XLR8 – Accelerating Mathematics Learning

XLR8 Unit 11

Describing location and movement

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.

Scope

This unit is based upon the **identification** of **location** according to key surrounding features or **landmarks** and the resulting **change** that occurs from **movement along a line** and in **space**. The **attributes** of **location** and **direction** of **movement** may be defined using either **internal or external frames of** reference. **Simple directions** of **left**, **right**, **forward**, **backward**, **up** and **down**, initially represented on **2D maps** using **alphanumeric grid references**, are extended to include **compass bearings** and **distance** to more accurately define **location in space**.

Location and **movement** of a **point** or **collection of points** on the **Cartesian plane** shares similar features to **locating an object** on a **2D map**. **Diagonal distances** of travel may be calculated when points are given with reference to an origin using **Pythagoras' Theorem** and mapping onto a **Cartesian plane**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts that is to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P11: Moving Around* accompanying Unit 11 engages students with plotting the points of shapes on a Cartesian plane and moving these through transformations to relocate the shape. Pythagoras' theorem is used to explore the point of no return of an aeroplane between Brisbane, Singapore and London.

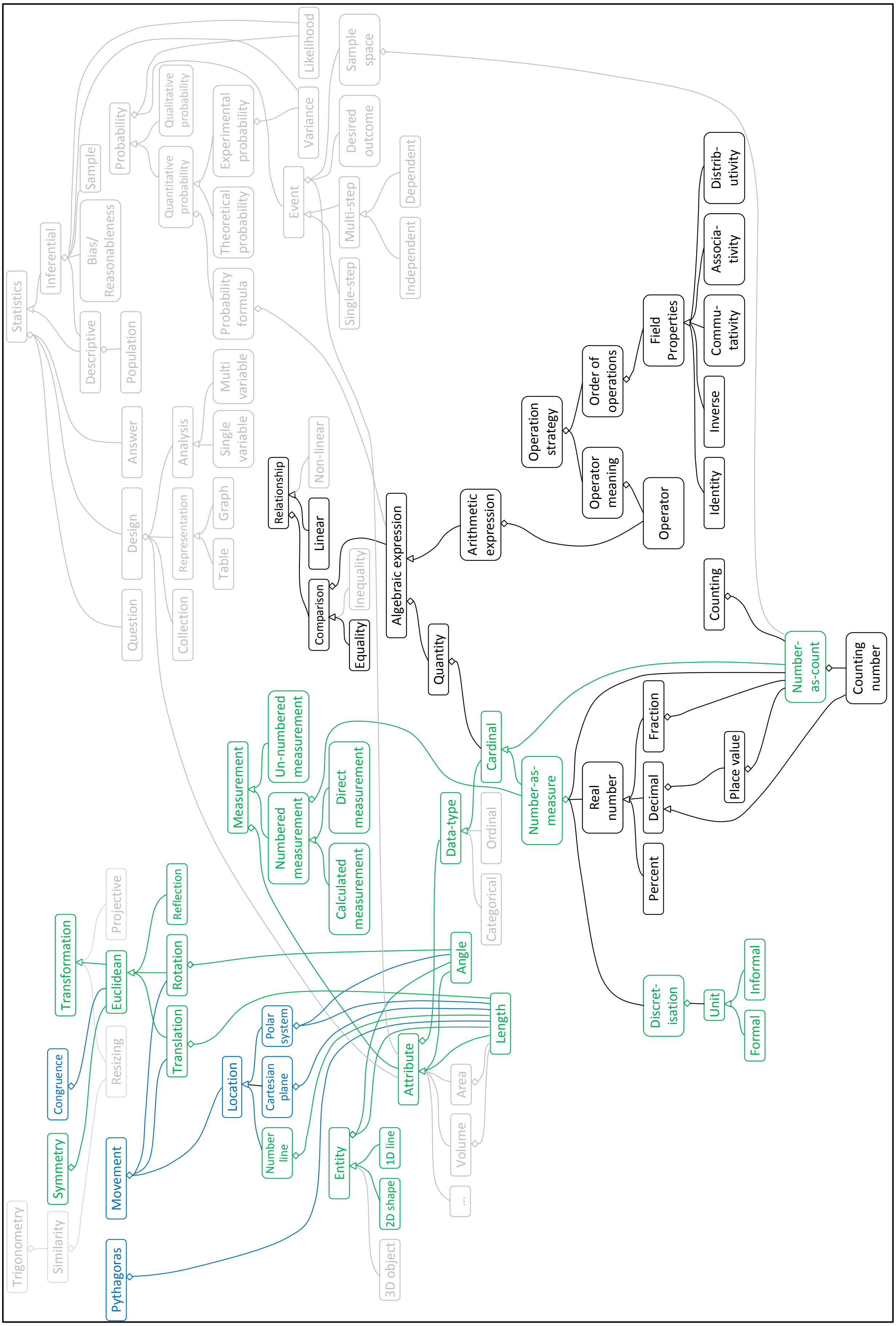


Figure 1. Scope of Unit 11

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Position and movement in a straight line

This cycle introduces integers and integer operations in the context of position and movement along a line. Locating a point along a line relative to its distance from the origin or a central point is the beginning idea that extends to locating a point on a 2D grid. This cycle will also introduce the idea of drawing a line to represent an actual distance in 2D space using simple scale (e.g., 1cm represents 1m). These ideas extend to creating and reading local maps and plans in Cycle 2.

Cycle 2: Location and movement: Alphanumeric coordinates

The focus of this cycle is to extend students' understanding of location and movement of single points along a line to location and movement of points in 2D space. Simple navigation in 2D space will be represented on alphanumeric grids (used in maps and games where moves can be plotted on a grid like battleship or chess). This cycle will introduce the notion of two components (horizontal and vertical) to identify the location of a region on a grid map.

Cycle 3: Cartesian plane

In this cycle, students' understanding of alphanumeric grids and compass directions using four quadrants is extended to location and movement in two dimensions using x- and y-coordinates. Location and movement on the two dimensional plane may be described by comparing ordered pairs of points (Cartesian coordinates).

Cycle 4: Measuring turn on the Cartesian plane

This cycle extends the idea of angle from measurement of turn used to describe vertices of shape, to measurement of turn applied to movement and location in space and its representation on the Cartesian plane.

Cycle 5: Polar coordinates

This cycle explores simple navigation using distance and compass bearings in familiar environments like the school ground and local area and continues exploration of representations of scale on maps and ways to describe direction of movement from simple egocentric directions to compass bearings.

Cycle 6: Pythagoras

Students have previously found the distance between two points by measuring. This cycle explores the use of Pythagoras' Theorem to calculate the distance between two points when these are stated in reference to an origin and can be mapped on a Cartesian plane.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially as it stands.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Position and movement in a straight line

Location, position, movement, direction, linear, origin, starting point, distance, length, represent, multiple, scale, landmarks, integers, negative, positive

Cycle 2: Mapping location and movement using alphanumeric coordinates

Point, region, coordinate, axis, x-axis, y-axis, coordinate pairs, ordered pairs, direction, compass points, compass bearings, polar coordinates, origin, scale, frame of reference, landmarks, key, towards, away from, next to, near, far, over, close, beside, left, right, forwards, backwards, North, South, East, West (also called cardinal points), intermediate bearings of Northwest, Northeast, Southwest, Southeast

Cycle 3: Cartesian plane location and movement

X-axis, y-axis, origin, ordered pair, x-coordinate, y-coordinate, point, collection of points, line, shape, Euclidean transformations, reflection, translation, rotation

Cycle 4: Measuring turn on the Cartesian plane

Angle, acute, obtuse, reflex, right angle, perpendicular, parallel, equilateral triangles, isosceles triangles, scalene triangles, protractor, degrees, vertex, vertices, lines, rays, points, angle labelling conventions, sharp, blunt, full turn, half turn, quarter turn, three-quarter turn, alternate angles, co-interior angles, corresponding angles, transversals, angle from the x-axis,.

Cycle 5: Compass bearings and distance

Left turn, right turn, full turn, half turn, quarter turn, three-quarter turn, one-eighth turn, angles of turn, degrees, 45^o, 90^o, 180^o, 270^o, North, South, East, West, compass points, cardinal points, distance, estimate, measurement, location, origin, starting point, compass

Cycle 6: Pythagoras

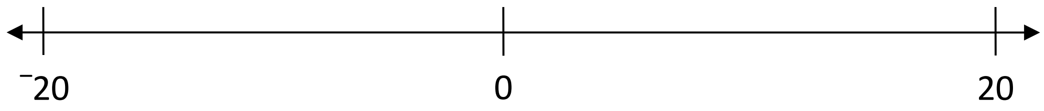
Mid-point, distance, coordinates, x coordinate, y coordinate, Pythagoras, one third points, quarter points

Name:____ Date:________

Can you do this? #1

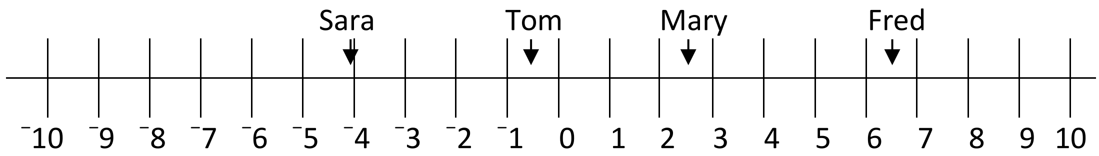
1. Position the following numbers on the number line:

   ¯1, 15, ¯13, ¯4, 2, 8, ¯8, 17



Each unit on the number line represents 10m.

2. What distance from the origin is each person located?



   (a) Sara __________ (c) Mary __________

   (b) Tom __________ (d) Fred __________

3. Fred moved 5m along the line towards the origin. What distance from the origin is his new location? ______________________________

4. Sara moved along the line to Fred's beginning location. How far did she move in total? ______________________________

5. List these numbers in order from largest to smallest:

   a) 23, 5, ¯5, 3, ¯3, ¯10, ¯2, 7, 0, ¯12

   ____

   b) Write down a number that is greater than the biggest number on the list. ______________________________

   c) Write down a number that is less than the smallest number on the list. ______________________________

Obj. 11.1.2
i☐ ii☐ iii☐ iv☐ v☐ vi☐ vii☐ viii☐

Obj. 11.1.1
2.a) ☐ b) ☐ c) ☐ d) ☐

Obj. 11.1.5
3. ☐

Obj. 11.1.4
4. ☐

Obj. 11.1.3
a) i ☐ ii ☐ iii ☐ iv ☐ v ☐ vi ☐ vii ☐ viii ☐ ix ☐ x ☐ b) ☐ c) ☐

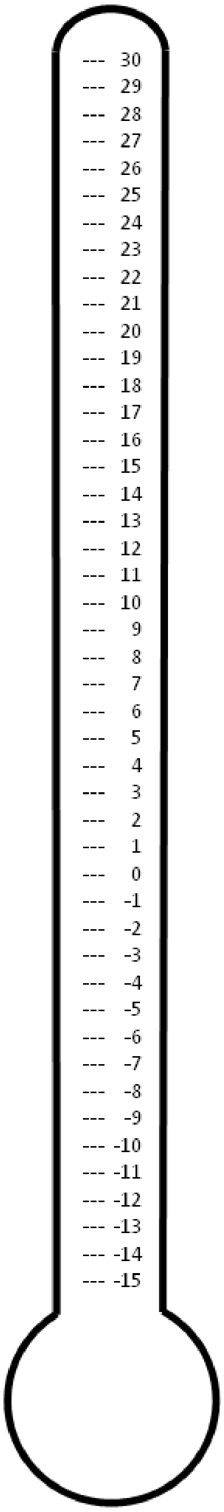
6. The following questions relate to temperatures and reading a thermometer. For each question, write the equation using a symbol for the unknown, and then solve the equation. (Use the thermometer to help you think.)

   (a) It was 3 degrees at midnight. Then, the temperature dropped by 5 degrees. What is the new temperature?

   (b) If the temperature changed from $^{-}3$ degrees by $^{-}4$ degrees, what is the new temperature?

   (c) After sunrise, the temperature rose from $^{-}5$ to 10 degrees. What is the change in temperature?

   (d) Wednesday evening at midnight the temperature was $^{-}3$ degrees. At 2am it was $^{-}6$ degrees. What was the change in temperature?



Obj. 11.1.5
a)i ☐
a)ii ☐
a)iii ☐
b)i ☐
b)ii ☐
b)iii ☐
d)i ☐
d)ii ☐
d)iii ☐

Obj. 11.1.4
c)i ☐
c)ii ☐
c)iii ☐

Cycle 1: Position and Movement in a Straight Line

Overview

Big Idea

This cycle introduces integers and integer operations in the context of position and movement along a line. This cycle will also introduce the idea of drawing a line using simple scale (e.g., using 1cm to represent 1m).

It is important to connect clearly the unit of count that is represented on the line (steps, metres, desks), rank and order of numbers, the change operation (additive or multiplicative), and the direction of movement (linked to the sign of the integer). Parts of the whole and representations of these on a number line are also connected when fractional distances are used.

Objectives

By the end of this cycle, students should be able to:

11.1.1 Investigate everyday situations that use integers. [6NA124]

11.1.2 Locate and represent integers on a number line. [6NA124]

11.1.3 Compare and order integers. [7NA280]

11.1.4 Add integers. [7NA280]

11.1.5 Subtract integers. [7NA280]

Conceptual Links

Locating a point along a line requires students to know the number names, symbols, and their order, and the ability to place these along a number line with reference to zero. Zero needs to be understood as a starting point or origin rather than 'nothing'. Students should also be able to use a ruler to draw a line and partition it into same size segments. Additive and multiplicative number facts will also be needed, and may be reinforced through this cycle.

Locating a point along a line relative to its distance from the origin or a central point is the beginning idea that later extends to locating a point on a 2D grid in Cycle 2. These ideas lead to creating and reading local maps and plans.

Materials

For Cycle 1 you may need:

- Large number line
- Measuring tape or metre ruler
- counter per student
- Number lines (A4 paper)

Key Language

Location, position, movement, direction, linear, origin, starting point, distance, length, represent, multiple, scale, landmarks, integers, negative, positive

Definitions

Integers: the set of whole numbers and their opposites. The integer zero (0) is neither positive or negative.

Negative integers: whole numbers less than zero. Negative integers are found to the left of zero on horizontal number lines or the section below zero on vertical number lines (e.g., thermometers).

Positive integers: whole numbers greater than zero. Positive integers are found to the right of zero on horizontal number lines or the section above zero on vertical number lines (e.g., thermometers).

Scale: multiplicative relationship between marks on a number line.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What do you notice about the number line?
- Make sure the marks are all the same distance apart.
- What scale will you use to make sure the information is represented on the line and fits on the page?
- Where is your starting point or origin? How have you labelled this?
- How can we tell the difference between numbers on opposite sides of the zero?
- Show me where negative numbers will be. Show me where positive numbers will be.
- What position on the number line did you start? Where did you finish up? How did your position change? What is the difference between the start and the finish?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the student portfolio task *P11: Moving Around*, but are not overtly included as standalone items.

RAMR Cycle

Reality

Discuss location and direction with students. Challenge them to find as many ways as possible to define their location in the classroom using everyday language (e.g., in front of the whiteboard, beside the window, near the door, at the back of the room, beside *Fred*). Draw a rough mud map (birds-eye view) of a line across the classroom on the whiteboard. Discuss how the location of objects or people along the line might be described more accurately. Explore location language from the perspective of different students along the line.

Consider the idea of a starting point or reference point to describe position from. Have students redefine their location as a number of steps from the origin. Discuss how students might define the direction of travel from the origin as well as the number of steps.

Abstraction

The abstraction sequence for this cycle builds student understanding of location and movement along a line from an individual's (internal) frame of reference to a common (external) frame of reference or origin. For clarity, this abstraction phase has been broken into positive integer location and positive integer change in positive and negative directions. Negative integer location and change will be addressed in the Mathematics phase. A suggested sequence of activities is as follows:

Positive integer location

1. *Kinaesthetic activity – number track*. Use alternating colours of paper to define a path across the classroom. Using one side of the room as a starting point, number each piece of paper (1, 2, 3, … similar to gameboard paths). Determine the location of items along the pathway as a number of pages or spaces across the room.
2. *Kinaesthetic activity – number line*. Use a length of rope or masking tape to define a line across the classroom. Discuss using one side of the room as the starting point or origin (label with zero) and an informal unit to measure the location of items along the line from the start. Have students describe their location as a distance from the origin. Since the focus is to be on integers, stay with whole numbers for this activity. **Note:** Ensure that students understand that the number line has 0 as the start and each number represents the end of a step (as for measurement). This is different from the number track where the centre of the page is the step.
3. *Represent/model with materials.* Represent the classroom line on the whiteboard (students use *Resource 11.1.1 Empty number lines*). Mark units across the number line, the 'origin' using 0, and number the unit marks. Label the location of each item and person along the number line. **Note:** The focus is a representation of the classroom line but is not 'to scale'.

Resource Resource 11.1.1 Empty number lines

4. *Connect to language and symbols.* Describe the location of items around the classroom in terms of their location on the number line. Connect the location on the number line to the distance from the origin.

Positive integers: Change in location (movement)

5. *Kinaesthetic activity – number line*. Using the previously defined line across the classroom, send a student to a point along the line (e.g., starting at 0, walk to 4 units from the origin). Model the change in location on a function machine. Identify the walk from 0 to 4 as a change of +4.

6. *Represent/model with materials.* Have students represent the classroom line on empty number lines. Bring students' attention to the fact that they have walked to the right along the number line from the start and walked from left to right through the function machine.

7. *Connect to language and symbols.* Represent the function machine walk as Arrowmath: $0 \xrightarrow{+4} 4$ Connect students' thinking to the direction of the change (walking to the right of the origin is a positive direction). Write the change as an equation: 0 + 4 = 4

8. *Explore walking in the other direction.* Starting from location 4, discuss with students where they might end up if they took two more steps. Students should easily identify that they would end up at location 6 going forward or right. Represent this as an equation: (4 + 2 = 6).
   Discuss what location they reach if they took their two more steps from 4 in the opposite direction (to the left). Represent this as an equation: (4 + 2 steps left = 2). Discuss how else 2 steps left might be written. Lead students' thinking towards representing the direction of motion using "+" or "–" symbols.

   Model with function machine and Arrowmath as: $4 \xrightarrow{+{}^{-}2} 2$. Write the equation as $4 + {}^{-}2 = 2$.

   Note: Ensure that students understand that the superscript negative sign denotes going the other way on the number line, just as subtraction is the opposite (inverse) operation for addition.

Positive integers: Difference in location

9. *Kinaesthetic activity – number line*. Using the previously defined line across the classroom, locate students on points along the line. Discuss the relationships between locations of items (e.g., Fred is 8 units from the origin, Sara is 4 units from the origin, the distance between Fred and Sara is 4 units). This is a static difference problem that compares locations to find the distance between. On a function machine this is effectively a known input and output, with an unknown change.

10. *Represent/model with materials.* Have students represent the classroom line on empty number lines. Label the location of each item and person along the number line.

11. *Connect to language and symbols.* Discuss how to represent these problems using equations. For example: Sara is 4 units from the origin; Fred is 8 units from the origin; what is the difference between their locations? is represented as 4 + ? = 8.

    Note that the operation to find difference is subtraction. Ensure students are able to use balance and inverse operation to rearrange the equation to find a solution.

12. *Extend thinking to include direction of difference. Explore examples where one* location and the difference are known. For example, Fred is 8 units from the origin; Sara is 2 units from Fred; what is Sara's location? Sara could be either 10 or 6 units from the origin. Discuss how the difference may be clearly communicated to locate Sara specifically. Connect to direction on number line. Write the equations for Sara's possible locations as follows: Sara 2 units right of Fred: $8 + {}^{\square}2 = 10$ and Sara 2 units left of Fred: $8 + {}^{\square}2 = 6$. Fill the boxes with the appropriate symbol to show the direction of travel.

    Note: Discuss what needs to happen with the "2" in each case to make the equation true. If Fred were to walk to Sara, what direction would he walk? Students may identify a need to count forward or backward 2 spaces, go 2 to the right or left. Lead students' thinking towards the need to add ${}^{+}2$ to 8 to reach 10, and ${}^{-}2$ to 8 to reach 6. Connect symbols to language ("+" to the right and "–"to the left). Reinforce with students that inverse or opposite direction of "+" is "–".

Mathematics

Language/symbols and practice

Negative integer location

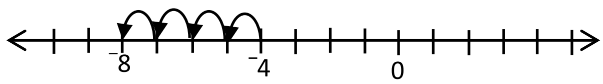
1. *Kinaesthetic activity – number line*. Return to the previously defined number line. Starting at position 8, explore taking steps in a negative direction. For example, $8 + {}^{-}5$ reaches 3. Discuss what will happen if 5 more steps in a negative direction are taken. Model starting at 3 and counting 5 steps in a negative direction. Discuss possible labels for this position. **Note:** Students may make the connection to 2 below zero (temperature) or 2 below ground (basement parking). Discuss labelling these locations below zero as negative integers.
2. *Represent/model with materials.* Represent the classroom line with a number line on the whiteboard (students should have an A4 sheet of empty number lines). Have students mark in the 'origin' using 0 and number the unit marks in each direction along the number line.
3. *Connect to language and symbols.* Describe the location of items around the classroom in terms of their location on the number line. Connect the location on the number line to the distance from the origin in each direction.

Resource Resource 11.1.1 Empty number lines

Negative integers: Change in location (movement)

The same sequence of steps may be used as in the abstraction phase for change in location with positive integers. Work through a sequence of problem types with students starting with a negative position, then continuing to walk in a negative direction. Model this on the number line and through the function machine. Note that for these initial examples, the input and the change are known, the output is unknown. For example, start at ${}^{-}4$, walk 4 steps in a negative direction (${}^{-}4$). Reinforce with students that the "–" means to go the opposite way to "+". So the function machine says to change by adding 4 in the opposite direction. This can also be modelled on the number line as jumps in the negative direction.

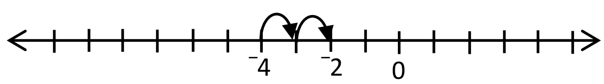
$${}^{-}4 \xrightarrow{+{}^{-}4} {}^{-}8$$



Write the change as an equation (${}^{-}4 + {}^{-}4 = {}^{-}8$).

Once students can add steps in a negative direction, explore adding steps in a positive direction when starting at a negative location. For example:

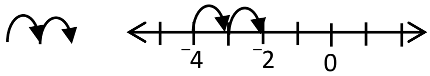
$${}^{-}4 \xrightarrow{+{}^{+}2} {}^{-}2$$

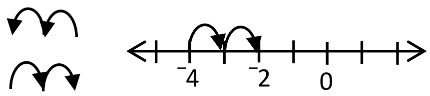


Extend to examples where the change results in a positive final location from a negative starting location. **Note:** Ensure that students understand the superscript negative sign denotes the direction of travel on the number line. Where there is no direction shown, the direction is positive.

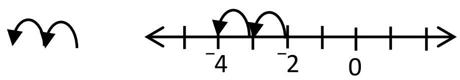
Negative integers: Difference in location

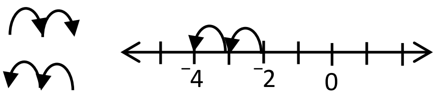
Using a similar sequence as for positive integers, explore the static difference between locations (change unknown). Model on function machines and number lines to develop clear understanding. For example, an input of $^-4$ and an output of $^-2$ could be reached in two different ways.

$^-4 \xrightarrow{+\,?} {}^-2$ How many steps in what direction?  $^-4 + 2 = {}^-2$

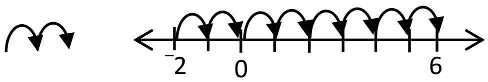
$^-4 \xrightarrow{-\,?} {}^-2$ How many steps in what direction? then do the opposite  $^-4 - {}^-2 = {}^-2$

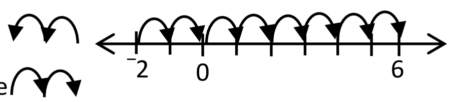
Also explore examples starting at $^-2$ and finishing at $^-4$.

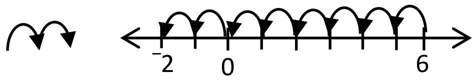
$^-2 \xrightarrow{+\,?} {}^-4$ How many steps in what direction?  $^-2 + {}^-2 = {}^-4$

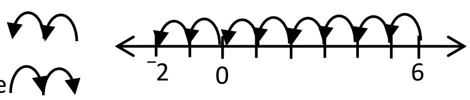
$^-2 \xrightarrow{-\,?} {}^-4$ How many steps in what direction? then do the opposite  $^-2 + {}^-2 = {}^-4$

Follow with examples that cross the origin. For example, start at $^-2$ and finish at 6 or start at 6 and finish at $^-2$.

$^-2 \xrightarrow{+\,?} 6$ How many steps in what direction?  $^-2 + 8 = 6$

$^-2 \xrightarrow{-\,?} 6$ How many steps in what direction? then do the opposite  $^-2 - {}^-8 = 6$

$6 \xrightarrow{+\,?} {}^-2$ How many steps in what direction?  $6 + {}^-8 = {}^-2$

$6 \xrightarrow{-\,?} {}^-2$ How many steps in what direction? then do the opposite  $6 - 8 = {}^-2$

Consolidate additive integer operations

Further problems not explored here so far include input unknown problems. These should be connected to algebra using balance and inverse ideas to generate an equation with the unknown input on its own and modelled with function machines and number lines as required. Give students a selection of starting points (inputs), changes or differences, and finish points (outputs) to explore.

Resource Resource 11.1.2 Additive input change output with integers

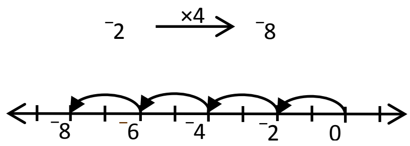
Practice number line applications of location and movement in a direction. For example, movement up and down large buildings with levels above and below ground. Connect up and down movement with positive and negative on a vertical number line and change as moving up the building for positive changes and moving down the building for negative changes.

Resource Resource 11.1.3 What level?

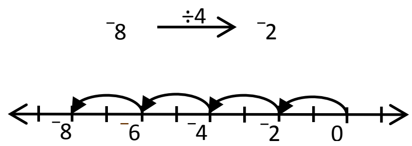
Multiplicative integer operations

Consolidate multiplication (as repeated addition) and division (as repeated subtraction) of integers with respect to change in location in positive and negative directions on the number line. For each of the problem types below, explore input-change-output using the function machine, act out and record steps with direction on the number line. As students develop confidence with finding unknown output from known input and known change, vary the problems to include unknown input from known change and known output as well as unknown change from known input and known output (multiplicative inverse problems).

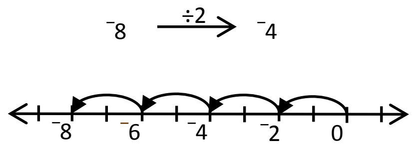
(a) Multiples of positive or negative quantities (repeated addition). For example, 4 jumps of $^{-}2$ ($4 \times {}^{-}2$ (pictured): each jump has a negative direction) or $^{-}4$ jumps of 2 ($^{-}4 \times 2$: the jump is positive, the direction of the multiplier is negative or the inverse direction of 4×2).



(b) Compare locations and break the distance between locations into a known number of equal jumps (division as partitioning: unit size of jumps is not known). The direction of the jumps may be negative ($^{-}8 \div 4$) or the direction of the multiple of jumps may be negative ($^{-}8 \div {}^{-}4$).



(c) Compare locations and break the distance between locations into an unknown number of jumps of a known unit size (division as quotitioning or repeated subtraction). The direction of the jumps may be negative ($^{-}8 \div {}^{-}4$) or the direction of the multiple of jumps may be negative ($^{-}8 \div 4$).



(d) Negative multiples of negative numbers (act these out for clarity). These entail a multiple of jumps in a negative direction that are then inverted because the direction of the multiplier is also negative or opposite. The associative law may be useful here. For example, $^{-}4 \times {}^{-}2$ is the same as $^{-}1 \times 4 \times {}^{-}2$. This can then be interpreted as making 4 jumps of 2 in a negative direction and then multiplying the result by $^{-}1$ which has the result of shifting the whole result into the opposite direction.

Resource Resource 11.1.4 Multiplicative input change output with integers

Directed number operations with fractional measures

Extend students' understanding of recording location and movement on the number line to include fractional steps or locations. For example, identify a location that is 1.5 steps from the origin in each direction. Model this on physical and drawn number lines to assist students to correctly determine which side of the whole number the fraction goes on the number line. Explore what it means to travel 3.5 steps in a positive direction from an initial location or 3.5 steps in a negative direction (additive operations). Also explore multiple fractional jumps in a direction and how many fractional jumps to reach a given location.

Use the function machine to connect change in location to input-output tables. For example, starting from the origin, move 2.5 steps in a positive direction (forwards or to the right). Move multiples of $^{+}2.5$ steps (e.g., 3 lots of $^{+}2.5$). Use input-output tables to record inputs and changes and have students record input, change and output on the number line. Ensure that students are able to identify the change from a sequence of outputs (e.g., 3.5, 2, 0.5, $^{-}1$, $^{-}2.5$ … the change is $^{-}1.5$).

Resource Resource 11.1.5 Fractional input change output with directed numbers

Reflection

Check the idea

Check understanding of location and movement by playing a dice game. As the game progresses, have students record equations for difference (starting point + or – finishing point = distance moved. Alternatively, play the game and record change in location to identify new location (starting point + or – distance moved = finishing point). Extend to include a multiplier to repeat a change in the positive or negative direction. *Resource 10.1.6 Location Change Game* has more detailed instructions for this type of game.

Resource Resource 11.1.6 Location Change Game

Apply the idea

Provide alternative contexts that require a facility with integer operations that are not connected to location and direction. For example, explore increases and decreases in temperature locally and in colder climates (consider using current local temperatures in other states of Australia, New Zealand and the Pacific Islands). Determine the differences between day and night temperatures, winter and summer temperatures, winter night time temperatures in a range of locations across the Pacific including Australia and New Zealand.

Other contexts may include incomes and expenses for a household and net worth following each transaction or business incomes and expenses over time.

Units of health or damage are also represented in game environments although these applications may prove too distracting to discuss. If students are able to be engaged with this context without causing extensive distraction it may be beneficial as a further context.

Extend the idea

Location and movement can also be represented on number line drawings more accurately using scale. Revisit the activities in the abstraction. Use a metre ruler or measuring tape for partitioning the line and identifying the location of the objects and people (instead of informal units) and define each location as a metric measure from the origin. Partition the classroom line on the whiteboard into 10cm segments. Label these as metre units. Locate each object and student along the line according to its distance. Discuss the relationship between the actual measurement and the scale drawing measurement. Connect the location along the line and convert to a distance using extended tens facts (convert metres to centimetres and divide by 10 in this case). Discuss with students how the scale of the drawing can be communicated (e.g., 10cm represents 1m, 10cm:100cm, 10:100, 1:10). Use blank number lines for students to practise making their own scale drawing of the number line.

Resource Resource 11.1.7 What level? (2)

Explore the relationships between places on the train line in the topological train map in *Resource 11.1.8 How far from home?* These relationships show order of stops not distance between stops.

Resource Resource 11.1.8 How far from home

Use a map to identify key locations between school and the local shopping centre or park (go both ways from the school). Mark these on a topological line map for your school area. Discuss the possible stops and relationships between landmarks with the school as the origin or starting point.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

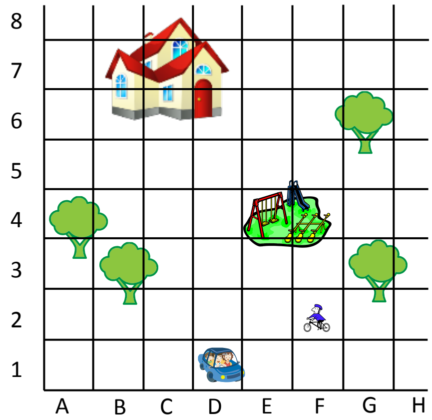
Can you do this? #2

1. Using the grid, identify the object that is at grid reference:

   (a) E4 ________________________

   (b) G3 ________________________

   (c) What is the grid reference for the cyclist? ______________

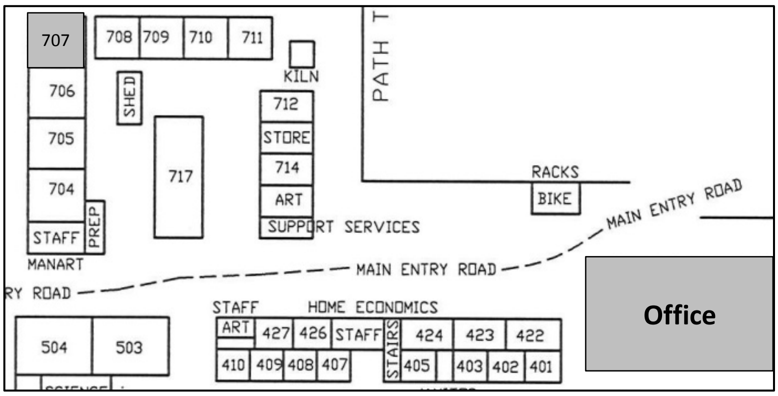


   (d) If the cyclist travels 3 squares in the direction he is facing, turns to the right one quarter turn, and travels 4 squares in this direction, what will be the new grid reference? ____________________

   (e) What other object is at this location? ____________________

2. The following map is part of a High School.

   (a) Draw how you would walk from the shaded office to the shaded classroom (707).



   (b) Give directions to describe how to get there.

   ____

   ____

   ____

Obj. 11.2.2
a) ☐
b) ☐
e) ☐

Obj. 11.2.4
c) ☐
d) ☐

Obj. 11.2.1
a)i ☐
ii ☐
Obj. 11.2.3
b)i ☐
ii ☐
iii ☐

Cycle 2: Location and Movement: Alphanumeric Coordinates

Overview

Big Idea

The focus of this cycle is to extend students' understanding of location and movement of single points along a line to location and movement of points in 2D space. Simple navigation in 2D space will be represented on alphanumeric grids (used in maps and games where moves can be plotted on a grid like battleship or chess). This cycle will introduce the notion of two components (horizontal and vertical position with respect to the origin) to identify the location of a region on a grid map.

Objectives

By the end of this cycle, students should be able to:

11.2.1 Create simple grid maps to show position and pathways. [3MG065]

11.2.2 Interpret information contained in basic maps. [4MG090]

11.2.3 Describe routes using landmarks and directional language. [5MG113]

11.2.4 Use an alphanumeric grid to describe locations. [5MG113]

Conceptual Links

Ideas in this cycle connect to and extend from the previous cycle which defines a location as its distance from an origin in one dimension. Students need to be able to create and interpret both vertical and horizontal scales on number lines.

This cycle provides an introduction to recording location and movement on a 2D alphanumeric grid reading references horizontally then vertically. These skills will be extended to reading and recording ordered pairs in *Cycle 3: Location and Movement: Cartesian Plane*, *Cycle 5: Polar Coordinates*, and *Cycle 6: Pythagoras*.

Materials

For Cycle 2 you may need:

- Maths mat
- Local area maps
- 1.5cm grids for creating own maps

Key Language

Location, movement, point, region, coordinate, alphanumeric, axis, x-axis, y-axis, coordinate pairs, ordered pairs, direction, compass points, compass bearings, origin, scale, frame of reference, landmarks, key, towards, away from, next to, near, far, over, close, beside, left, right, forwards, backwards, North, South, East, West (also called cardinal points), intermediate bearings of Northwest, Northeast, Southwest, Southeast

Definitions

Alphanumeric grids: grid references are used to locate particular points or regions on a map. Alphanumeric grids use letters to label one axis on the grid and numbers to label the other axis on the grid. This alleviates confusion as to which axis is referred to by which coordinate.

Compass bearings/cardinal points: specific directions using an external frame of reference. North remains north regardless of the orientation of any person or object.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What do you notice about the grid?
- Are the numbers/letters located in the middle of the square or the on the line?
- Are the scales marked at consistent distances vertically and horizontally?
- Where is your starting point or origin? How have you labelled this?
- How many quadrants are there?
- How can we show movement all around the origin?
- How can we number the lines so that we know where the location is relative to the origin?
- How can we be consistent with how we record coordinate pairs?
- Which is the horizontal coordinate? Which is the vertical coordinate?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the student portfolio task *P11: Moving Around*, but are not overtly included as standalone items.

RAMR Cycle

Reality

Discuss ways of recording location on local area maps (2D space). Discuss similarities and differences to defining location along a line from Cycle 1.

Explore a variety of maps with gridlines to see how these points or landmarks are referenced using a combination of capital and lower case letters, capital letters and numbers, or two sets of numbers. Elicit what what conventions students know for reading and using these coordinate systems.

Abstraction

The abstraction sequence for this cycle starts from students' experience of location to create a map on a 2D grid that is labelled using alphanumeric conventions. Students will use this map to record movements and directions. Students will then use their mapping skills to create a map and directions for other students to follow. Language used should progress from egocentric language and direction to external frames of reference. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Start from the Reality activity of describing location within the classroom. Have students describe the location of another point in the room with reference to their location (e.g., Fred's desk is three desks to the left and two desks forward from my desk). To informally develop consistent conventions, define locations across the room, then up or down the room. Record some of these verbal descriptions. Discuss how clear these are to someone coming into the room. What do they need to know to determine the exact location (where to start from)? If students are lacking a wide variety of vocabulary, activities in *Resource 11.2.1 Activities and games for location and direction: Early* may be useful.

Resource Resource 11.2.1 Activities and games for location and direction: Early

2. *Represent/model with materials.* Create more specific descriptions of location of desks and features in the room by labelling a back corner of the room as the origin. Draw a grid on the whiteboard (or use a maths mat or a grid marked on the floor with chalk or masking tape). Use student steps to pace from the corner across the room, then forward to the desired location. Replicate this location on the grid by starting at the origin and moving across the grid (x-axis) the required number of squares, then move up the grid (y-axis) to the target space. Record a selection of these points on the recording grid. Provide students with a grid for recording. Students should record locations of the remaining classroom items on their grids.

Resource Resource 11.2.2 A4 1.5cm Grid paper

3. *Connect to language and symbols.* Discuss with students the need for more concise descriptions of places on the grid. Students should be able to identify letters and numbers as convenient labels for each square. As for Refidex and game conventions like Battleships or Chess, these will be marked to reference the squares (labelling the lines or intersections between lines as points will be the focus of Cycle 3). *Resource 11.2.3 Coordinate Games: Bear Pits* is a game which uses a grid labelling areas connected to rolls of dice. This will be useful practise for finding locations on a grid.

Resource Resource 11.2.3 Coordinate games: Bear pits

Mathematics

In this part of the cycle the focus of activities is to progress from identifying locations to moving between locations using simple left, right, forward, back directions. Mapping activities in the Abstraction phase introduced students to a single quadrant, alphanumeric grid. These ideas need to be extended to include four quadrants and description of location and direction in terms of the cardinal directions and distance from a given origin. It is appropriate to include kinaesthetic activities within the Mathematics phase due to the practical and physical nature of the mathematics.

Connections

Make explicit connections to multiplication and division facts. For example, scaling axes and representing distances on grids require distances to be partitioned. When students are generating maps from informal step counts, ensure they understand how to convert their steps to distances. When working from maps provided by students, they should divide distances by 10m and then multiply by their step count to determine how many steps to take.

Language/symbols and practice

Introducing movement around the classroom map in one quadrant

Once the classroom has stationary objects located on the map and students are comfortable with the naming conventions for locations, introduce movement of location. Use the map to create directions for pathways around the room. Combine the language of left, right, forwards and backwards with coordinate labels. In order to reinforce conventions, move across the map/grid (x-axis) before moving forwards or backwards on the grid (y-axis).

Connect additive operations to movement in a direction. For example, Start at A2, move right 3 spaces (+3), move forward 2 spaces (+2) to D4. Progress to moving backward (e.g., back 3 spaces would be –3).

Use the map to create directions for pathways around the room. Have students give precise directions for the shortest path between two locations on the map, and a longer path between the same two locations on the map. Have students give directions from one part of the classroom to another, record the directions on the board, and then give and record directions to go back. Explicitly discuss what is the same and different about the directions (landmarks and positions stay the same but the order is reversed and left, right turns are opposite. Explore examples of directions in local area maps for going from school to the local shops and back. Compare the two sets of directions to consolidate the inverse idea with directions.

Resource Resource 11.2.4 Examples of directions

Introducing movement around the classroom map in four quadrants

So far the classroom grid map has made use of one quadrant only. Consider moving the origin to the middle of the room. Discuss with students items that are in the quadrant they have been using with alphanumeric coordinates. Practice naming the location of items and movement around the first quadrant. Connect additive operations to change in location, for example, stand at the origin and face in a positive direction on the y-axis, walk 3 steps forward (+3). Introduce facing a negative direction as an option to walking backwards (or negative direction, e.g., –3 steps). Extend to movements that involve students walking past the origin into the other quadrants. Label the quadrants (x-axis with lower case letters and y-axis with negative numbers). Practise identifying location and movement in four quadrants.

Consolidate location and movement on a larger scale

Consolidate students' mapping and direction skills by plotting paths around the school using location and direction language and simple maps. Identify an origin, locate a landmark that represents North, and engage students with constructing a map of the school grounds (or section of the school). In order to include the component of measured distance and scale, it is useful to spend some time having students practice walking 10m a number of times to determine their personal referent for 10m distances. Set up a 10m distance between two lines and have students count their normal walking step across this distance a number of times until they reach a consistent value. Students should note this value down so that they can convert their directions in number of steps to metres for more accurate directions for other students. *Resource 11.2.5 Activities and games for location and direction: Direction* may be useful to engage students' interest as they provide directions with starting points, turns and distances to new points.

Resource Resource 11.2.5 Activities and games for location and direction: Direction

Reflection

Check the idea

Ask students to make up directions to and from parts of the school using their school ground map created in Mathematics. Generate directions that use alphanumeric grid systems and directions using compass bearings and distances.

Apply the idea

Discuss with students following and creating lists of directions. Explore some examples of these from Google Maps (e.g., from school to local business district, collate the location and direction language used. How might walking directions differ from driving directions?

Use a map of the local area for students to create directions to places of interest. Exchange directions with another student to see if they can follow the directions to determine the location and give directions to return. Provide a selection of locations for students to devise directions to. Explore how many different paths there are to each location.

Resource Resource 11.2.4 Examples of directions

Extend the idea

Movement so far has been focusing on across and up/down the map to consolidate x-axis, y-axis conventions. Broaden students' range of compass directions to include North East, North West, South East, and South West.

If students are interested or able it may be possible to consider the directions between these although this level of finesse is less applicable to navigation within the school grounds.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

Can you do this? #3

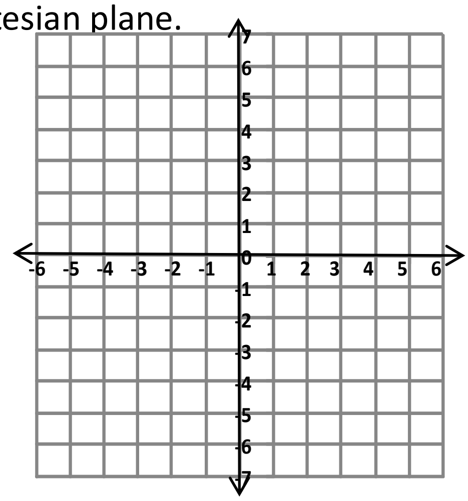
1. Plot and label these points on the Cartesian plane.

   a) A (6, 2)

   b) B (4, ¯4)

   c) C (¯3, ¯5)

   d) D (¯1, 2)



Obj. 11.3.1 a)i. □ ii. □

Obj. 11.3.3 b)i. □ ii. □ c)i. □ ii. □ d)i. □ ii. □

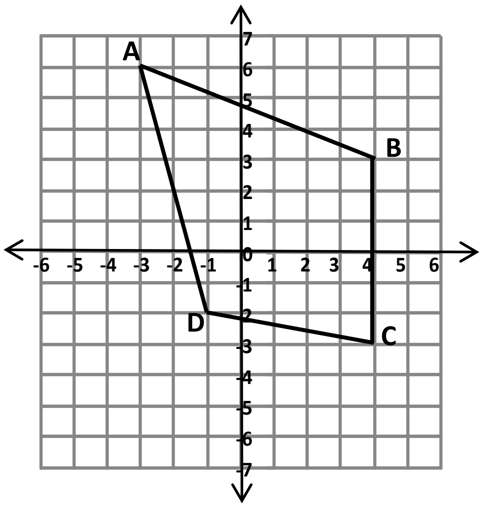
2. Write the coordinates for each vertex of the shape ABCD.

   a) A (______ , ______)

   b) B (______ , ______)

   c) C (______ , ______)

   d) D (______ , ______)



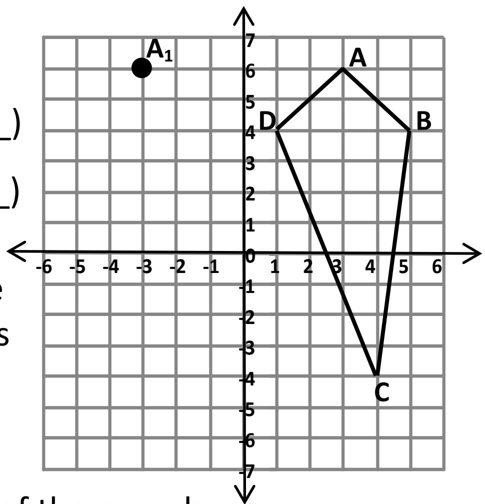
Obj. 11.3.2 b)i. □ ii. □

Obj. 11.3.4 a)i. □ ii. □ c)i. □ ii. □ d)i. □ ii. □

3. Write the coordinates for each vertex of the shape ABCD.

   a) A (____ , ____) b) B (____ , ____)

   c) C (____ , ____) d) D (____ , ____)



Obj. 11.3.1 3ai□ii□ bi□ii□ di□ii□

4. Slide the shape ABCD to the left of the y-axis. Label each vertex and rule lines (A_1 to B_1, B_1 to C_1, C_1 to D_1, D_1 to A_1) to draw the new shape .

Obj. 11.3.5 4i□ii□ iii□iv□

5. Write the coordinates for each vertex of the new shape.

   a) A_1 (____ , ____) b) B_1 (____ , ____)

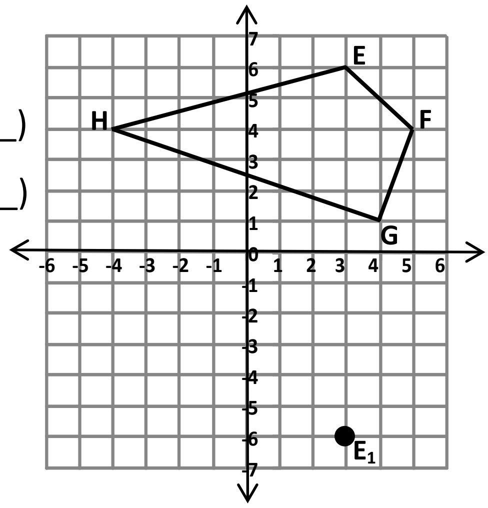
   c) C_1 (____ , ____) d) D_1 (____ , ____)

Obj. 11.3.4 3ci□ii□ 5ai□ii□ bi□ii□ ci□ii□ di□ii□

6. Write the coordinates for each vertex of the shape EFGH.

   a) E (____ , ____)   b) F (____ , ____)

   c) G (____ , ____)   d) H (____ , ____)



7. Flip the shape EFGH over the x-axis. Label each vertex and rule lines (E_1 to F_1, F_1 to G_1, G_1 to H_1, H_1 to E_1) to draw the new shape.

8. Write the coordinates for each vertex of the new shape.

   E_1 (____ , ____) F_1 (____ , ____)

   G_1 (____ , ____) H_1 (____ , ____)

9. Use a ruler to draw the line of symmetry between the two shapes.

Obj. 11.3.1
6ai☐ii☐
bi☐ii☐
ci☐ii☐
Obj. 11.3.6
7ai☐ii☐
iii☐iv☐
Obj. 11.3.4
6di☐ii☐
8ai☐ii☐
bi☐ii☐
ci☐ii☐
di☐ii☐
Obj. 11.3.7
9 ☐

Cycle 3: Location and Movement on the Cartesian Plane

Overview

Big Idea

Cartesian plane location and movement is related to previous explorations of alphanumeric grids in Cycle 2. However, while alphanumeric grid systems commonly used for local area maps, Battleship and chess games label the space between grid markings, the focus for mathematical Cartesian planes is on the lines and intersections between lines. This is the same as the difference between the number track understanding of number and the number line and allows for finer definition of location as fractional distances may be included.

In this cycle, students' understanding of alphanumeric grids and compass directions using four quadrants is extended to Cartesian plane location and movement using x- and y-coordinates. Location and movement on the Cartesian plane may be described by comparing ordered pairs of points, or by combining a measured distance with an angle measured from the positive x-axis. These distinctions need to be clearly differentiated from simple mapping representations of location and direction using the compass direction of North as the central focus for locating position.

Objectives

By the end of this cycle, students should be able to:

11.3.1 Plot points on the Cartesian plane from given positive coordinates. [7NA178]

11.3.2 Find coordinates for a given point on the Cartesian plane (positive quadrant). [7NA178]

11.3.3 Plot points on the Cartesian plane including negative coordinates. [7NA178]

11.3.4 Find coordinates for a given point on the Cartesian plane (all quadrants). [7NA178]

11.3.5 Describe translations in an axis on the Cartesian plane. [7MG181]

11.3.6 Describe reflections in an axis on the Cartesian plane. [7MG181]

11.3.7 Identify line symmetries on the Cartesian plane. [7MG181]

Conceptual Links

Ideas in this cycle connect to and extend from the Cycle 1 which defines a location as its distance from an origin in one dimension and from Cycle 2 which uses Alphanumeric conventions for labelling grids. Students need to be able to create and interpret both vertical and horizontal scales on number lines as well as identify fractional points on these scales. Conventions for reading horizontal axis before vertical axis are reinforced in this cycle.

Ideas developed in this cycle will be extended to mathematical equations to describe linear relationships in Unit 12.

Materials

For Cycle 3 you may need:

- A4 1cm grid paper
- Bear Pits game boards
- Rulers
- Seek and Destroy game boards

Key Language

Location, direction, x-axis, y-axis, origin, ordered pair, x-coordinate, y-coordinate, angle from the x-axis, point, collection of points, line, shape

Definitions

Ordered pair: pair of coordinates in parentheses separated by a comma. By convention the x-coordinate is first, the y-coordinate is second.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you show me the x-axis?
- Can you show me the y-axis?
- Which coordinate do you need first in an ordered pair?
- Do you go across first or up first?
- Which direction on the Cartesian plane do you need to go (+ or –)?
- What happens to the coordinates of the vertices of a shape when you flip it on the Cartesian plane?
- What happens to the coordinates of the vertices of a shape when you slide it on the Cartesian plane?
- What happens to the coordinates of the vertices of a shape when you rotate it on the Cartesian plane?
- Is there a pattern to how the coordinates of the vertices change when it is transformed?
- What changes? What stays the same?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the first section of the portfolio task *P11: Moving Around* which relocates and transforms a shape plotted on a Cartesian plane.

RAMR Cycle

Reality

Discuss uses other than navigation for grid referenced locating systems, for example, seating plans for a theatre, plane seats, and sprinkler systems in orchards or large gardens. Discuss the difference between locating seats (focus on spaces) and locating sprinklers (precise points of reference from an origin).

Discuss labels used for these arrangements (may still be alphanumeric or may be numeric connected to distance from origin). Elicit from students what they understand already about Cartesian coordinates and ordered pairs. Many students will have used these to draw pictures or designs in primary school experiences.

Abstraction

The abstraction sequence for this cycle starts from students' previous experience defining location of spaces on an alphanumeric 2D grid. Students will extend conventions explored with these maps to the mathematical conventions of the Cartesian plane where labels on the x- and y-axis define specific points on the plane. Students will explore locating and moving points and connecting collections of points on the plane. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Use the maths mat as a ready-made grid, or lay out a grid on the floor with masking tape, wool/string or draw with chalk. Connect to students' experiences in Cycle 1 with location on the number line and the use of 0 as the start point or origin. Place (0,0) at the lower left corner of the maths mat or grid. Discuss with students how to label the x-axis (connect to number line with positive numbers). Discuss options for the y-axis. Position a student at (0,0) and instruct them to move +3 along the x-axis. Discuss with students the name for this location. Instruct the student to move +2 up from the y-axis. Discuss with students possible names for this location. Label the y-axis to facilitate further movement in 2D space.

2. *Represent/model with materials/symbols.* Return to the classroom grid from previous activities. Measure from the back corner, across the room and up the room to give ordered pairs of distances for key items. Record a selection of these on the whiteboard.

3. Prepare a grid for the classroom on the whiteboard with gridlines 10cm apart. Discuss the labelling of the axes with students. Provide students with graph paper so that they may create their own Cartesian maps. Locate key items on the grids using the coordinate pairs.

Resource

Resource 11.3.1 Activities for Cartesian planes
Resource 11.3.2 A4 1cm Grid paper

4. *Connect to language/symbols.* Reinforce the language and symbol conventions for Cartesian coordinates. *Resource 11.3.3 Coordinate Games: Seek and Destroy* is a game which uses a grid labelling points connected to rolls of dice. This will be useful practise for finding locations on a grid.

Resource Resource 11.3.3 Coordinate games: Seek and destroy

Mathematics

Connections

Extend single quadrant Cartesian plane to four quadrants

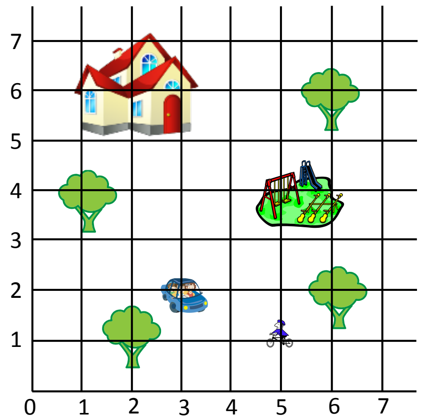
Connect the ideas developed with alphanumeric grids in four quadrants to Cartesian plane axes. Explicitly connect the horizontal and vertical axes to horizontal and vertical number lines. Ensure students understand the labelling of these axes to name the lines not the spaces.

Explicitly discuss similarities and differences between the two Coordinate Games of Bear Pits and Seek and Destroy. Bear Pits focus is on the location of regions on the grid, Seek and Destroy has a game grid where the focus is on the location of points on the grid. Discuss the accuracy of the two systems of locating items in 2D space.

Practise describing location on the four quadrants of the Cartesian plane. Reinforce the convention of naming points by x-coordinate then y-coordinate. Extend students to the continuous nature of axes and the ability to define decimal fractions on the Cartesian plane. Identify where the point (5.5, 3.5) might be; identify other locations that are located on points where the distance from the origin is other than a whole number.

Language/symbols and practice

Describe change in location with direction and distance



Discover and plot points with students standing on coordinate points. This can be varied by using the Maths mat to model animal parks, amusement parks, towns with pictures of items to place on coordinates. Discuss directions to each location on the mat in terms of coordinates (start in positive quadrant only and extend to include all four quadrants).

Rearrange the plan of the model you have created. Describe each change in location using equations as in Cycle 1. For example,

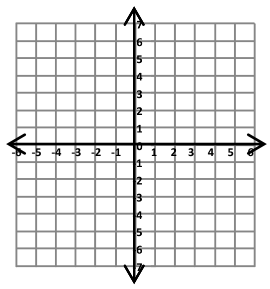
- move the swing set from location (5,4) to (3, 3);
- change the location of the swing set by moving across 2 and down 1 to (3,3);
- change the location of the swing set by ($^{-}2$,$^{-}1$);
- describe the movement of the swing set from (5,4) to (3,3).

Explicitly connect to additive integer operations.

Extend students' thinking of position and location to include decimal point within locations. Discuss how best to plot these points.

Connections

Connect movement on the Cartesian plane with compass bearings



Movement on Alphanumeric grids using compass bearings of N, S, E, W can be connected to movement on the Cartesian plane parallel to the y-axis or the x-axis. Ensure students can flexibly describe movement as across the plane in a positive or negative direction (or East or West) as well as up or down the plane in a positive or negative direction (or North or South).

Connections

Euclidean transformations on the Cartesian plane: Translations

Practise plotting coordinates on the Cartesian plane to generate 2D shapes. Explore translations of shapes across the Cartesian plane. Students should note that the orientation and the size of the shape remain the same as does the position of points on the boundary of the shape relative to each other. What changes is the location of the shape represented by changes in the coordinates or ordered pairs. Generate input-output tables of coordinates of vertices before and after slides. Look for a pattern in the coordinates that may be used to identify the slide. For example, sliding a shape six units horizontally on the Cartesian plane will result in a change in the x-coordinates of each vertex with a magnitude of 6 units, and no change in the y-coordinates.

Extend to sliding shapes vertically and diagonally on the Cartesian plane to identify what changes and what stays the same. Use input-output tables to assist with identifying patterns of change. For example, sliding a shape six units vertically on the Cartesian plane will result in a change in the y-coordinates of each vertex with a magnitude of 6 units, and no change in the x-coordinates. Identifying the change in x-coordinates and change in y-coordinates is a useful precursor to activities in Unit 12 that explore Pythagoras and trigonometry.

Euclidean transformations on the Cartesian plane: Reflections

Practise plotting coordinates on the Cartesian plane to generate 2D shapes. Explore reflections of shapes across the Cartesian plane. Students should note that the size of the shape remains the same as does the position of points on the boundary of the shape relative to each other. What changes is the location of the shape represented by changes in the coordinates or ordered pairs and the location of points on the boundary relative to the original shape. Generate input-output tables of coordinates of vertices before and after flips. Look for a pattern in the coordinates that may be used to identify the flip. For example, flipping a shape across a horizontal line on the Cartesian plane will result in a change in the y-coordinates of each vertex, and no change in the y-coordinates. The distance from the line of reflection to each point will remain the same but in the opposite direction, and the order of points around the shape when both read clockwise (or both read anticlockwise) will be reversed.

Extend to flipping shapes across a vertical line of reflection on the Cartesian plane to identify what changes and what stays the same. Use input-output tables to assist with identifying patterns of change in the coordinates. Identifying the change in x-coordinates and change in y-coordinates is a useful precursor to activities in Unit 12 that explore Pythagoras and trigonometry.

Line symmetry on the Cartesian plane

Explore lines of symmetry on the Cartesian plane. Identify shapes with line symmetry, place these on the Cartesian plane and identify the relationship between reflected points. As a reversing activity to the previous reflecting shapes across a line on the Cartesian plane, provide students with symmetrical shapes or designs on the Cartesian plane and have them identify and mark in the line of reflection or line of symmetry.

Reflection

Check the idea

Plot the points below on the grid. Join consecutive points (A, B, C etc.) with straight lines.

A (6, 2); B (4, 2); C (4, 4); D (6, 6); E (6, 13); F (7, 15); G (8, 13);
H (8, 6); I (10, 4); J (10, 2); K (8, 2); L (8, 1); M (6, 1), A (6, 2).

Make up your own shape. Draw it on a grid with corners where two grid lines intersect. Translate points to coordinates and provide coordinates to other students to draw the resulting shape.

Have students construct a Cartesian plane on grid paper and write their initials on it in block capitals. Identify a collection of points to define the location of each letter on the Cartesian plane. Generate a set of coordinates for these points. Alter the coordinates to move the letters down and across the grid (Euclidean transformation of translation) to a new location.

Have students write their initials in block capitals on a Cartesian plane. Generate a reflected image and mark in the line of symmetry on the resulting design. **Note**: if students colour their block letters, colouring should also be reflected or mirror reverse.

Apply the idea

Use the Cartesian plane to create more accurate maps of the classroom where corners of desks could be located by using ordered pairs of distance across and up the room from the origin. If a desk were to be moved 0.25m across the room, what would its new location be? Use a simple scale of 1cm represents 1m to accurately map the classroom on a Cartesian plane.

Extend the idea

Translations and reflections in a diagonal line

Explore sliding and reflecting shapes in a diagonal line. As for horizontal and vertical translations and reflections, it is useful to use input-output tables to organise information so that patterns of change may be identified.

Sliding diagonally will result in change of both the x-coordinates and the y-coordinates of each vertex. Identifying the change in x-coordinates and change in y-coordinates is a useful precursor to activities in Unit 12 that explore Pythagoras and trigonometry.

Reflections across a diagonal line will result in similar changes to flipping across vertical or horizontal lines of reflection; however changes will be noted in both x-coordinates and y-coordinates. Again, identifying these changes are useful in preparation for Unit 12.

Resource Resource 11.3.2 A4 1cm Grid paper

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #4

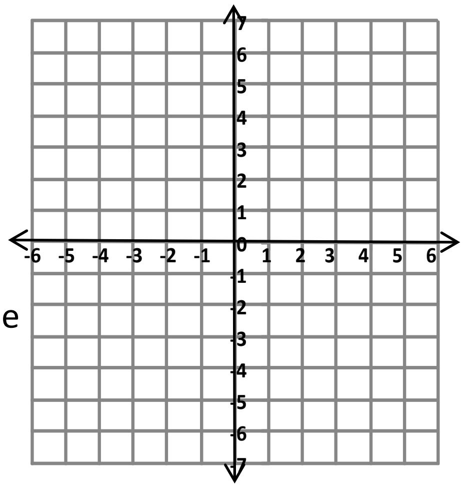
1. Draw a triangle on the Cartesian plane that has:

   (a) A base parallel to the x-axis

   (b) A side perpendicular to its base

   (c) A 30^{o} angle

   (d) Write down the measurement of the other angle in the triangle. _____



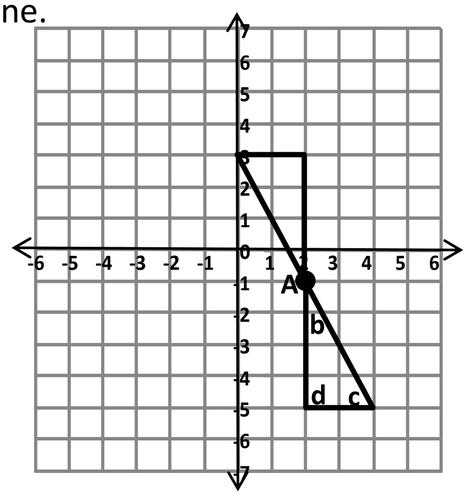
2. Look at the shape on the Cartesian plane.

   (a) How many degrees of rotational symmetry does the shape have?

   ____

   (b) Draw a copy of the shape rotated 90^{o} about its centre (A).

   (c) How many degrees of rotational symmetry does the finished shape have? ________________________



   (d) Angle (b) is 26^{o}. Draw a cross (✘) on the other 26^{o} angles.

   (e) Write down the measurement of angle (c). __________________

   (f) Draw a tick (✓) on the other angles the same size.

   (g) Write down the measurement of angle (d). __________________

   (h) Draw a (□) on the other angles that are the same size.

Obj. 11.4.2 a) □
Obj. 11.4.3 b) □
Obj. 11.4.1 c) □
Obj. 11.4.4 d) □
Obj. 11.4.5 a) □ c) □
Obj. 11.4.6 b) □
Obj. 11.4.4 d)i □ d)ii □ d)iii □ e) □ f)i □ f)ii □ f)iii □ g) □ h)i □ h)ii □ h)iii □

Cycle 4: Measuring Turn on the Cartesian plane

Overview

Big Idea

Angle is the amount of turn between two directions (arms) represented as two arrows from a common starting point. Angle is used to describe the corner of 2D or 3D shapes as well as physical turn to describe movement and location.

The measurement of angle is measured in terms of full turn (360 degrees). It is 360 degrees because the Babylonians visualised a full turn as 6 equilateral triangles meeting at a point and gave each corner of the equilateral triangles an angle of 60 degrees (thus angle, like time, has a base 60. Small angles are also measured in minutes (1/60 of a degree) and seconds (1/60 of a minute).

This cycle extends the idea of angle from measurement of turn used to describe vertices of shape, to measurement of turn applied to movement and location in space and its representation on the Cartesian plane.

Objectives

By the end of this cycle, students should be able to:

11.4.1 Draw and identify angles at a point on the Cartesian plane. [6MG141]

11.4.2 Draw and identify parallel lines on the Cartesian plane. [7MG164]

11.4.3 Draw and identify perpendicular lines on the Cartesian plane. [7MG164]

11.4.4 Find unknown angles on the Cartesian plane. [6MG141]

11.4.5 Describe rotations on the Cartesian plane. [7MG181]

11.4.6 Identify rotational symmetries on the Cartesian plane. [7MG181]

Conceptual Links

This cycle requires students to understand concepts related to measurement of turn (angle). Students need to understand that the greater the count of the unit, the greater the magnitude of the angle; and that smaller units will result in a greater count of units for a given measure than larger units. It is also necessary to connect to geometry concepts of point, line and ray, parallel and perpendicular lines, vertically opposite, alternating, corresponding and co-interior angles. Euclidean transformations explored previously in Unit 04 provide background understanding for exploring transformations of shapes and objects on the Cartesian plane.

This cycle provides an opportunity to extend students' thinking about shapes to measuring angles on shapes and objects and rotating shapes and objects about a point on the Cartesian plane. Future units exploring symmetry, transformational geometry, triangle geometry and Pythagoras extend from ideas explored in this cycle.

Materials

For Cycle 4 you may need:

- String
- Geoboards
- Rubber bands
- Rotagrams
- Protractors

Key Language

Angle, right angle, perpendicular, parallel, protractor, degrees, vertex, vertices, lines, rays, points, full turn, half turn, quarter turn, three-quarter turn, alternate angles, co-interior angles, corresponding angles, transversal.

Definitions

Transversal: straight line that crosses another line or set of parallel lines. Transversals across parallel lines create angles that have recognisable and consistent relationships between them such as alternating angles, corresponding angles, and co-interior angles.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you identify the angle here? Is it acute, obtuse or reflex?
- When you measure the angle, does the measure match up with its description (acute, obtuse, reflex)? Have you read the measurement of the angle on the correct part of the protractor scale?
- What is the angle measured from the x-axis?
- If it is an acute angle, which quadrant will it be in?
- If it is an obtuse angle, which quadrant will it be in?
- If it is a reflex angle, which quadrant will it be in?
- Are the coordinates in that quadrant positive or negative?
- When the shape is rotated about a point, what stays the same? What is different?
- Can you measure the angle of rotation?
- Is there rotational symmetry in this shape?
- How many times does the shape ‘match’ as it is rotated about its centre? How many degrees of rotational symmetry does the shape have?
- If you have to turn it fully around (360^o), is it really symmetrical?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the portfolio task *P11: Moving Around* but are not overtly included within questions.

RAMR Cycle

Reality

The attribute of angle in location and direction relates very closely to the attribute of angle in geometric shape. When two lines radiate from a common point they form an angle. Angles are formed at the corners of 2D and 3D shapes and the study of trigonometry combines measurement of angles and sides in right angle triangles. For students, the attribute of angle is best experienced as a measure of the amount of turn (developed in Unit 04). Briefly revise things that turn, experience turning bodies, connect turning bodies with direction faced so that students understand that angles measure turn about a point. Define angle as the amount of turn from one direction to another. Students may ‘do a 360’ on a skateboard or scooter, turn the pages of a book, observe the movement of the hands of clocks and investigate the properties of shapes by comparing and measuring the corners. These ideas were addressed in *Unit 04: Investigating, measuring and changing shapes*.

There are a range of words used to describe angles in particular size ranges. An acute angle is an angle between 0° and 90°. An obtuse angle is an angle between 90° and 180°. A reflex angle is an angle between 180° and 360°. A revolution angle is a full 360° circle. Connect these to quarter turn, half turn, three quarter turn and full turn. Students should act these out to further consolidate these connections.

Abstraction

The abstraction sequence for this cycle starts with a rich understanding of angle as turn, and the vertices of shapes and extends to development of the use of angles to describe location and movement on the Cartesian plane. The abstraction sequence is as follows:

1. *Kinaesthetic activity*. Have a student stand on a large grid or maths mat with both arms forward. Revise angle as turn by having student move their right arm to point to a quarter turn, then turn their body to face in the new direction. Repeat for half turn, three quarter turn and full turn. N**ote:** for three quarter turn and full turn it is not practical to leave the first arm in the same place. Remaining students should recreate these angles on a Cartesian plane and label them with their names as fractions of a turn, angle type and measurements in degrees.
2. *Kinaesthetic activity.* Use the maths mat or large grid and elastics to reproduce the turns described. Starting on the x-axis, identify a starting coordinate and direction to face, and have a student walk the following path: 2 units forward, quarter turn left, 2 units forward, quarter turn left, 2 units forward, quarter turn left, 2 units forward, quarter turn left. Recreate the path with elastics or wool on the maths mat or grid. Discuss what shape has been created by the path walked. Determine coordinates for each vertex to describe the location.
3. *Represent with drawings.* Recreate the path walked on a Cartesian plane. Identify the starting point and write the directions for the path walked.
4. Discuss with students the important features of the directions for the path walked. That is, a starting point, direction to face, distance to walk and angle to turn each time.
5. Discuss defining points with relation to an origin without using coordinates. Consider what may be needed for consistent interpretations of locations. Introduce the convention of measuring angles on the Cartesian plane anticlockwise from the x-axis. Define each vertex of a shape with a distance from the origin and angle from the x-axis. Generate paths on the maths mat and drawn grids with defined distances and angles from the x-axis. Identify Cartesian coordinates for each point.

Mathematics

Once students have explored defining a line or path on the Cartesian plane using ordered pairs of coordinates for vertices and distances from the origin with angles from the x-axis, extended practice can be obtained by exploring line and angle and connecting measures of angle to earlier work in 2D shapes (Unit 04).

Language/symbols and practice

Angle properties on the Cartesian plane

Explore straight lines, parallel lines and transversals on the Cartesian plane using the large grid or maths mat as well as drawn Cartesian planes. Can students describe a straight line and the attributes that distinguish it from a curved line? Identify parallel lines on the Cartesian plane starting with parallel to the x-axis and the y-axis.

Explore points that have the same angle from the x-axis but differing distances. These points should all appear on the same straight line from the origin. Explore changing the angle from the x-axis with the same distance. These points should generate a curved path around the origin.

Connections

Euclidean transformations on the Cartesian plane: Rotations

Use elastic or wool to create a square on a maths mat or large grid with both diagonals included. Define each vertex of the square using an ordered pair of coordinates. Define each side of the square by its angle from the x-axis and distance. Have a student hold the position of the centre of the square. Have the corner students rotate the square 45^{o}. Discuss with students what has changed and what has stayed the same. For example, the centre of the square is in the same location, the sides are the same length and the corner angles are the same; the angle of each side relative to the x-axis has changed. Find the new coordinates for each corner.

Repeat the exercise on Cartesian planes. Initially, it may be beneficial to have a small card square to trace around as it is rotated through varying degrees and coordinates for the vertices and angles from the x-axis recorded. Use input-output tables to record the changes in angles from the x-axis and starting coordinates to facilitate looking for patterns. Extend to rotating shapes around their centres by relocating vertices.

Rotational symmetry

Explore rotational symmetry of shapes other than squares on the Cartesian plane. Construct 2D shapes on the Cartesian plane, use cut out copies to rotate over the centre of the shapes with a mark to indicate the starting position. Where shapes match on the way around, record where the top mark has moved to. A shape has rotational symmetry if the shape matches twice or more in a full rotation. Draw lines from the centre of the shape on the Cartesian plane to each edge mark. Measure the angles between the lines (they should be the same angle). This is the angle of rotational symmetry. For example, a shape that matched twice on a full rotation would have a straight line between each point of matching and the centre. This would be 180^{o} rotational symmetry. Explore identifying and constructing shapes with rotational symmetry on the Cartesian plane.

Rotation about a point other than the centre

Explore rotation of shapes on the Cartesian plane with one vertex remaining constant. Students should note that sides radiating from the constant vertex will keep the same length, but the angle from the x-axis changes.

Use rotation of a shape about a vertex to create a shape with rotational symmetry. Explore generating shapes with rotational symmetry of 180°, 90°, 60° and so on. See if students can detect a link between how many times the shape matches and the degrees of rotational symmetry. For example, rotational symmetry of 120° should match 3 times in a full rotation and 3 × 120o = 360°.

Connect rotational symmetry and line symmetry

Explore shapes with rotational symmetry on the Cartesian plane. Identify how many rotations and how many lines of symmetry they have. See if students can generalise a rule to connect the symmetries of shapes.

Find examples of shapes with rotational symmetry but no line symmetry and examples of shapes with line symmetry and no rotational symmetry.

Reflection

Check the idea

Engage students with creating a personal logo that has rotational symmetry. Ensure that they can identify the angle associated with the rotational symmetry. Have students create their logo on a Cartesian plane and identify coordinates and angles from the x-axis for critical points within their logo. Generate a set of instructions for the creation of the logo as coordinates, angles and distances to swap with another student. Students should test the instructions to see if they can recreate others' logos with just the instruction set.

Apply the idea

Diagonal distances

Extend students to describing points on the Cartesian plane as a direct distance from the origin, diagonally across the plane. Discuss what other information might be needed to accurately find the desired location (either end point coordinates or orientation from the x-axis). Practise finding locations using a protractor to measure the angle, then a ruler to find the distance away from the origin that fits the description. An accurate Cartesian map of the classroom created to 1cm:1m scale would be appropriate to use to measure angles and distances. Locations recorded as (distance, angle) from the scale drawing will be in cm; ensure students can read these distances as metres for the actual classroom dimensions.

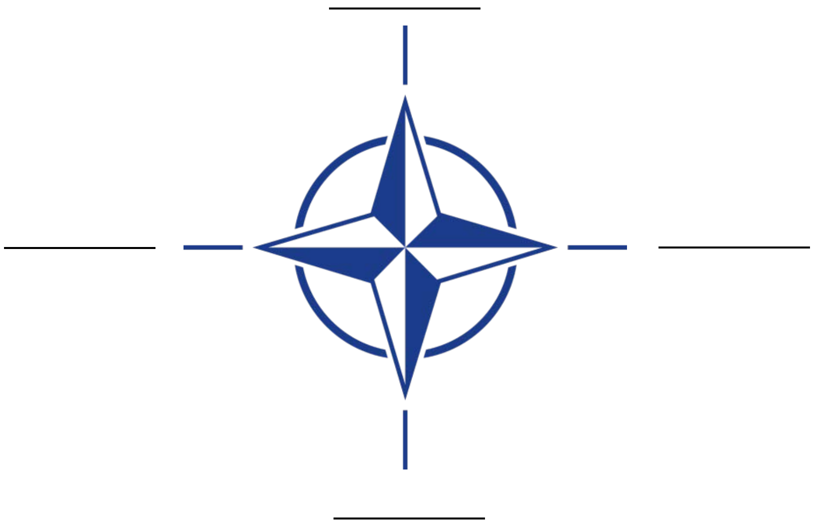
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

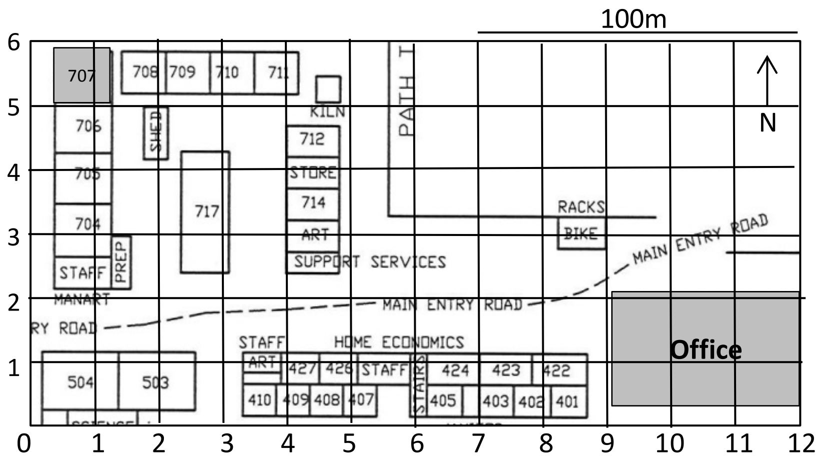
Name:____ Date:________

Can you do this? #5

1. Label the points on the compass.



2. (a) Draw a pathway from the office to classroom 707.



(b) Write directions for your pathway using compass bearings and coordinates to describe distance to walk and turns.

____

____

____

Obj. 11.5.1
i. ☐
ii. ☐
iii. ☐
iv. ☐

Obj. 11.5.2
a) ☐

Obj. 11.5.3
b)i ☐
b)ii ☐
b)iii ☐

Cycle 5: Polar Coordinates

Overview

Big Idea

Cartesian plane coordinates can be extended to navigation in the real world using polar coordinates and distances. Similar to diagonal movement on the Cartesian plane explored in the *Reflection* phase for *Cycle 4: Measuring turn on the Cartesian plane*, navigation in the real world also uses a starting point or origin, direction as an angle or bearing from North and distance to travel.

This cycle explores simple navigation using distance and compass bearings in familiar environments like the school ground and local area.

Objectives

By the end of this cycle, students should be able to:

11.5.1 Identify Cardinal points for compass bearings. [5MG113]

11.5.2 Use a Cartesian grid to describe locations. [5MG113]

11.5.3 Describe routes using distance and turn. [5MG113]

Future Links

Understanding of grid referencing systems, points on a number line as representing distance from the origin, conventions for reading x and y coordinates are needed for this cycle.

Ideas for this cycle can be connected to walking a path using compass bearings and distances that may comprise a 2D shape or part of a larger design.

Materials

For Cycle 5 you may need:

- Silva compasses or Basic rough compass handout to measure turn
- Measuring tape

Key Language

Left turn, right turn, full turn, half turn, quarter turn, three-quarter turn, one-eighth turn, angles of turn, degrees, 45°, 90°, 180°, 270°, North, South, East, West, compass points, cardinal points, distance, estimate, measurement, location, origin, starting point, compass

Definitions

Compass bearings/Cardinal points: directional names used for navigation that use the poles of the Earth as frames of reference. Comprised of four main directions of North, East, South and West.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Which direction are you facing?
- Is the compass aligned correctly?
- How many degrees from North do you need to turn?
- What distance will you travel?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the portfolio task *P11: Moving Around* but are not overtly included within questions.

RAMR Cycle

Reality

Connect to work students have done describing location horizontally and vertically from a starting point or origin. Consider more usual ways of moving from location to location (directly and diagonally). Ascertain from student experience how directions to key points may be written for navigation for boats, cycles, planes, cars or walking. Connect to previously introduced Cardinal points for direction (North, South, East, West).

Abstraction

The abstraction sequence for this cycle starts from students' previous experience of moving location horizontally and vertically on a 2D grid. Students will explore describing locations using distance travelled and angles or bearings from North. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Return to the mapping the classroom activity. Instead of stepping across the room, then towards the front, step out the distance diagonally (turn to face diagonally and count steps directly to the new location). Nominate the front of the room as North. Describe the amount of turn starting from North in everyday language, for example, turn partway right (towards NE), turn only half the way to face right (NE), turn one-eighth of a turn to the right (NNE).
2. *Model/represent with materials*. Mark the vertical axis as North. Record the turn on the whiteboard grid as a dotted line. Measure the distance from the origin along the line and mark the point on the whiteboard grid. Students should measure and mark points on their grid.
3. *Connect to language*. Discuss what a full turn from North would be, half a turn to the right, a quarter turn to the right or left, three-quarter turns right or left, one-eighth turn to the right. Estimate these and connect reference angles to them (e.g., one-eighth of a turn to the right from North would be equivalent to 45^o, a quarter turn from North would be equivalent to 90^o).
4. Define other locations in the classroom by turning an angle from North then measuring.
5. *Refine measurements*. Use a measuring tape to measure distances instead of steps for more accuracy in location on the classroom grid. Use *Resource 11.5.1 Basic Rough Compass Handout* to determine the angle of turn more accurately.

Resource Resource 11.5.1 Basic rough compass

6. *Represent on grid*. Record the more accurate measures on the whiteboard grid and students' grids using rulers and protractors. As before, 1cm can be used to represent 1m.
7. Create instructions for navigating the classroom using distance and turn. Note: Each location becomes the origin for the next distance and turn, North remains in a consistent direction.

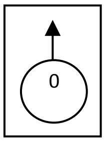
Mathematics

Connections

To extend practice in using distance and direction for finding locations, it is useful for students to be able to estimate 10m effectively. Students should practise walking 10m while counting their steps to determine how many steps they do in every 10m. Ensure students understand that to convert their steps to distances they must divide the total number of steps by their step count for 10m, and then multiply by 10 to arrive at the actual distance in metres. Spend some time connecting the number of steps for 10m with division facts to identify an estimate for 5m, 2m, 25m and so on. Discuss why it is important for each student to know their step count for 10m and why they need to give distances in metres rather than a simple step count (i.e., students take different length steps as they walk).

Language/symbols and practice

Develop Language and apply skills to Compass directions



Explicitly teach students to use directional tools for compass bearings (Silva compasses, analogue watches, *Resource 11.5.1: Basic rough compass handout*).

Always align the arrow with North (use the side of a building or a tree in the distance which is close to North). Then follow the direction given by the angle 0° to 360° from North. Use simple directions to provide students with practise finding a direction and walking a distance.

Practise finding locations when distance and direction are given. Set up a map for students to follow (give starting point and then distances and angles to follow). *Resource 11.5.2 Orienteering Activities* may provide some useful ideas.

Resource Resource 11.5.2 Orienteering activities

Engage students with creating directions for pathways around the school using distances and angles. Swap maps with another group and test the instructions to see if the pathway has been described effectively. When working from maps provided by students, they should take the total distance, divide by 10m and then multiply by their step count for 10m to determine how many steps to take.

Connect compass points other than North with their angle of turn clockwise from North, for example, East is 90°, South is 180°, West is 270°.

Resource Resource 11.5.3 Examples of directions

Reflection

Check the idea

Provide students with compass bearings to create regular shapes on the basketball court or oval. Have one student stand on a length of string at the origin, students should take turns navigating to the next point and standing on the turn in the string (or mark the line with chalk and move the string for the next part of the shape. Complete and identify the created shape.

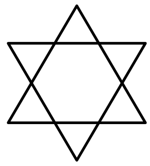
Engage students with planning a set of instructions using compass bearings to create a specific shape for scaling up on the basketball court or oval. Exchange instructions with another group to test out.

Apply the idea

Use a local area map to define a pathway from the school to locations of students' choice and write directions using compass bearings. For example, local parks, shops, train station. Consider creating a course that could be used for a 10km cross country run that avoids high traffic areas. Students will need to revise their multiplication and division facts, map reading, measurement and use of simple scale for this activity.

Extend the idea

Return to the activity in Check the idea. Engage students with creating instructions for designs that use multiple shapes. For example, a set of instructions that generate a string or chalk drawing of a simple house using a combination of triangles, squares and rectangles; create a six-pointed star from two equilateral triangles facing opposite directions. Test the instructions.



Try reversing the instructions to test if the created designs are the same or different. Discuss whether walking the directions in reverse will need the same or different angles; will the distances change or stay the same?

Connecting diagonal distances on the Cartesian plane with polar coordinates

Connect diagonal distances and angles from the x-axis on the Cartesian plane to the generation of polar coordinates and distances. Discuss with students the difference between the two systems. The major difference is the reading of Cartesian plane angles from the x-axis and the reading of polar coordinates as angles from North (essentially the y-axis). Discuss how these angles might be related. Extend students' thinking using complementary angle rules to convert between systems.

Use the Cartesian plane classroom map generated in Cycle 4 as an example to convert to a set of polar coordinates. Locations recorded as (distance, angle) from the scale drawing will be in cm; ensure students can read these distances as metres for the actual classroom dimensions. Recreate this map on the oval using polar coordinates, string and paper. Photograph the recreated space and compare with the classroom space.

Resource Resource 11.5.4 Polar coordinates on the Cartesian plane

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_______________________________________ Date:________

Can you do this? #6

1. Joshua is at the Shops (2, 4) and Selena is at the netball courts (6, 1). Selena said to Joshua 'Meet you half way, so we can share the load of groceries'.

   Use the first quadrant of the grid to answer the following questions:

   (a) Mark and label the location of the shops.

   (b) Mark and label the location of the netball courts.

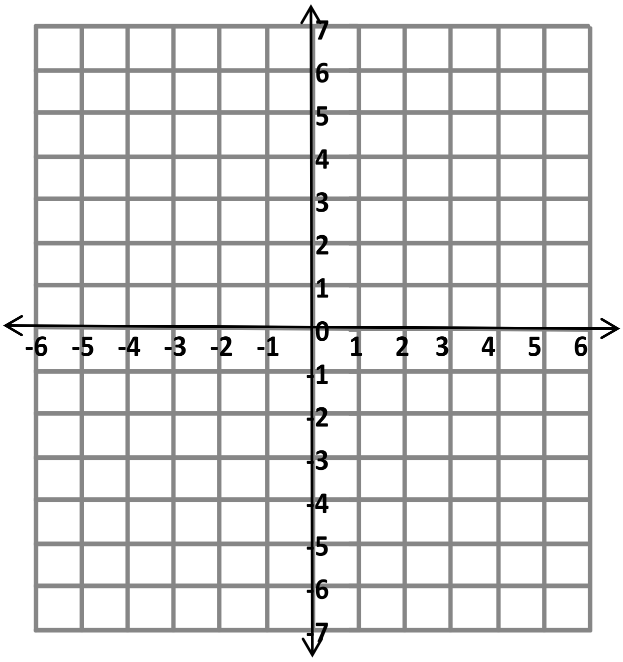
   (c) Calculate the change in the x-coordinates between the shops and the netball courts. ______________________________

   (d) Calculate the change in the y-coordinates between the shops and the netball courts. ______________________________

   (e) Use Pythagoras' theorem to find the distance between the shops and the netball courts in a straight line. ________________

   ____

   (f) Find the coordinates for the midpoint between the shops and the netball courts. ______________________________



Obj. 11.6.1
a) □
b) □
e)i. □
e)ii. □

Obj. 11.6.2
c) □
d) □
f)i. □
f)ii. □

Cycle 6: Pythagoras

Overview

Big Idea

So far students have been able to find distances by measuring and counting. It is also possible to use Cartesian coordinates to calculate the distance between two points by using knowledge of right angle triangles and Pythagoras' theorem. This idea can be further extended to find half way or mid-point of a line as well as third or quarter points. The key focus of this cycle will be to explore Pythagoras' theorem and apply it to finding distance between two points.

Objectives

By the end of this cycle, students should be able to:

11.6.1 Find the distance between two points on the Cartesian plane using Pythagoras. [9NA214]

11.6.2 Find the midpoint of a line segment (interval) on the Cartesian plane using (x,y) coordinates. [9NA294]

Conceptual Links

This cycle requires students to use their multiplication and division facts, knowledge of square numbers and square roots, and properties of triangles to develop Pythagoras' theorem. Measurement of length and area ideas and manipulation of algebraic equations may also be reinforced through this cycle.

Ideas from this cycle will connect to ideas within Unit 12: Modelling with linear relationships.

Materials

For Cycle 6 you may need:

- Large grid or maths mat
- Length of cord or elastic
- Grid paper
- Rulers
- Calculators

Key Language

Mid-point, distance, coordinates, x coordinate, y coordinate, gradient, slope, Pythagoras, one third points, quarter points, hypotenuse

Definitions

Hypotenuse: longest side of a triangle. In a right-angle triangle, the hypotenuse is the side opposite to the right angle.

Pythagoras' theorem: a relationship between the sides of a right-angle triangle. It states that the square on the hypotenuse (the side opposite the right angle) is equal to the sum of the squares on the other two sides.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Is this a right angle triangle?
- Which side is the hypotenuse?
- Which values do you know?
- What is the formula you can use?
- Can you rearrange the formula to find the unknown side?
- What coordinates do you have?
- Can you make a right angle triangle from those coordinates? Draw the other sides.
- Can you work out the side lengths from the information you have?
- What length will that line be?
- So what will be halfway?
- Is there another way you can find halfway on the Cartesian plane?

Portfolio Task

Skills developed through this cycle will contribute to understandings needed to complete the second section of the portfolio task *P11: Moving Around* to determine the point of no return for the aeroplane.

RAMR Cycle

Reality

Discuss the diagonal distances between points. Use the maths mat or grids drawn outside to step out distances vertically or horizontally on the grid. Have students investigate whether they can just count the squares when measuring diagonally or if this is actually a different distance on the grid. Identify the need to be able to calculate diagonal distances when measuring devices are not available.

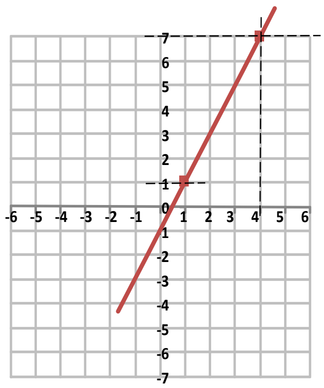
Abstraction

The abstraction sequence for this cycle starts from students' experience of representing locations on a Cartesian plane. Pythagoras' theorem is developed as an alternative to measurement for finding the distance between two points. A suggested sequence of activities is as follows:

1. *Kinaesthetic.* Start with the mat and plot a simple straight line. Discuss with students how they might find the distance between two points on the line.

2. *Connect to language.* Connect mathematical language of distances and Cartesian planes. Discuss with students how coordinates identify the location of each end-point but not the distance between them. Ensure students understand that just counting will not work as the diagonal across squares is not the same length as the horizontal and vertical distance across the squares.

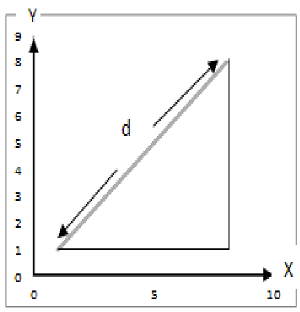
3. *Represent/model with materials.* Plot two points (say, x=1, y=1 and x=4, y=7) as on right. Draw a line between the points (and ongoing). Discuss with students what this line might represent if it was part of a shape, for example, the diagonal across a rectangle or the long side of a triangle (seen in bracing struts in a roof truss or across a gate).



4. *Discover Pythagoras' theorem for right angle triangles.* Use the triangles in *Resource 11.6.1: Discovering Pythagoras' theorem* to scaffold students' exploration of the relationship between the sides and hypotenuse of right angle triangles. Use the data in the table to help students generalise Pythagoras' theorem.

Resource Resource 11.6.1 Discovering Pythagoras' theorem

5. *Apply theorem to Cartesian plane problem.* Return to the initial Cartesian plane problem. See if students can suggest a way to find the measurement of the distance between the two points. This process proceeds as follows:



a) Make a right-angle triangle with the sides parallel to the x and y axes. The sides parallel to the x and y axes have lengths (here, 4–1=3 for x and 7–1=6 for y).

b) Using Pythagoras's theorem, $\mathrm{d}^2 = 4^2 + 7^2$ therefore $\mathrm{d} = \sqrt{16 + 49}$.
Repeat this for other points; ask students for pattern or generalisation.
Elicit that: (a) distance is calculated by Pythagoras's theorem; (b) it is based on the x-*difference* and the y-*difference*; and (c) the actual distance is:

$$\sqrt{(x\text{-}difference\ )^2 + (y\text{-}difference\ )^2}$$

Mathematics

Connections

Connect to and reinforce multiplication and division facts, geometry and measurement aspects of sum of angles in a triangle, right angles and square numbers.

Language/symbols and practice

Explore the use of Pythagoras' theorem to find the distance between points along a line. Practise finding the diagonal distance between points on a line when the horizontal and vertical distances are known. Extend students to applying this idea to find the vertical or horizontal distances when the diagonal and the other distance are known.

Resource Resource 11.6.2 Applying Pythagoras' theorem

Reflection

Check the idea

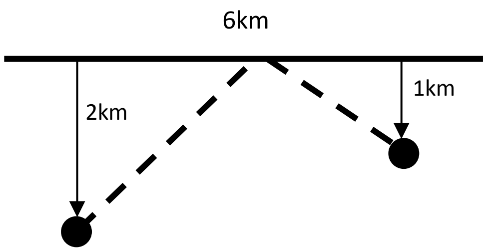
Provide students with examples where they need to find the distance between two points where they are given the distance, direction, slope and one end point and have to find the other end point.

Resource Resource 11.6.3 Practice for finding distances

Apply the idea

Provide students with real world examples of applications for finding distances between points and mid-points. For example,

> A freeway and two towns are positioned as on right. Where will the turn-off be placed so the new roads (dotted line) are as short as possible (and therefore the least expensive)?



Resource Resource 11.6.4 Applications of mid-point

Extend the idea

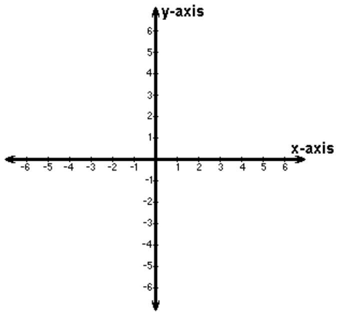
Extend students' work with finding distances between two points to finding the mid-point, third points or quarter points between two points. Give students problems where they know the mid-point and one end point and have to find the other end point.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 11 Portfolio Task – Teacher Guide

Moving Around



Content Strand/s:

Measurement and Geometry
Number and Algebra

Resources Supplied:

- Task sheet
- Teacher guide

Other Resources Needed:

- Rulers

Summary:

The first student task asks the students to use transformations (reflection, rotation, translation) on a Cartesian plane.
The second task uses the reality situation of plane flights to practice using a Cartesian plane.

Variations:

- Repeat the activity but ask students to plan a trip between three locations of their choosing.
- Have students to plot a shape on a Cartesian plane and then ask a friend to use transformations.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Measurement and Geometry

11.5.2 Use a Cartesian grid to describe locations. [5MG113]

11.5.3 Describe routes using distance and turn. [5MG113]

Number and Algebra

11.3.1 Plot points on the Cartesian plane from given positive coordinates. [7NA178]

11.3.2 Find coordinates for a given point on the Cartesian plane (positive quadrant). [7NA178]

11.3.3 Plot points on the Cartesian plane including negative coordinates. [7NA178]

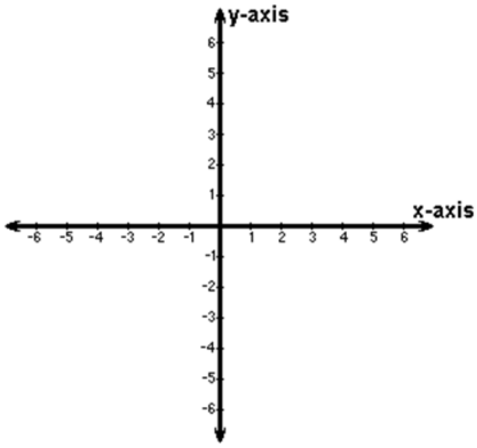
11.3.4 Find coordinates for a given point on the Cartesian plane (all quadrants). [7NA178]

11.6.1 Find the distance between two points on the Cartesian plane using Pythagoras. [9NA124]

11.6.2 Find the midpoint of a line segment using (x, y) coordinates. [9NA294]

Moving Around

Name	
Teacher	
Class	



Task 1:

You will be drawing a shape on a Cartesian plane and moving using given transformations.

Task 2:

Using a Cartesian plane calculate how far an aeroplane will fly.

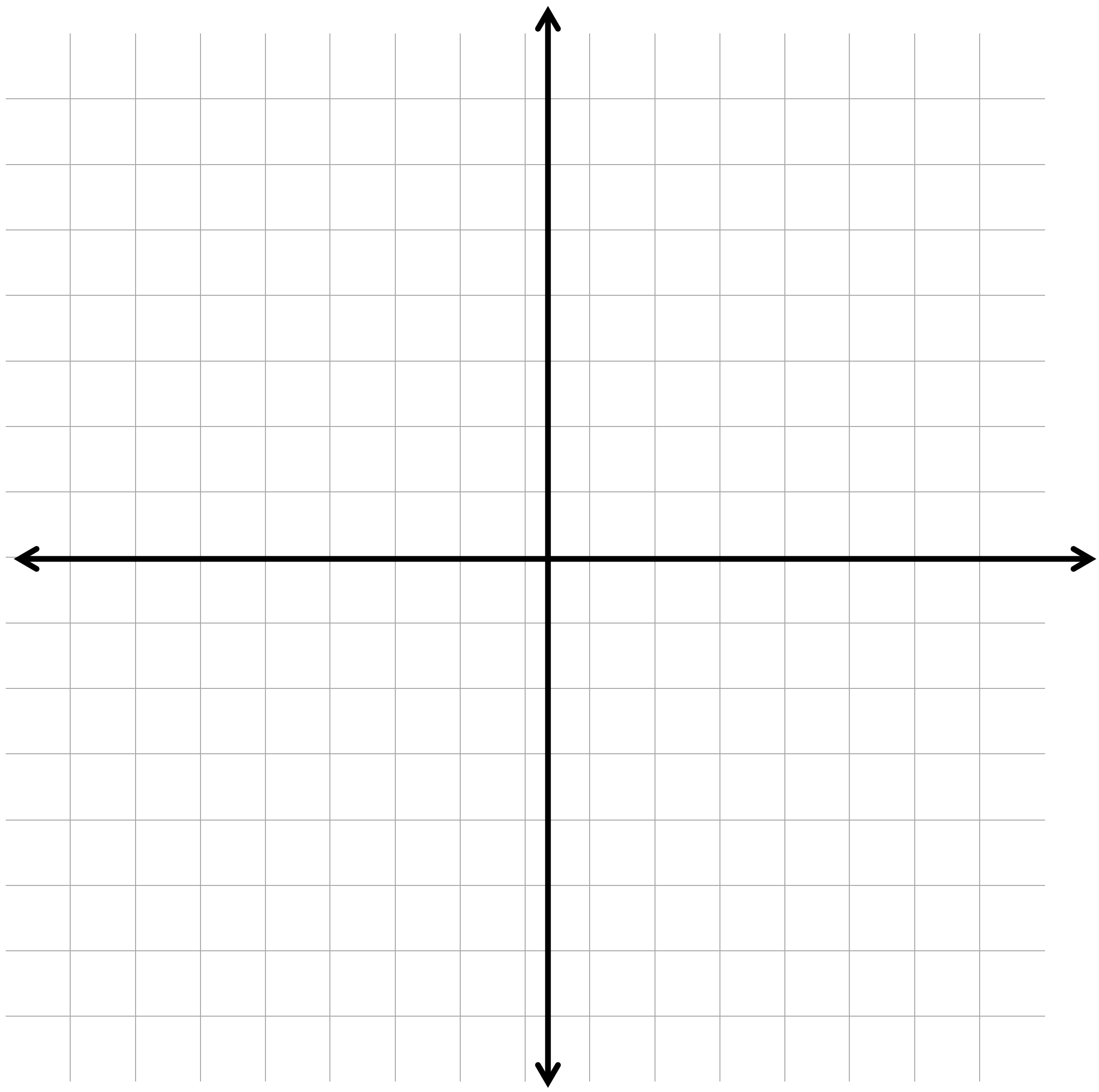
Within Portfolio Task 11, your work demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	**Procedural fluency**	11.3.1 Plot points on the Cartesian plane from given positive coordinates 11.3.2 Find coordinates for a given point on the Cartesian plane (positive quadrant) 11.3.3 Plot points on the Cartesian plane including negative coordinates 11.3.4 Find coordinates for a given point on the Cartesian plane (all quadrants)	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	11.5.2 Use a Cartesian grid to describe locations 11.5.3 Describe routes using distance and turn	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	**Mathematical modelling**	11.6.1 Find the distance between two points on the Cartesian plane using Pythagoras 11.6.2 Find the midpoint of a line segment using (x, y) coordinates	Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar	Development of mathematical models and representations in complex familiar or simple unfamiliar situations	Development of mathematical models and representations in simple familiar situations	Statements about simple mathematical models and representations	Isolated statements about given mathematical models and representations

Comments:

Task 1 – Transforming a shape

1. Label the axes of the following Cartesian plane



2. Use a ruler to draw a rectangle with the vertices:

   A (2, 2), B (5, 2), C (5, 6), D (2, 6).

3. Complete the following transformations:
   a) Turn 180° about point A in an anticlockwise direction. Draw the new position of the rectangle.
   b) Slide the rectangle 4 left and 3 up. Draw the new position of the rectangle.
   c) Flip the rectangle over the x-axis. Draw the new position of the rectangle.
   d) Write the coordinates of the four vertices of the final position of the rectangle:
   e) A (___, ___), B (___, ___), C(___, ___), D (___,___)

4. Write two transformations that will put the rectangle back into its original position.
   a) ____

      ____

   b) ____

      ____

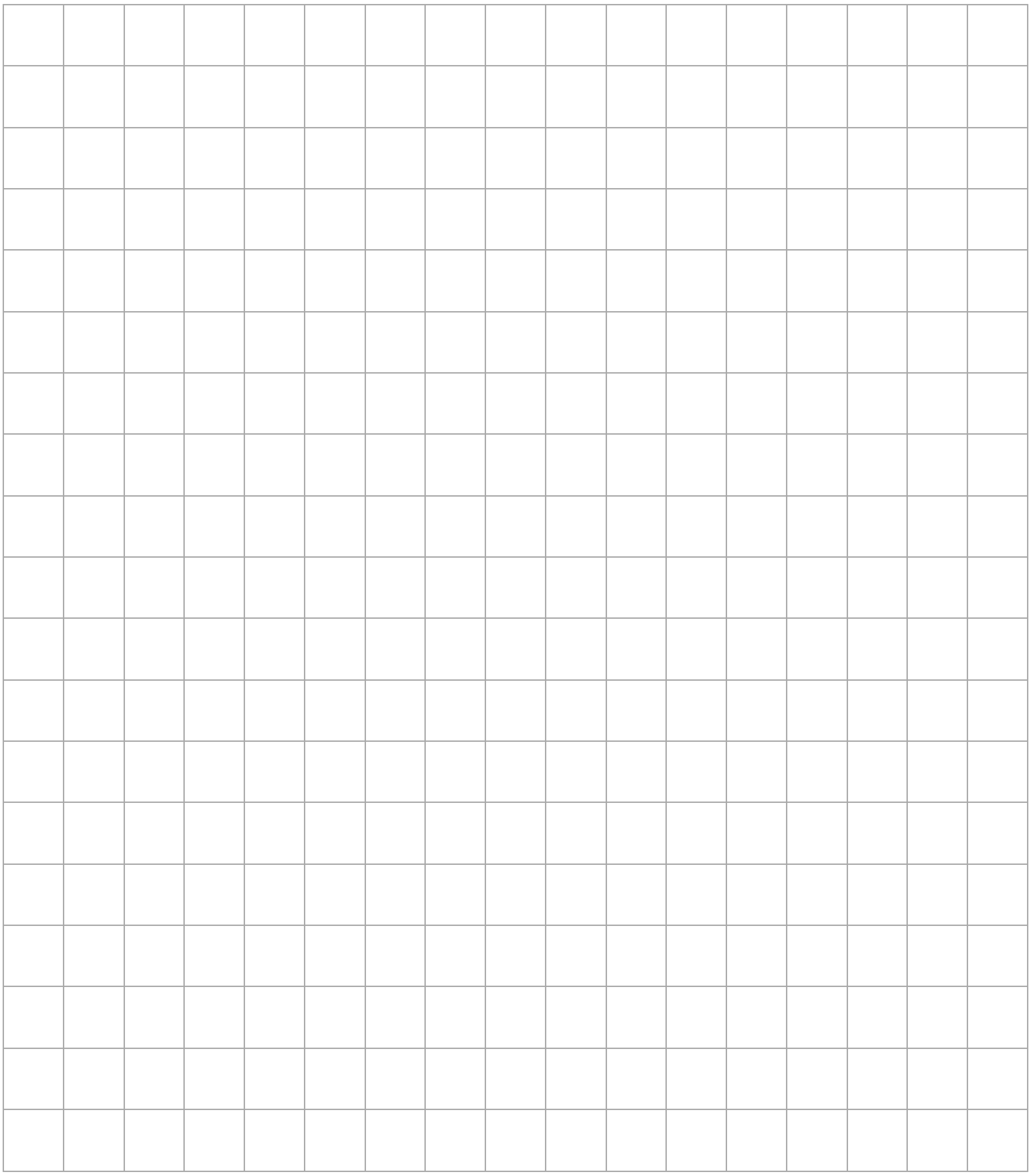
Task 2 – How far will the plane fly?

5. An aeroplane is set to fly from Brisbane to London, with a stopover in Singapore. Use the grid below to draw a Cartesian Plane and mark the location of each city and draw the route.

   a. Brisbane (0, 0)

   b. Singapore (0, 16)

   c. London (3.5, 5)



6. Using Pythagoras' Theorem calculate the distance the plane travelled when

   1 = 1000km

7. If the flight was direct (straight from Brisbane to Heathrow)

   a. What distance would the plane travel? ______________________

   b. Is this longer or shorter than the trip via Singapore?____________

   c. By how much?

   ____

8. For each trip (Brisbane to Singapore, Singapore to Heathrow, and Brisbane to Heathrow) the plane has just enough fuel. If something happens to the plane and it has to land, it will go to the closest airport.

   Calculate where the point of no return is (i.e., the midpoint of each trip).

   Brisbane to Singapore

   Singapore to Heathrow

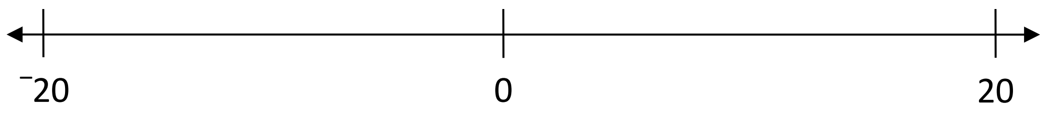
   Brisbane to Heathrow

Name:____________________________________ Date:________

Can you do this now? Unit 11

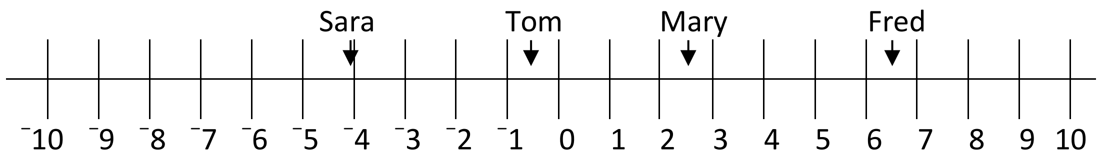
1. Position the following numbers on the number line:

   $^{-}3$, 16, $^{-}12$, $^{-}2$, 5, 9, $^{-}6$, 19



Each unit on the number line represents 100m.

2. What distance from the origin is each person located?



   (a) Sara __________ (c) Mary __________

   (b) Tom __________ (d) Fred __________

3. Fred moved 50m along the line towards the origin. What distance from the origin is his new location? ______________________

4. Sara moved along the line to Fred's beginning location. How far did she move in total? ______________________

5. List these numbers in order from largest to smallest:

   a) 25, 2, $^{-}6$, 1, $^{-}1$, $^{-}14$, $^{-}7$, 9, 0, $^{-}15$

   ____

   b) Write down a number that is greater than the biggest number on the list. ______________________________

   c) Write down a number that is less than the smallest number on the list. ______________________________

Obj. 11.1.2
i☐
ii☐
iii☐
iv☐
v☐
vi☐
vii☐
viii☐

Obj. 11.1.1
2.a) ☐
b) ☐
c) ☐
d) ☐

Obj. 11.1.5
3. ☐

Obj. 11.1.4
4. ☐

Obj. 11.1.3
a) i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐
ix ☐
x ☐
b) ☐
c) ☐

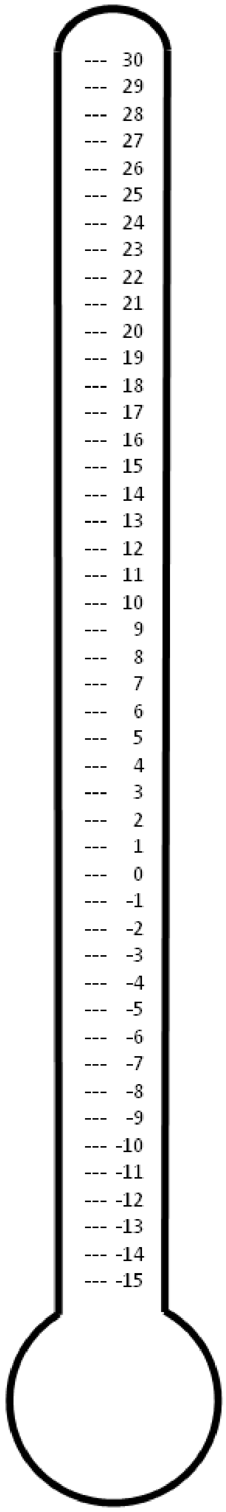
6. The following questions relate to temperatures and reading a thermometer. For each question, write the equation using a symbol for the unknown, and then solve the equation. (Use the thermometer to help you think.)

   (a) It was 5 degrees at midnight. Then, the temperature dropped by 8 degrees. What is the new temperature?

   (b) If the temperature changed from ¯4 degrees by ¯3 degrees, what is the new temperature?

   (c) After sunrise, the temperature rose from ¯2 to 10 degrees. What is the change in temperature?

   (d) Wednesday evening at midnight the temperature was ¯1 degrees. At 2am it was ¯4 degrees. What was the change in temperature?

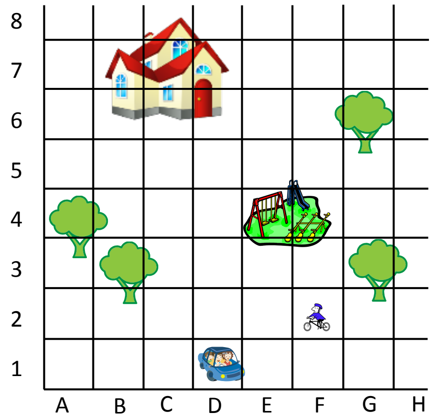


Obj. 11.1.5
a)i ☐
a)ii ☐
a)iii ☐
b)i ☐
b)ii ☐
b)iii ☐
d)i ☐
d)ii ☐
d)iii ☐

Obj. 11.1.4
c)i ☐
c)ii ☐
c)iii ☐

7. Using the grid, identify the object that is at grid reference:

(a) B3 ____________________



(b) F4 ____________________

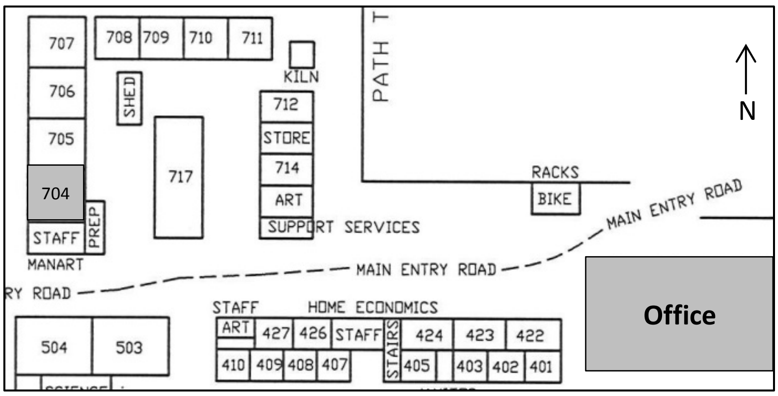
(c) What is the grid reference for the car? ____________________

(d) If the car does a u-turn and travels 3 squares forwards, turns to the left one quarter turn, and travels 2 squares in this direction, turns right one quarter turn, and travels 2 squares in this direction, what will be the new grid reference? ____________________

(e) What other object is nearest this location? ____________________

8. The following map is part of a High School.

(a) Draw how you would walk from the shaded office to the shaded classroom (704).



(b) Give directions to describe how to get there.

____

____

____

Obj. 11.2.2
a) ☐
b) ☐
e) ☐

Obj. 11.2.4
c) ☐
d) ☐

Obj. 11.2.1
a)i ☐
ii ☐
Obj. 11.2.3
b)i ☐
ii ☐
iii ☐

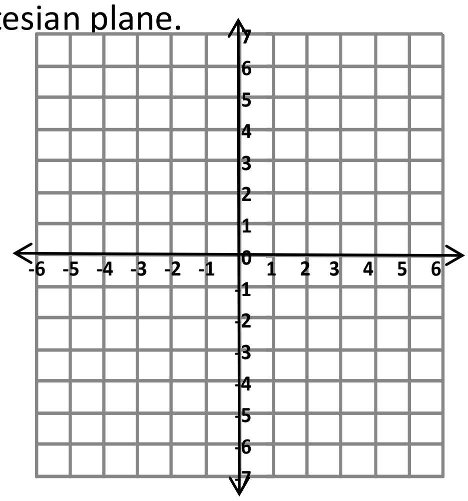
9. Plot and label these points on the Cartesian plane.

   a) A (5, 2)

   b) B (3, ¯4)

   c) C (¯5, ¯3)

   d) D (¯2, 3)



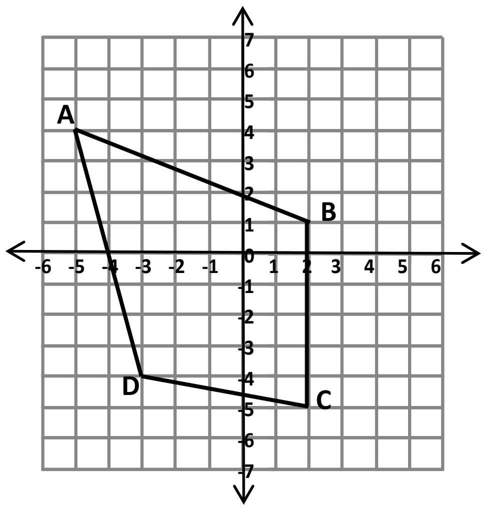
10. Write the coordinates for each vertex of the shape ABCD.

   a) A (______ , ______)

   b) B (______ , ______)

   c) C (______ , ______)

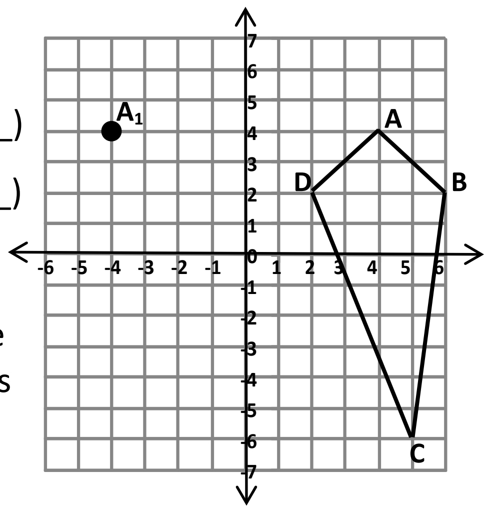
   d) D (______ , ______)



11. Write the coordinates for each vertex of the shape ABCD.

   a) A (____ , ____) b) B (____ , ____)

   c) C (____ , ____) d) D (____ , ____)



12. Slide the shape ABCD to the left of the y-axis. Label each vertex and rule lines (A_1 to B_1, B_1 to C_1, C_1 to D_1, D_1 to A_1) to draw the new shape .

13. Write the coordinates for each vertex of the new shape.

   a) A_1 (____ , ____) b) B_1 (____ , ____)

   c) C_1 (____ , ____) d) D_1 (____ , ____)

Obj.
11.3.1
a)i. □
ii. □

Obj.
11.3.3
b)i. □
ii. □
c)i. □
ii. □
d)i. □
ii. □

Obj.
11.3.2
b)i. □
ii. □

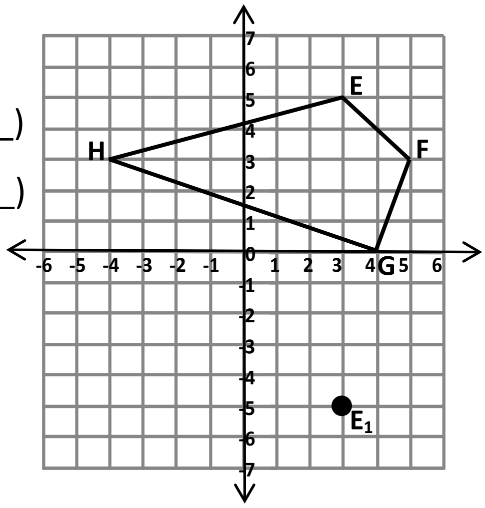
Obj.
11.3.4
a)i. □
ii. □
c)i. □
ii. □
d)i. □
ii. □

Obj.
11.3.1
11ai□
ii□
bi□
ii□
di□
ii□
Obj.
11.3.5
12i□
ii□
iii□
iv□
Obj.
11.3.4
11ci□
ii□
13ai□
ii□
bi□
ii□
ci□
ii□
di□
ii□

14. Write the coordinates for each vertex of the shape EFGH.

a) E (____ , ____) b) F (____ , ____)

c) G (____ , ____) d) H (____ , ____)



Obj. 11.3.1
14ai□
ii□
bi□
ii□
ci□
ii□

15. Flip the shape EFGH over the x-axis. Label each vertex and rule lines (E_1 to F_1, F_1 to G_1, G_1 to H_1, H_1 to E_1) to draw the new shape.

Obj. 11.3.6
15ai□
ii□
iii□
iv□

Obj. 11.3.4
14di□
ii□

16. Write the coordinates for each vertex of the new shape.

E_1 (____ , ____) F_1 (____ , ____)

G_1 (____ , ____) H_1 (____ , ____)

16ai□
ii□
bi□
ii□
ci□
ii□
di□
ii□

17. Use a ruler to draw the line of symmetry between the two shapes.

Obj. 11.3.7
17 □

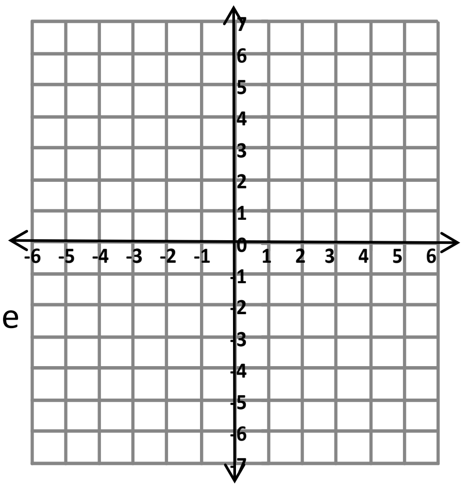
18. Draw a triangle on the Cartesian plane that has:

(a) A base parallel to the y-axis

(b) A side perpendicular to its base

(c) A 40° angle

(d) Write down the measurement of the other angle in the triangle. _____



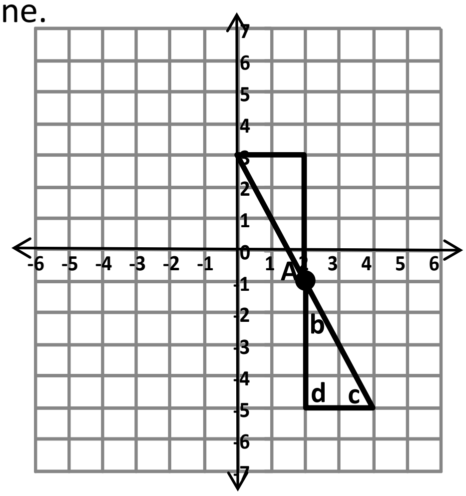
Obj. 11.4.2
a) □

Obj. 11.4.3
b) □

Obj. 11.4.1
c) □

Obj. 11.4.4
d) □

19. Look at the shape on the Cartesian plane.



(a) How many degrees of rotational symmetry does the shape have?

____

(b) Draw a copy of the shape rotated 90° about its centre (A).

(c) How many degrees of rotational symmetry does the finished shape have? ________________________

(d) Angle (b) is 26°. Draw a cross (✗) on the other 26° angles.

(e) Write down the measurement of angle (c). __________________

(f) Draw a tick (✓) on the other angles the same size.

(g) Write down the measurement of angle (d). __________________

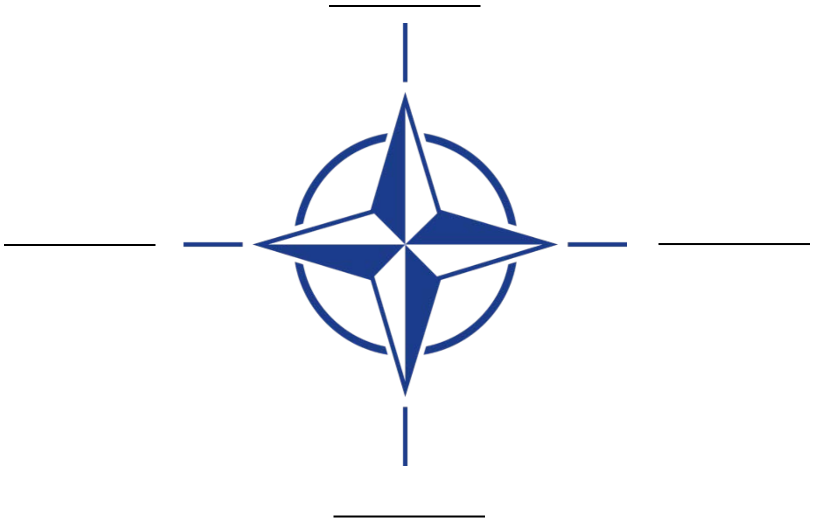
(h) Draw a (□) on the other angles that are the same size.

Obj. 11.4.5
a) □
c) □

Obj. 11.4.6
b) □

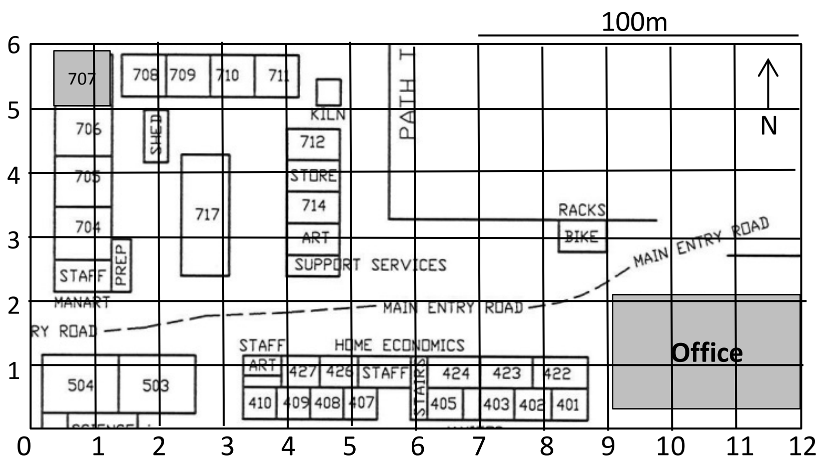
Obj. 11.4.4
d)i □
d)ii □
d)iii □
e) □
f)i □
f)ii □
f)iii □
g) □
h)i □
h)ii □
h)iii □

20. Label the points on the compass.



Obj. 11.5.1
i. ☐
ii. ☐
iii. ☐
iv. ☐

21. (a) Draw a pathway from the office to classroom 707.



(b) Write directions for your pathway using compass bearings and coordinates to describe distance to walk and turns.

Obj. 11.5.2
a) ☐

Obj. 11.5.3
b)i ☐
b)ii ☐
b)iii ☐

22. Joshua is at the Shops (2, 4) and Selena is at the netball courts (6, 1). Selena said to Joshua 'Meet you half way, so we can share the load of groceries'.

Use the grid to answer the following questions:

(a) Mark and label the location of the shops.

(b) Mark and label the location of the netball courts.

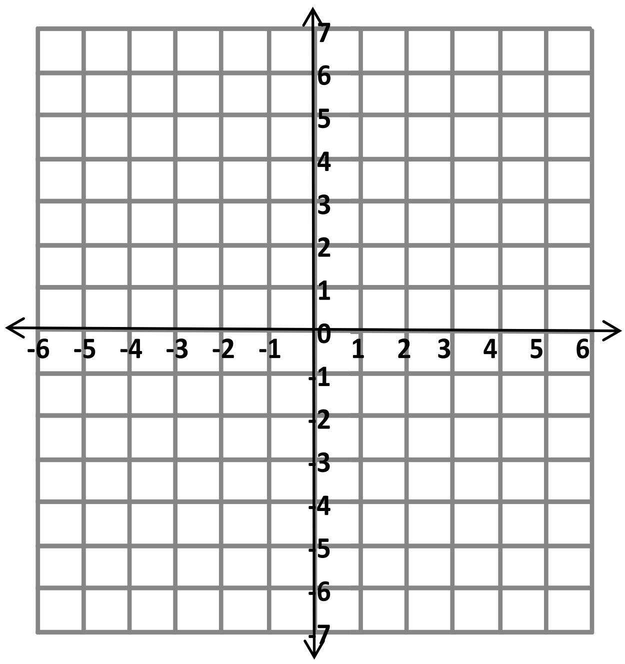
(c) Calculate the change in the x-coordinates between the shops and the netball courts. ____

(d) Calculate the change in the y-coordinates between the shops and the netball courts. ____

(e) Use Pythagoras' theorem to find the distance between the shops and the netball courts in a straight line. ______________________

____

(f) Find the coordinates for the midpoint between the shops and the netball courts. ____



Obj. 11.6.1
a) ☐
b) ☐
e)i. ☐
e)ii. ☐

Obj. 11.6.2
c) ☐
d) ☐
f)i. ☐
f)ii. ☐

XLR8 – Accelerating Mathematics Learning

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