XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

XLR8 Unit 01

Comparing, counting and representing quantity

01

2016

Prepared by YuMi Deadly Centre
Faculty of Education, QUT, Kelvin Grove

ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this module's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of discrete items or measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric rations between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.

Scope

This unit is based upon developing the basis for **comparing** and **ordering** and the **number-as-count** meaning of **cardinal** number. An **attribute** of a **population** can be identified and then the quantity of discrete entities with a similar-valued attribute can be counted; **Counting numbers** are used for this **counting**. Through such counting, the **decimal number** system is developed (limited to whole 3-digit numbers) along with the various embedded concepts including **place-value** and the **additive structure** and **multiplicative structure** of the number system.

A population can be studied: various attributes of the individuals in the population can be identified and each can be counted. The size of each part can be represented in **tables** and **graphs**, and these simple **descriptive statistics** can be used to conduct **analysis** and to provide **answers** to **questions**.

Another **attribute** that can be counted is the number of **possible outcomes** for an **event**. The **likelihood** of an event can be described **qualitatively** using a range of **informal** and **formal probability terms** or **quantitatively** as a **discrete number** of countable **chances**.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this module is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be used as evidence of students' demonstration of learning outcomes including:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio Task:* The portfolio task, *Planes, Planes, Planes!* at the end of Unit 01 engages students with exploring the angles on paper planes, gathering and representing data on the distance and accuracy of their plane's flight and generating an opinion on the best paper plane based on the results of their data.

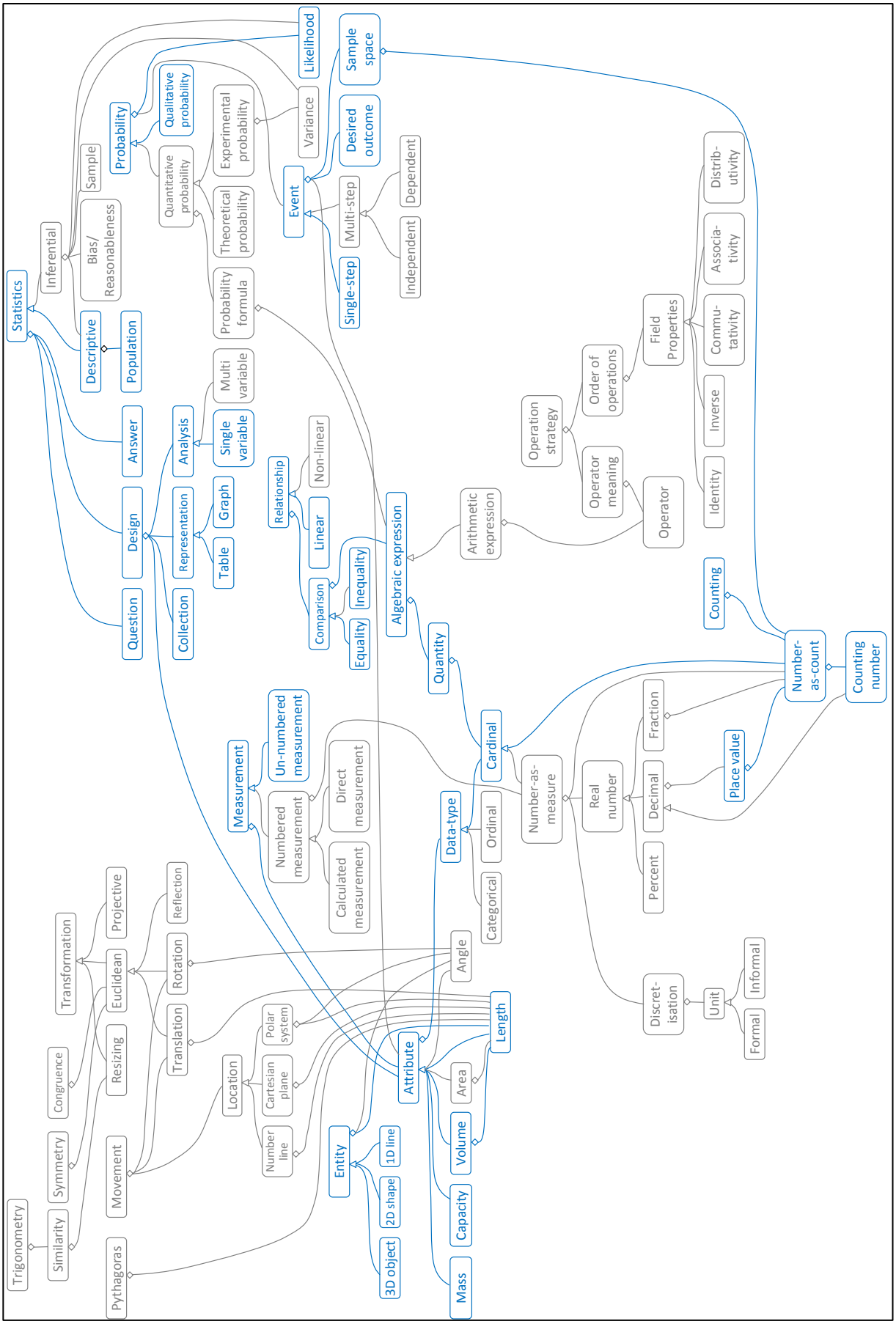


Figure 1. Scope of this cycle

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following sequence:

Cycle 1: Comparison and Order

This cycle develops understanding of informal comparison and ordering of attributes to describe more than, same as and less than. These ideas developed physically with measures can then be applied informally to the attribute of quantity before introducing the idea of counting and place-value.

Cycle 2: Counting Units and Place Value

This cycle develops students' understanding and skills using place-value to read numbers (symbols to language) and to write numbers (language to symbols). This requires: (a) knowledge of the role of zeros; (b) understanding that digits in numbers follow a strict pattern, even if language does not; and (c) understanding that any number is the sum of its components. This idea is explored using discrete items counted as single units, or grouped and renamed countable groups of that unit. For example, students (singles), rows of students (groups), class set of rows of students (groups-of-groups).

Cycle 3: Multiplicative Structure and Regrouping

This cycle relies on the place-value and additive structure concepts of number developed in previous cycles. The Multiplicative place-value relationship enables understanding that numbers can be renamed or regrouped as necessary to assist with calculations. For example, 362 can be thought of (i.e. renamed) as "3 hundreds, 6 tens and 2 ones", but can also be renamed as "36 tens and 2 ones", etc.

Cycle 4: Place Value with Larger Numbers

Within this cycle the notion of large numbers is developed as the integration of ideas developed in the previous cycle, that is, place-value/unit, additive structure, multiplicative structure, order and equivalence. This cycle extends place-value to incorporate the pattern-of-threes inherent within place-value naming of larger numbers. Each newly introduced place should be addressed as both a representation of quantity (to develop an understanding of magnitude) as well as a visual image of the place within a sequence of digits.

Cycle 5: Gathering and Representing Data

This cycle explores gathering data using simple questions or surveys, organisation of data in tables and ways of recording count, simple means of displaying data gathered and initial interpretation of data sets. There are occasions when the count of an object has more relevance when connected to its relationship with other objects or variables and clues as to the initial context. For example, "3" and "5" mean there are 3 things and 5 things. This tells us little compared to "3 people with black shoes" and "5 people with blue shoes". Further information about context is still needed to interpret this data, if the question is about students in uniform, then more students are not complying with the rules. However, if the question is what shoes must be stocked for sale, there is a clear preference for blue shoes. In this instance, a graphical representation is used requiring additional skills developed for the display and interpretation of these counts (data). Picture and column graphs represent number as a comparison between lengths.

Cycle 6: Comparing Likelihood

This cycle explores qualitative descriptions of likelihood. Students are engaged with identifying impossible, possible and certain events as well as the continuum of everyday language used to describe events with likelihood or chances of occurring that fall between these (many of the everyday words for probability describe various degrees of possibility). When linked to data sets, likelihood can provide valuable clues to assist with inferential decision making.

Notes on Cycle Sequence:

The proposed cycle sequence may be completed sequentially as it stands. However, it may be beneficial to alter the placement of *Cycle 6: Comparing Likelihood* within the sequence to directly after *Cycle 1: Comparison and Order* to continue the work of simply comparing before beginning to count to establish more than or less than. It may also be possible to complete *Cycle 4: Gathering and Representing Data* earlier in the sequence as a context requiring counting and place-value skills.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Comparison and Order

More than, less than, same as, greater than, less than, equal to, equivalent to, attribute language (wide, wider, widest, narrow, narrower, narrowest, long, longer, longest, short, shorter, shortest, …)

Cycle 2: Counting Units and Place Value

Cardinal numbers, ordinal numbers, nominal numbers, singles, groups, place, value, place-value, place-value charts, units, ones, tens, hundreds, digits, symbols, numerals, place value

Cycle 3: Multiplicative Structure and Regrouping

Ones, tens, hundreds, place value, renaming, regrouping, trading, more than, less than, multiply by, divide by, multiples of, groups of

Cycle 4: Place Value with Larger Numbers

Cardinal numbers, singles, groups, place, value, place-value, place-value charts, units, ones, tens, hundreds, digits, symbols, numerals, place value, renaming, regrouping, trading, more than, less than, multiply by, divide by, multiples of, groups of, thousands, millions, billions, pattern-of-three.

Cycle 5: Gathering and Representing Data

Categorical data, tallies, table, pictograph, one-to-one, axis, axes, x-axis, y-axis, horizontal, vertical, count, question, survey, scale, frequency, frequency graph, frequency table, title, axis title, label

Cycle 6: Comparing Likelihood

Impossible, possible, certain, likelihood, chance, everyday probability language (collect student examples first for local colloquialisms and to identify range of vocabulary), trial, event, outcome, experiment

Name:_____ Date:_____

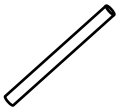
Can you do this? #1

1. Draw a line from the wider flag to the shorter pole.




Obj. 1.1.1 ☐

2. Number the straws in order from longest to shortest.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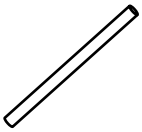
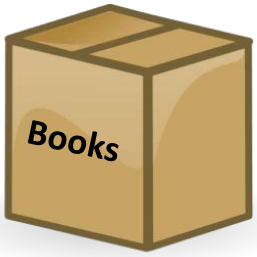

_____

_____

Obj. 1.1.1
i. ☐
ii. ☐

3. Circle the box that will be heavier.



Obj. 1.1.2 ☐

Cycle 1: Comparison and Order

Overview

Big Idea

This cycle develops understanding of informal comparison and ordering of attributes to describe more than, same as and less than. These ideas developed physically with measures can then be applied informally to the attribute of quantity before introducing the idea of counting and place-value.

Objectives

By the end of this cycle, students should be able to:

1.1.1 Use direct and indirect comparisons to decide which is longer and explain reasoning in everyday language. [FMG006]

1.1.2 Use direct and indirect comparisons to decide which is heavier and explain reasoning in everyday language. [FMG006]

1.1.3 Use direct and indirect comparisons to decide which holds more and explain reasoning in everyday language. [FMG006]

Conceptual Links

Comparison and order are qualitative ideas that underpin much of measurement and number. Comparison and order of attributes other than quantity will provide background understanding for notion of unit, counting and place value in Cycle 2.

Materials

For Cycle 1 you may need:

- Butchers' paper
- A4 paper
- Variety of items with different lengths, capacities, areas, masses
- Marking pens
- Noticeboard
- Scissors

Key Language

More than, less than, same as, greater than, less than, equal to, equivalent to, attribute language (wide, wider, widest, narrow, narrower, narrowest, long, longer, longest, short, shorter, shortest, …)

Definitions

Direct comparison: comparison of an attribute common to two or more items. For example, direct comparison of length involves holding two or more items together with a common baseline to see which is longer. For example, students may stand back to back to directly compare height to determine who is taller, a handful of pencils may be held with their base on a level surface and the longest or shortest pencil determined

Indirect comparison: comparison of an attribute common to two or more items that are compared using an intermediary. For example, a cupboard and a door may be compared by cutting a length of string to match the cupboard height and comparing the length of the string with the door. If the door is higher than the length of the string, then the door is higher than the cupboard (and the cupboard can fit through). This is a useful forerunner to indirect comparison using non-standard units.

Non-standard units: Any of the same item that can be repeated (or iterated) and counted in order to make an informal measurement of another object. For example, scissors, pencils, pens, paddle-pop sticks, erasers, books, marbles ...

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How can you tell if an item is longer/heavier/holds more than another?

  To ascertain if students understand measure and the significance of units, they must recognise that where units are used, the count of the unit will be greater if an item is longer, heavier, holds more. They must also understand that the same unit must be repeated with no gaps nor overlaps for the measure to be accurate. When comparing two items using a non-standard unit, students must recognise that they must use the same unit for measuring in order to be able to compare. Make sure that students always attach a unit name to the count when measuring (e.g., the table is 12 pencils long and 6 pencils wide).

Portfolio Task

This cycle provides students with comparing and ordering length, mass and capacity experiences which may assist with the task of informally and formally measuring the distance the planes fly. Comparison of size and ordering ideas can be extended to the alternative context of comparing angles within the portfolio task.

RAMR Cycle

Reality

Explore real-life comparison, sorting and ordering. Ensure students can identify an attribute for comparison, for example, length, mass, area or capacity. Other attributes may be discussed that are not easily ordered mathematically (e.g., colour, gender, hair colour). Discuss those attributes that may compared, sorted and ordered. (Note: In this first cycle it is not expected that attributes will be formally measured, the focus is on comparison and ordering into more than, same as or less than.)

Abstraction

The abstraction sequence for this cycle explores the concept of comparison for sorting and ordering starting with physical objects and attributes and extends this idea to comparing and ordering quantity. An abstraction sequence to build the concept of comparing and ordering is as follows:

1. *Kinaesthetic activity / Connect to language.* Start by identifying the attribute to be directly compared. Ensure students have the necessary vocabulary to describe the attribute. Compare and order the attribute. Height is an easy starter. Stand two students back to back to determine tall and taller or short and shorter. Introduce a third student to build to language of tall, taller, tallest.
2. *Model with materials.* Extend students' ability to compare lengths from direct comparison to using an intermediary to compare items. For example, the whiteboard is longer than a metre ruler, a desk is shorter than a metre ruler, the desk is then shorter than the whiteboard (move to less obvious examples).

Mathematics

Language/symbols and practice

Explore other attributes such as capacity or mass. A selection of identical boxes filled with different items can be effective for comparing and sorting mass by hefting. For example, fill sultana boxes with a selection of items (beads, fish tank gravel, hobbyfill, rice, toothpicks, empty box) and have students compare boxes with one in each hand to sort heaviest to lightest. Simple balance scales can be constructed from a coat hanger, plastic boxes and string to check for heavier or lighter. The objective of these activities is to develop comparison vocabulary and that attributes can be compared, sorted and ordered.

Connections

Extend students to comparing and ordering quantities of items. Initially, ensure there are too many small items in each pile for students to subitise (typically more than ten). Have students visually sort and order the piles into more than to less than series. Then use counting to check their response. The key idea is that quantity is another attribute of an item that can be compared, sorted and ordered.

Create growing patterns where piles of items increase in an observable pattern from one pile to the next. Provide a range of these as cards for students to sort into order. For example,

Growing patterns are a simple form of linear relationship, however, it is not expected that formal, symbolic representations of the linear relationship (e.g., y=mx+c) will be introduced in this cycle.

Resource Resource 1.1.1 Comparing and ordering pattern cards

Reflection

Check the idea

Provide students with a range of items to be sorted. Encourage them to identify the attribute they are sorting by. This can also be completed as a comparing and ordering guessing game where one student compares and orders a range of items and other students need to identify from the sorted collection what was the student's attribute of comparison.

Apply the idea

Use a range of countable non-standard units (e.g., handspans, paces, pencils …) to determine the length of items in the classroom. Compare and order lengths of items in the classroom according to the count of the non-standard unit. As students complete this, ensure they understand that items must be end-to-end with no gaps or overlaps. For example, length of desk in straws, length of chair in straws, lengths measured using paddle pop sticks.

Comparison and order of mass can be explored with non-standard units using a simple balance and objects such as marbles, unifix cubes or pens to compare and order masses.

Comparison and order of capacity can be explored with non-standard units such as plastic spoons or small cups and sand or rice (or water) to determine how many are needed to fill various containers. Ensure that students estimate which container will hold more or less before they start to count the number of units required to fill the container. Consider a range of containers that may be deceptive for students (e.g., some short but wide containers may hold more than some tall and narrow containers).

Extend the idea

Explore quantities of discrete items as informal collections. Order these intuitively.

Check by arranging each collection as a line and comparing the lengths of the lines. Note that greater quantities make a longer line.

Check by arranging each collection as an array (i.e., in rows and columns). Note that greater quantities cover a greater area.

These foundational ideas will translate to ordering on a number line and the comparison areas in later units.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_____ Date:_____

Can you do this? #2

1. Find the number words in the sentences below and write them as numbers:

   (a) A rugby union team has fifteen members.

   _____

   (b) Mr Jones is celebrating his thirty-seventh birthday.

   _____

Obj. 1.2.1
a) ☐
b) ☐

2. Find the numbers in the sentence, and write them in words:

   (a) There are 365 days in a year.

   _____

   (b) A chess board has 64 squares.

   _____

Obj. 1.2.2
a) ☐
b) ☐

3. Write a number that has:

   (a) 3 ones and 9 tens _____

   (b) 6 hundreds, 5 ones and 8 tens _____

Obj. 1.2.3
a) ☐
b) ☐

4. In 768 the 6 has a value of 6 hundreds/tens/ones (circle the correct answer)

Obj. 1.2.3
☐

5. Expand this number to show its place values: 472

   _____

Obj. 1.2.4
i. ☐
ii. ☐
iii. ☐

6. Write the missing number

   (a) 327 3 hundreds, _____ tens and 7 ones.

   (b) 260 2 hundreds, 6 tens and _____ ones.

Obj. 1.2.5
a) ☐
b) ☐

Cycle 2: Counting Units and Place Value

Overview

Big Idea

This cycle explores the underlying structure of our number system which includes the notion of a countable 'unit', use of same 9 digits to represent count of units with place value position of singles, groups, groups-of-groups, and the additive structure of the number. This underlying structure requires development of the following skills:

1. Flexibility in changing the unit that is counted, i.e., singles, groups, or groups-of-groups can all be counted as units. For example:

   If ones are the unit, then the tens are groups, and the hundreds are groups-of-groups etc.

   $$782 = (7\times100) + (8\times10) + (2\times1)$$
   7 groups-of-groups + 8 groups + 2 singles

   If instead the tens were the unit, then the hundreds are groups of ten etc.

2. Student place-value understanding should be developed by changing their perception of unit (i.e., gaining a multi-faceted perception of unit). Reading and writing numbers by place-value involves firstly positioning digits with ones on the right-hand-side, tens to the left of the ones and hundreds to the left of the tens. Empty places are filled with zeros.

3. Once established, quantities represented in each place are added to create the number. For example, 3 hundreds, 4 tens, 6 ones = 300 + 40 + 6 = 346.

Throughout this cycle underlying place value ideas are used to read numbers (symbol to language) and to write numbers (language to symbols) to represent quantities. This requires: (a) knowledge of the role of zeros; (b) understanding that digits in numbers follow a strict pattern, even if language does not; and (c) understanding that any number is the sum of its components. For example, when combined with the flexible choice of unit, students will realise that in the number 324, the 2 is two tens (when the ten is the unit) and also twenty ones (when the one is the unit), and that 3 hundreds, 2 tens and 4 ones are added to create the number.

Objectives

By the end of this cycle, students should be able to:

1.2.1 Read number names and write numbers to at least 1000. [2NA027]

1.2.2 Read numbers and write number names to at least 1000. [2NA027]

1.2.3 Write numbers using place value to at least 1000. [2NA027]

1.2.4 Count collections to 1000 by partitioning numbers using place-value. [2NA028]

1.2.5 Apply place value to partition, rearrange and regroup numbers to at least 1000 using additive structure. [4NA073]

Conceptual Links

In Cycle 3 of this unit, the concepts of counting, seriation and the odometer principle are applied to 3-digit numbers. Place value names for ones, tens and hundreds are combined with the Pattern of Threes to name larger (including 6 and 9 digit) numbers in Cycle 4.

Materials

For Cycle 2 you may need:

- 2 and 3 column place-value charts per student
- bundles of sticks per student
- multi-base arithmetic blocks (MAB) per student
- 3 sets A5 digit cards (0-9)
- thinkboard template per student
- money note sets ($10 and $1) per student
- flip cards
- 'Island Action' cut outs with box

Key Language

Cardinal numbers, ordinal numbers, nominal numbers, singles, groups, place, value, place-value, place-value charts, units, ones, tens, hundreds, digits, symbols, numerals, place value

Definitions

Additive structure of place-value: Each digit is given a place which indicates its value (ones, tens, hundreds …). Adding the value of each place together gives the whole number (e.g., 4 hundreds, 3 tens, 9 ones = 400 + 30 + 9 = 439).

Odometer principle: The decimal system has 9 digits that are repeated in sequence in each place. Once 9 is reached, a group of 10 is recorded as 1 in the tens place and 0 in the ones place. Counting then continues in the ones place up to 19 and so on.

Seriation: the skill of increasing/decreasing by one in each place within the decimal system. Later number fact strategies of count on/back and extended tens facts rely on this skill. Related to the odometer principle.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you read this number out loud?
- Write the number "three hundred forty-five" on the board/your book.
- Tell me the place value of each digit in this number.
- What number is in the "hundreds" place? (vary examples)

Portfolio Task

Counting and number representation skills from this cycle will be used to count steps to measure the distance planes fly within the portfolio task.

RAMR Cycle

Reality

Think of everywhere numbers are used. Think of as many situations as possible. What do these numbers mean? Are the meanings all the same? Have students create a class concept map/poster for the three types of numbers: Nominal (unique labels, names for things, e.g., phone numbers, house numbers); Ordinal (what order, place in a sequence, e.g., first, second); Cardinal (quantity, how much or how many). Students may like to draw or bring pictures or visuals next lesson to add to poster. For example:

> *Money, cards, addresses, back of sporting jerseys, board games, measuring tapes, barcodes, credit cards, temperature, TV channels, etc.*

Focus students' attention on Cardinal or Counting numbers. Discuss discrete objects that may be counted as singles, groups or groups-of-groups. The following resource may provide a useful activity.

Resource Resource 1.2.1 Finding singles, groups and groups-of-groups activity.

Choose a context relevant to students to follow through the Abstraction phase. For example, use a camera to photograph a line of students to copy and paste in PowerPoint to provide an easily transferable image of a group which can also be useful when creating groups-of-groups (either in small groups or as a line of ten in preparation for later activities with tens and hundreds).

Abstraction

In this abstraction sequence the focus is on moving from concrete representations of quantity to abstract symbolic representations of quantity, using tools such as place-value charts (PVCs) to scaffold students' understanding of place-value. Start with 2-digit numbers (singles and groups) for this sequence then extend to 3-digit numbers (groups-of-groups). The abstraction sequence is as follows:

1. *Kinaesthetic activity.* Real-world incidences of singles and groups (i.e., draw upon activities/contexts completed in Reality phase).
2. *Model with materials.* Represent singles and groups with counters or multilink cubes on informal place value charts (as in *Form student groups activity or Resource 1.2.2 Island action activity*).

Activity – Form student groups
a) Teacher instructs students into equal-sized groups of a number less than 10.
b) Students describe how many groups and how many left over.
c) Teacher repeats with another group size.

Resource Resource 1.2.2 Island action activity.

3. *Represent with symbols.* Read the associated 'number sets from left to right as, for example, 3 groups and 4 singles.
4. Focus more closely on contexts where groups and groups-of-groups are naturally in tens. *Form student groups* activity may be used here if pictures of ten students are available.

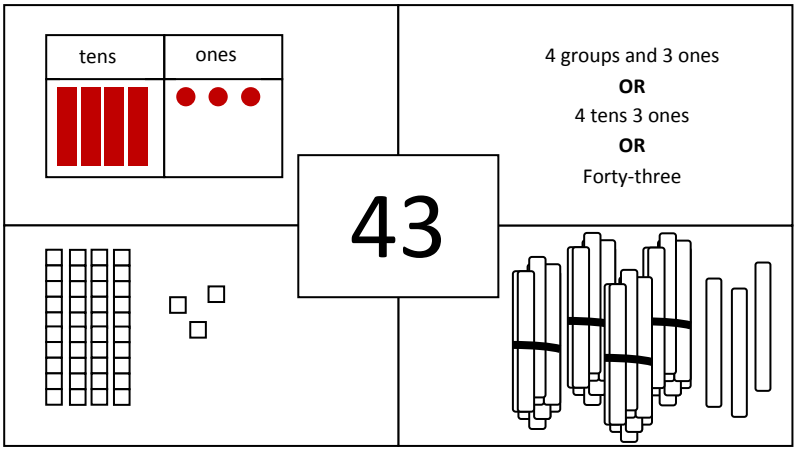
5. *Connect to language.* Connect language and representations of singles and groups with mathematical place-value language of ones and tens. Ensure that students recognise that as a unit is added to the ones place, the value of the number increases by one (seriation in the ones place). Extend to increasing by one in the tens place (count in tens or seriation in the tens place).

Resource Resource 1.2.3 Clap grouping activity.

6. *Connect to symbols.* Represent ones and tens on place-value charts and read the numbers from left to right as, for example, 3 tens, and 4 ones. Write these also as 3 tens + 4 ones.

7. *Connect to symbols.* Reading and writing numbers without place value charts as scaffolds (e.g., thirty-four). As students develop proficiency with verbalising concretely represented quantities scaffolded using PVCs, gradually remove the scaffold and become more reliant upon reading and writing numbers.

The numbers can be written on the PVC, on calculators and also on paper, (i.e., no PVC). As students become adept at informal "groups and ones" language, transition to normal numbers, e.g., from 2 tens and 5 ones to (eventually) twenty-five. Initially begin this transition using normal number specific language, e.g., "four tens and two ones", then "forty-two". For zeros in the ones position, start with "four tens and zero ones", then "forty-zero", and finally "forty". For the teens, first use "one ten and four ones", then "onety-four" and finally "fourteen".



A **thinkboard** can be used to demonstrate a range of representations of the same quantity.

Resource Resource 1.2.4 Number representation thinkboard or concept map

8. *Abstraction sequence introducing hundreds.* Brainstorm situations where 3-digit numbers are used. For example: Address, emergency phone numbers (000, 911), cricket score.

9. *Repeat the previous 2-digit activities using 3-digit numbers.* Gradually increase the difficulty of number types. Order of difficulty is as follows: a) simple numbers (e.g., 456); b) numbers with zero ones (e.g., 450); c) numbers with zero tens (e.g., 406); d) numbers with zero ones and zero tens (e.g., 400); e) teen-type numbers (e.g., 416).

10. *Explore numbers with similar language and symbols.* Consolidate student understanding by practising using numbers with language and symbols that are easily confused, e.g., "four hundred and seventy" and "four hundred and seventeen". *Resource 1.2.5 Reading numbers with zeros, tens and teens activity* provides detailed instructions for this activity.

Resource Resource 1.2.5 Reading numbers with zeros, tens and teens activity

11. *Explore the role of zero:* Explicitly examine how zeros do/do not influence the quantity when inserted within a series of digits in a number. Give some digits on a PVC and ask where extra zeros change things (e.g., 42→042, 42→420, 42→402, 267→0267, 267→2607, 267→2670). Ensure that students understand that leading zeros do not alter the place value of any other digits, zeros inserted within a series will change place values of some digits, zeros inserted in the ones place will change the place value of all digits within the number. *Resource 1.2.3 Clap grouping activity* may be easily adapted to assist with this exploration.

Resource Resource 1.2.3 Clap grouping activity.

Consider using language that focuses on pattern, puts in zeros, and does not use teens (i.e., non-standard number names of 11-19): 207 is 2 hundred and zeroty-seven; 270 is 2 hundred and seventy-zero; 217 is 2 hundred and onety-seven.

Mathematics

Language/symbols and practice

Students need to know, understand and be flexible with the written and verbal representations of whole numbers and how these relate to the concepts of unit and place-value.

Resource 1.2.6 Connecting representations is a worksheet activity for students to practise connecting representations of quantity. The worksheet provides empty rows at the end of the worksheet to allow students to include personally relevant data such as number of year 8s in school, size of collections of trading cards and so on.

Resource Resource 1.2.6 Connecting representations activity

In any activity when a teacher is calling out numbers, be sure to give the place-values in the wrong order at times. It is important that students see that a number is determined by 100s, 10s and 1s and that the order the place-value positions are given does not matter. For example:

"What number is 6 tens, 8 hundreds and 7 ones?"

It is important that this process is also reversed:

Get students to give all the different ways that the number 587 could be given – 5 hundreds, 8 tens, 7 ones; 8 tens, 7 ones, 5 hundreds; 7 ones, 8 tens, 5 hundreds; and so on.

Read-write calculator and *Wipe-out* are useful activities for this practice.

Activity: Read-write calculator

1. Teacher calls out three digits.
2. Students enter into their calculators and say the number aloud.

It is important to reverse this by calling out the number and after students enter it into calculator they say the digits used.

Resource Resource 1.2.7 Wipe-out game activity

Comparing and ordering numbers

Provide students with activities to compare and order symbolic representations of quantity. Reinforce with students that they need to compare the largest place value first, then look at successively smaller places. Start with 2- and 3-digit numbers.

Reflection

Check the idea

Apply the notion of unit back to the real world. Where in the world do we have interchangeable groups and ones of objects depending on how we think of them?

Apply the idea

The *Crack the code activity* could help students to demonstrate their understanding of the structure of a number system and its naming of numbers.

Resource Resource 1.2.8 Crack the code activity

Extend the idea

Generalise

Place-value names in the decimal number system (ones, tens and hundreds) can all be thought of as a unit of count. Ask students for other systems that count unit names, for example:

Wholes and fifths; weeks and days; hours, minutes, seconds; km, m, mm, and so on.

Can the idea of unit be used in algebra? Can variable *x* be considered as a countable unit even though it can be any number? For example, is it possible to count 1 group of x, 2 groups of x, 3 groups of x? Explore this idea using different group sizes of x with unifix cubes or straws. Consider using a cup which could have any number of counters in it and then can be repeated with that same amount as representing 1x, 2x, 3x and so on.

This is why the notion of unit is a big idea – it occurs right through mathematics from early years to senior secondary school.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_____ Date:_____

Can you do this? #3

1. The number 285 has:

   27 tens and _____ ones

Obj. 1.3.1 ☐

2. The number 863 has:

   7 hundreds, _____ tens and _____ones

Obj. 1.3.1 a) ☐ b) ☐

3. Write as a number:

   34 tens and 5 ones _____

Obj. 1.3.1 ☐

4. Write as a number:

   3 hundreds and 16 ones _____

Obj. 1.3.1 ☐

5. Fill in the following table:

Obj. 1.3.1 a)i. ☐ a)ii. ☐ b)i. ☐ b)ii. ☐ c)i. ☐ c)ii. ☐ d)i. ☐ d)ii. ☐

	Number	Action	Hundreds	Tens	Ones	Answer
	23	*×10*	*2*	*3*	*0*	*230*
a)	450	÷10				
b)	15		1	5		
c)	260	÷10				
d)	3					300

Cycle 3: Multiplicative Structure and Regrouping

Overview

Big Idea

The big idea in this cycle is that adjacent place-value positions relate to each other through multiplication and division as a direct result of the grouping nature of the number system. That is, moving one position to the left corresponds to multiplication by 10 and one place to the right corresponds to division by 10. Extending this, two places to the left is ×100 and two places to the right is ÷100, and so on. This cycle relies on the place-value and additive structure concepts of number developed in the previous cycles. It should be noted that many structured place-value materials are additive (singles, groups, and groups-of-groups are fixed units that are repeated within a place and the total values are added to form the number). In order to use these to teach the multiplicative relationship, it is necessary to focus students on the relationship between the single and the group as it can be seen on the material and then the relationship between the group and the group-of-group. Once this physical relationship has been recognised, materials such as digit cards on place-value charts can be less cumbersome and facilitate the movement to larger quantities than can be easily represented using materials.

The Multiplicative place-value relationship enables understanding (and calculating) that numbers can have more than one meaning in terms of place-value positions. For example, 362 can be thought of (i.e., renamed) as “3 hundreds, 6 tens and 2 ones” or “36 tens and 2 ones”, and so on. There are two types of such renaming: (a) simple renaming of PV position (e.g., 362 can be “36 tens, 2 ones”); and (b) more complex renaming which renames only part of each place-value (e.g., 362 can also be “2 hundreds, 14 tens, 22 ones”. In this cycle we will avoid considering problems that lead to fractional units (i.e., 270 is 27 tens, but we will not think of it as 2.7 hundreds).

Objectives

By the end of this cycle, students should be able to:

1.3.1 Apply place value to partition, rearrange and regroup numbers to at least 1000 using multiplicative structure. [4NA073]

Conceptual Links

In Cycle 2 the concept of addition was explored using discrete objects (set models). In the future, addition calculation, whether mental or recorded in an algorithm, requires regrouping of values within place values. Mental calculation is often more efficient if students have the ability to flexibly rename and regroup place values. These skills will be further built upon and applied in future units.

Materials

For Cycle 3 you may need:

- PVC per student
- $100, $10, $1 notes/coins
- MAB per student
- PV cards set (math mat and holding)
- math mat
- A7 digit cards per student
- A5 digit cards
- HTO number expander per student
- calculator per student

Key Language

Ones, tens, hundreds, place value, renaming, regrouping, trading, more than, less than, multiply by, divide by, multiples of, groups of

Definitions

Regrouping: Additive structure of place value denotes each digit a place value that may then be added to generate the whole number. For example, all or part of a place may be interpreted as the next place down by multiplying the digit by ten: 4 tens, 6 ones = 40 ones + 6 ones = 3 tens, 16 ones.

Renaming: As a consequence of regrouping, digits in consecutive place values may be renamed. Multiplicative structure enables 146 to be considered as 1 hundred, 4 tens, 6 ones or regrouped as 14 tens , 6 ones or 1 hundred, 46 ones or 146 ones.

Trading: Trading is a term used to describe the act of changing or regrouping a place to another place. Its use is largely related to working with structure place value materials such as MAB blocks. In order to mirror the work with language and symbols using MAB blocks, a long MAB block can be "traded" for ten single units. This language used with materials is often translated to work with symbols and language and reappears in additive operation strategies.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What happens to the place value of a number when it is multiplied/divided by 10?
- What happens to the place value of a number when it is multiplied/divided by 10 twice in a row?
- What happens to the place value of a number when it is multiplied/divided by 100?
- What happens to the place value of a number when it is multiplied/divided by 10 three times in a row? What happens to the place value of a number when it is multiplied/divided by 1000?
- Can you see a pattern here? Can you predict what might happen if you multiply/divide by 10 four times in a row? What number might you multiply by to make this happen in one go?

Portfolio Task

This cycle is not directly linked to activities within the portfolio task but contributes to overall understanding of representing quantity and is required for conversion between metric measures.

RAMR Cycle

Reality

The objective of this cycle is to be able to express numbers in different ways based upon the multiplicative structure of the decimal number system (e.g., 362 could be 3 hundreds, 6 tens, 2 ones or 36 tens, 2 ones).

Connect this cycle of activity back to previous cycles' work with singles, groups and groups-of-groups. Discuss with students how a multipack of smarties boxes could be also described as a number of smarties boxes which could also be described as a number of smarties. Discuss how each number is a multiple of the previous place. For example, if there are 30 smarties in a box and there are 10 boxes in a multipack, then there will be 30 × 10 = 300 single smarties in a multipack.

Consider some of the informal (not base 10) place value contexts discussed in Cycle 2. The focus is on the fact that trading groups for singles results in a quantity of singles that is a multiple of the quantity of singles in a group. To experience this physically, form students into a group of 9, a group of 3 and two singles in designated places at the front of the class. Explore moving a group of 9 to the groups of 3 place (the group reforms into three groups of 3). Explore moving a group of 3 to the groups of 1 place (the group disbands into 3 singles). Trade singles into groups and groups-of-groups to simplify. Discuss how the groups-of-groups can be 'traded' for groups, or groups 'traded' for singles. Also discuss how singles can be collated into complete groups and so on.

Resource 1.3.1 How many ways is a similar activity involving setting up the float for a shop with $100 notes, $10 notes and $1 coins. This will lead into more formal (base 10) place value contexts.

Resource Resource 1.3.1 How many ways?

Abstraction

First, simple forms of renaming must be considered and from there the combination of part-whole additive structure and multiplicative structure is introduced to solve harder renaming problems. The sequence of abstraction is as follows:

1. *Kinaesthetic activity.* Reinforce place value understanding developed in previous cycles by acting out place value ideas with a human place value chart (PVC) on the maths mat.

Resource Resource 1.3.2 Human place value chart.

2. *Model with materials.* Use a place value chart with digit cards and calculator to explore movements of digits across the place value chart when the calculator is used to multiply by 10.
3. Use the place value chart with digit cards and calculator to explore movements of digits across the place value chart when the calculator is used to divide by 10.
4. *Connect to language.* Use MAB materials and an A4 sized place value chart to explore trading hundreds for tens, tens for ones, ones for tens and tens for hundreds (see *Resource 1.3.3 Regrouping and number expanders*).

Resource Resource 1.3.3 Regrouping and number expanders worksheet.

5. *Connect to symbols.* Use number expanders to scaffold the reading and renaming process when reading and saying numbers (*see Resource 1.3.3 Regrouping and number expanders*).

Mathematics

Language/symbols and practice

Provide students with practice regrouping and renaming numbers using place value when multiplying and dividing by tens. These should operate in both directions, giving the correct multiplication or division factor and asking for the movement, and giving the movement and asking for the multiplication and division factor. A sample worksheet is provided in *Resource 1.3.4 Regrouping and renaming activity*.

Resource Resource 1.3.4 Regrouping and renaming activity

Use number expanders and worksheets, and worksheets alone, to practise trading all of one place-value position for another (e.g., 4T 5O = 45O). Make sure examples go both ways: from number to expanded form, and from expanded form to number. For example:

> *3H 2T 7O = 2H ___T 170: Work out how many tens.*
> *2 hundreds, 5 tens and 12 ones= 1 hundred,_____tens and 2: Work out how many tens.*
> *8H 6T 3O = 7H ___O: Work out how many ones.*

These skills provide an important base for later computation strategies and techniques.

Reflection

Check the idea

Students refer back to reality and the example of the till to communicate their understandings of patterns. They could do this by acting out getting change using $1, $10 and $100, and exchanging denominations (e.g., $100 → 8 × $10 + 20 × $1).

Apply the idea

Investigate where else multiplicative renaming is used in students' world. Make up a generality for any grouping (e.g., week → days is ×7). Students should see that other number areas are related in a similar way to whole numbers but possibly with different factors. For example weeks and days relate by factors of seven, hours and minutes relate by factors of sixty, metrics relate via factors of 10 but, of course, the mm ↔ m, the g ↔ Kg and mL ↔ L are related by factors of 1000.

Extend the idea

Generalising

How does multiplicative structure extend to 5-digit numbers? The understanding that move left is ×10 and move right is ÷10 has to be generalised to the understanding that any adjacent place-value positions are related by factors of ten, and that this relationship applies to all adjacent place-values and is bi-directional.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_____ Date:_____

Can you do this? #4

1. Find the number name in the sentence below and write it using numbers:

   The Queensland Sport and Athletics Centre can seat up to forty-nine thousand people. _____

Obj. 1.4.1 ☐

2. Find the numbers in the sentence, and write them in words:

   The distance to the moon is between 36 3104km and 406 696km.

   a) _____

   b) _____

Obj. 1.4.2
a) ☐
b) ☐

3. Write a number that has:

   6 hundred thousands, 8 hundreds and 90 millions

   _____

Obj. 1.4.3 ☐

4. In 768 543, what is the place-value of the 6?

   6 hundred thousands/ten thousands/one thousands (circle the correct answer)

Obj. 1.4.3 ☐

5. Expand this number to show its place values: 1 092 563

   _____

   _____

Obj. 1.4.4
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

6. Write the missing numbers

(a) 592 327 592 thousands, 3 hundreds, _____ tens and 7 ones.

(b) 295 603 2 hundred thousands, ____ thousands, 6 hundreds and 3 ones.

Obj. 1.4.5
a) ☐
b) ☐

7. Write down numbers can you make using: 1, 0, 2, 7, 8, 4

(a) Write the largest number you can find using all these numerals.

   _____

(b) Write the smallest number you can find using all these numerals.

   _____

Obj. 1.4.3
a) ☐
b) ☐

Cycle 4: Place Value with Larger Numbers

Overview

Big Idea

Within this cycle the notion of large numbers is developed as the integration of ideas developed in the previous cycle, that is, place-value/unit, additive structure, multiplicative structure, order and equivalence. This cycle extends place-value to incorporate the pattern-of-threes inherent within place-value naming of larger numbers. Each newly introduced place should be addressed as both a representation of quantity (to develop an understanding of magnitude) as well as a visual image of the place within a sequence of digits.

Objectives

By the end of this cycle, students should be able to:

1.4.1 Read number names and write numbers to at least 10 000. [4NA072]

1.4.2 Read numbers and write number names to at least 10 000. [4NA072]

1.4.3 Write numbers using place value to at least 10 000. [4NA072]

1.4.4 Apply place-value to partition, rearrange and regroup numbers to at least 10 000 using additive structure. [4NA073]

1.4.5 Apply place value to partition, rearrange and regroup numbers to at least 10 000 using multiplicative structure. [4NA073]

Conceptual Links

This cycle provides a basis for reading, writing and saying numbers and understanding place-value that will provide regrouping, trading and renaming skills necessary for the development and application of mental and written computation in Unit 2.

Materials

For Cycle 4 you may need:

- hand held PV cards
- A5 digit cards
- A7 digit cards per student
- party hat per student
- masking tape
- math mat
- pattern of threes PV chart 2 per student
- calculator
- numbers ranging from 1 to 1 billion, two per student
- dominoes set per pair/small group
- scrap paper per student

Key Language

Cardinal numbers, singles, groups, place, value, place-value, place-value charts, units, ones, tens, hundreds, digits, symbols, numerals, place value, renaming, regrouping, trading, more than, less than, multiply by, divide by, multiples of, groups of, thousands, millions, billions, pattern-of-three.

Definitions

Pattern-of-threes: Pattern of naming places in numbers that breaks large numbers into sets of three digits. Repetition of hundreds, tens and ones used to describe ones, thousands, millions, billions and so on. For example, a large number broken for ease of interpretation, 1 235 987 629 has place value sets of billions, millions, thousands, ones and is read as one billion, two hundred and thirty-five million, nine hundred and eighty-seven thousand, six hundred and twenty-nine (or 1 billion, 235 million, 987 thousand, 629).

Assessment

Anecdotal Evidence

Similar prompting questions to Cycle 2 and Cycle 3 may be used with attention to larger place values:

- What do you notice about the number names in the thousands and the millions?
- How is this similar to place value names in the ones, tens and hundreds?
- How might numbers beyond hundreds of millions be named (billions and beyond)?
- What happens to the place value of a number when it is multiplied/divided by 10 four times in a row? What happens to the place value of a number when it is multiplied/divided by 10 000?
- Does this pattern continue through bigger numbers?

Portfolio Task

This cycle is not directly linked to activities within the portfolio task but is a necessary understanding for representing quantity and is required for conversion between metric measures.

RAMR Cycle

Reality

Have students contribute or write down what is a very big number for them. Discuss situations in which numbers larger than 3 digits are used (e.g., How much does a person earn working in the mines? How much is their favourite singer worth? What does a new car cost?)

Visit the Worldometers website (http://www.worldometers.info/) and look at the many large numbers reported upon. For Queensland regional population figures and numbers of schools visit the Australian Bureau of Statistics (http://www.abs.gov.au/AUSSTATS/). Discuss what '000 might mean in the category labels for Queensland regional population figures. Compare with figures which do not have this multiplier.

Other interesting statistics can be found on Wikipedia by searching List of best-selling ... or List of million-selling … For example,

- Music artists http://en.wikipedia.org/wiki/List_of_best-selling_music_artists
- Video games http://en.wikipedia.org/wiki/List_of_best-selling_video_games
- Books http://en.wikipedia.org/wiki/List_of_best-selling_books
- Game consoles http://en.wikipedia.org/wiki/List_of_million-selling_game_consoles

These lists have numbers followed by the word millions. Discuss with students why this might be. The game consoles figures include decimals. What might this mean?

Discuss words used to name large numbers (e.g., thousands, millions, billions).

Abstraction

In this abstraction sequence the focus is on extending students' place-value understanding and ability to recognise, read, write and name numbers larger than 999 while also developing awareness of the magnitude of quantities represented by each place. These skills should flow naturally from the smaller number place-value understanding developed in Cycle 3. A suggested sequence of abstraction incorporating previous knowledge and new ideas of the repetition of hundreds, tens, ones (pattern-of-threes) combined with a multiplier (ones, thousands, millions, billions) is as follows:

1. *Kinaesthetic activity.* Establish magnitude of quantity represented in a place and the convention for naming the place. Revise unit of count with singles, groups and groups-of-groups. Ask students what might come next (groups-of-groups-of-groups). Introduce the thousands place and explore 1000 as a quantity (photograph 10 students in a row, arrange 10 copies of the picture on a PowerPoint slide, duplicate the slide nine times and look at the ten slides in the slide viewer to see 1000 children).
2. *Focus students' attention on thousands.* If ten slides with ten pictures of ten students was 1000, what might be the number of students on ten sets of ten slides? Ask students what the next higher places than 1000 might be called. Represent these on a human place-value chart. Practice reading and saying numbers in the thousands.
3. *Connect to language.* Ask what they notice about the place-value names compared to the units in the smaller places. Draw students' attention to the repeating hundreds, tens, ones pattern in the number names (i.e., the pattern of threes).

4. *Apply this sequence to millions.* For each larger number name ensure that students establish an idea of the size of the unit. (See *Resource 1.4.1 Notion of unit for larger numbers*).

Resource Resource 1.4.1 Notion of unit for larger numbers

5. *Very large numbers.* Once students have a grasp of the naming convention, see how far they can go with naming larger numbers and labelling an empty place-value chart. Large colourful place value charts for noticeboards are available online http://www.adrianbruce.com/math/place-value.html.
6. *Model with materials/Connect to symbols.* Follow up this investigation with individual A4 place value charts and digit cards. Practise reading and saying numbers. See how high students can go naming numbers. Students may like to Google search for larger place names (past millions are billions, trillions, quadrillions, quintillions, and so on …)

Resource Resource 1.4.2 Student place-value chart and pattern of threes

Mathematics

Once students have established the ability to recognise, read, write and say larger numbers and understand the value of units in each place, other aspects of the decimal number system can be extended to from previous understanding developed with smaller place-values. Students should be able to recognise and apply additive structure, multiplicative structure and renaming from Cycle 3 to quantities in the thousands, millions and billions.

Language/symbols and practice

Connect to additive structure.

Expand numbers according to place-value. Play wipe-out game with calculators to reinforce the place-value of a digit within a larger number. Identify numbers wiped out as thousands, hundreds, tens, ones etc. Students should record the quantity they subtracted in words and with symbols.

Resource Resource 1.4.3 Wipe-out game for larger numbers

Explore multiplicative structure.

Use calculators and place-value charts with digit cards to model multiplying numbers by ten through places beyond a million. Explicitly discuss the role of zero as a place-holder for empty places (connect back to the role of zero explored in Cycle 2. Repeat activity to model dividing numbers by ten back to ones. Discuss the effect of placing zero within a number, before a number and after a number.

Investigate regrouping, trading and renaming in larger places. Rename millions as thousands, ten thousands as hundreds and so on. Number expanders can be used here. Reinforcing the pattern of threes is also useful here as a precursor to metric conversion in later units.

Resource Resource 1.4.4 Multiplicative structure and renaming for larger numbers

Seriation skills.

Ensure students can count forwards and backwards in a place (e.g., count on in millions, thousands, hundred thousands as units. Pay particular attention to bridging the next place up and back.

Resource Resource 1.4.5 Counting, seriation and odometer for larger numbers

Compare and order quantities.

Students should have sufficiently developed understandings of the quantity of each place to recognise from the name of a number when it is larger or smaller than another quantity. Ensure students can compare and order quantities represented by symbols. Students need to clearly recognise that it is the place-value of the number that matters not the length of the number. This basic idea becomes important when comparing fractional quantities in later units.

Resource Resource 1.4.6 Chance number/chance order games
Resource 1.4.7 Comparing and ordering domino game

Reflection

Check the idea

Adapt the *Chance number/Chance order* activity by incorporating additive and multiplicative relationships using game boards such as:

□□□□ × 1000 < □□□□□□□

In this case a new deck of cards would need to be created containing a greater variety of multiplicative relationships.

Resource Resource 1.4.8 Chance number/Chance order extension:

Apply the idea

Conduct investigations into statistics that result in very large numbers. Find large numbers of interest to students and read and say these numbers. (Some ideas can be found within 'In the next 3 minutes' statistics or revisit the Worldometers website for examples (http://www.worldometers.info/). As a class categorise the numbers into millions, thousands or ones. Create a list of students' numbers; order the numbers on this list from smallest to largest.

Extend the idea

Encourage students to take their new understanding of large numbers back into their lives and look at where such numbers affect them (e.g., gigabytes of digital storage). Also revisit websites explored in the Reality phase and discuss the changing numbers.

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_____ Date:_____

Can you do this? #5

1. A class of 15 students tallied their pets. This table shows the results. Using the axis below, create a column graph of class pets.

Tally	IIII	III	IIII	II	II
Pet	Dog	Cat	Bird	Fish	No

2. This pictograph shows the birth month of 15 mathematics students.

 = 1 birthday

January		July	
February		August	
March		September	
April		October	
May		November	
June		December	

From reading the graph:

(a) Which month has most birthdays? _____

(b) Which months have the least birthdays? _____

(c) _____

(d) How many students are born in May? _____

Obj. 1.5.5

i. ☐

ii. ☐

iii. ☐

iv. ☐

v. ☐

Obj. 1.5.6

a) ☐

b) ☐

c) ☐

d) ☐

Cycle 5: Gathering and Representing Data

Overview

Big Idea

This cycle explores gathering data using simple questions or surveys, organisation of data in tables, ways of recording count, simple means of displaying data gathered and initial interpretation of data sets. There are occasions when the count of an object has more relevance when connected to its relationship with other objects or variables and clues as to the initial context. For example, “3” and “5” mean there are 3 things and 5 things. This tells us little compared to “3 people with black shoes” and “5 people with blue shoes”. Further information about context is still needed to interpret this data, if the question is about students in uniform, then more students are not complying with the rules. However, if the question is what shoes must be stocked for sale, there is a clear preference for blue shoes. In this instance, a graphical representation is used requiring additional skills developed for the display and interpretation of these counts (data).

Picture and column graphs represent number as a comparison between lengths. For comparison, we are restricted to data that are numbers or count of items. Initial graphing types increase in complexity from simple yes/no or comparisons of only two categories to picture graphs to column and frequency graphs. The suggested sequence of development is as follows:

Stage 1: One-to-one correspondence – two columns – students, physical objects or pictures;
Stage 2: More columns – students, physical objects or pictures;
Stage 3: Stick on – using paper squares instead of students, physical objects and pictures;
Stage 4: Squared paper stage – shading squares on graph paper;
Stage 5: Abstract representation stage – drawings of rectangles on axes.

Objectives

By the end of this cycle, students should be able to:

1.5.1 Pose questions and collect data by observation. [5SP118]

1.5.2 Pose questions and collect data by survey. [5SP118]

1.5.3 Organise data into categories using lists or tables. [3SP069]

1.5.4 Create displays using picture graphs. [5SP119]

1.5.5 Create displays using column graphs. [5SP119]

1.5.6 Describe and interpret different data sets in context. [5SP120]

Conceptual Links

This cycle provides base understandings for gathering and collating categorical data using tables and tally marks. Simple representations of data in one-to-one pictographs, frequency graphs and column graphs will develop initial understanding of scaled axes (to be extended on in Unit 2). Data gathering and graphs will provide quantities and contexts for applying counting skills (previously developed in Cycle 2), and for developing addition facts and operating strategies (Unit 2).

Materials

For Cycle 5 you may need:

- Multi-link cubes
- 1cm squared graph paper
- Small sticky notes
- A3 paper

Key Language

Categorical data, tallies, table, pictograph, one-to-one, axis, axes, x-axis, y-axis, horizontal, vertical, count, question, survey, scale, frequency, frequency graph, frequency table, title, axis title, label

Definitions

Frequency graph: Similar to a single bar of a stacked column graph that shows the complete sample in identifiable blocks.

Assessment

Anecdotal Evidence

Gather evidence of students ability to pose questions for data collection, organise data into categories and generate lists or tables to assist in the creation of graphic displays.

Possible questions may include:

- Can you suggest a question that could be used to collect data? What is something we may want to observe or survey people about?
- How could you collect that data? What might be the best way to organise the collection of data?
- What might be the best graph to display these results for people?

Portfolio Task

The portfolio task provides students with opportunities to apply graphing skills developed within this cycle.

RAMR Cycle

Reality

Generate a set of data that can be used to provide a comparison between a seemingly random list of numbers and a graphical representation of data. For example, write numbers on the board to represent the rows in the class, write the number of students in each row next to the row number (without labels). Ask students what these numbers could represent. A simpler example might be to simply write two numbers on the whiteboard to represent number of boys and girls in the class on the day and ask students what the numbers could mean.

Abstraction

Students need to experience all stages of data gathering, creation of graphical representations and interpretation of graphs to develop initial statistical literacy skills.

The abstraction phase for this cycle involves students in data gathering and graph creation activities that progress from pictographs to column graphs in the following sequence:

1. *Kinaesthetic activity.* Gather data using simple questions that can be represented by standing in line on mats. Provide each student with a piece of paper to draw/write their answer on so that this can be left on the mat as students step away. Have labels for title and category labels to complete the large graph.
2. *Model with materials.* Create a picture graph using sticky notes.
3. *Represent symbolically.* Discuss the comparison between the categories and the importance of each item to be represented by the same size piece of paper or picture on the graph. Discuss the comparison between categories. Progress from simple comparative language of more than and less than to numerical count of comparison. Ask students for an indication of a simpler way of determining the count. Explicitly link the fact that the sticky notes are all the same size for comparison to the scale. As each sticky note represents one item, the units on the axis also go up in ones.

Resource Resource 1.5.1 Simple picture graphs

4. Recreate the sticky note picture graph as a column graph using grid paper.

Resource Resource 1.5.2 From picture graphs to column graphs

5. Recreate the sticky note graph as a column graph using a ruler to mark off intervals on the axes for categories and count of items in each category. Ensure students recognise the importance of including a title for their graph and axes so that their visual display is easily interpreted by their audience.

Mathematics

Language/symbols and practice

Frequency graphs and Simple circle graphs

Picture graphs and column graphs can be easily converted to frequency graphs. This can be most easily demonstrated by using multilink cubes to generate a model of a column graph with separate colours for each category. Join the cube stacks end to end to create a frequency graph. This can be copied onto squared grid paper.

Extend students to creating a circle graph. Frequency graphs can be created on long strips that can be joined into a circle with colours on the inside. Have students draw around the inside of the circle and mark ends of colour bands. Join these marks to the centre of the circle and label each section according to its category. Discuss with students the value of this representation of the data. Note that it is difficult to identify actual numbers for each category unless these are written on the sectors.

Practise gathering data to answer questions

Explore creating graphs from survey questions that will involve the classification of data and creation of categories. Ensure students can develop lists and tables using category labels and tally marks to organise collected data, and then follow graphing principles already developed to create an appropriate graphical representation. If students struggle with these, consider *Resource 1.5.3 Data collection, tables and tallying* as organisers to assist with data collection, table creation and tallying skills.

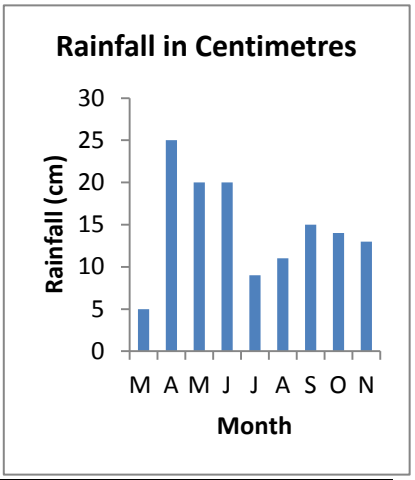
Resource Resource 1.5.3 Data collection, tables and tallying

As a class, gather ideas for data gathering that will help create a class profile. Use ideas from the students to construct some survey questions to practise gathering, collating and representing data.

Once there are several class graphs created (students could generate graphs on different topics in pairs), display the graphs on the wall. Discuss 'debugging' the graphs with students. For example, is it easy to see what each of the axes represent, are there clear titles on the graphs, is the scale appropriate? How could some of these aspects be improved?

Use the graphs as stimulus for interpretation activities. Ensure that students can answer questions from the data, between the data and beyond the data. Encourage students to pose their own questions to interrogate the data. Practise articulating and writing responses to questions that interpret the data displays. For example:

Bar Graph: Students constructed the following rainfall graph for one school year.



From the data:

- Which month had the most rain?
- What were the approximate amounts of rainfall in March?

Between the data:

- Which months had the same amount of rain?
- How much more rain was there in April than in March?
- What part of the school year would you call the rainy season?

Beyond the data:

- Are there months that might record higher rainfall?
- Why might these months not be recorded?

Connections

Explore graphs and statistics where the axis label includes a multiplier to reduce the magnitude of the numbers on the axis (e.g., Population ('000) or Population (millions)). Engage students with interpreting graphs of this type by converting numbers from the truncated form to full numbers (use digit cards on place-value charts to assist with correct placement until students are confident). Connect to reading and writing larger numbers and the pattern of threes explored in Cycle 4.

Reflection

Check the idea

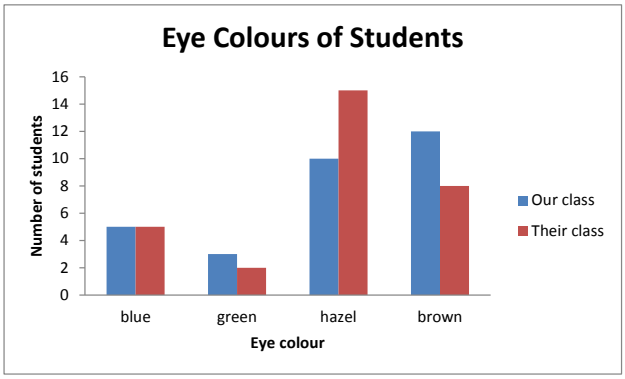
Give students a collection of badly drawn graphs with missing elements to critique and fix.

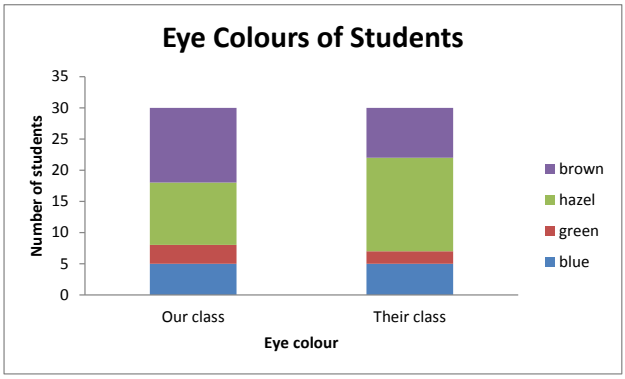
Resource Resource 1.5.4 Improving picture graph and column graph drawings

Have students create their own graphs from collected data and pose questions to swap with class members for answering.

Extend the idea

Extend students to explore comparative column graphs and stacked column graphs where more than one sample is represented for visual comparison. For example, the students may generate an eye colour graph for students in their class, gather data about eye colours in another class and represent this data on the same graph for comparison. The use of a legend to indicate which sample should also be introduced.





Discuss what each of these graphical representations show. Consider what audience might find this data useful.

Explore comparative graphs and data available on the Internet. Practice interpreting the data available in various media (Internet, magazines, newspapers, television reports).

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_____ Date:_____

Can you do this? #6

1. Circle the words to make the sentences correct.

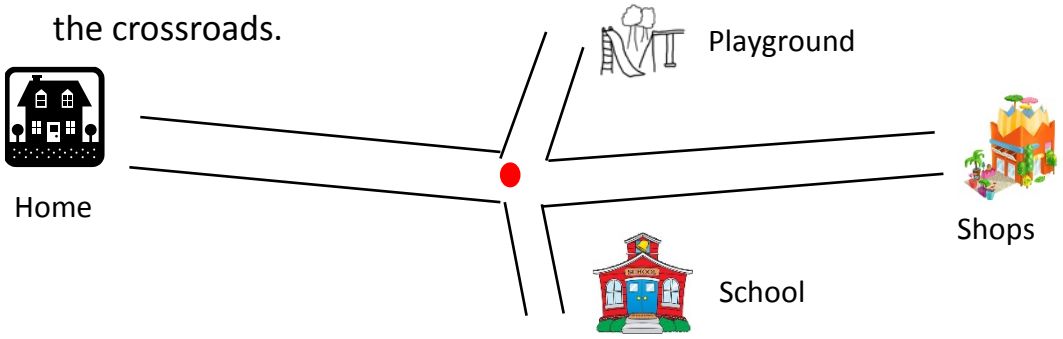
(a) It is certain/likely/impossible that the sun will rise in the morning.

(b) It is certain/likely/impossible that maths class will be on Sunday.

(c) It is certain/possible/impossible that you will roll a 5 on a die.

Obj. 1.6.1
a) ☐
b) ☐
c) ☐

2. You are standing at a crossroads. List the places you could go from the crossroads.



Obj. 1.6.3
a) ☐
b) ☐
c) ☐
d) ☐

a) _____ b) _____

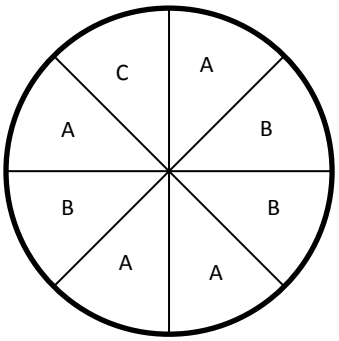
c) _____ d) _____

3. What is the chance of spinning:

(a) A _____

(b) B _____

(c) C _____



Obj. 1.6.3
a) ☐
b) ☐
c) ☐

4. Draw marbles in the bag so that the probability of getting a red marble is:

(a) 3 chances out of 5 (b) 1 out of 2 (c) certain

Obj. 1.6.4
a) ☐
b) ☐
c) ☐

Cycle 6: Comparing Likelihood

Overview

Big Idea

This cycle explores qualitative and informal quantitative representations of likelihood. Students are engaged with identifying impossible, possible and certain events as well as the continuum of everyday language used to describe events with likelihood or chances of occurring that fall between these (many of the everyday words for probability describe various degrees of possibility).

Probability is also represented quantitatively using 0 to describe impossible or no chance, and 1 to describe certainty or every chance. Although this idea formally involves fractions, for this cycle the focus will be on simply identifying the possible outcomes and counting the number of desirable outcomes out of the number of possible outcomes in order to compare likelihood.

There are a variety of ways to collate the understandings that are needed for students to be successful with probabilistic thinking. One of these is to consider probability as a set of skills for students to develop. This cycle focuses on ensuring students are able to:

- Classify events as impossible, possible or certain.
- List all possible outcomes of a chance process.
- State which outcome is most likely in a single trial of a chance process.
- Can correctly choose in what situation an event is more likely to occur.
- Can assign a numerical probability to an event as a number of chances out of the total.

(Note: Formal representation of probability as a fraction, decimal or percentage will be the focus of a later unit.)

Objectives

By the end of this cycle, students should be able to:

1.6.1 Describe possible everyday events. [4SP092]

1.6.2 Order the chances of everyday events occurring. [4SP092]

1.6.3 List outcomes of chance experiments involving equally likely outcomes. [5SP116]

1.6.4 Construct sample spaces for single-step experiments. [7SP167]

Conceptual Links

This cycle provides foundation ideas of the informal representation of likelihood that also provide opportunities to practice counting and comparing. Calculation and representation of probability as a fraction, decimal and percentage will be continued in a later unit.

Materials

For Cycle 6 you may need:

- Probability continuum language cards
- Washing line & pegs
- Spinners
- Dice
- Cards
- Counters
- Brown paper bags
- Probability games
- Attribute or pattern blocks
-

Key Language

Impossible, possible, certain, likelihood, chance, everyday probability language (collect student examples first for local colloquialisms and to identify range of vocabulary), trial, event, outcome, experiment

Definitions

Contiguous: Segments on a spinner where there is only one of each type of segment or like segments are together

Event: A chance occurrence that may have more than one outcome.

Non-contiguous: Segments on a spinner where one or more types of segment are separated from other like segments

Outcome: Possible results from a chance event

Example Questions

Anecdotal Evidence

Ensure that students are able to identify everyday events that involve likelihood, can identify all the possible outcomes attached to an event and have an understanding of certain, likely and impossible.

- What are some everyday events?
- Are you more likely to see a rain cloud than a fossil?
- You have a bag with red marbles, green marbles and blue marbles in it. What are the possible outcomes if you put your hand in without looking and drew out a marble?
- What would make it equally likely for you to pull out a red, green or blue marble?

Portfolio Task

The portfolio task provides students with opportunities to apply concepts of likelihood skills developed within this cycle.

RAMR Cycle

Reality

During this Reality phase, it is important to ensure that students are able to classify events as impossible, possible or certain; that is, recognising when situations involve chance. This is an opportunity to start from the local context and culture of the students. Get students to discuss when events are uncertain and have possible outcomes. Most students have many experiences with chance, for example, card games, sport, weather. Find out the important ones in your community.

Discuss characteristics of chance events by considering the focus (e.g., weather, sport, games), and consider whether chance involves activity or a question; consider whether there has to be more than one outcome, look at randomness and whether this is always present. Discuss chance events like buying a Lotto ticket or a scratchie. The question, *will I win?*, always involves chance. Are there questions that involve Lotto tickets that do not involve probability? For example, *How much is a 'Quickpick'?* Explore changing situations from chance to certainty and back again.

Classify events. Create three locations across the room (locations on the floor, desks, ...) and label them *impossible, possible* and *certain.* Have a selection of events for students to place in the relevant location (students could make up events and take them to the position). Some examples of situations/events could be: *It will rain tomorrow; drop a rock in water and it will sink; a flower seed planted today will flower tomorrow; the sun will rise tomorrow morning; if you ask someone who was the first Mayor of their town, they will know; you will have two birthdays this year*; and *you will be in bed by 10 pm*. Ask students to write three stories, one for each area. Use local stories. Record where the stories go on a table divided into columns by headings impossible, possible and certain.

Create a continuum of probability language. Use a rope across the room with cards and pegs, or a line on the whiteboard, denote one end as impossible, and the other end as certain. At this stage do not assign numerical values to represent probability. Students' situations from the previous activity can be assigned a position along the line. A similar approach can be used to generate a continuum of everyday probability language. Start with student language and broaden with examples from *Resource 1.6.1 Continuum of probability language as necessary.* Explicitly discuss the order of these from more likely to least likely.

Resource Resource 1.6.1 Continuum of probability language

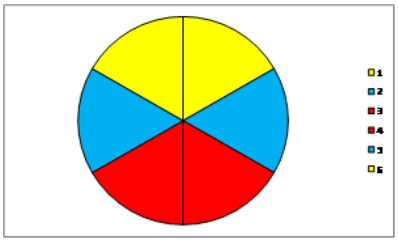
Abstraction

The abstraction sequence for this cycle starts from students' previous experience of qualitative descriptions and classification of chance events and continues through identifying all possible outcomes to stating which is most likely in the event that outcomes are not equally likely. A suggested sequence of activities is as follows:

1. *Identify and list possible outcomes.* Pose examples of chance events to students. Ask them to identify the possible outcomes. For example, the fire alarm will sound has two possible outcomes (it will make sound or it will not). Tossing a fair coin also has two outcomes (heads or tails).
2. *Consecutive trials of the same event.* Discuss with students what will happen if the experiment in step 1 is repeated. For example, if four heads come up from four tosses of a coin, which is more likely on the next toss, a head or a tail (check students don't believe that the tail becomes more likely).

3. *Explore consecutive trials with and without replacement.* This step is not useful for probability with coins but is applicable to cards or drawing items from mystery bags. Check that students know that replacement leaves things the same but no replacement changes the likelihood for successive draws.

4. *Explore contiguous and non-contiguous models.* This is particularly relevant when using spinners. Ensure students understand area models of probability and that more does not mean more likely if it applies to the number of regions but the area is the same. For example, in the diagram on the right, blue is still equally likely as red or yellow.



5. *Consider the likelihood of each possible outcome.* To return to the coin toss example, there are two possibilities that are equally likely. See if students can describe this likelihood (50-50, even chance).

6. Repeat Steps 1 – 5 with other common chance generators like drawing items from a bag, fair dice, playing cards, spinners. *Resource 1.6.2 Probability Skills Explained* has some useful small activities and questions that may be used throughout the sequence to focus on specific skills and to provide background probability information for teachers.

Resource Resource 1.6.2 Probability skills explained

Note: When students are competent, it is possible to progress from qualitative comparisons of more or less likely to numeric comparison and ordering. Students should have experiences comparing two or more sample spaces with the same number of outcomes and two or more sample spaces with different numbers of outcomes but the same number of favourable outcomes to ensure robust understanding.

Note: If students hold the misconception that it is more difficult to roll a 6 than any other number, *Resource 1.6.3 Beetle Game* will be useful to play here.

Resource Resource 1.6.3 Beetle Game

Note: If students struggle with identifying possible likelihood using area models or spinners, *Resource 1.6.4 Spinners* provides a range of spinners with increasing complexity.

Resource Resource 1.6.4 Spinners

Mathematics

Connections

Once students can confidently identify and list all the possible outcomes of an event (sample space), introduce the language and symbols for describing the likelihood quantitatively. For example, a coin toss has a sample space of two possible outcomes. Introduce the language of 1 chance in 2, 1 chance out of 2. (In later units, this will be connected to the fractional representation of the likelihood of heads being ½, 0.5 or 50% and the likelihood of tails being ½, 0.5 or 50%.)

Explore theoretical probability for fair dice as 1 chance out of 6 of any given number occurring. Extend thinking to simple, equal likelihood spinners before considering unbalanced varieties. Also consider the likelihood of a specific playing card, value of a playing card (any suit), picture card (any suit).

Using *Resource 1.6.3 Beetle Game*, connect back to graphing skills from Cycle 5. Students may generate either a column graph to display the number of rolls of each value on the dice to complete their beetle, or a comparative column graph to display the number of rolls of each value on the dice for both of their beetles. Compare graphs across the class to see which values on the dice were more/less frequently rolled. Create a class stacked column graph to record all groups' data on the one graph. Determine which number overall was rolled more/less frequently.

Language/symbols and practice

To engage students with more robust understanding of sample space and probability events it is beneficial to reverse the activity so that students construct their own sample space for a probability experiment. *Resource 1.6.5 Constructing sample space activities* may be useful.

Resource Resource 1.6.5 Constructing Sample Space Activities

Reflection

Check the idea

Consider games that rely on equal likelihood of chance events like Snakes and Ladders. Engage students with determining the likelihood of going up a ladder on the first roll of the dice. Consider further options such as:

- If your first roll was a 2, what are the possible next squares you might land on?
- Of these squares, how many will take you up a ladder (or down a snake)?
- What is the chance of getting ahead further than just the roll of the dice on the second roll?
- Is this the same for all Snakes and Ladders boards?

Resource Resource 1.6.6 Snakes and Ladders Boards

Apply the idea

Encourage students to devise their own game of chance where they identify the probability of outcomes as equally likely.

Extend the idea

Consider games (like Monopoly) that use added values of two dice. For the example of Monopoly, identify which squares are more likely to be landed on in the first throw, which are impossible. Identify the likelihood of landing on each of the first twelve squares. Are these equally likely? If players are more likely to buy properties on their first go around the board (when everyone has money), which property/ies is/are more likely to be bought first each time?

Resource Resource 1.6.7 Monopoly Board

Connect to students' previous representation of data from Cycle 5. Generate a class eye colour column graph, frequency graph and circle graph. This circle graph may be used to construct a class spinner that may be used for a class game of chance. Explore whether students are equally likely to be chosen or, for example, if brown eyes is more likely to be spun than blue eyes. Order the eye colours from most likely to least likely. Have students predict which eye colour is most/least likely to win if this spinner is used in a game of chance.

Play a game of chance on the basketball court where the class lines up on one side of the court and students are able to take a step forward each time their eye colour is spun. The first eye colour across the basketball court wins.

Discuss how the number of steps for each eye colour could be varied to make the chances more even (e.g., one step for brown or hazel, two steps for blue or green).

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 01 Portfolio Task – Teacher Guide

Planes, Planes, Planes!

Content Strand/s: Number and algebra
Measurement and geometry
Statistics and probability

Resources Supplied:

- Instruction sheet
- Recording sheet
- Web addresses for printable paper plane instructions

Other Resources Needed:

- A4 paper
- Graph paper
- Protractors
- Target

Simple plane instructions:

- Paper Plane Instructions downloadable and printable from: http://www.paperplanesmovie.com.au/#!/make
- Historic Paper Darts Story 1910: http://www.northdevongazette.co.uk/news/new_lease_of_life_for_historic_barnstaple_building_1_1477977

Advanced and complicated plane instructions:

- Video of world record airplane flight at The Paper Airplane Guy: https://www.youtube.com/user/ThePaperAirplaneGuy
- How to fold the world record airplane: https://www.youtube.com/watch?v=EDiC9iMcWTc

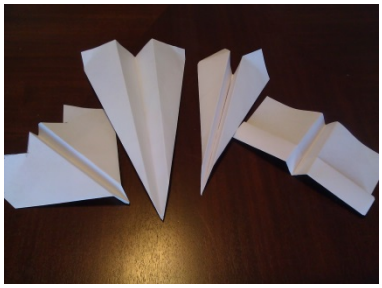
Summary:

As a context for counting and likelihood that also incorporates comparison of angles, paper planes are designed and flown at a target and across a distance. Angles on planes can be compared informally and classified as acute, obtuse or reflex; and less than 90^{o}, more than 90^{o}, more than 180^{o}. Planes are thrown across a distance to see which plane travels furthest (distance counted in steps and in counted metres using tape measure). Planes are thrown along a runway and at a target to determine accuracy as very accurate, some accuracy, not accurate.

ACARA Proficiencies Addressed:	**Content Strands:**
Understanding Fluency Problem Solving Reasoning	*Measurement and geometry* 1.1.1 Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language. [FMG087] *Number and algebra* 1.3.2 Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting. [2NA028] *Statistics and Probability* 1.5.1 Pose questions and collect categorical data by observation or survey. [5SP118] 1.5.2 Collect data, organise into categories and create displays using lists, tables, picture graphs and simple column graphs. [5SP119] 1.5.3 Describe and interpret different data sets in context. [5SP120]

Planes, Planes, Planes!

Name	
Teacher	
Class	



Your Task:

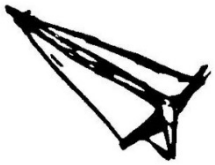
In this activity you will make some different paper airplanes and gather data to decide which ones fly best.

Within Portfolio Task 1, your work has demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Conceptual understanding	1.1.1 Use direct and indirect comparisons to decide which is longer and explain reasoning in everyday language.	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
	Procedural fluency	1.3.2 Group, partition and rearrange collections up to 1000 in hundreds, tens and ones.	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	1.5.1 Pose questions and collect categorical data. 1.5.2 Collect data, organise into categories and create displays.	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language
Problem Solving and Reasoning	Reasoning and justification	1.5.3 Describe and interpret different data sets in context.	Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions

Comments:

Section 1: Describe your plane.



1. In a group of three, choose a different plane each. Build one paper plane each.
2. Sketch your plane in the space below.
3. Find at least five different angles on your plane. Use letters to label the angles on your plane and your sketch.
4. Compare and classify the angles. Record your answers in the table below.

Angles on the plane

	Acute Angle (Less than 90^o)	Right Angle (Exactly 90^o)	Obtuse Angle (More than 90^o)	Reflex Angle (More than 180^o)
Label	**Acute/Right Angle/Obtuse/Reflex**			
A				
B				
C				
D				
E				

1. What is the largest angle on your plane? _____

2. What is the smallest angle on your plane? _____

Compare your plane to others in your group.

3. Which plane has the biggest angle? _____

4. Which plane has the smallest angle? _____

Section 2: How far can your plane fly?

Now we will test how far your plane can fly.

1. Predict how far your plane might fly along a straight line.
2. Throw your plane and then estimate how far it actually flew.
3. Measure how far your plane flew in footsteps (count how many steps from the start to your plane).
4. Measure distance with a tape measure (how many metres and how many centimetres).
5. Repeat this two times.

Distance along the runway

	Predict (no. of steps)	Estimate (no. of steps)	Measured Distance (no. of Steps)	Measured Distance (m or cm)
Throw 1				
Throw 2				
Throw 3				

1. What was your longest throw? _____

   How many steps long was the throw? _____

2. How many metres and centimetres was your longest throw? _____

Compare your distances to your group members'.

3. Whose plane flew the furthest? _____

4. How far did the plane fly? _____

5. Why do you think this plane flew furthest?

_____

_____

_____

_____

_____

Section 3: How accurate is your plane?

In this activity you will test the accuracy of your plane in two different ways.

1. Check the accuracy of your plane.
2. Over ten throws, record how many times your plane lands on the runway, close to the edge of the runway, or off the runway? Record your data in a table below.
3. Over ten throws at a target, record how many times your plane hits the centre, hits around the edge of the centre, or misses the target?
4. Graph the accuracy of your plane along the runway.
5. Graph the accuracy of your plane thrown at a target.

Make graphs of your accuracy:

Accuracy Along the Runway	Accuracy At a Target

Compare the Runway Accuracy and Target Accuracy.

1. Is your plane more accurate along a runway or at a target? _____
2. Compare your plane with other planes in your group. Which plane is most accurate along a runway or at a target? _____
3. Why do you think this plane flies more accurately?

   _____

   _____

   _____

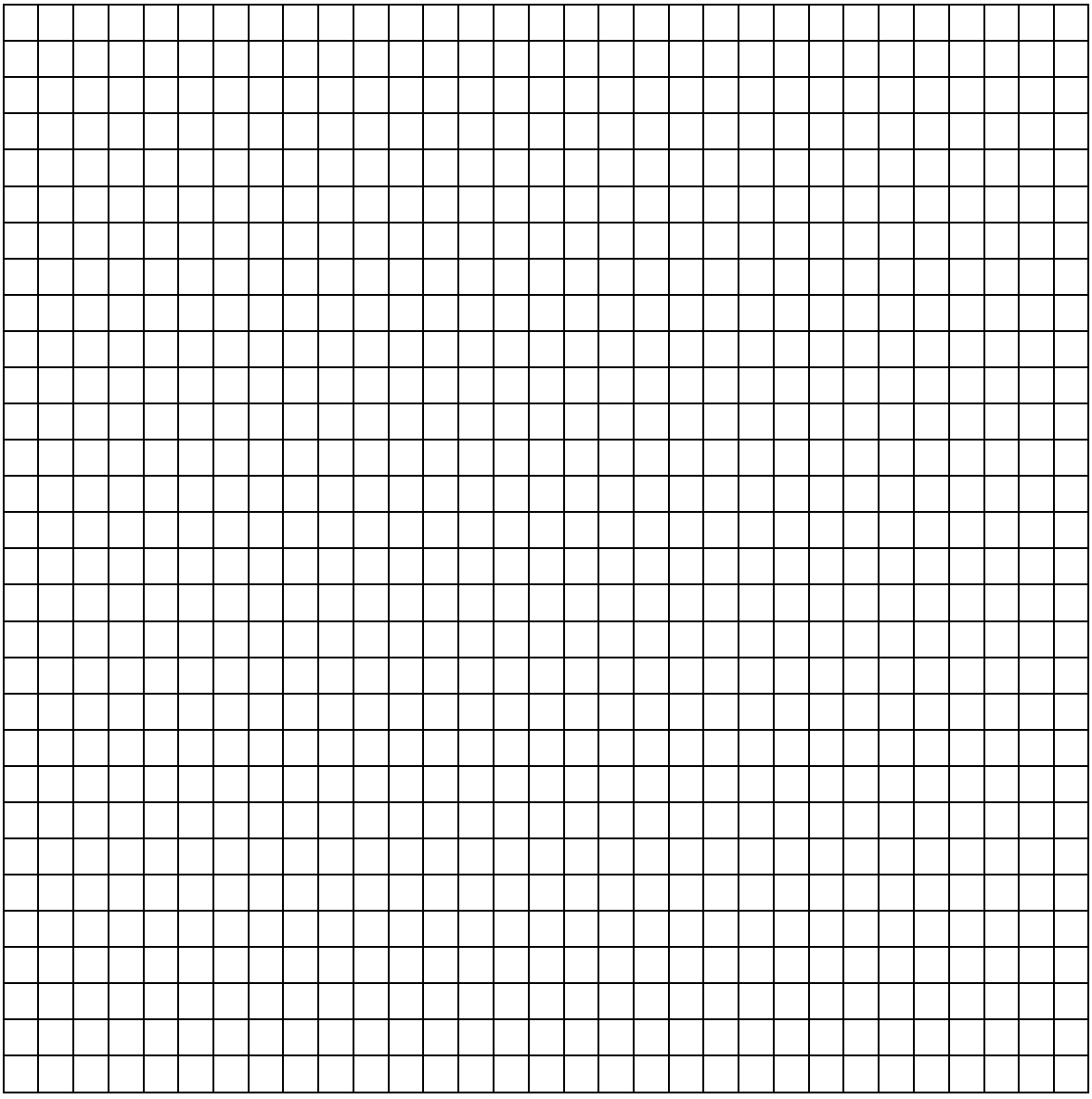
Section 4: Design a better plane.

In this activity you will try to improve on the plane design to make it fly more accurately and use your data to explain why it is a better design.

1. Sketch and describe your plane using what you know about angles.

2. Test your plane.

(**Hint:** How far can it fly and how accurate is it? How will you present your data to explain why your design is better?)



3. Compare your design to the planes you created earlier. Is your plane better? Why/why not?

_____

_____

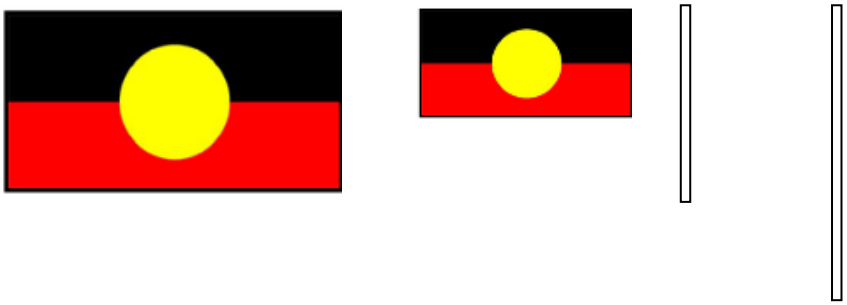
_____

_____

Name:_____ Date:_____

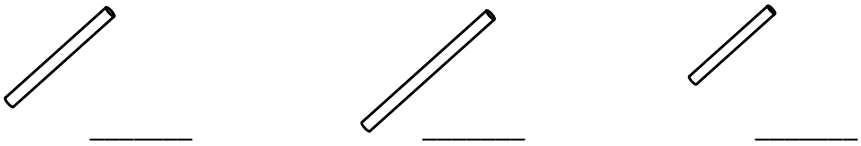
Can you do this now? Unit 01

1. Draw a line from the wider flag to the shorter pole.



Obj. 1.1.1 ☐

2. Number the straws in order from longest to shortest.



_____ _____ _____

Obj. 1.1.1
i. ☐
ii. ☐

3. Circle the box that will be lighter.

Obj. 1.1.2 ☐

4. Find the number words in the sentences below and write them as numbers:

(a) There are eighty-eight keys on a standard piano.

_____

Obj. 1.2.1
a) ☐
b) ☐

(b) There are twenty-nine days in February in leap year.

_____

5. Find the numbers in the sentence, and write them in words:

   (a) There are 52 weeks in a year.

   _____

   (b) There are 64 squares on a chess board.

   _____

Obj. 1.2.2
a) ☐
b) ☐

6. Write a number that has:

   (a) 5 ones and 8 tens _____

   (b) 8 tens, 6 ones and 4 hundreds _____

Obj. 1.2.3
a) ☐
b) ☐

7. In 976 the 6 has a value of 6 hundreds/tens/ones
   (circle the correct answer)

Obj. 1.2.3
☐

8. Expand this number to show its place values: 543

   _____

Obj. 1.2.4
i. ☐
ii. ☐
iii. ☐

9. Write the missing number

   (a) 507 5 hundreds, _____ tens and 7 ones.

   (b) 692 _____ tens and 2 ones.

Obj. 1.2.5
a) ☐
b) ☐

10. The number 285 has:

    27 tens and _____ ones

Obj. 1.3.1
☐

11. The number 963 has:
    8 hundreds, _____ tens and _____ones

Obj. 1.3.1
a) ☐
b) ☐

12. Write as a number:
    43 tens and 6 ones _____

Obj. 1.3.1
☐

13. Write as a number:
    5 hundreds and 18 ones _____

Obj. 1.3.1
☐

14. Fill in the following table:

Number	Action	Hundreds	Tens	Ones	Answer
23	*×10*	*2*	*3*	*0*	*230*
350	÷10				
15		1	5		
960	÷10				
5					500

15. Find the number name in the sentence below and write it using numbers:

    The Queensland Sport and Athletics Centre can seat up to forty-nine thousand people. _____

16. Find the numbers in the sentence, and write them in words:

    The distance to the moon is between 363 104km and 406 696km.

    _____

    _____

17. Write a number that has:

    6 hundred thousands, 8 hundreds and 90 millions

    _____

18. In 736 543, what is the place-value of the 6?

    6 hundred thousands/ten thousands/one thousands (circle the correct answer)

19. Expand this number to show its place values: 12 602 563

    _____

    _____

Obj. 1.3.1
a)i. ☐
a)ii. ☐
b)i. ☐
b)ii. ☐
c)i. ☐
c)ii. ☐
d)i. ☐
d)ii. ☐

Obj. 1.4.1
☐

Obj. 1.4.2
a) ☐
b) ☐

Obj. 1.4.3
☐

Obj. 1.4.3
☐

Obj. 1.4.4
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

20. Write the missing numbers

(a) 592 327 592 thousands, 3 hundreds, _____ tens and 7 ones.

(b) 295 603 2 hundred thousands, ____ thousands, 6 hundreds and 3 ones.

Obj. 1.4.5
a) ☐
b) ☐

21. Write down numbers can you make using: 3, 0, 5, 7, 6, 4

(a) Write the largest number you can find using all these numerals. _____

(b) Write the smallest number you can find using all these numerals. _____

Obj. 1.4.3
a) ☐
b) ☐

22. A class of 11 students tallied their favourite game. This table shows the results. Using the axis below, create a column graph of class favourite games.

Tally	III	II	IIII	II
Favourite game	Halo	Fable	Mine -craft	Mario Kart

Obj. 1.5.5
i. ☐
ii. ☐
iii. ☐
iv. ☐
v. ☐

23.This pictograph shows the sport played by 17 mathematics students.

 = 1 student

					
					
					
					
					
Soccer	Basketball	Swimming	Running	Baseball	Netball

From reading the graph:

(a) Which sport do most students play? _____

(b) Which sport do least students play? _____

(c) How many students play netball? _____

(d) Which other sport is played by two students? _____

24.Circle the words to make the sentences correct.

(a) It is certain/likely/impossible that the sun will set in the afternoon.

(b) It is certain/likely/impossible that maths class will be on Saturday.

(c) It is certain/possible/impossible that you will toss heads on a coin.

Obj. 1.5.6

a) ☐

b) ☐

c) ☐

d) ☐

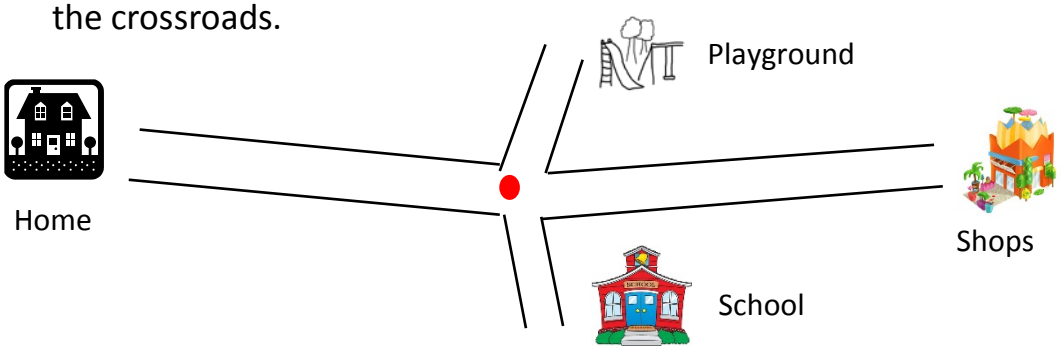
Obj. 1.6.1

a) ☐

b) ☐

c) ☐

25. You are standing at a crossroads. List the places you could go from the crossroads.



a) _____ b) _____

c) _____ d) _____

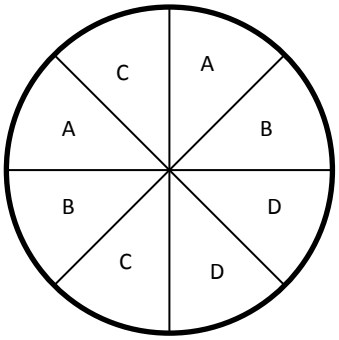
Obj. 1.6.3
a) ☐
b) ☐
c) ☐
d) ☐

26. What is the chance of spinning:

(a) B _____

(b) C _____

(c) D _____



Obj. 1.6.3
a) ☐
b) ☐
c) ☐

27. Draw marbles in the bag so that the probability of getting a red marble is:

(a) 3 chances out of 5 (b) 1 out of 2 (c) certain

Obj. 1.6.4
a) ☐
b) ☐
c) ☐

XLR8 – Accelerating Mathematics Learning

YuMiDeadly
Growing community through education

Faculty of Education
School of Curriculum
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059
CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
E-mail: ydc@qut.edu.au
Website: ydc.qut.edu.au