XLR8 – Accelerating Mathematics Learning

QUT

YuMiDeadly
Growing community through education

XLR8 Unit 09

Measuring and maintaining ratios of quantities

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend their ability to measure (including duration), convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.

Scope

This unit is based upon **proportional relationships** between **discrete items** and **measures** described using **ratio notation**. **Proportional relationships** using **ratio** describe **part to part relationships** (fractions describe part to whole relationships).

Ratio may be used to describe the **subsets or parts within a collection**. Once the relationship between the parts is described as a **ratio**, **equivalent collections** may be created that maintain **equivalent relationships** between the parts of the collection as the in the original collection. For two different collections to be **proportional**, the relationship between the parts within each collection must be equivalent. There is also a corresponding **multiplicative relationship between** the like parts of the two collections.

Ratio in measurement contexts may be used to create **similar mixtures** within collections of discrete objects or measures. To present ratios of measures without units, quantities must be represented in the same unit which may necessitate conversion between units of length, capacity, time and so on.

Common strategies to scaffold **proportional thinking** include **proportion tables** and **dual-scale number lines**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts that is to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task P09: Mixing paint consistently accompanying Unit 09 engages students with exploring the use of ratios to describe mixtures and converting these ratios to quantities of each component to create consistent colour across applications or dye lots.

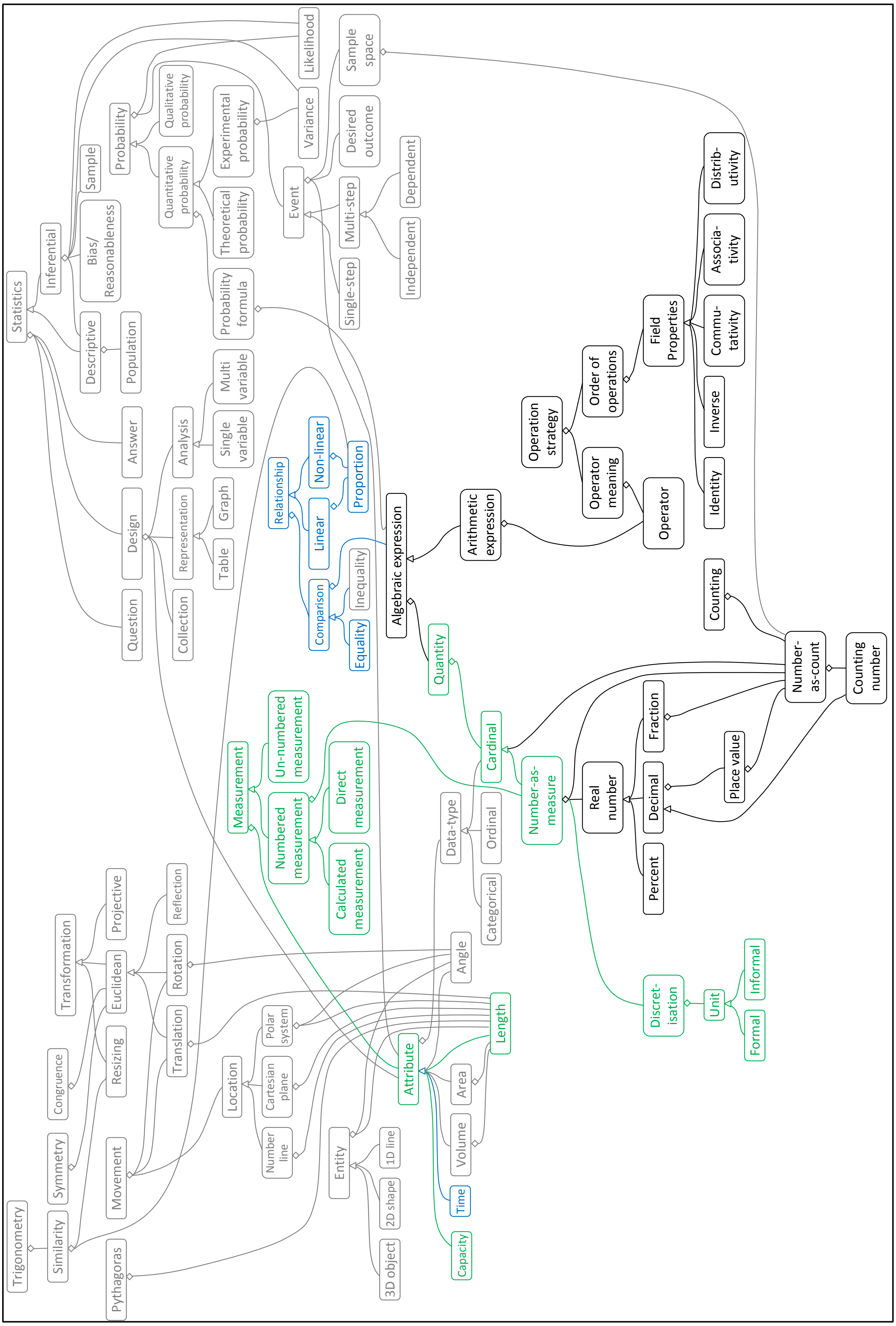


Figure 1. Scope of Unit 09

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Measuring duration

There are three aspects of time to develop, namely, point of time (e.g., reading a clock); sequence of time (e.g., knowing order and names for days of a week); and duration of time (e.g., how much time has elapsed). Measuring duration relies on an ability to read time, know the units used to describe the passage of time, and how times are recorded on timetables or time logs. Determining duration also requires an ability to calculate the difference between a starting time and a finishing time.

Cycle 2: Unit conversions

Students have already explored metric measures and conversion between units. This unit provides an opportunity to further consolidate and revise the conversion of metric measures while focussing on the notion of units, groups and groups of groups. Consolidating metric measure within this unit provides a timely opportunity to reinforce place value ideas, number facts and metric measures in preparation for exploring ratio relationships between quantities of discrete objects and continuous measures.

Cycle 3: Representing ratio

This cycle develops the idea of ratio as the relationship between parts that make up an identifiable whole and uses set, length and area models. Early ratio and proportion ideas are similar to fractions because they describe parts of an identifiable whole. However, whereas in earlier units a fraction represented either a number of parts of a partitioned whole (e.g., ¾ of an apple) or a static multiplicative relationship between two measures (e.g., the amount of cordial is ¼ of the amount of water). In this cycle ratio is introduced to describe the static relationship between quantities within a system.

Cycle 4: Ratio – Relationships between systems

This cycle extends beyond representing relationships within a single system to exploring the equivalence of ratios between systems including when quantities are scaled up or down by a common factor. Students will explore generating and comparing systems to determine whether or not the systems are proportional. Ratio comparisons may also be simplified by a common factor or to a unit comparison in order to facilitate effective comparison and ordering of ratio relationships to determine whether systems are proportional or not proportional (in the case of mixtures, this equates to determining weaker or stronger mixtures). In this cycle the focus will be upon measured quantities in mixtures and will not cover the scaling of geometric figures (which will be covered in Unit 12).

Cycle 5: Ratio – Relationships within systems

This cycle explores the multiplicative relationship between the parts within a system. For example, cordial is mixed with water in a ratio of 1 part : 4 parts, there is four times as much water as cordial or one quarter as much cordial as water. This multiplicative relationship within the system may be compared with the multiplicative relationship within a second system to compare systems for equivalence and determine unknown quantities.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially as it stands. However, teachers may choose to complete the first two units in either order within this sequence.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Measuring duration

Time, century, decade, year, month, fortnight, week, day, hour, minute, second, elapsed time, duration, o'clock, past, to, analogue, digital, time zones, Eastern Standard Time, timetables, time logs, morning, noon, afternoon, evening, night

Cycle 2: Unit conversions

Metric length measures, prefixes and abbreviations: metre (m), centimetre (cm), millimetre (mm), kilometre (km), ruler, tape measure, place value, measuring tape, tape measure, units of measure, capacity, holds more, holds less, cups, spoons, teaspoons, tablespoons, Litres (L), millilitres (mL), mass, weighs more, weighs less, kilograms (kg), grams (g), milligrams (mg)

Cycle 3: Representing ratio

Part-to-part, part-to-whole, ratio, per, mixture, part, whole

Cycle 4: Ratio – Relationships between systems

Part-to-part, ratio, per, equivalent ratio, mixture, part, whole, proportion, unit ratio, between-systems ratio, measures, attributes

Cycle 5: Ratio – Relationships within systems

Part-to-part, ratio, per, equivalent ratio, mixture, part, whole, within-system relationships, factors, multipliers

Name:______________________________________ Date:________

Can you do this? #1

1. (a) Fill in the blanks to list in order all the months of the year:
_____________ , February, _____________ , April, May, _____________,
July, _____________ , September, _____________ ,November, December

Write down how many days in:

(b) March ________ (c) June ________

(d) August ________ (e) November ________

2. Write down the time shown on the clock:

(a) In words ______________________________

(b) In digital format ______________________________

3. Draw the hands on the clock to show 7:40.

4. Michael catches a bus to school at 07:25 and a bus home at 14:50. Write the times Michael catches the bus in 12-hour time?

(a) ____________________ (b) ____________________

(c) How much time (in hours and minutes) passes between when Michael catches the bus in the morning and when he catches the bus in the afternoon? ______________________________

(d) How many minutes pass between when Michael catches the bus in the morning and when he catches the bus in the afternoon?

5. Draw a circle around the time I need to catch a bus from the depot to arrive at the shop between 4:15 and 4:25 pm.

Departs bus depot	**Stops at shop**
3:40pm	4:00pm
3:50pm	4:10pm
4:00pm	4:20pm
4:10pm	4:30pm

6. Western Australia is 2 hours behind Eastern Standard Time.
At 1:00 am Saturday in Queensland, what time and day is it in Perth?

____

7. If a 2½ hr film starts at 15:50, what time will it finish? ___________

Obj. 9.1.1
a) i□
a) ii□
a) iii□
a) iv□
a) v□
b)□c)□
d)□e)□

Obj. 9.1.2
a) □
b) □

Obj. 9.1.2
i. □
ii. □

Obj. 9.1.4
a)i□ii□
b)i□ii□

Obj. 9.1.6
c) i. □
c) ii. □

Obj. 9.1.3
d)i□ii□

Obj. 9.1.5
□

Obj. 9.1.6
i. □
ii. □
iii. □

Obj. 9.1.7
□

Cycle 1: Measuring Duration

Overview

Big Idea

Measuring duration or elapsed time relies on students being able to read time, know the units used to describe the passage of time, and how these times are recorded on timetables or time logs. Determining duration also requires an ability to calculate the difference between a starting time and a finishing time. This is complicated by the fact that time is not in base 10, and does not use a consistent base throughout. As a result, addition and subtraction algorithms can be less effective than mental computation methods when calculating duration.

Objectives

By the end of this cycle, students should be able to:

9.1.1 Describe duration using months, weeks, days and hours. [1MG021]

9.1.2 Tell time to the minute, using the language of "past" and "to". [3MG062]

9.1.3 Convert between units of time. [4MG085]

9.1.4 Convert between 12- and 24-hour time systems. [5MG110]

9.1.5 Interpret and use timetables. [6MG139]

9.1.6 Solve problems involving duration, using 12-hour time. [8MG199]

9.1.7 Solve problems involving duration, using 24-hour time. [8MG199]

9.1.8 Investigate very small and very large time scales and intervals. [9MG219]

Conceptual Links

Calculating duration requires students know the names of common units of time, the quantities these are grouped in, basic number facts and computation strategies. Other skills that may be applied through this cycle include reading graphical representations of data (e.g., transport timetables, television guides).

This cycle reinforces students' knowledge of time telling, units of time and duration of time. These ideas will be useful when exploring rates that involve time (for example, km/hr, m/sec, $/hr).

Materials

For Cycle 1 you may need:

- Geared clocks
- Paper plates and bobby pins
- Empty clock faces
- Timetables
- Australian time zones

Key Language

Time, century, decade, year, month, fortnight, week, day, hour, minute, second, elapsed time, duration, o'clock, past, to, analogue, digital, time zones, Eastern Standard Time, timetables, time logs, morning, noon, afternoon, evening, night

Definitions

Duration: length of time between the start and finish.

Elapsed time: similar to duration.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many seconds in a minute? (Use for any other pairing of units. Extend students to consider non-adjacent units (e.g., seconds in an hour; minutes in a day).)
- What is the base you are using for conversion?
- Check your answer – is your answer reasonable? (Where students are calculating duration across hours they may inadvertently lose an hour or more.)
- What strategy can you use to help you? (Some students may find it easier to compute time if they work from a start time, calculate minutes to the next hour, add hours, then the remaining minutes.)

Portfolio Task

Duration is not specifically included in the portfolio task.

RAMR Cycle

Reality

Determine students' existing understanding of time by discussing with them what is meant by time. Most students should have experience with point of time (reading a clock and telling time), duration of time (how much time has elapsed), sequence of time (months, days of the week, order of hours in a day) and standard notations for representing dates (dd/mm/yyyy for Australia). Identify with students when each perspective of time is relevant. For example, point in time is important for arrival, break and leaving times; sequence of time is important for ordering events; duration of time is important for determining how long an activity has taken (time in the oven for a cake, time taken to complete a race or length of a movie). Ensure students can read a clock to tell time, use time notation (hh:mm:ss) and understand sequence of time. If needed, activities are included in *Resource 9.1.1 Time telling skill, Resource 9.1.2 Sequence of time* and *Resource 9.1.3 Sequencing time of day*.

Resource

Resource 9.1.1 Time telling skill
Resource 9.1.2 Sequence of time
Resource 9.1.3 Sequencing time of day

Ask students where they most commonly see time recorded. Discuss the format used to represent time as point in time and elapsed time. Examples from student experience may include TV programs, bus/train timetables, school timetables, race times (elapsed time). Establish the units of time that students are familiar with and make sure they can order these from longest time to shortest time. Check if they know how many of a smaller unit are grouped for the next unit up. For example, 60 seconds in a minute, 60 minutes in an hour, and so on. Also ensure students understand am and pm.

Abstraction

The abstraction sequence for this cycle starts from students' understanding of telling the time and extends to strategies for determining duration or elapsed time. A suggested sequence of activities is:

1. *Kinaesthetic activity*. Experience activities that take a short or a long time. Have half the students hold a pose while others do things (e.g., hop 10 times, walk across the room). Swap students over and repeat. Discuss which seemed longer, standing still or moving. Relate the passing of time to another activity (e.g., count how many bounces of a ball it takes to walk across the room or to write your name and address).

2. *Represent with materials.* Relate common events (e.g., eating breakfast takes longer than a short cartoon on TV but dressing takes less time). Label strips of paper with events; the length of the strips of paper is determined by the length of time the event takes. Have students list all daily activities and create strips for each, compare relative time spent on each task.

3. *Connect to informal units.* Use a regular action to time events (e.g., bouncing a ball, counting, pulse, a pendulum's swing, etc.). Repeat the kinaesthetic activity, this time recording the number of bounces for each activity. Identify which activity took longer (has a greater count of units).

4. *Experience standard units for measuring duration.* Students sit with eyes closed as teacher sets a timer for 20 seconds. Challenge students to raise their hands when they think 20 seconds is up. Try again with 30 seconds. Teacher secretly sets the timer for a particular time, for example 28 seconds, and students listen until buzzer rings, try to estimate and write down how long it was. Check to see who was the closest. **Note:** It may be necessary to cover the classroom clock.

5. *Represent duration in symbols.* Repeat the kinaesthetic activity, this time using a stopwatch to time events. Practice reading the stopwatch and recording duration in minutes and seconds using standard time notation (hh:mm:ss).

Mathematics

Connections

Relate time to the movement of the sun

Discuss alternative measuring tools for duration including water clocks, sand clocks, egg timers, sundials. To link recording duration of time to the movement of the sun, at the start of the lesson place a large sheet of paper (weighted down) at the base of a pole that casts a shadow. Mark the shadow line and the time on the paper. After 30 minutes, check the shadow and mark in the new line. After another 30 minutes, check the shadow and mark in the new line. Discuss with students what might happen across the rest of the day. Discuss terms such as midday, morning, afternoon, evening (use worksheets and relate times to drawings). Discuss how to tell the difference between morning and afternoon when recording 12-hour time.

Recognise the linear scale of analogue clocks

The change in position of clock hands on analogue clocks should also be linked to fractions of a full circle turn. Connect the fraction of a circle turn to the language of quarter past, half past and quarter to when telling time on an analogue clock. Record times using numbers and units. Make sure to connect time on digital clocks to time on analogue clocks, for example, 5:43 on the computer clock to 17 minutes to 6 on an analogue clock.

Use paper plates, protractors and rulers to create individual paper plate clocks. Determine how many degrees of turn would show half past the hour on a circle, repeat for quarter past and quarter to. Mark in these major quarters on the paper plate clocks. Determine how many degrees of turn would show 5 minutes (30^o) and mark these in. Label each of the 30^o marks with numbers (1 – 12). Students should determine how many degrees of turn for each minute on the clock and make smaller marks for these (6^o). Bobby pins may be used for hands or cardboard hands secured with a split pin.

Language/symbols and practice

Converting between standard units for time, very small and very large intervals of time

The most commonly used units of time are not base 10. Hours, minutes and seconds are related by base 60. The relationships between units of time need careful consideration and exploration to be understood. Using place value charts may be useful to assist students in converting between units for time. An empty place value chart is included in *Resource 9.1.4 Converting standard units for time* to assist with this practice.

Decimal fractions are used for measuring durations less than one second. Stopwatches measure time in minutes, seconds, tenths of seconds, hundredths of seconds, thousands of seconds. Metric prefixes are used to refer to fractions of a second just as they are with other metric measurement units, for example one thousandth of a second (0.001sec) is one millisecond. So, one tenth of a second would be 100 thousandths of a second and 100 milliseconds.

Similarly, decimal conversions and prefixes are used for very large intervals of time such that ten years is one decade, one hundred years is one century and one thousand years is one millennium. Students may find it interesting to consider where the count of a year begins. Consider that years with the prefix 19xx were all within the twentieth century. Consider and discuss whether the 21st century started January 01, 2000 or January 01, 2001.

Resource Resource 9.1.4 Converting standard units for time

Timetables (not to be confused with times tables)!

Discuss the purpose of a timetable in everyday life. Consider this from multiple points of view – the students, the adults in their community, people living in rural areas, and people living in a busy city.

Have a number of timetables available for the students to practise reading. Make up some stories telling the student they need to be at a sport game by 2pm, what bus or ferry should they catch from the bus stop or jetty near their house? Try to use authentic timetables that are specific to the local environment. When students are comfortable with these, explore a timetable for a local but larger town or city (like Gold Coast or Brisbane). As an extension activity, consider a tube timetable for London, having the student pretend they are travelling to see an event or visiting sites within the city. Let the students select the event and locate the timetable on the Internet.

Resource Resource 9.1.5 Local transport timetables

24-hour time

Discuss where students might have seen 24-hour time used. Responses might include on digital clocks, on some TV guides, train or airline ticketing information. Some students may have heard it on TV shows from America, or in use by the military.

Discuss why 12-hour time might have been invented. What are the advantages? What are the disadvantages?

Present a clock face that shows both 12- and 24-hour time. If students created their own paper plate clock face, they could adapt this to show 24-hour time as well (this may help them see the connection between 12- and 24-hour time more clearly. Have the students create a table of corresponding times (e.g., 1pm = 1300 hrs, 2pm = 1400 hrs and so on). Discuss strategies (other than memorising) for the relationships between 12- and 24-hour times. Engage students in creating a story using 24-hour times and illustrate with representations of the clock times.

Time logs

Get students to fill in time logs – time in and time out – ask them to calculate the time spent on each job (this could also be used to determine total journey from combined modes of transport). Try to get examples of actual logs so students can see relevance in filling them in.

If students have trouble working out how long spent on a job, try the following:

1. **Number line**. Develop a number line for 7 am to 4 pm – mark in hours, ¼ hours. Get students to mark in starting and ending times on number line. Use the line to work out how long was taken. Look at the hours and the minutes between start and finish. To assist getting over the 12 noon restart of the numbers – do the difference in 2 steps – up to the 12 noon and after the 12 noon, or translate the number line to 24-hour time.
2. **Additive subtraction**. Do subtractions using the "shopkeeper's algorithm", that is, starting from smaller and building to larger. For the example start 8:35 am and finish 1:20 pm, make a series of jumps from 8:35 to 1:20 – first go to the next hour, then to 12, then to finish hour and finally to finish minutes. Then add up all the jumps – this is the difference.
3. Explore with students what will happen if sudden advancements in technology enable train and bus travel to double in speed. What would this do to the time taken for those parts of the trip? What effect would this have on the total travel time?

8:35 am	
	0 h 25 m
9:00 am	
	3 h
12:00 noon	
	1 h
1: 00 pm	
	0 h 20 m
1:20 pm	
	4 h 45 m

Reflection

Check the idea

Check local cinema starting times for movies along with their duration. Use the duration of the movie to work out what time the movie will finish.

Apply the idea

Have students use paper copies of local bus and train timetables to plan three options for travel from school to Brisbane Central Station to arrive before 9:00am. They should estimate walking time where necessary between stops and calculate the total time including waiting time for connecting services.

Consider changing the parameters of the problem. If a bicycle is used instead of walking between stops, what effect will this have on the total travel time? Students may identify that the bicycle cannot go on the bus. Calculate the effects on travel time for either case. Assume that a bicycle is available for use between the bus and the train if necessary.

Extend the idea

Time zones

Print out a copy of Australia and its time zones map. Choose a map that clearly illustrates the time zones, as well as the different states. Photocopy the map – one for each pair of students. Discuss the concept of time zones with students. Explain that time zones exist because the sun rises and sets at different times in different parts of Australia and the world. Discuss the implications of this in terms of TV coverage of big sporting events like the Olympics (events are held in the daytime in London, but viewed 'LIVE' during the night time in Australia) and operating businesses across the Queensland/New South Wales border on the Gold Coast during daylight savings time.

Shine a torch and hold it over a globe. Rotate the globe and explain that the torch represents the sun. As you rotate the globe, children will see how the sun illuminates different parts of the earth at different times. It is important to rotate the globe correctly, from left to right. (Australia has time generally 10 hours ahead of London, so we need to get the torchlight before London). Discuss what happens as the torchlight shines across Australia. Have students look at their map and relate the gradual lighting of Australia on the globe to the regions of the different time zones as follows:

- Australia has three time zones: Eastern Standard Time (EST) for the eastern states, Central Standard Time (CST) for the Northern Territory and South Australia and Western Standard Time (WST) for Western Australia. CST is half an hour behind EST and WST is two hours behind EST.
- Daylight Saving Time: New South Wales, Australian Capital Territory, Victoria, Tasmania and South Australia wind their clocks forward an hour during the Daylight Saving period from the beginning of October to the beginning of April. Western Australia, the Northern Territory and Queensland do not observe the practice of Daylight Saving. Discuss reasons for this (consider the length of twilight and sunrise times as you travel away from the equator).

Ask students to determine the times in different time zones. State different times in different time zones and ask them what time it would be in another time zone. For instance, you could ask, "If it's 3 o'clock in the Western time zone, what time is it in the Eastern time zone?" or "If it's midnight in Brisbane, what time is it in Perth?" Rolling a die may be a useful way to randomize times chosen.

Resource Resource 9.1.6 Australian time zones map

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________ Date:________

Can you do this? #2

1. Fill in the following conversions:

   (a) 15 cm = __________ mm

   (b) 3 m = __________ cm

   (c) 5.5 km = __________ m

   (d) 13 000 mL = __________ L

   (e) 2 000 L = __________ kL

   (f) 250 mL = __________ L

   (g) 5 t = __________ kg

   (h) 600 kg = __________ t

   (i) 600mg = __________ g

2. Name something that would be reasonable to measure using the following units.

   (a) L ______________________

   (b) mL ______________________

   (c) kL ______________________

   (d) g ______________________

   (e) kg ______________________

   (f) t ______________________

3. Would you use Litres or millilitres to measure:

   (a) a cup of cordial? ______________________

   (b) a car radiator? ______________________

   (c) a large saucepan? ______________________

4. Would you use kilograms or grams to measure:

   (a) your mass? ______________________

   (b) a loaf of bread? ______________________

   (c) a watermelon ______________________

Obj. 6.7.2
a) ☐
b) ☐
c) ☐

Obj. 9.2.2
d) ☐
e) ☐
f) ☐

Obj. 9.2.3
g) ☐
h) ☐
i) ☐

Obj. 9.2.1
a) ☐
b) ☐
c) ☐
d) ☐
e) ☐
f) ☐

Obj. 9.2.1
a) ☐
b) ☐
c) ☐

Obj. 9.2.1
a) ☐
b) ☐
c) ☐

Cycle 2: Unit Conversions

Overview

Big Idea

Students have previously explored metric length measures and conversion between units. This cycle provides an opportunity to further consolidate and revise the conversion of metric measures while focussing on the notion of units, groups and groups of groups. Consolidating metric measure within this unit provides a timely opportunity to reinforce place value ideas, number facts and metric measures in preparation for exploring ratio relationships between quantities of discrete objects and continuous measures.

Objectives

By the end of this cycle, students should be able to:

6.7.1 Connect decimal representations to the metric system. [6MG135]

6.7.2 Convert between common familiar metric units of length. [6MG136]

9.2.1 Choose appropriate units of measurement for length, capacity and mass. [5MG108]

9.2.2 Convert between common familiar metric units of capacity. [6MG136]

9.2.3 Convert between common familiar metric units of mass. [6MG136]

Conceptual Links

Metric measure and conversion between units are closely connected to notion of unit, counting, place value, number facts (particularly extended tens facts) and computation strategies.

This cycle provides the basis of metric conversion for length units that can be applied to most units of metric measure. Identification of attributes and the development of the measuring process is a skill that can be applied to measurement of any attribute.

Materials

For Cycle 2 you may need:

- Metre ruler
- Standard ruler (cm and mm)
- Dressmakers' tape measure
- Specialist measuring devices
- Tape measure (possibly from PE)
- Height stick (possibly from PE)
- Pocket tape measure (2m or 5m)

Key Language

Metric length measures, prefixes and abbreviations: metre (m), centimetre (cm), millimetre (mm), kilometre (km), ruler, tape measure, place value, measuring tape, tape measure, units of measure, capacity, holds more, holds less, cups, spoons, teaspoons, tablespoons, Litres (L), millilitres (mL), mass, weighs more, weighs less, kilograms (kg), grams (g), milligrams (mg)

Definitions

Capacity: the amount an item will hold, usually expressed in liquid measures.

Informal unit: any unit that may be repeated to measure an attribute. For example, spoonsful, cupsful (capacity); marbles, pens, dice (mass)

Standard prefixes: prefixes used to describe multiples of a standard unit. For example, milli, kilo, centi

Standard units: metric measure units. For example, Litre (capacity); grams (mass).

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many grams in a kilogram? (Use for any other pairing of units. Extend students to consider non-adjacent units (e.g., milligrams in a kilogram; grams in a tonne).)
- Check your answer – is your answer reasonable?
- Are you changing from a small unit to a big unit or a big unit to a small unit?
- If you are changing from a small unit to a big unit, will there be more or less of them?
- Do you need to multiply or divide?
- What factor will you multiply or divide by?

Portfolio Task

Within the portfolio task, students will need to convert between units of capacity.

RAMR Cycle

Reality

This cycle begins with the familiar context of length measurement. Choose situations where it is appropriate to measure in millimetres, centimetres, metres and kilometres. Reinforce the magnitude of each unit and connect to the decimal fraction representations.

A task that requires students to measure mass, length, width of a box to determine how many may safely be stacked on a shelf will provide an opportunity to check these skills.

Resource Resource 9.2.1 Shelf stacking task

Abstraction

The abstraction sequence for this cycle extends from students' previous experience with metric measures. This sequence needs to connect standard measuring units with place value ideas, standard prefixes and abbreviations. If students are weak in metric measures for length, capacity or mass and place value, it may be beneficial to repeat this cycle for each attribute. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Revise students' physical sense of the magnitude and relative size of standard units. Find examples in the real world that are 1mm, 1cm, 1m and 1km long. Have students estimate 10cm or a decimetre, then use a ruler to check.
2. *Represent with materials*. Using different colours of 1cm grid paper, cut ten strips that are 10cm in length. Tape alternating colours together to form a folding 1m measuring strip. This can be used to identify centimetres and groups of 10 centimetres to make up a metre. Reinforce the count with unit name. Students should be encouraged to count each decimetre as they go as one-tenth of a metre, two-tenths of a metre and so on to one metre.
3. *Connect to language*. Reinforce connections between metric measure and decimal place value using decimal fractions. Use the maths mat as a large place value chart and lay out place value cards from thousandths to millions. Place a Metre card under the ones. Research the meaning of prefixes kilo, centi, milli. Place these cards under their respective place values respective to the metre. Fill in non-standard metric prefixes to cover empty places (these exist although not used within the SI unit set). See *Resource 9.2.2 Metric prefixes chart for student desks* and *Resource 9.2.3 Connecting place value to metric measure* for more detail.
4. *Reinforce place value link to conversion*. Construct a larger copy of *Resource 9.2.4 Metric expanders* (kilometres, metres and millimetres) and cut it out. Fold as for number expanders. Use them to relate km, m and mm as for place-value cards. The connection needs to be made between the unit of measure and the ones place. If measuring in metres, then the unit for metres is placed under the ones place on the place value chart. The decimal fraction then represents centimetres and millimetres. To convert to millimetres, slide the chart across the top of the digits until the ones place is above the number for the millimetres. Focus on the movement of the chart three places to change from metres to millimetres. Connect to the multiplication for this conversion. *Resource 9.2.5 Metric slide rules* may also be useful here.

Resource

Resource 9.2.2 Metric prefixes chart for student desks
Resource 9.2.3 Connecting place value to metric measure (large cards)
Resource 9.2.4 Metric expanders length
Resource 9.2.5 Metric slide rule length

5. Familiarise students with a range of measuring tools/devices (rulers are simplest, dressmakers' tape measures and building tape measures are more complicated to read). Identify which unit name corresponds with each marking on the measuring tool/device to assist with correct reading of the measuring device. Discuss which unit will be most appropriate to measure an item. For example, is it more efficient to measure the width of the classroom (desk, notebook, ruler, eraser, whiteboard, school fence) in metres or millimetres? Practice representing measures using decimal fractions of a chosen unit and converting between units. *Resource 9.2.6 Determining personal referents for length* may be used or dolls, model cars, other items of interest may be measured as practice.

Resource Resource 9.2.6 Determining personal referents for length

Mathematics

Conversion of metric measures

Find contexts for students to practice measuring and converting between measures using metric measure and decimal fractions. Clearly make connections between conversions between kilometres and metres and the use of extended tens facts and place value understanding.

Revise perimeter

Explore addition of decimals in the context of measuring dimensions of items and finding the associated perimeter. Simple multiplication of decimals can also be explored here by adding a length to a width and then doubling.

Explore capacity

Check that students are aware of the attribute of capacity and related metric measures. If necessary, repeat the Reality and Abstraction phases to clearly establish the attribute of capacity, standard measures and conversion between these measures. Ensure during abstraction activities that students experience short and fat containers compared to tall and narrow containers to extend their estimation skills in ordering capacities of common containers.

Resource

Resource 9.2.2 Metric prefixes chart for student desks
Resource 9.2.3 Connecting place value to metric measure (large cards)
Resource 9.2.7 Metric expanders capacity
Resource 9.2.8 Metric slide rule capacity

Explore mass

Check that students are aware of the attribute of mass and related metric measures. If necessary, repeat the Reality and Abstraction phases to clearly establish the attribute of mass, standard measures and conversion between these measures. It is important for students to experience hefting and estimate the masses of items. Interesting activities can include estimating the mass of a pumpkin or watermelon, ordering common fruit or vegetable items by mass, ordering same size packets containing different items for unexpected differences in mass. Typical brainteasers to identify which is heavier, a kilogram of feathers or a kilogram of books can be used here to effect. Consider also, which takes up more space (or requires greater capacity) a kilogram of feathers or a kilogram of books. It can be interesting to explore the mass of 1L of water to connect these measures.

Resource

Resource 9.2.2 Metric prefixes chart for student desks
Resource 9.2.3 Connecting place value to metric measure (large cards)
Resource 9.2.9 Metric expanders mass
Resource 9.2.10 Metric slide rule mass

Reflection

Check the idea

Use a Thinkboard or Concept map to connect equivalent measures in kilometres, metres, centimetres, millimetres to a measurement of a common item. Have students highlight the most accurate unit to use for the item.

Check that students are also able to connect equivalent measures in kilograms, grams, milligrams and kilolitres, litres and millilitres. These Thinkboards may also be created as puzzles to be recreated.

Resource Resource 9.2.11 Equivalent metric measure Thinkboards

Apply the idea

Engage students with real measuring tasks that require them to convert between measurement units. Ensure that students are able to convert between measures of length, capacity and mass.

For example, find the mass of 1L of water. Explore making a box that is 10cm × 10cm × 10cm. Fill the box with rice from a 1L jug. What do you notice?

If a shelf can hold 150kg, how many 1L boxes of fruit juice could be stacked on it?

In a small group, mould play dough into a 10cm × 10cm × 10cm block. If you change the length or width of this block, what do the other dimensions become (e.g., doubling its width will make a 10cm × 20cm × 5cm block). Measure different play dough blocks to find other combinations of measures that will make a 1L box? How many mL would these boxes hold?

If a shelf is 1m long and 600mm wide, how many 1L boxes of fruit juice could be stacked in one layer on it?

What combination of measures for the 1L box will allow for most boxes of juice to fit in one layer on the shelf?

If this shelf can hold 50kg, how many layers of boxes of fruit juice could be stacked on it?

Extend the idea

Explore other instances of measures that are related and expressed using fractions and decimals. For example, scales on measuring jugs connect Litres and millilitres, scales for mass connect grams and kilograms. Consider the relationships between these measures, their fractions, decimal fraction representations and conversions between units. Explore how Scientific notation might be used to record very large quantities of small units in these instances.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #3

1. During one day (24 hours), the local radio station played 9 hours of rock and roll and the rest of the time was country music. What is the ratio of rock and roll to country music?

   Rock and Roll : Country = ______________ : ________________

Obj. 9.3.1 i. ☐ ii. ☐

A packet of M&Ms contains three colours: 12 red, 7 blue and 16 brown.

2. (a) Write the ratio of red to blue to brown M&Ms.

   ____________________ : ________________ : ____________________

Obj. 9.3.1 i. ☐ ii. ☐ iii. ☐

   (b) What is the ratio of red M&Ms compared to the whole packet?

   ____________________ : ____________________

Obj. 9.3.2 i. ☐ ii. ☐

3. Cordial and water are mixed to make 1000mL of drink. 300mL of the drink is cordial. What is the ratio of cordial to water?

   Cordial : Water = ____________:____________

Obj. 9.3.3 i. ☐ ii. ☐

4. Sand and cement are mixed to make concrete. The ratio of sand to cement is 7 : 2. What is the quantity of parts in the whole concrete mix? ____

Obj. 9.3.4 ☐

5. (a) Count how many boys and girls are in your class.

   Boys ___________________ Girls ______________________

   (b) What is the ratio of girls to boys in your class?

   ____________________ : ____________________

Obj. 9.3.1 i. ☐ ii. ☐ iii. ☐

   (c) What is the ratio of girls compared to all students in your class?

   ____________________ : ____________________

Obj. 9.3.2 i. ☐ ii. ☐ iii. ☐

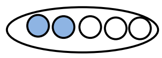
Cycle 3: Representing Ratio

Overview

Big Idea

This cycle develops the idea of ratio as the relationship between parts that make up an identifiable whole and uses set and length models. Early ratio and proportion ideas are similar to fractions because they describe parts of an identifiable whole. However, whereas in earlier units a fraction represented either a number of parts of a partitioned whole (e.g., ¾ of an apple) or a static multiplicative relationship between two measures (e.g., the amount of cordial is ¼ of the amount of water). In this cycle ratio is introduced to describe the static relationship between quantities within a system.

For example, in the following diagram 2 parts out of 5 are shaded and 3 parts out of 5 are unshaded. The fractions $\frac{2}{5}\ and\ \frac{3}{5}$ represent the shaded and unshaded parts respectively. It is also possible to identify the ratio of $shaded\ parts\!:\!unshaded\ parts$ as 2:3.



Objectives

By the end of this cycle, students should be able to:

9.3.1 Represent part-part relationships in simple ratios. [7NA173]
9.3.2 Represent part-whole relationships in simple ratios. [7NA173]
9.3.3 Can identify missing part given other part and whole. [7NA173]
9.3.4 Can identify the quantity in the whole given the parts. [7NA173]

Conceptual Links

Like fractions, ratio focuses on identifying parts of a whole or collection. It is important for students to understand the notion of unit and partitioning a whole or collection. From their work with fractions, students should be adept at identifying part – whole relationships. Basic number facts and computation strategies may also be reinforced throughout this cycle. This cycle will also provide an opportunity for added practice with finding fraction of a whole or collection.

Identifying and describing the relationship between parts of a collection using ratio notation leads to understanding proportion and unit ratio, ratio between systems (scaling on maps and plans), and ratio within systems (similar shapes and figures).

Materials

For Cycle 3 you may need:

- Double number line (drawn or elastic)
- Counters or unifix cubes
- Cordial, cups, water

Key Language

Part-to-part, ratio, per, equivalent ratio, mixture, part, whole

Definitions

Part-part: describes the relationship between the quantities within a system

Part-whole: describes the relationship between a quantity and the whole system. Similar to the part-whole relationship shown with common fractions

Ratio: relationship between quantities within a system

System: a collection of related quantities. For example, the water and cordial in a glass of drink, the boys and girls in a particular class.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the whole system?
- What are the quantities that make up the parts of the system?
- How are the quantities related to each other?
- How is each quantity related to the whole system?
- Can you write that in symbols?
- If you see 1 : 4, what does it mean? How can you describe this in words? (e.g., if you are mixing cordial, what would you use?)

Portfolio Task

Within the portfolio task, students have an opportunity to practice identifying ratio relationships and converting ratios to and from fractions.

RAMR Cycle

Reality

Discuss contexts where ratio is used in students' realities. Some common examples are in mixtures like cordial, concentrated juice, chemicals for weed spraying, grain mixes for animal feed, base recipes for cakes, scones, biscuits or fruit punch. Highlight the language students have experienced in these contexts (e.g., one part cordial to four parts water; one part in four; one part for every four; 30mL chemical in 5L water; 185g butter, 1 cup sugar, 3 eggs, 2 cups flour, $\frac{1}{4}$ cup milk). Discuss with students how recipes or mixtures have a defined relationship between the parts. Ensure that students understand that to make up the mixtures consistently requires an understanding of the relationship between each of the parts and the relationship between these parts and the whole. Also, if the relationships between each of the parts, and between each part and the whole is known, the amount of each part within a given whole or collection may be determined.

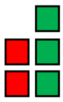
Abstraction

The abstraction sequence for this cycle builds student understanding of ratio as the relationship between each of the parts of a whole or mixture. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Identify a context relevant to students to follow through. For example, make up a mixed group of boys and girls in the class (e.g., 2 girls, 3 boys) is simple to experience and act out. Other relevant and interesting ideas to follow through can be mixing cordial or ingredients for drinks, simple biscuits or grain mixes (e.g., simple wild bird food equal parts of sunflower seeds, cracked corn, raisins, peanut butter).

Resource Resource 9.3.1 Mixture ideas

2. *Represent with materials and connect to language.* Represent the parts of the whole using unifix cubes or counters. For example, 2 girls (part), 3 boys (part), group of 5 (whole).



3. *Record in proportion table.* Use a table format to connect the language of the ratio to the symbolic parts.

   This table may be read as: The ratio of girls is to boys is 2 is to 3 (2 : 3).

girls	boys
2	3

   Ensure that students can also record this ratio in its symbolic form of 2 : 3, and that they recognise that it is a different ratio from the ratio of boys is to girls which would be 3 : 2.

   Note: This proportion table format will be useful to scaffold investigation of equivalent ratios.

4. *Repeat steps 1 – 3 with other contexts* to reinforce the ratio representation (part : part) and language. For example, chemical : water; seeds + corn + raisins : peanut butter; cordial : water.

chemical	water
30mL	1000mL

Seeds + corn + raisins	Peanut butter
3 parts	1 part

cordial	water
1 part	4 parts

Mathematics

Language/symbols and practice

Provide students with additional opportunities to practice:

- Expressing ratio using ratio notation
- Working with part and whole quantities to express part : part and part : whole ratios
- Multiplicative comparison problems. For example, The local bakery always sells twice as many chocolate chip cookies as it sells oatmeal cookies. What is the ratio of chocolate chip cookie sales to oatmeal cookie sales?

Extend to include examples with decimal fractions instead of whole numbers within the ratio (e.g., one and a half times as many chocolate chip as oatmeal). *Resource 9.3.2 Ratio problems* includes some links to Internet-based worksheets for ratio. Note that some problem types will be more suited to Cycle 4 and Cycle 5.

Resource Resource 9.3.2 Ratio problems

Connections

Ratio expressed as fraction notation

Part : part and part : whole comparisons should also be connected to fractions of the group. For example, if there are two girls for every three boys in a group, the whole group is five students, two fifths of which are girls and three fifths of which are boys.

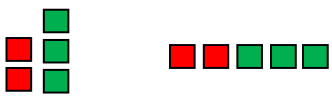
2 girls
for every
3 boys

2 girls
out of
5 students

Ratio notation, proportion tables, double number lines

Reinforce the connection between ratio notation, set materials and proportion tables. Explore set materials arranged as lengths (similar to frequency graphs). Express the lengths as ratios. For example, 2 girls for every 3 boys may be represented in all of these ways:

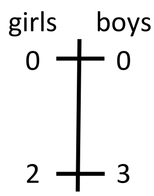
2 girls : 3 boys



girls	boys
2	3

Connect the length model and proportion table to a double number line. Demonstrate for students how parts expressed in a proportion table might be expressed on a double number line.

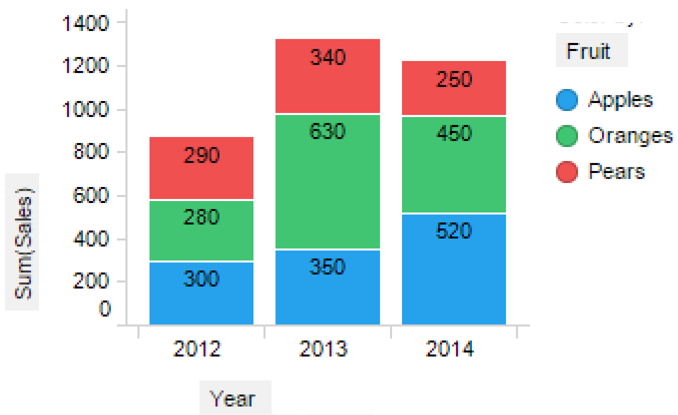
girls	boys
2	3



Stacked bar graphs

Stacked bar graphs may provide another context for exploring and expressing ratio relationships. Analysis of the data on the graph for each bar can provide quantities of attributes for expression as ratios. For example, in the graph below, fruit sales of apples to oranges to pears for 2014 could be expressed as 520 : 450 : 250.

Fruit Sales

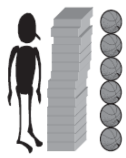


Reflection

Check the idea

Use models developed throughout the cycle to solve the following problems:

1. 117.5 kg of sand is mixed with 47 kg of cement, what is the ratio?
2. Mr. Brown is 6 basketballs tall. He is also as tall as a pile of 42 pizza boxes. What is the ratio of basketballs to pizza boxes for measuring height?



3. 1L juice concentrate is added to 4L water to make fruit juice drink. What capacity container is needed to put this in?
4. What fraction of the fruit juice drink is juice concentrate?
5. What fraction of the fruit juice drink is water?
6. If 5L of the fruit juice drink is 100%, what percentage of the fruit juice drink is juice concentrate?
7. If 5L of the fruit juice drink is 100%, what percentage of the fruit juice drink is juice concentrate?

Apply the idea

Encourage students to select a favourite recipe for analysis. Have them identify the relationship between selected parts of the recipe. For example, scones could be considered as relationships of flour to milk; flour to eggs; flour to sugar; or, dry ingredients to wet ingredients.

Engage students in constructing their own ratio comparison problems to share with the class for solving. Find how many different ratio comparisons can be made from quantities in the classroom. For example, ratio of desks to windows, ratio of students to desks, ratio of desks to chairs, ratio of students to teachers, ratio of students with long hair to students with short hair and so on.

Resource Resource 9.3.1 Mixture ideas

Extend the idea

Extend students' thinking to mixtures with ratios that use three or more terms. For example, fertiliser may have an NPK mix of nitrogen to phosphorous to potassium = 16 : 4 : 8 or concrete may be mixed in ratio of cement to sand to gravel = 1 : 3 : 5.

Extend the connection between ratio and fractions to include percentage. For example, a cordial mix of 1 part cordial to 4 parts water will have one fifth cordial and four fifths water. This can be represented as percentages in the final mixture of 20% cordial and 80% water.

Explore problems which will require comparison of ratios. Consider these intuitively and qualitatively rather than working with symbols and generating a precise response (this is the focus for Cycle 5). For example, if I have mixed a cup of cordial (250mL) in the ratio of cordial to water = 1 : 4, then read the bottle and find that this mix is supposed to be 1 : 5, will the cordial be too strong or too weak? How might you fix this problem?

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Can you do this? #4

1. The ratio of water to cordial in a drink is 4 parts : 1 part. What ratio of water to cordial will be needed to make a double amount of drink?

   ______________________ : ________________________

2. Circle the ratios that are equivalent to 3:5

   (a) 12:25
   (b) 21:35
   (c) 24:40
   (d) 32:52

3. List the following mixes of cordial from weakest to strongest.

   1 : 5 , 1 : 1.5 , 1 : 8 , 2 : 4 , 3 : 4

   __________ , __________ , ____________ , _________ , _________

4. There must be 1 instructor for every 6 students on a kayaking course.
   (a) If 24 students want to do the course, how many instructors will be needed?

Instructors		
Students		

   (b) If there are 5 instructors available, what is the maximum number of students that can do the course?

Instructors		
Students		

Obj. 9.4.1
i) ☐
ii) ☐

Obj. 9.4.2
a) ☐
b) ☐
c) ☐
d) ☐

Obj. 9.4.3
i) ☐
ii) ☐
iii) ☐
iv) ☐
v) ☐

Obj. 9.4.4
a) i ☐
ii ☐
iii ☐
iv ☐

Obj. 9.4.4
b) i ☐
ii ☐
iii ☐
iv ☐

Cycle 4: Ratio Relationships – Between Systems

Overview

Big Idea

This cycle extends beyond representing relationships within a single system to exploring the equivalence of ratios between systems including when quantities are scaled up or down by a common factor. Students will explore generating and comparing systems to determine whether or not the systems are proportional. Ratio comparisons may also be simplified by a common factor or to a unit comparison in order to facilitate effective comparison and ordering of ratio relationships to determine whether systems are proportional or not proportional (in the case of mixtures, this equates to determining weaker or stronger mixtures). In this cycle the focus will be upon measured quantities in mixtures and will not cover the scaling of geometric figures (which will be covered in Unit 12). Similarly, ratio notation to describe the multiplicative relationship between two proportional systems will not be used in this unit.

Objectives

By the end of this cycle, students should be able to:

9.4.1 Scale ratios up or down to generate proportional relationships between systems. [8NA188]
9.4.2 Compare between ratios to determine if systems are equivalent. [8NA188]
9.4.3 Order non-proportional ratios according to relative 'strength'. [8NA188]
9.4.4 Find unknown values in ratio comparisons using relationships between equivalent systems. [8NA188]

Conceptual Links

This cycle relies on student understanding of ratio and part-part relationships. Number facts and computation strategies may also be reinforced throughout this cycle.

Identifying and describing proportion and unit ratio is further applied in contexts of ratio between measures, similar shapes and figures, and scaling on maps and plans.

Materials

For Cycle 4 you may need:

- Double number line (drawn or elastic)
- Cordial, cups, water
- Proportion sticks
- Counters or unifix cubes

Key Language

Part-to-part, ratio, per, equivalent ratio, mixture, part, whole, proportion, unit ratio, between-systems ratio, measures, attributes

Definitions

Between-systems ratio relationships: compares relationships between measures of like attributes across systems to determine if systems are in proportion, or uses known proportionality to determine unknown quantities. For example, are two batches of cordial the same (equivalent) if one mixture of cordial to water is 250mL : 750mL and the other mixture of cordial to water is 1000mL : 3000mL? Putting these values into a proportion table and looking for a multiplicative relationship shows that the second mixture has 4 times the cordial and 4 times the water. Since the factor between the systems is the same for both parts, the systems are proportional and ratios of part : part are equivalent.

×4

cordial	250mL	1000mL
water	750mL	3000mL

×4

Unit ratio: part : part relationship where one of the parts is equal to 1 (e.g., 1 : 2, 3 : 1)

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What are the quantities that make up the parts of each system?
- How are the quantities in each system related to each other?
- How much is each quantity in this system multiplied by to make the other system?
- Is each quantity multiplied by the same amount? Are the systems in proportion?
- Can you write that in symbols?
- If one system is 1 : 4, and the other system is 3 : 12, are they in proportion?
- Can you simplify that system?

Portfolio Task

Within the portfolio task, students have the opportunity to practise generating equivalent and unit ratios in the context of mixing paint colours consistently.

RAMR Cycle

Reality

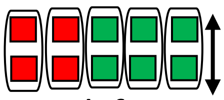
Discuss mixtures students explored in the previous cycle like cordial, concentrated juice, chemicals for weed spraying, grain mixes for animal feed or recipes. Discuss with students what is important when making up these mixtures for more or less than the stated amount. Ensure that students understand that to make up the mixtures in greater or smaller amounts involves keeping the proportions between the parts of the mixture consistent. Consider as a group why this consistency or proportion is important.

Abstraction

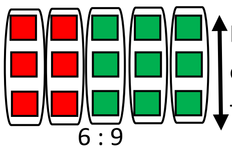
The abstraction sequence for this cycle builds student understanding of proportion as equivalent ratio for increasing or scaling up a mixture. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Identify a context relevant to students to follow through. For example, make up a mixed group of boys and girls in the class (e.g., 2 girls, 3 boys). Increase the size of the group while maintaining the proportion by having another group stand behind the first group (e.g., 2 girls, 3 boys, 2 girls, 3 boys = 4 girls, 6 boys).
2. *Represent with materials and connect to language.* Represent the parts of the whole using unifix cubes or counters or paper squares. For example, 2 girls (part), 3 boys (part), group of 5 (whole).

3. *Represent increased group with materials and connect to language.* Repeat each part of the ratio to model the equivalent ratio of 4 : 6. Ensure students are able to see clearly that the equivalent ratio of 4 : 6 has double the red squares and double the green squares.



4. *Repeat* with the equivalent ratio of 6 : 9. Ensure that students are able to see the equivalence between the systems. It may help to place each successive repeat of the comparison behind the first so that students can look and see the same relationship from the front, but also recognise that the second system has 3 times as many rows.



5. *Record ratios in a proportion table*. Explore relationships between equivalent ratios. For example, there are 2 reds in the original collection which become 6 reds if multiplied by 3. To keep collections in proportion, 3 greens needs to be multiplied by the same amount.

red	green
2	3
4	6
6	9

×3 ×3

More detail with respect to modelling equivalent ratios can be found in *Resource 9.4.2 Modelling proportion with materials*.

Resource Resource 9.4.1 Modelling proportion with materials

Mathematics

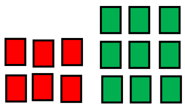
The abstraction sequence worked through an example of increasing the size of a group of students in the classroom to create a larger equivalent group. Other contexts to repeat the abstraction sequence as needed or to use within practice activities may include increasing the quantities when mixing cordial or ingredients for drinks, simple biscuits or grain mixes (e.g., simple wild bird food).

Resource Resource 9.4.2 Mixture ideas

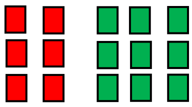
Language/symbols and practice

Simplifying ratios

Once students are comfortable with proportion and equivalent ratios for increasing quantities, it is important to follow through with decreasing quantities while maintaining the same or consistent mixtures (proportion), modelling these using pictures or double number line representations. It is also important to reinforce the use of proportion tables to compare between like values within the ratios to determine equivalence. The initial increasing example from the abstraction phase may be used in reverse to explore finding a common factor. For example:



Can be rearranged to show



Red : green
= 6 : 9

⇨

When same size groups are made, ratio of 6 : 9 can be simplified to 2 : 3.

The relationship between the systems should also be modelled using a proportion table to compare the quantities of red squares and the quantity of green squares to determine equivalence.

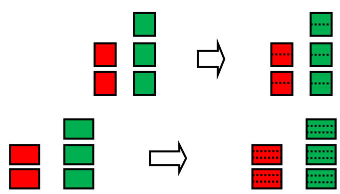
red	green
2	3
÷3	÷3
6	9

Changing the unit of the measure

So far, scaling of ratio has explored increasing or decreasing the quantities in proportion between systems. Some mathematical contexts involve no actual change in overall quantity, but the unit of count may increase or decrease through a change in unit of measure. For example, a quantity of liquid may be expressed in litres (L) and converted to millilitres (mL), or a layer cake with two layers of chocolate and one layer of vanilla may be cut into halves and the number of half sized pieces expressed as a ratio.

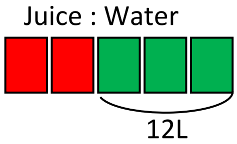
This type of problem may also be represented with materials using paper squares as follows:

1. Represent initial system with paper squares.
2. Cut each square in half to model the equivalent ratio of 4:6.
3. Cut squares in thirds to model the equivalent ratio of 6:9.



Problem applications

Consider problems that use ratio in real contexts. For example, *If juice is mixed with water in a ratio of 2:3, how much juice should be added to 12 L of water?*



1. This problem may be modelled with materials or solved using a proportion table. Draw the ratio/fraction diagram (as on right) to show the ratio of the parts. Place all the information from the problem on the diagram.
2. Determine the size of each part of water. Students may like to write the size of each part for the water on the diagram but should focus on the fact that

3 parts of water = 12L
so 1 part of water = 12 ÷ 3 = 4L.

3. Multiply the size of the part by the number of parts of juice to find the answer.

Extend students to model problems on a proportion table to model and solve the problem. This can be a vertical proportion table as previously demonstrated, or, make the connection to horizontal proportion table as shown below. Students should develop the ability to work in either format. For example:

Juice	2 parts			?L
Water	3 parts			12L

×?L per part

Work out the multiplier for the water (3 × ? = 12; ? = 4).

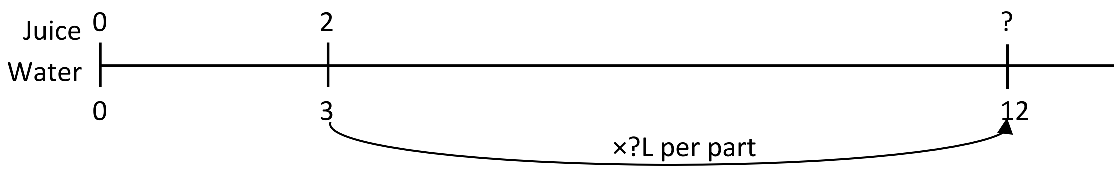
×4L per part

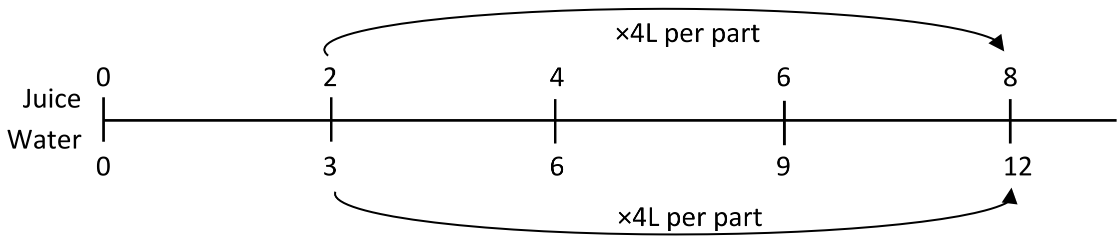
Juice	2 parts			?L
Water	3 parts			12L

×4L per part

Apply the same multiplier to the juice (2 × 4 = 8). So, 8L Juice are needed to add to 12L of water.

Ratio problems can also be modelled on a double number line which is similar to a proportion table without the outer framing. For example:





Note: Double number lines may also be presented vertically as in Cycle 3 to match a vertically presented proportion table.

Provide students with problems and have students construct problems where the ratio is given for a mixture and the total amount of the finished product for them to find the corresponding parts. For example, a class was formed with a ratio of 3 girls:2 boys. If there were 30 students in the class, how many were girls? If there were 30 students in the class how many were boys?

Resource Resource 9.4.3 Ratio problems
Resource 9.4.4 Problem construction and problem solving

Unit ratio

Once students are comfortable with increasing and simplifying ratios to determine equivalent systems, they should be able to extend to finding unit ratios (one of the parts is reduced to 1 part). Note: Start with examples where the remaining part stays a whole number and extend to examples that result in a fraction. Proportion tables will be useful tools to scaffold this process.

Comparing and ordering ratios

Once students are confident using proportion tables to determine increasing equivalent ratios and for simplifying ratios, provide examples of systems that are not equivalent and engage students with determining which is the "stronger" or "weaker" mix. For example, John mixed 250mL of cordial with 750mL of water (250mL : 750mL). Mark mixed 250mL of cordial with 1000mL of water (250mL : 1000mL). Which drink is "stronger" flavoured? Intuitively, John has stronger cordial because he has used less water. On a proportion table the comparison becomes:

John's drink

÷ 250mL

Cordial	250mL			1 part
Water	750mL			3 parts

÷ 250mL

Mark's drink				
	÷ 250mL			
Cordial	250mL			1 part
Water	1000mL			4 parts
	÷ 250mL			

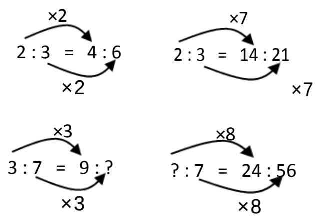
Students should recognise that drink mixed as 1 part cordial : 3 parts water will be "stronger" flavoured than drink mixed as 1 part cordial : 4 parts water.

Extend students to compare and order ratios gradually increasing the complexity. For example, comparing and ordering unit ratios (1 : 2, 1 : 3, 1 : 4, and so on) extending to comparing sets of ratios that need simplifying to unit ratios before comparison (1 : 2, 2 : 3, 7 : 8, 8 : 15, and so on).

Connections

Reinforcing multiplicative patterns in symbols

Once the visual image of equivalence or increasing/decreasing parts in proportion has been developed with suitable models to assist problem solving, patterning in the symbols may be generalised and reinforced using patterning tools like proportion sticks.



Looking at equivalent pairs of ratios, 2:3 = 4:6 = 6:9 = 8:12. Continuing this pattern would result in 14:21 = 24:36 and so on. Encourage students to explore multiples and notice patterns. Reverse problems and work out proportions by keeping multiples the same.

Connect this relationship to the double scale number line model and reinforce the symbol patterns using proportion tables or proportion sticks.

Resource Resource 9.4.5 Proportion sticks

Reflection

Check the idea

Use models developed throughout the cycle to solve the following problems:

(a) Sand and cement is 7:3. How much sand for 1 part of cement?
(b) Sand and cement is 7:3. How much cement for 1 part of sand?
(c) Concrete is mixed in the ratio of 7 parts sand to 3 parts cement. How much cement is needed if 0.5 m^3 of sand is used?
(d) 275 mL of mixture is needed for 2 m^2 of wall. How much mixture for a wall 2.5 m by 5.5 m?

Apply the idea

Encourage students to select a recipe to increase and reduce. Generate a recipe to make a single serving of a fruit drink. Increase the quantities by a multiplier to find quantities of ingredients to make an amount of fruit drink to serve the whole class.

Resource Resource 9.4.2 Mixture ideas

Extend the idea

Extend students' thinking and problem solving proportional increases and decreases to mixtures with ratios that use three or more terms.

For example, fertiliser is mixed in the ratio of nitrogen : phosphorous : potassium = 16 : 4 : 8 or concrete is mixed in the ratio of cement : sand : gravel = 1 : 3 : 5.

Extend the connection between ratio and fractions to include percentage. For example, a cordial mix of 1 part cordial to 4 parts water will have one fifth cordial and four fifths water. This can be represented as percentages in the final mixture of 20% cordial and 80% water.

Explore more difficult extensions of problems. For example, if I have mixed a cup of cordial (250mL) in the ratio of cordial to water = 1 : 4, then read the bottle and find that this mix is supposed to be 1 : 5, what can I do to fix the mixed cordial? Proportion tables can be useful for solving this problem as follows:

Original mix		
cordial	1	50mL
water	4	200mL

Correct mix		
cordial	1	50mL
water	5	250mL

Difference between amounts of water = 50mL. The mix can be corrected by adding 50mL water to the cup.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 09 Investigation: Changing a Recipe

Changing a recipe

Your class is going to have a party. Recipes for party food do not serve enough people.

1. Find a recipe.
2. Identify the ratio between the main ingredients.
3. Work out how many times the recipe needs to be increased by to make enough for your class to all share.
4. Adjust the recipe to create a class-sized ingredients list.

Name:____ Date:________

Can you do this? #5

1. You pour a small cup of cordial into a jug. To make a drink that is 4 parts water to 1 part cordial, how much water do you need to add?

   ____

Obj. 9.5.1
i. ☐
ii. ☐

2. You put a mix of frozen apple juice cubes and frozen blackcurrant juice cubes into drink bottles for school.

Bottle A 	**Bottle B** 
Blackcurrant : Apple 4 : 3	Blackcurrant : Apple 3 : 2

Obj. 9.5.1
a) ☐
b) ☐

(a) How many times more blackcurrant flavour than apple flavour is there in Bottle A? ______________________________

(b) How many times more blackcurrant flavour than apple flavour is there in Bottle B? ______________________________

(c) If you like stronger blackcurrant-flavour in your drink, which bottle should you take to school? ______________________

Obj. 9.5.2
c) ☐

3. In a group of people, the ratio of brown hair to blonde hair was 3 : 8.

Obj. 9.5.1
☐

(a) How many more times blonde haired people are there than brown haired people? ______________________________

(b) There are 24 blonde-haired people in the group. Write the equation that will help you work out how many brown-haired people there will be? ______________________________

Obj. 9.5.3
i. ☐
ii. ☐
iii. ☐

(c) How many brown-haired people will there be? ____________

Obj. 9.5.4
☐

Cycle 5: Ratio Relationships – Within Systems

Overview

Big Idea

This cycle explores the multiplicative relationship amongst the parts of a system. For example, cordial is mixed with water in a ratio of 1 part : 4 parts, there is four times as much water as cordial or one quarter as much cordial as water. This multiplicative relationship within a system may be compared with the multiplicative relationship within a second system to compare systems for equivalence and determine unknown quantities.

Objectives

By the end of this cycle, students should be able to:

9.5.1 Identifies multiplicative relationship between parts of a system. [8NA188]
9.5.2 Uses multiplicative relationship within systems to compare between systems. [8NA188]
9.5.3 Translate ratios within proportional systems to simple linear equations. [8NA188]
9.5.4 Use linear equations to solve for unknown quantities in proportional systems. [8NA188]

Conceptual Links

This cycle relies on student understanding of ratio, part-part relationships, part-whole relationships, multiplicative comparison and equivalence. Number facts and computation strategies may also be reinforced throughout this cycle.

Identifying and describing proportion and unit ratio is further applied in contexts of ratio between measures, similar shapes and figures, and scaling on maps and plans.

Materials

For Cycle 5 you may need:

- Double number line (drawn or elastic)
- Counters or unifix cubes
- Cordial, cups, water

Key Language

Part-to-part, ratio, per, equivalent ratio, mixture, part, whole, within-system relationships, factors, multipliers

Definitions

Unit ratio: part:part relationship where one of the parts is equal to 1 (e.g., 1:2, 3:1)

Within-systems ratio relationships: compares relationships between measures of attributes within systems to determine the multiplicative relationship between parts of the systems, or uses known relationship between the parts of systems to determine unknown quantities. For example, if a batch of cordial is mixed with 1 part cordial to 4 parts water, there is 4 times as much water as cordial within the mix, or one quarter as much cordial as water. Putting these values into a proportion table and looking for a multiplicative relationship shows that the water is 4 times as much as the cordial.

cordial	1 part
water	×4 4 parts

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How are the quantities in the system related to each other? What can you multiply this part of the ratio by to get the other part?
- Are the parts in this system related by the same multiplier?
- Are these systems in proportion?
- If one system is missing a quantity and they are in proportion, how can you find the other quantity?
- Can you make this relationship into an equation?
- If you have values for these parts of the equation, can you find the unknown part?

Portfolio Task

Within the portfolio task, students have the opportunity to practise analysing parts within a ratio to determine whether one colour will be a shade different from or the same as another.

RAMR Cycle

Reality

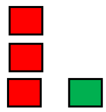
Discuss mixtures students explored in the previous cycles like cordial, concentrated juice, chemicals for weed spraying, grain mixes for animal feed, recipes, paint colours or groupings of an attribute within a population. Discuss with students what is important when comparing these mixtures with others to ensure consistency. Discuss the relationship between the parts of one mixture. For example, three times as many boys as girls in a group. Consider as a group why this consistency or proportion is important.

Abstraction

The abstraction sequence for this cycle builds student understanding of the multiplicative relationship between parts of a whole using whole number multipliers. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Identify a context relevant to students to follow through. For example, make up a mixed group of boys and girls in the class (e.g., 3 boys, 1 girl) is simple to experience and act out. Other relevant and interesting ideas to follow through can be mixing cordial or ingredients for drinks, simple biscuits or grain mixes (e.g., simple wild bird food equal parts of sunflower seeds, cracked corn, raisins, peanut butter).

2. *Represent with materials and connect to language.* Represent the parts of the whole using unifix cubes or paper squares. For example, 3 boys to 1 girl. Discuss the relationship between the number of boys and the number of girls. Ensure that students can see there are three times as many boys as girls.



3. *Create another group in proportion.* Generate another group of boys and girls starting with 2 girls. Discuss how many boys are needed to make an equivalent group. Discuss the use of the multiplier from the previous relationship (3). Create the equivalent group of 6 boys and 2 girls. Discuss other groups that would be proportional. Modelling on a proportion table may help students with this pattern (both vertical and horizontal are shown here).

boys		girls
3	×3	1
?	×3	2

boys	3 ×3	? ×3
girls	1	2

4. *Represent with symbols.* Write the relationship for all groups that are proportional as an equation. For example, number of boys = 3 times number of girls. Extend to y = 3x (where y = number of boys and x = number of girls).

boys		girls
3	×3	1
?	×3	2
3x	×3	x

boys	3 ×3	? ×3	3x
girls	1	2	x

Mathematics

Connections

Once students have grasped finding multiplicative relationships that are whole number multipliers, they may be extended to consider not simple ratios (e.g., 3:2), ratios involving real numbers (whole and fractional quantities, e.g., 1.5:1, 2:2.5), factors that relate quantities within the system forward and backward (e.g., Boys : Girls = 3 : 1, there are 3 times as many boys as girls or there are one-third as many girls as boys), and connect the multiplying factor within the system to the m coefficient of the simple linear relationship (y = mx + c).

Language/symbols and practice

Not simple ratios

The abstraction phase essentially explored unit ratios where the second element in the ratio is one (similar to simple rates). Extend the range of problems to include ratio relationships where both elements are greater than one. For example, make up class groups with 3 boys and 2 girls = 3 : 2.

Ratio with real numbers

So far all examples explored have been whole number values. Extend the range of problems to include ratio relationships where one or both of the elements includes a fractional quantity. For example, recipes may require wet ingredients to dry ingredients in the ratio of 1 : 1.5. Explore how to determine the multiplicative relationship between these two values.

Fractional factors

Extend students thinking to consider the backwards and forwards relationships between parts within a system. For example, in a group of 3 boys and 1 girl, there are 3 times as many boys as girls. This can also be expressed as one third as many girls as boys.

Simple linear equations

Reinforce the conversion of ratio relationships to simple linear equations using ratios with real numbers. For example, in a recipe with wet ingredients to dry ingredients in the ratio of 1 : 1.5, the quantity of dry ingredients will be 1.5 times the wet ingredients or y = 1.5x where y = dry ingredients and x = wet ingredients.

Ratio relationships as fractions

Discuss the fractions of each part in the whole. For example, juice drink mixed 1 part juice to 1 part water has $\frac{1\ part}{2\ parts\ drink}$ of juice and $\frac{1\ part}{2\ parts\ drink}$ water; juice drink mixed 1 part juice to 8 parts water has $\frac{1\ part}{9\ parts\ drink}$ of juice and $\frac{8\ parts}{9\ parts\ drink}$ of water. Reinforce the part to part relationships and the $\frac{part}{whole}$ relationships for each mixture. Reinforce modelling the representation with unifix cubes or counters or using graph paper squares to show the relationship between each of the parts of the mixture and the whole. Ensure that students are able to construct a proportion table to also represent these relationships. For example,

juice	water
1 part	8 parts

Reflection

Check the idea

Explore problems in real contexts that can be solved using within-system ratio relationships. For example:

John has red and blue marbles. He has 32 marbles altogether. He has 3 times as many red marbles as blue marbles. How many red marbles does John have altogether? How many blue marbles does John have altogether?

Apply the idea

Explore real world applications of within system ratio to solve problems. For example:

Mark is 1.5m tall. When he stands next to an electricity pole, his shadow is 5m long. The electricity pole shadow was 35m long. How tall is the electricity pole?

This problem could be scaffolded by exploring either the ratio comparison between the two shadows, or the ratio comparison between Mark and his shadow, to determine the height of the electricity pole.

The students may find it interesting to explore the heights of trees or structures in the school ground using a similar process.

Extend the idea

Extend the connection between ratio and fractions to include percentage. For example, a cordial mix of 1 part cordial to 4 parts water will have one fifth cordial and four fifths water. This can be represented as percentages in the final mixture of 20% cordial and 80% water. Consider how ratios with percentage may be expressed as linear equations.

Explore more difficult extensions of problems. For example, if I have mixed a cup of cordial (250mL) in the ratio of cordial to water = 1:4, then read the bottle and find that this mix is supposed to be 1:5, what can I do to fix the mixed cordial? Proportion tables can be useful for solving this problem as follows:

Original mix		
cordial	1	50mL
water	4	200mL

Correct mix		
cordial	1	50mL
water	5	250mL

Difference between amounts of water = 50mL. The mix can be corrected by adding 50mL water to the cup.

Teacher Reflective Notes

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Unit 09 Portfolio Task – Teacher Guide

Mixing Paint

Content Strand/s: Number and Algebra
Measurement and geometry

Resources Supplied:
- Task sheet
- Teacher guide

Other Resources Needed:
None

Summary:
Using the context of mixing paint, students will use multiplication, division and ratios. Students will determine how much of each base colour needs to be mixed to form a variety of paint colours.

Variations:
- Students can find out is all paint is mixed using the same base colours,
- Students could collect data on the most common paint colours which could then be graphed.

ACARA Proficiencies Addressed:
Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Measurement and Geometry

6.7.1 Connect decimal representations to the metric system. [6MG135]
6.7.2 Convert between common familiar metric units of length. [6MG136]
9.2.1 Choose appropriate units of measurement for length, capacity and mass. [5MG108]
9.2.2 Convert between common familiar metric units of capacity. [6MG136]

Number and Algebra

9.3.1 Represent part-part relationships in simple ratios. [7NA173]
9.3.2 Represent part-whole relationships in simple ratios. [7NA173]
9.3.4 Can identify the quantity in the whole given the parts. [7NA173]
9.4.2 Compare between ratios to determine if systems are equivalent. [8NA188]

Mixing Paint

Name	
Teacher	
Class	

Your Task:

It is your task to work out the quantities of base colour you need to consistently mix paint. To do this you will have to use:

- multiplication
- division
- ratio

Within Portfolio Task 9, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Conceptual understanding	9.2.1 Choose appropriate units of measurement for length, capacity and mass.	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
	Procedural fluency	9.3.4 Can identify the quantity in the whole given the parts.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	9.2.2 Convert between common metric units of capacity.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Problem solving approaches	9.4.2 Compare between ratios to determine if systems are equivalent.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches

Comments:

Genesis paint colour mixing chart:

With these 5 Genesis base colours...

Burnt Umber (U)	**Genesis Red (R)**	**Genesis Yellow (Y)**	**Titanium White (W)**	**Ultramarine Blue (B)**

You can create all 30 of these colours!...

	Cadmium Yellow Pale	White + Genesis Yellow =2:1
	Cadmium Yellow Light	White + Genesis Yellow = 1:1
	Cadmium Yellow Deep	White + Genesis Yellow + Genesis Red = 1:2:1
	Cadmium Orange	Genesis Yellow + Genesis Red =2:1
	Flesh	White + Genesis Red + Genesis Yellow = 1:1:1
	Dark Flesh	Genesis Red + Burnt Umber + Genesis Yellow + White = 1:1:1:1
	Cadmium Red Pale	Genesis Red + Genesis Yellow = 2:1
	Red Oxide	Genesis Red + Burnt Umber + Genesis Yellow = 3:1:1
	Cadmium Red Medium	Genesis Red + Genesis Yellow = 6:1
	Cadmium Red Deep	Genesis Red + Burnt Umber = 3:1
	Rose Madder	Genesis Red + Ultramarine Blue = 5:1
	Alizarin Crimson	Genesis Red + Ultramarine Blue = 4:1
	Dioxazine Purple	Ultramarine Blue + Genesis Red -= 2:1
	Prussian Blue	Ultramarine Blue + Genesis Red = 6:1
	Payne's Grey	Ultramarine Blue + Burnt Umber = 2:1
	Dark Teal	Ultramarine Blue + Genesis Yellow= 4:1
	Dark Leaf Green	Ultramarine Blue + Genesis Yellow = 3:1
	Medium Leaf Green	Genesis Yellow + Ultramarine Blue = 2:1
	Light Leaf Green	Genesis Yellow + Ultramarine Blue = 4:1
	Sap Green	Genesis Yellow + Ultramarine Blue = 1:1
	Olive Green	Burnt Umber + Genesis Yellow + Ultramarine Blue = 2:1:1
	Avocado	Burnt Umber + Genesis Yellow = 2:1
	Burnt Sienna	Genesis Red + Burnt Umber + Genesis Yellow = 1:1:1
	Raw Umber	Burnt Umber + Ultramarine Blue = 2:1
	Mars Black	Burnt Umber + Ultramarine Blue = 6:5
	Carbon Black	Ultramarine Blue + Burnt Umber = 1:1
	Neutral Gray	White + Ultramarine Blue + Burnt Umber = 1:1:1
	Light Ivory	White + Genesis Yellow = 1:1
	Antique White	White + Genesis Yellow = 12:1
	Ivory	White + Genesis Yellow + Genesis Red = 12:1:1

1. You need to mix paint consistently for a model project. Choose four different colours to paint your model project.

   Colour 1: ________________________

   Colour 2: ________________________

   Colour 3: ________________________

   Colour 4: ________________________

2. Write each colour as a ratio and as fractions of the whole colour.

   (An example has been provided)

Colour	Ratio	Fractions
Avocado	2U:1Y	U = $\frac{2}{3}$ Y = $\frac{1}{3}$
1		
2		
3		
4		

3. Represent two of the ratios as pictures in this space.

4. You need 1L of each paint colour. How many millilitres of each basic colour will you need for each of your paint colours?

Colour 1	**Colour 2**
Colour 3	**Colour 4**

5. What is the total quantity of each base colour you need?
(List the quantities in the space, then find the total.)

Colour	**Burnt Umber**	**Genesis Red**	**Genesis Yellow**	**Titanium White**	**Ultramarine Blue**
Total	___________mL	___________mL	___________mL	___________mL	___________mL

6. To complete your project, you need to make a new colour by combining Cadmium Yellow Pale and Cadmium Red Pale.

   a. What are the ratios of base colours in each mixed colour?

   Cadmium Yellow Pale: ______________________________

   Cadmium Red Pale: ______________________________

   b. What will the ratio of base colours in the new mixed colour be (when Cadmium Yellow Pale and Cadmium Red Pale are mixed)?

   c. What ***could*** you call this colour? ____________________________________

   d. Does this ratio of base colours already exist in the 'Genesis Paint Colour Mixing Chart?' ______________________________ If so, what is this colour called in the 'Genesis Paint Colour Mixing Chart'?

   e. You need 1.5L of this paint colour, how many millilitres of each base colour will you need?

7. Which base colour will you use:

   a. The most of: ___________

   b. The least of: ___________

8. Order the use of the five base colours from smallest to largest:

   ___________, ___________, ___________, ___________, __________

Name:____ Date:________

Can you do this now? Unit 09

1. (a) Fill in the blanks to list in order all the months of the year:
January, ____________ , March, April,________________, June, July, ____________ , ____________ ,October, __________ , December

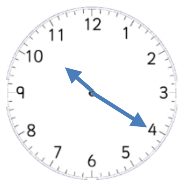
Write down how many days in:

(b) May _______ (c) September ________

(d) January _______ (e) April ________

Obj. 9.1.1 a) i□ a) ii□ a) iii□ a) iv□ a) v□ b)□c)□ d)□e)□

2. What time is shown on the clock?

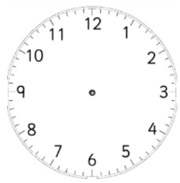


(a) In words __________________

(b) In digital format ____________

Obj. 9.1.2 a) □ b) □

3. Draw the hands on the clock to show 6:50.



Obj. 9.1.2 i. □ ii. □

4. Michael catches a bus to school at 08:25 and a bus home at 15:00. Write the times Michael catches the bus in 12-hour time?

(a) _______________ (b) __________________

Obj. 9.1.4 a)i□ii□ b)i□ii□

(c) How much time (in hours and minutes) passes between when Michael catches the bus in the morning and when he catches the bus in the afternoon? __________________________________

Obj. 9.1.6 c) i. □ c) ii. □

(d) How many minutes pass between when Michael catches the bus in the morning and when he catches the bus in the afternoon?

____

Obj. 9.1.3 d)i□ii□

5. Draw a circle around the time I need to catch a bus from the depot to arrive at the shop between 4:05 and 4:15 pm.

Departs bus depot	**Stops at shop**
3:40pm	4:00pm
3:50pm	4:10pm
4:00pm	4:20pm
4:10pm	4:30pm

Obj. 9.1.5 □

6. Western Australia is 2 hours behind Eastern Standard Time.
At 1:30 am Friday in Queensland, what time and day is it in Perth?

____

Obj. 9.1.6 i. □ ii. □ iii. □

7. If a 2½ hr film starts at 18:35, what time will it finish? ___________

8. Fill in the following conversions:

(a) 15 mm = ___________ cm

(b) 350 cm = ___________ m

(c) 5 500 m = ___________ km

(d) 15 L = ___________ mL

(e) 12 kL = ___________ L

(f) 5.603 L = ___________ mL

(g) 5 kg = ___________ t

(h) 6.304 t = ___________ kg

(i) 6.5 g = ___________ mg

9. Name something that would be reasonable to measure using the following units.

(a) mL ___________________________

(b) kl ___________________________

(c) L ___________________________

(d) kg ___________________________

(e) t ___________________________

(f) g ___________________________

10. Would you use Litres or millilitres to measure:

(a) a can of soft drink? ___________________________

(b) a watering can? ___________________________

(c) a small saucepan? ___________________________

11. Would you use kilograms or grams to measure:

(a) a laptop computer? ___________________________

(b) a packet of cereal? ___________________________

(c) a large pumpkin? ___________________________

Obj. 9.1.7 □

Obj. 6.7.2 a) □ b) □ c) □

Obj. 9.2.2 d) □ e) □ f) □

Obj. 9.2.3 g) □ h) □ i) □

Obj. 9.2.1 a) □ b) □ c) □ d) □ e) □ f) □

Obj. 9.2.1 a) □ b) □ c) □

Obj. 9.2.1 a) □ b) □ c) □

12. During one day (24 hours), the local radio station played 7 hours of rock and roll and the rest of the time was country music. What is the ratio of rock and roll to country music?

Rock and Roll : Country = __________ : ____________

Obj. 9.3.1 i. □ ii. □

A packet of M&Ms contains three colours: 6 red, 13 blue and 5 brown.

13. (a) Write the ratio of red to blue to brown M&Ms.

___________ : ____________ : ____________

Obj. 9.3.1 i. □ ii. □ iii. □

(b) What is the ratio of red M&Ms compared to the whole packet?

____________________ : _____________________

Obj. 9.3.2 i. □ ii. □

14. Cordial and water are mixed to make 2000mL of drink. 600mL of the drink is cordial. What is the ratio of cordial to water?

Cordial : Water = ____________:____________

Obj. 9.3.3 i. □ ii. □

15. Sand and cement are mixed to make concrete. The ratio of sand to cement is 6 : 2. What is the quantity of parts in the whole concrete mix? ____

Obj. 9.3.4 □

16. (a) Count how many boys and girls are in your class.

Boys _______________ Girls _______________________

(b) What is the ratio of girls to boys in your class?

_______________________ : _____________________

Obj. 9.3.1 i. □ ii. □ iii. □

(c) What is the ratio of girls compared to all students in your class?

_______________________ : _____________________

Obj. 9.3.2 i. □ ii. □ iii. □

17. The ratio of water to cordial in a drink is 5 parts : 1 part. What ratio of water to cordial will be needed to make a double amount of drink?

____________________ : _____________________

Obj. 9.4.1 i) □ ii) □

18. Circle the ratios that are equivalent to 3 : 5

(a) 12 : 25
(b) 21 : 35
(c) 27 : 50
(d) 27 : 45

Obj. 9.4.2 a) □ b) □ c) □ d) □

19. List the following mixes of cordial from weakest to strongest.

1 : 2.5 ,   1 : 3 ,   2 : 8 ,   2 : 4 ,   3 : 5

_________ , ___________ , _________ , __________ , _____________

Obj. 9.4.3
i) □
ii) □
iii) □
iv) □
v) □

20. There must be 1 instructor for every 4 students on a kayaking course.
(a) If 24 students want to do the course, how many instructors will be needed?

Instructors		
Students		

Obj. 9.4.4
a) i □
ii □
iii □
iv □

(b) If there are 6 instructors available, what is the maximum number of students that can do the course? ___________________________

Instructors		
Students		

Obj. 9.4.4
b) i □
ii □
iii □
iv □

21. You pour a small cup of cordial into a jug. To make a drink that is 5 parts water to 2 parts cordial, how much water do you need to add?

____

Obj. 9.5.1
i. □
ii. □

22. You put a mix of frozen apple juice cubes and frozen blackcurrant juice cubes into drink bottles for school.

Bottle A	**Bottle B**
	
Blackcurrant : Apple 5 : 3	Blackcurrant : Apple 3 : 2

Obj. 9.5.1
a) □
b) □

(a) How many times more blackcurrant flavour than apple flavour is there in Bottle A? _______________________________

(b) How many times more blackcurrant flavour than apple flavour is there in Bottle B? _______________________________

(c) If you like stronger blackcurrant-flavour in your drink, which bottle should you take to school? _______________________

Obj. 9.5.2
c) □

23. In a group of people, the ratio of brown hair to blonde hair was 4 : 7.

(a) How many more times blonde haired people are there than brown haired people? ______________________________

(b) There are 28 blonde-haired people in the group. Write the equation that will help you work out how many brown-haired people there will be? _________________________

(c) How many brown-haired people will there be? _________

Obj. 9.5.1 ☐

Obj. 9.5.3

i. ☐

ii. ☐

iii. ☐

Obj. 9.5.4 ☐

XLR8 – Accelerating Mathematics Learning

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