dimensions of the shape to be transformed by the same multiplier. The ratio between corresponding sides on similar shapes can be described using ratio notation and will be consistent across the shapes. Angles in similar shapes remain constant when enlargement and reduction transformations are performed. This cycle will explore the use of ratio relationships to determine similarity of shapes.

### Cycle 3: Ratio relationships on the Cartesian plane

Scaling on the Cartesian plane is similar to enlarging and reducing shapes using grid squares. However, instead of changing the grid size, enlargement and reduction on the Cartesian plane requires multiplicative change to the coordinates of each point, or to the length of the lines drawn, meanwhile, the angles between lines must remain constant.

### Cycle 4: Trigonometric ratios

Right-angle triangles (right triangles) form an interesting subclass of similar shapes. Because one angle is  $90^\circ$ , determination of the angle at one other corner determines all angles; all right triangles of the same first angle are similar (e.g., a right triangle with an angle of  $30^\circ$  is similar to all other right triangles with an angle of  $30^\circ$  no matter what the size). Because of the nature of proportion, this means that sides are also in ratio within a triangle.

### Cycle 5: Ratio relationships between areas of similar shapes

In the previous cycles, scale factor has been used to describe relationships between measures in one dimension of similar figures. This results in a linear, multiplicative relationship between corresponding dimensions in similar figures. As perimeter and circumference are linear measures, the multiplicative relationship between the corresponding dimensions of similar figures will also apply. Scale can also be applied to enlarging and reducing shapes and figures in more than one dimension. However, like conversion between area measures, this relationship is not linear. That is, if both dimensions of an area are doubled, the resulting area will be multiplied by 4.

### Notes on Cycle Sequence:

The proposed cycle sequence may be completed as outlined above. However, *Cycle 4: Trigonometric Ratios* can be addressed before *Cycle 3: Ratio Relationships Within Shapes*.

## Literacy Development

The consistent use of mathematical language is essential to the development of number and operation concepts. This language is integral to the expression of mathematical concepts at varying levels of representational abstraction (from concrete-enactive through to symbolic). In this unit, the following key language should be explicitly developed with students ensuring that they understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

### **Cycle 1: Scales and scale factors**

Scale, scale factor, ratio, enlarging, reducing, magnifying, zooming in, zooming out, added detail, plan, map, model, scale drawing

### **Cycle 2: Ratio and angle relationships between similar shapes**

Projection, parallel rays of light, enlargement, reduction, similar shapes, similarity, ratio, scale, scale factor, angle, opposite sides, corresponding sides, circles, radius, diameter,  $\pi$ ,  $\pi$

### **Cycle 3: Ratio relationships on the Cartesian plane**

Coordinates, Cartesian plane, x-axis, y-axis, x-coordinate, y-coordinate, scale, scale factor, ratio

### **Cycle 4: Trigonometric ratios**

Ratio, similar triangles, ratio, angle, tangent, sine, cosine, tan, sin, cos, proportion, opposite, adjacent, hypotenuse

### **Cycle 5: Ratio relationships between areas of similar shapes**

Scale, scale factor, ratio, enlarging, reducing, area, two dimensions, squared, circles, radius, diameter,  $\pi$ ,  $\pi$ , circumference

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## *Can you do this? #1*

The rectangle is a scale drawing of a large block of land. It has been drawn at the scale of 1 : 100 000.

1. Measure the length and width of the rectangle in centimetres.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

2. Write down the scale factor. \_\_\_\_\_

3. Calculate the real-life dimensions of the block of land in centimetres.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. Convert the measures to metres and write them on the diagram.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

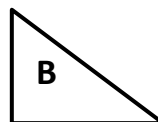
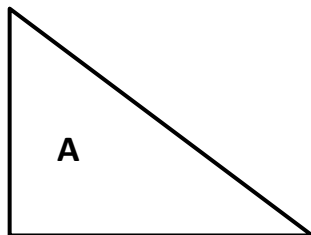
\_\_\_\_\_

(a) \_\_\_\_\_



(b) \_\_\_\_\_

- 5.



Triangle A is twice as long and twice as high as triangle B.

- (a) Write down the scale factor used to transform triangle B to triangle A? \_\_\_\_\_

- (b) Write down the scale of triangle B : triangle A as a ratio.

\_\_\_\_\_ : \_\_\_\_\_

Obj.

12.1.3

1a)i ☐

a)ii ☐

1b)i ☐

b)ii ☐

Obj.

12.1.1

2 ☐

Obj.

12.1.3

3a)i ☐

a)ii ☐

a)iii ☐

3b)i ☐

b)ii ☐

b)iii ☐

4a)i ☐

a)ii ☐

a)iii ☐

4b)i ☐

b)ii ☐

b)iii ☐

Obj.

12.1.1

a) ☐

Obj.

12.1.2

b)i ☐

b)ii ☐

---

# Cycle 1: Scales and Scale Factors

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## Overview



### Big Idea

Ratio notation is useful to describe the scale used to reduce full size constructions and objects to plan drawings and maps that provide a smaller overview of objects and their dimensions. These plans and maps serve to facilitate the planning process. Checking details on a small scale model assists planners to better prepare for the challenges and expenses of a full-scale project. Alternatively, large scales serve to elucidate items that are too small for the human eye to perceive. Unit ratio is frequently used to describe the scales of maps and plans. Scale and ratio can also be used for the comparison of dimensions in graphs of physical objects.



### Objectives

By the end of this cycle, students should be able to:

12.1.1 Identify scale factors as quantities used to enlarge/reduce measures. [5MG115]

12.1.2 Represent scale and scale factors as ratios. [5MG115]

12.1.3 Apply the enlargement transformation to familiar two dimensional shapes. [5MG115]



### Conceptual Links

Scaling and scale factors extend on previous understandings of ratio and proportion explored with discrete items and mixtures. Part-part relationships, number, fraction, multiplication and division concepts are also reinforced through this cycle. Opportunities also exist to reinforce measurement and map interpretation skills and conversion between measurements.

This cycle consolidates the use of ratio to describe relationships between scaled shapes and figures by considering its use in the creation of plans and maps and scale models.



### Materials

For Cycle 1 you may need:

- measuring tape
- calculators
- 1 cm grid paper
- rulers
- paper



### Key Language

Scale, scale factor, ratio, enlarging, reducing, magnifying, zooming in, zooming out, added detail, plan, map, model, scale drawing, congruent



## **Definitions**

*Scale factor:* multiplier used to create a proportional change in the size of a shape.



## **Assessment**

### ***Anecdotal Evidence***

Some possible prompting questions:

- When you flip, slide or turn your shape, what stays the same/changes?
- What are the measurements you have?
- How many times do you want to enlarge/reduce your copy by?
- Do you need to multiply your measures by the scale factor or divide them?
- Do you have the same units of measure?
- Can you write the scaling relationship as a ratio from the original to the copy?
- Can you write the scaling relationship as a ratio from the copy to the original?

### ***Portfolio Task***

Skills developed throughout this cycle will contribute to the student portfolio task *P12: Measuring Shapes*. Task 1 engages students in simple application and interpretation of scale to a drawing of an aircraft. Tasks 2 and 3 also provide opportunities to reinforce student understanding of simple scale and ratio.

# RAMR Cycle



## Reality

Revisit Euclidean transformations of shapes (Unit 04, Unit 11). Discuss what properties of shapes stay the same and what properties change. Students should be able to identify that the orientation and position of the shape changes, while the side lengths and angles remain the same. Consolidate the concept of congruence.

Discuss other ways that shapes may be transformed (students should be able to identify projective change – enlargement or reduction). Ask students what properties of shapes/objects change/stay the same when they are enlarged or reduced. Differentiate between these similar shapes and congruent shapes. Consider how the relationship between the original shape/object and the copy are commonly represented.

Revisit ratio ideas from Unit 8 where students explored ratio and proportion with discrete items and mixtures. Ask students where else in reality they might find or see ratios. Search for examples of maps, plans, scale drawings and models that students may recognise. Connections to science may be useful if scale drawings of microscopic life forms are available. Discuss how relationships between reality and scaled copies are communicated using scale factors and ratios (e.g., 1:12, 1:100 000).



## Abstraction

The abstraction sequence for this cycle starts from students' previous experience with ratios of mixtures and extends these ideas to scale and scale factors for maps and plans. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity and model/represent with materials.* Ask students to make measurements and draw their face on a piece of paper (actual size). Discuss what they would need to do to create a scaled drawing of their face that is accurate. Students should recognise the need for all measures to be changed or scaled by the same amount. Ensure students understand the need to use multiplicative thinking (division or multiplication by a fraction) for scaling rather than additive thinking (take away or subtraction).
2. Complete the activity for drawing faces in *Resource 12.1.1 Scale drawings of people*. Have students redraw their face on another piece of paper using scaled measures to create a more accurate representation. Alternatively, *Resource 12.1.2 Simple shapes for scaling*, or *Resource 12.1.3 Superhero shield initials* may be of greater interest for students.
3. *Connect to language.* Discuss the accuracy of the scaled drawing compared to the initial drawing. Connect language to the scale of the representation (if the measures were halved, the scale will be 1:2). Discuss with students how measures were multiplied by a consistent scale factor to create the drawn representation.
4. Explicitly connect the ratio notation of 1:2 to the measures used. Ensure students understand that where the units of the initial item and the scale drawing are the same, they do not need to be listed in the ratio.
5. Explore map scale notations with and without including units of measures. Ensure students understand the need to convert to common units before expressing ratios without units.



## Resource

Resource 12.1.1 Scale drawings of people  
Resource 12.1.2 Simple shapes for scaling  
Resource 12.1.3 Superhero shield initials



## Mathematics



### **Language/symbols and practice**

Extend students' experience of scaling measures by exploring scale drawings for models. *Resource 12.1.4 Scale drawings for models*, uses a plan drawing of a Mini and doll house to connect the previous activity to specific scaling used in model dollhouses, cars and railways.



### **Resource** Resource 12.1.4 Scale drawings for models

Previous examples involve scaling reality down to a plan drawing. Explore enlarging a smaller item to a larger drawing to provide additional detail. *Resource 12.1.5 Zoom scaling* has some pictures from the book *Zoom* by Istvan Banyai. This shows an image as it is gradually zoomed out. Several Youtube versions are available (it is best to leave the sound off <http://youtu.be/lhYblhdhQ1M>).



### **Resource** Resource 12.1.5 Zoom scaling

This idea can be explored further by gradually zooming in on a local area map of the school. Start so that only the suburb is listed on the map and gradually increase the zoom until students can identify individual streets. Use the street view feature to really zoom in to see the front of the school. Discuss how the scale of each view might be communicated.



### **Connections**

Ratio and scaling of landmarks and features can also be used to generate comparative graphs. The activities in *Resource 12.1.6 Scale drawings as comparisons*, engage students in making scaling decisions in order to generate comparative graphs of building heights in Australia, and mountain heights and ocean depths. In each case, students use calculators, and the measures and scale factor provided to determine the drawn measures and to create graphs. This activity could be extended by finding suitable already created graphs and interpreting the actual measures of items using the scale provided.



### **Resource** Resource 12.1.6 Scale drawings as comparisons



## Reflection



### ***Check the idea***

Provide packets of grocery items in different sizes (e.g., single serve cereal packet and large size cereal packet). Ask students to measure key logos/characters on the packets to determine if these have been scaled up or down in proportion.

Engage student groups by asking them to measure their classroom and then ask them to create a scaled down drawing in their books.



### ***Apply the idea***

Use of scaling, measurement and proportion can be used in reverse to solve problems. *Resource 12.1.7 Scale and proportion problem*, investigates the possible height of a thief from the length of an incriminating footprint.



**Resource** Resource 12.1.7 Scale and proportion problem



### ***Extend the idea***

Engage students with creating their own sequence of Zoom pictures by scaling up a section of the picture to show added detail.



## ***Teacher Reflective Notes***

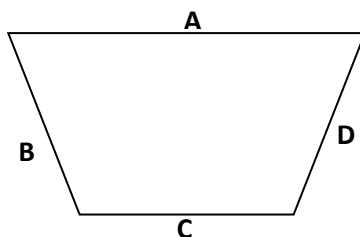
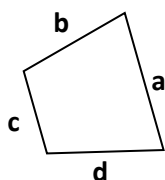
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Can you do this? #2



1. Measure and record the side lengths of each shape in millimetres.

A \_\_\_\_\_ B \_\_\_\_\_ C \_\_\_\_\_ D \_\_\_\_\_

a \_\_\_\_\_ b \_\_\_\_\_ c \_\_\_\_\_ d \_\_\_\_\_

2. Calculate the scale factor that has been used to enlarge the side lengths of the shapes. \_\_\_\_\_

3. Use your measurements to express the matching side lengths as ratios (small shape : large shape).

a : A = \_\_\_\_\_ : \_\_\_\_\_

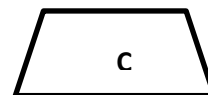
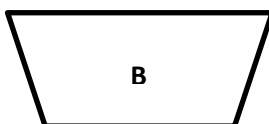
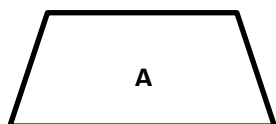
b : B = \_\_\_\_\_ : \_\_\_\_\_

c : C = \_\_\_\_\_ : \_\_\_\_\_

d : D = \_\_\_\_\_ : \_\_\_\_\_

4. Fill in the blanks to make these sentences true.

Shape A and shape B are congruent because they have \_\_\_\_\_ angles and matching \_\_\_\_\_. Shape C is \_\_\_\_\_ to shape A and shape B because its angles are \_\_\_\_\_ size but its \_\_\_\_\_ are different.

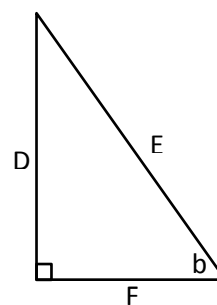
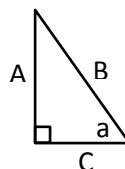


5. Fill in the blanks to make these sentences true.

Here are two similar right-angle triangles.

Angle a and angle b will be \_\_\_\_\_ size.

The ratio of A:D = B:\_\_\_\_\_ = \_\_\_\_\_:F.



Obj.  
12.2.1

1a)i ☐

ii ☐

1b)i ☐

ii ☐

1c)i ☐

ii ☐

1d)i ☐

ii ☐

2 ☐

3a)i ☐

ii ☐

3b)i ☐

ii ☐

3c)i ☐

ii ☐

3d)i ☐

ii ☐

Obj.  
12.2.2

i ☐

ii ☐

iii ☐

Obj.  
12.2.3

iv ☐

v ☐

vi ☐

Obj.  
12.2.4

i ☐

ii ☐

iii ☐

---

# Cycle 2: Ratio and Angle Relationships Between and Within Similar Shapes

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## Overview



### Big Idea

Enlargement and reduction of shapes to create similar figures requires all dimensions of the shape to be transformed by the same multiplier. The ratio between corresponding sides on similar shapes can be described using ratio notation and will be consistent across the shapes. Angles in similar shapes remain constant when enlargement and reduction transformations are performed. This cycle will explore the use of ratio relationships to determine similarity of shapes.



### Objectives

By the end of this cycle, students should be able to:

- 12.2.1 Explore the relationships between side lengths and angles of an enlarged/reduced image compared with the original to determine unknown scale factors. [5MG115]
- 12.2.2 Identify congruent shapes. [8MG201]
- 12.2.3 Identify similar shapes. [9MG200]
- 12.2.4 Identify the conditions for triangles to be similar. [9MG220]
- 12.2.5 Solve problems using similar shapes, ratio and scale factor. [9MG221]



### Conceptual Links

Ratio and angle relationships extend on previous understandings of ratio and proportion explored with discrete items, mixtures and scaled measures. Part-part relationships, number, fraction, multiplication and division concepts are also reinforced through this Cycle. Opportunities also exist to reinforce measurement of angles and angle relationships within shapes.

This cycle extends the previous cycle's work on ratio to explore its application to determine if shapes are similar and its use in enlarging and reducing shapes and figures. This cycle explores foundational concepts for future cycles where students will investigate plan drawings and the effects of enlargement and reduction transformations on perimeter and area of shapes.



### Materials

For Cycle 2 you may need:

- data projector and PowerPoint
- mirror
- grid papers (1 cm, 1.5 cm, 2 cm, 2.5 cm)
- cardboard shapes
- selection of circular lids, containers, wheels
- ruler
- protractor
- string
- geoboards and rubber bands
- torch



## **Key Language**

Projection, parallel rays of light, enlargement, reduction, similar shapes, similarity, ratio, scale, scale factor, angle, opposite sides, corresponding sides



## **Definitions**

*Similarity:* similar shapes have corresponding angles that are the same size while corresponding side lengths are different. The ratios of corresponding side lengths will be consistent between similar shapes.



## **Assessment**

### ***Anecdotal Evidence***

Some possible prompting questions:

- What do you notice about these shapes' angles?
- Is there a relationship between the lengths of corresponding sides?
- Can you see a pattern?
- If you look at the relationship between the corresponding sides, can you find the scale factor?
- If you know the side lengths of one shape and the scale factor, can you find missing lengths on another shape?
- Can you represent the relationship as a ratio?
- Can you use the ratio between the shapes to help you solve this problem?
- Can you use the ratio within the shapes to help you solve this problem?

### ***Portfolio Task***

Skills developed throughout this cycle will contribute to the student portfolio task *P12: Measuring Shapes*. Tasks 2 and 3 provide opportunities to apply and practice ratio and scale factor with similar shapes.

# RAMR Cycle



## Reality

Discuss real world instances of enlargement and reduction from students' experiences. Discuss how enlargement and reduction work without distorting the image. Use of a data projector with an image in PowerPoint may be useful here to directly demonstrate enlarging and reducing an image (with aspect ratio locked). Also explore what happens when an image is enlarged or reduced in only one dimension (with aspect ratio unlocked). Alternatively, use a torch or data projector to create shadows and discuss the resulting change in the properties of the object.



## Abstraction

The abstraction sequence for this cycle starts from students' experience of ratio relationships between discrete quantities and extends the idea to include relationships between measurements of similar shapes and figures. Ratio is also used to describe variation in length measures as a result of the enlargement/reduction transformation. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Select a group of students to hold a loop of elastic or string to create a quadrilateral (or pentagon, hexagon, etc). Draw around this shape on the floor with chalk. Ask all students to take a same-sized step backwards. Draw around the new shape on the floor with a different colour chalk. Discuss what is the same and different about the shape (check for same size angles, different length sides, cover different areas, different perimeters). Discuss the fact that while the shapes are the same, the sizes are different. Introduce the term "similar shapes". (Alternatively, use a string projection as described in *Resource 12.2.1 String projection, animation and computers.*) Discuss how smaller similar shapes could be created.
2. *Represent/model with materials.* Repeat the activity using geoboards (virtual geoboards can be a viable alternative to rubber bands on geoboards: Utah State University National Library of Virtual Manipulatives: Geometry: Geoboard: <http://nlvm.usu.edu>). Have students create larger and smaller similar shapes. As before, discuss same and different properties of the shapes.
3. *Draw on paper.* Draw similar shapes using a pencil and ruler. Use a protractor to verify that the angles are the same in the similar shapes created. The animation technique described in *Resource 12.2.1 String projection, animation and computers* may also be useful for this activity.
4. *Connect to language and symbols.* Reinforce with students the properties of the shapes that make them similar. Discuss the meaning of similar in everyday language as being sort of the same in some way (so a stretched or distorted image may be similar in everyday language). During this cycle it is important to establish that mathematical similarity is more specific and relies on establishing a mathematical relationship between properties of the shape.

This sequence of activities has focussed on creating larger similar shapes with a common centre (or point within a shape) without a focus on specific scale or relative size.

5. Repeat Steps 1 – 4 with a focus on creating larger similar shapes starting from a common corner.
6. Repeat Steps 1 – 4 with a focus on increasing lengths by a common factor (e.g., double, triple).



**Resource** Resource 12.2.1 String projection, animation and computers



## Mathematics



### **Language/symbols and practice**

Ensure students understand similarity and equivalent ratios. Practice enlarging and reducing shapes and diagrams by measuring the shape, determining new measurements using ratios, and redrawing the shape. Connect to ratio language and notation when enlarging and reducing shapes using scale.

Ask students to practice comparing similar shapes, determining the scale of enlargement or reduction and describing this using ratio notation.



### **Resource** Resource 12.2.2 Scaled shapes



### **Connections**

Ensure the relationship between similarity as enlarging and reducing, and similarity as properties of angle and ratio of sides are clearly connected for students. One connection is to relate general similarity to specific rules for shapes. An example for triangles is as follows.

#### **Similar triangles**

Consider you have drawn a small map of a triangular garden and you wish to provide the minimum information that would enable another to make an exact copy of your triangle or to make an enlargement (use paper, tracing paper, rulers, and protractors). Proportion tables could be useful to organise data.

1. Make a triangle that is scalene (no equalities).
2. Find a partner. Provide the partner with three pieces of information (made up from side lengths and angles) so that your partner can make an exact copy of your map. They use your data to make this copy on tracing paper so that it can be directly compared to the original.
3. Determine whether the data given is enough for an accurate copy. Data should state how angles and side lengths relate.

#### **Mirror method for measuring height.**

Explore using similar triangles to measure the height of tall items using the mirror method.



### **Resource** Resource 12.2.3 Mirror method for measuring height

#### **Other similar shapes**

Explore scaling shapes other than triangles. Is it possible to enlarge and reduce rectangles, irregular pentagons, irregular hexagons, and so on? Are all rectangles similar shapes and in proportion with each other or do the proportions of rectangles differ?

Is it possible to enlarge and reduce squares, regular pentagons, regular hexagons and so on? What do you notice about these shapes? Is it possible to make squares that are not similar or in proportion?

Think about circles. Are all circles similar shapes?



## Reflection



### Check the idea

Engage students with enlarging and reducing images using 2 cm and 1 cm graph paper.

- Draw a design with straight line sides and corners where lines meet (not too complicated – better if a simple shape or diagram) on the 2 cm graph paper. Get your partner to copy this onto the 1 cm graph paper by marking corners so that it is reduced in size.
- Repeat (a) above but put the design on the 1 cm graph paper and enlarge to the 2 cm graph paper. Take turns being the designer.
- Repeat (b) above but with a simple picture that has curved edges and does not necessarily have corners on places where lines meet.

Discuss how to make an image that is 1.5 times the size or 3 times the size?

Discuss how to take a simple tag or design and enlarge it many times so it could be, with permission, painted (with easily removable paint) on an oval or drawn on the basketball court as a very large design?



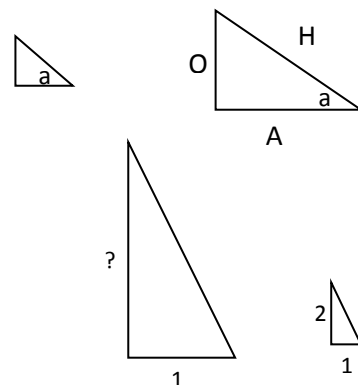
### Apply the idea

Use the ideas from finding the height of an object using a mirror and extend to using shadows to find height. This is pre-emptive trigonometry. It uses similar right-angled triangles and the ratios between corresponding sides.

Looking at the triangles on the right, if  $H$  is opposite the right angle,  $O$  is opposite angle  $a$  and  $A$  is adjacent to angle  $a$ , then the  $O$  sides are in the same ratio as the  $A$  sides and as the  $H$  sides.

This strategy allows students to find unknown lengths from known lengths. If the shadow of a 2 m post is 1 m long and the tree's shadow is 15 m long, then the fact that shadows are cast at the same angle means they are similar shapes and the height of the tree can be worked out as 30 m high. If corresponding sides are in ratio then ratios between pairs of corresponding sides are also the same.

This method is useful for measuring heights of mountains from sea level, heights of objects above your position, and distances by observation from two points.







### ***Extend the idea***

Explore the Golden ratio and Golden rectangles for other examples of ratio and proportion in nature, product design, logos, art, architecture and body ratios (waist to hip, waist to height). Challenge students to combine design ideas explored with the Golden ratio for design and creating a representation of their initials on grid paper.

Students should identify a suitable dimension for each letter in their design which conforms to the Golden ratio in height and width. Do not leave spaces between the letters. Mark rectangles for letters lightly in pencil on grid paper. Draw block letters within each rectangle and colour.

Identify major coordinates for each letter and use these to scale the image up and down (double and half will be sufficient).

Measure the length and width of each rectangle. Use these dimensions to check if the scaled images still conform to the Golden ratio.

If students have an interest in Golden ratio, challenge them to test the proportions of superhero and product logos from earlier activities to see if they conform to the Golden ratio. Images may be grouped on a noticeboard or the whiteboard according to Golden ratio/not Golden ratio.



### ***Resource***

Resource 12.2.4 Golden rectangles and the Golden ratio

Resource 12.2.5 A4 1 cm grid paper

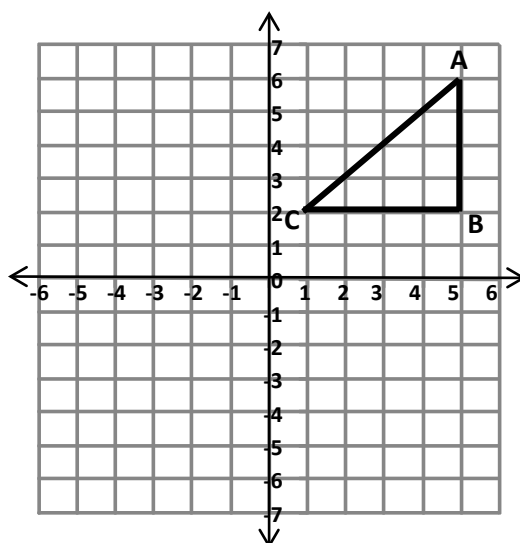
## ***Teacher Reflective Notes***

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Can you do this? #3



1. Write the coordinates for each vertex of the triangle ABC?

A (\_\_\_\_, \_\_\_\_), B (\_\_\_\_, \_\_\_\_), C (\_\_\_\_, \_\_\_\_)

2. (a) Write down the length of line AB. \_\_\_\_\_ units

(b) Write down the length of line BC. \_\_\_\_\_ units

3. Starting at A, draw a triangle on the Cartesian plane with sides that are twice the length of the drawn triangle. Label its corners  $B_1$  and  $C_1$ .

4. Write the coordinates for each vertex of the new triangle.

A (\_\_\_\_, \_\_\_\_),  $B_1$  (\_\_\_\_, \_\_\_\_),  $C_1$  (\_\_\_\_, \_\_\_\_)

5. (a) Write down the length of  $AB_1$ . \_\_\_\_\_ units

(b) Write down the length of  $BC_1$ . \_\_\_\_\_ Units

6. Write down the ratio of the side lengths for triangle ABC :  $AB_1C_1$ .

(a) AB :  $AB_1$

(b) BC :  $B_1C_1$

\_\_\_\_\_ :

\_\_\_\_\_ :

Obj.  
12.3.1  
1a)i ☐  
a)ii ☐  
1b)i ☐  
b)ii ☐  
1c)i ☐  
c)ii ☐

2a) ☐  
b) ☐

3 i ☐  
3 ii ☐

4a)i ☐  
a)ii ☐  
4b)i ☐  
b)ii ☐  
4c)i ☐  
c)ii ☐

5a) ☐  
b) ☐

6 a)i ☐  
a)ii ☐  
6 b)i ☐  
b)ii ☐

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# Cycle 3: Ratio Relationships on the Cartesian Plane

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## Overview



### Big Idea

Scaling on the Cartesian plane is similar to enlarging and reducing shapes using grid squares. However, instead of changing the grid size, enlargement and reduction on the Cartesian plane requires multiplicative change to the coordinates of each point, or to the length of the lines drawn, meanwhile, the angles between lines must remain constant.

Scaled maps of the local area, and Google maps can be scaled up or down in a similar fashion. When considering detail in maps, students should recognise and discuss how increasing the scale or zooming in on a map, allows for more detailed views. Enlarging images where finer detail is not available results in grainy or pixelated images.



### Objectives

By the end of this cycle, students should be able to:

12.3.1 Explore the Cartesian coordinates and side lengths of an enlarged image compared with the original. [5MG115]



### Conceptual Links

Ideas in this cycle connect to previous experiences plotting locations and distances on the Cartesian plane. This cycle may also be connected back to angles and angle properties including vertically opposite angles, corresponding angles, alternate angles and co-interior angles explored in Unit 10.

This cycle extends the previous cycle's work on ratio and angle relationships to explore ratio relationships within shapes. Exploring the properties of enlargement and reduction transformations on simple shapes will provide a basis for further exploring the effects of enlargement and reduction transformations on the perimeter and area of shapes.



### Materials

For Cycle 3 you may need:

- A4 1 cm grid paper
- rulers
- pencils



### Key Language

Coordinates, Cartesian plane, x-axis, y-axis, x-coordinate, y-coordinate, scale, scale factor, ratio



## Assessment

### ***Anecdotal Evidence***

Some possible prompting questions:

- Can you show me the x-axis? Can you show me the y-axis?
- Which coordinate do you need first in an ordered pair?
- Do you go across first or up first?
- What happens to the distances between the vertices of a shape when you enlarge/reduce it on the Cartesian plane?
- What happens to the coordinates of the vertices of a shape when you enlarge/reduce it on the Cartesian plane?
- Is there a pattern to how the coordinates of the vertices change when it is transformed?
- What changes? What stays the same?
- What do you notice about the change in the distance between vertices and the change in the coordinates when a shape is enlarged or reduced?

### ***Portfolio Task***

The skills students develop through this cycle will assist them in completing the student portfolio task *P12: Measuring Shapes*, although there are not identifiable standalone items that relate to this cycle only. The similar triangles and shapes tasks within the Portfolio Task could be extended to engage students with plotting the shapes' coordinates on Cartesian planes.

# RAMR Cycle



## Reality

Discuss students' experiences of maps on computer games with locations marked on. These can usually be slid or rotated for a different view of the landscape. The relationships between points stay consistent. The computer locates each point on the map using coordinates which are varied mathematically and zooming in or out on the maps changes the distances between points proportionally.



## Abstraction

The abstraction sequence for this cycle begins with students' previous experience of scaling the dimensions of a shape to enlarge or reduce its size. Students will extend their repertoire of enlargement and similarity understanding to include transforming coordinates on the Cartesian plane to generate similar figures. Students will explore scaling shapes using a multiplier to generate new coordinates followed by focusing on scaling the line lengths of shapes to find new coordinates. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Set up a maths mat or drawn grid on the concrete as the positive quadrant of a Cartesian plane. Select a group of students to hold a loop of elastic or string to create a quadrilateral (or pentagon, hexagon, etc) on the maths mat or a drawn grid on the concrete. Identify the coordinates for each corner of the shape. Leaving the shape on the grid, have students double their coordinates and stand in the new position on the grid. Use another length of elastic or string to define the new shape. Discuss the relationships between the lengths of the sides on the shapes and the angles within the shapes.
1. *Represent/model with materials.* Repeat the activity using geoboards (virtual geoboards are also available with a coordinate feature: Utah State University National Library of Virtual Manipulatives: Geometry: Geoboard: <http://nlvm.usu.edu>). Have students create scaled shapes by multiplying coordinates by whole numbers for enlarging and fractions for reducing. As before, discuss same and different properties of the shapes.
2. *Draw on paper.* Draw similar shapes using a pencil and ruler. Use a protractor to verify that the angles are the same in the shapes created. Focus students attention on the relationship between the multiplier or scale factor used to increase or decrease the coordinates and the ratio between the lengths of the lines in the original shape and the scaled shape.
3. *Connect to language and symbols.* Reinforce with students the properties of the shapes that they have created, the scale factor and the ratios between the created shapes. Discuss with students that these shapes are similar, but not the same (same shape, same angles, different lengths).

This sequence of activities has focussed on creating larger similar shapes by multiplying the coordinates of each point (or point within a shape). This results in similar shapes that share a common centre or similar shapes where one has increased in size and been translated on the Cartesian plane.

4. *Kinaesthetic activity.* Return to the maths mat and the created shape on the Cartesian grid. Stand a student on each corner of the shape and identify the coordinates for the position. Count the distance across the x-axis and up the y-axis to the next coordinate. Scale these measures and reposition a student for the corner of the new shape. Repeat this for each line segment that creates the shape to generate a shape which shares a common point with the original shape but is also scaled. Again, check the ratio between the line lengths in the original shape and the scaled shape.

5. As before, recreate the kinaesthetic activity on geoboards and paper and connect to language and symbols. This process can be scaffolded using a proportion table to record initial and scaled line lengths. Use *Resource 12.3.1 Simple shapes for scaling* and *Resource 12.3.2 A4 grid paper* to enlarge and reduce the size of simple shapes.



### Resource

Resource 12.3.1 Simple shapes for scaling

Resource 12.3.2 A4 1 cm grid paper



## Mathematics



### Language/symbols and practice

Engage students with practice identifying coordinates and scaling images on the Cartesian plane. Superhero crests and logos may be appropriate images that capture students' interest. The symmetrical shape of these forms allows students to easily identify and self-correct any coordinate errors. A selection of twelve superhero crests and logos are included in *Resource 12.3.3 Scaled superheroes on the Cartesian plane*. Discuss the scale factor used to enlarge or reduce images and express this as a ratio in the corner of each drawing.

As a practice exercise, discuss with students what ratio or scale would be needed to generate their logo to fit the height of the whiteboard. Ask students to work in pairs to generate coordinates that suit this scale.



### Resource

Resource 12.3.3 Scaled superheroes on the Cartesian plane

For practice working from an already generated list of coordinates, WorksheetWorks.com (<http://www.worksheetworks.com/math/geometry/graphing.html>) will generate a variety of picture puzzles for students to draw from given coordinates. To adapt these to the scaling context, provide additional grid paper for students so they can adapt the list of coordinates by multiplying by a scale factor (whole number for enlarging, fraction for reducing). Students can then redraw the image in a different scale. Ensure that students label the scaled drawing with its scale as a ratio. An example is available in *Resource 12.3.4 From coordinates to images*.



### Resource

Resource 12.3.4 From coordinates to images



### Connections

#### Maps

Provide students with a small map of their school. Identify specific points on the map to locate with coordinates. Measure these distances on the map and use the scale ratio to work out the distances in reality. Check these distances by pacing or measuring as appropriate (a pair of students could check each distance). Project a Cartesian grid onto the whiteboard (scale large enough for students to double the coordinates). Ask students to generate scaled coordinates by counting the distance across and up for each line on their map and doubling. Work out the new scale ratio for the enlarged map.

#### Similar triangles

Explore the properties of similar triangles scaled on the Cartesian plane. Clearly connect to Unit 11 work on angle properties. Identify where lines are parallel and instances of alternate angles, corresponding angles, vertically opposite angles and co-interior angles. Reinforce the properties of similar triangles and prove that similar triangles have the same size angles with side lengths that remain in proportion.



## Reflection



### ***Check the idea***

Provide students with a simple image for scaling (any logos unused from earlier activities will do). Have students determine the coordinates for each point on the image. Pass through a scaling function machine and scale the image by changing coordinates directly or by scaling lengths of line segments on the image.

Provide students with scaled logos on a Cartesian plane. Have students compare the coordinates and line segments on the shapes to identify the ratio relationships between line segments in the shapes and identify the scale relationship between the two shapes.



### **Resource**

Resource 12.3.1 Simple shapes for scaling

Resource 12.3.5 Scaled shapes for ratio comparison



### ***Apply the idea***

Engage students with creating a superhero shield (like Superman's) using their own initials. This image needs to be traced onto a Cartesian grid, coordinates generated, then scaled to fit A5, A7 and A4 paper. Students should select a scale factor to use and multiply the coordinates accordingly to create the list for the new size then test the drawing instructions.



**Resource** Resource 12.3.6 Superhero shield initials



## ***Teacher Reflective Notes***

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.



Name: \_\_\_\_\_

Date: \_\_\_\_\_

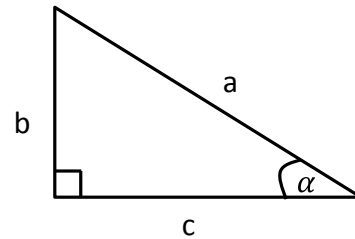
## Can you do this? #4

1. What are the three ratios of trigonometry for this triangle?

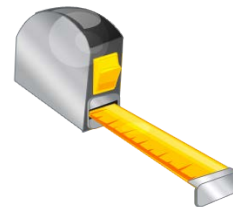
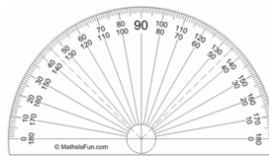
(a)  $\sin \alpha =$  \_\_\_\_\_

(b)  $\cos \alpha =$  \_\_\_\_\_

(c)  $\tan \alpha =$  \_\_\_\_\_



2. Circle the items you could use to find the height of a tree without climbing it?



3. How could you use trigonometry to find the height of a tree?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. A 2.4 m ladder is placed against a wall. The angle between the top of the ladder and the wall is  $20^\circ$  (draw a diagram to help you.)

(a) How far up the wall does the ladder reach?

(b) How far from the wall is the base of the ladder?

Obj.  
12.4.1  
a) ☐  
b) ☐  
c) ☐

Obj.  
12.4.2  
i ☐  
ii ☐  
iii ☐

Obj.  
12.4.2  
☐

Obj.  
12.4.2  
a) i ☐  
ii ☐  
iii ☐  
b) i ☐  
ii ☐  
iii ☐

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## Cycle 4: Trigonometric Ratios

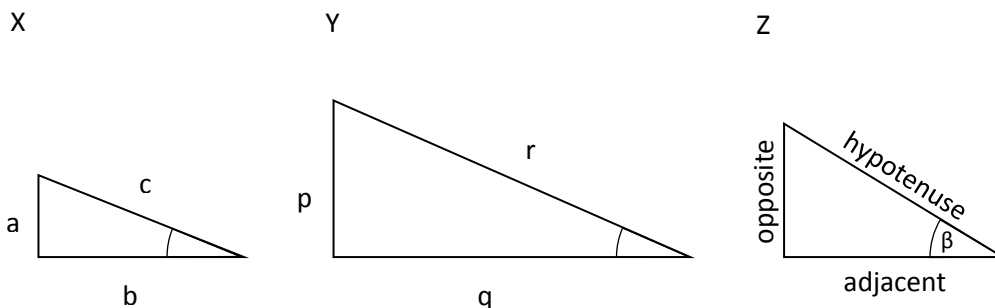
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### Overview



#### Big Idea

Right-angle triangles (right triangles) form an interesting subclass of similar shapes. Because one angle is  $90^\circ$ , determination of the angle at one other corner determines all angles; all right triangles of the same first angle are similar (e.g., a right triangle with an angle of  $30^\circ$  is similar to all other right triangles with an angle of  $30^\circ$  no matter what the size). Because of the nature of proportion, this means that sides are also in ratio within a triangle. In a right triangle the longest side is called the hypotenuse, the side next to angle is called adjacent, and the side opposite to angle is called opposite.



The two right triangles X and Y above are similar – they have the same angles. Thus  $\frac{p}{a}$ ,  $\frac{q}{b}$  and  $\frac{r}{c}$  are in proportion, they share the same ratio ( $\frac{p}{a} = \frac{q}{b} = \frac{r}{c}$ ). This idea, explored in Unit 9-2, is extended here to include trigonometric ratios in right-angled triangles. Because of the nature of proportion, the sides are also in ratio within a triangle, that is,  $\frac{p}{q} = \frac{a}{b}$ ,  $\frac{a}{c} = \frac{p}{r}$  and  $\frac{b}{c} = \frac{q}{r}$ . It also means that these sides are in the same ratio for any right triangle with the same angle as X and Y.

Therefore, if we know the angles of a right triangle, we can find the three proportions described below. For triangle Z with angle  $\beta$  above:

sine  $\beta$  = opposite/hypotenuse, cosine  $\beta$  = adjacent/hypotenuse, tangent  $\beta$  = opposite/adjacent.



#### Objectives

By the end of this cycle, students should be able to:

12.4.1 Identify sine, cosine and tangent ratios for a given angle in right-angled triangles. [9MG223]

12.4.2 Use trigonometry to solve right-angled triangle problems. [9MG224]



#### Conceptual Links

Ratio ideas, similar triangles, angle and side relationships are prerequisite ideas for this cycle.

Extending ratios within triangles to trigonometric ratios provides students with the necessary tools to find lengths of triangles given angles, and angles of triangles given lengths of sides. These have real-world problem solving applications in contexts including flight, sport and construction.



## Materials

For Cycle 4 you may need:

- mirror
- string
- masking tape
- 30 m tape measure
- card



## Key Language

Ratio, similar triangles, ratio, angle, tangent, sine, cosine, tan, sin, cos, proportion, opposite, adjacent, hypotenuse



## Definitions

*Cosine of an angle:* In a right angled triangle, the length of adjacent side divided by the length of the hypotenuse.

*Sine:* In a right angled triangle, the length of the opposite side divided by the length of the hypotenuse.

*Tangent:* In a right angled triangle, the length of the opposite side divided by the length of the adjacent side.

*Hypotenuse:* The longest side of a triangle. In a right angled triangle, the hypotenuse is the side opposite to the right angle.



## Assessment

### Anecdotal Evidence

Some possible prompting questions:

- Are these similar shapes?
- If the figures are similar, what does that tell you about the angles?
- If the figures are similar, what does that tell you about the relationships between the sides when you compare the shapes?
- Are these right-angled triangles?
- Do you know the value of the angle? Do you have measurements for any of the sides?
- Can you draw a diagram to help answer the question?
- Which is the hypotenuse/adjacent/opposite side?
- Which trigonometric ratio uses the side lengths you have/want?
- Can you use the calculator to find the answer?

### Portfolio Task

The skills students develop through this cycle will provide them with the knowledge necessary to complete the portfolio task *P12: Measuring Shapes* but the task does not overtly reference these skills.

# RAMR Cycle



## Reality

Find real contexts that can be used to experience and explore trigonometry. For example, contexts where distances are known but angles must be calculated (without drawing a scale diagram); contexts where the angles are known but distances must be determined. Other contexts include squaring construction (can be done by finding diagonal lengths to make the associated corners square); determining lengths of steel to cut so that roof truss pieces meet at the required angles; finding lengths of roofing to order for sloping roof sections; angles of slope for landscaping walls and banks. Students can calculate the heights of cliffs for tactical manoeuvres or bungee jumps; lengths of ropes and wires to support stunt teams in movies and circus acts; estimate the heights of trees for safety in felling trees so they miss houses and power lines.

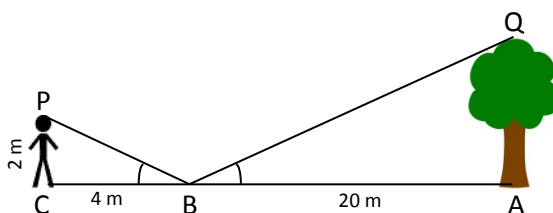
The simplest of these to act out or test in a school environment are squaring garden beds or hypothetical bases for playing courts or estimating the heights of trees and buildings. Students can create an engaging investigation by calculating and testing the length of rope needed to drop a Barbie doll or action figure from a second story railing so that the head does not hit the concrete (use non-stretching rope – do not forget to allow for the length of the doll – actual bungee calculations need to take in stretch of elastic cord and mass of jumper). This investigation could be used throughout the abstraction instead of the tree activity or revisited in the conclusion of the cycle to check student learning.



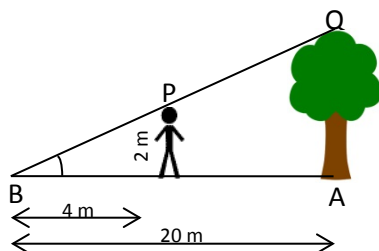
## Abstraction

The abstraction sequence further explores ratio and angle relationships within right-angle triangles. This cycle builds on student experience with similar triangles in Cycle 2. A suggested sequence of activities is as follows:

1. Remind students of the activity completed in Cycle 2 where they estimated the height of a tree using a mirror and ratio of sides using similar triangles.



*Kinaesthetic activity.* Instead of placing the mirror between the person and the tree, stand the person between the mirror and the tree. Once a straight line has been established between the top of the tree, the top of the person and the mirror, use a length of string to create a straight line from the top of the person's head to the top of the image in the mirror. Use masking tape to secure the string to a piece of card so that the angle can be measured with a protractor.



2. *Represent with pictures.* Draw a sketch of the context as above to help with thinking and relationships. Identify which dimensions of the triangles are in proportion. Measure the angle created with the string and the bottom of the card to find what the angle is at the point of the triangle. Ensure that students can understand that this angle is the same for the triangle made by the person to the mirror and the similar triangle made by the tree to the mirror.
3. *Explore relationships.* Generate a proportion table to record the distances in the sketch.

	Person	Tree
<b>Height</b> (side opposite to angle)	2 m	? m
<b>Distance</b> (side adjacent to angle)	4 m	20 m

Previously the focus was on finding the proportion between same measures on different similar triangles so that  $4:20 = 2:?$  In this case the focus is on finding the proportional relationship within the triangle and using this to find the unknown value so that  $2:4 = ?:20$

4. Find other similar triangles between these two triangles (e.g., what height matches with 5 m from the mirror, 10 m?). Use a measuring tape and string to create these triangles. Discuss with students what they notice about the angles in these triangles.
5. Place the measurements of the triangles on a table as below (values provided for teacher reference only).

Shape	Angle 1	Angle 2	Angle 3	Side 1 (opp angle 1)	Side 2 (adj angle 1)	Side 3 (hypotenuse)	Ratio $\frac{\text{side 1}}{\text{side 3}}$	Ratio $\frac{\text{side 2}}{\text{side 3}}$	Ratio $\frac{\text{side 1}}{\text{side 2}}$
Triangle	26°	90°	64°	2 m	4 m	4.47 m	$\frac{2}{4.47} = 0.4$	$\frac{4}{4.47} = 0.89$	$\frac{2}{4} = 0.5$
Triangle	26°	90°	64°	5 m	10 m	11.18 m	$\frac{5}{11.18} = 0.4$	$\frac{10}{11.18} = 0.89$	$\frac{5}{10} = 0.5$
Triangle	26°	90°	64°	10 m	20 m	22.36 m	$\frac{10}{22.36} = 0.4$	$\frac{20}{22.36} = 0.89$	$\frac{10}{20} = 0.5$

6. Extend the table to explore the ratios with Angle 3 as the focus (so Side 1 is opp Angle 3 and so on).



#### **Resource** Resource 12.4.1 Exploring Trigonometric Ratios Table 1

7. Discuss with students what they notice about ratios depending on which angle is the focus.

**Note:** Students should recognise that the ratio of the side opposite Angle 1 to the hypotenuse is the same as the ratio of the side adjacent Angle 3 to the hypotenuse. Similarly, they should also recognise that the ratio of side opposite to Angle 1 to side adjacent to Angle 1 is the inverse of the ratio of the side opposite to Angle 3 to side adjacent to Angle 3.

8. Explore these patterns further by constructing sets of right triangles of different sizes, but with the same angles. (Take the lengths of the three sides. Multiply them by the same number, or increase them by the same ratio. Keep the same angles.) Put the lengths back together to form a triangle. Complete the table. Encourage students to compare the values as before.
9. *Introduce mathematical language.* Discuss the formal mathematical terms that are used to describe the relationships students explored. Use calculators to check the sine, cosine and tangent values of angles with the measured values. Are they the same (or near enough)?



#### **Resource** Resource 12.4.2 Exploring Trigonometric Ratios Table 2

10. *Connect to mathematical language.* Repeat step 10 but use a table which has opposite, adjacent, hypotenuse, sin, cos and tan in place of side 1, side 2, side 3 and the resulting ratios. Introduce SOHCAHTOA if desired.



**Resource** Resource 12.4.3 Exploring Trigonometric Ratios Table 3



## Mathematics



### *Language/symbols and practice*

Practice and consolidate using trigonometric ratios to find sides when angles are known or angles when sides are known. This practice should involve three types of problems as both symbolic problems and real life application problems. Encourage students to draw a sketch as needed to identify opposite, adjacent and hypotenuse effectively.

1. Take right triangles that are similar – have all lengths in the first triangle known and only one in the second triangle known – use trigonometry to calculate the remaining sides.
2. Use a maths calculator to determine the size of the angles using sine, cosine and tangent. Take a right triangle with one angle and one side known. Use trigonometry to find the other sides.
3. Reverse the above. Take a triangle with two sides known. Use calculator in reverse to find the angle and state the location of the angle (or name the angle).



**Resource** Resource 12.4.4 Sources for useful practice sheets



### *Connections*

Exploring trigonometric ratios with students provides opportunities to connect to the ideas of inverse and the use of algebra to solve for unknowns.

This idea was highlighted during the Abstraction phase of this cycle. Ensure that students can recognise that for angles that add to  $90^\circ$ , the sine of one is the same as the cosine of the other.

Reinforce with students how to sketch diagrams and write equations that allow them to find solutions. For example,  $\tan a = \frac{\text{opp}}{\text{adj}}$  can be rearranged if the angle and opposite side are known:

$$\begin{aligned}\tan a \times \text{adj} &= \text{opp} \\ \text{adj} &= \frac{\text{opp}}{\tan a} \\ \text{adj} &= \text{opp} \times (\tan a)^{-1}\end{aligned}$$

Alternatively, it may be simpler for students to find a value for  $\tan a$  to substitute into their equation and then rearrange to find what they need to put into the calculator to determine the answer.

For example,  $\tan a = \frac{\text{opp}}{\text{adj}}$  with an angle of  $30^\circ$  and an opposite side of 10 m.

$$\begin{aligned}\tan 30 &= \frac{10}{\text{adj}} \\ 0.577 &= \frac{10}{\text{adj}} \\ 0.577 \times \text{adj} &= 10 \\ \text{adj} &= \frac{10}{0.577} = 17.33 \text{ m}\end{aligned}$$





## Reflection



### Check the idea

Provide students with a problem of each type from the Mathematics phase to ensure that they can solve these problems using trigonometry. Students may like to check how much length is needed to drop a figure from a railing without hitting the ground. Use an inclinometer to determine the angle up to the railing. Measure the distance away from the building where the inclinometer measurement was taken, use tan ratio to find the height from the railing to the inclinometer level and then add the height measure from the inclinometer to the ground.



**Resource** Resource 12.4.5 Simple inclinometer



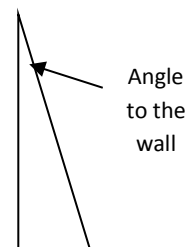
### Apply the idea

Use trigonometric ratios to determine solutions to real world problems

To be most stable, a ladder should be angled to the wall  $15^\circ$  to  $25^\circ$ . Use trigonometry to calculate the stable range of distances that a 2.3 m ladder should be pulled out from the floor. Use a calculator.

To act out this situation, cut a 2.3 m length of string and use it to look at different angles of the ladder – check the trigonometry with the string by measuring the angle to the wall with a protractor and the distance from the wall with a measuring tape.

Try this for different length ladders (for example, 900mm, 1.2 m, 1.5 m, 1.8 m, 3 m).



**Resource** Resource 12.4.6 Trigonometry problems



### Extend the idea

Combine mathematical concepts and processes to solve real world problems

Uncle Ernie needs to send a special 4.5 m spear to Darwin for a ceremonial event. He needs to get it there in time for the ceremony, so decides to send it by courier. The boxes need to be rectangular prisms where length to breadth is 7:1. To keep the cost down, the box must be as short as possible. Design a box to fit the 4.5 m spear neatly and record the dimensions. Use a calculator.

## ***Teacher Reflective Notes***

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

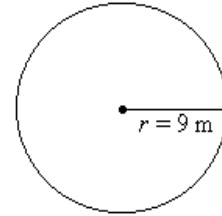
Name: \_\_\_\_\_

Date: \_\_\_\_\_

## *Can you do this? #5*

1. Consider this circle.

(a) Calculate the circle's circumference ( $C = 2\pi r$ ).



(b) Calculate the area of the circle ( $A = \pi r^2$ ).

2. A circle with a circumference of 5 metres is drawn on the ground. If the radius is enlarged by a scale factor of three, what will be the circumference of the new circle? (Circle your answer.)

(a) 12 m

(b) 8 m

(c) 15 m

(d) 10 m

3. Three triangular shade sails are cut with the same angles, but different side lengths. The second shade sail is 1.5 times the length and width of the first shade sail. The third shade sail is 2 times the length and width of the first sail. A 12 m long binding is needed around the edge of the first sail.

(a) How much binding will be needed for the second sail? \_\_\_\_\_

(b) How much binding will be needed for the third sail? \_\_\_\_\_

4. If the sun shines straight down on the shade sails, how many times greater will the shade from the third shade sail be compared to the first shade sail? \_\_\_\_\_

5. A circle has an area of 5 square metres. If the radius is enlarged by a scale factor of two, what will be the area of the enlarged circle?

Obj.  
12.5.1

a)i. ☐

ii. ☐

iii. ☐

b)i. ☐

ii. ☐

iii. ☐

Obj.  
12.5.2

a) ☐

b) ☐

c) ☐

d) ☐

Obj.  
12.5.2

a)i ☐

ii ☐

iii ☐

b)i ☐

ii ☐

iii ☐

Obj.  
12.5.3

☐

Obj.  
12.5.3

i ☐

ii ☐

iii ☐

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# Cycle 5: Ratio Relationships

## Between Areas of Similar Shapes

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### Overview



#### Big Idea

In the previous cycles, scale factor described relationships between measures in one dimension of similar figures. This results in a linear, multiplicative relationship between corresponding dimensions in similar figures. As perimeter and circumference are linear measures, the multiplicative relationship between the corresponding dimensions of similar figures will also apply. Scale can enlarge and reduce shapes and figures in more than one dimension. However, like conversion between area measures, this relationship is not linear. If both dimensions of an area are doubled, the resulting area will be multiplied by 4. This cycle will explore circle relationships between radius, circumference and area and how scaling results in the proportional change of these measures.



#### Objectives

By the end of this cycle, students should be able to:

- 12.5.1 Identify the relationship between features of circles such as circumference, area, radius and diameter. [8MG197]
- 12.5.2 Identify the relationship between perimeters of similar shapes and connect to ratio and scale factor. [9MG221]
- 12.5.3 Identify the relationship between areas of similar shapes and connect to ratio and scale factor. [9MG221]



#### Conceptual Links

This cycle relies on and reinforces previous experiences in measurement and calculation of perimeter and area along with ratio relationships and scaling concepts.

Ideas from this cycle will connect to surface area and volume calculations in a future unit.



#### Materials

For Cycle 5 you may need:

- data projector and PowerPoint
- 1 cm grid paper
- 2 cm grid paper
- string
- cardboard shapes
- selection of circular lids, containers, wheels



#### Key Language

Scale, scale factor, ratio, enlarging, reducing, area, two dimensions, squared, circles, radius, diameter, pi,  $\pi$



## **Assessment**

### ***Anecdotal Evidence***

Some possible prompting questions:

- Can you draw a diagram of the problem to help you?
- Are you looking for the distance around or the area covered?
- Can you use ratios to work this out?
- How could you work out the scale factor from what you know?

### ***Portfolio Task***

Ideas from this cycle may be applicable to *P12: Measuring shapes* Task 2 where applications to area and pixel size may be considered.

# RAMR Cycle



## Reality

Previous cycles have focussed on changing length and width separately and exploring the relationships between them. Discuss changing the dimensions of a shape in proportion. What will this mean for the amount of area the object covers? For example, if a window is doubled in length and width, what will this mean for the wall area to be painted?



## Abstraction

The abstraction sequence for this cycle starts from students' previous experience of the effects in one dimension of enlarging and reducing objects by scaling. In this sequence the focus is on the area covered by 2D shapes and the effects of scaling. A suggested sequence of activities is as follows:

1. *Kinaesthetic.* Place a sheet of newspaper on a grid on the floor. Determine how many square units it covers. Ask students to predict how many square units will be covered if the length and width are doubled. Test their predictions by placing a sheet of newspaper to double the length and allow them to revise their predictions if necessary. Use two more sheets of newspaper to double the width and complete the transformation. Check the new area in square units.
2. *Represent/model with materials.* Draw the previous activities in workbooks. Start with a rectangle and calculate its area. In a different colour, double its length and width and calculate the area of the enlarged shape.
3. *Repeat the activity by reducing the area.* Start with an A3 sheet, halve its width and length. Open the sheet back out to see the fraction of the initial area. Discuss explicitly with students how changing the length by a scale factor of 2, transforms the area by a factor of 4 or  $2^2$ .



## Mathematics



### Language/symbols and practice

Explore instances where scaling objects will result in changes to area. For example, if the area of sheet metal is known for creating a model at scale 1:12, how many models can be created from the same sheet of metal if the scale of 1:24 is used instead?

Measure the area of a pair of shorts and t-shirt that fit an average student (measure front of each, treat as a rectangle, and double). If replicas of these are to be created to dress the model people designed in the previous cycle, determine how many pairs of shorts and t-shirts could be created from the same area of cloth?

Tables may be useful with these types of problems to record initial measures and scaled measures.



## Connections

### Circumference of a circle

The concept of circumference can be explored by finding the ratio relationship between diameter and perimeter of circles. Circumference can be found by first “measuring and comparing” the circumferences of different objects using an intermediary as a means to developing the formula. This can provide memorable experiences to assist with remembering the formula and pi. *Resource 12.5.1 Circumference of circles* contains useful sequences for exploring the circumference of circles.



### Resource Resource 12.5.1 Circumference of circles

Connect the idea of circumference of circles to similar shapes and figures by exploring the relationships between features of circles as they are enlarged or reduced. Discuss how all circles are in fact similar shapes.

### Area of a circle

Calculating area of circles relies on the ratio between diameter and circumference ( $\pi$ ). The formula for the area of a circle can also be connected to previously explored area calculations (parallelogram and rectangle) by cutting the circle into wedges and reforming.



### Resource Resource 12.5.2 Area of circles

### Proportional relationships between perimeters and areas of shapes

Use a range of rectangular packets, lids, containers. Explore enlarging and reducing a selection of 2D faces of these shapes. Record dimensions, perimeters and areas in a table to facilitate looking for patterns within the data. Discuss the relationships with students. For example:

Shape	Attribute		$\times 2$	$\times 3$	$\times \frac{1}{2}$	
Rectangle	length	20 cm	40 cm	60 cm	10 cm	Values increase by scale factor.
	width	30 cm	60 cm	90 cm	15 cm	Values increase by scale factor.
	perimeter	100 cm	200 cm	300 cm	50 cm	Values increase by scale factor.
	area	600 cm <sup>2</sup>	2400 cm <sup>2</sup>	5400 cm <sup>2</sup>	150 cm <sup>2</sup>	Values increase by scale factor squared.
Circle	radius	10 cm	20 cm	30 cm	5 cm	Values increase by scale factor.
	circumference	20 $\pi$ cm	40 $\pi$ cm	60 $\pi$ cm	10 $\pi$ cm	Values increase by scale factor.
	area	100 $\pi$ m <sup>2</sup>	400 $\pi$ cm <sup>2</sup>	900 $\pi$ cm <sup>2</sup>	25 $\pi$ cm <sup>2</sup>	Values increase by scale factor squared.



## Reflection



### ***Check the idea***

Ask students to explore *Resource 12.5.3 Pizza problem* to determine if they are actually receiving value for money on the pizza deal.



### **Resource**     Resource 12.5.3 Pizza problem



### ***Apply the idea***

Draw scale pictures of the pizzas in *Resource 12.5.3 Pizza problem* on grid paper. Use the scale pictures to determine the dimensions of a box for the small pizza. Identify the scale factor for enlarging the small pizza box to the new size for the larger pizza. Determine how much more cardboard will be needed to create this box.



### ***Extend the idea***

Extend students' problem solving to include scaling recipes for different sized cake tins. Even though these are volume problems, keeping the depth or height of the cake the same changes the problems to area problems.

Provide students with a recipe to make a basic butter cake in a 20 cm round tin. Ask them to work out how to scale the recipe to suit a 25 cm, 30 cm, 40 cm, 15 cm round tin (dimensions are diameters).

Challenge students further to determine what size square tin the original mix will make. Scale the results to suit other size square tins (e.g., 20 cm, 25 cm, 30 cm square tins).

How much cake mix would be needed to make a rectangular cake for the class so that everyone in the room can have a piece that is 5 cm square?



## ***Teacher Reflective Notes***

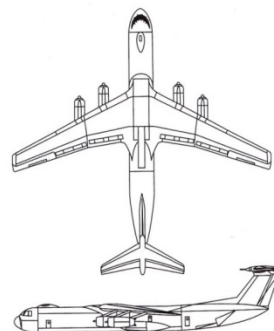
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

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## Unit 12 Portfolio Task – Teacher Guide

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### Measuring Shapes



**Content Strand/s:** Measurement and Geometry

**Resources Supplied:**

- task sheet
- teacher guide

**Other Resources Needed:**

- none

**Summary:**

There are three tasks within this workbook. The first asks students to use their knowledge of ratios, the second explores ratios and similar triangles, and the third explores enlargements.

**Variations:**

- Find additional scaled down objects and ask students to find their exact measurements.
- Explore how an image is projected onto a movie screen.

**ACARA Proficiencies**

**Addressed:**

Understanding

Fluency

Problem Solving

Reasoning

**Content Strands:**

*Measurement and Geometry*

12.1.3 Apply the enlargement transformation to familiar two dimensional shapes. [5MG155]

12.2.1 Explore the relationships between side lengths and angles of an enlarged/reduced image compared with the original to determine unknown scale factors. [5MG115]

12.2.4 Identify the conditions for triangles to be similar. [9MG220]

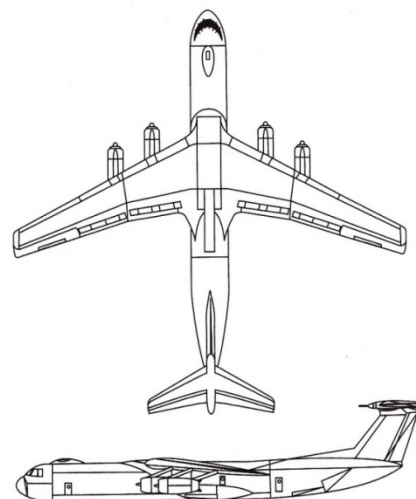
12.2.5 Solve problems using similar shapes, ratio and scale factors. [9MG221]

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## *Measuring Shapes*

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<b>Name</b>	
<b>Teacher</b>	
<b>Class</b>	



In this portfolio task, the measurement of shapes is presented in three ways:

**Task 1:** Explore the dimensions of an aircraft.

**Task 2:** Investigate image projection.

**Task 3:** Compare lengths within triangles.

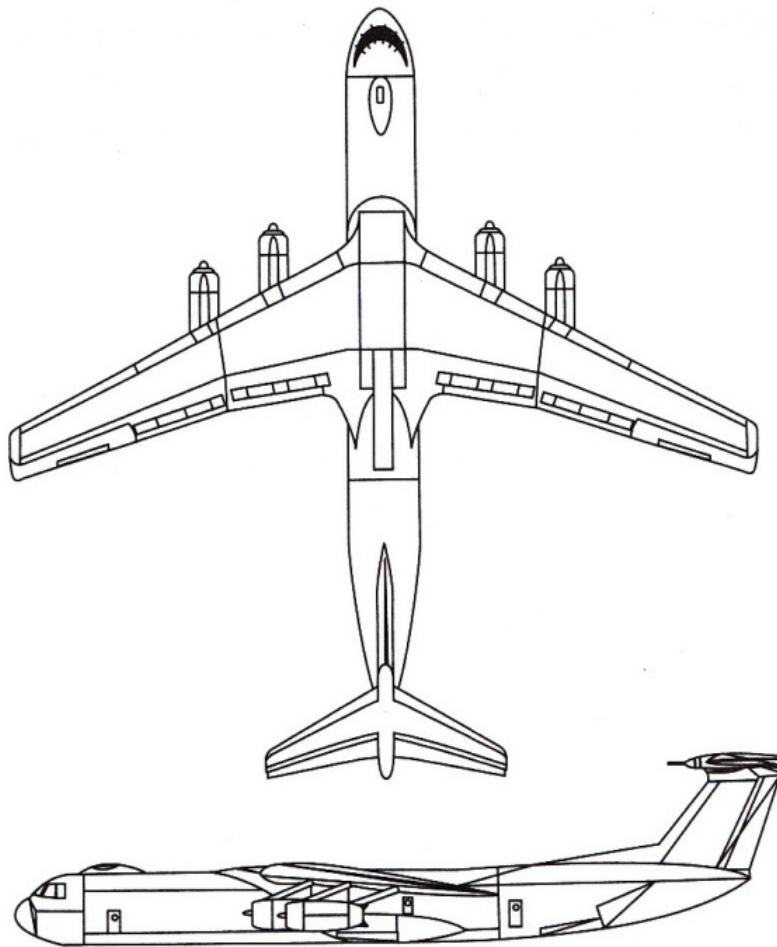
Within Portfolio Task 12, your work demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Conceptual understanding	12.2.1 Explore the relationships between side lengths and angles of an enlarged/reduced image compared with the original to determine unknown scale factors.	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
	Problem solving approaches	12.1.3 Apply the enlargement transformation to familiar two dimensional shapes.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches
Problem Solving and Reasoning	Mathematical modelling	12.2.5 Solve problems using similar shapes, ratio and scale factors.	Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar	Development of mathematical models and representations in complex familiar or simple unfamiliar situations	Development of mathematical models and representations in simple familiar situations	Statements about simple mathematical models and representations	Isolated statements about given mathematical models and representations
	Reasoning and justification	12.2.4 Identify the conditions for triangles to be similar.	Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions

Comments:

## Task 1 – Aircraft investigation

The diagram shows a large military transport aircraft drawn to a scale of 1:800.



1. Measure the wing span (length from one wing tip to the other) and length (nose to tail):

Wing span: \_\_\_\_\_ Length: \_\_\_\_\_

2. Find the real-life dimensions of the aircraft, to the nearest tenth of a metre, using the 1:800 scale.

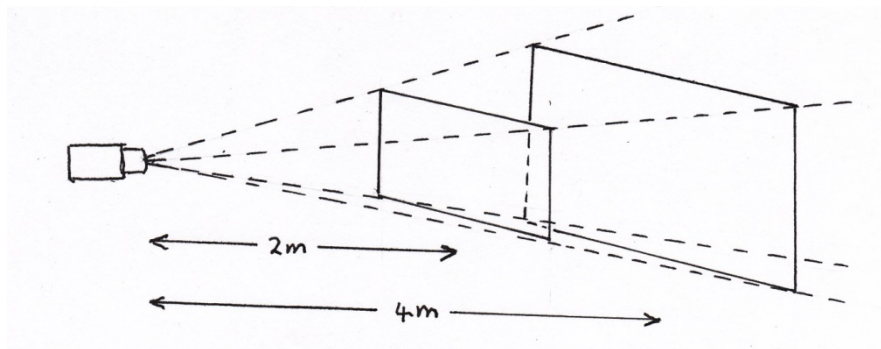
Wing span: \_\_\_\_\_ Length: \_\_\_\_\_

## Task 2 – Projected digital images investigation

Digital images (photographs, TV screens, projectors) are often described in terms of the numbers of **pixels** that make up the image. A pixel is an individual colour dot and a two dimensional array (rows and columns) of pixels makes up the rectangular image.

When buying a digital projector, there are many different models available. This projector produces a rectangular image 2000 pixels by 1500 pixels.

*Note: The size of the projected image depends on how far the screen is from the projector, but whatever the size, the image will be 2000 pixels across and 1500 pixels high.*



3. At a screen distance of 2 m, the image is 1600 cm wide and 1200 cm high. What total number of pixels will make up the projected image?
  
  
  
  
  
  
  
  
  
  
4. Calculate the number of pixels per square centimetre in the projected image at 2 metres.

5. If the distance from the projector to the screen is doubled to 4 metres, what will be the dimensions of the image?

*Hint: it may help to draw a diagram.*

6. Calculate the number of pixels per square centimetre in the projected image at 4 metres.

7. How does this number compare with the number of pixels per centimetre in the image at 2 metres?

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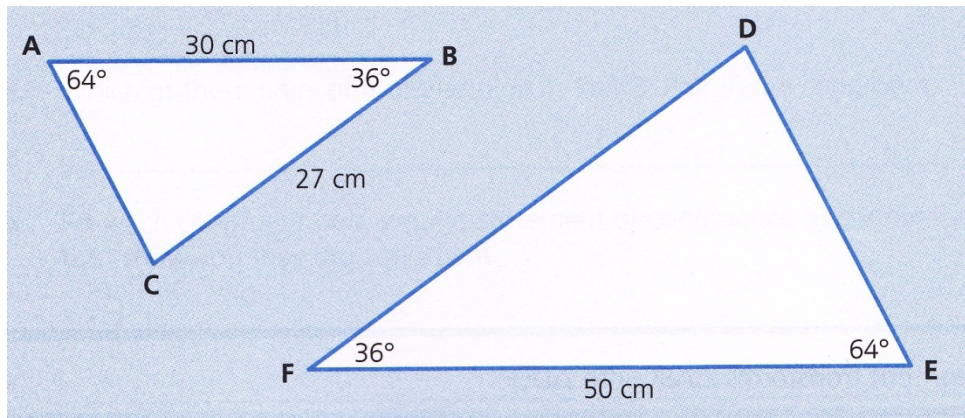
8. What effect do you think doubling the distance to the screen would have on the quality of the projected image?

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### Task 3 – Triangle investigation

This diagram shows two triangles: triangle ABC and triangle DEF.



9. When talking about corners and lines in triangles, they are identified by the letters. So, line AB is 30 cm and Angle E is  $64^\circ$ . What can you tell me about:

- Angle A: \_\_\_\_\_
- Line EF: \_\_\_\_\_
- Line BC: \_\_\_\_\_
- Angle F: \_\_\_\_\_

10. Write three points about how these two triangles are similar:

- 
- 
- 

11. Does this prove they are similar triangles? \_\_\_\_\_

12. Name the two sides in triangle DEF that correspond to the sides AB and BC in triangle ABC.

*Hint: corresponding sides are the sides that are in the same position in similar triangles.*

Side corresponding to AB: \_\_\_\_\_ Side corresponding to BC: \_\_\_\_\_



13. How can you find a ratio?

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14. Find the ratio AB : BC using the smallest possible whole numbers.

15. Use ratios to find the length of DF.

16. Find the scale factor that transforms triangle ABC into triangle DEF.

*Hint: another word for scale factor is ratio*

17. Use a protractor and a ruler to draw a similar triangle to ABC that has the corresponding line length to AB of 5 cm.



## *Can you do this now? Unit 12*

The rectangle is a scale drawing of a large block of land. It has been drawn at the scale of 1 : 100 000.

1. Measure the length and width of the rectangle in centimetres.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

2. Write down the scale factor. \_\_\_\_\_

3. Calculate the real-life dimensions of the block of land in centimetres.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. Convert the measures to metres and write them on the diagram.

Width (a) \_\_\_\_\_ Length (b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

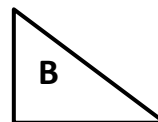
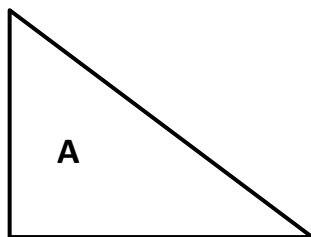
\_\_\_\_\_

(a) \_\_\_\_\_



(b) \_\_\_\_\_

- 5.



Triangle A is twice as long and twice as high as triangle B.

- (a) Write down the scale factor used to transform triangle B to triangle A? \_\_\_\_\_

- (b) Write down the scale of triangle B : triangle A as a ratio.

\_\_\_\_\_ : \_\_\_\_\_

Obj.

12.1.3

1a)i ☐

a)ii ☐

1b)i ☐

b)ii ☐

Obj.

12.1.1

2 ☐

Obj.

12.1.3

3a)i ☐

a)ii ☐

a)iii ☐

3b)i ☐

b)ii ☐

b)iii ☐

4a)i ☐

a)ii ☐

a)iii ☐

4b)i ☐

b)ii ☐

b)iii ☐

Obj.

12.1.1

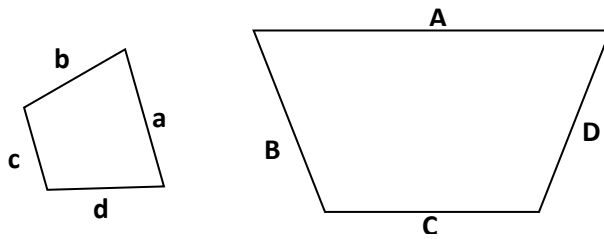
a) ☐

Obj.

12.1.2

b)i ☐

b)ii ☐



6. Measure and record the side lengths of each shape in millimetres.

A \_\_\_\_\_ B \_\_\_\_\_ C \_\_\_\_\_ D \_\_\_\_\_  
a \_\_\_\_\_ b \_\_\_\_\_ c \_\_\_\_\_ d \_\_\_\_\_

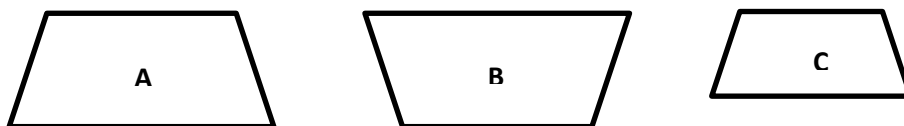
7. Calculate the scale factor that has been used to enlarge the side lengths of the shapes. \_\_\_\_\_

8. Use your measurements to express the matching side lengths as ratios (small shape : large shape).

a : A = \_\_\_\_\_ : \_\_\_\_\_  
b : B = \_\_\_\_\_ : \_\_\_\_\_  
c : C = \_\_\_\_\_ : \_\_\_\_\_  
d : D = \_\_\_\_\_ : \_\_\_\_\_

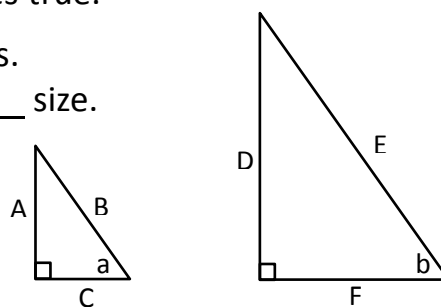
9. Fill in the blanks to make these sentences true.

Shape A and shape B are congruent because they have \_\_\_\_\_ angles and matching \_\_\_\_\_. Shape C is \_\_\_\_\_ to shape A and shape B because its angles are \_\_\_\_\_ size but its \_\_\_\_\_ are different.



10. Fill in the blanks to make these sentences true.

Here are two similar right-angle triangles.  
Angle a and angle b will be \_\_\_\_\_ size.  
The ratio of A:D = B:\_\_\_\_\_ = \_\_\_\_\_:F.



Obj.  
12.2.1

6a)i ☐

ii ☐

6b)i ☐

ii ☐

6c)i ☐

ii ☐

6d)i ☐

ii ☐

7 ☐

8a)i ☐

ii ☐

8b)i ☐

ii ☐

8c)i ☐

ii ☐

8d)i ☐

ii ☐

Obj.  
12.2.2

9i ☐

9ii ☐

9iii ☐

Obj.  
12.2.3

9iv ☐

9v ☐

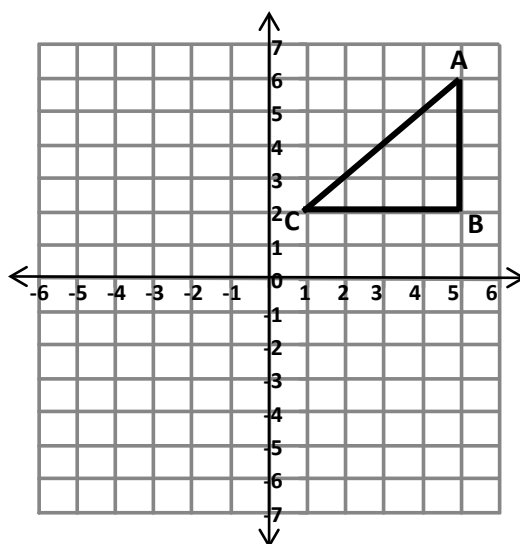
9vi ☐

Obj.  
12.2.4

i ☐

ii ☐

iii ☐



11. Write the coordinates for each vertex of the triangle ABC?

A (\_\_\_\_, \_\_\_\_), B (\_\_\_\_, \_\_\_\_), C (\_\_\_\_, \_\_\_\_)

12.(a) Write down the length of line AB. \_\_\_\_\_ units

(b) Write down the length of line BC. \_\_\_\_\_ units

13. Starting at A, draw a triangle on the Cartesian plane with sides that are twice the length of the drawn triangle. Label its corners B<sub>1</sub> and C<sub>1</sub>.

14. Write the coordinates for each vertex of the new triangle.

A (\_\_\_\_, \_\_\_\_), B<sub>1</sub> (\_\_\_\_, \_\_\_\_), C<sub>1</sub> (\_\_\_\_, \_\_\_\_)

15.(a) Write down the length of AB<sub>1</sub>. \_\_\_\_\_ units

(b) Write down the length of BC<sub>1</sub>. \_\_\_\_\_ Units

16. Write down the ratio of the side lengths for triangle ABC : AB<sub>1</sub>C<sub>1</sub>.

(a) AB : AB<sub>1</sub>

(b) BC : B<sub>1</sub>C<sub>1</sub>

\_\_\_\_\_ :

\_\_\_\_\_ :

Obj.  
12.3.1  
11a)i ☐  
a)ii ☐  
11b)i ☐  
b)ii ☐  
11c)i ☐  
c)ii ☐

12a) ☐  
b) ☐

13 i ☐  
13 ii ☐

14a)i ☐  
a)ii ☐  
14b)i ☐  
b)ii ☐  
14c)i ☐  
c)ii ☐

15a) ☐  
b) ☐

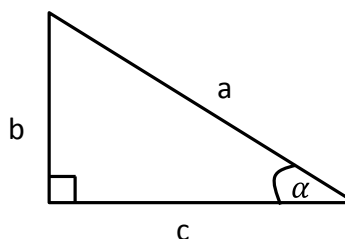
16a)i ☐  
a)ii ☐  
16b)i ☐  
b)ii ☐

17. What are the three ratios of trigonometry for this triangle?

(a)  $\sin \alpha =$  \_\_\_\_\_

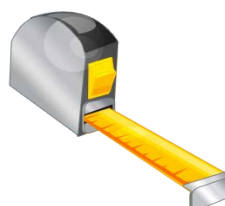
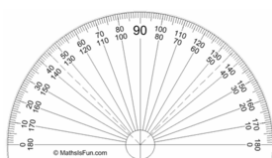
(b)  $\cos \alpha =$  \_\_\_\_\_

(c)  $\tan \alpha =$  \_\_\_\_\_



Obj.  
12.4.1  
a) ☐  
b) ☐  
c) ☐

18. Circle the items you could use to find the height of a tree without climbing it?



Obj.  
12.4.2  
i ☐  
ii ☐  
iii ☐

19. How could you use trigonometry to find the height of a tree?

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Obj.  
12.4.2  
☐

20. A 2.4 m ladder is placed against a wall. The angle between the top of the ladder and the wall is  $20^\circ$  (draw a diagram to help you.)

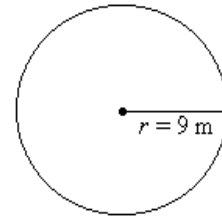
(a) How far up the wall does the ladder reach?

(b) How far from the wall is the base of the ladder?

Obj.  
12.4.2  
a) i ☐  
ii ☐  
iii ☐  
b) i ☐  
ii ☐  
iii ☐

21. Consider this circle.

(a) Calculate the circle's circumference ( $C = 2\pi r$ ).



(b) Calculate the area of the circle ( $A = \pi r^2$ ).

22. A circle with a circumference of 5 metres is drawn on the ground. If the radius is enlarged by a scale factor of three, what will be the circumference of the new circle? (Circle your answer.)

- (a) 12 m                      (b) 8 m                      (c) 15 m                      (d) 10 m

23. Three triangular shade sails are cut with the same angles, but different side lengths. The second shade sail is 1.5 times the length and width of the first shade sail. The third shade sail is 2 times the length and width of the first sail. A 12 m long binding is needed around the edge of the first sail.

(a) How much binding will be needed for the second sail? \_\_\_\_\_

(b) How much binding will be needed for the third sail? \_\_\_\_\_

24. If the sun shines straight down on the shade sails, how many times greater will the shade from the third shade sail be compared to the first shade sail?

25. A circle has an area of 5 square metres. If the radius is enlarged by a scale factor of two, what will be the area of the enlarged circle?

Obj.  
12.5.1

a)i. ☐

ii. ☐

iii. ☐

b)i. ☐

ii. ☐

iii. ☐

Obj.  
12.5.2

a) ☐

b) ☐

c) ☐

d) ☐

Obj.  
12.5.2

a)i ☐

ii ☐

iii ☐

b)i ☐

ii ☐

iii ☐

Obj.  
12.5.3

☐

Obj.  
12.5.3

i ☐

ii ☐

iii ☐



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