XLR8 – Accelerating Mathematics Learning

XLR8 Teacher Guide

Principles and Practices

2016

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We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Introduction

The low socioeconomic status (SES) urban secondary schools who are involved in this project have significant numbers of junior secondary (Year 7/8/9) students whose mathematics ability level is low (nominally equivalent to a Year (3/4 level). If taught mathematics at Year 8 level, these students are likely to learn little mathematics, disengage from school and enter post-compulsory years with mathematics understanding that is inadequate to access meaningful employment or tertiary education, thus becoming dependent on social welfare. Over-represented in this group are Aboriginal, Torres Strait Islander, migrant and refugee students and other students from low-income families. Thus, the project represents a convergence of social, economic and educational benefit; it is designed to give underperforming students the opportunity to change their future and to increase the number of mathematically trained people. It represents a win-win solution for government, community and individuals in relation to "closing the gap".

The focus of the project is to develop, trial, evaluate and refine a mathematics curriculum and associated resources that will accelerate the mathematics learning of these students from their initial low level of ability level to their appropriate year level ability. The project is an extension of other projects which have shown that acceleration is possible if: (a) mathematics is taught in a sequence that develops structural understanding of major ideas through activities that show how the ideas grow across the primary and middle school years; (b) mathematics is taught using a physically active pedagogy), with actions and material leading to ideas in the mind (i.e., mental models) and with discussion relating real-world instances; and (c) mathematics teaching is integrated with whole school change that encourages positive student attitude, behaviour and confidence, high teacher expectations, contextualisation of teaching into local culture and language, and local community involvement and support.

With these ideas in mind, the project provides a curriculum, resources and professional learning and support that will be trialled and refined by the teachers and students in their XLR8 classrooms. Using this action research-based approach to theory building, the anticipated outcomes of the project are:

(a) students with improved mathematics learning and life chances;
(b) theory on how students' mathematics learning can be accelerated; and
(c) innovative mathematics learning materials that support accelerated learning.

In 2016, the curriculum resources and professional learning and support will include:

- A series of 15 unit booklets that define the *structured sequence* of ideas that has been designed to promote acceleration. Nominally, each unit has been designed to be taught over a minimum of 5 weeks. Each unit is comprised of a sequence of cycles. For each cycle a suggested teaching sequence is provided which exemplifies the suggested pedagogy.

- A set of resources that can be used in class and which are referred to throughout the unit booklets.

- Assessment tasks, including diagnostic worksheets to be administered before each cycle, summative tests to be administered after each unit, and assignment-style portfolio tasks to be administered during each unit.

- Regular classroom visits in which researchers will act as co-teachers and/or teacher-aides and out-of-class meetings in which teachers and researchers will collaborate to plan and develop teaching resources.

- Regular off-site professional learning sessions in which all teachers will gather to share and discuss their participation in the XLR8 project.

This Teacher Guide presents the following:

- An introduction to the YuMi Deadly Centre, YuMi Deadly Mathematics and the ideas that underpin it.
- The XLR8 project is based upon three pillars: the structured sequence, the RAMR pedagogy and professional learning and support for teachers.
- An overview of the XLR8 curriculum that follows the structured sequence using the RAMR pedagogy.
- On overview of the research project's implementation plan, including the gathering and analysis of data.
- Appendices that discuss additional ideas related to XLR8, including the big ideas of mathematics and whole-school cultural change.

The YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

The mission of the YuMi Deadly Centre (YDC) is to enhance Indigenous and low SES students learning and to improve employment and life chances. Most of the present projects of YDC focus on mathematics and are built around a mathematics program called YuMi Deadly Mathematics (YDM). This program has been developed as a way to meet a crisis in both mathematics and in students' learning of mathematics that is present in Australia today. YDC presents mathematics and its teaching and learning in a way that enables powerful learning and supports this with a pedagogy that facilitates effective teaching.

From a mathematics perspective, there has been a decline in the number of mathematics trained people and Australia's participation in mathematics. There is a shortage of students wishing to undertake careers in mathematics, in the mathematical sciences and other disciplines based on mathematics, or in the teaching and learning of mathematics (Thomas, 2009). This has resulted in Mathematics Departments within Universities closing down or being subsumed into other (emerging) discipline areas such as Complex Systems. As a consequence, there is a shortage of mathematics teachers to the point that nearly half the teachers of mathematics in secondary schools have no pre-service training in mathematics or mathematics education. The YDC, through its YDM program, aims to enable teachers to teach and students to learn mathematics in a manner that encourages them to stay in mathematics and learn more about it.

From a student perspective, whole groups of students are missing out on learning mathematics. Aboriginal and Torres Strait Islander students are generally two years behind their non-Indigenous counterparts and low SES students are similarly behind when compared against students from high socio-economic families (Marginson, 2008). A major factor in Indigenous and low SES disengagement and underachievement is that students are confronted with a Eurocentric mathematics curriculum. From this perspective, mathematics is presented in a purely abstract form with very little connection to Indigenous and low SES people, culture and world view. This disassociation with the subject usually manifests itself in statements from students like "Why do I need to learn this?" and "What will I use this for?". YDM offers content and pedagogy that meets underperforming students' needs and can close the gap between these and other students in terms of mathematics learning.

Thus, to enhance engagement and achievement and to improve both mathematics and its learning, YDM is based on the answers to fundamental questions such as "What is mathematics?" and "How does culture relate to mathematics?" The exploration of answers to these questions has resulted in the development of a philosophy and pedagogy that aims to allow Indigenous and low SES students to explore mathematics on their own terms, through their world view, and that values and utilises the cultural and social capital that these students bring to the classroom (Bourdieu, 1973). Given that the emphasis is on allowing students to be creative and self-expressive, we would argue that the philosophy will not only allow Indigenous and low SES students to connect with mathematics but also allow these students to excel in learning mathematics.

Positioning of YDM

There are many mathematics programs that have been design to attempt to close the gap between Aboriginal, Torres Strait Islander and low socio-economic status (SES) students and mainstream students. These programs vary in many ways.

1. In terms of the direction and freedom given to teachers, some programs supply scripts of what to say from which teachers cannot deviate; others provide sequences of lesson plans, back-up resources and materials from which teachers have some freedom to choose and in terms of which teachers have some freedom to tailor to their students' needs; and some provide knowledge and exemplar lessons in order for teachers to plan their own sequences and fashion their own script.
2. In terms of the focus of their instruction, some programs see their role in meeting the functional needs of their students, so that they can recall, and use in standard situations, the content and procedures that have been specified for certain vocations; other programs believe that it is important to understand the mathematics behind the specific content and procedures so that this knowledge is portable and can be translated to new situations.
3. In terms of the scope of the program, some programs restrict themselves to only what happens in the mathematics classroom; others provide support for some out of classroom activities such as homework, excursions and clubs; while others see that their role extends to the school as a whole and the school's relationships with parents/carers and community.

These variations between programs can also be described in terms of the nature of mathematics, the nature of the teaching of mathematics, the nature of the learning of mathematics, and the nature of the school-community relationship. With regards to these four perspectives, YDM is designed to adopt a distinct position which is summarised by the quadrant diagrams of *Figure 1*.

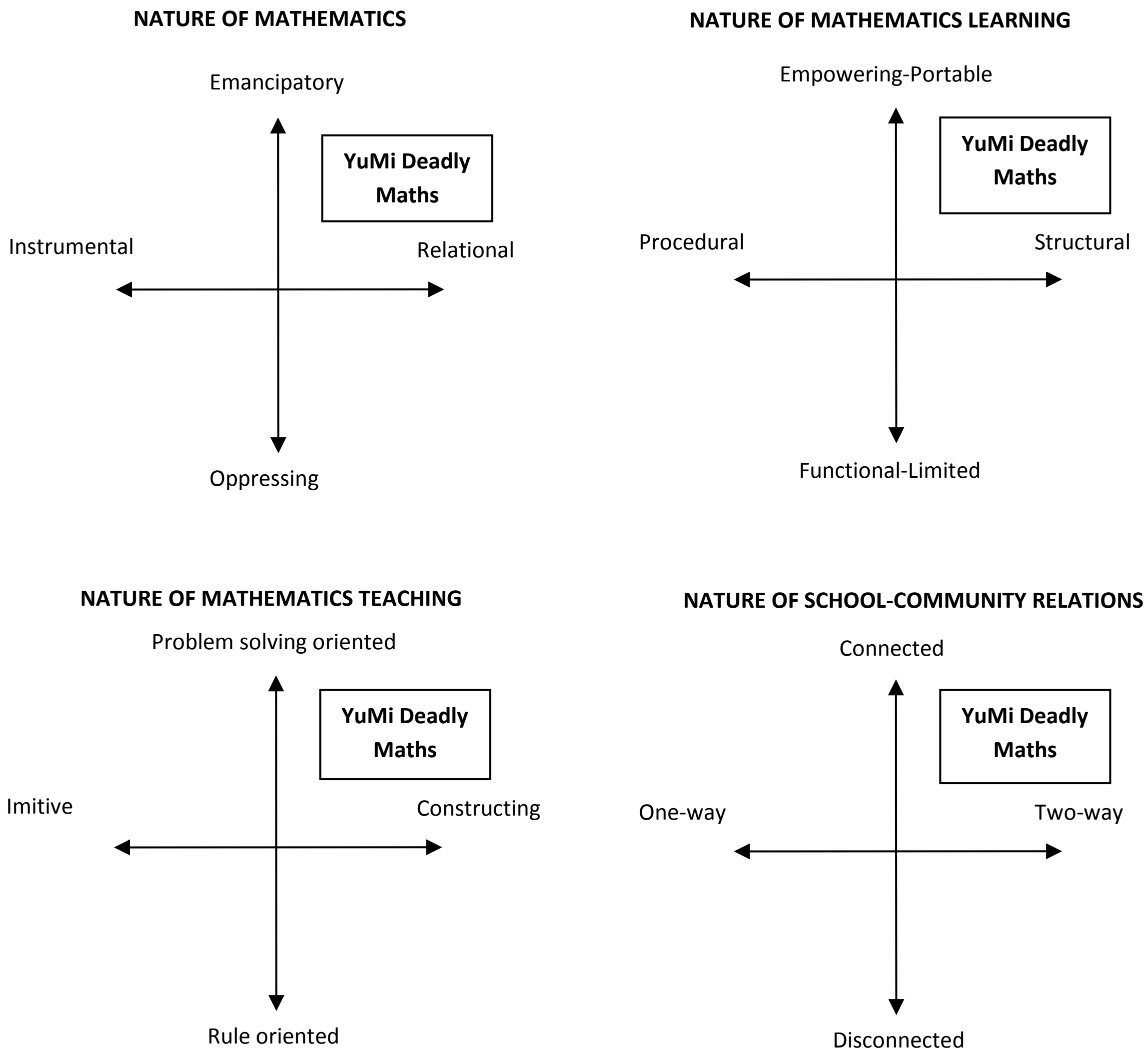


Figure 1. Four perspectives of mathematics programs

Principles of YDM

Over the last 12 years, YDC staff have been trialling teaching ideas with Indigenous and low SES students. Looking back, these ideas have been based on beliefs with respect to what students are capable of learning, what teachers are capable of teaching and how these relate to school and community. These ideas were summarised in the previous section. In general, YDC staff have found that success is the greatest when teaching encompasses both the knowledge required to understand mathematics as well as to do mathematics.

The YDC's experiences have been distilled into the following six principles upon which the YDM program is based:

1. that all people deserve the deepest mathematics teaching and learning that empowers them to understand their world mathematically and to solve their problems in their reality;
2. that all people can be empowered in their lives by mathematics if they understand it as a conceptual structure, life-describing language, and problem-solving tool;
3. that all people can excel in mathematics and remain strong and proud in their culture and heritage if taught actively, contextually, with respect and high expectations and in a culturally safe manner;

4. that all teachers can be empowered to teach mathematics with the outcomes above if they have the support of their school and system and the knowledge, resources and expectations to deliver effective pedagogy;
5. that all communities can benefit from the mathematics teaching and learning practices above if school and community are connected through high expectations in an education program of which mathematics is a part; and
6. that a strong empowering mathematics program can profoundly and positively affect students' future employment and life chances, and have a positive influence on school and community.

These six principles have formed the base for the design of the XLR8 project.

Pillars of XLR8

The XLR8 project is based upon three pillars: 1) The use of a **structured sequence** to carefully and strategically explore the structure of mathematical knowledge; 2) Use of the **RAMR pedagogy** as the basis for designing instruction; and 3) Providing timely **professional learning** and support to ensure teachers are able to follow the structured sequence using the RAMR pedagogy. Underlying these pillars is the belief that the accelerated learning of students is possible when four imperatives are satisfied. In the following sections, these imperatives are firstly introduced and then each of the three pillars are discussed. This broad view of the RAMR project is then refined in the following chapter when finer grained details of the structured sequence and use of the pedagogy are provided.

Imperatives for acceleration

To achieve acceleration, the XLR8 project believes the following four imperatives need to be considered.

Imperative 1: Structure

Mathematics topics are to be taught as a structure (i.e., a connected set of ideas) both *within domains* – as per place value of number; and *across domains* (e.g., place value and metric measures). The important focus is on abstraction (generalisation) (Cooper & Warren, 2008) and on application to the world; aiming to reveal the big ideas of mathematics (concepts, strategies and principles that apply across year levels, topics and different content – the abstract schema of Ohlsson, 1992) and to develop a "mathematical eye" through which to view and interpret the world. For example, conversion of metric units is connected to place value (same multiplicative base, 1000); multiplication and division would be related to measuring, fractions and chance. With respect to the last example, students are led to recognise the big ideas of mathematics and to understand that the following mathematics ideas are not different; they are all examples of *inverse relation principle* (big idea) – large divisors give small quotients in division, large units give small numbers in measurement, large denominators give small fractions, and many possibilities gives small chances of winning in probability.

Imperative 2: Chunking

Mathematics topics are to be taught in large chunks, not as small pieces spread over years. For example, place value from Years 3-8 was planned to be taught, not as individual places (e.g., ones, then tens, then hundreds and so on) but in terms of place value periods (e.g., ones, tens, hundreds of *ones*; ones, tens, hundreds of *thousands*; ones, tens, hundreds of *millions*; and so on). Thus, the focus is on building a holistic understanding around whole numbers as a complete multiplicative system − the approach to teaching that YDC staff have found to be effective with Indigenous students.

Imperative 3: Active pedagogy

Mathematics topics are to be taught actively, with actions and material leading to ideas in the mind (i.e., teaching is to follow the sequence body→hand→mind). Activity is based on social constructivism – use of materials and discussion with peers and teachers. Mathematics topics are to relate to real-world problems (from everyday experiences) and relate back to real-world problems, integrating real-world situations, physical, virtual and pictorial and symbolic representations, and language. Lessons are to entail moving between these representations that require knowledge of teaching methods at sequenced levels: the *technical* (knowing how to use appropriate materials to model mathematical ideas), *domain* (knowing the specific methods and materials for the mathematical idea) and *generic* level (knowing pedagogies that lead to structural understanding and which can be used throughout mathematics teaching).

Imperative 4: Culture, community and school focus

The cultural implications of Western mathematics are to be made visible, Indigenous frameworks for learning are to be used, and mathematics is to be contextualised into culture and related to home language. As well, the local community is to actively participate in the school and local knowledge is to be made legitimate in the classroom. Finally, schools are to adopt change processes similar to those advocated by the *Stronger Smarter Institute*, so that school processes build pride in heritage, change identity to where there is a belief that can learn, confront behaviour, ensure cultural safety and high expectations and are based on local leadership.

Pillar 1 – Structured sequence

Fundamental to the YDM approach is a focus upon the organisation, or structure, of mathematical ideas. YDM argues that knowledge of the detailed structure of mathematics can assist teachers be effective and efficient in teaching mathematics. This is because it enables teachers to:

1. ***determine what mathematics is important to teach*** (mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present);
2. ***link new mathematics ideas to existing known mathematics*** (mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem solving);
3. ***choose effective instructional materials, representations and strategies*** (mathematics that is connected to other mathematics or based around a big idea commonly can be taught with similar materials, representations and strategies); and
4. ***teach mathematics in a manner that makes it easier for later teachers to teach more advanced mathematics*** (by preparing the connections to other ideas and the foundations for later teaching).

Thus it is essential that teachers know the mathematics that precedes and follows what they are teaching, because they are then able to carefully design instruction which builds on the past and prepares for the future. In this way, teaching and learning develops students understanding of mathematics as richly connected schema – a network of connected nodes which facilitates recall (it is easier to remember a structure than a collection of individual pieces of information) and problem solving (connections between like problems leads to the realisation and transfer of appropriate solution strategies).

In the following two sub-sections, the major sequence of mathematical development promoted by YDM and the big ideas of mathematics are elaborated upon. Also, the important role that representations play in terms of developing students connected understanding is also discussed. This leads to a summary of the structured sequence employed in XLR8, further details of which are elaborated upon in the Curriculum Overview chapter.

Major sequences and connections

European mathematics grew out of two views of reality: the first was number, the amount of discrete objects present; and the second was the world around us, the shapes and the spaces we live in. The basis of number was the unit, the one. Large numbers were formed by grouping these ones, and small numbers (e.g., fractions) by partitioning these ones into equal parts. The operations of addition and multiplication, and the inverse operations of subtraction and division, were actions on these ones which joined and separated sets of numbers.

Algebra was constructed by generalising number and arithmetic, and representing general results with letters. *Figure 2* illustrates this relationship diagrammatically.

Number, Operations and Algebra, with input from Geometry, gave rise to applications within Measurement, and Statistics and Probability. This relationship is diagrammatically represented by *Figure 3*. This gives a framework for Years PP-9 mathematics that enables mathematics as a whole to be considered. It provides an overview and sequence for the connections upon which teaching should be built (e.g., Number and Geometry before Measurement; relationship between Probability and Fractions).

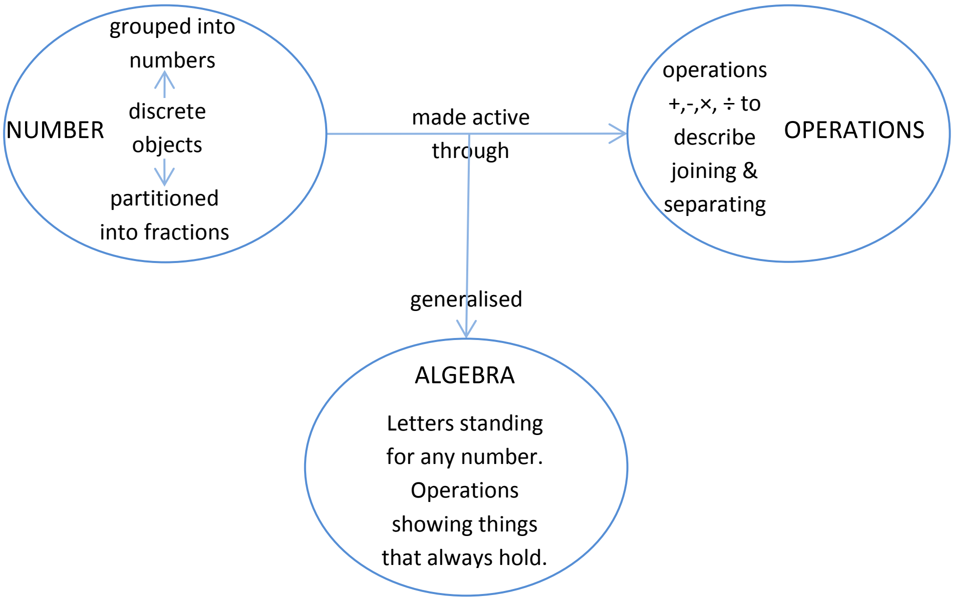


Figure 2. Sequences/connections between Number Operations and Algebra

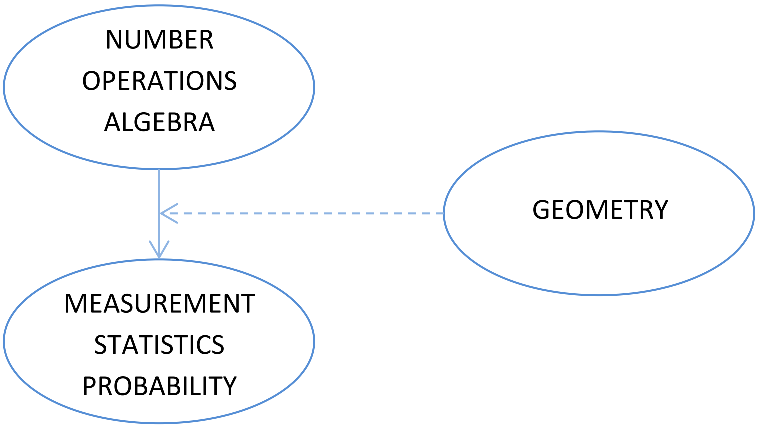


Figure 3. Sequence/connections between mathematics strands

Big ideas in mathematics

In mathematics there are ideas that last across many years of a mathematics curriculum and apply to many different contexts – YDM refers to these as the *big ideas*. YDM promotes the big ideas as an excellent focus for the teaching of mathematics, since the learning a big idea covers so much of mathematics. For example, the concept of addition holds for whole numbers, decimal numbers, common fractions, algebra and functions – it can be taught in Year 1 and still be applicable in Year 12. The big ideas of mathematics can themselves be organised into three groups: the concepts encountered within the various domains of mathematics; the strategies used to solve problems (usually applicable across the domains of mathematics); and the abstract principles that span within and between the mathematical domains. These three groups of big ideas are introduced in the following subsections.

Concepts

These include concept of number, concept of place value, concept of addition, concept of fraction and so on. Many concepts can be described in terms of sub-concepts. For example, the concept of fraction is composed of several sub-concepts (or meanings) – fraction as part of a whole (3/4 is a whole partitioned into 4 parts and taking 3 of these) , fraction as part of a set (3/4 is considering a set of objects is one whole partitioned into 4 parts and taking 3 of these), fraction as a single point on a number line (3/4 is a point on the line between 0 and 1 that is 3 parts out of 4 along the line), fraction as division (3/4 is 3÷4), and fraction as multiplier or operator (3/4 means to multiply by 3 and divide by 4). Concepts tend to be related to particular topics (e.g., number, fractions) and are described in the relevant unit booklets.

Strategies

These include the ways in which problems and examples are solved. They consist of generic strategies such "make a drawing" or "break into parts"; but they also cover specific strategies related to strands, such as those related to addition (e.g., "separate into parts, do parts separately, and combine", "keep one section as is, and combine with parts for other section in sequence" and "do something simpler and then compensate". Both generic and specific strategies are discussed in the relevant sections of the XLR8 unit booklets.

Principles

Principles are the relationships that recur across the years of mathematics. For example, inverse shows how two things are opposite to, or undo, each other such as: forward – backward, right – left, clockwise – anticlockwise; expansion – contraction; addition – subtraction, multiplication – division; square – square root, x^n – x^{-n} (multiplication), x^n – $x^{1/n}$ (power), differentiation – integration; and so on. Thus, principles are predominantly independent of context or topic. A list of principles can be found in the appendix 'Principle Big Ideas'.

Role of representations

XLR8 emphasises the careful selection of representations to express mathematical concepts. The XLR8 curriculum, which is based upon the structured sequence-based development of mathematical concepts, will have the following characteristics with regards to the use of representations:

1. They will have strong isomorphism (or similarities) to the mathematical concept to be learnt, will not have features that distract from the mathematical concept to be learnt, and are able to be extended to express more complex concepts and/or be applied to new or different situations.
2. They will be sequenced such that: their use becomes increasingly flexible and follow the general sequence from concrete to symbolic (from informal to formal); they are increasingly de-contextualised; they are increasingly powerful (they provide increasing coverage, such that later representations compensate for the limitations of earlier representations); and they remain continuously connected to reality.

3. The development of complex concepts can be supported by integrating more than one representation to express the concepts.
4. The development of abstract mathematical concepts is facilitated by comparing the representations used to highlight commonalities that represent the kernel of the mathematical idea.

XLR8 Structured Sequence

The chunking imperative suggests that big ideas should be the basis of a unit of work that covers what would be typically covered in many years of mathematics. Big ideas are best developed across such a collection of instructional activities – as Warren and Cooper (2009) argue, big ideas are built through carefully structured sequences of activities that span within and between domains of mathematics. Big ideas emphasise the centrality of models (ways of thinking about abstract concepts, often metaphorically) and representations (ways of expressing the models, including concretely, pictorially and with written or spoken language). Big idea-based instruction leverages these models and representations to scaffold learners understanding of the big ideas.

The structured sequence-based design of the curriculum is the centrepiece of the XLR8 project. The proposed XLR8 curriculum is characterised as vertical, because unlike a traditional high school curriculum that considers all sub-domains of mathematics in a given year, and then revisits these again in the next year, the vertical curriculum will consider only some of the sub-domains in a given year and consider those sub-domains in depth, from a low to high level of complexity. Previous trials of such a vertical curriculum in other YDC projects has shown that early teaching and learning (that covers the mathematical ideas typically encountered in lower primary years of school) have to be completed slowly and carefully to build the connections that frame out the big mathematics ideas, but then the later stages of teaching and learning can be covered quickly, often resulting in gestalt-like leaps of understanding.

Metaphorically, the resultant organisation of the XLR8 curriculum is described as a tall tree trunk with lateral branches. The trunk of the curriculum develops number, operations and algebra concepts. The lateral braches connect the trunk to concepts from the other strands of mathematics: measurement, geometry, statistics and probability. In this way, the other strands provide the contexts to which number, operations and algebra concepts can be applied. This is consistent with the overarching description of the mathematics strands presented in *Figure 3*.

A summary of the tall trunk and the connections made through the lateral branches is summarised by the genetic decomposition presented in *Figure 4*. In that diagram, a collection of concepts and their connections with one another is presented. The open diamond notation is used to show how concepts are coordinated together to constitute more complex concepts. Through the vertical middle section of the diagram can be seen the structure of number, operations and algebra related concepts. Through that centre section can be seen the parallel co-development of number and operation ideas, leading to the concepts of real number and real number operations. These concepts in turn feed into the concepts of ratio relationships and linear relationships. To the right, measurement and geometry concepts develop and become more sophisticated as the number, operations and algebra concepts also become more sophisticated. Similarly, the probability and statistics ideas develop, with a strong reliance upon real numbers to quantify both probabilities and statistical measures. Thus, the diagram summarises the structure of mathematical ideas embedded in the XLR8 project and upon which the curriculum's structured sequence is based. Further details of the XLR8 structured sequence, including its implementation as a series of units, is presented in the following chapter.

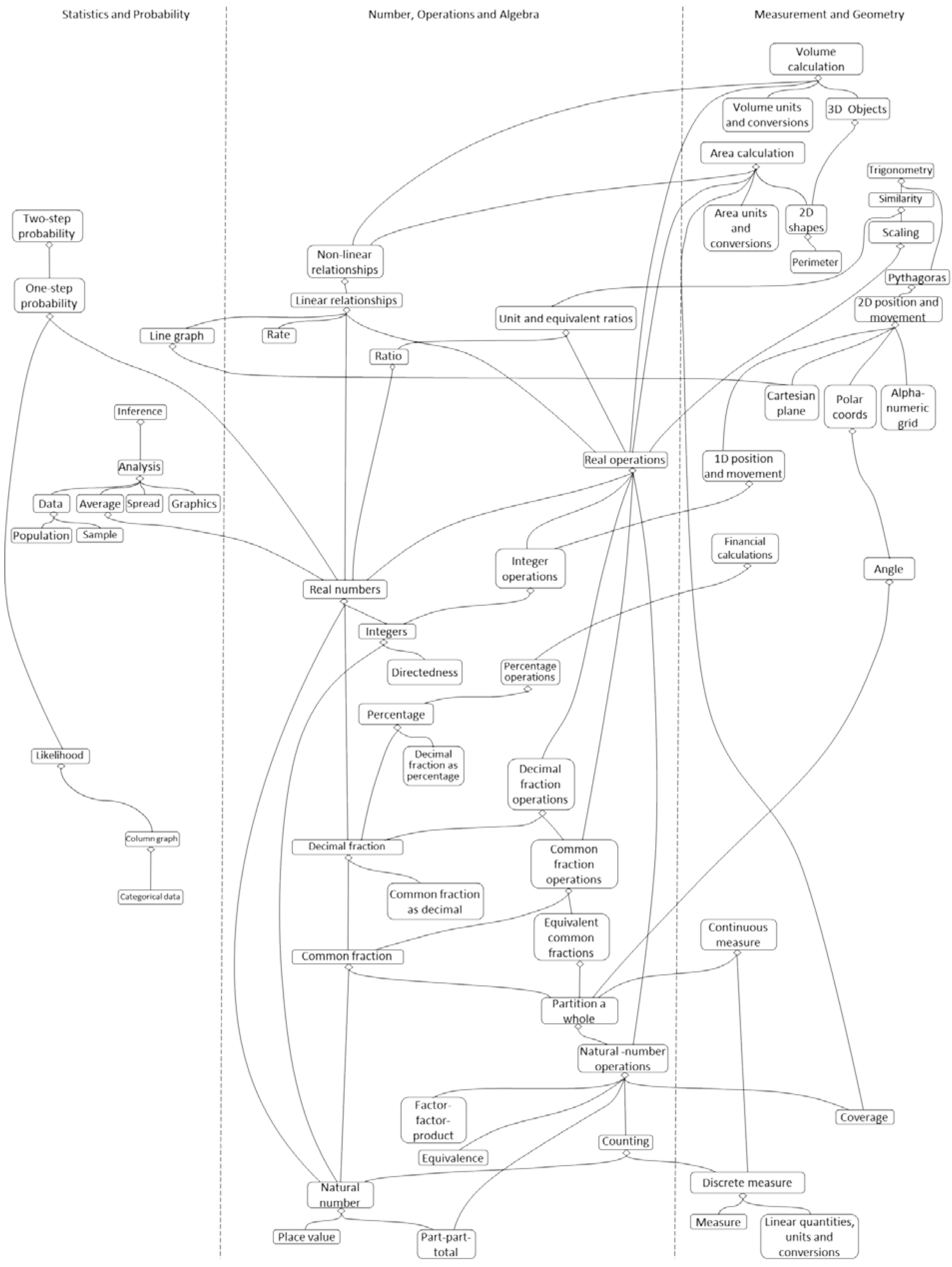


Figure 4. Genetic decomposition of XLR8 structure

Pillar 2 – RAMR Pedagogy

The following sections firstly introduce the underlying philosophy regarding being and learning mathematics. Then, the pedagogical framework promoted in XLR8 is introduced and discussed, including references to other pedagogical theories or frameworks upon which it is based.

Learning philosophy

The pedagogy promoted in XLR8 draws upon the YDC's extensive Indigenous research regarding the learning of mathematics has identified that Indigenous students prefer to experience mathematics within an authentic rather than school-like context (e.g., Cooper, Baturo, Ewing, Duus, & Moore, 2007; Martin, LaCroix, & Fownes, 2005; McGlusky & Thaker, 2006), that is, "doing real work that really matters" (Keller, 2007, p. 2). If a mathematics idea is embedded in something of value to the learner's community, this something provides motivation, a community connection and, most importantly, a framework in which to understand the idea, particularly in vocational contexts (Ewing, Cooper, Baturo, Matthews, & Sun, 2010; O'Callaghan, 2005). This is in line with growing global recognition of context-based initiatives (Cooper et al., 2007) and it is evident that this also applies to many of the populations that make up underperforming low SES students (Baker et al., 2006).

The YDM pedagogy has been influenced by the work of Dr Chris Matthews (Matthews, 2006) who is an Indigenous Australian applied mathematician. Matthews provided a description of what 'doing maths' meant for him. He described doing mathematics as starting from observations in a perceived *reality*. An aspect of a real-life situation is selected and *abstracted* and expressed using a range of *symbols*. The resulting *mathematics* is used to explain reality and solve problems. It is validated and extended by being *critically reflected* back to reality. The cycle from reality to mathematics and back means that abstraction and reflection are *creative* acts. The invented mathematics as a structure, language and problem-solving tool is built around *symbols*. The mathematics and how it is used in to explain and problem solve reality is framed by the *cultural bias* of the person. The act of abstraction requires learners to move from reality to symbols, and the act of reflection requires learners to extend this knowledge by relating symbols back to reality. This cyclic process is encapsulated in *Figure 5*.

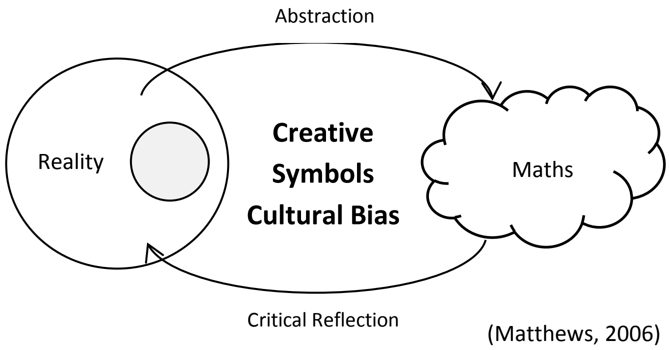


Figure 5. Relationship between perceived reality and invented mathematics

Creativity, symbols and cultural bias are central features of the process described in *Figure 5*. The first, *creativity*, is particularly evident in the acts of abstraction and critical reflection. In this regard, it is important to note that mathematics, as a way of working with the abstract, is similar to other artistic pursuits such as dance, music, painting and language. Therefore, mathematics can be considered as another art form and, in theory, relates to these other forms of abstractions. In essence, it is possible to develop empowering pedagogy that allows students to be creative and express themselves in the mathematics classroom. This would allow students to learn mathematics from their current knowledge (i.e., from the students' social and cultural background), thereby providing agency through creativity and ownership over their learning.

Secondly, as a product of the abstraction process, *symbols* and their meanings are important features of the process since they connect the abstract representation with reality. However, it is common that students do not make these connections easily and view mathematics as just sums with no real meaning. This is further exacerbated for students when they first learn algebra, and letters are suddenly introduced into mathematics without any obvious reason (except that "we are now learning algebra"). Interestingly, focusing on creativity within mathematics provides the opportunity for students to generate their own symbols to represent their understanding of the mathematical process. These symbol systems can then be compared to and assist in understanding the meanings of current symbols, symbolic language and their connection to reality. In addition, this can also lead to the teaching and learning of the underlying structure of mathematics, providing students with a holistic view of mathematics.

Thirdly, the feature of *cultural bias* exists in all aspects of the abstraction and critical reflection cycle. The observer expresses their cultural bias in the way they perceive reality and decide on which aspect of reality they wish to focus. In the abstraction process, the form a symbol takes and the meanings that are attached to this symbol or group of symbols is biased by a cultural perspective. Finally, the critical reflection processes are underpinned by the cultural bias within the abstraction process and the observer's perception of reality. If we have an understanding and appreciation of the cultural bias within mathematics, new innovative pedagogy can be developed that moves beyond some cultural biases so that students can relate to mathematics but also gain a deep understanding for the current form of mathematics and how mathematics is used.

The RAMR Pedagogical Framework

Consideration of Matthews' work, along with the experiences of the YDC researchers, led to the formulation of the RAMR pedagogical framework as a basis for YDM. The RAMR framework is the basis for the design of the XLR8 curriculum in terms of the activities used to explore the structured sequence. This framework uses the relationships between reality and mathematics as a cycle for planning. In this section the RAMR pedagogy is introduced and links are made to other theories of learning and pedagogy which have been incorporated into it.

> Not only will the RAMR model be central to the design of instructional activities, it will also be central to the XLR8 project from affective and cultural perspectives. Innumeracy, or the inability to use mathematics to effectively meet the general demands of life, arises when students have not experienced authentic contexts in which to develop a positive disposition to use mathematical concepts and skills, mathematical thinking strategies and more general problem-solving skills. Without such experience, the student is unlikely to develop a deep understanding of the structure of mathematical ideas.

The Wilson cycle (see Ashlock, et al., 1983) is a framework that has been used by YuMi Deadly Centre staff for many years with success. Within the Wilson cycle are six distinct types of learning activities, as depicted in *Figure 6*. Whilst teaching and learning can begin in any of the five activities on the perimeter of the model (and continue in a clockwise direction), assessment to establish prerequisite understanding and then ongoing assessment to check student understanding accompanies each of the other five activities.

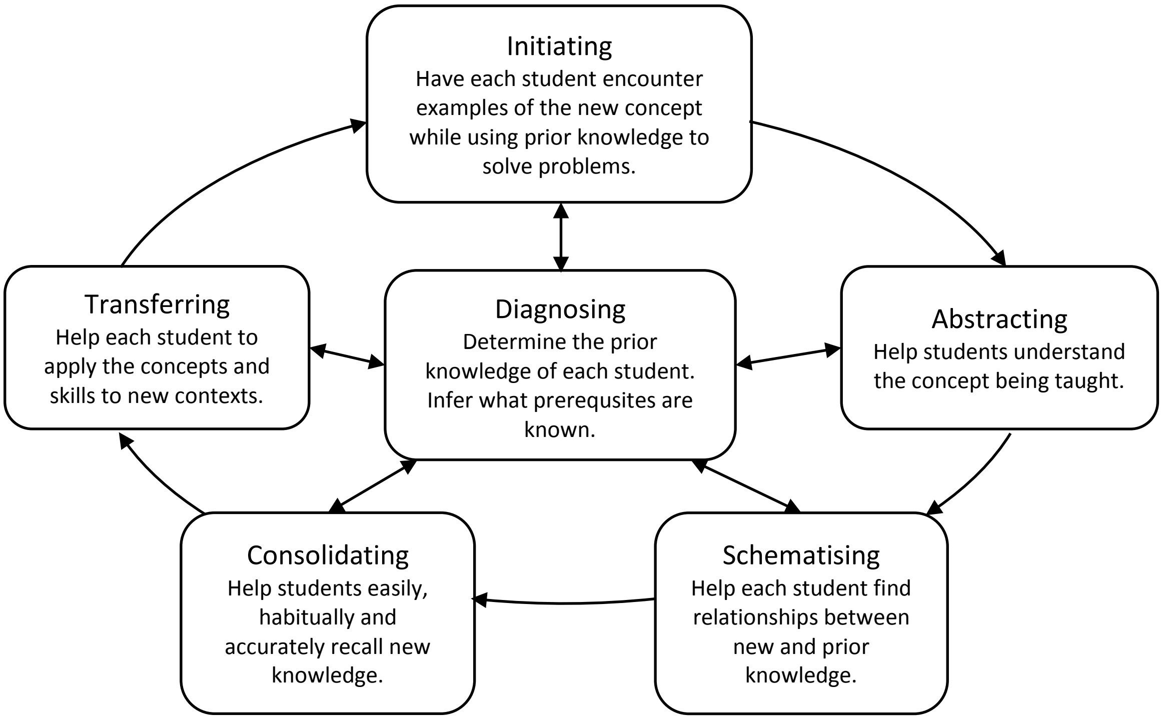


Figure 6. Wilson cycle

The RAMR framework has emerged from combining the activity types of the Wilson model with the process of doing mathematics described by Matthews. The resultant framework is comprised of four distinct phases, of instruction: 1) Reality; 2) Abstraction; 3) Mathematics; and 4) Reflection. This sequence of instruction is depicted in *Figure 7*, including a summary of what each phase involves. Further details of each phase in regards to the learning of a new idea (or organisation of related ideas) are then provided in the sub-sections that follow.

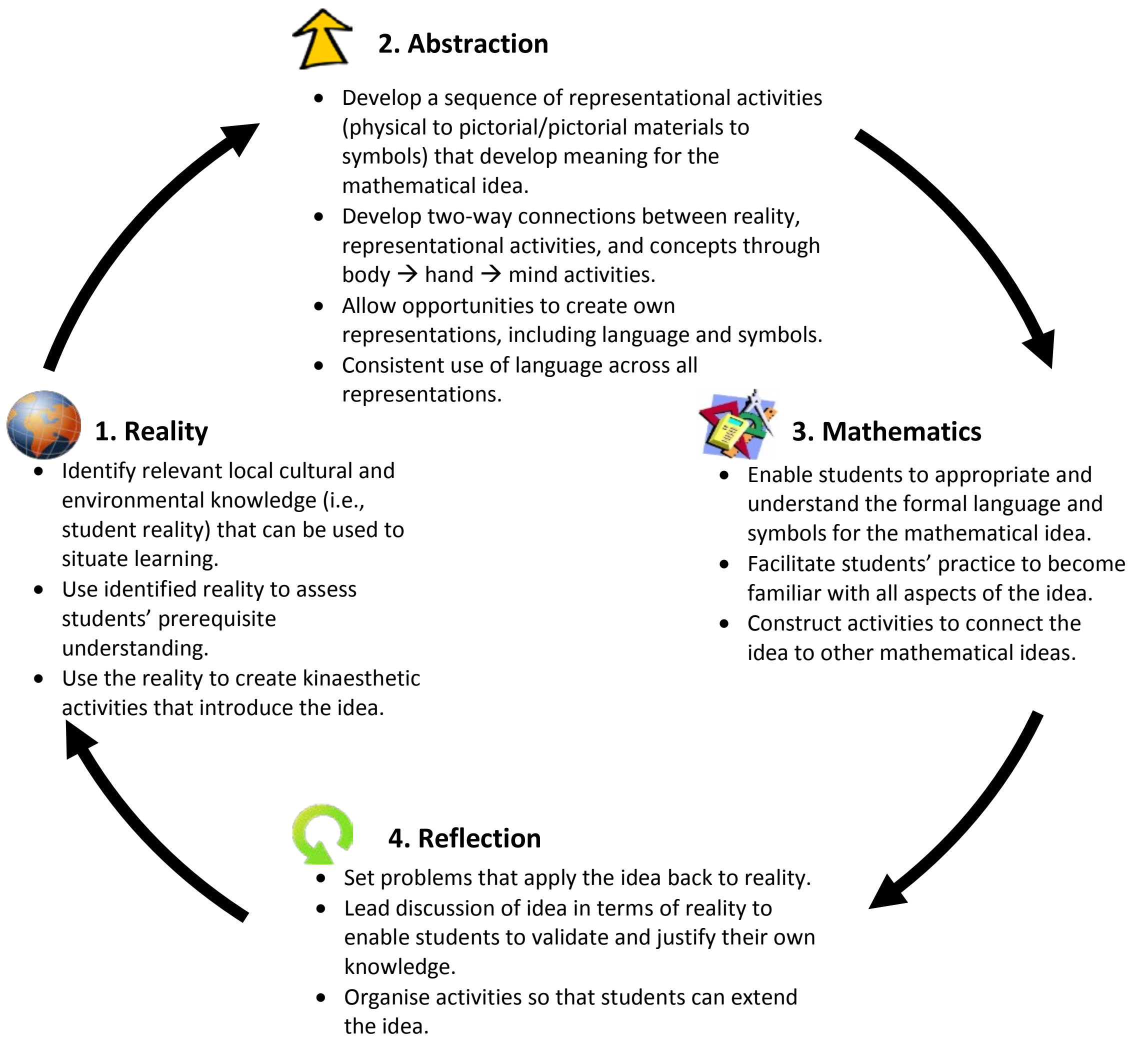


Figure 7. The RAMR Cycle

The RAMR framework also aligns to the ideas of Baturo (1998), who modified Leinhardt (1990) to identify four knowledge types: (1) entry (knowledge of mathematical ideas before instruction); (2) representational (knowledge of physical materials and pictures used to develop the ideas); (3) procedural (knowledge of definitions, rules and algorithms); and (4) structural (knowledge of relationships and concepts). All four types have to be developed in a sequence of learning, with the final goal being structural. The focus upon each of these knowledge types across the RAMR phases is presented in *Figure 8*.

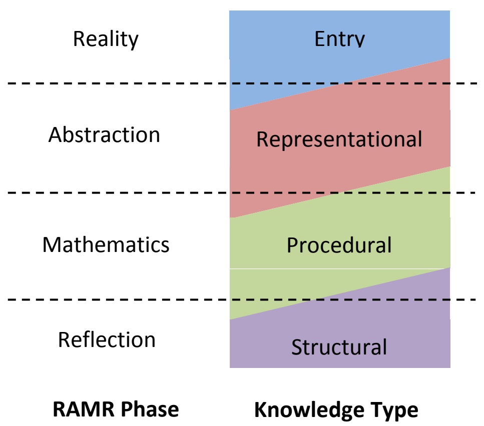


Figure 8. Comparison of RAMR phase and knowledge types

When considering the RAMR framework and the planning of instruction, three levels of instruction should be taken into account (Baturo et al., 2007):

- **Technical** – becoming familiar and proficient with the use of materials.
- **Domain** – knowing what materials and what activities will provide experiences effective for learning the topic being taught.
- **Generic** – knowing the reflective pedagogical strategies of:
    - **Flexibility** - experiencing the mathematical idea many ways (e.g., describing a collection using various numbers names, such as 1 hundred, 2 tens and 5 ones cf. 12 tens and 5 ones).
    - **Reversing** - teaching in the opposite direction (e.g., whole to a fraction and then fraction to a whole).
    - **Generalising** - developing the idea into a rule that can be applied to a variety of situations (e.g., part-part-total).
    - **Changing parameters** - considering what would happen if something changed (e.g.,).

Importantly, along with continuous assessment of learner's understanding and progress (as was central to the Wilson model), the generic strategies should be used throughout the RAMR cycle, especially during the development of structural knowledge in the mathematics and reflection phases. The centrality of these generic strategies, along with continuous assessment, in summarised in *Figure 9*.

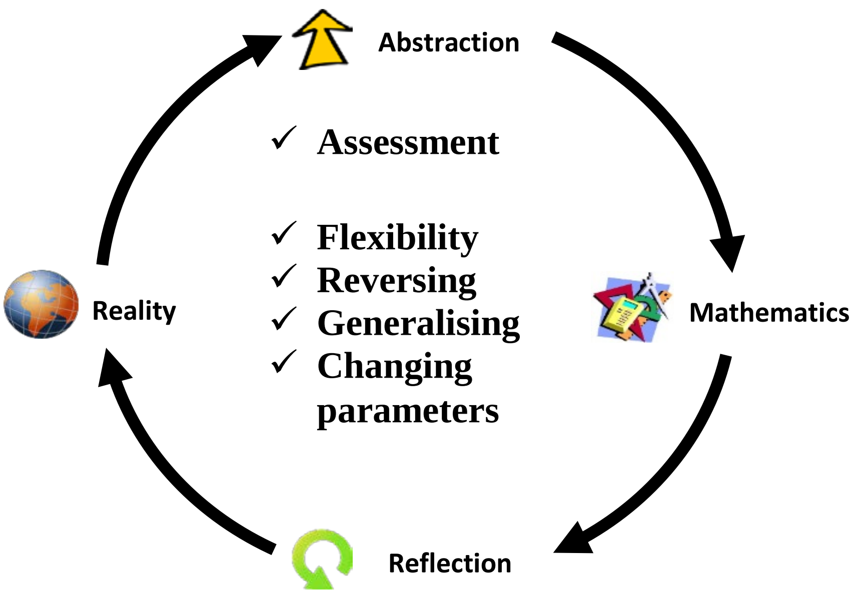


Figure 9. Central pedagogies of RAMR

The four phases of the RAMR framework are now discussed in further detail.

1. Reality

Closely aligned to the Initiate activity type in the Wilson model, the reality component of the cycle is where learners: (1) access knowledge of their environment and culture; (2) utilise existing mathematics knowledge prerequisite to the new mathematical idea; and (3) experience real-world activities related to the idea. The focus in this component is to situate the new idea in everyday experiences and provide an experiential base for building connections. A reality, and associated activity, should be selected such that a natural transition into the abstraction phase is permitted. That is, the physcial situation should have features that can be represented using a series of increasingly abstract representations. For example, talking about cars in a line waiting at the at the traffic lights naturally lends itself to representation using a length model (number line or number track) which in turn can be represented using a number sentence. Among the kinaesthetic, physical and visualisation activities that predominate in this component, it is vital that learners be provided with opportunities to generate their own experiences and verbalise their own actions. This generation and verbalisation provides the students with ownership over their understanding of the mathematical idea.

2. Abstraction

The abstraction phase is when learners experience a variety of representations, actions and language to expresses the new ideas in increasingly sophisticated or abstract ways and which enable meaning to be developed. That is, students begin by expressing their reality in relation to the new idea using physical context-specific representations and progress to using abstract (and so multi-purpose) representations. During such activity, the progressive abstracton of representations, actions and language will predominantly follow the pattern as shown in *Figure 10*.

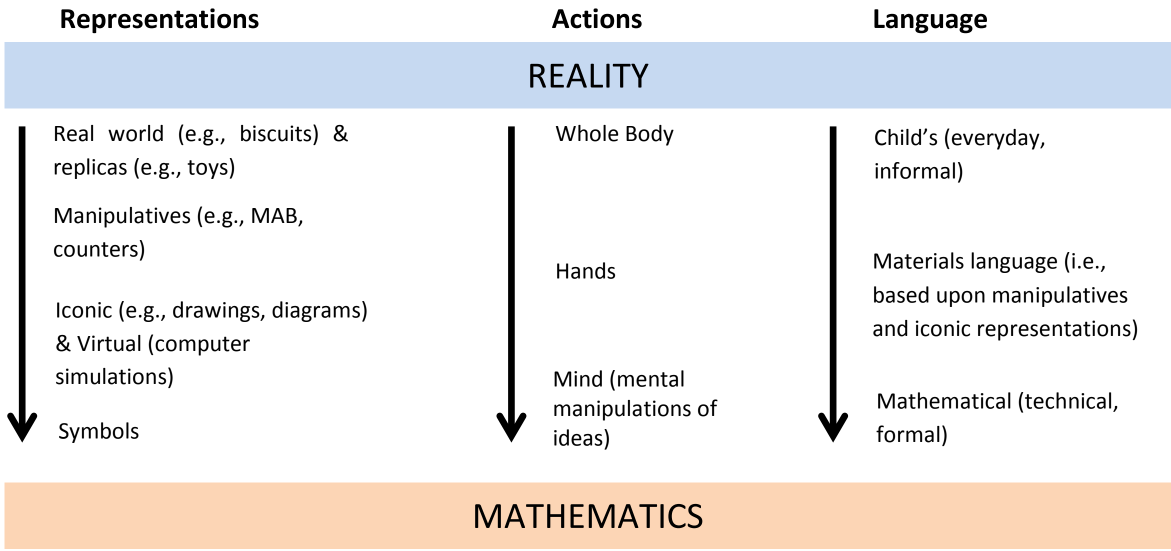


Figure 10. Abstraction sequence from reality to mathematics

During the abstraction phase, learners should also be provided with opportunities to create their own representations (including language and symbols) to express the mathematical idea that has been initially experienced through physical activity. This allows students to have a creative experience that will, firstly, develop meaning and, secondly, attach it to language and symbols. The sharing of other learners' representations provides learners with alternative views of the same idea attached to varied representations, including those that are more conventional (e.g., jumps on a number line, number sentences). Discussions on the use of different symbols enables learners to: (1) critically reflect on their journey (enabling them to justify and "prove" their ideas); (2) understand the role of symbols in mathematics (enabling them to understand the relation between symbol, meaning and reality); and (3) be ready to appropriate (Ernest, 2005) the commonly accepted or conventional symbols of Eurocentric mathematics.

The progressive abstraction from reality to mathematics is based upon several theories regarding the use of representations and language in mathematics education, which are now described.

As a framework, Payne & Rathmell's (1977) model, often referred to as the Payne-Rathmell triangle, relates real-world situations, graphical or materials-based representations, language and mathematical symbols. For example, as shown in *Figure 11*, the real-world problem that embodies the concept of a third can be expressed using a length model graphical representation, the symbol $\frac{1}{3}$ and the language 'one third'. Importantly, Payne and Rathmell emphasised the importance of learners moving between the various ways of representing the concept and that the language used, as learners make use of these various forms, should be consistent.

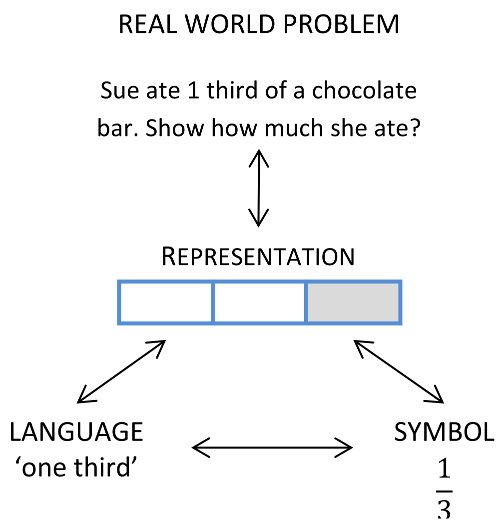


Figure 11. Payne and Rathmell triangle framework for teaching fraction 1/3

The seminal work of Bruner (1966) identified three distinct forms or ways by which a concept might be expressed: the concrete, action based enactive representation; the visual or sensory iconic representation; and, the abstract, context or action free symbolic representation. Enactive representations characterise the state of doing, they include specific details of action. Iconic representations are a synopsis of the enactment, formed after the activity has occurred (Presno, 1997). The symbolic representations are disconnected from reality, and make use of language, including words and numbers. At each level of abstraction, the representations are facsimiles of actual experience and are used by individuals to make sense of the world around them (Presno, 1997).

When the Payne-Rathmell triangle and Bruner's three levels of representational abstraction are considered together, the importance of language as the mediator of meaning can be seen. This bringing together of Payne-Rathmell and Bruner is summarised in *Figure 12*.

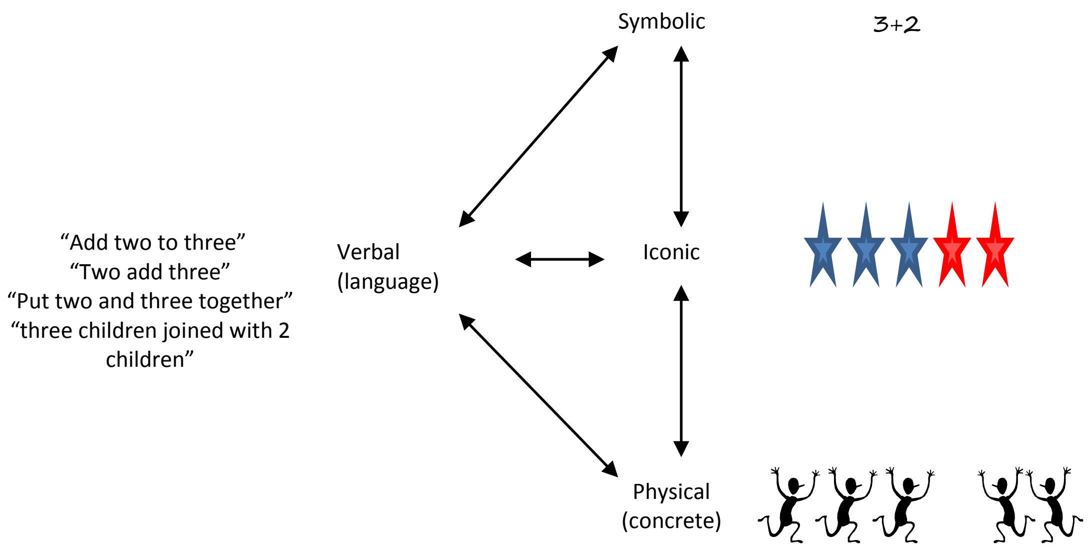


Figure 12. Language mediated abstraction of representation

In the example given in *Figure 11*, a fraction strip – perhaps a folded piece of paper – was used to represent the third. Duval (1999) argued that mathematics comprehension results from the coordination of at least two representational forms or registers: the multifunctional registers of natural language and figures/diagrams; and the mono-functional registers of symbols. Learning is deepest when students can integrate registers. This can be extended when a range of graphical representations (e.g., set, length, area) are used and learners are encouraged to flexibly move between them. For example, in *Figure 13* the one third of chocolate is expressed using a variety of representations.

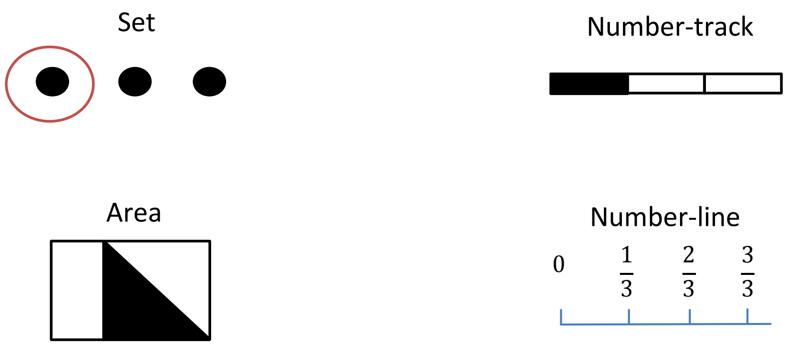


Figure 13. Various graphical representations of 'one third'

YDM uses a material called a **thinkboard** which is an A3 sheet divided into five regions – one for symbols, one for language, one for drawing, one for materials and one for a real-world story, as shown in *Figure 14*. The regions in the thinkboard could be change, to encourage students to use a variety of materials or graphical representations. For example, students could be asked to show how to represent "15 subtracted from 32 equals?" using both sets and number-lines and to write appropriate every-day stories.

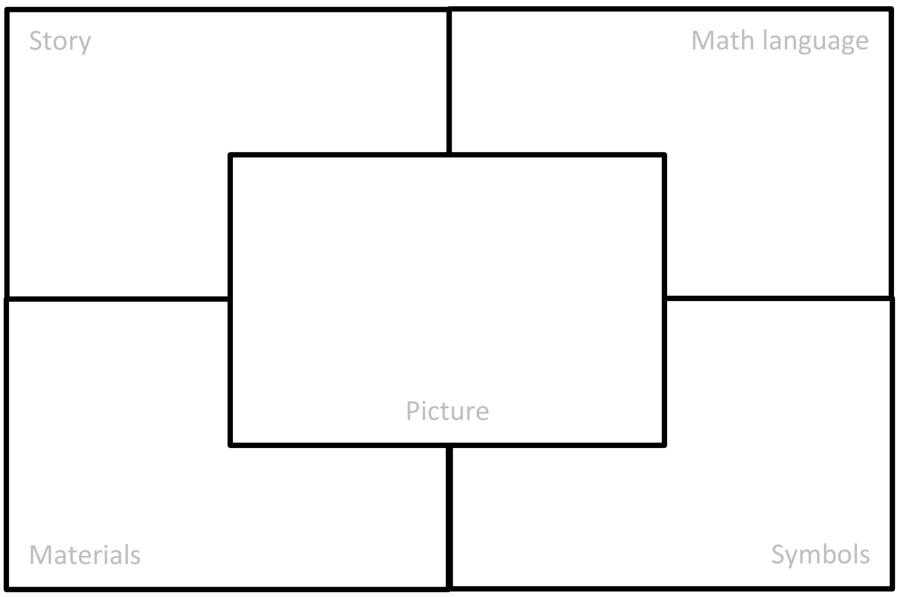


Figure 14. A Thinkboard

Early in the abstraction phase it makes sense to focus students on a single form of representation. However, later in the abstraction phase, alternate representations may be introduced to represent the learner's reality. The features of these representations should be compared and contrasted to aid students' flexible movement between them.

3. Mathematics

The mathematics component of the cycle is where learners: (1) appropriate and practice the formal or conventional language and symbols of mathematics; (2) reinforce the knowledge they have gained during the abstraction; and (3) build connections with other related mathematical ideas. The focus is to assist learners to construct their own set of tools (filling their “mathematical toolbox”) that will enable them to recognise and recall mathematical ideas from the language and symbols associated with the ideas, thus adding to their bank of accessible knowledge. The connections between new and existing ideas enable better recall of mathematical ideas and improve problem solving.

During this phase, as a part of the building of connections, it is appropriate to leverage the experiences of the preceding reality and abstraction phases to introduce additional new ideas. For example, in the preceding reality and abstraction phases, and then into the mathematics phase, the concept of adding fractions with like denominators may have been developed. Then, once this concept has been developed, the previously developed idea of equivalent fractions may be drawn upon to add fractions with unlike denominators. Rather than explicitly repeat the entire reality-abstraction-mathematics sequence, the previous experiences may be drawn upon to quickly move to working with these more complex fractional amounts. Alternatively, the new idea introduced during the mathematics phase could be an alternate representation: the reality and abstration phases maybe have used a set model to operations on counted quantities and then during mathematics a number line may be intoduced as an alternate way representation to express the same operations.

4. Reflection

The reflection process is critically important for three reasons:

- It provides an opportunity for learners to apply, or transfer, their new knowledge back to their reality in order to solve everyday life problems.
- It provides an opportunity for learners to validate/justify their knowledge and, when misconceptions are identified, refine their knowledge
- It provides an opportunity to extend understanding. For example, learners can reflect on 3+4=7 and see that if one addend, say the 3, was reduced by 2, then the sum, 7, has to be reduced by 2 to keep the equation equal, which marks the beginning of the balance rule.

As well as reflecting on the mathematics they have learnt in relation to the world they live in, the reflection phase should involve learners considering their journey from reality to mathematics via abstraction that they took in developing the mathematical ideas. It requires thinking about what they learnt, how they learnt it, and why they learnt it. It also requires them to justify their outcome. Such reflection on learning and the validation of knowedge against their everyday life is valuable because it leads learners towards ownership of their learning and their knowledge.

Pillar 3 – Professional learning

To achieve the goals of XLR8 will require teachers to be prepared and able to teach below the nominal year level and to focus their teaching on the topics presented in the XLR8 curriculum, sometimes disregarding or ‘putting on hold’ their ideas regarding what should be taught in the particular year level. To enable this requires effective and pertinent professional learning and support to equip teachers with the knowledges needed to achieve students’ accelerated learning.

Teacher knowledges for XLR8

Low SES students and, particularly, Indigenous students, tend to be holistic in learning style, moving from whole to parts, and not aligned with traditional procedural/algorithmic teaching which moves part to whole. To take advantage of this, the approach to mathematics teaching advocated in XLR8 focuses on big ideas (concepts, strategies and principles), vertically sequenced units, and the RAMR cycle. However, this is a teaching approach that requires a lot from teachers – understanding of mathematics structure, active pedagogy and classroom control of behaviour. The XLR8 pedagogy also relies on teachers making decisions in regard to instruction themselves, based on their understanding of mathematics and the knowledge of the individual students. XLR8 provides teaching ideas and activities but not in the form of scripts or "recipes".

Thus, XLR8 pedagogy requires teachers to know all three of Shulman's knowledge types for effective mathematics teaching: subject-matter (knowledge of mathematics content in terms of how it is structured, sequenced and connected), pedagogic content (knowledge of how to teach mathematics) and lesson planning (general knowledge of how to organise and run a lesson, including behaviour management). *Figure 15* visually illustrates these three knowledge types. To facilitate this, the activities of XLR8 are designed to build the capacity of teachers. However, in the past, it has taken time for teachers to come to terms with what was required to effectively implement an XLR8 type mathematics program.

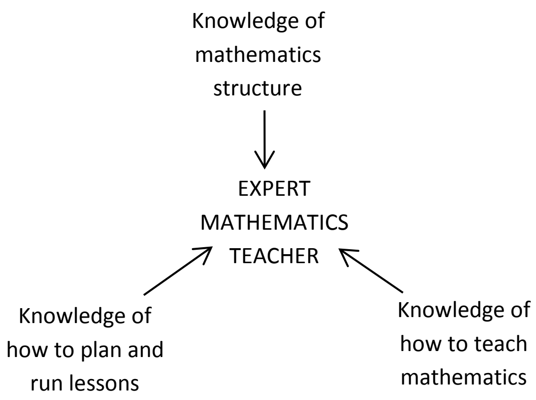
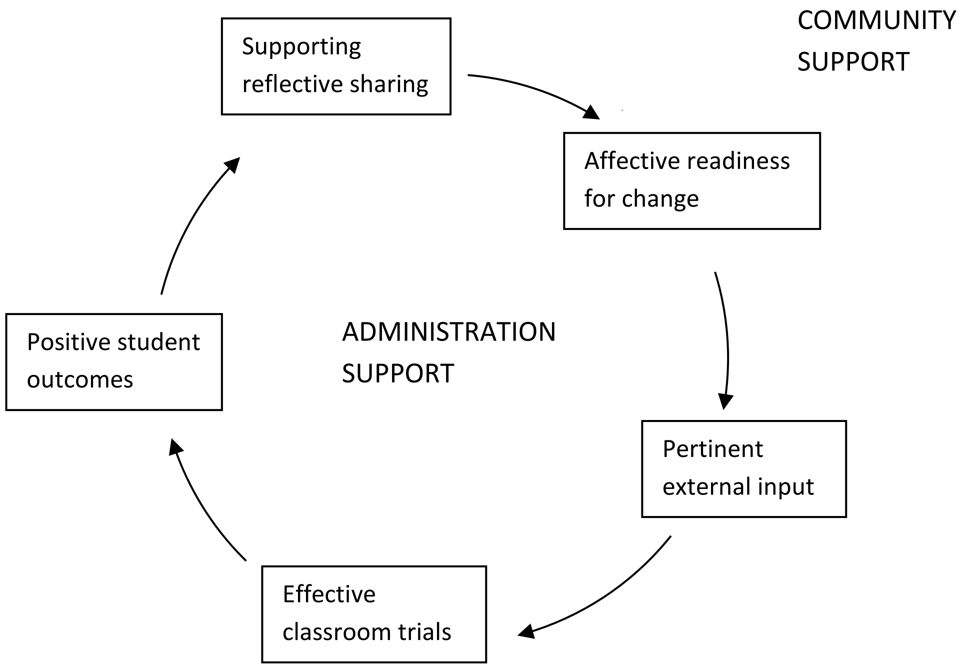


Figure 15. Shulmans's knowledge types

Similar to teachers, teacher aides need subject-matter, pedagogic-content and lesson-planning knowledge. XLR8 believes in providing teacher aides with appropriate PD, however, the knowledge is usually placed more within the framework of the particular units and within the framework of their role in the classroom. This does not mean simplification because the highest quality knowledge is required for 1:1 or 1:many tutoring.

The role of professional learning and support

XLR8 sees professional learning and changing teachers' classroom practices being a cycle of affective readiness, pertinent external input, effective classroom trials, positive student responses, and supportive reflective sharing leads to further readiness and so on (see *Figure 16*).



(adapted from Clarke & Peters, 1993).

Figure 16. YDC effective professional learning cycle

To build the capacity of the XLR8 project schools to teach mathematics effectively the development, trial and refinement of the XLR8 curriculum (as associated support of teachers) is sequenced to generally follow four steps (as represented in *Figure 17*): (a) mathematics resources are being developed (with appropriate tests), (b) teachers are being provided with professional learning and school visits, (c) the teachers are being asked to trial the resources and keep records of effectiveness, and (d) students are being tested at the start and end of the trials. Along with teacher feedback, the post test data results will be used as the basis for the improvement of the XLR8 curriculum.

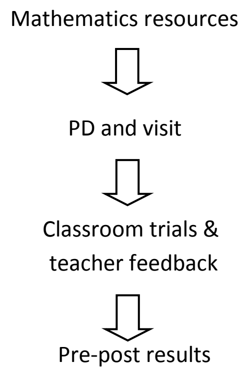


Figure 17. Student evaluation process

XLR8 acknowledges the importance of the interaction between researcher input and teacher need and readiness for this input, and the role of success (in terms of student outcomes) when trialling new ideas. It recognises that positive student responses along with initial teacher-readiness are crucial to successful pedagogical change. Factors that will influence the success of teachers success using the XLR8 curriculum (and the XLR8 curriculum itself) include: (a) pertinent, relevant and innovative ideas and resources as input (the XLR8 resources and PD workshops); (b) just-in-time support before and during classroom trials (assistance with planning, classroom visits to provide in-class support and feedback on teaching); (c) gathering data, in an action research process, which will assess student outcomes; and (d) providing opportunities for teachers to share their successes and to attribute them to their ability. XLR8 also recognises the important role of principal (and other administrative staff) and the local community. Without the support of the principal and other senior administrators, few if any interventions succeed in changing school practices. The same is true without the support of the local school community.

XLR8 recognises that input provided by YDC staff through resources and professional learning workshops is quite removed from student responses in assessment tasks. YDC staff have control over resources and professional learning and, through visits, some involvement in school policies and classroom teaching. However, much of the teaching that affects student outcomes will be undertaken by teachers without involvement of YDC staff. And it is this teaching that determines the central outcome of student success that determines effectiveness of intervention. Thus supporting the professional practice of XLR8 teachers is paramount to the project's success in terms of student learning outcomes.

XLR8 Curriculum Overview

Following on from the previous sections, the XLR8 curriculum is modular: each teaching term of the two-year curriculum (i.e., Year 8 and Year 9) participants will complete two units that each use a vertical approach to address the big ideas of mathematics. The units comprise cycles of work sequenced such that they build big ideas.

The vertical curriculum of the XLR8 project is based on the hypothesis that a structured sequence presentation of mathematics will enable the acceleration of mathematics learning. The XLR8 curriculum is presented as a sequence of 15 units spanning two or three years. The proposal is that by following a carefully designed sequence of units, each of which contains a vertical sequence of constituent units, students will be able to gain the mathematics knowledge normally covered in 4-6 years in 2-3 years. In this sequence earlier instruction will lay a strong foundation for later learning.

The XLR8 curriculum has five key features: 1) the structure of mathematical ideas embodied in each of the units; 2) the conceptual sequence by which the units and their cycles explore the structure; 3) the RAMR pedagogy that is described in each of the cycles with regard to the corresponding content (i.e., to follow the structured sequence); 4) the resources used to implement the structured sequence using the RAMR pedagogy; and 5) the assessment materials that generate diagnostic, formative and summative evidence of students' mathematical understanding.

In the following, the modular structure of the curriculum is summarised and then a single cycle taken from one unit is presented to highlight it's key features. Following that, the assessment used in XLR8 to inform teaching, for reporting and for research purposed is discussed.

Unit Structure

The XLR8 curriculum has been refined into a set of 15 units (labelled Unit 01 – Unit 15) to be taught over a 2 year period. Each unit has been designed to be nominally 5 weeks in duration, however it is anticipated that different classes may (and should) progress through these units at differing speeds, depending on the particular needs of each class. In addition to the 15 units, the XLR8 curriculum provides a set of diagnostic, summative and portfolio tasks. These have been provided as a basis for generating broader evidence of students' abilities and for encountering mathematical topics or ideas that span across the curriculum (e.g., problem-solving strategies).

The two year modular structure of XLR8 is summarised in *Figure 18*.

Year A		Year B	
Unit 01:	Comparing, counting and representing quantity	**Unit 08:**	Calculating coverage
Unit 02:	Additive change of quantities	**Unit 09:**	Measuring and maintaining ratios of quantities
Unit 03:	Multiplicative change of quantities	**Unit 10:**	Summarising data with statistics
Unit 04:	Investigating, measuring and changing shapes	**Unit 11:**	Describing location and movement
Unit 05:	Dealing with remainders	**Unit 12:**	Enlarging maps and plans
Unit 06:	Operations with fractions and decimals	**Unit 13:**	Modelling with linear relationships
Unit 07:	Percentages	**Unit 14:**	Volume of 3D objects
		Unit 15:	Probability

Figure 18. Summary of XLR8 units

Figure 18 summarises the units by their title. Within each of the corresponding unit booklets a more detailed description of the content and rationale for the structured sequence is given. A brief summary of the content and approach used in each cycle is provided. Then, each of the cycles in the unit are presented in detail.

The modular structure of the XLR8 curriculum potentially offers the following advantages:

1. They give teachers the freedom to teach at ability level knowing they will end up at age level and accelerate the students' knowledge.
2. It gives the students a sense of progress and makes them aware that they can reach the standard of others. For one school, this is already an outcome of their involvement – the growth in student confidence.
3. It is a really effective way of ensuring students learn in chunks – teachers will stay on a cycle of work around one topic if there is progress.
4. It enables big ideas to be experienced from early maths to later maths and this gives a strong generalisation of the big idea.
5. It enables teachers to see the growth of student ideas through stages and make on-the-run in-class assessment a reality – they can see what their students are learning.
6. Most importantly it allows for the most powerful form of acceleration – the beginning ideas and images build a strong foundation and the tall tree of knowledge leaps upward in an almost gestalt way. For more than one school, the power of this has been seen in how hundreds-tens-ones knowledge explodes up through thousands and millions.

Cycle Structure

Each unit in the XLR8 curriculum is defined in terms of a series of cycles that follow the structured sequence defined for that unit. That is, each successive cycle will build upon the previous to progressively develop the learners' understanding. This will allow teachers to initially teach to the ability level of the learner but then accelerate the learner up to an age-appropriate ability level.

In the following figures, an example cycle (Cycle 1 of Unit 1) is presented to highlight the key features presented in the unit booklets.

Cycle 1: Comparison and Order

Overview

Big Idea

This cycle develops understanding of informal comparison and ordering of attributes to describe more than, same as and less than. These ideas developed physically with measures can then be applied informally to the attribute of quantity before introducing the idea of counting and place-value.

Objectives

By the end of this cycle, students should be able to:

1.1.1 Use direct and indirect comparisons to decide which is longer and explain reasoning in everyday language. [FMG006]

1.1.2 Use direct and indirect comparisons to decide which is heavier and explain reasoning in everyday language. [FMG006]

1.1.3 Use direct and indirect comparisons to decide which holds more and explain reasoning in everyday language. [FMG006]

Conceptual Links

Comparison and order are qualitative ideas that underpin much of measurement and number. Comparison and order of attributes other than quantity will provide background understanding for notion of unit, counting and place value in Cycle 2.

Materials

For Cycle 1 you may need:

- Butchers' paper
- A4 paper
- Variety of items with different lengths, capacities, areas, masses
- Marking pens
- Noticeboard
- Scissors

Key Language

More than, less than, same as, greater than, less than, equal to, equivalent to, attribute language (wide, wider, widest, narrow, narrower, narrowest, long, longer, longest, short, shorter, shortest, …)

Definitions

Direct comparison: comparison of an attribute common to two or more items. For example, direct comparison of length involves holding two or more items together with a common baseline to see which is longer. For example, students may stand back to back to directly compare height to determine who is taller, a handful of pencils may be held with their base on a level surface and the longest or shortest pencil determined

Indirect comparison: comparison of an attribute common to two or more items that are compared using an intermediary. For example, a cupboard and a door may be compared by cutting a length of string to match the cupboard height and comparing the length of the string with the door. If the door is higher than the length of the string, then the door is higher than the cupboard (and the cupboard can fit through). This is a useful forerunner to indirect comparison using non-standard units.

Non-standard units: Any of the same item that can be repeated (or iterated) and counted in order to make an informal measurement of another object. For example, scissors, pencils, pens, paddle-pop sticks, erasers, books, marbles ….

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How can you tell if an item is longer/heavier/holds more than another?

  To ascertain if students understand measure and the significance of units, they must recognise that where units are used, the count of the unit will be greater if an item is longer, heavier, holds more. They must also understand that the same unit must be repeated with no gaps nor overlaps for the measure to be accurate. When comparing two items using a non-standard unit, students must recognise that they must use the same unit for measuring in order to be able to compare. Make sure that students always attach a unit name to the count when measuring (e.g., the table is 12 pencils long and 6 pencils wide).

Portfolio Task

This cycle provides students with comparing and ordering length, mass and capacity experiences which may assist with the task of informally and formally measuring the distance the planes fly. Comparison of size and ordering ideas can be extended to the alternative context of comparing angles within the portfolio task.

Overview – this first major section provides an overview of the cycle

Big Idea – provides a summary of the cycle's content and its sequencing

Objectives – these are learning objectives associated with the cycle. They will be reflected in the nature of the pre/post test questions. The language of these objectives will be used in the reporting of pre/post test results and in the Formative Assessment Tool. XLR8 learning objectives have been mapped to ACARA content descriptors.

Conceptual Links – this describes how this cycle fits in with ideas learnt in other cycles or ideas in other units.

Materials – List of materials that may need to be prepared prior to teaching this unit.

Key Language – list of terminology/ vocabulary used in this cycle.

Definitions – provides definitions of some key terms

Assessment – provides some ideas for collection of evidence of understanding.

Anecdotal Evidence - Where appropriate, provides some ideas of questions and observations that may provide alternative evidence of student learning.

Portfolio Task - Where appropriate, links to aspects of the portfolio tasks.

Figure 19. XLR8 Cycle features - Overview section

RAMR Cycle

Reality

Explore real-life comparison, sorting and ordering. Ensure students can identify an attribute for comparison, for example, length, mass, area or capacity. Discuss instances where comparison is necessary in order to sort and order items.

Abstraction

The abstraction sequence for this cycle explores the concept of comparison for sorting and ordering starting with physical objects and attributes and extends this idea to comparing and ordering quantity. An abstraction sequence to build the concept of comparing and ordering is as follows:

1. *Kinaesthetic activity / Connect to language.* Start by identifying the attribute to be compared. Ensure students have the necessary vocabulary to describe the attribute. Compare and order the attribute. Height is an easy starter. Stand two students back to back to determine tall and taller or short and shorter. Introduce a third student to build to language of tall, taller, tallest.
2. *Model with materials.* Extend students' ability to compare lengths from direct comparison to using an intermediary to compare items. For example, the whiteboard is longer than a metre ruler, a desk is shorter than a metre ruler, the desk is then shorter than the whiteboard (move to less obvious examples).

Mathematics

Language/symbols and practice

Explore other attributes such as capacity or mass. A selection of identical boxes filled with different items can be effective for comparing and sorting mass by hefting. For example, fill sultana boxes with a selection of items (beads, fish tank gravel, hobbyfill, rice, toothpicks, empty box) and have students compare boxes with one in each hand to sort heaviest to lightest. Simple balance scales can be constructed from a coathanger, plastic boxes and string to check for heavier or lighter. The object of activities is to develop comparison vocabulary and the idea that attributes can be compared, sorted and ordered.

Connections

Extend students to comparing and ordering quantities of items. Initially, ensure there are too many small items in each pile for students to subitise (typically more than ten). Have them visually sort and order the piles into more than to less than series. Then use counting to check their response. The key idea is that quantity is another attribute of an item that can be compared, sorted and ordered.

Create growing patterns where piles of items increase in an observable pattern from one pile to the next. Provide a range of these as cards for students to sort into order. For example,

Resource Resource 1.1.1 Comparing and ordering pattern cards

Reflection

Check the idea

Provide students with a range of items to be sorted. Encourage them to identify the attribute they are sorting by. This can also be completed as a comparing and ordering guessing game where one student compares and orders a range of items and other students need to identify from the sorted collection what the student's attribute of comparison was.

Apply the idea

Use a range of countable non-standard units to determine the length of items in the classroom. Compare and order lengths of items in the classroom according to the count of the non-standard unit. Ensure students understand that items must be end-to-end with no gaps or overlaps. For example, length of desk in straws, length of chair in straws, lengths measured using paddle pop sticks.

Similar ideas can be explored using a simple balance and marbles, unifix cubes, pens to compare and order masses.

Extend the idea

Explore quantities of discrete items as informal collections. Order these intuitively.

Check by arranging each collection as a line and comparing the lengths of the lines. Note that greater quantities make a longer line.

Check by arranging each collection as an array (in rows and columns). Note that greater quantities cover a greater area.

These ideas will translate to ordering on a number line and considering comparing areas in later units and modules.

RAMR Cycle – The second major section of the cycle description describes the actual teaching sequence using the RAMR cycle.

Reality – description of possible realities that are aligned to the concepts and representations to be used during the abstraction and mathematics phases.

Abstraction – description of a sequence of activities that are aligned to the body-hand-mind theory, which move from physical to abstract representations and which make consistent use of language to scaffold students' making of meaning.

Mathematics – description of the mathematics phase.

Language/symbols and practice – description activities that provide opportunities for students to practice using symbols and language and to practice procedures or skills

Connections – description of opportunities to build structural knowledge (connections) in regard to other mathematical concepts.

Resource – identification of resources (workseets, games, investigations). All resources are available via the Blackboard website, and are sorted by Unit and Unit.

Reflection phase – description of the reflection phase.

Check the idea – activities to validate students understanding, usually in familiar contexts.

Apply the idea – problem solving activities that allow students to transfer their knowledge to new (usually unfamiliar) situations.

Extend the idea – activities to extend new ideas and to incorporate further new ideas.

Figure 20. XLR8 Cycle features - RAMR Cycle section

Assessment

The modular, vertical curriculum allows teachers to accelerate learners from ability to age appropriate levels. To do this, teachers need to assess where to start a unit (i.e., not necessarily at the first cycle) and to assess how far along the structured sequence learners have progressed. Not only will the assessment regime inform teaching, analysis of the assessment data will also inform the critique of the structured sequences at the heart of the XLR8 project's design.

In the XLR8 project, assessment of a learner's ability level will be conducted in four ways:

- Diagnostic pre-testing will be administered prior to instruction. This diagnostic will inform the teacher as to where to begin in the structured sequence and/or which cycles will require more or less attention. This data will reported on a student-by-student basis and in a way to highlight the structured sequence of ideas presented in the unit.
- A formative assessment tool that will allow teachers to track to the progress of each student along the structured sequence of instruction
- Summative post-testing (either identical or matched to the pre-test) that will allow teachers to ascertain the final ability level of the learner.
- For each unit, an assignment-style portfolio task will be provided that will allow for the generation of additional data. The portfolio will assess not only procedural skills and conceptual understanding, but also provide a more authentic opportunity for students to demonstrate their mathematical ways of working.

Notes on pre/post testing

1. The pre/post testing will be conducted via a diagnostic worksheet to be administered before each cycle of teaching and a summative test to be administered after each unit.
2. Diagnostic worksheets should not take longer than 15 minutes to administer. The summative tests are designed to be completed within a one hour lesson. Whilst students should be encouraged to complete the test, test completion should not be laboured if this means that productive teaching time is sacrificed or if the accuracy of the test data is compromised.
3. Where appropriate, aid should be given to students (one-to-one or whole class) to assist students to overcome any literacy demands associated with the test that would otherwise mask the students' mathematical ability (e.g., reading out questions).
4. As well as the clean test papers, teachers will be provided with a marked-up copy of the test papers that identify:
    - Links to cycle objectives
    - Expected answers
    - Marking scheme

XLR8 Research Activities

Based on the YDC's prior experience, and in the local contexts of the high schools participating in this project, a process of iterative action research will be used to build and evaluate theories regarding how mathematics is learnt. Instrumental to the iterative action research methodology is the enactment and subsequent evaluation of theory through the practical development and trial of resources. Consequently, whilst building new theory the project will also develop practices (and associated artefacts) that in themselves are important and significant outputs, and provide participating schools with professional development, ongoing support and resources to immediately begin to improve the teaching and learning of mathematics.

Using this approach, the project will:

> **Objective 1:** Propose, critique and refine a conceptual framework that embodies cognitive, affective and cultural elements and which will inform the accelerated teaching and learning of mathematics that emphasises the structural organisation of mathematical ideas.
>
> **Objective 2:** Guided by the developing conceptual framework, propose, critique and refine theories related to specific classroom practices (including the design of resources and assessment instruments) that scaffold the accelerated learning of mathematics.
>
> **Objective 3:** Guided by the developing conceptual framework, propose, critique and refine theories regarding effective professional learning for teachers in the context of whole-school supported accelerated learning of mathematics.
>
> **Objective 4:** Reflect upon the theory building activity and the impact of the accompanying theory-based practices (including classroom pedagogy, the achievements with regards to accelerated learning, and the programs of professional learning) and identify implications for future research.

This project will develop theory through creating programs of accelerated learning which will advance underperforming students to levels normally associated with their year level in a compressed amount of time. The project will deliver a vertical curriculum, based upon the teaching of 'big ideas' to junior-secondary students over a two year period. In this two year period, the vertical curriculum will address the full range of mathematics, building the big ideas up to a level that aligns to the expectations for a Year 9 student. By theorising and putting into practice a program of accelerated learning, the participating students (and future students like them) will have enhanced employment and life chances.

Participants

The XLR8 project is conducted at several metropolitan state high schools. At each site, two distinct types of participants will be involved in the project: (1) the junior high school students who will study the XLR8 curriculum; and (2) the range of staff who will have various roles in the delivery of the XLR8 curriculum.

Student participants

The project will involve multiple cohorts of student participants. In this way, the project will build practice-based theory from the one cohort's experience, and then refine and further the practice and theory with the subsequent cohort(s).

The student participants will be involved in the research project in the following ways:

- Participate in pre-delivery diagnostic testing aimed at establishing a base-line of student ability prior to the delivery of instruction. Typically, this base-line testing will be administered at the beginning of each school term.
- Participate in classroom instruction, based upon the vertical curriculum, across each term of school.
- Complete surveys designed to ascertain student's beliefs and/or attitudes towards mathematics and the learning of mathematics.
- Provide samples of work (e.g., completed worksheets, artefacts from other in-class activities) that will be studied by teachers and researchers to identify alignment to the mathematics being taught.
- Where appropriate, participate in small group interviews to ascertain beliefs and values with regards to mathematics and mathematics learning. These interviews may be audio and/or video recorded for the purposes of transcription and analysis.
- Be observed in the naturalistic school setting participating in the planned instruction. These observations may be audio and/or video recorded, transcribed and analysed for their mathematical and affective content.
- Participate in post-instruction diagnostic assessment aimed at establishing the student's growth in mathematical ability as a result of instruction using the vertical curriculum.

Staff participants

The staff participants in the project will be primarily the classroom teachers of the participating students, as well as administration (Principals, HoDs) and Teacher-aides.

The classroom teachers participating in the project will be central to the project's successful achievement of its research objectives, in addition to the students' successful accelerated learning of mathematics. As with the student participants, it is desirable that continuity of the classroom teachers involved in the project be achieved. Ideally, a teacher would remain a participant for the entire 2013-2015 period. Perhaps more realistically, a teacher should remain in the program for at least one calendar year, both so that their mathematics and pedagogical knowledge has an opportunity to develop and so that there is minimal disruption for the students.

During their period of involvement in the project, classroom teacher participants will be involved in the following activities:

- Attend all professional learning activities associated with the project. This would include pre-project professional learning related to the YDC approach to teaching mathematics as well as professional learning events held during the school terms.
- Participate in face-to-face and/or online professional support sessions scheduled throughout school terms.
- Using the materials provided by the research team as a base, design classroom learning activities aligned to the structure of the proposed vertical curriculum and which align to the RAMR cycle and pedagogical model.
- Maintain an up-to-date and comprehensive journal of classroom activity. The content of this journal will be negotiated with the research team to reflect the changing imperatives or particular research focus of the project (this might be project wide, school or even classroom specific). The format of this journal will be determined in collaboration with the research

team to ensure it contains the required data but does not impose an inappropriate work load upon the teacher.
- Permit members of the research team to observe classroom learning. These observations may be audio and/or video recorded for the purposes of transcription and analysis.
- Participate in individual and/or focus group style interviews with the research team, which will explore themes emerging from the journal of classroom activity and/or researcher's observations of professional learning and/or classroom activities. These interviews may be audio and/or video recorded for the purposes of transcription and analysis.
- As needed, administer pre/post tests.

Principals and Heads of Department will be involved in the project in the following ways:

- As necessary, attend professional learning activities (such as initial training in the YDC approach to mathematics learning) so that they develop a suitable appreciation for the theoretical and practical underpinnings of the project.
- Participate in individual and/or focus-group styled interviews, in which themes emerging from the research data will be explored. These interviews may be audio and/or video recorded for the purposes of transcription and analysis.
- Provide mentoring support to Classroom Teacher, Teacher Aides and other Support Staff involved in the project.

Teacher-aides will be involved in the project in the following ways:

- Participation in individual and/or focus-group styled interviews, in which themes emerging from the research data will be explored. These interviews may be audio and/or video recorded for the purposes of transcription and analysis.
- Allow members of the research team to observe their interactions with students and classroom teachers during classroom learning. These observations may be audio and/or video recorded for the purposes of transcription and analysis.
- As appropriate, attend professional learning activities associated with the project. This may include pre-project professional learning related to the YDC approach to teaching mathematics as well as professional learning events held at the mid-point and end-point of each school term.

Timeline

The project can be divided into three distinct phases: the entry phase (late 2012), the intervention phase (2013-2015) and the exit phase (start of 2016). This 3-phase approach is illustrated in *Figure 21*.

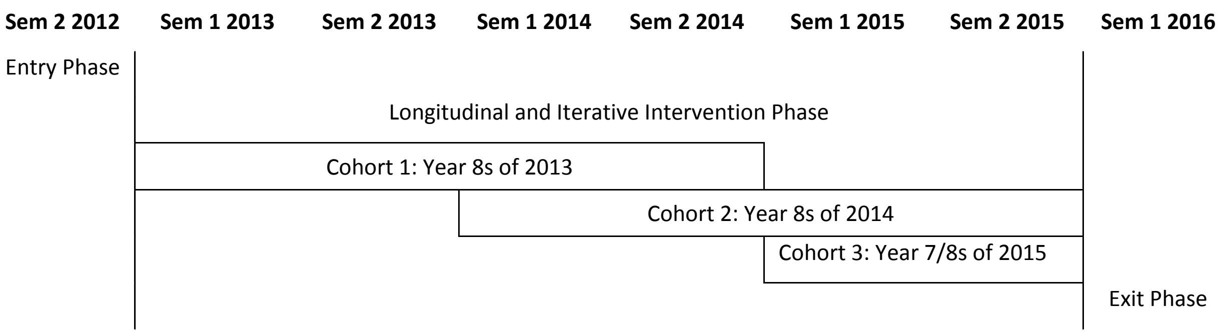


Figure 21. Three phases of the XLR8 research project

Data gathering and analysis

The XLR8 project has adopted a design-based approach to its research design. This approach seeks to firstly identify participating students' initial understanding, then apply an intervention that is aimed at achieving some sort of learning outcome, and then identify any subsequent change in learners' abilities. Along the way, data is collected about the implementation of the intervention that can then be analysed to explain any changes in learners' abilities. Such analysis would compare similarities and difference both within and between student cohorts in both their test achievements and the learning activities they encountered.

In the case of XLR8, the intervention will be the use of the structured sequence-based XLR8 curriculum (delivered using the RAMR pedagogy). Initial and final student understanding will be ascertained using pen and paper structural knowledge tests. A variety of tools, including teacher records of lesson activities and formative assessment, classroom observations and interviews and affective surveys of students' attitudes towards mathematics will be used. In the following subsections, each of the data gathering instruments to be used in the project are discussed.

Pre/Post-tests

To determine students' structural understanding of mathematics, pen and paper tests will be used. The diagnostic worksheet (pre-test) will be administered prior to each cycle of instruction. This diagnostic worksheet will cover work to be taught within the cycle.

Summative tests will be administered at the end of each unit. These will be comprised of the same questions as the diagnostic worksheets with changed numbers. Student results can be checked between the diagnostic worksheet and the summative test to determine change in understanding as demonstrated by pen and paper tests.

Teacher journal

The XLR8 curriculum is the basis for teaching – from it, participating teachers will use the RAMR pedagogy to design learning activities that develop the mathematical ideas in each of the units cycles. This structured sequence based teaching will lead to the students' accelerated learning of mathematics. For this reason, it is necessary to construct a detailed picture of exactly what activities the learners were involved in that lead to their growth in understanding.

The teacher journal will provide a hopefully convenient way for teachers to plan their lessons and to record, against those plans, exactly what occurred in class. It is anticipated that for each RAMR cycle (nominally corresponding to each sub-cycle in the XLR8 curriculum) the teachers will record: (a) the mathematical ideas they hoped their students would develop an understanding of; (b) the realistic situations in which they positioned those mathematical ideas; (c) the sequence of representations used to progress from working with concrete, kinaesthetic representations to abstract mathematical notations; (d) the mathematical activities the students will undertake to consolidate their understanding of the mathematical ideas to build connections to other mathematical ideas; and (e) the ways in which the students will be encouraged to reflect their new mathematical understanding back onto reality, and so validating its structure and extending that structure's application into a new context. Also, at the end each of cycle teachers are encouraged to reflect on their teaching, including their understanding of the structured sequences and the RAMR pedagogy.

If completed fully, the teacher journal will become a rich explanatory record of the activities that contributed to the students' accelerated learning of mathematics.

Formative assessment tool

In addition to the pre-post testing (i.e., diagnostic and summative testing), it is important to assess student progress in an ongoing fashion. To aid this, a formative assessment tool will be provided which will allow teachers to record each student's achievement with respect to the various mathematical ideas in the structured sequence. This presentation of this tool will align the reporting provided from the pre- and post-tests.

The research team may also review the formative assessment tool to gauge student learning, both at the individual and class level.

Student workbook

Students should be encouraged to keep a well organised workbook that is an accurate and chronologically ordered record of their learning. This will complement the teacher's journal – the journal records what the teacher did to encourage accelerated learning and the student workbook will be a record of each student's response to the learning activities. This workbook should include not only handwritten work, but also any worksheets etc., completed.

Interviews

Interviews of teachers and of students will provide insight into not only participants' understanding of the mathematical ideas, but also their attitudes towards the teaching and learning of mathematics. These interviews will be video recorded and transcribed for detailed analysis. These interviews may be conducted one-on-one or in small groups. These interviews may be conducted at various times throughout the program, and with a variety of participants. Some participants may be longitudinally tracked across the XLR8 project, whereas others might only be interviewed in 'once-off' situations.

Classroom observations

Classroom observations – that is, teachers and students working in their naturalistic setting – provides a means for the research team to gather an authentic and detailed picture of learning and the factors that might be contributing (either positively or negatively) to the success of accelerated mathematics learning. Like interviews, these observations maybe video recorded and transcribed for detailed analysis.

Model teaching

Model teaching, in which an experienced teacher contributes to the classroom activity, serves a twofold purpose. Firstly it can assist a teacher, and in turn the students, to become familiar and confident with the RAMR pedagogy and the use of the structured sequence approach. Secondly, model teaching provides the researcher/teacher with firsthand experience of the challenges of using the XLR8 curriculum and applying the RAMR pedagogy.

Student attitude survey

An important part of the XLR8 program is to encourage students to enjoy learning, especially mathematics. To understand students' attitudes towards mathematics, a simple survey will be administered twice each year – at the beginning and at the end – to try and ascertain students attitudes towards learning mathematics.

Appendix – Principle Big Ideas

The principle big ideas are categorised, as follows.

1. Global principles:

   Symbols tell stories. The symbols of mathematics enable the world to be described succinctly and in a generalised way (e.g., 2+3=5 means caught 2 fish and then caught another 3 fish, or bought a $2 chocolate and $3 drink, or joined a 2m length of wood to a 3m length, and so on).

   Change vs relationship. Everything can be seen as a change (e.g., 2 goes to 5 by +3) or as a relationship (e.g., 2 and 3 relate to 5 by addition).

   Interpretation vs construction. Things can either be interpreted (e.g., what operation for this problem, what properties for this shape) or constructed (write a problem for 2+3=5; construct a shape of 4 sides with 2 sides parallel).

   Probabilistic vs absolutist. Things are either determined by chance (e.g., will it rain?) or are exact (e.g., what is $2 + $5?).

   Accuracy vs exactness. Problems can be solved accurately (e.g., find 5,275+3,873 to the nearest 100) or exactly (5,275+3,873=9,148).

   Continuous vs discrete. Attributes can be continuous (smoothly changing and going on forever – e.g., a number line) or they can be broken into parts and be discrete (can be counted – e.g., a set of objects). Units break continuous length into discrete parts (e.g., metres) to be counted.

   Part-part-total/whole. Two parts make a total or whole, and a total or whole can be separated to form two parts (e.g., fraction is part-whole, ratio is part to part; addition is knowing parts, wanting total).

2. Grouping and Position principles:

   Notion of unit. Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts. (E.g., if there are 6 counters, each counter can be a unit, making 6 units, or the set of six can be a unit, making one unit.)

   Odometer. All positions change forward from 0 to base, then restart at 0 with position on left increasing by 1, and the opposite for counting back (e.g., $2^{3}/_{5}$, $2^{4}/_{5}$, 3, $3^{1}/_{5}$, and so on).

   Multiplicative structure. Adjacent positions are related by moving left (× base); moving right (÷ base).

3. Equivalence and Order principles:

   Reflexivity (and non-reflexivity). A=A but A is not $>$ A.

   Symmetry (and anti-symmetry). $A=B \rightarrow B=A$ while $A>B \rightarrow B<A$ and $A<B \rightarrow B>A$.

   Transitivity. $A=B$ and $B=C \rightarrow A=C$ and $A>B$ and $B>C \rightarrow A>C$.

   Balance. Whatever is done to one side of the equation is done to the other.

4. Field properties for operations:

   Identity. 0 and 1 do not change things (+/- and ×/÷ respectively).

Inverse. A change that undoes another change (e.g., +2 / −2; ×3 / ÷ 3).

Commutativity. Order does not matter (e.g., 8+6 / 6+8; 4×3 / 3×4).

Associativity. What is done first does not matter (e.g., (8+4)+2 = 8+(4+2), but (8÷4)÷2 does not = 8÷(4÷2).

Distributivity. ×/÷ act on everything (e.g., 2×(3+4) = 6+8; (6+8)÷2 = 3+4).

5. Extension of field properties:

   Compensation. Ensuring that a change is compensated for so answer remains the same – related to inverse (e.g., 5+5=7+3; 48+25=50+23; 61−29=62−30).

   Equivalence. Two expressions are equivalent if they relate by +0 and ×1 – also related to inverse (e.g., 48+25=48+2+25−2=73; 50+23=73; 2/3=2/3×2/2=4/6).

   Inverse relation. The higher the number the smaller the result (e.g., 12÷2=6 > 12÷3=4; 1/2 > 1/3).

   Triadic relationships. When three things are related, there are three problem-types; (e.g., 2+3=5 can have a problem for; ?+3=5, 2+?=5, 2+3=?).

   Backtracking. Using inverse to reverse and solve problems (e.g., 2*y*+3=11 means *y*×2+3, so answer is 11−3÷2=4).

6. Measurement principles:

   Common units. Must use same units when comparing and calculating (e.g., a 3m by 20cm rectangle does not have an area of 60).

   Inverse relation. Same as Extension of field properties principle (i.e., the bigger the unit, the smaller the number – e.g., 2m=200cm).

   Accuracy vs exactness. Same as Global principle (e.g., cutting a 20 cm strip usually does not give a length of exactly 20 cm).

   Attribute vs instrumentation. The meaning of an attribute leads to the form of measuring instrument (e.g., mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring).

7. Shape principles:

   Reflection and rotational relationship. Number of rotations equals number of reflections; rotation angle double reflection angle (holds for symmetry and Euclidean transformations).

   Euler's formulae. Nodes/corners plus regions/surfaces equals lines/edges plus 2 (holds for 3D shapes and maps).

   Transformational invariance. Topological transformations change straightness and length, projective change length but not straightness, and Euclidean change neither.

Appendix - Culture and School Change

An important component of XLR8 is to take account of the cultural differences between the under-performing students and the middle class Western culture of the school and to ensure there is a focus on school change and community involvement in relation to the professional development (PD) and other support sessions. This chapter focuses on cultural implications, PD and school change, and teacher knowledge for positive change.

Cultural implications

The XLR8 project is to improve mathematics performance of low SES students who have underachieved and who do not reflect the dominant culture of both school and Australian society. This includes students who are Indigenous, refugee, and immigrant as well as low SES students from single and unemployed parent homes.

Indigenous students

The underachievement of Aboriginal and Torres Strait Islander students is, in part, a consequence of being part of a dispossessed people who have been considered by the dominant culture as primitive with no value for a modern society. This has implications for the way mathematics is taught to Aboriginal and Torres Strait Islander students. The devaluing of Indigenous cultures still continues today; the notion perpetuated by the education system that "human-kind" evolved from hunter-gathers to technologically advanced societies does not provide a sense of pride for Indigenous students about their culture. It ignores the reality that Aboriginal and Torres Strait Islander people have powerful and sophisticated forms of mathematical knowledge that enable the complexity evident in total ecosystems to be understood. It leads to disengagement and non attendance.

Aboriginal and Torres Strait Islander students predominantly come to school with a home language which is not standard English and with knowledge, skills, and patterns of interaction that are not appreciated by schools and do not match what facilitates success in school. This mismatch is particularly evident in the way mathematics is taught in schools. Aboriginal and Torres Strait Islander students tend to be active holistic learners, appreciating overviews of subjects and conscious linking of ideas (Grant, 1998). In fact, Indigenous people have been characterised as belonging to "high-context culture groups" using a holistic (top-down) approach to information processing in which meaning is "extracted" from the environment and the situation. In contrast, mainstream Australian culture is characterised as a "low-context culture" and uses a linear, sequential building block (bottom-up) approach to information processing in which meaning is constructed (Ezeife, 2002). School mathematics is traditionally presented in a compartmentalised form where the focus is on the details of the individual parts rather than the whole and relationships within the whole, a form of presentation that disadvantages Aboriginal and Torres Strait Islander students.

Low SES students

Historically, educational institutions have favoured higher to middle-class backgrounds, beliefs and practices. This is due to a number of factors including the history of the purpose of schooling across its development and the socio-economic backgrounds of the majority of teachers and curriculum developers (Meadmore, 1999). As such, there are pre-existing patterns of communication and interactions (or discourses) endemic to education in Australia which are not favourable to lower SES students (Meadmore, 1999). Thus, the middle-class Eurocentric culture of Australian schools and implicitly understood patterns of communication and interactions serve to further marginalise students from low socio-economic backgrounds from school mathematics. The nature of discourses within school practices do not always successfully link to nor validate mathematical practices that may be part of low SES students' out-of-school experiences (Baker, Street, & Tomlin, 2006) leading to insufficient links being made between students' existing mathematical knowledge and practices and school mathematics. In these cases, students may disassociate from school mathematics and feel they cannot succeed, particularly if their home skills and knowledge are not valued nor actively sought (Thomson, 2002).

Expectations may also pose difficulties for low SES students as for Indigenous students. Low SES parents may perceive mathematics as alienating and unnecessary or too difficult for their children to learn; this can lead to students not expecting to succeed in mathematics, having low expectations of themselves and their future roles in society, and thus not participating in mathematics classrooms. Teachers may also have low expectations of low SES students and often believe that lower level or *life* numeracy is all that is needed for these students (Baker, Street, & Tomlin, 2006). The resulting emphasis of mathematics for these students becomes utilitarian, rote and procedural mathematics tasks that are not explicitly related to overarching mathematical structures.

Strengths and weaknesses of Eurocentric mathematics

Interestingly, the Eurocentric mathematics that is taught in Australian schools has weaknesses due to its cultural bias. Because of the way their culture was developing, European societies developed mathematics to help them explain their world and solve their problems, particularly to explain space, time, and eventually, number. When trading became a way of life, a need developed to be more precise in representing and quantifying value to have a shared agreement of how values could be compared ("is mine worth more than yours"), a more sophisticated process than quantification as it involves rate (e.g., 3 cows = 1 boat). Over time, the European mathematics' quantification and comparison system grew to encompass a variety of numbers (common and decimal fractions, percentages, rates and ratios), measures of time and shape (length, area, volume, mass and angle), and two operations (addition and multiplication) and their inverses (subtraction and division). The system was also generalised to findings that hold for all numbers and measures, and the resulting mathematics area of algebra has grown in importance as science and technology has expanded.

The weakness of European mathematics lies in its strength, the success of its quantifying and comparing systems in underpinning the growth of science and technology. This has resulted in longer and healthier lives and the devices that support work, home life and leisure. Western society now has the tools to change our environments to make living better; we can cool the hot, warm the cold, clear the land, bring in new plants and animals, and clothe, shelter, and feed large populations. However this triumph has affected European culture and society. Success has come to mean increasing numbers and continued growth; smaller numbers and negative growth are to be avoided (and are given names that signify failure, such as "recession"). The culture appears to have little ability to understand harmony and act sustainably; it tries to dominate the land, the sea, the weather, and the animals, birds and fish, with little understanding of how things interact to allow human life, resulting in poverty, hunger, war, pestilence and global warming. Mathematics can be developed which would reinforce planetary equity and sustainability (see *Figure 22*), however, such mathematics requires less dominance by number, less need for growth, and an emphasis on living in harmony with land and sea; a non-Eurocentric form of mathematics.

Status depends on number and size → Success means larger numbers → Over production and global warming

The progress of Eurocentric mathematics

Less emphasis on number and size → Success is caring for country → Planetary sustainability

A better mathematical progress

Figure 22. Comparison of two mathematical progressions

Implications for teaching

Traditional teaching of mathematics can confront Indigenous and low SES students' cultures and perpetuate the belief that success in mathematics classrooms requires the rejection of culture (i.e., that one has to become "white" or middle class). To be effective, mathematics teaching has to enhance mathematics outcomes but retain pride in culture and heritage. Approaches that seem to be effective are as follows.

1. Confront the Eurocentric nature of traditional school mathematics by making students aware of the cultural implications of mathematics teaching, and draw attention to which mathematics ideas change perceptions of reality. Discuss the role of mathematics in European culture and draw attention to the culture's strengths and weaknesses. Make mathematics available to all students.
2. Legitimise local Indigenous knowledge and integrate students' culture and mathematics instruction, to match the mathematics classroom to the cultural capital brought by the students (Bourdieu, 1973). Contextualise mathematics into the life and culture of students, use representations and activities from the everyday lives of the students. Use a local cultural framework for learning.
3. Take account of English not being the first or home language of Aboriginal and Torres Strait Islander students, develop language and be aware of different meanings for mathematics words
4. Present mathematics as a holistic structure that can empower the learner by focusing on big ideas and using instructional strategies that relate acting, creating, modelling and imagining.
5. Modify teaching pedagogies to include cultural perspectives.
6. Realise approaches to improve mathematics learning cannot stand alone, and they need to be allied with whole-school activities that include the local community, challenge attendance and behaviour, have high expectations, develop Indigenous leadership and give a strong role to Indigenous teacher aides.
7. School change and leadership
8. The provision of new mathematics teaching ideas is often insufficient for sustainable improvement in Aboriginal and Torres Strait Islander students' learning of mathematics. The new ideas have struggled to have positive effects when low attendance and negative behaviour are endemic across a school, when school practices and learning spaces disengage students, when positive partnerships are not formed between teachers and their Indigenous teacher aides, when classrooms do not involve community leaders or acknowledge local knowledge, and when teachers do not believe that the students are capable of the work. The ideas have been successful when they have been integrated into whole-school changes that challenge attendance

and behaviour, integrate and legitimise local community knowledge, build in practices to support the culture of the students, and change teacher attitudes towards and relationships with the students. Thus, the XLR8 project is much more than a set of new teaching ideas; it integrates: (a) a particular teaching learning approach to mathematics (i.e., RAMR) that is designed to empower students within their culture; and (b) an approach to PD, and school change designed to facilitate change to support community involvement and student engagement.

9. The XLR8 approach to school change and leadership is based on the belief that schools with students underachieving in mathematics can only enhance mathematics learning with a program that focuses on mathematics and on school change together. In simple terms, XLR8 believes that schools should see themselves as part of, not apart from, the communities in which its students live, and should see their role in terms of the slogan, as building community through education. It believes that school change can have a profound effect in creating emancipatory environments that actively seek to improve the educational outcomes and life chances for Aboriginal, Torres Strait Islander, and low SES students, and strong school leadership plays a critical role in acknowledging the existence of this student excellence.

10. XLR8 has been influenced by the philosophy and success of the Stronger Smarter Institute which argues that school change and leadership cycles through four requirements (see *Figure 23*): community-school partnerships; local leadership; positive student identity; and high expectations. It aims to develop not just new capabilities but also shifts in thinking individually and collectively. Maintaining the cycle of the four requirements ensures sustained growth towards enhanced learning. It creates and sustains emancipatory environments that enhance the opportunities of the students that attend, and challenges mechanisms and processes that continue to reproduce disengagement among Aboriginal and Torres Strait Islander students within the schooling system.

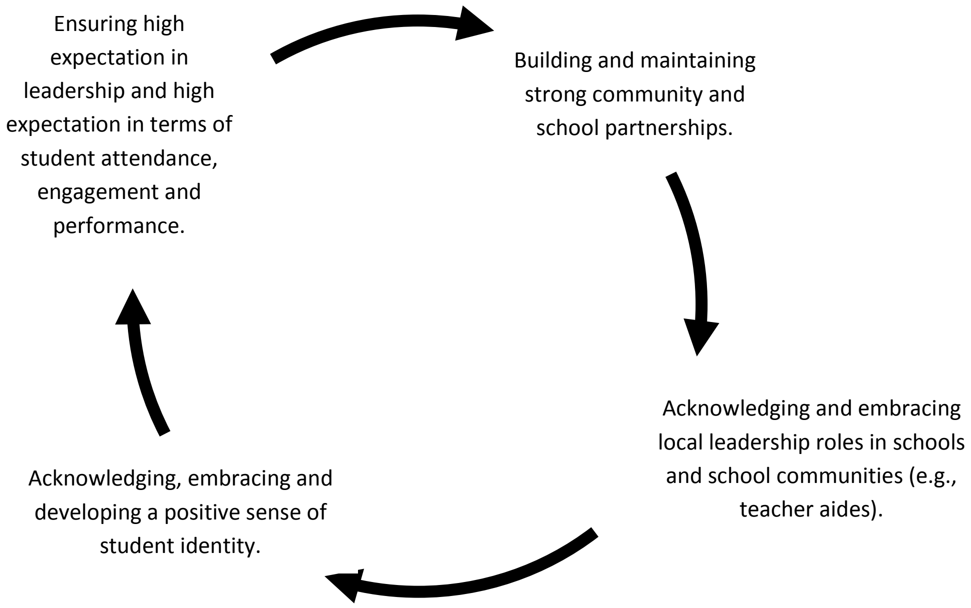


Figure 23. Cycle of school change and leadership

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XLR8 – Accelerating Mathematics Learning

YuMiDeadly

Growing community through education

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