XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

XLR8 Unit 07

Percentages

07

2016

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YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percent. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of discrete items or measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric rations between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively when describing quantity comparisons and recommended daily intake of nutrients. Percentage is also used to describe the change to a starting amount when applied to discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.

Scope

This unit is based upon the **representation** of **common fractions** and **decimal fractions** as **percent**. **Percent** represent fractions as **parts per hundred** where a **whole** is **partitioned** into **one hundred parts** and these parts are counted. The **symbol %** is used to indicate that the **number is the count** of **hundredth-sized parts of a whole**.

The use of **money** to represent **value or worth** provides a useful context for applications of **percentage as change** in terms of **discounts**, **markup**, **tax** and **simple interest**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this unit are highlighted in **black**.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *Daily Percentages* at the end of Unit 07 engages students with exploring nutritional information represented as percentages, use of percentages when shopping and considering finances.

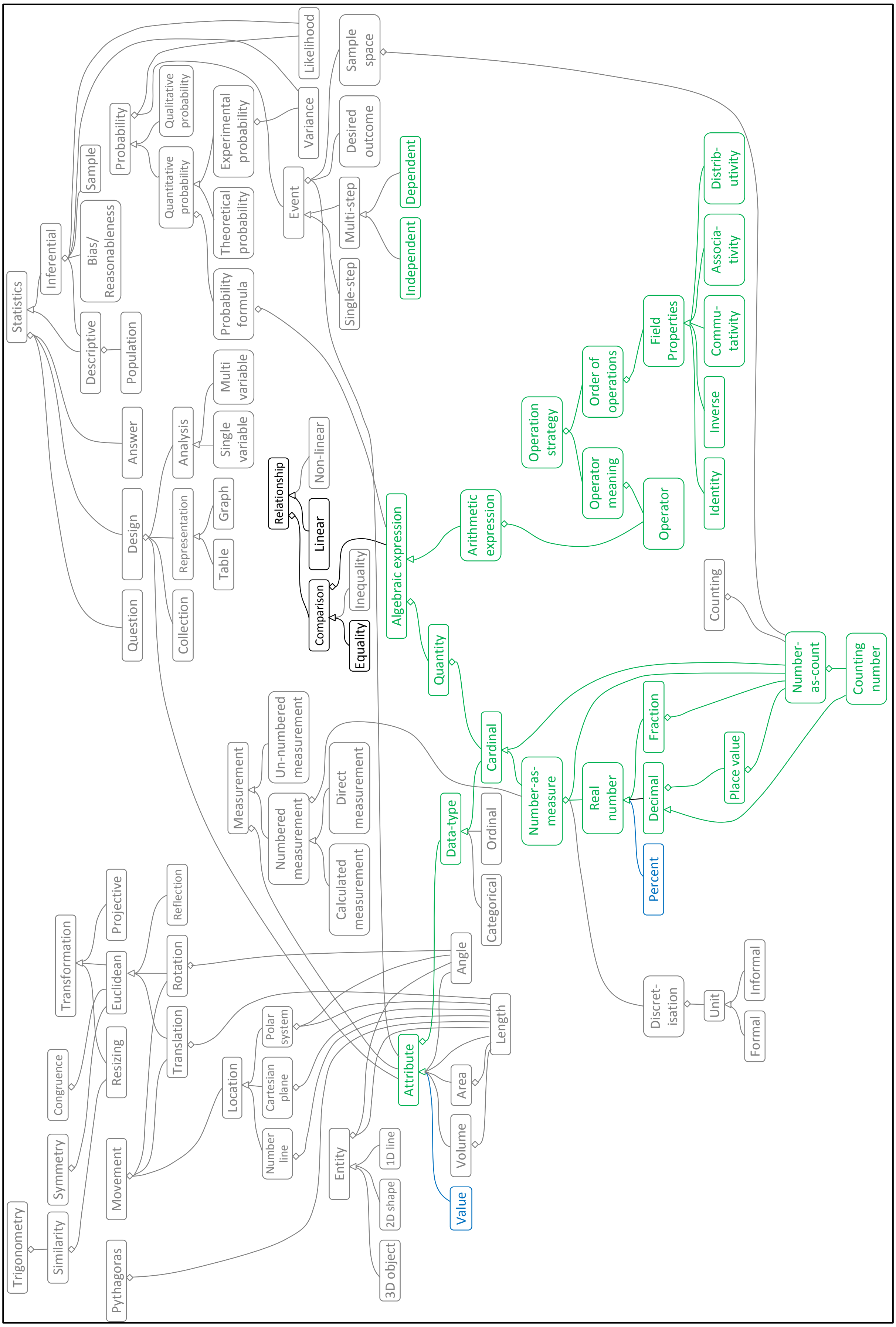


Figure 1. Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Measuring value

This cycle develops the idea of money and monetary amounts as a measure of value or worth of an item. Students may already have money-handling skills. However, determining alternative mixes of coins and notes to generate the same value, operating with monetary values and identifying how the values of goods and services are determined are useful practice of operations with decimals and percentages. Since cost is often one of the measures in common rates (e.g., cost/Litre, cost/kilogram) it is important to ensure that students understand money as a measure of value.

Cycle 2: Representing Fractions as Percent

In this cycle, activities extend student understandings of representations of common fractions and decimal fractions to include the representation of percent. Percent literally translates as per hundred. When counting in percent, students are counting hundredth-sized parts of wholes. The focus of this cycle is to connect representations of common fractions and decimal fractions to their equivalent representation as percent and to be able to convert flexibly between these representations.

Cycle 3: Finding Percentage of

The focus of this cycle is to compare quantities using the language and symbols of percentage, and to use the relationship between quantities to solve for unknown values. For example, the length of this table is 50% of the length of the other table; the number of students in this class is 75% of the number in the other class. Encourage students to make a quick sketch if it helps them to picture the parts of the problem. Percentage problems are multiplicative in nature so can be represented using area, number line or Arrowmath diagrams. Use the representation that works best for your students.

Cycle 4: Percentage Change

The focus of this cycle is to interpret and solve problems involving *percentage less* (e.g., discounts) and *percentage more* (e.g., mark-ups). These problems can be treated as two-steps, which highlight the parts and the thinking for the solution. It is also possible to solve these problems by finding the complementary percentage (e.g., 15% less calculated and subtracted is the same as calculating 85% of) or treating the percentage as 100% plus the mark-up amount (e.g., 10% more calculated and added on is the same as calculating 110% of) so that the calculation is completed in one step. This cycle is extended to explore simple interest and contexts where successive consistent percentage changes occur (e.g., population growth predictions or compound interest).

Notes on Cycle Sequence:

The proposed cycle sequence outlined is best completed sequentially as presented above.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Measuring Value

Value, cost, price, worth, demand, rare, dollars, cents, goods, services, attribute, measure, currency, barter, exchange, money

Cycle 2: Representing Fractions as Percent

Percent, percent, out of a hundred, parts per hundred, hundredths, tenths, whole, common fraction, decimal fraction

Cycle 3: Finding Percentage of

Per, percentage of, fraction of, compare, comparison, factor, product, double number line

Cycle 4: Percentage Change

Percentage, percent, %, discount, mark-up, GST, tax, percentage more, percentage less, interest, simple interest, per year, per annum, loan interest, savings interest

Name:_______________________________________ Date:________

Can you do this? #1

1. I need to pay the shop keeper 45 cents for a packet of lollies. One way I could pay would be 20c + 20c + 5c. Write down two other combinations of coins I could use to pay the exact amount.

   (a) ____

   (b) ____

2. How many cents in $2.00? ____

3. I have three Australian notes in my wallet.

   (a) Sketch three notes.

   (b) What is the total amount of money in my wallet?

   ____

4. I bought three things from the shop which cost $9.48 altogether. How much cash did I have to pay? ____

5. I went to the shop to buy a $3.50 carton of eggs. I paid with a $10.00 note. How much change will I get? (show your working)

6. I buy 2 cans of Fanta for $1.50 each, and a pie for $2.50. I give the shopkeeper $5.

   (a) Have I given the shopkeeper enough money? ________________________

   (b) If not, how much more money do I need? (show your working)

7. When buying a bag of jellybeans in the supermarket there are two options, circle the best buy.

Option 1: 350g bag for $2.10 Unit Price: 60c/100g	**Option 2:** 250g bag for $1.80 Unit Price: 72c/100g

Obj. 7.1.1
a) ☐
b) ☐

Obj. 7.1.1
☐

Obj. 7.1.1
a) ☐
b) ☐

Obj. 7.1.2
☐

Obj. 7.1.3
☐

Obj. 7.1.3
a) ☐
b) ☐

Obj. 7.1.4
☐

Cycle 1: Measuring Value

Overview

Big Idea

This cycle develops the idea of money and monetary amounts as a measure of value or worth of an item. Students may be well aware of local currency value and already have money-handling skills. However, determining alternative mixes of coins and notes to generate the same value, operating with monetary values and identifying how the values of goods and services are determined are valuable numeracy skills and useful practice of operations with decimals and percentages. Since cost is often one of the dimensions in common rates (e.g., cost/Litre, cost/kilogram, $/metre) it is important to ensure that students understand money as a measure of value.

Objectives

By the end of this cycle, students should be able to:

7.1.1 Represent money values in multiple ways. [3NA059]
7.1.2 Round change to the nearest five cents. [3NA059]
7.1.3 Solve problems involving purchases and the calculation of change. [4NA080]
7.1.4 Investigate and compare unit prices. [7NA174]

Conceptual Links

Value builds on measurement and number concepts. Students need to apply place value, operations and measurement ideas to the concept of value. Objectives from Unit 3 (Cycles 4, 5, 6) are reinforced using this context.

Facility with Australian decimal currency is a vital numeracy skill for everyday life. In Cycle 4 a facility with money and its decimal notation is necessary to understand the application of percentage discount and mark-up. This is extended in Cycle 5 when considering repeated percentage change as in population growth or the cost of borrowing money over time, which is calculated as interest.

Materials

For Cycle 1 you may need:

- Australian currency play money (optional)
- Printed sheets of Australian money
- Examples of other currency
- Exchange rates

Key Language

Value, cost, price, worth, demand, rare, dollars, cents, goods, services, attribute, measure, currency, barter, exchange, money

Definitions

Cost: Amount that is charged for an item or service

Value: What something is held to deserve in terms of importance or usefulness and may involve cost or monetary value.

Worth: Equivalent in value to the sum or item specified. Something may or may not be "worth" its "value".

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How can you tell if an item has greater value than another?
- What sorts of things do you consider when deciding on a purchase?
- How do you decide if an item is worth its price tag or cost?
- What coins or notes can we use to pay for these things?
- How much does it come to?
- If you do not have exact money, what notes/coins might you use? What change should you get/give?

Portfolio Task

While this cycle is not enough to specifically address any part of the task, any application of percentage discount or markup will also require a background understanding of value.

RAMR Cycle

Reality

Discuss money and transactions with students. Ensure they can all identify the coins and notes used in Australian currency. Further discuss what the price on items actually means. Connect to the idea that money is the measure of value or cost of an item.

Identify the attribute of value and how this is measured. Discuss changes in value and cost through history. For example, salt used to be incredibly rare in some areas and held great value, hence the saying “worth his weight in salt”. Discuss how the value or cost of water rises during drought, fruit and vegetables increase in cost when purchased out of season and decrease in cost in times of plenty. Discuss what things students value most (e.g. “what one thing would you take to be marooned with...”, etc.), why they value it, and how they could show its value. For example, you will value a small boat more if you are on an island than if you are in the desert. Discuss things that have little value. Make a collage of pictures of expensive and cheap things. Compare costs and consider equivalences, e.g., one burger at a restaurant costs the same as three burgers at McDonalds.

Extend students’ thinking to value or cost of services and entertainment. Consider how these things might have value attached to them. What things contribute to the value or cost of entertainment parks, movie cinemas, medical services, drivers’ licences or any other services students may add?

Abstraction

The abstraction sequence for this cycle builds student understanding of money as a unit of measure for value and operate with Australian currency (which includes a necessity for rounding). A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Students can be encouraged to barter and swap items or activities for other things. To make the swap or barter they will need to compare the value of their items. This comparison could be done directly or indirectly. A novel way to discuss the “value” of items would be to explore swapping of food that may occur at lunch time, for example, a meat and salad sandwich (tasty and filling) swapped with a small cup cake or biscuit (junk food).
2. Discuss with students how bartering might work. Identify a variety of items of student interest. Place pictures of these on PowerPoint slides. Discuss with students which items they would be willing to swap for other items. Try to engage students with determining a class price for items on the PowerPoint slides. Provide students with the opportunity to practice shopping for items, giving and receiving change to increase familiarity with Australian currency.
3. *Represent with symbols and connect to language.* Extend the shopping activity to engage students in recording the price, currency used to pay for the item and change received/required as they go. These should be written as equations but can be calculated mentally.

Explore relationships between values of currency. Create a fraction wall from *Resource 7.1.1 Australian currency fraction wall* (or *Resource 7.1.2 Australian currency fraction wall PowerPoint*) starting with $100 note at the top. Students should identify the number of smaller notes that combine to be equivalent to $100. This activity can be extended by encouraging students to identify the fraction of $100 that each smaller note represents. Repeat with coins. Explicitly connect representations of dollars and cents to decimal fractions and place value. Ensure students understand that 1c is one hundredth of a dollar.

Resource

Resource 7.1.1 Australian currency fraction wall
Resource 7.1.2 Australian currency fraction wall PowerPoint

Find different combinations of coins or notes to create the same value. Find as many different combinations of notes and coins to generate a given value. Discuss with students why we use the values of coins we do. Are there any combinations that cannot be created with the coins we have? Do we really need a 30c piece?

Resource Resource 7.1.3 Change the treasure

Mathematics

Language/symbols and practice

Counting mixed coins and notes

Counting money to find the total value of a mixture of different coins is difficult due to the need to change the count. Provide students with practice sorting and counting collections of coins. Find strategies for organising coins in multiples of $1 and multiple stacks of $10 to simplify counting.

Explore rounding

Australia does not use 1c or 2c coins, New Zealand does not use 5c coins. Practise adding lists of items then rounding to the nearest 5c value (as used when paying with cash). Discuss the benefit to shoppers/traders of the effects of rounding. Who does it really benefit? Discuss what it might mean in Australia if the 5c piece was discontinued (debate recently featured in news articles).

Best buys at the supermarket

Determining best buys at the supermarket is hugely simplified by unit pricing which states cost/mass on their displays. Connect "per" language to the diagonal line used to represent it. Connect this representation to fraction representations so that cost/mass is also understood as $\frac{cost}{mass}$.

For example, to determine best buy in a supermarket, two cans of beans (A and B) are compared. The label for Can 1 states 48c/100g, while the label for Can 2 states 52c/100g. Have students discuss the better value item. Compare the cost/100g to determine which the cheaper item is. Consider further, if Can 1 only comes in 1kg size, and only 440g is to be used with 560g to be wasted, which is the better value? What is the cost for each can? Critically discuss when a bargain is not a bargain.

One could look at a lot of other situations and use rates to work out best value (although value is more than money – if Can 1 is a brand with a flavour that is not to your liking, is it worth paying an additional 4c/100g for beans you will like?). For example, what are the costs of events (e.g., football games, concerts) in relation to how long they go for? This gives possibilities of working out cost/time (lower the better) and time/cost (higher the better).

Of course often there is more than one thing involved in determining value. For instance, a phone might have higher text costs and lower call costs or vice versa. This leads to more complex and richer investigations with results that require a level of interpretation to connect back to reality – what do the values *really* mean? **Note:** These basic rate problems are precursors to rates of change and linear relationships explored in Unit 13.

Auctions

Discuss with students how auctions work. Identify a variety of items of student interest. Place pictures of these on PowerPoint slides. Discuss with students what a minimum value or reserve price for each item should be. Provide students with a selection of notes and coins that they may use for bidding. Students must ensure before they bid that they have sufficient cash to do so. This activity will provide a context for students to practise identifying and using notes and coins, while also considering the effect of availability and demand on the price people are willing to pay for items.

Connections

Explore other currencies

Explore the currencies of other countries that use dollars and cents. How many of these are based on decimal number and base 100? Investigate the similarities and differences between Australian, New Zealand and Singapore currencies.

Canadian and American currency use 5c, 10c, 25c, $1, $2 coins. Does this still work for generating all values up to $1?

Explore a range of Asian currencies. Do these also use the decimal place value system? Thai Baht do not use hundredths. They use 1B, 2B, 5B, 10B … coins and 10B, 20B, 50B, 100B, 200B, 500B notes. What does this mean for recording these monetary values?

Resource Resource 7.1.4 Exploring other currencies

Reflection

Check the idea

Provide students with a tuckshop list and have them identify what notes and coins they might need to pay exact money for their lunch order. Challenge them to identify as many different combinations for their order as possible.

Extend this idea by asking them to determine how much change they should receive from $10 (or other values) and what possible denominations this change could be provided in.

Note: This is essentially a realistic activity. When working in a shop, if the till is low in a particular denomination of coin it is necessary to be able to use other coins and still provide customers with the correct change.

Apply the idea

Provide students with a range of catalogues and a shopping list that can be assembled from each catalogue. Have them add their shopping lists. As a class, order the total costs to determine which shop's catalogue represented the best value. Determine the difference between total costs.

Resource Resource 7.1.5 Shopping catalogues

School excursion

Prepare a budget for a hypothetical (or real) school excursion. Encourage students to brainstorm possible expenses. Provide brochures and/or allow Internet research to calculate the cost of each essential and optional activity. Tabulate the results. Students should aim to propose the cheapest and most desirable excursion.

Extend the idea

Explore relative value of different currencies

Exchange rates can involve some very complicated mathematics that can be better left to a calculator. For this cycle, introduce the language of per used in describing exchange rates as 16 Czech crowns per $1 Australian dollar and simple comparisons. Estimation and number sense can be useful when trying to determine what an item might cost in familiar money. For example, if there are 16 Czech crowns to $1 Australian, it is possible to take the Czech price of an item, divide by 4 and then divide by 4 again to check the equivalent value in Australian dollars. Converting Thai Baht is simple if the Thai price is divided by ten then divided by 3. Use some simple exchange problems to informally explore strategies for estimating relative cost.

Resource

Resource 7.1.6 Estimating relative cost
Resource 7.1.6 Estimating relative cost PowerPoint

Explore relative value of different phone plans

Collate data on plan costs with a range of carriers for students to compare. Ensure that they consider the cost of calls (connection fee + cost/minute), text messages, included data and access fees. Students may further consider the value of phone plans that include the cost of a handset compared with BYO phone plans. Is it better value to change phones at the end of a plan and negotiate a new plan, buy a new phone to connect to the old plan, or to keep the old phone and plan past the plan end date?

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________ Date:________

Can you do this? #2

1. Write the following percents as decimal fractions:

(a) 12% __________ (c) 125% __________

(b) 35% __________ (d) 86.5% __________

2. Write the following decimal fractions as percents:

(a) 0.25 ________ (c) 1.05 __________

(b) 0.76 __________ (d) 0.976 __________

3. Write the following percents as common fractions:

(a) 15% ________ (c) 81.6% __________

(b) 37% __________ (d) 100% __________

4. Write the following common fractions as percents:

(a) $\frac{23}{100}$ __________ (c) $\frac{79}{100}$ __________

(b) $1\frac{3}{10}$ __________ (d) 3 __________

5. Write the following common fractions as percents:

(a) $\frac{23}{50}$ __________ (c) $\frac{1}{3}$ __________

(b) $1\frac{3}{4}$ __________ (d) $\frac{85}{50}$ __________

Obj. 7.2.1
a) ☐
b) ☐
c) ☐
d) ☐
Obj. 7.2.2
a) ☐
b) ☐
c) ☐
d) ☐
Obj. 7.2.3
a) ☐
b) ☐
c) ☐
d) ☐
Obj. 7.2.4
a) ☐
b) ☐
c) ☐
d) ☐
Obj. 7.2.5
a) ☐
b) ☐
c) ☐
d) ☐

Cycle 2: Representing Fractions as Percent

Overview

Big Idea

In this cycle, activities extend student understandings of representations of common fractions and decimal fractions to include the representation of percent. Percent literally translates as per hundred. When counting in percent, students are counting hundredth-sized parts of wholes. The major difficulty with this representation is connecting the new language of "per" to meaningful contexts and converting percentage (parts per hundred) to common or decimal fractions. Once these connections are clearly understood, many percent problems become applications of decimal or fraction operations that students have already experienced.

The focus of this cycle is to connect representations of common fractions and decimal fractions to their equivalent representation as percent and to be able to convert flexibly between these representations.

Objectives

By the end of this cycle, students should be able to:

7.2.1 Express percent as a representation of hundredths as a decimal. [6NA131]

7.2.2 Express decimals as percent. [6NA131]

7.2.3 Express percent as a representation of hundredths as a common fraction. [7NA157]

7.2.4 Express common fractions of tenths or hundredths as percent. [7NA157]

7.2.5 Express common fractions other than tenths or hundredths as percent. [7NA157]

Conceptual Links

Students will need to have understanding of decimal place values and fractions as parts of wholes to be able to connect effectively between these and percent.

Representations of fractions as percent are used widely when describing percentage change (Cycle 4, e.g., discounts and mark-ups), when calculating simple interest on loans (Cycle 4), and when considering repeated change (e.g., population growth over subsequent years, compound interest). Percentage skills also have applications in geometry and measurement (scaling), representations of probability, and graphical representations (e.g., depending on the data and the purpose of the graph, percentages may be used as the unit of count on pie graphs, column graphs, line graphs).

Materials

For Cycle 2 you may need:

- Counters
- Calculators
- Bead strings
- Hundredths grids
- Common fraction, decimal fraction, percentage matching game cards

Key Language

Percentage, percent, out of a hundred, parts per hundred, hundredths, tenths, whole, common fraction, decimal fraction

Definitions

Percent: one part in every hundred

Percentage: a rate, number or amount in every hundred; any proportion or share in relation to a whole

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What decimal fraction do you have?
- Can you say that as hundredths?
- So how many percent would that be?
- Can you make that into an equivalent fraction with tenths or hundredths as the denominator?

  Ensure that students are able to enter a fraction into a calculator reliably to reach a decimal fraction. For example, ¾ can be entered as 3 ÷ 4 = 0.75. This can then be converted to 75%.

Portfolio Task

There are not specific sections that apply to this cycle although ideas from this cycle provide background connections between percentages and decimal fractions that are needed to successfully complete the task.

RAMR Cycle

This cycle introduces the language of percentage and its use as an equivalent representation of common and decimal fractions.

Reality

Consider the language of percent, in particular situations where students hear “per” … one spoon per person, one egg per egg cup, planning for a camping trip with food quantities and items per person, accommodation costs (e.g., campsites $80 per person per night), and so on. Extend to examples that are not unitary such as one cake per eight people, one cabin per six people, one bus per seventy students and so on. Ask students to explain what “per” means in these contexts. It may be useful to collate these on a concept map/poster for the classroom.

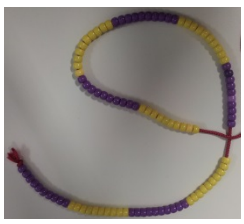
Discuss where students may have come across “percent” before. For example, marks, discounts, GST, humidity, likelihood, statistical data for populations, interest rates, or nutritional information. Check students’ prior knowledge of the symbol for percent (%).

Abstraction

The abstraction sequence for this cycle builds student understanding of the language of percent and its use as an equivalent representation for common and decimal fractions by using simple fractions that gradually increase in difficulty and complexity. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Act out simple reality examples within the classroom. For example, 4 students for every 10 desks (or 4 out of 10 students raise their hand).
2. *Connect to language.* Introduce the language “4 per 10” in place of “for every”.
3. *Connect to fractions.* Ask students what fraction of the desks are occupied if there are 4 students per 10 desks (or what fraction of the 10 students have their hand raised). Represent as common fractions and decimal fractions (e.g., $\frac{4}{10}$, 0.4).
4. *Model/represent with materials.* Draw pictures to show $\frac{4}{10}$ as shown. 
   Say, we could say this fraction as 4 per 10 as it is 4 shaded out of a whole of 10. State that 4 per 10 means that for every 10, we have 4. *Resource 7.2.1 Modelling part-whole problems* provides some examples of questions that involve real-world stories as a guide.

Resource Resource 7.2.1 Modelling part-whole problems



5. *Connect to equivalent fractions.* Discuss with students how else 4 per 10 can be represented (e.g., $\frac{4}{10}$, $\frac{40}{100}$, 0.4, 0.40). Bead strings may be useful to model 4 per 10 as 40 per hundred if 4 blocks out of 10 are separated from the remainder (see picture) and then also counted to demonstrate 40 out of 100 beads. Discuss what the equivalent fraction of $\frac{40}{100}$ might represent (40 students per 100 desks).
6. *Connect language and symbols.* Connect language of parts per hundred to the common fraction and decimal representations and the percent symbol. Highlight that the % symbol is shorthand for “parts per hundred” or “/100”.
7. *Explore with other examples* like 5 rulers per 10 students; 40 pages per 100 students; 3 students per row of 10; 1 book per 2 students; 1 row per 5 students; 2 students per 5 desks; 3 red counters per 5 counters. What would the equivalent fractions for these be if they were per 100? What percentage of the whole would be represented?

Mathematics

Connections

Use real-world examples of fractions and percent. Engage students with converting these to other representations using hundredths squares, common fractions, decimal fractions and percent. Ensure that students understand the meaning of percent as parts per hundred. Initial practice should involve fractions that are hundredths or tenths to convert to percent and whole number percent to convert to fractions.

Language/symbols and practice

Within this cycle, the emphasis is on percent as it represents parts of a whole. Thus, students need to understand that to be expressed as percent, the whole is partitioned into one hundred pieces and the fraction represents that many pieces of the whole. Bead strings may be useful here for students to visualise hundredths.

Where fractions are not already expressed as hundredths, it is necessary for students to be able to convert fractions other than hundredths or tenths to percents. Some examples are easily converted using equivalent fractions. Equivalent fraction sticks may be a useful resource to help students recognise the patterns in the symbols for equivalent fractions if this process is still not clear.

Resource Resource 7.2.2 Equivalent fraction sticks

Students should be able to "see" a common fraction as tenths or hundredths, convert to decimal and convert to a percentage. Similarly, given a percentage, students should be able to convert to a decimal and a common fraction. It may be useful for some students to connect to area models of tenths or hundredths to see parts per hundred. A selection of problems are available in *Resource 7.2.3 Hundredths squares.*

Resource Resource 7.2.3 Hundredths squares

Converting other fractions

As for equivalent fractions, there are patterns within the symbolic representations that help with converting fractions to percentages. For example, 4 per 10 becomes 40 per 100 which is also represented as $\frac{40}{100}$. Discuss what happens when this is entered into a calculator to create a decimal (40 ÷ 100 = 0.40) which is also 40%. Try with other common fractions that students may know, for example, a half (1 ÷ 2 = 0.50). Try some other examples that students may be familiar with to test the process. Practice connecting and converting pictures, language and symbols from common fractions to percent.

In addition to making connections, it is beneficial for students to practice connecting commonly occurring fractions and percents to their equivalent representations. *Resource 7.2.4: Equivalent representation games* can be used to facilitate student practice with equivalent representations.

Resource Resource 7.2.4 Equivalent representation games

Reflection

Check the idea

Continue percentage practice with examples of “per 1”, “per 10” and “per 100”questions.

Ensure that these questions take students to answering, “how many if there were 100” questions.

For example:

(a) 50 per 100 were boys, if there are 100 students, how many will be boys? (50)

(b) 50% of the group were boys, what fraction of the group are boys? ($\frac{50}{100}$ or 0.50)

(c) 25 per 100 were girls, if there are 100 students, how many will be girls? (25)

(d) 25% were girls, what fraction of the group are girls? ($\frac{25}{100}$ or 0.25)

(e) 7 per 10 were boys, if there are 100 students, how many will be boys? (70)

(f) If 7 per 10 were boys, in a group of 100 students, how many will be girls? (30)

(g) 3 out of 4 of the group were boys, what fraction of the group are boys? ($\frac{3}{4}$ or 0.75)

(h) If 75% were boys, what fraction of the group are girls? ($\frac{25}{100}$ or $\frac{1}{4}$ or 0.25)

Apply the idea

Explore the contents and nutritional information on food packets to convert between percent and fraction representations. Belvita breakfast biscuits and Sunbeam sultanas are included here. Students may locate other examples of their own interest.

Belvita Breakfast Biscuits (per 50g): Protein, 4g; Fat, 6.8g; Carbohydrate, 33.6g; Dietary Fibre, 3.2g; Sodium, 0.151g.

Sunbeam Sultanas (per 40g): Protein, 1g; Fat, 0.3g; Carbohydrate, 31.6g; Dietary Fibre, 1.3g; Sodium, 0.003g; Calcium, 0.025g; Magnesium, 0.016g; Iron, 0.0003g.

Engage students with determining what fraction of 40g each item represents. Sultanas, for example, protein = $\frac{1}{40}$ = 1 ÷ 40 = 0.025; or carbohydrate = $\frac{31.6}{40}$ = 31.6 ÷ 40 = 0.79. Students should then convert the decimal fraction to a percent (e.g., 2.5% protein or 79% carbohydrate).

Extend the idea

Consider contexts where percents greater than 100% are reported. For example, when flooding is imminent, dams may be described as 150% full. Consider what this means and work backwards to convert the percent to decimal and common fractions to better visualise the meaning. Find other contexts where greater than 100% may be considered a valid amount.

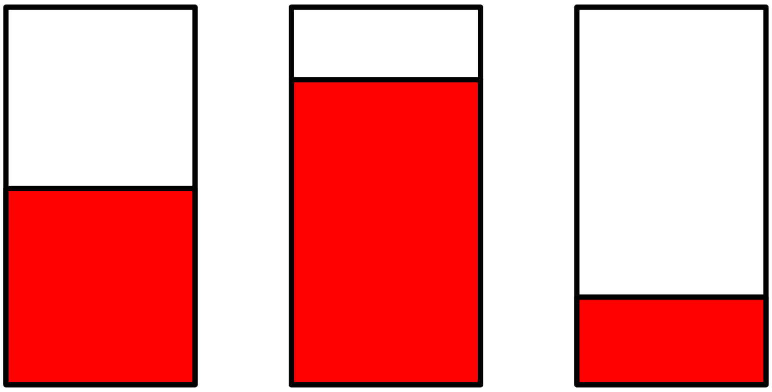
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

Can you do this? #3

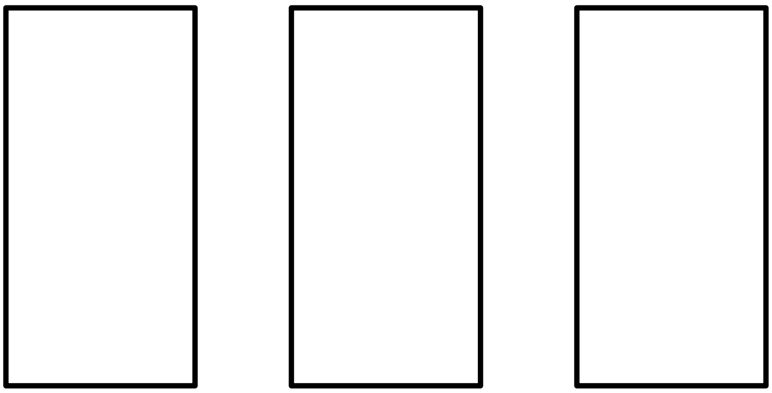
1. Estimate the amount of cordial as a percentage of the glass:



(a) _________% (b) _________% (c) _________%

Obj. 7.2.4
a) ☐
b) ☐
c) ☐

2. Draw the amount of cordial in each glass for the given percent:



(a) 10 % (b) 60% (c) 90%

Obj. 7.2.3
a) ☐
b) ☐
c) ☐

3. Out of a group of 50 students, 13 students walked to school, half rode bicycles and the rest arrived by bus.
   (a) What percentage of students rode bicycles to school? _______
   (b) What percentage arrived by bus? _______________________

Obj. 7.3.1
a) ☐
b) ☐

4. The length of the room is 15m. The width of the room is 50% of the length of the room. Calculate the width of the room in metres.

Obj. 7.3.2
i. ☐
ii. ☐

5. Shoes normally cost $40. They are discounted by 25% today. Calculate the amount of the discount.

Obj. 7.3.2
i. ☐
ii. ☐

Cycle 3: Finding Percentage of

Overview

Big Idea

In the previous cycle, percent was explored as a representation of fractional quantities. The focus of this unit is to calculate one quantity given another quantity and the percentage that describes the relationship between the quantities. Two different types of relationship may be represented using percentage, static multiplicative comparison or active change. For example, measures may be compared (e.g., the length of this table is 50% of the length of the other table; the number of students in this class is 75% of the number in the other class) so that the percentage describes the relationship between the measures or the proportions. These types of problems will be the focus of this cycle. Using percentage amounts to actively change the initial value (e.g., the cost of the item will be discounted by 25% of the marked price), will be the focus of Cycle 4. Encourage students to make a quick sketch if it helps them to picture the parts of the problem. Percentage problems are multiplicative in nature so can be represented using area, number line or Arrowmath diagrams. Use the representation that works best for your students.

Objectives

By the end of this cycle, students should be able to:

7.3.1 Convert between percentages and fractions to solve percentage problems. [7NA157]

7.3.2 Express one quantity as a percentage of another. [7NA158]

Conceptual Links

Finding percentage of a quantity relies heavily on students' ability to find a fraction of a quantity. Students need to understand that a fraction can represent parts of a whole, and when applied to a quantity the initial quantity needs to be divided by the denominator to find the size of each share of the quantity. This share then needs to be multiplied by the numerator or count of the parts to determine the fraction or percentage of. Finding a fraction of a collection needs to be revised before finding a percentage of to clearly connect these ideas.

The ability to find a percentage of a quantity is used widely when working out the cost of items after application of discounts and mark-ups (Cycle 4) and when calculating simple interest on loans or savings (Cycle 5). Percentage skills also have future applications in geometry and measurement (scaling), representations of probability, and graphical representations.

Materials

For Cycle 3 you may need:

- Hundredths squares
- Function machine
- Double number lines

Key Language

Per, percentage of, fraction of, compare, comparison, factor, product, double number line

Definitions

Percent: one part in every hundred

Percentage: a rate, number or amount in every hundred; any proportion or share in relation to a whole

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What information do you know?
- Can you represent that in a picture to help you?
- What is the total amount? What is 100%?
- What is the part? What percentage of the total is the part?
- What do you need to work out?
- Can you write down what you need to put into a calculator to work that out?

  Ensure that students are able to enter information they have onto a double number line to help them determine what they need to do to solve the problem.

Portfolio Task

Sections of the task that require students to make comparisons between quantities expressed as a percentage value or find the percentage of a quantity may be completed as part of or following this cycle.

RAMR Cycle

Reality

Discuss situations where a fraction, or part of a total needs to be calculated. For example, out of a whole test worth 50 marks, a student may have a fraction of 35 correct out of 50. The 35 represents 70% of the possible total. Discuss how many marks half or 50% of 50 marks might be. Other contexts that are applicable include comparing the length of tables (e.g., this table is 50% or half as long as that table which is 1m long), ¼ of the class are girls, if there are 10 boys in a group of 25 students they make up $\frac{10}{25}$ of the group. Consider also the meaning of 40% of the population, 25% of the recommended daily intake, 15% of the cost.

Abstraction

The abstraction sequence for this cycle starts from students' experience of representing parts of a whole as common fractions, decimals and percentages to finding a percentage of a collection or quantity. A suggested sequence of activities is as follows:

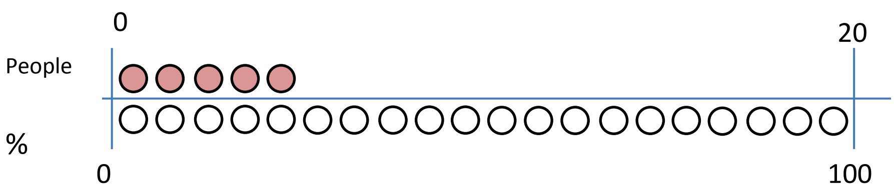
1. *Kinaesthetic activity*. Start from the reality discussion. Find how many students in the class, separate into two groups with 25% of the class in one group and 75% of the class in the other. Record how many in each group.

2. *Represent/model with materials.* Use counters, unifix cubes or bead strings to model the groups found as a number line model so that the two parts and the whole may be identified.

   Ensure that students can identify one quarter or 25% of the whole group, three quarters or 75% of the whole group and the whole group of students that make up 100%. Make sure that students are able to represent these as percentages, decimal and common fractions.

3. *Represent/model with materials.* This type of problem also lends itself to modelling on a double number line. Draw a number line with the left hand end labelled zero (0). Twenty counters or beads may be lined up under the line to represent the whole group and a mark put on the other end to represent 100%. Above the line, line up five counters or beads and determine the relevant fraction of the whole that is represented (in this case, one quarter or 25%).

4. *Connect to language/symbols.* Write equations to represent the parts and the whole. For example, 25% of 20 students = $\frac{25}{100}$ × 20 = 5. Students may also work this out using a calculator and decimal fractions with the equation, 25% of 20 students = 25 ÷ 100 × 20 = 5.

5. *Connect to language and symbols.* Discuss the process for finding the percentage of a quantity. Ensure students can make the connection to finding a fraction or part of a quantity.

6. *Repeat* this sequence but split the class in different ways (e.g., boys vs. girls, sporting teams favoured, pencil case/no pencil case).

Mathematics

In this part of the cycle the focus of activities is to consolidate finding percentage of a quantity. The number line model is provided as a possible means of sketching or modelling the problem to assist students with identifying the elements of situations as factor-factor-product.

Language/symbols and practice

Number Line Model

For percentage, rate and ratio problems that involve multiplicative or proportional relationships between quantities, double number lines may be used (as indicated in the Abstraction phase). Double number lines are actually single lines on which numbers appear above and below for different attributes and amounts and using different scales. The numbers above and below are equivalent when on the ends of lines that cross the number line. In this way, it is possible to represent the thing being partitioned on one side of the line and the equivalent percentage on the other side. This model is very useful for clearly identifying the elements of the problem that are known and the unknown element within the problem to be found. *Resource 7.3.1: Double number line models for percentage* steps out the process of modelling percentage of problems on the double number line and includes a worksheet for students.

Resource Resource 7.3.1 Double number line models for percentage

Number line model and Multiplicative comparison

The number line model is also useful for representing comparison types of problem (e.g., the width of the table is 50% of the length of the table). In these problems, one of the quantities assumes the role of the whole with the other quantity described as a percentage of the whole. Other comparison problems may include considering the class group as a percentage of the year group.

Change Model

In some instances, percentage of problems include active change components. In these instances, the percentage that is calculated is often used in a further step or operation to generate an end result. Contexts can include percentage increase or decrease (e.g., markup or discount). The Change Model using Arrowmath equations may be a useful model for students to use (although double number lines are also effective here). For example, using the earlier problem of twenty students, 25% of the class group may decide to walk away. To depict this as a change, the problem is modelled as below:

$20 \xrightarrow{\times 25\%} ?$ people who walked away or $20 \xrightarrow{\times 75\%} ?$ people who stayed in line.

Note: Whichever model is used, the key understanding students need to have is what their result represents. In the example modelled here, 25% is the number of people who walked away; 75% is the number of people who stayed.

Resource Resource 7.3.2 Change models for percentage

Reflection

Check the idea

Identify sub-groups within the class and represent each group as a percentage of the class. For example, group according to eye colours, glasses or not glasses, hair colour.

Consider a range of advertising material that displays percentage discount offers. Determine what money value is represented by each discount amount. For example, shoes advertised at $39.95 may be 15% discount. What amount of money will be saved? Note that students are only expected to calculate the discount here – the new, discounted price or percentage change is the topic for the next cycle.

Apply the idea

Explore other contexts for the application of percentages like nutritional value of foods and serving sizes. Ask students to bring in examples from food packets. Consider examples where 20% of the recommended daily intake is in 100g of a food. Obtain fast food indications of the amount of kJ contained in particular meals. Determine these as percentages of the recommended daily intake for good health. Explore the outcomes of different serving sizes or many of these meals in a day. For example, the Breakfast biscuits from Cycle 2 have the following percentage daily intake:

> ***Belvita Breakfast Biscuits (per 50g):*** Protein, 8% (4g); Fat, 10% (6.8g); Carbohydrate, 11% (33.6g); Dietary Fibre, 11% (3.2g); Sodium, 7% (0.151g).

Engage students with determining what these mean. For example, if 4g protein = 8% of daily intake, what is the total daily intake?

$$\begin{aligned}
8\% \times \text{daily intake} &= 4\text{g} \\
8 \div 100 \times \text{daily intake} &= 4\text{g} \\
0.08 \times \text{daily intake} &= 4\text{g} \\
0.08 \times \text{daily intake} \div 0.08 &= 4\text{g} \div 0.08 \\
\text{daily intake} &= 50\text{g}
\end{aligned}$$

Resource Resource 7.3.3 Nutritional facts

Extend the idea

Engage students with personal data collection of measures of height and arm span and assign to categories (e.g., 150cm-155cm). Identify the percentage of the class in each category. See if students can identify how many students of each height range might be in an equivalent class of more students (e.g., if there are 7 students that are 150cm-155cm tall in a class of 28 (25%), and an equivalent class were to be created with 40 students, how many students of this height range would there be?).

The above data can be used to generate column graphs or histograms. (**Note:** Histograms is a new graph type for students so salient features of the histogram will need to be discussed.) Consider generating stacked bar graphs that show the number of boys and girls in each category as part of the total. Explore representing these values as percentages. Extend students thinking to consider what these percentages mean if they are considered an average sample for a larger population. For example, if 20% of the class are a particular height category, what does this mean within a population of a thousand students their age?

Area Model

Area diagrams use squares to show, pictorially and visually, the amounts and the attributes of the problem. Students may prefer hundredths grids to start with but should progress to quick sketches of a square to represent 100% or the whole. Some students may like the area model as the hundredths can be seen although these can potentially become confusing when problems arise that are not neatly divisible by tens or hundreds.

Resource Resource 7.3.4 Area models for percentage

Area Model

Once students can calculate percentage of, they should be able to use this skill to generate circle graphs. If students have completed Unit 04, they should be able to extend their angle measuring skills using *Reource 7.3.5 Constructing circle graphs (protractor).* If students do not have reliable angle measuring and calculation skills, bead strings can be looped into a circle and the relevant number of beads counted around to determine the size of the graph sector. A suitable resource to use in this instance is *Resource 7.3.6 Constructing bead string circle graphs* (the circle in this resource is designed to suit a string of one hundred 6mm pony beads). Use student generated data for graphs (e.g., class favourite colours, pets, favourite foods).

Resource

Resource 7.3.5 Constructing circle graphs (protractor)
Resource 7.3.6 Constructing bead string circle graphs
Resource 7.3.7 Interpreting circle graphs

Teacher Reflective Notes

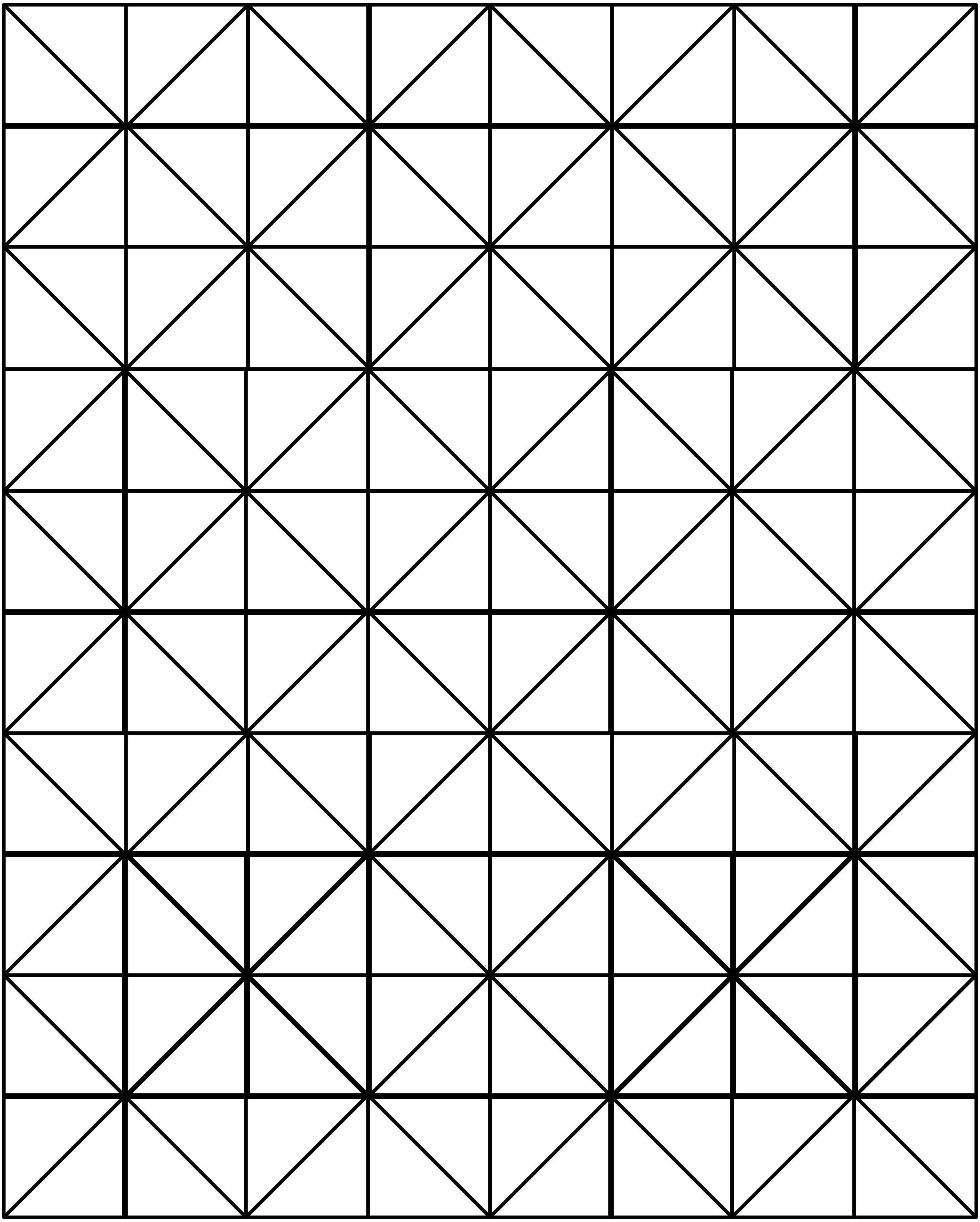
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:__________

Unit 07 Investigation: Coloured Percentages

8. Use coloured triangles to create a mosaic design to fill this space (20cm × 25cm).
9. Choose three colours to make your mosaic with.
10. Each colour must be used to cover at least 20% of the area given and no more than 60% of the area given.
11. In a table, record how many triangles of each colour you have used.
12. Work out what percentage of the total rectangle is covered by each colour.

Print this page on assorted colour paper (e.g., pink, blue, green, yellow, orange).



Name:________________________________ Date:________

Can you do this? #4

1. A pair of $39.95 shoes were marked 15% off.

   How much were the discounted shoes?

Obj. 7.4.1

i. ☐

ii. ☐

2. A TV is priced at $350. The shopkeeper has to add an additional 10% GST. What is the final price including GST?

Obj. 7.4.1

i. ☐

ii. ☐

3. A 250g chocolate bar is labelled as “New 20% bigger”.
   What would it have weighed before?

Obj. 7.4.2

i. ☐

ii. ☐

4. A new cycle path reduces the distance to the shops by 15%. The distance is now only 34km. What was the distance before?

Obj. 7.4.2

i. ☐

ii. ☐

5. Jill purchased items for $4.50 each and is now selling them for $4.00 each. What is her loss for each item as a percentage?

Obj. 7.4.3

i. ☐

ii. ☐

6. Fred aims to make 10% profit on sales in his business. If he purchases goods for $43 each, what price must he sell them for?

Obj. 7.4.3

i. ☐

ii. ☐

7. Mr Smith has $5500 in a bank account. He receives 8% simple interest each year. How much money does Mr Smith have altogether at the end of the year?

Obj. 7.4.4

i. ☐

ii. ☐

Cycle 4: Percentage Change

Overview

Big Idea

Activities so far have focussed on “percentage of”. The focus of this cycle is to extend working with percentages to interpreting and solving problems involving *percentage less* (e.g., discounts) and *percentage more* (e.g., mark-ups), i.e., multiplicatively transforming one quantity into another by a percentage. These problems can be treated as two-steps, which highlight the parts and the thinking for the solution. It is also possible to deal with these problems by considering *percentage less* as finding the complementary percentage (e.g., 15% less calculated and subtracted is the same as calculating 85% of) or treating the percentage as 100% plus the mark-up amount (e.g., 10% more calculated and added on is the same as calculating 110% of). In this case the percentage calculation is completed in one step. In addition to discounts and markups, other financial situations and repeated percentage change are also considered.

If numbers for percentage problems are chosen such that computations can be performed mentally, then students may be able to work without calculators. However, as real world contexts, percentage more and less are seldom simple amounts and students should develop the ability to use the calculator to obtain valid answers to assist with financial decisions.

Objectives

By the end of this cycle, students should be able to:

7.4.1 Calculate percentage discount or markup of 10%, 25%, 50% on sale items. [6NA132]

7.4.2 Solve percentage increase or decrease problems. [8NA187]

7.4.3 Solve problems involving profit or loss. [8NA189]

7.4.4 Apply percentages to problems involving simple interest. [9NA211]

7.4.5 Apply percentages to problems involving repeated change. [9NA211]

Conceptual Links

Percentage discount and markup essentially provide applications to consolidate finding percentage of from the previous cycle. Number, fraction, decimal, place value and operations skills are needed and consolidated through this cycle.

Ideas developed in this cycle can be applied to scaling and measurement problems in later units.

Materials

For Cycle 4 you may need:

- Hundredths grids
- Function machines
- Advertised prices with/without GST
- Double number lines
- Advertised discounts using percentages
- Play money

Key Language

Percentage, percent, %, discount, mark-up, GST, tax, percentage more, percentage less, simple interest, per year, per annum, loan interest, savings interest, compound interest, complementary percentage

Definitions

Compound Interest: percentage amount of increase that is repeated across time such that the percentage increase from one period of time is also increased by the same percentage in the next period of time

Discount: percentage amount taken off of the original money amount (usually sales discount)

GST: Goods and services tax. Percentage increase on selected goods and services paid to the government as tax (currently 10%).

Markup: percentage amount added to the original money amount (usually sales profit)

Per annum: each year

Simple Interest: percentage amount of increase applied once to a money amount (may be interest earned or interest charged)

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What information do you know?
- Can you represent that in a picture to help you?
- What is the total amount? What is 100%?
- What is the part? What percentage of the total is the part?
- Is this percentage increase or decrease? What do you need to work out?
- How much increase/decrease will it be?
- Do you need to add to the initial amount or subtract from it?
- Can you write down what you need to put into a calculator to work that out?

Ensure that students are able to model the information they have to help them determine whether the problem is a percentage increase or percentage decrease problem.

Portfolio Task

Any questions related to percentage increase (markup) or percentage decrease (discount) and simple interest earned or paid may be completed with or after this cycle.

RAMR Cycle

Reality

Consider shopping realities involving percentages. Ask students to identify instances of percentage discount or percentage tax. Ask students what they think happens to prices if a percentage discount is applied. Ask further what happens to prices if a percentage tax or mark-up occurs. Think of specific contexts where percentages occur. Categorise examples of *percentage less* or *percentage more* into groups (e.g., discounts, rebates, scaling down are usually *percentage less* contexts whereas profit margin, overheads, taxes, scaling up, interest rates on loans or bank balances are usually *percentage more* contexts).

Abstraction

The abstraction phase focuses on percentage increase only. Percentage decrease problems will be introduced in the mathematics phase, once students are confident with percentage increase. The abstraction sequence for this cycle starts from students' previous experience of determining percentage of a quantity. This is useful to start with as the percentage can be calculated then added or taken from the initial quantity. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Have ten students hold up all their fingers. Ask what percentage of the total of the fingers are up. Ask students what 20% of the total of the fingers that are up would be (this may be each student with 2 fingers raised or 2 students with all fingers raised). Return to 100%. Now ask students how they could show 20% more fingers. (Add two students to the line.)
2. *Represent/model with materials.* Represent the previous example with counters, unifix cubes or bead strings. Encourage students to identify the whole, the percentage increase, and the final amount. Ensure that students understand that for percentage increase they are adding the percentage to the initial whole.
3. *Represent using length model.* Use a fraction strip to represent the original whole and the percentage increase.

4. *Represent/model with double number line.* Connect length model to double number line to model percentage increase problems. For example:

   Have five students stand in a line. Discuss with students what they could do to make the line of people 20% longer. Discuss what fraction or percentage of the line of people results.

   Students may see this as a two-step problem where they calculate 20% increase to be added to the total. Connect to how this looks as an equation for calculation. For example,

20% of 5	= 0.2 × 5
	= 1 more person
5 people + 1 person	= 6 people in the line

Alternatively, a more algebraic approach can be used:

Final = 100% of Initial + 20% of initial
= (100% + 20%) of initial
= 120% of initial
= 1.2 × initial

Connect to how this looks as an equation on the calculator. For example,

Final = 120% of 5
= 1.2 × 5
= 6 people in the line

5. *Represent/model using change/Arrowmath diagram.* To model this as percentage change using Arrowmath diagrams students need to identify the start, the multiplier or percentage change and the total. Since this is percentage increase, the total will represent 120% of the initial quantity so the multiplier is 120%. The diagram is as follows:

$$5 \xrightarrow{\times 120\%} ?$$

Enter this on a calculator: Final = 5 × 120%
= 5 × 1.2
= 6 people in the line

Note: Whichever model is used, the key understanding students need to have is what their result represents. In the example modelled here, 20% is the number of people who joined the line; this is added to 100% to give 120% of the original line as the end result.

Mathematics

Language/symbols and practice

Percentage Decrease

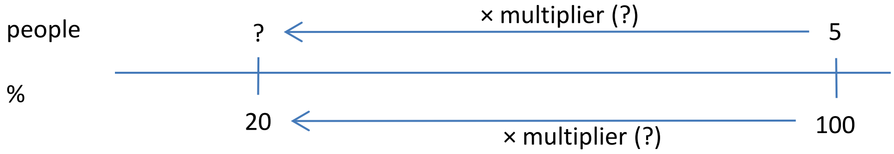
The steps from the Abstraction phase may be repeated to physically explore percentage decrease problems. In this instance, the percentage decrease is subtracted from 100% or the complementary percentage may be calculated to find the end result. Again, represent/model with double number lines or change models as in the following example:

Have ten students (or five students if the class is small) stand in a line. Discuss with students what they could do to make the line of people 20% shorter (or less). Discuss what fraction or percentage of the line of people is remaining.

Number Line Model

This problem could be modelled using the double number line to show the 20% decrease.

Students may see this as a two-step problem where they calculate 20% to be subtracted from the total. Connect to how this looks as an equation for calculation. For example,



20% of 5 = 0.2 × 5
= 1 person leaving the line

Original line of 5 people – 1 person leaving = 4 people remaining in line

Alternatively, this problem could be modelled to show the complementary percentage of 80% of people who stayed in line. This is calculated by determining 100% – 20% = 80% remaining in line.

This generates a one-step calculation where students find 80% of the initial total. Connect to how this looks as an equation for calculation. For example,

80% of 5 = 0.8 × 5
= 4 people remaining in the line

Change Model (Arrowmath Diagram)

To model this as percentage change using Arrowmath diagrams students need to identify the start, the multiplier or percentage change and the total. Since this is percentage decrease, the total will represent 100% – 20% of the original quantity leaving 80%. The diagram is as follows:

$$5 \xrightarrow{\times 80\%} ?$$

Enter this on a calculator: 5 × 80% = 5 × 0.8
= 4 people remaining in the line

Practice finding percentage increase and decrease in money contexts

Money is an ideal context for reinforcing percentage work and can be acted out in a very tangible way using coins. Starting with simple examples and 10%, provide students with notes and coins to represent the unit cost of an item. Have them work out 10% tax and add this amount of money as a pile beside their unit cost pile. Have them work out 5% mark-up on the original item cost and place in a pile beside the others. Calculate the total sale price for the item. A similar process may be used to experience and reinforce percentage discount.

Work some examples of finding *percentage increase* of common amounts or quantities of items familiar to students. For example, groups of people, football or music audiences, mark-ups on prices, mixtures like juice, cordial, paint. Practice imagining percentage questions as a whole (100%) and some more and identifying these on sketches (these can be hundredths boards to start with but progress to an open square to aid thinking; or double number line; or change diagram).

Work some examples of finding *percentage decrease* of common amounts or quantities of items familiar to students. For example, groups of people, discounts on prices, mixtures like juice, cordial, paint. Practice identifying the parts of questions and identifying these on sketches (double number line or change diagram).

Resource Resource 7.4.1 Percentage more or less problems

Simple Interest

Percentage increase/decrease problems can be extended to include simple interest earned/charged within financial contexts. Students may benefit from acting out these contexts using play money.

1. *Act out percentage more and simple interest using play money.* Consider the context of owing $120 dollars at 10% interest/year. We need to pay this back at the end of the year so how much do we save per month ($120 ÷ 12 = $10 per month).

   Students should realise that we also need to allow for the interest repayment. Ask students what they think they need to do to figure out how much extra to save.

   If students suggest that they need to save $12 per month act this out for students to see how much extra is saved this way.

   If students suggest that they need to save the interest for the year gradually each month, discuss how they might work out what this amount might be. Act out suggestions with play money. Compare the amounts that resulted.

2. *Model the earlier problem with drawings.* These can be Number Line or Change models. Note that for the interest calculated across a year, this is a simple percentage more calculation. Calculations can either work out 10% of $120 and add it to the original 100% or find 110% of $120. We can also do this using a double number line or change model. Use of any of these methods will indicate that after a year $12 interest will be owing and we will need to pay back a total of $132.

3. *Discuss the initial acting out.* If we put away an amount per month to pay back, what will it be now? ($132 ÷ 12 = $11).

4. *Practice a range of simple interest problems.* Once students are comfortable with simple interest over a year, see if they can work out the reverse problem of what the interest was from the amount repaid and the initial loan amount.

These examples can also be extended to contexts where loan amounts are paid in less time or extend beyond the year before repayment. Explore ways of working out how much interest is fair to pay back after 6 months.

Resource Resource 7.4.3 Simple interest problems

Reflection

Check the idea

Engage students in constructing their own *percentage increase/decrease* problems. The thinkboard or concept map may be a useful organiser for this activity or students may like to draw a model to swap with another class member to write the accompanying story context. Alternatively, draw a picture on the board and ask students to tell a suitable story for the picture.

Resource Resource 7.4.2 Thinkboard or concept map

Apply the idea

Use advertised discounts and prices as stimulus for students to pose and solve percentage increase/decrease problems. Have students work out the cost of a shopping list at regular prices as well as at discount prices. Depending on the stimulus material, students may need to work backwards from the advertised discounted price and the discount to find the regular price.

Extend the idea

Multiple Percentage Change

Explore situations where multiples of successive percentage changes area applied to items.
For example:

$$\text{cost price} \xrightarrow{\times\, 120\%} \text{shelf price} \xrightarrow{\times\, 110\%} \text{sale price including GST}$$

Compare final prices resulting from: a) percentage profit and percentage GST are calculated on the cost price then added together; with b) percentage profit calculated on cost price then GST calculated on top of both.

When time is different to a year

Since interest is a rate per year, then the interest and the repayment depend on how long the amount is borrowed. For 3 months or $\frac{1}{4}$ of a year, the rate is $\frac{1}{4}$ of 10% or 10% ÷ 4 = 2.5%. thus the amount paid back is:

$$\begin{aligned} \$120 + \$120 \times 2.5\% &= \$120 + \$120 \times 0.025 \\ &= \$120 + \$3 \\ &= \$123 \end{aligned}$$

Repeated Percentage Change

Consider the effects of repeated percentage change over time such as 10% population growth per year across 10 years. Discuss the effect of continuing to increase the annual total by 10% where the original quantity increases each year as well as the growth amount. For example, 1000 people increasing by 10% over 5 years would give the following results:

	Start (people)	**10% increase**	**Total (people)**
Year 1	1000	100	1100
Year 2	1100	110	1210
Year 3	1210	121	1331
Year 4	1331	133.1	1464.1
Year 5	1464.1	146.41	1610.51

The cumulative effect of the repeated interest could also be shown as follows:

	Start (people)	**10% increase**	**Total (people)**
Year 1	1000	100	1100
Year 2	1000+100	100+10	1210
Year 3	1000+100+100+10	100+10+10+1	1331
Year 4	1000+100+100+10+100+10+10+1	100+10+10+1+10+1+1+0.1	1464.1
Year 5	1000+100+100+10+100+10+10+1+100+10+10+1+10+1+1+0.1	100+10+10+1+10+1+1+0.1+10+1+1+0.1+0.1+0.01	1610.51

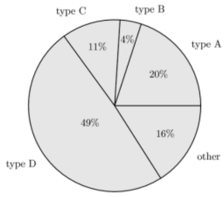
This non-linear, exponential growth can also be drawn on a graph.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 07 Portfolio Task – Teacher Guide

Daily Percentages



Content Strand/s: Number and algebra

Resources Supplied:

- Student task sheet
- Teacher guide

Other Resources Needed:

- Common breakfast foods nutritional information (either on packaging or online)

Summary:

Students use their percentage knowledge to explore shopping discounts, bank interest and nutritional information for dietary requirements.

Variations:

- It may save some time to generate a selection of common breakfast foods' nutritional information on cards so that students may select from these for the Investigation activity instead of relying on them looking for information for homework.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

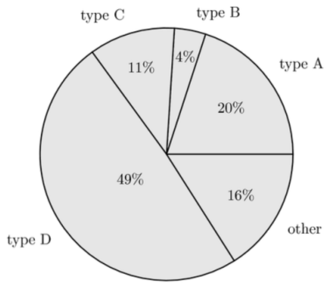
Content Strands:

Number and Algebra

6.3.1 Convert percentages to fractions to solve percentage problems. [7NA157]
6.3.2 Convert percentages to decimals to solve percentage problems. [7NA157]
6.3.3 Express one quantity as a percentage of another. [7NA158]
6.4.1 Calculate percentage discount of 10%, 25%, 50% on sale items. [6NA132]
6.5.1 Apply percentages to problems involving simple interest. [9NA211]

Daily Percentages

Name	
Teacher	
Class	



In this task you will encounter percentages in three ways:

- Dietary requirements
- Shopping
- Banking

To complete this task you will use your knowledge of:

- Multiplication
- Division
- Percentages
- Graphing
- Interest

Within Portfolio Task 7, your work demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Conceptual understanding	6.3.1 Convert percentages to fractions to solve percentage problems. 6.3.2 Convert percentages to decimals to solve percentage problems.	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
	Procedural fluency	6.3.3 Express one quantity as a percentage of another.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
Problem Solving and Reasoning	Problem solving approaches	6.4.1 Calculate percentage discount of 10%, 25%, 50% on sale items. 6.5.1 Apply percentages to problems involving simple interest.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches

Comments:

Task 1: Dietary requirements

In this task you are going to calculate the percentage of recommended daily intake for each of 5 types of nutrient that are contained in your breakfast. Here are the recommended daily intakes.

Nutrient	Recommended daily intake
Energy	8000 kJ
Protein	50 grams
Fat	70 grams
Carbohydrate	90 grams
Dietary fibre	30 grams
Sodium	2300 milligrams

1. Make a list in the space below of the foods you have for breakfast and estimate the quantity of each one. If you wish, you can imagine your ideal breakfast.

2. Find out the amount of each of the 5 nutrients listed above that your foods contain by : (a) looking at the packaging of the food and/or (b) searching on the internet.

   Add up the total for each nutrient and enter them in the table below. Then calculate the percentage of recommended daily intake for each one.

Nutrient	Quantity in breakfast	% Daily intake
Energy		
Protein		
Fat		
Carbohydrate		
Dietary fibre		
Sodium		

Now write a paragraph in which you assess the nutritional value of your breakfast. Do you think it contains too much fat or sugar? Does it have enough protein and fibre?

3. A breakfast cereal label says that one serve contains 100 micrograms of *folate*, which is 50% of the recommended daily requirement. From this information, what is the recommended daily intake of *folate*?

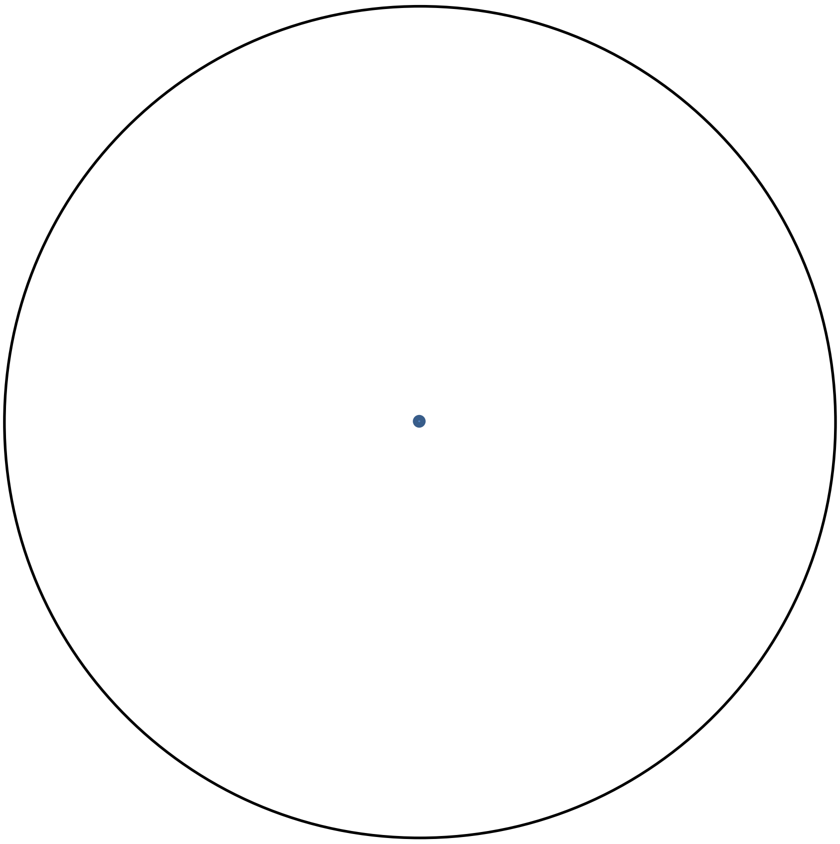
   Recommended daily *folate* intake = _______ micrograms

4. A muesli bar has 18 grams of sugar in a 45 gram bar. What percentage of the bar is sugar? Show your working.

5. The current population of Australia is approximately 23.5 million. Latest figures show that 63% of Australians are overweight or obese. How many people in Australia are overweight or obese?

6. The table below shows the recommended number of daily serves of the main food groups.
   Complete the percentage and angle columns of the table, then draw a pie graph to show the information.

Food group	Serves	Percentage	Angle
Grains/nuts/seeds	7		
Vegetables	5		
Fruits	3		
Meat/poultry/fish	1		
Low fat dairy	2		
Fats/oils	1		
Sweets	1		



Task 2: Shopping

When goods are on sale, the discounts are often expressed as a percentage of the original price, such as “20% off”. Calculate the discounts and sale prices in the following examples. Show your working.

7. A bike has a marked price of $880. In a sale its price is being reduced by 25%. What is the sale price?

8. A watch with a marked price of $250 was bought on sale for $200. What percentage discount was given?

9. A store advertised: ***Prices reduced by 40%*** An item was marked: ***Was $32, now $20***

   Is their claim of 40% off true?

Task 3: Banking

Interest rates at banks and other financial institutions are usually expressed as percentages. Calculate the interest and totals in the following examples. Show your working.

10. A bank has an interest rate of 6% p.a. on a deposit left for a year or more. How much interest would be earned by a deposit of $500 over 2 years?

11. A bank deposit earns interest at the rate of 5% p.a. calculated at the end of each year. The interest earned each year is added to the deposit straight away. That is, the total value of the deposit goes up each year. How much will $1000 amount to after 3 years?

Name:____ Date:________

Can you do this now? Unit 07

1. I need to pay the shop keeper 65 cents for a packet of lollies. One way I could pay would be 50c + 10c + 5c. Write down two other combinations of coins I could use to pay the exact amount. — Obj. 7.1.1 a) ☐ b) ☐

   (a) ____

   (b) ____

2. How many cents in $2.50? ____________________________________ — Obj. 7.1.1 ☐

3. I have three Australian notes in my wallet. — Obj. 7.1.1 a) ☐ b) ☐

   (a) Sketch three notes.

   (b) What is the total amount of money in my wallet?

   ____

4. I bought three things from the shop which cost $5.22 altogether. How much cash did I have to pay? ____ — Obj. 7.1.2 ☐

5. I went to the shop to buy a $2.99 carton of eggs. I paid with a $5.00 note. How much change will I get? (show your working) — Obj. 7.1.3 ☐

6. I buy 2 cans of Sprite for $1.20 each, and a pie for $3.50. I give the shopkeeper $5. — Obj. 7.1.3 a) ☐ b) ☐

   (a) Have I given the shopkeeper enough money? ____________________

   (b) If not, how much more money do I need? (show your working)

7. When buying a bag of jellybeans in the supermarket there are two options, circle the best buy. — Obj. 7.1.4 ☐

Option 1: 350g bag for $2.10 Unit Price: 60c/100g	**Option 2:** 250g bag for $1.80 Unit Price: 72c/100g

8. Write the following percents as decimal fractions:

(a) 22% __________ (c) 235% __________

(b) 75% __________ (d) 12.5% __________

Obj. 7.2.1 a) ☐ b) ☐ c) ☐ d) ☐

9. Write the following decimal fractions as percents:

(a) 0.20 ________ (c) 1.5 __________

(b) 0.79 __________ (d) 0.376 __________

Obj. 7.2.2 a) ☐ b) ☐ c) ☐ d) ☐

10. Write the following percents as common fractions:

(a) 10% ________ (c) 93.6% __________

(b) 37.5% __________ (d) 200% __________

Obj. 7.2.3 a) ☐ b) ☐ c) ☐ d) ☐

11. Write the following common fractions as percents:

(a) $\frac{25}{100}$ __________ (c) $\frac{3}{100}$ __________

(b) $1\frac{7}{10}$ __________ (d) 1 __________

Obj. 7.2.4 a) ☐ b) ☐ c) ☐ d) ☐

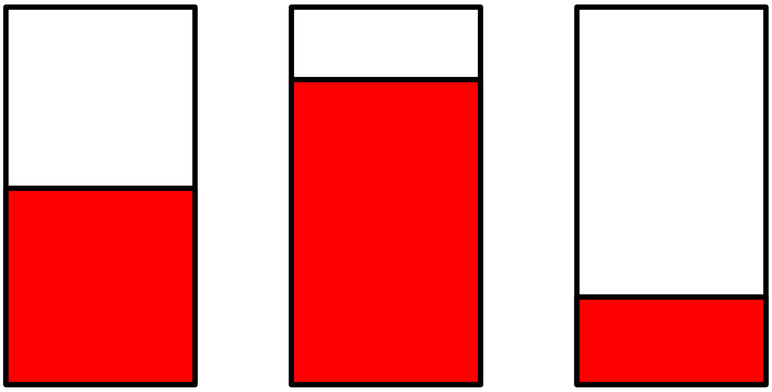
12. Write the following common fractions as percents:

(a) $\frac{15}{50}$ __________ (c) $\frac{2}{3}$ __________

(b) $1\frac{1}{4}$ __________ (d) $\frac{75}{60}$ __________

Obj. 7.2.5 a) ☐ b) ☐ c) ☐ d) ☐

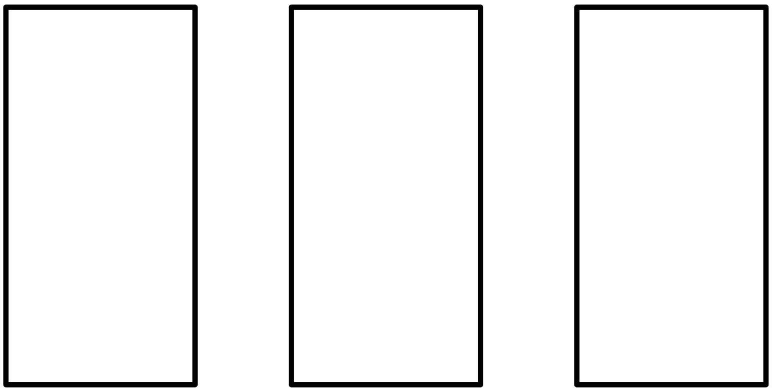
13. Estimate the amount of cordial as a percentage of the glass:



(a) _________% (b) _________% (c) _________%

Obj. 7.2.4
a) ☐
b) ☐
c) ☐

14. Draw the amount of cordial in each glass for the given percent:



(a) 10 % (b) 60% (c) 90%

Obj. 7.2.3
a) ☐
b) ☐
c) ☐

15. Out of a group of 25 students, 7 students walked to school, one fifth rode bicycles and the rest arrived by bus.
    (a) What percentage of students rode bicycles to school? _______________
    (b) What percentage arrived by bus? ________________________________

Obj. 7.3.1
a) ☐
b) ☐

16. The length of the table is 90cm. The width of the table is 60% of the length of the table. Calculate the width of the table in centimetres.

Obj. 7.3.2
i. ☐
ii. ☐

17. Shoes normally cost $60. They are discounted by 15% today. Calculate the amount of the discount.

Obj. 7.3.2
i. ☐
ii. ☐

18. A pair of $59.95 shoes were marked 25% off.

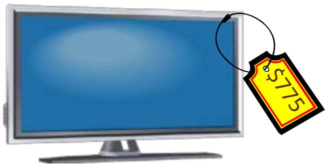
How much were the discounted shoes?

Obj. 7.4.1

i. ☐

ii. ☐

19. A TV is priced at $775. The shopkeeper has to add an additional 10% GST. What is the final price including GST?



Obj. 7.4.1

i. ☐

ii. ☐

20. A 185g chocolate bar is labelled as “New 30% bigger”. What would it have weighed before?

Obj. 7.4.2

i. ☐

ii. ☐

21. A new cycle path reduces the distance to the shops by 20%. The distance is now only 34km. What was the distance before?

Obj. 7.4.2

i. ☐

ii. ☐

22. Jill purchased items for $6.50 each and is now selling them for $5.00 each. What is her loss for each item as a percentage?

Obj. 7.4.3

i. ☐

ii. ☐

23. Fred aims to make 15% profit on sales in his business. If he purchases goods for $42 each, what price must he sell them for?

Obj. 7.4.3

i. ☐

ii. ☐

24. Mr Smith has $8500 in a bank account. He receives 8% simple interest each year. How much money does Mr Smith have altogether at the end of the year?

Obj. 7.4.4

i. ☐

ii. ☐

XLR8 – Accelerating Mathematics Learning

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