XLR8 – Accelerating Mathematics Learning

QUT

YuMiDeadly
Growing community through education

XLR8 Unit 06

Operations with fractions and decimals

06

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will connect the common fraction and decimal fraction representations of tenths, hundredths and thousandths. The cycles describe the mathematical concepts within contexts that are familiar to the students and include concepts of money, measurements and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.

Scope

This unit applies the **number-as-measure** meaning of a **cardinal** number. **Fractions** describe **parts of wholes** or **parts of units**, and may be represented as **common fractions** or **decimal fractions**.

Like whole numbers, **parts of wholes** that are alike can be counted and subject to **additive change**. For common fractions to be alike, they require **like denominators**. Similarly, in order to perform **additive change or comparison operations** with **decimal fractions**, **like place values** must be aligned.

The meaning of **multiplicative fraction and decimal operations** can be difficult to navigate. **Fraction and decimal multiplication** involves finding **how many** or **how much** when the **fractional amount** is **repeated** or finding a **fraction of an amount** (which is essentially **division by the reciprocal**). Conversely, **fraction and decimal division** involves finding the **how many shares** of the **fractional amount fit within** the **original quantity** as **repeated subtraction**.

Continuous attributes of an entity such as **length**, **mass** and **capacity** can be partitioned into **standard metric units** for **counting**. Each of these **units** can be further **partitioned** as a result of **division**. The **parts of the unit** can be **represented** using **fraction notation**. **Standard metric units** of measure use a common set of **prefixes** to nominate a **multiplier** for the **base unit**. This multiplier indicates how many of the **standard unit** are combined/partitioned into an alternative related unit. Using **place value understanding**, **decimal fractions**, and **extended tens facts** it is possible to **convert** between **standard metric units** of **measure** using understandings of fraction operations.

The organisation of these and other related concepts is shown in

Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P06: Calculating Costs* accompanying Unit 06, engages students with exploring fractions of discrete items and of collections generated when preparing for a BBQ.

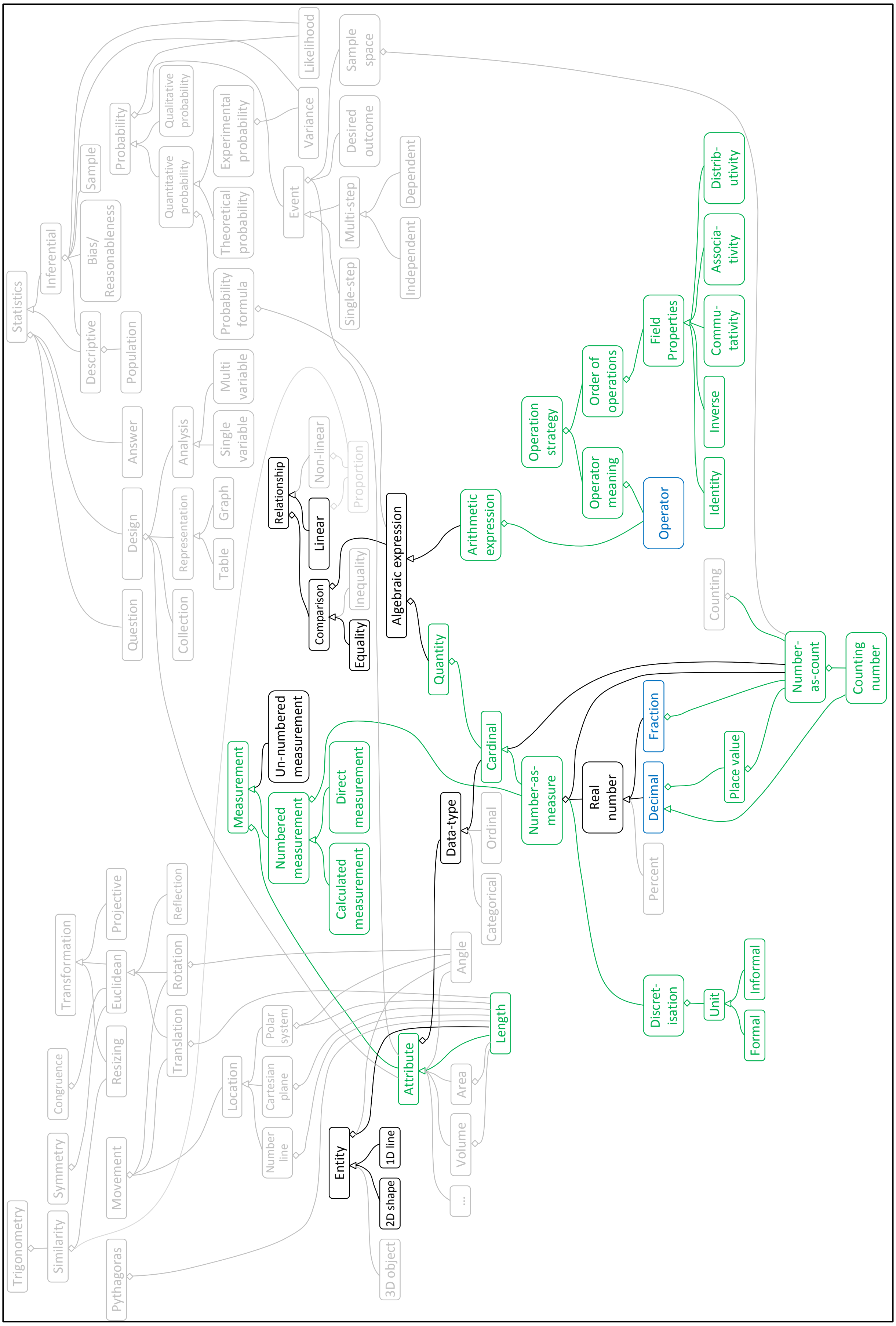


Figure 1 Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following suggested sequence:

Cycle 1: Additive Operations with Fractions

This cycle extends addition and subtraction concepts and processes explored with whole numbers to addition and subtraction of fractions. Additive operations with common fractions and mixed numbers rely on the use of equivalence to generate like denominators. This allows operations to be performed on the numerators as if they are whole numbers.

Cycle 2: Multiplication of Fractions

This cycle develops strategies for multiplication of fractions by connecting multiplication concepts and processes with fraction concepts. The most accessible meanings for multiplication of fractions are repeated addition and change. This section uses area and number line models to represent problems and assist thinking.

Cycle 3: Division of Fractions

This cycle connects division concepts and processes explored with whole numbers to fraction concepts. The same meanings and models for division of whole numbers are applicable to division of fractions. The most accessible meaning for division operations is repeated subtraction using number line and area models. When sharing or grouping fractional amounts, it is exceedingly difficult to conceptualise the processes involved. It is useful at this point to explore and connect to the abstract mathematical meaning of division which is multiplication by inverse or reciprocal.

Cycle 4: Representation of Fractions as Decimals

This cycle connects representations of common fractions to their decimal fraction counterparts starting with tenths, hundredths and thousandths. This is then extended to other common fractions and their conversion to decimal fractions. This exploration will include terminating and recurring decimals and irrational numbers (specifically pi).

Cycle 5: Additive Operations with Decimals

This cycle extends addition and subtraction concepts and processes explored with whole numbers and fractions to addition and subtraction of decimals. Decimal fraction operations can be explored as an extension of common fractions with denominators that are powers of ten. They can also be viewed as an extension and reinforcement of place value additive operation techniques using previously explored whole number strategies.

Cycle 6: Multiplicative Operations with Decimals

This cycle develops strategies for multiplicative operations with decimals by connecting multiplication concepts and processes with fraction concepts. The most accessible meanings of multiplicative operations for decimals are repeated addition/subtraction and change. Area and number line models represent problems and assist thinking. Decimal fraction operations can be explored both as an extension of common fraction operations with denominators that are multiples of ten, and as an extension and reinforcement of place value multiplicative operation techniques and strategies.

Cycle 7: Metric Measure Conversions and Scientific Notation

Units of metric measure, like the decimal number system, rely on place value groupings where places increase or decrease in multiples of ten. Knowledge of the base unit for a measured attribute, prefixes for standard units and the multiples that apply to standard units makes conversion between units of measure relatively simple if connections can be made to place value understandings.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students. This ensures they understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Additive Operations with Fractions

Same or like denominator, different or unlike denominator, common denominator, lowest common denominator, equivalent fractions, tenths, hundredths, thousandths, common fraction

Cycle 2: Multiplication of Fractions

Multiply, repeated addition/subtraction of fractions, fraction of a fraction, change, lowest common denominator

Cycle 3: Division of Fractions

Multiplicative inverse, how many fractions in, how many shares of, reciprocal

Cycle 4: Representation of Fractions as Decimals

Tenths, hundredths, thousandths, decimal fractions, divide, denominator, equivalent fractions, repeating decimals, recurring decimals, terminating decimals, irrational numbers, pi, π, rounding, significant figures, significant places, circumference, diameter

Cycle 5: Additive Operations with Decimals

Equivalent fractions, place value, tenths, hundredths, thousandths, decimal fraction, decimal point

Cycle 6: Multiplicative Operations with Decimals

Multiply, fraction of a fraction, change, how many fractions in, how many shares of, decimal fraction, tenths, hundredths, thousandths

Cycle 7: Metric Measure Conversions and Scientific Notation

Standard units, metre, centimetre, millimetre, kilometre, bigger and smaller units, prefixes for standard length measure units

Name:____ Date:________

Can you do this? #1

1. Continue the sequences and identify the changes

Example: $\frac{1}{9}, \frac{3}{9}, \frac{5}{9}, \frac{7}{9}, \frac{9}{9}$ **Change: +** $\frac{2}{9}$

(a) $\frac{1}{15}, \frac{5}{15}, \frac{9}{15}$, ________ , ________ , ________ Change: _________

(b) $\frac{30}{12}, \frac{25}{12}, \frac{20}{12}$, ________ , ________ , ________ Change: ________

(c) $1\frac{1}{10}, 1\frac{4}{10}, 1\frac{7}{10}$, ________ , ________ , ________ Change: ________

2. Solve the following equations:

(a) $\frac{3}{5} + \frac{4}{5} =$

(b) $\frac{7}{8} + \frac{3}{6} =$

(c) $\frac{24}{30} - \frac{2}{10} =$

3. Write the equation and solve the problem for the following stories:

(a) Julie drove $31\frac{1}{2}$ km from home to a shopping centre, then drove $12\frac{3}{4}$ km to a friend's place, then drove $26\frac{7}{8}$ km home. How many kilometres did she drive?

____

____

____

(b) Jim had $\frac{3}{4}$ of a chocolate bar. He gave $\frac{1}{3}$ of the chocolate bar to his friend. How much of the chocolate bar did Jim keep?

____

____

____

Obj. 6.1.1 a) □ □ □□□

Obj. 6.1.2 b) □□ □□□

Obj. 6.1.3 c) □□ □□□

Obj. 6.1.4 a) □

Obj. 6.1.6 b) □

Obj. 6.1.5 c) □

Obj. 6.1.4 a) i.□ ii.□ iii.□ iv.□ v.□

Obj. 6.1.7 b) i.□ ii.□ iii.□ iv.□ v.□

Cycle 1: Additive Operations with Fractions

Overview

Big Idea

This cycle extends addition and subtraction concepts and processes explored with whole numbers to addition and subtraction of fractions. Additive operations with common fractions and mixed numbers rely on the use of equivalence to generate like denominators. This enables operations on the numerators as if they are whole numbers.

Objectives

By the end of this cycle, students should be able to:

6.1.1 Describe, continue and create patterns with fractions resulting from addition and subtraction. [5NA107]

6.1.2 Describe, continue and create patterns with improper fractions resulting from addition and subtraction. [5NA107]

6.1.3 Describe, continue and create patterns with mixed numbers resulting from addition and subtraction. [5NA107]

6.1.4 Solve problems involving addition of fractions with the same or related denominators. [6NA126]

6.1.5 Solve problems involving subtraction of fractions with the same or related denominators. [6NA126]

6.1.6 Solve problems involving addition of fractions with unrelated denominators. [7NA153]

6.1.7 Solve problems involving subtraction of fractions with unrelated denominators. [7NA153]

Conceptual Links

Counting and additive operation strategies developed with whole numbers may be applied to additive operations with fractions that have the same or related denominators. Operating on fractions with unrelated denominators consolidates multiplicative skills of finding factors, common factors and multiples.

Additive operations with common fractions are used in contexts where fractions are not usually expressed as decimals (e.g., portion sizes, commonly encountered fractions such as quarters, halves, thirds and fifths). Similarly, likelihood is commonly expressed as a common fraction. Combined probability calculations may use additive operations with common fractions. For example, the likelihood of rolling any given number on a fair 6-sided die is $\frac{1}{6}$. The likelihood of rolling a 1, 2, or 3, is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. While it is possible to use a calculator to convert fractions to decimals and then add/subtract the decimals, it is still useful for students to develop the skills needed to work with fractions for algebraic work with formulae and for instances where the common fraction becomes a non-terminating or repeating decimal (e.g., items partitioned into sixths, ninths, sevenths).

Materials

For Cycle 1 you may need:

- fraction sticks
- part – Whole chart (similar to PVC)
- connect 5/cover the board games
- fraction base boards
- overlays
- thinkboard

Key Language

Same or like denominator, different or unlike denominator, common denominator, lowest common denominator, equivalent fractions, tenths, hundredths, thousandths, common fraction

Definitions

Like denominator: same denominator

Related denominators: denominators that are obviously related multiplicatively

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What fractions are you adding/subtracting?
- Are the denominators the same?
- Are the denominators related?
- What will be a useful common denominator to use?
- How can you find the equivalent fractions with common denominators?

Portfolio Task

Additive operations with fractions could be used to start *Portfolio Task 06: Calculating Costs*, although the task will be better conducted using multiplicative operations.

RAMR Cycle

The focus of this cycle is additive operations with common fractions, improper fractions and mixed numbers. To add and subtract common fractions, it is necessary for them to have common denominators. Area and number line models are used to model additive fraction operations. Students should continue interpreting story contexts that use fractional values and broaden their vocabulary for creating their own stories to suit equations. For this cycle, the focus of the abstraction will be additive operations with common fractions. Additive operations with improper fractions and mixed numbers will be included in the mathematics phase.

Reality

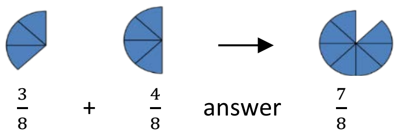
Real contexts involving joining or separating are ideal for operations with common fractions. Reinforce with students that fractions are the same size pieces of a whole. Contexts that use common fractions include joining and separating pieces of pizzas, pies or cakes, or any other items that are naturally shared as pieces, and are ideal for additive operations.

Abstraction

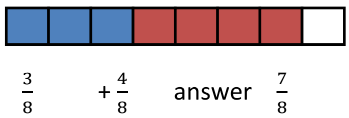
The abstraction sequence for this cycle begins with students' experience of finding equivalent fractions, then leads to joining or separating available fraction pieces to find a new total amount. A suggested sequence of activities is as follows:

Like denominators

1. *Kinaesthetic activity.* Act out sharing pie into eighths. Leave three pieces on one plate and four pieces on another plate. Ask students how many pieces if they are put on one plate. Identify the fractions of pie that each plate represents. Name the fraction of the whole pie that is on the plate (seven eighths).

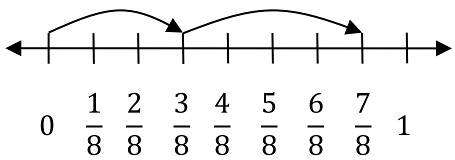


2. *Represent/model with materials.* Model with unifix cubes or circle pieces and connect to language and symbols. For example, three eighths add four eighths = seven eighths.

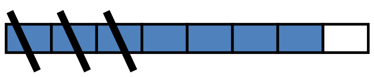


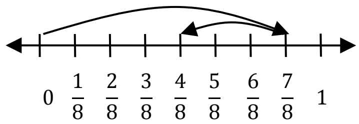
3. *Reinforce connection* to addition with whole numbers that need to be the same unit.

4. *Represent/model* on a number line to reinforce the like denominators and the jumps for addition of common fractions. Reinforce part-part-total as for whole number addition.



5. *Repeat* steps 1 – 4 to explore the inverse operation of subtraction with fractions with like denominators. Model subtraction with the area and number line models as shown.





Unlike denominators

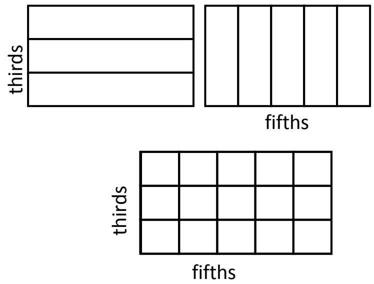
1. *Kinaesthetic activity.* Act out sharing pie, cutting one pie into fifths and the other into thirds (like the *Chocolate Cake Game*). Put $\frac{2}{3}$ on a plate and add $\frac{3}{5}$. Ask students how much pie they have. Reinforce that just adding pieces will not work as the units are different.

2. *Represent/model with materials.* Use virtual fraction strips to model adding $\frac{2}{3}$ to $\frac{3}{5}$. Connect to language and symbols and write the equation for the addition. Compare $\frac{2}{3} + \frac{3}{5}$ with a whole fraction strip to identify that this is equivalent to one whole with some more.

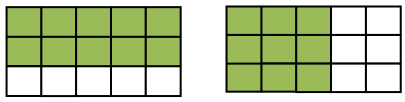
Resource Resource 6.1.1 Virtual fraction strips
Resource 6.1.2 Fraction wall

3. Discuss how the fractions were added together because the units were the same. Ask students for strategies to add these fractions. Suggest using equivalent fractions with like denominators that can be added together. It is simplest to continue with the area model.

4. Start with two congruent rectangles and divide one into thirds and the other into fifths and overlay them (overhead transparencies or virtual manipulatives are ideal for this representation) to create one rectangle with one dimension divided into thirds and the other into fifths. Using one rectangle to show both fractions assists with the maintenance of the whole and in seeing the equivalent fractions. One dimension of the rectangle is used to show each fraction.



   Using the combined diagram, the fractions in the computation can be represented. Students can use counting or multiplication facts to identify that two-thirds is the same as $\frac{10}{15}$ and that three-fifths is the same as $\frac{9}{15}$. Now that fractions have the same unit, they can be added.

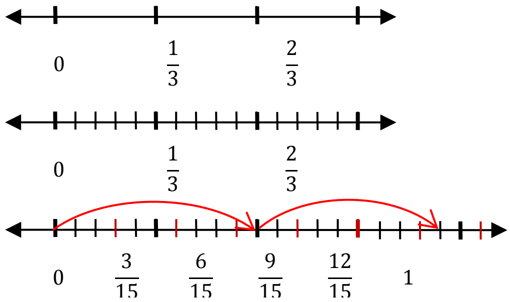

two-thirds
ten-fifteenths

three-fifths
nine-fifteenths

Resource Resource 6.1.3 Fraction baseboards and overlays
Resource 6.1.4 Virtual fraction baseboards and overlays

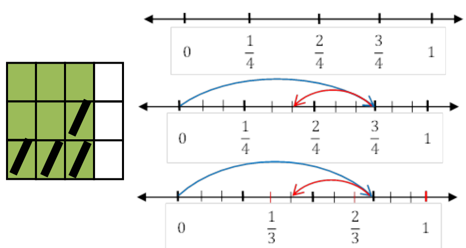
5. Reinforce part-part-total from operations with whole numbers. Ensure students can identify which are the parts in the story and which is the total – in this case, $\frac{2}{3}$ and $\frac{3}{5}$ are the parts and we are looking for the total of $\frac{19}{15}$ or $1\frac{4}{15}$.

6. *Model* with the number line using fraction strip pieces to partition the line to further cement the process and connect to the symbols as shown.



7. *Connect models to symbols.* Encourage students to look for the patterns in the symbols. Generalise the relationship between the original denominators and the equivalent common denominator. Use this student-generated rule to reduce reliance on materials or drawn representation.

8. *Repeat* steps 1-8 to explore subtraction of fractions with unlike denominators by separating pie onto plates. Wording of these problems is tricky. For example, *I have $\frac{3}{4}$ of a pie and give you $\frac{1}{3}$ of a whole pie, how much pie will I have left?* Model with area and numberline as shown. Continue to reinforce story contexts, part-part-total and the triad.



Mathematics

Language/symbols and practice

Practice additive operations with common fractions

Engage students with constructing their own problem situations. Ask them to use addition/subtraction of common fractions with like denominators and addition/subtraction of common fractions with unlike denominators.

Resource Resource 6.1.5 Additive problems with common fractions

Additive operations with improper fractions

Additive operations with improper fractions can be connected to the processes used for common fractions. The level of complexity increases if answers are written as mixed numbers. Reinforce conversion of improper fractions to mixed numbers (from Unit 05).

Discuss adding or subtracting improper fractions based on real story contexts such as:

> *Five pizzas were cut into six pieces and put out on platters. If a person took 4 pieces of pizza, what fraction of pizza was left on platters?* ($\frac{30}{6} - \frac{4}{6}$).

Engage students with constructing their own problem situations. Ask them to give examples of both addition and subtraction of improper fractions with like denominators, and addition and subtraction of improper fractions with unlike denominators.

Resource Resource 6.1.6 Additive problems with improper fractions

Additive operations with mixed numbers

Ask students to suggest strategies they can use to add mixed numbers before teaching strategies. There are two possible strategies:

- use part-whole thinking, add the wholes and the parts, then simplify any fractions that result; or
- convert mixed numbers to improper fractions and then add and follow by simplifying fractions back to mixed numbers.

Allow students to model these strategies with materials and use whichever strategy gives them the most consistent results.

Ensure students experiment and practise with both problem types: mixed numbers with like denominators and mixed numbers with unlike denominators.

Resource Resource 6.1.7 Additive problems with mixed numbers
Resource 6.1.8 Whole-part charts

Connections

Properties of numbers and operations

Explore properties of additive operations encountered with whole numbers such as: commutativity, associativity, inverse operations, part-part-total, and triadic relationships between parts of a story problem. Ensure that students understand these properties apply to common fractions as well as whole numbers.

Problem construction and interpretation

Practice writing equations with fractions from stories and writing stories from equations for additive operations. Focus on the parts and totals within the equations and ensure the stories have suitable unknown parts or totals for the operation the number sentence suggests.

Try to move students to understanding that a pronumeral can be any symbol or letter as it represents the unknown quantity of a unit not the unit itself. For example, 's' does not need to stand for spent, socks, shoes, sandals, seagulls … it can be all these things but also displacement, distance, dogs, cats, fish, … and so on. Discuss which of these items make sense as fractions and which do not (for example, half of a seagull is a little gruesome but a half metre long plank of wood makes sense).

Reflection

Check the idea

Return to Thinkboards or concept maps for students to generate and connect fraction representations and additive strategies. Challenge students to find fraction combinations that add to 1 whole, 2 wholes, and so on. Find fraction combinations that can be subtracted from 3 to get to a half.

Resource Resource 6.1.9 Connect 5/Cover the board games
Resource 6.1.10 Virtual Connect 5/Cover the board games

Apply the idea

Ask students to consider usage of cereal packets and other bulk items in terms of servings. Consider using these items for a week. Determine what fraction of the cereal students might eat in a week on their own, if a friend stayed for three meals and so on.

Extend the idea

When students are comfortable working with numeric representations of fractions, extend their thinking to unknowns or variables within fractions. Start with one fraction numerator represented as a symbol, progress to two or more fraction numerators represented as a symbol, then include examples where the denominator is a symbol.

For example:

$$\frac{m}{6}+\frac{4}{6}=\frac{m+4}{6} \qquad \frac{m}{6}+\frac{n}{6}=\frac{m+n}{6} \qquad \frac{m}{p}+\frac{n}{p}=\frac{m+n}{p}$$

$$\frac{m+4}{6}-\frac{m}{6}=\frac{4}{6} \qquad \frac{m+n}{6}-\frac{m}{6}=\frac{n}{6} \qquad \frac{m+n}{p}-\frac{m}{p}=\frac{n}{p}$$

Encourage students to generalise addition and subtraction of fractions with unlike denominators to algebra.

This means in general that $\frac{a}{b}+\frac{c}{d}$ can be added by converting each to the common denominator of $b \times d$, and that $\frac{a}{b}=\frac{(a\times d)}{(b\times d)}$ (multiplying top and bottom by d) and that $\frac{c}{d}=\frac{(b\times c)}{(b\times d)}$ (multiplying top and bottom by b), meaning that:

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d+b\times c}{b\times d}$$

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #2

1. What is $\frac{1}{6}$ of 36? ________

Obj. 6.2.1 ☐

2. What is 6 lots of $\frac{3}{8}$? ________

Obj. 6.2.2 i. ☐ ii. ☐

3. Solve the equations:

   a) $\frac{3}{8} \times \frac{7}{3} =$

   Obj. 6.2.3 i. ☐ ii. ☐

   b) $2\frac{3}{8} \times \frac{2}{3} =$

   Obj. 6.2.4 i. ☐ ii. ☐ iii. ☐ iv. ☐

   c) $\frac{3}{8} \times 3\frac{2}{3} =$

   Obj. 6.2.4 i. ☐ ii. ☐ iii. ☐ iv. ☐

4. Write the equation and solve the problem for the following story:

   Five friends each had $\frac{7}{8}$ of a pizza. How much pizza did they have altogether?

   ____

   ____

   ____

   ____

Obj. 6.2.2 i. ☐ ii. ☐ iii. ☐

Cycle 2: Multiplication of Fractions

Overview

Big Idea

This cycle develops strategies for multiplication of fractions by connecting multiplication concepts and processes with fraction concepts. The most accessible meanings for multiplication of fractions are repeated addition and change. Number line and area models can be used to represent problems and assist thinking.

Objectives

By the end of this cycle, students should be able to:

6.2.1 Find a simple fraction of a quantity where the result is a whole number. [6NA127]

6.2.2 Find whole number multiples of same-size fractions. [7NA154]

6.2.3 Multiply a fraction by a fraction. [7NA154]

6.2.4 Multiply a mixed number by a fraction or mixed number. [7NA154]

Conceptual Links

This cycle links to multiplicative operations with whole numbers and builds on ideas explored with equivalent fractions. Multiplicative operations with fractions are necessary to work with percentage problems, engage with financial mathematics, explore multiplicative relationships in measurement, statistics and probability, and algebraic applications of fractions.

It is possible to convert common fractions and mixed numbers to decimals for calculations. This might appear to render pen and paper calculations with fractions unnecessary. However, the processes and the understanding behind these kinds of calculations are useful for common fractions that do not have terminating decimal forms. Pen and paper fraction calculations are also useful for working with algebraic formulae, manipulating problems with unknowns, and connecting to geometry, scale and proportion.

Materials

For Cycle 2 you may need:

- PVC per student
- calculators
- hundredths grids
- fraction baseboards and overlays

Key Language

Multiply, repeated addition of fractions, fraction of a fraction, change, lowest common denominator, highest common factor

Definitions

Highest common factor: when factors are determined for a pair of numbers, common factors may arise. The highest common factor is the largest of these.

Lowest common denominator: the smallest number that can be used as a denominator for 2 or more fractions.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Finding a fraction of a quantity where the result is a whole number is clearly linked to finding a fraction of a collection explored in Unit 05. This connection should be highlighted.
- Draw students' attention to the fact that if they are finding a fraction of a collection or whole number, they are effectively partitioning the collection into a quantity of groups represented by the denominator, and then adding up the total number in the quantity of these groups represented by the numerator.
- Clearly link finding a fraction of a whole number to dividing the whole number by the denominator of the fraction and multiplying by the numerator.
- Are you finding a fraction of a whole number or a fraction of a fraction?
- What strategy will you use to multiply?
- Can you simplify your answer?
- Can you find a simpler equivalent fraction to your answer?
- Is there a common factor between the numerator and the denominator?
- Is your answer an improper fraction?
- How many times can you subtract the denominator from the numerator?
- What is left over?
- Can you write that as a mixed number?

Portfolio Task

Sections of the *Portfolio Task 06: Calculating Costs* that combine multiples of an item in the shopping list or recipe tasks may be completed following this cycle.

RAMR Cycle

The focus of this cycle is the multiplication of common fractions, improper fractions and mixed numbers. Using an array or area model, multiplication of common fractions is relatively straightforward as it extends on equivalent fraction ideas from Unit 05. Mixed numbers may be converted to improper fractions and multiplied like common fractions or can be treated in parts using an array or area model.

Reality

Real contexts for common fractions arise when there are multiple pieces of the same size fraction or when further sharing already partitioned items. For example, there may be three halves of pizzas of different flavours left over; overall this is $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\, or\, \frac{3}{1}\times\frac{1}{2}$. Alternatively, half a pizza may be further shared with 5 people so that each person receives one fifth of half a pizza. Note the language shift from “by” to “of”. Both these forms of language are useful for linking to models of multiplication of fractions.

Multiples of same size fractions can be addressed using examples that multiply a whole number by a fraction and can use repeated addition ideas and strategies already developed. Further partitioning a fraction into smaller pieces requires a little more thought.

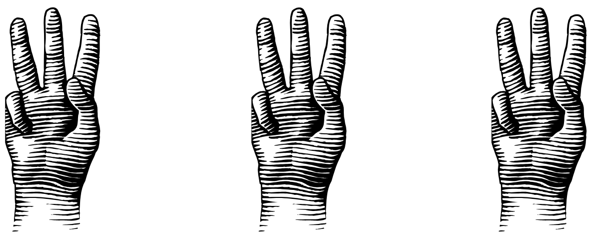
Identifying the areas on a netball court or soccer pitch by position may work if students are sports-minded. For example, if each team “owns” half the court/pitch, players in wing positions tend to cover from the edge across to the middle. If they are expected to cover half of a half, how much of the whole court/pitch is this? Discuss with students the real world effect of partitioning a fraction.

Abstraction

The abstraction sequence for this cycle takes students from physical experiences of multiples of the same size fractions to develop strategies for solving these types of problems using language and symbols. Understandings are then extended to develop processes for calculation of a fraction of a fraction using array and area models through to symbolic representations of fractions. These ideas and processes are extended in the mathematics phase to include strategies for improper fractions and mixed numbers. A suggested sequence of activities is as follows:

Multiples of same-sized fractions

1. *Kinaesthetic activity.* Explore examples of multiple fractions with students. For example, if three students hold up three fingers each, they are each holding up three fifths of a hand of fingers. Collectively, they are holding up nine fingers which would be nine fifths if collected into hand-sized groups.
2. *Connect to language and symbols.* Represent the fingers example with drawings and symbols to show $\frac{3}{1}\times\frac{3}{5}=\frac{9}{5}$.



Partitioning a fraction

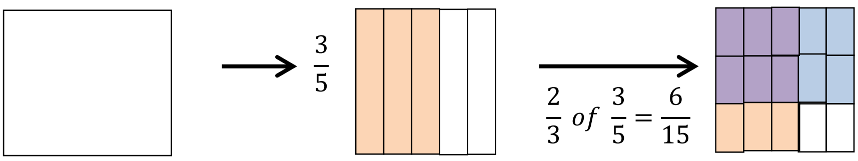
1. *Kinaesthetic activity.* Explore the sections of a netball/basketball/football court/field as fractions of a whole. If physically visiting the field is not practical, a diagram on PowerPoint may be used to highlight the parts of the field and connect these to fractions of the whole. Discuss individual player positions and the area they cover in play as a part of these other fractions. Alternatively, consider fractions of a fraction with pizza slices or cake pieces.

Resource

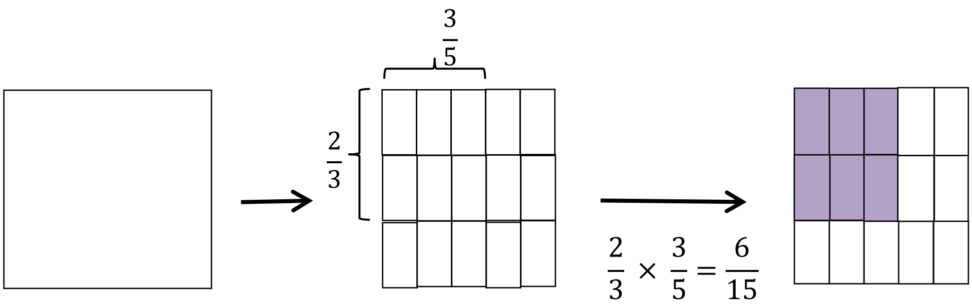
Resource 6.2.1 Sports field fractions
Resource 6.2.2 Sports field fractions PowerPoint

2. *Represent/model with materials.* There are two methods of exploring multiplication of fractions using an array or area model: a fraction of a fraction; or a fraction by a fraction. The models for these are almost identical but use different processes and rely on different thinking.

   Fraction of a fraction: Multiplying fractions involves finding a part of a part. In this understanding, it is helpful to think of multiplication in relation to fractions using 'of' instead of 'by'. For example, to model $\frac{2}{3} \times \frac{3}{5}$, a whole is first partitioned to show $\frac{3}{5}$, then the whole is partitioned the other way to show $\frac{2}{3}$. In this example, the whole is finally partitioned into fifteenths. The section that is double shaded represents $\frac{2}{3} \times \frac{3}{5}$ or six fifteenths. This is similar to the process students used to find equivalent fractions.



   Fraction by a fraction: This method represents the whole to be partitioned as a square unit that is partitioned into thirds one way and fifths the other. The array that we are interested in is the part of the whole represented by a shaded array of $\frac{2}{3} \times \frac{3}{5}$ or area of $\frac{2}{3}$ unit $\times \frac{3}{5}$ unit. The rectangle $\frac{2}{3}$ by $\frac{3}{5}$ can be calculated as a fraction of one square unit. This method breaks the unit into thirds by fifths then counts 2 by 3 to find six fifteenths.



3. *Connect to language and sysmbol patterns.* As a result of either method, it is important to engage students in discussion around the pattern that works with the symbols. Using the example $\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5}$ ask students for the pattern or connection between the digits in the multiplication and the counted result. Test this out with a few examples to establish the rule or generalisation. This pattern can then be used without models.

Mathematics

Within the Mathematics phase, it is important to practice problems that involve finding multiples of same-size fractions using set (considered in Abstraction), number line and array or area models. These are most easily conceptualised as repeated addition. Similarly, problems exploring fraction "of" or "by" a fraction should be represented using set, number line and array or area (explored in Abstraction) models. These ideas should be further extended to include multiplication of mixed numbers and multiplication of decimal fractions.

Language/symbols and practice

Multiples of fractions

Consider examples like collating multiple pieces of pizza or cake into boxes, identifying multiples of subgroups of a collection, finding multiple distances or measurements expressed as fractions, or doubling a recipe which uses fractional measures. These examples all describe multiplication of a fraction by a whole number.

Resource Resource 6.2.3 Multiples of fractions

Fraction of or by a fraction

Practise examples of finding a part of an already partitioned item that might be represented by set, number line and array or area models. For example, find half of a quarter-sized group, travel half a fractional distance, find a part of an already partitioned area, or reduce a recipe.

Resource Resource 6.2.4 Fractions of or by fractions

Multiplying mixed numbers

Multiplication of mixed numbers can be addressed in two ways. The first is to convert the mixed numbers to improper fractions and then work through as for "Fraction of or by a fraction".

Alternatively, students can apply the separation strategy. In this case, students use an area model to work through the multiplication. For example, $1\frac{3}{5} \times 3\frac{1}{4}$ can be represented as a rectangle with a length of one mixed number, a width of the other mixed number, from which the area is calculated. This method allows students to work with smaller operations that they can calculate simply and provides a picture that ensures that all parts of the problem are addressed.

	1 +	1 +	1 +	$\frac{1}{4}$	
1 +	1	1	1		$1 \times \frac{1}{4}$
$\frac{3}{5}$	$\frac{3}{5} \times 1$	$\frac{3}{5} \times 1$	$\frac{3}{5} \times 1$		$\frac{3}{5} \times \frac{1}{4}$

The areas of each square are then added together. $1 + 1 + 1 + \frac{1}{4} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{20} = 3\frac{44}{20} = 5\frac{1}{5}$

Further notes relating to this method of multiplication of mixed numbers can be found in *Resource 6.2.5 Multiplication of mixed numbers.*

Resource Resource 6.2.5 Multiplication of mixed numbers

Reflection

Check the idea

Sketch a netball, basketball, soccer or football field (depending on student interest). Partition into thirds or quarters and determine what fraction of a third or quarter of the field each position covers.

Explore contexts where half a cake or pizza is cut into quarters or thirds. Find what fraction of the whole cake or pizza is represented by each piece.

Apply the idea

Practice increasing and reducing quantities in recipes and mixtures. For example:

Find the quantities for three and a half of the following recipe:

1 Cup of flour — *Answer:* $1 \times 3\frac{1}{2} = 3\frac{1}{2}$

1 Cup of milk — *Answer:* $1 \times 3\frac{1}{2} = 3\frac{1}{2}$

$\frac{1}{2}$ *teaspoon of bicarb soda* — *Answer:* $\frac{1}{2} \times 3\frac{1}{2} = \frac{1}{2} \times \frac{7}{2} = \frac{7}{4} = 1\frac{3}{4}$

$\frac{1}{3}$ *Cup of sugar* — *Answer:* $\frac{1}{3} \times 3\frac{1}{2} = \frac{1}{3} \times \frac{7}{2} = \frac{7}{6} = 1\frac{1}{6}$

1 teaspoon of vanilla — Answer: $1 \times 3\frac{1}{2} = 3\frac{1}{2}$

Alternative contexts exist with mixtures of chemicals for weed spraying, insect repellent for stock, cordials, non-alcoholic cocktails, paint colours. Explore several contexts that use fractions where multiples or parts of these are needed.

Extend the idea

The King's Chessboard story explores what happens when an initial quantity of 1 is continually doubled. This investigation could be extended to start with a large whole and continually halve. For example, in provincial France (a very long time ago) property was shared equally between surviving sons. If an initial piece of land is one whole and is consistently halved across successive generations (assuming each generation only has two sons), what fraction of the original land will each son in the 5th or 6th generation receive? What if each generation has three sons? If the original piece of land is 1000 m by 1000 m, when will partitioning the pieces no longer be useful? Is there a point where a generation will receive no land?

Provide students with a range of fractions. See if they can determine what they need to multiply each fraction by to reach a whole number. Extend this idea further to see if they can determine what they can multiply a fraction by to make one whole.

Consider multiplication of fractions that include variables or unknown amounts. Can these be multiplied the same as fractions with numerals? For example, $\frac{a}{6} \times \frac{3}{5}$; $\frac{a}{6} \times \frac{b}{5}$; $\frac{5}{a} \times \frac{b}{5}$; $\frac{a}{b} \times \frac{c}{d}$.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #3

1. How many quarters in:

   (a) $1\frac{3}{4}$ ___________ quarters

   (b) $5\frac{1}{2}$ ___________ quarters

2. Solve the equations:

   (a) $\frac{1}{2} \div 4 =$

   (b) $\frac{7}{5} \div \frac{1}{2} =$

3. Write the equation and solve the problem:

   (a) $4\frac{1}{2}$ cakes were shared evenly between 9 students. How much cake did each student receive?

   (b) Eight students in the class had mobile phones at school.
   There were 24 students in the class.
   What fraction of the class had mobile phones at school?
   ____

Obj. 6.3.1 a) ☐

Obj. 6.3.1 b) ☐

Obj. 6.3.2 a)i. ☐ ii. ☐

Obj. 6.3.3 b)i. ☐ ii. ☐ iii. ☐

Obj. 6.3.2 i. ☐ ii. ☐ iii. ☐

Obj. 6.3.4 i. ☐ ii. ☐

Cycle 3: Division of Fractions

Overview

Big Idea

This cycle connects division concepts and processes explored with whole numbers to fraction concepts. The same meanings and models for division of whole numbers are applicable to division of fractions. The most accessible meaning for division operations is repeated subtraction using number line and area models. When sharing or grouping with fractional amounts, it is exceedingly difficult to conceptualise the processes involved. It is useful at this point to explore and connect to the abstract mathematical meaning of division which is multiplication by inverse or reciprocal.

Objectives

By the end of this cycle, students should be able to:

6.3.1 Find how many of a fractional quantity in a whole number. [7NA154]

6.3.2 Divide a fraction by a whole number. [7NA154]

6.3.3 Find how many of a fractional quantity in a fraction. [7NA154]

6.3.4 Express one quantity as a fraction of another. [7NA155]

Conceptual Links

This cycle links to multiplicative operations with whole numbers, repeated addition of fractions, and equivalent fraction concepts. This cycle also consolidates understanding of multiplicative relationships in measurement, statistics and probability, and algebraic applications of fractions and decimals.

Concepts explored within this cycle provide initial skills in multiplicative operations with fractions and decimals. These skills are required in future units to work with percentage problems, and engage with financial mathematics.

Materials

For Cycle 3 you may need:

- PVC per student
- shopping catalogues (or access to online)
- calculators
- hundredths grids
- fraction baseboards and overlays

Key Language

Multiplicative inverse, reciprocal, how many fractions in, how many shares of, decimal fraction, tenths, hundredths, thousandths,

Definitions

Reciprocal: The reciprocal of a number is 1 divided by the number, also known as multiplicative inverse. Multiplying a number by its reciprocal equals one. For example, $3 \times \frac{1}{3} = 1$.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the total amount that you have?
- What size pieces or how many pieces are you sharing the total amount by? What number is the total amount being divided by?
- What is dividing a quantity the same as? (Finding a fraction of or multiplying by the fraction or reciprocal).

Portfolio Task

Sections of the *Portfolio Task 06: Calculating Costs* that partition an item in the shopping list or recipe tasks may be completed following this cycle.

RAMR Cycle

Reality

The simplest attribute to use when exploring division of fractions is length. For example, where a length has already been partitioned, it is possible to explore how many pieces there are of a certain fraction size. So, if sharing a Killer Python jelly snake, how many quarter-sized pieces are there in a whole snake? How many quarter-sized pieces in half a snake? If half a snake is shared with four people, what fraction of a whole snake would they have? Other real contexts for sharing using fractions might include sharing lengths of ribbon, chocolate logs, kabana, buckets or containers of water, distances between places, mass, capacity and volume. Other possible stories could include:

> ***Area/sharing:*** *I have $\frac{3}{4}$ cake to share amongst 6 students, how much of a whole cake will each student receive? ($\frac{3}{4} \div 6$) (Any food item could be used in this scenario.)*
>
> ***Length:*** *How many pieces of ribbon $\frac{2}{5}$ of a metre long can I cut from 2 m of ribbon? ($2 \div \frac{2}{5}$)*
>
> ***Volume:*** *My bucket holds $\frac{1}{4}$ L. How many times will I need to fill my bucket to fill a tank that holds $35\frac{1}{2}$ litres of water? ($35\frac{1}{2} \div \frac{1}{4}$)*

Using contextual problems helps students to experience the connection to using the inverse when dividing fractions. It makes sense to realise that when sharing quantities with 6 people, we are actually finding $\frac{1}{6}$ of the quantity; and that a greater quantity of smaller buckets are needed to fill a container than if larger buckets are used.

Abstraction

The abstraction sequence for this cycle starts from student understanding of multiplication "of" or "by" fractions and works towards understanding division of or by fractions. Development of the process to work with division of fractions requires students to understand multiplicative inverse or multiplication by inverse as it applies to fractions. Area, length or volume contexts and models may be used to explore division of or by fractions according to students' interests. A suggested sequence of activities using length or area models is as follows:

Whole number divided by a fraction

1. *Kinaesthetic activity.* Choose the most practical length or area context to use from reality that can be adapted to students' interests. For example, explore how many $\frac{1}{2}$ m intervals there are on the running track. Students could step this out and count to determine 100m $\div \frac{1}{2}$ m. They should find that they have $200 \times \frac{1}{2}$ m intervals on the running track (28 m $\div \frac{1}{2}$ m connects to $56 \times \frac{1}{2}$ m intervals on the netball court or soccer field). Try to lead students to see that there are double the original number when they find how many halves (or dividing by a half). Explore this with other distances to consolidate. Alternatively, share an area model like cake or pizza. Ask students how many halves there will be in one cake, extend this to how many halves in two cakes and so on until students recognise that they are doubling the number of cakes to find how many halves.

2. *Represent/model with materials*. Encourage students to draw the story previously acted out. Ensure they can visualise the running track, basketball court, or cake/pizza, the size of the pieces it is being partitioned into and the resulting multiplier for the original value. For example, the basketball court is 28 m partitioned into half metre sections, so the number of sections is double the length of the court; there are 6 cakes cut into halves so there will be double 6 pieces.

3. *Connect to symbols*. Encourage students to label their drawings with the length, size of the partitions and the equation that represents the story. For example, $28 \div \frac{1}{2} = 56$. Also have them write the corresponding relationship they have noticed: $28 \times 2 = 56$
4. Test this idea with other unit fractions (for example, how many thirds, quarters or fifths in a whole number). See if students can identify a general rule that dividing by a unit fraction is the same as multiplying by the denominator as a whole number.
5. *Connect to language.* Discuss reciprocal as the mathematical word for the inverse of a half ($\frac{1}{2}$ is the reciprocal of $\frac{2}{1}$ or 2). See what happens when a number and its reciprocal are multiplied together, for example, $2 \times \frac{1}{2}$, $3 \times \frac{1}{3}$, $4 \times \frac{1}{4}$, and so on (always equals one).

Resource Resource 6.3.1 How many fractions in a whole number?

Fraction divided by a whole number

1. *Kinaesthetic activity.* Choose the most practical length or area context to use from reality and students' interests. For example, explore sharing $\frac{3}{4}$ cake amongst 6 students. Discuss how many pieces the amount of cake needs to be split into for 6 people and what an equivalent fraction for $\frac{3}{4}$ cake might be ($\frac{6}{8}$ *or* $\frac{18}{24}$ are possibilities). Share the resulting fractions of cake so that each person receives $\frac{1}{6}$ of the $\frac{3}{4}$ cake each. Connect the idea that sharing, partitioning or dividing a cake into six pieces means they have one sixth of the available cake each. Discuss what fraction of a whole cake each person has. Alternatively, explore how far up a netball court or soccer field a player might travel as three quarters of the court. If this were broken into six sections, what fraction of the court would each section be? A length of wool that started as three quarters of the court long cut into six equal pieces may be a suitable representation.
2. *Represent/model with materials*. Have students draw the story previously acted out. For example, draw a rectangular cake and name as $\frac{3}{4}$ of a cake. Dot in where a whole cake might be. Partition the $\frac{3}{4}$ of a cake into six pieces. Identify each partition as $\frac{1}{6}$ of $\frac{3}{4}$. Discuss with students how many pieces would be in a whole cake if it was partitioned into these size pieces. Discuss what fraction of a whole cake each of the 6 people are receiving.
3. *Connect to symbols*. Label the drawing as $\frac{3}{4}$ shared with 6. Write the equation that represents the story. For example, $\frac{3}{4} \div 6$. Also write the corresponding relationship they noticed as $\frac{3}{4} \times \frac{1}{6}$. Connect this idea to the idea of multiplication by reciprocal previously explored.
4. Test this idea with other whole number divisors (for example, what happens when a fraction is shared further with 2, 3, 4 and so on). See if students can identify a general rule that dividing a fraction by a whole number is the same as multiplying by a unit fraction with that denominator.

Resource Resource 6.3.2 Fraction shared by a whole number

Fraction divided by a fraction

1. *Kinaesthetic activity.* Choose the most practical length or area context to use from reality or students' interests. For example, explore sharing $\frac{3}{4}$ cake into eighth-sized pieces. Discuss how many pieces the original cake was split into to have $\frac{3}{4}$ cake and what eighth-sized pieces look like. Identify how many eighth-sized pieces in $\frac{3}{4}$ cake. Connect to equivalent fractions. Discuss the fact that there are more of a smaller size fraction in a bigger fraction. Alternatively, explore how far up a netball court or soccer field a player might travel as three quarters of the court. If this were

broken further into eighths for several positions to cover, what fraction of the side of the court would each section be? A length of wool that started as three quarters of the basketball court long, partitioned into three for quarters, then each of these halved again for eighths might be a suitable representation. Discuss with students how many pieces long the whole court or field would be and how many there are in three quarters when they are represented as eighths.

2. *Represent/model with materials*. Have students draw the story previously acted out. For example, draw a rectangular cake partitioned to show three quarters. Partition again to make the cake into eighths and count how many eighths are in the cake they have. Identify and name the cake as $\frac{3}{4}$ of a cake, and how many eighths in $\frac{3}{4}$ of a cake.
3. *Connect to symbols*. Label the drawing as $\frac{3}{4}$ shared by $\frac{1}{8}$. Write the equation that represents the story. For example, $\frac{3}{4} \div \frac{1}{8}$. Connect to previous examples that involved division of fractions and the idea of reciprocals. Also write the corresponding relationship as $\frac{3}{4} \times 8$. Connect to the idea of multiplication by reciprocal explored when dividing a whole number by a fraction.
4. Test this idea with other fractional divisors (for example, what happens when a fraction is shared further with $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and so on). See if students can identify a general rule that dividing a fraction by a unit fraction is the same as multiplying by that fraction's reciprocal. Explore with students what happens when finding what part of a larger fraction there is in a smaller fraction. They should find they end up with a smaller number or a fraction as an answer. For example, there is half of a half in a quarter; there are two quarters in one half.

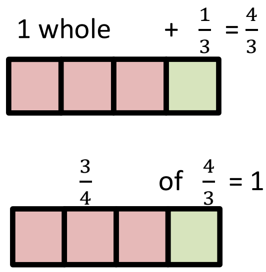
Resource Resource 6.3.3 How many fractions in a fraction?

Mathematics

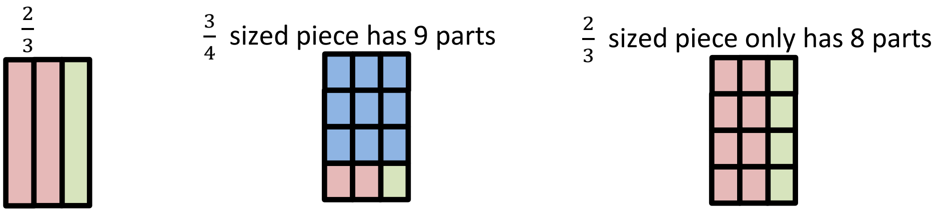
Language/symbols and practice

Dividing fractions other than unit fractions

Once students have mastered the basics of multiplication by reciprocal, practice examples like $\frac{2}{3} \div \frac{3}{4}$. Start from student understanding of the reciprocal of whole numbers and unit fractions, for example $2 \times \frac{1}{2} = 1$. Also use area models to explore $\frac{3}{5} \times \frac{5}{3}, \frac{3}{4} \times \frac{4}{3}$, and so on to see what the result is. They should find that each of these examples multiply to 1.



Use area models to explore how many $\frac{3}{4}$ sized pieces are in $\frac{2}{3}$ to test if the rule generated for unit fractions extends to non-unit fractions. This means $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.



So there are $\frac{8}{9}$ of a $\frac{3}{4}$ sized piece in $\frac{2}{3}$

Resource Resource 6.3.4 Dividing fractions other than unit fractions

Volume model

Explore sharing a volume model filling jugs or cups from a bucket. Act out the example from reality to see how many of various sized containers can be filled from a standard container (e.g., use a 1 L jug to fill 4 cup measures, 8 smaller cups, 2 larger cups and so on). This is a smaller version of the problem, *My bucket holds* $\frac{1}{4}$ *L. How many times will I need to fill my bucket to fill a tank that holds* $35\frac{1}{2}$ *litres of water?* This is $35\frac{1}{2} \div \frac{1}{4}$.

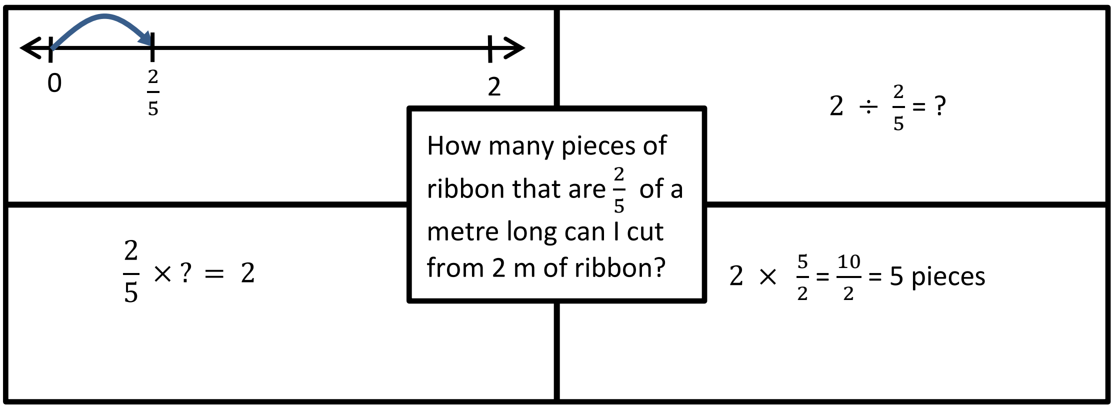
Encourage students to represent with drawings if necessary. If students are struggling with this idea, discuss the reality they have experienced. They should intuitively recognise that they will need to fill more of the smaller buckets than larger buckets. For example, $8 \times \frac{1}{8}$ L buckets to make 1 L, and the number of buckets needed to fill $35\frac{1}{2}$ L can be found by completing the computation $35\frac{1}{2} \times 8$ (which is multiplication by reciprocal). Using a contextual problem helps reinforce the connection to multiplying by inverse when dividing fractions.

Resource Resource 6.3.5 Exploring fractions using volume models

Reflection

Check the idea

There are many connections between common fraction operations including area or number line representations, inverse operations and reciprocal (or inverse) fractions. Construction of Thinkboards that depict equivalent fraction operations connecting multiplication and division to story problems are beneficial. For example:



Extend the idea

Change Parameters

Examples given so far have focused on division by common proper fractions. See if students can identify strategies that will allow them to divide by mixed numbers for consistent results. **Note:** Students may work with part-whole thinking or by converting the mixed number to an improper fraction and then working as for common fractions.

Generalise

See if students can generalise from division of fractions to manipulating and simplifying algebraic formula that include division of fractions. For example, $\frac{a}{b} \div \frac{c}{b} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________ Date:________

Can you do this? #4

1. Write the number words as decimal numerals for each sentence:

   (a) To make Asian noodle salad, half a wombok is needed. ________

   (b) The probability of rolling a 6 on a die is one tenth and sixty-seven thousandths. ______________________________

2. Write the numerals in number words for each sentence:

   (a) The table was 4.25 m long. ______________________________

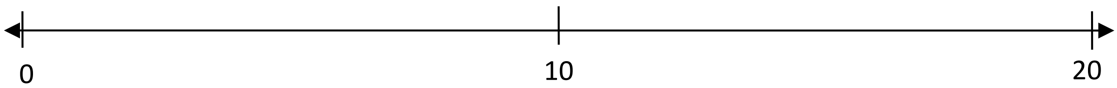
   ____

   (b) The distance to the shops is 12.35 km. ______________________

   ____

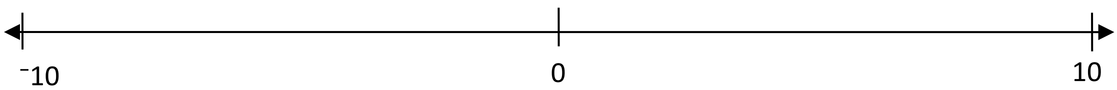
3. Rank the following numbers on a number line:

   17.6, 8.1, 4.7, 12.5, 18.9, 0.3, 14.6, 7.3



4. Rank the following numbers on a number line:

   $^{-}5.6$, 3.1, $^{-}2.7$, 2.5, 7.9, $^{-}0.3$, 0.6, $^{-}4.3$



5. Are the following numbers rational or irrational?

   (a) $1.\overline{36}$ □ rational □ irrational

   (b) 0.75 □ rational □ irrational

   (c) $1.\overline{024831}$ □ rational □ irrational

   (d) π □ rational □ irrational

Obj. 6.4.4
a) □

Obj. 6.4.1
b) □

Obj. 6.4.1
a)i. □
ii. □
iii. □

Obj. 6.4.1
b)i. □
ii. □
iii. □

Obj. 6.4.2
□□
□□
□□
□□

Obj. 6.4.2
□□
□□
□□
□□

Obj. 6.4.6
a) □
b) □
c) □
d) □

6. Are the following decimals terminating or recurring?

(a) $1.\overline{36}$ □ terminating □ recurring

(b) 0.75 □ terminating □ recurring

(c) $1.\overline{024831}$ □ terminating □ recurring

(d) 0.543 278 □ terminating □ recurring

Obj. 6.4.5 a) □ b) □ c) □ d) □

7. Round these numbers to three decimal places:

(a) 3.14159 ________________ (b) 0.16161616 __________________

Obj. 6.4.3 a) □ b) □

8. Write these common fractions as decimal fractions:

(a) $\frac{1}{4}$ __________________ (b) $\frac{2}{3}$ __________________

(c) $\frac{12}{8}$ __________________ (d) $\frac{6}{12}$ __________________

Obj. 6.4.4 a) □ b) □ c) □ d) □

9. Write these decimal fractions as common fractions:

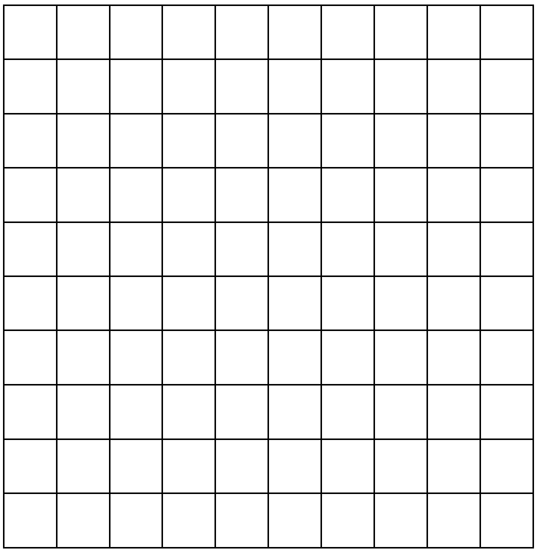
(a) 0.35 __________________ (b) 1.43 __________________

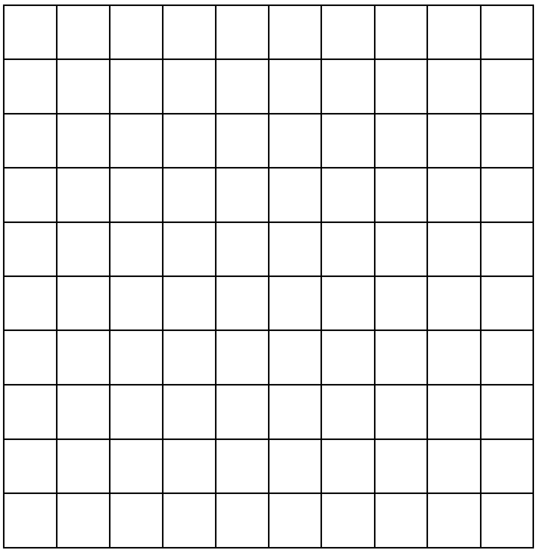
(c) 2.06 __________________ (d) 0.001 __________________

Obj. 6.4.1 a) □ b) □ c) □ d) □

10. Use 10×10 grids to show:

(a) 25 hundredths (b) 5 tenths and 6 hundredths





Obj. 6.4.1 a) □ b) □

Cycle 4: Representation of Fractions as Decimals

Overview

Big Idea

This cycle connects common fraction representations to their decimal fraction counterparts starting with tenths, hundredths and thousandths, before extending to other common fractions and their conversion to decimal fractions. This exploration will include terminating and recurring decimals and irrational numbers (specifically π).

Objectives

By the end of this cycle, students should be able to:

6.4.1 Represent common fractions of tenths, hundredths and beyond using decimal notation. [5NA104]

6.4.2 Compare, order and represent decimals on a number line. [5NA105]

6.4.3 Round decimals to a specified number of decimal places. [7NA156]

6.4.4 Represent common fractions other than tenths, hundredths and so on as decimals. [8NA184]

6.4.5 Identify terminating and recurring decimals. [8NA184]

6.4.6 Identify irrational numbers including π. [8NA186]

Conceptual Links

Decimal fraction representations of quantities connect and extend place value understanding developed in Unit 01.

Representation of fractional quantities as decimals is commonly used when dealing with money, metric measures and later for conversion to and from percentages.

Materials

For Cycle 4 you may need:

- chocolate cake game recording and working sheet
- equivalent fraction (cover the board game)
- equivalent fraction Connect 5
- fraction cards
- fraction mats
- equivalent representation (cover the board game)
- equivalent representation Connect 5
- fractions cards (happy families / go fish / snap / concentration / memory)
- fraction strips
- calculators

Key Language

Tenths, hundredths, thousandths, decimal fractions, divide, denominator, equivalent fractions, repeating decimals, recurring decimals, terminating decimals, irrational numbers, pi, π, rounding, significant figures, significant places, circumference, diameter

Definitions

Irrational number: a real number that cannot be written as a common fraction. The resulting decimal goes on forever without repeating

Rational numbers: a number that can be written as a common fraction where the numerator and denominator are both whole numbers

Repeating/recurring decimals: a number in which the same sequence of digits repeats indefinitely

Significant figures/significant places: number of places or digits to round non-terminating decimals to. Usually used in measurement contexts where decimals may extend beyond the accurate range of the measuring instrument and become irrelevant in practicality.

Terminating decimals: decimals which do not repeat or have a finite number of digits. All terminating decimals can be rewritten as common fractions.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the denominator? Does that sound like a place value that you know?
- Can you write that fraction as a decimal?
- Is the denominator a multiple of ten?
- If the denominator is not a multiple of ten, how else can we interpret the fraction?
- What operation do we use on the calculator?
- Is that decimal terminating – does it end?
- Can you see a pattern or repeat in the numerals in the decimal? Is it a recurring or repeating decimal?

Portfolio Task

This cycle contributes to sections of the *Portfolio Task 06: Calculating Costs* that represent fractions as decimals.

RAMR Cycle

This RAMR cycle focuses on connecting common fraction and decimal fraction representations of parts of a whole and converting between these representations. Exploration of the relationship between diameter and perimeter of a circle is included as an introduction to irrational numbers.

Reality

Discuss how fractions are generated and represented in the real world. Connect back to contexts explored during Unit 05. Discuss which division/sharing scenarios are usually expressed as fractions and which are not. Extend the conversation to mixed numbers (whole numbers and common fractions as well as whole numbers and decimal amounts).

Discuss decimal fractions and how these are represented in the real world. For example, partitioning lengths or distances, volume and capacity, money amounts (when do we see money amounts as whole numbers and when do we see money amounts as mixed numbers). Is there a connection between decimal fractions and common fractions? How do we move between these?

Abstraction

The abstraction sequence for this cycle connects common fraction representations of parts of wholes to decimal fraction representations. Once the decimal representation of fractions is established, finding equivalent fractions, comparing and ordering fractions, the notion of density, and types of decimal numbers (terminating, recurring, irrational) can be developed in the Mathematics phase of learning. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Find examples of items that come in packets or groups of ten, using students' fingers can be a simple and accessible option. Ask students to hold up 3 fingers. Identify what fraction of all their fingers they are holding up (three out of ten, three tenths, $\frac{3}{10}$).
2. *Represent/model with materials.* Use ten unifix cubes or counters to represent ten fingers. Separate 3 unifix cubes from the set of ten to represent 3 fingers. Alternatively, use bead strings and separate the first 3 beads from the first set of ten.
3. *Connect to language/represent with symbols*. Connect the name of the common fraction (three tenths) to its place value representation as a decimal part of a unit.
4. *Connect to symbol system*. Introduce place value names of decimal numbers. Explore what happens to a number's position on the place value chart when multiplying and dividing by ten. Act this out holding number cards on a human place value chart at the front of the room. Connect the symbolic representation of the decimal (e.g., 0.3) to the language of three tenths and the place on the place value chart. Ensure students have clearly established the quantity represented by three tenths.

Resource

Resource 6.4.1 Regrouping and renaming decimals
Resource 6.4.2 Regrouping and renaming number expander
Resource 6.4.3 Decimal place value charts

5. *Reinforce counting in symbol system.* Use a large number line (masking tape or cotton tape) on the floor for students to position decimal fraction and matching common fraction cards. Have pictorial representations for students to link to if necessary.
6. Ask students to practice representing digits on a place value chart for decimals. Ask them to write these as a decimal, say the decimal fraction, write them as a common fraction and represent them on a number line.

7. *Repeat Steps 1–6* to establish the size of parts for hundredths. Ten fingers on ten students can be used as a kinaesthetic hundred, one students' fingers are a tenth of the total, one finger of one student is one hundredth of the total. Use bead strings to identify a tenth (one set of coloured beads within the whole string) and parts of tenths. When reading decimal number names, group the digits to the right of the decimal point as a complete number and name according to the smallest place. For example, 2.35 is read as two and thirty-five hundredths; 2.05 is read as two and five hundredths. This enables students to focus on the value of the decimal fraction rather than the digit value of each place.
8. *Repeat Steps 1–6* to establish the size of parts for thousandths. Lay ten bead strings together so that one bead string is now one tenth, one set of coloured beads is one hundredth, and a single bead is one thousandth. When reading decimal number names, group the digits to the right of the decimal point as a complete number and name according to the smallest place. For example, 2.355 is read as two and three hundred, fifty-five thousandths; 2.055 is read as two and fifty-five thousandths; 2.005 is read as two and five thousandths. This enables students to focus on the value of the decimal fraction rather than the digit value of each place.

Mathematics

Language/symbols and practice

Flexible conversion between representations of tenths, hundredths, thousandths

Practise flexibly moving from common fraction, mixed number and improper fraction to decimal representations. Fraction and decimal matching and memory games may be useful here.

Resource Resource 6.4.4 Fraction and decimal representation games

Using a calculator to create decimal fractions

Remind students of the *Chocolate Cake Game* from Unit 05. Use *Resource 6.1.5 Chocolate cake game with decimals* to engage students with practice converting common fractions to decimals. Act out the first six seatings for the game and record the resulting common fractions. Discuss the meaning of each fraction as the number of cakes shared by a number of people. Connect this idea to division and enter into a calculator. For example, 2 cakes shared with 5 people would be 2 ÷ 5, pressing the = key will give a fraction of 0.4. Ensure students are able to name the fraction and check that four tenths is an equivalent fraction of two fifths. Use calculators to divide the cakes as decimal fractions for other seatings in the game. Read and write these fractions.

Resource Resource 6.4.5 Chocolate cake game with decimals

Comparing and ordering decimal fractions

Extending from students' experience with placing decimal fractions on the number line, connect to comparing and ordering of decimal fractions. Ensure students understand they need to compare like place values starting with the largest place so that 2.2 is larger than 2.1; 2.20 is smaller than 2.21; 2.200 is smaller than 2.201 and 2.211.

Use the set of fractions from the previous iteration of the *Chocolate Cake Game* to compare and order fractions on a dual number line with common fractions above the line and decimal fractions below the line.

Connections

Role of zero

Discuss the use of zero to fill empty place values and where it makes no difference to the value of a number. For example, in the number 2.35, placing zeros before the 2 will not change the value of the number and placing zeros after the 5 will not change the value of the number, because these placings do not alter the place of any of the digits within the number. Placing zeros anywhere between these digits will change the value of the number because digits will move between places.

Density with fractions

Reinforce density with decimal fractions by using a long length of rope (or a long line on the whiteboard or floor). Label each end with a whole number (e.g., 3 and 4). Find the halfway point on the string or line and mark, identify the fraction (e.g., 3.5). Place the rest of the tenths along the line. Choose one section to continue partitioning and labelling fractions to include hundredths. Discuss with students where thousandths would fit. Repeat the process with successively smaller decimal place values to consolidate students' understanding that there are an infinite number of fractions between any two locations on a line (although they do become smaller than human perception).

Smaller decimal place value names

Discuss smaller place value names than thousandths. Explore the symmetrical nature of place value names so that students are able to read and name smaller fractions. This can be acted out using place name cards along a human place value chart. Stand students in a line from millions down to ones. Put a hat on the student holding the ones card. Stand additional students with tenths, hundredths and thousandths cards. Have students with ten and tenths cards take a step forward; students with hundred and hundredths cards take two steps forward; and students with thousands and thousandths cards take three steps forwards. Ask students to predict what the matching decimal place will be for ten thousands, give this card to a student and have this pair take four steps forward. Repeat with hundred thousands, hundred thousandths, and millions and millionths. Ensure students notice the symmetry in the place names pivoting around the ones place (not the decimal). Reinforce with students that the pattern pivots around the ones and the role of the decimal place in always being positioned to the right of the ones place to separate whole numbers from decimal fractions.

Resource Resource 6.4.6 Symmetry of decimal place value

Classifying decimal fractions

While generating fractions in the *Chocolate Cake Game*, students may have noticed that some fractions fill the screen of the calculator and others do not. Create a poster with headings of *Terminating Decimals*, *Repeating Decimals* and *Irrational Numbers*. Gather the set of fractions generated during the game and sort out fractions that terminate such as (quarters, eighths, halves, and so on). See if students can suggest other fractions that will terminate, check with a calculator and add to the grouping. Work backwards from terminating decimals to their lowest common denominator common fraction (e.g., $0.356 = \frac{356}{1000} = \frac{178}{500} = \frac{89}{250}$).

Follow up with repeating decimals such as $0.\overline{3}$ ($\frac{1}{3}$), $0.\overline{285714}$ ($\frac{2}{7}$), $0.8\overline{3}$ ($\frac{5}{6}$). Ensure that students can identify the repeating part of the decimal. Students should be able to generate other repeating decimals without using too much imagination (e.g., $0.\overline{1}$, $0.\overline{2}$, $0.\overline{4}$, $0.\overline{5}$, and so on). Introduce students to the other names sometimes used for these decimals (recurring and infinitely recurring). Explore how to convert these decimals to common fractions using algebra.

Resource Resource 6.4.7 Terminating and repeating decimals

Irrational numbers – rounding

Rounding. Students should have encountered a range of decimals that are neither terminating nor repeating decimals. Discuss with students how to round these numbers to a number of significant places. Discuss the meaning of *significant* as *enough to make a difference*. Discuss where accurate measures may be needed and where they are less important. For example, most measurements are accurate enough to millimetres. Think of examples where smaller measures are needed. *Cary and Michael Huang's: The Scale of the Universe* can be useful to explore very small and very large measures http://htwins.net/scale2/ (very slow to load).

Discuss with students how to terminate an irrational number to significant places. Discuss rounding numbers (students should be familiar with this idea from shopping). Explore rounding kinaesthetically using a number track on the floor. Place ten at one end of the track and twenty at the other end. Discuss with students where to round up to twenty and where to round down to ten. Explore with other values. Ensure students also experience determining what numbers to place at each end of the track in order to round up or down. Use a number line to reinforce more than halfway and less than halfway as criteria for rounding. Reinforce and apply to decimal numbers.

Reflection

Check the idea

Represent fractions as common fractions, pictures, and decimals. Demonstrate as many different combinations of representations as possible. Identify families of common fractions that result in the same decimal.

Apply the idea

Explore decimal fraction operations and rounding using money transactions. For example, when paying cash, money amounts are rounded to nearest 5c or 10c; petrol prices are advertised as cents per litre with a decimal fraction resulting in further rounding to the nearest cent while also converting from cents to dollars. For example,

- Generate lists of items for purchase in a shop with decimal fraction values for students to add. Alternatively, use online shopping catalogues for students to select items from. Add the lists of items and determine what the total cost will be in dollars rounded to the nearest 5c or 10c.
- Explore petrol pricings. Convert these from cents to dollars. Calculate what 10L or 20L of fuel will cost in total (extended tens facts and place value can be consolidated here). Round this amount to two decimal places for keycard payment. Round this amount further to 5c or 10c for cash purchases. This may be extended to 50L or 100L where larger capacity vehicles are relevant for students.

Extend the idea

Extend students' work on repeating decimals to examples where there is more than a 2 digit repeat. These examples need to use a different multiplier depending on the length of the repeating part. Give students a few examples with 2 digit repeats, 3 digit repeats and 4 digit repeats.

Resource Resource 6.4.8 Terminating and repeating decimals extension

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 06 Investigation: Irrational Numbers

Exploring pi

Using measurement as a context, explore finding the distance around a circle (or circumference). Use a variety of circles to compare the diameters with the corresponding circumferences. Measure these as accurately as possible to determine how many diameters fit around the circumference (express as a fraction $\frac{circumference}{diameter}$). Compare fractions to approximate a constant value for pi (π). *Resource 6.4.8 Circumference of circles* has guidelines for the activity to assist with scaffolding student understanding.

Discuss how approximations for pi are only as accurate as the measuring implements usedto determine circumference and diameter measures.

Classify pi as an irrational number on the classroom chart. Discuss commonly used values for pi (e.g., 3.14, 3.14159, $\frac{22}{7}$, and so on).

Resource Resource Investigation Unit 06 Circumference of circles

Applications for pi

Find the best diameter to use for a trundle wheel to measure 1 m.

Determine the distance travelled with one turn of different sized bicycle wheels (e.g., diameters of 700 mm, 559 mm, 406 mm).

What size bicycle wheel is better for long distances (which one travels furthest for each turn of the wheel)?

Original bicycles were not geared. Each time the pedal crank completed a full turn, the wheel only travelled the same circumference as the pedal crank. (Google images: Penny farthing bike).

Explore the circumference of pedal crank turns compared to full wheel turns on bicycles. How many turns of the pedal crank will it take for the wheel to complete a full turn (assuming there is no gearing)?

Is there a maximum practical length for a pedal crank? Ask students to stand on one foot against a wall with the other foot raised so that the upper part of the leg is roughly parallel to the floor. Measure the distance from the base of the foot to the floor. How does this measurement relate to the measurement of a pedal crank?

Look at some pictures of penny farthing bikes. Some of these appear to have pedal cranks that are half the radius of the wheel, some seem to be about a third of the radius of the wheel and some are about a quarter of the radius of the wheel. What will these differences mean for the number of pedal turns to make a full turn of the wheel?

Name:____ Date:________

Can you do this? #5

1. Complete the patterns and write the change rule for each pattern.

   (a) 1.04, 1.24, 1.44, _____, _____, _____ The change is _____

   (b) 0.3, 0.35, 0.4, _____, _____, _____ The change is _____

   (c) 0.5, 0.48, 0.46, _____, _____, _____ The change is _____

   (d) 1.3, 1.27, 1.24, _____, _____, _____ The change is _____

2. Solve the equations:

   (a) $0.34 + 0.54 =$

   (b) $2.45 - 0.29 =$

   (c) $15.8 - 4.9 =$

   (d) 0.305 + 0.35 =

3. Write the equations and solve the problems:

   (a) The table was 4.25m long. A tablecloth needs to hang 0.5m over each end. How long does the tablecloth need to be?

   (b) Jack has to walk 12.35km to the shops from home. The swimming pool is 15km away from home. How much further away from home is the swimming pool?

Obj. 6.5.1
a)□□□ □□
b)□□□ □□
c)□□□ □□
d)□□□ □□

Obj. 6.5.2
a) □

Obj. 6.5.3
b) □

Obj. 6.5.3
c) □

Obj. 6.5.2
d) □

Obj. 6.5.2
a)i □
ii □
iii □

Obj. 6.5.3
b)i □
ii □
iii □

Cycle 5: Additive Operations with Decimals

Overview

Big Idea

This cycle extends addition and subtraction concepts and processes explored with whole numbers and fractions to addition and subtraction of decimals. Decimal fraction operations can be explored as an extension of common fractions with denominators that are powers of ten. Decimal fractions can also be explored as an extension and reinforcement of place value additive operation techniques using previously explored whole number strategies.

Objectives

By the end of this cycle, students should be able to:

6.5.1 Describe, continue and create patterns with decimals resulting from addition and subtraction. [5NA107]

6.5.2 Add decimals. [6NA128]

6.5.3 Subtract decimals. [6NA128]

6.5.4 Use estimation and rounding to check reasonableness of answers. [6NA128]

Conceptual Links

Decimal fraction operations rely on students' previous development of place value and operations with whole numbers developed in Unit 01, Unit 02 and Unit 03.

Skills developed in additive operations with decimal fractions are further used in contexts that involve money/finance or budgeting, measurement contexts where values are added (such as perimeter), complementary angles in geometry, and statistics and probability calculations.

Materials

For Cycle 5 you may need:

- PVC per student
- PV cards set (math mat and holding)
- math mat
- fraction sticks
- Part – Whole chart (similar to PVC)
- Connect 5/cover the board games
- shopping catalogues (or access to online)
- A7 digit cards per student
- A5 digit cards
- calculator per student
- fraction base boards
- overlays
- thinkboard

Key Language

Equivalent fractions, place value, tenths, hundredths, thousandths, decimal fraction, decimal places

Definitions

Decimal place: place values smaller than a whole unit. For example, tenths, hundredths, thousandths

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What values are you adding/subtracting?
- What place values are in each number?
- Are you adding/subtracting the same place values?
- Will it change the value of the number if you put a zero in the place after the last decimal place value (to the right of the last digit after the decimal point)?
- Will it change the value of the number if you put a zero in the place before the first whole number place value (to the left of the first digit in the number before the decimal point)?

Portfolio Task

The student portfolio task may be given to students as an independent task to complete in class, or as a teaching/application activity. This cycle contributes to sections of the task that involve additive operations with decimals like adding or subtracting money amounts and metric measures.

RAMR Cycle

The focus of this cycle is additive operations with decimal fractions. Additive operations with decimal fractions rely on place value understanding so that like places are added or subtracted. Area and number line models are used to model additive fraction operations. Students should continue interpreting story contexts that use fractional values and broaden their vocabulary for creating their own stories to suit equations.

Reality

Real contexts involving joining or separating are ideal for working with decimal fractions. Reinforce with students that fractions are the same size pieces of a whole. Some contexts are naturally found as decimals, for example, any contexts involving money, metric measure (length, capacity, mass). These measures can be joined or separated when: planning a trip to several places, totalling a shopping list or subtracting discounts, adding dry ingredients to a bowl or adding liquids to a mixture when cooking, and when spraying chemicals or mixing fuel. These contexts can be used within the Mathematics phase to explore additive decimal operations.

Abstraction

The abstraction sequence for this cycle starts from students' experience of additive operations with whole numbers to include calculations with decimal place value quantities. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Start by acting out a problem using a number ladder or line on the floor. This can depict as distances along a tape for an authentic context. For example, fabric 0.3 m wide is laid edge to edge with a piece of fabric 0.6 m wide; how much length will it cover? If I have covered 0.9m and only need to cover 0.7 m, what width do I need to cut off? However, few tape measures are constructed with decimal numbers on them in a way that does not confuse students. A 5 m length of wide bias tape with markings (Nikko pen is great) that correspond to 0.1 m and so on can be more useful and will roll up again ready for next usage. This could also be created with 0 in the middle so that negative numbers are included on the other half. Alternatively, if the 100 m running track is considered as a full length, decimal addition and subtraction could be considered with how far up or down the track a person runs if the length is broken into equal parts for 10 runners when practising baton changes.
2. *Represent/model with materials.* Continue to model addition and subtraction stories with drawn representations (number lines). Ensure that students recognise that inverse operation from whole numbers still works with decimal numbers. Ensure when students are working with subtraction of decimal numbers or negative decimal numbers, that they interpret the placement of the numbers correctly. For example, if placing $^{-}3.5$, students need to locate $^{-}3$, then continue to the left to place $^{-}3.5$.
3. *Represent with symbols.* Continue to build student vocabulary and flexibility with worded problems by modelling a range of problem situations using a wide range of addition and subtraction language. Record these as equations to be solved. Continue to highlight part-part-total with addition and subtraction stories and connect to the triads to reinforce student connections to interpreting and creating real-world problems. Ensure that students generalise that in additive problems, when given both the parts, students can find the total if they add the parts together. Meanwhile, if students know the total and one part, then they have to find the other part and need to subtract.

4. *Reinforce place value calculation strategies.* Encourage students to apply their previous strategies with whole number operations to decimal place values. Ensure that students understand that decimal place values work the same way as whole number place values even though they represent parts of the whole.

Use relevant story contexts where decimal fractions are normally encountered like money and metric measures. Encourage students to construct their own problem situations. Addition and subtraction of decimal numbers can be thought of as operations with the equivalent common fractions or as an extension of whole-number operations using place value. These ideas are demonstrated further in *Resource 6.5.1 Additive operations with decimal fractions*.

Resource

Resource 6.5.1 Additive operations with decimal fractions
Resource 6.5.2 Place value charts

Mathematics

Language/symbols and practice

Use relevant story contexts where decimal fractions are normally encountered like money and metric measures. Encourage students to construct their own problem situations. Addition and subtraction of decimal numbers can be thought of as operations with the equivalent common fractions or as an extension of whole-number operations using place value.

Resource 6.5.3 Decimats game is a game context that connects area models of decimal fractions and their common fraction and decimal fraction representations while also providing opportunities to practice addition of decimal fractions based on place value amounts.

Resource

Resource 6.5.3 Decimats game
Resource 6.5.4 Virtual Decimats game

Connections

Properties of numbers and operations

Explore properties of additive operations encountered with whole numbers such as commutativity, associativity, inverse operations, part-part-total, and triadic relationships between parts of a story problem. Ensure that students can see that all these properties apply to common fractions and decimal fractions as well as whole numbers.

Problem construction and interpretation

Practice writing equations with fractions from stories and writing stories from equations for additive operations. Focus on the parts and totals within the equations and ensure the stories have suitable unknown parts or totals for the operation the number sentence suggests.

Try to move students to understanding that a pronumeral can be any symbol or letter as it represents the unknown quantity of a unit not the unit itself. For example, ‘s’ does not need to stand for spent, socks, shoes, sandals, or seagulls. The symbol can represent any quantity of these things, but also displacement, distance, dogs, cats, fish, … and so on. Discuss which of these items make sense as fractions and which do not (for example, 0.5 of a seagull is a little gruesome but 0.5 m long plank of wood makes sense).

Broaden students' understanding to include the variety of contexts that any given number sentence could represent. For example, 0.5 + 0.3 = s could represent stories like:

- I measured 0.5 m along the wall and then another 0.3 m; what was the distance along the wall altogether?
- I walked 0.5 km from home and then 0.3 km back; how far did I walk?
- There was 0.5 kg of flour in the packet and I added another 0.3 kg; how much flour did I have?
- I bought a freddo frog for $0.50 and a jelly snake for $0.30; how much did I spend?

Reflection

Check the idea

Return to Thinkboards or concept maps so students can generate and connect fraction representations and additive strategies.

Use Decimats game to check students' abilities to understand and add decimal place values.

Challenge students to find decimal combinations that add to 1 whole, 2 wholes, and so on. Find fraction combinations that can be subtracted from 3 to get to a half.

Apply the idea

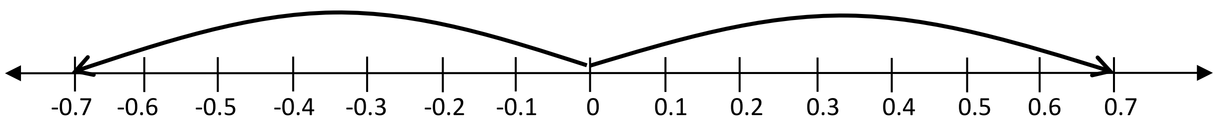
Use a variety of shopping catalogues or grocery shopping online sites so that students can engage with monetary values that are current and real to practise addition and subtraction with decimal fractions.

Extend the idea

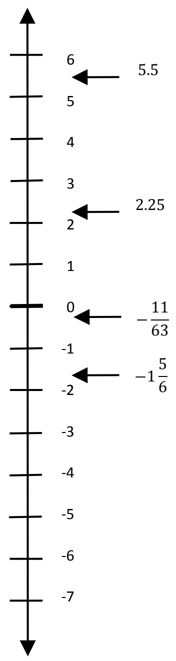
Directed decimal

As for whole numbers, directed decimals represent decimals with both quantity and direction. The directions are opposites and described as either positive or negative. An example of negative decimals can be with temperature where the temperature drops whole and decimal degrees below zero.

As with whole numbers, directed numbers are opposite to the corresponding positive decimal. It is important to make the distinction between negative numbers and the operation of subtraction. For example, the number opposite 0.7 is negative 0.7 and is written as $^{-}0.7$. A number line is an effective tool for demonstrating how these numbers are opposites. The numbers are the same distance or the same number of jumps in each direction from zero.



There is a potential difficulty with horizontal number lines if they become confused with place value charts. It is useful to show the diagram on the right. Students sometimes confuse place value with directed number. Meanwhile, some students start to see decimals as similar to negative numbers and feel that negative numbers start after the ones instead of after zero.



If students are displaying this tendency and the diagram on the right does not help, it may be beneficial to return to concrete paper folding activities to show that amounts less than one are still positive amounts less than 1 and greater than 0.

Using profit and loss or income and expense contexts for positive and negative money amounts may also be useful if this context was successful when students investigated whole numbers.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #6

1. Solve the equations:

   (a) $1.57 \times 12 \ =$

   Obj. 6.6.1
   a) ☐

   (b) $0.3 \div 0.5 =$

   Obj. 6.6.5
   b) ☐

2. Write the equations and solve the problems:

   (a) The whiteboard is 2.4m long. What is the measurement to mark halfway along the board?

   Obj. 6.6.2
   a)i ☐
   ii ☐
   iii ☐

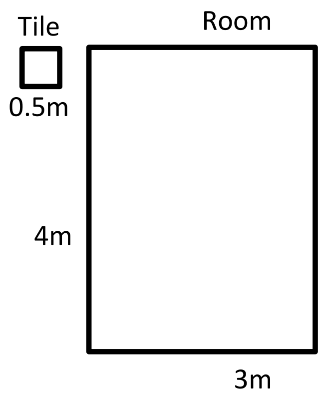
   (b) A wall is 5.4m long. Nine plaster sheets are covering the wall. How wide is each sheet of plaster?

   Obj. 6.6.4
   b)i ☐
   ii ☐
   iii ☐

3. A room is 3m wide and 4m long.
   Square floor tiles are 0.5m across.

   (a) How many tiles will fit across the room?

   (b) How many tiles will fit along the room?



Obj. 6.6.3
a)i ☐
ii ☐
iii ☐
b)i ☐
ii ☐
iii ☐

4. Jellybeans in bulk cost $7.50 for 1000 g.

   (a) What fraction of 1000 g is 350 g?

   (b) If you buy 350 g of jellybeans, how much will they cost?

Obj. 6.6.6
a) ☐
b)i ☐
ii ☐
iii ☐

5. Another market sells jellybeans in 250g bags for $1.80.

   (a) What fraction of 1000 g is 250 g?

   (b) How much will 1000 g of jellybeans cost from this market?

   (c) Is it cheaper to buy 1000 g of jellybeans in bulk or in 250 g bags?

   ____

Obj. 6.6.6
a) ☐
b)i ☐
ii ☐
iii ☐
c) ☐

Cycle 6: Multiplicative Operations with Decimals

Overview

Big Idea

This cycle develops strategies for multiplicative operations with decimals by connecting multiplication concepts and processes with fraction concepts. The most accessible meanings of multiplicative operations for decimals are repeated addition/subtraction and change. Area and number line models can represent problems and assist thinking. Decimal fraction operations can be explored as an extension of common fraction operations with denominators that are multiples of ten, and as an extension and reinforcement of place value multiplicative operation techniques and strategies.

Using an array or area model, multiplicative operations with decimal fractions are relatively straightforward as they use extended tens facts strategies or may be treated in parts as for whole number operations. Conceptual understanding needs to be developed that tenths × tenths (or tenths of tenths) are hundredths, tenths of hundredths gives thousandths and so on. Base boards and overlays that link tenths to hundredths are useful here. Careful connection to language and number names is also necessary to connect to place value understandings.

Objectives

By the end of this cycle, students should be able to:

6.6.1 Multiply decimals by whole numbers. [6NA129]

6.6.2 Multiply fractions and decimals. [7NA154]

6.6.3 Find how many of a decimal quantity is in a whole number. [7NA154]

6.6.4 Divide decimals by whole numbers. [7NA154]

6.6.5 Divide decimals by decimal quantities. [7NA154]

6.6.6 Investigate and calculate ‘best buys’. [7NA174]

Conceptual Links

Decimal fraction operations rely on students’ previous development of place value and operations with whole numbers developed in Unit 01, Unit 02 and Unit 03.

This cycle links to multiplicative operations with whole numbers and builds initial skills in multiplicative operations with decimals. It provides the skills necessary to work with percentage problems, engage with financial mathematics, multiplicative relationships in measurement, statistics and probability, and algebraic applications of fractions and decimals.

Materials

For Cycle 6 you may need:

- PVC per student
- shopping catalogues (may be online)
- calculators
- hundredths grids
- fraction baseboards and overlays

Key Language

Multiply, fraction of a fraction, change, how many fractions in, how many shares of, decimal fraction, tenths, hundredths, thousandths, dividend, divisor, quotient

Definitions

Dividend: total quantity to be shared or divided

Divisor: the number of shares to break the total quantity into (partitioning: size of share unknown) or the size of each share to be created (quotitioning: number of shares unkown).

Quotient: the size of each share to be created (partitioning) or the number of shares to be created (quotitioning)

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Are you finding a fraction of a fraction or how many pieces in?
- Do you need to multiply or divide?

Portfolio Task

This cycle contributes to sections of the *Portfolio Task: Calculating Costs* that involve multiplicative operations with decimals like finding multiples of money amounts and metric measures.

RAMR Cycle

Reality

Shopping contexts are ideal for engaging students using catalogues for prices or online shopping catalogues to plan and work out the total cost of a shopping list with multiples of items. Where items are available singly or in multiple lots (e.g., single cans drink and boxes of 6 cans, twin packs and four packs of jelly), there are rich opportunities to compare value for money decisions. Similarly, division may be used to determine how many of a given item may be purchased with a set amount of money, and to determine unit cost or pricing for best buys. This is an appropriate opportunity to introduce and explore the language of “per” and its meaning as “for each”.

Other appropriate contexts are measurement of mass, capacity or length where decimal amounts of kilograms, litres, metres, or kilometres can be explored as multiples or partitioned. Examples might include: a single seat is 0.48 m wide, how wide will a row of six occupy; a bag of rice is 1.5 kg and 12 of these fit in a shopping bag, how heavy will the shopping bag be to carry; we are using 1.5 L bottles of soft drink and juice to make fruit punch, if there is a bottle of lemonade, a bottle of ginger ale and a bottle of fruit juice, what quantity of fruit punch will be the total?

Encourage students to construct and interpret story contexts and to create equations with decimal values to solve. This will also broaden students’ vocabulary. Ensure that question patterns involve a range of formats so that the product is not always the last or first quantity stated.

Abstraction

The abstraction sequence for this cycle takes students from physical experiences of multiples of decimal fractions to develop strategies for solving these types of problems using language and symbols. A suggested sequence of activities is as follows:

Multiplication of decimals

1. *Kinaesthetic activity.* Experiences from multiplication of common fractions may be extended here and connected to common fractions where the denominator is a multiple of ten. Explore and consolidate that a fraction of a fraction is a smaller fraction. As for Cycle 4, it can be useful to connect to ten students holding up ten fingers. One student has 0.1 of the fingers held up, one finger on this student is 0.1 × 0.1 of the fingers held up which is 0.01 (one hundredth).
2. *Connect to language and symbols.* Reinforce the connection between language used with multiple of tens common fractions and decimal place values.
3. *Model with materials.* Gradually increase the complexity of examples to abstract decimal × decimal rules so that conceptual understanding and number sense are reinforced along with technique. Start with whole × decimal examples, extend to decimal × decimal examples.
4. *Connect, reinforce and consolidate.* Use calculator patterns to verify place value position and to assist students to generalise a way of working with decimal × decimal examples is consistent. *Resource 6.6.4 Decimal fractions “of” or “by” decimal fractions* explains this further.

Resource

Resource 6.6.1 Tenths and hundredths base boards and overlays
Resource 6.6.2 Tenths and hundredths base boards and overlays PowerPoint
Resource 6.6.3 Multiples of decimal fractions
Resource 6.6.4 Decimal fractions “of” or “by” decimal fractions
Resource 6.6.5 Multiplication of fractions games
Resource 6.6.6 Multiplication of fractions games PowerPoint

Division of or by decimals

Conceptual understanding needs to be reinforced that division is finding how many in a group or how many groups – this does not change with decimals. What is different is the size of the pieces that make up the group. If the divisor is a whole number, division can be an extension of whole-number division.

4. *Kinaesthetic activity.* As for division of common fractions, contexts can include volume models, area models and length models where metric measure is used. For example, capacity, area covered, and distances. An ICT based activity could involve the use of distances segmented equally, for example:

   Six friends drive to Sydney from Brisbane. Use Google Maps to find this distance (920.7 km).

   If each friend drives an equal amount of kilometres, how far will each friend drive?

5. *Model with materials and connect to language and symbols.* Model the previous activity with materials (number line model will show the problem best but area model for place value will show the partitionin best). This could first be demonstrated using a number line, showing the 6 equal jumps. Then question the students asking how each jump can be calculated. However, to model the process for calculation, it is necessary to break the number into place value segments for working.

   Note that the following language should be used alongside the written algorithm so that students may understand the sharing process and the regrouping.

   The whole distance is *920.*7 km. This is 9 hundreds, 2 tens, 0 ones, 7 tenths. Each of these places is shared by or divided by 6 in turn.

   For this example, out of 9 hundreds, 6 people travel 1 hundred each, leaving 3 hundreds. These 3 hundreds are regrouped as tens to give a total of 32 tens. Out of 32 tens, 6 people travel 5 tens each leaving 2 tens. These are regrouped as ones to give 20 ones. Sharing these, each person travels 3 ones leaving 2 ones which are regrouped as tenths to give 27 tenths. Each person travels 4 tenths and the remaining 3 tenths are regrouped as 30 hundredths of which each person travels 5 hundredths. This means that each individual travels 1 hundred, 5 tens, 3 ones, 4 tenths and 5 hundredths. Written as a complete digit this becomes 153.45 km each to drive.

   This activity can be repeated with various distances common to the students' interests.

Resource Resource 6.6.7 Whole-part sharing to traditional algorithm

Mathematics

Within the Mathematics phase, it is important to practice problems that involve finding multiples of same-size fractions using set (considered in Abstraction), number line and array or area models. These are most easily conceptualised as repeated addition. Similarly, decimal fraction of or by a decimal fraction problems should be explored using contexts that can be represented using set, number line and array or area (explored in Abstraction) models.

Connections

Ensure that generalisations of ways of working to solve multiplicative operations with decimal fractions are clearly connected to place value understanding.

Language/symbols and practice

Calculation when the divisor is a decimal number

For examples like 0.378 ÷ 0.7, using an extension of whole number techniques requires changing the divisor without changing the quotient. There are two related methods of exploring this with students which will lead to a similar pattern for working with symbols.

1. **Using reality**. Where metric measure reality situations have been used it is possible to consider conversion to a smaller unit to change the decimal component of the problem. For example, if the problem is to find 0.378 cm ÷ 0.7 cm, then converting to mm it becomes 3.78 ÷ 7. This can be done by whole-number methods.

   Note: this method works best when translation to reality is simple. For example 0.21 ÷ 0.07, changing it from m to cm changes the example to 21 ÷ 7, making it easy to solve. This is related to the number-size compensation principle which may also be used in the same way.

   Discuss with students how this idea works. How are they changing the measures for the division to be simpler? Ensure they realise that they are multiplying both the dividend and the divisor by the same amount. Test these examples using calculators to allow students to verify that the quotients (answers) for each pair of equations remain the same.

2. **Using fraction equivalence**. Connect to students' understanding of fractions as a representation of division and finding equivalent fractions. The example 0.378 ÷ 0.7 is the same as fraction $\frac{0.378}{0.7}$, multiplying top and bottom by 10, this is equivalent to fraction $\frac{3.78}{7}$ which is the same as 3.78 ÷ 7. Again, verify that equations solved this way result in the same outcomes.

Once the divisor has been converted to a whole number, students can calculate using mental computation or pen and paper calculations with a traditional algorithm. Practice division with decimals from a range of story contexts as well as writing stories from given equations.

Resource Resource 6.6.8 Division of or by decimal fractions

Both dividend and divisor are decimal numbers

For examples like 0.378 ÷ 0.7, using an extension of whole number techniques requires changing the example without changing the answer. There are four methods to explore which will all lead to a similar pattern for working.

1. **Using reality**. Where metric measure reality situations have been used it is possible to consider conversion to a smaller unit to remove the decimal component of the problem. For example, if the problem is to find 0.378 cm ÷ 0.7 cm, then converting to mm it becomes 3.78 ÷ 7. This can be done by whole-number methods.

   Note: this method works best when translation to reality is simple. For example 0.21 ÷ 0.07, changing it from m to cm changes the example to 21 ÷ 7, which is easy to solve.

   Discuss with students how this idea works. How are they changing the measures for the division to be simpler? Ensure they realise that they are multiplying both the dividend and the divisor by the same amount. This leads naturally into the next two methods for working.

2. **Using the number-size compensation principle**. In this method, 0.7 changes to 7 by × 10, so to compensate change 0.378 the same way, that is, 0.378 ÷ 0.7 = 3.78 ÷ 7, and this can be solved by whole-number methods. Explore a number of these on calculators to see that the results for these equations are the same.

3. **Using fraction equivalence**. Connect to students' understanding of fractions as division and finding equivalent fractions from Unit 05 and Cycle 1.The example 0.378 ÷ 0.7 is the same as fraction $\frac{0.378}{0.7}$, multiplying top and bottom by 10, this is equivalent to fraction $\frac{3.78}{7}$ which is the same as 3.78 ÷ 7.
4. **Using calculator patterns**. Connect place value ideas from working with hundredths grids to calculator patterns explored in the number-size compensation principle method. Use the same steps as for multiplication of decimals (use calculators to relate 0.345 ÷ 0.5 to 345 ÷ 5 and find the pattern, practise the pattern, describe the pattern, and use the pattern). See if students can generalise that for decimal division 0.345 ÷ 0.5, we do the whole-number division and subtract the decimal place-value positions to find the ones in the whole-number answer. That is, 345 ÷ 5 is 69 (it is three decimal places subtract one decimal place = two decimal places) so the ones place is three from the right and the decimal division answer is 0.69.

Consolidate and practice

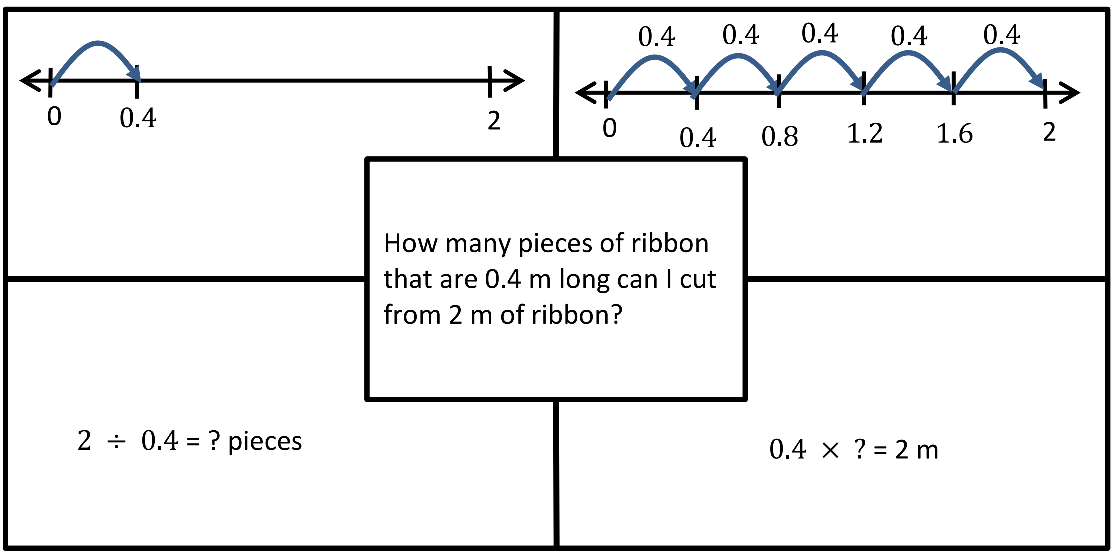
Practise multiplicative operations within a range of story contexts. Also, practice writing stories from given equations. Students need to experience, interpret and construct stories that have one factor or the other unknown as well as an unknown product. Ensure that a variety of story patterns are used that have the product at points in the story other than the ending.

Colour by number activities, bingo games where the factors are called for products on the bingo board and Connect 5 activities can all be used as ways to consolidate multiplicative operations.

Reflection

Check the idea

Use shopping list contexts to generate examples of multiplicative operations with decimal fractions. Decimal by decimal contexts may be generated by calculating 1.5 kg prices for fresh fruit and vegetables or meat products. There are many connections between common fraction and decimal operations including area or number line representation, inverse operations and reciprocal (or inverse) fractions. Construction of Thinkboards that depict equivalent fraction and decimal operations connecting multiplication and division to story problems will be beneficial. For example:



Apply the idea

Rates and best buys in supermarket

A good application of division with decimals is to visit a supermarket (or online shopping catalogue). Ask students to look at costs and calculate the best buy. **Note**: Supermarkets have already done a lot of the work for us by stating cost/weight on their displays.

For example: To determine best buy in a supermarket two cans of beans (A and B) are compared. This can be done using two methods, as presented in the table below.

Note: These are rate problems. Connect "per" language to the diagonal slash used to represent it. Connect this representation to fraction representations so that cost/mass is also understood as $\frac{cost}{mass}$.

Can 1 - Mass: 440 g, Cost: $2.10

Can 2 - Mass: 300 g, Cost: $1.55

Method A	Method B
Rate 1: Cost/mass ($/g)	Rate 2: Mass/cost (g/$)
A gives $2.10 ÷ 440 g = $0.0048/g	A gives 440 g ÷ $2.10 = 210g/$
B gives $1.55 ÷ 300 g = $0.0052/g	B gives 300 g ÷ $1.55 = 194g/$
B is more expensive because each gram costs more.	B is more expensive because you get less grams for each dollar.

Note: Lower is better for Rate 1, $/g; while higher is better for Rate 2, g/$.)

One could look at a lot of other situations and use rates to work out best value. It is important to recognise that value is more than just money; something may be more expensive but tastes better – are we happy to pay a few cents more for something that tastes better?. For example, what are the costs of events (e.g., football games, concerts) in relation to how long they go for? This gives possibilities of working out cost/time (lower the better) and time/cost (higher the better).

Of course often there is more than one thing involved in determining value. For instance, a phone might have higher text costs and lower call costs or vice versa. This leads to more complex and richer investigations with results that require a level of interpretation to connect back to reality – what do the values *really* mean?

Extend the idea

Directed decimals

Explore multiplicative operations with negative and positive decimal amounts. Ensure students understand that principles that apply to integers can also be applied to directed decimals.

Change parameters

Revisit scientific notation and indices. Explore multiplicative operations with decimal numbers "by" numbers expressed as powers of ten. For example, 3.2×10^3; 0.6×10^3; $3.2 \div 10^3$; $0.6 \div 10^3$.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #7

1. Fill in the following conversions:

   (a) 15 cm = __________ mm

   (b) 3 m = __________ cm

   (c) 4600 m = __________ km

   (d) 60 mm = __________ cm

2. Write the following numbers using scientific notation.
   For example: 364 000 = 3.64×10^5

   (a) 3 200 ________

   (b) 0.085 ________

3. What is the value of:

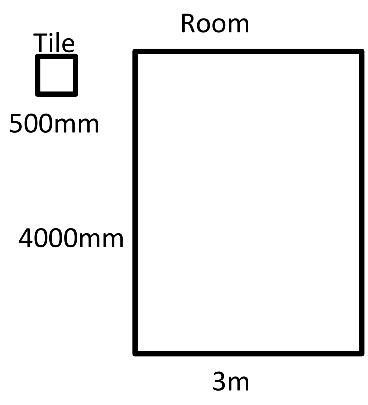
   (a) 3.65×10^2 ____________

   (b) 45×10^{-1} ____________

   (c) $0.1 \div 10^2$ ____________

   (d) $10\ 000 \div 10^2$ ____________

4. A room is 3 m wide and 4000 mm long.
   Square floor tiles are 500 mm across.



   (a) How many tiles will fit across the room?

   (b) How many tiles will fit along the room?

   (c) How many tiles needed to cover the floor?

Obj. 6.7.1
a) □
b) □
c) □
d) □

Obj. 6.7.2
a) □
b) □

Obj. 6.7.3
a) □
b) □

Obj. 6.7.4
c) □
d) □

Obj. 6.7.1
a)i □
ii □
iii □
b)i □
ii □
iii □
c)i □
ii □
iii □

Cycle 7: Metric Measure Conversions

Overview

Big Idea

Units of metric measure, like the decimal number system, rely on place value groupings where places increase or decrease in multiples of ten. Conversion between units of measure is relatively simple if connections are made to place value understanding. For example, understanding the meaning of prefixes used for standard units, knowledge of the base units for a measured attribute and an understanding of the multiples that apply to standard units, provide students with a fundamental understanding of the connectivity between measures.

Objectives

By the end of this cycle, students should be able to:

6.7.1 Convert between common metric units. [6MG136]

6.7.2 Express numbers in scientific notation. [9NA210]

6.7.3 Multiply decimals by powers of 10. [6NA130]

6.7.4 Divide decimals by powers of 10. [6NA130]

Conceptual Links

This cycle provides the basis of metric conversion for length units that can be applied to most units of metric measure.

Materials

For Cycle 7 you may need:

- metre ruler
- standard ruler (cm and mm)
- dressmakers' tape measure
- specialist measuring devices
- tape measure (PE for measuring long jump)
- height stick (PE for measuring high jump)
- pocket tape measure (2 m or 5 m)

Key Language

Standard units, metre, centimetre, millimetre, kilometre, bigger and smaller units, prefixes for standard length measure units, scientific notation, International System of Units (SI)

Definitions

International System of Units: international conventions for decimal metric measure prefixes

centi: one hundredth of base unit (10^{-2})

deci: one tenth of base unit (10^{-1})

kilo: one thousand of base unit (10^{3})

mega: one million of base unit (10^{6})

milli: one thousandth of base unit (10^{-3})

Further International System Units: http://physics.nist.gov/cuu/Units/prefixes.html

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What are the units of measure you have?
- If you are calculating, are the measures the same unit?
- Which measure will it make more sense to convert?
- Are you changing from a larger unit to a smaller unit or from a smaller unit to a larger unit?
- Will there be more or less of the unit to measure the same distance?
- Do you need to multiply or divide?
- What is the relationship between the units? How many places apart are they? How many powers of ten difference?
- What do you need to multiply by?
- Note: for most metric conversions multiply/divide by 1000 (differences in places is 3 powers of 10). Between centimetres and metres the difference is only 2 powers of 10; multiply/divide by 100. Between millimetres and centimetres the difference is only 1 power of 10; multiply/divide by 10.

Portfolio Task

Skills developed and reinforced in this cycle may assist students will measurement conversions for calculations in *Portfolio Task 06: Calculating Costs.*

RAMR Cycle

Reality

Where possible, find real-life contexts in which to embed the activities. Extend from situations explored in Unit 4 that were represented as common fractions. Choose situations where it is appropriate to measure in millimetres, centimetres, metres and kilometres. Reinforce the magnitude of each unit and connect common fraction representations to the decimal fraction representations.

Abstraction

The abstraction sequence for this cycle extends from common fractions developed in Unit 05 for measuring length. This sequence needs to connect standard measuring units with place value ideas, standard prefixes and abbreviations, to complete students' ability to measure with and convert between standard metric units. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Reinforce students' physical sense of standard units. Experience the magnitude of each unit physically. Find examples in the real world that are 1 mm, 1 cm, 1 m and 1 km long. Have students estimate 10 m or a decimetre, then take a trundle wheel along to check. Students should be encouraged to count each metre as they go as one-tenth, two-tenths and so on to one decimetre. Record these fractions and counting sequence as decimals. This could be repeated using the 100 m track as one tenth of a kilometre to reinforce the counting and recording of the distance walked as tenths of a kilometre up to 1 km walked.
2. *Represent with materials*. Revisit the activity from Unit 05 using different colours of 1 cm grid paper, cut ten strips that are 10 cm in length. Tape alternating colours together to form a folding 1 m measuring strip. This can be used to identify centimetres and groups of 10 centimetres to make up a metre.
3. *Connect to language*. Reinforce connections between metric measure and decimal place value using decimal digits instead of fractions. Use the maths mat as a large place value chart and lay out place value cards from thousandths to millions. Place a Metre card under the ones. Research the meaning of prefixes kilo, centi, milli. Place these cards under their respective place values respective to the metre. Fill in non-standard metric prefixes to cover empty places (these exist although not used within the SI unit set). See *Resource 6.7.1 Metric prefixes chart for student desks* and *Resource 6.7.2 Connecting place value to metric measure* for more detail. Reinforce the connection between the common fraction name and the decimal fraction place.
4. *Reinforce place value link to conversion*. Construct a larger copy of *Resource 6.7.3 Metric expanders* (kilometres, metres and millimetres) and cut it out. Fold as for number expanders. Use them to relate km, m and mm as for place-value cards. The connection needs to be made between the unit of measure and the ones place. If measuring in metres, then the unit for metres is placed under the ones place on the place value chart. The decimal fraction then represents centimetres and millimetres. To convert to millimetres, slide the chart across the top of the digits until the ones place is above the number for the millimetres. Focus student attention on the fact that the chart slid three places to the right to change the thousandths place for metres to the ones place for millimetres. Connect to the symbolic multiplication that accompanies this conversion. *Resource 6.7.4 Metric slide rules* may also be useful here.

Resource

Resource 6.7.1 Metric prefixes chart for student desks
Resource 6.7.2 Connecting place value to metric measure (large cards)
Resource 6.7.3 Metric expanders
Resource 6.7.4 Metric slide rule

5. Additional measuring practice is always useful. It may be beneficial to repeat measures of personal referents. Connect these to decimal measures (e.g., represent measures as centimetres with millimetres as decimal fractions of centimetres). If students are not keen to repeat measures of personal referents, this could be varied to consider equivalent measures using dolls or models to measure. Students may be engaged by measuring lengths and wheels of model cars and real cars, lengths and widths of collections of leaves from different trees, any items of interest that can be measured and recorded using metric measure and decimal fractions.

Resource Resource 6.7.5 Determining personal referents

Mathematics

Conversion of metric measures

Find contexts for students to practice measuring and converting between measures using metric measure and decimal fractions. Explore addition of decimals in the context of measuring dimensions of items and finding the associated perimeter. Simple multiplication of decimals can also be explored here by adding a length to a width and then doubling.

Scientific notation

Encourage students to practice writing conversion equations (e.g., 2 m × 1000 = 2000 mm). Use a calculator to connect repeated multiplication (10 × 10 × 10) to 10^3. Rewrite the equation (e.g., 2 m × 10^3). Connect each place to its corresponding power of 10. Practice converting between large numbers and their corresponding representation in scientific notation.

Reflection

Check the idea

Use a Thinkboard or Concept map to connect equivalent measures in kilometres, metres, centimetres, millimetres to a measurement of a common item. Ask students to highlight the most accurate unit to use for the item. Also connect to Scientific notation for each measure where values include larger whole numbers. These Thinkboards may also be created as puzzles to be recreated.

Resource Resource 6.7.6 Equivalent metric measure Thinkboards

Apply the idea

Once students have reached the point where they understand a need for standard units, engage them with tasks to practice measuring items in centimetres or metres to compare lengths. Familiarise students with a range of measuring tools/devices (rulers are simplest, dressmakers' tape measures and building tape measures are more complicated to read). Identify which measure is the unit for each marking to assist with correct reading of the measuring device.

Extend the idea

Explore other instances of measures that are related and expressed using fractions and decimals. For example, scales on measuring jugs connect litres and millilitres, scales for mass connect grams and kilograms. Consider the relationships between these measures, their fractions, decimal fraction representations and conversions between units. Explore how Scientific notation might be used to record very large quantities of small units in these instances.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 06 Portfolio Task – Teacher Guide

Calculating Costs

Content Strand/s: Number and Algebra

Resources Supplied:

- Task sheet
- Teacher guide

Other Resources Needed:

- None

Summary:

The two tasks within this workbook each require a student to use additive and multiplicative operations with fractions and decimals to explore the cost of catering a school camp, and the cost involved with taking a road trip.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Number and Algebra

6.2.1 Find a fraction of a quantity where the result is a whole number. [6NA127].

6.2.2 Find whole number multiples of same-size fractions. [7NA154]

6.4.1 Represent common fractions of tenths, hundredths and beyond using decimal notation. [5NA104]

6.4.3 Round decimals to a specified number of decimal places. [7NA156]

6.6.1 Multiply decimals by whole numbers. [6NA129]

6.6.4 Divide decimals by whole numbers.[7NA154]

Calculating Costs

Name	
Teacher	
Class	

Your Task:
This portfolio task has two tasks. You will

- Explore the cost of catering for a school camp, and
- Explore the cost of financing a road trip.

You will use your knowledge of adding, subtracting, multiplying and dividing fractions and decimals to complete the tasks.

A calculator will be required for this task.

Within Portfolio Task 06, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Procedural fluency	6.2.1 Find a simple fraction of a quantity where the result is a whole number. 6.2.2 Find whole number multiples of same-size fractions. 6.4.1 Represent common fractions of tenths, hundredths and so on using decimal notation. 6.4.3 Round decimals to a specified number of decimal places.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
Problem Solving and Reasoning	Problem solving approaches	6.6.1 Multiply decimals by whole numbers. 6.6.4 Divide decimals by whole numbers.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches

Comments:

Task 1: Catering for Camp

In this task you will be working out the main foods and drinks for a weekend camp. As well as these foods, people will be bringing their own snacks and extras. There will be ***24 people*** going to the camp.

1. Calculate the required amounts of each food and the total cost. Show working in the spaces provided.

a. Frozen pizzas ¼ of a pizza per person $6 per pizza

Number of pizzas needed = ____________ Cost of pizzas = ___________

b. Sausages 200 g (0.2 kg) per person $7.60 per kg

Mass of sausages needed = ____________ Cost of sausages = ___________

c. Sliced bread 1/8 of a loaf per person $2.98 per loaf

Number of loaves needed = ____________ Cost of bread = ____________

d. Salad bags 1/3 bag per person $4 per bag

Number of salad bags needed = _________ Cost of salad bags = _________

e. Eggs 2 eggs per person $3.49 per dozen

Number of dozen eggs needed = _________ Cost of eggs = ___________

f. Soft drink (2 litre bottles) 500 mL per person $2.60 per bottle

Number of bottles needed = ______________ Cost of soft drinks = _________

As well as the foods you have already calculated, the organisers will be providing sandwich fillings which cost $30 and breakfast cereals that cost $25.

2. Calculate the total cost of all the foods above, including the sandwich fillings and breakfast cereals.

   Total cost of foods = ______________

3. What is the cost per person (24 people).

   Cost per person = _____________

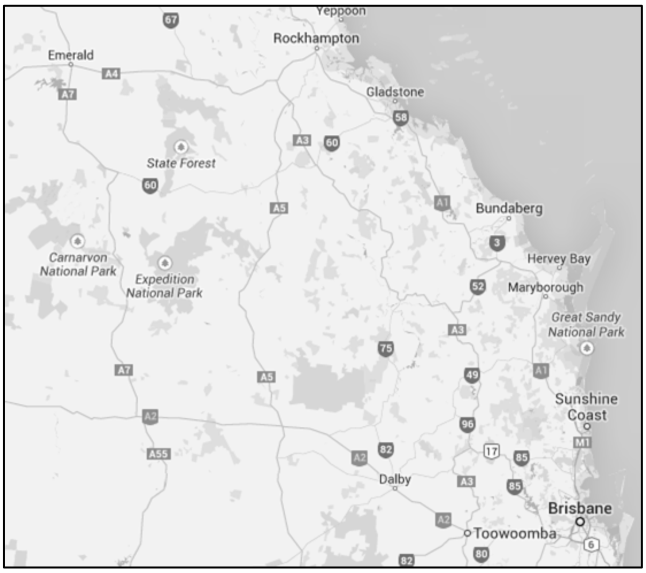
4. To find the most convenient amount of money to charge each person, round the cost per person up to the nearest 50 cents.

Charge per person = ____________

Task 2: Road trip

In this task you will be planning a road trip. This includes calculating the distances and times of travel, as well as some costs.

5. Calculate the total distance of the journey from Brisbane to Emerald from the sector distances from the map as shown below. Mark the journey on the map.



Brisbane – Maryborough	166 km
Maryborough – Gladstone	366 km
Gladstone – Rockhampton	107 km
Rockhampton – Emerald	263 km

Total distance of journey = _____________

6. If the driver can average 80 km/h on the trip, how many hours of driving will the trip take?

7. It is recommended that drivers take breaks during long trips, and a minimum of 10 minutes break for every hour of driving is necessary. This can include a meal break and other shorter breaks.

   For this trip, what total break time should be included?

8. Now find the total minimum time that should be allowed for the trip.

9. If the trip is started at 7 am, and it is not intended to drive right through to Emerald in one day, where would it be convenient to have an overnight stop? Why?

Challenge

10. The fuel consumption of a car is given in the rate unit ***litres per 100 kilometres*** (L/100 km). The car for this trip has a highway average fuel consumption of 8.2 L/100 km.

    At this rate of fuel consumptions, calculate the quantity of fuel in litres that would be used on the trip (round your answer to the nearest tenth of a litre).

11. If the price of fuel is 159.9 cents per litre, what is the total cost of the fuel for the trip?

12. How much will be spent on petrol there **and back**?

Name:____ Date:________

Can you do this now? Unit 06

1. Continue the sequences and identify the changes

   Example: $\frac{1}{9}, \frac{3}{9}, \frac{5}{9}, \frac{7}{9}, \frac{9}{9}$ **Change:** $+\frac{2}{9}$

   (a) $\frac{1}{16}, \frac{5}{16}, \frac{9}{16}$, ________ , ________ , ________ Change: _________

   (b) $\frac{25}{12}, \frac{22}{12}, \frac{19}{12}$, ________ , ________ , ________ Change: ________

   (c) $1\frac{1}{11}, 1\frac{5}{11}, 1\frac{9}{11}$, ________ , ________ , ________ Change: ________

2. Solve the following equations:

   (a) $\frac{3}{7} + \frac{2}{7} =$

   (b) $\frac{6}{8} + \frac{3}{6} =$

   (c) $\frac{26}{30} - \frac{3}{10} =$

3. Write the equation and solve the problem for the following stories:

   (a) Julie drove $25\frac{1}{4}$ km from home to a shopping centre, then drove $12\frac{3}{8}$ km to a friend's place, then drove $22\frac{1}{2}$ km home. How many kilometres did she drive?

   ____

   ____

   ____

   (b) Jim had $\frac{3}{8}$ of a chocolate bar. He gave $\frac{1}{3}$ of the chocolate bar to his friend. How much of the chocolate bar did Jim keep?

   ____

   ____

   ____

Obj. 6.1.1 a) □□ □□□

Obj. 6.1.2 b) □ □ □□□

Obj. 6.1.3 c) □□ □□□

Obj. 6.1.4 a) □

Obj. 6.1.6 b) □

Obj. 6.1.5 c) □

Obj. 6.1.4 a) i.□ ii.□ iii.□ iv.□ v.□

Obj. 6.1.7 b) i.□ ii.□ iii.□ iv.□ v.□

4. What is $\frac{1}{4}$ of 24? ________

5. What is 6 lots of $\frac{3}{9}$? ________

6. Solve the equations:

   (a) $\frac{3}{8} \times \frac{3}{7} =$

   (b) $2\frac{1}{9} \times \frac{2}{5} =$

   (c) $\frac{3}{7} \times 3\frac{1}{3} =$

7. Write the equation and solve the problem for the following story:

   Five friends each had $\frac{5}{8}$ of a pizza. How much pizza did they have altogether?

   ____

   ____

   ____

   ____

8. How many quarters in:

   (a) $2\frac{1}{4}$ __________ quarters

   (b) $4\frac{1}{2}$ __________ quarters

Obj. 6.2.1 ☐

Obj. 6.2.2 i. ☐ ii. ☐

Obj. 6.2.3 i. ☐ ii. ☐ iii. ☐

Obj. 6.2.4 i. ☐ ii. ☐ iii. ☐ iv. ☐

Obj. 6.2.4 i. ☐ ii. ☐ iii. ☐ iv. ☐

Obj. 6.2.2 i. ☐ ii. ☐ iii. ☐

Obj. 6.3.1 a) ☐

Obj. 6.3.1 b) ☐

9. Solve the equations:

   (a) $\frac{3}{8} \div 4 =$

   (b) $\frac{7}{8} \div \frac{1}{2} =$

Obj. 6.3.2
a)i. ☐
ii. ☐

Obj. 6.3.3
b)i. ☐
ii. ☐
iii. ☐

10. Write the equation and solve the problem:

   (a) $3\frac{1}{2}$ cakes were shared evenly between 14 students. How much cake did each student receive?

   (b) Six students in the class had mobile phones at school.
   There were 32 students in the class.
   What fraction of the class had mobile phones at school?

   ____

Obj. 6.3.2
i. ☐
ii. ☐
iii. ☐

Obj. 6.3.4
i. ☐
ii. ☐

11. Write the number words as decimal numerals for each sentence:

   (a) To make Asian noodle salad, half a wombok is needed. ________

   (b) The probability of rolling a 5 on a die is one tenth and sixty-seven thousandths. ______________________________

Obj. 6.4.4
a) ☐

Obj. 6.4.1
b) ☐

12. Write the numerals in number words for each sentence:

(a) The table was 2.4 m long. ________________________________

____

(b) The distance to the shops is 15.65 km. ______________________

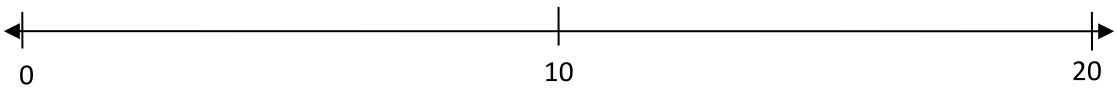
____

Obj. 6.4.1 a)i. □ ii. □ iii. □

Obj. 6.4.1 b)i. □ ii. □ iii. □

13. Rank the following numbers on a number line:

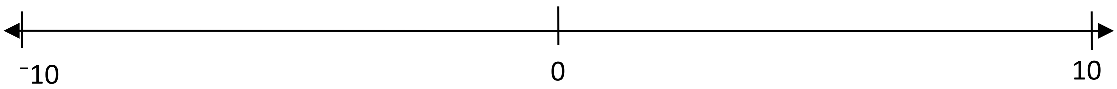
17.6, 8.1, 4.7, 12.5, 18.9, 0.3, 14.6, 7.3



Obj. 6.4.2 □□ □□ □□ □□

14. Rank the following numbers on a number line:

$^{-}5.6$, 3.1, $^{-}2.7$, 2.5, 7.9, $^{-}0.3$, 0.6, $^{-}4.3$



Obj. 6.4.2 □□ □□ □□ □□

15. Are the following numbers rational or irrational?

(a) $1.\overline{36}$ □ rational □ irrational

(b) 0.75 □ rational □ irrational

(c) $1.\overline{024831}$ □ rational □ irrational

(d) π □ rational □ irrational

Obj. 6.4.6 a) □ b) □ c) □ d) □

16. Are the following decimals terminating or recurring?

(a) $1.\overline{36}$ □ terminating □ recurring

(b) 0.75 □ terminating □ recurring

(c) $1.\overline{024831}$ □ terminating □ recurring

(d) 0.543 278 □ terminating □ recurring

Obj. 6.4.5 a) □ b) □ c) □ d) □

17. Round these numbers to three decimal places:

(a) 3.14159 ________________ (b) 0.16161616 ________________

Obj. 6.4.3
a) □
b) □

18. Write these common fractions as decimal fractions:

(a) $\frac{1}{3}$ __________________ (b) $\frac{3}{4}$ __________________

(c) $\frac{11}{8}$ __________________ (d) $\frac{3}{12}$ __________________

Obj. 6.4.4
a) □
b) □
c) □
d) □

19. Write these decimal fractions as common fractions:

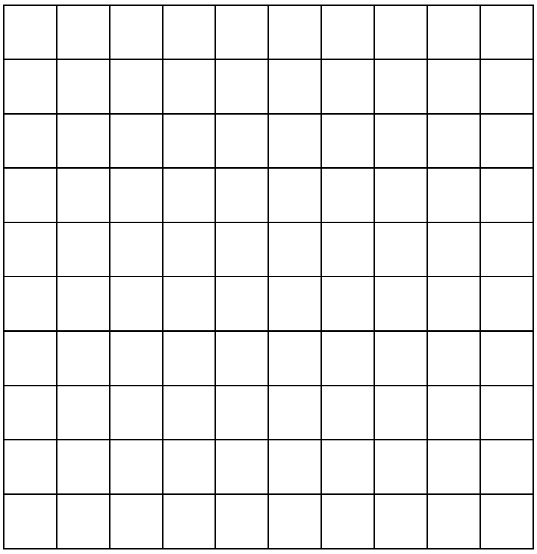
(a) 0.65 __________________ (b) 1.33 __________________

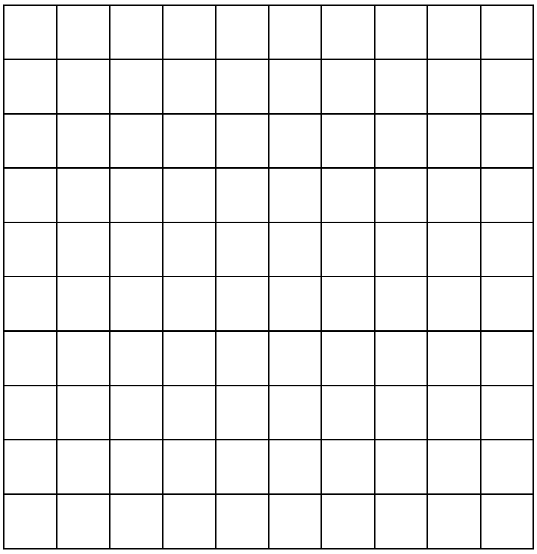
(c) 2.07 __________________ (d) 0.016 __________________

Obj. 6.4.1
a) □
b) □
c) □
d) □

20. Use 10×10 grids to show:

(a) 15 hundredths (b) 6 tenths and 8 hundredths





Obj. 6.4.1
a) □
b) □

21. Complete the patterns and write the change rule for each pattern.

(a) 1.03, 1.23, 1.43, ______, ______, ______ The change is ______

(b) 0.2, 0.25, 0.3, ______, ______, ______ The change is ______

(c) 0.6, 0.49, 0.38, ______, ______, ______ The change is ______

(d) 1.4, 1.28, 1.16, ______, ______, ______ The change is ______

Obj. 6.5.1
a)□□□□□
b)□□□□□
c)□□□□□
d)□□□□□

22. Solve the equations:

(a) $0.44 + 0.64 =$

(b) $2.43 - 0.29 =$

(c) $15.6 - 5.9 =$

(d) 0.205 + 0.75 =

23. Write the equations and solve the problems:

(a) The table was 2.25m long. A tablecloth needs to hang 0.5m over each end. How long does the tablecloth need to be?

(b) Jack has to walk 13.25km to the shops from home. The swimming pool is 18km away from home. How much further away from home is the swimming pool?

24. Solve the equations:

(a) $1.47 \times 12 =$

(b) $0.6 \div 0.5 =$

Obj. 6.5.2 a) ☐

Obj. 6.5.3 b) ☐

Obj. 6.5.3 c) ☐

Obj. 6.5.2 d) ☐

Obj. 6.5.2 a)i ☐ ii ☐ iii ☐

Obj. 6.5.3 b)i ☐ ii ☐ iii ☐

Obj. 6.6.1 a) ☐

Obj. 6.6.5 b) ☐

25.Write the equations and solve the problems:

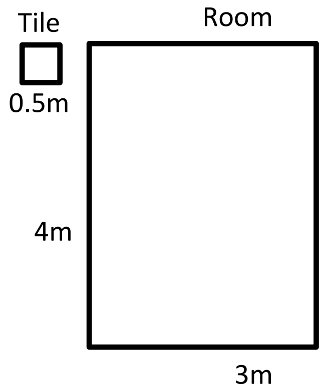
(a) The whiteboard is 2.2m long. What is the measurement to mark halfway along the board?

Obj. 6.6.2
a)i □
ii □
iii □

(b) A wall is 5.4m long. Six plaster sheets are covering the wall. How wide is each sheet of plaster?

Obj. 6.6.4
b)i □
ii □
iii □

26.A room is 3m wide and 4m long.
Square floor tiles are 0.5m across.



(a) How many tiles will fit across the room?

(b) How many tiles will fit along the room?

Obj. 6.6.3
a)i □
ii □
iii □
b)i □
ii □
iii □

27. Jellybeans in bulk cost $7.50 for 1000 g.

(a) What fraction of 1000 g is 350 g?

Obj. 6.6.6
a)i ☐
ii ☐
iii ☐
b)i ☐
ii ☐
iii ☐

(b) If you buy 350 g of jellybeans, how much will they cost?

28. Another market sells jellybeans in 250g bags for $1.80.

(a) What fraction of 1000 g is 250 g?

Obj. 6.6.6
a)i ☐
ii ☐
iii ☐
b)i ☐
ii ☐
iii ☐
c) ☐

(b) How much will 1000 g of jellybeans cost from this market?

(c) Is it cheaper to buy 1000 g of jellybeans in bulk or in 250 g bags?

____

29. Fill in the following conversions:

(a) 25 cm = __________ mm

(b) 6 m = __________ cm

(c) 4300 m = __________ km

(d) 45 mm = __________ cm

Obj. 6.7.1
a) ☐
b) ☐
c) ☐
d) ☐

30. Write the following numbers using scientific notation.
For example: 364 000 = 3.64×10^5

(a) 3 350 ________

(b) 0.095 ________

Obj. 6.7.2
a) □
b) □

31. What is the value of:

(a) 2.45×10^2 ____________

(b) 43×10^{-1} ____________

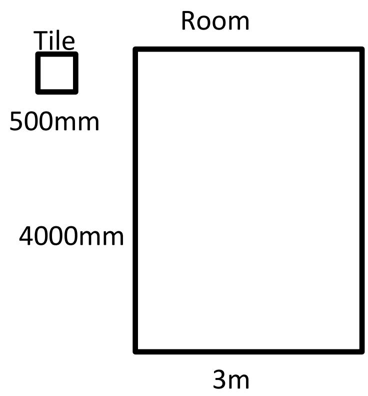
(c) $0.2 \div 10^2$ ____________

(d) $10\ 450 \div 10^2$ ____________

Obj. 6.7.3
a) □
b) □

Obj. 6.7.4
c) □
d) □

32. A room is 3 m wide and 4000 mm long.
Square floor tiles are 500 mm across.



(a) How many tiles will fit across the room?

(b) How many tiles will fit along the room?

(c) How many tiles needed to cover the floor?

Obj. 6.7.1
a)i □
ii □
iii □
b)i □
ii □
iii □
c)i □
ii □
iii □

XLR8 – Accelerating Mathematics Learning

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