XLR8 – Accelerating Mathematics Learning

XLR8 Unit 02

Additive change of quantities

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).

Scope

This unit is based upon the **number-as-count** meaning of **cardinal** number. An **attribute** of a **population** can be identified and then the quantity of discrete entities with a similar-valued attribute can be counted; **Counting numbers** are used for this **counting**. Through such counting, the **decimal number** system is developed (extending to whole numbers 3-digits and more) along with the various embedded concepts including **place-value** and the **additive structure** and **multiplicative structure** of the number system.

Once counted, the quantity of entities can be included in simple **additive relationships** that describe the population in terms of the sum of each similar-value entity part. **Addition strategies** can be used to compute the total if the size of the parts is known. **Subtraction strategies** can be used to compute the unknown part if the size of the other part and the total is known.

A population can be studied: various attributes of the individuals in the population can be identified and each can be counted. The size of each part can be represented in **tables** and **graphs**, and these simple **descriptive statistics** can be used to conduct **analysis** and to provide **answers** to **questions**. Large quantities represented on graphs as smaller numbers provide a link to **place-value** understandings where the **axis label** includes a **multiplier** (e.g. 1000s).

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts that is to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task *P2: Exploring Apple Inc.* accompanying Unit 02 engages students with exploring additive operations in the context of interpreting large data sets presented in tables.

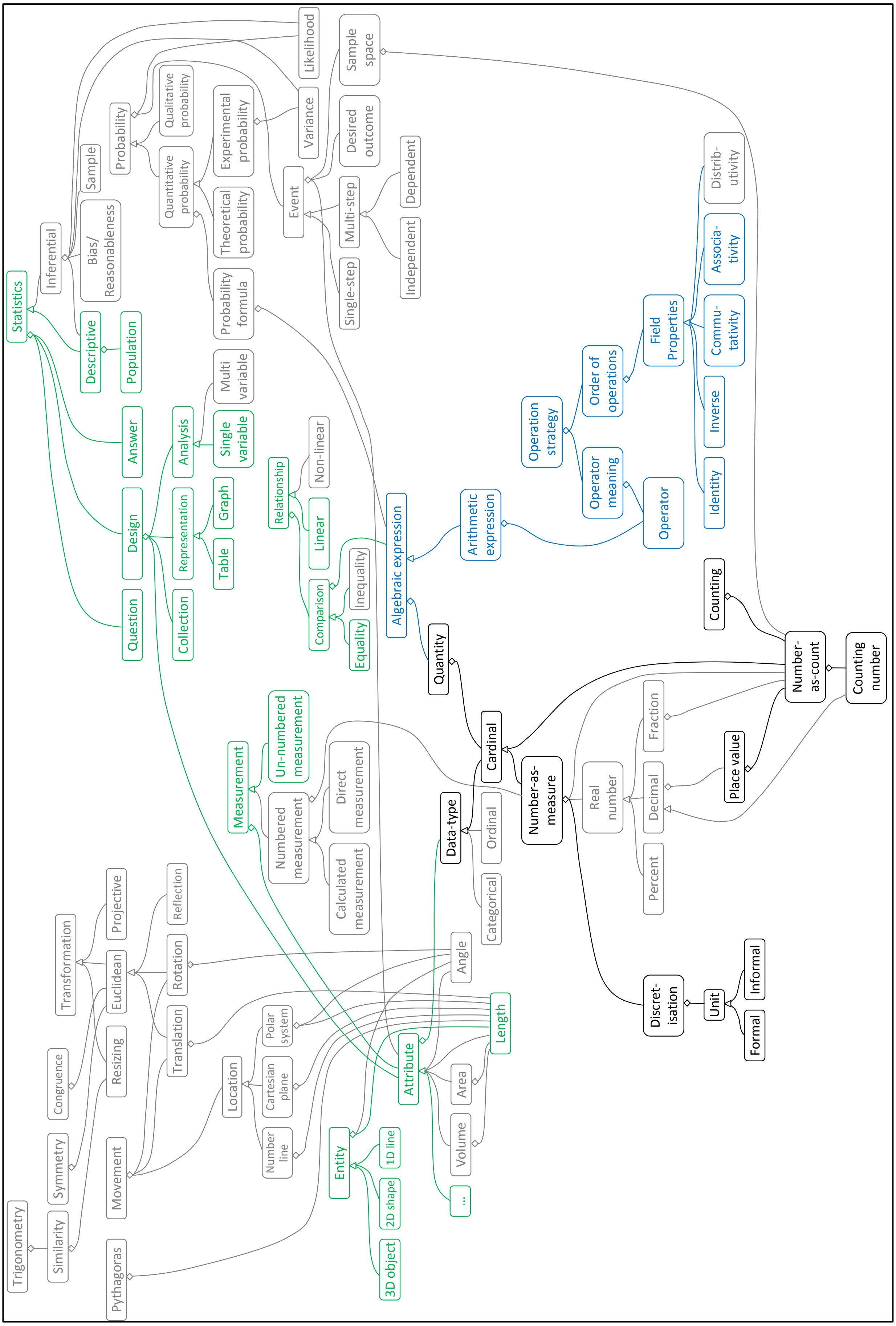


Figure 1 Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following suggested sequence:

Cycle 1: Unnumbered Balance

In this cycle, the meaning of the equals sign is developed i.e., that it shows two equivalent expressions. In this cycle the emphasis will be more general – using the equals sign to show that one thing is the 'same as' or balances another in unnumbered contexts. This thinking will provide a base for understanding equivalent arrangements or representations of quantity and for extending to symbolic equations with numeric quantities.

Cycle 2: Change and Inverse

In this cycle, initial ideas of change and inverse will be developed using the function machine. This will lead onto considering operations as a series of inputs and outputs that are changed by a rule and the representation of these using backtracking diagrams, Arrowmath notation and equations.

Cycle 3: Additive Operations

In this cycle, real-world stories are used to develop the part-part-total structure of addition and subtraction. These real-world stories can be categorised as 'active' (bringing two or more groups together or separating a group into parts), 'static' (usually involves renaming, e.g., 5 dogs + 3 cats = 11 animals) or 'comparing' (you have $4, I have $2 more or $2 less). This cycle focuses on additive operations using discrete objects so set models (counters, unifix cubes, multilink cubes, MAB blocks) are used to represent anything that can be counted as separate items.

Cycle 4: Basic Fact Strategies

The focus of this RAMR cycle is to develop number fact strategies. Scenarios involving discrete objects are represented by set model materials (such as bundling sticks, MAB or money) on place value charts (PVCs). Students will explore number patterns created by additive change, properties of numbers, and basic fact strategies. Seriation (counting on or back by ones in any place), number fact strategies and computation techniques take time to develop with students. Students may be familiar with these from primary school, but not all may be remembered or applied effectively.

Cycle 5: Addition with Larger Numbers

The focus of this RAMR cycle is to develop addition computation strategies and techniques (separation, sequencing and compensation) with two- and three-digit quantities, extending to operating on numbers with higher place-values.

Cycle 6: Subtraction with Larger Numbers

The focus of this RAMR cycle is to develop subtraction computation strategies and techniques (separation, sequencing and compensation) with two- and three-digit quantities, extending to operating on numbers with higher place-values, alongside the further development of the concept of subtraction as additive inverse.

Notes on Cycle Sequence:

The proposed cycle sequence should be completed sequentially as it stands. Contexts from other strands may be beneficial to reinforce throughout this sequence. For example,

- Perimeter as the distance around a shape provides a context for addition;
- Categories of data may be combined or separated when interpreting graphs;
- Angles within shapes may be combined to find totals.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Unnumbered Balance

Same as, balances, equals, equivalent, equation, expression.

Cycle 2: Change and Inverse

Change, input, output, function machine, undo, opposite, inverse, Arrowmath notation, backtracking diagram, equation.

Cycle 3: Additive Operations

More than, less than, adding, plus, equals, equivalent, equation, number sentence, inverse, additive inverse, opposite, difference, take away, subtracting, minus, separate

Cycle 4: Basic Fact Strategies

Number facts, triadic relationship, number fact families, counting on and counting back, use tens, near ten, doubles, doubles plus one, odd, even.

Cycle 5: Addition with Larger Numbers

Place-value, renaming, regrouping, trading, algorithms, adding, plus.

Cycle 6: Subtraction with Larger Numbers

Place-value, renaming, regrouping, trading, inverse, additive inverse, difference, take away, subtracting, minus.

Name:____ Date:_________

Can you do this? #1

1. Circle the words that you think mean ‘equals’ in the equation:

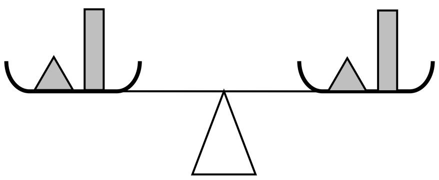
$$5 + 3 = 1 + 7$$

the same as	balances	makes
totals	adds up to	answers
measures	leaves	matches

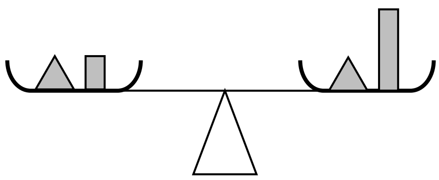
Obj. 2.1.1
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐
vi. ☐
vii. ☐
viii. ☐
ix. ☐

2. Circle the word below (true or false) to describe the following:

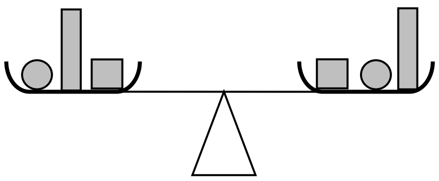
(a) True / False



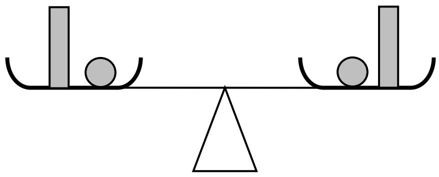
(b) True / False



(c) True / False



(d) True / False

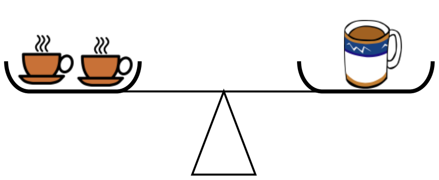


Obj. 2.1.1
a) ☐
b) ☐
c) ☐
d) ☐

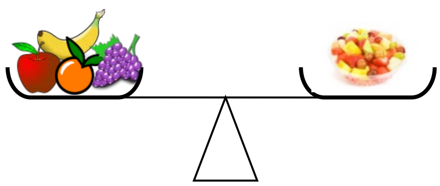
3. Write equations for each of the following balance relationships.

The first one has been done for you.

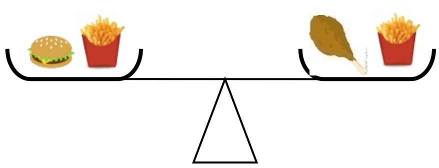
(a) Small cup + small cup = large cup.



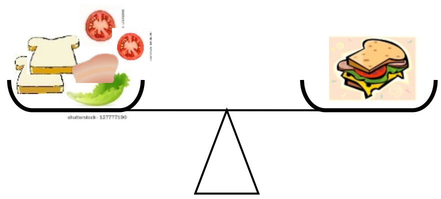
(b) ______________________________.



(c) ______________________________.



(d) ______________________________.



Obj. 2.1.2
b) ☐
c) ☐
d) ☐

Cycle 1: Unnumbered Balance

Overview

Big Idea

For many students the equals sign has become a symbol for ‘put the answer here’ or ‘do something’ when its real meaning is ‘same value as’ or ‘balance’. Although the process of 7 subtract 3 equals 4 is represented with symbols as 7 – 3 = 4, it must also be seen as 7 – 3 is the same value as 4. This means that it is possible and equally correct to show the balance relationship between this triad of numbers as 7 – 3 = 4 and 4 = 7 – 3.

In this cycle, the meaning of the equals sign is developed i.e., that it shows two equivalent expressions. The emphasis will be general – using the equals sign to show that one thing is the ‘same as’ or balances another in unnumbered contexts. This thinking will provide a base for understanding equivalent arrangements or representations of quantity and for extending to symbolic equations with numeric quantities.

Objectives

By the end of this cycle, students should be able to:

2.1.1 Describe equivalence as balance. [7NA177]

2.1.2 Use equals in informal equations to describe ‘same as’. [7NA177]

Conceptual Links

This cycle consolidates ideas of comparison of quantity from Unit 01 in instances where one quantity is the same as or equal to another quantity.

Basic conceptual understanding from Cycle 1 is necessary for understanding and expressing operations as written equations for the remaining cycles in this unit.

Materials

For Cycle 1 you may need:

- Variety of everyday items to use to demonstrate sameness or equivalence
- Input, output and change cards
- Simple pan balance
- Function machine

Key Language

Same as, balances, equals, equivalent, equation, expression.

Definitions

Equation: a statement that two quantities are equal. An equation has two sides which are balanced or equal separated by an equals sign.

Expression: numbers, symbols and operators grouped together that show the value of something. Expressions may not contain the equal sign or any type of inequality.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you show me a balanced scale?
- What does a balanced scale mean?
- What other words can be used to describe equals?
- What does it mean when we use the symbol for equals?
- Can we 'read' the balance scale from right to left as well as left to right when it is in balance or equals?
- Can we 'read' the balance scale from right to left as well as left to right when it is not in balance or not equals?
- Does it matter what order the things are on the balance scale?

Portfolio Task

Balance contributes to student understanding necessary to complete the task successfully, but does not directly appear within *P2: Exploring Apple Inc.*

RAMR Cycle

This RAMR cycle focuses on the identification and description (ultimately using the equals sign) of the static equivalence relationship between un-numbered quantities.

Reality

Use descriptive stories that include key phrases such as ‘equal to’ or ‘the same as’. These do not need to be mathematical or numbered stories, simply stories in which two entities are compared as equal. Example contexts may be equally matched cricket teams, tennis players, bands, theme parks. The idea is to identify and determine criteria and/or attributes of these that make them the same.

Abstraction

The abstraction sequence for this cycle takes students from an informal understanding of ‘same as’ to a mathematical understanding of equals as ‘balance’. A suitable sequence of activities may be:

1. Identify a variety of synonyms and everyday language for ‘same as’ or equals. For example, same as, equivalent to, equal to, balances, as good as, just like, and so on.
2. From the ‘balances’ idea, use a simple physical pan balance and materials to explore things that balance or have the same mass as each other. Grocery items work here, as things with the same mass will visually balance. For example, ‘macaroni balances spaghetti’ or ‘soup and tuna balances tinned tomatoes’ and so on.

   Note: Students will initially only put same things on each side of the balance. Encourage them to extend their thinking to find different combinations of things that balance.
3. Introduce drawn balances (*Resource 2.1.1 Drawn balance worksheet*). For example, equally matched teams, musicians, nutritional values of foods, any ideas that can be defined as equal.
4. Informally represent balance scenarios with equations. Progress from drawn balances with words to represent items in the scenarios using an equals symbol to connect; introduce representing items in an equation with a symbol (which may be a letter) for a more ‘maths-like’ equation.

Resource 2.1.1 Drawn balance worksheet.

Mathematics

Introduce the ‘=’ sign to show equivalence. Give students a variety of scenarios that can be represented in symbols and vice versa. These scenarios must emphasise the “balance” meaning of equals, as this is the long-term meaning used in algebra (*Resource 2.1.1 Drawn balance worksheet*). Extend students’ range of attributes (can include quantity) that can be represented on the balance picture as ‘the same as’ or balance.

Reflection

Check the idea

Ask students to create their own equivalency scenario that they express using a diagram and (written) words or letter symbols featuring the use of ‘equals’.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:________________________________ Date:________

Can you do this? #2

1. Dry soil is changed by Adding water to make Mud. This change could be represented using the following diagram:

   Dry soil $\xrightarrow{\text{Add water}}$ Mud

   Draw a diagram to represent:

   dry cereal changed by adding milk to make soggy cereal

Obj. 2.2.1
i. ☐
ii. ☐
iii. ☐

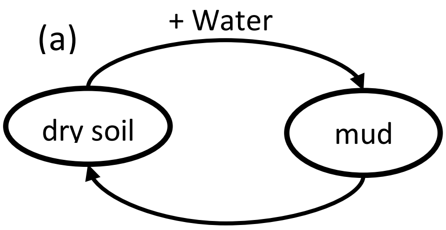
2. Fill in the blanks:

	Input	Change	Output
(a)	‘p’	Join with ‘at’	
(b)		Cook	Fried eggs
(c)	mad		dam

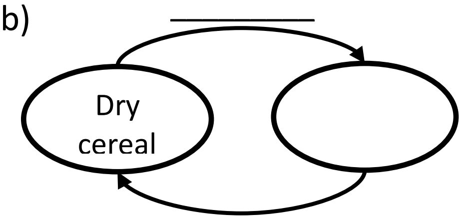
Obj. 2.2.2
a) ☐
b) ☐
c) ☐

3. Fill in the blanks on the backtracking diagrams:

(a)



(b)



Obj. 2.2.5
a) ☐

Obj. 2.2.4
b)i ☐
b)ii ☐
b)iii ☐

The backtracking diagram of Question 3(a) could be represented using the following two equations:

For example: (1) Dry soil + water = mud (2) Mud – water = dry soil

Write equations to represent the backtracking diagram of Question 3(b) on the lines below.

(c) ______________________________

(d) ______________________________

Obj. 2.2.3
c) ☐
d) ☐

3. Fact families show different equations for the same change. For example: (i) Dry soil + water = mud (ii) Water + dry soil = mud
   (iii) Mud – water = dry soil (iv) Mud – dry soil = water

   Write the other two equations from the fact family for:
   Dry cereal changed by Adding milk to make Soggy cereal.

   (e) ______________________ (f) ______________________

Obj. 2.2.6
e) ☐
f) ☐

4. Circle the changes that can be undone in reality:

(a)		flat tyre	$\xrightarrow{\text{pump up}}$	inflated tyre	
(b)		apple	$\xrightarrow{\text{cook}}$	apple pie	
(c)		water	$\xrightarrow{\text{freeze}}$	ice	

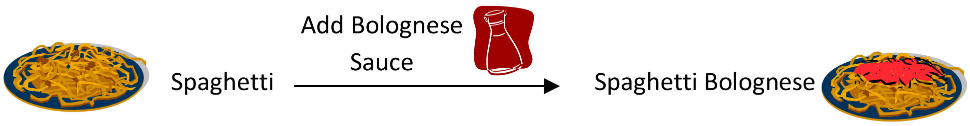
Obj. 2.2.5
a) ☐
b) ☐
c) ☐

Cycle 2: Unnumbered Change and Inverse

Overview

Big Idea

Operations involve real-world stories that describe relationships between values or change. Thus, any equation can be seen as a balance relationship where the sum of parts on one side **balances** or **is the same as** the other (as in Cycle 1), or as an input is changed by a process **to result in** an output. These relationships can be explored in unnumbered situations to develop an understanding of the structure that can then be applied to numbered situations (in Cycle 3). For example, Spaghetti add Bolognese Sauce results in Spaghetti Bolognese. Arrowmath notation can be used to record this change simply as:



The focus here is on the parts (Spaghetti and Bolognese Sauce) that go together to make Spaghetti Bolognese. Similarly, if you have an output of Spaghetti Bolognese and you know your change was to add Bolognese Sauce, working backwards or backtracking from the output by using the inverse operation will result in the input of Spaghetti.

Objectives

By the end of this cycle, students should be able to:

2.2.1 Represent unnumbered change using Arrowmath notation. [7NA176]
2.2.2 Represent unnumbered change using input/output tables. [7NA176]
2.2.3 Represent unnumbered change as equations. [7NA176]
2.2.4 Represent unnumbered change and inverse using backtracking diagrams. [7NA177]
2.2.5 Identify additive inverse relationships. [3NA054]
2.2.6 Identify unnumbered relationships as fact families. [7NA177]

Conceptual Links

Previous knowledge of comparing and ordering including recognition of same as, equivalent and equals contributes to understanding the recording of operations as equations.

Basic conceptual understanding from this cycle is necessary for understanding and expressing operations as backtracking diagrams, Arrowmath notation and written equations for the remaining cycles in this unit, including the understanding of inverse as opposite.

Materials

For Cycle 2 you may need:

- Variety of everyday items to use to demonstrate sameness or equivalence
- Input, output and change cards
- Simple pan balance
- Function machine

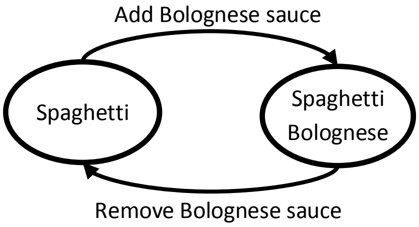
Key Language

Change, input, output, function machine, undo, opposite, inverse, Arrowmath notation, backtracking diagram, equation, input/output table.

Definitions

Arrowmath notation: notation that represents change situations as an input, labelled arrow to identify change, and output (see spaghetti Bolognese example on previous page).

Backtracking diagram: diagram that represents change situations as an input, labelled arrow to identify change, and output with the added facility to indicate the undoing of the change or inverse action



Function machine: device to explore inputs, change and outputs

Input/Output table: recording mechanism for changes experienced using function machines. Useful for identifying patterns of change when both input and output are known. Precursor to proportion tables and tables of values in later units

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What do you know? (Input, Change or Output)
- What do you need to find out?
- Can you write this as an equation?
- How will you represent the missing information?
- Can this change be undone in the real world?
- Can this change be imagined undone?
- What is the inverse of this change?
- Can you write a story for this equation?

Portfolio Task

There are not specific sections in *P2: Exploring Apple Inc.* that apply to this cycle.

RAMR Cycle

This RAMR cycle focuses on developing understanding of mathematical problems as change processes that may be reversed to return to an initial state. Understanding mathematical problems in terms of inputs, change, outputs can be powerful ways to approach operations and algebra.

Reality

Use descriptive stories that involve change. These should be stories in which one entity becomes another entity through a change process. Ideally, introduce stories for which there is the possibility of a physically opposite action or undoing (inverse) action.

Build a list of opposite things and actions that also can be related to parts becoming whole or wholes being separated into parts. For example, car parts are assembled to make cars and cars can be disassembled into car parts, players are collected together to make teams and teams can be dispersed into individual players, packets of cereal are packed into cartons for shipping and cartons of cereal packets are separated into individual boxes for sale, and so on.

Abstraction

The abstraction sequence for this cycle explores notions of change in the real world and ways of representing these changes symbolically. Function machines are useful models for acting out change. These can take the form of a box, robot or drawn machine that can sit on a whiteboard ledge. Electronic versions of these may also be created simply using PowerPoint. While exploring change, it is also beneficial to explore undoing the change or opposite actions as these lead to understandings of inverse operations and resulting fact families. The abstraction sequence from acting out change to symbolic representations is as follows:

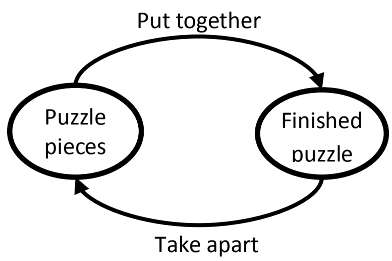
1. Use function machine to act out unnumbered change stories. For example, 'cook it', 'make bigger', 'make smaller', 'growing up', 'add at', 'capitalise', 'reverse order' (*Resource 2.2.1 Function machines*). Ensure students walk from one side of the machine to the other to physically enact the change.

Resource Resource 2.2.1 Function machines

2. Record changes with Arrowmath notation. For example,

creased $\xrightarrow{\text{iron}}$ uncreased

3. Progress from providing input and change to determine output, to providing change and output to determine input. Discuss with students what it means to **go backwards** through the machine. What happens to the change? What undoes change? Can all changes be undone physically? This is an important precursor to the idea of **inverse**.
4. Record changes with pictures progressing to backtracking diagrams to record the inverse change. For example,



Resource Resource 2.2.2 Backtracking

5. Record changes using input/output tables (*Resource 2.2.3 Input/Output tables*). These can be particularly useful when recording a series of inputs and outputs in order to determine a pattern that leads to identifying the change. Extend focus to examples that give the output and the change so that students need to identify the inverse change and input.

Resource Resource 2.2.3 Input/Output tables

6. Discuss with students how these changes might be recorded as equations using equals. Clearly connect examples of backtracking diagrams and Arrowmath notation to their equivalent representation as equations. Explore all the possible equations that might be written for a relationship as fact families (include informal representations of inverse and turnaround ideas). For example, it is possible to add water to dry soil, or it is possible to add dry soil to water to create mud. The idea is not to discuss commutativity here, but to provide an unnumbered experience that may later provide a basis for commutativity.

Mathematics

Language/symbols and practice

Practise using backtracking diagrams to show change and identify inverse. Explore some of the other examples suggested in Reality or Abstraction to provide students with breadth of experience. Encourage students to generate a wide variety of scenarios that can be represented in drawn symbols and vice versa. These scenarios must emphasise the “change” meaning of processes where parts are assembled into wholes or wholes are disassembled into parts. Ensure students can also represent these changes and their inverse changes in equations.

Try applying multiple changes and determining interim outputs. This would be the equivalent of using multiple function machines and is applicable to multi-step problems.

Connections

Although the focus of this cycle is unnumbered change, it may be useful to explicitly connect the idea of change to regrouping or trading ideas developed in the previous unit while exploring place value. For example, 1 group of a hundred can be regrouped to be considered as 10 groups of tens. This will assist to consolidate students’ understanding that the changes multiplying or dividing by ten when regrouping from one place to another on the place value chart is the consequence of a change to the grouping of quantity as it is represented. This will also provide a useful background experience before exploring computation strategies for addition and subtraction in coming cycles.

For example, 26 ones can be changed to 1 ten and 16 ones or 2 tens and 6 ones. This change is usually referred to as grouping and renaming place values. Similarly, 3 tens can be changed to 2 tens and 10 ones without changing the quantity. This change is usually referred to as trading or regrouping. The ability to flexibly regroup number in this way is needed for operation work with numbers.

Reflection

Check the idea

Ask students to create their own change scenarios that they express using diagrams and (written) words or symbols of own choice featuring the use of backtracking diagrams and as 'maths-like' equations. Encourage students to use a symbol of some sort for the part of the diagram and equation they do not know (this may be an empty box, circle, question mark or letter symbol). Ensure that students are able to phrase change scenarios where the input is unknown, the output is unknown and the change is unknown. The ability to construct each type of scenario is an important step towards successfully interpreting worded problems into equations in later units. A thinkboard may be useful to collate these ideas (*Resource 2.2.4 Thinkboard or concept map*).

Resource Resource 2.2.4 Thinkboard or concept map

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

Can you do this? #3

1. In each story, underline the parts and draw a circle around the total.

(a) Our team has 16 players and the other team has 11 players. There are 27 players altogether.

(b) There were 22 balls in the sports room. Eight balls were taken out and 14 are left.

2. Complete the following:

(a) 15 dogs $\xrightarrow{\text{add 6 dogs}}$ ______________

(b) 33 people on the bus $\xrightarrow{\_\_\_\_\_\_\_\_\_\_}$ 29 people on the bus

3. Complete the input/output tables and identify the change.

(c)

Change: ______	
Input	**Output**
66	76
76	86
86	
	106
116	

(d)

Change: ______	
Input	**Output**
878	858
858	838
838	
	798
758	

4. Is this an addition or subtraction problem? Circle the correct symbol.

a) A 1m length was cut from a plank of wood. There was 2m left. How long was the plank of wood at the start? + –

b) Jack had $13 and his mum gave him $6. How much money does he have altogether? + –

c) I was given 7 books more, this made 11 books. How many did I start with? + –

Obj. 2.3.2
a)i ☐
a)ii ☐
a)iii ☐
b)i ☐
b)ii ☐
b)iii ☐

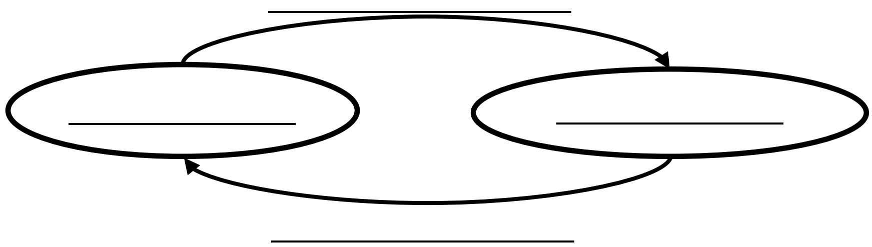
Obj. 2.3.3
a) ☐
b) ☐

Obj. 2.3.4
a) i☐
a) ii☐
a) iii☐
b) i☐
b) ii☐
b) iii☐

Obj. 2.3.5
a)☐
b)☐
c)☐

5. Write a backtracking diagram for the following story:

A 3 carriage train had 5 carriages linked to it making it an 8 carriage train.



Obj. 2.3.7
i □
ii □
iii □
iv □

6. Write a story for the equation : $6 + 15 = ?$

Obj. 2.3.6
i □
ii □
iii □

7. Write the story as an equation using variables.

I was given money and then spent $10. How much did I have left?
(m is how much money I was given; s is how much money was left)

Obj. 2.3.6
i □
ii □
iii □

8. A fish and chip shop prices their meal deals as follows:

Cost of fish (f) + cost of chips (c) + profit (p) = meal deal price (d)

(a) Write the story as an equation using variables.

(b) What is the meal deal price if fish is $5, chips are $3, profit is $2?

(c) If the cost of fish is only $3, what will be the new meal deal price?

Obj. 2.3.8
a) i □
a) ii □
a) iii □
a) iv □
b) i □
b) ii □
b) iii □
b) iv □
c) □

Cycle 3: Additive Operations

Overview

Big Idea

In this cycle, real-world stories are used to develop the concept of addition and the additive inverse operation of subtraction. These real-world stories can be categorised as ‘active’ (actively bringing two or more groups together or separating groups), ‘part-part-whole’ (may involve renaming or redefining super sets of animals into smaller categories, e.g., 5 dogs + 3 cats = 11 animals) or ‘comparing’ (you have $4, I have $2 more, I have $6). This cycle focuses on addition and subtraction of discrete objects, so set models (counters, unifix cubes, multilink cubes, MAB blocks) are used to represent anything that can be counted as separate items.

Objectives

By the end of this cycle, students should be able to:

2.3.1 Act out, interpret and informally represent additive stories. [2NA036]
2.3.2 Interpret and identify the parts and total within additive stories. [2NA036]
2.3.3 Represent numbered change using Arrowmath notation. [7NA176]
2.3.4 Represent numbered change using input output tables. [7NA176]
2.3.5 Identify appropriate operation to create equations from additive word problems. [6NA123]
2.3.6 Represent numbered additive change as equations with a symbol for unknowns. [7NA176]
2.3.7 Represent numbered change and inverse using backtracking diagrams. [7NA177]
2.3.8 Evaluate algebraic additive equations by substituting a value for each variable. [7NA176]

Conceptual Links

This cycle extends on the concepts of balance, change and inverse explored in the preceding cycles of this unit and understanding of place value and the notion of unit of count from Unit 01.

This cycle introduces addition and subtraction as a concept, interpretation of word problems to select an appropriate operation and form a suitable equation in order to facilitate finding a solution. This facility to generate questions and equations will lead into basic fact strategies and calculation skills explored in Cycle 4, Cycle 5, and Cycle 6.

Materials

For Cycle 3 you may need:

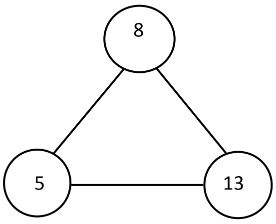
- Everyday items (pencils, pegs, bottle tops)
- Counters
- Maths mat
- Function machine
- calculator per student
- Straws or paddle pop sticks
- Place Value Charts
- Drawn number lines
- ‘Island Action’ cut outs with box
- Thinkboards
- Input, output and operation cards

Key Language

More than, less than, adding, plus, equals, equivalent, equation, number sentence, inverse, additive inverse, opposite, difference, take away, subtracting, minus, separate

Definitions

Triadic relationships: triangle with a quantity at each corner. Quantities are related such that two points of the triad add to the third quantity. Fact families may be generated by exploring relationships between the values around the triad.



Fact families:

8 + 5 = 13	***Note:*** also	13 = 8 + 5
5 + 8 = 13		13 = 5 + 8
13 – 8 = 5		5 = 13 – 8
13 – 5 = 8		8 = 13 – 5

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Do the quantities in the story change or are you comparing quantities?
- Is there an input? Is there an output? Is there a change?
- What are the parts in the story? What is the total?
- Which operation do you need to use – addition or subtraction? Which symbol will that be?
- Can you write this story as an equation? What symbol will you use to represent your unknown quantity (what you need to find)?

Portfolio Task

There are no specific aspects that apply to skills developed in this cycle although skills from this cycle will underpin operations with larger numbers within *P2: Exploring Apple Inc*.

RAMR Cycle

The focus of this RAMR cycle is the development of the addition and subtraction concepts, interpretation and construction of worded problems to generate equations and the use of a symbol to represent the unknown quantity within equations. Mental and written calculation strategies will be the focus of the following three cycles.

Reality

Act out addition problems with the students drawing on a range of problems, for example:

Active: *There were 3 students sitting at a table and 2 more students joined them. How many students altogether?*

Static: *There are 4 boys and 2 girls in this group. How many students altogether?*

Comparison: This class has 24 students. How many students in the next class if there are 2 more than in this class?

Briefly discuss opposites with students (remind them of change and backtracking from Cycle 1). Act out addition problems with the students drawing on a range of problems. Immediately follow each problem with the inverse or opposite action (subtraction). Continue to connect the idea of inverse from Cycle 1 to subtraction as the inverse of addition. For example:

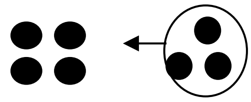
Addition: There were 3 students sitting at a table and 2 more students joined them. How many students altogether?

Subtraction: The group of 5 students chatted for a while, and then the 2 students walked away. How many students left at the table?

Abstraction

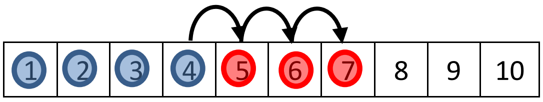
In this abstraction sequence the focus is on developing conceptual understanding of addition using the active or change meaning of addition, moving from acting out operations on whole numbers to creating abstract symbolic representations of quantity that are recorded as equations (number sentences). Other meanings of addition are explored in the Mathematics phase. A sequence using set models (counters, unifix cubes, multilink cubes, straws) is as follows:

1. *Kinaesthetic activity.* Start by physically acting out the problem. The focus is on the action. Ensure students can identify the parts being joined and the total. For example, *4 students were sitting together, 3 students sat with them; there are now 7 students.* Use key questions to identify parts and total: *How many students were there to start with? This is one part. How many students sat with them? This is the other part. What was the question asked? How many altogether? This is the total.* This acting out can be reinforced with the use of a function machine to identify the input, the change and the output.
2. *Represent with materials.* Repeat the acting out process with materials.
3. *Represent with pictures.* Students should draw a picture that symbolises the process of joining the groups for them (example on right).



4. *Represent with symbols.* Act out the addition process as change with a function machine and represent the process using Arrowmath notation. Use a question mark or symbol for the unknown total or output.

5. *Represent on number tracks.* Active change problems may also be represented on number tracks (a useful precursor to number lines). As with representing materials with counters, squares on a number track form a discrete, countable entity. To scaffold a shift from representing problems with counters to working along a number track it is possible to use both representations together. For example,



6. *Represent with symbols.* Represent the question as an equation. Use a question mark or symbol for the unknown total or output.

7. *Connect to language.* Ask students to generate their own questions that they can model with materials and write as equations. Work on increasing vocabulary for addition problems and vary question structures so that the total does not always appear at the end.

8. Generate thinkboards or concept maps with students that combine the possible representations of stories (*Resource 2.3.1: Thinkboard or concept map*). *Resource 2.3.5 Addition-Subtraction Compilation* has some useful games and resources for developing and practising the addition concept with students.

Resource

Resource 2.3.1 Thinkboard or concept map
Resource 2.3.5 Addition-Subtraction compilation

Mathematics

Connections

Subtraction or Additive inverse

Once the meaning of addition as actively changing a quantity has been established and connected to symbolic representations, it is possible to explore subtraction as the inverse operation for addition. It is beneficial to return to the Reality and Abstraction phases when starting to explore subtraction as additive inverse. A recommended sequence for subtraction as actively taking away is as follows:

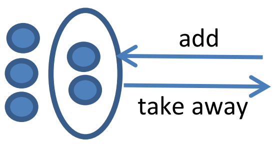
1. *Extend representation of addition to representation of subtraction.* Start by physically acting out the problem. The focus is on the action. Ensure students can identify the total, the known part and the unknown part. For example,

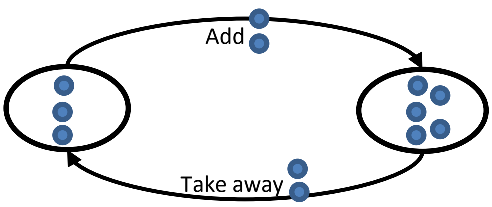
   5 students were sitting together, 2 students walked away; how many students stayed sitting?

   Use key questions to identify parts and total: *How many students were there to start with? What happened next? How many students walked away? What was the question asked? How many students stayed sitting? What was the total of the students? (5) What part do we know? (2) What is the unknown part? (How many students stayed sitting?)*

2. *Reinforce inverse action.* Walking backwards through the function machine is a beneficial activity to clearly emphasise the undoing of the addition action for subtraction.

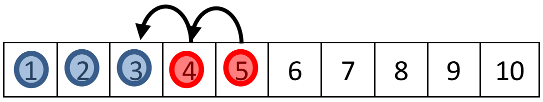
3. *Represent with pictures.* Link to students' experiences with backtracking diagrams in Cycle 2. Represent an action and its inverse or subtraction in pictures. For example,





4. *Represent with symbols.* Act out the subtraction process as change with a function machine and represent the process using Arrowmath notation. Use a question mark or symbol for the unknown total or output. If students struggle with the format of the equation and where to write the parts and the total, encourage them to write the addition equation using an unknown symbol for the missing part.

5. *Represent on number tracks.* Revisit addition representation on number track and use inverse or backtracking to represent subtraction. For example,



6. *Represent with symbols.* Represent the question as an equation. Use a question mark or symbol for the unknown total or output.

   Using inverse operation, balance and backtracking ideas encourage the rearrangement of the equation to find the subtraction equation for the story. For example, 2 + ? = 5 or 5 = 2 + ? may make more sense to students from the story example above. This can be rearranged as 2 + ? – 2 = 5 – 2, then simplified to ? = 5 – 2.

7. *Connect to language.* Encourage students to generate their own questions that they can model with materials and write as equations.

8. As for addition, generate thinkboards or concept maps with students that combine the possible representations of stories (*Resource 2.3.1: Thinkboard or concept map*). *Resource 2.3.5 Addition-Subtraction Compilation* has some useful games and resources for developing and practising the subtraction concept with students.

Resource Resource 2.3.1 Thinkboard or concept map
Resource 2.3.5 Addition-Subtraction compilation

Language/symbols and practice

Once both addition and subtraction concepts have been developed as active change, other meanings for additive operations may be explored. These include static situations where parts are statically joined to determine a total quantity (parts stay as separate entities) or statically separated into smaller categories within a superset (for example a group of animals may be separated into cats and dogs). Other question types involve the comparison of quantities to determine the total effect of a difference in quantity or to find the difference between two quantities.

Part-part-whole and comparison meanings of addition and subtraction

So far problem types have been situations which are easy to act out and involve one part of a collection or group actively changing. Other common problem situations that are solved by addition and subtraction include part-part-whole (a group comprised of two or more identifiable sections), and comparisons between collections or groups (finding the difference between groups). It is important that in all these cases students build familiarity with the vocabulary used and an ability to determine what is known, what is unknown, which quantities are the parts and which quantity is the total or what quantities are being compared. Identifying these will assist students in writing equations. The abstraction sequence can be applied to each new problem type to ensure students develop competency with decoding routine word problems. *Resource 2.3.2: Problem interpretation and construction* has some useful suggestions in the teacher notes for varying wording of problems for students.

Resource Resource 2.3.2 Problem interpretation and construction

Problem interpretation and construction

Provide students with practice posing their own problems for their peers to solve. Ensure that they can work flexibly from worded problems to equations and from equations to possible worded problems. It is important that students understand that an equation is a symbolic representation of real world activity and, if the unit names of the original problem are removed, the equation can apply to any problem situation with the same digits. For example, 5 + 2 = 7 can represent 5 dogs joined by 2 dogs, 5 pizzas and another 2 pizzas, 5 cars and 2 more cars, 5m + 2m, and so on.

Encourage students to use a symbol they like for the unknown quantity to be found and ensure that they can provide a suitable unknown element that shows their understanding of the unknown part or total in worded problems they construct. *Resource 2.3.5: Addition-Subtraction Compilation* has some useful resources for developing and practising problem interpretation and construction.

Resource Resource 2.3.5 Addition-Subtraction compilation

Reflection

Check the idea

Provide students with a variety of addition or subtraction stories. Ask them to complete an empty thinkboard with the story, set representation or number line representation of action, own drawing, equation/symbols and language. Ask students to identify the basic fact strategy to solve the story.

Provide a variety of thinkboards for addition or subtraction stories cut into "mix and match puzzle cards" for students to recreate. Ask students to identify the basic fact strategy to solve the story.

Resource Resource 2.3.3 Subtraction mix and match puzzle cards

Apply the idea

Explore the distance around the edge of desks, whiteboard, assorted irregular shapes using informal units or simple measures. Add the number of units of measure for each side of the shape to determine the perimeter. Reinforce with students that perimeter is the distance around. Focus on this as an application of addition, not to use formulae for perimeter of regular shapes.

Consider some of the class data collection graphs from Unit 1. What comparisons between the data can students see and represent? What numbers will combining categories generate? What conclusions or generalisations can be drawn about the class population? For example, four-legged pets are most common for people in the class; number of cats + number of birds = number of dogs.

Consider the class data collection from Unit 1. Students should be able to use the differences between categories as a stimulus to construct addition and subtraction questions and equations. Encourage students to determine the differences between categories represented in their data sets, how many more or less of one category is needed to be the same as another category?

Compare class data sets with similar data collected on Australian Bureau of Statistics. For example, students may complete a survey and generate a table of how long they spend sleeping, eating, at school. Small number facts can be practised as they compare similarities and differences between their use of time and the Australian average use of time.

Resource Resource 2.3.4 How Australians use their time

Extend the idea

Generalise

Reinforce the representation of additive stories using symbols for unknowns.

Generalise rearrangement of equations using backtracking and balance ideas in parallel. Balance activities demonstrate that whatever is altered on one side of an equation needs to also be changed on the other side of the equation. Backtracking ideas explain which operator to alter.

For example,	c + 2 = 5	Backtracking: inverse change is – 2
	c + 2 – 2 = 5 – 2	Balance: inverse change applied to both sides
	c = 5 – 2	

Using symbols for unknowns, explore the relationship between addition and subtraction stories and the types of questions that may be asked. For example,

Active stories:

There are 3 cars in the parking lot, 2 drive in, there are now 5 cars in the parking lot.

Possible questions can be:

(a) output unknown:	How many cars in the parking lot now?	3 + 2 = o
(b) input unknown:	How many cars in the parking lot to start with?	i + 2 = 5
(c) change unknown:	How many cars drove in to the parking lot?	3 + c = 5

Note that questions (b) and (c) use subtraction to solve and can be rearranged as:

(b) How many cars in the parking lot to start with?	5 – 2 = i
(c) How many cars drove into the parking lot?	5 – 3 = c

Part-part-total stories:

There are 3 girls and 2 boys on the basketball team.

(a) Total unknown:	How many children on the team altogether?	3 + 2 = t
(b) Part unknown:	How many girls on the basketball team?	g + 2 = 5
(c) Part unknown:	How many boys on the basketball team?	3 + b = 5

Note that questions (b) and (c) can be rearranged as:

(d) How many cars in the parking lot to start with?	5 – 2 = g
(e) How many cars drove into the parking lot?	5 – 3 = b

Comparison stories:

You have 3 cars. Difference is 2 cars. I have 5 cars.

(a) My total unknown:	How many cars do I have?	3 + 2 = m
(b) Your total unknown:	How many cars do you have?	y + 2 = 5
(c) Difference unknown:	What is the difference (how many more or less)?	3 + d = 5

Note that questions (b) and (c) can be rearranged as:

(b) How many cars do you have?	5 – 2 = y
(c) What is the difference between how many cars we have?	5 – 3 = d

See if students are able to generalise further to state equivalent equations using inverse and algebra:

A + diff = B   or   A = B - diff

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #4

1. Continue the number patterns:

(a) 63, 73, 83, ____________ , ___________ , _________

(b) 924, 824, 724, _________ , ___________ , _________

(c) 1946, 1936, 1926, __________ , ___________ , _________

(d) 10 054, 11 054, 12 054, __________ , ___________ , _________

2. Solve the following:

(a) 23 + 78 =

(b) 47 – 38 =

3. Complete the following:

(a) 13 = ______ + 5

(b) 8 + 9 = ________

(c) 7 + ________ = 14

(d) 23 – 7 = _______

(e) 36 – ______ = 25

(f) 112 + _________ = 115

4. Complete the following:

(a) 60 + 50 = ______

(b) 900 – _________ = 200

(c) 7 000 + 7000 = _______.

(d) ______ = 18 000 – 9000

(e) ______ + 600 = 1100

(f) 630 + 80 = _________

5. Use the numbers 3, 5 and 8 to write a fact family of equations:

(a) ______ + _______ = _______

(b) ______ + _______ = _______

(c) ______ – _______ = _______

(d) ______ – _______ = _______

6. Mark the following equations as true or false:

(a) 78 + 96 = 96 + 78 □ True □ False

(b) 5 + 12 – 5 = 12 □ True □ False

(c) 15 + 3 = 6 + 11 □ True □ False

(d) 25 + 16 = 50 – 8 □ True □ False

Obj. 2.4.1
a)i□ii□ iii□
b)i□ii□ iii□
c)i□ii□ iii□
d)i□ii□ iii□

Obj. 2.4.2
a) □
b) □

Obj. 2.4.2
a) □
b) □
c) □
d) □
e) □
f) □

Obj. 2.4.2
a) □
b) □
c) □
d) □
e) □
f) □

Obj. 2.4.3
a)□b)□
c)□d)□

Obj. 2.4.4
a) □
b) □
c) □
d) □

Cycle 4: Basic Fact Strategies

Overview

Big Idea

The focus of this RAMR cycle is to develop number fact strategies. Scenarios involving discrete objects are represented by set model materials (such as bundling sticks, MAB or money) on place value charts (PVCs). This is extended to the representation of quantity on number lines.

Students will explore number patterns created by additive change, properties of numbers, and basic fact strategies. Seriation (counting on or back by ones in any place), number fact strategies and computation techniques take time to develop with students. Students may be familiar with these from primary school, but not all may be remembered or applied effectively.

Objectives

By the end of this cycle, students should be able to:

2.4.1 Describe, continue and create additive number patterns. [3NA060]
2.4.2 Solve additive stories using mental computation strategies. [2NA030]
2.4.3 Identify fact families of numbered relationships. [7NA177]
2.4.4 Identify equivalent number sentences involving additive operations. [4NA083]

Conceptual Links

This cycle extends on the concepts of balance, change and inverse explored in the preceding cycles of this unit and understanding of place value and the notion of unit of count from Unit 01.

This cycle introduces a range of strategies for performing whole number addition or subtraction and aims to build a degree of recall-based answering of simple number facts since this will assist more complex calculations used in algebra, measurement, probability and statistics.

Materials

For Cycle 4 you may need:

- Everyday items (pencils, pegs, bottle tops)
- Tens strips
- Counters
- Maths mat
- Function machine
- flip cards
- calculator per student
- Straws or paddle pop sticks
- Place Value Charts
- Drawn number lines
- ‘Island Action’ cut outs with box
- Thinkboards
- Input, output and operation cards
- 3 foam cups per student
- 3 sets A5 digit cards (0-9)
- A7 digit cards, 2 10-sided dice

Key Language

Number facts, triadic relationship, number fact families, counting on and counting back, use tens, near ten, doubles, doubles plus one, odd, even.

Definitions

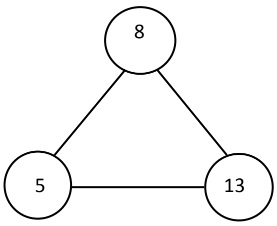
Count all: Addition strategy where collections are joined and then all items counted.

Count on / back: More efficient than count all and recommended for small count on/back amounts (0, 1, 2, 3). Start from the larger number and count on for the smaller number for addition; start from the total and count back for subtraction.

Fact families: Basic facts that group together to represent all the additive relationships that exist between two parts and a total (see Triadic relationships). Also connected to turnarounds (commutative property).

Use doubles: Strategies that include doubles, doubles plus or minus 1, doubles plus or minus 2. Students seem to develop a facility with doubles quite quickly that can be leveraged to determine doubles plus or minus 1 (e.g., 4 + 4 = 8; 4 + 3 = 4 + 4 – 1 = 8 – 1 = 7)

Triadic relationships: Depicted as a triangle with a quantity at each corner. Quantities are related such that two points of the triad add to the third quantity. Fact families may be generated by exploring relationships between the values around the triad.



Fact families:

8 + 5 = 13	***Note:*** also	13 = 8 + 5
5 + 8 = 13		13 = 5 + 8
13 – 8 = 5		5 = 13 – 8
13 – 5 = 8		8 = 13 – 5

Turnaround: Commutative property of addition whereby any two numbers may be added in any order. The commutative property does not apply to subtraction.

Use ten: When one of the numbers is near 10, the other number is broken into parts such that the start number and one of the parts add to make 10 (e.g., 8 + 5 = 8 + 2 + 3 = 10 + 3 = 13). Similarly, for subtraction, the number being subtracted is broken into parts such that the first subtraction takes the start number back to 10, then the rest of the number is subtracted (e.g., 13 – 5 = 13 – 3 – 2 = 10 – 2 = 8). These strategies are effectively represented on number lines and extend to mental computation strategies for larger numbers.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What strategy can you use to make this calculation quicker?
- How can you break this number into parts to add it onto that number?
- Will a number line help you?
- Can you rearrange the equation to look like a number fact you know?

Portfolio Task

There are no specific aspects that apply to skills developed in this cycle although skills from this cycle will underpin operations with larger numbers within *P2: Exploring Apple Inc*.

RAMR Cycle

The focus of this RAMR cycle is the development of a range of basic fact strategies for addition and subtraction including seriation, count on/back, use doubles, use tens, triadic relationships and fact familiies, turnarounds and think addition.

Reality

Explore real world scenarios where basic facts are useful, for example, adding two dice for board games, finding totals in Yahtzee!, sporting scores, or mentally estimating shopping total cost. Act out possible Monopoly moves using first 6-sided and then 10-sided dice. Have students determine the number of squares to move each time. Placing a virtual counter on a Monopoly PowerPoint slide may be beneficial. Record the equations for each rolled move on the board. Students may also record equations in their books for later activities.

Resource Resource 2.4.1 Monopoly board

Abstraction

In this abstraction sequence the focus is on exploring and developing a range of basic fact strategies to assist with quicker mental calculations. A sequence to begin this exploration is as follows:

1. *Kinaesthetic activity.* Select three students and ask them to stand in two rows. Discuss the arrangement with students. Ask them what they notice about the group. If necessary, guide the discussion towards the fact that there is one pair and one left over. Add students in pairs. After each addition, discuss, students need to observe that they are adding an even number each time to an odd number and ending up with an odd number. Note whether students are counting all students each time, counting on, or are able to say how many from recall of facts.
2. *Represent with materials.* Test the theory discovered using materials by adding larger even numbers to a starting odd number. Explore adding two odd numbers. Discuss the results. Ask students to predict what will happen when adding two even numbers. Discuss with students that this is a useful check for answers with operations to ensure that their answer is correct – if they have added an odd number to an even number and their answer is even, they have a mistake.
3. *Discuss.* Discuss with students how they knew how many students were in the group each time. Did they count all, count on from how many students were there, or just know the answer?
4. *Connect back to reality.* Consider the list of equations generated during the Reality phase. Discuss with students how these are solved when rolling dice to move around a gameboard (e.g., move the number of spaces shown on each die constitutes a count all strategy, adding the two dice together may need alternative strategies). Discuss with students what strategies they use to add two dice (counting on from the larger number may be a common strategy for finger or ruler counters – do not discourage these – they are good starting strategies).
5. *Represent on number tracks.* Model representing a roll of two dice on a number track as in Cycle 3 (e.g., 4 + 3).
6. *Connect to language.* Discuss the name of this strategy as counting on. Ask students how they might use this strategy to count back. Discuss when this strategy might become difficult (adding or subtracting more than 3 this way can be time consuming). Create a category for Counting on and list the dice generated equations that fit into this strategy category. Lead into consideration of further strategies in the mathematics phase.

Mathematics

Teaching basic facts is about developing ways to calculate answers more quickly than relying upon the representation of the operation with counters and counting to get the answer. It is still important to provide materials to model basic fact strategies.

Language/symbols and practice

Basic fact strategies

There are a variety of strategies that can be used to perform addition:

- Counting
- Turnarounds
- Doubles and near doubles
- Near ten or Make ten
- Seriation skills (units in every place use count)
- Higher decade facts

When viewed together, these strategies cover (sometimes with repetition or in combination with one-another) all of the 100 basic addition facts (i.e., 0+0 … 9+9). To develop students' proficiency in using number fact strategies, begin instruction with materials (e.g., counters) accompanied by symbolic representations, then progress to 'in the mind' use of the strategy.

There are fewer strategies for subtraction number facts than for addition facts. These strategies are:

- Counting back (use Seriation skills)
- Think addition (use number triad)

Diagnosing level of basic facts

The first step is to diagnose what facts are not known and to set up regular speed practice for the unknown facts. *Resource 2.4.2: Diagnosing basic facts* is a resource designed to diagnose students' facility with the basic facts. If there are noticeable strategy gaps, work through relevant strategy development activities in *Resource 2.4.3: Developing basic fact strategies*.

Resource
Resource 2.4.2 Diagnosing basic facts
Resource 2.4.3 Developing basic fact strategies

Suggestions for practising facts

Use the student tracking worksheet to aid students with the process. Set up a regular daily practice program – 10 mins per day (e.g., 4 minute mile, flash cards, bingo). Each student should graph the number of correct answers each day to compare with previous results. Record errors to identify strategies not developed for special practice.

Connections

Commutativity – turnaround strategy

Remind students of the unnumbered activity in Cycle2 that explored fact families and turnarounds (dry soil + water = water + dry soil). Connect back to the Abstraction phase activity. When an odd number was added to an even number, would it make any difference to start with an even number and add odd numbers? Test this out if students are not sure. This introduces the turnaround strategy for addition facts. It nearly halves the number of facts to be learnt by showing that 'bigger + smaller' (e.g., 5+2) is the same as 'smaller + bigger' (e.g., 2+5). For example, 4 + 7 is the same as 7 + 4 is the same as 11. This is based on the commutative principle.

Identify which equations from the Reality activity might be rewritten to start with the larger number. List these under a category heading of turnarounds. Highlight the equations that fit in both categories generated so far as these are using a combination of strategies for their solution.

It is beneficial to explore **generalising** this strategy. When students are comfortable with 3+6 = 6+3, pose a number of additions for them using bigger and bigger numbers (e.g., is 36+53 = 53+36). Follow up with exploring any number + any other number = any other number + any number, a + b = b + a. This is a useful point to consolidate the use of a letter in place of an unknown. If necessary, scaffold this using shapes other than letters to start with (e.g., ♦, ♥, ♣, ♠, β, α). In students' word banks, connect the turnaround strategy with its mathematical name; commutative law or commutativity.

Doubles, Doubles+1

Connect back to Abstraction phase activity where students and materials were arranged in two lines. Discuss with students how even numbers were arranged as two rows of the same number (e.g., double 3 for 6). Categorise Reality activity dice equations into doubles. Discuss the fact that using turnarounds on these has little point.

Consider the remaining dice equations. See if students can see any examples that they might be able to use doubles with if one number was different. Guide students if necessary to recognise doubles+1 examples (e.g., 4 + 5 = 4 + 4 + 1). Categorise these examples near to doubles as doubles+1 or near doubles. Examine what examples are left, students may like to include doubles+2 in their near doubles set. **Note:** It may be beneficial to augment the initial dice generated equation set with examples that may not be represented.

Near ten or Make ten

Remaining facts may include facts that can use the make ten strategy (e.g., 9+1, 8+2, 7+3 … these may also be known as rainbow facts). Investigate how the make ten strategy may be used for more difficult combinations of 6, 7, 8, 9 addition facts. This strategy can be modelled using counters on number tracks or tens frames as described in *Resource 2.4.3: Developing basic fact strategies*.

Resource Resource 2.4.3 Developing basic fact strategies

Subtraction - Use inverse or Think addition

The think addition strategy for basic facts requires students to use their understanding of inverse operations and rearrangement of equations. This is best modelled initially using the backtracking diagrams and balance ideas developed in Cycle 1 and Cycle 2. Remind students of these unnumbered activities to connect these ideas and explore changing addition equations to subtraction equations. Introduce the number triad and fact families. Understanding the relationships embodied in the triad is most important as it is a process that assists students with decoding routine worded problems into symbolic equations.

With this strategy the idea is not to do subtraction but to think of the problem in terms of addition (i.e., the strategy is based upon the additive inverse principle). For example, 8 − 3 is thought of as *"what is added to 3 to make 8"*.

To develop this strategy, it needs to be shown that subtraction and addition are inverses of each other. For each addition/subtraction fact, there are 4 members of the family: 3+5=8, 5+3=8, 8−5=3, and 8−3=5. Thus, if the subtraction problem can be re-thought of as an addition problem then students' knowledge of the addition facts can be drawn upon. (*Resource 2.4.4 Triad and fact families*).

Resource Resource 2.4.4 Triad and fact families

Once students can use basic fact strategies dealing with single digit addends, the strategies may be extended to mental computation with two digit numbers starting with higher decade facts. Students need to access place value understanding and recognise that digits in each place can be counted as they would be for units (seriation skills). This provides a base for large number additive operations.

Exploring seriation skills

The decimal place value system uses nine digits to represent quantity in each place and fills empty places with a zero. Within each place, units are counted forwards until 9 units, or counted backwards until 1 unit is in the place. Building seriation skills assists students with simple calculation when they recognise that they can work with each place as units of that place and can use single-digit number fact strategies for solving higher decade facts and operating flexibly with larger quantities.

For single-digit operands, counting up and back are associated with students counting on their fingers. Explicitly link the counting back aspect of seriation to subtracting as students count back in each place. If students need additional practice with this skill, revise *Resource 2.4.5: Seriation skills* activities and *Resource 2.4.3: Developing basic fact strategies: Count on and Count back*.

Resource

Resource 2.4.5 Seriation skills
Resource 2.4.3 Developing basic fact strategies

Higher decade facts

Once students demonstrate competence with single digit number facts, explore connecting known facts with seriation skills to find higher decade facts in the tens, hundreds, and thousands places. For example, if students can solve 3 + 5, they can also solve 30 + 50, 300 + 500, and so on.

Reflection

Check the idea

Use the triad and fact families to generate possible stories from sets of related numbers. Ensure that students can identify all possible additive relationships between three numbers. These may be recorded on thinkboards with the triad in the centre and related number facts with a suitable story in each corner square and the matching equation with an unknown value.

Resource

Resource 2.4.6 Thinkboard or concept map
Resource 2.3.5 Addition-Subtraction compilation

Apply the idea

Use snakes and ladders with ten-sided dice to apply additive strategies. Ensure students record equations and calculate the next square to go to directly instead of counting. Encourage students to find the difference between starting and ending squares for snakes and ladders encountered.

Resource

Resource 2.4.7 Snakes and ladders

Resource 2.4.8: Counting on in tens and ones is a counting on racetrack game. In this game students add the dice roll value to either the ones or the tens place according to the space landed on, the calculation is filled in within the square. Highest value at the end of the track wins. This game may be varied by starting at 10 000 and counting back in tens and ones with lowest value needed to win.

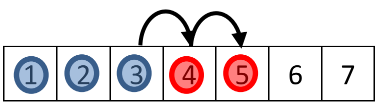
Resource

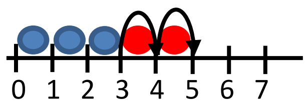
Resource 2.4.8 Counting on in tens and ones

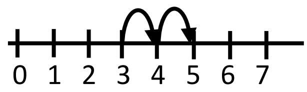
Extend the idea

Representing change on a number line

So far students have experienced discrete items and counting along a number track. Measurement contexts and future representations rely on number line representations of quantity and change. To introduce the number line it is possible to use a number track representation. However, rather than step or jump into the centre of each number track square, number lines mark the end of the jump or step and have a starting point or zero.







Students may already use a ruler to help them with calculations which is the same strategy or model. Use of this representation requires that students identify the first quantity, then make jumps to the other quantity. Explore the use of this strategy for subtraction as well as addition.

Associativity

Using a list of items for addition, explore the effect of adding items in a different order than in the original list to see if it makes a difference. This is leading to thinking needed to understand Associativity. After testing a variety of examples see if students are able to generalise in words (when you have a list of items to add it does not matter what is added first). In algebraic terms this is often written as (a+b)+c = a+(b+c). Introduce the notation of brackets to mark operations to be done first.

While considering a list of items for subtraction, explore the effect of subtracting items in a different order than the original list. After testing a variety of examples see if students are able to generalise. Introduce the order of operations for addition and subtraction. Students should understand that the initial number in subtraction equations represents the total from which known parts are subtracted to find the unknown part. This may help them remember why subtraction is not commutative.

Relationships between parts and totals and identity

Create a set of similar subtraction number sentences to explore the pattern that arises from changing the parts or total for subtraction and for addition.

For example,	10 – 3 = 7	10 – 3 = 7	7 + 3 = 10	7 + 3 = 10
	10 – 4 = 6	11 – 3 = 8	7 + 4 = 11	6 + 4 = 10
	10 – 5 = 5	12 – 3 = 9	7 + 5 = 12	7 + 3 = 10
	10 – 6 = 4	13 – 3 = 10	7 + 6 = 11	8 + 2 = 10

Explore the patterns with students – as the part being subtracted is increased and the total stays the same the other part (difference) is decreased. For addition, as an addend is increased, the total is increased. If an addend is increased and the total stays the same the other addend is decreased.

Discuss with students what could be added to or subtracted from a number to result in no change. How many different ways can identity (adding or subtracting 0) be achieved? (e.g., 5 – 5, a – a, etc.)

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 02 Investigation: Additive Card Games

Engage students in rotational activities to practise addition facts. Many card games are available online (e.g., [Dice and Card Games to Practice Math Facts.pdf]). Some sample games are:

Teaching Addition Math Facts to Kids With Go Fish!

This new twist on the old classic Go Fish! helps kids to learn addition by mentally working out simple math problems. Each round played practices math facts for a specific number, making it easy to stick with one set of facts for as long as needed to solidify them in the players' mind. All that's needed to play this game is a standard deck of playing cards. It is best enjoyed with 2-4 players.

How to Play Go Fish!

1. Sort through the deck to remove all cards that are higher than that featured number for the math game. For example, if the goal is to learn addition facts for the number seven, the game will be played with ones (aces) through sevens.
2. Deal out five cards to each player and place the remaining cards in a draw pile.
3. Have each player look through his or her hand of cards to find any pairs that add up to the featured number and place them face up in their discard pile. For example, if learning addition facts for the number seven, appropriate pairs would be 6+1, 5+2 or 4+3. The 7 card would also be laid aside as a correct solution that doesn't require a pair.
4. The person to the left of the dealer may now ask any other player for a card that will help create the sum required. If the person asked has the card in his hand, he must give it up to the player that made the request. A player can keep asking for cards until no further matches are able to be made, at which point he is told to Go Fish! from the draw pile and the next player takes a turn trying to make a match.
5. If a player runs out of cards he can choose five more cards from the draw pile to stay in the game.
6. Continue playing until all the cards in the deck have been matched into pairs. The player with the highest number of pairs at the end of the game is the winner.

Learning Addition Facts by Playing Memory

The card game Memory, or Concentration, is another great game that can be modified to teach addition facts to kids. As with the instructions for Go Fish! above, each game focuses on math facts for a specific number. All that's needed to play this game is a standard deck of playing cards. It can be played alone or with a group.

How to Play Memory

1. Sort through the deck to remove all cards that are higher than that featured number for the math game. For example, if the goal is to learn addition facts for the number six, the game will be played with ones (aces) through sixes.
2. Shuffle the deck and turn all the cards face down in a grid pattern.
3. Taking turns, have each player flip two cards to look for a matching pair. For example, if learning addition facts for the number six, appropriate pairs would be 5+1, 4+2 or 3+3. The 6 card would also be laid aside as a correct solution that doesn't require a pair.
4. Continue playing until all the cards in the deck have been matched into pairs. The player with the highest number of pairs at the end of the game is the winner.

Subtraction "War"

Besides strengthening subtraction skills, this game also provides practice in comparing numbers.

What You Need:

- Deck of cards
- Kitchen timer

How to play:

1. Shuffle the deck of cards and deal them face down, giving each player an equal number of cards until the deck runs out. Each player keeps his cards in a stack. Assign picture cards, such as jacks, queens, and kings, a value of 10. Give aces a value of 1.
2. Each player turns two cards face up, reads the number sentence and supplies the answer. For example, if a player draws a 5 and a 4, s/he says 5 - 4 = 1. If the other player draws a 7 and a 2, then their number sentence is 7-2 = 5. The player with the larger result (5), wins the four cards and puts them at the bottom of their pile.
3. If each player has a number sentence with the same answer, then it's war! At this point, reverse the math "operation" and do an addition problem. Each player puts four cards face down and turns up two of them. The player with the highest sum wins all eight cards.
4. Set up the timer and play the game for 10 to 15 minutes. When the bell goes off, each player counts his cards. The player with the most cards wins. If one player runs out of cards before time is up, then the other player wins.

Close Call:

Challenge students to create sums as close to 100 as possible, without going over. This requires each student to evaluate all possible sums, based on the numbers they are given.

What You Need:

- Deck of cards
- Paper and pencils (for scratch paper)

How to play:

1. Remove 10s and face cards from the deck. Shuffle the deck and deal each player 6 cards.
2. Each player selects four of their cards and creates two 2-digit numbers from them. The goal is to create two numbers that have a sum as close to 100 as possible, without going over. (For example, a player may use the cards 4, 6, 8, and 1, creating the problem 14 + 86 = 100.)
3. After players have made their selections, they place their cards face up in front of them, arranging them so other players can see which two numbers they have created.
4. The player with the numbers adding closest to 100, without going over, wins a point. In the case of a tie, a point is awarded to each team.
5. Shuffle the cards before dealing another round.
6. Play continues for 5 rounds. The player with the most points after the last round wins the game.

Variations:

- Change the number of cards dealt, the number of cards used, or the goal.
- For younger players, restrict the number of cards dealt to 4 per player, allow them to use only 2 of the cards, create single-digit numbers, and set the goal to 10.
- To make the game more challenging, deal 8 cards to each player, let them choose 6, create 3-digit numbers, and set the goal to 1,000.

Name:______________________________ Date:________

Can you do this? #5

1. Solve the following. Show your working in the boxes.

(a) 573 + 26 =	(b) 573 + 609 =
(c) 627 people at the football were joined by another 254. How many people watched football?	(d) Joy had 1 265 stamps in an album. Pat had 324 more stamps than Joy. How many stamps did Pat have?

Obj. 2.5.1
a) i. □
a) ii. □
b) i. □
b) ii. □
c) i. □
c) ii. □
d) i. □
d) ii. □

2. Use the data table to answer the questions. Use the boxes to show your working.

Mobile Game Revenue Forecasts – including phones, smartphones, tablets (Million $)					
	2012	**2013**	**2014**	**2015**	**2016**
Asia Pacific	3 654	4 360	4 965	5 582	6 169
Europe, MiddleEast, Africa	2 075	2 555	2 970	3 321	3 561
Latin America	358	449	525	568	598
North America	1 751	1 979	2 123	2 224	2 294
Total Worldwide	**7 839**	**9 342**	**10 583**	**11 695**	**12 622**

Obj. 2.5.1
a) i. □
a) ii. □
a) iii. □

(a) What are the 2015 Mobile Game Revenue Forecast numbers for America (combine North America and Latin America)?

(b) What are the 2016 Mobile Game Revenue Forecast numbers for America?

Obj. 2.5.1
b) i. □
b) ii. □
b) iii. □

(c) What are the 2016 Mobile Game Revenue Forecast numbers for the rest of the world?

Obj. 2.5.1
c) i. □
c) ii. □
c) iii. □

Cycle 5: Addition with Larger Numbers

Overview

Big Idea

The focus of this RAMR cycle is the development of addition computation strategies to calculate two- and three-digit addition. These strategies can be extended to larger operands although it is also recommended to use estimation and calculators for operating on numbers with higher place-values. These activities provide opportunities to continue practice with interpretation and construction of worded problems, and strategy development using understanding of the place-value system.

Three computation techniques may be taught, separation, sequencing and compensation. The separation strategy leads to the traditional algorithm. It is useful to learn because as a strategy, separation (separate into place-value components, calculate separately, and combine) is used in a variety of contexts. Sequencing and compensation techniques tend to be used more commonly as mental computation strategies but can also be supported by recording. Accuracy of computation will rely on building confidence with number facts and practice.

Objectives

By the end of this cycle, students should be able to:

2.5.1 Solve addition stories using computation strategies. [4NA073]

Conceptual Links

Previous understanding of the addition concept, place value additive structure and place value multiplicative structure is needed for success in this cycle. This cycle provides continued opportunities to reinforce place value structural understandings, seriation skills and number facts. Commutative and associative properties of operations introduced in the previous cycle contribute to computation strategies.

This cycle extends on strategies for performing whole number addition with simple problems to working with larger place-values. These skills will be useful in later measurement units when calculating perimeter, area and volume using smaller units. Activities in this cycle also reinforce place-value understanding useful for conversion of metric measures.

Materials

For Cycle 5 you may need:

- Maths mat
- Function machine
- flip cards
- calculator per student
- A7 digit cards, 2 10-sided die, or spinner per pair
- PVC per student
- Straws or paddle pop sticks
- Place Value Charts
- Addition thinkboards
- Input, output and operation cards
- 3 foam cups per student
- 3 sets A5 digit cards (0-9)

Key Language

Place-value, renaming, regrouping, trading, algorithms, adding, plus.

Definitions

Algorithm: written procedure to perform calculation

Regrouping / trading: making groups of ten from a collection of units (usually encountered in addition) or deconstructing a group of tens into units (usually to facilitate subtraction).

Renaming: a process that involves the flexible naming of numbers according to their constituents (e.g., 335 may be 33 tens and 5 ones or 32 tens and 15 ones).

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How could you solve this problem?
- Is the question a subtraction or addition?
- Can you use number fact strategies to help you?
- How will you write this equation to solve it?
- What will the answer be roughly? Can you estimate it?
- Check your answer. Does it make sense? Is it close to what you would estimate?

Portfolio Task

The Portfolio Task: *Exploring Apple Inc.* task involves students' interpretation of a data table which has data sets described in '000s. Questions require students to collate data categories expressed in large numbers to generate totals. Aspects of this task can be completed following this cycle.

RAMR Cycle

Reality

Remind students of previously acted out addition problems, for example:

> *There were 3 students sitting at a table and 2 more students joined them. How many students altogether?*

Check students' retention of higher decade facts and place-value by changing numbers in problems to hundreds (e.g., 300 students on school oval joined by 200 more students (fire drill perhaps)).

Ask students how they might solve the problem if the numbers were 3000 and 2000. See if students can pose situations and problems with big numbers where they can apply addition number facts.

Abstraction

In this abstraction sequence the focus is on extending conceptual understanding of addition and the addition process to find ways to solve addition problems with larger numbers. There are three main strategies that may be used for written or mental computation. This abstraction phase explores the separation strategy (traditional addition algorithm) as this is likely most familiar to students. The sequencing and compensation strategies are included within the Mathematics phase. Problems should be connected to students' interests and realities where possible. A suggested sequence is as follows:

Separation strategy

1. *Kinaesthetic activity.* Revise acted out addition problems (can be done using digit cards on a large place value chart on the maths mat or whiteboard). Extend these to higher decade facts to reach large numbers that can be computed using simple number facts and place-value understanding. Ensure students can create stories from equations and equations from stories. Encourage students to use a symbol they are comfortable with for the unknown part of their equation. Part-part-total and the triad are both useful tools to connect these ideas for students.
2. *Model with materials.* Use MAB materials or bundling straws to model addition of 2-digit numbers on the Maths mat place value chart. Ask students to share their strategy for adding the numbers (most likely students will recall the separation algorithm). Focus students' attention on adding digits with like place-values. This connects back to the notion of unit from Unit 1. Explicitly connect counting and adding like named things (e.g., dogs and dogs, cats and cats, ones and ones, tens and tens).
3. *Recording with symbols.* Practise recording addition problems to obtain a total. Reinforce constructing and interpreting real-world stories for addition that use larger quantities. Ask students to generate their own questions that they write as equations. Work on increasing vocabulary for addition problems and vary question structures so that the known part, unknown part and total are not always in the same order.

Resource Resource 2.5.1 Computation techniques: Addition

Mathematics

Once the active meaning of addition as joining has been established and the separation strategy and algorithm practised with larger numbers, provide students with opportunities to explore further meanings of addition and their decoding into equations and to establish alternative strategies for computing with larger numbers.

Connections

Ensure that students recognise the connections between additive and multiplicative structure of place-value and the addition algorithm. Students must recognise that when adding items together, they need to be adding the same type of unit together so that units are added to units, tens to tens and so on. Linking to the name of the unit that they are adding will reinforce the notion of unit explored in Unit 01 and consolidate an understanding that needs to be accessible when exploring fraction and measurement concepts. This contributes to more robust understanding than a rote rule of aligning place values in the algorithm.

Reinforce the connection between place-value and grouping units in tens when adding quantities that bridge to the next place up and result in 'carrying' over. Ensure that students recognise that the digit they 'carry' is a consequence of having one or more groups of a place when joining the units during the addition process which is represented as a quantity of groups (tens, hundreds, and so on).

Language/symbols and practice

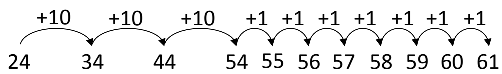
Sequencing strategy

Revisit subtraction strategies, potentially revisiting a similar abstraction sequence, to develop the Sequencing strategy for mental computation.

1. *Kinaesthetic activity.* Revise acted out addition problems (can be done using digit cards on a large number line on the floor or whiteboard). Extend these to higher decade facts to reach large numbers that can be computed using simple number facts and place-value understanding. Ensure students can create stories from equations and equations from stories. Encourage students to use a symbol they are comfortable with for the unknown part of their equation. Part-part-total and the triad are both useful tools to connect these ideas for students.

2. *Model with materials.* Use large number line and digit cards to model addition of 2-digit numbers. Focus students' attention on locating the first number in the addition story. Separate the number to be added into tens and ones and model the jumps along the number line to complete the addition. For example, 24 + 37 will start at 24 on the number line. The 37 can be separated into 3 tens and 7 ones. Use seriation strategies to count up in tens on the number line and then up in ones to find the total.

   Note: an informal sketch of an empty number line is appropriate for this strategy.

3. *Recording with symbols.* Practise recording addition problems to obtain a total. Reinforce constructing and interpreting real-world stories for addition that use larger quantities. Ask students to generate their own questions that they write as equations. Work on increasing vocabulary for addition problems and vary question structures so that the known part, unknown part and total appear in different positions within the worded problem.

Resource Resource 2.5.1 Computation techniques: Addition

Compensation strategy

Consider ways to separate numbers other than place value (as used in Separation and Sequencing strategies) that may help with mental compuation of addition of larger numbers. This will extend students to the Compensation Strategy (see *Resource 2.5.1 Computation Techniques: Addition*). Explore combinations of number fact strategies, computation strategies and recording techniques that can be used to find solutions.

Resource Resource 2.5.1 Computation techniques: Addition

Practice addition skills

Opportunities to practice and apply addition skills and place-value understandings may be found using real investigations of data sets with large numbers where comparisons can be made between categories. Consider exploring wildlife estimates where subcategories can be combined to super-categories (e.g., kangaroos are separated into reds, eastern greys, western greys, wallaroos; these could be re-categorised into reds, greys, wallaroos). Alternatively, data categorised by state could be combined into Western, Central and Eastern Australia.

Resource Resource 2.5.2 Addition from datasets

Estimation

Explore the use of rounding to estimate whether or not a total is correct. For example, 245 + 364 should have a total that is between 200 + 300 = 500 and 300 + 400 = 700. Encourage students to refine this technique (e.g., this could be refined to 250 + 350 = 600 for a ballpark total). Discuss when an estimate might be appropriate instead of an exact amount.

Reflection

Check the idea

Provide students with a variety of numbers and contexts to create addition stories from. Swap these with a partner for solving.

Use 6 dice to generate random numbers. Students should arrange these to create the largest 6-digit number they can and the smallest 6-digit number they can and then add these together. Variations could include creating three of the biggest 6-digit numbers possible and adding them together. Greater challenge can be achieved using 10-sided dice or greater numbers of dice. Consider rolling dice in sets of three to create millions, thousands, ones … again, biggest number, smallest number but with different constraints.

Apply the idea

Data tables

Explore examples of data tables and column graphs with large numbers in the data sets. Have a variety of axis labels so that students are working in place-values. Use data from the graph to ask questions for comparisons in the data that use addition. For example, look at the average weekly earnings for part-time workers from the Australian Bureau of Statistics. If a household had several people earning, what would be the combined household income? Which occupations combined within a household will give the highest income? Which give the lowest income?

Resource Resource 2.5.3 Average weekly earnings

Perimeter of familiar items

Measure the dimensions of classroom items in millimetres. Add these together to find perimeters.

Plan a trip

Select a number of destinations on Google Maps that may make a possible itinerary for a trip. Collate the distances (use whole numbers) between the destinations and calculate the total distance for the trip. This could be represented as a strip map, and the similarities to a number line highlighted for students.

Fill a boat or truck

Boats and trucks have load limits. Investigate the masses of a variety of items that may be loaded onto a boat or truck. Add these masses together to check whether more may be loaded or if the total is over the accepted limit. For example, how many cars may be shipped by road to a destination on a truck; by rail on a railcar; across water on a ferry?

Extend the idea

Flexibility

Discuss creative ways to partition very large numbers using place-values to solve addition problems without every step of an algorithm. For example, 34 596 000 + 23 402 322. The answer will have 322 ones because there are all zeros in the other number. 34 million + 23 million will be 57 million. 596 thousand + 402 thousand = 998 thousand (no carrying into the millions) so the final answer will be 57 998 322.Explore instances where calculators cannot work with larger numbers (8-digits or more). Encourage students to devise and share strategies to solve these problems.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:__________________________________ Date:________

Can you do this? #6

1. Solve the following. Show your working in the boxes.

(a) 57 – 29 =	(b) 603 – 36 =
(c) 627 people were at the football until 254 people went home. How many people were left watching football?	(d) Joy had 1 265 stamps in an album. Pat had 428 less stamps than Joy. How many stamps did Pat have?

Obj. 2.6.1
a) i. ☐
a) ii. ☐
b) i. ☐
b) ii. ☐
c) i. ☐
c) ii. ☐
d) i. ☐
d) ii. ☐

2. Use the data table to answer the questions. Use the boxes to show your working.

Mobile Game Revenue Forecasts – including phones, smartphones, tablets (Million $)					
	2012	**2013**	**2014**	**2015**	**2016**
Asia Pacific	3 654	4 360	4 965	5 582	6 169
Europe, MiddleEast, Africa	2 075	2 555	2 970	3 321	3 561
Latin America	358	449	525	568	598
North America	1 751	1 979	2 123	2 224	2 294
Total Worldwide	**7 839**	**9 342**	**10 583**	**11 695**	**12 622**

Obj. 2.6.1
a) i. ☐
a) ii. ☐
a) iii. ☐

(a) What is the 2016 Mobile Game Revenue Forecast for Asia Pacific?

____

(b) What would be the 2016 Mobile Game Revenue Forecast for the rest of the world (not including Asia Pacific)?

Obj. 2.6.1
b) i. ☐
b) ii. ☐
b) iii. ☐

Cycle 6: Subtraction with Larger Numbers

Overview

Big Idea

The focus of this RAMR cycle is the development of subtraction computation strategies to calculate two- and three-digit subtraction. These strategies can be extended to larger operands although it is also recommended to use estimation and calculators for operating on numbers with higher place-values. These activities provide opportunities to continue practice with interpretation and construction of worded problems and strategy development using understanding of the place-value system.

Three computation techniques may be taught, separation, sequencing and compensation. The separation strategy leads to the traditional algorithm. It is useful to learn because as a strategy, separation (separate into place-value components, calculate separately, and combine) is used in a variety of contexts. Sequencing and compensation techniques tend to be used more commonly as mental computation strategies but can also be supported by recording. Accuracy of computation will rely on building confidence with number facts and practice.

Objectives

By the end of this cycle, students should be able to:

2.6.1 Solve subtraction stories using computation strategies. [4NA073]

Conceptual Links

Previous understanding of the subtraction concept, addition concept, place value additive structure and place value multiplicative structure is needed for success in this cycle. This cycle provides continued opportunities to reinforce place value structural understandings, seriation skills and number facts.

This cycle extends on strategies for performing whole number subtraction with simple subtraction problems to working with larger place-values. These skills can be applied to measurement when calculating dimensions from perimeter, area and volume using smaller units. Activities in this cycle also reinforce place-value understanding useful for conversion of metric measures.

Materials

For Cycle 6 you may need:

- Maths mat
- Function machine
- Input, output and operation cards
- flip cards
- calculator per student
- counters
- Place Value Charts
- Thinkboards
- 3 foam cups per student
- 3 sets A5 digit cards (0-9)
- A7 digit cards, 2 10-sided dice per pair

Key Language

Place-value, renaming, regrouping, trading, inverse, additive inverse, difference, take away, subtracting, minus.

Definitions

Algorithm: written procedure to perform calculation

Regrouping / trading: making groups of ten from a collection of units (usually encountered in addition) or deconstructing a group of tens into units (usually to facilitate subtraction).

Renaming: a process that involves the flexible naming of numbers according to their constituents (e.g., 335 may be 33 tens and 5 ones or 32 tens and 15 ones).

Anecdotal Evidence

Some possible prompting questions:

- How could you solve this problem?
- Is the question a subtraction or addition?
- Can you use number fact strategies to help you?
- How will you write this equation to solve it?
- What will the answer be roughly? Can you estimate it?
- Check your answer. Does it make sense? Is it close to what you would estimate?

Portfolio Task

This task involves students interpretation of a data table which has data sets described in '000s. Questions require students to compare differences between data categories expressed in large numbers. Aspects of this task can be completed following this cycle.

RAMR Cycle

Reality

Remind students of previously explored inverse of addition operations with smaller numbers. Act out addition problems with the students. Immediately follow each problem with the inverse or opposite action. For example:

Addition: *There were 3 students sitting at a table and 2 more students joined them. How many students altogether?*

Subtraction: *The group of 5 students chatted for a while, then the 2 students walked away. How many students left at the table?*

Check students' retention of higher decade facts and place-value by changing numbers in problems to hundreds (e.g., 300 students on school oval; 200 moved to buildings (bell rang)). Ask students how they might solve the problem if the numbers were 3000 and 2000. See if students can pose situations and problems with big numbers where they can apply the think addition strategy.

Abstraction

In this abstraction sequence the focus is on extending conceptual understanding of subtraction and the subtraction process to find ways to solve subtraction problems with larger numbers connected to students' interests and realities. There are three strategies that may be used for written or mental computation. This abstraction phase explores the separation strategy (traditional addition algorithm) as this is likely most familiar to students. The sequencing and compensation strategies are included within the Mathematics phase. A suggested sequence is as follows:

Separation strategy

1. *Kinaesthetic activity.* Revise acted out subtraction problems (can be done using digit cards on a large place value chart on the maths mat or whiteboard). Extend these to higher decade facts to reach large numbers that can be computed using simple number facts and place-value understanding. Ensure students can create stories from equations and equations from stories. Encourage students to use a symbol they are comfortable with for the unknown part of their equation. Part-part-total and the triad are both useful tools to connect these ideas for students.

2. *Model with materials.* Use MAB materials to model subtraction of 2-digit numbers on the Maths mat place value chart. Focus students' attention on subtracting digits with like place-value names. Explicitly connect to counting and subtracting like-named things (e.g., dogs and dogs, cats and cats, ones and ones, tens and tens) as explored in Unit 01 (notion of unit).

   Note: When subtracting, represent the total on the place value chart with materials, separate the amount to be subtracted, and then interpret what is left. The action needs to reinforce the separation of the parts from the total amount.

Resource Resource 2.6.1 Computation techniques: Subtraction

3. *Checking strategy.* Ensure that students can see that the parts generated by the subtraction can be added back together to the original total. Highlight that subtraction is the inverse operation and inverse action from addition. In addition, materials on the place value chart are joined to make a total, with subtraction the total is separated into parts.

4. *Recording with symbols.* Practise recording subtraction problems to find the unknown part. Reinforce constructing and interpreting real-world stories for subtraction that use larger

quantities. Ask students to generate their own questions that they write as equations. Work on increasing vocabulary for subtraction problems and vary question structures so that the known part, unknown part and total are not always in the same order.

5. *Check answer.* Discuss with students the use of adding the answer to the amount subtracted to check that their answer matches the amount of the beginning total.

Mathematics

Once the active meaning of subtraction as taking away has been established and the separation strategy and algorithm practised with larger numbers, provide students with opportunities to explore further meanings of subtraction and their decoding into equations and to establish alternative strategies for computing with larger numbers.

Connections

Ensure that students recognise the connections between additive and multiplicative structure of place-value and the subtraction algorithm. Students must recognise that when subtracting items, they need to be operating on the same type of unit so that units are subtracted from units, tens from tens and so on. Linking to the name of the unit that they are operating on will reinforce the notion of unit explored in Unit 01 and consolidate an understanding that needs to be accessible when exploring fraction and measurement concepts. This contributes to more robust understanding than a rote rule of aligning place values in the algorithm.

Reinforce the connection between place-value and grouping units in tens when subtracting quantities that bridge to the next place up and result in trading or regrouping. Ensure that students recognise that the unit they trade from a place up is worth ten of the unit in the next place down as a consequence of moving one group of a place.

To highlight the inverse relationship between addition and subtraction, compare the action of grouping places in tens and carrying as a unit to the next place up in addition with the inverse action of using a unit of a higher place to trade for ten of the unit in the next place down when subtracting.

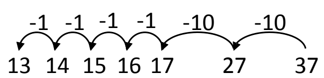
Language/symbols and practice

Sequencing strategy

1. *Kinaesthetic activity.* Revise acted out subtraction problems (can be done using digit cards on a large number line on the floor or whiteboard). Extend these to higher decade facts to reach large numbers that can be computed using simple number facts and place-value understanding. Ensure students can create stories from equations and equations from stories. Encourage students to use a symbol they are comfortable with for the unknown part of their equation. Part-part-total and the triad are both useful tools to connect these ideas for students.

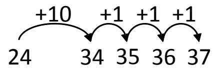
2. *Model with materials.* Use large number line and digit cards to model subtraction of 2-digit numbers. Ask students to share their strategy for subtracting the numbers (most likely students will recall the separation algorithm). Focus students' attention on locating the first number in the addition story. Separate the number to be added into tens and ones and model the jumps along the number line to complete the subtraction. For example, 37 – 24 will start at 37 on the number line. The 24 can be separated into 2 tens and 4 ones. Use seriation strategies to count back in tens on the number line and then back in ones to find the unknown part.

   Note: an informal sketch of a number line or an empty number line is appropriate for this strategy.



Alternatively, a counting up strategy may be used to find the difference between the two quantities. This is also known as the shopkeeper's method of subtraction. For example, 37 – 24 will start at 24 on the number line. A jump of 10 counts on to 34, three more jumps of 1 will reach 37. The difference or unknown part is thus 10 + 3 = 13. Again, an informal sketch of an empty number line is appropriate to represent this strategy.

Note: this strategy encapsulates the think addition strategy for subtraction where, instead of soving for 37 – 24, the focus is on what amount is added to 24 to total 37.



3. *Recording with symbols.* Practise recording subtraction problems to find the unknown part. Reinforce construction and interpretation of real-world stories for subtraction that use larger quantities. Ask students to generate their own questions that they write as equations. Work on increasing vocabulary for subtraction problems and vary question structures so that the known part, unknown part and total appear in different positions within the worded problem.
4. *Flexibility.* Consider other ways to separate numbers other than place value that may help with subtraction of larger numbers. This will extend to the Compensation Strategy (see *Resource 2.6.1 Computation Techniques: Subtraction*).
5. *Estimation.* Explore the use of rounding to estimate whether or not a total is correct. For example, 364 – 245 should have a total that is around 100. Encourage students to refine this technique as their confidence with mental computation increases. Discuss when an estimate of a total might be appropriate in preference to an exact amount.

Resource Resource 2.6.1 Computation techniques: Subtraction

Compensation strategy

Consider ways to separate numbers other than place value (as used in Separation and Sequencing strategies) that may help with mental compuation of subtraction of larger numbers. This will extend students to the Compensation Strategy (see *Resource 2.6.1 Computation Techniques: Subtraction*). Explore combinations of number fact strategies, computation strategies and recording techniques that can be used to find solutions.

Resource Resource 2.6.1 Computation techniques: Subtraction

Practice subtraction skills

Students need opportunities to practice and apply their subtraction skills and place-value understandings. Real investigations of data sets with large numbers where comparisons can be made and categories separated can be useful. Consider exploring wildlife estimates where annual change can be compared and difference determined (e.g., 2011 kangaroo populations compared with 2010 populations).

Resource Resource 2.6.2 Subtraction from datasets

Estimation

Explore the use of rounding to estimate whether or not a total is correct. For example, 245 + 364 should have a total that is between 200 + 300 = 500 and 300 + 400 = 700. Encourage students to refine this technique (e.g., this could be refined to 250 + 350 = 600 for a ballpark total). Discuss when an estimate might be appropriate instead of an exact amount.

Reflection

Check the idea

Provide students with a variety of numbers and contexts to create subtraction stories from. Swap these with a partner for solving.

Use 6 dice to generate random numbers. Students should arrange these to create the largest 6-digit number they can and the smallest 6-digit number they can and subtract one from the other. Variations could include creating three of the biggest 6-digit numbers possible and finding the differences between them. Greater challenge can be achieved using 10-sided dice or greater numbers of dice. Consider rolling dice in sets of three to create millions, thousands, ones … again, biggest number, smallest number but with different constraints.

Apply the idea

Explore examples of data tables and column graphs with large numbers in the data sets. Have a variety of axis labels so that students are working in place-values. Use data from the graph to ask questions for comparisons in the data that use difference. For example, look at the average weekly earnings for part-time workers from the Australian Bureau of Statistics. What is the difference in income between part-time and full-time earnings? What is the difference between the highest and the lowest wages? What are the differences between male earnings and female earnings?

Resource Resource 2.6.3 Average weekly earnings

Perimeter of familiar items

Work from known perimeters of familiar items and one side measure to find the missing side.

Plan a trip

Select a number of destinations on Google Maps that may make a possible itinerary for a trip. Collate the distances (use whole numbers) between the destinations and calculate the total distance for the trip. Work backwards from a total trip distance to find how far left to go by subtracting completed sections of the journey.

Fill a boat or truck

Boats and trucks have load limits. Start with a total load mass and reduce it progressively to within limits by subtracting items. For example, a number of cars have been loaded onto a ferry. Rough weather means that the ferry is now overloaded. How many cars must be removed to travel safely?

Extend the idea

Flexibility

Discuss creative ways to partition very large numbers using place-values to solve subtraction problems without every step of an algorithm. For example, 34 596 000 – 23 402 322. The answer will have 11 millions (no need for trading in the thousands). 593 thousands – 400 thousands will be 193 thousands (allowing 3 thousands to take 2322 from 3000). 3000 – 2322 will be 678 so the final answer will be 11 193 678.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 02 Portfolio Task – Teacher Guide

Exploring Apple Inc.

Content Strand/s: Number and Algebra

Resources Supplied:
- Task sheet
- Teacher guide

Other Resources Needed:
None

Summary:
Using the context of Apple products and the World's population, students solve addition and subtraction problems, interpret data from a table and order numbers on a Place Value Chart.

When calculating question 11 an assumption is made that each person does not own more than one Apple product. This could be used to have students question the reasonableness of the statement that '413,679,000 people own an apple product'.

Variations:
- Students can create a column graph of the data for the 3 years and use this to compare the amounts of products sold.

ACARA Proficiencies Addressed:
Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Number and Algebra
2.3.2 Interpret and identify the parts and total within additive stories. [2NA036]
2.3.5 Identify appropriate operation to create equations from additive word problems. [6NA123]
2.3.6 Represent numbered additive change as equations with a symbol for unknowns. [7NA176]
2.3.7 Represent numbered change and inverse using backtracking diagrams. [7NA177]
2.5.1 Solve addition stories using computation strategies. [4NA073]
2.6.1 Solve subtraction stories using computation strategies. [4NA073]

Exploring Apple Inc.

Name	
Teacher	
Class	

Your Task:

You are an employee of a new computer company conducting market research into Apple Inc. It is your job to look at the different statistics of Apple Inc. around the world. You will need to:

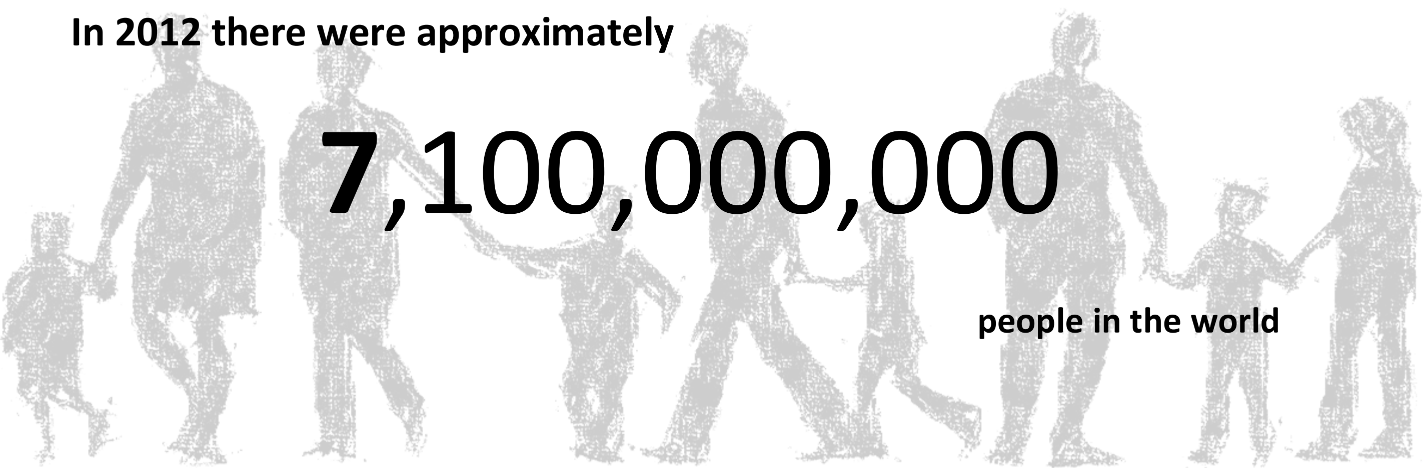
- Compare numbers
- Total numbers
- Find the difference between numbers, and
- Justify choices

You may use a calculator from **page 4.**

Within Portfolio Task 2, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Procedural fluency	2.3.2 Interpret and identify the parts and total within additive stories.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
Understanding and Fluency	Mathematical language and symbols	2.3.6 Represent addition stories as equations using symbols for unknowns. 2.5.1 Solve addition stories using computation strategies. 2.6.1 Solve subtraction stories using computation strategies.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Reasoning and justification	2.3.7 Represent numbered change and inverse. 2.3.5 Identify appropriate operation to create equations from additive word problems.	Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions

Comments:



In 2012 there were approximately

7,100,000,000

people in the world

In 2012 Apple sold a total of 236,679,000 units, including, Mac computers (desktop and laptop), iPad, iPod and iPhone.

1. Calculate the total number of products sold in each year

Product	*2012*	*2011*	*2010*
Desktop Computers (1000's)	4,656	4,669	4,627
Laptop Computers (1000's)	13,502	12,066	9,035
iPod (1000's)	35,165	42,620	50,312
iPhone (1000's)	125,046	72,293	39,989
iPad (1000's)	58,310	32,394	7,458
TOTAL (1000's)			

a. What strategy could you use to double check your answers?

____

b. What does the 1000's next to each product name mean?

____

2. Use the table below answer the following questions:

Product	*2012*	*2011*	*2010*
Desktop Computers (1000's)	4,656	4,669	4,627
Laptop Computers (1000's)	13,502	12,066	9,035
iPod (1000's)	35,165	42,620	50,312
iPhone (1000's)	125,046	72,293	39,989
iPad (1000's)	58,310	32,394	7,458

a. Which product has become less popular from 2010 to 2012? ___________

b. Why do you think this has happened?

____

3. Compare the total number of products sold:

a. In 2012, 4 656 000 desktop computers were sold. Write and solve the equation for the total (T) number of products found in 2012 and 2011.

*Be careful to write the whole number **including** the thousands.*

Equation: ____

Answer:_________________________

b. What is the difference (D) in the number of products sold in 2012 and 2011?

Equation: ____

Answer:____________________

c. How many more products were sold in 2011 than in 2010?

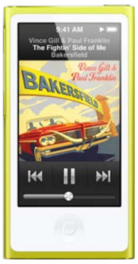
Equation: ____

Answer:____________________

d. Is this larger or smaller than the difference in product sales between 2012 and 2011?

____

e. By how much? ___________________



4. There was a recording error in 2010 at Apple and the number of iPods sold was actually 22, 141 more than is recorded in the table above.

   a. Will there be more or less iPods sold after the error is fixed? ________

   b. How many iPods were actually sold in 2010? ________________

5. If **413,679,000** people own an Apple Product, and there are a total of **7,100,000,000** people in the world. What is the difference?

   a. Try to put this number into a calculator. Does it fit? __________ Why? _____________

   b. Break the numbers up into two sections, billions and millions, and thousands and ones:

	Billions			**Millions**		
	100s	10s	1s	100s	10s	1s
World's Population						
Apple Products						

	Thousands			**Ones**		
	100s	10s	100s	10s	100s	10s
World's Population						
Apple Products						

   c. Find the difference between the billions and millions using a calculator:

   ___________________ Millions

   d. Find the difference between the thousands and one using a calculator:

   _________________________ Ones

   e. Using your answers from c. and d. what is the difference between 7, 204, 863, 544 and 513, 679, 000?

6. Compare the number of people who own an Apple product, and the number of people who do not own an Apple product:

   a. What is larger? Tick the correct answer.

      □ number of people who own an Apple product

      □ number of people that do not own an Apple product

   b. How did you work that out?

      ____

      ____

   c. Explain how you could check your answer to make sure it is correct?

      ____

      ____

      ____

Name:____ Date:________

Can you do this now? Unit 02

1. Circle the words that you think mean ‘equals’ in the equation:

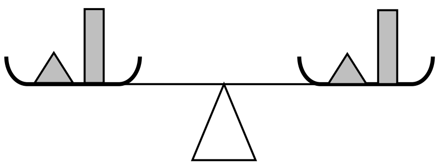
$$5 - 3 = 1 + 1$$

the same as	balances	makes
totals	adds up to	answers
measures	leaves	matches

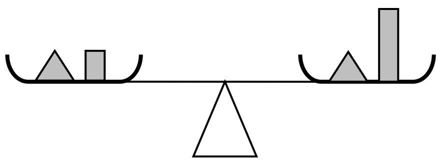
Obj. 2.1.1
i ☐
ii ☐
iii ☐
iv ☐
v ☐
vi ☐
vii ☐
viii ☐
ix ☐

2. Circle the word below (true or false) to describe the following:

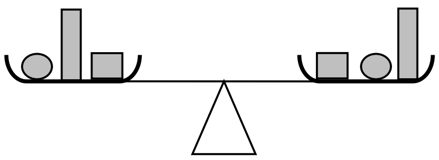
(a) True / False



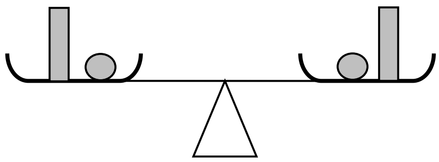
(b) True / False



(c) True / False



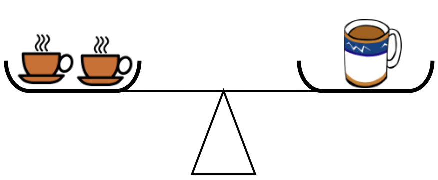
(d) True / False



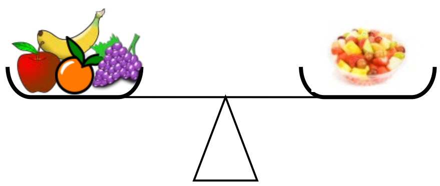
Obj. 2.1.1
a) ☐
b) ☐
c) ☐
d) ☐

3. Write equations for each of the following balance relationships.
   The first one has been done for you.

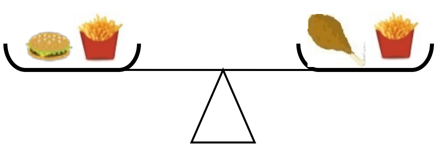
(a) <u>Small cup + small cup = large cup.</u>



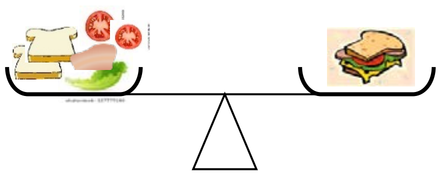
(b) ______________________________.



(c) ______________________________.



(d) ______________________________.



Obj. 2.1.2
b) ☐
c) ☐
d) ☐

4. Dry cereal is changed by adding milk to make soggy cereal. This change could be represented using the following diagram:

$$\text{Dry cereal} \xrightarrow{\text{Add milk}} \text{Soggy cereal}$$

Draw a diagram to represent:

Dry soil changed by adding water to make mud.

____

Obj. 2.2.1 i ☐ ii ☐ iii ☐

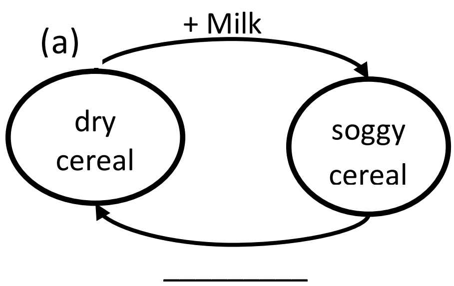
5. Fill in the blanks:

	Input	Change	Output
(a)	‘p’	Join with ‘op’	
(b)		Cook	Apple pie
(c)	pat		tap

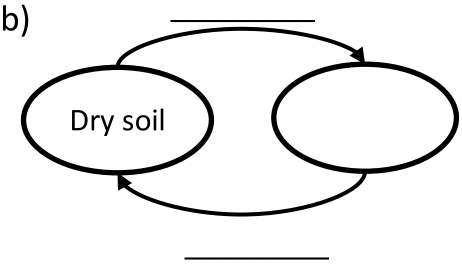
Obj. 2.2.2 a) ☐ b) ☐ c) ☐

6. Fill in the blanks on the backtracking diagrams:

(a)



(b)



Obj. 2.2.5 a) ☐

Obj. 2.2.4 b)i ☐ b)ii ☐ b)iii ☐

The backtracking diagram of Question 6(a) could be represented using the following two equations:

For example: (1) Dry cereal + milk = soggy cereal

(2) Soggy cereal – milk = dry cereal

Write equations to represent the backtracking diagram of Question 6(b) on the lines below.

(c) ____

(d) ____

Obj. 2.2.3 c) ☐ d) ☐

Fact families show different equations for the same change. For example: (i) Dry cereal + millk = soggy cereal

(ii) Milk + dry cereal = soggy cereal

(iii) Soggy cereal – milk = dry cereal

(iv) Soggy cereal – dry cereal = milk

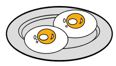
Write the other two equations from the fact family for:
Dry soil changed by Adding water to make Mud.

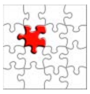
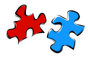
(e) ______________________ (f) ______________________

Obj. 2.2.6
e) □
f) □

7. Circle the changes that can be undone in reality:

(a)  inflated tyre $\xrightarrow{\text{let down}}$ flat tyre 

(b)  eggs $\xrightarrow{\text{cook}}$ fried eggs 

(c)  completed jigsaw $\xrightarrow{\text{break up}}$ jigsaw pieces 

Obj. 2.2.5
a) □
b) □
c) □

8. In each story, underline the parts and draw a circle around the total.

(a) Our team has 15 players and the other team has 12 players. There are 27 players altogether.

(b) There were 20 balls in the sports room. Eight balls were taken out and 12 are left.

Obj. 2.3.2
a)i □
a)ii □
a)iii □
b)i □
b)ii □
b)iii □

9. Complete the following:

(a) 16 chickens $\xrightarrow{\text{add 6 chickens}}$ ____________

(b) 27 people on the bus $\xrightarrow{\text{\_\_\_\_\_\_\_\_}}$ 29 people on the bus

Obj. 2.3.3
a) □
b) □

10. Complete the input/output tables and identify the change.

a)

Change: ___________	
Input	**Output**
57	67
77	87
	117
127	

b)

Change: ___________	
Input	**Output**
978	958
938	918
898	
	858

Obj. 2.3.4
a)i ☐
a)ii ☐
a)iii ☐
b)i ☐
b)ii ☐
b)iii ☐

11. Is this an addition or subtraction problem? Circle the correct symbol.

a) A 2m length was cut from a plank of wood. There was 1m left. How long was the plank of wood at the start? + −

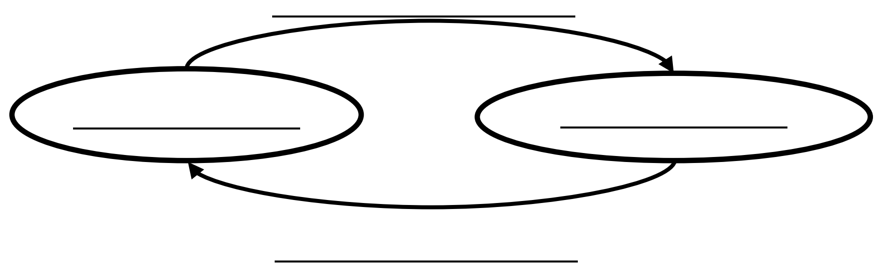
b) Jack had $23 and his mum gave him $6. How much money does he have altogether? + −

c) I was given 7 books more, this made 11 books. How many did I start with? + −

Obj. 2.3.5
a) ☐
b) ☐
c) ☐

12. Write a backtracking diagram for the following story:

A 3 carriage train had 5 carriages linked to it making it an 8 carriage train.



Obj. 2.3.7
i ☐
ii ☐
iii ☐
iv ☐

13. Write a story for the equation : 6 + 15 = ?

____

____

Obj. 2.3.6
i ☐
ii ☐
iii ☐

14. Write the story as an equation using variables.
I was given money and then spent $10. How much did I have left?
(m is how much money I was given; s is how much money was left)

____

Obj. 2.3.6
i ☐
ii ☐
iii ☐

15. A fish and chip shop prices their meal deals as follows:

Cost of fish (f) + cost of chips (c) + profit (p) = meal deal price (d)

(a) Write the story as an equation using variables.

____

(b) What is the meal deal price if fish is $5, chips are $3, profit is $2?

____

____

(c) If the cost of fish is only $3, what will be the new meal deal price?

____

____

Obj. 2.3.8
a)i □
a)ii □
a)iii □
a)iv □
b)i □
b)ii □
b)iii □
b)iv □
c) □

16. Continue the number patterns:

(a) 65, 77, 89, __________ , __________ , __________

(b) 624, 524, 424, __________ , __________ , __________

(c) 1936, 1926, 1916, __________ , __________ , __________

(d) 10 154, 11 254, 12 354, __________ , __________ , __________

Obj. 2.4.1
a)i □
ii □
iii □
b)i □
ii □
iii □
c)i □
ii □
iii □
d)i □
ii □
iii □

17. Solve the following:

(a) 43 + 78 =

(b) 57 – 38 =

Obj. 2.4.2
a) □
b) □

18. Complete the following:

(a) 18 = ______ + 5

(b) 6 + 9 = ________

(c) 8 + ________ = 16

(d) 25 – 7 = ________

(e) 35 – ______ = 24

(f) 106 + _________ = 115

Obj. 2.4.2
a) □
b) □
c) □
d) □
e) □
f) □

19. Complete the following:

(a) 70 + 50 = ______

(b) 900 – __________ = 300

(c) 7 000 + 7000 = ________.

(d) ______ = 16 000 – 8000

(e) ______ + 600 = 1100

(f) 640 + 80 = __________

20. Use the numbers 4, 9 and 13 to write a fact family of equations:

(a) ______ + _______ = _______

(b) ______ + _______ = _______

(c) ______ – _______ = _______

(d) ______ – _______ = _______

21. Mark the following equations as true or false:

(a) 78 + 96 = 96 + 78 □ True □ False

(b) 5 + 12 – 5 = 12 □ True □ False

(c) 15 + 3 = 6 + 11 □ True □ False

(d) 25 + 16 = 50 – 8 □ True □ False

22. Solve the following. Show your working in the boxes.

(a) 583 + 26 =	(b) 583 + 609 =
(c) 627 people at the basketball were joined by another 354. How many people watched basketball?	(d) Joy had 1 365 trading cards in an album. Pat had 324 more trading cards than Joy. How many trading cards did Pat have?

Obj. 2.4.2
a) □
b) □
c) □
d) □
e) □
f) □

Obj. 2.4.3
a) □
b) □
c) □
d) □

Obj. 2.4.4
a) □
b) □
c) □
d) □

Obj. 2.5.1
a)i □
a)ii □
b)i □
b)ii □
c)i □
c)ii □
d)i □
d)ii □

23. Use the data table to answer the questions. Use the boxes to show your working.

Mobile Game Revenue Forecasts – including phones, smartphones, tablets (Million $)					
	2012	**2013**	**2014**	**2015**	**2016**
Asia Pacific	3 654	4 360	4 965	5 582	6 169
Europe, MiddleEast, Africa	2 075	2 555	2 970	3 321	3 561
Latin America	358	449	525	568	598
North America	1 751	1 979	2 123	2 224	2 294
Total Worldwide	**7 839**	**9 342**	**10 583**	**11 695**	**12 622**

Obj. 2.5.1
a)i □
a)ii □
a)iii □

(a) What are the 2015 Mobile Game Revenue Forecast numbers for Asia Pacific and Europe, Middle East and Africa?

(b) What are the 2016 Mobile Game Revenue Forecast numbers for America?

Obj. 2.5.1
b)i □
b)ii □
b)iii □

(c) What are the 2016 Mobile Game Revenue Forecast numbers for the rest of the world (not including America)?

Obj. 2.5.1
c)i □
c)ii □
c)iii □

24. Solve the following. Show your working in the boxes.

(a) 57 – 39 =	(b) 603 – 37 =
(c) 627 people were at the basketball until 107 people went home. How many people were left watching basketball?	(d) Joy had 2 457 trading cards in an album. Pat had 428 less trading cards than Joy. How many trading cards did Pat have?

Obj. 2.6.1
a)i □
a)ii □
b)i □
b)ii □
c)i □
c)ii □
d)i □
d)ii □

25. Use the data table to answer the questions. Use the boxes to show your working.

Mobile Game Revenue Forecasts – including phones, smartphones, tablets (Million $)					
	2012	**2013**	**2014**	**2015**	**2016**
Asia Pacific	3 654	4 360	4 965	5 582	6 169
Europe, MiddleEast, Africa	2 075	2 555	2 970	3 321	3 561
Latin America	358	449	525	568	598
North America	1 751	1 979	2 123	2 224	2 294
Total Worldwide	**7 839**	**9 342**	**10 583**	**11 695**	**12 622**

Obj. 2.6.1
a)i ☐
a)ii ☐
a)iii ☐

(a) What is the 2015 Mobile Game Revenue Forecast for Asia Pacific?

____

(b) What would be the 2015 Mobile Game Revenue Forecast for the rest of the world (not including Asia Pacific)?

Obj. 2.6.1
b)i ☐
b)ii ☐
b)iii ☐

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