XLR8 – Accelerating Mathematics Learning

XLR8 Unit 05

Dealing with remainders

05

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

Contents

List of Figures

XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend their investigations of partitioning and quotitioning to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning. This exploration will use features from the immediate environment of the students and of larger populations in their state, country or world. Continuous measures such as length provide useful contexts for partitioning and quotitioning.

Scope

This unit is based upon the **number-as-measure** meaning of **cardinal** number. **Partitioning** or **quotitioning** of **collections** or **entities** that do not result in whole numbers may be annotated as **remainders.** Alternatively, they can be further partitioned into **parts** or **fractions** or **smaller units**.

Continuous attributes of an entity such as **length**, **mass** and **capacity** can be partitioned into **units** for **counting**. These units can be **non-standard units** or **standard metric units**. Each of these **units** can be further **partitioned** as a result of **division**. The **parts of the whole** can be **represented** using **fraction notation**.

Collections of items can also be **partitioned** into **part-sized groups**. **Part-sizes** can also be **represented** as **fractions** of the **whole collection**. **Parts of wholes** that are the **same-size fraction** can be **counted** or **grouped in multiples of the same size** (as experienced with whole numbers).

One-to-many pictographs also result in part-sized groups when representing data. When linked to measurement and number-line representations, this idea may be connected to **scaling of graph axes**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P05: Time for a BBQ* accompanying Unit 05, engages students with exploring fractions of discrete items and fractions of collections generated when preparing for a BBQ.

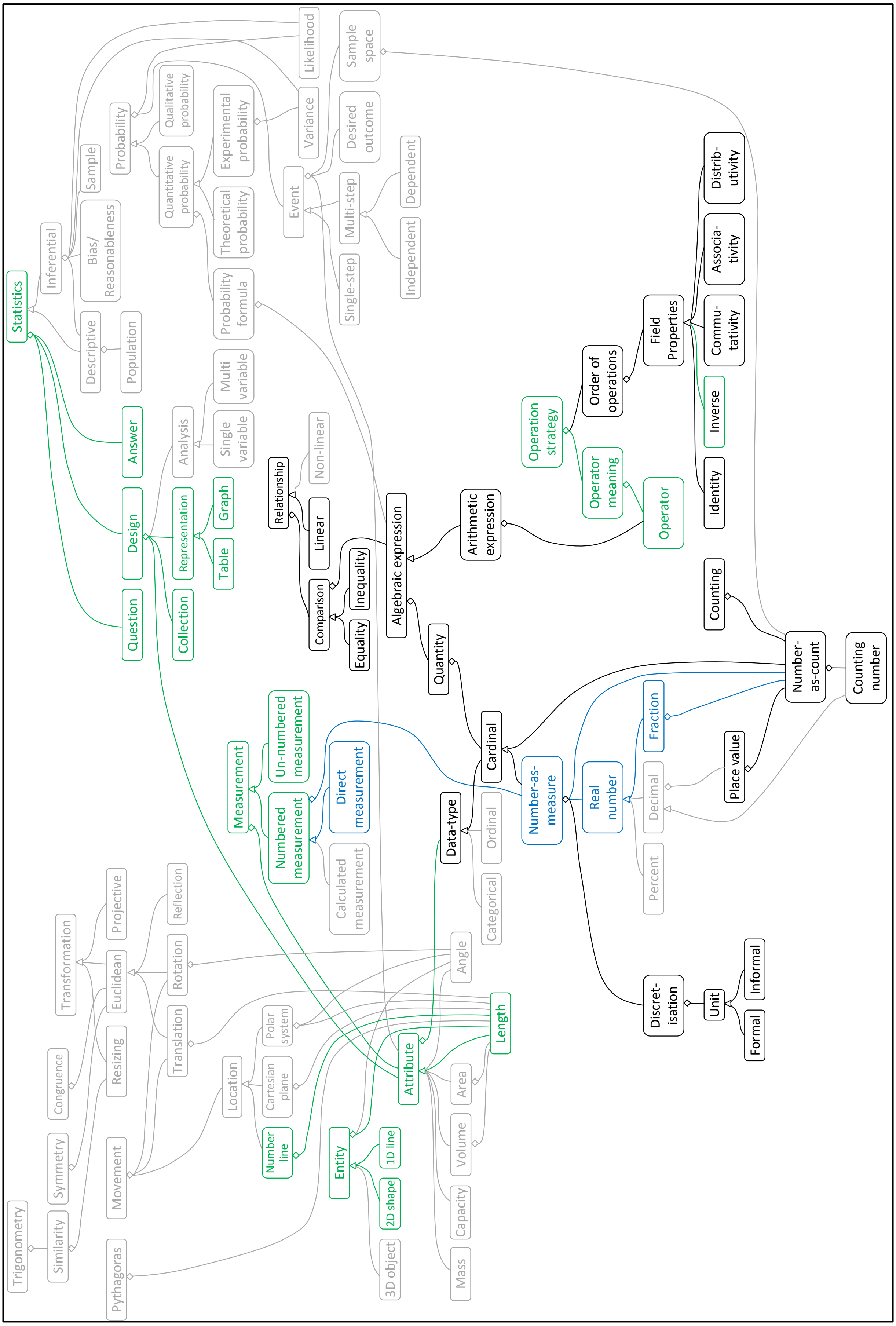


Figure 1 Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following suggested sequence:

Cycle 1: Partitioning One Whole

In this cycle, the partitioning or sharing of a whole (using area and length models) will be explored as a means to deal with remainders. These remainders represent part of one thing that can be further shared by partitioning (for example, 1 cake left over). This will lead to the consideration of contexts where the remainder is more than one thing to be partitioned (e.g., 2 or more cakes left over) which will then extend to considering multiples of unit fractions, counting in fractions to improper fractions and mixed numbers.

Cycle 2: Comparing and Ordering Fractions Using Equivalence

As for whole numbers, fractional quantities can be compared and ordered. Equivalent fractions result when a multiple of a unit fraction of one denominator coincides with a multiple of a unit fraction with another denominator. This is most commonly seen and recognised with related denominators such as halves, quarters and eighths, although there are strategies to make equivalent fractions for any given selection of unrelated fractions. This is also related to the phenomenon of density that occurs with fractional quantities, where there are infinite possibilities for fractional quantities between any two given whole numbers.

Cycle 3: Dividing Sets

In this cycle, remainders of collections are explored as parts of a whole group (using set models). Where the number of groups is known and the size of the groups is unknown (e.g., when separating 27 people into 6 groups, there will be 6 groups of 4 with 3 out of 6 left over), the fractional representation of the remainder connects to partitioning division. For example, 27 people shared into 6 groups is equivalent to 27 ÷ 6 is equivalent to $\frac{27}{6}$ or 4 people in each group with 3 out of 6 extra or $4\frac{3}{6}$ people per group). We don't want to have half a person per group, but 3 out of the 6 groups can potentially have an extra person or 3 people are left over. Where the size of the group is known and the number of groups is unknown, the fractional representation of the remainder connects to quotition division. Remainders resulting from this separation may be represented as a fraction of the unit sized group so we have 4 groups of 6 and 3 people out of a group of 6 left over or $4\frac{3}{6}$. In this instance we can happily express the remainder as half a group left over.

Cycle 4: Dividing Continuous Quantities

The focus of this RAMR cycle is the separation of a continuous whole into unit-sized pieces that may then be counted and the remainder described as a fraction of the unit-sized piece. This idea is the basis of measurement of length into units that may be described using fraction notation. When comparing continuous attributes like length, area, mass, capacity and volume, only qualitative judgements can be made unless items' attributes are quotitioned into uniform units to facilitate a counting process. Indirect comparison using uniform informal units is the third stage in the sequence of development of measurement understanding.

Unit 5: Metric Measure

The focus of this RAMR cycle is the use of breaking a continuous whole into unit-sized pieces that may then be counted, and the remainder further broken into smaller size units. This idea is the basis of measurement of length into units of a set size, with the remainder further broken into smaller units. This idea becomes the basis of metric measure (e.g., $5\frac{1}{2}$m, is equivalent to 5m and 50cm or 5m 500mm).

Literacy Development

Language that is consistent with the organisation of the mathematical concepts is core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic). In this unit, the following key language should be explicitly developed with students to ensure that they understand both the everyday and mathematical uses of each term and where applicable, the differences and similarities between these categories.

Cycle 1: Partitioning One Whole

Fraction, whole, part, sharing, partitioning, numerator, denominator, vinculum, division, same-size pieces, ordinal number names for denominators (e.g., third, fourth, fifth …), half, quarter

Cycle 2: Comparing and Ordering Fractions Using Equivalence

Greater than (>), less than (<), equivalent, equivalence, multiples, lowest common denominator, factors

Cycle 3: Dividing Sets

Fraction, whole, part, sharing, partitioning, numerator, denominator, vinculum, division, multiplication, same-size groups, part of a group, mixed numbers, improper fractions

Cycle 4: Dividing Continuous Quantities

Attribute, length, width, height, breadth, compare, direct, indirect, measure, continuous, distance, perimeter, side, edge, long-short-tall-wide(er, est), etc.

Unit 5: Metric Measure

Metric length measures, prefixes and abbreviations: metre (m), centimetre (cm), millimetre (mm), kilometre (km), ruler, tape measure, place value, measuring tape, tape measure, units of measure

Name:______________________________________ Date:________

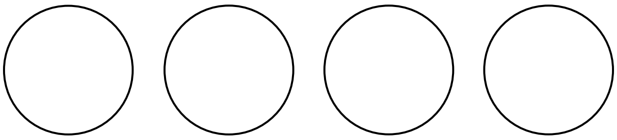
Can you do this? #1

1. Write down or draw two more examples of fractions. One has been done for you.

   (a) half a cup in cooking (b) ______________ (c) ______________

Obj. 5.1.1
b) □
c) □

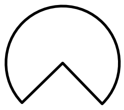
2. If these four cakes were shared amongst 5 people what fraction of cake would each person get? ___________________



Obj. 5.1.1
i.□
ii.□

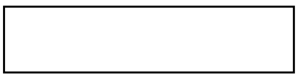
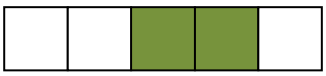
3. Represent one third ($\frac{1}{3}$) on each of the following diagrams:

(a) 

(b) 

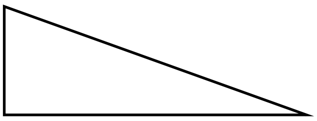
Obj. 5.1.2
a) □
b) □

4. What fraction is shown by each of the following diagrams? Fill in the missing parts on the table.

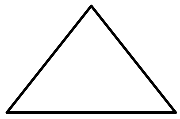
(a) 	Words one quarter	Fraction __________
(b) 	Words ___________	Fraction $\frac{1}{8}$
(c) 	Words ___________	Fraction __________

Obj. 5.1.2
a)i. □
b)i. □
Obj. 5.1.3
a)ii. □
b)ii. □
Obj. 5.1.4
c)i. □
c)ii. □

5. (a) This shape is half $(\frac{1}{2})$ of a whole. Draw the whole shape.



(b) This shape is two thirds $(\frac{2}{3})$ of a whole. Draw the whole shape.



Obj. 5.1.4
a) □
b) □

6. Write the fraction or number that is one fifth $(\frac{1}{5})$ more than:

(a) $\frac{2}{5}$ __________ b) $\frac{4}{5}$ __________

Obj. 5.1.5
a) □
b) □

7. Complete the counting sequence and write down the change as a fraction.

(a) $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, ________, ________, ________ __________

(b) $3\frac{2}{4}$, $3\frac{3}{4}$, 4, ________, ________, ________ __________

Obj. 5.1.6
a)□□□
b)□□□
Obj. 5.1.7
a)iv□
b)iv□

Cycle 1: Partitioning One Whole

Overview

Big Idea

In this cycle, initial ideas of partitioning or sharing a whole (using area and length models) will be explored as a means to deal with remainders of one thing that can be further shared by partitioning (for example, 1 cake left over). This will lead onto considering contexts where the remainder is more than one thing to be partitioned (e.g., 2 or more cakes left over) which will then extend to considering multiples of unit fractions, counting in fractions to improper fractions and mixed numbers.

Objectives

By the end of this cycle, students should be able to:

5.1.1 Recognise and interpret common uses of fractions of whole items. [2NA033]

5.1.2 Represent unit fractions as a picture. [3NA058]

5.1.3 Represent unit fractions symbolically. [3NA058]

5.1.4 Represent multiples of unit fractions to a complete whole. [3NA058]

5.1.5 Count by fractions. [05NA078]

5.1.6 Continue and create sequences involving whole numbers and fractions (mixed numbers). [6NA133]

5.1.7 Identify the change used to continue and create counting sequences in fractions. [6NA133]

Conceptual Links

This cycle relies on previously developed counting skills as well as the ability to identify an item or attribute that may be partitioned into smaller pieces.

In Cycle 2 of this unit, fractions will be represented on a number line (partitioned length model) and extended to explore equivalent fractions. These ideas connect to division of continuous quantities in Cycle 4 and metric measure in Cycle 5.

Materials

For this cycle you may need:

- paper materials (A4, square, circle, strip)
- chalk
- masking tape
- fraction wheels
- fraction mat
- fraction representation games
- counters
- chocolate cake game
- think board template
- fraction sticks
- unifix
- whole part charts

Key Language

Fraction, whole, part, sharing, partitioning, numerator, denominator, vinculum, division, same-size pieces, ordinal number names for denominators (e.g., third, fourth, fifth …), half, quarter

Definitions

denominator: represents the number of pieces the whole has been partitioned into; names the fraction; numeral under the vinculum

numerator: the count of pieces of the whole within the share or remainder; numeral above the vinculum

partitioning: breaking or sharing of a whole into same-size parts

vinculum: line between the numerator and denominator on a common fraction

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Are the pieces the same size?
- How many pieces are there in the whole? What number is the whole being divided by? How many are you sharing the whole with? How many pieces do you need to partition the whole by?
- What name would you give that fraction? What is the denominator?
- How many pieces out of the whole do you have? What is the numerator?
- What is the fraction name? What are you counting by?

Portfolio Task

Ideas from this unit will enable students to recognise common fractions used to partition whole food items within *Portfolio Task P05: Time for a BBQ*.

RAMR Cycle

This RAMR cycle focuses on dividing a whole into equal parts where the whole is an area. To do this, we use two notions that underlie the teaching of fraction, *partitioning*, making parts out of a whole; and *unitising*, making a whole out of parts. This leads to formal fraction notation which can be extended to include improper fractions and mixed numerals.

Reality

Use contexts that can be modelled with lengths or areas from the local culture of students or the local environment as things that can be cut into fractions. For example, engage students to identify instances where they use fraction language like half a glass of water, half way home, a quarter of a sandwich, cut up fruit, pizzas, cakes, a half in a sporting fixture, half-way lines on the sporting field, over a third in netball. Discuss what these mean.

Also discuss colloquial uses of fraction language that are not strictly fractions like the top half of a car being from the bottom of the windows up, the bigger half of an apple, and other instances where the pieces are not the same size but are referred to as a fraction based on the number of pieces. Clarify that for pieces of something to be fractions, each piece must be exactly the same size.

Abstraction

The abstraction sequence for this cycle begins with physically covering a fraction of an area or finding a fraction of a length. Students then represent this part of a whole quantity symbolically and with language to establish the meaning and representation of fractions. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Position students to cover the shooting circle on the basketball court. Next, shift students to cover only half the shooting circle on the basketball court. Alternatively, act out sharing a cake/pizza into equal sized pieces. Discuss what are fair/unfair places to make cuts to share with 2, 4, 8 … whole class. Discuss what each share represents (e.g., 1 whole cake shared with 4 people is 1 piece each out of 4 pieces).
2. *Represent/model with materials.* Use newspaper sheets to cover half a maths mat (or drawn grid, whiteboard, window). Alternatively, use paper to model sharing cake/pizza into equal parts.
3. *Connect to language.* Stress the whole and the fraction part of the whole. Discuss that numbers always need a label or name to clearly identify the unit or thing to be counted. Discuss the naming conventions for fractions in language (e.g., 1 cake shared with 4 people means we cut 4 pieces of cake the same size and each person has 1 piece out of 4 pieces).
4. *Represent with symbols.* Connect to the symbolic representation of fractions. Introduce the mathematical language for each part of the fraction (denominator: number of pieces in the whole; numerator: number of pieces you have; vinculum: line in the middle that divides or separates the numerator from the denominator). Count the pieces as counting number and unit name (e.g., one quarter, two quarters, three quarters, four quarters or one whole).

Resource Resource 5.1.1 Creating Fractions by Paper Folding

5. Repeat the steps to create a range of fractions. Start with unit fractions and progress to situations where you might have 2 out of 5 pieces.
6. *Reverse the activity.* Engage students with activities where they are given a fraction and need to find or show the whole. For example, draw half a cake and ask them to show the whole cake.

Mathematics

Language/symbols and practice

Sharing more than one thing

Explore sharing where there is more than one of the same item to be shared. For example, sharing two pizzas with four people; how many pieces each? There is more than one way to partition pizzas in this case. Cut each pizza in half so there are four pieces in total or share each pizza into four pieces and give each person a piece from each pizza. Discuss real world situations with students that may be relevant (e.g., if pizzas are the same flavour it makes sense to halve each one; if pizzas are different flavours, it makes sense for each person to have a piece of each). When sharing multiple items, it is often simpler to partition each whole, give each person a share from each whole, then count how many pieces each person has.

Connections

Connect sharing more than one thing to remainders found in division problems. Explore some of these problems with students and practice sharing wholes and then partitioning the remainder into fractions. For example, 6 cakes shared with 4 people, how many slices per person? Discuss strategies for working with this problem. Strategies may include sharing wholes first and then partitioning remainders, or partitioning everything, sharing the parts and counting to name the fraction (which may then need to be simplified). *Resource 5.1.2 Whole-part charts* are useful for reunitising improper fractions to mixed numbers. *Resource 5.1.3 The Chocolate cake game* is an effective way to engage students with sharing multiple wholes among people.

Resource

Resource 5.1.2 Whole-Part Charts
Resource 5.1.3 The Chocolate Cake Game

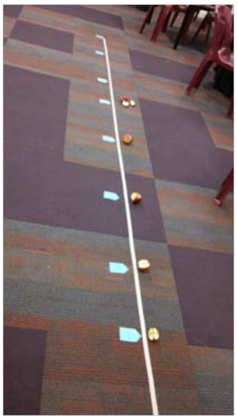
Counting in fractions

As for whole numbers, fractions can be counted. For example, $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$. Ensure that students can continue counting sequences forward and backward when counting in fractions. Also extend this idea by counting in mixed numbers.

Activity: Apples for the Teacher

Materials: Tape, 6-8 small apples each almost cut into quarters but not separated, cards for each of the quarter fraction names up to 2 wholes.

1. Begin with 0 apples, putting the card at the base of the line. Ask students to find one quarter and place both the quarter apple and the fraction numeral along the line.
2. Once over one, mixed numbers are required and can be discussed. Continue like this to place all the pieces of apple until you run out. When the supply of apples runs out, encourage the students to identify what pieces of apply they need to create the mixed number. For example, $2\frac{3}{4}$ apples can be made up from 2 whole apples, a half apple and a quarter apple (other combinations also viable).



Some additional teacher notes and student activities for counting with mixed numbers and improper fractions are included in *Resource 5.1.4 Mixed numbers* and *Resource 5.1.5 Improper fractions*.

Resource

Resource 5.1.4 Mixed numbers
Resource 5.1.5 Improper fractions

Connect division stories to fractions

Connect division problems to fractions. Consider the problem of sharing 5 cakes with 4 people as a division problem, 5 ÷ 4, which results in 1 cake each with 1 cake left over. The left over cake can then be partitioned as one cake shared with four people, which is 1 quarter of a cake each. Using the idea of fractions, introduce the connection between division and fractions. Represent the division story of sharing 1 cake with 4 people as $\frac{1}{4}$. Connect the language and the idea that sharing or dividing by 4 is the same as finding a quarter of the whole. Extend this idea to other division/fraction connections. Ensure that students can work between recognising one quarter as 1 ÷ 4 as well as $\frac{1}{4}$ of an item.

Reflection

Check the idea

Thinkboards may be used by students to identify context or story, language, symbol and representation with materials for a given fraction.

Resource 5.1.6 Fraction representation games may be used to assist students with their practice of connecting language, symbol and representation.

Resource Resource 5.1.6 Fraction representation games

Apply the idea

Find items other than pizzas and cake that can be partitioned. Consider partitioning lengths of wood, ribbon or wire for specific tasks; other ideas may be partitioning a garden area into sectors for planting or partitioning a yard into smaller enclosures for livestock.

Extend the idea

Extend students' understanding of specific numeric fractions to more general cases. Use stories and real contexts to connect the idea of an unknown numerator shared into parts. For example, 3 items shared with 4 gives $\frac{3}{4}$ each, 1 item shared with 4 gives $\frac{1}{4}$ each, 7 items shared with 4 gives $\frac{7}{4}$ each, any number (*n*) of items shared with 4 gives $\frac{n}{4}$ each. Once students are comfortable with a variable or algebraic unknown for the numerator, use the same process to explore an unknown or variable denominator. Extend students' thinking to the complete generalisation; if you break a whole into q equal parts and shade p of them, the fraction is p $q$ths or $\frac{p}{q}$.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

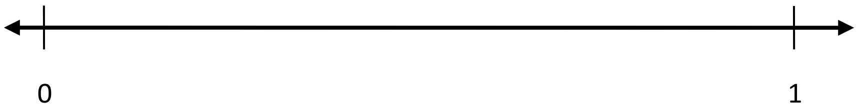
Can you do this? #2

1. How many quarters in:

   a) $3\frac{1}{4}$________ quarters b) $4\frac{1}{2}$___________ quarters

Obj. 5.2.6 a) ☐ b) ☐

2. Place these fractions on the number line: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$



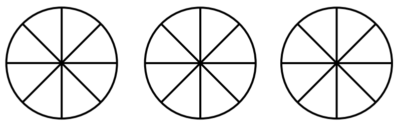
Obj. 5.2.1 i. ☐ ii. ☐ iii. ☐ iv. ☐ v. ☐

3. Draw a diagram to show $1\frac{2}{5}$ as chocolate bars.

Obj. 5.2.3 i. ☐ ii. ☐

4. Sammy ordered 10 slices of pizza. A whole pizza has eight slices.

   (a) Shade the number of pizza slices Sammy has on the outlines.



   (b) Write down as a mixed number, how much of a pizza Sammy has. ________________________

Obj. 5.2.3 a) ☐ b)i. ☐ ii. ☐

5. Write the following fractions in words:

   (a) $3\frac{3}{4}$ ____________________ (b) $\frac{2}{6}$ ______________________

Obj. 5.2.3 a) ☐ Obj. 5.2.1 b) ☐

6. Look at the point on the number line. Name this fraction in words and as a common fraction.

____________________ ________

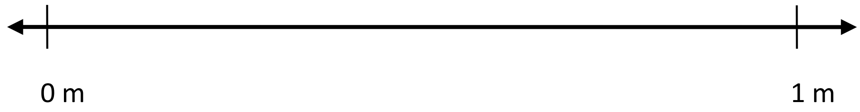
Obj. 5.2.1 i. ☐ ii. ☐

7. A 1m long string of liquorice was shared equally between 5 friends.

   (a) What fraction of the string of liquorice would each friend receive?

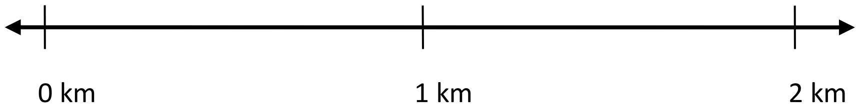
   ____

   (b) Use the number line to show the pieces of liquorice.



Obj. 5.2.3
a) ☐
b) ☐

8. Ben drove one and a half kilometres from his house to the park. Mark this distance on the number line.



Obj. 5.2.3
☐

9. Write the following as mixed numbers, i.e. 4 thirds = $1\frac{1}{3}$ or $\frac{4}{3} = 1\frac{1}{3}$

   (a) 7 fifths = ______________ b) $\frac{8}{7}$ = ______________

   (c) 10 quarters = ______________ d) $\frac{7}{2}$ = ______________

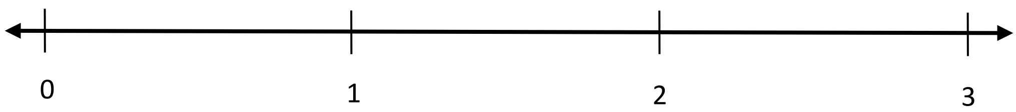
Obj. 5.2.6
a)☐
b)☐
c)☐
d)☐

10. (a) Write the following mixed numbers and improper fractions in order from smallest to largest:

$$\frac{12}{5}, \quad 1\frac{2}{5}, \quad \frac{6}{2}, \quad 1\frac{2}{4}, \quad \frac{11}{4}$$

____

   (b) Locate each of the mixed numbers and improper fractions on the number line.



Obj. 5.2.5
i.☐
ii.☐
iii.☐
iv.☐
v.☐

Obj. 5.2.4
i.☐
ii.☐
iii.☐
iv.☐
v.☐

Cycle 2: Comparing and Ordering Fractions Using Equivalence

Overview

Big Idea

As for whole numbers, fractional quantities can be compared and ordered. Equivalent fractions result when a multiple of a unit fraction of one denominator coincides with a multiple of a unit fraction with another denominator. This is most commonly seen and recognised with related denominators such as halves, quarters and eighths, although there are strategies to make equivalent fractions for any given selection of unrelated fractions. This is also related to the phenomenon of density that occurs with fractional quantities, whereby there are infinite possibilities for fractional quantities between any two given whole numbers.

Objectives

By the end of this cycle, students should be able to:

5.2.1 Name, locate and represent common fractions on a number line. [6NA125]

5.2.2 Compare and order common fractions. [7NA125]

5.2.3 Name and represent improper fractions and mixed numerals using set or area models. [7NA152]

5.2.4 Locate and represent improper fractions and mixed numerals on a number line. [7NA152]

5.2.5 Compare and order improper fractions and mixed numerals. [4NA078]

5.2.6 Convert between mixed numbers and improper fractions. [6NA133]

Conceptual Links

An understanding of common fractions is an important prerequisite to comparing, ordering and representation of fractions on a number line and equivalent fractions. Students must understand the concept of equal-sized parts of a whole. Students must also understand that the count of pieces is represented by the numerator, and that the count of pieces in the whole is represented by the denominator.

Equivalent fraction understandings are used to facilitate additive operations with fractional quantities. Fraction concepts are also used to describe probabilities where likelihood of events are expressed using values between 0 and 1.

Materials

For Cycle 2 you may need:

- counters or unifix cubes
- rope
- fraction mats
- fraction baseboards
- pegs
- fraction cards
- fraction sticks

Key Language

Greater than (>), less than (<), equivalent fractions, equivalence, multiples, lowest common denominator, highest common factor, factors, unit fractions

Definitions

Equivalent fractions/equivalence: have the same value even though they may look different

Factor: numbers which multiply together to give another number

Highest common factor: when factors are determined for a pair of numbers, common factors may arise. The highest common factor is the largest of these.

Lowest common denominator: the smallest number that can be used as a denominator for 2 or more fractions.

Unit fractions: fractions with a numerator of 1.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Is there a closed boundary?
- What geometric 2D shape does that surface look like?
- How many sides does the shape have?
- How many angles does the shape have?
- What could you name this shape?

Portfolio Task

Student portfolio task *P4: Time for a BBQ* may be given to students as an independent assessment task to complete in class or as homework. This task involves students comparing and ordering fractional quantities of ingredients to determine which ingredient represents the greater fraction within the recipe.

RAMR Cycle

This RAMR cycle focuses on comparing and ordering fractions starting with unit fractions. Where fractions have different denominators and are not unit fractions, it is necessary to develop an understanding of equivalence to effectively compare and order fractions.

Reality

Use contexts that can be modelled with lengths or areas from the local culture of students or the local environment as things that can be partitioned into fractions. Ensure that students understand that the relative size of a fractional piece will depend on the relative sizes of the wholes. For example, half of a small pizza will not be the same size as half of a large pizza. This understanding is important when considering comparison and ordering of fractions.

Once students understand the importance of comparing fractions of a same size whole, explore real instances of comparison of unit fractions. For example, identify one half, one third and one quarter on a netball field. Record these fractions on a sketch for later consideration. Alternatively, partition the length of the whiteboard into halves, thirds, quarters, fifths. Identify which fractional pieces are larger or smaller.

Abstraction

The abstraction sequence for this cycle takes students from an experience of physically comparing unit fractions, to representing this part of a whole quantity symbolically and with language. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Experience physical comparison of unit fractions by marking halves, thirds, quarters on the netball court. Alternatively, these fractions can be marked along the window section of the classroom or the whiteboard.
2. *Represent with symbols.* Label each of these unit fractions symbolically.
3. *Connect to language.* Discuss the relative size of each fractional piece. Ensure that students can generalise a pattern with respect to the denominator of the fraction and the size of the piece. Students should intuitively recognise that the larger the denominator, the smaller the piece.
4. *Model/represent on a number line.* Provide students with strips of paper to create fractional lengths from. Use the folded paper to measure and mark along a number line. Extend students to partitioning a line into the appropriate number of pieces and labelling fractions along a line.
5. *Reinforce working with symbols.* Provide students with opportunities to practice ordering unit fractions according to size (e.g., $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5}$ and so on). Attaching fraction cards to a washing line or placing cards along the whiteboard can be a means of practising this skill. Fraction dominoes (*Resource 5.2.1: Comparing and ordering fractions*) can be played to practise this skill but gamecards should be limited to unit fractions at this stage.
6. Extend the activity to compare and order fractions with like denominators and unlike numerators (e.g., $\frac{5}{6} > \frac{4}{6} > \frac{3}{6} > \frac{2}{6}$ and so on).
7. Extend the activity to compare and order fractions with like numerators and unlike denominators (e.g., $\frac{4}{6} > \frac{4}{7} > \frac{4}{8} > \frac{4}{9}$ and so on).

Resource Resource 5.2.1 Comparing and ordering fractions

Mathematics

Language/symbols and practice

Density with fractions

Density is a phenomenon that occurs with fractional quantities and applies most easily to fractions and decimals. Explore *density* with fractions by starting with a long length of rope (or use the length of a playing field as a unit length, or a long line on the whiteboard or floor). Label each end with a whole number (e.g., 3 and 4). Find the halfway point along the line and mark this point, identify the fraction (e.g., $3\frac{1}{2}$). Halve one section again and label the marked fractions of the whole. Choose one section only to continue partitioning and labelling fractions. Discuss with students if there is a limit to the fractions that can be made. Repeat the process with thirds, fifths, mixed fractions to consolidate students' understanding that there are an infinite number of fractions that may be found between any two locations on a line (although they do become smaller than human perception).

Finding equivalent fractions

Start with a real context like cutting cake. If we have a cake and cut it into halves, two people can have one piece each. If we cut the same cake into four pieces, two people can have two pieces each. Does each person end up with the same total amount of cake? Students will have already experienced this as it is quite natural when creating quarters to create a half and then halve again. To capitalise on this experience, intentionally create a half by folding paper and shade one half. Fold the paper again and record the new fraction that is shaded ($\frac{2}{4}$). Fold again to find the equivalent fraction in eighths. Explore a range of these fractions each time naming and labelling the equivalent fractions. Ask students to explain what happened each time they refolded their paper (e.g., when we folded the paper we doubled the number of pieces the whole was broken into and also doubled the number of pieces that were shaded). Another way to explore this is using fraction baseboards and fraction walls. Look for the pattern in the fractions and develop a generalised rule to create equivalent fractions. Ensure students can multiply a fraction to find equivalent fractions with larger denominators. They must also be able to use division of the numerator and denominator to find equivalent fractions with smaller denominators. Baseboards and overlays are also useful resources for generating and exploring equivalent fractions.

Activity: Bases and Overlays

Materials: square, plastic overlays (number of fractions=number of overlays)

1. Take a square, with part of it shaded (e.g., half), this is a 'base'.
2. Use a number of plastic see-through overlays of the square that are marked into halves, thirds, quarters, fifths, etc. but not shaded.
3. If the overlays are put on top of the base, a sequence of equivalent fractions is determined by how the overlay cuts up the half.

This can also be done using a digital representation (PowerPoint).

Resource

Resource 5.2.2 Fraction PowerPoint
Resource 5.2.3 Fraction Baseboards

Resource 5.2.4 Equivalent fractions provides some further ideas and resources for teaching equivalent fractions. *Resource 5.2.5 Fraction sticks* are useful symbolic patterning devices to assist with generating equivalent fractions from symbols and generalising a rule for working symbolically.

Resource Resource 5.2.4 Equivalent Fractions
Resource 5.2.5 Fraction Sticks

Connections

Lowest common denominators

Connect students' exploration of equivalent fractions and comparison of common denominators to finding common factors from *Unit 3: Multiplicative change of quantities*. Provide students with practice finding equivalent fractions using common denominators, and extend this to finding the lowest common denominator. For example, finding a common denominator for halves and tenths, it is possible to use twentieths to consistently apply a rule, but the lowest common denominator will be tenths.

Some useful worksheet resources for practising comparison of fractions, equivalent fractions and lowest common denominators may be found on the Math-aids website (http://www.math-aids.com/Fractions/).

Reflection

Check the idea

Once students can determine equivalent fractions, ensure they can apply this skill to comparing and ordering fractions with unlike denominators and numerators. The earlier fraction dominoes game can now be played with all gamecards in use. Encourage students to use working paper to jot down working to facilitate comparison of fractions.

Resource 5.1.6 Fraction representation games may be extended to include number line cards to ensure that students can successfully switch between symbols, language and models for fractions.

Resource Resource 5.2.6 Number line cards for fraction representation games

Apply the idea

Identify a real life application of common fractions that requires students to understand equivalence, order and comparison. For example, ask students to explore imperial measures for spanners and drill sizing. Investigate problems where these need to be sorted into size order, or give a specific spanner size and ask students to identify a spanner size up or down.

Extend the idea

Cycle 1 included exploration of mixed numbers and improper fractions. Extend students' work with equivalent fractions, comparing and ordering to include mixed numbers and improper fractions. For example, $1\frac{1}{2}$ is equivalent to $\frac{3}{2}$ is equivalent to …

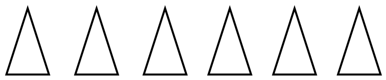
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

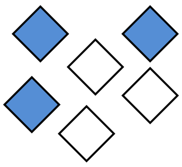
Can you do this? #3

1. Show one third ($\frac{1}{3}$) of this collection of triangles.



Obj. 5.3.1 ☐

2. Look at the collection of shapes. Name the fraction of the collection that is shaded in words and as a common fraction.



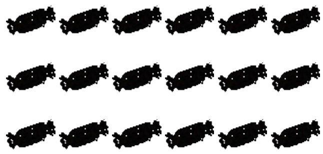
______________________________ ______________

Obj. 5.3.1 i. ☐ ii. ☐

3. This is 1 and a half ($1\frac{1}{2}$) packets of pencils. Circle how many pencils are in 1 packet.

Obj. 5.3.1 ☐

4. This is three fifths ($\frac{3}{5}$) of a bag of lollies. Draw the missing lollies to make one whole bag of lollies.



Obj. 5.3.1 ☐

5. What is $\frac{1}{5}$ of 25? ______________

Obj. 5.3.2 ☐

6. There are groups of 4 students in the classroom.
   Each group of students is $\frac{1}{5}$ of the whole class.
   How many students in the class? ______________________________

Obj. 5.3.2 ☐

7. The school has 24 basketballs in the storeroom. You take 6 basketballs out for a class game. What fraction of the total number of basketballs did you take? ______________________________

Obj. 5.3.3 i. ☐ ii. ☐

8. There are 24 basketballs in the school. Only 18 basketballs are left in the storeroom. What fraction of the total number of basketballs are left in the storeroom? ______________________________

Obj. 5.3.3 i. ☐ ii. ☐

Cycle 3: Dividing Sets

Overview

Big Idea

In this cycle, remainders of collections are explored as parts of a whole group (using set models). These should be connected to partitioning division and quotition division from Unit 3.

Where the number of groups is known and the size of the groups is unknown (e.g., when separating 27 people into 6 groups, there will be 6 groups of 4 with 3 out of 6 left over), the fractional representation of the remainder connects to partitioning division explored in Unit 3. This idea can also be connected to mixed numbers, improper fractions and the division meaning of fractions (e.g., 27 people shared into 6 groups is equivalent to 27 ÷ 6 is equivalent to $\frac{27}{6}$ or 4 people in each group with 3 out of 6 extra or $4\frac{3}{6}$ people per group). We don't want to have half a person per group, but 3 out of the 6 groups can potentially have an extra person or 3 people are left over.

Where the size of the group is known and the number of groups is unknown, the fractional representation of the remainder connects to quotition division explored in Unit 3. Remainders resulting from this separation may be represented as a fraction of the unit sized group (e.g., 27 people separated into groups of 6 would be 4 groups of 6 and 3 people out of a group of 6 left over or $4\frac{3}{6}$). In this instance we can happily express the remainder as half a group left over.

Objectives

By the end of this cycle, students should be able to:

5.3.1 Recognise and interpret fractions of collections of items. [2NA033]

5.3.2 Find a simple fraction of a quantity where the result is a whole number. [6NA127]

5.3.3 Express one quantity as a fraction of another. [7NA155]

Conceptual Links

Sharing collections into groups links directly to Unit 3 division operations, providing opportunities to explore how to deal effectively with remainders resulting from partitioning or quotition division problems.

Sharing collections into groups and representing the remainder as a part of the group extends further into quotition division for measuring length with units in Cycles 4 and 5. Representing part of a collection as a fraction is also an important understanding for the representation of chance or likelihood as a fraction ($\frac{desired\ outcome}{sample\ space}$).

Materials

For Cycle 3 you may need:

- unifix cubes
- chalk
- counters
- fraction cards (2 colours)
- think board template – per student

Key Language

Fraction, whole, part, sharing, partitioning, quotitioning, numerator, denominator, vinculum, division, multiplication, same-size groups, part of a group or collection, mixed numbers, improper fractions

Definitions

Partitioning: sharing division where the number of groups for division is known and the group size is not.

Quotitioning: division where the size of groups for division is known and the number of possible groups from the total is unknown.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many in the total?
- How many groups are you sharing into?
- What is the size of each group once you have finished sharing?
- So what fraction of the total is each group?
- What is the size of each group supposed to be?
- How many of that size group can you make?
- So what fraction of the total is each group?
- Can you represent the number in each group/number of groups and the total group size as a fraction?
- Can you simplify that fraction?

Portfolio Task

This task requires students to alter a recipe by applying fractions to measures and collections.

RAMR Cycle

This RAMR cycle focuses on dividing a whole into equal parts where the whole is a set of objects. To do this, we use two notions that underlie the teaching of fraction, *partitioning* (making parts out of a whole); and *unitising* (making a whole out of parts). This links to formal fraction notation.

Reality

Use contexts that can be modelled with discrete items from the local culture of students or the local environment as collections that can be separated into a number of groups with some left over. For example, identify with students instances where they talk about collections and groups using fraction language like half a class, a quarter of a population, half the team, a third of a busload, a sixth of the smarties, half the fish caught, a quarter of the $10 notes. Discuss what leftovers or remainders from these contexts mean.

Further discuss examples like a sixth of the smarties. Ensure students understand that it is a sixth of the number of smarties in the packet, which will not necessarily be all the smarties of a given colour. Clarify that for groups within a collection to be represented with the same fraction, each group must be comprised of the same quantity of items or shared out equally.

Also explore contexts that can be modelled with discrete items as collections that can be separated into groups of a known or set size with some left over. For example, making up car loads for transport, creating sporting teams from a pool of players, a quarter of a population, half the team, a third of a busload, ten smarties each from a packet, three of the $10 notes each. Discuss what leftovers or remainders from these contexts mean.

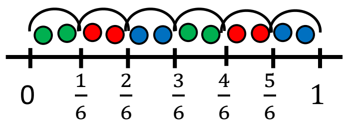
Abstraction

Partition Division: Finding fraction of a collection

The abstraction sequence for this cycle takes students from an experience of physically showing a collection that can be partitioned into groups where each group is a fraction of the whole collection, to representing this part of a whole quantity symbolically and with language. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Gather a collection of nine students, define as one whole collection, then separate into three parts (groups). As the focus is partitioning or sharing evenly, designate an area for each group, and send students one at a time to each group (similar to dealing cards, "one for you, one for you and one for me"). Describe each group as one part out of three parts altogether. Alternatively, use hands and fingers, for example, 4 students can hold up 5 fingers to make eighths or twentieths.
2. *Represent/model with materials.* Use counters or items to model the kinaesthetic activity of creating a whole collection and partitioning into equal sized parts of the whole collection. Identify possible fractions (e.g., if partitioning the collection into 3 parts, identify one part as 1 part out of 3).
3. *Connect to language.* Connect to language identifying one part out of the collection (e.g., one part out of three, one third).
4. *Represent with symbols.* Connect to the symbolic representation of fractions. Introduce the mathematical language for each part of the fraction (denominator: total number of groups or parts in the whole; numerator: number of parts or groups you have; vinculum: line in the middle that divides or separates the numerator from the denominator).

5. Repeat the steps to create a range of fractions and consolidate student understanding. Start with unit fractions and progress to situations where you might have 2 out of 5 parts or more.
6. *Reverse the activity.* Engage students with activities where they are given a group of items as a fraction and ask them to find or show the whole collection. For example, draw a group of 5, name it as one seventh and ask them to show the whole collection. Progress to examples that are not unit fractions (e.g., a group of 10 representing two sevenths of a collection would give a collection of 35).
7. *Represent/model with materials.* Use one of the partitioning activities to explore arranging the collection in a line and marking between the resulting fractions. Transfer this thinking to representing fractions on a number line. Use this activity to connect sharing one whole from Cycle 1 to representing a partitioned collection on a number line.



8. These examples have not used remainders. Extend students by asking them to consider division stories that result in a remainder that may be expressed as a fraction.

Mathematics

Language/symbols and practice

Quotition Division

An abstraction sequence is appropriate to engage students with the difference between sharing contexts (partitioning) and contexts where the group size is known, but the number of shares is not. This sequence begins by inviting students to experience a physical demonstration of how a collection can be separated into groups. Each of these groups has a known set-size and represents a fraction of the whole collection. Then students represent this part of a whole quantity symbolically with language. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Gather a collection of twelve students and define them as one whole collection, then separate into groups of three. In this case each group will have three out of twelve students or $\frac{3}{12}$. Counting the groups created will result in 4 groups, which is explained by the fact that the group size of 3 is one quarter of 12. Alternatively, use hands and fingers; for example, 4 students can hold up 5 fingers to separate 20 into groups of 5 (also creates quarters or fourths).
2. *Represent/model with materials.* Use counters or items to model the kinaesthetic activity of creating a whole collection and separating into groups of a set size from the whole collection.
3. *Connect to language.* Language can be used to identify one part out of the collection (e.g., one part out of four, one quarter or fourth).
4. *Represent with symbols.* Students can connect to fractions by using symbolic representations. Connect the mathematical language for each part of the fraction (denominator: total number of groups or parts in the whole; numerator: number of parts or groups you have; vinculum: line in the middle that divides or separates the numerator from the denominator).
5. *Represent/model with materials.* Represent fractions on a number line. Connect to repeated subtraction for division.
6. Repeat the steps to create a range of fractions and consolidate student understanding. Start with unit fractions and progress to situations where you might have 2 out of 5 parts or more.

7. *Reverse the activity.* Engage students with activities where they are given a group of items as a fraction and ask them to find or show the whole collection. For example, draw a group of 5, name it as one seventh and ask them to show the whole collection. Progress to examples other than unit fractions (e.g., a group of 10 as two sevenths of a whole would give a collection of 35).

Formal language and symbols

Consolidate formal language and symbols including *denominator* and *numerator*, and so on. Students need to be able to create different representation of fractions: diagrams ↔ word descriptions ↔ fraction notation. Consolidate finding fractions of sets through the use of a think board.

Providing various fractions, ask students to mark them on a number line. For each fraction ask students to represent them in a number line, in word descriptions, and in fraction notation. Reverse this by indicating a point on the number line from 0 to 1 and asking for the fraction. Ensure students can use division to partition and generate fractions on their drawn number lines.

Resource Resource 5.3.1 Fraction Thinkboards

Practice

It is important to play games and complete worksheets that interrelate the three models (area, set, line) with language and symbols such as "Fraction Uno", "Fraction Dominoes" and "Fraction Memory" including all representations.

Use a variety of practice activities: worksheets with four columns (e.g., rectangle/strip shaded, circle shaded, language, symbol, or two models together) and fill in only one column (different for each example) – students fill in the other columns.

Resource

Resource 5.3.2 Find the missing representations
Resource 5.3.3 Find the missing representations 2
Resource 5.3.4 More cards for fraction representation games

Consolidate mixed numbers and improper fractions

Explore mixed numbers and improper fractions with set models (as for Cycle 1). If students find this difficult, create the mixed number or improper fractions with an area model and whole-part chart and then distribute the collection to be partitioned between the area parts. For example, one and three quarters of twenty-four could be modelled with one circle and three quarters of a circle. Twenty-four items on the whole circle, another twenty-four distributed between four segments of a circle and then one part removed to leave three quarters of the circle with 3 groups of 6 items arranged on its pieces.

Connections

Connect fractions to division

The set model is useful to explore fractions as representations of division. This can be emphasised by:

(a) using examples that relate division to fraction (e.g., share twelve cupcakes amongst 4 people, this is 12 ÷ 4 and can be shown, by sharing the cakes, to equal $\frac{1}{4}$ of the cakes to each person);

(b) look at similarities between division and fractions (e.g., more people to share with means less to each person – larger denominator means smaller fraction).

Reflection

Check the idea

Thinkboards may be used for students to identify context or story, language, symbol and representation with materials for a given fraction.

Fraction representation games may be used to assist students with their practice of connecting language, symbol and representation.

Refer back to the students' reality and check students are able to solve problems:

> *In a restaurant cake are pre-cut into eighth size pieces and put onto plates. There are 13 plates of cake in the fridge, how much cake is there? (13 eighths, $\frac{13}{8}$)*
>
> *Share 7 cupcakes amongst 2 people. How many cupcakes does each person receive?*
>
> *Four people went out fishing and caught 20 fish between them. They each took home an equal share for their families. How many fish did each person take home?*
>
> *A long distance runner runs 1500m in a race. What fraction of the race did the athlete run if they tripped after running 900m.*

Ensure students are able to reverse the process to go from identifying a part of a collection and reconstruct the whole collection.

Apply the idea

Extend on probability and likelihood from earlier units. Likelihood is represented as a fractional quantity where the number of possible outcomes is divided by the sample space. Explore the likelihood of rolling a specific number and represent this as a fraction. Explore the likelihood of attaining a colour on a fair spinner and represent these as fractions. *Resource 1.6.3: The Beetle Game, 1.6.4: Spinners* and *Resource 1.6.5: Constructing Sample Space Activities* from *Unit 1: Cycle 6: Probability* may be useful to revisit here to clearly link to fractional representations of probability.

Extend the idea

Spend time looking at other mathematics topics where fractions can be used. For example, length: $\frac{3}{4}$m is 75 cm; percent: $\frac{2}{5}$ is 40%; time: $\frac{2}{3}$ hour is 40 minutes; and angle: $\frac{5}{6}$ of a rotation is 150 degrees; plus applications in other measures. Relate time as a fraction of an hour to the fraction of the area of the clock face covered or the fraction of the circumference marked out by the minute hand.

Try to get across the generalisation that if you take a whole and break into q equal parts and shade p of them that the fraction is p $q$ths or $\frac{p}{q}$.

Students should be able to identify how they would change 'n' wholes and $\frac{b}{c}$ parts of 'n' to an improper fraction. In this case, the denominator would be 'c and numerator would be 'n x c + b' to give the improper fraction: $\frac{n \times c + b}{c}$.

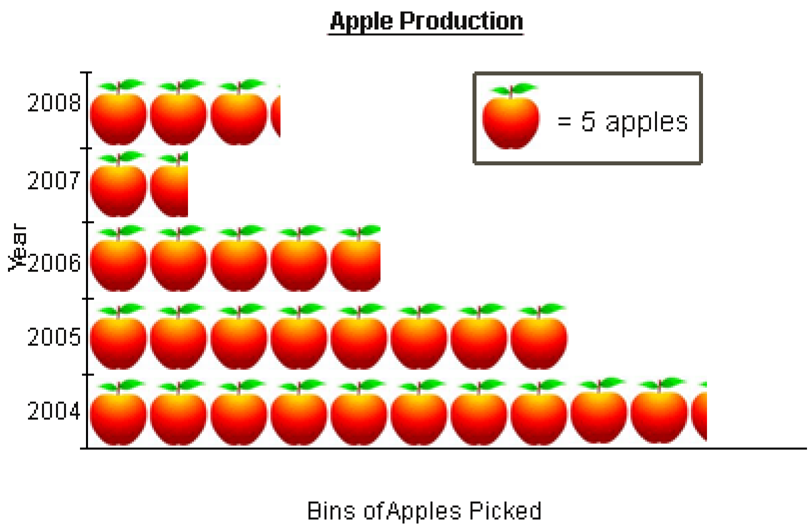
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

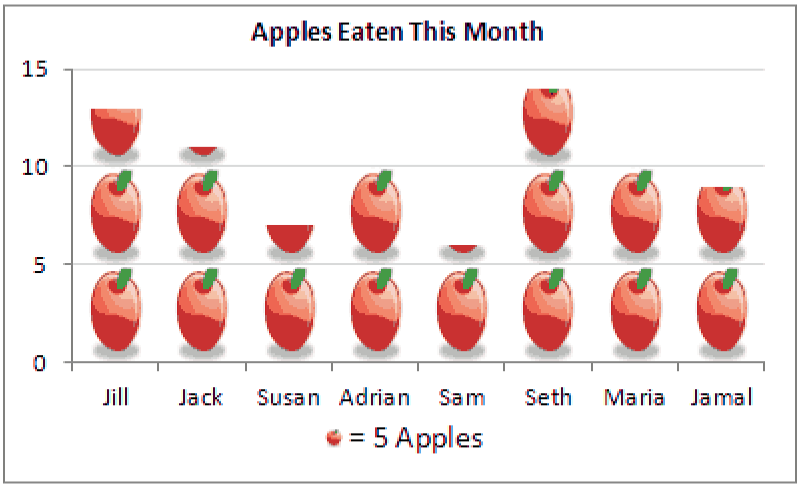
Unit 05 Investigation: One-to-Many Pictographs

One-to-many pictographs provide a further opportunity to apply fraction understandings.

Explore pictographs where one image represents a group of a set size.



Discuss the meaning of part apples on this graph with students. Consider the precision of interpretation of values and how they might represent data sets where values occur that are not exact multiples of the chosen image.



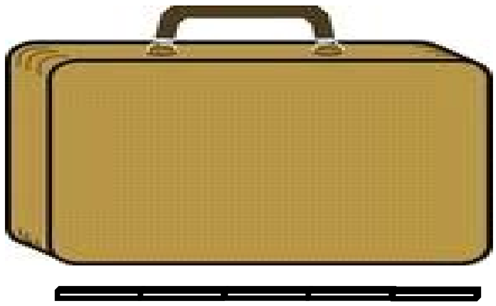
Engage students with generating their own pictographs where they need to partition data sets into groups, with a remainder to be represented with a partial or fractional image. Consider reworking some of the graphs produced from data gathering activities during Unit 01.

Name:________________________________ Date:________

Can you do this? #4

1. Consider the following objects.

   (a) Estimate the width of each of the following objects in matchsticks.



i.____________________ ii.____________________ iii.____________________

Obj. 5.4.1 a)i. ☐ ii. ☐ iii. ☐

   (b) Order the objects from shortest to longest in width.

i. ________________, ii. __________________, iii. _________________

Obj. 5.4.1 b)i. ☐ ii. ☐ iii. ☐

2. It is half as far to walk from the shops to home as it is to walk from school to the shops. If you take 2400 steps between school and the shops, how many more steps will you take to walk home from the shops? (Draw a picture to help you solve the problem.)

Obj. 5.4.2 i. ☐ ii. ☐ iii. ☐

Cycle 4: Dividing Continuous Quantities

Overview

Big Idea

The focus of this cycle is the separation of a continuous length into pieces. The length may be partitioned into fractions as in Cycle 1, or quotition division may be used where pre-determined unit-sized pieces are counted and the remainder described as a fraction of the unit-size piece. This is the same as division stories where the group size is known and the number of groups is to be found. The quotition division idea is the basis of measurement of length into units that may be described using fraction notation.

Number is naturally applied to counting discrete objects to determine quantity. However, when comparing continuous attributes like length, area, mass, capacity and volume, only qualitative judgements can be made unless an intermediary is used to facilitate the comparison. Alternatively, items' attributes are partitioned by uniform units to facilitate a counting process. To count measures accurately, units need to be an appropriate size to measure the item, be repeated without gaps and without overlaps.

In this cycle, early measurement concepts are reinforced using the context of length. Understanding of the measurement process is developed through a sequence starting with direct comparison, followed by indirect comparison using an intermediary and indirect comparison using uniform informal units. Throughout the sequence it is important to highlight the inverse relationship between the size of units and the quantity (smaller units result in a bigger count while larger units result in a smaller count).

Objectives

By the end of this cycle, students should be able to:

5.4.1 Compare and order several shapes and objects based on length, using appropriate uniform informal units. [2MG037]

5.4.2 Solve problems involving the comparison of lengths. [6MG137]

Conceptual Links

This unit is built on students' previous experiences of identifying attributes for comparison. Comparisons between continuous attributes like length, mass, temperature, capacity, area and volume are easier and more accurate to make if informal or formal units are used to break the continuous into discrete and countable pieces. This process applies quotitioning skills.

Where continuous measures are quotitioned into countable pieces, remainders may occur. These remainders can be expressed using fraction language and notation.

Materials

For Cycle 4 you may need:

- strips of paper
- informal units (scissors, pegs, pencils, paddle pop sticks, tooth picks, match sticks)

Key Language

Attribute, length, width, height, breadth, long-short-tall-wide(er, est), etc., compare, direct, indirect, measure, continuous, distance, perimeter, side, edge

Definitions

Continuous attributes: attributes which are not countable unless first broken into units. For example, length, mass, capacity, area, volume, temperature, value

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What unit are you using to break this distance up?
- How many of the unit have you used?
- What is the length you have in <units>?
- Is there a part length not measured?
- About how much of the unit do you think that is leftover?
- What happens when you use a smaller unit to measure the same thing?
- What happens when you use a larger unit to measure the same thing?
- Which unit is more accurate for this length?

Portfolio Task

There are no specific aspects of *Portfolio Task 05: Time for a BBQ* that relate to this cycle.

RAMR Cycle

Reality

For this cycle the focus is on the attribute of length. Focus students' attention on contexts where length is compared using appropriate vocabulary: short or long, wide or narrow, thick or thin, high or low, deep or shallow, near or far, up or down, distance around, perimeter. Relevant real-life contexts can be distance, heights, legal limits for fish, length of trucks or dimensions of fencing.

Abstraction

The abstraction sequence for this cycle starts with a rich understanding of the attribute to be measured, followed by reinforcement of the foundations of measurement concepts. Students need to understand that continuous measures are partitioned into sections to make the continuous countable. For units to be useful in countable measure they need to be of uniform size, repeated without gaps or overlaps, and appropriate for the context. Students need to progress to understanding the need for standard units of measure. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity.* Remind students of the activities from the start of the year where they counted the length of items using non-standard units. As an example, briefly revisit directly comparing students' heights or foot lengths.
2. *Measure with non-standard units and represent with symbols.* Introduce counted measure by using uniform items for measuring such as pegs, pencils, scissors, paper clips, paddle pop sticks, spoons or matchsticks. Find half the length of an item by dividing the number of units by two. Engage students with dividing the number of units evenly into four. Discuss what fraction of the length of the whole is represented by one of the groups of units.
3. Connect previous thinking with fractions to partition parts of units when counting length.
4. Use a pair of units that are directly related in length (e.g., a peg might be half the length of a pencil). Explore how many of each unit is used to measure the same length. Ensure students can identify that if the unit is half the length, there will be twice as many used to measure.

Resource Resource 5.4.1 Measuring length with non-standard units

Mathematics

Language/symbols and practice

Provide opportunities for students to practise measuring familiar items using their own rulers made with non-standard units and identify which unit of measure is most appropriate for different contexts. For example, which of their units will be most appropriate for measuring the length of the oval, length of an exercise book, width of a desk, distance around the edge of a desk, distance around the edge of an exercise book? This experience will also engage students with an opportunity to develop deeper understanding of the working of scale on measuring tools. Connect the idea of scale with scales on graphs developed in earlier Units to reinforce the notion of same size partitions and units for display of data and comparison.

Connections

This cycle has touched on perimeter or the distance around in some places. Explore this idea more fully. Is it necessary to measure the distance right around something or is it possible to add up dimensions around something to find perimeter? Discuss strategies for dealing with addition of remainders or fractional units.

Reflection

Check the idea

Check students' understanding of the principles of measurement with non-standard and standard units with *Resource 5.4.2 Strange rulers.* This activity requires students to identify which of the rulers shown would be valid measuring devices and which would not. Ensure students can articulate their reasoning.

Resource Resource 5.4.2 Strange rulers

Apply the idea

Measure the perimeter of the basketball court (or whiteboard, desks, classroom, window edge) using students' constructed rulers. Discuss what comprises the perimeter of the basketball court (the fence around the outside or the white line of the court).

How many metres of white line are there on the basketball court? How could the shooting circles be measured (without exploring circle formulae – these are a problem for another day)?

Extend the idea

Establish the need for standard units so that measures are comparable without extra description of the context, unit of measure and errors that come from use of personal referents.

Resource Resource 5.4.3 Measuring length with standard units
Resource 5.4.4 The apprentice's problem

Teacher Reflective Notes

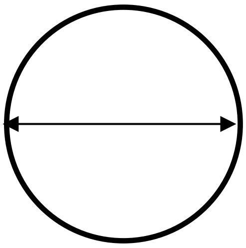
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_______________________________________ Date:________

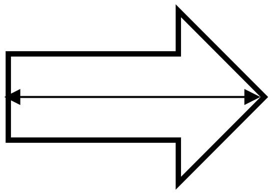
Can you do this? #5

1. Consider the following objects.

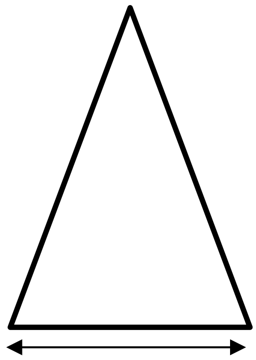
   (a) Measure the width of each of the following objects in millimetres.



________mm



________mm



________mm

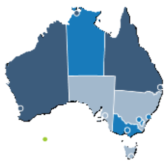
Obj. 5.5.1
a)i. □
ii. □
iii. □

   (b) Order them from smallest to largest in width.

   i. ________________, ii. ______________, iii. __________________

Obj. 5.5.2
b)i. □
ii. □
iii. □

2. Match the unit of length to the appropriate item to measure:

(a)		m
(b)		cm
(c)		mm
(d)		km

Obj. 5.5.3
a) □
b) □
c) □
d) □

Cycle 5: Metric Measure

Overview

Big Idea

The focus of this RAMR cycle is the use of breaking a continuous whole into unit-sized pieces that may then be counted, and the remainder further broken into smaller size units. This idea is the basis of measurement of length into units of a set size, with the remainder further broken into smaller units. This idea becomes the basis of metric measure (e.g., $5\frac{1}{2}$m, is equivalent to 5m and 50cm or 5m 500mm).

Objectives

By the end of this cycle, students should be able to:

5.5.1 Measure objects using familiar metric units of length. [4MG084]

5.5.2 Order and compare objects using familiar metric units of length. [3MG061]

5.5.3 Choose appropriate units of measurement for length. [5MG108]

Conceptual Links

Metric measure relies on previously developed skills in determining attribute, counting, comparison and order. Place value ideas including singles, groups and groups of groups and multiplicative relationships between place values are also required in order to understand metric measure.

Ideas from this cycle consolidate and extend basic measurement concepts and will connect to decimals and metric conversions in later units.

Materials

For Cycle 5 you may need:

- metre ruler
- standard ruler (cm and mm)
- dressmakers' tape measure
- specialist measuring devices
- tape measure (PE for measuring long jump)
- height stick (PE for measuring high jump)
- pocket tape measure (2m or 5m)

Key Language

Metric length measures, prefixes and abbreviations: metre (m), centimetre (cm), millimetre (mm), kilometre (km), ruler, tape measure, place value, measuring tape, tape measure, units of measure.

Definitions

Metric abbreviations: metric measure uses a common base unit which may be grouped using prefixes to make units larger or smaller than the base unit. For example, length uses the metre as the base unit. Groupings of 1000 m = 1 km. Smaller units based on the metre are quotitioned into set piece sizes of decimetres (one tenth of a metre); centimetres (one hundredth of a metre); millimetres (one thousandth of a metre).

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What unit are you using to measure this distance?
- How many of the unit have you used?
- What is the length you have in <units>?
- Is there a part length not measured?
- About how much of the unit do you think that is leftover?
- What happens when you use a smaller unit to measure the same thing?
- What happens when you use a larger unit to measure the same thing?
- Which unit is more accurate for this length?
- Which unit is more practical for this length?

Portfolio Task

There are no specific aspects of *Portfolio Task 05: Time for a BBQ* that relate to this cycle. However, it may be possible to extend the investigations to include the need to measure tablecloths for tables at the BBQ, consider liquid measures of ingredients as fractions of a litre, or determine the fraction of a whole block represented by a cube of cheese and what fraction of a kilogram this might represent.

RAMR Cycle

Reality

Where possible, find real-life contexts in which to embed the activities. The situations selected should require the application of measures in millimetres, centimetres, metres and kilometres.

Abstraction

The abstraction sequence for this cycle extends from the non-standard units developed in Cycle 2 for measuring length. This sequence needs to connect standard measuring units with place value ideas and standard prefixes and abbreviations to complete students' ability to measure with these standard metric units. A suggested sequence of activities is as follows:

1. *Kinaesthetic activity*. Develop a physical sense of standard units. Experience the magnitude of each unit physically. Find examples in the real world that are 1mm, 1cm, 1m and 1km long. Find a way for students to walk 1km (e.g., around the school fence line, an appropriate number of circuits of the running track, 10 repeats of the 100m sprint track). Have students estimate how far 1km is first then take a trundle wheel along to check.
2. *Represent with materials*. Using different colours of 1cm grid paper, cut ten strips that are 10cm in length. Tape alternating colours together to form a folding 1m measuring strip. This can be used to identify centimetres and groups of 10 centimetres to make up a metre. Practise estimating distances using paces. Count normal walking steps to pace out 10m. Multiply by 100 to estimate a kilometre and check the step count against the previous 1km walked.
3. Use a measuring tape to measure and record personal measures (e.g., handspans, armspans, foot lengths). Identify personal referents for standard measures. Use these personal referents to estimate measures of items around the room.

Resource Resource 5.5.1 Determining personal referents

4. Estimate, using a variety of techniques – internal measures, time, paces, etc. a variety of items or lengths – don't forget perimeters! Then measure the same lengths and see how close you get.

   Estimate larger distances that students may walk every day – classroom to canteen, school to shop or significant landmarks in the community and then measure these distances. Try to develop the distance of a kilometre.
5. *Connect to Language*. Make connections between metric measure and decimal place value. Use the maths mat as a large place value chart and lay out place value cards from thousandths to millions. Place a Metre card under the ones. Research what prefixes kilo, centi, milli mean. Place these cards under their respective place values respective to the metre. Fill in non-standard metric prefixes to cover empty places (these exist, they are just not part of the Standard International unit set). See *Resource 5.5.2 Metric prefixes chart for student desks* and *Resource 5.5.3 Connecting place value to metric measure* for more detail. Decimals are not introduced until Unit 5, but fraction names can be used for places that are parts of a metre. For example, 1cm is one hundredth of a metre; 1mm is one thousandth of a metre. These may be written in their fraction form and make useful connections to names of decimal places.

Resource Resource 5.5.2 Metric prefixes chart (fractions) for student desks
Resource 5.5.3 Connecting place value to metric measure (large cards)

6. Explore using a unit to measure an item, then counting the smaller units for fractional parts. For example, a table might be 79cm and 6mm wide. Represent the smaller unit as a fraction of the larger unit, that is, 6mm = $\frac{6}{10}$ cm. The table width would be $79\frac{6}{10}$ cm.

Mathematics

Since conversion between metric measures is consistent with place value ideas and language, it is possible to use number expanders and place value charts to reinforce metric conversion. *Resource 5.5.4 Metric expanders* and *Resource 5.5.5 Metric slide rule* can be useful here.

Metric Expanders. Construct a larger copy of Expander A (kilometres, metres and millimetres) and cut it out. Fold the expander like number expanders. Use them to relate km, m and mm as for place-value cards.

Resource Resource 5.5.4 Metric expanders

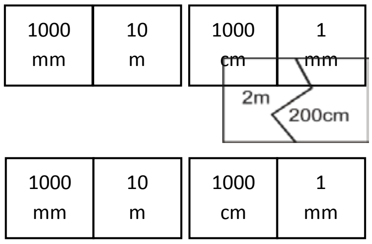
Metric Slide Rule. Copy the metric slide rule. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale. Use the slide rule to relate metrics and decimal numeration.

Resource Resource 5.5.5 Metric slide rule

Give students experience with specialist measuring instruments such as callipers, verniers, etc. Undertake outdoor activities such as height measurement and surveying.

Consolidate the metric conversions through drill – some examples:

- Dominoes
- Bingo
- Mix and Match cards
- Card decks (for Concentration, Gin Rummy, Snap etc)



Connections

Metrics should be introduced along with decimals but can be introduced with common fractions first to consolidate the place names and the fraction as parts of a whole. They apply decimal understanding and reinforce decimal concepts. For instance: hundredths are related to money (dollars and cents) and length (m and cm); and thousandths are related to length (m and mm), mass (Kg and g, and t and Kg) and volume (L and mL). Making these connections here reinforces the number of smaller units within the larger unit and will contribute to enhanced understanding of conversions between units using decimal number representations in Unit 06.

For example, when reading a tape measure, identify the unit of measure for each marking. Determine how many of the smaller unit is in each unit (e.g., count the number of millimetres in a centimetre) and identify what fraction of the larger unit the smaller unit is (e.g., each millimetre is one tenth of a centimetre; each centimetre is one hundredth of a metre).

Reflection

Check the idea

Use standard units to measure the width, length and perimeter of common items in the classroom and order these according to length from smallest to largest.

If the length of the room is measured with desk lengths, how many would there be? What fraction of the length of the room is the length of a desk?

Apply the idea

Once students have reached the point where they understand a need for standard units, engage them with tasks to practice measuring items in centimetres or metres to compare lengths. Familiarise students with a range of measuring tools/devices (rulers are simplest, dressmakers' tape measures and building tape measures are more complicated to read). Identify the unit of measure for each marking to assist with correct reading of the measuring device.

Extend the idea

Explore other instances of measures that are related and expressed using fractions. For example, scales on measuring jugs connect Litres and millilitres, cups and fractions of cups; measuring spoons include fractions of teaspoons. Consider the relationships between these measures and their fractions. Discuss how many quarter teaspoons are contained in a teaspoon.

Consider the meaning of fractions when used with spanners, nuts, bolts and drill bit sizes. What is the relationship between these values? Are they just names or is there a measurement that goes with them?

Where else do fractions appear other than cooking measures and spanner sizes?

Consider time in terms of one quarter of an hour, half an hour, quarter past, half past. What do these terms represent and how do we change from fractions of hours to minutes? How would these be represented as equivalent fractions (e.g., $\frac{1}{4}$ hour = $\frac{15}{60}$ hour)? What does $\frac{15}{60}$ hour mean (15 minutes out of 60 minutes)? So, $\frac{1}{4}$ hour = 15 minutes.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 05 Portfolio Task – Teacher Guide

Time for a BBQ

Content Strand/s: Number and Algebra

Resources Supplied:
- Task sheet
- Teacher guide

Other Resources Needed:
- None

Summary:

Students use their fractional knowledge to explore food eaten for a barbeque. The students need to work through the activity sequentially.

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Number and Algebra

5.1.3 Represent unit fractions symbolically. [3NA058]

5.2.2 Compare and order common fractions. [7NA125]

5.2.5 Compare and order improper fractions and mixed numerals. [4NA078]

5.3.1 Recognise and interpret fractions of collections of items. [2NA033]

5.3.3 Express one quantity as a fraction of another. [7NA155]

Time for a BBQ

Name	
Teacher	
Class	

Your Task:

It is your task to plan a barbeque for yourself and 5 friends.

You will use your fractional understanding to answer questions to calculate the amount of food eaten at your BBQ.

Within Portfolio Task 05, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Procedural fluency	5.2.2 Compare and order common fractions. 5.2.5 Compare and order improper fractions and mixed numerals.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	5.1.3 Represent unit fractions symbolically.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Reasoning and justification	5.3.1 Recognise and interpret fractions of collections of items. 5.3.3 Express one quantity as a fraction of another.	Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions

Comments:

Planning a BBQ

When planning a BBQ, you need to consider a variety of things, such as the amount of food needed, and how much each person can eat.

At your BBQ you are going to eat sausage sandwiches with salad. In preparation for this BBQ you have bought the following ingredients:

- Sausages
- Bread
- Onion
- Tomato
- Cucumber
- Lettuce
- Capsicum

1. Using the salad recipe, and the amount of food purchased, calculate the fraction of each item that was used:

Salad Recipe	***Purchased food***
3 tomatoes, sliced	5 tomatoes
1 cucumber, cubed	2 cucumbers
$\frac{3}{4}$ lettuce, shredded	2 lettuce
$\frac{1}{2}$ capsicum, sliced	3 capsicum
$\frac{1}{2}$ cup cubed cheese	2 cups of cubed cheese

e.g., The fraction of cucumber used can be found using this formula: $\frac{(amount\ used)}{(amount\ bought)} = \frac{1}{2}$

Therefore, the fraction of cucumber used was $\frac{1}{2}$.

a. Tomato

b. Lettuce

c. Capsicum

d. Cheese

2. Which ingredient had the highest fraction used? _______________

3. What fraction of the salad could you make with the leftover ingredients?

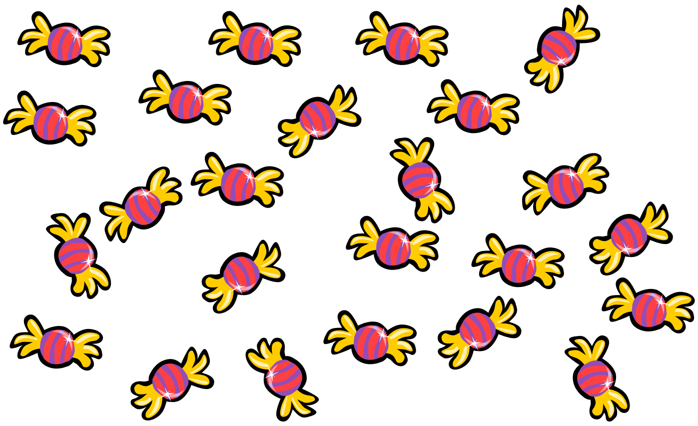
4. The sausages came in a pack of 20, and the loaf of bread had 18 slices. If all the bread is eaten with one sausage per slice of bread, what fraction of the pack of sausages remains?

5. $\frac{4}{6}$ of the people attending the BBQ enjoy sliced onion with their sausages. If 2 cups of onion were cooked, and all the onion was eaten, what fraction of onion did each person eat on **one sausage sandwich?**

6. Two sausages were not eaten. What fraction of sausage would each person get if the two sausages were shared evenly?

7. As a snack after the BBQ, you and your friends shared one packet of lollies. The lollies from the packet can be seen below.

   a. If these lollies were shared evenly, what fraction of the packet would each person receive? ______________________



   b. Circle the number of lollies that 4 people would eat.

   c. What is the fraction with the lowest possible denominator for this number of lollies? ____________________

8. How many lollies would there be in $2\frac{1}{2}$ packets of lollies? Draw a picture to help you.

Name:________________________________ Date:________

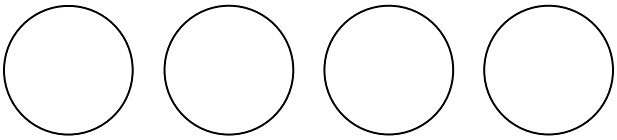
Can you do this now? Unit 05

1. Write down or draw two more examples of fractions. One has been done for you.

   (a) half a cup in cooking (b) ______________ (c) ______________

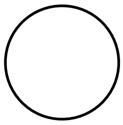
Obj. 5.1.1
b) ☐
c) ☐

2. If these four cakes were shared amongst 6 people what fraction of cake would each person get? __________________



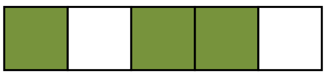
Obj. 5.1.1
i.☐
ii.☐

3. Represent one quarter ($\frac{1}{4}$) on each of the following diagrams:

(a)  (b) 

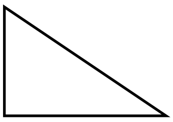
Obj. 5.1.2
a) ☐
b) ☐

4. What fraction is shown by each of the following diagrams? Fill in the missing parts on the table.

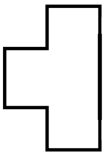
(a) 	Words one fifth	Fraction __________
(b) 	Words ___________	Fraction $\frac{1}{6}$
(c) 	Words ___________	Fraction __________

Obj. 5.1.2
a)i. ☐
b)i. ☐
Obj. 5.1.3
a)ii. ☐
b)ii. ☐
Obj. 5.1.4
c)i. ☐
c)ii. ☐

5. (a) This shape is one third ($\frac{1}{3}$) of a whole. Draw the whole shape.



(b) This shape is three fifths ($\frac{3}{5}$) of a whole. Draw the whole shape.



Obj. 5.1.4
a) ☐
b) ☐

6. Write the fraction or number that is one sixth ($\frac{1}{6}$) more than:

   (a) $\frac{2}{6}$ _________ b) $\frac{5}{6}$ _________

Obj. 5.1.5
a) ☐
b) ☐

7. Complete the counting sequence and write down the change as a fraction.

(a) $\frac{2}{5}, \frac{3}{5}, \frac{4}{5},$ ________, ________, ________ ________

(b) $3\frac{4}{6}, 3\frac{5}{6}, 4,$ ________, ________, ________ ________

Obj. 5.1.6
a)□□□
b)□□□
Obj. 5.1.7
a)iv□
b)iv□

8. How many fifths in:

a) $3\frac{1}{5}$ ____________ fifths b) $4\frac{6}{10}$ __________ fifths

Obj. 5.2.6
a) □
b) □

9. Place these fractions on the number line: $\frac{3}{4}, \frac{7}{8}, \frac{1}{2}, \frac{2}{3}, \frac{4}{5}$

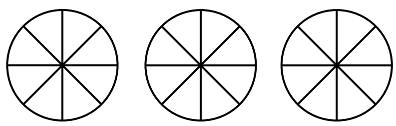
Obj. 5.2.1
i. □
ii. □
iii. □
iv. □
v. □

10. Draw a diagram to show $1\frac{2}{7}$ as chocolate bars.

Obj. 5.2.3
i. □
ii. □

11. Sammy ordered 12 slices of pizza. A whole pizza has eight slices.

(a) Shade the number of pizza slices Sammy has on the outlines.



(b) Write down as a mixed number, how much of a pizza Sammy has. ________________________

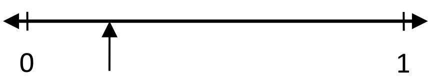
Obj. 5.2.3
a) □
b)i. □
ii. □

12. Write the following fractions in words:

(a) $3\frac{4}{5}$ ____________________ (b) $\frac{1}{6}$ ______________________

Obj. 5.2.3
a) □
Obj. 5.2.1
b) □

13. Look at the point on the number line. Name this fraction in words and as a common fraction.



____________________ ________

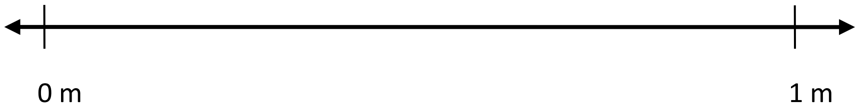
Obj. 5.2.1
i. □
ii. □

14. A 1m long string of liquorice was shared equally between 3 friends.

(a) What fraction of the string of liquorice would each friend receive?

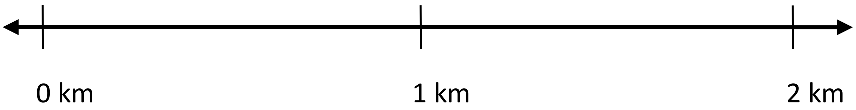
____

(b) Use the number line to show the pieces of liquorice.



Obj. 5.2.3
a) □
b) □

15. Ben drove one and a quarter kilometres from his house to the park. Mark this distance on the number line.



Obj. 5.2.3
□

16. Write the following as mixed numbers, i.e. 4 thirds = $1\frac{1}{3}$ or $\frac{4}{3} = 1\frac{1}{3}$

(a) 7 sixths = ______________ b) $\frac{10}{7}$ = ______________

(c) 13 quarters = ______________ d) $\frac{7}{3}$ = ______________

Obj. 5.2.6
a)□
b)□
c)□
d)□

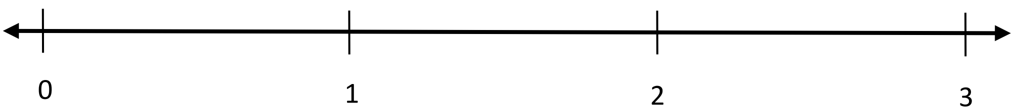
17. (a) Write the following mixed numbers and improper fractions in order from smallest to largest:

$$\frac{12}{5}, \quad 1\frac{2}{5}, \quad \frac{6}{2}, \quad 1\frac{2}{4}, \quad \frac{11}{4}$$

____

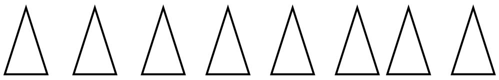
Obj. 5.2.5
i.□
ii.□
iii.□
iv.□
v.□

(b) Locate each of the mixed numbers and improper fractions on the number line.



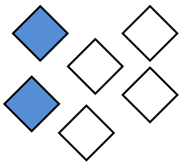
Obj. 5.2.4
i.□
ii.□
iii.□
iv.□
v.□

18. Show one quarter ($\frac{1}{4}$) of this collection of triangles.



Obj. 5.3.1 ☐

19. Look at the collection of shapes. Name the fraction of the collection that is shaded in words and as a common fraction.



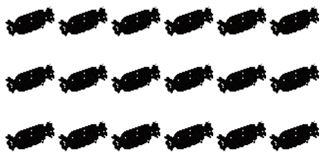
______________________________ ______________

Obj. 5.3.1 i. ☐ ii. ☐

20. This is 1 and a fifth ($1\frac{1}{5}$) packets of pencils. Circle how many pencils are in 1 packet.

Obj. 5.3.1 ☐

21. This is six sevenths ($\frac{6}{7}$) of a bag of lollies. Draw the missing lollies to make one whole bag of lollies.



Obj. 5.3.1 ☐

22. What is $\frac{1}{6}$ of 30? ______________

Obj. 5.3.2 ☐

23. There are groups of 5 students in the classroom.
Each group of students is $\frac{1}{5}$ of the whole class.
How many students in the class? ______________________________

Obj. 5.3.2 ☐

24. The school has 20 basketballs in the storeroom. You take 5 basketballs out for a class game. What fraction of the total number of basketballs did you take? ______________________________

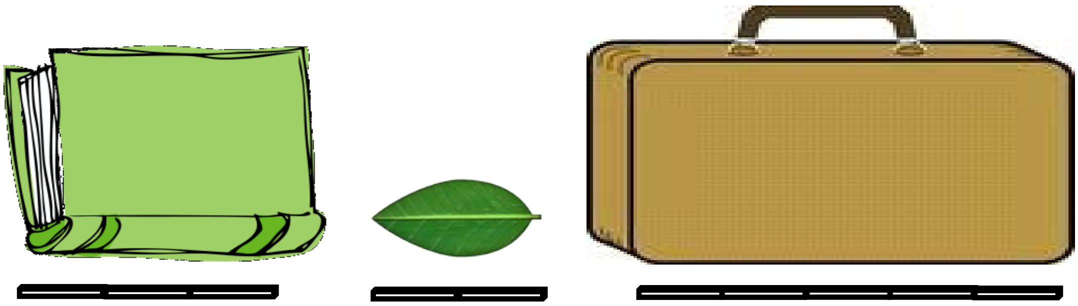
Obj. 5.3.3 i. ☐ ii. ☐

25. There are 20 basketballs in the school. Only 15 basketballs are left in the storeroom. What fraction of the total number of basketballs are left in the storeroom? ____________________

Obj. 5.3.3
i. □
ii. □

26. Consider the following objects.

(a) Estimate the width of each of the following objects in matchsticks.



i.__________ ii.__________ iii.__________

Obj. 5.4.1
a)i. □
ii. □
iii. □

(b) Order the objects from shortest to longest in width.

i. __________, ii. __________, iii. __________

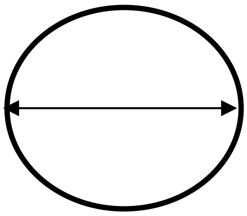
Obj. 5.4.1
b)i. □
ii. □
iii. □

27. It is half as far to walk from the shops to home as it is to walk from school to the shops. If you take 2400 steps between school and the shops, how many more steps will you take to walk home from the shops? (Draw a picture to help you solve the problem.)

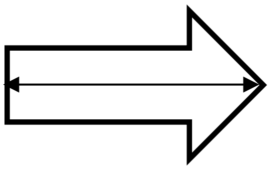
Obj. 5.4.2
i. □
ii. □
iii. □

28. Consider the following objects.

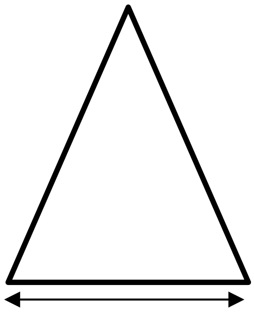
(a) Measure the width of each of the following objects in millimetres.



________mm



________mm



________mm

Obj. 5.5.1
a)i. ☐
ii. ☐
iii. ☐

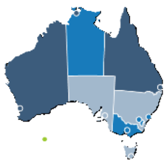
(b) Order them from smallest to largest in width.

i. ________________, ii. ______________, iii. __________________

Obj. 5.5.2
b)i. ☐
ii. ☐
iii. ☐

29. Match the unit of length to the appropriate item to measure:

(a)  m

(b)  cm

(c)  mm

(d)  km

Obj. 5.5.3
a) ☐
b) ☐
c) ☐
d) ☐

XLR8 – Accelerating Mathematics Learning

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