XLR8 – Accelerating Mathematics Learning

QUT

YuMiDeadly
Growing community through education

XLR8 Unit 08

Calculating coverage

08

2016

Prepared by YuMi Deadly Centre
Faculty of Education, QUT, Kelvin Grove

ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of discrete items or measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric rations between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.

Scope

This unit is based upon the **identification** and **measurement** of **area**. The **attribute** of **area** is defined as the **coverage** of a **region** within an **enclosed boundary**. This attribute needs to be carefully differentiated from perimeter which is the distance around an enclosed boundary.

Comparison of area in this unit includes areas of **simple irregular plane shapes**, **simple regular plane shapes**, **composite shapes** and **surface area** of **three-dimensional prisms and pyramids (not spheroids)**.

The **area model** is useful for assisting students with developing the concept of multiplication and provides a useful scaffold for **large number multiplication and division strategies**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *How to Build an Economical House* at the end of Unit 08 engages students with exploring the relative costs of external and internal walls when designing square, rectangular or L-shaped houses to determine the most economical shape for housing.

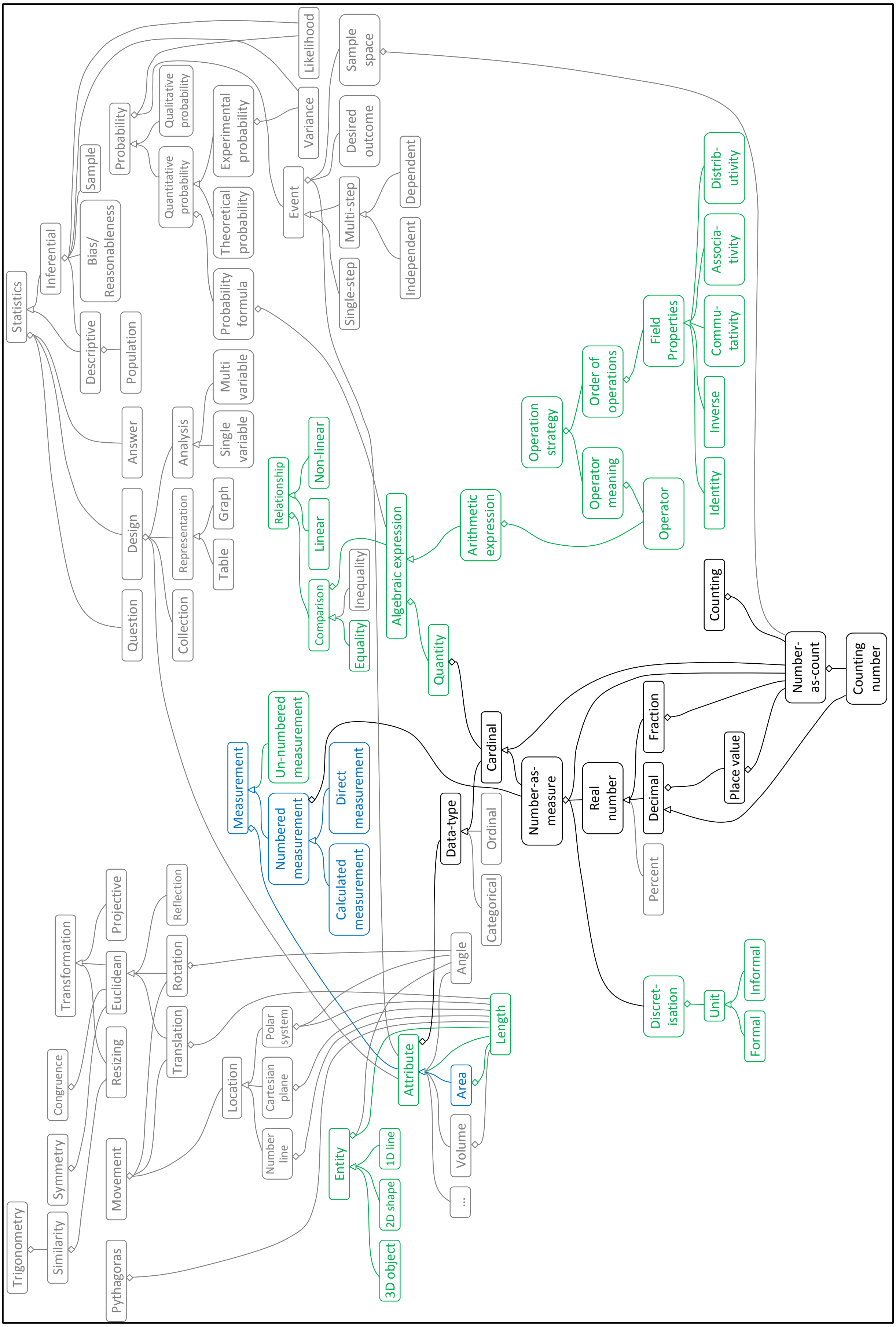


Figure 1. Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Area Comparison

This cycle develops understanding of the attribute of area as coverage of two-dimensional space enclosed within the boundary of a two-dimensional shape and ways to directly and indirectly compare and order area without reference to number or quantity of area.

Cycle 2: Counting Coverage

This cycle extends indirect comparison of coverage to using the count of a uniform repeated item to compare coverage. Students develop notion of non-standard repeated units that tessellate to cover an item to be measured with no gaps or overlaps and that an item's area is identified by a number and the unit of measure. The idea of a square unit is introduced.

Cycle 3: Calculating Rectangular Areas

This cycle extends from counted area to calculated area of rectangular shapes. This idea makes use of the array model of multiplication, where a number of equal rows can be multiplied by the number of elements in each row (in preference to counting), which leads to a formula for calculating rectangular area. The attributes of area and perimeter are also explicitly differentiated so that students develop the understanding that shapes with the same area may have different perimeters and visa versa.

Cycle 4: Unit Conversion for Area

This cycle connects use of a square unit for measuring, calculated area of rectangular shapes and metric measure to explore unit conversions for area starting with relationships between related non-standard unit squares and continuing to metric unit squares (mm^2, cm^2, m^2, km^2).

Cycle 5: Area of Parallelograms and Triangles

This cycle extends from calculated area of rectangular shapes to calculated area of parallelograms. This idea requires students to make a connection between parallelograms and rectangles which leads to the development of a formula for calculating the area of a parallelogram. A further connection between parallelograms and triangles leads to the formula for calculating area of triangles.

Cycle 6: Area of Circles

This cycle extends from calculated area of parallelograms to calculated area of circles. This idea requires students to make a connection between parallelograms and circles which further leads to the development of a formula for calculating the area of a circle.

Cycle 7: Area of Composite Shapes

This cycle challenges students to see composite shapes as combinations of shapes they know. Examples explored should enable students to apply area of rectangles, parallelograms, triangles and circles to real world problems involving composite shapes. As an extension of composite shapes, this cycle also challenges students to perceive the surface area of prisms and pyramids as a combination of the area of the faces which form the nets of these objects. Examples explored should enable students to apply area of rectangles, parallelograms and triangles to real world problems involving surface area.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Area Comparison

Coverage, surface, region, enclosed boundary, 2-dimensions, attribute, area, covers more, covers less, larger area than, smaller area than

Cycle 2: Counting Coverage

Coverage, surface, region, enclosed boundary, 2-dimensions, attribute, area, units, unit names, square units

Cycle 3: Calculating Rectangular Areas

Coverage, surface, 2-dimensions, attribute, area, units, unit names, square units, length, width, formula

Cycle 4: Unit Conversion for Area

Metric measures of area, units, unit names, square units, centimetre squares, cm^2, metre squares, m^2, millimetre squares, mm^2, kilometre squares, km^2

Cycle 5: Area of Parallelograms and Triangles

Coverage, surface, 2-dimensions, attribute, area, units, unit names, formula, length, width, parallelogram, rhombus, perpendicular width, standard metric measures and units, accepted mathematical symbols to indicate perpendicular lines, parallel lines and same length sides, triangle, right angle triangle, isosceles triangle, equilateral triangle, scalene triangle

Cycle 6: Area of Circles

Coverage, surface, area, units, unit names, pi, square units, formula, diameter, radius, circumference, standard metric measures and units,

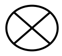
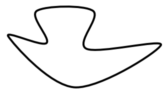
Cycle 7: Area of Composite Shapes

Coverage, surface, surface area, composite shapes, length, width, perpendicular width, standard metric measures and units, 2D nets of 3D prisms, rectangular prisms, cubes, square based prisms, triangular based prisms, pyramids, composite 3D objects

Name:____ Date:________

Can you do this? #1

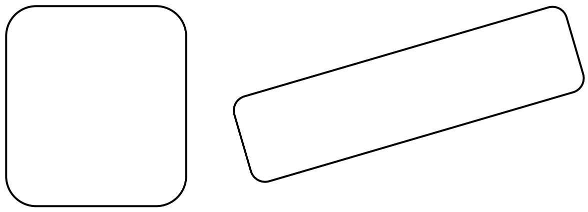
1. Circle the shapes that have an area

(a)  (b)  (c)  (d) 

Obj. 8.1.1

a) ☐
b) ☐
c) ☐
d) ☐

2. Describe how to compare the **areas** of these two shapes without counting, measuring lengths or calculating.



____

____

____

Obj. 8.1.1

☐

Cycle 1: Area Comparison

Overview

Big Idea

Area is a measure of the coverage of two-dimensional space enclosed within the boundary of a two-dimensional shape. As for other measures, students need to develop understanding of the attribute of coverage and ways to compare this attribute between items.

This cycle develops understanding of the attribute of area as coverage and ways to compare and order area without reference to number or quantity of area.

Objectives

By the end of this cycle, students should be able to:

8.1.1 Compare and order areas of regular and irregular shapes by informal means. [4MG087]

Conceptual Links

Comparing coverage connects ideas from Geometry and Measurement concepts. Conceptual links should be made to consolidate students' previous experiences in classifying 3D objects, 2D shapes and properties of 3D objects and 2D shapes. Also applicable here is the use of direct or indirect comparison to determine larger than, the same as or smaller than when considering measurement of an attribute.

Cycle 1 provides foundational conceptual understanding of the attribute of area and its comparison which underpins all the cycles in this unit. The tangram activity within the abstraction phase provides a foundation for determining area of composite shapes in Cycle 7. Considering area of surfaces touching the ground in the reflection phase provides a foundation for determining surface area in Cycle 7.

Materials

For Cycle 1 you may need:

- Butchers' paper
- A4 paper
- Paint or coloured paper and glue
- Tangrams (can be printed and cut out)
- Variety of items with different areas
- Marking pens
- Noticeboard
- Scissors
- Tangram picture outlines

Key Language

Coverage, surface, region, enclosed boundary, 2-dimensions, attribute, area, covers more, covers less, larger area than, smaller area than

Definitions

Direct comparison: comparison of an attribute common to two or more items. For example, direct comparison of area involves holding two or more items together with a common baseline to see which is larger. For example, a book may be set on a desktop with a common corner (and accompanying edges) and compared to see which has greater area. Note that with area the comparison needs to be made in two dimensions and may be more complicated to compare using this method if one item is longer but not wider than the other.

Indirect comparison: comparison of an attribute common to two or more items that are compared using an intermediary. For example, the top surface of a desk and the whiteboard can be compared by creating a sheet of paper the size of the desktop and comparing it with the whiteboard. If the whiteboard is larger than the sheet of paper, then the desktop has less area than the whiteboard. This is a useful forerunner to indirect comparison using non-standard units.

Non-standard units: Any of the same item that can be repeated (or iterated) and counted in order to make an informal measurement of another object. For example, serviettes, napkins, A4 paper, books, pencil cases, iPads ...

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How can you tell if an item has greater area than another?

  To ascertain if students understand measure and the significance of units for area, they must recognise that the attribute is coverage (the amount of surface the item covers or the amount of space enclosed in the boundary). To make comparison with coverage, students must consider two dimensions (both length and width). Where an item is longer but not wider, judgements about which item has the greater coverage become more difficult.

Portfolio Task

This cycle provides students with underlying understanding of the attribute of area to assist with their completion of the overall portfolio task.

RAMR Cycle

Reality

Explore real-life instances of area. Ensure students understand that area is the measure of two-dimensional space enclosed by a boundary. For example, a floor can be covered with tiles or carpet; the area of a wall or a ceiling can be painted; the surface of a window; the grass on the oval are all examples of area. Clearly differentiate the attribute of area from length or perimeter. The skirting board around the edge of the floor; the cornice around the edge of a ceiling; the frame around a window; the fence around the oval (with a connecting line that could be fenced) are all examples of perimeter or the distance around.

Abstraction

The abstraction sequence for this cycle explores the concept of coverage while building ways of comparing areas to determine which covers more or covers less. An abstraction sequence to build the concept of area as coverage and the comparison and ordering of objects by their area is as follows:

1. *Kinaesthetic activity.* Start by experiencing and identifying the attribute of area as coverage. Stand students against a large sheet of paper (feet together and arms by side). Trace around a student and discuss covering the area of the paper within the boundary (enclosed space) with items. The paper outline can be covered by one student. Explore other items that can be used to cover the area within the boundary. Ensure that students understand that area is contained within a boundary and is the same as coverage.

2. *Consolidate the attribute.* Choose predefined irregular and regular shapes (some closed shapes and some open) in PowerPoint. Ask students to predict which of these have area. Test each shape by copying and pasting into Paint and selecting *Fill*. Closed shapes will fill the shape only; open shapes will spill out and fill the drawing space.

Resource Resource 8.1.1 Regular and irregular shapes PowerPoint slide

3. *Explore with materials.* Use materials, computers and pictures in worksheets to experience area. Get students to colour inside shapes that have area and circle shapes that do not have area – hexagon yes, hexagon with part of a side missing no, rolled up long thin rectangle yes, spiral no (see below). Use pictures of surfaces as well.

Resource Resource 8.1.2 Area or not area?

4. *Compare and order areas directly.* For example, stand two students back to back and look from both sides to see if the areas of their backs are the same or different (be obvious, choose different sized chidren). Once larger area and smaller area have been identified, compare a third student with each of the first two to determine order. Reinforce the terms coverage and area. Alternatively, lay two shirts (identical except for size) on the floor and compare to determine larger or smaller area. Compare a third shirt with each of the other two to determine order of area covered by the shirts. Record comparisons using larger than (>), smaller than (<), equal (=) language and notation.

5. *Explore with materials.* Trace around closed hands and feet onto A4 sheets. Students should cut around the outlines and write their name on their outlines. Compare and order the class set of hands and feet. Pin to notice board in order from largest area to smallest area (or smallest area

to largest area). Discuss strategies to determine larger areas or smaller areas of hands where they may be longer and narrower or shorter and wider. Students may choose to colour overhanging parts and visually compare or take another tracing and cut the tracing to fit onto other parts for comparison. Record comparisons using larger than (>), smaller than (<), equal (=) language and notation.

Mathematics

Language/symbols and practice

Use comparison and ordering experiences from the examples developed above to practice comparing and ordering areas. Continue to discuss comparative language for area such as large, larger, largest, small, smaller, smallest, and so on. Present students with a variety of comparing and ordering problems, for example: find a surface larger/smaller than this one? Order pictures of objects or pictures of shapes by area.

Use dissection puzzles like tangrams to explore the notion of taking a shape apart and reforming to compare with another shape (pentominoes will also work). Discuss the fact that disassembling a tangram square does not change its area and that any shape formed with all the pieces will cover the same area. Compare the area of the medium triangle with the large and small triangles. What is the relationship between these? Compare the area of the small triangles with the square. What is the relationship between these? See if students can make a deduction about the relationship between the area of the square and the medium triangle.

Resource Resource 8.1.3 Tangram pieces and picture

Connections

Discuss what attribute makes a surface/shape have large or small area. Discuss how we can measure this. Look at a rectangle. What is it about the lengths of the sides that will affect area? If the length or width is increased, what will happen to the area? What if the length and width are decreased? It is not necessary to discuss these changes to length and width using numbers. Qualitative description of change will suffice to develop conceptual understanding.

Reflection

Check the idea

Relate understanding of area back to everyday life – what things in the world have area (e.g., cups, tables, walls, footballs, and so on) – what has large/small area? Explore area of parts of things. For example, a cup has a base and takes up only this area of the table (if it has straight sides); we can see the area of our footprint on the sand as the amount of ground our foot covers. If we stand against a wall the area is based on a different surface of our body. Think of as many things as you can that have an area. Walk around the school. Find things with the same or similar area.

What might be the best indication of a person’s area? Ground coverage when standing, sitting or lying down?

Apply the idea

Relate understanding of comparing and ordering area back to everyday life – find the largest park in your local area - investigate (e.g., Google Map and Internet) to find out. Is it bigger or smaller than your school grounds? How can you tell? Discuss ways to compare these accurately to determine which is bigger.

Extend the idea

Explore indirect comparison of areas using an intermediary. For example, a sheet of butchers' paper overhangs the desktop, but is smaller than the window. Indirectly compare and order items as larger area than or smaller area than.

What was important when we were comparing area? When shapes were longer and narrower or shorter and wider, what strategies were used to decide what order the shapes were from largest area to smallest area? Are there other strategies or ways to make this comparison? (This was done by colouring areas not covered and engaging in further comparison. Ideas from students here may lead to the need for a strategy to count area using units.)

Use the tangram pieces from *Resource 8.1.3 Tangrams* to explore areas of small items within the room. Explore covering surfaces with only middle-sized triangles.

- Will more or less of the big triangles cover the same area?
- Will more or less of the small triangles cover the same area?
- Is there another of the tangram shapes that can cover the same area?

Formal understanding of the relationship is not required here but this activity should lead into counting a repeated unit to compare area in Cycle 2.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #2

Here is a picture of Kingston Park Raceway.

1. Estimate the area enclosed within the red line in square units.

   Hint: Put crosses on full squares as you count them.

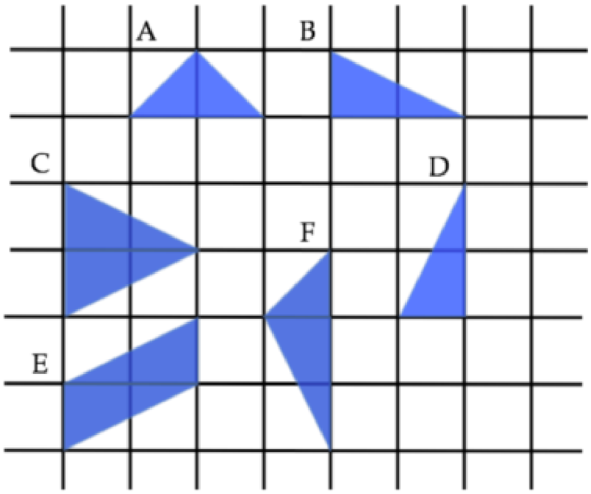
   ____

Obj. 8.2.1

i. ☐

ii. ☐

2. There are 6 shapes shown on the grid.
   Use the grid to answer the questions:



   (a) Find one shape that has the same area as D. __________________

   (b) Which shape has the same area as C? __________________

   (c) Order the shapes – A, C and F – from smallest to largest.

   ________,________,________

Obj. 8.2.1

a) ☐

b) ☐

c)i. ☐

c)ii. ☐

Cycle 2: Counting Coverage

Overview

Big Idea

Indirect comparison of coverage using an intermediary can be extended to indirect comparison of coverage using the count of a uniform repeated item. The notion of non-standard repeated units that cover a surface to be measured with no gaps or overlaps applies to area measurement in the same way that length measurement was encountered in previous units. Students should develop the understanding that area of a surface is identified by a number and the unit of measure.

Objectives

By the end of this cycle, students should be able to:

8.2.1 Compare and order areas of regular and irregular shapes by counting coverage. [4MG087]
8.2.2 Identify appropriate informal units for area. [4MG087]

Conceptual Links

Counting coverage connects ideas from Number and Geometry to Measurement concepts. Conceptual links should be made to consolidate students' previous experiences in classifying 3D objects, 2D shapes and properties of 3D objects and 2D shapes. Measurement is number driven, requires students' counting skills and provides opportunities to consolidate place value and operations. As coverage is made countable by repeating a square unit, tessellation ideas are also useful experiences to connect to.

This cycle builds the basis for standard units for measuring area. Counting area of regular shapes will connect to the array and area models for multiplication and provide a stimulus for development of the formula for calculating rectangular area in Cycle 3.

Materials

For Cycle 2 you may need:

- Multiple copies of everyday items that do not tessellate (students' handprints and footprints from Cycle 2 will work)
- Everyday items that will tessellate (books, A4 paper, newspaper sheets)
- Maths mat or large grid on floor
- Length of rope
- Overlay grids (various size squares)
- Irregular puddles (can be created on concrete with water or can be irregular shapes that can be placed on the maths mat)

Key Language

Coverage, surface, region, enclosed boundary, 2-dimensions, attribute, area, units, unit names, square units

Definitions

Geoboards: boards with regularly spaced pins or pegs that are used with rubberbands or string to generate geometric shapes with straight line edges. Can be physical or virtual (National Library of Virtual Manipulatives, USU: http://nlvm.usu.edu/en/nav/frames_asid_277_g_1_t_3.html)

Informal unit: any unit that may be repeated to cover an area. These need not be square. The count of this unit becomes the measure of the area. For example, area of 15 books.

Tessellation: any shape repeated exactly so there are no gaps and no overlaps

Square unit: any unit that has the same dimension along and across. These are useful for area as the unit is the same across and along the item making these lengths easily comparable.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the number of units for each area?
- Which area is bigger when you compare the count of units?
- Does it make a difference which way around you arrange the units if they are the same size?
- What will make it easier to compare areas – rectangular units or square units?

Students must also understand that the same unit must be repeated without gaps and overlaps in each dimension for the measure to be accurate. When comparing two items using a non-standard unit, students must recognise that they must use the same unit for measuring in order to be able to compare. Make sure that students always attach a unit name to the count when measuring (e.g., the table has an area of 12 exercise books). Where units are square this should also be stated. For example, a paddle-pop square unit can be created by gluing four paddle-pop sticks together in a square. Note that the paddle-pop square unit has two dimensions – paddle-pop by paddle-pop – or paddle-pop^2. As for other measures where units are used, the count of the unit will be greater if an item has greater area.

Portfolio Task

Counting of coverage can be used within Portfolio Task 8 although it is preferable that students are able to practise calculating coverage within the task.

RAMR Cycle

Reality

To ensure students develop counting of coverage it is best to start with irregular shapes that do not translate easily to formulae. For example, determine the number of students that will fit in a classroom by the area needed for a desk and chair leaving a walk space at the front of the room and around each doorway (this can be acted out by arranging desks and chairs and counting each desk and chair as one unit); explore the lunchtime seating areas that are not rectangles in terms of the number of pavers in size that are not rectangular; cover a desk with handprints.

Abstraction

The abstraction sequence for this cycle partitions an area using a repeated uniform unit to count the coverage of irregular and then regular shapes. This idea of unit is extended to a square unit which is tessellated to count coverage in both non-standard and standard unit squares. A suggested abstraction sequence to build counted coverage is as follows:

1. *Kinaesthetic activity.* Repeat uniform units to cover a set area. For example, students may use repeats of their traced hand from Cycle 1 to measure the area of the top of their desk. Discuss whether these are useful units for measuring and reasons why/why not (shape does not tessellate well for accurate measuring with no gaps or overlaps).

2. *Explore with materials as a class.* Introduce regular-shaped, tessellating units. Students should be encouraged to suggest a variety of better items to use as units, for example, dusters, blocks, sheets of paper, tessellating pattern blocks. Create a puddle on the concrete to measure with units or use a variety of 'puddle shapes' drawn on newspaper sheets to measure with these units. Either repeat measuring units or trace around measuring unit and repeat. The goal is for students to focus on area as count of units without the temptation to go straight to formulae. Discuss strategies to count parts of units (e.g., about half counted as half, over half counted as one, and under half not counted). Record the area of each puddle as a number followed by unit name (e.g., an area of 7 dusters).

3. *Explore with materials individually.* Provide a range of irregular shaped 'ponds or puddles' on A4 for students to measure individually or in pairs. Discuss appropriate units to use for the size of the shape to be measured. Estimate how many units the area of the 'pond or puddle' will be before counting areas.

Resource
Resource 8.2.1 Tessellating units to count area
Resource 8.2.2 Irregular shaped areas

4. *Establish appropriate sizes of unit and best shape for measuring.* Measure desktops, maths book, area taken up by a desk and chair with at least three different units (e.g., duster, A5 paper, calculator, square paper serviettes). Students should create a table to record item, unit, estimate area, counted area of each item as a number followed by unit name (e.g., an area of 7 dusters).

- Discuss whether the area will be the same if rectangular units are arranged a different way (e.g., along the desk instead of across the desk). Test this out to check.
- Discuss which shape of unit is most effective and why square units might be most useful.
- Discuss the relationship between the size of the unit and the number of units to count the area.
- Discuss which units will be more accurate for measuring and which are most appropriate depending on the size of the item being measured.

5. Use a length unit to create a square for counting area. Once students have established that square units are most appropriate for measuring area, explore the creation of a square unit from a length unit (e.g., paddle-pop sticks, matchsticks). Revisit the puddle measures on A4 from Resource 8.2.2 Irregular shaped areas. Use different sized overlay grids to count the area of these puddles in square units.

Resource Resource 8.2.3 Overlay grids

Mathematics

Once students' have understood the need for square units that can be counted to determine area, it is important to practice the language and recording of area measures in square units (or unit squares). Remember that the focus is still on counted area, although students may suggest 'short cuts' for determining area that are multiplicative.

Language/symbols and practice

Create regions inside string or rubber bands on geoboards (real or virtual) as irregular and regular shapes to practise estimating and counting areas (grids for counting are created by line segments between the nails or posts). Estimate areas first, then count (whole squares and half squares or part squares). Focus on describing the area as square units or unit squares.

Resource National Library of Virtual Manipulatives, USU:
http://nlvm.usu.edu/en/nav/frames_asid_277_g_1_t_3.html

Use plastic overlay grids (5mm, 10mm, 20mm grids) without explicitly mentioning the dimensions (refer to as small, medium and large grids), to determine the number of square units covered by local parklands on the Refidex or Google Maps (or use *Resource 8.2.4 Parkland maps*). Count the areas in square units or unit squares.

Note: Relative size of the square units as larger or smaller is the focus here in terms of count, not the size of the grids/metric measure/scale. The key idea that students need to consolidate is that fewer large units will be needed to measure an area than medium units, and more small units will be needed to measure the same area than medium units. That is, there is an inverse proportion between unit size and number of units when measuring.

Resource Resource 8.2.3 Overlay grids
Resource 8.2.4 Parkland maps

Connections

Connect Area and Perimeter

Since students have explored length, perimeter and area measure, it is important to ensure that they have a clear understanding of each of the attributes and the relationships between them. Length is directly related to perimeter and area. Doubling lengths of a shape will double the perimeter of the shape, while increasing the area by 2^2 or 4 times. Since expecting to double all dimensions of a shape to double the area is a common mistake, it is important to differentiate between the attributes of perimeter and area, and to clearly develop understanding that shapes with the same area may have different perimeters and shapes with the same perimeter may have different areas. For a cartoon clip that may introduce the idea try Cyberchase 202 Totally Rad (Part 1: http://youtu.be/P_dMf6QscfY; or Part 2: http://youtu.be/xE1lr7bKpCM; Part 3 gives the answer to the problems and should be left until after students have investigated problems on their own).

- Explore using a piece of string of a set length (e.g., 16cm) to make rectangles on 1cm grid paper. Record the side lengths and areas of each rectangle with the same perimeter.
- Reinforce by marking out rectangles with areas of 16cm^2 on grid paper. Find how many different rectangles can be marked out with the same area. Record the side lengths and perimeters of each rectangle.

Resource Resource 8.2.5 Tables for recording perimeter and area

Explore connections between area and perimeter of simple shapes. Using *Resource 8.2.6 Area and Perimeter*, record the perimeters of shapes with the same area and the areas of shapes with the same perimeter. Extend to related shapes where one dimension is doubled and both dimensions are doubled to reinforce the difference between area and perimeter as measures.

Resource Resource 8.2.6 Area and Perimeter

Relationship between changing side lengths and area

Lay paddle pop stick square units out on a surface. These may be made from paddle pop sticks joined in squares or by using *Resource 8.2.3 Overlay grids*. Sit half paddle pop stick square units on top of paddle pop stick squares. Explore how many smaller square units fit within the larger square unit. Explore further with quarter length paddle pop stick squares and the number of these that fit within each of the other units. Discuss what would happen if double paddle pop stick squares were used.

Students should note that when the side length of a square unit is increased by a factor of two, the new unit is the same as four (two by two) of the original unit. Similarly, if the side length of a square unit is decreased by a factor of two, the new unit is the same as one quarter (half by half) of the original unit.

Reflection

Check the idea

Find examples of areas that can be counted using non-standard units in the local community (e.g., measuring the area of paving in terms of large pavers). Try to get students to think of a variety of ways non-standard units are used in the world, local and otherwise, for area (e.g., describing an area as so many house blocks, 1.5 car space garages). Use history and look at other units used in the past (e.g., acres).

Apply the idea

Set up area problems based on non-standard units. Determine the area of the verandah outside the classroom in 'school bag units', determine the area in the classroom in 'desk, chair and walk-space' units, determine the area of the classroom in 'desk islands, chairs and walk-space' units. Relate this back to class size and number of students possible in a room.

Perimeter, Area and seating problem

A café seating area needs to seat as many people as possible within a small space. Investigate whether long tables to seat 6 or smaller tables to seat 4 are more space effective. Justify your response.

Extend the idea

Explicitly discuss critical ideas behind area measurement with students. Include the following ideas:

- Discuss how the area non-standard units have broken up a continuous surface into small parts that allows that area to be counted.
- Discuss that: (a) we cannot measure accurately if we vary the unit; and (b) we cannot know if a surface is larger/smaller than another unless we use the same unit. Reinforce what happens if a surface is measured sloppily; allowing units to have gaps/overlaps, and ask what is wrong?
- Look particularly at the best area unit in terms of shape – why square?
- Discuss the need for a common standard unit.
- Ensure students understand that they will use less large units to measure the same area than they will with small units. Connect this idea to accuracy of measurement.

Teacher Reflective Notes

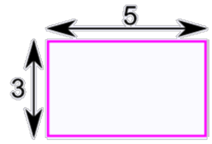
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________ Date:________

Can you do this? #3

1. Find the area of the rectangle.

   The units are hand spans.



Obj. 8.3.1
i. ☐
ii. ☐

2. Name something that would be reasonable to measure using the following units.

   (a) cm^2 ____________________

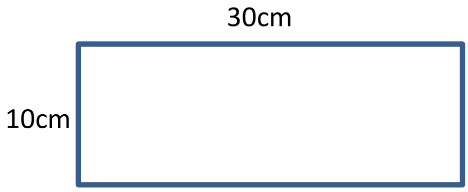
   (b) m^2 ____________________

   (c) mm^2 ____________________

   (d) km^2 ____________________

Obj. 8.3.2
a) ☐
b) ☐
c) ☐
d) ☐

3. Calculate the area of this rectangle.

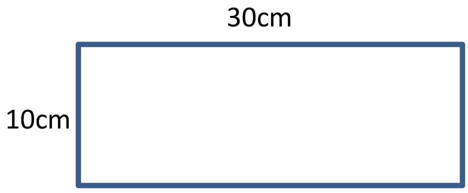


Obj. 8.3.3
i. ☐
ii. ☐

4. What might be the dimensions of another rectangle that would have the same area?

Obj. 8.3.3
☐

5. Calculate the perimeter of this rectangle.



Obj. 4.3.3
i. ☐
ii. ☐

6. What might be the dimensions of another rectangle that would have the same perimeter?

Obj. 4.3.3
☐

Cycle 3: Calculating Rectangular Areas

Overview

Big Idea

This cycle extends from counted area to calculated area of rectangular shapes. This idea makes use of the array model of multiplication where a number of equal rows can be multiplied in preference to counting. Rectangular area measure connects to students' measurement of lengths using metric measures, and should also be connected to square units of these measures (mm^2, cm^2, m^2, km^2). However, the conversion between metric units will not be addressed in this cycle.

Once students see that they can multiply two dimensions of a rectangular area they should also be able to generate a formula for calculating rectangular area. The area model for multiplication is also used to explore strategies for large number multiplicative operations. It is also important to explicitly differentiate area from perimeter. Students develop the understanding that shapes with the same area may have different perimeters and shapes with the same perimeter may have different areas.

Objectives

By the end of this cycle, students should be able to:

8.3.1 Calculate the area of rectangles using familiar units. [5MG109]
8.3.2 Identify appropriate metric units for area. [5MG108]
8.3.3 Calculate the area of rectangles using familiar metric units. [5MG109]

Conceptual Links

Calculating coverage connects ideas from Number and Geometry to Measurement concepts. Conceptual links should be made to consolidate students' previous experiences in classifying 3D objects, 2D shapes and properties of 3D objects and 2D shapes. Measurement is number driven, requires students' counting skills and provides opportunities to consolidate place value and operations. As coverage is made countable by repeating a square unit, tessellation ideas are also useful prior experiences to connect to.

This cycle provides a basis for calculating the area of a rectangular shape which forms the basis for understanding conversion of square units in Cycle 4, discovering the formula for area of parallelograms and triangles in Cycle 5, and area of circles in Cycle 6. Calculating the area of rectangular shapes will also contribute to calculating the area of composite shapes and surface area of prisms in Cycle 7.

Materials

For Cycle 3 you may need:

- Math mat
- Newspaper sheets
- A4 paper
- Matchsticks/paddle pop sticks
- Counters
- Number lines
- MAB blocks
- Metre rulers
- Centimetre overlay grids
- Felt pens or markers
- 99 boards
- PVC (hundreds, tens, ones
- Calculators

Key Language

Coverage, surface, 2-dimensions, attribute, area, units, unit names, square units, length, width, formula

Definitions

Array model of multiplication: items arranged in rows and columns to facilitate simpler counting. Useful step between repeated addition and area model.

PVC: Place value chart

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many units along the length of the area? How many units across the width of the area?
- If you multiply these together what answer do you get? Is this the same as if you counted them separately? Does it matter which order you multiply these in?
- If you know the total number of units in the area, and how many along the length or width, can you work out the other dimension? What operation can you use to work this out?
- How can we write this down in symbols?

Reinforce with students that they are working in two dimensions (length by width) so their answer must have unit^2 or square units after it.

Portfolio Task

Within the task students are provided with the opportunity to calculate areas and perimeters and compare these in the context of building cheaper buildings.

RAMR Cycle

Reality

For this cycle, contexts need to involve rectangular shapes. For example, determine the area of the ceiling by looking at the ceiling tiles; consider the area of the walls in building block units; consider the area of the floor in carpet tiles; cover a noticeboard with enough pieces of paper for each student to display a poster. Discuss how the units are arranged on the ceiling, walls and noticeboard (as an array – in rows and columns).

Abstraction

The next step from counted coverage using grids is to use grids on regular shapes like rectangles where the grids form an array. Initially this array can be counted but students should quickly see the link between counting all and counting the number in a row and then the number of rows. This is the beginning of calculated area which enables students to discover the formula for area of a rectangle. A suggested sequence for this development is as follows:

1. *Choose a rectangular area to measure.* Start with the maths mat and count its area using the marked squares as in *Cycle 2: Counting coverage*.
2. *Connect to students' previous experience with the array model for multiplication.* Discuss quicker ways to determine the area than counting every square. Students should be able to identify the number of squares in a row and how many rows in the rectangle. Explicitly connect to multiplication. Ensure students provide a label for their measure as square units or unit squares.
3. *Reinforce the previous experience* by using square metres to measure the basketball court. Use a tape measure to mark out metres with chalk along the baseline of the court. Mark 1m up the sideline on each side. Use lengths of rope or wool to lay out the first 'row' of metre squares. Mark metre segments along the edge of the basketball court. Discuss how many rows of metre squares would fit along the side of the basketball court. Use these dimensions to calculate the area of the basketball court.
4. This process could be further reinforced by considering each section of the basketball court as separate areas for calculation.

Mathematics

Language/symbols and practice

Provide students with a variety of items to practise estimating and measuring the area of using 1 cm grid overlays. For example, book, paper clip box, pencil case, desk, whiteboard. Encourage students to identify the number of rows and columns on the grid paper and then use multiplication to determine the area of each item. Recording these on a table will assist students to discover a rule or formula for area calculation of rectangles. Students should progress from examples with the grid marked in, to examples with marks along the edges, to examples with measures only.

Resource Resource 8.3.1 Estimating and measuring area.
Resource 8.3.2 Discovering formula for area of a rectangle.

As for length measure, find personal referents for area (e.g., little fingernail with a square cm, palm of hand related to 10 square cm). Find everyday items that are 1cm^2, 1m^2, 1mm^2.

Look for square metres in the classroom – part or all of a door, a window, a blackboard, the teacher's desk top, and so on.

Resource Resource 8.3.3 Personal referents for area.

Attempt to experience a hectare (stand a student at each end of the 100m running track as one edge, measure or pace out another 100m at right angles from each student to place another two students. Spread the rest of the class within these boundaries evenly to experience standing inside this sized space). The hectare could take some discussion as it appears disconnected from other metric measures (not explicitly linked to a single unit).

Use a map of the local area – draw around areas that are a hectare or a square km. How many house blocks fit into the hectare? How many road blocks fit into a square km? Find areas on local maps that could be measured in square kilometres (use Refidex grids as a base for calculating; *Resource 8-4A Parkland maps* may be useful here).

Discuss with students what measures would be most appropriate to use to measure common items, for example, a desk, the classroom ceiling, the school ground, local parkland, the school's suburb.

Connections

Factor-Factor-Product and Inverse

Connect measurement of area to division. Ensure students can use factor-factor-product thinking to find the unknown dimension of a circle if they know the other dimension and the area.

Relationship between Length, Area and Perimeter

Where possible, find real life contexts for calculating areas and perimeters to embed the activities in using relevant objects or situations. For this cycle, these contexts can include determining the most cost effective shape to provide maximum area with a minimum of perimeter fencing for animal enclosures, most effective room dimensions to maximise living area with a minimum of wall costs, or making the most usable area from a set number of pavers and edging material. For this cycle exploration will be restricted to rectangles (circles are actually the biggest area for the smallest perimeter but are beyond the scope of this cycle).

Engage students with investigating and justifying the cheapest cat pen with the greatest area for Matt's cats. Students should demonstrate using diagrams what shaped cage will use the least wire around four sides and across the top to give Matt's cats a space to be outside without endangering local wildlife. A total length of wire should be calculated to aid Matt in shopping for materials for the cat cage.

Resource Resource 8.3.4 Matt's cat cage

Reflection

Check the idea

Challenge students to calculate the area of windows in the classroom and compare this to the area of solid wall. Which is larger? What is the difference between the two areas?

Apply the idea

Reinforce fraction and percentage calculations by finding the fraction of a wall that is taken up by windows. Convert this fraction to a percentage. If students are interested research the building regulations for percentage of wall space that must be windows. Is the classroom more or less than this value? Calculate the total area of wall in the classroom to be painted. Discuss where items should be removed from the wall to paint properly. Calculate what area of wall is covered by noticeboards, whiteboards, and windows.

Extend the idea

Extend the earlier investigation of area and perimeter connection using metric measures and calculations for perimeter and area.

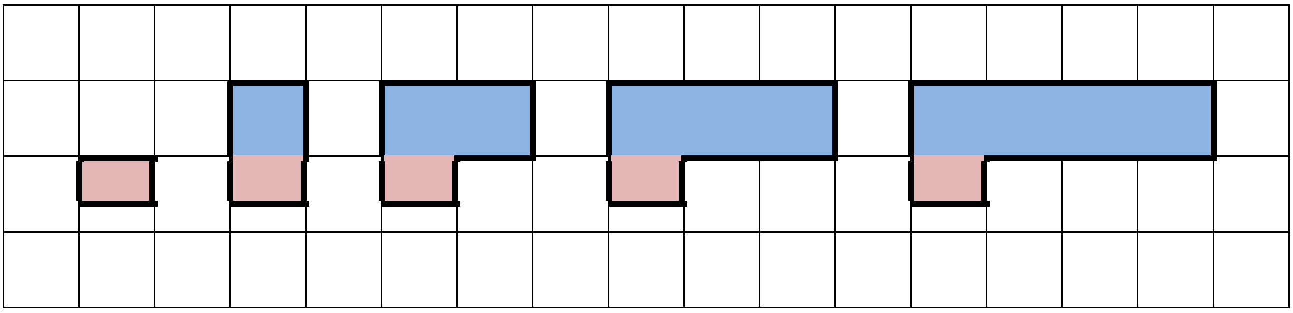
Skate Park Design

Design a layout for a skate park to include ramps and jumps that is safe for skateboards to turn and jump within. Justify the shape of the park in terms of distance for skating and tricks and area to turn around within for safety. Identify the best shape for your park.

Patterning in Area and Perimeter

Explore growing patterns involving area and perimeter.

Give students a sheet of grid paper each. Have them draw the following shapes.



- Have students create the next two shapes in the pattern.
- See if they can predict and draw the tenth shape in the pattern without including the intervening shapes.

Create a table to start abstracting a growing pattern rule.

n (no. of blue squares)	0	1	2	3	4	5	6	7	8	9	10
Area	red	1 × blue + red	2 × blue + red	3 × blue + red	4 × blue + red	5 × blue + red	6 × blue + red				

See if students can:

- Continue the pattern up to (or beyond) 10 blue squares
- Generalise the pattern into a rule: Area = number of blue squares + red square = nb + r
- Repeat for Perimeter

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:_______

Can you do this? #4

1. Would you use square metres or square centimetres to measure:
   (a) The cover of a book? ____________________
   (b) A CD cover? ____________________
   (c) Sports oval? ____________________
   (d) The classroom floor? ____________________

Obj. 8.3.2
a) ☐
b) ☐
c) ☐
d) ☐

2. Convert the following metric measures:
   (a) $1m^2$ = _______ cm^2
   (b) $1km^2$ = _______ m^2
   (c) 1ha = _______ m^2
   (d) $1cm^2$ = _______ m^2

Obj. 8.4.1
a) ☐
b) ☐
c) ☐
d) ☐

3. Use a ruler to draw a shape that has an area of 12 cm^2.
   Label your shape with measurements for each side.

Obj. 8.4.2
i. ☐
ii. ☐
iii. ☐
iv. ☐

4. How could you change your shape to double its area to 24 cm^2?
   ____
   ____

Obj. 8.4.2
☐

Cycle 4: Metric Conversion for Area

Overview

Big Idea

This cycle connects use of a square unit for measuring, calculated area of rectangular shapes and metric measure to explore unit conversions for area starting with relationships between related non-standard unit squares and continuing to metric unit squares (mm^2, cm^2, m^2, km^2).

Previous cycles (Cycle 2 and Cycle 3) explored the squaring relationship between side length and area when comparing scaled (proportional) shapes. In this cycle, rather than changing the side length, we are changing the unit of side length measure (count of unit goes up/down by a conversion factor) and so, in a related way, the count of the area units also changes by the square of the factor.

Objectives

By the end of this cycle, students should be able to:

8.4.1 Convert between metric units for area. [8MG195]
8.4.2 Establish general formulae for area and use these in problem solving. [7MG159]

Conceptual Links

This cycle requires students to be able to calculate area of rectangles and convert between familiar metric units of length. These ideas provide a base to build strategies to convert between familiar metric units of area. Place value understandings and operations are also consolidated throughout activities in this cycle.

This cycle provides a basis for converting areas from larger to smaller units and smaller to larger units. This will consolidate area understanding and calculations and further develop understanding of metric measure for future extension to volume understanding and calculations in Unit 14.

Materials

For Cycle 4 you may need:

- Math mat
- Large square units, small square units (e.g., paddle pop stick squares, half paddle pop stick squares from *Resource 8.4.1 Overlay Grids*)
- MAB blocks
- Metre rulers
- Overlay grids

Key Language

Metric measures of area, units, unit names, square units, centimetre squares, cm^2, metre squares, m^2, millimetre squares, mm^2, kilometre squares, km^2

Definitions

Acre: Imperial unit of measure that students may have heard or seen. 1 ha is about 2.5 acres.

Hectare (ha): Area of 100m by 100m; 10 000 m^2; 0.01 km^2; easily experienced by students on the oval by placing a student at each end of 100m running track, measuring 100m at right angles to the running track and placing students at other corners.

Standard units: Fact Monster has an attractive page with simply stated information for students. http://www.factmonster.com/ipka/A0769580.html

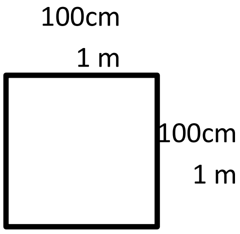
Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the unit of measure of the sides?
- Is the unit you are converting to larger or smaller?
- If you are converting to a larger unit, will you have more or less units to cover the area?
- If you are converting to a smaller unit, will you have more or less units to cover the area?
- Can you draw a picture to help you?
- Which operation will you need to use; multiply or divide?
- What are you going to multiply/divide by?

It may help students with conversions if they draw a labelled picture of a unit square, to work out the conversion factors for area. For example,



Thinking process:

$1\text{ m}^2 = 1\text{m} \times 1\text{m}$

$= 100\text{cm} \times 100\text{cm}$

$= 10\,000\text{ cm}^2$

So to convert from:

$m^2 \rightarrow cm^2$ multiply by 10 000

$cm^2 \rightarrow m^2$ divide by 10 000

Portfolio Task

Ideas from this cycle may be used to complete sections of the task that require calculation of rectangular areas and include conversion between metric units of measure.

RAMR Cycle

Reality

For this cycle, contexts need to be rectangular shapes. For example, consider the area of the floor using carpet or ceramic tiles; what would the count of units for the area be if smaller carpet or ceramic tiles were used. Discuss how the number of tiles across the room would change if the tiles were half the width they are. Discuss how the number of tiles along the room would change if the tiles were half the length they are.

Abstraction

While length conversion is simply linked to place value, the use of square units in area requires multiplication to calculate area conversions. That is, when converting area units both the length and the width need to be converted resulting in a multiplicative relationship (e.g., $1m^2 = 100cm \times 100cm = 10\ 000cm^2$). This abstraction sequence develops this idea by starting with non-standard units (that are related multiplicatively), before connecting to metric unit conversion and place value. A suggested sequence for this development is as follows:

1. *Choose a rectangular area to measure.* Start with the maths mat and count or calculate its area using the marked squares as in *Cycle 2: Counting coverage*.
2. Halve one dimension of each square by laying lengths of wool or string across the mat from edge to edge. Count or calculate the number of units for each dimension (should be doubled for one and the same for the other). Discuss that these units are not square units anymore. Calculate the area in these units and compare to the original (should be doubled).
3. Halve the other dimension of each square by laying lengths of wool or string across the mat from edge to edge (recreating square units). Count or calculate the number of units for each dimension (both dimensions should now be doubled). Calculate the area using these units and compare to the original (should be four times or double^2).
4. *Reinforce the previous experiences* by using paddle pop stick square units to measure a desk and half-paddle pop stick square units. Estimate how many of each first, measure and record with the larger unit, revise the estimate for the smaller unit, then measure and record with the smaller unit. If desired this can be extended to include the quarter-paddle pop stick square units. *Resource 8.4.1 Overlay grids* may be useful here.

Resource Resource 8.4.1 Overlay grids

5. *Discuss the relationships between the square units.* Ensure that students understand that the larger square unit is double the dimension of the smaller square unit both ways. This means that there are twice as many units across and along the square which makes four smaller square units in the larger square unit.
6. *Extend to metric units.* Use metre rulers to define $1m^2$ on the floor. Explore how many square centimetres fit within a square metre (MAB flats are useful here if a physical representation is needed). Discuss the differences in size and how many of the smaller unit squares fit into the larger unit square ($10\ 000cm^2$ in $1m^2$).

7. *Connect metric units to place value.* Set up place value cards as follows up to a square metre:

		m^2		dm^2		cm^2		mm^2
H	T	O	H	T	O	H	T	O
	Millions			Thousands			Ones	

8. Use multiplicative relationships between PV positions to discuss the conversion rate between the area units, from m^2 to cm^2; $1m^2 = 10\ 000cm^2$.

   Note: The decimetre (10cm; one tenth of a metre) square has been included here to fill a space within the sequence. It is useful to discuss the decimetre so that students are familiar with this terminology because of its value in connecting volume and capacity (i.e., $1dm^3 = 1L$).

9. Use a movable place value strip to show how place values change when units are converted. For example, to show the conversion from $2.45\ m^2$ to $24\ 500\ cm^2$, set up the chart as follows with the ones of ones label under the unit name of m^2.

								m^2		dm^2		cm^2		mm^2
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
	Millions			Thousands			Ones			thousandths			millionths	
								2	4	5				

   To convert to square centimetres, move the place value strip only so that the ones of ones label is under the unit name of cm^2 and fill in extra place values with zeros.

				m^2		dm^2		cm^2		mm^2	
H	T	O	H	T	O	H	T	O	H	T	O
	Millions			Thousands			Ones			thousandths	
				2	4	5	0	0			

   If students struggle here ask them to visualise the number of square centimetres in a square metre. Connect to the 'square' so that they see 100cm long by 100cm wide. Let them draw a labelled sketch if it helps. This should help reinforce that there are $10\ 000cm^2$ in $1m^2$.

10. Use metric expanders and metric slide rules to relate/connect metrics to PV and to each other (conversions). Metric expander is PV set up above with blanks for spaces between metrics: It folds like ordinary number expanders with the pleat fold at the units.

				m^2			dm^2			cm^2			mm^2	

Resource Resource 8.4.2 Metric expanders and slide rules for area

Mathematics

Language/symbols and practice

Discuss the units used for length measurement. Revise what a square based on each of these units will look like and the relationships between the lengths. Revisit the puddle measures on A4 from

Resource 8.4.3 Irregular shaped areas. Use a 1cm overlay grid to estimate (combine calculation and count) the area of these puddles in square centimetres (cm^2). Convert these estimates to square millimetres (mm^2) or square decimetres (dm^2).

Resource Resource 8.4.1 Overlay grids
Resource 8.4.3 Irregular shaped areas

Connections

Place value and decimal connections

Extend the connections between place value and decimals started in the Abstraction phase. Set up the place value chart as follows:

		km^2		ha				m^2		dm^2		cm^2		mm^2
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
	Trillions			Billions			Millions			Thousands			Ones	

Use multiplicative relationships between PV positions to discuss the conversion rate between the area units, mm^2, cm^2, m^2, ha, and km^2: 100 mm^2 = 1 cm^2; 10 000 cm^2 = 1 m^2; 10 000 m^2 = 1 ha; 100 ha = 1 km^2. Place value charts can be used as described in the Abstraction phase. If students struggle here, encourage them to draw a labelled sketch of a square of the unit and connect to the conversion rate for the side length. For example, to visualise millimetres in a square centimetre, connect to the 'square' so that they see 10mm long by 10mm wide. This should help reinforce that there are 100mm^2 in 1cm^2.

Extend metric expanders and metric slide rules to relate/connect larger metric units to PV and to each other (conversions). Metric expander is PV set up above with blanks for spaces between metrics: It folds like ordinary number expanders with the pleat fold at the units.

	km^2			ha					m^2					cm^2			mm^2

Explore the hectare as a 'fill in measure' between square metres and square kilometres. Discuss why this might be useful.

Resource Resource 8.4.2 Metric expanders and slide rules for area

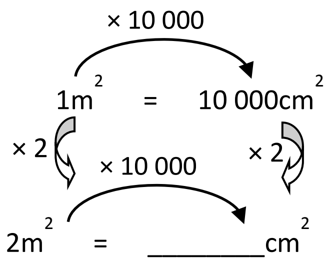
Metric conversions

Metric conversions should be explicitly connected to decimals. They apply decimal understanding and reinforce decimal concepts. Area can be seen to have very large conversions, therefore we have to use a large number of PV positions to get from mm^2 to km^2.

Use patterns to develop conversion calculations:

1m^2 = 1m × 1m = 100cm × 100cm = 10 000cm^2

Arrowmath diagrams may make this clearer as a single step to see the conversion pattern:



This diagram may be used for other comparisons as shown below:

$$\begin{array}{ccc} & \times\,10\,000 & \\ 1\text{m}^2 & = & 10\,000\text{cm}^2 \\ \times\,3 \downarrow & \times\,10\,000 & \downarrow \times\,3 \\ 3\text{m}^2 & = & \_\_\_\_\_\_\_\_\text{cm}^2 \end{array}$$

$$\begin{array}{ccc} & \times\,10\,000 & \\ 1\text{m}^2 & = & 10\,000\text{cm}^2 \\ \times\,? \downarrow & \times\,10\,000 & \downarrow \times\,? \\ 11\text{m}^2 & = & \_\_\_\_\_\_\_\_\text{cm}^2 \end{array}$$

Try saying the conversion aloud with the students – “For every 1 square metre there are ten thousand square centimetres”. Repeat the process to explore cm^2 to mm^2.

The emphasis is on the students knowing and understanding the relationship between the units not memorising a multiplier. This multiplier can be worked out using a quick sketch when needed.

Reflection

Check the idea

Explore common packaging of items (e.g., jelly crystals, sultanas). Ask students to estimate and calculate the area of each surface of their box in square centimetres and convert to square millimetres.

Alternatively, challenge students to calculate the area of windows in the classroom in square centimetres and convert this to square metres.

Apply the idea

Builders’ dimensions on floor plans are usually written in millimetres. Calculate the areas of walls for painting from a floor plan for a 2400mm ceiling height. What would be the difference in area if the ceiling height were 2.7m?

Consider where windows are marked on the plan. Calculate the area of each window from its given dimensions (usually millimetres). Calculate how much less area of the wall needs to be painted because of windows (again, square metres when considering painting). What is the resulting area of wall for painting for each ceiling height?

If each 1L of paint will cover $16m^2$ of wall (wall area for painting is usually in square metres), how many litres of paint will be needed to paint the walls?

Resource Resource 8.4.4 Floor plan

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #5

1. Use a ruler to sketch a parallelogram and label it with the length of 12cm and the perpendicular height of 3 cm.

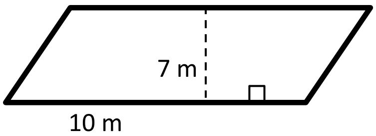
Obj. 8.5.1 i.☐ ii.☐ iii.☐

2. Calculate the area of your parallelogram: _________cm^2

Obj. 8.5.1 i. ☐ ii. ☐ iii. ☐

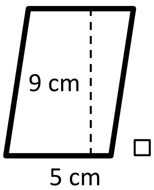
3. Find the area of the following parallelograms:

(a)



_________m^2

(b)

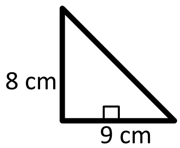


_________cm^2

Obj. 8.5.1 a)i.☐ a)ii.☐ b)i.☐ b)ii.☐

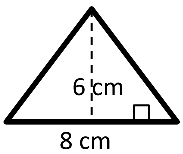
4. Find the area of the following triangles:

(a)



_________cm^2

(b)



_________cm^2

Obj. 8.5.2 a)i.☐ a)ii.☐ b)i.☐ b)ii.☐

5. A triangular floor tile has an area of $40cm^2$, and perpendicular height of 8 cm.
   (a) Sketch the triangle and label it with the information provided.
   (b) What is the length of the triangle's base? ______________________
   (c) Explain how you calculated this? ___________________________
   ____

Obj. 8.5.2 a)i.☐ a)ii.☐ b)i.☐ b)ii.☐ c)i.☐

Cycle 5: Area of Parallelograms and Triangles

Overview

Big Idea

This cycle extends from calculated area of rectangular shapes to calculated area of parallelograms and triangles. This idea requires students to make a connection between parallelograms, rectangles and triangles. Once students see that they can create a rectangular area from a parallelogram, they should also be able to generate a formula for calculating area of a parallelogram. Similarly, once students recognise that a triangle is half a parallelogram or rectangle, they should be able to generate a formula for calculating area of a triangle.

Objectives

By the end of this cycle, students should be able to:

8.5.1 Establish the formula for areas of parallelograms and use these in problem solving. [7MG159]
8.5.2 Establish the formula for areas of triangles and use these in problem solving. [7MG159]

Conceptual Links

This cycle requires students to be able to calculate area of rectangles as a basis for exploring area of other 2D shapes. Students need to be able to identify 2D shapes of parallelograms and triangles as well as have an understanding of the properties of these shapes to succeed in this cycle.

This cycle provides a basis for calculating the area of parallelograms and triangles which forms a basis for discovering the formula for area of circles in Cycle 6. Calculating the area of parallelograms will also contribute to calculating the area of composite shapes in Cycle 7.

Materials

For Cycle 5 you may need:

- A4 paper
- Selection of parallelograms
- Protractors
- Rulers
- Scissors
- Centimetre overlay grids

Key Language

Coverage, surface, 2-dimensions, attribute, area, units, unit names, formula, length, width, parallelogram, rhombus, perpendicular height, standard metric measures and units, accepted mathematical symbols to indicate perpendicular lines, parallel lines and same length sides, triangle, right angle triangle

Definitions

Apex: Point on a triangle; usually opposite to the base

Parallelogram: Quadrilateral with two pairs of parallel sides

Rhombus: Special case of a parallelogram with all sides the same length

Perpendicular height (⊾): Height of a triangle measured from base to apex at right angles to the base, or height of a parallelogram measured between two parallel sides at right angles to the base

Right-angle triangle: Triangle with one angle that measures 90^{o}

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you draw a line perpendicular to the base and up to the top/apex?
- What is that distance?

Portfolio Task

Ideas from this cycle may be used to complete sections of the task that require calculation of triangular areas.

RAMR Cycle

Reality

For this cycle, contexts need to include parallelograms and triangles. (A Google Images search for Parallelograms in the real world is helpful here.) For example, floor tile patterns to give the illusion of 3D blocks, tumbling blocks quilts, floor mosaics with a symmetrical star (although some of these are actually kites), rectangles viewed from an angle.

Abstraction

In geometry, rectangles and squares are a specific subset of parallelograms. However, the right angles within rectangles and squares make these easier to use when discovering relationships and determining formulae. To extend students' thinking from these specific parallelograms to the more general with respect to calculating area, it is necessary to provide students with experiences that (a) allow students to see that a rectangle with the same area can be created from all the pieces of a parallelogram, (b) identify critical attributes of the shapes that lead to classification, and (c) refine students' understanding of the critical attributes of the dimensions of the rectangle that transfer to the formula that can then be applied to the area of the parallelogram. A suggested abstraction sequence for this progression of thinking is as follows:

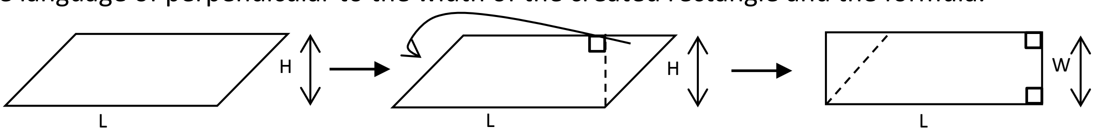
1. *Revisit classification of shapes by attributes* (Unit 04). Sort a collection of shapes that includes variably-sized parallelograms. Identify the critical attributes of a parallelogram: two pairs of parallel sides, each pair the same length, relationships between angles. Discuss the attributes of rectangles/squares for students to see how these are special cases of parallelograms. Explicitly identify the right angle in a rectangle as a critical attribute of a rectangle.
2. *Revisit dissecting shapes* to make other shapes of the same area from *Cycle 1: Area Comparison*. Encourage students to explore arrangements of the square and two small triangles. Can these be made into a parallelogram? Can these be made into a rectangle? Do these shapes have the same area? Can students measure and calculate the area of the rectangle that has been created?

Resource Resource 8.5.1 Tangram pieces and picture

3. Explore a variety of parallelograms. Initially, these can be created on grid paper to connect to counted area and then progress to blank parallelograms for measuring. Tabulate the lengths of the sides, whether they can be made into rectangles (allow students to cut the parallelograms and reform as they did with the tangram pieces), the lengths and widths of the rectangles and the calculated areas.

Resource Resource 8.5.2 Parallelogram exploration

4. Explicitly discuss with students the relationship between the dimensions of the parallelograms and the rectangles that allow them to calculate the area of parallelograms without physically cutting. Explore parallelograms without grid markings. Encourage students to pencil in where they would cut the parallelogram and where the offcut would shift to for a rectangle. Connect the language of perpendicular to the width of the created rectangle and the formula.



Mathematics

Language/symbols and practice

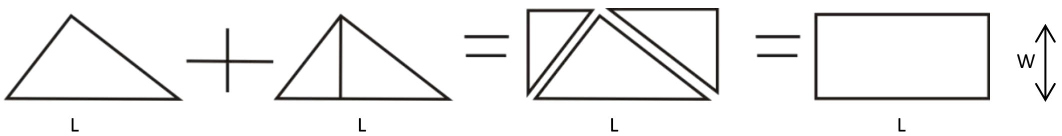
Once students have explored area of parallelograms by connecting to rectangular areas, they should be given further opportunities to practise calculating areas. Generate tiling problems that use parallelograms for designs. What area parallelogram will be needed to fill the space? From a given area to tile, how many parallelograms of a given size will be needed to tile the area? What possible dimensions of tiles will cover the given area?

Resource Resource 8.5.3 Parallelogram tiling problems

Exploring area of triangles

Triangles are frequently used in construction because of their rigidity. It is easy to see that right angle triangles can be doubled to create a rectangle. While this is a simple example to use which can be quickly connected to rectangular formulae, it does not translate as readily to a formula that works for all triangles unless students understand that the perpendicular height of the triangle is what the width of the rectangle becomes. Using triangles other than right angle triangles, finding a perpendicular width, and then connecting to parallelograms by doubling the triangle or cutting a copy of the triangle to create a rectangle will provide a more robust understanding for students. A suggested sequence for developing the area of a triangle is as follows:

1. *Revisit classification of shapes by attributes*. Identify the critical attributes of a triangle as having three sides and three angles. Discuss the attributes of a variety of triangles to ensure students have retained the properties of different classes of triangles.
2. *Revisit dissecting shapes* to make other shapes of the same area from *Cycle 1: Area Comparison*. Use a range of triangles and tracing paper. Make a copy of the triangle, use a protractor to create a perpendicular line from the base to the point of the triangle. Cut the copy along the line and check that the pieces still have the same area as the original triangle.
3. Ask students to create a parallelogram using the original triangle and the pieces of the copy. See if they can also create a rectangle (as below).



4. *Explore a variety of triangles.* These can be created on grid paper initially to connect to counted area and progress to blank triangles for measuring. Tabulate the lengths of the sides, whether they can be made into rectangles (allow students to cut the parallelograms and reform as they did with the tangram pieces), the lengths and widths of the rectangles and the calculated areas.

Resource Resource 8.5.4 Triangle exploration

5. *Explicitly discuss* with students the relationship between the dimensions of the parallelograms and the rectangles that allow them to calculate the area of triangles without physically copying and cutting. Explore triangles without grid markings. Make a copy of the triangle, so that students have two triangles to work with. Encourage students to pencil in where they would cut the second triangle and where the pieces would be moved to make a rectangle. Connect the language of perpendicular height to the width of the created rectangle and the formula. Ensure students understand that the area of the triangle is half the area of the corresponding rectangle.

Once students have explored area of triangles by connecting to rectangular areas, they should be given further opportunities to practise calculating areas. Generate tiling problems that use triangles for designs. What area triangle will be needed to fill the space? From a given area to tile, how many triangles of a given size will be needed to tile the area? What possible dimensions of tiles will cover the given area?

Resource Resource 8.5.5 Triangle tiling problems

Connections

Factor-Factor-Product and Inverse

Connect measurement of area to division. Ensure students can use factor-factor-product thinking to find the unknown dimension of triangles and parallelograms if they know the other dimension and the area. (*Resource 8.5.5 Triangle tiling problems* and *Resource 8.5.3 Parallelogram tiling problems* include some examples of these types of problems.)

Reflection

Check the idea

Consider the use of parallelograms in construction of robots to create moving joints or combinations of triangles in biplane wing supports. Determine the area of 'skin' needed to cover the section of leg on the robot or the biplane wing section that is a parallelogram.

Consider the use of triangles in construction of roofs and bridge supports. Determine the area of cladding needed to cover the triangular ends of roofs.

Apply the idea

Challenge students to create their own tumbling blocks design. This design uses three rhombuses to create a hexagon as demonstrated in *Resource 8.5.6 Tumbling blocks design for quilts or floor tile patterns*. Students should create their own rhombus, calculate its area and the resulting area of the hexagon it will create when combined with its partner rhombuses. How many of these will cover an A4 or A3 sheet of paper? If making a cushion cover, bedspread, chair throw, how many of these rhombuses will be needed to cover the area?

Resource Resource 8.5.6 Tumbling blocks design for quilts or floor tile patterns.

Explore creative shapes for tiling or mosaics that can be created from tessellating triangles. What areas will they cover?

Challenge students to find triangles in shapes other than rectangles. See if they can apply ideas learnt so far to finding the areas of these shapes. For example, trapeziums, hexagons, octagons.

Revisit Matt's cat cage investigation. Will hexagons or octagons be more cost effective shapes for the cage? Justify your answer.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

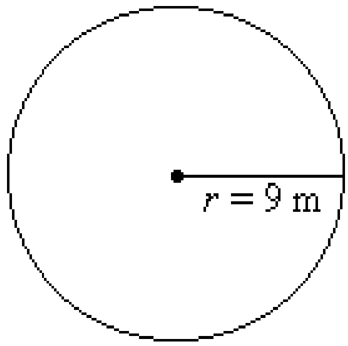
Name:____ Date:________

Can you do this? #6

$\pi = 3.14$

Look at this circle.

1. What is the circumference of the circle?



Obj. 5.8.1
i. ☐
ii. ☐
iii. ☐

2. What is the area of the circle?

Obj. 8.6.1
i. ☐
ii. ☐
iii. ☐

3. A circular garden has an area of 12.56m^2.

   (a) Sketch the garden bed and label it with the information provided.

   (b) Calculate the radius of the garden bed.

   (c) What is the diameter of the garden bed? ____________________

Obj. 8.6.1
a)i. ☐
b)i. ☐
b)ii. ☐
b)iii.☐
c) ☐

4. What length of garden edge will be needed to go around the garden bed?

Obj. 5.8.1
i. ☐
ii. ☐
iii. ☐

Cycle 6: Area of Circles

Overview

Big Idea

This cycle extends from calculated area of rectangular shapes and parallelograms to calculated area of circles. This idea requires students to make a connection between area of a parallelogram and area of a circle. Once students see that they can create a rectangular area from sections of a circle, they should also be able to generate a formula for calculating circular area.

Objectives

By the end of this cycle, students should be able to:

8.6.1 Use formulae to find area of circles. [8MG197]

Conceptual Links

This cycle requires students to be able to calculate area of parallelograms as a basis for exploring area of circles. Students need to be able to identify properties of parallelograms, identify properties of circles, recognise pi and calculate circumference of a circle (within Unit 06) to be able to complete this cycle.

This cycle provides a basis for calculating the area of circles based on the rules developed for area of parallelograms which contribute to calculating the area of composite shapes in Cycle 7. Calculation of area of a circle is also a prerequisite idea for some volume calculations to be explored in Unit 14.

Materials

For Cycle 6 you may need:

- A4 paper
- Selection of circles
- Protractors
- Rulers
- Scissors
- Centimetre overlay grids

Key Language

Area, two dimensions, squared, circles, radius, diameter, pi, π

Definitions

Circumference: $C = 2\pi r$ *or* πD

Diameter: Distance from one side of a circle to the other running through the centre; double radius

Pi (π): Irrational number; ratio of the circumference of a circle to its diameter; 3.14 or 3.14159
Maths is Fun: https://www.mathsisfun.com/numbers/pi.html

Radius: Distance from centre of a circle to its circumference

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Show me the radius?
- Which measure do we need for Area? (Radius)
- Which measure have you been given?
- If you have diameter of the circle, what do you need to do?

Portfolio Task

Ideas from this cycle may be used to complete sections of the task that require calculation of circular areas.

RAMR Cycle

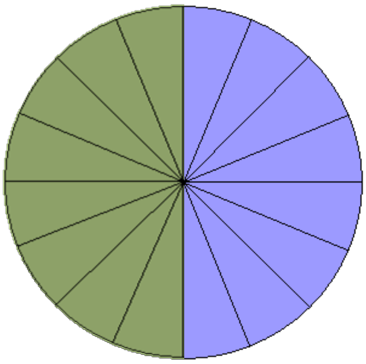
Reality

For this cycle, contexts need to be circular. For example, circles are used as the bases of cylinders (tins, rainwater tanks, cross-sections of pipes, clock faces). Discuss what areas of cladding might be needed to cover circles. Consider, why are circles used for manhole covers? (Circular covers cannot drop through the hole if placed on their edge, square or rectangular covers can drop through if put vertically and across the diagonal.) Discuss the properties of circles students understand already, in particular diameter, radius, circumference and pi.

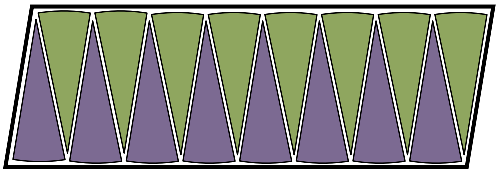
Abstraction

Calculating area of circles relies on the ratio between diameter and circumference (π). Area of circle formula can also be connected to previously explored area calculations (parallelogram and rectangle) by cutting the circle into wedges and reforming. A suggested sequence for developing the area of a circle is as follows:

1. *Revisit classification of shapes by attributes from Unit 04*. Identify the critical attributes of a circle as having no straight edges and no corners, and where every point on the circumference is the same distance from the centre. This length is known as the radius (previously explored in Unit 06).
2. Cutting and reforming shows that a circle can be considered as a rectangle of length $\pi \times r$ and width r, and thus the area of a circle is $\pi \times r \times r$ or πr^2. To do this, we have to remember that the circumference is $2 \times \pi \times r$ (previously explored in Unit 06).
3. Use paper circles, a large lid or paper plate, base from under a frozen pizza (at least 12cm in diameter), a ruler and scissors. Measure the diameter of the circle and find the radius (r).
4. Place the lid firmly on the piece of the paper, and draw a circle around it.
5. If using paper, cut out the circle and fold in half, quarters, eighths, sixteenths (as shown on the right). Card, lids, paper plates, pizza bases will likely need to be marked using a ruler and pencil before cutting.



6. Cut out the 16 sectors. Arrange the sectors in the configuration shown below. Note that this configuration looks very much like a parallelogram. Note: It can be beneficial to make students' circles different colours and have them trade half their pieces with another student. This allows the arrangement of one colour on each side of the parallelogram.



7. Draw students' attention to the perpendicular height of the parallelogram. Connect this to the dimension it represents on the circle (radius). Draw students' attention to the length of the parallelogram and connect this to the dimension it represents on the circle (half the circumference).

8. Connect the area of the parallelogram to the circle formulae using algebra.

Area of parallelogram	=	length by perpendicular height
	=	$\frac{1}{2} \times 2\pi r \quad \times \quad r$
	=	πr^2

Note: If the circle piecing activity is repeated for a range of circle sector sizes (e.g., quarters, eighths, sixteenths, and so on it highlights the idea that as the number of pieces increases, the shape becomes more and more like a parallelogram.

9. As a check that the formula works, measure the perpendicular height and length of the parallelogram. Calculate the area of the parallelogram. Divide the length of the parallelogram by the radius. This should give an answer that is close to π (approximately equal to 3.14).

Resource Resource 7.6.1 Area of circles

Mathematics

Language/symbols and practice

Once students have explored area of circles by connecting to area of rectangles or parallelograms, they should be given further opportunities to practise calculating areas. Generate problems that use areas of circles.

Connections

Factor-Factor-Product and Inverse

Connect measurement of area to division. Ensure students can use factor-factor-product thinking to find the unknown dimension of a circle if they know the other dimension and the area.

Reflection

Check the idea

Consider the use of circles in construction of water tanks and cylindrical containers. Determine the area of cladding needed to cover the circular ends of water tanks or to make lids for cylinders.

Apply the idea

Explore creating boxes for circular objects. What shapes could they be? What would their dimensions be? What areas would they cover?

Extend the idea

Revisit Matt's cat cage investigation. Will circles be more cost effective shapes for the cage? Justify your answer. Why might circles not be a practical choice for the cat cage?

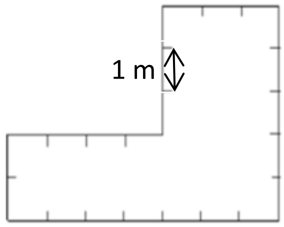
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #7

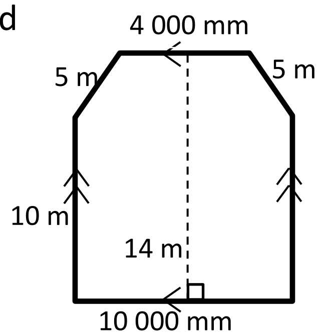
1. What is the area of this shape if each of the spaces between marks measure 1 m?



Obj. 8.7.1
i.☐
ii.☐
iii.☐

2. A new house has a living room which is shaped like the picture. Carpet needs to be laid in the living room.

   (a) How many square metres of carpet are needed to cover the area of the floor?



   (b) List the different shapes you broke the room into to calculate the area? ____

Obj. 8.7.1
a)i.☐
a)ii.☐
a)iii.☐
a)iv.☐
a)v.☐
b)i.☐
b)ii.☐
b)iii.☐

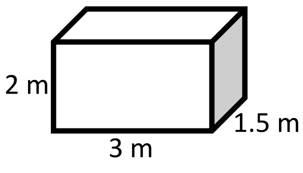
3. The walls of this room are 3m high.

   (a) What area of wall is there to paint (without allowing for doors or window openings)?

   (b) The 4 000 mm wall will be painted blue. All other walls will be painted white. What is the area of the white walls?

Obj. 8.7.2
a)i.☐
a)ii.☐
a)iii.☐
a)iv.☐
b)i.☐
b)ii.☐
b)iii.☐
b)iv.☐

4. This block needs to be painted. Calculate the surface area of the block to be painted. Show all your calculations. (Hint: Sketch the net for the shape first, then work out the surface area.)



Obj. 8.7.2
a)i.☐
a)ii.☐
a)iii.☐
a)iv.☐
a)v.☐
a)vi.☐

Cycle 7: Area of Composite Shapes

Overview

Big Idea

This cycle challenges students to see composite shapes as combinations of shapes they know and can work with. This extends students' application of ideas developed when calculating areas and parallelograms and triangles.

This cycle challenges students to see faces of prisms and pyramids as combinations of shapes they know and can work with to find surface area. Examples explored should enable students to apply area of rectangles, parallelograms and triangles to real world problems involving surface area.

Objectives

By the end of this cycle, students should be able to:

8.7.1 Calculate the areas of composite shapes. [9MG216]
8.7.2 Solve problems involving the surface area of right prisms. [9MG218]
8.7.3 Solve problems involving the surface area of right cylinders. [9MG217]
8.7.4 Solve problems involving the surface area of right pyramids. [10MG242]

Conceptual Links

This cycle requires students to be able to dissect a composite shape into its simpler components for separate area calculations then add these together for the overall area.

This cycle provides a basis for calculating the area of composite shapes which contribute to calculating the surface area of prisms. This cycle provides a basis for calculating the surface area of prisms which can be applied to wrapping or packaging problems and future modules which explore rate.

Materials

For Cycle 7 you may need:

- A4 paper
- Selection of composite shapes
- Dissection puzzles
- Protractors
- Selection of prisms and pyramids
- Nets for prisms and pyramids
- Rulers
- Scissors
- Centimetre overlay grids

Key Language

Coverage, surface, surface area, composite shapes, length, width, 2D shape language, perpendicular height, standard metric measures and units, 2D nets of 3D prisms, rectangular prisms, cubes, square based prisms, triangular based prisms, pyramids, composite 3D objects

Definitions

Composite shape/composite object: Shape or object that can be dissected into simpler known shapes

Prism: 3D object with two congruent surfaces parallel to each other joined by parallelograms

Pyramid: 3D object with a base joined to an apex by triangular surfaces

Rectangular prism: 3D object with two congruent rectangular surfaces parallel to each other joined by parallelograms

Right prism: prism that has two congruent surfaces parallel to each other joined by rectangles

Right pyramid: pyramid that has its apex directly above the centre of its base

Square-based prism: 3D object with two congruent square surfaces parallel to each other joined by parallelograms

Triangular-based prism: 3D object with two congruent triangular surfaces parallel to each other joined by parallelograms

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What shapes can you find area for?
- How can you break this shape up to make it shapes we know?
- What faces can you see? What shapes are they? Make a list.
- Can you find the area of each of these shapes?

Portfolio Task

Ideas from this cycle may be used to complete sections of the task that require calculation of areas of composite shapes and surface areas.

RAMR Cycle

Reality

For this cycle, contexts need to be irregular shapes that have straight edges and can be dissected into rectangles, parallelograms and triangles. For example, explore carpeting odd shaped rooms with bay windows, L-shaped hallways, painting a ceiling and a feature wall the same shade, tiling pathways that take corners through gardens, covering surfaces of 3D objects. Note: Cylinders may be used as the net is a rectangle and two circles, cones and spherical objects may be too difficult to explore.

Abstraction

Finding area of composite shapes involves students looking flexibly at shapes and visualising what possible shapes they can be broken into. Activities and learning in this sequence can be real-world and problem driven as visualising shapes within shapes is a skill that requires flexibility and takes practice. A suggested sequence for developing strategies to find areas of composite shapes is as follows:

1. *Revisit dissecting shapes* to make other shapes of the same area from Cycle 1: Area Comparison. Tangrams may be used as in *Cycle 1: Area Comparison* or other dissection puzzles such as Pentominoes. Instead of starting from puzzle pieces, start with outlines of shapes and have students mark in pencil where they might break the shape to use the pieces. These can then be tested with puzzle pieces.

Resource Resource 8.7.1 Tangram picture outlines

2. Rather than use physical materials, provide scaled drawings of shapes and practise dissecting and determining useful dimensions for calculating areas.
3. Explore a carpeting problem for a room with a bay window. Students should pencil in where they can break the shape first, determine the measurements they need, calculate the separate areas and then add these together for the final shape.

Mathematics

Language/symbols and practice

Once students have explored strategies to break composite areas into known shapes for area calculations, they should be given further opportunities to practise calculating areas.

Explore creative shapes for tiling or mosaics that can be created from tessellating polygons. What areas will they cover?

Resource Resource 8.7.2 Composite shape problems

Connections

Surface area of prisms and pyramids

Where possible, find real life contexts for surface area to embed the activities in using relevant objects or situations. For example, explore painting sets of children's blocks for construction, or coating cardboard shapes with plasticiser.

Finding surface area of prisms and pyramids involves students looking at these solid objects flexibly and visualising the nets. Once prisms and pyramids are visualised as nets, they become composite shapes that can be solved using strategies students already know. Visualising shapes within shapes is a skill that requires flexibility and takes practice. Some activities are as follows:

1. Using a Toblerone box as a stimulus, discuss with students how they might find the surface area of the box for painting.
2. Explore suggested strategies. If students suggest tracing around the box onto paper, follow this idea through on the board. See if students connect this idea to finding nets of 3D prisms from previous explorations. Have students visualise and sketch the net for the Toblerone box.
3. Ask students what dimensions they need to mark on their net sketch to calculate the areas of each face. Measure these.
4. Calculate the area of each face of the box. Add these for the surface area.

Resource Resource 8.7.2 Composite shape problems

Once students have explored strategies to decompose prisms and pyramids into nets and then into known shapes for area calculations, they should be given further opportunities to practise calculating surface areas. Generate painting or coverage problems for pyramids and prisms. Egyptian pyramids originally had facing tiles to cover each face so that they were smooth. What would be the surface area of the faces that show? If the Eiffel Tower were to be clad in Perspex to keep it dry, what surface area of Perspex would be needed?

Reflection

Check the idea

Provide a variety of composite shapes with some areas marked and some dimensions. Students can determine missing areas, total areas and missing dimensions from information given.

Apply the idea

Challenge students to find triangles in shapes other than rectangles. See if they can apply ideas learnt to finding the areas of these shapes. For example, trapeziums, hexagons, octagons.

Extend the idea

Challenge students to find the external surface areas of more complex, composite, 3D objects like buildings or tents. These may need surfaces calculated for painting, surface pest spraying, or replacement canvases. See if students can apply the strategies from area of composite shapes to surface area of composite solid objects.

Consider curved shapes like soccer balls that are initially made up of flat surfaces. Have students explore the construction of a soccer ball using paper pentagons and hexagons that can be joined using sticky tape. If students are given a range of different sized pentagons and hexagons to create their paper balls, a range of sizes may be created. Count the number of faces used to create the balls and calculate the overall surface area of the paper soccer balls. It may be interesting for students to determine the radius/diameter of their ball and compare this with the surface area.

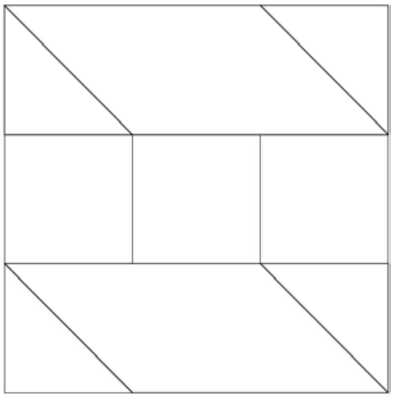
Resource Resource 8.7.4 Nets for soccer balls

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 08 Investigation: Tessellation

Design Challenge – Quilt or Floor Tiles



A set of quilt pieces can be arranged to cover a larger area. Square blocks made up of other shapes can be arranged in repeating patterns to create intricate designs for quilts or flooring. For example, the template on the right has been used to create the coloured design pictured below that could be used as a quilt pattern or floor tile pattern.

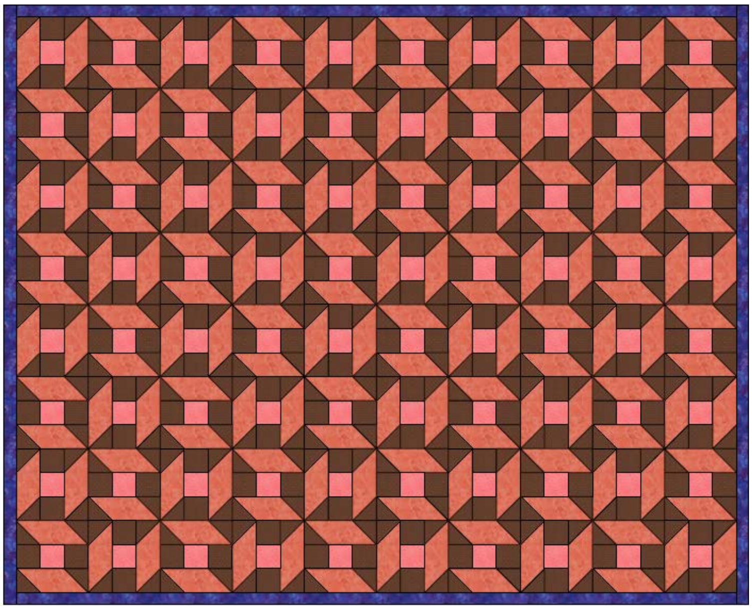
1. Design your own basic block using multiple shapes.
   - Measure the block and find its total area.
   - List how many of each shape and colour pieces you use in a block.
   - Calculate the area of each different piece used in the block.
2. a) Work out how many repeats of your block you will need to cover an area that is 2.4 m square.

   b) Calculate the number of each different shape you will need to complete the job.

   c) Calculate how much area of each colour is needed.
3. a) If you were to use this design with a blue border strip that is 30mm wide along each edge, to cover an area that is 1260mm wide and 2460mm long, how many repeats of your block would you use?

   b) How many of each different shaped piece will you use?

   c) What will be the total area needed in each shape?
4. Write instructions for someone else so that they can recreate your shape if they do not have a picture.



Unit 08 Portfolio Task – Teacher Guide

Build an Economical House

Content Strand/s: Number and Algebra
Measurement and geometry

Resources Supplied:

- Task sheet
- Teacher guide

Other Resources Needed: String

Summary:

Using the context of building an economical house, students solve length and area problems to discover what makes a house more economical to build.

Variations:

- Students can create a diagram of their own homes and work out the approximate cost to rebuild their own home.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Measurement and Geometry

8.3.1 Calculate the area of rectangles using familiar units [5MG109]
8.3.3 Calculate the area of rectangles using familiar metric units [5MG109]
8.3.4 Calculate the perimeter of rectangles using familiar metric units [5MG109]

Number and Algebra

3.7.1 Solve for unknown values in multiplication stories [7NA176]
3.7.2 Solve for unknown values in division stories [7NA176]

Build an Economical House!

Name	
Teacher	
Class	

Your Task:

To build an economically priced house you need to know what makes the house cheaper to construct. A lot of cheaper houses are nearly always square in their overall shape. Work through the portfolio task to determine why.

It is your task to design and draw an economical house, and identify how much it will cost to build. To do this you will have to use:

- area
- perimeter
- length
- cost calculations

This task is in three parts:

1. Design the shed and estimate the cost.
2. Design the house and estimate the cost.
3. Design the garden and estimate the cost.

Within Portfolio Task 8, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Procedural fluency	8.3.3 Calculate the area of rectangles using familiar metric units.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	8.6.1 Use formulae to find the area of circles.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Problem solving approaches	8.5.2 Establish the formula for areas of triangles and use these in problem solving.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches
	Reasoning and justification		Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions

Comments:

1. The Shed

The shed needs to hold two cars. Cars can be parked side by side or end to end. Each car needs a space 6 m long by 3 m wide.

1. On graph paper, draw a floor plan for each shed. Each square represents 1 m × 1 m.

End-to-end design	**Side-by-side design**

2. Walls and doors for the shed are 2.5 m high. Each shed roof has a triangular face at each end above the garage doors/end walls. If the triangle is 1m high, what will the area of the back wall be for each shed? (The back wall will be a rectangle with a triangle on top.)

End-to-end doors/end walls area:	**End-to-end side walls area:**
Side-by-side doors/end walls area:	**Side-by-side side walls area:**

3. Walls and doors for the shed cost the same amount per square metre. If walls cost $300 per square metre what is the cost of the walls and doors of each shed?

End-to-end doors/end walls cost:	**End-to-end side walls cost:**
Side-by-side doors/end walls cost:	**Side-by-side side walls cost:**

4. The shed roof also costs $300 per square metre. To allow for overhang, the distance from the peak of the roof to the gutter is 2m for an end-to-end shed or 3.5m for a side-by-side shed. What is the cost of the roof of each shed?

	Roof area	**Roof cost**	**Total shed cost**
End-to-end			
Side-by-side			

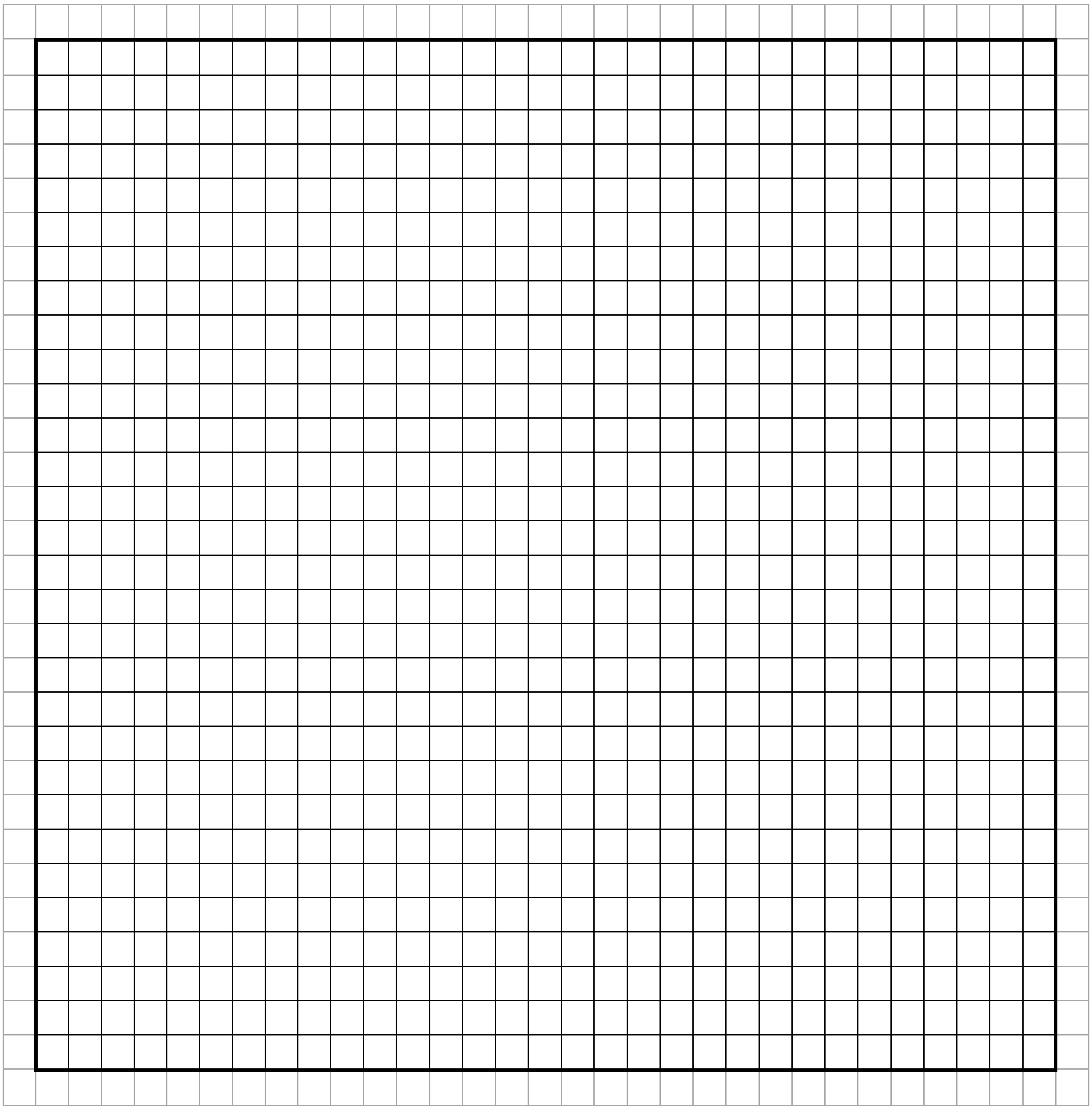
5. Is it cheaper to park the cars side by side or end to end? ___________________________

6. Why does this work out cheaper? ____

____

2. The House

1. Locate your house on the land shown below. The house front must be at least 6 m from the front fence, sides must be no closer than 1 m to each side fence.



2. Draw a floor plan for your house. Label each room, the dimensions of each wall and the location of doors and windows. Your house must have the following rooms:

   (a) Kitchen 3m × 4m

   (b) Lounge 6m × 4m

   (c) Dining Room 4m × 4m

   (d) 3 Bedrooms 3m × 3m

   (e) 1 Bathroom 2m × 2m

   (f) Allow for access hallways where necessary

3. Foundations cost $250 per square metre of floor area. Calculate the cost of the foundations for your house design.

$_______________

4. Calculate the total length of external walls you need to build for your house.

5. If all the external walls are 2.5m high, what is the total external wall area of your house to clad with brick?

6. If external walls cost $1500 per square metre to build, what is the cost of external walls for your house?

$_______________

7. Calculate the total length of internal walls you need to build for your house.

8. If all the internal walls are 2.5m high, what is the total internal wall area of your house?

9. If internal walls cost $1500 per square metre to build, what is the cost of internal walls for your house?

$_______________

10. Roofing costs $150 for each square metre of roof area. Calculate the cost of roofing for your house design. Make sure that you add 0.6m to the length and width of the floor for your house to allow for slope and eaves on the roof.

$_______________

11. What is the total cost for your house design? (add up all the questions with $____________).

12. Compare your house cost with other houses in the class. How much does the cheapest house cost?

____

13. Why do you think this house cost the least?

____

____

3. The Garden

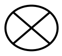
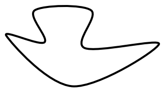
1. Quarter circle garden beds are included in the back corners of the yard with a radius of 3m each. What area of weedmat will need to be laid underneath the gardens? (Sketch these on your house block to help you)

2. The rotary clothes line has an arm length of 2.4m (from the centre pole to the outside edge). What size circle of ground needs to be left clear under the washing line?

Name:____ Date:________

Can you do this now? Unit 08

1. Circle the shapes that have an area

(a)  (b)  (c)  (d) 

Obj. 8.1.1

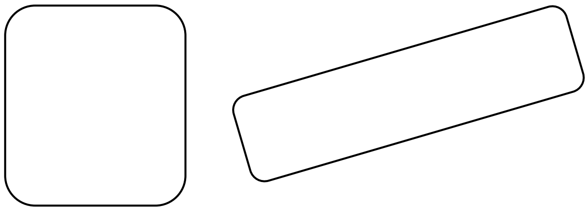
a) ☐

b) ☐

c) ☐

d) ☐

2. Describe how to compare the **areas** of these two shapes without counting, measuring lengths or calculating.



____

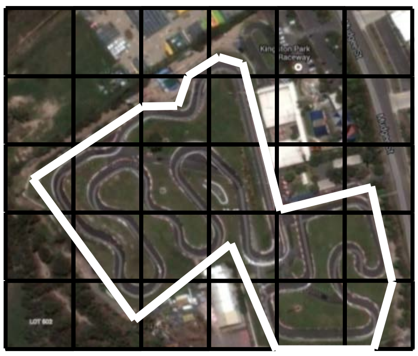
____

____

Obj. 8.1.1

☐

Here is a picture of Kingston Park Raceway.



3. Estimate the area enclosed within the red line in square units.

   Hint: Put crosses on full squares as you count them.

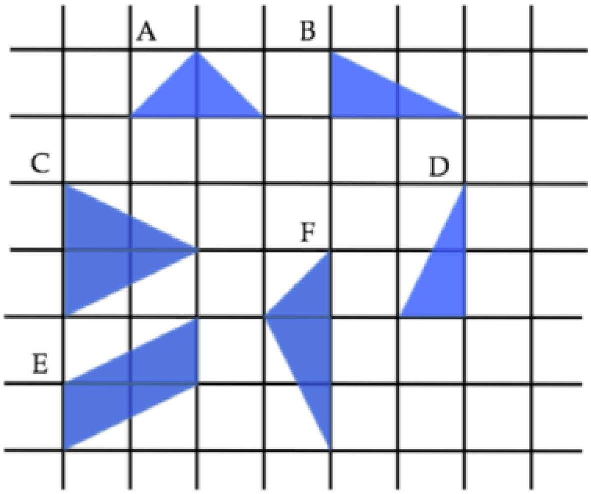
____

Obj. 8.2.1

i. ☐

ii. ☐

4. There are 6 shapes shown on the grid.
   Use the grid to answer the questions:



(a) Find one shape that has the same area as B. ________________

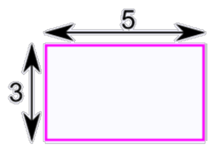
(b) Which shape has the same area as E? ________________

(c) Order the shapes – A, C and F – from smallest to largest.

________,________,________

5. Find the area of the rectangle.

   The units are foot lengths.



6. Name something that would be reasonable to measure using the following units.

(a) cm^2 ____________________

(b) m^2 ____________________

(c) mm^2 ____________________

(d) km^2 ____________________

Obj. 8.2.1

a) ☐

b) ☐

c)i. ☐

c)ii. ☐

Obj. 8.3.1

i. ☐

ii. ☐

Obj. 8.3.2

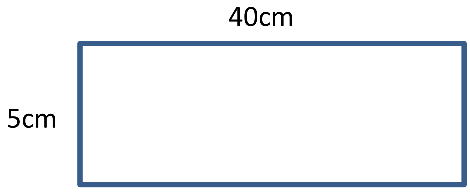
a) ☐

b) ☐

c) ☐

d) ☐

7. Calculate the area of this rectangle.



Obj. 8.3.3

i. ☐

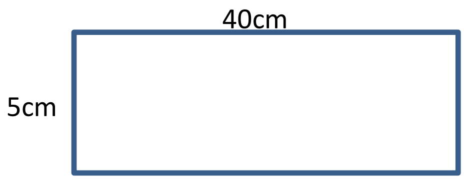
ii. ☐

8. What might be the dimensions of another rectangle that would have the same area?

Obj. 8.3.3

☐

9. Calculate the perimeter of this rectangle.



Obj. 4.3.3

i. ☐

ii. ☐

10. What might be the dimensions of another rectangle that would have the same perimeter?

Obj. 4.3.3

☐

11. Would you use square metres or square centimetres to measure:

(a) The cover of a book? ____________________

(b) A CD cover? ____________________

(c) Sports oval? ____________________

(d) The classroom floor? ____________________

Obj. 8.3.2

a) ☐

b) ☐

c) ☐

d) ☐

12. Convert the following metric measures:

(a) $2m^2$ = _______ cm^2

(b) $1km^2$ = _______ m^2

(c) 5ha = _______ m^2

(d) $6.4cm^2$ = _______ m^2

Obj. 8.4.1

a) ☐

b) ☐

c) ☐

d) ☐

13. Use a ruler to draw a shape that has an area of 16 cm^2.
Label your shape with measurements for each side.

Obj. 8.4.2

i. ☐

ii. ☐

iii. ☐

iv. ☐

14. How could you change your shape to double its area to 32 cm^2?

____

____

Obj. 8.4.2

☐

15. Use a ruler to sketch a parallelogram and label it with a base length of 15cm and a perpendicular height of 6 cm.

Obj. 8.5.1

i.☐

ii.☐

iii.☐

16. Calculate the area of your parallelogram (show working):

__________ cm^2

Obj. 8.5.1

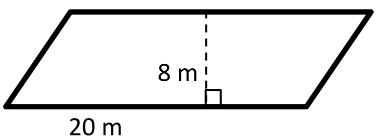
i. ☐

ii. ☐

iii. ☐

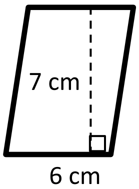
17. Find the area of the following parallelograms (show working:

(a)



_________ m^2

(b)



_________ cm^2

Obj. 8.5.1

a)i.☐

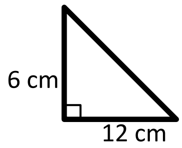
a)ii.☐

b)i.☐

b)ii.☐

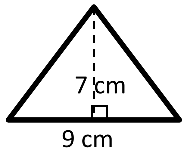
18. Find the area of the following triangles:

(a)



________ cm^2

(b)



________ cm^2

Obj. 8.5.2

a)i.☐

a)ii.☐

b)i.☐

b)ii.☐

19. A triangular floor tile has an area of $30cm^2$, and perpendicular height of 6 cm.

(a) Sketch the triangle and label it with the information provided.

(b) What is the length of the triangle's base? ____________________

(c) Explain how you calculated this? _________________________

____

Obj. 8.5.2

a)i.☐

a)ii.☐

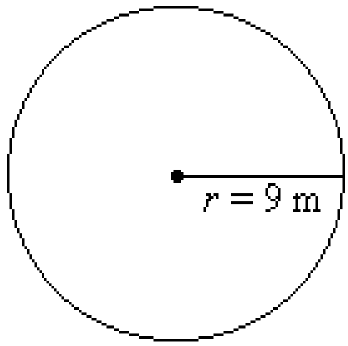
b)i.☐

b)ii.☐

c)i.☐

$\pi = 3.14$

Look at this circle.



20. What is the circumference of the circle?

Obj. 5.8.1

i. ☐

ii. ☐

iii. ☐

21. What is the area of the circle?

Obj. 8.6.1

i. ☐

ii. ☐

iii. ☐

22. A circular garden has an area of $25.12m^2$.

(a) Sketch the garden bed and label it with the information provided.

(b) Calculate the radius of the garden bed.

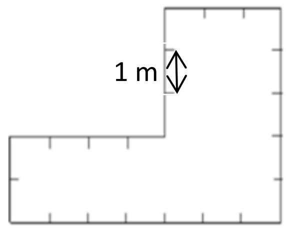
(c) What is the diameter of the garden bed? ____________________

Obj. 8.6.1
a)i. ☐
b)i. ☐
b)ii. ☐
b)iii. ☐
c) ☐

23. What length of garden edge will be needed to go around the garden bed?

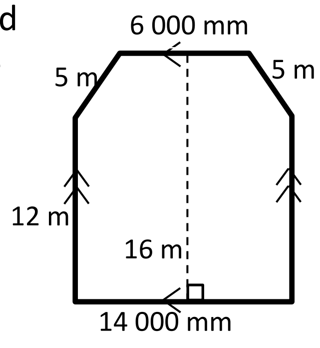
Obj. 5.8.1
i. ☐
ii. ☐
iii. ☐

24. What is the area of this shape if each of the spaces between marks measure 1m?



Obj. 8.7.1
i. ☐
ii. ☐
iii. ☐

25. A new house has a living room which is shaped like the picture. Carpet needs to be laid in the living room.



(a) How many square metres of carpet are needed to cover the area of the floor?

(b) List the different shapes you broke the room into to calculate the area?

____

Obj. 8.7.1
a)i. ☐
a)ii. ☐
a)iii. ☐
a)iv. ☐
a)v. ☐
b)i. ☐
b)ii. ☐
b)iii. ☐

26. The walls of this room are 3m high.

(a) What area of wall is there to paint (without allowing for doors or window openings)?

(b) The 6 000mm and 5m walls will be completely glass. All other walls will be painted white. What is the area of the white walls?

Obj. 8.7.2

a)i.☐

a)ii.☐

a)iii.☐

a)iv.☐

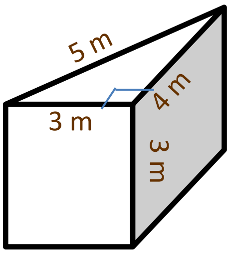
b)i.☐

b)ii.☐

b)iii.☐

b)iv.☐

27. This block needs to be painted. Calculate the surface area of the block to be painted. Show all your calculations. (Hint: Sketch the net for the shape first, then work out the surface area.)



Right-angle triangle based prism

Obj. 8.7.2

a)i.☐

a)ii.☐

a)iii.☐

a)iv.☐

a)v.☐

a)vi.☐

XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

Faculty of Education
School of Curriculum
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059
CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
E-mail: ydc@qut.edu.au
Website: ydc.qut.edu.au