



XLR8 Unit 04

Investigating, Measuring and Changing Shapes

2016

ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <http://ydc.qut.edu.au> and staff can be contacted at ydc@qut.edu.au.

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Contents

XLR8 Program: Scope and Sequence	iv
Overview	1
Context.....	1
Scope.....	1
Assessment	1
Cycle Sequence	3
Literacy Development	4
Can you do this? #1	5
Cycle 1: 3D Objects to 2D Shapes	6
Overview	6
RAMR Cycle	8
Can you do this? #2	11
Cycle 2: Creating 2D Shapes	14
Overview	14
RAMR Cycle	16
Can you do this? #3	21
Cycle 3: Exploring Perimeter	22
Overview	22
RAMR Cycle	24
Can you do this? #4	27
Cycle 4: Geometric Reasoning	30
Overview	30
RAMR Cycle	32
Can you do this? #5	35
Cycle 5: Flips, Slides and Turns	38
Overview	38
RAMR Cycle	40
Unit 04 Investigation: Teacher Notes	46
Unit 04 Investigation: Exploring Design with Tessellating Shapes	47
Unit 04 Portfolio Task – Teacher Guide	54
Packaging Mayhem	1
Can you do this now? Unit 04	1

List of Figures

<i>Figure 1</i> Scope of this unit.....	2
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XMLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will identify common 3D objects and their corresponding 2D faces. Students will further explore line and angle as attributes of 2D shapes and the calculation of perimeter. Students will use Euclidean transformations of 2D shapes, symmetry and tessellation to create a design.

Scope

This unit begins by considering **3D objects** and **2D shapes** and their relationships with one another. Once these geometric objects have been identified, then **length** and **angle** related attributes of the 2D shapes can be **quantified**. This unit builds on students' understandings of **un-numbered attribute comparison** (explored in Unit 01), to extend to the use of **non-standard units**, and then **standard units** of measure to characterise the attributes. Initially, length measure begins with a limited set of measurement units, which are then expanded to include other units of the **metric system**. The multiplicative relationships between these units are developed and connected to place value understanding. Once the concept of measurement is developed, it can then be applied to the **calculation of perimeter of 2D shapes**, linking to and reinforcing additive and multiplicative operations.

Symmetry of shapes and their **transformations** have many applications in design. Students explore **symmetry** and **Euclidean transformations of shapes** within the context of **tessellation** and its application to design. Students make further connections between the **line** and **angle attributes** of **2D shapes** and the **suitability** of **2D shapes** for **tessellation designs** as single shape designs or multiple shape designs.

The organisation of these and other related concepts is shown in *Figure 1*, in which the scope of concepts **to be developed** in this unit is highlighted in **blue**, concepts that may be **connected to and reinforced** are highlighted in **green** and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets*: The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence*: Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet*: The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P04: Packaging mayhem!* accompanying Unit 04 engages students with choosing the most economical shape for packaging a small item, creating the net and the 3D package, and determining how many of these may fit safely on a specified shelf.

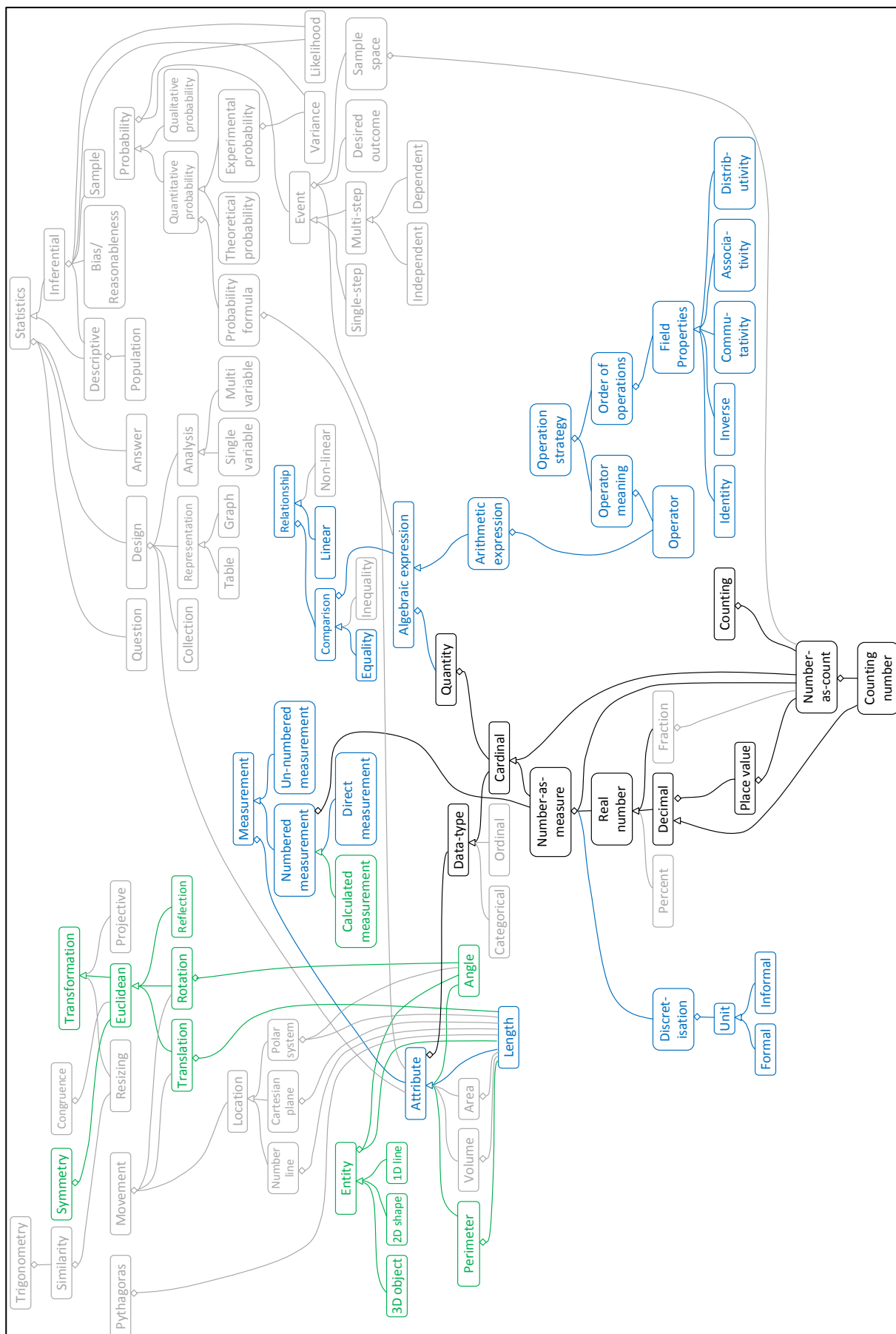


Figure 1 Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following suggested sequence:

Cycle 1: 3D Objects to 2D Shapes

To understand and to modify our environment has required the use of logic, the development of language and number, an understanding of the space that the environment exists in and an understanding of shape, size and position that enables these to be visualised (what we call geometry). The key focus for this unit is the classification of 3D objects, identification of critical attributes of objects, and the relationships between them.

Cycle 2: Creating 2D Shapes

This cycle explores creating polygons from 3D objects and defining polygons in terms of their properties (side lengths and turns) and critical attributes. Exploration of comparing and measuring angle as turn is also included as one of the critical attributes for defining and naming 2D shapes. These ideas are consolidated by exploring the properties of shapes that enable them to repeat or tessellate to create a design.

Cycle 3: Exploring Perimeter

This cycle extends the development of measurement concepts using the context of length to explore distance around a shape (perimeter). Building from students' experience with comparing and ordering lengths and indirect comparison using informal units, students will develop an understanding of the calculation of perimeter of shapes by adding the lengths of the sides. Students will also be encouraged to consider the conversion of additive equations for perimeter to multiplicative equations where applicable (e.g., perimeter of a square = $4 \times$ length of side; perimeter of a rectangle = $2 \times$ (length + width); perimeter of a regular pentagon = $5 \times$ length of side. This cycle does not extend to metric measure of length (explored in Unit 06).

Cycle 4: Geometric Reasoning

Angles in real life will be either static or dynamic. A static angle does not change and a dynamic one does. The corner of a room or corner of a window is an example of a static angle. The hands of a clock are an example of a dynamic angle which changes during the day. Where angles are formed by transversals (straight lines that cross other lines), consistent relationships between the magnitudes of resulting angles may be observed. This cycle extends from measurement of angle as the magnitude of turn to explore properties of angles within geometric figures.

Cycle 5: Flips, Slides and Turns

The focus of this cycle is to explore Euclidean transformations: slides (translations), turns (rotations) and flips (reflections). Slides may be vertical, horizontal, diagonal or a combination of these. Turns are about a point located in the centre of the shape, on a corner or a side of the shape. Rotating a shape may result in an image that has rotational symmetry. Flips may be horizontal, vertical or diagonal resulting in a mirror image or, when the original image is left in place, result in images with line symmetry. Similarly, shapes can be perceived as having lines of symmetry where a distinct line could be imagined as separating the shape into two congruent halves. Euclidean transformations may be completed in isolation or in combination to generate a variety of tessellating designs.

Notes on Cycle Sequence:

The proposed cycle sequence outlined may be completed sequentially as it stands.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: 3D Objects to 2D Shapes

3D objects, polyhedra, non-polyhedra, prism, cube, cylinder, sphere, triangular, pyramid, cone, flat surface, curved surface, edge, vertex, vertices

Cycle 2: Creating 2D Shapes

2D shapes, region, boundary, polygon, triangle, quadrilateral, parallelogram, rectangle, square, rhombus, trapezium, trapezoid, pentagon, hexagon and so on, line, angle, right angle, acute angle, obtuse angle, parallel, perpendicular, congruent, point, tessellate, tessellating, design, equilateral triangles, isosceles triangles, scalene triangles, protractor, degrees, vertex, vertices, interior angles, exterior angles, complementary angles

Cycle 3: Exploring Perimeter

Perimeter, boundary, edge, length, width, breadth, sides, formula, formulae

Cycle 4: Geometric Reasoning

Lines, rays, points, angle labelling conventions, sharp, blunt, full turn, half turn, quarter turn, three-quarter turn, alternate angles, co-interior angles, corresponding angles, complementary angles, transversals, angle, right angle, acute angle, obtuse angle, parallel, perpendicular, congruent

Cycle 5: Flips, Slides and Turns

2D shapes, turn, rotate, rotation, point, centre of rotation, rotation around a point, rotational symmetry, orientation, slide, translate, translation, direction of movement, tessellate, tessellating shape, tessellation, angles, order of rotation, flip, reflection, mirror, mirror image, straight lines, points, curves, angles, line of reflection, line of symmetry, line symmetry, congruent, congruence

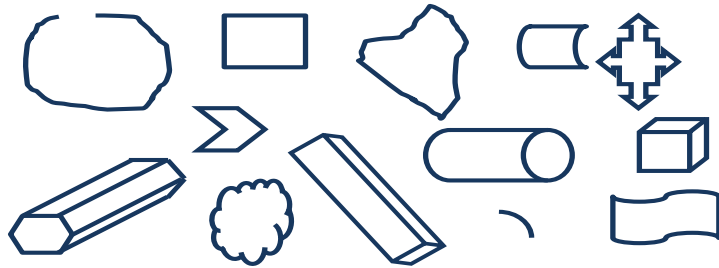
Name: _____

Date: _____

Can you do this? #1

1. Circle the 2D shapes.

Cross the 3D objects.



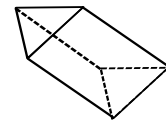
Obj.
4.1.1

☐ ☐ ☐ ☐
☐ ☐ ☐ ☐
☐ ☐ ☐ ☐
☐

2. (a) How many faces does this 3D object have? _____

(b) How many edges does this 3D object have? _____

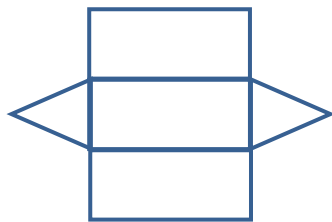
(c) What might you name this 3D object?



Obj.
4.1.2

a) ☐
b) ☐
c) ☐

3. What object will this 'net' make when it is folded up?

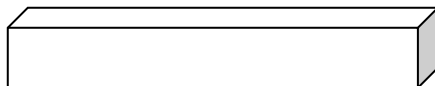


- (a) Matchbox
- (b) Tent
- (c) Pyramid
- (d) Cube

Obj.
4.1.3

a) ☐
b) ☐
c) ☐
d) ☐

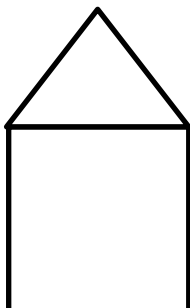
4. Sketch the net for this object.



Obj.
4.1.4

i. ☐
ii. ☐
iii. ☐

5. A triangular prism is placed on top of a rectangular prism. The picture shows the view from the front. Draw the view of this combination of prisms from the side.



Obj.
4.1.5

i. ☐
ii. ☐

Cycle 1: 3D Objects to 2D Shapes

Overview



Big Idea

Human thinking has two aspects: verbal logical and visual spatial. Our senses have enabled both these forms of thinking to evolve and develop. To understand and to modify our environment has required the use of logic, the development of language and number, an understanding of the space that the environment exists in and an understanding of shape, size and position that enables these to be visualised (what we call geometry).

The key focus for this cycle is the classification of 3D objects and 2D shapes, identification of critical attributes, and the relationships between them. The shape sequence begins with 3D shape experiences, uses these to introduce 2D shape and then uses 2D shape experiences to introduce line and angle. At this point the direction of learning changes and line and angle are used to rebuild 2D shape, and subsequently 3D object, at a deeper level.



Objectives

By the end of this cycle, students should be able to:

- 4.1.1 Describe and draw two-dimensional shapes. [2MG042]
- 4.1.2 Make models of three-dimensional objects and describe key features. [3MG063]
- 4.1.3 Connect three-dimensional objects with their nets. [5MG111]
- 4.1.4 Construct simple prisms and pyramids. [6MG140]
- 4.1.5 Draw different views of prisms and solids formed from combinations of prisms. [7MG161]



Conceptual Links

This cycle builds from students' experiences of 3D objects and 2D shapes within their environment.

Ideas from this cycle will provide a base for measuring attributes of 3D objects and 2D shapes in following cycles, and language to describe 3D objects and 2D shapes. In later cycles, this language base is used to explore and describe symmetry, transformations and geometric reasoning.



Materials

For Cycle 1 you may need:

- Variety of 3D boxes
- Card/paper
- Nets for 3D objects
- Geoboards
- Scissors / Sticky tape
- 3D solids
- Maths mat and elastics
- Rubber bands



Key Language

3D objects, polyhedra, non-polyhedra, prism, cube, cylinder, sphere, triangular, pyramid, cone, flat surface, curved surface, edge, vertex, vertices



Definitions

2D shape: a shape with only two dimensions (such as width and length) and no thickness or height.

3D objects: an object with three dimensions (such as length, width and height).

Isometric drawing: method for visually representing 3D objects in two-dimensions.

Polygon: a plane figure with at least three straight sides and angles.

Non-polyhedron/non-polyhedra: three-dimensional geometric figures where one or more sides are not polygons

Polyhedron/polyhedra: three-dimensional geometric figures where all surfaces are polygons.

Prism: a three-dimensional object whose two ends are congruent and parallel, joined by sides that are parallelograms.

Pyramid: a three-dimensional object with a base that is a plane shape and sloping triangular sides that meet in a point at the top.



Assessment

Anecdotal Evidence

Some possible prompting questions:

- What geometric 3D object does that item look like?
- How many faces does your object have?
- What 2D shapes are the faces of your object?
- What could you name this object?
- How many edges does the object have?
- How many vertices does the object have?
- Is this a composite object? What 3D objects could you put together to make this object?
- Does this object look the same from all sides/views?

Portfolio Task

The portfolio task *P4: Packaging Mayhem!* engages students with the creation of nets for 3D objects from which to construct the packaging for their item.

RAMR Cycle

This RAMR cycle explores the classification of 3D objects by their critical attributes, then focussing on polyhedral, and the connection between 3D objects and their 2D faces. These attributes or properties are then used to construct 2D nets for 3D objects.



Reality

Look around school and home – what 3D objects can be identified – houses, shed, cupboards, clocks, tanks, balls, and so on. What properties do they have – which objects roll, which have flat surfaces, which stack easily? What are the features of these shapes which allow shapes to roll or stack? Of these, which have pointy corners, which pack with best use of space, and so on? For example, objects with flat surfaces as their base and top stack easily and do not roll easily; objects with curved surfaces roll; objects with pointy tops do not stack well unless every second one is upside down.

Draw these objects. Look for similarly-shaped objects in the classroom. Discuss the need for shared vocabulary to describe objects, shapes and their properties to streamline planning and description for construction, decorating, art, packaging and so on.



Abstraction

Using the environmental approach, the abstraction sequence for this cycle starts with identifying objects in the students' reality and classification of these shapes according to their 2D faces. 2D faces are further classified by the quantity and arrangement of lines and angles. The abstraction sequence suggested for 3D objects and 2D shapes is as follows:

1. Identify 3D objects in reality and the names of these objects. Make sure students are able to connect everyday names with mathematical names. For example, ball \leftrightarrow sphere, tin/can \leftrightarrow cylinder, box/block \leftrightarrow cube and so on. Use 3D object models if available.
2. Classify shapes according to attributes. For example, students may choose to put all the four-sided objects together or collate all objects with curved surfaces together. Students must be able to articulate their reasoning for their classification. Discuss mathematical naming conventions for 3D objects (usually according to number and type of surfaces). Hula hoops provide convenient organisers for classification (or create circles from maths mat elastics or washing line).
3. Create common 3D objects using the maths mat and large elastic loops (this enables a focus on edges and vertices. Count and record the name of the object, number of vertices and number of edges (emphasised by elastic lengths). If available, use blocks, LEGO, multilink cubes or polydrons to construct common 3D objects (this enables a focus on the surfaces). Alternatively, students may be able to imagine the surfaces of the 3D objects created with elastic bands or can explore classroom models of 3D objects. Count and record the number of faces for each shape.



Resource Resource 4.1.1 Exploring 3D objects

4. Focus on the 2D surfaces that make 3D objects. These can be highlighted by tracing around solid objects or creating shapes with elastic on the maths mat. Find relationships between shapes of cross sections and 3D shapes.
5. Deconstruct 3D objects to 2D nets, deconstruct 2D nets into 2D shapes, and reconstruct 2D nets and 3D objects from 2D nets.



Resource

Resource 4.1.2: Deconstructing 3D objects into 2D nets

Resource 4.1.3: Constructing 3D objects from 2D nets



Mathematics

There are relationships between 3D shapes and their 2D flat surfaces. For example, the two ends/bases of prisms are joined by rectangles, while the end/base of pyramids are joined to a point by triangles. Ensure students can relate 3D shapes to their surfaces and the faces on 2D nets. Explore the arrangement of 2D faces on nets for 3D objects. Is there only one arrangement of 2D faces that will create a net for a given object?



Resource

Resource 4.1.4: Cut nets problem

Resource 4.1.5: What 3D shape is this?



Reflection



Check the idea

Students recognise that 3D shapes are put together using 2D shapes as faces. Different combinations of 2D faces will result in different 3D shapes but only certain combinations will work together. Why? Which edges of a net need to be congruent (the same) to make the shape work?



Apply the idea

Students can explore the 3D shapes that can be made from the following combinations of 2D shapes.

Draw a net for the following shapes that would fold into a 3D shape:

- 2 congruent squares and 4 rectangles
- 2 congruent triangles and 3 rectangles
- 2 triangles and 3 trapeziums

Combinations can be given where there is a surplus of 2D shapes requiring students to determine which one to omit and, alternatively, given combinations with one shape missing where they have to determine what the omitted 2D shape would be.



Extend the idea

Show students a 3D object (e.g., a cube) with patterns on each face. Engage students with finding a suitable net for the object, and then draw the patterns on the net so that it folds to be the same as the sample object.

Explore drawing isometric projections of 3D objects. *Resource 4.1.6: Isometric grid paper* and *Resource 4.1.7: Isometric dot paper* may be used to facilitate simple construction of isometric drawings.



Resource

Resource 4.1.6: Isometric grid paper

Resource 4.1.7: Isometric dot paper

Construct 3D objects given orthogonal drawings of top and side views (using multilink cubes, MAB blocks, any similar construction items).

Draw top and side views of 3D objects and swap with other students as construction challenges.



Resource

Resource 4.1.8: 3D construction challenges

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name: _____

Date: _____

Can you do this? #2

1. Circle whether statement is true or false.

- | | | |
|--|------|-------|
| (a) A plane shape is a solid shape. | True | False |
| (b) A 2D shape is always flat. | True | False |
| (c) A 2D shape always has straight sides. | True | False |
| (d) All faces of 3D shapes are plane shapes. | True | False |

Obj.
4.2.1
a) ☐
b) ☐
c) ☐
d) ☐

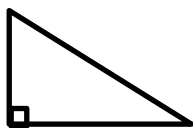
2. (a) How many sides does this polygon have? _____

(b) What is the name of this polygon? _____

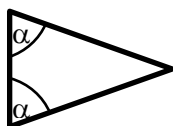


Obj.
4.2.1
a) ☐
b) ☐

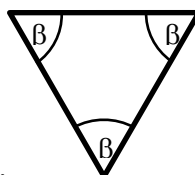
3. Name the following triangles:



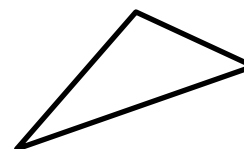
(a) _____



(b) _____



(c) _____



(d) _____

Obj.
4.2.2
a) ☐
b) ☐
c) ☐
d) ☐

4. Fill in the blanks in these sentences:

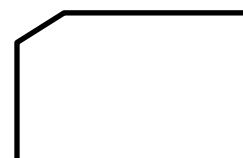
This shape is a _____.



The opposite sides of a _____ are _____
and the same _____. The angles are all _____.

Obj.
4.2.3
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

5. (a) This shape is a _____ because it has _____
sides and _____ angles.



(b) If this shape was a rectangle to start with, the
cut off piece would be a _____.

Obj.
4.2.3
a) i. ☐
a) ii. ☐
a) iii. ☐
Obj.
4.2.4
b) ☐

6. Use a ruler to draw an octagon.

Obj.
4.2.3
i. ☐
ii. ☐

Name: _____

Date: _____

7. This angle is a right angle

(a) How many right angles do you need to make a straight line? _____

(b) How many right angles do you need to make a complete turn? _____

(c) Draw an angle smaller than a right angle.

(d) Draw an angle that is larger than a right angle.



Obj.
4.2.5

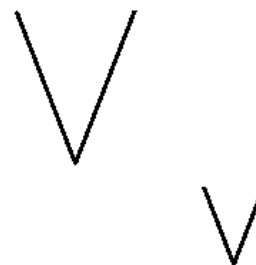
- a) ☐
b) ☐
c) ☐
d) ☐

8. Here are two angles.

(a) Draw a circle around the largest angle.

(b) Use a protractor to measure each angle.

i _____ ii _____



Obj.
4.2.5

- a) ☐

Obj.
4.2.6

- b) i. ☐
b) ii. ☐

9. (a) Use a protractor to draw a triangle with one angle of 33° and one right angle.

Obj.
4.2.7

- a) i. ☐
a) ii. ☐

Cycle 2: Creating 2D Shapes

Overview



Big Idea

This cycle explores creating polygons from 3D objects and defining polygons in terms of their properties (side lengths and turns) and critical attributes. Exploration of comparing and measuring angle as turn is also included as one of the critical attributes for defining and naming 2D shapes. These ideas are consolidated by exploring the properties of shapes that enable them to repeat or tessellate to create a design.



Objectives

By the end of this cycle, students should be able to:

- 4.2.1 Describe and draw two-dimensional shapes. [2MG042]
- 4.2.2 Classify triangles according to their side and angle properties. [7MG165]
- 4.2.3 Describe shapes according to their side properties. [7MG165]
- 4.2.4 Describe 2D shapes that result from combining and splitting common shapes. [4MG088]
- 4.2.5 Compare angles and classify them as equal to, greater than or less than a right angle. [4MG089]
- 4.2.6 Estimate, measure and compare angles using degrees. [5MG112]
- 4.2.7 Construct angles using a protractor. [5MG112]



Conceptual Links

This cycle requires students to be able to apply basic shape recognition strategies explored in Cycle 1. Ideas from this cycle will provide a base for measuring attributes of shapes in following cycles. In later units, ideas from this cycle provide bases for transformations and geometric reasoning on the Cartesian plane.



Materials

For Cycle 2 you may need:

- Card/paper
- Geoboards
- Rubber bands
- Scissors
- Maths mat and elastics or large grid and loops of wool



Key Language

2D shapes, region, boundary, polygon, triangle, quadrilateral, parallelogram, rectangle, square, rhombus, trapezium, trapezoid, pentagon, hexagon and so on, line, angle, right angle, acute angle, obtuse angle, parallel, perpendicular, congruent, point, tessellate, tessellating, design, equilateral triangles, isosceles triangles, scalene triangles, protractor, degrees, vertex, vertices, interior angles, exterior angles, complementary angles, full turn, half turn, quarter turn, three-quarter turn



Definitions

Acute angle/Right angle/Obtuse angle/Straight line/Reflex angle/Revolution: angles may be grouped and labelled generally according to their relative size: acute angles $< 90^\circ$; right angles $= 90^\circ$; obtuse angles $> 90^\circ$; straight lines $= 180^\circ$; reflex angles $> 180^\circ$; a revolution $= 360^\circ$.

Congruent: identical to, the same as. In shapes, the same size lengths of sides and angles.

Parallel: lines that maintain the same distance apart for their entire length.

Perpendicular: at right angles to.

Tessellate: repeat a shape with no gaps or overlaps in a tiling pattern.



Assessment

Anecdotal Evidence

Some possible prompting questions:

- Is there a closed boundary?
- What geometric 2D shape does that surface look like?
- How many sides does the shape have?
- How many angles does the shape have?
- What could you name this shape?
- Is this a composite shape? What 2D shapes could you put together to make this shape?
- Can you estimate the amount of turn in that angle?
- Is it more than or less than a right angle?
- Can you measure the angle with a protractor?
- Does the number you have read from the protractor make sense with your angle size (e.g., this is an acute angle, is the number you have less than 90° ; this is an obtuse angle, is the number you have more than 90°)?

Portfolio Task

The portfolio task *P4: Packaging Mayhem!* engages students with the creation of packaging for their item. Students may be able to engage with sections of the task requiring students to generate 2D nets for their packaging following this cycle.

RAMR Cycle

This RAMR cycle explores the classification of 2D shapes by their critical attributes and properties. These attributes or properties are then used along with multiplicative relationships to determine how many shapes or repeats of rows of shapes are needed to create a design of a specific size.



Reality

Remind students of the 3D objects, 2D nets and 2D faces on 2D nets explored in Cycle 1. Look around the school and local area to identify 2D shapes on 3D objects in the real world. Gather a selection of examples from the Internet or magazine pictures. Sort and classify shapes according to known names and recognisable properties (e.g., curved edges or straight edges, number of sides, number of angles). Focus students' attention on the edges and corners (sides and vertices) of shapes. Walk pathways focusing on lines and turns making sure to close the boundary for 2D shapes. This can be done with wet footprints on concrete, walking shape paths in the long jump pit, or laying down lengths of string or elastic as a walking guide in the classroom. Large copies of 2D shapes as walking paths can be made by using lengths of masking tape on the classroom floor. Discuss how simple closed paths enclose regions and are 2D shapes. Identify and walk straight lines, curved lines, parallel lines, intersecting lines, paths with turns. The Australian Edinborough Military Tattoo 2016 has many clear examples of bands and marching teams walking all these types of paths (<https://www.youtube.com/watch?v=1OkrxDqgrJ8>).

Note: If there are gaps in students' shape vocabulary, aim to fill these gaps during activities.



Abstraction

The abstraction sequence for this cycle starts from the identification of defining properties of 2D shapes (using the sub-concept approach) to identify points, lines and angles of turn that create shapes. The abstraction sequence suggested for exploring attributes of 2D shapes is as follows:

1. *Kinaesthetic activity.* For students, the attribute of angle is best experienced as a measure of the amount of turn. Discuss things that turn, experience turning bodies, connect turning bodies with direction faced so that students understand that angles measure turn about a point. Define angle as the amount of turn from one direction to another. Students may 'do a 360' on a skateboard or scooter, turn the pages of a book, and observe the movement of the hands of clocks.

Focus students' attention on turn to create vertices of shapes. Engage students with acting out angle as turn by standing with arms together in front of their body aimed in one direction. Have them turn their whole body to face a new direction. Connect change in direction facing to angle of turn (as below). Explore making a full turn, half turn, quarter turn.



2. *Connect to language.* Relate the change from one direction to another to the drawing of angle – stress that angle is the turn between the two directions as before. Ensure students understand that the **point** where the **arms** or **rays** of the two directions meet at a **corner** (represented by their body) is also known as a **vertex** when it creates the corner of a 2D shape.

3. *Connect to language.* Use the body as the vertex to make a **right angle** – put hands out forward, turn one hand so it aims along shoulders, then complete this turn – this is close to a right angle. Connect this experience to the previous acting out of full, half and quarter turns. Identify which turn this is (quarter turn). Make angles smaller than right angles – identify these as **acute angles**; make a straight line by starting with a right angle and moving the forward arm to the side – repeat with the other arm – consider this as two right angles or a **straight line** (connect to two quarter turns or a half turn); identify angles greater than a right angle and less than a straight line as **obtuse angles**.
4. *Record* right angles, acute angles, straight lines and obtuse angles with a drawing and own explanation/description.
5. *Find an angle in your classroom.* Describe what makes the angle (e.g. the top of the door and the wall make arms of an angle, the angle is between the wall and the door, the angle shows how far the door has turned when opening).



Resource

Resource 4.2.1 Exploring angle as turn

Resource 4.2.2 Comparing and ordering angles

6. *Introduce counted measure by using uniform items for measuring.* Since angle is measuring turn, define a predetermined distance from the origin (e.g., a ruler length) and count how many widths of a pencil or paddle pop stick or ruler width can be placed between the starting line and the finishing line. Students could make their own protractors here from their chosen non-standard unit and an angle wheel for use in measurement activities.



Resource

Resource 4.2.3: Measuring angle with non-standard units

7. *Establish the need for standard units* so that measures are comparable without description of the context and unit of measure. Connect informal language for turn with formal units of degrees.



Resource

Resource 4.2.4: Measuring angle with standard units



Mathematics

Once the sub-concepts of point, line, angle, and path have been established it is possible to combine these properties to consolidate identification and classification of 2D shapes. Activities can be completed on Geoboards (or Dot paper) to define and classify shapes.



Language/symbols and practice

2D Shapes with Geoboards/dot paper

Using rubber bands and Geoboards, explore combinations of straight line segments and angles (acute, right angle and obtuse) to create 2D shapes. (If using dot paper line segments and angles may be drawn by connecting dots with ruled lines. Explore shapes with specific numbers of sides, numbers of angles, parallel sides, and so on. *Resource 4.2.5 Constructing 2D shapes with geoboards* details a sequence for exploring lines and shapes with increasing levels of complexity.



Resource

Resource 4.2.5 Constructing 2D Shapes with Geoboards

Resource 4.2.6 Dot paper

Identifying Properties of 2D Shapes

Once students have had experience constructing 2D shapes with materials, and developed some naming conventions, explore the critical properties of familiar 2D shapes that contribute to a shape's classification. In particular, ensure students understand the minimum properties needed to classify a shape before exploring the special cases. For example, a triangle has 3 sides and 3 angles; a triangle that has a right angle is a right angle triangle; triangles with 2 sides of equal length and 2 equal angles are isosceles triangles; triangles with 3 sides of equal length and 3 equal angles are equilateral triangles; all other triangles are scalene triangles. By these classifications, a triangle may be both a right angle triangle and an isosceles triangle, all equilateral triangles are also isosceles triangles but not all isosceles triangles are equilateral triangles, none of these triangles are scalene triangles. See *Resource 4.2.7 Classifying 2D shapes* for a similar description of special classes of quadrilaterals.



Resource

Resource 4.2.7 Classifying 2D shapes

Resource 4.2.8 Properties of 2D shapes



Reflection



Check the idea

Challenge students to draw or construct shapes from given properties. For example, draw a shape that has 5 sides, 5 interior angles that include 3 acute angles, 1 obtuse angle and 1 reflex angle; what could you call this shape?

Pose problems for students that combine shape and multiplication. For example, if we are going to make pentagons from straws and sticky tape, how many straws will we need to make 5 pentagons? If there are only 30 straws, how many quadrilaterals could we make; how many triangles; how many hexagons; will there be any straws left over? If we start with 28 straws and made one of each shape starting with a triangle, then a quadrilateral and so on, what shapes could we make before we don't have enough straws?

Play Shape Celebrity Heads. Make bands for students to wear with shape names on. Students wearing the band may ask questions of the class to determine the critical attributes of their shape towards guessing the shape. Reverse this activity by playing 'What shape am I?' Use a series of statements to describe a shape for students to identify.



Apply the idea

Investigate a variety of different 3D packaging options. For example, Toblerone, packet soups, golf balls, coffee tins and so on. Categorise the packaging options as polyhedra or non-polyhedra. For each package, identify the 2D faces that make up the 3D object. List the properties of and classify each 2D face. Identify parallel lines, perpendicular lines and measure angles.



Extend the idea

Interior and exterior angles

So far angles have been simply considered as a measured amount of turn. However, when walking a path, the angle turned is the complementary angle of the interior angle of the shape.

Explore the difference between the interior angle and the amount of turn from the previous direction of travel to create a 2D shape (complementary angles to interior angles). Measure the resulting interior angles. See if students can generalise the rule that complementary angles add to 180° . Practise giving directions of path to generate shapes. For example, to walk an equilateral

triangle it is necessary to walk forward three steps, turn 120° , walk forward three steps, turn 120° , walk forward three steps (turn 120°) to complete the closed boundary. These directions will give interior angles of 60° . Explore what shape is generated if the directions only turn 60° from the direction of travel.

Explore measuring the exterior angles of shapes. Determine if there is a pattern or rule to adding interior and exterior angles of 2D shapes (total turn is 360°).

Tessellations

We exist in a society that puts shapes together to build and cover, and that packs shapes together to carry them around. With this in mind, it is important to investigate shapes that fit together to cover surfaces and pack well. Shapes that fit together without gaps or overlaps are called *tessellations*. Squares and rectangles tessellate, as shown in the images on the right.



Circles do not tessellate, as shown in the images on the right. When attempting to pack or tessellate circles, there will always be either a gap or an overlap.

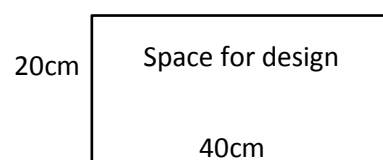


Explore familiar and regular 2D shapes to see which ones will tessellate on their own, which ones tessellate with another shape, and which ones do not tessellate. Investigate the side and angle properties of shapes that will tessellate (i.e., sides need to be straight or complementary curves, angles around a point need to add to 360°).

Note: This will provide useful background on the repetition of a shape to create a tessellation or tiling pattern. In the investigation activity, students will explore means of generating tessellating shapes other than regular polygons.

Tessellations of 2D shapes are frequently used in tiling and mosaic designs. If the dimensions of the shape or tessellating design are known, it is possible to determine how many will fit within a given space using multiplicative understandings.

For example, if tiles within a tessellating design are 2cm across and 4cm long, and the place where the design is to be created is 20cm across and 40cm long, it is possible to use division to work out how many tiles across and how many tiles long the design will be. Using the combining or product meaning of multiplication and an array model, it is also possible to ascertain how many tiles will be needed to create the tessellating design.



If the tile is placed so that it has the same orientation as the space for the design, then $20\text{cm} \div 2\text{cm} = 10$ tiles across; $40\text{cm} \div 4\text{cm} = 10$ tiles along; $10 \text{ tiles across} \times 10 \text{ tiles along} = 100$ tiles.



Explore other examples including designs that are not square.

Note: The focus here is not on Area calculation but on estimating how many will fit across and along the shape to be covered. The important ideas here are finding how many by breaking side lengths into pieces as long as or as wide as the tessellating shape.

Teacher Reflective Notes

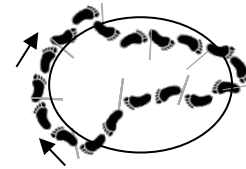
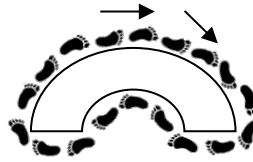
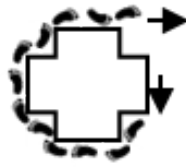
This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name: _____

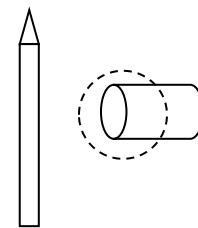
Date: _____

Can you do this? #3

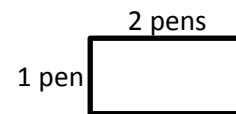
1. The footprints show where John walked. Circle the shapes where he walked a perimeter:



2. Without counting anything, how could you work out if the length of the pencil is longer than the distance around the rim of the can? _____



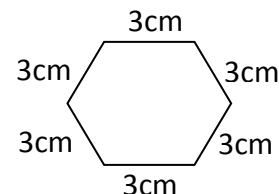
3. Calculate the perimeter of this rectangle.



4. This shape has 6 sides, it is not drawn to scale. Each side is 3cm long.

(a) Describe how to find the perimeter of the hexagon. _____

(b) Calculate the perimeter of the hexagon?



Obj.
4.3.1
i. ☐
ii. ☐
iii. ☐
iv. ☐

Obj.
4.3.1
☐

Obj.
4.3.3
i. ☐
ii. ☐
iii. ☐

Obj.
4.3.4
a) ☐
b) i. ☐
b) ii. ☐
b) iii. ☐

Cycle 3: Exploring Perimeter

Overview



Big Idea

This cycle extends the development of measurement concepts using the context of length to explore distance around a shape (perimeter). Building from students' experience with comparing and ordering lengths and indirect comparison using informal units, students will develop and understanding of the calculation of perimeter of shapes by adding the lengths of the sides. Students will also be encouraged to consider the conversion of additive equations for perimeter to multiplicative equations where applicable (e.g., perimeter of a square = $4 \times$ length of side; perimeter of a rectangle = $2 \times$ (length + width); perimeter of a regular pentagon = $5 \times$ length of side. This cycle does not extend to metric measure of length (explored in Unit 06).



Objectives

By the end of this cycle, students should be able to:

- 4.3.1 Identify the perimeter of a shape as the distance around its boundary. [5MG109]
- 4.3.2 Compare and order several shapes based on distance around, using appropriate uniform informal units. [2MG037]
- 4.3.3 Calculate perimeter of rectangles. [5MG109]
- 4.3.4 Calculate perimeters of parallelograms, trapeziums, rhombuses and kites. [8MG196]



Conceptual Links

Identification of attributes and the development of the measuring process is a skill that can be applied to measurement of any attribute. Measuring skills provide information needed to determine calculated measures such as perimeter, area and volume.



Materials

For Cycle 3 you may need:

- String
- Strips of paper
- Informal units (scissors, pegs, pencils, paddle pop sticks, tooth picks, match sticks)



Key Language

Perimeter, boundary, edge, length, width, breadth, sides, formula, formulae



Definitions

Perimeter: distance around the boundary of a 2D shape.



Assessment

Anecdotal Evidence

Some possible prompting questions:

- Is there a closed boundary?
- Do you have measurements for all the sides?
- What 2D shape does that look like?
- Can you work out what some of the sides will be from other sides?
- How many sides does the shape have?
- Is this a composite shape?
- What 2D shapes could you put together to make this shape?
- Will that help you to decide what the missing lengths should be?

Portfolio Task

The portfolio task provides students with an opportunity to demonstrate measurement skills as they measure the item to be packaged and determine the dimensions of their box.

RAMR Cycle



Reality

For this cycle the focus is on the attribute of perimeter (distance or length around a 2D shape). Focus students' attention on contexts where perimeter is compared. Relevant real-life contexts can be boundary fences, borders around garden beds, distance around the outside of buildings, length around a window for insect screen edging, and so on.



Abstraction

The abstraction sequence for this cycle starts with a rich understanding of the attribute to be measured, followed by development of understanding the foundations of measurement concepts. Students need to understand that perimeter is the distance around a shape. The abstraction sequence is as follows:

4. *Kinaesthetic activity.* Identify the attribute for comparison by physically walking around boundary fences or tracing around the edges of items using a fingertip (e.g., desktops, pages, whiteboard, window, posters or brick on wall).
5. *Use indirect comparison with an intermediary to measure.* A piece of string or a third item can be used to compare items indirectly.
6. *Indirectly compare* the distance around items to determine long, longer (or other appropriate words). Small items can be 'rolled' along a line or string with the starting and finishing point marked and then informal units laid along to measure (e.g., pencils, erasers, envelopes, unifix cubes, paddle pop sticks, match sticks). The most important concept here is the idea of distance around the boundary.
7. *Record.* Record the perimeters of a variety of items within a table. Ensure that students record the unit with the measure. Compare and order items from least to most perimeter.
8. Focus students on the need for appropriately sized units to count and measure each side of a shape. Add these measures together for perimeter. Compare this result with measures determined by using a piece of string and then measuring the string. Discuss which is more accurate. Discuss which is more practical in most circumstances.



Mathematics



Language/symbols and practice

Engage students with exploring the perimeter of shapes in their surroundings.

Irregular shapes

Start with irregular shapes where it is possible to focus on the total distance around by counting the length of each side and then adding these together. Encourage students to write these as addition equations with a symbol for the unknown so that they have the opportunity to practice creating algebraic equations. Ensure that students are comfortable with putting the unknown first followed by the equals sign (e.g., $P = \text{side} + \text{side} + \text{side} \dots$). Ensure that students always record a unit name with their counted measure of perimeter.

Regular shapes

Continue exploration of perimeters of regular shapes such as squares, rectangles, regular pentagons and so on. As students gain competence with forming equations for perimeter calculations, see if they are able to shift to multiplicative equations for regular shapes. For example, a square with a side length of 3 pencils could be written as $P = 3 \text{ pencils} + 3 \text{ pencils} + 3 \text{ pencils} + 3 \text{ pencils} = 4 \times 3 \text{ pencils}$.

Explore optimisation problems

Consider applications like fenced enclosures for animals. Find how many different sized enclosures can be created from the same length of fencing (stay within whole number parameters). For example, a 16m length of mesh could create an enclosure that is 1m wide by 7m long, 2m wide by 6m long, 3m wide by 5m long, 4m wide by 4m long. These can be acted out with pencils, paddle pop sticks or matchsticks if necessary.



Reflection



Check the idea

Check students' understanding of perimeter by comparing the distance around common classroom items (estimate and order items before measuring and calculating perimeters). Measure and calculate perimeters and then reorder where necessary.



Apply the idea

Drying glasses activity (an idea from the 'I hate mathematics' book)

When you dry a glass with a towel, you dry up the side and around the top rim. Which is longer, the distance up the side (the height) or the distance around the top (the circumference of the circle)? Use your towel. Mark off the height of the glass on the towel. See if this distance will wrap around the top. Is this always true? Are there glasses that do not do this?



Extend the idea

See if students are able to generalise a rule for calculation of perimeter of irregular shapes. Extend this rule to more specific applications like squares, other regular polygons, rectangles, parallelograms, rhombuses, trapeziums and kites.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name: _____

Date: _____

Can you do this? #4

1. Draw pictures of the following:

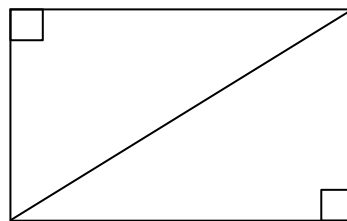
(a) Parallel lines

(b) Perpendicular lines

Obj.
4.4.4
a) ☐
b) ☐

2. This is a rectangle with a diagonal drawn across it.

(a) How many triangles are in the rectangle?



(b) What do the angles inside each triangle add up to? _____

(c) What do the angles inside the rectangle add up to? _____

(d) Tick (✓) a pair of alternate angles.

(e) Draw a line across the middle of the rectangle from top to bottom.

(f) Use a protractor to measure one of the angles in the centre of the rectangle. Label the angle with its measurement. _____

(g) Use a cross (×) to label another angle that measures the same.

Obj.
4.4.2
b) ☐

Obj.
4.4.8
a) ☐
c) ☐

Obj.
4.4.6
d)i ☐
d)ii ☐

Obj.
4.4.3
e) ☐
g) ☐

Obj.
4.2.6
f) ☐

3. Draw a shape that has 5 sides, 2 acute and 3 obtuse interior angles.

Obj.
4.4.9
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

4. (a) Use a protractor to draw a triangle with one angle of 33° and one right angle.

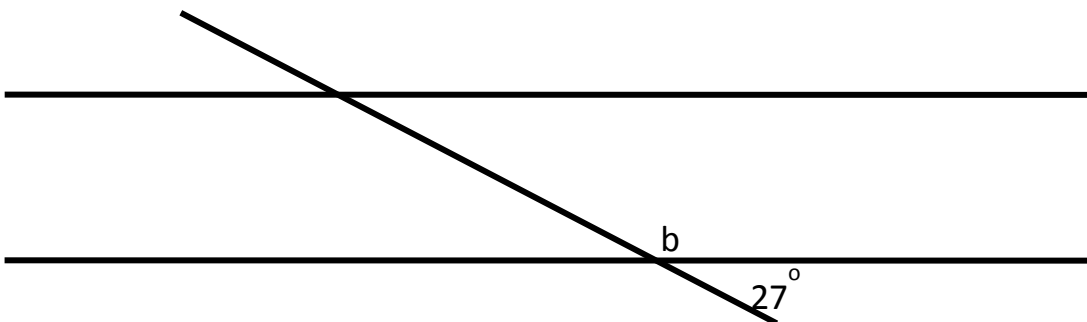
Obj.
4.2.6
a) i. ☐
a) ii. ☐

(b) Calculate the value of the other angle. _____

Obj.
4.4.2
b) ☐

5. A beam fell across a railway track. The angle between the beam and the rail on one side was estimated at 27° .

- (a) On the diagram for the police report, mark with a cross (x) the corresponding 27° angle.



Obj.
4.4.5
a) ☐
Obj.
4.4.1
b) ☐

(b) Calculate the angle marked with a letter "b". _____

Cycle 4: Geometric Reasoning

Overview



Big Idea

Angles in real life will be either static or dynamic. A static angle does not change and a dynamic one does. The corner of a room or corner of a window is an example of a static angle. The hands of a clock are an example of a dynamic angle which changes during the day. Where angles are formed by transversals (straight lines that cross other lines), consistent relationships between the magnitudes of resulting angles may be observed. This cycle extends from measurement of angle as the magnitude of turn to explore properties of angles within geometric figures.



Objectives

By the end of this cycle, students should be able to:

- 4.4.1 Use investigation of angles on a straight line to find unknown angles. [6MG141]
- 4.4.2 Demonstrate that the angle sum of a triangle is 180 degrees. [7MG166]
- 4.4.3 Use investigation of vertically opposite angles to find unknown angles. [6MG141]
- 4.4.4 Investigate conditions for two lines to be parallel. [7MG164]
- 4.4.5 Identify corresponding angles and use to find unknown angles. [7MG163]
- 4.4.6 Identify alternate angles and use to find unknown angles. [7MG163]
- 4.4.7 Identify co-interior angles and use to find unknown angles. [7MG163]
- 4.4.8 Use the angle sum of a triangle to find the angle sum of a quadrilateral. [7MG166]
- 4.4.9 Establish properties of quadrilaterals using congruent triangles and angle properties. [8MG202]



Conceptual Links

This cycle incorporates previous concepts of shape, line and the measurement of angle as turn.

This cycle provides an opportunity to extend students' reasoning using the properties of geometric figures. This contributes to students' understanding of symmetry and transformational geometry explored in Cycle 5, location and direction on the Cartesian plane in Unit 10 and triangle geometry and Pythagoras in Unit 12.



Materials

For Cycle 4 you may need:

- String
- Geoboards
- Rubber bands
- Rotagrams
- Protractors



Key Language

Lines, rays, points, angle labelling conventions, sharp, blunt, alternate angles, co-interior angles, corresponding angles, complementary angles, transversals, angle, right angle, acute angle, obtuse angle, parallel, perpendicular, congruent



Definitions

Alternate angles/Co-interior angles/Corresponding angles: where parallel lines are crossed by a straight line: alternate angles will be the same size (drawn as a letter “Z”); co-interior angles will add to 180° (drawn as a letter “U”); corresponding angles will be the same size (drawn as a letter “F”)

Equilateral triangle/Isosceles triangle/Scalene triangle/Right angle triangle: triangles can be more specifically classified according to their side and angle attributes: equilateral triangles have 3 sides all the same length with all 60° angles; isosceles triangles have 2 sides the same length with 2 same-size angles; right angle triangles have one angle at 90° ; scalene triangles have all sides different lengths and all angles different sizes

Transversals: straight lines that cross other lines

Vertically opposite angles: when two straight lines cross, angles opposite each other are the same size (drawn as a letter “X”)



Assessment

Anecdotal Evidence

Some possible prompting questions:

- What is the relationship between these angles?
- Can you use this to work out the missing angle?
- Are these lines parallel or perpendicular or do they cross at a measurable angle?
- Which other angles will be the same magnitude?

Portfolio Task

The portfolio task provides students with an opportunity to demonstrate shape, line and angle understanding as they measure the item to be packaged, determine the shape and construct the net for their box.

RAMR Cycle



Reality

Angles in real life will be either static or dynamic. A static angle does not change and a dynamic one does. The corner of a room or corner of a window is an example of a static angle. The hands of a clock are an example of a dynamic angle which changes during the day. Discuss with students other examples of static and dynamic angles. Explore different sized dynamic angles created by opening swinging doors wider or narrower. Discuss whether these angles are measurable.

Focus students' attention on items with static angles (e.g., doors, windows, reinforcing struts, chain wire fences). Try to locate some examples that are not solely comprised of rectangles (Internet images may be necessary here). If necessary, revise angle measurement using a protractor (Cycle 2). Identify the edges and vertices of the 2D geometric figures that are formed by these shapes. Revise measuring interior and exterior angles of these shapes.



Abstraction

The abstraction sequence for this cycle starts from students' understanding of angle as measurement of turn, extending to an exploration of angle properties of geometric figures. The abstraction sequence is as follows:

1. *Kinaesthetic activity.* Create a range of triangles on the floor with masking tape (or draw chalk triangles on the concrete with pavement chalk). Have a student walk the path of the triangle. Stop at each vertex and identify the change in direction as the angle turned and its complementary angle. Add the complementary angle measures together to determine the interior angle sum of the triangle. Test a few triangles to generate a table of data of interior angle sums. Look for a pattern in the angle sums (should be close to 180° depending on accuracy of measure).
2. *Explore with materials.* Have students cut a triangle from a piece of A4 paper and place a pen mark in each of the interior vertices. Tear the corners off of the triangle (ensures that the original angle of interest is obvious). Lay the corners with their points together along a straight line in an exercise book and glue down. Identify what angle the interior angles added to.
3. *Explore with symbols.* Measure each angle with a protractor. Add the measured angles together to reinforce the total measure. Add to the larger table of angle sums and identify the most frequently occurring angle sum. Generalise the rule for the angle sum of a triangle.



Mathematics



Connections

Connect measurement of angles and properties of 2D shapes from Cycle 2. Explore the sums of interior angles of polygons by measuring and adding and by testing as described in *Resource 4.4.1: Exploring interior angles of polygons*. Explore relationships between angles in quadrilaterals formed by combinations of parallel lines and transversals (trapeziums, parallelograms, rhombuses, rectangles and squares). Measure and identify which angles have the same measures and which are complementary angles. Highlight the parallel lines and the matching and complementary angles.



Resource Resource 4.4.1: Exploring interior angles of polygons



Language/symbols and practice

Explore straight lines, parallel lines and transversals. Ensure students can describe a straight line in contrast with a curved line? Identify where parallel lines exist in reality? What angles are created when a transversal is drawn across parallel lines and what patterns are there in their measurements? Connect to previous data generated from exploring angle properties within quadrilaterals. Generalise vertically opposite, alternate, co-interior and corresponding angle rules.



Resource Resource 4.4.2: Exploring angle properties



Reflection



Check the idea

Students should have completed sufficient measuring of angles in the previous activities to have demonstrated ability to measure angle using a protractor.

Consider using the paper planes constructed in *Portfolio Task 01* as contexts for practising and demonstrating the measurement of angles. How do the angle measurements of the most effective paper planes compare with the angle measurements of less effective planes?



Apply the idea

Clock hands are an example of dynamic angles (the angle of turn between the hour and minute hands and the vertical starting position change over time). Investigate the construction of a clock. Calculate how many degrees between the hour hand and vertical to evenly space the twelve numbers around a clock face. How many degrees needed to place minute marks?



Extend the idea

As an extended application of angle and shape, explore rigidity, diagonals and angles with geostrips. This idea is the foundation of construction of support in bridges, roof sections and walls.



Resource Resource 4.4.3: Exploring rigidity, diagonals and angles

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

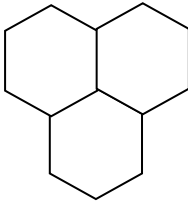
Name: _____

Date: _____

Can you do this? #5

1. (a) Look at this shape. Does it have rotational symmetry? _____

(b) If so, what order of rotational symmetry does it have? _____



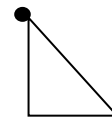
(c) Trace the shape. Rotate the tracing 180° , slide it to the right, and transfer to the page to extend the tessellating pattern.

(d) Rotate the tracing 180° , slide it to the right, and transfer to the page to extend the tessellating pattern. (You should have 9 hexagons drawn on the page).

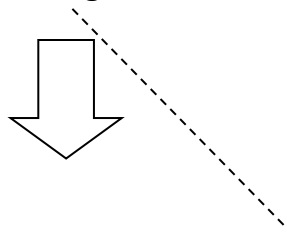
(e) Does the new shape have rotational symmetry? _____

(f) If so, what order of rotational symmetry does it have? _____

2. (a) Rotate the triangle 90° , clockwise, about the dot.



(b) Show the arrow after a slide along the dotted line.



3. Draw a shape that has rotational symmetry.

Obj.
4.5.3

a) ☐

b) ☐

Obj.
4.5.7

c) i ☐

c) ii ☐

d) i ☐

d) ii ☐

Obj.
4.5.3

e) ☐

f) ☐

Obj.
4.5.3

a) i ☐

a) ii ☐

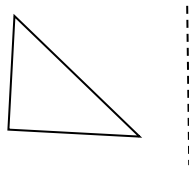
Obj.
4.5.2

b) ☐

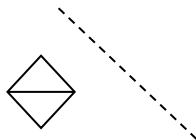
Obj.
4.5.1

☐

4. a) Draw a flip of the shape, over the dotted line.

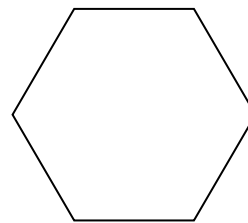


- b) Draw a flip of the shape, over the dotted line.



5. Look at this shape:

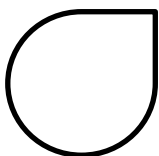
How many lines of symmetry does this shape have? _____



6. Reflect this image to create a picture with one line of symmetry.



7. (a) Trace the shape. Flip the shape vertically, and transfer to the page.
(b) Trace the new shape, flip the shape horizontally, and transfer to the page.



- (c) How many lines of symmetry does the final shape have? _____
(d) Colour or shade the shape so it has only one line of symmetry.
(e) Rewrite the instructions using a different transformation to make the same shape? _____

Obj.
4.5.4
a) ☐
b) ☐

Obj.
4.5.6
☐

Obj.
4.5.1
☐

Obj.
4.5.7
a) ☐
b) ☐
e) ☐

Obj.
4.5.6
c) ☐
d) ☐

Cycle 5: Flips, Slides and Turns

Overview



Big Idea

Change is frequently thought of as changing quantity or size of an entity. However, changes in location and orientation brought about by movement do not affect the dimensions and shape of 3D objects and 2D shapes. These changes are collectively described as Euclidean changes, flip (reflection), slide (translation), and turn (rotation). When 2D shapes are subjected to Euclidean change, related properties may be observed in line symmetry (related to flips) and rotational symmetry (related to turns).

The focus of this cycle is to explore Euclidean turns (rotations), slides (translations) and flips (reflections). Turns are about a point which may be located anywhere on the shape (not just the centre) or even outside the shape. Rotating a shape may also result in an image that has rotational symmetry. Slides may be vertical, horizontal, diagonal or a combination of these. Flips over a line may be horizontal, vertical or diagonal resulting in a mirror image or, when the original image is left in place, result in images with line symmetry. Similarly, shapes that can be imagined to have been created in this way, or may be visually dissected into matching pieces, may be defined as having lines of symmetry. Flips, slides or turns may be completed in isolation or in combination to generate a variety of tessellating designs.



Objectives

By the end of this cycle, students should be able to:

- 4.5.1 Create symmetrical patterns, pictures and shapes using reflections, rotations or translations. [4MG091]
- 4.5.2 Describe translations of two-dimensional shapes. [5MG114]
- 4.5.3 Describe rotations of two-dimensional shapes. [5MG114]
- 4.5.4 Describe reflections of two-dimensional shapes. [5MG114]
- 4.5.5 Identify rotational symmetries. [5MG114]
- 4.5.6 Identify line symmetries. [5MG114]
- 4.5.7 Investigate combinations of reflections, translations and rotations. [6MG142]



Conceptual Links

This cycle aims to begin from students' existing capacity to describe 2D shapes and 3D objects using geometric language of line, angle, face, edge and vertex and students' ability to measure angles.

This cycle introduces transformation of shapes using rotations (turns), translations (slides), or flips (reflections), tessellation of shapes and objects and dissections. These ideas are used when calculating area of shapes and volume of objects. Dissection of shapes connects to and consolidates ideas used when creating or deconstructing nets of 3D objects (Unit 04) and when determining area of complex shapes (Unit 08) and surface area of 3D objects (Unit 14). Angle of rotation experiences will further connect to transformations of shapes on the Cartesian plane (Unit 11).



Materials

For Cycle 5 you may need:

- Long lengths of string, wool or coloured elastic
- Tessellating puzzles
- Mira mirrors or computer
- Lengths of string, wool or coloured elastic
- Coordinate grids
- Maths mat or large grid on floor
- Squared paper or grids
- Tracing paper or OHT plastic
- Pavement chalk



Key Language

2D shapes, turn, rotate, rotation, point, centre of rotation, rotation around a point, rotational symmetry, orientation, slide, translate, translation, direction of movement, tessellate, tessellating shape, tessellation, angles, order of rotation, flip, reflection, mirror, mirror image, straight lines, points, curves, angles, line of reflection, line of symmetry, line symmetry, congruent, congruence



Definitions

Euclidean transformations: reflections (flips), translations (slides), or rotations (turns) of a shape or object change the orientation or position without changing its shape or size.

Line of reflection: line along which an image may be reflected (flipped) to create a mirror image.

Order of rotation: rotational symmetry may be given a number to denote how many rotations result in a matching image. Rotational symmetry is also described by the number of degrees an image may be turned to make it match each time.

Symmetry: when one shape becomes exactly like another if you flip, slide or turn it.



Assessment

Anecdotal Evidence

Some possible prompting questions:

- If you look at the repeat of this shape, what stayed the same and what changed?
- Did the shape's orientation or position change? In what way?
- Does this image match if you turn it? How many times can you make it match in one full turn (360° rotation)? What type of symmetry is that? How many degrees has it turned each time to make it match?
- Does the new shape look like a mirror image of the first shape? What type of symmetry is that?
- Can you make a line through the shape so that it is the same each side?
- Can you make a line between the shapes so that each corresponding point on the shape is the same distance from the line but on opposite sides?
- Can you make a tessellation of this shape? Do you need to slide the shape, rotate the shape or flip the shape to make it tessellate? Do you need to use a combination of these transformations?
- If you flip this shape horizontally, then vertically, what does it look like?
- Is there another transformation you could use to make the same change?

Portfolio Task

The portfolio task provides students with an opportunity to apply transformation skills as they slide or rotate their package net to tessellate it upon a single piece of paper/card.

RAMR Cycle



Reality

Discuss changing location of shapes without changing the size or straightness of the shape such as in Tetris. Tetris shapes are made up of blocks that slide downwards. The column the blocks are sliding down may be shifted left or right using keys.

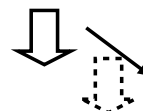
Simple slides or turns can be physically enacted and connected to students experiences in Cycle 2 where they walked in a line (slide) and changed direction (turn). These can be reinforced by having students strike an asymmetric pose and walk sideways or diagonally without changing their orientation for a slide, around in a circle on the spot, or stepping sideways keeping a fixed centre in view and the same distance away (holding a string looped around a central pivot to maintain the same distance may be beneficial). This is also why we only ever see one side of the moon. As it orbits the Earth it also rotates on its own axis at the same speed as its orbit so that one face always faces the Earth and the other always faces away.



Abstraction

The abstraction sequence for this cycle starts from students' previous experience of shapes and explores what aspects of a shape change/stay the same when Euclidean transformations of slides are applied. Transformations involving rotations (turns) and reflections (flips) will be explored in the Mathematics phase. A suggested sequence of activities is as follows:

1. *Kinaesthetic.* Use a length of string (or maths mat elastics) and 3 students to create a large triangle. Keeping the lengths of string and the angles consistent, move in straight lines within the classroom to different locations. Discuss the features of the shape that stay the same (lengths of sides, sizes of angles, relationships between points, orientation of the shape); and what features changed (location of each point within the room).
2. *Represent/model with materials.* Draw a copy of the triangle on squared paper. Mark in an arrow to guide the slide. Trace the shape and arrow, slide along the arrow for a distance and glue the shape down in its new location. Discuss what changed and stayed the same about the two shapes.
3. *Property noticing.* Identify properties of translations as students are constructing slides. Ensure students notice that all parts/points of the object move in parallel for the same distance and in the same direction (i.e., lines drawn from initial positions of each arm to final positions of each arm will be parallel). This can be reiterated using tracing paper to create the slide along a line and determine that points have remained parallel and the same length away.



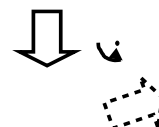
Resource Resource 4.5.1 Slides



Mathematics

Turns (rotations)

Repeat the activity sequence for slides with a focus on rotations (turns). Initially it is simplest to make one of the vertices of the shape the centre of rotation. Use a length of string and 3 students to create a large triangle. Keeping the lengths of string consistent, have one student stay still as the pivot point as the other 2 students walk in an arc. Discuss the features of the shape that stay the same (lengths of sides, sizes of angles, relationships between points); and what features changed (orientation of the shape, location of each point within the room).



Draw a copy of the triangle on squared paper. Mark in a point to guide the rotation. Trace the shape and turn around the point for a distance and glue the shape down in its new location. Discuss what changed and stayed the same about the two shapes.

Identify properties of rotations as students are constructing turns. Ensure students notice that all parts/points of the object turn but parts close to the centre of the turn stay close to the centre and parts away from the centre stay away from the centre by the same distance. This can be reinforced using tracing paper to demonstrate that corresponding points at the start and the finish are joined by curves, the change is circular and the same angle.

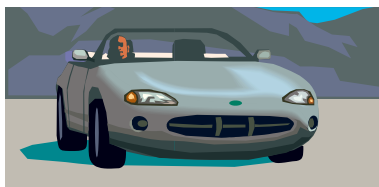
These steps should be repeated to explore rotation of shapes about a centre point instead of a corner and rotation about a point external to the shape.



Resource Resource 4.5.2 Turns

Flips (reflections)

The concept of reflections and mirror images should be familiar to students. Discuss more deeply with students what constitutes a mirror image or reflection. Establish the extent of students' understandings, are mirror images always horizontal or vertical; can they also be created diagonally; do the parts of the reflected image always touch or can shapes or objects be reflected so that they do not become melded into one shape? Consider also flips or reflections where the object or shape is flipped without the original staying in place. This can be easily demonstrated on the computer when flipping a shape vertically or horizontally. Students may include the notion of flipping a car when thinking of flips. This is actually a rotation in the vertical plane which is different from a reflection or flip. Ensure that this distinction is understood and connected through language that turning a car over so that the wheels are up not down is a rotation. A true reflection or flip of a car would be more accurately seen when comparing a left-hand drive car with a right-hand drive car.



Original car image.



Car reflected/flipped.



Reflected car rotated.

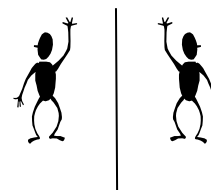


Resource Resource 4.5.3 Reflections

Flips can be the most difficult of the transformations to act out. Reflection can be practised by having two students sitting and/or standing facing each other. In turn, one of the students strikes asymmetric poses or moves around and the other student acts as a mirror. Getting a student to experience the act of reflection on their own requires them to strike an asymmetric pose, walk towards an imaginary mirror (e.g., a line on ground), and imagine passing through the mirror to become the reflection. This would require the student turning around, facing the other way, and changing the pose so left becomes right and right becomes left. However, it is the important way for learning because it requires the student to be active in achieving the change. Any of these could be done by having students lie on the ground in a pose, with other students looking over the top and have a second student perform a flip in relation to the first student.

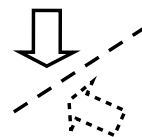
First student lying down or standing against the whiteboard

Second student making flip (or reflection)



Use 3 different coloured lengths of string (or maths mat elastics) and 3 students to create a large triangle. Place a line on the floor to reflect across. Place 3 students in mirror reverse on the other side of the line. Keeping the lengths of string and the angles consistent (or using a matching set of string or elastics), create a reflected image on the other side of the line. Discuss the features of the shape that stay the same (lengths of sides, sizes of angles, relationships between points); and what features changed (location of each point within the room, orientation of the shape).

Draw a copy of the triangle on squared paper. Mark in a line for the reflection. Trace the shape and line, flip the page over and line up the reflection line. Glue the shape in its new location. Discuss what changed and stayed the same about the two shapes.



Identify properties of reflections as students are constructing flips. Ensure students notice that left and right interchange with all points near the flip line staying near the flip line and points away from the flip line staying away from the flip line. Using tracing paper and measurement, students should be able to recognise that the distance from a point to the line of reflection will be the same as the distance from the line of reflection to the matching point on the reflected shape.



Resource Resource 4.5.4 Flips



Connections

Creating designs using reflections, translations and rotations

A design can be copied onto tracing paper and then the tracings used to reflect the design horizontally and vertically to make a larger design, which is better than the original. Other variations may be achieved by combining reflections, rotations and translations.

The 'image' is the starting image and the 'reflect' is in reference to this original image.

image	reflect	reflect	reflect	reflect	reflect
reflect	reflect	reflect	reflect	reflect	reflect

Mira mirrors are Perspex mirrors that are useful tools for exploring reflections. A Mira mirror can be used to superimpose images and to draw reflections. Its purpose is to develop *spatial visualisation* (the ability to mentally manipulate, twist, rotate, reflect, slide or invert shapes), an understanding of the role of angles in reflection, and an understanding of the relationship between reflections and symmetry.



Resource Resource 4.5.5 Reflection Designs using Mira Mirrors

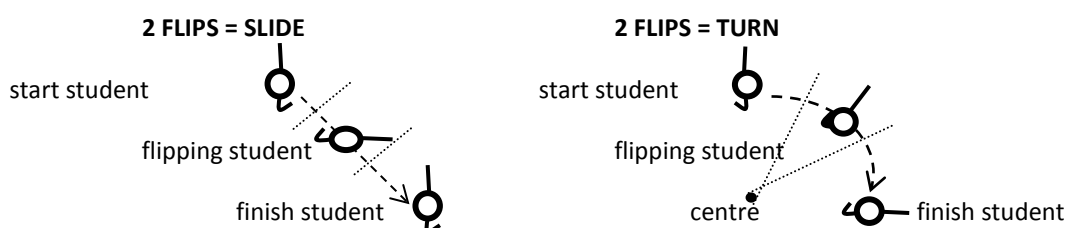
Similarly, tracings may be used to translate or turn the image to make a larger design, as below. Other variations may be achieved by combining transformations. Wallpaper, fabrics for upholstery, curtaining and dressmaking often have *one-way* designs or *two-way* designs that are created this way. Older houses with decorative cornice or border prints around the wall in bathrooms or kitchens are also real-world applications of translations. Where one tile is a quarter of a design and placed with 3 identical tiles rotated around a corner, the overall design is created using rotations.

The 'image' is the starting image and the 'translate', or 'rotate' are in reference to this original image.

image	rotate	image	translate	translate	translate	translate
rotate	rotate	translate	translate	translate	translate	translate

Relationships between Euclidean transformations

Explore combinations of Euclidean transformations. For example, *two flips make a slide* or *two flips make a turn*. To act these out, get two students to strike the same pose (one in front of the other) and the front one to complete a slide (resulting in the slide being depicted by a start student and a finish student). Then get a third student to copy the pose, stand in front of the start and do two flips in the direction of the slide. The second flip will end with this student in exactly the same pose as the finish student for the slide so, if flip lines are the right distance apart (half the slide distance), it shows that two flips is the same as one slide. This process can be repeated for the turn but here the flip lines will go through the centre and be half the angle of the turn. Repeat both processes with objects.



Exploring line symmetry

Line symmetry is where a shape can be folded in half so that both sides match. The very simplest symmetrical pictures are created by putting paint onto one half of a piece of paper and then folding over and pressing down to create a symmetrical paint blob.



This idea can also be explored by asking students to rule a line on the page and write their name in block letters that all touch the line. Students should then reflect each letter across the baseline to create a mirror image or reflection of their name. Selected use of colour can create an interesting symmetrical design.



Note: Mira mirrors can make the copying of the reflected letters easier.

Students should experience constructing symmetrical figures, completing symmetrical figures and identifying line symmetry on figures. *Resource 4.6.4: Exploring Line Symmetry* includes activities that encompass these ideas. Mira mirrors may be used, or tracing paper, or careful copying.



Resource Resource 4.5.6 Exploring Line Symmetry

Exploring rotational symmetry

Rotational symmetry is where a copy of a shape can be turned on top of itself and matches in a part turn. Using tracing paper is a powerful way to test for rotational symmetry as the tracing of the shape can be rotated over the original shape for easy comparison. Rotational symmetry is also be described by the number of degrees of rotation completed for the shape to match.



Resource Resource 4.5.7 Rotational symmetry

Properties of and relationships between line and rotational symmetry

It is important for students to understand the differences between line and rotational symmetry. The cards used to identify rotational symmetry can be reused as an activity to identify line symmetry and to explore the properties and relationships that exist within shapes that have line and/or rotational symmetry. *Resource 4.5.8: Symmetry* includes a table that can be used to explore relationships between the number of lines and rotations of symmetry in symmetrical 2D shapes.



Resource Resource 4.5.8 Symmetry



Reflection



Check the idea

Use visual imagery activities to provide students with practice determining which shape is the same as another to fill a space. Ask students to identify how the shape needs to be rotated or translated to fill the space. Give students a shape and ask which of four shapes match.



Resource Resource 4.5.9 Visualisation problems

Use dissection puzzles such as *Resource 4.5.10 Tangrams* to engage students with identifying which transformations to puzzle pieces are applied to create the new shape.



Resource Resource 4.5.10 Tangrams



Apply the idea

Create a design using rotational symmetry that could be used as the basis of a logo for a new junior secondary uniform pocket at your school. Find a way to incorporate the initials of your school in the design that retains line symmetry in your logo.



Extend the idea

Tessellations of shapes may be created using turns and slides of 2D shapes. Some shapes will tessellate naturally due to the configuration of their sides and angles, others will have complementary sides which will enable them to tessellate. These include Pentominoes, Egg Puzzles and McMahon's Colour Triangles. Explore with students the changes to each piece to make it fit its new location. Extend students' thinking to the properties of angles at the point where shapes meet with no gaps or overlaps (angles add to 360°). *Resource 4.5.11 Tessellating puzzles* has ideas to engage students in the exploration of tessellating shapes and the creation of shapes that will tessellate.



Resource Resource 4.5.11 Tessellating puzzles

Congruent shapes are when one shape changes to the other by flips, slides and turns **only**. Explore the shapes in *Resource 4.5.12: Congruence* to determine which shapes are congruent. Prisms, pyramids and cylinders have some congruent faces. Test a variety of 3D objects to identify which faces are congruent. Connect this exploration to 2D nets of 3D objects.



Resource Resource 4.5.12 Congruence

Three-dimensional puzzles are very important as these act as preparation for our 3-D world in the realms of architecture, brain surgery, packaging and so on. Two examples are:

- Soma cubes – 7 pieces that make a $3 \times 3 \times 3$ cube and many other shapes; and
- wooden cube pentominoes – 12 3-D pentomino pieces that can be used for 3-D as well as 2-D puzzles



Resource Resource 4.5.13 3D Tessellation Puzzles

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 04 Investigation: Teacher Notes

Introduce the lesson with computer images of Escher drawings and fabric design.

- Art of Escher: <http://www.tessellations.org/eschergallery1thumbs.shtml>
- Tessellating Fabric Designs: <http://www.spoonflower.com/tags/tessellation>

Engage students with discussion around the features of the designs. Lead them to recognise that the same shapes have been repeated with no gaps or overlaps. Students may also identify that the colours of the designs are alternating. This is a stylistic or artistic choice to enhance the appearance of the artwork.

Provide students with regular polygons to try to generate their own tessellation design or artwork using translation, rotation or reflection.

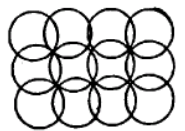
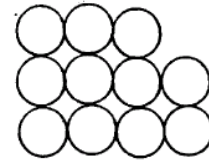
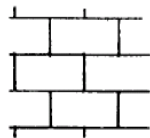
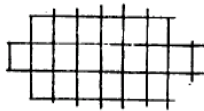
Extend students by providing them with a circle to transform into a tessellation design or artwork.

Ensure students can identify places in reality where tessellation is common. For example, honeycomb, paving, brickwork, fabric design.

Encourage students to estimate how many of their shape will fit on a page, count how many fit on a page and calculate how many will fit on a page. These ideas reinforce multiplicative thinking ideas from Unit 03 and underpin the measure of coverage in Unit 08.

Unit 04 Investigation: Exploring Design with Tessellating Shapes

Tessellation designs repeat the same shapes without gaps or overlaps to create an overall design. They are most often found in paving and tiling. When the tessellating objects are 3D, items pack together like in boxes with no gaps or overlaps.



Tessellations

Not Tessellations

1. Write a list of things you can think of that use tessellating shapes.

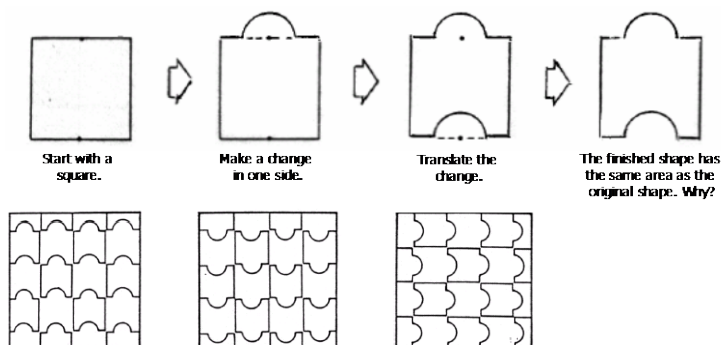
Most shapes can be used as a starting point to make shapes that will tessellate.

Try changing a square to make a tessellating shape for a design.

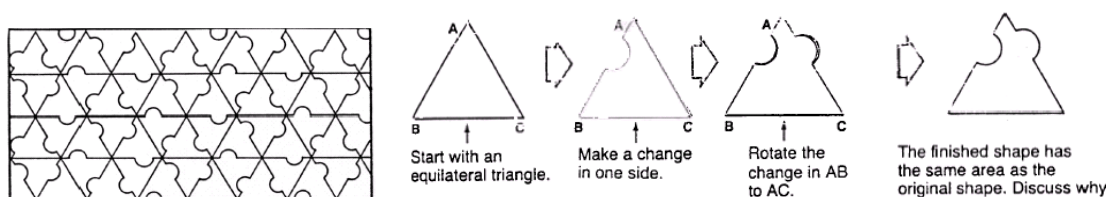
Shapes can be changed by translation (sliding), rotation (turning) or reflection (flipping) to make tessellating designs.

1. Start with a cardboard square that is 5cm long and wide.
2. Cut a piece from one side and add it to the other with sticky tape. This changes the shape by translation.
3. Trace around the shape on paper many times to make a tessellation design.

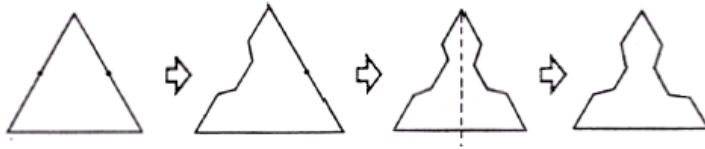
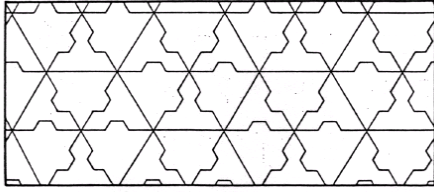
Changing shapes by sliding (i.e., translating):



Changing shapes by rotation



Changing shapes by reflection:



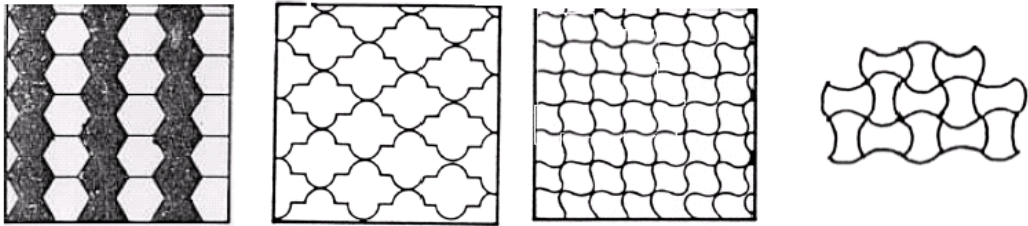
Estimate how many of your base shape you can draw on this page as a tessellating design using reflection.

Draw your tessellating design here.

Start with a circle and try to use the same techniques to make a tessellating design.
Draw your design on this page.



These shapes tessellate by themselves:



These shapes do not tessellate by themselves.

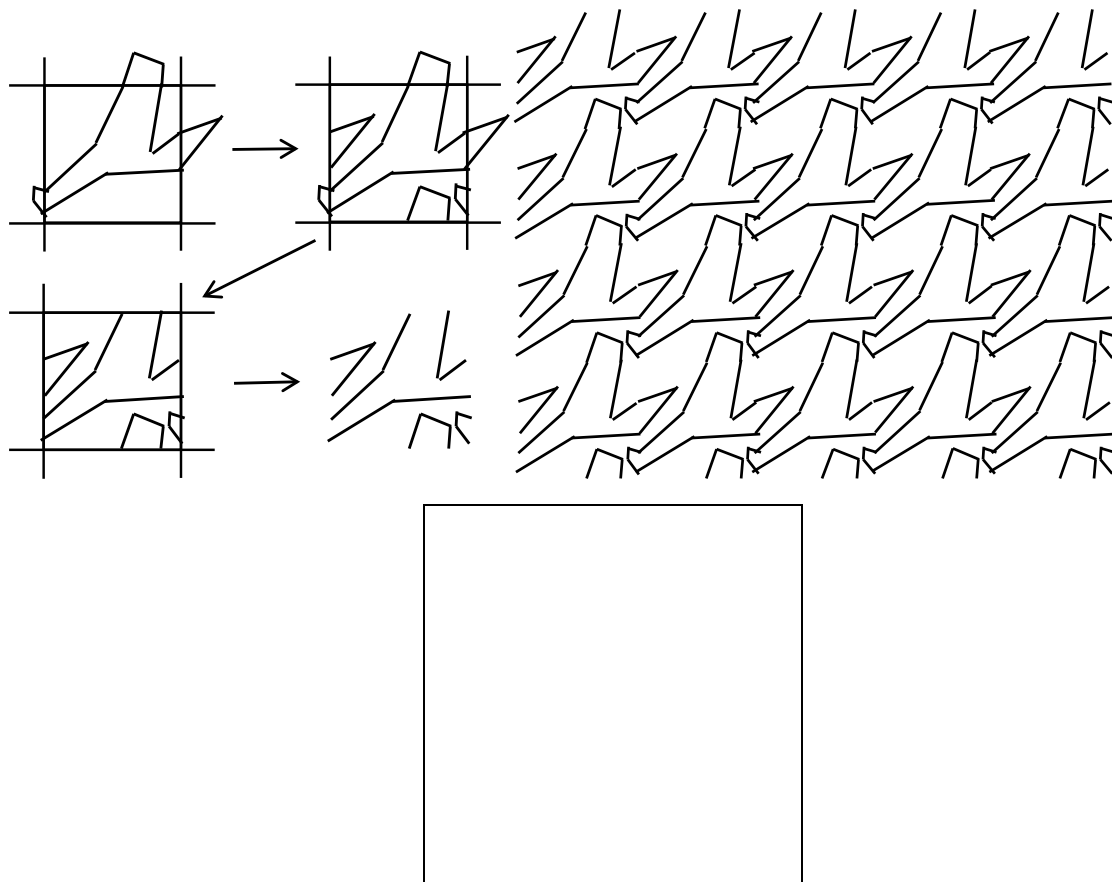


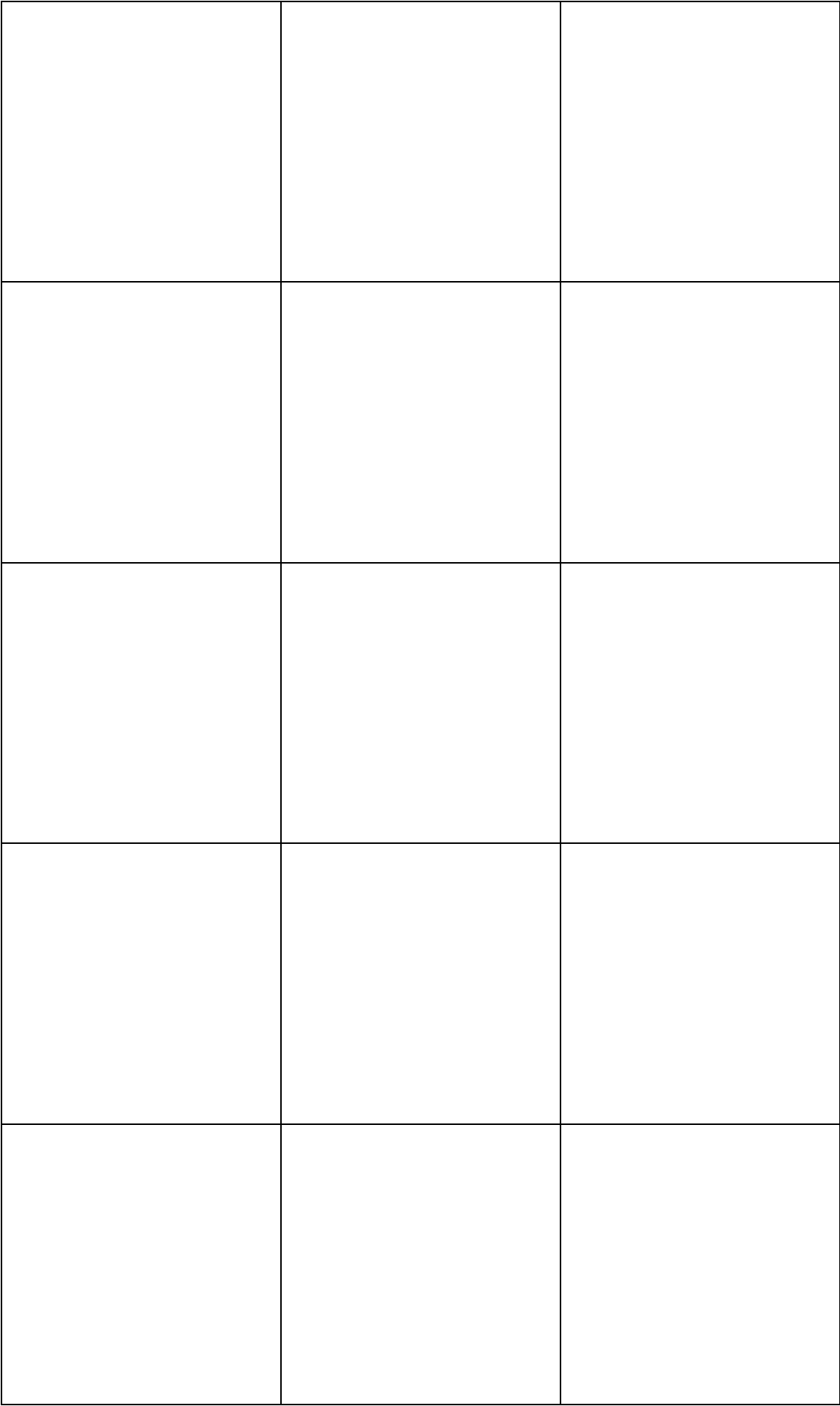
Design your own tessellation that uses more than one shape.

Fabric design

Fabric patterns sometimes also use tessellating shapes to make the design.

1. Choose a tessellating shape.
2. Draw a design in it. If the design goes outside the shape, this outside part is redrawn inside the shape on the opposite side.
3. Repeat the design to make the pattern.
4. Use the grid on the next page to make your own design.





Unit 04 Portfolio Task – Teacher Guide

Packaging Mayhem



Content Strand/s: Measurement and geometry

Resources Supplied:

- Task sheet
- Teacher guide

Other Resources Needed:

- Item for packaging
- Card and decorations for box creation

Summary:

Within this task students are expected to do the following:

- Choose an item for packaging
- Take dimensions of the package
- Create possible nets of a box for the item, considering:
 - how multiple nets will economically fit onto one piece of card if the product were mass produced, and
 - how the boxes will stack together economically.
- Create the chosen box and design the cover.

NB: Ensure the item is quite small, as multiple nets need to be able to fit onto one sheet of A4 paper.

This task can be done over time throughout the unit, or as an activity across 2 lessons. This is up to the classroom teacher.

ACARA Proficiencies Content Strands:

Addressed: *Measurement and Geometry*

Understanding 4.1.2 Make models of three-dimensional objects and describe key features.
Fluency [3MG063]

Problem Solving 4.5.1 Create symmetrical patterns, pictures and shapes using rotations or
Reasoning translations. [4MG091]

4.1.3 Connect three-dimensional objects with their nets. [5MG111]

4.1.4 Construct simple prisms and pyramids. [6MG140]

4.6.4 Investigate combinations of reflections, translations and rotations.
[6MG142]

Packaging Mayhem

Name	
Teacher	
Class	



Your Task:

It is your new job to design a package of your choice. You will choose a small item, and design a box to package it. You must take the following things into consideration, the:

- size of the item
- size of the package that must fit the item
- economical design of the net
- way the packages stack together

Within Portfolio Task 04, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Procedural fluency	4.1.3 Connect three-dimensional objects with their nets. 4.1.4 Construct simple prisms and pyramids.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	4.1.2 Make models of three-dimensional objects and describe key features.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Problem solving approaches	4.5.1 Create symmetrical patterns, pictures and shapes using rotations or translations. 4.6.4 Investigate combinations of reflections, translations and rotations.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches

Comments:

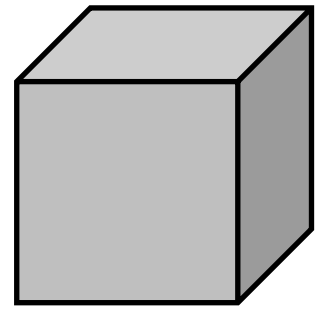
1. Decide what item you want to package. Draw a large picture of it in the space below, and label it.

2. Measure the dimensions of the item and write them on the picture you have drawn.

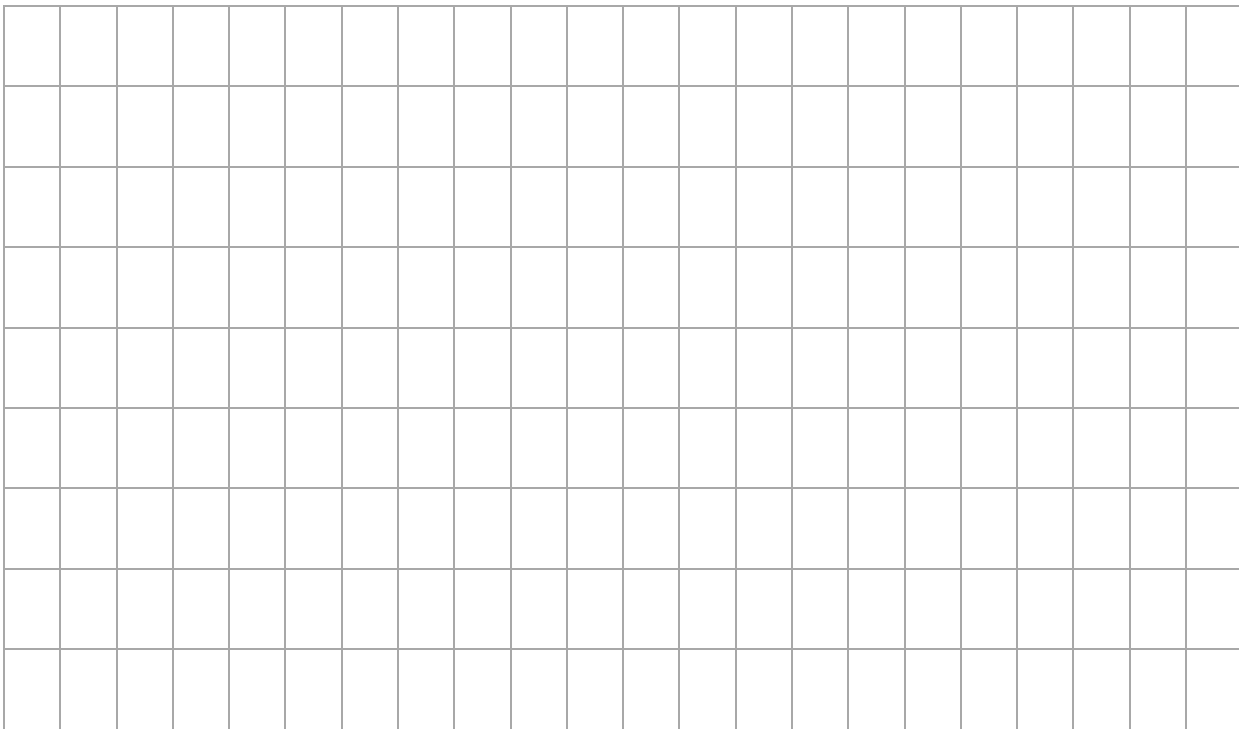
You need to do this ***accurately***; otherwise the item might not fit into the package you create.

3. How did you measure the item? _____
4. How else could you have measured it? _____
5. What unit of measurement did you use to measure it? _____

Next you need to figure what type of 3D object you are going to use as a package for your item, and design different nets. But first, let's think about the cube.

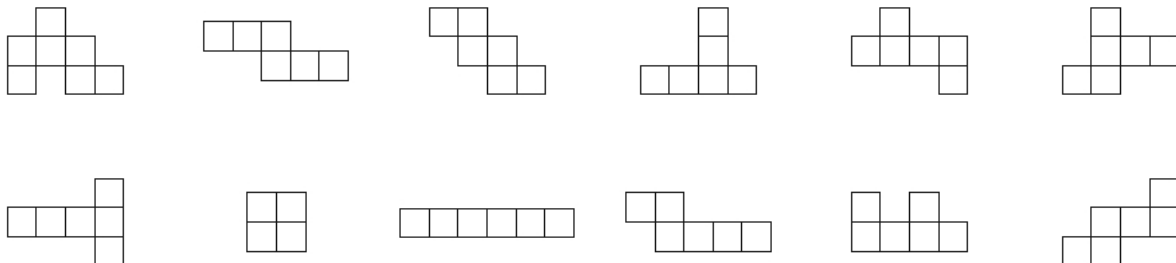


6. Draw as many possible nets of a cube as you can think of on the grid below:



7. How many could you make? _____

8. Look at the following images, circle the ones that do not make a cube:

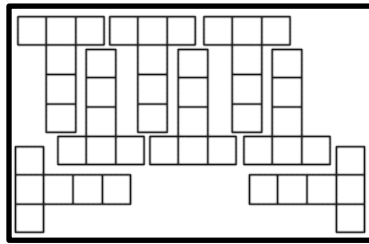


9. Can 6 squares be put together in any order to make a net? What else did you notice? _____

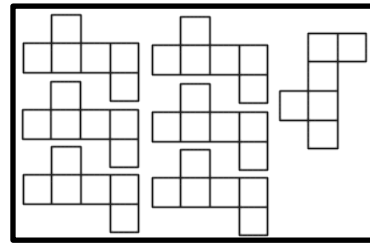
10. Keeping the dimensions of your object in mind, create 2 different packages and their matching nets below that could be used as a package for your object. The nets could be prisms or pyramids.

Try to fit as many nets onto a sheet of paper as you can. For example:

8 cube
nets per
sheet



Better – more economical



7 cube
nets per
sheet

Also think about the shape of the 3D package, and how it can stack if mass produced. For example cubes, and rectangular prisms stack well, but a sphere does not stack as efficiently.

11. Draw the net that you have chosen in the space below, write in the dimensions.

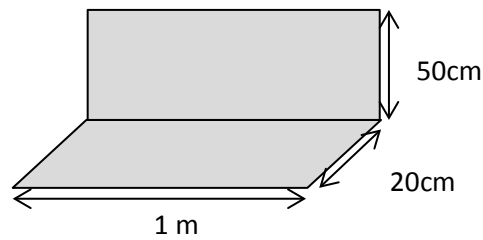
12. Why is this choice better than the other package you designed?

13. What size piece of paper would you need to fit 4 of your nets onto?

14. Using thin cardboard (e.g., a cereal box), create the package for your object and design the cover.

***Glue a photograph
of the finished
product here***

15. You have a shelf that is 1m long, 20cm deep, and 50 cm high. Showing working, answer the following questions.



a. How many of your package can you fit on the base of the shelf?

Number of packages: _____

b. How many packages can you stack on top of one another?

Number of packages: _____

c. How many packages can fit altogether?

Number of packages: _____

d. The shelf holds a maximum weight of 30kg, how many packages can you fit onto the self?

Number of packages: _____

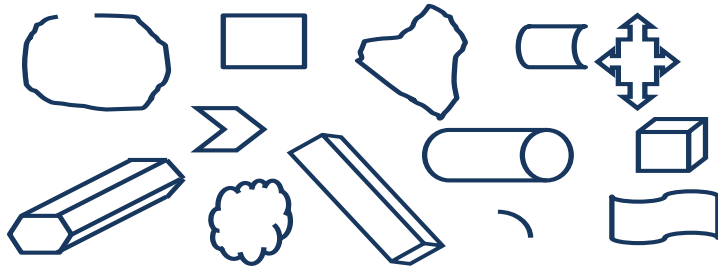
Name: _____

Date: _____

Can you do this now? Unit 04

1. Circle the 2D shapes.

Cross the 3D objects.



Obj.
4.1.1

☐ ☐ ☐ ☐

☐ ☐ ☐ ☐

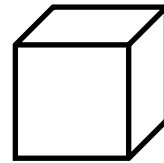
☐ ☐ ☐ ☐

☐

2. (a) How many faces does this 3D object have? _____

(b) How many edges does this 3D object have? _____

(c) What might you name this 3D object?



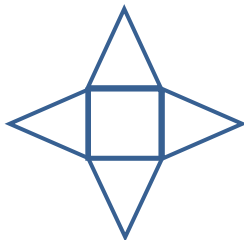
Obj.
4.1.2

a) ☐

b) ☐

c) ☐

3. What object will this 'net' make when it is folded up?



(a) Matchbox

(b) Tent

(c) Pyramid

(d) Cube

Obj.
4.1.3

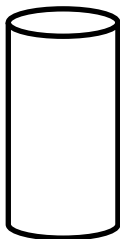
a) ☐

b) ☐

c) ☐

d) ☐

4. Sketch the net for this object.



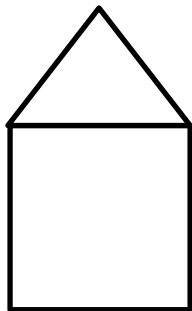
Obj.
4.1.4

i. ☐

ii. ☐

iii. ☐

5. A triangular prism is placed on top of a rectangular prism. The picture shows the view from the front. Draw the view of this combination of prisms from the side.



Obj.
4.1.5
i. ☐
ii. ☐

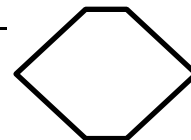
6. Circle whether statement is true or false.

(a) A plane shape is a solid shape.	True	False
(b) A 2D shape is always flat.	True	False
(c) A 2D shape always has straight sides.	True	False
(d) All faces of 3D shapes are plane shapes.	True	False

Obj.
4.2.1
a) ☐
b) ☐
c) ☐
d) ☐

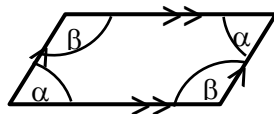
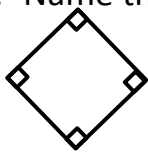
7. (a) How many sides does this polygon have? _____

- (b) What is the name of this polygon? _____



Obj.
4.2.1
a) ☐
b) ☐

8. Name the following quadrilaterals:

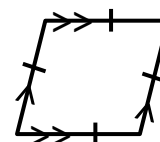


(a) _____ (b) _____ (c) _____ (d) _____

Obj.
4.2.2
a) ☐
b) ☐
c) ☐
d) ☐

9. Fill in the blanks in these sentences:

This shape is a _____.



The opposite sides of a _____ are _____ and the same _____. The interior angles add up to _____.

Obj.
4.2.3
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

10.(a) This shape is a _____ because it has _____ sides and _____ angles.



(b) If the cut off piece was a right angle triangle, the original shape would have been a _____.

11. Use a ruler to draw a pentagon.

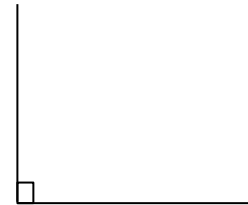
12. This angle is a right angle

(a) How many right angles do you need to make a straight line? _____

(b) How many right angles do you need to make a complete turn? _____

(c) Draw an angle smaller than a right angle.

(d) Draw an angle that is larger than a right angle.

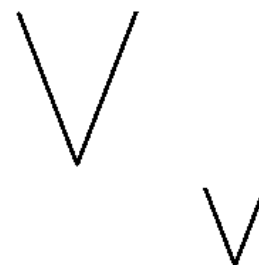


13. Here are two angles.

(a) Draw a circle around the largest angle.

(b) Use a protractor to measure each angle.

i _____ ii _____



Obj.
4.2.3
a) i. ☐
a) ii. ☐
a) iii. ☐

Obj.
4.2.4
b) ☐

Obj.
4.2.3
i. ☐
ii. ☐

Obj.
4.2.5
a) ☐
b) ☐
c) ☐
d) ☐

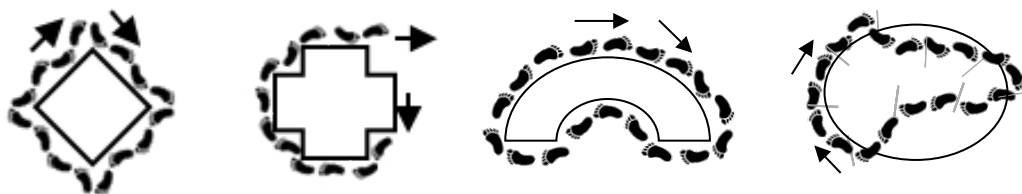
Obj.
4.2.5
a) ☐

Obj.
4.2.6
b) i. ☐
b) ii. ☐

14.(a) Use a protractor to draw a triangle with one angle of 27° and one right angle.

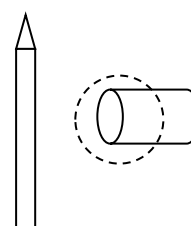
Obj.
4.2.7
a) i. ☐
a) ii. ☐

15. The footprints show where John walked. Circle the shapes where he walked a perimeter:



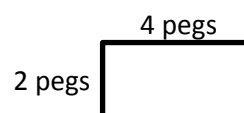
Obj.
4.3.1
i. ☐
ii. ☐
iii. ☐
iv. ☐

16. Without counting anything, how could you work out if the length of the pencil is longer than the distance around the rim of the can? _____



Obj.
4.3.1
☐

17. Calculate the perimeter of this rectangle.

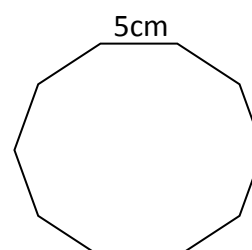


Obj.
4.3.3
i. ☐
ii. ☐
iii. ☐

18. This shape has 10 sides, it is not drawn to scale. Each side is 5cm long.

(a) Describe how to find the perimeter of the decagon. _____

(b) Calculate the perimeter of the decagon?



Obj.
4.3.4
a) ☐
b) i. ☐
b) ii. ☐
b) iii. ☐

19. Draw pictures of the following:

(a) Parallel lines

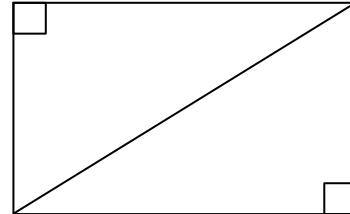
(b) Perpendicular lines

Obj.
4.4.4
a) ☐
b) ☐

20. This is a rectangle with a diagonal drawn across it.

(b) How many triangles are in the rectangle?

(c) What do the angles inside each triangle add up to? _____



Obj.
4.4.2
b) ☐

(d) What do the angles inside the rectangle add up to? _____

(e) Tick (✓) a pair of alternate angles.

Obj.
4.4.8
a) ☐
c) ☐

(f) Draw a line across the middle of the rectangle from top to bottom.

Obj.
4.4.6
d)i ☐
d)ii ☐

(g) Use a protractor to measure one of the angles in the centre of the rectangle. Label the angle with its measurement. _____

Obj.
4.4.3
e) ☐
g) ☐

(h) Use a cross (x) to label another angle that measures the same.

Obj.
4.2.6
f) ☐

21. Draw a shape that has 6 sides, 1 reflex, 2 acute and 3 obtuse interior angles.

Obj.
4.4.9
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

22.(a) Use a protractor to draw a triangle with one angle of 33° and one right angle.

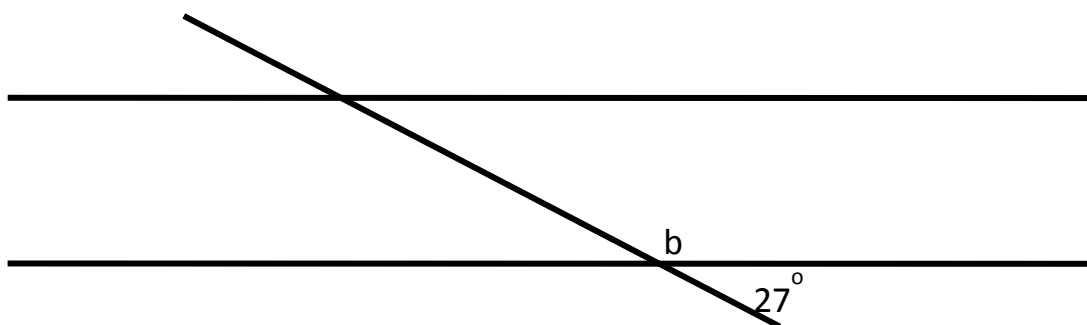
Obj.
4.2.6
a) i. ☐
a) ii. ☐

(b) Calculate the value of the other angle. _____

Obj.
4.4.2
b) ☐

23.A beam fell across a railway track. The angle between the beam and the rail on one side was estimated at 27° .

(a) On the diagram for the police report, mark with a cross (x) the corresponding 27° angle.

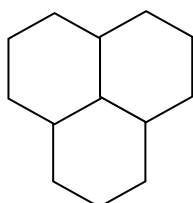


Obj.
4.4.5
a) ☐
Obj.
4.4.1
b) ☐

(b) Calculate the angle marked with a letter "b". _____

24.(a) Look at this shape. Does it have rotational symmetry? _____

(b) If so, what order of rotational symmetry does it have? _____



Obj.
4.5.3
a) ☐
b) ☐

(c) Trace the shape. Rotate the tracing 180° , slide it to the right, and transfer to the page to extend the tessellating pattern.

(d) Rotate the tracing 180° , slide it to the right, and transfer to the page to extend the tessellating pattern. (You should have 9 hexagons drawn on the page).

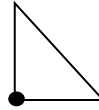
Obj.
4.5.7
c) i ☐
c) ii ☐
d) i ☐
d) ii ☐

(e) Does the new shape have rotational symmetry? _____

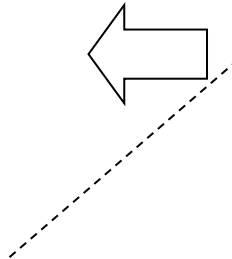
(f) If so, what order of rotational symmetry does it have? _____

Obj.
4.5.3
e) ☐
f) ☐

25.(a) Rotate the triangle 90°, clockwise, about the dot.

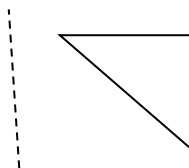


(b) Show the arrow after a slide along the dotted line.

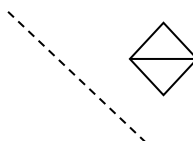


26. Draw a shape that has rotational symmetry.

27.a) Draw a flip of the shape, over the dotted line.



b) Draw a flip of the shape, over the dotted line.



Obj.
4.5.3

a) i ☐

a) ii ☐

Obj.

4.5.2

b) ☐

Obj.
4.5.1

☐

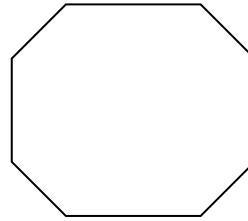
Obj.
4.5.4

a) ☐

b) ☐

28. Look at this shape:

How many lines of symmetry does this shape have?



Obj.
4.5.6



29. Reflect this image to create a picture with one line of symmetry.

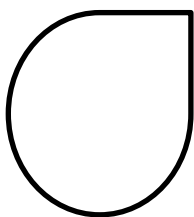


Obj.
4.5.1



30.(a) Trace the shape. Flip the shape vertically, and transfer to the page.

(b) Trace the new shape, flip the shape horizontally, and transfer to the page.



Obj.
4.5.7

a) ☐

b) ☐

e) ☐

Obj.
4.5.6

c) ☐

d) ☐

(c) How many lines of symmetry does the final shape have? _____

(d) Colour or shade the shape so it has only one line of symmetry.

(e) Rewrite the instructions using a different transformation to make the same shape? _____



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