ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre (YDC) is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of all students to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YDC’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates YDC’s vision: Growing community through education.

YDC can be contacted at ydc@qut.edu.au. Its website is https://research.qut.edu.au/ydc.

ABOUT THE MITI PROJECT

Mathematicians in Training Initiative (MITI) is a YDC mathematics project designed to improve Years 7–12 students’ mathematics learning and thereby employment and life chances. The focus of MITI is to enrich and extend mathematics teaching to build confidence and interest in, and deep understanding of, powerful mathematics ideas in order to improve participation in high-level senior secondary mathematics subjects, university entrance and mathematically based careers, particularly for low-SES schools. It is a two-stage project: Stage 1 – investigations, problems, and seamless sequencing of powerful mathematics; and Stage 2 – deep applications in futures contexts. The overarching aim of MITI is to develop the capacity of high schools to build strong, high-level mathematics classes and university opportunities for their students.

To join a project, contact ydc@qut.edu.au.

CONDITIONS OF USE AND RESTRICTED WAIVER OF COPYRIGHT

Copyright and all other intellectual property rights in relation to this book (the Work) are owned by the Queensland University of Technology (QUT). Except under the conditions of the restricted waiver of copyright below, no part of the Work may be reproduced or otherwise used for any purpose without receiving the prior written consent of QUT to do so.

The Work may only be used by schools that have received professional development as part of a YuMi Deadly Centre project. The Work is subject to a restricted waiver of copyright to allow copies to be made, subject to the following conditions:

1. all copies shall be made without alteration or abridgement and must retain acknowledgement of the copyright;
2. the Work must not be copied for the purposes of sale or hire or otherwise be used to derive revenue;
3. the restricted waiver of copyright is not transferable and may be withdrawn if any of these conditions are breached.

© 2017 Queensland University of Technology through the YuMi Deadly Centre
# Contents

<table>
<thead>
<tr>
<th>Preamble</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>MiTi resources</td>
<td>1</td>
</tr>
<tr>
<td>Structure of this resource</td>
<td>2</td>
</tr>
<tr>
<td>Assessment, achievement and affect</td>
<td>2</td>
</tr>
<tr>
<td>Summary of investigations and links to Australian Curriculum</td>
<td>3</td>
</tr>
<tr>
<td><strong>Year 7 Investigations</strong></td>
<td>7</td>
</tr>
<tr>
<td>7A1 “Reflecting on Rotations”</td>
<td>7</td>
</tr>
<tr>
<td>7A2 “Connected Expressions”</td>
<td>12</td>
</tr>
<tr>
<td>7G1 “Flippin Congruence”</td>
<td>18</td>
</tr>
<tr>
<td>7G2 “Finding the Winning Strategy”</td>
<td>30</td>
</tr>
<tr>
<td>7M1 “Rocking Around the World”</td>
<td>37</td>
</tr>
<tr>
<td>7M2 “Cheap Houses”</td>
<td>42</td>
</tr>
<tr>
<td>7N1 “Tangled Fractions”</td>
<td>48</td>
</tr>
<tr>
<td>7N2 “Directing Numbers”</td>
<td>55</td>
</tr>
<tr>
<td>7N3 “What Are You Worth?”</td>
<td>59</td>
</tr>
<tr>
<td>7N4 “How Tall Is the Criminal?”</td>
<td>69</td>
</tr>
<tr>
<td>7N5 “Rating Our World”</td>
<td>78</td>
</tr>
<tr>
<td>7P1 “Dice Doubles”</td>
<td>84</td>
</tr>
<tr>
<td>7P2 “Fair Game”</td>
<td>91</td>
</tr>
<tr>
<td>7S1 “Are Older Actors Better?”</td>
<td>96</td>
</tr>
<tr>
<td>7S1 “Assuming Too Much”</td>
<td>99</td>
</tr>
<tr>
<td><strong>Year 8 Investigations</strong></td>
<td>103</td>
</tr>
<tr>
<td>8A1 “Growing Patterns and Growing Graphs”</td>
<td>103</td>
</tr>
<tr>
<td>8A2 “Algebraic Caterpillars”</td>
<td>113</td>
</tr>
<tr>
<td>8A3 “Building a Mathematics Structure”</td>
<td>124</td>
</tr>
<tr>
<td>8G1 “Maths in a Box”</td>
<td>130</td>
</tr>
<tr>
<td>8M1 “Accuracy and Precision”</td>
<td>138</td>
</tr>
<tr>
<td>8M2 “Designing a Kitchen”</td>
<td>142</td>
</tr>
<tr>
<td>8M3 “Crazy Bird Boxes”</td>
<td>146</td>
</tr>
<tr>
<td>8N1 “Diminishing Fractions”</td>
<td>148</td>
</tr>
<tr>
<td>8N2 “Challenging Fractions”</td>
<td>153</td>
</tr>
<tr>
<td>8N3 “Square Percentages”</td>
<td>158</td>
</tr>
<tr>
<td>8N4 “Consecutive Sums”</td>
<td>163</td>
</tr>
<tr>
<td>8P1 “The Lucky Prince”</td>
<td>166</td>
</tr>
<tr>
<td>8P2 “And Not Or”</td>
<td>176</td>
</tr>
<tr>
<td>8S1 “Distributing Distributions”</td>
<td>193</td>
</tr>
<tr>
<td>8S2 “Topological Oddities”</td>
<td>200</td>
</tr>
<tr>
<td><strong>Year 9 Investigations</strong></td>
<td>209</td>
</tr>
<tr>
<td>9A1 “Dividing Diagonals”</td>
<td>209</td>
</tr>
<tr>
<td>9A2 “Taking the Guesswork Out of Maths”</td>
<td>216</td>
</tr>
<tr>
<td>9G1 “Constructive Constructions”</td>
<td>224</td>
</tr>
<tr>
<td>9M1 “Look Out for the Baby!”</td>
<td>232</td>
</tr>
<tr>
<td>9M2 “Square Angles”</td>
<td>238</td>
</tr>
<tr>
<td>9M3 “Three-Fact Triangles”</td>
<td>243</td>
</tr>
<tr>
<td>9M4 “How High is that Tree?”</td>
<td>246</td>
</tr>
<tr>
<td>9N1 “Power-ful Mathematics”</td>
<td>254</td>
</tr>
<tr>
<td>9N2 “It’s All Greek to Me”</td>
<td>259</td>
</tr>
<tr>
<td>9N3 “Which Card?”</td>
<td>265</td>
</tr>
<tr>
<td>9P1 “Is Greed Good?”</td>
<td>267</td>
</tr>
<tr>
<td>9P2 “Monopolising Monopoly”</td>
<td>273</td>
</tr>
<tr>
<td>9S1 “How to Lie with Statistics”</td>
<td>284</td>
</tr>
</tbody>
</table>
Abbreviations

AIM Accelerated Inclusive Mathematics
AIM EU Accelerated Inclusive Mathematics – Early Understandings
ICT information and communication technology
MITI Mathematicians in Training Initiative
QCAA Queensland Curriculum and Assessment Authority
RAMR Reality–Abstraction–Mathematics–Reflection
SES socio-economic status
XLR8 Accelerating Mathematics Learning
YDC YuMi Deadly Centre
YDM YuMi Deadly Maths
Preamble

The YuMi Deadly Centre (YDC) is a research and service centre in QUT’s Faculty of Education dedicated to improving mathematics teaching and learning, and thus students’ employment and life chances. To achieve its purpose, YDC has developed a pedagogy for teaching mathematics which it calls YuMi Deadly Maths (YDM). YDC has developed three forms of teacher-training projects based on the YuMi Deadly Maths pedagogy: (a) a general Years P–9 training for teachers of mathematics (called YDM Teacher Development Training); (b) three acceleration/remedial projects for underachieving students titled Accelerated Inclusive Mathematics (AIM), Accelerating Mathematics Learning (XLR8), and AIM Early Understandings (AIM EU); and (c) an extension and enrichment project, the focus of this book, called Mathematicians in Training Initiative (MITI).

The YDM mathematics pedagogy was originally developed for students who were typically of low socio-economic status (SES) and/or from Aboriginal, Torres Strait Islander, migrant or refugee backgrounds. However, the pedagogy works equally well with all students, mainstream to elite private schools. This particularly includes the enrichment and extension pedagogy used in the MITI project, which is the focus of this book.

MITI resources

As with all YDM projects, the MITI project focuses on equipping teachers with the knowledge to write their own lessons and units of work rather than providing textbook materials – to provide a “fishing rod” rather than a “fish”. However, exemplar activities are provided to show what is possible. It is expected that these resources will be adapted by teachers to fit the special needs of their students before they are used with those students.

As the MITI Overview book describes, MITI teacher training is in two stages. Stage 1 focuses on Years 7 to 9 extending to 10 to 12 and looking at the Reality–Abstraction–Mathematics–Reflection (RAMR) teaching cycle, problem solving and investigations, and using technology to develop powerful mathematics ideas. This is supported by two sets of resources covering investigations and technology applications. Stage 2 focuses more on the transition to Years 10 to 12, looking at how understanding of powerful mathematics can be built through extending the RAMR model, seamless sequencing from Years 7–9 to 10–12 and using instructional sequences that build big mathematics ideas from applications. This stage is also supported by two sets of resources, covering seamless transitions and industry applications.

This means that MITI training is built on five collections of resources:

(a) MITI Overview – the introductory book that describes the YDM pedagogy, and three specialised smaller booklets: Big Ideas of Mathematics, Problem Solving, and Literacy in Mathematics.

(b) MITI Investigations – a book of 45 investigations that cover Years 7–9 mathematics curriculum content as set out in the Australian Curriculum (this book).

(c) MITI Technology – a book of exemplar activities where technology is used to build deep understanding of powerful mathematics.

(d) MITI Transitions – a book of examples of topics which are taught in Years 7 to 9 in ways that can be extended to Years 10 to 12 (pre-empting activities).

(e) MITI Applications – a book of applications in industry and business that can be used in Years 11 and 12 to motivate and contextualise important Year 11 and 12 mathematics topics.
Structure of this resource

This investigations resource comprises 45 investigations covering all topics in Years 7 to 9. The investigations are designated by:

- Year level (7, 8 or 9);
- topicstrand (N – Number, A – Algebra, M – Measurement, G – Geometry, S – Statistics, P – Probability, PS – Problem-Solving); and
- order for the topic (for example, 7A2 is the second Algebra investigation for Year 7).

Each investigation consists of:

- task description – designed to be given to students to do the investigation;
- optional or extra student materials (if any) for completing the investigation;
- essential vocabulary – language the students should understand;
- teaching information – information for teachers about the investigation, including different ways of completing the investigation; and
- links to the Australian Mathematics Curriculum.

Information on assessment, achievement standards and affective criteria is the same for all investigations and is described in this preamble and in Appendices A and B.

The investigations are designed to:

- involve the students in inquiry and problem solving, that is, working out what to do to arrive at a solution; and
- enable the students to learn the mathematics that is related to the investigation in finding the solution.

To this end, the task description is in a form to leave as much as possible to the students (no hints, no questions, and no background). However, as some of the investigations have much to offer as a sequence of more directed sub-activities, this way of presenting the investigation is included in teaching activities to give the teachers options with how they use the investigation idea with their students.

Assessment, achievement and affect

Assessment rubrics

Any assessment rubrics are modelled on the suggested Queensland Curriculum and Assessment Authority (QCAA) assessment rubric in continuum format. The continuum format is used in preference to the matrix format because it allows more scope for teacher judgement, especially in cases where the nature of the students’ work does not exactly match the descriptors in a matrix. It also responds to the QCAA advice that focuses particularly on the differences between the A, C and E levels.

Achievement standards

Achievement standards are included in Appendix A. They follow those determined by the Australian Curriculum and are based on the revised draft Standards Elaborations for Year 9 mathematics recommended by the QCAA (https://www.qcaa.qld.edu.au/downloads/p_10/ac_maths_yr9_se.pdf).
Affective criteria

It is recommended that tasks be used formatively, that is, as assessment for learning. The formative approach generally requires that students are given whatever assistance they need to ensure that their progression through the various elements of the task provides opportunities for learning. However, it is not expected that all students will progress through all stages of the task, nor would they master items of equal complexity, even with assistance.

The practicalities of student assessment often require that teachers also want to use this task summatively, that is, for grading and reporting purposes. The challenge for teachers is to assess students equitably, when some students may have received more assistance with the task than others or when some group members have made a greater contribution than others.

One way of maximising the opportunities for learning through the formative use of the task, but to also obtain a summative result that is equitable to all students, is by the inclusion of affective criteria alongside the usual achievement standards. They allow the teacher to assess the student on how they undertook the task as well as their responses to the task items. For example, students can be assessed on the extent of the independence, diligence, teamwork, compliance, and so on, displayed while undertaking the task, in addition to the quality of the work submitted. The use of affective criteria has the advantage of improving classroom behaviour as well as ensuring equity.

A rubric for assessing some affective criteria that may be relevant to each task is presented in Appendix B. Teachers should tailor the criteria to the circumstances of the class and the conditions of each task. Additional criteria can be included.

The weighting given to the affective criteria is a decision for schools and/or teachers. However, it is suggested that affective criteria could represent from 30% to 50% of the overall assessment.

Summary of investigations and links to Australian Curriculum

A summary table of the 45 MITI investigations in this book is presented on the following two pages. Appendix C maps the Australian Curriculum Mathematics Content Descriptors and General Capabilities to each MITI investigation. Appendix D does the reverse, listing each investigation and the areas of the Australian Curriculum it covers.
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>7A1</td>
<td>Reflecting on Rotations</td>
<td>Representation of transformations on the Cartesian plane, drawing on existing understandings of transformations.</td>
</tr>
<tr>
<td>7A2</td>
<td>Connected Expressions</td>
<td>Application of the solutions to sets of six connected linear equations to identify patterns. As a challenge, investigation of the connections between the equations that result in these patterns.</td>
</tr>
<tr>
<td>7G1</td>
<td>Flippin’ Congruence</td>
<td>Investigation of the common transformations (translations, rotations and reflections), leading to the exploration of congruence and patterns.</td>
</tr>
<tr>
<td>7G2</td>
<td>Finding the Winning Strategy</td>
<td>Development of spatial awareness, logic and planning through playing a variety of games of strategy.</td>
</tr>
<tr>
<td>7M1</td>
<td>Rocking Around the World</td>
<td>Development of an itinerary for a ten-day world tour by a rock band, taking into account time zones, travelling time, and the time taken for set up and performance.</td>
</tr>
<tr>
<td>7M2</td>
<td>Cheap Houses</td>
<td>Investigation of how to build a functional house at lowest cost.</td>
</tr>
<tr>
<td>7N1</td>
<td>Tangled Fractions</td>
<td>Application of calculations with fractions to a practical activity involving four students and two skipping ropes.</td>
</tr>
<tr>
<td>7N2</td>
<td>Directing Numbers</td>
<td>Investigation of directed number, including situations where negative numbers arise, arithmetic operations with directed number, and the appropriate use of calculators.</td>
</tr>
<tr>
<td>7N3</td>
<td>What Are You Worth?</td>
<td>Investigation of personal net worth, relating it to the addition of positive and negative numbers. Exploration of different ways of representing very large numbers.</td>
</tr>
<tr>
<td>7N4</td>
<td>How Tall is the Criminal?</td>
<td>Investigation of ratios, how they are expressed and some practical applications, including to develop a description of a wanted criminal based on clues left at the crime scene.</td>
</tr>
<tr>
<td>7N5</td>
<td>Rating Our World</td>
<td>Application of ratios and rates to find the costs of household water and electricity and the impact of installing solar panels and/or rainwater tanks.</td>
</tr>
<tr>
<td>7P1</td>
<td>Dice Doubles</td>
<td>Application of probability to identify and report on the unfairness in a game based on simulated throws of two dice.</td>
</tr>
<tr>
<td>7P2</td>
<td>Fair Game</td>
<td>Application of probability to identify the unfairness in a game based on spinners, and to propose rule changes to make the game fair.</td>
</tr>
<tr>
<td>7S1</td>
<td>Are Older Actors Better?</td>
<td>Comparison of the ages of the best actor/actress Academy Award recipients to argue whether male or female actors get better as they age.</td>
</tr>
<tr>
<td>PS1</td>
<td>Assuming Too Much</td>
<td>A series of problems that challenge students to question the assumptions that they make when solving problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8A1</td>
<td>Growing Patterns and Growing Graphs</td>
<td>Exploration of linear and nonlinear patterns (relations) and the different methods of representing those relations: visually; tables; pattern rules; equations; and graphs.</td>
</tr>
<tr>
<td>8A2</td>
<td>Algebraic Caterpillars</td>
<td>Introduction to the distributive law by generalising from the calculation of areas using arithmetic methods to an algebraic approach.</td>
</tr>
<tr>
<td>8A3</td>
<td>Building a Mathematics Structure</td>
<td>Introduction to the fundamental properties of our number system (associativity; commutativity; closure; identity; inverse; distributive law) and to some key structures such as groups and fields.</td>
</tr>
<tr>
<td>8G1</td>
<td>Maths in a Box</td>
<td>Investigation of the relations between the size of a sheet of paper used and the height and volume of a box made from that sheet of paper.</td>
</tr>
<tr>
<td>8M1</td>
<td>Accuracy and Precision</td>
<td>Investigation of the concepts of accuracy and precision in measurements. Exploration of the various representations of measurement error, and the impact of that error on the appropriate number of significant figures in a measurement.</td>
</tr>
<tr>
<td>8M2</td>
<td>Designing a Kitchen</td>
<td>Development of a design and quotation of a kitchen in a new home, including details of the total area of bench space and volume of cupboards.</td>
</tr>
<tr>
<td>8M3</td>
<td>Crazy Bird Boxes</td>
<td>Development of a design and costing of a bird box that does not contain any right angles, including details of the floor area and volume.</td>
</tr>
<tr>
<td>8N1</td>
<td>Diminishing Fractions</td>
<td>Practical application of fractions, ratios and patterns to the “A” series of paper sizes and other diminishing sequences.</td>
</tr>
<tr>
<td>8N2</td>
<td>Challenging Fractions</td>
<td>Through completion of several activities, development of deep thinking about fractions and enhanced understandings of common and decimal fractions; and identification of the strengths and limitations of each type of fraction.</td>
</tr>
<tr>
<td>8N3</td>
<td>Square Percentages</td>
<td>Application of geometry to analyse the areas of pieces resulting from the dissection of squares into up to ten different pieces. Calculation of the percentage and/or fraction of the square represented by each piece.</td>
</tr>
<tr>
<td>8N4</td>
<td>Consecutive Sums</td>
<td>Working with natural numbers, an investigation of the ways that each number can be expressed as the sum of consecutive numbers.</td>
</tr>
<tr>
<td>8P1</td>
<td>The Lucky Prince</td>
<td>Application of probability to find the optimum strategy in two puzzles.</td>
</tr>
<tr>
<td>8P2</td>
<td>And Not Or</td>
<td>Development of an understanding of attributes, Venn diagrams, the logical connectives (and, or, not), and their applications to probability.</td>
</tr>
<tr>
<td>8S1</td>
<td>Distributing Distributions</td>
<td>Introduction to the collection of data, the nature of a statistical distribution, the preparation of frequency distribution tables, and the use of those tables to calculate measures of central tendency and dispersion.</td>
</tr>
<tr>
<td>PS2</td>
<td>Topological Oddities</td>
<td>Investigation of different aspects of topology: topological change and homeomorphism; the Möbius strip and Klein bottle; and map colouring.</td>
</tr>
</tbody>
</table>

**Year 9**

| 9A1 | Dividing Diagonals | Application of the midpoint formula to construct a figure on the Cartesian plane. Investigation of the properties of that figure by calculating the distance and gradient of key line segments. |
| 9A2 | Taking the Guesswork Out of Maths | Introduction to algebraic methods of solving simultaneous equations. |
| 9G1 | Constructive Constructions | Investigation of geometric construction techniques, leading to an exploration of: triangle congruence; angle theorems; deductive geometry; and geometric proofs. |
| 9M1 | Look Out for the Baby! | Investigation of the relationship between surface area and volume in order to understand why body temperature regulation can be more difficult for babies. |
| 9M2 | Square Angles | Investigation of how carpenters can use a framing square to construct a range of angles. |
| 9M3 | Three-Fact Triangles | Investigation of various combinations of side and angle facts in triangles, leading to the development of tests of congruence. Application of trigonometry to solve problems and develop generalisations about the area of a triangle. |
| 9M4 | How High is that Tree? | Application of a variety of methods, including shadow sticks, to estimate the height of a tree. |
| 9N1 | Power-ful Mathematics | Investigation of powers with integer exponents, including their properties and laws. Application of powers to scientific notation. |
| 9N2 | It’s All Greek to Me | Investigation of irrational numbers by the completion of several activities. |
| 9N3 | Which Card? | Investigation of the different debit and credit cards available. Comparison of the cards with each other and with cash transactions. Determination of the best card for various spending patterns and other circumstances. |
| 9P1 | Is Greed Good? | Application of probability to investigate a game based on throwing a die and to develop a strategy to maximise the probability of success. |
| 9P2 | Monopolising Monopoly | Exploration of some of the probabilities associated with the game of Monopoly to determine whether skill or luck has the greater influence on the outcome of the game. |
| 9S1 | How to Lie with Statistics | Through the preparation of arguments from the same set of data that support and reject an assertion, investigate how statistics, both numbers and graphs, can be used to mislead. |
| 9S2 | A vs B | Collection of data about the careers of two celebrities and the presentation of this data to support a conclusion about who has the more successful career. |
| PS3 | Investigating Investigations | An open-ended investigation of a given geometric figure, providing opportunities to pose and respond to questions in the areas of angles, length, perimeter, area, patterns, etc. |

Key: A = Algebra; G = Geometry; M = Measurement; N = Number; P = Probability; S = Statistics; PS = Problem-Solving
7A1 “Reflecting on Rotations”

Task description

Activity 1

Construct a design for a shield:
(a) Divide the paper into four parts by folding.
(b) Draw a design in the top right-hand quarter, as shown on right. Take a photocopy of your design – needed for step (d).
(c) Use Mira to reflect the design about the horizontal and vertical folds (three reflections altogether), as shown on near right.
(d) Take your photocopied original design and use tracing paper to rotate the design anticlockwise 90°, as shown on far right.
(e) Compare your two designs. Which do you like better? Why?
(f) Compare your designs with others in your class.

Activity 2

Using graph paper, repeat Activity 1:
(a) Divide the graph paper into four parts by drawing in $x$ and $y$ axes.
(b) Draw a design in first quadrant of the Cartesian plane by joining points together using coordinates to describe the points in order (that is, like a “join the dots” puzzle).
(c) Use Mira to reflect the design in the $x$ and $y$ axes (three reflections altogether).
(d) Note the coordinates of the points in the reflected design.
(e) Find the rules for reflecting an image using coordinates. You might find it easier to focus on a reflection into the fourth quadrant first (as shown in the diagram on the right), before looking at the reflections into the second and third quadrants.

Activity 3

Repeat Activity 2, but this time rotating the design anticlockwise 90° to complete the other three quadrants.
(a) Note the coordinates of the points in the rotated design.
(b) Find the rules for rotating an image using coordinates.
(c) Write rules for a point to be reflected three times about the axes (that is, resulting in an image in all four quadrants).
(d) Find the rule for a point to be rotated by 90° three times about the axes (that is, resulting in an image in all four quadrants). As before, you might find it easier to focus on a rotation into the fourth quadrant first, before looking at the rotations into the second and third quadrants.

Activity 4

Construct a pattern using only coordinates to flip, slide and turn.

Make sure you understand the meanings of any words in italics.
Label each axis from -10 to 10 (the positive direction of the x-axis is to the right and the positive direction of the y-axis is up).

Plot the following points, joining them up with straight line segments as you go:

Start, (0, 1½); (1, 2); (4, 2); (7, 1); (7½, 3); (7, 5); (4, 5); (3, 6); (3, 7); (4, 8); (6, 9); (8, 9); (9, 8); (10, 4); (10, 0); (9, -5); (7½, -5½); (8, -8); (5½, -8½); (5, -6); (2, -6); (½, -5½); (½, -8); (-2, -8); (-2, -5); (-5, -7); (-7, -7); (-10, -4½); (-7, -5); (-5, -3½); (-2, 0); (0, 1½); End

Draw your own design (or find one on the internet) and turn it into a join-the-dots activity for the others in your group.
Essential vocabulary

**Cartesian plane:** The Cartesian plane is a flat space divided by an $x$-axis (horizontal) and a $y$-axis (vertical) that meet orthogonally (at right angles) at the origin. The plane and the axes continue infinitely in all directions. It is also known as the coordinate plane or the number plane.

**Coordinates:** A set of values that show an exact position. They are used on maps and the Cartesian plane. Cartesian coordinates have two values where, by convention, the first value is the $x$-value that shows the distance from the origin in a horizontal direction, and the second value is the $y$-value that shows the distance from the origin in a vertical direction (to remember this, think about a baby that learns to *crawl* before it *climbs*). There are other types of coordinates, for example, three-dimensional coordinates and polar coordinates.

**Quadrant:** The four sections of the Cartesian plane separated by the $x$-axis and the $y$-axis. They are labelled quadrant 1 (where $x$ and $y$ values are both positive), quadrant 2 (where $x$ values are positive and $y$ values are negative), quadrant 3 (where $x$ and $y$ values are both negative), and quadrant 4 (where $x$ values are negative and $y$ values are positive).

**Reflection:** A change (transformation) in which a shape or design is flipped across a line, creating a mirror image. That line is called the axis of reflection. Also described as a flip.

**Rotation:** A change (transformation) in which a shape or design is moved in a circular direction through a specified angle (often 90°) around a point. That point is called the point of rotation. Also described as a turn.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of Cartesian coordinates and simple transformations. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

If students are unfamiliar with plotting coordinates in all four quadrants of the Cartesian plane, you may wish to include the optional join-the-dots activity that develops the skill of plotting points in all four quadrants (resulting in a dinosaur), before Activity 3.

Students can do Activities 2 and 3 on graph paper or a large grid such as a Maths Mat or a geoboard. You can simplify the activity, if needed. Students could be given this design to start with rather than asking them to develop their own. They may find it easier to focus on a rotation or reflection into one quadrant first, before looking at the other two quadrants.

In the case of rotations, students may first have to find the rule that line XA (2 up, 1 across) is perpendicular to XB, if B is 1 down and 2 across (or the other way, 1 up and 2 across).

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA178</td>
<td>ACMMG181</td>
</tr>
<tr>
<td></td>
<td>Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point</td>
<td>Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries</td>
</tr>
</tbody>
</table>
### 7A2 “Connected Expressions”

**Task description**

<table>
<thead>
<tr>
<th>A: 3x - 4</th>
<th>B: 4x - 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Connected Expressions</strong></td>
<td></td>
</tr>
<tr>
<td>D: 2x - 3</td>
<td>C: 6x - 19</td>
</tr>
</tbody>
</table>

The screen shot above was taken from an Excel spreadsheet.

1. Copy the four expressions into the boxes below. Follow the arrows to form an equation from every possible pair of expressions. Write your six equations in the first column of a two-column table.

2. Solve the six equations you formed. Write the solutions in the second column of your table. What do you notice?

3. Open the Excel file called “Connected Expressions” (your teacher will have a copy of the file).

4. Use the expressions in the spreadsheet to set up and complete another table with the six equations formed. What do you notice?

5. Generate another set of expressions by simultaneously pressing Ctrl-Alt-F9. Create another table with your equations and their solutions. Develop a hypothesis about the solutions to the equations. Try some more sets of equations from the spreadsheet to test your hypothesis.

6. Make up four expressions of your own, without using the spreadsheet. Create and solve the six equations. What do you notice?

7. (Challenge) Can you explain what is occurring and why it occurs? Can you create your own sets of equations with solutions that are similar to those from the spreadsheet?

*Make sure you understand the meanings of any words in italics.*
### Sets of connected expressions

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-5x - 2</td>
<td>B</td>
<td>-4x - 5</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>-6x - 1</td>
<td>C</td>
<td>-2x - 17</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-3x - 3</td>
<td>B</td>
<td>-2x - 6</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>-4x - 2</td>
<td>C</td>
<td>-18</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-x + 1</td>
<td>B</td>
<td>-2</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>-2x + 2</td>
<td>C</td>
<td>2x - 14</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2x - 2</td>
<td>B</td>
<td>3x - 5</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>x - 1</td>
<td>C</td>
<td>5x - 17</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>x + 2</td>
<td>B</td>
<td>2x - 1</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>+ 3</td>
<td>C</td>
<td>4x - 13</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5x + 3</td>
<td>B</td>
<td>6x</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>4x + 4</td>
<td>C</td>
<td>8x - 12</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-5x + 1</td>
<td>B</td>
<td>-4x - 2</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>-6x + 2</td>
<td>C</td>
<td>-2x - 14</td>
<td>D</td>
</tr>
</tbody>
</table>
Essential vocabulary

**Expression:** An *algebraic* expression is made up of three things: numbers, variables, and operation signs such as + and −. Some examples are $2a$, $a + b$, $a^2$, $ab$. A *numeric* expression does not include variables, for example, $3 + 2$, $4 \times 7 - 1$.

**Equation:** A statement that the values of two mathematical expressions are equal (indicated by the sign $=$). The expressions may comprise numbers and/or variables (e.g. $3 + 1 = 4$ and $3x + 1 = 4$ are both equations).

**Hypothesis** (plural hypotheses): A proposed explanation for a phenomenon. In the case of mathematical hypothesis, it is most commonly a generalisation of an observed pattern. A single contrary case is sufficient to disprove a hypothesis. However, a hypothesis cannot be proved by examples, no matter how many there are.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

**General**

This activity is based on an idea presented in Foster, C. (2014), “Mathematical fluency without drill and practice”, *Mathematics Teaching*, 239(5).

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of simple linear equations. It has deliberately not been written for a “set and forget” teaching approach. If this task is used with students in Years 7 or 8, it may be sufficient for them to explore the patterns formed by the solutions of the equations created by the combinations of expressions A, B, C and D. This gives them valuable practice of solving simple linear equations with variables on both sides. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The challenge activity may be beyond most Year 7 or 8 students. It requires students to develop and test a hypothesis and then consider ways of explaining, or even proving, their hypothesis. However, it would be of value to students nearing the end of Year 9 or in Year 10.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

**Generating the equations**

The equations used in this task are not chosen at random. Expression A (top left) is generated randomly and the other expressions (B, C and D) are generated from A. If students generate their own expressions, the pattern in the solutions to the equations is unlikely to arise.

A spreadsheet accompanying this task allows students to generate an unlimited number of sets of four expressions that when paired in all possible combinations have the integers 1 to 6 as their solutions. It does not require students to enter any data. A program is also available for the TI–Nspire that allows students to generate the equations.

Alternatively multiple sets of equations have been included in the task. This page can be copied and cut into individual sets of equations for distribution to students.

Students should be encouraged to experiment with and explore these expressions before any structured discussion takes place.

**Prompts for students**

Enabling prompts – for use with groups/individuals having difficulty starting the task:

- Try to create just two expressions that have a solution of \( x = 1 \) when joined to form an equation.
- Start with one expression; what must be added to form an equation with a solution of \( x = 1, x = 2 \) etc.?
- Look for a pattern. If you have solved several sets of equations can you spot the connection between the expressions? Expression A (top left) is generated randomly and the other expressions (B, C and D) are generated from A.

Extension prompts — for students that have spotted the pattern:

- Why does this work? What mathematical property is being used to create these equations?
• If you had six expressions how many equations could be formed?
• Can you create a set of four expressions that will form equations with \(x = 2, 4, 6, 8, 10, 12\) or some other pattern or sequence as solutions.

**Mathematical theory**

The process used to create the equations makes use of the property of the additive identity (zero) and the fundamental concept of what constitutes an equation. To illustrate this, we can consider an equation where the solution is \(x = 3\). Since \(x = 3\), then \(x - 3 = 0\). In other words, \(x - 3\) is the additive identity for that equation. Additive identities have the property that adding the identity (zero) to an input results in an output that is the same as the input. In this example, if zero (or \(x - 3\), an expression that is equal to zero) is added to one side of the equation, it does not change the value of that side and the equation continues to be true. Similarly, adding \(n(x - 3)\), where \(n\) is a random integer, does not change the truth of the equation (since, in this example, \(n(x - 3) = n \times 0 = 0\)).

So, in the example on the task description sheet, the equation formed by expressions A and B is \(3x - 4 = 4x - 7\), with a solution of \(x = 3\). The spreadsheet has been set up so that the equation formed from expressions A and B will always have \(x = 3\) as a solution. In this example expression A is \(3x - 4\) and \(x - 3\) has been added to A to form \(4x - 7\), that is, expression B. Similarly adding \(n(x - 3)\) where \(n\) is a random integer to expression A to form expression B will always result the equation A = B having a solution of \(x = 3\).

The solutions for every pair equation formed by a pair of expressions are shown below.

Since \(x = 6\) is the solution formed by \(B = C\), to form expression C, we can add \(x - 6\), or a multiple of add \(x - 6\), to expression B. However, since \(x = 5\) is the solution formed by \(A = C\), expression C must also be the same as expression A plus \(x - 5\) or a multiple of \(x - 5\). This gives the result that

\[
m(x - 3) + n(x - 6) = p(x - 5), \quad \text{where } m, n \text{ and } p \text{ are integers}
\]

Expanding

\[
mx - 3m + nx - 6n = px - 5p
\]

Equating like terms,

\[
(m + n)x = px \quad (1) *
\]

and

\[
-3m - 6n = -5p \quad (2)
\]

Substituting eqn (1) into eqn (2) gives

\[
3m + 6n = 5m + 5n
\]

So, \(n = 2m\)

(* this step may need to be discussed with students). This means that if a random value of \(n\) is used to form expression B by adding \(n(x - 3)\) to A, then expression C will be formed by adding \(2n(x - 6)\) to expression B. By showing that

\[
n(x - 3) + 2n(x - 6) = n(x - 5),
\]

students can verify algebraically that the equation formed by expressions A and C will have a solution \(x = 5\).
A similar approach can be taken to form expression D from C and B (or A). The detail of how this is done is left to the student and teacher.

**Use of technology**

Two spreadsheet files accompany this task. They allow students to generate the expressions needed for the task. The spreadsheets are password protected to prevent corruption of the spreadsheet by students. The password is “YuMi Deadly” (with capitals and space). The calculations used to generate the expressions are hidden in columns N to Y of the spreadsheets. However it is recommended that a copy of the original spreadsheet is made before attempting to make any changes. The Excel spreadsheet ‘Connected Expressions.xlsx’ generates expressions for any value of the multiplier, $n$.

As an alternative, two TI–Nspire CAS CX files also accompany this task. CAS technology in the form of the TI–Nspire can also be used to produce unlimited sets of four expressions. If you do not have access to the TI–Nspire CAS calculator or computer software then these files will run in a web browser with the Nspire Document Player add-in installed. The document player can be found at [http://education.ti.com/en/us/document-player](http://education.ti.com/en/us/document-player). Like the Excel file described in the previous paragraph, the CAS CX file ‘Connected Expressions.tns’ generates expressions for any value of the multiplier, $n$.

The spreadsheet ‘Connected Expressions – Simple.xlsx’ (Excel) and the TI–Nspire CAS CX file ‘Connected Expressions — Simple.tns’ generate simpler sets of equations, making it easier for students to spot the pattern. In these files, the random multiplier has been set at $n = 1$. All sets of expressions follow exactly the same pattern. In the sets of equations on page 4, the first 8 sets have $n = 1$.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA150 Compare, order, add and subtract integers</td>
</tr>
<tr>
<td></td>
<td>ACMNA176 Create algebraic expressions and evaluate them by substituting a given value for each variable</td>
</tr>
<tr>
<td></td>
<td>ACMNA179 Solve simple linear equations</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA192 Simplify algebraic expressions involving the four operations</td>
</tr>
<tr>
<td></td>
<td>ACMNA194 Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution</td>
</tr>
</tbody>
</table>
7G1 “Flippin’ Congruence”

Task description

Background

Two shapes are congruent when they have the same shape and size, but can have different orientations. Congruence is important in our modern lives where angles and measurements matter.

There are two ways to think of congruence between two shapes – by relationship (where we look for the corresponding sides and angles to be equal) and by change (where we get from the first shape to second shape by one or more flips, slides and turns).

In this task, we will explore flips, slides and turns, show how they show congruence, look at artistic patterns that emerge from them, and finally show that, in the words of the great hit song, “all you need is flips”.

The investigations were designed to be conducted using tracing paper and Mira mirrors but the investigations can also be done with a computer or tablet. You will have to get to know how to use a Mira if this is new to you. This will require exploring how a Mira can be used to construct the image of a shape after a flip. Your teacher will give you worksheets of shapes, patterns and drawings that you can use with this investigation.

Activity 1

Experience flips slides and turns – use tracing paper and a Mira (or a computer or tablet). What are the mathematical names for flips, slides and turns?

Activity 2

Show that, for flips, slides and turns, the original shape and its image are congruent.

Activity 3

Use flips, slides and/or turns to create a repeating design for a wall frieze or a pattern to be printed on fabric.

Activity 4

Investigate the properties of flips, slides and turns. What are the rules that always apply to each process?

Activity 5

Investigate the relationship between flips, slides and/or turns. Is there more than one way to create the same image? Can you write any rules to show the relations between flip, slides and/or turns?

Activity 6

(Challenge) Divide a square into two triangles by adding a diagonal. Are these two triangles congruent? Now divide the same square into four triangles by adding a second diagonal. Are these four triangles congruent? What can you conclude about the properties of a square? Repeat this task for other special types of quadrilaterals (rectangle, parallelogram, rhombus, kite and trapezium). Present your conclusions in a table that summarises the properties of the various quadrilaterals. What can you conclude about the relationships between the special types of quadrilaterals?

Make sure you understand the meanings of any words in italics.
Additional student information – “Flippin’ good activities” worksheets

MAKE THE PLANE TAKE OFF, FLY AROUND AND LAND BACKWARDS!

PUT THE EARS ON THE MOUSE!

PUT THE BONE IN BRUNO’S MOUTH!

Bruno
FLIP THE SHAPES AROUND THE LINE!

FLIP THE FROG AROUND THE LINE!
Construct your own shape from the 3×3 grid above and see what looks better – a pattern that flips or a pattern that reflects. Also try your pattern as a slide. [I think the flip is better of the patterns above.]

Try a 4×4 grid as a start (or a 5×5 or more). Make up your own way to get a starting pattern – don’t make it too complicated. (What about a spiral? Or some curved lines?)
Fabric design. Make a design within a square – put all protrusions back into the square, remove the square and flip the design. You will have a fabric design – see below.

- Make up your own design.
- Make the design on another tessellating shape (e.g. hexagon or triangle).

Nets

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Design 1" /></td>
<td><img src="image2.png" alt="Design 2" /></td>
<td><img src="image3.png" alt="Design 3" /></td>
<td><img src="image4.png" alt="Design 4" /></td>
<td><img src="image5.png" alt="Design 5" /></td>
<td><img src="image6.png" alt="Design 6" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="Design 7" /></td>
<td><img src="image8.png" alt="Design 8" /></td>
<td><img src="image9.png" alt="Design 9" /></td>
<td><img src="image10.png" alt="Design 10" /></td>
<td><img src="image11.png" alt="Design 11" /></td>
<td><img src="image12.png" alt="Design 12" /></td>
</tr>
<tr>
<td><img src="image13.png" alt="Design 13" /></td>
<td><img src="image14.png" alt="Design 14" /></td>
<td><img src="image15.png" alt="Design 15" /></td>
<td><img src="image16.png" alt="Design 16" /></td>
<td><img src="image17.png" alt="Design 17" /></td>
<td><img src="image18.png" alt="Design 18" /></td>
</tr>
<tr>
<td><img src="image19.png" alt="Design 19" /></td>
<td><img src="image20.png" alt="Design 20" /></td>
<td><img src="image21.png" alt="Design 21" /></td>
<td><img src="image22.png" alt="Design 22" /></td>
<td><img src="image23.png" alt="Design 23" /></td>
<td><img src="image24.png" alt="Design 24" /></td>
</tr>
</tbody>
</table>

© QUT YuMi Deadly Centre 2017
Essential vocabulary

Change: If a thing is different from what it once was then it has changed. In mathematics, it usually refers to a process that alters shapes, locations, orientations, values, or measurements.

Congruent: To be identical in all respects. Usually used in two-dimensional geometry to describe shapes where the corresponding sides and angles are equal.

Diagonal: A line segment joining two non-adjacent vertices of a polygon or polyhedron. As the vertices cannot be adjacent, the diagonal will pass through the inside of the polygon or polyhedron.

Orientation: The position or alignment of a shape relative to compass points, the positive direction of the x-axis in a Cartesian plane, the top of the page, or some other direction.

Properties: The basic and/or essential attributes shared by all elements (members) of a set. For example, a description of the properties of a particular type of quadrilateral might include information about whether the shape is closed, the size of the angles, the length of the sides, the length of the diagonals, angles at which the diagonals intersect, and whether the diagonals bisect each other (cut each other in half).

Relationship: A connection between two ideas or things. In mathematics, the word relation is often used to describe the connection(s) between an object, pattern, or the value of a variable, before and after a change.

Vertex (plural vertices): A point formed by the intersection of two straight lines. It is typically used to describe the corners of a polygon or polyhedron or the point of an angle.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This investigation can introduce students to basic geometric transformations. This section provides information on completing the investigation activities. This information can be used by the teacher as a basis for hints to students to keep them working or it can be used as the basis of worksheets. Please see the “Flippin good activities worksheets” for further information. General information will be given first and then this section will look at each activity on the task description page of this investigation.

Flips, slides and turns are also known as reflections, translations and rotations, respectively. During this investigation, students should discover that additional information is needed to accurately describe each transformation. For example:

- Flips: we need to know if the flip is horizontal, vertical, diagonal, or at some other angle, and where the axis of reflection is (passing through the shape or outside the shape) – we use a dotted line.
- Slides: we need to know how far and in what direction.
- Turns: we need to know the number of degrees the shape is rotated through and the location of the point of rotation (pivot point).

For activities involving reflections and rotations, students should not use shapes with too many lines of symmetry, otherwise they may draw false conclusions. An arrow head such as shown at the right is quick and easy to draw. More capable students could construct a 5 cm square, number each vertex, cut it out, turn it over and place the same numbers on the underside of each vertex. They can then explore the translations by considering how they change the order of the numbered vertices. Capable students could also examine the relation between symmetry and transformations.

An important outcome of this task is for students to recognise that the properties of different orientations of the same shape are the same. If you draw, for example, a right-facing right-angled triangle, students must recognise that the left-facing and inverted versions of that right-angled triangle are congruent and so any conclusions about them (such as side length or angle size) will be the same.

The challenge task allows students to investigate the properties of the special quadrilaterals and the relationships between them. In addition to identifying the similarities and differences between the various quadrilaterals, they should also be able to identify the relationships between these quadrilaterals, for example that the square is a special case of both the rectangle and the rhombus, and squares, rectangles and rhombuses are special cases of parallelograms.

The information for students includes some essential mathematical words that students should understand and be able to spell. Included before this sub-section is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
**Activity 1: Experience**

The first activity is to get to know the Mira and to experience flips, slides and turns with tracing paper (and Mira).

A Mira is a red plastic mirror as shown on right. Its strength is that the red plastic allows you to look through the Mira to what is behind and, at the same time, to see the reflection of what is in front of the Mira. This means that a Mira can be used to superimpose and to draw reflections. The diagram below shows how to use the Mira to draw reflections and superimpose a shape. Its purpose is to develop *spatial visualisation* (the ability to mentally manipulate, twist, rotate, reflect, slide or invert shapes), an understanding of the role of angles in reflection, and an understanding of the relationship between reflections and symmetry. Use the Mira by placing it on the dotted line or between the two objects if moving something.

![Diagram of Mira use](image)

**FLIP**

To construct a flip: (a) copy shape and line onto tracing paper, (b) fold tracing paper along line (fold away), (c) copy shape through onto other side of shape – draw with a dotted line, (d) open tracing paper and place on top of drawing and dotted line, (e) press hard and copy back onto original page (shading back of drawing of shape with pencil will help), and (f) draw in flipped shape as a dotted line. (Note: The flip can be done with the Mira.)

**SLIDE**

To construct a slide: (a) copy shape onto tracing paper and place a dot at start of arrow, (b) holding original page fixed, move tracing paper so that dot slides along arrow to end without turning the tracing paper, (c) press hard and copy shape back onto original page, and (d) draw in slid shape as a dotted line. As you are in pairs, one can hold the paper and the other can turn the tracing paper.

**TURN**

To construct a turn: (a) copy shape and centre dot onto tracing paper and draw a dotted line from the centre to the start of arrow, (b) holding original page fixed and placing pencil point on centre, turn the tracing paper so that line slides along arrow to end without moving the pencil point, (c) press hard and copy shape back onto original page, and (d) draw in turned shape as a dotted line. Again, as you are in pairs, one can hold the paper and the other can turn the tracing paper.

**Activity 2: Congruence**

If you superimpose the image on the original shape in all the examples above, you can see that they are congruent. However, some other ideas are as follows.

- Draw a shape, use Miras and tracing paper to flip, slide and turn the shape to a new position, check with protractor and ruler that length of sides and measure of angles remains the same. Repeat this with a different shape.
- Take a plastic shape, copy this onto paper, move it around and recopy in a new place. Can you get from the first to the second shape by flipping, sliding and turning with Mira and tracing paper?
There are many instances where we need to see if one shape can replace or is the same as another – we need to flip, slide and turn the first shape and get the second. An example – how many different shapes can you make from 5 squares or 6 equilateral triangles?

**Activity 3: Frieze patterns and fabric design**

**Frieze pattern.** Use flips and turns to construct your own frieze pattern as shown on the *Flippin’ good activities* worksheet. Interesting extensions are to: (a) construct it, cut copies out of coloured paper, and stick them on paper, flipping or turning as you put the next one on; and (b) make them out of fabric and sew headbands. These frieze patterns were used in temples around windows – where else are they used or could they be used?

**Fabric design.** Use the techniques on the *Flippin’ good activities* worksheet to create a fabric design. Draw a square as in the worksheet (with lines extending) – draw with pencil so it can be rubbed out. Draw a design in the square – it can go outside the square. Place the outside bits back into the square – this sometimes means changing the design so there is no overlap. Remove the square. Use Mira to flip the design so that it covers the space and we have a fabric design. (Note – this is a tessellation activity – you can make the design on another tessellating shape such as a hexagon or triangle.)

**Activity 4: Properties of flips, slides and turns**

Do the following for the flips, slides and turns that your students construct in Activity 2.

- **FLIP**
  - 1. Join three corresponding points.
  - 2. Write down properties for the flip change.

- **SLIDE**
  - 1. Join corresponding points.
  - 2. Write down properties for the slide change.

- **TURN**
  - 1. Join corresponding points.
  - 2. Write down properties for the slide change.
  - 3. Repeat above for concentric circles.

In working out the properties:

- Look for parallel lines.
- Look at lengths of movements.
- Look at relation of movement lines to the flip line, slide line or turn line.
- Look at whether the definition should refer to straight or curved lines.

Write your properties down under headings – flip properties, slide properties, and turn properties:

- **Flip properties** – Points move parallel to each other perpendicular to flip line. flip line is perpendicular bisector of movement lines (close to flip line stays close and far way stays far away). Left goes to right and vice versa.
- **Slide properties** – all points move parallel to each other the same distance.
- **Turn properties** – all points move the same angle in concentric circles around centre of turn.
**Activity 5: Relationships**

Use the Mira to show that slide and turn from Activity 2 can be done by two flips and answer the questions.

- What is the relation between the flips and the slide? [2 flips equal one slide if flip lines parallel, perpendicular to slide, and half slide length distance apart.]
- What is the relation between the flips and the turn? [2 flip lines equal one turn if flip lines pass through centre of turn and angle between flip lines is half turn angle.]
- Write these two relations as a rule for flips slides and turns. [Any flip, slide and turn movement can be completed by flips alone.]
- How many flips will be needed for congruence? Demonstrate this! [Three flips give congruence.]

**Activity 6: Properties of quadrilaterals**

A worksheet as below can assist.

1. Divide the quadrilateral into two triangles by drawing a straight line from opposite vertices (corners).
2. Cut out the triangles. Use flips and turns to determine whether the two triangles are congruent.
3. If the triangles are congruent, what can you conclude about:
   - The relationship between the angles of the quadrilateral (for example, are they equal?)
   - The relationship between the sides of the quadrilateral?
   - The relationship between the diagonals of the quadrilateral?
4. Divide the quadrilateral into four triangles by drawing a straight line from each pair of opposite vertices.
5. Cut out the triangles. Use flips and turns to determine whether the four triangles are congruent.
6. If the triangles are congruent, what can you conclude about how the diagonals intersect? For example do they bisect each other (cut each other in half exactly), are the diagonals perpendicular?
7. Repeat steps 1 to 6 for each of the special types of quadrilateral: square, rectangle, parallelogram, rhombus, kite, trapezium.
8. Summarise your results in a table like the one below:

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Angles</th>
<th>Sides</th>
<th>Diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size</td>
<td>Which are equal?</td>
<td>Which are equal?</td>
</tr>
<tr>
<td>Square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rhombus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trapezium</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. You can conclude from your table that a square has the same properties as a rectangle, but there is an additional property that all sides must be equal. This means that the square is a special type of rectangle. Can you find similar relationships between other quadrilaterals in your table?
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMMG114 Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries</td>
</tr>
<tr>
<td>6</td>
<td>ACMMG142 Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMMG200 Define congruence of plane shapes using transformations ACMMG202 Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning</td>
</tr>
</tbody>
</table>
Task description

Board games can provide hours of fun. Often the winning strategy relies on logic and thinking ahead. Problem solving trains mathematicians to think logically and to plan ahead. Most can apply this skill to board games. Consequently, mathematicians usually enjoy, and are successful at, board games.

There are five types of board games: (a) war games like Chess or Draughts; (b) race games like Snakes and Ladders; (c) hunt games like Kaooa; (d) alignment games like Noughts and Crosses (Tic-tac-toe); and (e) accumulation games like Kalah. This task looks at: (1) the role of strategies for winning logical games – looking at board games; and (2) the role of knowledge and experience in winning games.

1. **Investigate** the following games:
   - Noughts and Crosses
   - Kaooa
   - Nim
   - the French Military Game
   - Kalah
   (Your teacher will give you game boards for Kaooa and the French Military Game).

2. Find out the rules of each game and play them with a partner. The focus of the task is not to beat your partner, but to **work together as a group to figure out the winning strategies**.

3. When you think you have worked out the winning strategy in a game, try **modifying the rules** of the games (for example, what if three in a row in Noughts and Crosses was a loss, not a win?). **What affect does this have on the winning strategy?**

4. **Write out the winning strategy as a procedure for each game** (as series of steps using language such as first, next, if ... then, etc).
Additional student information

Game board for Kaooa
Game board for the French Military Game

FRENCH MILITARY GAME
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of spatial awareness, logic and planning. It has deliberately not been written for a “set and forget” teaching approach. It is expected that some students will require assistance in understanding the rules of the various games (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with hints to keep them working. How this occurs is a matter for teacher judgement. This section provides a description of the games and some questions that could be posed to students. These detailed instructions could also be used as the basis of student worksheets. The most important is, for each game, ask the students to write out the winning strategy as a procedure (a series of steps using language such as first, next, if ... then, etc).

Noughts and Crosses

Rules. Noughts and Crosses is a simple alignment game and we are starting with it because it is possible to completely analyse it. It is played on a simple board which has nine cells and can be quickly drawn on any piece of paper (see on right). It is played by two players who take turns. To begin, one player takes the O and the other the X. The rules are that players in turn place their O or X in one of the nine cells that is vacant. The winner is the first player to get three of their O’s or X’s in a row, column or diagonal. Otherwise the game is a draw.

Instructions for students. Play the game, trying to work together to work out winning moves. They will need to work out which strategies provide the best chance of winning and of achieving a draw.

1. Ask students to share strategies. If not much is being achieved, ask these questions:
   
   (a) What is the most important square or cell to fill? If you fill this cell, what mistake by your opponent will ensure you win? What can you do to maximise your chances of this error?
   
   (b) If your opponent fills this cell, what do you have to do to ensure you draw? Is there a way to win without filling this most important cell first? Why does it work?

2. Ask them to try these “end games” – it will help students to develop strategies.

   (a) X plays next and can win no matter what O does! How?

   \[
   \begin{array}{c|c|c}
   X & O \\
   \hline
   \end{array}
   \]

   (b) O plays next and if places O wrongly, X wins no matter what. Why is this cell wrong for O?

   \[
   \begin{array}{c|c|c}
   O & \ \ & \ \\
   X & \ \\
   \hline
   \end{array}
   \]

   (c) O plays next and if places O wrongly, X wins no matter what. Why is this cell wrong for O?

   \[
   \begin{array}{c|c|c}
   \ & O \\\n   X & \ \\
   \hline
   \end{array}
   \]
3. Write down the following strategies:

(a) Strategy for how to win starting from middle cell.
(b) Strategy for how not to lose when opposition starts from middle cell.
(c) Strategy for how to win from a corner cell.
(d) Strategy for how not to lose when opposition starts from a corner cell.

4. Why does a corner cell player often win if the other player is a regular middle cell player?

**Kaooa**

**Rules.** Kaooa is a hunt game played on a board of 10 circles presented in the diagram on the right. (A large game board is provided in “Additional student information”.) It is a game for two players – one is the seven Kaooas and the other is the tiger they hunt. The seven Kaooas are placed on the circles that have the little circles inside, the tiger is then placed anywhere on the other three circles. One Kaooa moves first, then the tiger, and so on. Kaooas can move along any line to a vacant circle. The tiger moves in the same way but can eat (jump) a Kaooa by going over one along a line to a vacant circle. The Kaooas win if they trap the tiger (there is no legitimate move for the tiger) while the tiger wins if it can eat one Kaooa. The game should be won by the Kaooas.

**Instructions for students.** Play the game, trying to work together to work out winning strategies. If the students have difficulty with Kaooa, ask the following questions:

(a) How can you stop being eaten (where can you not leave a cell open)?

(b) What are the important cells to fill? Can your strategy work with the tiger starting in any of the three cells available?

**Nim**

**Rules.** There are many games of Nim, which is a war game. A simple game for two players starts with 14 counters in a line. Each player in turn, takes one or two from either end of the line. The winner is the player who does not take the last counter. (Note: The game can be played by drawing 14 circles in a row and crossing them out.) The game should always be won by the first player as there is a strategy that always wins.

**Instructions for students.** Play the game, trying to work together to work out the winning strategy. If the students have difficulty, ask the following questions:

(c) What number of counters would you like/not like to be left with? How can you ensure you leave your opponent with the number of counters that means they lose?

(c) If there were 15 counters, the second player would always win. Why?

**The French Military Game**

**Rules.** This game is a hunt game which uses a board as on the right. (A large game board is provided in “Additional student information”.) The board has 11 cells – the three with small black circles are where the three English soldiers are placed, the one with the small white circle is where Napoleon is placed. One soldier moves first – soldiers can move one cell forward/sideways along any line to a vacant cell but not backwards (even diagonally). Napoleon can move one cell in any direction along a line to a vacant cell. The English soldiers win if they can pin Napoleon (he has no legitimate move). All other results, continuous moving between spaces (“stalemate”) or Napoleon getting behind the English soldiers to where they cannot go back to get him, are a win for Napoleon.
Instructions for students. Play the game, trying to work together to work out winning strategy. Analyse the game. Who wins and how? What are good strategies – for Napoleon and for the soldiers? Who should always win? (Remember – this is a French game!)

On the right is an end game (English to move). How can the English win? How can Napoleon get away?

Kalah (Challenge)

Allow the students to find out for themselves how to play the accumulation game Kalah (or Marcala). This game was not known in Europe. Ask the following questions: (a) What are the strategies to win this game? (b) How are these strategies different from war, hunt and alignment games?

All games – Modifications that change strategies

1. What if in the game of Noughts and Crosses, you could put either a nought or a cross when it was your turn to play? What is the strategy now?

2. What if three in a row was a loss?

3. What if the game of Nim was changed so that you could remove 1, 2 or 3 counters from either end of the line of counters? If the start was still 14 counters in a line, how would this change the strategy? Would the first player still win? With what strategy?

4. If the 14 counters of Nim were put in a circle and you were allowed to remove 1 counter or 2 counters that were adjacent (a counter had not been removed from between them), would this change the strategy? Would the first player still win? With what strategy? Or would this give the game to the second player (and with what strategy)? Remember, the player who takes the last counter loses.

5. What if the games of Nim were changed to “the player who takes the last counter wins”?

Links to the Australian Curriculum

Mathematics

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMMG114 Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries</td>
</tr>
<tr>
<td>6</td>
<td>ACMMG142 Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies</td>
</tr>
</tbody>
</table>
### Critical and creative thinking

<table>
<thead>
<tr>
<th>Critical and creative thinking</th>
<th>By the end of Year 8</th>
<th>By the end of Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inquiring – identifying, exploring and organising information and ideas</strong></td>
<td>Pose questions: pose questions to probe assumptions and investigate complex issues</td>
<td>Pose questions: pose questions to critically analyse complex issues and abstract ideas</td>
</tr>
<tr>
<td></td>
<td>Identify and clarify information and ideas: clarify information and ideas from texts or images when exploring challenging issues</td>
<td>Identify and clarify information and ideas: clarify complex information and ideas drawn from a range of sources</td>
</tr>
<tr>
<td></td>
<td>Organise and process information: critically analyse information and evidence according to criteria such as validity and relevance</td>
<td>Organise and process information: critically analyse independently sourced information to determine bias and reliability</td>
</tr>
<tr>
<td><strong>Generating ideas, possibilities and actions</strong></td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals</td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals</td>
</tr>
<tr>
<td></td>
<td>Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting</td>
<td>Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting</td>
</tr>
<tr>
<td></td>
<td>Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
<td>Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
</tr>
<tr>
<td><strong>Reflecting on thinking and processes</strong></td>
<td>Think about thinking (metacognition): assess assumptions in their thinking and invite alternative opinions</td>
<td>Think about thinking (metacognition): give reasons to support their thinking, and address opposing viewpoints and possible weaknesses in their own positions</td>
</tr>
<tr>
<td></td>
<td>Reflect on processes: evaluate and justify the reasons behind choosing a particular problem-solving strategy</td>
<td>Reflect on processes: balance rational and irrational components of a complex or ambiguous problem to evaluate evidence</td>
</tr>
<tr>
<td></td>
<td>Transfer knowledge into new contexts: justify reasons for decisions when transferring information to similar and different contexts</td>
<td>Transfer knowledge into new contexts: identify, plan and justify transference of knowledge to new contexts</td>
</tr>
<tr>
<td><strong>Analysing, synthesising and evaluating reasoning and procedures</strong></td>
<td>Apply logic and reasoning: identify gaps in reasoning and missing elements in information</td>
<td>Apply logic and reasoning: analyse reasoning used in finding and applying solutions, and in choice of resources</td>
</tr>
<tr>
<td></td>
<td>Draw conclusions and design a course of action: differentiate the components of a designed course of action and tolerate ambiguities when drawing conclusions</td>
<td>Draw conclusions and design a course of action: use logical and abstract thinking to analyse and synthesise complex information to inform a course of action</td>
</tr>
<tr>
<td></td>
<td>Evaluate procedures and outcomes: explain intentions and justify ideas, methods and courses of action, and account for unexpected outcomes against criteria they have identified</td>
<td>Evaluate procedures and outcomes: evaluate the effectiveness of ideas, products and performances and implement courses of action to achieve desired outcomes against criteria they have identified</td>
</tr>
</tbody>
</table>
7M1 “Rocking Around the World”

Task description

This task requires you to think about how time zones work given that the planet Earth is (nearly) a sphere.

Scenario

A rock band, “The Crazy Hermits”, wishes to tour the world. You have been hired to organise an itinerary for the band. They want to play in as many of the cities listed below in just 10 days to set a world record. This will gain free advertising for the band.

To complete the task you will need to take account of world time zones and the time taken to travel by air between the major cities. You will also need to consider the time needed to: get from airports to venues; set up; perform; pack up; and return to the airport to get to the next city. The band wants to start in Brisbane on a Sunday night and to play a “We conquered the world” concert in Brisbane 10 days later on Tuesday night.

Instructions

Plan a schedule for “The Crazy Hermits” which allows them to visit as many cities around the world as possible. The handout includes a list of cities that have agreed to accept “The Crazy Hermits” to perform a concert. You may choose other cities not in the list. Make sure you check with your teacher. The city needs to have an international airport.

1. You can use the following assumptions to plan the travel arrangements:
   - it takes 3½ hours from when the plane touches down until the concert starts;
   - all concerts can commence as soon as the band arrives;
   - each concert is of two hours duration;
   - it will take 2½ hours from the end of each concert to be on the plane flying out to the next destination.

2. To estimate the flight times use the websites in the handout. Likewise, if you need airport codes for a city not in the list, use the airport codes website given in that handout.

3. After you have constructed your schedule, write a paragraph or two for the band members explaining to them the choices you have made, taking into account the geography, the distance and the direction of the travel.

4. Challenge: Write, for the members of the band, an explanation of how and why the world is divided into time zones.
### Additional student information

#### List of cities for the Crazy Hermits tour

<table>
<thead>
<tr>
<th>City</th>
<th>Country</th>
<th>Airport Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antwerp</td>
<td>Belgium</td>
<td>ANR</td>
</tr>
<tr>
<td>Athens</td>
<td>Greece</td>
<td>ATH</td>
</tr>
<tr>
<td>Auckland</td>
<td>New Zealand</td>
<td>AKL</td>
</tr>
<tr>
<td>Beijing</td>
<td>China</td>
<td>PEK</td>
</tr>
<tr>
<td>Boston</td>
<td>USA</td>
<td>BOS</td>
</tr>
<tr>
<td>Brisbane</td>
<td>Australia</td>
<td>BNE</td>
</tr>
<tr>
<td>Budapest</td>
<td>Hungary</td>
<td>BUD</td>
</tr>
<tr>
<td>Buenos Aires</td>
<td>Argentina</td>
<td>AEP</td>
</tr>
<tr>
<td>Capetown</td>
<td>South Africa</td>
<td>CPT</td>
</tr>
<tr>
<td>Chicago</td>
<td>USA</td>
<td>ORD</td>
</tr>
<tr>
<td>Copenhagen</td>
<td>Denmark</td>
<td>CPH</td>
</tr>
<tr>
<td>Dublin</td>
<td>Ireland</td>
<td>DUB</td>
</tr>
<tr>
<td>Edinburgh</td>
<td>Scotland</td>
<td>EDI</td>
</tr>
<tr>
<td>Edmonton</td>
<td>Canada</td>
<td>YEG</td>
</tr>
<tr>
<td>Frankfurt</td>
<td>Germany</td>
<td>FRA</td>
</tr>
<tr>
<td>Geneva</td>
<td>Switzerland</td>
<td>GVA</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>Hong Kong</td>
<td>HKG</td>
</tr>
<tr>
<td>Honolulu</td>
<td>USA</td>
<td>HNL</td>
</tr>
<tr>
<td>Jakarta</td>
<td>Indonesia</td>
<td>HLP</td>
</tr>
<tr>
<td>Kingston</td>
<td>Jamaica</td>
<td>KIN</td>
</tr>
<tr>
<td>Lahore</td>
<td>Pakistan</td>
<td>LHE</td>
</tr>
<tr>
<td>Lima</td>
<td>Peru</td>
<td>LIM</td>
</tr>
<tr>
<td>London</td>
<td>England</td>
<td>LHR</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>USA</td>
<td>LAX</td>
</tr>
<tr>
<td>Madrid</td>
<td>Spain</td>
<td>MAD</td>
</tr>
<tr>
<td>Mumbai</td>
<td>India</td>
<td>BOM</td>
</tr>
<tr>
<td>New York</td>
<td>USA</td>
<td>JFK</td>
</tr>
<tr>
<td>Oslo</td>
<td>Norway</td>
<td>OSL</td>
</tr>
<tr>
<td>Ottawa</td>
<td>Canada</td>
<td>YOW</td>
</tr>
<tr>
<td>Paris</td>
<td>France</td>
<td>CDG</td>
</tr>
<tr>
<td>Perth</td>
<td>Australia</td>
<td>PER</td>
</tr>
<tr>
<td>Rio De Janeiro</td>
<td>Brazil</td>
<td>GIG</td>
</tr>
<tr>
<td>Rome</td>
<td>Italy</td>
<td>FCO</td>
</tr>
<tr>
<td>Rotterdam</td>
<td>Holland</td>
<td>RTM</td>
</tr>
<tr>
<td>San Francisco</td>
<td>USA</td>
<td>SFO</td>
</tr>
<tr>
<td>Seoul</td>
<td>South Korea</td>
<td>ICN</td>
</tr>
<tr>
<td>Singapore</td>
<td>Singapore</td>
<td>SIN</td>
</tr>
<tr>
<td>Stockholm</td>
<td>Sweden</td>
<td>ARN</td>
</tr>
<tr>
<td>Suva</td>
<td>Fiji</td>
<td>SUV</td>
</tr>
<tr>
<td>Sydney</td>
<td>Australia</td>
<td>SYD</td>
</tr>
<tr>
<td>Tel Aviv</td>
<td>Israel</td>
<td>JRS</td>
</tr>
<tr>
<td>Vienna</td>
<td>Austria</td>
<td>VIE</td>
</tr>
<tr>
<td>Warsaw</td>
<td>Poland</td>
<td>WAW</td>
</tr>
<tr>
<td>Wellington</td>
<td>New Zealand</td>
<td>WLG</td>
</tr>
</tbody>
</table>
Website information

The following websites will give time zone information:

- http://www.timeanddate.com/worldclock/
- http://www.timeticker.com/
- http://wwp.greenwichmeantime.com/

The following website will give flight time information:

- http://www.quicktrip.com/

The following website will give airport codes:

- http://gc.kls2.com/
**Teaching information**

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of time zones. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

You will need to check the websites in the “Additional student information” section, which give time zone and other information, to ensure that they are up-to-date before duplicating the information for student use.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMMG110 Compare 12- and 24-hour time systems and convert between them</td>
</tr>
<tr>
<td>6</td>
<td>ACMMG139 Interpret and use timetables</td>
</tr>
<tr>
<td>8</td>
<td>ACMMG100 Solve problems involving duration, including using 12- and 24-hour time within a single time zone</td>
</tr>
</tbody>
</table>
7M2 “Cheap Houses”

Task description

In engineering and building the objective is usually to get the best design for the least cost. In this task you are asked to explore how to build a functional house at the lowest cost.

Investigate why cheap houses are nearly always square in overall shape. Use this information to design the cheapest possible 144 square metre house with three bedrooms, one bathroom, a kitchen and a lounge room.

Additional student information

To calculate the cost of your house, you should use these components

- External walls: $1500 per metre
- Internal walls: $1200 per metre
- Foundations and roof: $700 per square metre
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of area and perimeter in Year 7. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Detailed instructions

Activity 1: Constructing houses

Materials: 1 cm graph paper

Set-up: Groups of three – Builder (decision-maker), Architect (drawing), Inspector (checking)

1. On graph paper, draw plans for 3 houses of 144 square metres (use 1 cm on the graph paper to represent 1 m):
   - 12 × 12 square
   - 8 × 18 rectangle
   - L-shaped (see right)

2. Draw internal walls for same-size kitchen, lounge, three bedrooms, bathroom and hall (if necessary), for each house (make sure all components are the same size for each house).

3. Work out cost of walls if external walls cost $1500 per metre and internal walls cost $1200 per metre. Add on cost of foundations and cost of roof at $700 per square metre.

4. Which house is cheaper?

Activity 2: Why is it cheaper?

Materials: 1 cm graph paper

Set-up: Groups of three (same roles as Activity 1 but swap roles among students)

1. On graph paper, draw different polygon shapes – square, rectangles of different types, triangles, hexagons, pentagons, and shapes made up of different length sides. Have all corners of the shapes on intersection points of the graph.

2. Calculate the area (count squares ½ and more than ½ in the shape or use technique below) and calculate the perimeter (use a ruler).

Technique

Area is ½ of:

Area is found by subtracting shaded parts of:
3. Complete the table:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>Perimeter</th>
<th>Area/Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (draw shape)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and so on</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. What shape is cheapest to build for a given area?

5. Prepare a display, giving data, conclusions and reasons why.

6. **Challenge:** Find out what Pick’s Theorem is. Would it work for the above activity?

**Activity 3: Cheaper still**

**Materials:** String, 1 cm graph paper

**Set-up:** Groups of three (same roles as Activity 1 but swap roles among students)

1. Cut and join a piece of string so it is 36 cm in a knotted circle:

2. Use this string to construct different types of curved shapes on graph paper. Include an ellipse and a circle but other shapes as well:

3. Calculate area (perimeter is always 36 cm) – remember if more than ½ the shape is in a square, the square is counted; less than ½ the shape in the square, it is not.

4. Complete the table:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>Perimeter</th>
<th>Area/Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (draw shape)</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and so on</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. In theory, what shape is cheapest to build for a given area?

6. Why might this not be true in practice?

7. Extend your display from Activity 2.

8. **Challenge:** Find out what the “Earthbag construction” technique is. Why would this be really a cheap way to build a house?

**Activity 4: Challenge – the cheapest house possible**

1. Suppose we are using normal construction methods (straight walls), what would be the cheapest possible 144 square metre house, with three bedrooms, one bathroom, a kitchen and a lounge room?

2. Remember that the cost of the roof and foundation is fixed. Where you have to save is with respect to the walls, particularly the internal walls. However, the bedrooms and bathrooms must have four walls.

3. See who can design the cheapest house.
# Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMNA106 Create simple financial plans</td>
<td>ACMMG109 Calculate the perimeter and area of rectangles using familiar metric units</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA132 Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies</td>
<td>ACMMG136 Convert between common metric units of length, mass and capacity ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA159 Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies</td>
<td>ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA187 Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies</td>
<td>ACMMG195 Choose appropriate units of measurement for area and volume and convert from one unit to another</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA208 Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems</td>
<td>ACMMG216 Calculate the areas of composite shapes</td>
</tr>
</tbody>
</table>
7N1 “Tangled Fractions”

Task description

Calculating with common fractions is an important skill for mathematicians. We use them daily in everyday life, for example, using a clock, slicing a cake or pizza, or following a recipe. When looking at the fuel gauge in a car it shows if there is three quarters, a half, or a quarter of a tank left. Common fractions are often easier to use than many decimal fractions, for example, \( \frac{1}{3} \) is easier to use than 0.3333..., and \( \frac{5}{8} \) is easier to use than 0.625. Fractions are also used by many people at work. Scientists and chemists use fractions to measure the right amount of a solution or chemical to use. Fractions are particularly important in algebra, where they allow us to see patterns that are not as obvious when using decimals. This activity provides you with practice in calculating with fractions.

Read the attached instructions and ensure that you have correctly understood how to Twist and Turn according to the rules for this activity. These are the only two allowable moves and they must be carried out correctly.

Tangles are scored as follows:

- for every twist, add one to your score; and
- for every turn take the negative reciprocal of your score.

In your group, obtain two lengths of rope and create a tangle using only Twists and Turns. Record the sequence of moves and determine the tangle score.

Answer the following questions.

1. Using only the two allowable moves, can you generate a sequence of twists and turns to produce a tangle score of (a) \( \frac{2}{3} \), (b) \( \frac{7}{10} \), (c) \( -\frac{5}{7} \).

2. We call these rational tangles because they are represented by a rational number. Is it possible to create a tangle for any rational number?

3. Can you undo your tangle using only the same two moves (Twists and Turns) as defined? Can you devise a generalised procedure to untangle any rational tangle?

4. Challenge: The Ancient Egyptians thought that all fractions had to have a numerator of 1, for example, \( \frac{1}{5} \), \( \frac{1}{13} \), \( \frac{1}{45} \). To show a fraction with a different numerator, they had to use a sum of Egyptian fractions, for example \( \frac{7}{12} \) had to be written as \( \frac{1}{4} + \frac{1}{3} \). Could the Egyptians write any fraction using this method, or were there some fractions that they could not show? Justify your answer.

Make sure you understand the meanings of any words in italics.
Additional student information

**Twisting and Turning instructions**

Start with four students and two lengths of rope arranged as shown (right). This is the starting position.

Two moves are allowable: a TWIST and a TURN

---

**TWIST**

![Diagram showing a twist](image)

A Twist is shown in the diagram to the left. Students in the A and B positions swap places with student A passing his/her rope **under** the rope held by student B.

The diagram at the right shows how two twists would look.

---

**TURN**

![Diagram showing a turn](image)

All students rotate clockwise through a quarter of a turn.

i.e. A→B; B→C; C→D and D→A
**Keeping score**

Call the starting position zero. Each twist increases the tangle so add one. For example, Start followed by three Twists (S → Twist → Twist → Twist) would be represented by 0 → 1 → 2 → 3.

Each time you do a Turn, take the negative reciprocal of the number. For example, Start followed by three Twists and then a Turn (S → Twist → Twist → Twist→ Turn) would be represented by 0 → 1 → 2 → 3 → $-\frac{1}{3}$.

Consider the sequence of moves and scores below:

<table>
<thead>
<tr>
<th>Move</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>0</td>
</tr>
<tr>
<td>Twist</td>
<td>1</td>
</tr>
<tr>
<td>Twist</td>
<td>2</td>
</tr>
<tr>
<td>Twist</td>
<td>3</td>
</tr>
<tr>
<td>Turn</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{5}{3}$</td>
</tr>
<tr>
<td>Turn</td>
<td>$-\frac{3}{5}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{2}{5}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{7}{5}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{12}{5}$</td>
</tr>
<tr>
<td>Turn</td>
<td>$-\frac{5}{12}$</td>
</tr>
<tr>
<td>Twist</td>
<td>$\frac{7}{12}$</td>
</tr>
</tbody>
</table>

Make sure you understand the scoring system then create your own tangled fraction. Remember to always record the moves and the score.

**Essential vocabulary**

**Common fractions**: A rational number written in the form $\frac{p}{q}$, that is as a numerator and denominator (which are both integers, $q \neq 0$) separated by a horizontal or slanted line (called a vinculum), for example, $\frac{1}{2}$ or $\frac{3}{4}$.

**Decimal fractions**: A rational number where the denominator is a power of ten and the numerator is expressed by figures placed to the right of a decimal point, for example 1.456.

**Rational numbers**: Any number that can be expressed as the quotient or fraction $\frac{p}{q}$ of two integers, $p$ and $q$, with the denominator $q$ not equal to zero. Since $q$ may be equal to 1, every integer is a rational number.

**Reciprocal**: A reciprocal is one of a pair of numbers that, when multiplied together, equal 1. If you write the number as a common fraction, finding the reciprocal is simply a matter of transposing the numerator and the denominator. Also known as the multiplicative inverse.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides hands-on approach for teaching reciprocals of common fractions and calculations with fractions. It has deliberately not been written for a “set and forget” teaching approach. Up until recently I had only ever used the activity as a mechanism to get students doing lots of fraction and negative calculations in a fun setting. Working with YuMi Deadly Maths and the concepts of reversals and big ideas encouraged the development of this task.

It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

This activity is based on an interesting trick developed by the mathematician John Conway that can be done with two skipping ropes. It was first demonstrated in a lecture given by Conway at Cambridge in 1998. The original John Conway lecture can be viewed at http://www.uctv.tv/shows/Tangles-Bangles-and-Knots-23319

For this task, each group of students needs two skipping ropes. Students should start by practising the two moves, labelled Twists and Turns. Ensure students are moving correctly, particularly during a twist, where the rope held by the student in the top right position crosses over the rope held by the student in the top left position. Only the students in the top positions move during a twist.

Determining a sequence of moves for a particular rational number is a good example of the “working backwards” problem-solving strategy and the concept of an inverse. Think of an inverse twist (−1) as the opposite of a twist as defined in the activity. Start at the desired number and work back to zero.

For example:

<table>
<thead>
<tr>
<th>Score</th>
<th>Move</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} )</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>(-\frac{1}{3})</td>
<td>Turn</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Inverse twist (-1)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Inverse twist (-1)</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Inverse twist (-1)</td>
<td>0</td>
</tr>
</tbody>
</table>

The required sequence of moves is therefore: Twist; Twist; Twist; Turn; Twist
Some more examples:

<table>
<thead>
<tr>
<th>Score</th>
<th>Move</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{7}{10})</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{7}{10})</td>
</tr>
<tr>
<td>(-\frac{3}{10})</td>
<td>Turn</td>
<td>(\frac{10}{3})</td>
</tr>
<tr>
<td>(\frac{10}{3})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{7}{3})</td>
</tr>
<tr>
<td>(\frac{7}{3})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{4}{3})</td>
</tr>
<tr>
<td>(\frac{4}{3})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>(\frac{1}{3})</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{2}{3})</td>
</tr>
<tr>
<td>(-\frac{2}{3})</td>
<td>Turn</td>
<td>(\frac{3}{2})</td>
</tr>
<tr>
<td>(\frac{3}{2})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>(-\frac{1}{2})</td>
<td>Turn</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Inverse twist (-1)</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Inverse twist (-1)</td>
<td>0</td>
</tr>
</tbody>
</table>

The required sequence of moves is therefore: Twist; Twist; Turn; Twist; Twist; Turn; Twist; Twist; Twist; Turn; Twist

---

<table>
<thead>
<tr>
<th>Score</th>
<th>Move</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{5}{7})</td>
<td>Turn</td>
<td>(\frac{5}{7})</td>
</tr>
<tr>
<td>(\frac{7}{5})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{2}{5})</td>
</tr>
<tr>
<td>(\frac{2}{5})</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{3}{5})</td>
</tr>
<tr>
<td>(-\frac{3}{5})</td>
<td>Turn</td>
<td>(\frac{5}{3})</td>
</tr>
<tr>
<td>(\frac{5}{3})</td>
<td>Inverse twist (-1)</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>Inverse twist (-1)</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>(-\frac{1}{3})</td>
<td>Turn</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Inverse twist (-1)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Inverse twist (-1)</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Inverse twist (-1)</td>
<td>0</td>
</tr>
</tbody>
</table>

The required sequence of moves is therefore: Twist; Twist; Twist; Turn; Twist; Twist; Turn; Twist; Twist; Turn

---

Once a whole number is reached then a series of inverse twists will reduce the score to zero. A two-step algorithm for the process is:

- if the number is positive reduce it to a negative value with a series of inverse twists; and
- if the number is negative apply a turn to make it the positive reciprocal.

Repeat these two steps as often as required.

Exploring this pattern brings up an interesting point for consideration by students. If we need to introduce the concept of an inverse twist why don’t we need an inverse turn? The turn is an example of a self-inverse. Consider a tangle with a score of \(\frac{5}{3}\) and applying a turn would bring the score to \(-\frac{5}{3}\). Applying a second turn would turn the score back to \(\frac{5}{3}\).
This provides another challenge task. If we have a complicated tangle say $\frac{34}{65}$ and we apply two turns in a row then the tangle score will have gone $\frac{34}{65} \rightarrow -\frac{65}{34} \rightarrow \frac{34}{65}$ but the students holding the ropes are clearly not back in the positions they were in when the score first reached $\frac{34}{65}$. Is it the same tangle, that is, is a turn really a self-inverse? Students should be encouraged to start with a simpler tangle to verify that this is in fact the case.

Undoing a tangle is a little different to generating a tangle of a particular value because we are restricted to the two allowable moves: a twist and a turn. Although we can imagine an inverse twist and apply a value of $-1$ to an inverse twist we cannot actually undo a twist. The procedure for undoing a tangle is similar to that devised above.

- If the number is negative make it positive – this can be achieved by applying as many twists as necessary.
- If the number is positive make it negative – this can be achieved by applying a turn.
- Repeat these steps as necessary until a score of zero is reached.

Example: Starting with $\frac{7}{12}$ as in the student instructions, this sequence of steps should undo the tangle.

<table>
<thead>
<tr>
<th>Score</th>
<th>Move</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{7}{12}$</td>
<td>Positive, therefore make negative by applying a turn ($-\text{reciprocal}$)</td>
<td>$-\frac{12}{7}$</td>
</tr>
<tr>
<td>$-\frac{12}{7}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$\frac{5}{7}$</td>
</tr>
<tr>
<td>$-\frac{5}{7}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$\frac{2}{7}$</td>
</tr>
<tr>
<td>$\frac{2}{7}$</td>
<td>Positive, therefore make negative by applying a turn ($-\text{reciprocal}$)</td>
<td>$-\frac{7}{2}$</td>
</tr>
<tr>
<td>$-\frac{7}{2}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$-\frac{5}{2}$</td>
</tr>
<tr>
<td>$-\frac{5}{2}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$-\frac{3}{2}$</td>
</tr>
<tr>
<td>$-\frac{3}{2}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$-\frac{1}{2}$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>Positive, therefore make negative by applying a turn ($-\text{reciprocal}$)</td>
<td>$-2$</td>
</tr>
<tr>
<td>$-2$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$-1$</td>
</tr>
<tr>
<td>$-1$</td>
<td>Negative, therefore move towards the positive by applying a twist (+1)</td>
<td>$0$</td>
</tr>
</tbody>
</table>

You can add a little extra fun to the activity by having a student stand in between the ropes at the start of the activity and become caught in the tangle which his/her fellow students have to undo.

Alternatively the teacher can stand in the middle and let themselves become caught up in the student tangle and then demonstrate “powerful mathematics” by directing students to make the moves that will release him/her from the tangle. Students then have the task of determining how the teacher worked out the sequence of moves to be followed.

Further information can be obtained at: Twisting and Turning on the nrich website at http://nrich.maths.org/5776
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td>ACMMG142</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA153</td>
<td>ACMNA154</td>
</tr>
<tr>
<td></td>
<td>Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACMNA154</td>
<td>Multiply and divide fractions and decimals using efficient written strategies and digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA183</td>
<td>ACMNA183</td>
</tr>
<tr>
<td></td>
<td>Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies</td>
<td></td>
</tr>
</tbody>
</table>
7N2 “Directing Numbers”

Task description

In 200 BCE Chinese mathematicians represented positive numbers in black and negative numbers in red (a system still used by accountants today). An Indian mathematician, Brahmagupta (598–670 CE) used the idea of “fortunes” and “debts” for positive and negative. He used a special sign for negatives and listed the rules for operations with positive and negative quantities. However, as late as 1758, some mathematicians did not accept that negative numbers could exist.

In this task, we will investigate operations with positive and negative numbers (also called directed numbers). We will start by considering how numbers behave in a variety of situations.

Activity 1
We use negative numbers when we measure temperatures below freezing point. Find and explain four more examples where we use negative numbers (remember that explain means what, how and why).

Activity 2
Addition and subtraction can be thought of as movements along a number line. For example,
- $3 + 4$ is start at $3$, then 4 steps to the right;
- $-3 + 4$ is start at $-3$, then take 4 steps to the right;
- $3 + -4$ is start at $3$, then take 4 steps to the left; and
- $-3 + -4$ is start at $-3$, then take 4 steps to the left.

Using numbers other than $\pm 3$ and $\pm 4$, illustrate each process on a separate number line. Can you find a rule that predicts the finishing position in each case? Make up some other examples to see if your rule is correct.

Activity 3
Multiplication can be thought of as repeated movements along a number line. For example,
- $3 \times 4$ is face right, then take 3 lots of 4 forward steps;
- $-3 \times 4$ is face left, then take 3 lots of 4 forward steps;
- $3 \times -4$ is face right, then take 3 lots of 4 backward steps; and
- $-3 \times -4$ is face left, then take 3 lots of 4 backward steps.

Using numbers other than $\pm 3$ and $\pm 4$, illustrate each process on a separate number line. Can you find a rule that predicts the finishing position in each case? Make up some other examples to see if your rule is correct.

Activity 4
By thinking of division as multiplying by the reciprocal, can you find a rule that predicts the outcome of division of positive and negative numbers? Make up some examples and check on your calculator to see if your rule is correct.

Activity 5
Complete activities 6 to 9 on the “Some More Activities” handout.

Challenge
Calculators have two similar keys: one usually marked $\pm$, and one marked $\mp$. How are they similar? How are they different? Why do you think that calculators have two such keys?
Some more activities

1. Use four $-4$s and any operations you want to write expressions that are equivalent to as many of the integers between 1 and 20 as you can. Are any impossible?

2. Write as many expressions as you can that equal 3 using three operations and at least one negative number.

3. Five numbers added together in pairs give $-9$, $-6$, $-5$, $1$, $2$, $3$, $4$, $5$, $7$, $14$. What are the five numbers? A different five numbers multiplied together in pairs give $-30$, $-20$, $-18$, $-12$, $-6$, $3$, $5$, $15$, $24$. What are the five numbers?

4. Make up some more examples similar to those in no. 8 for your fellow students to try.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of directed number. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

Activity 1 asks students to identify contexts where positive and negative numbers are used to describe values. Examples include: height above and below sea level; time measured as CE (AD) and BCE (BC); scores above and below par in golf; profit and loss; borrowing and saving; floors above and below the ground floor; latitude (E or W); longitude (N or S); and time differences (compared to GMT). These contexts could be used for students to explore how negative numbers behave. Students should observe that, in every case, the digits of negative values run in the reverse order compared to the digits in positive numbers (so, –8 is smaller than –6). Students should be able to distinguish the use of + and – as values and their use as labels, for example, the poles on a battery or magnet, and their use in the grading of student work (e.g. A+). In each of these examples it would not make sense to use these symbols in a calculation.

Students might be amused by the experience of the British Lottery described at http://www.manchestereveningnews.co.uk/news/greater-manchester-news/cool-cash-card-confusion-1009701

The focus of teaching directed number should be on students’ own experimentation with a variety of situations, including the contexts identified in Activity 1, leading to the development of principles or rules that they can apply when calculating with directed number. These rules should come from the student, not the teacher.

There are some more activities for students on page 4. The solutions to Activity 8 are –5, –4, –1, 6, 8 and –5, –3, –1, 4, 6.

Student confusion over directed number often arises because the same symbol (+/−) and word (plus/minus) are used for two different ideas: the operation (addition or subtraction) and the object (a positive or negative number). To minimise this confusion, some teachers and books (and this task) write positive and negative signs as superscripts (⁺ and ⁻) and the symbols for addition and subtraction in normal font (+ and −). In spoken forms it is recommended that the words “positive” and “negative” are the only words used to describe the object, and the words “plus” and “minus” be used exclusively for the operation. Thus, −−3 should be verbalised as “minus negative three”.

The confusion between “subtract” and “negative” can be eliminated entirely if students know that the operation of subtraction does not exist. Subtraction is a form of addition, where the value being added is negative. This leads to fruitful discussions about additive inverses (the additive inverse of a is –a), and the fact that addition, unlike subtraction, is commutative (so –a + b = b + –a, but a – b ≠ b – a). Since an understanding of this underlying structure of mathematics is essential for students’ future development as mathematicians, capable students should be encouraged to view subtraction as adding a negative as early as possible. These are some of the ideas that students should be proposing in response to the Challenge activity.
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA280: Compare, order, add and subtract integers</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA183: Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies</td>
</tr>
</tbody>
</table>
7N3 “What Are You Worth?”

Task description

Almost everybody has assets, and many people have debts. Learning how to manage assets and debts sensibly is important preparation for adult life. Many of these skills are learnt in your school mathematics course. This task introduces you to some of the ideas needed to manage your personal finances.

What is net worth? How would we calculate the net worth of a person? The net worth of the Commonwealth of Australia, many businesses and some people, such as Kylie Minogue, is measured in the millions. Investigate the idea of personal net worth and show how it can relate to the addition of positive and negative numbers. Along the way learn different ways of representing very large numbers.

Make sure you understand the meanings of any words in italics.
Consider the article from the Courier Mail on 27 April 2009 about the net worth of Kylie Minogue:

**KYLIE Minogue is singing the recession blues, having lost a sizeable chunk of her wealth in the global financial crisis.**

The 2009 rich list of musos compiled by The Sunday Times has revealed that the pop princess has seen her net wealth drop from $83.5 million last year to $71.25 million.

Minogue’s wealth may have slipped more had she not sung away the recession blues to earn $1.5 million in a Bollywood movie music clip just months after being paid $5 million to perform at the opening of a Dubai hotel.

She also recently sold a property in Melbourne worth $1 million and is one of the world’s richest female singers.

---

1. Show your understanding of the very large numbers in this article by showing how they can be written in at least three (or, for more challenge, four) different ways.

2. Compare the net worth of three different people. Which would you prefer? Justify your decision.

3. People make decisions with regard to their net worth – they buy assets, they go into debt. What kind of decisions with regard to assets and debts are good for net worth? What are not good for net worth?

4. How do calculations of net worth relate to addition and subtraction? Explain, with the help of some number line examples.

5. How do calculations of net worth relate to the addition of positive and negative numbers? Make up or research some written descriptions of personal net worth (such as in the Kylie Minogue article) and rewrite them as number sentences where positive and negative numbers are added. When would it be better to use the verbal descriptions and when would you use the number sentences. Why?
**Essential vocabulary**

**Asset:** A useful or valuable thing. It can include money (in cash or in the bank), possessions, equipment, trademark, or property. In some circumstances it can include knowledge (such as a patent or copyright) or goodwill (customers that keep coming back to the business or a reputation for quality, excellence or reliability).

**Debt:** Something that is owed to (or borrowed from) another person or business and must be returned or replaced in the future. It represents a future reduction in assets. Accountants use the word *liabilities* to describe debts.

**Net worth:** The value of a person or business after all the debts are repaid. It is calculated by adding the value of the assets and subtracting the value of any debts or liabilities.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching the addition and subtraction of integers, the management of personal finances, and representations of large numbers. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

The task is intended to check on students’ understandings of very large numbers and the various ways of representing them (including an introduction to scientific notation for the more capable students). It also provides a concrete opportunity to introduce students to the ideas of addition and subtraction of directed numbers.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words (you may not want to give it to students at the beginning of the task). The glossary could be used as the basis of supporting language and literacy activities for students.

Detailed instructions

Activity 1: What is Kylie worth?

Materials: Pen, paper, calculator

1. The Courier Mail reported on 27 April 2009 the following about the net worth of Kylie Minogue:

**KYLIE Minogue is singing the recession blues, having lost a sizeable chunk of her wealth in the global financial crisis.**

The 2009 rich list of musos compiled by The Sunday Times has revealed that the pop princess has seen her net wealth drop from $83.5 million last year to $71.25 million.

Minogue’s wealth may have slipped more had she not sung away the recession blues to earn $1.5 million in a Bollywood movie music clip just months after being paid $5 million to perform at the opening of a Dubai hotel.

She also recently sold a property in Melbourne worth $1 million and is one of the world’s richest female singers.
(a) What does it mean to have a “net worth” of $71.25 million?
(b) Write down the net worth of Kylie Minogue as a number using all place values from units to tens of millions.
(c) Write down the worth of the $71.25 million in words and as a number using all place values from units up to the required value.
(d) Write down the worth of the $71.25 million in extended form:
   ... ten millions, ... millions, ... hundred thousands, ... ten thousands, ... thousands, ... hundreds. ... tens, ... ones
(e) Write 16.5 billion in two other forms.
(f) (Challenge): Investigate how 16.5 billion could be written using scientific notation.

2. (a) What sorts of assets could Kylie Minogue own to have this net worth?
   (b) Make a list of possible assets she might own.

3. Despite her total worth, Kylie Minogue will have debts.
   (a) What sort of debts could she have?
   (b) Make a list of possible debts Kylie Minogue may have.

**Activity 2: Which bachelor would you date?**

**Materials:** Pen, paper, calculator

1. Which of the following three bachelors has the best net worth? Show all your calculations.

<table>
<thead>
<tr>
<th>Net Worth Statement: Bachelor 1</th>
<th>Net Worth Statement: Bachelor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name:</strong> Kevin John Smith</td>
<td><strong>Name:</strong> Abdul Kartik</td>
</tr>
<tr>
<td><strong>Cash Assets</strong></td>
<td><strong>Cash Assets</strong></td>
</tr>
<tr>
<td>Bank balance</td>
<td>Bank Balance</td>
</tr>
<tr>
<td>$2 500</td>
<td>$7 600</td>
</tr>
<tr>
<td>Retirement fund</td>
<td>Retirement fund</td>
</tr>
<tr>
<td>$35 000</td>
<td>$20 000</td>
</tr>
<tr>
<td><strong>Personal Assets</strong></td>
<td><strong>Personal Assets</strong></td>
</tr>
<tr>
<td>House</td>
<td>House</td>
</tr>
<tr>
<td>$365 000</td>
<td>$35 000</td>
</tr>
<tr>
<td>Shares</td>
<td>Shares</td>
</tr>
<tr>
<td>$2 175</td>
<td>$10 500</td>
</tr>
<tr>
<td>Car</td>
<td>Car</td>
</tr>
<tr>
<td>$13 600</td>
<td>$35 000</td>
</tr>
<tr>
<td><strong>Total Assets</strong></td>
<td><strong>Total Assets</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Debts</strong></td>
<td><strong>Debts</strong></td>
</tr>
<tr>
<td>Car loan</td>
<td>Car loan</td>
</tr>
<tr>
<td>$14 500</td>
<td>$8 750</td>
</tr>
<tr>
<td>House mortgage</td>
<td>Avocado farm investment</td>
</tr>
<tr>
<td>$225 000</td>
<td>$12 670</td>
</tr>
<tr>
<td><strong>Total Debts</strong></td>
<td><strong>Total Debts</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Net Worth</strong></td>
<td><strong>Net Worth</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. If you were making a decision based on financial worth only, which bachelor would you date?

*Note*: From now on, please use a table similar to these when working out net worth.

**Activity 3: Good and bad decisions**

**Materials**: Pen and paper

1. People make decisions with regard to their net worth – they buy assets, they go into debt.
   
   (a) What kind of decisions with regard to assets and debts are good for net worth?
   (b) What are not good for net worth?

2. Complete the table by writing “good” or “bad” beside each decision and explaining your reasoning.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Good or bad</th>
<th>Reason(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Ann: She took away an asset of (+$200) from her net worth.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Bradley: He added an asset of (+$3000) to his net worth.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Christian: He took away an asset (+$50) from his net worth.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) Devon: He added a debt of (−$650) to his net worth.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) Ernie: He took away a debt of (−$5400) from his net worth.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f) Fran: She took away an asset of (+$201) from her net worth.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(g) Gracie: She added a debt of (+$67) to her net worth.

(h) Herbert: He took away an asset of (+$450) from his net worth.

3. In working out net worth, we say an asset is a plus (e.g. an asset of $500 is +$500) and a debt is a minus (e.g. a debt of $400 is −$400). Thus, we can write gaining and losing assets and debts as number sentences using + and −, like this:
   - +($500) is gaining an asset
   - +($400) is gaining a debt
   - −($500) is losing an asset
   - −($400) is losing a debt

   Complete the table by writing what the decision is as a number sentence.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Number sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Ann: She took away an asset of (+$200) from her net worth.</td>
<td></td>
</tr>
<tr>
<td>(b) Bradley: He added an asset of (+$3000) to his net worth.</td>
<td></td>
</tr>
<tr>
<td>(c) Christian: He took away an asset (+$50) from his net worth.</td>
<td></td>
</tr>
<tr>
<td>(d) Devon: He added a debt of (−$650) to his net worth.</td>
<td></td>
</tr>
<tr>
<td>(e) Ernie: He took away a debt of (−$5400) from his net worth.</td>
<td></td>
</tr>
<tr>
<td>(f) Fran: She took away an asset of (+$201) from her net worth.</td>
<td></td>
</tr>
<tr>
<td>(g) Gracie: She added a debt of (+$67) to her net worth.</td>
<td></td>
</tr>
<tr>
<td>(h) Herbert: He took away an asset of (+$450) from his net worth.</td>
<td></td>
</tr>
</tbody>
</table>

**Activity 4: Adding and subtracting**

**Materials:** Pen and paper

1. Activity 3 can be reversed. You can change the number sentence into a description of an asset or debt being gained or lost. For example:
   - net worth +(+$300) says that net worth was increased by gaining or adding an asset of $300
   - net worth +(−$800) says that net worth was decreased by gaining or adding a debt of $800
   - net worth −(+$200) says that net worth was decreased by losing or taking away an asset of $200
   - net worth −(−$200) says that net worth was increased by losing or taking away a debt of $200.

   Use the words add, take away, debt and asset to describe each of the following transactions. Complete the table.

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Net worth −(+$300)</td>
<td></td>
</tr>
<tr>
<td>(b) Net worth +(−$340)</td>
<td></td>
</tr>
<tr>
<td>(c) Net worth +(+$525)</td>
<td></td>
</tr>
<tr>
<td>(d) Net worth −(+$550)</td>
<td></td>
</tr>
<tr>
<td>(e) Net worth +(+$37)</td>
<td></td>
</tr>
<tr>
<td>(f) Net worth −(−$2915)</td>
<td></td>
</tr>
</tbody>
</table>
2. Act out and illustrate your descriptions of the six transactions above on the number lines below.

(a) Net worth \(-(+\$300)\)

(b) Net worth \(+(-\$340)\)

(c) Net worth \(+(+\$525)\)

(d) Net worth \(-(+\$550)\)
Activity 5: Positive and negative operations

1. Solve the problems below. Your task is to represent the problems as number sentences. The problems will finish with all symbols as they become increasingly symbolic.

   (a) Monica has a net worth of −$6500. An asset of $2500 is taken away. Is this good or bad? What is her net worth now?

   (b) Net worth: $1750; transaction: adds a debt of $600

   (c) $540: add an asset of (+$55)

   (d) $360: add (−$35)

   (e) $95 − (−$110)

   (f) 60 − (−70)

   (g) −3 + −4 + 17 − (−23) − 9

2. If we can think of adding a positive number as (for example) earning money, and subtracting a positive number as (for example) making a donation to charity, give some examples of how you could describe in words

   (a) subtracting a positive number, and

   (b) subtracting a negative number.

3. (a) Make up or research six asset, debt and net worth problems (verbal description) and translate them to number sentences involving positive and negative numbers.

   (b) Make up six number sentences using positive and negative money amounts and translate them into asset, debt and net worth problems (verbal descriptions).

   (c) When would it be better to use the verbal descriptions and when would you use the number sentences. Why?
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA149 Investigate index notation and represent whole numbers as products of powers of prime numbers</td>
</tr>
<tr>
<td></td>
<td>ACMNA280 Compare, order, add and subtract integers</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA189 Solve problems involving profit and loss, with and without digital technologies</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA209 Express numbers in scientific notation</td>
</tr>
</tbody>
</table>
7N4 “How Tall Is the Criminal?”

Task description

A ratio is a comparison of two numbers of the same type. We use them in percentages (when one of the numbers is 100), fractions (when we compare the numerator with the denominator), maps and scale drawings (where the scale is often written as the ratio of the map distance to the actual distance), when adapting in recipes, and many other areas of life. This task introduces you to the idea of a ratio and gives you the opportunity to explore some ways that we can use them. Perhaps you can think of other uses?

Investigate what a ratio is and the various ways of expressing a ratio, giving several examples. Use your knowledge of ratios to explore the following:

- Flags come in many shapes and sizes, but the ratios do not vary for a particular country. Explore the use of ratios in flags of different countries, for example Australia, USA, France, Germany, Denmark, and Belgium.
- Comparisons of the length and width of different cuts of diamonds. Investigate the ratios used in the marquise cut of diamonds.

Finally, answer the following.

The Police investigating a crime scene found a footprint and some clothing that had not been there previously. A drawing of the footprint is on the right. The scale of the drawing is 1:4.65. The depth of the footprint in clay soil was given as an average of 3.05 mm. A shirt was found that was XL and the hat was imported from the United States of America and had a size of 7 1⁄8 under the brim.

Should the Police be looking for one criminal or two? Give your reasons. Write a description of the criminal(s) that the Police can release to the press and media.

Make sure you understand the meanings of any words in italics.
## Additional student information

Some useful websites:

**Flags:** [http://www.crwflags.com/fotw/flags/xf-rati.html](http://www.crwflags.com/fotw/flags/xf-rati.html)

**Diamonds:** [http://www.thediamondbuyingguide.com/diamondglossary.html](http://www.thediamondbuyingguide.com/diamondglossary.html)

Other useful information:

<table>
<thead>
<tr>
<th>Body part</th>
<th>Ratio to height</th>
<th>Footprint depth (mm)</th>
<th>Weight per cm² (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Femur</td>
<td>1:4</td>
<td>2.90</td>
<td>0.5475</td>
</tr>
<tr>
<td>Ulna</td>
<td>1:7 (same as a foot length)</td>
<td>3.00</td>
<td>0.5875</td>
</tr>
<tr>
<td>Tibia</td>
<td>1:5</td>
<td>3.10</td>
<td>0.6250</td>
</tr>
<tr>
<td>Cranium</td>
<td>1:3</td>
<td>3.20</td>
<td>0.6600</td>
</tr>
<tr>
<td>Hand span</td>
<td>1:8.5</td>
<td>3.25</td>
<td>0.6750</td>
</tr>
<tr>
<td>Arm span</td>
<td>1:1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrist circumference</td>
<td>1:6 (similar to length of ulna)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head circumference</td>
<td>1:3 (3.5 × wrist circumference)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Hat sizes:

<table>
<thead>
<tr>
<th>cm</th>
<th>Inch</th>
<th>US</th>
<th>UK</th>
<th>Approx.</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>20 7/8</td>
<td>6 5/8</td>
<td>6 1/2</td>
<td>XSMALL</td>
</tr>
<tr>
<td>54</td>
<td>21 1/4</td>
<td>6 3/4</td>
<td>6 5/8</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>21 5/8</td>
<td>6 7/8</td>
<td>6 3/4</td>
<td>SMALL</td>
</tr>
<tr>
<td>56</td>
<td>22</td>
<td>7</td>
<td>6 7/8</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>22 1/2</td>
<td>7 1/8</td>
<td>7</td>
<td>MEDIUM</td>
</tr>
<tr>
<td>58</td>
<td>22 7/8</td>
<td>7 1/4</td>
<td>7 1/8</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>23 1/4</td>
<td>7 3/8</td>
<td>7 1/4</td>
<td>LARGE</td>
</tr>
<tr>
<td>60</td>
<td>23 5/8</td>
<td>7 1/2</td>
<td>7 3/8</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>24</td>
<td>7 5/8</td>
<td>7 1/2</td>
<td>XLARGE</td>
</tr>
<tr>
<td>62</td>
<td>24 3/8</td>
<td>7 3/4</td>
<td>7 5/8</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>24 3/4</td>
<td>7 7/8</td>
<td>7 3/4</td>
<td>XXLARGE</td>
</tr>
</tbody>
</table>

### Shirt sizes:

<table>
<thead>
<tr>
<th>Shirt size</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
<th>XXL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chest (cm)</td>
<td>120</td>
<td>130</td>
<td>140</td>
<td>150</td>
<td>160</td>
</tr>
<tr>
<td>Neck (cm)</td>
<td>39</td>
<td>42</td>
<td>44</td>
<td>47</td>
<td>49</td>
</tr>
<tr>
<td>Centre back length (cm)</td>
<td>82</td>
<td>84</td>
<td>86</td>
<td>93</td>
<td>93</td>
</tr>
<tr>
<td>Shoulder to shoulder (cm)</td>
<td>50</td>
<td>53</td>
<td>56</td>
<td>59</td>
<td>62</td>
</tr>
<tr>
<td>Sleeve length (cm)</td>
<td>86</td>
<td>89</td>
<td>91.5</td>
<td>94.5</td>
<td>97</td>
</tr>
<tr>
<td>Cuff width (cm)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of ratios. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Once students have mastered the concept of a ratio, they could be encouraged to search widely for examples of ratios. The examples in this task are but a few of what they might find. Students could go on a “ratio hunt”, bringing into class any examples of ratios that they find outside the classroom. They should be able to explain their example to you, or the class, and how ratios are used in their example.

Other ideas for teaching ratios include:

- Students could apply their knowledge of ratios to the information in the nutrition information panel on packaged food, converting the measurement to ratios, discuss how much of each ingredient they would consume if they ate the entire contents of package, compare it to recommended daily intakes, consider advertising claims such as “provides half the daily allowance of…” (including how much fat or sugar they would consume as they get that daily allowance), consider if the implied serving size is realistic, etc.

- Investigate the use of ratios in the enlargements and reductions available on a school photocopy machine. This could lead to a comparison of ratios in length and area (for example, if we increase the sides in the ratio 2:1, in what ratio does the area increase?).

- Using lemonade, orange juice concentrate, dry ginger ale and water, experiment with various punch recipes and develop the ratios needed for the best punch recipe.
Activity 1: Calculating with ratios

1. A ratio is a comparison of two numbers of the same type. Example: In a class of 26 students there are 14 boys. The ratio of boys to girls is 14:12 (verbally, “14 is to 12”). The ratio can be written in many ways.

<table>
<thead>
<tr>
<th>No. of students</th>
<th>No. of boys</th>
<th>No. of girls</th>
<th>Ratio of boys to girls</th>
<th>Type of ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>14</td>
<td>12</td>
<td>14:12</td>
<td>Non-simplified</td>
</tr>
<tr>
<td>14 ÷ 2</td>
<td>12 ÷ 2</td>
<td></td>
<td>7:6</td>
<td>Simplified to lowest terms</td>
</tr>
<tr>
<td>14 ÷ 12</td>
<td>12 ÷ 12</td>
<td></td>
<td>1.16recurring:1</td>
<td>Unitary</td>
</tr>
<tr>
<td>14 ÷ 2</td>
<td>12 ÷ 2</td>
<td></td>
<td>(\frac{7}{6})</td>
<td>Fractional</td>
</tr>
<tr>
<td>Round the decimal</td>
<td>Leave as 1</td>
<td></td>
<td>1.17:1</td>
<td>Approximate</td>
</tr>
<tr>
<td>7 × 3</td>
<td>6 × 3</td>
<td></td>
<td>21:18</td>
<td>Equivalent</td>
</tr>
<tr>
<td>7 × 5</td>
<td>6 × 5</td>
<td></td>
<td>35:30</td>
<td>Equivalent</td>
</tr>
</tbody>
</table>

Ratios are used in calculations in the following way. Example: In a school of 442 students, the ratio of boys to girls is 7:6. Calculate the number of boys and girls in the school. (The solution below is only one possible method of solution.)

The ratio is 7:6. There are \(7 + 6 = 13\) parts to the ratio. In the number 442 there are \(442 ÷ 13 = 34\) students in each part of the ratio. There are 7 parts boys. Number of boys = \(34 \times 7 = 238\). There are 6 parts girls. Number of girls = \(34 \times 6 = 204\). Check the answer: 238 boys + 204 girls = 442 students.

New problem: The ratio of the number of students in state schools to the number of students in private schools in the Ipswich area is 35:20.

(a) Draw a table similar to the one above for the Ipswich data.
(b) Calculate the simplified ratio (showing how the calculation was done as in the table above).
(c) Calculate the exact unitary ratio (again showing how the calculation was done).
(d) Calculate three equivalent ratios (again showing how the calculation was done).

2. Answer the following questions:

(a) If there are 10 885 state school students in the Ipswich area, how many private school students are there?
(b) How many students are there in the Ipswich area, both state and private?
(c) 22 Ipswich students are selected to represent the area in a forum in Canberra. How many state school students and how many private school students should be in the selected group?
Activity 2: Ratios and flags

1. Flags of each country have a set length to width ratio. Look up the website: http://www.crwflags.com/fotw/flags/xf-rati.html and check the following.

   (a) British flags have a 1:2 ratio (United Kingdom, Australia, Bahamas, Canada, Ireland, and with the little correction of 10:19 United States, and of course Liberia).

   (b) French flags have a 2:3 ratio (France, Italy, Cameroon, Ivory Coast, Algeria, Spain and most of the Latin-American flags).

   (c) German flags have a 3:5 ratio.

   (d) Some nations have unusual ratios, for example Denmark (28:37) or Belgium (13:15).

2. Answer these questions (give any approximate answers to the nearest millimetre):

   (a) If the width of an Australian flag is 660 mm, what should the length of the flag be?

   (b) An Italian flag is 868 mm long. How wide is the Italian flag?

   (c) A German flag has a length : width ratio of 351 mm : 585 mm. If a flag is to be made that is 30% wider, what will the new dimensions of the flag be?

   (d) A Belgian flag (from Belgium) and a Danish flag (from Denmark) are flying side by side on adjacent flagpoles. The Belgian flag is 1845 mm long. Both flags have the same width. How long is the flag from Denmark?
Activity 3: “Diamonds are forever” ratios

1. Look up the website below and check the following information:
   http://www.thediamondbuyingguide.com/diamondglossary.html

   “Length-to-width ratio: A comparison of how much longer a diamond is than it is wide. It is used to analyze the outline of fancy shapes only; it is never applied to round diamonds. There’s really no such thing as an ‘ideal’ ratio; it’s simply a matter of personal aesthetic preferences. For example, while many people are told that a 2 to 1 ratio is best for a marquise, most people actually tend to prefer a ratio of around 1.80 to 1 when they actually look at marquises. And though the standard accepted range for the length-to-width ratio of a marquise generally falls between 1.70 to 1 and 2.05 to 1, there are customers who insist on having ‘fatter’ marquises of about 1.60 to 1 and other customers who want longer, thinner marquises of 2.25 to 1.”

2. Below are images of 4 marquise diamonds.

   (a) Use a ruler to find the length-to-width ratio of each diamond below. Express the ratio as a unitary ratio as in the diamond buying guide above.

   (b) Which of the above diamonds fit into the “standard accepted range” of length-to-width ratio?
Activity 4: Finding the criminal’s height

When police investigate a crime scene many pieces of information may go into providing clues to the identity of the criminal. This task requires the use of ratios in providing the information that may be used in identifying a suspect. The table (right) of body part ratios will be needed.

The diagrams below will help you find the position of the bones listed in the table.

<table>
<thead>
<tr>
<th>Body part</th>
<th>Ratio to height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Femur</td>
<td>1:4</td>
</tr>
<tr>
<td>Ulna</td>
<td>1:7</td>
</tr>
<tr>
<td></td>
<td>Same as a foot length</td>
</tr>
<tr>
<td>Tibia</td>
<td>1:5</td>
</tr>
<tr>
<td>Cranium</td>
<td>1:3</td>
</tr>
<tr>
<td>Hand span</td>
<td>1:8.5</td>
</tr>
<tr>
<td>Arm span</td>
<td>1:1</td>
</tr>
<tr>
<td>Wrist circumference</td>
<td>1:6</td>
</tr>
<tr>
<td></td>
<td>Length of ulna</td>
</tr>
<tr>
<td>Head circumference</td>
<td>1:3</td>
</tr>
<tr>
<td></td>
<td>3.5 wrist circumference</td>
</tr>
</tbody>
</table>

A footprint was found at a crime scene. A drawing of the footprint is on the right. The scale of the drawing is 1:4.65

The depth of the footprint in clay soil was given as an average of 3.05 mm. The table below gives the footprint depth in clay soil for weight per cm² of foot area.

<table>
<thead>
<tr>
<th>Footprint depth (mm)</th>
<th>Weight per cm² (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.90</td>
<td>0.5475</td>
</tr>
<tr>
<td>3.00</td>
<td>0.5875</td>
</tr>
<tr>
<td>3.10</td>
<td>0.6250</td>
</tr>
<tr>
<td>3.20</td>
<td>0.6600</td>
</tr>
<tr>
<td>3.25</td>
<td>0.6750</td>
</tr>
</tbody>
</table>
1. Use the information above to find the following data on the owner of the footprint.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>cm</th>
<th>Inch</th>
<th>US</th>
<th>UK</th>
<th>Approx.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Height</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Foot length</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) Head circumference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) Arm span</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f) Wrist circumference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Some items of clothing and a hat were found near the crime scene. The tables for hat and shirt sizes are given below.

<table>
<thead>
<tr>
<th>cm</th>
<th>Inch</th>
<th>US</th>
<th>UK</th>
<th>Approx.</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>20 7/8</td>
<td>6 5/8</td>
<td>6 1/2</td>
<td>XSMALL</td>
</tr>
<tr>
<td>54</td>
<td>21 1/4</td>
<td>6 3/4</td>
<td>6 5/8</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>21 5/8</td>
<td>6 7/8</td>
<td>6 3/4</td>
<td>SMALL</td>
</tr>
<tr>
<td>56</td>
<td>22</td>
<td>7</td>
<td>6 7/8</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>22 1/2</td>
<td>7 1/8</td>
<td>7</td>
<td>MEDIUM</td>
</tr>
<tr>
<td>58</td>
<td>22 7/8</td>
<td>7 1/4</td>
<td>7 1/8</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>23 1/4</td>
<td>7 3/8</td>
<td>7 1/4</td>
<td>LARGE</td>
</tr>
<tr>
<td>60</td>
<td>23 5/8</td>
<td>7 1/2</td>
<td>7 3/8</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>24</td>
<td>7 5/8</td>
<td>7 1/2</td>
<td>XLARGE</td>
</tr>
<tr>
<td>62</td>
<td>24 3/8</td>
<td>7 3/4</td>
<td>7 5/8</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>24 3/4</td>
<td>7 7/8</td>
<td>7 3/4</td>
<td>XXLARGE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shirt size</th>
<th>S</th>
<th>M</th>
<th>L</th>
<th>XL</th>
<th>XXL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chest (cm)</td>
<td>120</td>
<td>130</td>
<td>140</td>
<td>150</td>
<td>160</td>
</tr>
<tr>
<td>Neck (cm)</td>
<td>39</td>
<td>42</td>
<td>44</td>
<td>47</td>
<td>49</td>
</tr>
<tr>
<td>Centre back length (cm)</td>
<td>82</td>
<td>84</td>
<td>86</td>
<td>93</td>
<td>93</td>
</tr>
<tr>
<td>Shoulder to shoulder (cm)</td>
<td>50</td>
<td>53</td>
<td>56</td>
<td>59</td>
<td>62</td>
</tr>
<tr>
<td>Sleeve length (cm)</td>
<td>86</td>
<td>89</td>
<td>91.5</td>
<td>94.5</td>
<td>97</td>
</tr>
<tr>
<td>Cuff width (cm)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

The shirt was XL and the hat was imported from the United States of America and had a size of $7 \frac{1}{8}$ under the brim.

Use all the information available to answer the following question, “Is it possible that the shirt and hat belong to the same person who left the footprint?”
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td>ACMMG108 Choose appropriate units of measurement for length, area, volume, capacity and mass</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA173 Recognise and solve problems involving simple ratios</td>
<td>ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA173 Solve a range of problems involving rates and ratios, with and without digital technologies</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>ACMNA188</td>
<td></td>
</tr>
</tbody>
</table>
7N5 “Rating Our World”

Task description

Water and electricity are a substantial, and increasing, part of the cost of running a household. What options do we have for reducing these costs?

This task requires you to apply ratios and rates to problems involving the costs associated with running a household to find out how much your home, or an average home, pays for water and electricity.

How much water can be collected from the roof of a typical house in your area? How does this compare to the water usage for a typical household? How does that affect the size of the rainwater tank that should be bought? Are water tanks a worthwhile purchase?

Would putting solar panels on the roof reduce electricity costs? By how much? Are the savings enough to justify the cost of the panels?

What advice would you give to someone thinking of installing a water tank or solar panels at their house? Prepare an argument to persuade them why you are right. Explain your advice and justify your arguments. You can present this argument in any genre you choose.

Make sure you understand the meanings of any words in italics.
Essential vocabulary

**Grid (electricity):** The usual mathematical meanings of grid are another name for a table, or a horizontal and vertical overlay on a map to assist in determining location. However, in the context of electricity, it means the network of poles and wires that carry electricity to and from the power station to the consumer (for example, the household). Thus, “taking electricity from the grid” is another way of saying “purchasing electricity from the power company” and “returning electricity to the grid” is an alternative to “selling electricity to the power company”.

**Household:** A group of people sharing a dwelling and, consequently, also sharing the costs of running the dwelling. For example, a family or a group of friends.

**Rate:** A comparison of quantities of different kinds, for example, distance with time. Since the quantities are different, a rate must include units, usually separated by the word “per”, for example, kilometres per hour. Some rates have special names, for example, speed and velocity (comparisons of distance and time), flow (a comparison of capacity and time), acceleration (a comparison of speed and time), and power (a comparison of energy and time).

**Ratio:** A comparison of quantities of the same kind, for example, the ratio of girls to boys is 3:4. Since the quantities are of the same kind, there is no need to include units with a ratio.

**Tariff:** A cost or a charge, for example the tariff in a taxi is the cost per kilometre travelled and the electricity tariff is the cost per kilowatt hour. As tariffs compare two quantities of different kinds, they are particular examples of rates.

Some words have several meanings. These definitions give the meanings of the words in the way that they are used in this task. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of ratios and rates. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Students should be able to research most of the information needed for this task, but some information is included in the activities below. When working with rates, students need to ensure that the time components are compatible before calculating or making comparisons. For example, calculations of water use might be per day, whilst most rainfall statistics are provided per month. However, since rainfall varies seasonally, it could be easier to make all comparisons on an annual basis.

Students may need some assistance in understanding the detail of domestic solar electricity generation. As surplus electricity cannot be stored in the solar panels, any unused electricity is fed back to the electricity grid where it is purchased by the power supplier for sale elsewhere. A household will typically purchase electricity at night or on cloudy days (when household usage exceeds the amount generated) and sell power on sunny days (when household usage is less than the amount generated). Thus solar panels reduce household costs by (a) reducing the power that they need to purchase from the grid, and (b) selling surplus power back to the grid. For simplicity, in this task students should assume that the purchase price equals the selling price (feed-in tariff), although this may not always be the case. Further, students may find that there is a range of electricity tariffs available, depending on the nature and time of day of the usage. For further simplicity, students should use the standard domestic tariff (called Tariff 11 in Queensland). More capable students may be able to explore what happens if some of these assumptions are varied.

In this task students are asked to justify the use (or otherwise) of water tanks and solar panels. They are expected to take a position on one side or the other of the debate and provide arguments and justification for their position. They could select their own genre (for example, poster, report, feature article for a newspaper, brochure, blog, web page, etc).

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
**Activity 1: Water costs**

1. South-east Queensland has had drought and floods. During drought, we are urged to use less water. The price of water has been increasing in recent years. The table below gives the amount of water used for a typical household. All values are approximate.

Source

<table>
<thead>
<tr>
<th>Water Use</th>
<th>Volume (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual flush toilet Single flush</td>
<td>5</td>
</tr>
<tr>
<td>Dual flush toilet Double flush</td>
<td>11</td>
</tr>
<tr>
<td>Shower (7.5 L/minute) for low water flow shower head</td>
<td>30 per 4 min shower</td>
</tr>
<tr>
<td>Shower (12 L/minute) regular shower head</td>
<td>48 per 4 min shower</td>
</tr>
<tr>
<td>Bath</td>
<td>96</td>
</tr>
<tr>
<td>Washing machine water efficient AAAA front loading per load</td>
<td>40</td>
</tr>
<tr>
<td>Regular washing machine per load</td>
<td>130</td>
</tr>
<tr>
<td>Meal preparation</td>
<td>5</td>
</tr>
<tr>
<td>Washing up</td>
<td>10</td>
</tr>
<tr>
<td>Jug</td>
<td>2</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>40</td>
</tr>
<tr>
<td>Bucket for garden watering</td>
<td>9</td>
</tr>
<tr>
<td>Swimming pool top up for 50 000L pool</td>
<td>720</td>
</tr>
<tr>
<td>Teeth cleaning with tap running</td>
<td>5</td>
</tr>
<tr>
<td>Teeth cleaning with set amount</td>
<td>1</td>
</tr>
<tr>
<td><strong>Other water uses</strong></td>
<td></td>
</tr>
</tbody>
</table>

(a) Use the table above to calculate the volume of water used in your household every week. Remember to take account of all the people who live in the house in a typical week.

(b) Look up the present cost of water per kilolitre. How much would you expect your household to pay for a quarter (3 months)?

(c) List ways your household could save water. Calculate the reduced amount of water as a percentage of your current use.

**Activity 2: Rainwater tanks**

1. Many people have installed rainwater tanks. How much water can they collect per square metre?

   (a) What is the average rainfall in your area? Use the internet.

   (b) If 5 mm of rain fell on one square metre, calculate the volume of rain that has fallen on this area.

   (c) Use (a) and (b) above to calculate how much rain we can collect in a typical year from one square metre of roof.

2. How much water can be collected on your roof?

   (a) Select a suitable method to calculate the area of your roof. Your answer should include a diagram.

   (b) Use this answer and the information above to calculate the average volume of water which could be collected by your area of roof.

3. What percentage of your typical household use, as calculated in your answer to Activity 1, Question 1(a), could be accommodated by a rainwater tank?
4. If your local area average rainfall fell to 60% of its current level what percentage of your household use could be accommodated by a rainwater tank?

5. Construct a table to show the percentage of household water a tank would hold if the average rainfall became 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100% of the current average.

6. Challenge: Are rainwater tanks a worthwhile purchase? What do you need to consider when deciding to switch from town water (that is, the water that comes to a house through the pipes) to tank water? What would you recommend to your parents? What reasons would you give them?

**Activity 3: Electricity costs**

1. A ratio is a comparison of two numbers of the same type. A rate is a comparison of quantities of a different kind.

   With energy use in the news, it is an opportune time to investigate the cost of electricity in our homes. Power is the rate at which energy is used compared to time.

   On every electrical appliance there is a power rating. For example, the power rating for the laptop computers that teachers use is 65 Watts. If the laptop was used for 4 hours, the energy use is 65 Watts \times 4 \text{ Hours} = 260 \text{ Watt hours} = 0.260 \text{ kilowatt hours (kWh)}. The unit of energy used to charge household use is the kilowatt hours (kWh).

   Go on the internet and find the tariff for your area from your local energy company.

2. Complete the table below using the following steps.

   (a) Make a list of all the electrical appliances in your home. You may find it easier to list all the appliances by placing them into categories: heating and cooling devices (air conditioner reverse cycle, bar heater, etc.), cooking (jug/kettle, microwave, fan-forced oven, etc.), household use (refrigerator, water heater, pool pump, etc.), entertainment (radio, TV, computer, etc.), and so on.

   (b) Record the power rating for each appliance.

   (c) Estimate the number of hours each appliance is used in your home each week.

   (d) Calculate the energy consumed by each electrical appliance in your home each week.

   (e) Use this information to calculate the total energy used.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Power Rating(W)</th>
<th>Hours/week</th>
<th>Energy used (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jug</td>
<td>1500</td>
<td>2.5</td>
<td>3.75</td>
</tr>
<tr>
<td>... and so on</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Using the information from 1 and 2 above:

   (a) Calculate the quarterly cost of electricity for your household.

   (b) Compare this with your usual household quarterly bill.

   (c) Give likely reasons for any differences in the amounts.

   (d) List ways your family could reduce electricity use.

   (e) Calculate the reduced energy use as a percentage of original total energy use.

4. Challenge: Prepare an argument, using the above data, for reducing electricity use. Include actual estimates of the money that could be saved from a reduction in electricity use.
Activity 4: Solar panels and electricity

1. Some people are putting solar panels on their roof to reduce costs. Look up internet advertisements for these panels to find the following.
   (a) The most popular area (size) of solar panels sold to households.
   (b) The average cost of purchasing and installing solar panels of this size on a household roof.

2. Continue researching and find the following.
   (a) The amount of energy that the solar panels from 1(a) above can produce in a year.
   (b) The electricity generated by the solar panels is used first to supply the household’s needs. This saves the household money by reducing the amount of electricity that needs to be purchased from the grid (that is, from the company supplying the electricity). If the solar panel is generating more electricity than the household is using (for example in the middle of a sunny day), the surplus is sold back to the grid, so that the household can earn money from the sale of the electricity. The amount earned is deducted from the household’s electricity bill. If we assume that the rate for purchasing electricity from the grid is the same as the rate at which it could be sold back to grid, estimate the annual reduction in an electricity bill if the solar panels from 1(a) are installed?

3. Using the data above (and any other data from your research), calculate the length of time (in years) that it would take for the reduction in electricity bills to pay for the cost of the solar panels.

4. Challenge: If there was enough room on the roof, would extra solar panels be more effective?

5. Challenge: Prepare an argument (written or a poster) for installing (or not installing) solar panels.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td>ACMMG108 Choose appropriate units of measurement for length, area, volume, capacity and mass</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA173 Recognise and solve problems involving simple ratios</td>
<td>ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA174 Investigate and calculate ‘best buys’, with and without digital technologies</td>
<td>ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA174 Investigate and calculate ‘best buys’, with and without digital technologies</td>
<td>ACMMG160 Calculate volumes of rectangular prisms</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA188 Solve a range of problems involving rates and ratios, with and without digital technologies</td>
<td>ACMMG198 Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume</td>
</tr>
<tr>
<td>9</td>
<td>ACMMG217 Calculate the surface area and volume of cylinders and solve related problems</td>
<td></td>
</tr>
</tbody>
</table>
Task description

You have been provided with a spreadsheet that simulates the rolling of two six-sided dice. Use the spreadsheet to play a game with the following rules:

- Use the spreadsheet to roll the dice 36 times.
- If the two dice show different numbers then you win five points.
- If the two dice show the same number (that is, a double) then you lose five points.
- If you reach a total of 100 points in any single game then you win a chocolate frog.

Play the game as often as you like to gather data. The spreadsheet will keep track of the number of dice rolls and the total score for each game.

Record your data and prepare a mathematical report on the game. You must have evidence to claim a chocolate frog from your teacher. Make sure you also have evidence to justify any claims you make about the game.
Writing mathematical reports

A mathematical report requires several different writing skills. Each section of the report should be drafted separately, with the different sections put together into one document just before the report is finalised. The graphic organiser on page 39 assumes that the essential sections of the report are: introduction, method/procedure, results/analysis, discussion and conclusion. Other sections, such as abstract, recommendations, and appendices, are optional.

The organiser can help in planning the report. Each section links back to the writing skills detailed earlier in this book. The layout of the graphic organiser suits the order in which the sections are usually drafted: the procedure, results and discussion sections should be drafted first. Then the conclusion and recommendations (if any) should be developed. The introduction should be drafted after the other sections have been written. The last section to be written is the abstract (if required). Any research required for the project might be included in different parts of the report: introduction, procedure, results or discussion.

The main sections of a report (introduction, procedure, results, discussion, recommendations and conclusions) should be as short as possible. All details, such as calculations, tables, graphs, and diagrams, should be placed in appendices. However, the information in each appendix must be summarised in the main body of the report, so that the reader can understand what the report is saying without having to read the appendices. Each appendix must be mentioned somewhere in the main body of the report so that the reader knows that further supporting information is available.

Headings (and possibly sub-headings) should be used to signpost the different sections of a report. It is usual to number the sections.

**GRAPHIC ORGANISER FOR PLANNING A REPORT**

<table>
<thead>
<tr>
<th>Title:</th>
</tr>
</thead>
</table>

**What is the name of the report?**

<table>
<thead>
<tr>
<th>Research:</th>
</tr>
</thead>
</table>

**Procedure:** (sequencing, see page 28)
- Consider numbering these steps.
- Where/when were the data collected?
- Put copies of survey forms in an appendix.

<table>
<thead>
<tr>
<th>Appendices:</th>
</tr>
</thead>
</table>

**Results:** (analysing, see page 30)
- Put raw data in an appendix.
- Use tables and graphs to summarise the results.

**Discussion:** (discussing, see page 32)

**Conclusion(s):** (concluding, see page 36)

**Recommendation(s):** (recommending, see page 34)

---

**What information is needed?**
Collect information for the bibliography during the research.

---

**Introduction:** (Introducing, see page 26)

*Write this section after the conclusion and any recommendations, but put it before the procedure.*

---

**Abstract:**

*Write this section last, but put it before the introduction.*

---

**Other Details:**
- Does the report contain your name, class, teacher's name?
- Are any of the following needed: title sheet (put this on top), title page, table of contents?
- Has the report been proof read?
- Are all the pages numbered and collated in the correct order? Are they all stapled together?

The shaded sections may be left out, depending on the requirements of the task.

---

---
**Essential vocabulary**

**Certain:** An *outcome* is certain when it is the only possibility. In these cases, the *probability* of the outcome is said to be one.

**Die** (plural *dice*): A small throwable object with multiple resting positions, used for generating random numbers. Traditionally, a die is a cube with rounded edges, with each of its six faces showing a different number of dots (pips) from 1 to 6. When thrown or rolled, the die comes to rest showing on its upper face a random number from one to six, with each value being equally likely. However, other polygons can be used, for example a decagon could be used to generate random numbers from 0 to 9.

**Equally likely:** *Outcomes* are equally likely when they have the same probability of occurring. In the cases of two outcomes, the *probability* of each outcome is said to be 0.5. Synonyms are: even chance, fifty-fifty.

**Event:** An event is one or more *outcomes* of an *experiment*.

**Experiment:** In probability, an experiment is a process involving chance that leads to results called outcomes. It can have one or more steps (*trials*). An example would be tossing a coin ten times.

**Impossible:** An *outcome* is impossible when it cannot occur in any circumstances. In these cases, the *probability* of the outcome is said to be zero.

**Likely:** An *outcome* is likely when it is expected to occur more often than not. In these cases, the *probability* of the outcome is said to be greater than 0.5 but less than one. Synonyms are: odds on, probable, good chance.

**Outcome:** An outcome is the result of a single *trial* of a probability *experiment*.

**Probability:** A measure of how likely an event is. While a probability can be described using words such as certain, likely, and impossible, it is measured by assigning a value between 0 (impossible) and 1 (certain). Probabilities can also be expressed as percentages. Synonyms: chance, likelihood.

**Sample space:** A listing of all of the possible *outcomes* of an experiment.

**Trial:** A single stage of a probability *experiment*. An experiment can consist of one or more trials. For example, in an experiment of tossing a coin ten times, a trial would be tossing the coin once.

**Unlikely:** An *outcome* is unlikely when it is not expected to occur in the majority of situations. In these cases, the *probability* of the outcome is said to be greater than zero, but less than 0.5. Synonyms are: long odds, improbable, poor chance.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, requires students to analyse the fairness of a game based on the simulated rolling of two dice. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students.

Make the spreadsheet available to students. The spreadsheet uses macros. Ensure students have rights to enable macros if running on a school network.

Be prepared for an outcry from your students. The spreadsheet has been created so that the probability of a double (a loss) is far greater than would normally be expected. No chocolate frogs have ever been earned by students using this spreadsheet.

Students very quickly claim that it is rigged or unfair and this then becomes the focus for the report required. To initial claims that it is unfair you can respond with statements like: “Everyone has the same chance of winning”; or “No guarantees were given that every/any player wins a prize”. This leads to a class discussion on “fairness”.

Student claims of the spreadsheet being rigged/unfair/biased etc. need to be countered with a need for proof. Ask students/groups how they are going to prove it. To prove that this game is “rigged” they must gather data from other sources. Alternative data might be collected by:

- playing the game using traditional dice
- playing the game using alternative virtual dice
- exploring theoretical probability.

Virtual dice can be found at:

- [www.bgfl.org/virtualdice](http://www.bgfl.org/virtualdice) (click on the 6-sided die twice to open two pop-up windows)
- [http://nrich.maths.org/6717](http://nrich.maths.org/6717)

Data can be collected working in groups and reports completed individually. This activity can best be used at the start of a unit on probability. The controversy surrounding the game gets students engaged in the topic. After the initial data collection students can build their report as they learn more about probability and the outcomes of rolling two dice. The game is designed to be played with 36 rolls of the dice to simplify the student analysis.

The first time I used this task it was intended only to be used as a mechanism to prompt group/class discussion. The outcry from the Year 8 class when no chocolate frogs were won was significant and claims that I had “rigged” the activity were vigorously expressed. Feigning outrage at being accused of unfairness by students I said I was going to report the matter to the school ethics committee. Next lesson I informed the class that the ethics committee required a written report from each student and then presented the requirements for both the content and format of the report. Students recognised that they had been “conned” but entered into the task with enthusiasm. The Principal and one of the deputies were co-opted as an “ethics committee” and one member of each group presented the results of their investigation.

In my “defence” members of the ethics committee were invited to play the game (returned to the correct probability) and naturally won a chocolate frog. The reports produced by students were extensive and of high quality. The efforts students made to “prove” that they had been cheated of a chocolate frog had no limitations. The quality of the reports in terms of both their written presentation,
use of tabulated results, graphs etc. and the analysis of the theoretical probabilities far exceeded expectations.

A graphic organiser to assist students in preparing a mathematical report is included with this task.

There are some essential mathematical words associated with this task that students should understand and be able to spell. Included as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP167</td>
</tr>
<tr>
<td></td>
<td>Construct sample spaces for single-step experiments with equally likely outcomes</td>
</tr>
<tr>
<td></td>
<td>ACMSP168</td>
</tr>
<tr>
<td></td>
<td>Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
</tbody>
</table>
7P2 “Fair Game”

Task description

*Probability* is an important part of our world. We use it to decide on risk (for example, which is the best investment for our retirement). It determines our chances of winning Lotto and is a big part of many games – helping us to find the best strategy. In gambling, we can use probability to decide how *fair* a game is before placing a bet. This challenge requires you to use probability to evaluate the fairness of a spinner game.

Your task is to play the spinner game, collect data, and analyse the game using the results of your data set, your understanding of spinners and the rules of the game. Is it a *fair* game?

Play the spinner game with two friends. Assign roles of game host, Player A and Player B. Record the results as you play (this set of data will be needed for the questions below). Play the game three times. Each time rotate the player roles (host, Player A, Player B) so that each person plays one complete game in each role.

1. Who won the most games, Player A or Player B? What were the scores?
2. What is the probability of each player winning a point on any given spin? What makes you say that?
3. Do you think the winner (Player A or B) would be the same for most games? Is the game fair? Justify your answer (the language of probability that you can use in your justification is provided in the handout called Essential vocabulary).
4. If you think the game is not fair, how could you change the rules (not the spinner) to give each player an equal chance of winning? Justify your answer by showing how the probability of winning on a given spin would be the same for each player in the new game.
5. Challenge: Using the same spinner, design a new set of rules where Player A has twice the chance of winning a point as Player B. (Hint: don’t try to make this too complicated.) What are some of the simplest ways to create this situation using the spinner? Justify your answer using probabilities.

*Make sure you understand the meanings of any words in italics.*
Use the spinner template shown above with a pencil and a paperclip (as shown in the photograph on the right) to randomly generate numbers from 1 to 8. Alternatively, you could use a die with eight faces (octahedron).
**Rules of the Spinner Game**

- Three people are needed for this game.
- The game “host” always spins the spinner.
- There are two contestants, Player A and Player B.
- For each spin, Player A always multiplies the number spun by 2 and Player B adds 4 to the number spun (the same number). The player whose total is greater gets one point.
- If the two values are equal no one scores on that spin. For example, a 4 is spun. Player A multiplies 4 times 2 to get 8 and Player B adds 4 and 4 to get 8. No one scores for that round.
- The first player to get 20 points wins.

**Essential vocabulary**

**Justify:** To show or prove a decision, action or idea about something is reasonable or necessary by giving sound, plausible and logical reasons for it; to answer the question “why”.

**Certain:** An outcome is certain when it is the only possibility. In these cases, the probability of the outcome is said to be one.

**Equally likely:** Outcomes are equally likely when they have the same probability of occurring. In the cases of two outcomes, the probability of each outcome is said to be 0.5. Synonyms are: even chance, fifty-fifty (if there are only two outcomes).

**Event:** An event is one or more outcomes of an experiment.

**Experiment:** In probability, an experiment is a process involving chance that leads to results called outcomes. It can have one or more steps (trials). An example would be tossing a coin ten times.

**Fair:** A process is fair when there is an equally likely chance of each outcome. So, a game is fair if each player has an equal chance of winning.

**Impossible:** An outcome is impossible when it cannot occur in any circumstances. In these cases, the probability of the outcome is said to be zero.

**Likely:** An outcome is likely when it is expected to occur more often than not. In these cases, the probability of the outcome is said to be greater than 0.5 but less than one. Synonyms are: odds on, probable, good chance.

**Outcome:** An outcome is the result of a single trial of a probability experiment.

**Probability:** A measure of how likely an event is. While a probability can be described using words such as certain, likely, and impossible, it is measured by assigning a value between 0 (impossible) and 1 (certain). Probabilities can also be expressed as percentages. Synonyms: chance, likelihood.

**Sample space:** A listing of all of the possible outcomes of an experiment.

**Trial:** A single stage of a probability experiment. An experiment can consist of one or more trials. For example, in an experiment of tossing a coin ten times, a trial would be tossing the coin once.

**Unlikely:** An outcome is unlikely when it is not expected to occur in the majority of situations. In these cases, the probability of the outcome is said to be greater than zero, but less than 0.5. Synonyms are: long odds, improbable, poor chance.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an application of probabilities to allow students in Year 8 to determine if a game is fair. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

Students are asked to make decisions about the fairness of the game and justify their decisions by reference to probabilities. This provides a good opportunity to teach the meaning of justify and how to write a justification. In planning their written justifications, it can help students to ask themselves the following questions: “What is the decision that I am attempting to justify?”, “What are the reasons for the decision?”, and “How can I use mathematics to provide evidence in support of the decision?”. The information for students includes some essential mathematical words (marked in italics) that students should understand, be able to spell and incorporate into their written justifications. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP168 Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP205 Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’.</td>
</tr>
</tbody>
</table>
**Task description**

The Academy of Motion Picture Arts and Sciences annually presents awards for outstanding performances in the film industry. The awards are popularly known as the Academy Awards (or the Oscars). An award for Best Actor and for Best Actress has been presented annually since the first Academy Awards ceremony for 1927–1928.

It has been *claimed* that male actors take longer to develop their skills and dramatic ability than their female counterparts. The claim was based on the evidence that in most years the winner of the Academy Award for Best Actor was older than the winner of the Academy Award for Best Actress.

Your teacher will provide you with a spreadsheet containing the data on all the winners of the Best Actor and Best Actress Academy Awards since the inception of the awards.

1. Your first task is to *analyse* the data and present *evidence* in a range of different formats to investigate the claim. The evidence should include calculated statistical measures (*summary statistics* such as averages and range) as well as data displays such as tables and graphs. Clearly state your conclusion. If you do not believe that the evidence supports the claim in the first paragraph, then propose an alternative claim that is supported by the evidence.

2. Propose some possible explanations for your conclusion.

3. The conclusion in part (1) is a *fact* (provided that it is supported by the evidence). The possible explanations in part (2) are generally not facts as you do not have evidence to support them (you can, of course, collect more data to investigate and provide evidence for a hypothesis). They are called *hypotheses*. When we discuss facts we use vocabulary such as: *is*; *has been found to be*; *definitively*; *shown to be*; *evidence*; *supports the conclusion that*; *incontrovertible*; *reasons*; *conclusively*; *proof*; *verified*. On the other hand, when we discuss hypotheses we use vocabulary that provides less certainty and allows for other possibilities, for example: *could be*; *possible/possibly*; *infer/inference*; *might*; *perhaps*; *suggests*; *supports the hypothesis that*; *proposed that*; *interpret/interpretation*.

Write a paragraph explaining your conclusions and explanations. Ensure that you select vocabulary that allows the reader to distinguish facts from hypotheses.

*Make sure you understand the meanings of any words in italics.*
**Essential vocabulary**

**Analyse**: Examine something in detail, often by identifying its component parts and the relationships between them, in order to explain and interpret it.

**Claim**: A statement or assertion that something is true, usually without providing evidence or proof.

**Conclusion**: A judgement or decision reached by reasoning and supported by evidence.

**Data display**: A visual summary of statistical information in a table or graph. Common tables include frequency distribution tables and stem-and-leaf plots. Common graph types are frequency histograms, frequency polygons, pie (sector) graphs, divided bar graphs, picture graphs, box-and-whisker plots, and scatter graphs.

**Data**: Statistical information.

**Evidence**: The available body of facts or information indicating whether a belief or proposition is true or valid.

**Fact**: An event, item of information, or situation that exists, has been observed, or known to have happened, and which is confirmed, verified, or validated to such an extent that it is considered to be true.

**Hypothesis** (plural hypotheses): A supposition or explanation (theory) that seeks to interpret something, and to provide guidance for further investigation. A hypothesis may be proven correct or refuted (disproved). If a hypothesis is proved correct, it is said to be **verified** or **corroborated**.

**Mean**: The average of all the items in a data set. To compute a mean, add up all the values and divide by the total number of items in the data set.

**Median**: The middle number in a series of numbers when numbers are placed in order from lowest to highest. If there is an even number of items in the data set, the median is the average of the two middle values.

**Mode**: The most frequently occurring value in the data set.

**Range**: The difference between the highest and lowest values in a data set.

**Summary statistics**: Numbers calculated from a statistical distribution that give an indication of the characteristics of that distribution. They can include measures of location (averages) such as mean, median, and mode, or measures of spread, such as range, inter-quartile range, standard deviation, and variance.

Some words have several meanings. These definitions give the statistical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of statistics. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The data for this task has been provided in an Excel spreadsheet, showing the awards up to and including 2015. These data sets were obtained from Wikipedia (http://en.wikipedia.org/wiki/List_of_Academy_Award_Best_Actress_winners_by_age and http://en.wikipedia.org/wiki/List_of_Academy_Award_Best_Actor_winners_by_age). Students may need to undertake research to include more recent data. The Wikipedia page contains hyperlinks to further information about the movies and actors/actresses. These have been removed from the spreadsheet data. It is a teacher decision whether or not to restrict students to the spreadsheet data or direct them to the online source.

The size of the data sets are such that ideally students should have access to statistics software or a spreadsheet for manipulating the data and constructing the graphs required. The task has been left open with no stipulation given as to the number or type of graphs required. Teachers should provide additional guidance to students on this issue. See the Australian Curriculum links relevant to this task for additional guidance.

This task was first used with a claim that “The judges of the Academy Awards are biased towards older actors for males and favour the younger female actresses”. Teachers should modify the task to fit the target group of students. Make the claim fit the local situation if possible. For example, “maths teacher A says the statistical evidence clearly shows...” and “drama teacher B says this is rubbish and everyone knows that ...”.

In addition to the analysis of the data, the task seeks to develop students’ understanding of the difference between factual conclusions and hypothesised explanations. It is important that students understand these differences, and are able to make these distinctions in their vocabulary choices when discussing or writing about their work. This is an important aspect of literacy in mathematics.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words.

As the glossary defines some terms that students are asked to research, it is for the teacher to decide when to make this glossary available to students. The glossary could be used as the basis of further language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACMSP169</td>
<td>Identify and investigate issues involving numerical data collected from primary and secondary sources</td>
</tr>
<tr>
<td>ACMSP170</td>
<td>Construct and compare a range of data displays including stem-and-leaf plots and dot plots</td>
</tr>
<tr>
<td>ACMSP171</td>
<td>Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data</td>
</tr>
<tr>
<td>ACMSP172</td>
<td>Describe and interpret data displays using median, mean and range</td>
</tr>
</tbody>
</table>
PS1 “Assuming Too Much”

Task description

Not every maths problem gives you all the necessary information needed to be solved easily. Some problems are not straightforward, and contain hidden assumptions that may make them difficult or impossible to solve. This task looks at the strategy “Check for hidden assumptions” and explores problems that need that strategy. It contains 21 problems for you to try.

After completing each problem, discuss with your group, or ask yourself, “What makes this problem difficult?” Ask other classmates, friends and family to try to complete the problems and see if they have trouble with them. When they do work it out, ask them what stopped them from doing it more quickly?

When you have completed all 21 problems, find some more problems that require the strategy to “check for hidden assumptions”.

© QUT YuMi Deadly Centre 2017
Additional student information

1. Without lifting your pencil from the paper, draw four straight line segments to pass through all 9 dots in a 3 by 3 array.

2. How many right-angled triangles can be drawn with their vertices on a 4 by 4 array of dots?

3. Three men pay $10 each (total $30) for a hotel room. The manager says the cost was really $25 and returns $5 with the bell boy. The bell boy pockets $2 and returns $3 to the men. The men therefore paid $9 each, total $27. The bellboy’s $2 makes $29. What happened to the other $1?

4. Two ski tracks come down a snow-covered slope. One track goes around a tree on one side and the other track goes around the tree on the opposite side. Give 10 different physical explanations to fit these facts.

5. You want to know whether the way to a town is on your left or right. You meet two people. One always lies, the other always tells the truth but you don’t know who is who. What question can you ask that ensures you will get the correct direction from either person?

6. What are the next four terms in each of these sequences?
   (a) J, F, M, A, __ __ __ __
   (b) S, M, T, W, __ __ __ __
   (c) O, T, T, F, F, __ __ __ __ __
   (d) M __ 8 __

7. Use toothpicks or matchsticks to show that half of ELEVEN is SIX.

8. If you drop a paper match it will always land on its side. What do you have to do to the match to ensure that when you drop it, the match will land on its edge instead?

9. A father and son were driving home. The car crashed and the father died instantly. His son was injured and rushed by ambulance to the hospital. The doctor on duty came in and took one look at him and said “I can’t operate, this is my son!” What is going on here?

10. Fred’s target was to drive 400 km at an average speed of 80 km/h. With his engine overheating, he travelled the first 200 km at an average speed of only 40 km/h. How fast must he travel the second 200 km to reach his target?

11. A desk calendar is composed of two cubes that can be lifted up and positioned together with any side facing the front. It is shown here on May 24. What are the remaining numbers on the two cubes? Remember all dates from 01 to 31 must be possible.

12. How can you balance a hardboiled egg on its point?

13. How can two people stand on a single sheet of newspaper without being able to touch each other?

14. The number 7 can be displayed on a calculator by pressing only the buttons for 4, +, −, x, ÷ and = (in any order). For example, $4 + 4 + 4 + 4 + 4 + 4 ÷ 4 = 7$ is found by pressing 16 buttons, can you do it in 7?

15. A man went for a walk. It started to rain. He did not have a hat. He was not carrying an umbrella. He kept walking. His clothes got wet. His shoes got wet. Still, his hair did not get wet. How come?

16. A woman unwrapped a lump of sugar. She put it into her coffee. The sugar did not get wet. How can this be?

17. A man was running home. Near home he met a masked man. He stopped. Then he turned around and ran back to where he started. Why?

18. Peter, Mary, Bill and Sally live in the same house. One night Peter and Mary went out to the movies. When they got back they found
Sally beat up and dead on the floor. Bill was not arrested. He was not questioned for any crime. Why not?

19. Bill was perplexed. When he first entered the room, six glasses were arranged so that three glasses containing water (F) were on the left and three empty glasses (E) on the right (F, F, F, E, E, E). When he entered the second time, the glasses containing water were alternating with the empty glasses (F, E, F, E, F, E). Forensic evidence said that only one glass had been moved. How could that be?

20. Jane listened entranced. “The bear walked 2 kilometres south, 2 kilometres east”, said her teacher. “Then the same distance north and ended up where he started from.” “How can this be?” asked her teacher. “I don’t know! But I know the colour of the bear!” exclaimed Jane. Why was Jane able to do this?

21. Sherlock Holmes was passing a house at night. A man screamed from inside, “Don’t Bill!” There was a shot. Sherlock Holmes rushed inside to find a dead man and a smoking gun being watched by a lawyer, a priest and a doctor. Sherlock Holmes arrested the priest immediately and took him to the police where he confessed to the murder. How did Sherlock Holmes solve the case without seeing the murder?
### Teaching information

This task, with some teacher involvement, provides an opportunity for students to question the assumptions that they make when solving problems. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others).

### Links to the Australian Curriculum – critical and creative thinking

<table>
<thead>
<tr>
<th>Critical and creative thinking</th>
<th>By the end of Year 8</th>
<th>By the end of Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inquiring – identifying, exploring and organising information and ideas</td>
<td>Pose questions: pose questions to probe assumptions and investigate complex issues Identify and clarify information and ideas: clarify information and ideas from texts or images when exploring challenging issues Organise and process information: critically analyse information and evidence according to criteria such as validity and relevance</td>
<td>Pose questions: pose questions to critically analyse complex issues and abstract ideas Identify and clarify information and ideas: clarify complex information and ideas drawn from a range of sources Organise and process information: critically analyse independently sourced information to determine bias and reliability</td>
</tr>
<tr>
<td>Generating ideas, possibilities and actions</td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
</tr>
<tr>
<td>Reflecting on thinking and processes</td>
<td>Think about thinking (metacognition): assess assumptions in their thinking and invite alternative opinions Reflect on processes: evaluate and justify the reasons behind choosing a particular problem-solving strategy Transfer knowledge into new contexts: justify reasons for decisions when transferring information to similar and different contexts</td>
<td>Think about thinking (metacognition): give reasons to support their thinking, and address opposing viewpoints and possible weaknesses in their own positions Reflect on processes: balance rational and irrational components of a complex or ambiguous problem to evaluate evidence Transfer knowledge into new contexts: identify, plan and justify transference of knowledge to new contexts</td>
</tr>
<tr>
<td>Analysing, synthesising and evaluating reasoning and procedures</td>
<td>Apply logic and reasoning: identify gaps in reasoning and missing elements in information Draw conclusions and design a course of action: differentiate the components of a designed course of action and tolerate ambiguities when drawing conclusions Evaluate procedures and outcomes: explain intentions and justify ideas, methods and courses of action, and account for expected and unexpected outcomes against criteria they have identified</td>
<td>Apply logic and reasoning: analyse reasoning used in finding and applying solutions, and in choice of resources Draw conclusions and design a course of action: use logical and abstract thinking to analyse and synthesise complex information to inform a course of action Evaluate procedures and outcomes: evaluate the effectiveness of ideas, products and performances and implement courses of action to achieve desired outcomes against criteria they have identified</td>
</tr>
</tbody>
</table>
8A1 “Growing Patterns and Growing Graphs”

Task description

This task allows us to investigate different types of patterns and how we can represent these patterns in a variety of ways: pictorially; a table; a pattern rule; an equation; and a graph. The patterns can produce linear and nonlinear graphs. We will explore these relationships using the patterns in the handout.

For each of the patterns, prepare a table of values with two rows. Write the number of the pattern (shown underneath each of the patterns) in the top row and the number of parts to the pattern in the bottom row.

Activity 1

Find the pattern rule for linear patterns A, B and C. Can you reverse this process? Show with an example.

Activity 2

Draw the graph for linear patterns D and E. What is the relationship of the graph to the pattern? Can you reverse this process? Show with an example.

Activity 3

What is the relationship between the pattern rule and the linear equation that describes the graph?

Activity 4

Linear growing patterns have growing parts and fixed parts. Use patterns F to K to show how the growing and fixed parts relate to the graph and the linear equation. What is different when the graph starts at 1 or at any other number (consider patterns L, M and N)?

Activity 5

Challenge: Can this rule be extended to nonlinear patterns – to cubics and higher order functions? Find nonlinear patterns to work with – O and P are a start. Make up your own rule!

Make sure you understand the meanings of any words in italics.
Additional student information

A.

B.

C.

D.

E.

F.  G.  H.

I.  J.  K.
Essential vocabulary

**Equation**: A statement that the values of two mathematical expressions are equal (indicated by the sign =). The expressions may comprise numbers and/or variables (e.g. $3 + 1 = 4$ and $3x + 1 = 4$ are both equations). Examples of equations used to describe patterns could be $y = 3x + 5$ (a linear equation) or $y = x^3 - 2$ (a nonlinear equation). In these examples, $x$ is the term number (that is, the position: 1$^{st}$, 2$^{nd}$, 3$^{rd}$, etc.) and $y$ is the pattern value (that is, the number of dots, matches, or whatever else is being counted, in that position).

**Linear**: A pattern that when graphed produces a straight line.

**Nonlinear**: A pattern that when graphed produces a curved line.

**Pattern rule**: A description of a pattern, often in words. For example, “start with 3 and add 5 each time” would describe the pattern 3, 8, 13, 18, 23, ...

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of linear and nonlinear patterns and the different ways of representing these patterns. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The basis of this task is that growing patterns can be used to introduce the concepts of variables and graphing, particularly for linear functions of the type $3x - 2$, $\frac{x}{3} + 4$ (where there is a variable but it is not raised to a power such as $x^2$, $x^3$ or $x^4$). Furthermore, there is a relationship between growing patterns and graphs of linear functions that enables graphs to be imagined and drawn from the first steps of pattern analysis. Finally, there are similar relationships between patterns and graphs of nonlinear functions (where there is $x^2$, $x^3$ or $x^4$). The question is, what are these relationships between patterns and graphs – for linear and nonlinear functions?

This section provides detailed instructions to complete the task questions. These instructions can be used by the teacher as a basis for hints to students to keep them working or they can be used as a basis of worksheets.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
Activity 1: Building algebraic generalisations (patterns A, B and C)

Materials: Sticks, counters, pen, paper

1. Construct the pattern below from sticks. It begins with the starting position 0 and then grows as shown by the end of periods 1, 2 and 3, and so on.

![Pattern Diagram]

(a) Construct the pattern for positions 4 and 5.
(b) Count the number of sticks for positions 0, 1, 2, 3, 4 and 5.
(c) Look at the visual way the pattern is developed and the relation of the number of sticks to the position number, and identify the fixed and growing parts of the pattern.

2. Use the results of 1 above to complete the table below.

<table>
<thead>
<tr>
<th>Position</th>
<th>Number</th>
<th>Position</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>If given any position, write how you would work out the number of objects</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>If given any number of objects, write how you would work out the position</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table looks at position to number (the position-pattern rule) and number to position (this is reversing the normal direction for the position-pattern rule). It does this for small numbers and large numbers, in terms of language, and in terms of variables $n$ and $k$.

3. Compare the generalisation in algebraic terms using the letter $n$ with the fixed and growing parts of the pattern.

4. Challenge: Reverse the whole direction of this activity and construct a pattern that has the position pattern rule of $2n + 3$.

5. Repeat 1 to 3 above for the following patterns. Construct a table similar to that in 2 and fill it in.

(a)
(b)

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

(c)

\[
\begin{array}{cccccc}
X & XX & XXX & XXX & X XX & X X \\
# & ### & #### & ##### & ###### & \\
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

6. Challenge: Repeat 4 above for:

(a) \(4n - 1\)

(b) \(2n - 3\)

*Hint*: Start the pattern at 1.

**Activity 2: Extension to graphs (patterns D and E)**

**Materials**: Sticks, counters, pen, paper, graph paper, objects to make patterns with, small number cards

1. Look at the first pattern from Activity 1.

   (a) Construct a simple table and place early values in it, e.g.

<table>
<thead>
<tr>
<th>Number of sticks</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
<th>and so on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

   (b) Use the table to draw a graph.

2. Reverse everything: Make up a pattern to match the graph on the right.

3. Repeat 1 above for the following patterns:

   (a)

   \[
   \begin{array}{cccccc}
   X & XX & XXX & XXX & X XX & X X \\
   0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}
   \]

   (b)

   \[
   \begin{array}{cccccc}
   X & X & X & X & X & X \\
   0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}
   \]
4. Repeat 2 above for the graphs below:

(a) ![Graph 1](image1)

(b) ![Graph 2](image2)

**Activity 3: Pattern rules and functions**

**Materials:** Pen, paper, graph paper, small counters such as MAB units, objects to make patterns with, small number cards

1. A line graph is a straight line. Therefore it only needs two points to define it. To plot a graph: (a) a Cartesian coordinate system is drawn; (b) points on this line are determined and plotted on the graph; and (c) the graph is represented by an equation or function relating \( y \) and \( x \).

<table>
<thead>
<tr>
<th>( y )</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) Draw the Cartesian coordinate system and plot the points above.

(b) Do you get the graph on the right? The equation is \( y = 2x + 1 \).

2. **Relating patterns to linear equations.** For the pattern below:

<table>
<thead>
<tr>
<th>OO X</th>
<th>OO XX</th>
<th>OO XXX</th>
<th>OO XXXX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Construct positions 4 and 5.

(b) Determine fixed and growing parts.

(c) Determine position rule in terms of \( n \) and plot graph.

(d) Rename position as \( x \) and pattern value as \( y \), then redraw graph and rewrite the rule.

3. **Reverse the situation.** Start with a linear equation e.g. \( y = 3x - 2 \) and then:

(a) Draw a table of \( x \) and \( y \) values as on right and fill in the table for \( x = 0, 1, 2, 3 \) and so forth.

(b) Draw the graph.

(c) Change \( x \) to \( n \) and construct a pattern to reflect the graph.

(d) Looking at exercise 2 above as well as this exercise 3, what does this mean?

4. **Explore how position pattern rules change positions on a graph.** Use counters, graph paper and small counters (e.g. MAB units) to do the following:
(a) Start with a linear position pattern, e.g. $2n + 1$. What linear function is this?

(b) Act out this linear function as a change by placing counters on bottom of graph paper, numbering each counter 0, 1, 2, 3, and so on (left to right), and moving each counter forward double the number given plus one (see diagram below):

(c) Draw the graph.

(d) What is the slope and what is the $y$ intercept?

**Activity 4: Relationships between patterns and graphs for linear equations (patterns F to N)**

**Materials:** Pen, paper, objects to make patterns from, small number cards, graph paper

1. Look at the patterns below and do the following for each one:

   (1) 
   (2) 
   (3) 
   (4) 
   (5) 
   (6) 

(a) Continue the patterns and draw positions 4 and 5.

(b) Determine the composition of the pattern (growing part, fixed part and 0 position number).

(c) Determine the position pattern rule.

(d) Draw the graph and determine the graph characteristics – slope and $y$-intercept.

(e) Complete the table below – the first row has been completed.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Composition</th>
<th>Position rule</th>
<th>Graph characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Grows by 2, no fixed part, starts with 0</td>
<td>$2n$</td>
<td>Slope 2, $y$-intercept 0</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. For the patterns in 1 above and the line graphs that emerge from them:
   (a) Look for relationships between composition of pattern, position rule, and characteristics of graph.
   (b) Write a general rule for relating slope \((m)\) to growing part \((g)\).
   (c) Write a general rule for relating \(y\)-intercept \((c)\) to fixed part \((f)\).
   (d) Challenge: The general formula for a line graph is \(y = mx + c\). If \(g\) is growing part and \(f\) is fixed part, what would the formula for the line graph be?

3. Look at the following patterns starting at 1:

   (1) 
   (2) 
   (3) 

   (a) Determine the growing part, the fixed part, and the position rule for each pattern.
   (b) Use the pattern backwards to find position 0 for each pattern.
   (c) Draw the graphs and find the slope and \(y\)-intercept.
   (d) Challenge: What has to change, if anything, in the rules in 2 above for these patterns? If \(g\) is the growing part and \(f\) is the fixed part, can we write a formula for these patterns? How does this formula relate to the fixed part?

**Activity 5: Challenge: Nonlinear relations and graphs (patterns O and P)**

**Materials:** Pen, paper, objects to make patterns from, small number cards, graph paper

1. Look at the following triangle-shaped pattern:

   (a) Construct positions 4 and 5.
   (b) Draw a table with position and number of counters as headings.
   (c) Determine the pattern rule (Hint: Form a rectangle out of two copies of the triangles in positions 2 and 3; the triangle is \(\frac{1}{3}\) this number).
   (d) Is there a growing and fixed part? Calculate the difference between each term. Calculate the difference between the differences and so on – when do we get a growing part that is always the same?
   (e) Look up information on triangular numbers like this.

2. Repeat 1 for the square pattern below.
(a) Look at the differences between each term, as in 1(d) above. What is similar between the square and the triangle pattern?

(b) Is this rule true for all patterns with a rule that contains $x^2$?

(c) What is the rule for $x^3$? What helps find coefficients of $x^3$, $x^4$ and so on?

3. Check your rule for (c) above on other nonlinear patterns.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMNA107</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA166</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA175</td>
</tr>
<tr>
<td></td>
<td>ACMNA176</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA193</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA296</td>
</tr>
</tbody>
</table>
8A2 “Algebraic Caterpillars”

Task description

Algebra is the *generalisation* of arithmetic. It allows us to change from acting like a calculator to becoming a mathematician who can investigate patterns and relationships without having to depend on particular *values* (numbers). It is a powerful tool that all mathematicians should master.

Algebra and arithmetic are presented so differently that it can be hard to see how algebra comes from arithmetic. So the question is: does arithmetic contain the methods that we can use in algebra? In other words, is arithmetic the caterpillar that grows into the algebra butterfly?

We cannot explore everything in algebra in one task, so we will focus on an important section – expansion and factorisation of expressions. We will begin by focusing on the area of rectangles.

1. What is the connection between the area of rectangles and multiplication? Divide a large rectangle into several smaller rectangles. Investigate the relationship between an *expression* for the combined areas of the small rectangles and an expression for the area of the large rectangle. Extend this to rectangles of different sizes.

2. How can we connect what we learned in step 1 to the *algorithm* used for long multiplication?

3. How can we extend the ideas in step 1 to negative numbers?

4. Investigate what happens if we replace the values (numbers) used in steps 1 and 3 with *variables* (letters)?

5. Find out about the distributive law, including *expansion* and *factorisation*. How does it relate to what we have been doing with the rectangles? Why do you think it might be an important principle of mathematics? **Challenge**: How can we use the ideas in this task to understand factorisation?

*Make sure you understand the meanings of any words in italics.*
**Additional student information**

Here are some suggestions to start you off.

**Step 1:**

Here are some rectangles to get you started. We want to calculate the area of the large rectangle (the one with the thick border) in two ways.

<table>
<thead>
<tr>
<th>Total width: 11 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width: 7 units</td>
</tr>
<tr>
<td>Width: 4 units</td>
</tr>
<tr>
<td>Height: 5 units</td>
</tr>
</tbody>
</table>

Write a *relation* (number sentence) for the area of each of the two smaller rectangles and for the large rectangle. Can you write a relation that connects the method of calculating the area of the two small rectangles with the method of calculating the area of the large rectangle?

<table>
<thead>
<tr>
<th>Total width: 10 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width: 4 units</td>
</tr>
<tr>
<td>Width: 3 units</td>
</tr>
<tr>
<td>Total height: 5 units</td>
</tr>
<tr>
<td>Height: 2 units</td>
</tr>
<tr>
<td>Height: 3 units</td>
</tr>
<tr>
<td>Width: 3 units</td>
</tr>
</tbody>
</table>

Write a relation for the area of each of the six smaller rectangles and for the large rectangle. Now write a relation that connects the two methods of calculating the area. How are they connected?

Make up some examples of your own.
Step 2:
Repeat step 1, but divide your rectangles into tens and ones. For example, to find the area of a rectangle that is 68 units long and 23 units wide, split the length into 68 = 60 + 8 and the width into 23 = 20 + 3. Compare your results with the long multiplication algorithm used to calculate 68 \times 23.

Try some examples of your own, possibly including three and four digit numbers.

Step 3:
Extend step 1 to negative numbers. Here is an example.

You could start with a simpler example, such as width a + 7 units and height 5 units.

(Challenge) If you have understood and been able to do everything so far, you could try investigating:

- **trinomial factors**, for example \((2x + y - 1)(x + 3y + 4)\). (Hint: you will need to divide the rectangle into nine parts);
- **expressions with triple factors**, for example \(3(x - 2)(3x + 5)\). (Hint: you will have to think about volumes of rectangular prisms).

Look up factorisation. How would the ideas in this task help you to understand that?
Essential vocabulary

Algebra: A generalisation of arithmetic in which letter symbols are used to represent unknown quantities so that we can generalise specific arithmetic relationships and patterns. For example, the arithmetic facts $3 + 3 + 3 + 3 = 4 \times 3$ and $5 + 5 + 5 + 5 = 4 \times 5$ are special cases of the algebraic statement $z + z + z + z = 4z$.

Algorithm: A sequence of defined steps or processes to solve a particular problem. It is most commonly used in mathematics to describe the processes of manually calculating with multiple digit numbers.

Expansion: The process of multiplying where at least one of the products is a sum of two terms. The result is a sum of a series of terms. For example, $2(x + 3)$ can be expanded to $2x + 6$.

Expression: An algebraic expression is made up of three things: numbers, variables, and operation signs such as $+$ and $–$. Some examples are $2a$, $a + b$, $a^2$, $ab$. A numeric expression does not include variables, for example, $3 + 2$, $4 \times 7 – 1$.

Factorisation (in USA the spelling is factorization): The process of writing a number or an algebraic expression as the product of its factors. For example, $2x + 6$ can be factorised as $2(x + 3)$.

Generalisation (in USA the spelling is generalization): To take a specific idea and apply it more broadly is a generalisation. For example, it is a generalisation to say all dogs chase cats. In mathematics, the arithmetic facts $3 + 3 + 3 + 3 = 4 \times 3$ and $5 + 5 + 5 + 5 = 4 \times 5$ can be generalised to say that $z + z + z + z = 4z$.

Value: Any fixed number, for example, $3$, $–7$, $3647$, $5^2$, $\pi$, $1.5$ million.

Variable: In algebra, a symbol that stands for an unknown quantity, usually represented as a letter (in the English or Greek alphabets). Unknown and pronumeral are other names that can be used to describe a variable.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for introducing the distributive law. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions. The detailed instructions could also be used as the basis of student worksheets.

The task introduces students to the distributive law, which states that:

\[ a(b + c) = ab + ac \]

When applying the law from left to right, the process is described as expansion. When applying it from right to left, it is called factorisation. Students typically find factorisation more challenging than expansion.

The distributive law can be applied to values, for example, \(2(4 + 7) = 2 \times 4 + 2 \times 7\). However, in this case there is little point in using the law because we would usually perform the addition in the brackets before the multiplication. However, it becomes more useful as the basis of the multiplication algorithm, for example, \(9 \times 68 = 9 \times 60 + 9 \times 8\).

The distributive law becomes particularly important in algebra, when it is often not possible to add the terms in the brackets before multiplying, for example \(2(p - 3) = 2 \times p - 2 \times 3 = 2p - 6\). It can become particularly challenging where all of the factors have two or more terms or where there are more than two factors, for example, \((x + 4)(4x - 1)\) or even \((2a + 5b + 4)(a - 3b + 6)(5a + b - 2)\). Whilst we would not normally expect Year 8 students to master expressions of this complexity, the task allows students to venture that far if they are able.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
Activity 1: Rectangular area and multiplication

Materials: Graph paper, pen and paper

1. Draw four different rectangles (A, B, C and D) on graph paper. Find the areas of these rectangles by counting squares and record this information with length and width on a table as below.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (a) What is the relation between area of rectangles and multiplication?

(b) What does this mean for calculating multiplications?

3. Draw two rectangles (E and F) on graph paper and break one into two parts and the other into three parts – similar but not the same as on right.

(a) Complete the following table from your two rectangles.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Overall rectangle</th>
<th>1st part rectangle</th>
<th>2nd part rectangle</th>
<th>3rd part rectangle</th>
<th>Total part areas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>W</td>
<td>A</td>
<td>L</td>
<td>W</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What does this mean for multiplication?

(c) Complete the multiplications for your rectangles in a similar manner to right.

4. Repeat 3 above for a rectangle divided into four parts – similar but different to the rectangle G on right.

The area relation for G is 13×19 = 8×7 + 8×12 + 5×7 + 5×12. The multiplication relation is the same.

(a) What is the area/multiplication relation on your rectangle?

5. Write down multiplication relations for the two rectangles below.

(a) The first rectangle is 24×28: 24 is divided into 9 and 15; and 28 is divided into 11 and 17.

(b) The second rectangle is 43×21: 43 is divided into 18, 14 and 11; and 21 is divided into 7 and 14.

6. (a) Make up two more rectangles, break them into parts and write down the multiplication.

(b) Can you make one of them so that it is divided into 6 parts?
7. Reverse the process – draw rectangles for:

(a) \(15 \times 17 = 8 \times 9 + 8 \times 8 + 7 \times 9 + 7 \times 8\)
(b) \(34 \times 26 = 14 \times 18 + 14 \times 8 + 5 \times 18 + 5 \times 8 + 15 \times 18 + 15 \times 8\)
(c) \(26 \times 32 = 20 \times 30 + 20 \times 2 + 6 \times 30 + 6 \times 2\)

Activity 2: Connecting to place value

Materials: Graph paper, pen and paper, calculator

1. Construct a rectangle for \(6 \times 27\) based on place value (\(27 = 20 + 7\)) as below.

\[
\begin{array}{c|c}
6 & 20 \\
\hline
27 & 7 \\
\end{array}
\]

The multiplicative relation that is associated with this rectangle and its area is \(6 \times 27 = 6 \times 20 + 6 \times 7\), and this can be related to the vertical algorithm as per the setting out on the above right.

(a) Construct a rectangle and vertical algorithm for \(8 \times 46\). Use a calculator for the \(6 \times 7\) and \(6 \times 20\) type calculations if you need to.
(b) Construct a rectangle and vertical algorithm for \(9 \times 78\).

2. More complicated algorithms can be represented and calculated as on right:

Show how the rectangles and setting out can help in the following algorithms (use a calculator to do the \(20 \times 20\) and \(3 \times 20\) type parts of this, if you need to):

(a) \(8 \times 73\)
(b) \(34 \times 28\)
(c) \(87 \times 64\)
(d) Challenge: \(362 \times 238\)
Activity 3: Using negatives

Materials: Graph paper, pen and paper, calculator

1. Look at $6 \times 37$ – this can be solved with rectangles as on right. The $6 \times 37$ is thought of as $6 \times 30 + 6 \times 7$.

Now think of the $37$ as $40-3$. Then $6 \times 37$ would be $6 \times (40-3)$. In terms of the algorithm 37 is 3 tens and 7 ones, 40–3 could be written as $4 -3$ standing for 3 less than 40.

It should be noted that the same answer is achieved with $6 \times 37$ as $6 \times 4 -3$.

Use rectangles to solve the following pairs of algorithms and to show they are the same:

(a)  
\[
\begin{array}{c}
28 \\
\times 4
\end{array} 
\begin{array}{c}
3 -2 \\
\times 4
\end{array} 
\begin{array}{c}
69 \\
\times 6
\end{array} 
\begin{array}{c}
7 -1 \\
\times 6
\end{array} 
\begin{array}{c}
128 \\
\times 9
\end{array} 
\begin{array}{c}
13 -2 \\
\times 9
\end{array}
\]

2. More complicated algorithms can be solved this way:

The traditional method on the left looks at $38 \times 26$, while the use of negatives on the right focuses on $4 -2 \times 3 -4$ or $(40 -2) \times (30 -4)$. The negatives are not a problem as both versions have the same answer.

Use rectangles to solve the following pairs of algorithms and to show they are the same:

(a)  
\[
\begin{array}{c}
26 \\
\times 57
\end{array} 
\begin{array}{c}
3 -4 \\
\times 6 -3
\end{array} 
\begin{array}{c}
87 \\
\times 29
\end{array} 
\begin{array}{c}
9 -3 \\
\times 3 -1
\end{array} 
\begin{array}{c}
439 \\
\times 86
\end{array} 
\begin{array}{c}
44 -1 \\
\times 9 -4
\end{array}
\]
**Activity 4: Algebraification**

**Materials:** Graph paper, pen and paper, calculator

1. The approach in Activity 3 also works for algebra. On the right is the rectangle for \(a(a + b)\); using the rectangle, this can be seen as \(a \times a\) and \(a \times b\) and, using the same approach as with numbers, this leads to \(a(a + b) = a^2 + ab\).

   (a) Use the approach to find \(y(y + 2)\): draw the rectangles, give the vertical setting out.

   (b) Use the approach to find \(2a(3a + 2)\).

   **Note:** We do not use the \(\times\) sign, so \(2a\) means \(2 \times a\) and \(a(a + b)\) means \(a \times (a + b)\).

2. Extend the rectangle idea to negative numbers, as we did for numbers in Activities 2 and 3, to show how to multiply the following; draw the rectangles and show the setting out and the four part multiplications.

   (e) \(p(p - q)\)

   (f) \(y(2y - 3)\)

3. Extend the idea even further, as we did when we went from \(6 \times 27\) to \(23 \times 27\) in Activity 2, to solve the following (still use the vertical setting out):

   (a) \((p + 2q)(2p + q)\)

   (b) \((y + 4)(3y + 2)\)

4. Bring in the directed numbers:

   (a) \((3p - q)(4p - 2q)\)

   (b) \((2y - 5)(3y + 7)\)

5. Challenge:

   (a) Look up the mathematics principle called the distributive law. What does this principle do? Is it important for what we have been doing?

   (b) How would this law help in multiplying \(4\ m\ 67\ cm\) by \(23\)? What about \(7 \times 5\ hr\ 43\ mins\)?

   (c) Look up factorisation. How would the ideas in this task help you to understand that:

   - \(3y + 6 = 3(y + 2)\),
   - \(3pq + q^2 = q(3p + q)\), and
   - \(y^2 - 5y + 6 = (y - 3)(y - 2)\)
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ACMNA190 Extend and apply the distributive law to the expansion of algebraic expressions</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA191 Factorise algebraic expressions by identifying numerical factors</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA213 Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate</td>
</tr>
<tr>
<td>10</td>
<td>ACMNA230 Factorise algebraic expressions by taking out a common algebraic factor</td>
</tr>
</tbody>
</table>
Task description

Mathematics has structure and beauty. There are fundamental properties that dictate how our arithmetic operations behave. We see these same properties repeated in other situations. Understanding how these properties work can help us make sense of many other situations. Good mathematicians see mathematics as a large interconnected tapestry into which we weave our growing understanding of different aspects of the subject.

In this task we will examine some of these fundamental properties and consider how they can arise in other situations.

We define the properties of mathematical operations in terms of what they do (for example, how addition works) and which sets they apply to (for example the set of whole numbers, or the set of rational numbers [fractions]).

The handout entitled “Essential vocabulary” defines those fundamental properties and how they can be combined to create groups and fields (words that have very specific meanings in mathematics). Read this handout, focusing on the commutative law, associative law, closure, identity and inverse. The explanations of each of these properties include some examples. Give three more examples of where the property applies and three examples where the property does not apply. Remember, you will have to specify both the operation and the set of numbers to which the operation applies.

Now read the explanation of groups and fields. The structure of arithmetic and algebra means that addition and multiplication (without 0) form groups (and together form fields) on the rational numbers. Why do we need to exclude zero from the set of numbers that multiplication applies to?

What other groups are there? That is the key question for this task.

Investigate the operations of the symmetries of the triangle and modulo arithmetic (described in the handout “Some new operations”). In each case, consider which of the principles of mathematics apply. Do these examples form groups? If you need more help to answer these questions, ask your teacher for the handout called “Additional student information”.

Make sure you understand the meanings of any words in italics.
**Essential vocabulary**

**Operation:** An action or procedure that produces a new value (output) from one or more input values. There are two common types of operations: unary and binary. Unary operations involve only one input value, such as negation and trigonometric functions. Binary operations, on the other hand, require two input values, and include addition, subtraction, multiplication, and division. Whilst operations are commonly thought of as involving numbers as inputs, other inputs are possible, for example, flips, twists and turns can be thought of as operations on shapes. The symbol * is often used to refer to an operation without using a particular example.

**Set:** A set is a collection of distinct objects. However, a set is also considered as an object in its own right. For example, the numbers 2, 4, and 6 are distinct objects when considered separately, but when they are considered collectively they form a single set of size three, written [2,4,6]. The objects within a set are called elements. Sets can be empty (written { } or φ), finite or infinite. Some examples of infinite sets include the set of whole numbers, the set of integers, and the set of rational numbers (fractions). Sets are one of the most fundamental concepts in mathematics.

**Associative law:** An operation is associative if it does not matter which part we do first. For example multiplication is associative because $(3 \times 2) \times 4 = 3 \times (2 \times 4)$. Algebraically we can say that $(a*b)*c = a*(b*c)$.

**Commutative law:** An operation is commutative if changing the order of the inputs does not change the outcome. For example, addition is commutative because $2 + 3 = 3 + 2$. Algebraically we can say that $a*b = b*a$. However, division is not commutative because $2 \div 3 \neq 3 \div 2$.

**Closure:** An operation on a set is closed if the output of the operation belongs to the same set as the inputs. For example, addition of whole numbers is closed because $3 + 2 = 5$ and $2, 3$ and $5$ are all whole numbers.

**Identity:** An operation has an identity if there is an element (number) that does not change the value of the input. For example, the identity for addition is 0, demonstrated by $4 + 0 = 4$. The symbol $e$ is often used to refer to an identity without using a particular example. Then we can describe the algebraic rule for an identity as $a*e = a$ and/or $e*a = a$.

**Inverse:** An operation has an inverse if there is an element (number) that undoes the process of the operation. For example, in addition the inverse of $+3$ is $-3$. Applying the operation to a number and its inverse results in the identity, for example, in addition $+3 + -3 = 0$. The symbol $a^{-1}$ is often used to refer to an identity without using a particular example. Then we can describe the algebraic rule for an identity as $a*a^{-1} = e$ and/or $a^{-1}*a = e$.

**Group:** A set of numbers forms a group under a particular operation if (a) the operation is closed, (b) the associative law applies, (c) there is an inverse for every member of the set, and (d) the set has an identity. For example, addition forms a group on the integers, but not on the whole numbers (why not?). If the operation on a group also obeys the commutative law, we say it is an Abelian group.

**Distributive law:** The distributive law applies to two operations on the same set of numbers and states that: $a(b + c) = ab + ac$. For example, $2(3 + 4) = 2 \times 3 + 2 \times 4 = 14$. In this case we say that multiplication distributes over addition.

**Field:** If two operations defined on a set of numbers form groups, and the distributive law applies, then we say that the set of numbers forms a field. For example, the real numbers (all fractions and integers) forms a group under addition and multiplication, and as multiplication distributes over addition, we say that the real numbers form a field.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Some new operations

**Symmetries of the triangle:** The symmetries of the triangle are formed by flips and turns (rotations) of an equilateral triangle. They can be explored by cutting out an equilateral triangle and labelling it as on right. Turn the triangle over and write the same letters in each corner on the reverse side. There are six translations of the triangle that can be made:

- **I:** Rotating 0°
- **P:** Rotating 120°
- **Q:** Rotating 240°
- **R:** Flipping around A position
- **S:** Flipping around B position
- **T:** Flipping around C position

![Triangle Symmetries Diagram]

**Challenge:** Can you list the symmetries of the square?

**Modulo (modular) arithmetic:** A system of arithmetic for integers, where numbers “wrap around” upon reaching a certain value – the modulus. It can also be described as the remainder when the number is divided by the modulus. An analogue clock face is an example of modulo 12, in the case of hours (on reaching 12 o’clock, the numbers go back to 1), or modulo 60, in the case of minutes. Modulo is abbreviated mod, so modulo 12 is abbreviated to mod 12.
Additional student information

Activity 1: Describing an operation using a Cayley table

1. Instead of describing an operation as a process (such as addition or multiplication), if the number of elements in the set is small, we can define an operation by listing all of the inputs (as row and column headings of a table) and the outputs (as the cell entries).

2. The symmetries of the triangle described in the “Some New Operations” handout, under an operation that we call “followed by”, can be defined in a table. It works as follows:

(a) Start from

(b) Perform first operation, say P:

(c) Follow with second operation, say T, so P followed by T:

The second operation acts on the positions, not on A, B and C. For example, in the above, T is flipping around the third angle. Thus, even if the capital letters change, the flip is still around the third angle (now B, not C).

(d) Look at the changes in “Some New Operations” and we can see that this is R:

So P followed by T is R

3. Complete the table using (a) to (d) above for each two operations (the first two rows have been done).

4. This sort of table is called a Cayley table.

5. An operation defined by a Cayley table forms a group if we find that:

   • it is closed – there are no new letters in the table;
   • there is an identity I – a row and column which leaves everything unchanged;
   • everything has an inverse – there is an I in each row and column;
   • the associative principle holds – each letter appears once and once only in each row and column;
   • the commutative law may hold – the table is symmetrical about the leading diagonal.
6. Does the operation “followed by” in the Cayley table that you created in this question form a group? Is it an Abelian (commutative) group?

7. Challenge: Consider the symmetries of the square. Do they form a group?

**Activity 2: Modulo arithmetic**

1. We can add and multiply modulo. We do this as follows:

   (a) Take two numbers, e.g. 7 and 4. Add them to get 11 or multiply them to get 28.

   (b) Take a modulo number, e.g. 8. Divide 8 into the addition and multiplication from step (a) and find remainder (R):

   e.g. \[ 11 \div 8 = 1 R 3 \]
   \[ 28 \div 8 = 3 R 4 \]

   (c) Modulo arithmetic gives remainder

   e.g. \[ 7 + 4 \text{ modulo } 8 = 3 \]
   \[ 7 \times 4 \text{ modulo } 8 = 4 \]

2. Complete the Cayley tables of modulo operations.

   \[
   \begin{array}{c|ccccc}
   \text{× mod 6} & 0 & 1 & 2 & 3 & 4 \\
   \hline
   0 & & & & & \\
   1 & & & & & \\
   2 & & & & & \\
   3 & & & & & \\
   4 & & & & & \\
   5 & & & & & \\
   \end{array}
   \]

   (a) Which is a non-commutative group – A, B or C?

   (b) Which is an Abelian (commutative) group – A, B or C?

   (c) Which is not a group at all – why?

3. Challenge: Some modulo numbers form groups under multiplication and some do not. Find the rule for which numbers do form groups.

   Note: In arithmetic, addition and multiplication separately form groups. When they are put together as in Task 8A2 Algebraic Caterpillars, they gain a further property (called the distributive law), which means that together they form a bigger structure called a field.

**Activity 3: Artistic challenge**


Use one of them to make an artistic pattern (i.e. a work of art).
Teaching information

This task, with some teacher involvement, introduces students to the fundamental properties of our number system. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide students with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The first part of the task is the handout of essential vocabulary. Mastery of this handout is critical to the understanding of this task. Students are asked to consider the properties of various operations on various sets of numbers and to investigate groups. The concept of an operation is then extended beyond the number system to Euclidean transformations of a triangle.

The handout “Additional student information” can be provided to students requiring additional scaffolding to undertake this task.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA151 Extend and apply the associative, commutative and distributive laws to aid mental and written computation</td>
</tr>
<tr>
<td></td>
<td>ACMNA177 Extend and apply the laws and properties of arithmetic to algebraic terms and expressions</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA190 Extend and apply the distributive law to the expansion of algebraic expressions</td>
</tr>
</tbody>
</table>
**8G1 “Maths in a Box”**

**Task description**

When industries make packaging (boxes), they cut and fold containers of the size they want from paper or light or heavy cardboard. How do they work out what size to cut the cardboard to get the right volume box?

In this task, we will explore the *relationship* between size of paper and volume of resulting box. We use special folding to do this. Directions for construction are on the next page.

1. Construct boxes from different length paper squares.
2. What is the *relation* between length of paper square and height of box?
3. What is the relation between length of paper square and volume of box?
4. Is there a relation between ratios of lengths of paper squares and ratios of volumes of boxes?
5. Unfold the box, identify the net and determine a way to calculate the actual volume for your boxes.
6. Challenge: Construct a formula for calculating the box volume for a paper square of length L. Construct a formula for calculating the length of paper needed to make a box of volume V.

*Make sure you understand the meanings of any words in italics.*

**Essential vocabulary**

**Relationship:** A connection between two ideas or things. In mathematics, the word *relation* is often used to describe the connection(s) between an object, pattern, or the value of a variable, before and after a change.

Some words have several meanings. This definition is the mathematical meaning of the word. You should know how to use and spell this word.
Additional student information

Construction directions

Materials: Scissors, paper

1. Cut a 16 cm × 16 cm square piece of paper.

2. Fold the square in half horizontally and unfold, and then fold it vertically and unfold.

3. Fold each corner of the square in to meet the centre.

4. Fold each of the original corners under so that they touch the midpoints of their respective sides.

5. Turn the paper over. Fold the left and right edges in so that they meet in the centre of the paper.

6. Fold the left trapezoidal flap over to meet the right edge. Fold the small triangles on the top and bottom of the left side in to the midline. Fold the trapezoidal flap back along the midline to meet the left edge.
7. Repeat Step 6 using the right trapezoidal flap so that the paper looks like the shape below.

8. Crease where indicated by the dashed lines. Pull out the sides while pushing the top and bottom in to finish the box.

9. Repeat for other size paper to make different volume boxes.
**Teaching information**

This section provides more detailed instructions to complete the task questions. They can be used by the teacher as a basis for hints to students to keep them working or they can be used as a worksheet.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

**Activity 1: Construction of box**

Suggestion – let students construct the box without teacher input. Get those who work it out to help others.

Note: Turning over the paper is what most students miss in construction. However, it is useful to let them work this out themselves.

**Activity 2: Length of side and relationship with height of box**

**Materials:** Ruler, calculator, graph paper, pen

1. (a) Measure the height of the box made from 16 cm × 16 cm paper.

   (b) Find the volume of the box by measuring sides and using the formula for the volume of a rectangular prism.

2. Construct boxes for other paper lengths and complete the table below, displaying the dimensions and volume of the boxes.

<table>
<thead>
<tr>
<th>Length of paper square (cm)</th>
<th>Length/width of the folded box (cm)</th>
<th>Height of the folded box (cm)</th>
<th>Volume found by $V = L \times W \times H$ (cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose a square of side less than 16 cm and a square of side greater than 20 cm and construct new boxes to complete the table.

3. Use graph paper to prepare a graph relating length of side of square with height of box

   (a) Use the data from the table above to plot points on a graph as below.

   (b) Draw a smooth curve or straight line through the points on your graph.

   (c) What is the relationship this line shows between, length of square and height of box?
Activity 3: Length of side and relationship with volume of box

Materials: Ruler, pen, calculator, graph paper

1. Use graph paper to prepare a graph relating length of side of square with volume of box
   (a) Use the data from the table above to plot points on a graph relating length of side to volume of box.
   (b) Draw a smooth curve or straight line through the points on your graph.
   (c) What is the relationship this line shows between, length of square and height of box?

2. Can you see a pattern that relates length of side of paper square to volume of box? Would it help to construct some new boxes to support your pattern?

Activity 4: Ratios of sides and volumes

Materials: Pen, calculator

1. Find the ratio of the volume of the box made from 20 cm × 20 cm square paper to the volume of the box made from 16 cm × 16 cm square paper.

2. Calculate ratios between length of sides of paper squares and volumes of boxes from these squares. Construct a table as below (use you lengths if different to below)

<table>
<thead>
<tr>
<th>Squares being compared</th>
<th>Volumes of boxes</th>
<th>Ratio of sides</th>
<th>Ratio of volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 and 16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 and 20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 and 24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>... and so on</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Can you see a pattern in the volume ratios? Is it same or different to length ratios?

4. Is there a relation between length ratios and volume ratios? [Hint: Useful to compare boxes for 12 cm and 24 cm paper squares? Why?]
**Activity 5: Pythagoras’ theorem**

**Materials:** Ruler, calculator, graph paper, pen

1. Unfold the box made from the 16 cm × 16 cm square. The folds should create a pattern as on right.

2. Can you identify a net in the diagonal lines that would fold into the box?

3. Can you use this net to state a way that the side length of the square determines the length, width, height and volume of the box?

4. Does this hold for the other squares?

5. Can you use Pythagoras’ theorem to find the length, width and height of the box for the 16 cm × 16 cm square? Use this to calculate the volume of the box.

6. How close was your calculation using ruler measurements to the Pythagoras calculation?

**Activity 6: Algebra challenge**

**Materials:** Pen and paper, calculator

1. Looking across your four boxes, can you find a rule for height of the box in terms of the length of the squared paper? Test your rule using the values in the table and the graph. State your rule as a formula.

2. Start with a box made from any length square of paper (say, L cm x L cm square piece of paper), Write a formula to calculate the volume of the box in terms of length L?

3. Calculate the length of square that would be needed to make a box that was 256 cm³?

4. Start with a box of volume V. Calculate the length (L) of square in terms of V needed to make this box.
### Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
</table>
| 5          | ACMNA107 | Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction | ACMMMG109  
Calculate the perimeter and area of rectangles using familiar metric units  
ACMMG111  
Connect three-dimensional objects with their nets and other two-dimensional representations |
| 6          | ACMNA123  
ACMNA133 | Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers  
Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence | ACMMMG138  
Connect volume and capacity and their units of measurement |
| 7          | ACMNA175 | Introduce the concept of variables as a way of representing numbers using letters | ACMMMG159  
Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving  
ACMMG160  
Calculate volumes of rectangular prisms  
ACMMG161  
Draw different views of prisms and solids formed from combinations of prisms |
| 8          | ACMNA183  
ACMNA188 | Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies  
Solve a range of problems involving rates and ratios, with and without digital technologies | ACMMMG198  
Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume |
<table>
<thead>
<tr>
<th>ACMNA208</th>
<th>Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACMMG216</td>
<td>Calculate the areas of composite shapes</td>
</tr>
<tr>
<td>ACMMG218</td>
<td>Solve problems involving the surface area and volume of right prisms</td>
</tr>
<tr>
<td>ACMMG222</td>
<td>Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles</td>
</tr>
</tbody>
</table>
**Task description**

All measurements are approximations—no measuring instrument can give perfect measurements. Are accuracy and precision in measurement the same? How do accuracy and precision affect results? That is what you will investigate in this task.


2. Use different measuring instruments to measure a fixed quantity. For example, you could measure the length of your classroom using a trundle wheel, a 30 cm ruler, a metre stick, the length of your pace, and a tape measure. What can you conclude about the accuracy and precision of each measuring instrument? What things might you consider when selecting the best measuring instrument for a particular purpose?

3. If a measuring instrument is inaccurate, we can show this by giving the measurement as a range, for example, between 85 and 95 cm, or 90 ± 5 cm. In this example, 5 cm is called the absolute error. Research relative errors and percentage errors, giving examples and an explanation of how they are calculated.

4. How do we find the measurement error for a particular measuring instrument? For example, explain the differences in the measurement error of a trundle wheel marked in 10 cm units, a tape measure marked in cm, and a ruler marked in mm. (Remember that an explanation includes what, how and why.)

5. How do errors in measurements affect the results of calculations using those measurements? Investigate and explain what happens to the measurement error when we (a) add or subtract and (b) multiply or divide two measurements? Does it matter if the measurement errors in the two measurements are different (for example if we add 90 ± 5 cm to 20 ± 0.5 cm)?

6. How does measurement error affect the number of significant figures used in the measurements? After researching significant figures, can you suggest a rule that we can use to decide how many significant figures should be used in a particular measurement?

7. **Challenge:** Indirect (or secondary) methods of measurement involve measuring something by measuring something else. Confused? Try some examples. We can measure the thickness of a piece of paper by measuring the thickness of a ream of paper and dividing by 500 (the number of pages in the ream). In aviation, altitude (height above sea level) is often calculated by measuring air pressure. Find some more examples of indirect methods of measurement. How do indirect methods of measurement affect measurement error?

*Make sure you understand the meanings of any words in italics.*
Essential vocabulary

**Accuracy**: The closeness of the measured value to the true value. For example, a clock is accurate only if it is set to the correct time. Accuracy errors can usually be corrected by recalibrating the measuring device (in the case of the clock example, resetting the clock to the correct time). A measurement that is not accurate is said to be **inaccurate**.

**Indirect methods of measurement**: Measuring something by measuring something else, for example, measuring the thickness of a piece of paper by measuring the thickness of a ream of paper and dividing by 500 (the number of pages in the ream).

**Measurement error**: The difference between the actual measurement and the measured value. It takes account of both accuracy and precision. Repeating the measurement will reduce the error caused by a lack of precision, but will not change errors caused by a lack of accuracy. A measurement error can be given in several ways: a **range** (for example, between 85 and 95 cm); an **absolute error** (for example, 90 ± 5 cm); a **relative error** (for example 90 cm ± 0.1), or a **percentage error** (for example 90 cm ± 10%).

**Measuring instrument**: A device for measuring a physical quantity, usually compared to an external, fixed standard such as those defined for the metric system.

**Precision**: The closeness of two or more measurements to each other. If something is measured several times, with the same result each time, then the measurement is very precise. A measurement that is not precise is said to be **imprecise**.

**Rounding (rounding off)**: A method of limiting the number of decimal places in a decimal fraction. Decimals can be rounded up (if the right-hand digit to be removed by the rounding process is at least 5) or rounded down (if the right-hand digit to be removed by the rounding process is less than 5). A decimal that has been rounded is an **approximation** of the exact value. For example, in Australia, the absence of one-cent coins means that cash payments must be rounded to the nearest five cents.

**Significant figures**: The number of digits in a number that are known with some degree of reliability. The number 13.2 is said to have three significant figures. The number 13.20 is said to have four significant figures. As exact values are considered to have an infinite number of significant figures, the number of significant figures used in a measurement indicates the size of the measurement error.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of measurement error and the rounding of decimals. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

Measurement error, and its implication for the number of significant figures used, is an important concept, especially when calculators are involved. Some students, when presented with six decimal places in a calculator display, will reproduce all these digits in their work, unaware of the accuracy in measurement implied by the use of a large number of decimal places. Consequently, students give (and teachers may require) solutions to mathematics problems that imply an unwarranted degree of accuracy. On the other hand, few students (and adults) appreciate that 1 means “at least 0.5 and less than 1.5” whereas 1.000 means “at least 0.9995 and less than 1.0005”. Too often the latter number is rounded to 1, with a consequent loss of important information.

Students should understand the accuracy of measurements that they use and be able to make informed judgements about the number of significant figures presented in solutions to mathematical and real-world problems. That is the purpose of this task. It also provides a good foundation for students who study the sciences, especially physics and chemistry.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. In this task, it is suggested that students should have attempted up to step 6 of the task before they are given the glossary.

The glossary could be used as the basis of supporting language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>ACMMG084 Use scaled instruments to measure and compare lengths, masses, capacities and temperatures</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA156 Round decimals to a specified number of decimal places</td>
<td>ACMMG135 Connect decimal representations to the metric system</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA157 Connect fractions, decimals and percentages and carry out simple conversions</td>
<td>ACMNA158 Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies</td>
</tr>
<tr>
<td></td>
<td>ACMNA183</td>
<td>ACMNA187</td>
</tr>
<tr>
<td>---</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td></td>
<td>Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies.</td>
<td>Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies.</td>
</tr>
</tbody>
</table>
8M2 “Designing a Kitchen”

Task description

You are a tradesperson working for Kitchens Galore, specialising in designing and remodelling kitchens. The kitchen is probably the most used room in the house, so it makes sense to spend time deciding the layout, storage areas and workbenches.

You have been asked to design a kitchen for the new home of Mr and Mrs Jones, who are building their second house, and to prepare a quote for the new kitchen. Mr and Mrs Jones are not happy with the kitchen design on the plan and want you to design an alternative. You are provided with the plan of the house that Mr and Mrs Jones intend to build and some information about kitchen layouts.

Mr and Mrs Jones’ requirements include:

- as much counter space as possible to create working space for cooking;
- as many cupboards as possible;
- a full-length pantry;
- space for a dishwasher; oven (width 600 mm, height 600 mm, depth 600 mm); and stove top;
- room for a refrigerator (it is your choice the dimensions of the refrigerator); and
- a Clarke two-bowl, single drainer flush-line sink – width 600 mm, depth 150 mm.

You are required to

1. Prepare a scale plan of the kitchen with all the kitchen appliances shown on the plan to scale. Redraw the kitchen floor from the floor plan given to you, using a scale of 1:20.

2. Provide Mr and Mrs Jones with calculations of the total area of bench space for food preparation and the total volume of cupboard space available.

3. Generate a detailed and accurate quotation for the total cost of a new kitchen for your clients Mr and Mrs Jones, listing all costs and including GST as a separate item. You should assume the following:

   - Cupboards cost (including labour) $178.50 per 1000 cm² of shelf area.
   - Under-bench cupboards (standard height of 1.1 m) have two shelves, whilst full-length pantries (standard height of 1.8 m) have five shelves. The standard depth of cupboards is 600 mm.
   - Your suppliers will offer you a trade discount of 5.2%.
   - The tiles Mr and Mrs Jones have chosen for their kitchen cost $35.50 per m², including laying. You need to allow for 5% wastage when laying the tiles. Tiles are required underneath the dishwasher and refrigerator but not underneath any cupboards.
   - Labour costs for installing the kitchen are $55.00 per hour for a fully qualified tradesperson and $26.00 per hour for an assistant. This job should take two days to complete (10 hours per day).
   - You will need to find the cost of each appliance from brochures or the internet (attach copies to your quote).
Additional student information

House plan
**Kitchen layouts**

The shape and design of your room dictates your kitchen layout. The layout of your kitchen needs to consider adequate work and storage space. Don’t forget to allow room for doors and windows, plus curtains and blinds.

The layout of your kitchen will be determined by both the physical properties of the space and your own preferences. Here we look at the advantages and disadvantages of the most common kitchen designs.

1. **U-shape kitchen**

   ![U-shape kitchen diagram]

   This versatile design suits both small and large kitchens. Its efficient shape prevents household traffic from moving through the kitchen and offers maximum bench and storage space. One of the legs of the U can be used as a breakfast bar.

   If used in a small space the U design may leave you cramped for space and feeling closed in. Time also needs to go into thoughtful storage planning so that hard-to-access corner cupboards are used effectively.

2. **L-shape kitchen**

   ![L-shape kitchen diagram]

   Best suited to narrow rooms, long rooms or open-plan living areas, the L-shaped kitchen frees up workspace and provides a good amount of storage below the countertop. The design also minimises walking distances between main working areas.

   While some storage may be lost at the corner of the cabinets, there are products on the market that help overcome this problem. This kitchen is often integrated with the meals area in open-plan designs.

3. **Single-line kitchen**

   ![Single-line kitchen diagram]

   Works well in small spaces and provides extra floor space. Most of the storage should be located overhead, while the appliances are situated under the bench top, leaving a large workspace for chopping and other kitchen activities. Position workstations and appliances so that work flows along it for efficiency and safety.

   This layout can be inefficient for serious cooks however, and if there is a doorway at either end of the kitchen family members will use it as a traffic area. It is advised to position the stove so it gets the least possible through-traffic.

4. **Two-way galley**

   ![Two-way galley diagram]

   This design is an efficient use of space, with cabinets down each side of the kitchen. Walking distance is minimal as cupboards are located behind you and in front of you.

   There needs to be enough space left between the two sides, so that you can cook effectively and so that people can walk around you. The two-way galley can suffer from the same problems with household traffic as the single-line kitchen.

5. **Island**

   ![Island diagram]

   This design combines any kitchen design with a separate workbench (the island) which can be used for food preparation. This works best in large areas and can be helpful when more than one person is cooking at any time.
**Teaching information**

This hands-on task draws on content from several different strands of mathematics. It develops the following skills: scale drawing; conversion of metric units; calculations of areas and volumes of complex shapes; percentages; and costings. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMNA106 Create simple financial plans</td>
<td>ACMMG109 Calculate the perimeter and area of rectangles using familiar metric units</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA132 Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies</td>
<td>ACMMG136 Convert between common metric units of length, mass and capacity ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA187 Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies</td>
<td>ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA208 Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems</td>
<td>ACMMG195 Choose appropriate units of measurement for area and volume and convert from one unit to another ACMMG198 Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA216 Calculate the areas of composite shapes</td>
<td>ACMMG216</td>
</tr>
</tbody>
</table>
8M3 “Crazy Bird Boxes”

Task description

A bird box is a man-made enclosure for wild birds to nest in. Placing bird boxes in the environment can help to maintain and build populations of particular bird species in an area.

Bird box design

A well-designed bird box has the following features:

- an overhanging, sloped roof to keep out the wind and rain
- a floor, with drainage holes
- an entrance hole for the birds
- ventilation holes
- a way for humans to access the interior for cleaning
- no outside perches which could assist predators.

The size of the bird box affects the bird species likely to use the box. Very small boxes attract sparrows and very large ones may attract cockatoos. The entrance hole for the bird must be large enough to allow the bird to come and go, but too small to admit predators (many small birds select boxes with a hole only just large enough for an adult bird to pass through). Bird boxes are usually designed for a particular species of bird.

Some bird boxes can be highly decorated and complex, sometimes mimicking human houses or other structures.

Crazy structures

Our modern world is composed of right angles. Occasionally we choose to create interest by doing something different. Examples of structures that do not rely on right angles include crazy paving (see Figure 2) and the new building at the University of Technology in Sydney (see Figure 3).

Your task

Your task is to research, design and cost a crazy bird box (that is, a box that has no right angles) suitable for a species of bird that lives in your area. Your submission must include the following:

- a description of the species of bird that your box is intended to attract and how that affects the size of the box and its opening;
- scale drawings of your bird box, from several perspectives;
- calculation of the floor area and volume of your bird box;
- calculation of the quantities of materials (for example, timber, hinges, glue, screws, paint) that you must purchase to complete the bird box (think about the way that timber is sold and how small items are sold in packets); and
- detailed costings of all of the materials needed to build your bird box.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of areas and volumes of irregular shapes. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

Students’ designs must be a suitable size for the bird species selected. They must show the supporting calculations for the floor area and volume. Students should be encouraged to make use of familiar shapes, such as rhombuses, parallelograms, trapeziums and polygons. Curved surfaces should be discouraged as they are likely to require calculations that are too complex for students in Years 7 to 9. The scale drawings must include sufficient detail for another person to be able to construct the bird box. The calculation of required building materials must allow for the fact that timber is purchased by length and often has a minimum length that must be purchased, with all lengths being multiples of 300 mm, screws come in packets, etc. Students could be encouraged to visit a hardware shop (in their own time) or its website.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>ACMCG108 Choose appropriate units of measurement for length, area, volume, capacity and mass</td>
</tr>
<tr>
<td>7</td>
<td>ACMCG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td></td>
<td>ACMCG160 Calculate volumes of rectangular prisms</td>
</tr>
<tr>
<td></td>
<td>ACMCG161 Draw different views of prisms and solids formed from combinations of prisms</td>
</tr>
<tr>
<td>8</td>
<td>ACMCG195 Choose appropriate units of measurement for area and volume and convert from one unit to another</td>
</tr>
<tr>
<td></td>
<td>ACMCG196 Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites</td>
</tr>
<tr>
<td></td>
<td>ACMCG198 Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume</td>
</tr>
<tr>
<td>9</td>
<td>ACMCG216 Calculate the areas of composite shapes</td>
</tr>
<tr>
<td></td>
<td>ACMCG221 Solve problems using ratio and scale factors in similar figures</td>
</tr>
<tr>
<td>10</td>
<td>ACMCG242 Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids</td>
</tr>
</tbody>
</table>
8N1 “Diminishing Fractions”

Task description

The A series of cut paper sizes (also known as the international paper sizes) are now in common use throughout the world apart from North America. In Europe the A paper sizes were adopted as the formal standard in the mid 20th century and from there they spread across the globe. The A4 paper size is now the standard size of business letters in English-speaking countries such as Australia, New Zealand and the UK that formerly used British Imperial sizes (which had interesting names such as Duchess, Duke, Foolscap, Quarto, and Octavo).

1. Examine the diagram showing the international paper sizes arranged in a square.
   
   (a) If A4 paper is 201 mm × 297 mm, what are the dimensions of A0 paper?
   
   (b) Prepare a table showing the dimensions, perimeter and area of each sheet size from A0 to A12. Examine the relationship between the perimeters and areas as the paper size decreases. How are they similar and how are they different?
   
   (c) What fraction is A1 of A0? What fraction is A2 of A0? What happens as you extend this pattern? What about A10? A15? A100? Is there a way of working out the fraction of any size in relation to A0?
   
   (d) What fraction is A7 of A2? What about A9 of A3? A20 of A10? Is there a way of working out any size in relation to any other size of paper?

2. Examine the diminishing patterns. The overall shape in each case is a square.

   (a) In pattern (a) what fraction of the whole square is the first darker rectangle? What about the first two darker rectangles? The first three? What happens when you keep going?

   (b) Now try the same approach with patterns (b) and (c).

Challenge 1: What is the aspect ratio of A0, A1, A2, ... sized sheets of paper (the aspect ratio is short side: long side)?

Challenge 2: Research the Fibonacci sequence. What is the ratio of the first number to the second? What about the second number to the third? What happens as you keep going?
Additional student information

*International paper sizes*

![Diagram of international paper sizes]

*Diminishing patterns*

(a)  ![Diagram of diminishing patterns]
Teaching information

This task, with some teacher involvement, provides students with the opportunity to examine the patterns associated with diminishing sequences. It has been adapted from some ideas in a resource written by Nyima Drayang (see www.tesaustralia.com/Download.aspx?storycode=6303904&type=X&id=6683862). It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice.

The first task provides students with a practical opportunity to use fractions, to explore the relationships between areas and perimeters, and to extrapolate sequences. The second task builds on the approach of the first, but requires students to examine the relationship between diminishing areas. Each shape allows students to discover that infinite diminishing series converge to a finite sum, and to estimate the magnitude of that sum [$\frac{2}{3}$ in the case of (a) and (b) and $\frac{1}{2}$ in the case of (c)]. While not formally part of the task, it also provides the opportunity to discuss the concept of similarity.

In Challenge 1 students should find that the aspect ratio of paper sized in the A0, A1, A2, ... system is $1: \sqrt{2}$. This ratio guarantees that cutting a sheet in half along a line parallel to its short side results in the smaller sheets having the same ratio as the original sheet. Students will require an understanding of Pythagoras’ Theorem to complete this challenge.

In Challenge 2 students examine similar issues in the context of the Fibonacci sequence. The first two terms in the sequence are 1 and 1. Thereafter, each term is the sum of the previous two terms, so the sequence becomes 1, 1, 2, 3, 5, 8, 13, ... . The ratio between each term forms a diminishing sequence that converges to 1.6180339887... (the approximation of the irrational number known as phi [$\phi$] with an exact value of $\frac{1+\sqrt{5}}{2}$). The ratio is known as the Golden Ratio. Interested students can research the rich array of information about this ratio.
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMNA107 Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction</td>
<td>ACMMG109 Calculate the perimeter and area of rectangles using familiar metric units</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA126 Solve problems involving addition and subtraction of fractions with the same or related denominators</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACMNA133 Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>ACMNA153 Solve problems involving addition and subtraction of fractions, including those with unrelated denominators</td>
<td>ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA183 Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACMNA188 Solve a range of problems involving rates and ratios, with and without digital technologies</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>ACMMG222 Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles</td>
</tr>
</tbody>
</table>
8N2 “Challenging Fractions”

Task description

Fractions are an important part of mathematics. The ability to manipulate and calculate with fractions, using both manual and digital methods, is essential for success in the higher levels of mathematics. It is also important to understand the strengths and limitations of both common fractions and decimal fractions, in order to decide which method will be most efficient in a particular situation. This task involves several activities designed to make you think deeply about fractions and improve your understanding of both common and decimal fractions.

There is an “Additional student information” handout to accompany this task that gives more detail about some of these activities.

1. There are four ways of thinking of common fractions: part of a whole, a quotient, a ratio, and a measurement. Explain each method (recall that “explain” means what, how and why), giving examples. How do the different approaches relate to each other?

2. The language in English that we use to describe common fractions is complicated. How would you explain this language to a student in Year 5?

3. What are equivalent fractions? Explain how and why they are important to the idea of simplifying common fractions and the methods that we use to add, subtract, multiply and divide with common fractions.

4. To divide common fractions, we multiply by the reciprocal. What is a reciprocal? Explain why this method of division works.

5. When we multiply by a whole number, the result is a larger number, and when we divide by a whole number, the result is a smaller number. This does not always occur when we multiply and divide by fractions. Explain why. Is there a general rule that we can use to explain whether multiplying and dividing increases or decreases the original value?

6. Common fractions cannot a have denominator of zero, but can have a numerator of zero. Why is there a difference?

7. How do we find the square and the square root of a common fraction?

8. A compound fraction occurs when we have a fraction inside another fraction. How can we simplify compound fractions?

9. When is it easier to use common fractions compared to decimal fractions? Give examples.

10. Challenge 1: How can you use your knowledge of common fractions to solve simple equations that include common fractions?

11. Challenge 2: Read the examples of how to convert a repeating decimal to a common fraction. Make up some examples of decimal fractions that repeat after two, three and four places. Convert them to common fractions without using a calculator.
Additional student information

Activity 1

There are four ways of thinking of common fractions: part of a whole, a quotient, a ratio, and a measurement. Explain each method (recall that “explain” means what, how and why), giving examples. How do the different approaches relate to each other?

A fraction is part of a whole:

\[ \frac{3}{5} = \] [Diagram of five equal parts with three shaded]

A fraction can be written as a quotient:

\[ \frac{3}{5} = 3 \div 5 \]

A ratio can be presented as a fraction:

\[ \frac{3}{5} = 3 \text{ parts out of } 5: \]

A fraction can be presented as a measurement:

[Measurement diagram]

Activity 2

The language in English that we use to describe common fractions is complicated. How would you explain this language to a student in Year 5?

The numerator is expressed as a cardinal number, e.g. “three”, “seven”, “twenty-four”.

The denominator is expressed as an ordinal number, e.g. “thirds”, “sevenths”, “twenty-fourths”. But there are exceptions: we say “half” instead of “second” and “quarter” as an alternative to “fourth”.

However, we can also talk about a fraction as the numerator over the denominator, e.g. “four over five” or “four on five”.

Activity 4

To divide common fractions, we multiply by the reciprocal. What is a reciprocal? Explain why this method of division works.

For example, \( \frac{1}{5} \div 2 = \frac{1}{5} \times \frac{1}{2} \)
and \( \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} \)

Activity 5

When we multiply by a whole number, the result is a larger number, and when we divide by a whole number, the result is a smaller number. This does not always occur when we multiply and divide by fractions. Explain why. Is there a general rule that we can use to explain whether multiplying and dividing increases or decreases the original value?

For example, \( 2 \times 4 = 8 \) and \( 2 \div 4 = \frac{1}{2} \)
but \( 2 \times \frac{1}{4} = \frac{1}{2} \) and \( 2 \div \frac{1}{4} = 8 \)

What happens if the fractions are negative?
**Activity 6**

Common fractions cannot have a denominator of zero, but can have a numerator of 0. Why is there a difference?

Enter $\frac{0}{3}$ into your calculator using the fraction key. What do you see? Compare it to the result when you enter $0 ÷ 3$.

Now enter $\frac{3}{0}$ into your calculator using the fraction key. What do you see? Compare it to the result when you enter $3 ÷ 0$.

**Activity 7**

How do we find the square and the square root of a common fraction?

For example, $(\frac{2}{3})^2$ and $\sqrt{\frac{9}{16}}$.

**Activity 8**

A compound fraction occurs when we have a fraction inside another fraction. How can we simplify compound fractions?

For example, \( \frac{2}{3}, \frac{2}{5}, \frac{2}{3}, \frac{2}{4} \).

**Activity 10**

(Challenge 1) How can you use your knowledge of common fractions to solve simple equations that include common fractions?

For example:

\[
\begin{align*}
\frac{3x}{5} &= 12 \\
\frac{2x}{5} &= \frac{3x}{8} \\
\frac{4}{5}x &= \frac{3 - x}{2} \\
\frac{2x + 1}{3} - 2 &= \frac{3x - 1}{7}
\end{align*}
\]
Activity 11

(Challenge 2) Read the examples below of how to convert a repeating decimal to a common fraction. Make up some examples of decimal fractions that repeat after two, three and four places. Convert them to common fractions without using a calculator.

Convert $0.\overline{5}$ to a common fraction:

Let $x = 0.5555555...$
Then $10x = 5.5555555...$

Subtract the first equation from the second:

$10x - x = 5.5555555... - 0.5555555...$

$9x = 5$

$x = \frac{5}{9}$

So, $0.\overline{5} = \frac{5}{9}$

Now convert $3.\overline{471}$ to a common fraction:

Let $x = 3.471471471...$
Then $1000x = 3471.471471471...$

Subtract the first equation from the second:

$1000x - x = 3471.471471471... - 3.471471471...$

$999x = 3468$

$x = \frac{3468}{999}$

$= 3 \frac{471}{999}$

So, $0.3.\overline{471} = 3 \frac{471}{999}$

Enter $0.5555555...$ and $3.471471471...$ into your calculator and convert it to a decimal fraction. What results do you see?
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of fractions and decimals. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice about this task. The “Additional student information” handout provides further information.

The task is designed for students who have mastered the basic operations with fractions, both common and decimal. It gives them an opportunity to develop a deeper understanding of fractions and to link their knowledge to the overall structure of mathematics.

The challenge activities require an understanding of algebra, particularly solving linear equations.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA152 Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line</td>
</tr>
<tr>
<td></td>
<td>ACMNA153 Solve problems involving addition and subtraction of fractions, including those with unrelated denominators</td>
</tr>
<tr>
<td></td>
<td>ACMNA154 Multiply and divide fractions and decimals using efficient written strategies and digital technologies</td>
</tr>
<tr>
<td></td>
<td>ACMNA155 Express one quantity as a fraction of another, with and without the use of digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA184 Investigate terminating and recurring decimals</td>
</tr>
<tr>
<td></td>
<td>ACMNA194 Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution</td>
</tr>
</tbody>
</table>
Task description

1. Examine Square Dissection 1 in the “Additional student information” handout. Piece A is one eighth or 12.5% of the entire square. Why? What percentage of the whole square are the other pieces? How do you know? Summarise your answer in a table with the following column headings: Piece, Fraction, Percentage, Reasons, and a row for each piece. Include the information for Piece A in the table. What should the Fraction column add up to? What should the Percentage column add up to?

2. Repeat Step 1 with the other square dissections in the handout. In Square Dissection 3, also calculate the percentage of the whole square taken up by the star.

3. Make up your own square dissection for your partner to complete.

4. **Challenge:** Examine the design of the Greek flag in the handout. What percentage of the flag is blue? Would using a fraction instead of a percentage give a better answer? Why?

*Make sure you understand the meanings of any words in italics.*
Additional student information

Square Dissection 1

Square Dissection 2

Square Dissection 3

Square Dissection 4 (the curves are circular arcs)

Square Dissection 5
Greek flag

The shaded sections represent the blue parts of the Greek flag. The length and height of the Greek flag are in the ratio 3:2. All stripes are equally wide.

Essential vocabulary

Arc: A curved section of a line.

Circular arc: Part of the circumference of a circle.

Dissect: To cut up, or divide into sections (do not confuse with bisect).

Dissection: The result of cutting up or dividing into sections.

Per cent (or percent): The number of parts per hundred (per means out of or divided by, cent means hundred); a ratio where the second term is 100. It may also be written as one word: percent. It is used to follow a number, in place of a unit of measurement, for example, six per cent. Represented by the symbol %.

Percentage: A portion or share, measured in parts per hundred. It is generally used without an associated number, for example, “the percentage of boys is higher ...”.

Ratio: A comparison of quantities of the same kind, for example, the ratio of girls to boys is 3:4. Since the quantities are of the same kind, there is no need to include units with a ratio.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides a hands-on activity, suitable for Year 7 or 8 students, based on fractions and percentages, but also draws on a knowledge of geometry, areas (squares, rectangles, triangles and circles) and the skill of logical deduction. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students.

Students should be able to calculate the fractions of the squares through their knowledge of areas and geometry and logical deduction. Estimating the areas by counting the number of squares covered by the piece should be used as a last resort and only by those students incapable of using more precise methods. In some cases, student will need to choose carefully which piece of the square to start with – they do not have to deal with the pieces in alphabetic order. In some cases the fractions and percentage will be complex. However, it is suggested that manual calculations with the fractions should be encouraged where possible as it will provide a useful review of students’ understanding of fractions. Students should recognise without prompting the change in the size of the grid in Square Dissection 5.

The Challenge activity is based on a question in a past Queensland Core Skills Test. Students may need to investigate the meaning of a ratio.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
### Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>ACMMG088 Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>ACMMG109 Calculate the perimeter and area of rectangles using familiar metric units</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA123 Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>ACMNA157 Connect fractions, decimals and percentages and carry out simple conversions</td>
<td>ACMMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td></td>
<td>ACMNA158 Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>ACMNA184 Investigate terminating and recurring decimals</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACMNA187 Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>ACMMG216 Calculate the areas of composite shapes</td>
</tr>
</tbody>
</table>
8N4 “Consecutive Sums”

Task description

Can 15 be expressed as the sum of two or more consecutive whole numbers?

Numbers are *consecutive* if each number in the sequence is one more than the previous number. For example, the numbers 2, 3, 4 are consecutive, the numbers 8, 9, 10, 11 are consecutive, and the numbers 23 and 24 are consecutive. On the other hand, the numbers 6, 8, and 10 are not consecutive because each number is two more than the previous number. A single number by itself is not considered to be consecutive.

A consecutive sum is a sum of a sequence of consecutive numbers: \(2 + 3 + 4 = 9\) or \(8 + 9 + 10 + 11 = 38\) or \(23 + 24 = 47\). Can 9 be expressed as the sum of any other sequence of consecutive numbers? Can 38? What about 47?

Your task is to explore the idea of consecutive sums. Work with your group to try to find patterns and make generalisations.

Find ways for your group to work together to investigate this. You might want to start with a certain list of numbers, say 1 to 36, and find all the ways that those numbers can be expressed as consecutive sums. Are there some numbers that are easy to do? Why? Are there any that are impossible?

Your group should prepare a display of some kind on a large sheet of paper. It should show your work and your results. It should also include conjectures (statements about the patterns that you notice that you think might be true) as well as patterns that you are certain are true. Include your justifications for why you are certain.

Also, if you make a conjecture and then discover that it is false, include this conjecture and provide the evidence that later convinced you that it was false.

**Challenge**

Can you find a way to explore the patterns of consecutive sums algebraically, that is, without using a guess and check approach?
Teaching information

This task, with some teacher involvement, provides Year 7 and 8 students with the opportunity to investigate patterns in the natural numbers. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice.

The task requires students to add several one- and two-digit numbers. It is suggested that most students should not need a calculator for the task, but it is a teacher decision.

If students are investigating the counting numbers up to 36, they could be asked how many consecutive numbers they need to consider. They would need to start with two consecutive numbers, but, clearly, there is no need to try 36 consecutive numbers. 35 would also be too high (why)? How low can they go? The answer is eight (because $36 = 1 + 2 + 3 + \ldots + 8$).

Some students should be capable of the challenge task where they are asked to model the task algebraically. For example, if we consider that the consecutive numbers are $n$, $n+1$, $n+2$, … then the numbers that are the sums of three consecutive numbers could be modelled as follows:

$$N = n + (n+1) + (n+2)$$

leading to the conclusion that only multiples of three can be expressed as a sum of three consecutive numbers. To find the three consecutive numbers that add to give 15, we can solve the equation

$$15 = 3n + 3$$

$$3n = 12$$

$$n = 4$$

showing that the three consecutive numbers would be 4, 5 and 6.

Similarly, the numbers that are the sums of four consecutive numbers could be modelled as follows:

$$N = n + (n+1) + (n+2) + (n+3)$$

$$= 4n + 6$$

The numbers that can be written as the sum of four consecutive numbers clearly must be even numbers: $10 = 1 + 2 + 3 + 4$ (when $n = 1$), $14 = 2 + 3 + 4 + 5$ (when $n = 2$), $18 = 3 + 4 + 5 + 6$, $22 = 4 + 5 + 6 + 7$, etc.

If the whole numbers that can be expressed as a sum of three or more consecutives can be modelled as $3n + 3$ (for three consecutive numbers), $4n + 6$ (four consecutives), $5n + 15$ (five), $6n + 21$ (six), $7n + 28$ (seven), and so on, all of which are clearly not prime numbers, students might conjecture that prime numbers cannot be expressed as a sum of three or more consecutives. How could they prove/disprove this?
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>ACMNA071 Investigate and use the properties of odd and even numbers</td>
</tr>
<tr>
<td>5</td>
<td>ACMNA098 Identify and describe factors and multiples of whole numbers and use them to solve problems</td>
</tr>
<tr>
<td></td>
<td>ACMNA107 Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction</td>
</tr>
<tr>
<td>6</td>
<td>ACMNA122 Identify and describe properties of prime, composite, square and triangular numbers</td>
</tr>
<tr>
<td></td>
<td>ACMNA123 Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers</td>
</tr>
<tr>
<td>7</td>
<td>ACMNA175 Introduce the concept of variables as a way of representing numbers using letters</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA192 Simplify algebraic expressions involving the four operations</td>
</tr>
</tbody>
</table>
8P1 “The Lucky Prince”

Task description

*Probability* is an important part of our world. We use it to decide on risk (for example, which is the best investment for our retirement). It determines our chances of winning Lotto and is a big part of many games – helping us to find the best strategy. This challenge requires you to use probability to find the best strategy in two puzzles.

**Lucky Prince 1:** The prince climbed the wall to meet the emperor’s daughter. He was captured. The emperor gave him a chance. “Here are 50 black balls and 50 white balls”, he said to the prince. “Put them any way you like into these three identical urns. In the morning the urns will be rearranged; you can pick an urn and then pick a ball from that urn”, continued the emperor. “If it is white, you live and can marry the princess. If black, you die.” The prince was clever and lucky. He picked white, lived and married the princess. How did he put the balls in the urns to improve his chances of living? What is the highest probability that the prince could set up?

**Lucky Prince 2:** In another version of this story, the emperor puts the prince in front of three doors – he has to choose a door. Behind two are tigers, behind one is the princess. The emperor says: “You choose a door and then I’ll show you one of the other doors that has a tiger. Then you can stick with your choice or change to the other door.”

Your task is to find, in each case, the strategy with the highest probability of the prince finding his princess. You then should be able to justify your strategy mathematically (in other words, use probability to explain why your strategy is best).

*Make sure you understand the meanings of any words in italics.*
Essential vocabulary

Certain: An outcome is certain when it is the only possibility. In these cases, the probability of the outcome is said to be one.

Equally likely: Outcomes are equally likely when they have the same probability of occurring. In the cases of two outcomes, the probability of each outcome is said to be 0.5. Synonyms are: even chance, fifty-fifty.

Event: An event is one or more outcomes of an experiment.

Experiment: In probability, an experiment is a process involving chance that leads to results called outcomes. It can have one or more steps (trials). An example would be tossing a coin ten times.

Impossible: An outcome is impossible when it cannot occur in any circumstances. In these cases, the probability of the outcome is said to be zero.

Likely: An outcome is likely when it is expected to occur more often than not. In these cases, the probability of the outcome is said to be greater than 0.5 but less than one. Synonyms are: odds on, probable, good chance.

Outcome: An outcome is the result of a single trial of a probability experiment.

Probability: A measure of how likely an event is. While a probability can be described using words such as certain, likely, and impossible, it is measured by assigning a value between 0 (impossible) and 1 (certain). Probabilities can also be expressed as percentages. Synonyms: chance, likelihood.

Sample space: A listing of all of the possible outcomes of an experiment.

Trial: A single stage of a probability experiment. An experiment can consist of one or more trials. For example, in an experiment of tossing a coin ten times, a trial would be tossing the coin once.

Unlikely: An outcome is unlikely when it is not expected to occur in the majority of situations. In these cases, the probability of the outcome is said to be greater than zero, but less than 0.5. Synonyms are: long odds, improbable, poor chance.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of the probability of two-step events. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides an explanation of the solutions to the two challenges, some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

In the three-urn problem, the trick is to set up two of the urns with the greatest chance of selecting a white ball (call these Urns 1 and 2). This can be done by placing only one white ball and no black balls in each of Urns 1 and 2. Urn 3 will have the rest of the balls: 48 white and 50 black – a total of 98 balls. After the urns have been shuffled about, the prince will select either Urn 1 or Urn 2 two-thirds of the time, and thus be certain of selecting a white ball. The remaining third of the time, he will select Urn 3 and, even then, he will have a probability of \( \frac{48}{98} \) (nearly half) of selecting a white ball. The mathematics would look like this:

\[
P(\text{white}) = \left( \frac{1}{3} \times \frac{1}{1} \right) + \left( \frac{1}{3} \times \frac{1}{1} \right) + \left( \frac{1}{3} \times \frac{48}{98} \right) = 0.83 \text{ (approximately)}
\]

In the three-door problem, the prince should always change to the other door. That will result in a probability of \( \frac{2}{3} \) of selecting the princess. The solution can be thought of this way:

- One third of the time the prince will first select the door with the princess (call his first choice Door 1). Then there will be tigers behind the other two doors. If he is shown one of the tigers (call this Door 2) and he switches to Door 3, he will find the other tiger.

- Two thirds of the time the prince will first select a door with a tiger (call his first choice Door 1). Then the princess will be behind one of the other two doors and the second tiger behind the other. If he is shown where the second tiger is (call this Door 2) and he switches to Door 3, then he will find the princess.

To summarise, one third of the time it will be wrong to change from Door 1 to Door 3. But two thirds of the time changing from Door 1 to Door 3 will be correct. So the strategy of always switching to the other door will result in a probability of \( \frac{2}{3} \) of finding the princess. The mathematics would look like this:

\[
P(\text{Princess}) = \left( \frac{1}{3} \times \frac{0}{1} \right) + \left( \frac{1}{3} \times \frac{1}{1} \right) + \left( \frac{1}{3} \times \frac{1}{1} \right) = \frac{2}{3}
\]

In this task students will encounter some essential mathematical words that they should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
**Activity 1: Getting up to speed on probability**

**Materials:** Bags, coloured counters (red and blue)

1. Make up the following bags of counters:

![Images of bags A, B, and C with red and blue counters]

What is the chance of getting a red counter (after shaking and selecting) from the bags above? Give probability as a fraction and as a %.

(a) Bag A ________ ________%
(b) Bag B ________ ________%
(c) Bag C ________ ________%

2. For each bag, in turn, select and replace counters (ensuring bags are shaken between selections) 20 times. Record results in a table as follows.

<table>
<thead>
<tr>
<th>Bag A</th>
<th>Bag B</th>
<th>Bag C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 R</td>
<td>1 B</td>
<td>1</td>
</tr>
<tr>
<td>2 B</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3 etc.</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total red</th>
<th>Total red</th>
<th>Total red</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction red /20</td>
<td>Fraction red /20</td>
<td>Fraction red /20</td>
</tr>
<tr>
<td>% red</td>
<td>% red</td>
<td>% red</td>
</tr>
</tbody>
</table>

3. (a) Are your answers for questions 1 and 2 the same – are they close?

(b) Should they be different?

(c) Why could they be different?
**Activity 2: Reversing the direction**

**Materials:** Balls (or counters) of 3 different colours, bags

1. Put 6 balls in a bag (3 different coloured balls – red, blue, yellow). Shaking the bag before each selection, select and replace a ball 20 times. Record results in a table as follows.

<table>
<thead>
<tr>
<th>Draw</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball</td>
<td>R</td>
<td>B</td>
<td>etc.</td>
<td></td>
<td></td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total</th>
<th>R</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use the above to determine how many of each colour in the bag. (Hint: what % chance of getting one ball out of 6?)

3. Make up the following bags:

Which bag is most likely to give you a red?

4. Test your answer by selecting from each bag (after shaking and with replacement) 20 times. Record your results in a table as follows.

<table>
<thead>
<tr>
<th>Bag P</th>
<th>Bag Q</th>
<th>Bag R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>Tally</td>
<td>Colour</td>
</tr>
<tr>
<td>R</td>
<td></td>
<td>R</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

(a) Did the bag you picked give the most reds?

(b) Should it have?

(c) Are there reasons why it might not?

Questions 3 and 4 above represent the reverse of Activity 1 – they are “finding the situation most likely to give desired outcome” not “finding the most likely outcome”.
Activity 3: Two-step probability

Materials: Balls (or counters) of 3 different colours, bags

1. Make up the following two bags (identical bags) of counters.

What is the chance of getting a red counter (after shaking and selecting) from each of the bags above? Give probability as a fraction and as a %.

(a) Bag X ___________ %
(b) Bag Y ___________ %

2. If we have to choose a bag and then select a counter from that bag (with the bags being interchanged and shaken before each selection), what are the probabilities:

(a) Selecting Bag X or Bag Y ___________ ___________%
(b) Selecting a red counter overall ___________ ___________%

Hint: consider the two-step process as a tree diagram as on right.

3. Set up a process whereby someone selects a bag and chooses a counter. Repeat 20 times. Record results in a table as follows.

<table>
<thead>
<tr>
<th>Draw</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball/Counter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td></td>
</tr>
</tbody>
</table>

(a) What probability did you get? _______. Is this the same as your probability from 2(b)? Yes/No

(b) How could you increase the probability of a red by redistributing the colours in the bags?

4. What is the probability of a red if you have to choose one of the bags below and then choose a counter?

(a) What is the probability of a red? ___________ = ___________ %

(b) What is the probability of a yellow? ___________ = ___________ %

(c) How could you increase the probability of a red by redistributing the colours in the bags?
Activity 4: Lucky prince 1

Now try to work out how the lucky prince distributed his black and white balls across the three urns. Remember he is distributing the balls so he is most likely to get his desired outcome by picking any urn and any ball from the chosen urn. Also work out to what he raised his probability of getting a white ball.

1. To help, try this easier problem:

5 black and 5 white counters, 2 bags (identical)

You have to put the counters in the bags so that when you choose a bag and pick a counter from that bag you have the most chance of getting a white ball.

(a) How would you distribute the balls?

(b) What is the best probability?

Hint: Use a tree diagram to help.

2. Trial a few distributions of the counters in the two bags. Take 20 selections and replacements and see which of the distributions gives you the most chance. Record your results in a table as follows:

<table>
<thead>
<tr>
<th>Distribution</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total white/20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Is the trial giving the same result as 1(b)?

(b) Why or why not?

3. Think you have it for the easier problem? Now solve the original problem.

(a) How did the prince distribute the 50 white balls and 50 black balls to have the best chance to get a white?

(b) What chance of getting a white did the lucky prince set up? ___________ = __________%
**Activity 5: Lucky prince 2 – challenge**

1. In another version of this story, the emperor puts the prince in front of three doors – he has to choose a door. Behind two are tigers, behind one is the princess. The emperor says: “You choose a door and then I’ll show you one of the other doors that has a tiger. Then you can stick with your choice or change to the other door.”

   The prince was clever and lucky. He chose to change his choice to the other door and he survived and married the princess.

   The prince chose his door (call this Door 1) and the emperor showed a door with a tiger behind it (call this Door 2). The prince changed his choice to the other door (Door 3).

   ![Diagram showing three doors: Door 1, Door 2, and Door 3. Door 1 is the first chosen by the prince, Door 2 is the doorway shown by the emperor to have a tiger, and Door 3 is the other door.]

   (a) Should the prince have changed his choice to Door 3?

   (b) Does changing make the chance of picking the princess higher?

   (c) Why or why not?

2. Set up the situation – three doors, one princess, two tigers. This could be done by three upside down foam cups with a small counter under one to show where the princess is – the tigers can be the empty cups. Student A sets up the cups and student B chooses.

   When student B chooses a door, student A points to the cup with a tiger. Student B now has to stick with his/her original selected door or change. Run the activity 10 times with student B changing and 10 times with student B not changing. Record your results in a table as follows:

<table>
<thead>
<tr>
<th>Changing</th>
<th>Draw</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>10</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Princess</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not changing</td>
<td>Draw</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Princess</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   (a) Probability of picking the princess with change: _________/10

   (b) Probability of picking the princess with no change: _________/10

   (c) What appears to be the better strategy?

3. (a) Construct an argument of why it is better or not better to change.

   (b) What is the probability of getting the princess with no change?

   (b) What is the probability of getting the princess with change?
### Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP168 Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP204 Identify complementary events and use the sum of probabilities to solve problems</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP225 List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events</td>
</tr>
</tbody>
</table>
8P2 “And Not Or”

Task description

In this task we will explore attributes and the way they can be combined. We go on to examine how they apply to probability theory.

1. Use the domino and train games described in the handout “Additional student information” to become familiar with attribute cards.

2. Use the hoop games described in the handout to become familiar with Venn diagrams.

3. Use hoop activities to explore the mathematics meanings of the words “and”, “or”, and “not”.

4. Consider the applications of these concepts to probability, by completing Problems 1 to 4 in the handout.

5. Compose some probability problems relying on the use of Venn diagrams, tree diagrams, probability tree diagrams, and the words “and”, “or”, and “not” for others in your group. Complete the problems composed by the others in your group. Use appropriate mathematical notation when writing and completing the problems.

6. Challenge: Explain the logic behind the probability rule:

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Make sure you understand the meanings of any words in italics.
Additional student information

Attribute cards

Your teacher should have given you a set of 60 attribute cards. Look at them to identify the four attributes that can change from card to card. How many variations are there in each attribute?

Domino games

To become familiar with the attribute cards, try some domino games. Put all the cards into a pile face down and deal five cards to each player. Place the remaining cards in a pile face down on the table and turn one card over. Players take turns to place a card face up on the table following the rules. Players who cannot place a card have to take another card from the pile. The player who first uses all of their cards is the winner. There are a variety of games.

Domino Game 1: Make a line. Players have to place their card at either end of a line, starting to the left or right of the first card, so that each card differs in one attribute only from the previously placed card.

Domino Game 2: Make a cross. Players have to place their card at either end of two lines that cross where the first card was placed. Card placement is one attribute different for one line (as in the previous game) and two attributes different for the other line.

Domino Game 3: Three attributes. Players place their cards similar to Game 1 but the placed card has to be three attributes different to the previous card.

Domino Game 4: Matrix dominoes. This is similar to Game 2, but is played on a 10 × 6 grid. A card is placed near the centre. Then players place cards so that the card is one attribute different left to right and two attributes different up and down.
**Domino Game 5: Dominoes solitaire.** This is a one-player game. A card is placed in the centre of a large board and then other cards are placed so that they are one attribute different along the lines of a network. The objective is to place all 60 cards.

![Diagram of network]

**Trains**

Train logic cards have tracks as on right below, and small cards on which attributes are written – the train logic cards say which attribute card goes on which track. Arrange some cards in the top row – the cards that you want will be delivered at the top row, the “not” cards will be delivered at the bottom row.

For example, “red and large” is as follows:

```
All cards   red   large  red and large” cards
           “not red and large” cards
```

While “red or large” is as follows:

```
All cards   red   large  red or large” cards
           “not red or large” cards
```

**Venn diagram games**

In probability, it is important to be able to identify attributes when there are more than one involved (e.g. the difference between “even and a multiple of 7” and “even or a multiple of 7”). One way to do this is with attribute cards and hoops. If hoops are not available, then intersecting circles could be drawn on a sheet of butcher’s paper.

**Hoop Game 1: Two way.** Place two hoops so that they intersect (overlap). Label each hoop (see the diagram to the right) and then place cards inside or outside the regions marked by the hoops according to their intersecting attributes.

To play this as a game, put out the hoops and label them (change the labels for each new game). Divide all sixty attribute cards amongst the players, with each player taking a turn to place a card. Players score 1 point for a hoop and 2 points for an intersection of the two hoops. There are no points for cards placed outside both hoops. The highest score wins.
Hoop Game 2: Three way. Put out three hoops and label them (see the diagram to the right). Divide all sixty attribute cards amongst the players, with each player taking a turn to place a card. Players score 1 point for a hoop, 2 points for an intersection of 2 hoops and 3 points for an intersection of 3 hoops. There are no points for cards placed outside all three hoops. The highest score wins.

Hoop Game 3: Whysy. Play the games above but, to score a point, students have to correctly explain why they put the card where they do.

Hoop Game 4: Notty. Plays games 1, 2 and 3 but include the word “not” in the label of each hoop (for example, “not red”, “not large”, and “not circle”). You could start with only one “not” label in one hoop.

Logical connectives “not”, “and”, “or”, “if … then”

The logical connectives are “not”, “and”, “or”, and “if … then”. The attribute cards do a good job of introducing these. However, just games is not enough. You must discuss the meanings in your groups and record your definitions (the use of diagrams may help).

Hoop activities

“NOT”: Place a hoop and label it with an attribute (say, RED). Place cards inside and outside the hoop following the label. The ones outside are NOT the attribute.

Example:

![Diagram showing NOT RED]

Then label the hoop NOT … [e.g. NOT RED] and place the cards again.

“AND” and “OR”: Place two hoops so they intersect (overlap) and label them with different attributes (e.g. SMALL and RED). Place cards inside the hoops, the intersection of the hoops, and outside the hoops, following the labels.

Example:

![Diagram showing RED AND SMALL]

The cards in the intersection show “and”. The cards in both hoops and the intersection show “or”. Thus “and” means both attributes are present, whilst “or” means one or both are present.
**Probability applications**

**Venn diagrams**

**Problem 1:** A group of 90 visitors to Brisbane were surveyed about the other places in Queensland that they had visited:

- 20 visited the Gold Coast and the Barrier Reef but not the Sunshine Coast;
- 4 visited the Sunshine Coast and the Barrier Reef but not the Gold Coast;
- 12 visited the Gold Coast and the Sunshine Coast but not the Barrier Reef;
- 20 visited only the Gold Coast;
- 18 visited only the Sunshine Coast;
- 2 visited the Barrier Reef only;
- 4 visited all three locations; and
- 10 did not visit the Gold Coast, Sunshine Coast or Barrier Reef.

(a) Use a Venn diagram to represent this information.

(b) Calculate the probability of a visitor travelling to each location (as either the only place that they visited or combined with visits to other locations).

(c) If there are 2.1 million visitors to Queensland each year, estimate the number of visitors that each of the Gold Coast, Sunshine Coast and Barrier Reef can expect.

*Problem 1 provided all of the information needed to complete the Venn diagram. In some problems you have to deduce some of the information. On a clean sheet of paper, try Problem 2.*

**Problem 2:** A group of 90 visitors to Brisbane were surveyed on the other places in Queensland that they had visited:

- 20 visited the Gold Coast and the Barrier Reef but not the Sunshine Coast;
- 4 visited the Sunshine Coast and the Barrier Reef but not the Gold Coast;
- 78 visited the Gold Coast and the Sunshine Coast;
- 20 visited only the Gold Coast;
- 18 visited only the Sunshine Coast;
- 2 visited the Barrier Reef only;
- 4 visited all three locations; and
- 10 did not visit the Gold Coast, Sunshine Coast or Barrier Reef.

Use a Venn diagram to represent this information.

*You should have obtained the same Venn diagram as in Problem 1.*

Make up your own Venn diagram probability problems similar to Problem 2 (that is, where some information is missing and has to be deduced) and give them to your classmates to complete. You may decide to start with problems that need only two circles in the Venn diagram before trying the harder three-circle examples such as Problems 1 and 2.
**Tree diagrams and logical connectives**

**Problem 3:** Peter selected an attribute card at random and noted the colour and size of the shape and whether it was shaded or unshaded. He drew up the tree diagram below to describe all of the possible outcomes. What is the probability of selecting a shape that is:

- (a) red
- (b) small
- (c) shaded
- (d) blue and small
- (e) green and unshaded
- (f) red or shaded
- (g) large or unshaded
- (h) not green and small
- (i) blue and not shaded
- (j) not green or not red
- (k) not blue or not shaded
- (l) not red or not large or not shaded
- (m) not red and not large
- (n) not small and not shaded
- (o) not large and not small

When calculating probabilities, “and” means you should multiply the individual probabilities and “or” means that you should add the individual probabilities. If words such as “at least/most”, “less/more than” or similar are used, they may need to be translated into combinations of outcomes using the words “and” and “or”. For example, if a die is rolled, then the probability of getting a number less than three (that is a one or a two) is

\[ \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \]

If a coin is tossed twice, then the probability of both coins showing the same result (that, is a head and a head or a tail and a tail) is

\[ \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \]
**Problem 4:** A bag contains twelve marbles in three colours: four red, two green and six blue. Joan chooses one marble, puts it aside, and then chooses a second marble.

1. Complete the following statements:
   
   (a) The probability of Joan selecting a red marble first is ______
   
   (b) If Joan first picks a red marble then the probability of selecting another red marble is ______
   
   (c) If Joan first picks a red marble then the probability of the second marble being green is ______
   
   (d) If Joan first picks a red marble then the probability of the second marble being blue is ______

2. Complete the probability tree diagram at right by writing the probabilities of each branch in the boxes provided.

3. Remembering that when calculating probabilities, “and” means multiply and “or” means add, calculate the probability of selecting:
   
   (a) red and blue (in that order)
   
   (b) red and blue (in any order)
   
   (c) two green
   
   (d) two of the same colour
   
   (e) two of different colours
   
   (f) the first is not blue
   
   (g) the second is not red
   
   (h) no green
   
   (i) neither green nor red.
**Notation**

Probabilities are often written in symbols as \( P(A) \), \( P(x = A) \), \( P(x < A) \), \( P(x \geq A) \), \( P(x \neq A) \), where \( A \) describes the desired outcome. For example the probability of getting at least three when rolling a die would be written as \( P(x \geq 3) \) and then calculated as:

\[
P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)
\]

The word “not” in probability is often called the *complement*. It can also be shown in symbols by placing a bar above the outcome. For example, \( P(x = 3) \) means the “probability of not getting a three”.

**Challenge: An important probability rule**

An important rule used in probability says that

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).
\]

It can be rearranged to give

\[
P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B).
\]

This rule allows us to calculate probabilities of two-stage events without the need for a tree diagram.

Explain, with the use of Venn diagrams, why this rule works.

**Essential vocabulary**

*And*: Together with, indicating that all of the attributes must occur. Also described as the intersection of two sets.

*Attribute*: A quality or characteristic that is associated with something.

*Complement*: Something that completes a mathematical set. Often described using the word “not”.

*Not*: The opposite or absence of something.

*Or*: As an alternative to, indicating that any or all of the attributes must occur. Also described as the union of two sets.

*Venn diagram*: A diagram using a rectangle (to show the entire set) and one, two or three circles inside the rectangle (to show subsets) to show all possible logical relations (intersections, unions, and complements) between the subsets. Developed in about 1880 by John Venn.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of probability and logic. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

After students have developed an understanding of patterns the next step is using patterns to recognise attributes. Attributes can be complex when several attributes arise in the same process and when they are used with logical connectives. The activities in this task build attribute recognition and an understanding of logical connectives needed for two-stage probability tasks.

In this task, students develop an understanding of attributes, Venn diagrams, the logical connectives (and, or, not, if ... then), and their applications to probability. The logical connectives are also used extensively in computer programming and applications, including the efficient use of search engines. Students have a worksheet to assist them in developing an understanding of the concepts and some problems that illustrate the concepts. They are asked to demonstrate their grasp of the concepts by composing similar problems for others in their group. As a challenge, students are asked to explain the logic behind formulae for calculating the probabilities associated with the intersection and union of sets.

This task was originally designed for use with attribute blocks. However, as many secondary schools may not have class sets of this concrete manipulative, the task has been modified by substituting cards for the blocks. On the next five pages are photocopy masters that can be used to make up sets of the attribute cards and the train logic cards. Each attribute card set contains sixty cards with four attributes: shape (square, circle, triangle, pentagon and hexagon); colour (blue, red and green); size (large and small); and shading (shaded and unshaded). It is recommended that the attribute cards be printed or copied in colour (although the colours are written on the cards if that is not possible). There is also a set of 15 train logic cards. After printing onto card, laminate each page and then cut up the cards (students could do the cutting out). If multiple sets of cards are required, print each set on different coloured card stock so that the sets can be quickly restored if they become mixed during an activity.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP168 Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP204 Identify complementary events and use the sum of probabilities to solve problems</td>
</tr>
<tr>
<td></td>
<td>ACMSP205 Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’</td>
</tr>
<tr>
<td></td>
<td>ACMSP292 Represent events in two-way tables and Venn diagrams and solve related problems</td>
</tr>
<tr>
<td></td>
<td>ACMSP225</td>
</tr>
<tr>
<td>---</td>
<td>---------</td>
</tr>
<tr>
<td>9</td>
<td>List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Blue</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>Red</td>
</tr>
</tbody>
</table>
8S1 “Distributing Distributions”

Task description

Modern life revolves around statistics. For example, think about the variety of statistics that your school collects about you and the other students. It is not practicable to examine tables and graphs every time we want to use statistical information. Sometimes we want to understand what is going on by looking at just a few pieces of information. That is what measurements like mean, median, mode and deviation (often called summary statistics) can do for us. It is important that we understand what they are and how they can help. We also need to understand their limitations.

This task involves constructing different statistical distributions (sets of data) so we can consider what the mean, median, mode and standard deviation can tell us about a distribution.

1. Compare two sets of data about shoe sizes, observing the differences in data displays (frequency distribution tables and different types of graphs) and in the summary statistics (mean, median and mode). What differences do you see? Do you see the same patterns in the data displays as in the summary statistics? Can you describe what a typical shoe size would be?

2. How can you calculate the mean, median and mode when the data is presented in a frequency distribution table? Can you suggest a formula to use, or describe how to do it in a few steps?

3. Investigate what happens when we change some of the data:

   (a) What is the effect on the mean, median and mode of including a few very large or very small observations or measurements?

   (b) Can you construct two distributions of shoe size data with the same mean: one with the full range of sizes and the other with only very large and very small shoes? Does this substantially affect the median and mode?

4. Investigate the standard deviation. How does it change in the different distributions used so far in this activity? Why? How does it help us to understand the patterns of the distributions?

Note: Your teacher will give you some sets of data to use in parts of this task. In other places you will use data that you have collected yourself.

Make sure you understand the meanings of any words in italics.
**Additional student information**

**Activity 1**
You need to have some data to work with. Gather data from your classmates about their shoe sizes. Display your data by presenting it in a frequency distribution table and graphing it (which types of graphs might be best for this? – you could experiment with different graph types using Microsoft Excel or similar software). For the other set of data needed in this step, you could use the following:

<table>
<thead>
<tr>
<th>( x ) (shoe size)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) (frequency)</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Activity 2**
You need to have some more data to work with. Pick another topic to gather data on from your classmates, e.g. the length of their hand. Alternatively, you could use this data:

<table>
<thead>
<tr>
<th>( x ) (hand length in cm)</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) (frequency)</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Activity 3**
In this activity you are asked to make up your own data to see what happens as you change it.

**Activity 4**
While investigating standard deviation, choose your websites carefully. Some of them can be very technical, with lots of mathematical notation, not suitable for students in junior secondary school. Look for those that explain the concept in words rather than symbols.

You can use Google to help you to find websites that are easier to read, by following these steps:

1. Type [google.com](http://google.com) into the search box of your internet browser to find the Google home page.
2. On the Google home page, enter your search term(s) in the box. For this example, enter *standard deviation*.
3. On the results page, you will see a list of web pages (and a few advertisements) found by Google that match your search criteria. Click on *Search tools* (immediately above the list of results, on the right).
4. Click on *All results*.
5. In the drop down box, click on *Reading level*.
6. You will now see a bar graph showing how many of the search results are at the *basic, intermediate* and *advanced* reading level. You can see that in the case of standard deviation, most of them are at the advanced level. By clicking on the labels on the vertical axis of the bar graph, the results that you were shown previously will be filtered by the reading level you have selected. Try clicking on each of *basic, intermediate* and *advanced* in turn to see how your search results change.
7. It is suggested that in your search for information about standard deviation, you should start with websites rated to be at a basic reading level.
Essential vocabulary

**Data display:** A visual summary of statistical information in a table or graph. Common tables include *frequency distribution tables* and stem-and-leaf plots. Common graph types are frequency histograms, frequency polygons, pie (sector) graphs, divided bar graphs, picture graphs, box-and-whisker plots, and scatter graphs.

**Data:** Statistical information.

**Deviation:** A measure of difference between the *scores* and the *mean*; the extent to which the data deviates from the centre or mean.

**Frequency distribution table:** A table that organises data to show how often something happens. It must include columns headed *score* (what is measured or counted) and *frequency* (how often it occurs), but other columns can be added, for example, tally, cumulative frequency, relative frequency.

**Mean:** The average of all the items in a data set. To compute a mean, add up all the values and divide by the total number of items in the data set.

**Median:** The middle number in a series of numbers when numbers are placed in order from lowest to highest. If there is an even number of items in the data set, the median is the average of the two middle values.

**Mode:** The most frequently occurring value in the data set.

**Observation:** A statistical observation is a single piece of information. It can be collected by counting or measuring. In the statistical use of the word, it is not linked to the idea of sight and does not imply that the information was watched or seen.

**Range:** The difference between the highest and lowest values in a data set.

**Scores:** What is measured or counted.

**Standard deviation:** A measure of the spread of the data; that is, the distance, on average, of the individual observations from the mean. It averages the square root of the square of the differences between mean and scores. It uses the same units as the data; for example, if the data is measured in metres, then the units used for the standard deviation would also be metres.

**Statistical distribution:** A collection of statistical data, usually summarised in a frequency distribution table. May be called a *data set*.

**Summary statistics:** Numbers calculated from a statistical distribution that give an indication of the characteristics of that distribution. They can include measures of location (averages) such as mean, median, and mode, or measures of spread, such as range, inter-quartile range, standard deviation, and variance.

Some words have several meanings. These definitions give the statistical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching frequency distributions and the calculation of the mean, median and mode from those distributions. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Activities 1 and 2 of the task introduce students to the collection of data, the idea of a statistical distribution, the preparation of frequency distribution tables, and the use of those tables to calculate the measures of location, also referred to as averages, such as the mean, median and mode. The task assumes that students have a prior understanding of the mean, median and mode and how to calculate them from a list of data (that is, data that is not presented in a table). It also looks at the importance of allowing students to make up their own formulae before they are given the formal ones of mathematics.

Activities 3 and 4 introduce the idea of standard deviation as a measure of spread to complement the measures of location covered in Activities 1 and 2. While students will use calculators in the future to calculate the standard deviation, they can only develop a deep understanding of the concept by calculating it manually. Students should be encouraged to think deeply about each step in the process of calculating the standard deviation, possibly with the aid of diagrams or graphs, and then to look back at the completed process to summarise what the calculation has achieved and how they can use it.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Activity 1: Plotting frequencies

1. Pick a topic to gather data on, e.g. shoe sizes. Construct a physical graph from everybody’s shoe sizes like the one below (each 0 represents a student).

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>Cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>28</td>
<td></td>
</tr>
</tbody>
</table>

2. List the shoe sizes above in order, e.g. 3, 5, 5, 5, 5, 6, 6, ... and so on. Calculate mean, median and mode. Repeat this for your data.

3. Record data on a frequency table as below for the data in (1). Add cumulative frequencies. Repeat this for your data.

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>Cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>28</td>
<td></td>
</tr>
</tbody>
</table>
4. Graph the data from the frequency table as a bar graph and the cumulative frequency as a line graph. Repeat this for your data.

5. Compare your data with data from (1). What is the difference in the mean, median and mode? If there is a difference, why? If there is not, why not?

6. Describe and justify from your class’s data only (as the other is made up) what you would think a typical shoe size for a class at the same year level as yours would be. Do you need more data for this? Why?

**Activity 2: Building formulae**

1. Repeat (1) to (4) from Activity 1 for another set of similar data, e.g. hand length to nearest centimetre.

2. If you had to calculate the mean, median and mode from the frequency and cumulative frequency table, how would you do it? Try some ways to see if you arrive at the same mean, median and mode.

3. Here is some data in a table:

<table>
<thead>
<tr>
<th>x</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Cf</td>
<td>2</td>
<td>6</td>
<td>11</td>
<td>12</td>
<td>18</td>
<td>20</td>
<td>23</td>
<td>24</td>
</tr>
</tbody>
</table>

Calculate mean, median and mode from the table.

4. **Challenge:** In your group construct a formula for finding the following measures (if a formula is possible):
   (a) mean
   (b) mode
   (c) median

   Helpful information: Statisticians use some notation to assist in developing formulae:
   - $n$ is used to represent the total number of scores in the distribution;
   - the letter $x$ with a line across the top, written $\bar{x}$, can be used to represent the mean of all of the scores; and
   - the Greek upper case letter “sigma”, written $\Sigma$, is used as shorthand for “the sum of”.

5. Compare your formulae with the formal formulae. What is different? What is the same?

**Activity 3: Constructing distributions**

1. Suppose a class had small and large students and no in-between sizes.
   (a) Construct a frequency table for shoe size for this class which gives the same mean as in Activity 1 but with no 6, 7, or 8 shoe sizes.
   (b) What happens to mean, median and mode?
   (c) Draw a frequency graph. How is it different to your original graph from Activity 1?
   (d) Can you keep the median the same?
2. Suppose two giants joined the class with shoe sizes of 25 and 27.
   (a) Redo mean, median and mode for your data. What is the difference?
   (b) Draw a frequency graph. How is it different to your original graph?
   (c) How could you change the shoe sizes for the rest of the class so that the mean was reduced to the same number as in Activity 1?
   (d) Can you make the median the same as in Activity 1?

3. What if we had a lot of students with size 2 shoes?
   (a) Let’s add five size 2 students and remove two from each of 6, 7, and 8. What happens to the mean, median and mode?
   (b) Draw a frequency graph. How is it different to your original graph from Activity 1?
   (c) Can you increase the sizes of some of the other students so you get the same mean? What happens to mode and median?

4. The temperatures for a week were:

<table>
<thead>
<tr>
<th>Su</th>
<th>M</th>
<th>Tu</th>
<th>W</th>
<th>Th</th>
<th>F</th>
<th>Sa</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>24</td>
<td>23</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

   (a) What was mean, median and mode? Did you notice that all three are the same?
   (b) Can you get the mean to differentiate from the mode and median by changing some temperatures? Which ones and by how much? Can it be done with a simple change?
   (c) Modify the above to make a distribution with a low mean.
   (d) Modify the above to make a distribution with a high mean.
   (e) Draw the frequency graphs for (b), (c) and (d). How are they different?

**Activity 4: Why we need standard deviation**

1. Look up standard deviation (Wikipedia is a good place to start).
   (a) How do you work it out?
   (b) What is the formula?

2. Take the data from the original graph (Activity 1) and from steps 1, 2 and 3 in Activity 3 and the mean that you calculated from each distribution.
   (a) Put data from each distribution in a table, as in the example that has been started for you (the mean or \( \bar{x} \) in this example is 7):

<table>
<thead>
<tr>
<th>Score (x)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (f)</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference from the mean (x – ( \bar{x} ))</td>
<td>–4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square difference (x – ( \bar{x} ))^2</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) For each distribution, calculate the mean of the square differences (how will you allow for the fact that most scores occur more than once?) and then take the square root of that mean. The new statistic that you have just calculated for each distribution is called the standard deviation (SD).

(c) Why would we be interested in the difference of each score from the mean (row 3 in the table above)? How does it help us to understand more about the distribution?

(d) Why do you think that it was necessary to square the differences from the mean (row 4 in the above table)? (Hint: try calculating the mean of the differences without first squaring them – that is, try leaving out row 4 in table above.)

(e) What do you notice regarding standard deviation? Was one distribution’s standard deviation higher than the rest? Which distribution had this higher deviation? Why? Any interesting results in the other deviations?

(f) Write a few sentences explaining your understanding of the standard deviation, what it measures, and how we can use it.

3. In a new study, you gathered data on shoe size and found that the mean was 13.
   (a) What can you say about the distribution if the standard deviation is high?
   (b) What can you say about the distribution if the standard deviation is low?

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP170 Construct and compare a range of data displays including stem-and-leaf plots and dot plots</td>
</tr>
<tr>
<td></td>
<td>ACMSP171 Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data</td>
</tr>
<tr>
<td></td>
<td>ACMSP172 Describe and interpret data displays using median, mean and range</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP284 Investigate techniques for collecting data, including census, sampling and observation</td>
</tr>
<tr>
<td></td>
<td>ACMSP293 Explore the variation of means and proportions of random samples drawn from the same population</td>
</tr>
<tr>
<td></td>
<td>ACMSP207 Investigate the effect of individual data values, including outliers, on the mean and median</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP228 Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources</td>
</tr>
<tr>
<td></td>
<td>ACMSP282 Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’</td>
</tr>
<tr>
<td></td>
<td>ACMSP283 Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread</td>
</tr>
<tr>
<td>10</td>
<td>ACMSP249 Construct and interpret box plots and use them to compare data sets</td>
</tr>
<tr>
<td>10A</td>
<td>ACMSP278 Calculate and interpret the mean and standard deviation of data and use these to compare data sets</td>
</tr>
</tbody>
</table>
**PS2 “Topological Oddities”**

**Task description**

*Topology* is a modern version of geometry. It has been called “rubber sheet mathematics” because it requires us to visualise what happens as shapes, lines and surfaces stretch, shrink and bend in one or more directions. Topology can be used to simplify complex network problems. For example, the diagram of a train network shown in suburban trains is not the same as a scaled map of that network. However the diagram and scaled map are *topologically equivalent* because one can be obtained by stretching and shrinking the other. Using the diagram of the network simplifies the task of understanding how to get from one place to another by train. Topology also has applications in genetics, economics, computer science, physics and robotics.

In this task you can research some different aspects of topology:

- Investigate topological change and *homeomorphism* (your teacher can give you a page with some essential vocabulary and definitions to get you started). Make a list of lines, surfaces and shapes before and after some changes. Classify them as *topological* or *not topological*. Which numerals and letters of the alphabet are homeomorphic?

- Investigate the Möbius strip – an example of an object with a single surface. Start by making some Möbius strips and investigate what happens as you cut it along the length of the strip (not across). What else can you do with Möbius strips? **Challenge**: Investigate the Klein bottle.

- Investigate map colouring. Maps must be coloured so that no two states/countries with a common border have the same colour. A famous *postulate* was made that four colours would suffice for any map (and it took centuries to prove it). Investigate the relationship between the number of odd and even *vertices* in the map and the minimum number of colours needed (an odd vertex has an odd number of lines coming out of it; an even vertex has an even number of lines coming out of it). Why was the four-colour problem interesting and how was it proved?

*Make sure you understand the meanings of any words in italics.*
**Additional student information**

This task involves internet research. Some of the websites that you find will be very complex, containing theoretical mathematics way beyond the scope of junior secondary mathematics. You can use Google to help you to find websites that are easier to read, by following these steps:

1. Type `google.com` into the search box of your internet browser to find the Google home page.

2. On the Google home page, enter your search term(s) in the box. For this example, enter `topology`.

3. On the results page, you will see a list of web pages (and a few advertisements) found by Google that match your search criteria. Click on *Search tools* (immediately above the list of results, on the right).

4. Click on *All results*.

5. In the drop-down box, click on *Reading level*.

6. You will now see a bar graph showing how many of the search results are at the *basic*, *intermediate* and *advanced* reading level. You can see that in the case of topology, most of them are at the advanced level. By clicking on the labels on the vertical axis of the bar graph, the results that you were shown previously will be filtered by the reading level you have selected. Try clicking on each of *basic*, *intermediate* and *advanced* in turn to see how your search results change.

It is suggested that in your search for information about topology, you should start with websites rated to be at a basic or intermediate reading level.
Essential vocabulary

**Deformation:** In topology a deformation is the result of stretching, shrinking or bending lines, shapes or surfaces (called topological spaces), but not making holes, cutting or gluing. Shapes and/or surfaces can be distorted but not broken.

**Homeomorphic:** Two spaces are homeomorphic if one can be deformed into the other without cutting or gluing. For example, a coffee mug and a doughnut are homeomorphic because a sufficiently flexible doughnut could be reshaped to a coffee cup by creating a dimple and progressively enlarging it, while shrinking the hole into a handle. Another way of describing homeomorphic spaces is to say that they are topologically equivalent.

**Properties:** The basic and/or essential attributes shared by all elements (members) of a set. For example, a description of the properties of a particular type of quadrilateral might include information about whether the shape is closed, the size of the angles, length of the sides, the angles at which the diagonals intersect, and whether the diagonals cut each other in half.

**Topology:** An area of geometry concerned with the properties of space that are preserved under transformations such as stretching, shrinking and bending, but not cutting or gluing. It studies connectedness, continuity and boundary.

**Transformation:** Transformations can be classified into three categories:

- The simplest transformations are reflection in a line (flip), rotation (turn) and translation (slide). They are called Euclidean transformations because the properties that we focus on in Euclidian geometry (angle, size and straightness) do not change under these transformations.

- Projective transformations include dilation (enlarging and reducing in one or more directions), shear (moving some points in a fixed direction so that, for example, a square becomes a rhombus — think also of what occurs when the axes of a graph do not meet at right angles), and reflection in a curved surface (think about what occurs when light rays meet concave and convex mirrors). These types of transformations allow us to think of different shapes as being equivalent (for example, ellipses and circles or parallelograms and rectangles) and parallel lines that meet at a “vanishing point”. They became important as a result of the use of perspective in art and technical drawing and the development of different map projections. In projective transformations, length and angle can change, but straightness does not.

- Topological transformations can include all of the above, but introduce the deformations of stretching, shrinking or bending. Unlike the transformations mentioned above, they do not have to be applied in a uniform way. They can be likened to what happens as we mould a piece of wet clay (without making holes, breaking off pieces, or attaching new pieces). In topological transformations, length, angle and straightness can all change.

**Vertex (plural vertices):** A point where two or more straight lines meet, for example, the point of an angle or the corner of a polygon. In topology, where straightness is not important, the definition would be modified to be the place where two or more lines meet.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task provides an enrichment activity in geometry. It goes beyond the Australian Curriculum requirements, providing a holistic view of geometry beyond the Euclidean approach that is the focus of school mathematics courses. It has deliberately not been written for a “set and forget” teaching approach. Within the task there are many opportunities for the teacher to encourage discussion among students. This section provides some teaching advice and also more detailed instructions than those given to students. The detailed instructions could also be used as the basis of student worksheets.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Activity 1: Topological change

Topology is change where length and straightness can change (projective has length changing but not straightness and flips-slides-turns has neither length nor straightness changing) but there is no punching of holes or tearing in half or joining. Things can be very distorted but not broken.

Materials: Paper, scissors, glue or sticky tape, coloured pencils

1. Below is a collection of before and after changes. List which of them are topological and which are not (note that topological changes include flip-slide-turn changes and projective changes).

2. Think up some other changes that are topological – try and fool your friends. For example: sort numbers and letters into topologically the same groups – find things that are equivalent to a figure 8.
Activity 2: Möbius strip

In all the activities below, always do the activity with a normal cylinder/ring with no twists (as on right) and compare so you can see how the Möbius strip is different.

1. Make a Möbius strip. Cut out a flat strip of paper about 50 cm long. Twist the paper (once) and then join the two ends to make a closed ring.

2. Try to colour one side of the strip red and the other side green (use any two different colours). What do you notice?

3. Try to draw a line along the centre of the strip continuing until you come back to the same point from which you started. Are you convinced that this strip has only one side?

4. Predict what you think will happen if you cut along the middle of the strip as shown on right. Validate by cutting. (Cut along the line you have already drawn.) Were you surprised by what happened?

5. Make another Möbius strip. This time draw a line that is 1 third of the width. Continue to draw the line the same distance from the edge until you come back to the same point from which you started. Cut along the line you have just drawn. Do you have a chain of Möbius strips?

6. Make a Möbius strip where one end is twisted twice before it is glued to the other end. Repeat activities above. Make Möbius strips that have 3 or 4 twists and repeat activities above again. Can you discover a pattern emerging?

Activity 3: Map colouring

Maps are coloured so that no two states/countries with a common border have the same colour. A famous postulate was made that four colours would suffice for any map (and it took centuries to prove it).

1. Colour the maps in item 1 of the Map colouring worksheet. How many colours are needed?

2. Complete the table for item 2 of the map colouring worksheet from the maps in item 1. (Note: an odd vertex has an odd number of lines coming out of it; an even vertex has an even number of lines coming out of it.)

3. Challenge: This table is supposed to enable you to determine a pattern or rule that gives how many colours are needed. If you find this is the case, apply your pattern to item 3 on the map colouring worksheet. Count the vertices, predict the colours and check.

4. Challenge: If you find that the pattern in 3 does not give an answer, see if you can work out your own pattern. Maybe it is based on something other than odd and even vertices?

5. Look up the “four-colour problem” on the internet – what was interesting about this problem and how was it solved?
Map colouring worksheet

1. Map colouring:

   ![Map A](image1)
   ![Map B](image2)
   ![Map C](image3)

   ![Map D](image4)
   ![Map E](image5)
   ![Map F](image6)

   Number of different colours ...

2. Table:

<table>
<thead>
<tr>
<th>Map</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of odd vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of even vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of different colours</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Challenge:

   ![Challenge Map A](image7)
   ![Challenge Map B](image8)
   ![Challenge Map C](image9)
## Links to the Australian Curriculum

### Mathematics

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMMG114 Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries</td>
</tr>
<tr>
<td>6</td>
<td>ACMMG142 Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMMG200 Define congruence of plane shapes using transformations</td>
</tr>
</tbody>
</table>
## Critical and creative thinking

<table>
<thead>
<tr>
<th>Critical and creative thinking</th>
<th>By the end of Year 8</th>
<th>By the end of Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inquiring – identifying, exploring and organising information and ideas</strong></td>
<td>Pose questions: pose questions to probe assumptions and investigate complex issues Identify and clarify information and ideas: clarify information and ideas from texts or images when exploring challenging issues Organise and process information: critically analyse information and evidence according to criteria such as validity and relevance</td>
<td>Pose questions: pose questions to critically analyse complex issues and abstract ideas Identify and clarify information and ideas: clarify complex information and ideas drawn from a range of sources Organise and process information: critically analyse independently sourced information to determine bias and reliability</td>
</tr>
<tr>
<td><strong>Generating ideas, possibilities and actions</strong></td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action</td>
</tr>
<tr>
<td><strong>Reflecting on thinking and processes</strong></td>
<td>Think about thinking (metacognition): assess assumptions in their thinking and invite alternative opinions Reflect on processes: evaluate and justify the reasons behind choosing a particular problem-solving strategy Transfer knowledge into new contexts: justify reasons for decisions when transferring information to similar and different contexts</td>
<td>Think about thinking (metacognition): give reasons to support their thinking, and address opposing viewpoints and possible weaknesses in their own positions Reflect on processes: balance rational and irrational components of a complex or ambiguous problem to evaluate evidence Transfer knowledge into new contexts: identify, plan and justify transference of knowledge to new contexts</td>
</tr>
<tr>
<td><strong>Analysing, synthesising and evaluating reasoning and procedures</strong></td>
<td>Apply logic and reasoning: identify gaps in reasoning and missing elements in information Draw conclusions and design a course of action: differentiate the components of a designed course of action and tolerate ambiguities when drawing conclusions Evaluate procedures and outcomes: explain intentions and justify ideas, methods and courses of action, and account for expected and unexpected outcomes against criteria they have identified</td>
<td>Apply logic and reasoning: analyse reasoning used in finding and applying solutions, and in choice of resources Draw conclusions and design a course of action: use logical and abstract thinking to analyse and synthesise complex information to inform a course of action Evaluate procedures and outcomes: evaluate the effectiveness of ideas, products and performances and implement courses of action to achieve desired outcomes against criteria they have identified</td>
</tr>
</tbody>
</table>
9A1 “Dividing Diagonals”

Task description

Draw and label a set of Cartesian axes. On that Cartesian plane, using blue or black ink, draw a square with sides of twelve units. We will call this square the “original square”. The task will be easier if the coordinates of the vertices of the square are integers and if a ruler is used to ensure that all lines are straight. If you are working in a group, each person in the group should draw a different original square.

Draw the diagonals of the original square, also in blue or black ink. Find the midpoints of the four “half-diagonals”, that is, the line segments joining the vertices of the original square with the point of intersection of the diagonals.

Change to a red pen. Select one vertex of the original square and draw a line that passes through that vertex and a nearby midpoint (this line should not overwrite one of the diagonals of the original square). Select another vertex and draw a line that passes through that vertex and a different midpoint. Repeat this process with the other two vertices.

Examine the quadrilateral defined by the intersection of the four red lines. We will call this shape the “internal quadrilateral”.

1. What are the properties of the internal quadrilateral? Use appropriate calculations (not measurements) to justify any claims made about the length of the sides and the size of the angles.

2. What is the relationship between the area of the original square (in blue or black ink) and the area of the internal quadrilateral (in red ink)?

3. Is the relationship between the areas true for any square?

4. Challenge: Can you prove the relationship that you found between the original square and the internal quadrilateral? Remember that a proof cannot rely on particular coordinates and/or values.

Make sure you understand the meanings of any words in italics.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on activity to support the teaching of distance, midpoint and gradient in the Cartesian plane. The task assumes that students are already familiar with these concepts.

The task has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or prompts to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and information on completing the task.

Student prompts

The task is best left as open as possible for student investigation. However, some students may need prompting to get started and/or keep going.

The instructions for this task are deliberately provided in words, without a supporting diagram. Students in Year 9 should be able to develop diagrams from written instructions and/or descriptions; however it may be necessary to ensure that students understand the meaning of vertex and vertices. Students in need of prompting should be encouraged to commence this task by representing the information as a diagram.

If students need prompting in question 1 (considering the properties of the internal quadrilateral), they could be asked to develop a hypothesis about the nature of the quadrilateral. They can test their hypothesis by considering the properties of the quadrilateral (such as side lengths and angles). A further prompt could be to find the equations of the lines that form the sides of the internal quadrilateral. The equations, in $y$-intercept form, will give the gradient of each line and students should be able to demonstrate that these lines intersect perpendicularly (thus eliminating all quadrilateral options other than squares and rectangles). The equations of the lines can be used to find the points of intersection of the lines, allowing a calculation that will show that the sides of the internal quadrilateral are equal (thus demonstrating that the internal quadrilateral is a square).

The lengths of the sides of the original square and the internal quadrilateral can be used to find the areas of those figures and a ratio between those areas.

In question 3, students are asked to consider whether changing the original square (that is, varying the coordinates of the vertices) changes the nature of the solution in any way. If all students started with a different original square, they should find that they all reached the same conclusions in questions 1 and 2. Alternatively, students could be prompted to try questions 1 and 2 with a different original square. They could also consider what they know about the properties of a figure if it is translated (slides), rotated (turns), or dilated (enlarged/reduced).

Teachers may consider removing the specification of a side-length of 12 units for student groups with sufficient algebra skills.

Linking algebra and geometry

The problem as originally posed provides only one measurement (side length = 12 units) and that the figure is a square. The internal quadrilateral (in red ink) looks like a square but students need to realise that this property cannot be used as part of the solution. Placing the square on the Cartesian plane transforms the problem to favour an algebraic solution or analytic geometry.
Using technology

Dynamic Geometry Software (DGS): This problem can be easily represented using any of the DGS packages available in schools including Geometer’s Sketchpad, Autograph, TI-Nspire and the free web-based options such as GeoGebra. Figure 1 is an image captured from GeoGebra.

DGS allows students to manipulate points on the square and hence change the areas while the ratio remains constant. They can verify that the internal quadrilateral is a square by using the software to measure the sides and angles.

Students can also move the vertices of the original square to verify that even if the original shape is not a square the ratio of the areas remains constant at 0.4.

Computer Algebra System (CAS) calculators: While not difficult, the algebra involved in arriving at the final result is long and repetitive. This is one situation in which students can use computer algebra systems to advantage. A CAS will not solve this problem but it does provide the student with tools to complete the repetitive tasks such as simultaneous equations. Students can still demonstrate their knowledge of how to solve a problem such as this without having to repeat each step four times.

An example of how a CAS could be used to solve this problem is included as an appendix at the end of this section.

The proof

Discussions on the nature of what constitutes a proof flow directly from the use of DGS. The software clearly demonstrates that the ratio of the areas is constant but does not constitute a mathematical proof. The figure formed looks like a square so the discussion with students needs to identify what properties of a square (if shown to be true) are necessary and sufficient to prove that it is in fact a square. Teachers wanting to explore this concept further should consider the example provided in the Further investigation section.

Students could be prompted to consider how to generalise their solutions to questions 1, 2 and 3 to develop the proof. They should be asked to consider why a square of side-length 12 units was selected for the initial problem. As the problem is built around the repeated calculation of the midpoint, 12 was used as the starting value since it can be divided by 2 and divided by 2 again to give whole-number values for the coordinates of points.

Most students will adopt an unknown side length for the original square, $s$, when attempting to complete the proof in question 4. Using a side length of $s$ will produce midpoint coordinates such as ($\frac{s}{4}, \frac{s}{4}$) and ($\frac{s}{4}, \frac{3s}{4}$) making the subsequent calculations unnecessarily complex. Students should be encouraged to see that adopting an unknown side length of $12s$ is a valid generalisation but the resulting expressions will be simpler. The midpoint coordinates would be $(3s, 3s)$ and $(3s, 9s)$.

Students should also recognise that changing the size of the square leaves the gradient of the lines unchanged. For example, in Figure 1 above, the equation of RQ is $y = \frac{1}{3}x + 8$. Using a side of $12s$ the equation becomes $y = \frac{1}{3}x + 8s$. 
Even if using a CAS, keeping the expressions as simple as possible makes it easier to follow the output from the CAS operations.

**Further investigation**

Consider the following similar task.

Given a square, construct the midpoints of each side. Draw segments connecting each vertex to the two midpoints not on the sides that contain the vertex.

1. What is the best descriptor of the type of polygon that is formed?

2. Is there a relationship between the area of the square and the area of the polygon? If so, what is it?

Full investigation is left for the reader, however a diagram of the figure produced is shown in Figure 2 (left).

The internal figure IJKLMNOP is an octagon but it is not a regular octagon. Students may assume that it is regular. All sides are of equal length, opposite sides are parallel. However all the internal angles are not congruent. The angles are approximately $143^\circ$ and $127^\circ$.

While the algebra in this task is probably too much to ask of most Year 9 students, it is a valid investigation using DGS. It illustrates to students the need for all the necessary conditions to be satisfied.
Appendix: A solution using CAS

Define the equations of the sides of PQRS:

\[ y = \frac{x}{3} \]
\[ y = -\frac{x}{3} \]
\[ y = 12 \cdot a - 3 \cdot x \]
\[ y = 36 \cdot a - 3 \cdot x \]

\[
\text{LinSolv}\left(\begin{array}{cc}
\frac{x}{3} \\
y = -\frac{x}{3} \\
y = 12 \cdot a - 3 \cdot x \\
y = 36 \cdot a - 3 \cdot x
\end{array}\right) = \left\{ \begin{array}{cc}
34 \cdot a \\
18 \cdot a \\
5 \\
5 \\
5 \\
5
\end{array}\right\}
\]

\[
\frac{54 \cdot a}{5} = px \\
\frac{18 \cdot a}{5} = py
\]

\[
\text{LinSolv}\left(\begin{array}{cc}
\frac{1}{3} \\
y = 3 \cdot x + 12 \cdot a
\end{array}\right) = \left\{ \begin{array}{cc}
18 \cdot a \\
6 \cdot a \\
5 \\
5
\end{array}\right\}
\]

\[
\frac{6 \cdot a}{5} = px \\
\frac{6 \cdot a}{5} = py
\]

\[
\text{LinSolv}\left(\begin{array}{cc}
\frac{1}{3} \\
y = 3 \cdot x + 36 \cdot a
\end{array}\right) = \left\{ \begin{array}{cc}
42 \cdot a \\
54 \cdot a \\
5 \\
5
\end{array}\right\}
\]

\[
\frac{42 \cdot a}{5} = px \\
\frac{54 \cdot a}{5} = py
\]

Length of PQ:

\[
\sqrt{(pe - qe)^2 + (pe - qe)^2} = 12 \cdot |a| \cdot \sqrt{10} / 5
\]
© length of QR
\[
\sqrt{(qx-rx)^2 + (qy-ry)^2} = \frac{12 \cdot |a| \cdot \sqrt{10}}{5}
\]

© length of RS
\[
\sqrt{(rx-sx)^2 + (ry-sy)^2} = \frac{12 \cdot |a| \cdot \sqrt{10}}{5}
\]

© length of SP
\[
\sqrt{(sx-px)^2 + (sy-py)^2} = \frac{12 \cdot |a| \cdot \sqrt{10}}{5}
\]

© therefore all sides are same length

© slope of PQ
\[
\frac{py-qy}{px-qx} = -3
\]

© slope of QR
\[
\frac{qy-ry}{qx-rx} = \frac{1}{3}
\]

© slope of RS
\[
\frac{ry-sy}{rx-sx} = -3
\]

© slope of SP
\[
\frac{sy-py}{sx-px} = \frac{1}{3}
\]

© therefore opposite sides parallel, adjacent sides perpendicular

© PQRS is a square
## Literacy

<table>
<thead>
<tr>
<th>Literacy</th>
<th>By the end of Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Navigate, read and view learning area texts</td>
<td>Navigate, read and view a wide range of more demanding subject-specific texts with an extensive range of graphic representations</td>
</tr>
<tr>
<td>Interpret and analyse learning area texts</td>
<td>Interpret and evaluate information within and between texts, comparing and contrasting information using comprehension strategies</td>
</tr>
<tr>
<td>Compose spoken, written, visual and multimodal learning area texts</td>
<td>Compose and edit longer and more complex learning area texts</td>
</tr>
</tbody>
</table>
**9A2 “Taking the Guesswork Out of Maths”**

**Task description**

There are many problems where guessing and checking and then using this for a better guess is a good strategy. However, it only works if the solution is easily “guessable”. For example, if the solution was 452.6892 it might take a lot of guesses to find that value! We need a better method – one that will work easily every time.

We can use an algebraic model known as simultaneous equations. This task investigates two methods for solving the simultaneous equations.

Start with the rabbits and ducks problem (an easier example because the solution has to be a whole number).

1. Solve the equation by guess and check.
2. Reformulate the problem into two equations (one for heads and one for legs) using \( d \) and \( r \) as ‘shorthand’ for ducks and rabbits – this is your algebraic model.
3. To solve these equations:
   
   (a) Rearrange one of the equations (pick whichever equation looks easier to work with – let’s call it equation A) to isolate one of the variables (this means to apply inverse operations to both sides of the equation until you have one variable by itself on the left-hand side of the equation and the other variable and any constants on the right-hand side).
   
   (b) Substitute into the other equation (we’ll call that equation B) the expression on the right-hand side of equation A for the variable that you isolated; remember to put brackets around that expression.
   
   (c) Equation B should now contain only one variable, and can be solved using the standard methods of solving one-variable equations.
   
   (d) When you know the value of one variable, you can substitute it into equation A to calculate the value of the other variable.

4. Did your answer match the one that you found using the guess-and-check method?

5. Ask your teacher for a copy of the annotated solution to this problem. Go through it carefully to identify each of the four steps (a) to (d) described in part 3 above. Pay close attention to the setting out (formatting) of the solution because that is how you should present your solutions from now on.

This method is called solving simultaneous equations by substitution. Try this algebraic method with the other problems given to you.

**Challenge:** There is another method of solving simultaneous equations – by elimination. Good mathematicians use both methods and select the one that is easier for a particular set of equations. Research this method (or ask your teacher to explain it to you) and then try it out on the problems in this task.

*Make sure you understand the meanings of any words in italics.*
Additional student information

Simultaneous equation problems:

1. There are 52 feet and 17 heads, how many ducks, how many rabbits?

2. A small iceblock costs 40 cents; a small chocolate costs 60 cents. I spend $6.20 on 12 items, how many iceblocks and how many chocolates did I buy?

3. 26 young children, 64 wheels, how many bicycles, how many tricycles?

4. It costs $49 for 2 adults and 5 children to get into the fair and it costs $27 for 1 adult and 3 children to get into the fair. What is the price of an adult ticket and what is the price of a child ticket?

5. Of the 73 Year 6 students at Henry State School, there are seven more girls than boys. How many girls? How many boys?

6. In this two-digit number, the tens digit is greater than the ones digit. The sum of the two digits is 11 and the product is 28. What is the two-digit number?

7. Fred gave James a box of chocolates containing 30 small bite-size Mars Bars, KitKats and Cherry Ripes. There were twice as many Cherry Ripes as KitKats; there were three times as many Mars Bars as Kit Kats. How many of each chocolate?

8. Challenge: Sue threw a dice four times. The numbers were different each time. The product was 48, the sum was 13. What numbers did Sue throw?

9. Challenge: A three-room apartment followed these specifications. The apartment was designed as on the right (not to scale):

   • area of living room was 42 m²
   • shorter side of kitchen was 3 m
   • shorter width of bedroom and bathroom was 5 m
   • the entire area of the apartment was 110 m²

   [Your teacher should be able to find some more of this type of problem for you if you want more practice.]
Essential vocabulary

Algebraic model: One or more equations that represent a real-life situation. The equations are algebraic, that is, they contain variables to represent the unknowns in the real-life situation (which is what we are usually seeking to find out) and constants to represent the known information. For example, the equation $2d + 4r = 52$ could represent the total number of legs (52) in a group of ducks with two legs each (represented by the variable $d$) and rabbits with four legs each (represented by the variable $r$).

Constant: In algebra, a symbol that stands for a known, fixed quantity, that is, a number. It is usually represented by numerals, but other symbols can be used, for example $\pi$. In the equation $2d + 4r = 52$, $d$ and $r$ are variables and 4 and 52 are constants.

Equation: A statement that the values of two mathematical expressions are equal (indicated by the sign $=$). The expressions may comprise numbers and/or variables (e.g. $3 + 1 = 4$ and $3x + 1 = 4$ are both equations).

Inverse operations: An inverse operation is a process (operation) that reverses a previous process (operation). For example, subtraction and addition are inverse operations, as are multiplication and division. Inverse operations are used in the process of solving equations.

Isolate a variable: The process of rearranging an algebraic equation so that a selected variable is by itself (that is, with a coefficient of 1) on one side of the equation (usually the left) and all other information is on the other side of the equation (usually the right). The rearrangement of the equation occurs through the usual approach of applying inverse operations to both sides of the equation. For example, in the equation $2d + 4r = 52$, $r$ could be isolated to give $r = \frac{52-2d}{4}$ (or, more simply, $r = \frac{26-d}{2}$).

Simultaneous equations: Equations with more than one type of variable. Generally, to solve simultaneous equations, we need as many different equations as there are variables. For example, to solve a set of simultaneous equations with two variables, we need two different equations.

Substitute: To replace letters (variables) with values (numbers) in an algebraic expression. After a number is substituted for a variable in an expression, it is often possible to calculate an exact value of the expression, for example, $3x + 2$ becomes $3 \times 4 + 2 = 14$ when $x = 4$.

Variable: In algebra, a symbol that stands for an unknown quantity, usually represented as a letter (in the English or Greek alphabets). Variables are usually written in script form (for example, Times New Roman italic font). Unknown and pronumeral are other names that can be used to describe a variable.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
**Annotated solution for Ducks and Rabbits**

**There are 52 feet and 17 heads, how many ducks, how many rabbits?**

Let $d$ be the number of ducks and $r$ be the number of rabbits.

Then $d + r = 17$  \hspace{1cm} equation A

and $2d + 4r = 52$  \hspace{1cm} equation B

From equation A:  \hspace{1cm} $d = 17 - r$

Substituting for $d$ into equation B:

\begin{align*}
2(17 - r) + 4r &= 52 \\
34 - 2r + 4r &= 52 \\
-2r + 4r &= 52 - 34 \\
2r &= 18 \\
r &= 9
\end{align*}

Substituting for $r$ into equation A:

\begin{align*}
d &= 17 - r \\
&= 17 - 9 \\
&= 8
\end{align*}

There are 8 ducks and 9 rabbits.

We start by stating what each variable represents.

Equation A represents the number of heads, whilst equation B is the number of legs.

Note that we have labelled each equation and that the = signs in each equation are aligned.

We have rearranged equation A so that $d$ is isolated.

Note that $17 - r$ was placed inside brackets. Also the use of explanations to make it easier for the reader to follow what is going on.

We have now solved equation B for $r$, so we know that there are 9 rabbits.

Note that we have continued to keep the = signs aligned.

So now we know that there are 8 ducks.

Check the solution:
- 9 rabbits and 8 ducks have a total of 17 heads;
- 9 rabbits have 36 feet and 8 ducks have 16 feet, for a total of 52 feet.

Give the solution in a sentence because the question was asked in a sentence.

The solution would be similar if we chose to isolate $r$ instead of $d$ in the second step:

**Let $d$ be the number of ducks and $r$ be the number of rabbits**

Then $d + r = 17$  \hspace{1cm} equation A

and $2d + 4r = 52$  \hspace{1cm} equation B

From equation A:  \hspace{1cm} $r = 17 - d$

Substituting for $r$ into equation B:

\begin{align*}
2d + 4(17 - d) &= 52 \\
2d + 68 - 4d &= 52 \\
2d - 4d &= 52 - 68 \\
-2d &= -16 \\
d &= 8
\end{align*}

Substituting for $d$ into equation A:

\begin{align*}
r &= 17 - d \\
&= 17 - 8 \\
&= 9
\end{align*}

There are 8 ducks and 9 rabbits.
Teaching information

This task, with some teacher involvement, provides an approach for teaching the algebraic solution of simultaneous equations. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task. The detailed instructions could also be used as the basis of student worksheets.

The approach taken is to demonstrate to students that a guess-and-check approach is inefficient. They are then led through the process of solving simultaneous equations by substitution. After students have attempted the process themselves, an annotated solution is available for their inspection. Some additional problems are provided in this task although many others, if needed, could be copied from a mathematics textbook. Students should be encouraged to focus on the presentation of their solution as much as the process, since effective communication is as much a requirement of mathematics as efficient calculation.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Activity 1: Rabbits and ducks

The problem: “There are 52 feet and 17 heads, how many ducks, how many rabbits?”

1. Solve this by guess and check:
   (a) Make a guess for one of the animals.
   (b) Work out how many feet and heads this would give.
   (c) Make a better guess.
   (d) Keep going until you get the right number of heads and feet.

2. Let us rethink the problem:
   (a) Use letters for the number of ducks and rabbits – for example: $d$ for number of ducks and $r$ for number of rabbits.
   (b) What do we know about the numbers $d$ and $r$? (Hint: What do 52 feet and 17 heads mean in terms of $d$ and $r$?)
   (c) Write what you know about $d$ and $r$ as equations. Can you relate $d$ and $r$?

3. Let’s solve it with the letters:
   (a) How can we solve equations? We need one variable. Can we use the relation between $d$ and $r$ to do this?
   (b) Can you substitute into one variable so that there is only one variable in an equation?
   (c) Solve the equation. Did you get the same answers for that letter? What about the other letter?

Note: This process is called solving simultaneous equations.
Activity 2: Other problems

1. A small iceblock costs 40 cents; a small chocolate costs 60 cents. I spend $6.20 on 12 items, how many iceblocks and how many chocolates did I buy?

(a) Solve this by guess and check.

(b) Solve this by simultaneous equations. Remember, write letters for numbers to set the problem up. Use letters for the number of iceblocks and the number of chocolates. Then reduce to one letter.

2. 26 young children, 64 wheels, how many bicycles, how many tricycles?

(a) Solve this by guess and check.

(b) Solve it by simultaneous equations – remember to create letters for the numbers, set up the equations and substitute to get one letter.

3. It costs $49 for 2 adults and 5 children to get into the fair and it costs $27 for 1 adult and 3 children to get into the fair. What is the price of an adult ticket and what is the price of a child ticket?

(a) Solve this by guess and check.

(b) Solve it by simultaneous equations – remember to create letters for the numbers, set up the equations and substitute to get one letter.

(Use a calculator)

Activity 3: Second method


(a) It has the following equations (substituting $i$ for number of iceblocks and $c$ for number of chocolates, and converting $6.20 to 620 to represent total spent in cents):

\[ i + c = 12 \]
\[ 40i + 60c = 620 \]

(b) It is solved by realising that:

\[ i = 12 - c \]
\[ 40i + 60c = 40(12 - c) + 60c \]
\[ = 480 - 40c + 60c \]

Therefore: \[ 480 + 20c = 620 \]
\[ 20c = 140 \]
\[ c = 7 \] chocolates
\[ i = 12 - 7 \] iceblocks
\[ = 5 \] iceblocks

60 cents 40 cents
2. The equations can be solved another way:

(a) Write down both equations:

\[ i + c = 12 \] \quad \text{A} \\
\[ 40i + 60c = 620 \] \quad \text{B}

(b) Multiply equations so one variable is the same:

\[ A \times 60 \]
\[ 60i + 60c = 720 \]
\[ B \text{ as is} \]
\[ 40i + 60c = 620 \]
\[ \text{Subtract equations} \]
\[ 20i + 0 = 100 \]
\[ i = 5 \]
\[ c = 7 \]

3. Repeat method (2) above for:

(a) Activity 2, problem 2 (\(t\) – number of tricycles, \(b\) – number of bicycles)

\[ t + b = 26 \]
\[ 3t + 2b = 64 \]

(b) Activity 2, problem 3 (\(a\) – amount of adult ticket, \(c\) – amount of child ticket)

\[ 2a + 5c = \$49 \]
\[ a + 3c = \$27 \]

Activity 4: Further problems

1. Solve the following by algebra (simultaneous equations) by any of the methods above.

(a) Of the 73 Year 6 students at Henry State School, there are seven more girls than boys. How many girls? How many boys?

(b) In this two-digit number, the tens digit is greater than the ones digit. The sum of the two digits is 11 and the product is 28. What is the two-digit number?

(c) Fred gave James a box of chocolates containing 30 small bite-size Mars Bars, KitKats and Cherry Ripes. There were twice as many Cherry Ripes as KitKats, there were three times as many Mars Bars as Kit Kats. How many of each chocolate?

(d) Challenge. Sue threw a dice four times. The numbers were different each time. The product was 48, the sum was 13. What numbers did Sue throw?

(e) Challenge. A three-room apartment followed these specifications. The apartment was designed as on the right (not to scale):

- area of living room was 42 m\(^2\)
- shorter side of kitchen was 3 m
- shorter width of bedroom and bathroom was 5 m
- the entire area of the apartment was 110 m\(^2\)

2. Challenge. Operating on equations was the method used in the 1960s and 1970s, while substitution is the method currently used today. Why have the methods changed?
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution</td>
</tr>
<tr>
<td>ACMNA194</td>
<td>ACMNA215</td>
</tr>
<tr>
<td>ACMNA230</td>
<td>ACMNA235</td>
</tr>
<tr>
<td>8</td>
<td>Sketch linear graphs using the coordinates of two points and solve linear equations</td>
</tr>
<tr>
<td>9</td>
<td>Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology</td>
</tr>
<tr>
<td>10</td>
<td>Solve problems involving linear equations, including those derived from formulas</td>
</tr>
</tbody>
</table>
9G1 “Constructive Constructions”

Task description

In this task we will use geometric constructions and deduction to explore many aspects of Euclidean geometry.

1. Research geometric constructions on the internet. A good place to start is [http://www.mathopenref.com/tocs/constructionstoc.html](http://www.mathopenref.com/tocs/constructionstoc.html), but there are many other websites on this topic. Using only a straight edge and a pair of compasses, practise the basic techniques of geometric constructions.

2. Triangles can be classified in two ways: by angle and by side length. Use geometric construction methods to draw an example of each type of triangle. Quadrilaterals can also be classified by a combination of angle and side length. Research their names and use geometric construction methods to draw an example of each quadrilateral.

3. There are three different ways of constructing a triangle that leads to a unique triangle. What are they? Prepare your own diagrams showing how each method works.

4. There is another method of constructing a unique right-angled triangle. What is it? Prepare your own diagrams showing how the method works.

5. There are other methods of constructing triangles, but they all result in more than one type of triangle. To illustrate this, show that it is possible to construct two different triangles if you are given two sides and a non-included angle (that is, the angle is not in between the two sides).

6. The methods of constructing congruent triangles explored in steps 2 to 4 lead to four tests of congruence for triangles. What are they?

7. Construct a set of two parallel lines. Draw in an oblique straight line interval (called a transversal). The transversal creates a total of eight angles. Research the names given to the different sets of angles that appear in your diagram. Which angles appear to be congruent? What is the relationship between the sets of angles that are not congruent?

8. Prove that if co-interior angles are supplementary, then (a) corresponding angles are equal, (b) alternative angles are equal, and (c) vertically opposite angles are equal. If necessary, read the additional information in the handout about this activity.

9. Use the properties of angles in parallel lines to prove that the interior angle sum of a triangle is 180°. Hence prove that the interior angle sum of a quadrilateral is 360°.

10. Practise some proofs that triangles are congruent. There is an example of such a proof in the handout. Your teacher should be able to give you some examples to practise.

11. **Challenge:** Can you mount an argument, based on the five Euclidean postulates, that there is only one line through a given point that is parallel to a given line?

12. **Further challenge:** What are the circle theorems? Can you devise proofs of these theorems using what you know about lines, angles, and congruent triangles?

*Make sure you understand the meanings of any words in italics.*
Additional student information

Activity 1

Geometric constructions use a straight edge (ruler) and a set of compasses only. The ruler cannot be used to measure anything, other than to check the lengths in a completed construction. Similarly, a protractor cannot be used to draw an angle, only to check the angle size in a completed construction.

Accuracy is possible only if your pencil is sharp. It is best to use unlined paper. The construction process makes use of small arcs drawn with the compasses (for an example, see the diagram below). Do not erase them – that is how you show your working.

Practise the basic geometric constructions until you can do them confidently and accurately. They should include bisecting both a line and an angle, constructing a right angle, replicating both a straight line and an angle. Check your results using a protractor and/or a ruler.

Activity 2

Triangles can be classified by angle (acute, right and obtuse) and side length (equilateral, isosceles and scalene). There are six common sub-groups of quadrilaterals.

Activity 3

Uniquely constructing a triangle means that if the instructions are followed correctly only one triangle can be constructed. If the instructions can result in more than one type of triangle, then the triangle you have constructed is not unique.

Confused? Try constructing a triangle with one side measuring 7 cm and another side measuring 8 cm. You should be able to draw many different triangles because the length of the third side has not been specified. On the other hand, if the third side is 9 cm long, you can draw only one triangle. If you think that you have been able to construct two different triangles with sides of 7, 8 and 9 cm, you will find that the two triangles are congruent, although they may have different orientations. We can say, therefore, that the requirement that the sides must have lengths of 7, 8 and 9 cm results in a unique triangle.

More generally, the diagram below shows the construction of a triangle with three sides of given lengths. It is possible to construct only one triangle that has those three sides.

(Source: http://www.mathopenref.com/consttrianglesss.html)

This example shows that if the lengths of the three sides of a triangle are specified, only one triangle can be constructed. This method is usually abbreviated to SSS. There are two remaining methods of constructing unique triangles for you to discover.
Activity 4
This method should be different from the three methods identified in Activity 2.

Activity 5
It may help to start with three specific measurements; for example, construct two different triangles (that is, triangles that are not congruent) where two of the sides are 8 cm and 9 cm and the non-included angle is 30°. Now generalise this approach, by drawing triangles where the measurements of these two sides and one angle change. Are there any circumstances when only one triangle is possible?

Activity 6
Two shapes are congruent when they have the same shape and size. However, they can have different orientations. Congruence is important in our modern world where angles and lengths matter. In this activity we focus on congruence in triangles.

Clearly if all three corresponding angles and all three corresponding sides are equal in two triangles then those triangles must be congruent. However, we do not need that much information. For example, if we know that two corresponding angles are equal then the third set of angles must also be equal since we can deduce the size of the third angle in a triangle from the other two. The challenge of this activity is to find the smallest number of angle and side facts needed to show that two triangles are congruent. Considering the combination of angle and side facts that you needed to know to construct unique triangles (Activities 2 to 4) should help you.

Activity 7
The sets of angles formed in parallel lines are called corresponding, alternate and co-interior. Prepare some diagrams showing the location of these sets of angles. Can you find any vertically opposite angles and any angles on a straight line? How can the vertically opposite angles and any angles on a straight line help you deduce the relationship between the sets of corresponding, alternate and co-interior angles?

Activity 8
Euclid of Alexandria was a Greek mathematician who lived in approximately 300 BCE. He is recognised today as the “Father of Geometry”. Euclidean geometry is a mathematical system described in his textbook called “The Elements”. Euclid described geometry on a plane (a flat space spreading infinitely in all directions). He described the relationships between points, straight lines and circular arcs on a plane, based on five starting ideas (called axioms or postulates):

- A straight line segment (or interval) can be drawn joining any two points.
- Any straight line segment can be extended indefinitely, called a straight line.
- Given any straight line segment, a circle can be drawn with the segment as a radius and one endpoint as the centre.
- All right angles are congruent.
- If a straight line cuts two other straight lines to form two interior angles (on the same side of the line segment) and their sum is less than 180°, then the two lines, if extended far enough, will intersect. Turning this postulate around gives us the more familiar statement that, if the sum of the two interior angles is 180°, the two lines will not intersect; that is, they are parallel.

These postulates are the basic assumptions that underlie the Euclidean system of geometry. They are not capable of being proved. The fifth postulate, called the parallel postulate, turned out to be very important. Many mathematicians believed that it was not a true postulate because it could be proved from the other four postulates. There were many unsuccessful attempts over the millennia since Euclid
lived to prove the parallel postulate until 1826, when two mathematicians showed that it could not be proved. In Activity 7 you observed that co-interior angles are supplementary (add to 180°). You can see now that this geometry “fact” is Euclid’s fifth postulate.

Euclid’s method consisted of deducing many other propositions from the five postulates. These propositions are called theorems and corollaries (a corollary is a proposition that logically follows from a theorem, with little or no additional proof needed).

Now take a walk in Euclid’s shoes by proving three of his theorems. You must give reasons for (justify) all statements that are not assumed to be one of the basic postulates of geometry and/or mathematics.

(a) 

(b) 

(c) 

\[ \text{angle } a = \text{angle } b \] 

\[(a \text{ and } b \text{ are called corresponding angles)}\]

\[ \text{angle } c = \text{angle } d \] 

\[(c \text{ and } d \text{ are called alternate angles)}\]

\[ \text{angle } p = \text{angle } q \] 

\[(p \text{ and } q \text{ are called vertically opposite angles)}\]

**Activity 9**

To prove that the interior angle sum of a triangle is 180° (i.e. in the triangle at right, \[a + b + c = 180°\]), add in a line parallel to any one of the sides, as shown in the diagram below, and use what you know about angles in parallel lines.

Hence prove that the interior angle sum of a quadrilateral is 360°. 
**Hint:** Partition the quadrilateral into two triangles – does this help?
**Activity 10**

Having looked at some simple proofs, now try proving that two triangles are congruent.

In the diagram, \( PR \parallel QS \) and \( PQ \parallel RS \). The pairs of parallel lines intersect at \( P, Q, R \) and \( S \). A straight line joins \( P \) and \( S \), so that \( PS \) is a transversal for both pairs of parallel lines. Show that triangles \( PQS \) and \( SRP \) are congruent.

In \( \Delta PQS \) and \( \Delta SRP \):

\[
\angle RPS = \angle QSP \text{ (alternate angles)}
\]

\[
\angle PRS = \angle SQP \text{ (alternate angles)}
\]

\( PS \) is common (that is, it is a side shared by both triangles)

\[
\therefore \Delta PQS \equiv \Delta SRP \text{ (two angles and corresponding side are equal)}
\]

As a consequence of proving that the \( \Delta PQS \equiv \Delta SRP \), we can also state that \( \angle SPR = \angle PSQ \); \( PR = QS \); and \( PQ = SR \).

So, if we were asked to prove that the opposite sides of a parallelogram are equal, this would be one way of doing it.

**Activity 11**

Use indirect methods to mount your argument, based on the five Euclidean postulates, that there is only one line through a given point that is parallel to a given line. Start with the following diagram:

![Diagram of a line through a point parallel to a given line]

Assume there is a line through the dot parallel to the given line; now assume that there is a second one – what will happen?

**Activity 12**

There are eight theorems that deal with lines and angles associated with circles. Ensure that you understand the meaning of terms such as *centre*, *point of contact* (both are types of points), *radius*, *diameter*, *chord*, *secant*, *tangent* (all are types of straight lines), *circumference*, *major arc*, *minor arc* (all are types of curved lines), *hemisphere*, *quadrant*, *major segment*, *minor segment*, *sector* (all are types of regions) before you start work with the circle theorems.
Essential vocabulary

**Bisect:** To cut or divide an angle, line segment or plane shape into two equal parts. The line dividing the two equal parts is called a bisector.

**Congruent:** To be identical in all respects. Two angles are congruent if they are the same size. Similarly, two line intervals are congruent if they have identical lengths. However, the word congruent is more commonly used in two-dimensional geometry to describe shapes where the corresponding sides and angles are equal. The symbol $\equiv$ means “is congruent to”.

**Corresponding sides and angles:** Corresponding sides are sides that are in the same position in different plane figures. Corresponding angles are angles that are in the same position in different plane figures (although there is a different meaning given to “corresponding angles” when we discuss angles in parallel lines). For example, in triangles ABC and DEF, the corresponding sides are AB and DE, BC and EF, and AC and DF. The corresponding angles are A and D, B and E, and C and F. This example shows that when we give letter names to two congruent triangles, the arrangement of the letters is important.

**Generalise:** Make a general or broad statement (or formulate a rule) by inferring from specific cases; that is, applying the principles learned from an inspection of particular examples.

**Geometric construction:** The drawing of various geometric shapes following the methods developed by Euclid, using only a pair of compasses and a straight edge. The measurement of lengths (with a ruler) or angles (with a protractor) is not allowed.

**Geometric proof:** A step-by-step logical explanation that uses definitions, axioms, and previously proved theorems to draw a conclusion about a general geometric statement.

**Properties:** The basic and/or essential attributes shared by all elements (members) of a set. For example, a description of the properties of a particular type of quadrilateral might include information about whether the shape is closed, the size of the angles, the length of the sides, the length of the diagonals, the angles at which the diagonals intersect, and whether the diagonals bisect each other (cut each other in half).

**Orientation:** The position or alignment of a shape relative to compass points, the positive direction of the $x$-axis in a Cartesian plane, the top of the page, or some other direction.

**Relationship:** A connection between two ideas or things. In mathematics, the word *relation* is often used to describe the connection(s) between an object, pattern, or the value of a variable, before and after a change.

**Test of congruence:** Combinations of the smallest number of congruent sides and/or angles needed to show that two triangles are congruent.

**Vertex** (plural *vertices*): A point formed by the intersection of two straight lines. It is typically used to describe the corners of a polygon or polyhedron or the point of an angle.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of Euclidean deductive geometry. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. For this task, students will need a straight edge (ruler), a pair of compasses and a sharp pencil. Protractors may only be used for checking the size of any angles.

There are two ways to think of congruence between two shapes – by relationship (where we look for the corresponding sides and angles to be equal) and by change (where we get from the first shape to the second shape by one or more flips, slides and turns). The Flippin’ Congruence task (7G1) introduced students to congruence using flips, slides and turns. This task develops the concept of congruence by exploring relationships. To do this, it introduces students to geometric constructions.

For the geometric constructions part of this task, students will need a pair of compasses, a straight edge, a sharp pencil and some blank paper. Insist on work that is neat, accurate and well-labelled. As an alternative, students may be able to use dynamic geometry software such as Geometers Sketchpad or Autograph to perform the constructions, provided that they avoid using functions that copy and paste, or draw lines and angles of specified sizes.

As the task progresses, students are then asked to apply the knowledge obtained from the geometric constructions to develop and apply some general principles about congruence and angles in parallel lines. Eventually, they are asked to apply these principles to develop geometric proofs. This introduces students to the concepts of generality and proof and some ideas about axioms, postulates and theorems.

The proof that alternate angles are equal is shown below.

In the diagram, AB \parallel CD. Line EF is a transversal, cutting AB and CD at X and Y respectively.

\[ \angle AXY \text{ and } \angle DYX \text{ are alternate angles, similarly, } \angle BXY \text{ and } \angle CYX \text{ are alternate angles.} \]

\[ \angle AXY + \angle CYX = 180^\circ \text{ (co-interior angles)} \]

\[ \therefore \angle AXY = 180^\circ - \angle CYX \]

\[ \angle DYX + \angle CYX = 180^\circ \text{ (angles on a straight line)} \]

\[ \therefore \angle DYX = 180^\circ - \angle CYX \]

\[ \therefore \angle AXY = \angle DYX \]

By a similar method, it can be shown that \( \angle BXY = \angle CYX \)

Thus, alternate angles are equal. Strictly speaking, we should say that if co-interior angles are supplementary, then alternate angles are equal.

It is important that students understand that a proof cannot depend on any of the angles having a particular size. It must apply generally, that is, to angles of all sizes.

For Activity 10 students will need some exercises that require them to prove that triangles are congruent. They can be found in many textbooks (especially older textbooks written when geometric proofs received more emphasis in the junior mathematics curriculum) or online. Insist on appropriate
justification of the proofs, including a statement of what is to be proved, reasons for every step (excluding axioms) and a concluding statement – these skills are required for higher level mathematics.

Most of the answers to this task can be found online. Students have been asked to research many aspects of the task. However, ensure that students are not downloading responses to the activities that they have been asked to complete for themselves. For example, in Activity 10, they can download statements of the circle theorems, but not the proofs of them.

Many students will find this task challenging because it introduces them to unfamiliar concepts and ways of thinking. The “Additional student information” handout may assist those students who need more help to progress through the task.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles</td>
</tr>
<tr>
<td>7</td>
<td>Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal</td>
</tr>
<tr>
<td></td>
<td>Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning</td>
</tr>
<tr>
<td></td>
<td>Classify triangles according to their side and angle properties and describe quadrilaterals</td>
</tr>
<tr>
<td></td>
<td>Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral</td>
</tr>
<tr>
<td>8</td>
<td>Develop the conditions for congruence of triangles</td>
</tr>
<tr>
<td></td>
<td>Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning</td>
</tr>
<tr>
<td>10</td>
<td>Formulate proofs involving congruent triangles and angle properties</td>
</tr>
<tr>
<td></td>
<td>Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes</td>
</tr>
<tr>
<td>10A</td>
<td>Prove and apply angle and chord properties of circles</td>
</tr>
</tbody>
</table>
9M1 “Look Out for the Baby!”

Task description

This task requires you to investigate the relationship between volume and surface area of several familiar shapes. That relationship is important in many situations. For example, manufacturers of canned soft drinks need to choose a can size and shape that:

- Minimises the use of aluminium (surface area)
- Maximises the amount of soft drink that can be put in the can (volume)
- Withstands the pressure of the compressed carbon dioxide inside the can (surface area)
- Stacks into cartons for transporting to the seller with the least amount of wasted space between the cans (volume)
- Stacks easily in refrigerators (volume)
- Can be held comfortably in the drinker’s hand (length and width)

The solution to this problem involves complex mathematics, beyond what we can do in junior secondary years.

However, we can look at another situation that links volume and surface area: Why do we have to take more care of babies than adults when it is hot or cold?

Challenge: Explore the volume and surface area relationship further by finding the shape with the largest volume that can be made out of a single A4 sheet of paper.
Additional student information

Since a baby is not an even shape, working out the volume and surface area of a baby is complex. However, we can simplify the task by thinking of the baby as a combination of familiar solid shapes, as shown in the diagram below:

An adult body can be approximated by tripling the dimensions of the baby.

Challenge: What effect will simplifying the overall shape of the human body like this have on our conclusions?
**Teaching information**

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of volume and surface area in Year 9. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. For this task, students need to know, or to learn as they progress through the task, the methods of calculating the volume and surface of prisms, tetrahedrons, cylinders and spheres. This section provides detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

**Activity 1: Doing things by halves**

**Materials:** Plasticine, mass measurer (scales), ruler, calculator

**Set-up:** Students work in groups of three: Leader, Clocker, Recorder (rotate roles)

1. Use a piece of plasticine to make a cube as best as you can. Measure the side of the cube. Work out surface area and volume.

2. Repeat this for a rectangular prism (with length double the width of the rectangle), a tetrahedron, and a sphere (roll plasticine to make best sphere you can).

   Record results on table below. Use a calculator to work out the division.

   **Full-size plasticine:**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume (cm³)</th>
<th>Surface area (cm²)</th>
<th>Volume/Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tetrahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Cut plasticine in half – check with mass measurer (scales) that you have half. Repeat (1) and (2) above and record results on table below. Use a calculator to work out the division.

   **Half-size plasticine:**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume (cm³)</th>
<th>Surface area (cm²)</th>
<th>Volume/Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tetrahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Cut plasticine in half again – check with mass measurer that you have half. Repeat (1) and (2) above and record results on table below. Use a calculator to work out the division.

**Quarter-size plasticine:**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume (cm³)</th>
<th>Surface area (cm²)</th>
<th>Volume/Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tetrahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. What shape gives the highest volume for surface area? What shape gives the lowest?

6. What size plasticine (full, half or quarter) gives the highest volume for surface area? What size gives the lowest?

7. Write some sentences to argue in favour of your conclusions to questions 5 and 6.

**Activity 2: Doubling the net**

**Materials:** Nets, ruler, calculator

1. Make a series of nets for cubes, rectangular prisms and triangular pyramids as follows:

   - A. 5 cm × 5 cm × 5 cm
   - B. 10 cm × 10 cm × 5 cm
   - C. 5 cm × 5 cm × 5 cm

2. Repeat this for nets where the length of the sides is half the above.

3. Work out surface area and volume of each of A, B and C and put on the table below. Use a calculator to work out the division.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume (cm³)</th>
<th>Surface area (cm²)</th>
<th>Volume/Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A half length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B half length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C half length</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. What shape gives the highest volume for surface area? What shape gives the lowest?
5. What size shape gives the highest volume for surface area? What size gives the lowest? The original shape or the double-length shape?

6. Predict the results for another doubling of the net? And for three times the small net?

**Activity 3: Bigger the child**

**Materials:** Paper, ruler, calculator

1. Make a representation of a small child as on right:

2. Make an adult replica of the child that is three times as large.

3. Find surface area and volume of each and record as follows:

<table>
<thead>
<tr>
<th></th>
<th>Total Volume (cm³)</th>
<th>Total Surface area (cm²)</th>
<th>Volume/Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adult</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. What is the difference? Why is this important in hot and cold situations?

5. Present your data and arguments in a genre of your choice (for example, written persuasive text, poster, PowerPoint).

**Activity 4: Challenge**

1. What is the largest volume open box (rectangular prism) you can make from a net cut from an A4 sheet of paper?

2. What is the largest box you can make with a piece of paper of the same area as an A4 sheet but with different dimensions?

3. Make a cylinder out of an A4 sheet long ways and short ways:

   (a) Which of the two cylinders has more volume? (Work this out by formulae and check by pouring sand into cylinder 1. Then pour from cylinder 1 into cylinder 2.)

   (b) Can you make a cylinder from another piece of paper with the same area as A4 that would give an even bigger volume?

   (c) What is the biggest volume you can make with paper having the same area as A4 but with different dimensions?
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>ACMMG111 Connect three-dimensional objects with their nets and other two-dimensional representations</td>
</tr>
<tr>
<td>6</td>
<td>ACMMG137 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td></td>
<td>ACMMG138 Connect volume and capacity and their units of measurement</td>
</tr>
<tr>
<td></td>
<td>ACMMG140 Construct simple prisms and pyramids</td>
</tr>
<tr>
<td>7</td>
<td>ACMMG159 Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</td>
</tr>
<tr>
<td></td>
<td>ACMMG160 Calculate volumes of rectangular prisms</td>
</tr>
<tr>
<td></td>
<td>ACMMG161 Draw different views of prisms and solids formed from combinations of prisms</td>
</tr>
<tr>
<td>8</td>
<td>ACMMG195 Choose appropriate units of measurement for area and volume and convert from one unit to another</td>
</tr>
<tr>
<td></td>
<td>ACMMG197 Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area</td>
</tr>
<tr>
<td>9</td>
<td>ACMMG216 Calculate the areas of composite shapes</td>
</tr>
<tr>
<td></td>
<td>ACMMG217 Calculate the surface area and volume of cylinders and solve related problems</td>
</tr>
<tr>
<td>10</td>
<td>ACMMG242 Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids</td>
</tr>
</tbody>
</table>
9M2 “Square Angles”

Task description

Your task is to create a table to show carpenters and woodworking enthusiasts how to use their framing square to construct a range of angles.

Read the article about using the measurements on the blades of a framing square to construct angles. The table to the left was taken from the article.

1. Alternative versions of these tables can be found on the internet. They keep one measure constant, for example, they keep the blade length fixed at 300 mm and vary the tongue length.

   (a) Use a spreadsheet to construct a table like this.
   
   (b) Try other values for the blade length, e.g. 200 mm, 500 mm.
   
   (c) Does any blade length provide greater accuracy?

2. In the table above, all measurements are in whole millimetres, simplifying the measuring and making the guide easy to use.

   (a) Find similar pairs of measures for the other angles, using whole numbers only. Try to find a blade/tongue pair for an angle of 10°.
   
   (b) Can you repeat this method to complete the table?
   
   (c) Describe the method(s) you used to find this whole number pair.
   
   (d) Do the measurements provided in the table provide exact angle values?
   
   (e) What would be a reasonable degree of accuracy?

3. Which version is better?

   (a) Which table was the easiest to calculate?
   
   (b) Which version would be the easiest to use?
   
   (c) “A large square will provide greater accuracy for this method.” Comment.

4. Challenge: Research the tangent (tan) ratio used in trigonometry. Explain how it relates to this task.
Additional student information

Layout secrets for your Framing Square

Learn a few tips and tricks to get more from this common tool.

A framing square is often considered a rough-use tool — great for building a house, but not accurate enough for fine woodworking. In my opinion, this bad reputation comes from using a poor-quality, inaccurate tool.

Like any layout tool, it pays to spend a little extra money and choose one that’s truly square with crisp, precise markings. Then you can depend on your framing square for a variety of layout tasks.

My square gets the most use for laying out lines on panels and sheet goods. In addition, it works great for verifying that a large assembly is square. The long reference surfaces reveal the slightest inconsistency. But beyond these everyday tasks, there are lots of other handy ways to put your square to use.

**Lay Out Angles.** Carpenters use the various tables found on some framing squares to mark angles for rafters and other framing members. Although these tables don’t work for furniture-sized parts, there are ways to use a framing square to lay out angles. The key is to take advantage of the scales on both legs of the square.

The easiest way is to lay out an angle based on its “rise and run” (1:8, for example). You can plot both measurements at the same time with a framing square. However, you’re more likely to find angles referenced in degrees. To use a framing square here, you just need a little help. With a simple chart, you can use the graduations on the blade and tongue (the shorter arm of the square) to lay out a wide range of angles. In the left drawing, you can see how to lay out several common angles.

**LOCATE WHEELS.** Another way to pu
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching about angles, and an opportunity to introduce the tan ratio used in trigonometry. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice.

This task could become a lot of repetitious trial and error calculations; therefore students need to be encouraged to use appropriate technology to automate the procedure in some way and force them to consider a process to be followed. Spreadsheets are ideal for this investigation. The excerpt below shows a simple set up.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Blade</th>
<th>Tongue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>=TAN(A2*PI()/180)*B2</td>
</tr>
</tbody>
</table>

Notes:

(a) Excel uses angle values in radians when evaluating trig ratios so the conversion factor needs to be included in the formula.
(b) Students will need reminding about appropriate levels of rounding in this practical context.

The full table from the article is available below. It is unlikely that students will get exactly the same values. Some strategies that students might employ include:

1. Start with one of the spreadsheets from the question and select all those that are close to whole number values. Remove these from the spreadsheet and change the blade value and keep repeating the process until they have an acceptable value for all angles.

2. Some students have tried graphing a function and using the trace function to look for points with close to integral values. This type of approach will depend on the level of technology available to the students.

3. The repetitious nature of the task makes it an ideal programming task. Most graphics calculators will provide some programming capacity with the capability to handle this task.
To the left is a program created on the TI-Nspire. It is not very sophisticated and uses the repetitious processing power to test all possibilities for each angle. The program moves on to the next angle after the first pair of values for the blade and tongue length are less than the prescribed error margin, in this case 0.01.

To the right is some sample output from the program. Each row consists of the angle, the blade length and the tongue length. You may wish to have students check the accuracy of the values produced.

An extra task

Ask students to figure out how to draw a circle with a square! Naturally we mean a carpenter’s square. It is a simple practical application of circle geometry that relies on the fact that the angle at the circumference of a semicircle is a right angle, yet not often seen in a maths text.
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ACMNA188 Solve a range of problems involving rates and ratios, with and without digital technologies</td>
<td>ACMMG222 Investigate Pythagoras’ Theorem and its application to solving simple problems involving right-angled triangles</td>
</tr>
<tr>
<td>9</td>
<td>ACMMG223 Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles</td>
<td>ACMMG224 Apply trigonometry to solve right-angled triangle problems</td>
</tr>
</tbody>
</table>
9M3 “Three-Fact Triangles”

Task description

Triangles are the most important shape in engineering. A triangle cannot be deformed without changing the length of a side or breaking one of its joints, so triangles are used to make other shapes more stable. For example, a rectangle can be strengthened by adding a brace across its diagonal length, forming two triangles. If you understand triangles, you will understand any polygon, not just quadrilaterals, because all polygons are made up of triangles. However, mathematicians do not just study something because it is useful. Often they study something because it is fascinating. Triangle geometry is a rich area of geometry filled with beautiful results and unexpected connections with other areas of mathematics.

A three-fact triangle is one where three of the following four facts are true:

- One side is 3 cm
- One angle is 90°
- One side is 4 cm
- One angle is 30°

1. Can all four facts be true in the same triangle? Explain why or why not.
2. How many different three-fact triangles are there? Do any of the combinations of three facts give more than one possible triangle?
3. What extra information could be supplied so that your partner can draw the same triangle as you? You cannot use extra measurements because it still has to be a three-fact triangle.
4. What sets of three facts will enable a triangle to be exactly duplicated (that is, to ensure that the triangles are congruent)? Is there more than one set of facts that will ensure that the triangles are congruent? Can you devise some tests of congruence (that is, some rules that we could use to find out if two triangles are congruent)?
5. Can you use trigonometry to calculate the area and perimeter of the three-fact triangle formed by side lengths of 3 cm and 4 cm and an angle of 30°?
6. Challenge: Can you devise a method or rule to find, in one process, the area of any triangle with sides of length $a$ and $b$ and an angle $\theta$ ($\theta$)? What conditions would have to be imposed on your rule?

Make sure you understand the meanings of any words in italics.
**Essential vocabulary**

**Congruent**: To be identical in all respects. Usually used in two-dimensional geometry to describe shapes where the corresponding sides and angles are equal.

**Corresponding sides/angles**: If the relative position of two sides is the same in two figures, then they are called corresponding sides. Similarly, if the relative position of two angles is the same in two figures, then they are called corresponding angles. Because figures can be presented in a variety of orientations, corresponding sides and/or angles are not necessarily in the same position relative to the page on which they are drawn. However, they are in the same position compared to the other sides and angles.

**Tests of congruence**: Tests that describe the minimum combinations of equal sides and/or angles that are needed to determine if two triangles are congruent.

**Theta**: The eighth letter of the Greek alphabet, written uppercase Θ, lowercase θ or cursive ϑ. In Australia, it is pronounced to rhyme with “Peter”, but in the USA it may be pronounced to rhyme with “later”. Theta, like some other Greek letters, is commonly used in mathematics to label the size (magnitude) of angles.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of congruence in triangles. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

There are four tests of congruence:

- Three corresponding sides in the two triangles are equal (SSS).
- Two corresponding sides and the included corresponding angle in the two triangles are equal (SAS).
- Two corresponding angles and one corresponding side in the two triangles are equal (AAS).
- In right-angled triangles, the hypotenuse and one corresponding angle in the two triangles are equal (RHS).

In the second rule, it is necessary to stipulate that the angle is the included angle. If this does not occur, two different triangles can be constructed, as shown in the diagram to the right.

These are the understandings that students are expected to develop through investigation.

Question 5 of the task requires students to calculate the area of the triangle and missing side lengths by dividing the triangle into two right-angled triangles and the use of right-angled trigonometry. The challenge task invites students to generalise this approach to finding the area into the Cosine Rule, which states that

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>ACMMG042 Describe and draw two-dimensional shapes, with and without digital technologies</td>
</tr>
<tr>
<td>4</td>
<td>ACMMG088 Compare and describe two-dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMMG201 Develop the conditions for congruence of triangles</td>
</tr>
</tbody>
</table>
| 9          | ACMMG222 Investigate Pythagoras’ Theorem and its application to solving simple problems involving right-angled triangles  
ACMMG224 Apply trigonometry to solve right-angled triangle problems |
9M4 “How High is that Tree?”

Task description

It can be difficult to measure the height of a tall object such as a tree or a flagpole. In many cases it is not possible (or safe) to reach the top of the tall object in order to measure the distance to the ground. This means that we must use indirect methods that allow us to estimate the height. This task investigates methods of estimating the height of a tall object.

1. Ask your teacher for the handout on how to estimate heights using the “shadow sticks” method.

2. Select an object around your school where you already know (or can find out) the height (your teacher may be able to assist with this). Demonstrate that the “shadow sticks” method works by using it to confirm the height of that object. If your results are not accurate, suggest reasons for the inaccuracy.

3. Use the “shadow sticks” method to estimate the height of at least three other tall objects. Are your results reasonable? Why/why not?

4. Explain why the “shadow sticks” method works. What mathematical principle(s) does the method rely on? What assumptions do we make when relying on this method? Are the assumptions reasonable? This may require some research.

5. Challenge: Can you find other methods of estimating the height of a tall object (there are at least eight)? In each case, explain how the method works. An explanation includes what the method is, how it works, and why it works (that is, what mathematical principle(s) does the method rely on?). What assumptions are made when using the method and how reasonable are those assumptions?

6. Further Challenge: You are an advisor to the National Parks and Wildlife Service. What method would you recommend that they use to estimate the height of a tree in a national park? Explain the strengths and limitations of that method.

Make sure you understand the meanings of any words in italics.
Additional student information

“Shadow Stick” method

1. Measure the shadow of the tall object.

2. Get a straight stick (a ruler or metre stick will do) and measure the length of the stick.

3. Hold the stick upright and measure the length of the stick’s shadow.

4. Use the “shadow stick” rule:

\[
\frac{\text{length of stick}}{\text{length of stick's shadow}} = \frac{\text{height of tall object}}{\text{height of tall object's shadow}}
\]

5. Substitute the unknown \( x \) for the height of the tall object.

6. Substitute the three measurements from steps 1, 2, and 3.

7. Solve the resulting equation for \( x \) to obtain the height of the tall object.

Essential vocabulary

Accuracy: The accuracy of a measurement is the closeness of the measured value to the true value. A measurement that is not accurate is said to be inaccurate.

Estimate: (noun) an approximation of the value, number, or quantity of something, often used when it is not practicable or necessary to obtain an accurate value or measurement; (verb) the process of obtaining an estimate, usually by a rough calculation, judgement or indirect method of measurement.

Indirect methods of measurement: Measuring something by measuring something else; for example, measuring the height of a tree by measuring the length of its shadow.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of measurement error and the rounding of decimals. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

For steps 1 to 4 of this task, students will need a copy of the “Additional student information” sheet, a straight stick (a ruler or metre stick will do), a 25 or 50 metre tape measure, and a calculator. The “shadow stick” method requires a sunny day. Measurements are better taken at the beginning or end of the school day when shadows are longer. All measurements should be taken at approximately the same time of day (if the angle of the sun in the sky changes substantially between measurements then the results will be inaccurate). The best objects to measure are those with a clearly defined shadow (such as flagpoles, electrical poles, or aerials that are not close to other objects that cast a shadow). If the height of a spreading tree is to be estimated, it may be difficult to determine which part of the foliage is casting the longest shadow.

In step 2, students are asked to verify that the “shadow stick” method works by confirming the known height of an object. They will need teacher assistance in obtaining the height of that object. There may be information in the school (such as architectural drawings) that shows the height of a classroom block or tower in the school grounds. The height of a flagpole could be estimated by measuring the length of the halyard (rope) used to raise and lower the flag. In the absence of such information, the teacher may need to make his/her own estimate of the height to give to the students.

One of the key objectives of this task is for students to learn about similar triangles and their properties. The “shadow stick” method relies on two similar triangles formed by (a) the stick and its shadow and (b) the tall object and its shadow. The triangles have the following properties:

- the angle of the sun is assumed to be the same in both triangles;
- the level (gradient) of the ground in both triangles is assumed to be the same; and
- the stick and the tall object are assumed to form the same angle with the ground.

It follows that the two triangles used in the “shadow stick” method are mathematically similar because the three corresponding angles are equal. As a property of similar triangles is that corresponding sides are in the same ratio, it allows the use of ratios to estimate the height of the tall object.

Examining the assumptions made when using the “shadow stick” method is also an important aspect of the task. A knowledge of these assumptions assists students in applying the method as accurately as possible. It also allows them to assess the accuracy of the resulting estimate. The ability to recognise when assumptions have been made and to evaluate the effect(s) of those assumptions is a skill that is important for students’ future development as mathematicians.

In the first challenge task students are asked to propose alternative methods of estimating heights. Nine methods of estimating the height of a tall object (including, for the sake of completeness, the “shadow stick” method) are summarised on the second handout for students. As students should be able to research some of these methods themselves, it is suggested that the second handout is withheld until after students have conducted their own research.

The second challenge task requires students to evaluate the strengths and limitations of the nine methods of measuring the height of a tree that may possibly be in the middle of a forest. They should take account of practical issues; for example, it may not be possible to see the shadow of a particular
tree when it is surrounded by many other trees, even if the sun is shining. The ability to consider the strengths, limitations, and practicalities of a mathematical process is another skill that is important for students’ future development as mathematicians.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
Methods of estimating the height of a tall object


In this handout, the tall object is assumed to be a tree. However, these methods can be applied to any tall object.

<table>
<thead>
<tr>
<th>“Felling” the Tree Method</th>
<th>![Diagram of Felling the Tree Method]</th>
</tr>
</thead>
<tbody>
<tr>
<td>This is the method used by loggers and naturalists to estimate the height of a tree. Hold a pencil upright in front of you, moving your hand backward or forwards until the top and bottom of your pencil coincide with the tip and base of the tree. Without changing the distance of the pencil from your face, turn it through 90°, ensuring that the bottom of the pencil is still aligned with the base of the tree. Ask a friend to stand beside the tree and then to walk sideways away from the tree until he/she is aligned with the top of your pencil. The distance of your friend from the tree is the same as the height of the tree.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indian Method</th>
<th>![Diagram of Indian Method]</th>
</tr>
</thead>
<tbody>
<tr>
<td>This is a method used by the Indians in Canada. Turn your back to the tree, bend over and look through your legs. Move closer to, or further away from, the tree until you can just see the tip of the tree. If you are of average height, your distance from the tree will be equal to the height of the tree. If you are not of average height, take the mean of the estimates made by all students in the class.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Right-Angled Isosceles Triangle Method</th>
<th>![Diagram of Right-Angled Isosceles Triangle Method]</th>
</tr>
</thead>
<tbody>
<tr>
<td>This method uses the property that the two shorter sides of a right-angled isosceles triangle are equal. Cut a right-angled isosceles triangle out of cardboard and tape a drinking straw along the hypotenuse. Hold the bottom of the straw to one eye, keeping one of the short sides of the triangle in a horizontal position. Move closer to, or further away from, the tree until you can just see the tip of the tree through the straw. In this position your distance from the tree is equal to the height of the tree above your eye level. Add the height of your eye level to your distance from the tree to estimate the height of the tree.</td>
<td></td>
</tr>
</tbody>
</table>
**Shadow Stick Method**

Measure the length of the shadow of the tree. Find a straight stick and measure its length. Hold the stick upright and measure the length of the stick’s shadow. This forms two similar triangles: one involving the stick and the shadow of the stick and a second involving the tree and the shadow of the tree. The estimated height of the tree is calculated using the rule:

\[
\text{Height of tree} = \frac{\text{length of the stick}}{\text{length of the stick’s shadow}} \times \text{length of the tree’s shadow}
\]

**Sighting Along a Stake Method**

Drive a stake vertically into the ground. The stake must be taller than you or you will need to kneel or squat down so that you are shorter than the stake. Move to where the tip of the tree aligns with the top of the stake. This forms two similar triangles: one between you and the tree and another between you and the stake. Measure your distance along the ground from both the stake and the tree. Also measure the difference between your eye level and the top of the stake. The estimated height of the tree is calculated using the rule:

\[
\text{Height of tree} = \frac{\text{distance from tree} \times \text{length of stake above eye level}}{\text{distance from stake} + \text{distance of eye level above ground}}
\]

**Mirror Method**

Place a mirror on the ground and move so that the tip of the tree can be seen in the mirror. This forms two similar triangles: one between your eyes, the ground and the mirror and another between the tip of the tree, the ground and the mirror. Measure your distance along the ground from the mirror and the distance along the ground of the mirror from the tree. Also measure the distance of your eye level above the ground. The estimated height of the tree is calculated using the rule:

\[
\text{Height of tree} = \frac{\text{distance from tree to mirror} \times \text{distance of your eye level above the ground}}{\text{your distance from the mirror}}
\]
Scale Drawing Method

Measure your distance along the ground to the tree. Without changing your position, use an inclinometer to measure the angle of elevation to the tip of the tree. Using graph paper and a suitable scale (for example, 1 cm represents 1 m), draw a horizontal line, to scale, representing the distance along the ground to the tree. At one end of the horizontal line, construct an angle to match the angle of elevation that you measured. At the other end of the horizontal line draw a perpendicular line to represent the tree. Form a right angle triangle by extending the vertical and oblique lines until they intersect. Measure the height of the vertical line that represents the tree and use the scale to convert it to the actual height of the tree. If you measured the angle of elevation from your eye level, then you will need to add the distance from your eye level to the ground.

Trigonometric Method

Measure your distance along the ground to the tree. Without changing your position, use an inclinometer to measure the angle of elevation to the tip of the tree (we will call the size of this angle A). The estimated height of the tree is calculated using the rule:

\[
\text{Height of the tree} = \text{your distance from the tree} \times \tan A
\]

If you measured the angle of elevation from your eye level, then you will need to add the distance from your eye level to the ground.

Inaccessible Object Method

This method is used if it is not possible to measure the distance to the tree (for example, if there is a river in the way). At point A, use an inclinometer to measure the angle of elevation to the tip of the tree. Move to point B, closer to, or further away from, the tree and measure the angle of elevation again. Measure the distance between points A and B. Using graph paper and a suitable scale (for example, 1 cm represents 1 m), draw a horizontal line, to scale, representing the distance from points A and B. Construct the angle of elevation measured at each point and extend these lines until they intersect at point C. Drop a vertical line from point C to the level of the horizontal line – this represents the height of the tree. Measure the height of this vertical line and use the scale to convert it to the actual height of the tree. If you measured the angle of elevation from your eye level, then you will need to add the distance from your eye level to the ground.
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>ACMMA10 0</td>
<td>ACMMG08 4 Use scaled instruments to measure and compare lengths, masses, capacities and temperatures</td>
</tr>
<tr>
<td>5</td>
<td>ACMMA10 0 Use efficient mental and written strategies and apply appropriate digital technologies to solve problems</td>
<td>ACMMG13 7 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>6</td>
<td>ACMMA10 0</td>
<td>ACMMG13 7 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>7</td>
<td>ACMMA10 0</td>
<td>ACMMG13 7 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>8</td>
<td>ACMMA10 0</td>
<td>ACMMG13 7 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
<tr>
<td>9</td>
<td>ACMMA10 0</td>
<td>ACMMG13 7 Solve problems involving the comparison of lengths and areas using appropriate units</td>
</tr>
</tbody>
</table>

- ACMMA10 0: Recognise and solve problems involving simple ratios
- ACMMA10 0: Solve a range of problems involving rates and ratios, with and without digital technologies
- ACMMA10 0: Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar
- ACMMA10 0: Solve problems using ratio and scale factors in similar figures
- ACMMA10 0: Solve right-angled triangle problems including those involving direction and angles of elevation and depression
**9N1 “Powerful Mathematics”**

**Task description**

One of the first mathematicians to consider powers was Euclid, who used the word power to describe the square of a line. Archimedes proved that $10^a \times 10^b = 10^{a+b}$. Rene Descartes (after whom the Cartesian plane is named) was the first to use the notation that we are familiar with today.

In this task, we will examine the meaning of powers when the index or exponent is a natural number, zero, and a negative integer. All powers obey the same set of laws. The definitions, properties and laws of powers are summarised in the Powerful Handout.

Powers are used in scientific (or standard) notation, which allows us to write very large and very small numbers in a convenient form.

**Activity 1**

(a) Read the handout on scientific notation. Can you devise a simple rule to determine the size of the index?

(b) The handout explains how to write very large numbers in scientific notation. How could you modify this to show very small numbers (for example 0.000 000 1435)? (Hint: read the Powerful Handout)

(c) Use the index laws to explain how you can multiply, divide, square and cube numbers in scientific notation without expanding the numbers.

(d) Explain why, in scientific notation, zero is excluded as a value of the index and the coefficient.

**Activity 2 (calculator not permitted)**

You are offered a job, which lasts for seven weeks. You are invited to choose your salary.

- Option 1: You get $100 for the first day, $200 for the second day, $300 for the third day. Each day you are paid $100 more than the day before.

- Option 2: You get 1 cent for the first day, 2 cents for the second day, 4 cents for the third day. Each day you are paid double what you were paid the day before.

Which option will you choose? Why? Now check your answer by calculating how much you will earn (remember, no calculator!). How can you use scientific notation to make your calculations easier?

**Activity 3**

A rapidly evolving bacteria doubles its life span each generation. If the life span of the first generation of bacteria is one second, how many generations will it take until the bacteria can live for more than one billion years?

**Challenge**

By experimenting with some examples using numbers, and then generalising the pattern by replacing the numbers with variables, prove the first index law: $a^m \times a^n = a^{m+n}$. Remember that a proof must show that the law is true for every value of $a$, $m$ and $n$. Examples that rely on specific values, no matter how many of them there are, are demonstrations, not proofs. Use a similar approach to prove the following: $\frac{a^x}{a^y} = a^{x-y}$; $\frac{1}{a^x} = a^{-x}$; $\frac{1}{a^x} \times \frac{1}{a^y} = a^{-x-y}$; $(a^x)^n = a^{nx}$; and $a^0 = 1$.

*Make sure you understand the meanings of any words in italics.*
Powerful handout

Definitions

\[ a^n = a \times a \times a \times a \times ... \ (n \text{ times}) \]

For example, \(3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243\)

\[ a^{-n} = \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \times ... \ (n \text{ times}) = \frac{1}{a^n} \]

For example, \(3^{-5} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{3^5} = \frac{1}{243}\)

Laws

These laws can be proved using the definition of a power

\[ a^m \times a^n = a^{m+n} \quad \text{For example,} \ 3^5 \times 3^2 = 3^{5+2} = 3^7 = 2187 \]

\[ \frac{a^m}{a^n} = a^m \times a^{-n} = a^{m-n} \quad \text{For example,} \ \frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27 \]

\[ (a^m)^n = a^{mn} \quad \text{For example,} \ (3^2)^3 = 3^{2\times3} = 3^6 = 729 \]

Properties

These properties arise as a consequence of the power definitions and laws.

\[ a^1 = a \quad \text{For example,} \ 3^1 = 3 \]

\[ a^0 = 1 \quad \text{For example,} \ 3^0 = 1 \]

\[ \left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times ... \ (n \text{ times}) = \frac{a \times a \times a \times ...}{b \times b \times b \times b \times ...} \ (n \text{ times}) = \frac{a^n}{b^n} \]

For example, \(\left(\frac{2}{3}\right)^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{32}{243}\)

\[ \left(\frac{a}{b}\right)^{-1} = \frac{b}{a} \quad \text{For example,} \ \left(\frac{3}{2}\right)^{-1} = \frac{2}{3} \]

Some useful facts

\[ (ab)^n = a^n b^n \quad \text{For example,} \ (2 \times 3)^5 = 2^5 \times 3^5 \]

\[ (-a)^n = a^n \text{ if } n \text{ is even;} \quad (-a)^n = -a^n \text{ if } n \text{ is odd} \]

For example, \((-5)^2 = 5^2 = 25\), but \((-5)^3 = -5^3 = -125\)
Scientific notation

Scientific notation (also called standard notation) is a method of representing very large and very small numbers as the product of a decimal fraction and a power of 10:

\[ a \times 10^b \]

In this form, ‘\( a \)’ is called the coefficient and ‘\( b \)’ is the index or exponent. The coefficient is defined as a single digit integer (except zero) to the left of a decimal point plus a mantissa, which is significant digits to the right of the decimal point. For example, -569 531 in scientific notation is:

\[ -569531 = -5.69531 \times 10^5 \]

Very large numbers, such as the mass of the earth (5 976 300 000 000 000 000 000 000 kg) are very cumbersome numbers to work with. It is easier if we express it in scientific notation:

\[ 5.9763 \times 10^{24} \]

There are a few rules applying to scientific notation:

- the base is always 10;
- the index (exponent) is a non-zero integer
- the value of the coefficient must be more than −10, less than 10, and not equal to zero.
Essential vocabulary

**Power notation**: A form of mathematical notation, used to show the repeated multiplication of a number (called the *base*) by itself (the *index* or *exponent* shows how many times the multiplication occurs). The plural form of index is *indices*.

For example, $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the index is 5. Here, 3 is the base, 5 is the index, and 243 is the power or, more specifically, the fifth power of 3, 3 raised to the fifth power, or 3 to the power of 5. The word “raised” is usually omitted, and very often “power” as well, so $3^5$ is usually pronounced “three to the fifth”. Similarly $a^n$ is usually read as “$a$ to the $n$”. There are two exceptions to this practice: $a^2 = a \times a$, which is called the *square* of $a$ because it is the area of a square with side-length $a$. It is pronounced “$a$ squared”. The expression $a^3 = a \times a \times a$ is called the *cube* of $a$ because it is the area of a cube with side-length $a$. It is pronounced “$a$ cubed”.

**Integers**: The infinite set of rational numbers that are not fractions. They are the numbers {... −3, −2, −1, 0, 1, 2, 3, ...}. The set is often represented by the symbol $\mathbb{Z}$.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of powers and scientific notation. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. As the glossary defines some terms that students are asked to research, it is for the teacher to decide when to make this glossary available to students. The glossary could be used as the basis of supporting language and literacy activities for students.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ACMNA182 Use index notation with numbers to establish the index laws with positive integral indices and the zero index</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>ACMNA209 Apply index laws to numerical expressions with integer indices</td>
<td>ACMMG219 Investigate very small and very large time scales and intervals</td>
</tr>
<tr>
<td></td>
<td>ACMNA210 Express numbers in scientific notation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACMNA212 Extend and apply the index laws to variables, using positive integer indices and the zero index</td>
<td></td>
</tr>
</tbody>
</table>
9N2 “It’s All Greek to Me”

Task description

In ancient times, people who could calculate were highly in demand. They often became nearly as powerful as the Emperor. They were called “grand viziers”. They have had a bad press – they were nearly always depicted as the baddies. Their jobs were very influential; they looked after the Emperor’s money, determined taxes, charted movements of stars and planets, and built cities and temples. They were accountants, engineers, astronomers, philosophers, teachers, and controlled big businesses. Even today, the study of mathematics leads to these careers (and many others).

There is evidence that the organisations that taught calculation deliberately kept student numbers low to ensure their scarcity (and therefore job security and high salary). Pythagoras (approx. 569 BCE to 475 BCE) and his followers espoused logic as the mathematics of the people and kept calculation for their own members. Included in the many areas that Pythagoreans investigated was the mathematics of music. Remarkably for the times, the Pythagoreans included women.

The Pythagoreans also saw a relation between calculation and religion. They believed one stood for god, two for women, three for men, and so on. They believed that the world created by god depended on rationality and whole numbers. Fractions were included because they were made up of two whole numbers. But suddenly they discovered a number that was not a fraction, an “irrational” (explaining why the numbers were labelled irrational). In their world view, this was a problem and knowledge of it was kept a secret.

We are going to look at these irrational numbers, what they are and what they do.

We start in Activity 1 by recalling Pythagoras’ theorem and how useful it is.

In Activity 2, you are asked to demonstrate that $\sqrt{2}$ is irrational. The method that you will use is known as a proof by contradiction. It depends on an assertion by Aristotle that a logical statement is either true or false and there is no third possibility. Up until the 1950s, some mathematicians could not accept this assertion and, thus, did not accept that proofs by contradiction were valid. What do you think?

Activities 3 to 8 ask you to investigate irrational numbers further. You may need to research some of the concepts on the internet.

Make sure you understand the meanings of any words in italics.
Additional student information

Activity 1

If you have not already learnt about Pythagoras’ theorem, ask your teacher to explain it or research it on the internet. Your teacher might give you some exercises to practise or you can download one of the many worksheets on the internet.

If (when) you think that you are an expert in Pythagoras’ theorem, try the following:

1. Find a rule for the length of the internal diagonal of a cube. (Hint: start by assuming that the sides of the cube are 1 unit long; then find the length of the diagonal if the sides are \( x \) units long.)

2. List three different applications of Pythagoras’ theorem.

Activity 2

1. Some numbers cannot be represented by a fraction (or by \( \frac{\text{whole number}}{\text{whole number}} \)). They are called irrational numbers while fractions are called rational numbers.

2. Show that length \( x \) in the triangle below is not a fraction by assuming it is, and using Pythagoras to get a contradiction.

![Triangle with sides 1 and 1 and hypotenuse \( x \)]

To do this, show by using Pythagoras’ theorem that \( x = \sqrt{2} \).

Start by assuming that \( \sqrt{2} \) is rational. In other words, assume that \( \sqrt{2} = \frac{p}{q} \), where \( p \) and \( q \) are whole numbers that cannot be cancelled down further (that is, they have no common factor). Then square \( \sqrt{2} \) and \( \frac{p}{q} \) to get \( 2 = \frac{p^2}{q^2} \) and discuss how this can be if \( p \) and \( q \) are fully cancelled down.

Activity 3

Find other irrationals on the internet, for example, \( \pi \). What other irrational numbers are there?

Activity 4

2. What kind of decimals can you have?

3. Take these types of decimals and show which can be fractions.

4. Take fractions and show what kind of decimals they can become if they are divided out.

5. Argue that this shows the kind of fractions that are irrationals.

Activity 5

Look at equivalent fractions:

3. Can these be put into sequences all equal to the first one?

\( \frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \text{and so on} \)
6. Can you put all the fractions in a grid starting with \( \frac{1}{1} \) then \( \frac{1}{2} \) and so on?

\[
\begin{align*}
\frac{1}{1} &= 2 \\
&= \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots \\
\frac{1}{2} &= \frac{2}{4} = \frac{3}{6} = \ldots \\
\frac{1}{3} &= \ldots \\
\frac{2}{3} &= \ldots
\end{align*}
\]

7. Show that these can be counted by following the pattern:

\[\text{Diagram of counting pattern}\]

**Activity 6**

4. Can the irrational numbers be counted?

8. Can rational numbers be counted?

9. This makes two types of infinity – countable and uncountable. Are there other types of infinity?

**Activity 7**

We have seen that we cannot express an irrational number exactly as a fraction or decimal. Yet, we can construct a line that is exactly \( \sqrt{2} \) units long (this makes it a *real number*). Show how we can do this. What about the other irrational numbers?

**Activity 8**

What are *real numbers*? Can you draw a “family tree” that shows the relationships between the types of numbers you have encountered so far in your study of mathematics? Include counting numbers, whole numbers, positive and negative whole numbers, zeros, integers, rational numbers, common fractions, the different types of decimal fractions, and the different types of irrational numbers.
Essential vocabulary

**Algebraic numbers**: The infinite set of numbers that could be the solution of an equation in one variable with rational coefficients and constants.

**Common fractions**: The set of rational numbers written in the form \( \frac{p}{q} \), that is as a numerator and denominator (which are both integers, \( q \neq 0 \)) separated by a horizontal or slanted line (called a vinculum), for example, \( \frac{1}{2} \) or \( \frac{3}{4} \).

**Decimal fractions**: The set of rational numbers where the denominator is a power of ten and the numerator is expressed by figures placed to the right of a decimal point, for example 1.456.

**Infinity**: An abstract concept describing something without any limit. The concept of infinity is represented by the symbol \( \infty \). Infinity is not a number or value, nor is it measurable. It follows that it is incorrect to describe a value as being *equal to infinity* (= \( \infty \)); the best that can be said is that a value approaches infinity \( (\to \infty) \).

**Integers**: The infinite set of rational numbers that are not fractions. They are the numbers \{... –3, –2, –1, 0, 1, 2, 3, ...\}. The set is often represented by the symbol \( \mathbb{Z} \).

**Irrational numbers**: Any real number that cannot be expressed as a ratio of integers. Irrational numbers cannot be represented as terminating or repeating decimals. They are often divided into algebraic irrationals and transcendentals. The set of irrational numbers is often represented by the symbol \( \mathbb{Q} \).

**Natural numbers**: The infinite set of numbers \{1, 2, 3, 4, ...\}. The set is often represented by the symbol \( \mathbb{N} \). Also known as counting numbers.

**Proof by contradiction**: A form of proof, and more specifically a form of indirect proof, that establishes the truth or validity of a proposition by showing that if the proposition is true it would eventually result in a contradiction or impossibility. Also known as an apagogical argument, proof by assuming the opposite, and in Latin as *reductio ad absurdum*.

**Rational numbers**: The infinite set of numbers that could be written as a ratio of integers, that is, in the form \( \frac{p}{q} \), where \( p \) and \( q \) are whole numbers that have no common factor.

**Real numbers**: Any number that could be represented on an infinitely large number line, that is, they have a measurable value. The set of real numbers is often represented by the symbol \( \mathbb{R} \).

**Whole numbers**: The infinite set of numbers \{0, 1, 2, 3, 4, ...\}, that is, the set of natural numbers with the inclusion of zero.

Some words have several meanings. These definitions give the mathematical meanings of the words.

You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of irrational numbers. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

The task is designed for students who have mastered the basics of Pythagoras’ theorem. It gives them an opportunity to develop an understanding of irrational numbers and to link their knowledge to the overall structure of mathematics.

The first part of Activity 1 is intended to refresh students’ understanding of Pythagoras’ theorem. Students who have not previously encountered the theorem may need some additional teacher support at this point. The solution to question 1 in Activity 1 is $x\sqrt{3}$ (when the sides of the cube are $x$ units long). Some practical applications of Pythagoras’ theorem might include finding the distance of a short side or the hypotenuse of a right-angled triangle in a range of building or navigation applications; checking to see if an angle is a right angle (tilers need to know this before laying floor and wall tiles); and the formula for the distance between two points used in Cartesian/analytic geometry.

In Activity 2, for those unfamiliar with the proof that $\sqrt{2}$ is irrational it is shown below:

Assume that $\sqrt{2}$ is rational

Thus $\sqrt{2} = \frac{p}{q}$, where $p$ and $q$ are whole numbers that have no common factor

(this follows from the definition of a rational number).

Squaring both sides, $2 = \frac{p^2}{q^2}$

So, $2q^2 = p^2$

So if $p^2$ is an even number then $p$ must also be an even number, because the square of an odd number is always odd.

If $p$ is even, then we can say that $p = 2k$ where $k$ is a whole number.

But, from above, $2q^2 = p^2 = (2k)^2$

Thus, $2q^2 = 4k^2$

$q^2 = 2k^2$

So, $q^2$, and hence $q$, is also even.

If $p$ and $q$ are both even then they have a common factor of 2. This contradicts the original assumption that $p$ and $q$ have no common factor. So that assumption must be incorrect and, consequently, the assumption that $\sqrt{2}$ is rational is also incorrect.

Thus, $\sqrt{2}$ is irrational.

QED

In Activity 7, students are asked to construct a line that is $\sqrt{2}$ units long. As students require familiarity with geometric construction techniques, this task should be sequenced after task 9G1 Constructive Constructions.
The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. As the glossary defines some terms that students are asked to research, it is for the teacher to decide when to make this glossary available to students. The glossary could be used as the basis of supporting language and literacy activities for students.

**Links to the Australian Mathematics Curriculum**

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
<th>Measurement and Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>ACMNA186 Investigate the concept of irrational numbers, including π</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>ACMMMG222 Investigate Pythagoras’ Theorem and its application to solving simple problems involving right-angled triangles</td>
<td></td>
</tr>
</tbody>
</table>
9N3 “Which Card?”

Task description

Financial literacy is an important life skill. You should understand the advantages and disadvantages of using credit cards, debit cards and cash, and be able to make the best choice for your circumstances.

Investigate the different types of debit and credit cards available. Ensure that you consider the following:

- credit cards with and without an annual fee
- credit cards with reward programs
- credit cards offered by different types of businesses, for example, banks, supermarkets, petrol suppliers
- credit cards with an introductory offer
- credit cards that allow you to transfer your balance from another credit card at no/low interest
- debit cards.

Compare the similarities and differences between each type of card and also with cash purchases. Describe the spending patterns and other circumstances that would make each type of card the best option for a customer.
Teaching information

This task, with some teacher involvement, provides a hands-on activity based on financial mathematics. It draws on an understanding of fixed costs (such as annual fees) and variables costs (such as interest). Students are expected to compare each type of card and cash payments. As these comparisons are complex, typically involving many areas of difference, students may consider the use of a table to summarise the information. They will need to give careful consideration to the column headings that they choose.

Students are required to describe the typical expenditure patterns and other characteristics that would make each type of card (or cash) the best choice for an individual. They should also take into account intangible matters such as the willingness or ability to control spending decisions, security, convenience, etc.

This task is similar to work often undertaken by students who select numeracy-based courses in the senior years of schooling. It is included in this package because the students undertaking these extension tasks typically go on to select higher level mathematics courses in the senior years that do not have sufficient time available for a detailed study of financial literacy issues.

Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Number and Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMNA174 Investigate and calculate ‘best buys’, with and without digital technologies</td>
</tr>
<tr>
<td>8</td>
<td>ACMNA189 Solve problems involving profit and loss, with and without digital technologies</td>
</tr>
<tr>
<td>9</td>
<td>ACMNA211 Solve problems involving simple interest</td>
</tr>
<tr>
<td>10</td>
<td>ACMNA229 Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies</td>
</tr>
</tbody>
</table>
**9P1 “Is Greed Good?”**

**Task description**

*Probability* is an important part of our world. We use it to decide on risk (for example, which is the best investment for our retirement). It determines our chances of winning Lotto and is a big part of many games – helping us to find the best strategy. This challenge requires you to use probability to find the best strategy in the “Greedy Pig” game.

“Greedy Pig” (see <http://nzmaths.co.nz/resource/greedy-pig-0>) is a game that motivates the study of probability. The idea is that the teacher has a *die* and the students stand up. The teacher throws the die with the following results:

(a) If the die is 2 to 6, the students add the amount thrown to their score, and the teacher continues throwing.

(b) If the die is 1, students standing lose their score and go back to zero and the game ends.

(c) If a student sits down before the throw, he/she retains the score he/she has and may record it.

For each game, the teacher throws the die until all sit down or a 1 is thrown. More than one game is played and each student’s total score is the sum of their scores in each game. Throwing a 1 only affects the game in which the 1 is thrown.

This task, “Is Greed Good?”, is based on the “Greedy Pig” game.

After becoming familiar with the game by playing it, investigate:

1. How many throws are needed before you are more likely to lose than to win? What is the best time to sit down, that is, after how many throws of the die should we stop?

2. **Challenge**: Is there a strategy based on your score so far that you can use to maximise your chances of winning? For example, is there a particular score that is the best one to sit down at?

*Make sure you understand the meanings of any words in italics.*
**Essential vocabulary**

**Die** (plural dice): A small throwable object with multiple resting positions, used for generating random numbers. Traditionally, a die is a cube with rounded edges, with each of its six faces showing a different number of dots (pips) from one to six. When thrown or rolled, the die comes to rest showing on its upper face a random number from one to six, with each value being equally likely. However, other polyhedrons can be used, for example a decahedron could be used to generate random numbers from zero to nine.

**Equally likely:** Outcomes are equally likely when they have the same probability of occurring. In the cases of two outcomes, the *probability* of each outcome is said to be 0.5. Synonyms are: even chance, fifty-fifty.

**Expected value:** The weighted average of all outcomes, explained by the rule:

\[
\text{expected value of an outcome} = \text{value of the outcome} \times \text{probability of the outcome}
\]

If there are several possible outcomes, then the expected values of the individual outcomes are added. The value of a loss is usually negative. The expected value is represented symbolically as \( E(x) \), where \( x \) is the outcome.

**Experiment:** In probability, an experiment is a process involving chance that leads to results called outcomes. It can have one or more steps (trials). An example would be tossing a coin ten times.

**Likely:** An *outcome* is likely when it is expected to occur more often than not. In these cases, the *probability* of the outcome is said to be greater than 0.5 but less than one. Synonyms are: odds on, probable, good chance.

**Outcome:** An outcome is the result of a single *trial* of a probability experiment.

**Probability:** A measure of how likely an event is. While a probability can be described using words such as certain, likely, and impossible, it is measured by assigning a value between 0 (impossible) and 1 (certain). Probabilities can also be expressed as percentages. Synonyms: chance, likelihood.

**Trial:** A single stage of a probability experiment. An experiment can consist of one or more trials. For example, in an experiment of tossing a coin ten times, a trial would be tossing the coin once.

Some words have several meanings. These definitions give the mathematical meanings of the words.

You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching probability, especially applied to games of chance. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Before commencing this activity it is recommended that you read the “Greedy Pig” resource <http://nzmaths.co.nz/resource/greedy-pig-0>. That webpage provides the solutions to the activity and a discussion of those solutions.

To successfully undertake this task, students need to understand several concepts:

- The use of a mean result to represent a variety of outcomes. In the Greedy Pig game, if students can win 2, 3, 4, 5 or 6 points, then the mean value of a win is 4.

- Probability is multiplicative. In games that involve repeated rounds, the probability of winning \( n \) rounds is \( p^n \), where \( p \) is the probability of winning one round.

- The concept of expected value. The expected value of continuing to play for another round of the game can be summarised as follows:

  \[
  E(\text{playing}) = \text{outcome of a win} \times P(\text{win}) + \text{outcome of a loss} \times P(\text{loss})
  \]

  Note that the outcome of a loss is usually negative.

- (for the challenge task) In games of chance, if \( E(\text{playing}) \) is positive, then you should continue to play. In the case of the Greedy Pig game, the value of a win is +4 (on average), while the value of a loss depends on how many points you have accumulated. So, if you have accumulated 10 points already, then the value of the loss is \(-10\), and the expected value of continuing to play is:

  \[
  E(\text{playing}) = +4 \times \frac{5}{6} + (-10) \times \frac{1}{6} = 1.67
  \]

  Since, in this case, the expected outcome of the next round of the game is positive, it is worthwhile remaining standing for that round. When the \( E(\text{playing}) \leq 0 \), students should sit down.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
Activity 1: Learning the game
1. Play 5 games. See what total score you get across the 5 games. Were you greedy and got wiped out (lost your score) too many times? Were you too careful and did not get any large scores?
2. Repeat (1) and have another 5 games. Try different strategies. Can you identify better ways to play and get a larger score?
3. Discuss with the class the students’ strategies to maximise their scores. Ask the students if they think there is a best time to sit down either in terms of number of throws or in terms of score achieved.

Activity 2: Students experiment on their own
1. In a small group or individually, students are given a die to practise the game.
2. Students trial different times to sit down. What seems to be the best time?
3. What score is not greedy enough? What score is too greedy?
4. Ask students to work with other groups to gather data. Get someone to always sit down after 3 throws, 4 throws, 5 throws, 6 throws, 7 throws, 8 throws, 9 throws. See who scores the worst.

Record results in a table:

<table>
<thead>
<tr>
<th>Throws to sitting down</th>
<th>Scores for 5 sets of 5 games</th>
<th>Average score per game</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st set</td>
<td>2nd set</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Try to hypothesise the right time to sit down and trial these hypotheses.

Activity 3: Using tree diagrams
1. Ask students to look at this theoretically – draw a tree diagram as below.

```
1st throw
   1
   2–6
```

What is the probability of the first throw?
2. Extend this to two or more throws

```
1  
2–6  Throw 1

1  
2–6  Throw 2

1  
2–6  Throw 3

1  
2–6  Throw 4
```

What is the chance of getting here after 4 throws?

3. Complete a table as follows (use a calculator):

<table>
<thead>
<tr>
<th>Number of throws</th>
<th>Chance of not getting a 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{5}{6} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{5}{6} \times \frac{5}{6} = )</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

4. What number of throws has the best chance of not being a 1? When do the throws go below 50% probability? Is this the time to stop?

**Activity 4: Challenge**

1. What is the best time to stop? What is the best score to stop at?

2. Is experience or theoretical analysis the best way to get an answer?

3. Can you set up a simulation on a computer (perhaps using spreadsheet software) that will run the game many times so you can see the results?
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP168 Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP204 Identify complementary events and use the sum of probabilities to solve problems ACMSP205 Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’.</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP225 List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events ACMSP226 Calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’</td>
</tr>
<tr>
<td>10</td>
<td>ACMSP249 Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence</td>
</tr>
</tbody>
</table>
9P2 “Monopolising Monopoly”

Task description

Monopoly is a board game that was created during the depression of the 1920s. It has sold millions of copies all over the world and has spawned thousands of different-themed versions. The aim of the game is to survive and earn as much money as possible. The game ends when everyone is bankrupt (out of money) except one person. Players can earn money in many ways but the most common is to purchase properties and build houses and hotels on these properties. Players pay rent for using (that is, landing on) the properties owned by other players. The rent increases as the number of houses and hotels increase on each property. To move around the board each player rolls two dice. There are many pitfalls for players when they land on the wrong squares.

In business, a monopoly occurs when one person or company is the only operator in a particular line of business; for example, the AFL has a monopoly over the professional Australian Rules football competition (there are related words such as duopoly and oligopoly – what would they mean?). The aim of the game of Monopoly is for one player to develop a monopoly of all of the properties on the board (that is, to be the owner of all of the properties).

In this task we will explore some probabilities associated with the game of Monopoly.

1. Start by working together in groups of three or four to become familiar with the game by playing it. Explore the combinations possible when rolling two dice, how likely each combination is, and how this affects the probability of a player landing on particular properties. Write a game plan (strategy) for someone wanting to win at Monopoly (consider: when to buy and sell properties; which properties to buy; when to build houses and hotels).

2. To simplify the study of the probabilities in the game of Monopoly, you could start with a simplified version named after your school, called the [School Name] Property Game; answering the questions about that game on the handout that your teacher can give you.

3. Mayfair is the square on the Monopoly board with the highest rent but has the highest building costs. The timing of your purchase of houses and hotels is critical, so having an idea about when a player will land on Mayfair is critical. Use probabilities to explain when you would build a house on Mayfair. Your teacher can give you a table to complete to help you to do this.

4. Which group of properties would it be better to purchase: the green or the blue? Justify your answer using probabilities, rent amounts and other costs.

5. Finally, having studied the game of Monopoly in detail, decide which is the greater influence on the result in a game of Monopoly – the skill of the player or the roll of the dice? Write a detailed justification of your answer.

Make sure you understand the meanings of any words in italics.
Additional student information

[School Name] Property Game

Rules

The game is played with two dice but the 4, 5 and 6 of each dice count as 1, 2 and 3. The two dice are thrown and their numbers added, to give values between 2 and 6, to move each player’s counters around the board. Similar to Monopoly, the players start from $, they have money, there is a bank, they can buy numbers they land on, and charge rent. The rules are:

- Start the game with $400, and collect another $100 every time you pass $.
- If you land on a property, you can buy it. Costs are: $40 each for 1, 2, 3, 4 and 5; $60 each for 7 and 8; $80 each for 10, 11, 12, 13 and 14; $100 each for 16 and 17. Properties 6, 9 and 15 cannot be purchased. This gives you a chance of landing on a square without cost similar to “Chance” and “Community Chest” in the real game of Monopoly.
- You can mortgage bought properties to the bank for half their value (you cannot charge rent on mortgaged properties); you can sell properties to other players for whatever you can get.
- When players land on a property that has been sold to another player, they pay rent worth half the value of the property (rent is doubled if you own two properties side by side).
- The winner is the last person with money (if a player runs out of money they have lost and must leave the game).

Questions

You should have an A4-sized copy of the playing board. Play the game in pairs, focusing on how best to play the game. Think about the following questions.

1. Probability vs. strategy vs. maths:

   (a) What can you control and what can’t you control? Make lists.
   
   (b) Is there a best strategy?
   
   (c) What maths do you need (e.g. good mental arithmetic skills)?

2. Important probability/strategy decisions:

   (a) Is it best to buy any property that you land on or should you focus on certain properties? What happens if you never land on them?
   
   (b) Are some properties better than others? Which ones are they? Why?
   
   (c) Should you keep money in reserve for rent or go for broke, i.e. buy all you can?

3. Probability knowledge:

   (a) You’ve bought 17 and 16 and can charge $100 rent. What is the chance that people will land on one of these two as they go around the board?
   
   (b) Can skill help you overcome probability in the game?

   (This last question is very difficult, but your teacher can give you some ideas about how to investigate the probabilities).
**Monopoly game**

Use the table below to examine the probability of landing on Mayfair from each possible square on the board. Any square more than 12 spaces back from Mayfair will have a probability of zero (you cannot throw a value higher than 12 with two dice). From which square are you most likely to land on Mayfair? How does this affect the way you would purchase properties (houses or hotels) on Mayfair? Explain when you would purchase a house on Mayfair.

There is one other way of landing on Mayfair that has not been covered. What is it and what is the probability of landing on Mayfair in this way?

**Probability of landing on Mayfair**

<table>
<thead>
<tr>
<th>Number</th>
<th>Combinations</th>
<th>Number of combinations</th>
<th>Probability</th>
<th>Starting square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fraction</td>
<td>Decimal</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
[School Name] Property Game board

[SCHOOL NAME] PROPERTY GAME
Essential vocabulary

**Monopoly:** (1) A situation in which a single business owns all or nearly all of the market for a particular type of product or service. As there is no competition, monopolies often result in high prices and/or inferior products. (2) A board game in which players engage in simulated property and financial dealings using imitation money. It was invented in the US and the name was adopted by Charles Darrow in approximately 1935.

**Mortgage:** A legal agreement by which a bank, building society, etc. lends money (called a loan or debt) in exchange for taking title of the borrower’s real estate property. The borrower is obliged to pay back the mortgage with a predetermined set of payments. Mortgages are used by individuals and businesses to make large real estate purchases without paying the entire value of the purchase up front. Over a period of many years, the borrower repays the loan, plus interest, until he/she eventually owns the property free and clear.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of probability, based on the game of Monopoly. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

Students will need access to the game of Monopoly. The game involves a number of factors, including luck and skill, but students can use probability theory to get a better understanding of the game and develop strategies to improve their chances of winning. As the game is quite complex, they may find it easier to develop an understanding of the game by first playing a simplified version of the game (which you can name after your school), called the [School Name] Property Game. Included in this package is a game board template and a handout you can give students on how to play the game. (Note: the [School Name] Property Game is a made-up simpler version of Monopoly. The game is still being worked out. Feel free to tweak rules and money any way you like, to make the game work. There is some concern that the starting and passing-S amount of money is wrong. So you could set the students the task of improving the game when they play it.)

At the end of the game, students are asked to form an opinion as to whether Monopoly is a game of skill or chance. The requirement to justify their decision in writing provides an opportunity for students to learn how to plan and compose a piece of extended writing in a mathematics context, understanding the importance of relying on facts rather than opinion.
Activity 1: Getting to know the game

1. Play Monopoly in groups of 3 or 4. Get to know the rules and the things that players can do. Discuss the game – how to win, and how not to lose.

2. Roll the dice 30 times. For each roll of the dice, write down which squares you would have landed on, imagining every time that you are starting from GO. Remember you can’t roll a 1 with two dice. Record the data in a table like the one below.

<table>
<thead>
<tr>
<th>Property</th>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer the following questions using your findings.

(a) Which square is a player most likely to land on?
(b) Which property is a player most likely to land on?
(c) What property is a player most likely to purchase first?
(d) Draw a graph showing the number of times you land on a square.

3. Write a game plan for someone wanting to play Monopoly. Use the following headings and subheadings.

(a) Know the equipment
   • What is in the game of Monopoly? How much money does each player start with?

(b) Property purchasing
   • When is the best time to buy? Discuss when to buy, for example, at the start of the game in comparison to later in the game. When is the best time to sell? What are the reasons for mortgaging property? How can you trade property? What properties are best to aim for and why?

(c) Renting guide
   • What are the amounts of rent? How will you be able to increase the rent? What are the most commonly landed-on properties? Give reasons why.

(d) House/Hotel building
   • When is the best time to build? When is the ideal time to sell? What are the pitfalls of selling?

(e) Possible ways of earning/losing money
   • Jail – how do you get in? When to stay in and when and how to get out. How do the Tax squares work? What is the role of the Chance/Community Chest cards? Which cards will give you the best rewards?

(f) Factors out of your control when playing Monopoly. Discuss the following:
   • The roll of the dice, the human element of property buying/trading, how to conduct yourself as a player, other elements of the game that could affect the outcome.
**Activity 2: Playing a simpler game**

Monopoly is an interesting and complex game. As a board game, it is classified as a race game. Probabilities involve chances of falling on particular squares as two dice are thrown to determine the movements around the board. Maths knowledge relates to property finance and probability. It is useful to start by looking at a smaller version of the game. We shall call it the [School Name] Property Game. The board is below with a larger example in the Additional student information.

**School game rules:** The game is played with two dice but the 4, 5 and 6 of each dice count as 1, 2 and 3. The two dice are thrown and their numbers added, to give values between 2 and 6, to move each player’s counters around the board. Similar to Monopoly, the players start from $S$, they have money, there is a bank, they can buy numbers they land on, and charge rent. The rules are:

- Start the game with $400, get $100 every time you pass $S$.
- If you land on a property, you can buy it. Costs are: $40 each for 1, 2, 3, 4 and 5; $60 each for 7 and 8; $80 each for 10, 11, 12, 13 and 14; $100 each for 16 and 17. Properties 6, 9 and 15 cannot be purchased. This gives you a chance of landing on a square without cost similar to “Chance” and “Community Chest” in the real game of Monopoly.
- You can *mortgage* bought properties to the bank for half their value (you cannot charge rent on mortgaged properties); you can sell properties to other players for whatever you can get.
- When players land on a property that has been sold to another player, they pay rent worth half the value of the property (rent is doubled if you own two properties side by side).
- The winner is the last person with money (going broke means leaving the game).

**School game questions:** Play the game in pairs but tell students to focus on working out how best to play the game. Questions that could be asked are as follows.

1. **Probability vs. strategy vs. maths:**
   (a) What can you control and what can’t you control? Make lists.
   (b) Is there a best strategy?
   (c) What maths do you need (e.g. good mental arithmetic skills)?

2. **Important probability/strategy decisions:**
   (a) Is it best to buy any property that you land on or should you focus on certain properties? What happens if you never land on them?
   (b) Are some properties better than others? Which ones are they? Why?
   (c) Should you keep money in reserve for rent or go for broke, i.e. buy all you can?

3. **Probability knowledge:**
   (a) You’ve bought 17 and 16 and can charge $100 rent. What is the chance that people will land on one of these two as they go around the board?
   (b) Can skill help you overcome probability in the game?
Activity 3: Applying probability to landing on 17

Question 3 in Activity 2 is very difficult but we can do some work to help us out.

1. Work out all the possibilities in terms of moving. Two dice – 1, 2 and 3 are possible so the array of possibilities is:

<table>
<thead>
<tr>
<th>1 1</th>
<th>1 2</th>
<th>1 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 1</td>
<td>2 2</td>
<td>2 3</td>
</tr>
<tr>
<td>3 1</td>
<td>3 2</td>
<td>3 3</td>
</tr>
</tbody>
</table>

1 1 gives 2; 1 2 and 2 1 give 3; 1 3, 2 2, 3 1 give 4;
2 3 and 3 2 give 5; and 3 3 gives 6. So the probabilities are: 2 and 6 – \( \frac{1}{9} \) each; 3 and 5 – \( \frac{2}{9} \) each; and 4 – \( \frac{1}{9} \).

2. Work out from where you can land on 17. S, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are more than 6 away so you cannot land on 17 on your next move – probability of 0. This leaves 11 to 16. Use the probabilities from the array to work out chances from these numbers:

(a) 11 – need a 6 – probability \( \frac{1}{9} \)
(b) 12 – need a 5 – probability \( \frac{2}{9} \)
(c) 13 – need a 4 – probability \( \frac{3}{9} \)
(d) 14 – need a 3 – probability \( \frac{2}{9} \)
(e) 15 – need a 2 – probability \( \frac{1}{9} \)
(f) 16 – cannot throw a 1 so no chance – probability 0.

A table can be drawn up with all possibilities – possible moves (2 to 6), ways of making the move (e.g. for a move of 3, this is 1 2 and 2 1); numbers of combinations (e.g. for a move of 3, there are two ways), probabilities in fractions (e.g. \( \frac{7}{9} \)), probabilities in decimals, probabilities in %, and starting square that will get you to 17 with a move of 3 (e.g. 14). Bar graphs can also be drawn for each square S to 17 giving probabilities.

3. Initially be general in your thinking. Look at possible moves generally. When we roll only 1-2-3 on the two dice, we have five possibilities of movement (2, 3, 4, 5 and 6). **Now the 4 is most likely and also the average of the moves.** What does this mean? It means, in general, that, on average, you will move four spaces each throw. Of course in reality this will not happen but, if probabilities hold, it should average out at multiples of four – you throw 2, then you throw 6 (total 8) – you throw 5, then 3 (total 8) – you throw 6 and 5 and then 2 and 3 (total 16). Of course probabilities never hold in a few games – you could throw all sixes. But it does mean this – **on average**, over a lot of games:

(a) you will move around the board in fours;
(b) it will take five throws to pass S again (18 squares);
(c) it is more likely that you will land on multiples of four (so 16 is better than 17?);
(d) it is likely that one of your throws will take you into the four squares that include 17.

4. If you have time and inclination, you can work out all the possibilities that will get you to 17. It would be a huge tree diagram. First decision would be that it is possible to throw a 2, 3, 4, 5 or 6 with your first throw. Then for each of these, the next step would be another possible five moves between 2 and 6. You could do it as a complete list, starting from the largest possible number of throws to get to 17 and working towards the smallest possible number of throws, for each starting number from 2 to 6:

- eight throws (seven 2s and one 3, with eight possible positions for the 3)
- seven throws (six 2s and one 5, with seven possible positions for the 5)

You could work out the probabilities of each member of the list, for example:
• eight throws (seven 2s and one 3) is $\frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{2}{9} \times 8$.

• seven throws (six 2s and one 5) is $\frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{2}{9} \times 7$.

This needs a computer, and a lot of time (I wouldn’t do it) but in this way you could work out the probability of landing on 17 – that could be done.

**Activity 4: Probabilities in Monopoly – focusing on Mayfair**

1. Mayfair is the square with the highest rent but has the highest building costs. The timing of your purchase of houses and hotels is critical so understanding when a player will land on Mayfair is critical.

   Examine the probability of landing on Mayfair from each possible square on the board. Obviously any square more than 12 spaces back from Mayfair will have a probability of zero (you cannot throw a value higher than 12 with two dice).

   Use the table named *Probability of landing on Mayfair* in the Additional student information to work out your answer.

2. Draw a graph that represents the probability of landing on Mayfair from each square. From which square are you most likely to land on Mayfair?

3. How does the answer to question 2 affect the way you would purchase properties (houses or hotels) on Mayfair? Explain when you would purchase a house on Mayfair.

4. There is one other way of landing on Mayfair that has not been covered. What is it and what is the probability of landing on Mayfair in this way?

5. After researching the probabilities of landing on Mayfair, which group of properties would it be better to purchase: the green (Bond Street, Regent Street and Oxford Street) or the blue (Mayfair and Park Lane)? Justify your answer using probabilities, rent amounts and other costs.

**Activity 5: Conclusions**

1. What is the greater influence on the result in a game of Monopoly – the skill of the player or the roll of the dice?
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP168 Assign probabilities to the outcomes of events and determine probabilities for events</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP204 Identify complementary events and use the sum of probabilities to solve problems ACMSP205 Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’.</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP225 List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events</td>
</tr>
<tr>
<td>10</td>
<td>ACMSP246 Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence</td>
</tr>
</tbody>
</table>
9S1 “How to Lie with Statistics”

Task description

A British Prime Minister once said “Lies, damned lies and statistics!”, showing that he thought that they were all as bad as each other. It is true that statistical information can be used to “prove” just about anything you want, as the advertising industry knows well. That means that we need to recognise when people are attempting to mislead us with statistics. It also means that, if we want our work to be taken seriously, we need to know how to use statistics ethically, that is, without lying.

In this task you are asked to investigate how statistics, both numbers and graphs, can be used to mislead. Devise an assertion (for example, young drivers are more dangerous than old drivers) that you can collect data about. Prepare arguments that use the same set of data to both support and reject the assertion (include some graphs in both arguments). Prepare a double-sided poster display for your assertion, with the arguments in favour of the assertion on one side and the arguments against the assertion on the other. Make it really convincing and appealing on both sides.

Make sure you understand the meanings of any words in italics.
Additional student information

Statistical misinformation


1. Sample bias

*Statistics* showing location (central tendency) for a large population are nearly always based on small *sample* sizes. Such samples may not represent the *population* as a whole and so bias the statistic.

For example, Oxendorf University may advertise that its graduates of 10 years standing earn on average $145 165 per year. Is this really correct? A closer look at how this statistic was arrived at may provide insight.

It is likely that the information on salaries was collected by replies to an emailed survey. In this case, only those graduates whose email addresses were known and who bothered to reply were included in the result. Furthermore, the statistic was calculated not on their actual salary but on what they said their salary was. These factors mean that the statistics are open to problems of lying and bias towards more successful graduates, whose addresses are known and who have support staff to reply to the email (and who may inflate their salaries).

The statistic of average salary being $145 165 may also conceal large differences. What about the *spread* of the data (deviation or dispersion)? The statement seems to imply that such a salary is what *every* graduate can expect!

We should always realise that all samples have a bias – towards people with more money, more education, more information and alertness, better clothing and more conventional and settled appearance – because these are people that most interviewers feel more at ease with.

So when faced with a survey result, ask “how was the information collected?”.

2. Wrong average

In a factory or enterprise, there may be, for example:

- a manager/owner earning $900 000 per year;
- a partner earning $300 000 per year;
- two assistant managers earning $200 000 per year;
- a sales manager earning $114 000 per year;
- three sales people earning $100 000 per year;
- four information technology staff earning $74 000 per year;
- a foreperson earning $60 000 per year; and
- 12 workers/clerical staff earning $40 000 per year.

In this case, the *mode* (the wage/salary occurring most frequently) is $40 000 per year (the workers at the bottom of the range). The *median* (the wage/salary in the middle – 12 people earn more and 12 people earn less) is $60 000 per year (the foreperson) while the *mean* (the average) is $114 000 per year, but only four of the 25 people earn more than this. Depending on the data, mean, median and mode may be the same or differ widely. Where there is a large range of values which contains a few very large values and many close together low values (which is typical of income statistics), the mean is high and the mode low. The median is the best measure of location or central tendency.

So when faced with an average, ask “which average?”.
3. Missing information

Statistically inadequate samples (small ones) can produce just about any result. Therefore if we ignore unfavourable samples, we can end up with an “independent laboratory test” certified by a “public accountant” proving just about anything. Four out of five liking “Exo teeth liquorice” can be just that – groups of five people were asked if they liked “Exo” until one group was found where four out of five did. The thirty previous groups in which less than four liked “Exo” were excluded.

The average alone can be misleading. A town with cold nights and hot days can end up with a delightful average temperature. We need information on the spread of the data (deviation) as well as average.

Words have different meanings to different people. What does the statistician mean when he or she says that “Tuffo” cleans twice as bright? Is this twice as bright as other cleaners or twice as bright as before cleaning?

4. Irrelevant statistics

Darrell Huff gives the old adage: “If you cannot prove what you want, demonstrate something else and pretend it is the same”. Statistics about related matters are often used to support arguments for which there is no direct support. The statistic quoted may well be true but not for the situation to which it is directed.

For example, “laboratory controlled tests” may indeed show that “Basho” destroys 9 out of 10 germs when used in high concentration in a test tube, but will it do anything in your mouth in dilute concentrations? Young people aged from 16 to 21 may indeed have more car accidents than the 50 to 55 age range, but this may be because they drive more. Accidents per person per kilometres driven may show that it is safer, for a 100 kilometre drive, to be with a young person!

5. Direct misrepresentation

Statistical data can be directly misrepresented. For example, juvenile delinquency figures can take a large jump when the courts change their recording procedures to count charges for group activities to each individual. For example, five youths stealing from a house can change from one offence to five breaking and entering, five being unlawfully on premises and five incidents of stealing (15 offences in all).

Percentages can make increases smaller or larger, depending on what you want, by choosing the appropriate base. Percentage increases can also look different to absolute increases. For example, someone taking a 50% pay cut from $800 to $400 per week would not be happy if they were told that the 50% would be returned but then that return was based on the $400, i.e. to $600. We would not think it right if our 50% rabbit burger was made by mixing one rabbit with one bullock.

Graphical misinformation

One of the best ways to “lie” or “misrepresent” with data is with graphs. The vertical axis scale can be reduced to increase the slope of the graph. Graphs can be truncated (this suppression of the zero is a powerful tool in misinformation). Scales need not be given. The graph can only show the top of the scale to accentuate differences which are small in comparison to the numbers being considered.

“Trend” lines can be drawn through scattered sets of points in a scatter graph. Extrapolation which extends the graph well beyond the range of the data can be full of errors and very misleading if no indication (e.g. a dotted line) is shown that it is being done.

Here is an example of how to make a 10% increase in the gross national product more impressive – in three steps (idea from Darrell Huff, *How to lie with statistics*).
Step 1 – the actual graph

Step 2 – cut the bottom off – truncate

Step 3 – stretch the vertical axis scale

Look at these two graphs representing the same data. Do they convey the same message?
Picture graphs and drawing-type graphs (pictograms) can be very misleading. For example, the book by Darrell Huff *How to lie with statistics* has the following three ways to show that the workers of Rotundia earn half as much as the workers of USA.

The drawings in the last chart have the USA bag of money being twice as high as the Rotundia bag of money. It also appears twice as wide and twice as thick (deep). Hence the reader is left with the impression that the USA wages are eight times more than the Rotundia wages.

**Talking back to statistics**

Darrell Huff recommends that we form a general impression of the argument and then ask the following five questions.

1. **Who says so?**
   
   Look for bias, missing information, wrong measures, ambiguous statements and value-laden names (the prestigious university). What are the interests of the people making the claim?

2. **How do you know?**
   
   Look at how the statistic was calculated (the gathering of the data, sample size, type of data, calculations, and so on).

3. **Is there anything missing?**
   
   Is there a measure of spread (such as range and deviation) as well as average? What average? Try to look past percentages to raw scores. Look to see if important words are properly defined. Be wary of value-laden labels. Look at the groupings and categories. How were they selected?

4. **Did someone change the subject?**
   
   Watch for the switch from data to conclusion. More reports of cases do not mean more cases! What people say they do may not be what they actually do. Watch comparisons. Are they between different things?

5. **Does it make sense?**
   
   Many a statistic is false on its face. We often accept numbers without question. Be wary of impressive levels of accuracy. If the statistic was calculated from estimates then it is misleading to give its value to four decimal places. If a less accurate measuring device was used, highly accurate
measurements should not be quoted (for example, a trundle wheel measures to the nearest 10 cm, so you could not claim that the distance from your classroom to the tuckshop is 54.16 m if it was measured using a trundle wheel).

**Essential vocabulary**

**Measures of location (central tendency):** Statistics that attempt to give some idea of a typical score. There are three common measures of location: mean, median and mode. Measures of location may also be called *measures of central tendency or averages.*

**Measures of spread (dispersion):** Statistics that attempt to indicate how far the data is spread out, for example, range, inter-quartile range, standard deviation, and variance. Measures of spread are also known as *measures of dispersion.*

**Population:** The entire group being studied. In the statistical use of the word, a population can be made up of people (e.g. all school students in Australia), other living organisms (e.g. all green turtles in the Great Barrier Reef Marine Park), or things (e.g. all houses in Cairns). Therefore, there is no suggestion that a population must consist of people or animals (as might apply when the word is used in geography).

**Sample:** A subset of a population, often used when it is impossible, too difficult or too expensive to collect information about the entire population.

**Statistic:** A quantity that is computed from a sample, for example, mean, median, mode, standard deviation.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of the misleading use of data. It has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement. This section provides some teaching advice and also detailed instructions to complete the task questions given to students. The detailed instructions could also be used as the basis of student worksheets.

In this task students are asked to investigate how you can lie with statistics. They can do this through their own research or by reading the student information provided with the task. The activities in this section can be used to support students’ understanding of the misleading use of statistics.

The culminating product of this task is a two-sided poster that uses statistical techniques to argue both for and against an assertion. Some students may need assistance in selecting an assertion that can be argued both ways. You may wish to have some suggestions available for students struggling to develop their own assertion. For example, students could use the information in the nutrition information panel on a packet of biscuits to argue that consuming those biscuits is both a healthy and an unhealthy choice.

The information for students includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
**Activity 1: Five types of misinformation**

1. Suppose a university wanted to show that its students earned more by going to university.
   
   (a) What data would they need to gather?
   
   (b) How would this gathering be done?
   
   (c) Does this lead to bias? How?

2. We have the following data:
   
   - a manager/owner earning $900 000 per year;
   - a partner earning $300 000 per year;
   - two assistant managers earning $200 000 per year;
   - a sales manager earning $114 000 per year;
   - three sales people earning $100 000 per year;
   - four information technology staff earning $74 000 per year;
   - a foreperson earning $60 000 per year; and
   - 12 workers/clerical staff earning $40 000 per year.

   Calculate the mean, mode and median.
   
   (a) What do you notice?
   
   (b) How does this come about?
   
   (c) What is the best of the three measures (mean, mode, median) for giving the middle of the data?
   
   (d) What is wrong with the average or mean as the number giving the middle?

3. Answer the following statistical questions.
   
   (a) What does an advertisement mean when it says there is evidence that Tuffo cleans twice as bright? [Twice as bright as what?]
   
   (b) What does the advertisement mean when it says 4 out of 5 housewives recommend Tuffo? [Does it mean 80% of all housewives?]
   
   (c) The disinfectant ingredient in Basho cleaner has been shown in laboratories to kill 99% of all bacteria. What does this mean for Basho coming out of the spray can onto the kitchen table? [Does spray can Basho have the same concentrations as Basho’s ingredients in the laboratory?]
   
   (d) Do statistics showing drivers aged 22–30 have more accidents mean that they are worse drivers than drivers aged 52–60? Why or why not?
   
   (e) Is it a 50% rabbit burger if you mix the meat of one rabbit with the meat of one cow? Is there any way this could be looked at as true?

4. Huff describes five types of statistical misinformation. Read them and get to know them.
Activity 2: Recognising misinformation

1. The principal says that the class has a mathematics average of 6 out of 10. What could this mean? Is it useful information on its own? Does it mean that over half the students passed? What type of misinformation could this be?

2. Before she used “Mucho” hair lotion, the lady is pictured dowdy, lank and sad. Afterwards, she is pictured smiling with bouncy and glistening hair. What do these pictures tell us about “Mucho”? Anything of value? What type of misinformation could this be?

3. The factory has an increased wages bill of 10%, an increased advertising bill of 10% and increased material costs of 10%. Are they justified in raising their prices by 30%? Why/why not? What type of misinformation could this be?

4. A report states an average family of four requires $673.86 per week to survive. How is such a statistic calculated – from what information? Is the decimal necessary? Why is it always used? Does this mean that all families of four can survive on this amount? Why/why not? What type of misinformation could this be?

5. The price of bread rises from $1.60 to $2.40 and the price of milk drops from $2.00 to $1.20. Jenny and Jane calculate the percentages: Jenny gets overall 10% increase in cost of living (50% increase for bread and 40% decrease for milk) and Jane gets overall 33 1/3% decrease in cost of living (33 1/3% increase for bread and 66 2/3% decrease for milk). How is this possible? What type of misinformation could this be?

6. Investigation: There is a paradox in percent in that two sets of data can show a reduction yet the combined data show an increase. Find this paradox. Describe it. Why is it possible?

Activity 3: Graphing for misinformation

1. The number of cars made in Australia is as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cars</td>
<td>22 000</td>
<td>23 000</td>
<td>20 000</td>
<td>21 000</td>
<td>19 000</td>
<td>18 000</td>
</tr>
</tbody>
</table>

(a) Draw this on a normal line graph.

(b) Redraw it with the y-axis truncated at 16 000 and stretched.

(c) Redraw it with the x-axis compressed into half its length.

(d) Which graph is best for the advertisement “car sales plummet”? Which is best for the advertisement “car sales not falling as fast as thought”?

2. Zoom sells 12 000 cars a month and Vroom sells 6 000.

Graph this as follows:

(a) A basic two-column bar graph.

(b) A two-column graph where Zoom has two cars and Vroom has one.

(c) A two-column graph where Zoom has a car double the size of Vroom.

(d) Which graph supports Zoom the strongest?

3. Read the “Graphical misinformation” section of the Additional student information and be aware of the ways graphs can lie.
Activity 4: Recognising graphical misinformation

What is wrong with these graphs? How do they misrepresent? How can we modify them so they do not misrepresent?

1. Children going to camp

2. The shoe size children have

3. School revenue
4. My favourite TV program

![Favourite TV Program Graph]

5. Comparing size of schools

![Comparing size of schools Graph]

6. Salary changes

![Salary changes Graph]

7. **Investigation**: Obtain magazines and newspapers. Find misrepresentations in these magazines and newspapers. Are they common? Are there more on the internet or social media?
**Activity 5: Doing both sides**

1. Use the following data on government spending in the second half of the year to draw line graphs which emphasise that:

   (a) Government spending has increased.
   
   (b) Government spending has remained steady.

<table>
<thead>
<tr>
<th>Month</th>
<th>Spending ($m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>195</td>
</tr>
<tr>
<td>August</td>
<td>193</td>
</tr>
<tr>
<td>September</td>
<td>196</td>
</tr>
<tr>
<td>October</td>
<td>194</td>
</tr>
<tr>
<td>November</td>
<td>197</td>
</tr>
<tr>
<td>December</td>
<td>202</td>
</tr>
</tbody>
</table>

   (c) List the different ways that can be used to emphasise the side of an argument that we like!

   (d) Is it possible to overdo the misrepresenting? What is the best visual representation, (a) or (b)?

2. Prepare a double-sided poster display for an assertion.

   (a) Choose an assertion (e.g. Young drivers are more dangerous than old drivers) that is suitable for (b) below.

   (b) Gather data and prepare a poster presenting this data in a way that supports and rejects the assertion (make sure each side has a graph).

   (c) Think of opposing ideas and how you could be able to find opposing data – remember that there may need to be a different way of looking at the data. Think of different ways your data could be biased and your graphs look better in their support of bias. Try not to simply repeat the idea in (1) above.

   (d) Display your poster. Make it really attractive and appealing on both sides.

   How convincing can you make both sides of your poster? Are there some types of data that cannot be misrepresented easily?
<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP169 Identify and investigate issues involving numerical data collected from primary and secondary sources</td>
</tr>
<tr>
<td></td>
<td>ACMSP170 Construct and compare a range of data displays including stem-and-leaf plots and dot plots</td>
</tr>
<tr>
<td></td>
<td>ACMSP171 Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data</td>
</tr>
<tr>
<td></td>
<td>ACMSP172 Describe and interpret data displays using median, mean and range</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP206 Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes</td>
</tr>
<tr>
<td></td>
<td>ACMSP293 Explore the variation of means and proportions of random samples drawn from the same population</td>
</tr>
<tr>
<td></td>
<td>ACMSP207 Investigate the effect of individual data values, including outliers, on the mean and median</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP227 Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians</td>
</tr>
<tr>
<td></td>
<td>ACMSP228 Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources</td>
</tr>
<tr>
<td></td>
<td>ACMSP282 Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’</td>
</tr>
</tbody>
</table>
9S2 “A vs B”

Task description

Identify two prominent sportspersons, actors, musicians, bands or groups from the last 30 years. If you want to determine which has had a better career, what data might you consider and how might you present this data to best advantage? Use web resources as your source of data.

Present a written argument, based on data, outlining why one career has been better than the other.

*Make sure you understand the meanings of any words in italics.*
**Essential vocabulary**

**Data display:** A visual summary of statistical information in a table or graph. Common tables include *frequency distribution tables* and stem-and-leaf plots. Common graph types are frequency histograms, frequency polygons, pie (sector) graphs, divided bar graphs, picture graphs, box-and-whisker plots, and scatter graphs.

**Data:** Statistical information.

**Deviation:** A measure of difference between the *scores* and the *mean*; the extent to which the data deviates from the centre or mean.

**Frequency distribution table:** A table that organises data to show how often something happens. It must include columns headed *score* (what is measured or counted) and *frequency* (how often it occurs), but other columns can be added, for example, tally, cumulative frequency, relative frequency.

**Mean:** The average of all the items in a data set. To compute a mean, add up all the values and divide by the total number of items in the data set.

**Median:** The middle number in a series of numbers when numbers are placed in order from lowest to highest. If there is an even number of items in the data set, the median is the average of the two middle values.

**Mode:** The most frequently occurring value in the data set.

**Observation:** A statistical observation is a single piece of information. It can be collected by counting or measuring. In the statistical use of the word, it is not linked to the idea of sight and does not imply that the information was watched or seen.

**Range:** The difference between the highest and lowest values in a data set.

**Scores:** What is measured or counted.

**Standard deviation:** A measure of the spread of the data; that is, the distance, on average, of the individual observations from the mean. It averages the square root of the square of the differences between mean and scores. It uses the same units as the data; for example, if the data is measured in metres, then the units used for the standard deviation would also be metres.

**Statistical distribution:** A collection of statistical data, usually summarised in a frequency distribution table. May be called a *data set*.

**Summary statistics:** Numbers calculated from a statistical distribution that give an indication of the characteristics of that distribution. They can include measures of location (averages) such as mean, median, and mode, or measures of spread, such as range, inter-quartile range, standard deviation, and variance.

Some words have several meanings. These definitions give the mathematical meanings of the words. You should know how to use and spell these words.
Teaching information

This task, with some teacher involvement, provides an alternative, hands-on approach for teaching of statistics in Year 9. It is based on an idea first published in “Math Teaching in the Middle School” (NCTM, Feb 2014). The original article proposed that students compare two actors, but the scope has been extended so that students can tap into an area of interest to them. Some students may wish to compare two female actors, others may select two sports stars or singers/musicians/bands.

You will need to monitor students’ choices. They need to have access to sufficient data to make the task worthwhile. Apart from the need for appropriate websites, both careers need to be of sufficient duration to generate enough data. For example, the latest Australian Idol winners with only one album released is not a suitable subject for this task. The comparison should relate to the same field of endeavour. The individuals/teams being compared must be sufficiently similar to make the comparison challenging. For example, comparing cricketer Michael Clarke to a cricketer playing in the Australian team for the first time, although they are both cricketers, is unlikely to pose any challenges in deciding who is the more successful. On the other hand, making comparisons from different eras (for example, actors Tom Hanks and Laurence Olivier) could provide interesting challenges for more capable students in making adjustments for inflation, increased population, different opportunities, and so on. Be prepared to modify the students’ choices, if needed. Alternatively, you may wish to select the people/teams to be compared and restrict students to a spreadsheet with data that you download.

The level and sophistication of the data analysis will depend on the students. Vary the instructions if you wish students to use particular statistical tools (for example, box plots) in their comparison. Different students can use the same data sources and even produce the same graphs but reach different conclusions based on their interpretation of the data (for example, different students may select different criteria for success).

Sources of data include:

- Golf players: www.pgatour.com/stats.html
- Football (soccer): www.whoscored.com/statistics (team stats); www.statbunker.com/ (team stats); fantasyfootball.telegraph.co.uk/premierleague/players/all (player stats)
- Athletes: www.forbes.com/athletes/

Data can be copied from websites and pasted into Excel or alternative graphing/statistics packages for analysis. Given the quantity of the data that may be involved it is almost imperative that students have access to some form of technology to complete this task. Students may need to remove any formatting ($ signs, comma separators, etc.) before using the data.

The task has deliberately not been written for a “set and forget” teaching approach. It is expected that students will require assistance in completing aspects of the task (some more than others). Within the task there are many opportunities for the teacher to provide groups or the whole class with direct instruction or hints to keep them working. How this occurs is a matter for teacher judgement.

The information for students also includes some essential mathematical words (marked in italics) that students should understand and be able to spell. Included in the task as an optional handout for students is a glossary of these words. The glossary could be used as the basis of supporting language and literacy activities for students.
## Links to the Australian Mathematics Curriculum

<table>
<thead>
<tr>
<th>Year Level</th>
<th>Statistics and Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ACMSP170: Construct and compare a range of data displays including stem-and-leaf plots and dot plots. ACMSP171: Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data. ACMSP172: Describe and interpret data displays using median, mean and range.</td>
</tr>
<tr>
<td>8</td>
<td>ACMSP207: Investigate the effect of individual data values, including outliers, on the mean and median.</td>
</tr>
<tr>
<td>9</td>
<td>ACMSP228: Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources. ACMSP282: Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’.</td>
</tr>
<tr>
<td>10</td>
<td>ACMSP249: Construct and interpret box plots and use them to compare data sets. ACMSP250: Compare shapes of box plots to corresponding histograms and dot plots.</td>
</tr>
</tbody>
</table>
PS3 “Investigating Investigations”

Task description

Investigating involves more than just finding out about something. We need to be able to pose questions that we can then find the answers to. Some questions could be:

- Does it make sense? If not, what can we do about it?
- What has been assumed?
- What if the assumptions change?
- How do we know that it is true (can we prove it)? [The answer is not “Because the teacher/internet said so”.
- Can we show that it is not true in some situations (can we disprove it)?
- How can we apply the theory to a practical situation?
- How does it relate to other mathematical concepts?
- What are the limitations of this idea?
- Can we generalise (propose a rule that applies beyond this specific situation)?
- How can we improve on what has been done?
- Can we break this idea down into its components?
- Can we simplify a complex situation?
- What if ...?

Good mathematicians should be asking these questions all the time.

In this task, we begin with a diagram and a single direction: “Investigate!” Can you pose questions and solve an investigation from such a restricted start? The idea is to try something yourself.

Below is a shape. Investigate!
Teaching information

In this task booklet, we begin with a diagram and a single direction: “Investigate!” Can students pose questions and solve an investigation from such a restricted start? This section provides the start and some investigative options which could be used as the basis of student worksheets. The idea is to get the students to do something themselves. Only proffer options if there is no progress, and then a bit at a time!

Activity 1: The drawing

1. On the right there is a drawing of a shape. Investigate!
2. If you cannot get anywhere, ask:
   (a) Could there be something that involves angles that this leads you to?
   (a) Could perimeter and/or area lead somewhere?
   (b) Anything else?

Activity 2: One option – angles

If we were to drive around the shape, we would have angles that turned left or right. Investigate!

1. (a) Could look at how many of each angle turn type.
   (b) Could look at difference between turns left or right.
   (c) Could look at position of turns.
2. Could see if still the same or different if there are more angles (more complex shape).
3. Could see if still the same or different if angles are any size.
4. Could look and see if there are changes if the shape changes like A or B below:

5. We can make up generalisations, rules and patterns for:
   (a) the original shape with more angles added in;
   (b) shapes that have any number of angles but do not cross;
   (c) shapes that have any type of angle or are continuously curved but do not cross;
   (d) shapes that cross like A or B with any angle;
   (e) shapes like A or B that cross more than once;
   (f) shapes that have both A and B type crossings; and
   (g) shapes of any type you can imagine.
6. This investigation is important in at least one sport. Why? What has to be done to circumvent problems it causes?

**Activity 3: Second option – perimeter**

1. If we were to compare the shape in Activity 1 to the rectangle below right with the same dimensions, what has changed, perimeter or area?

2. We could investigate adding in more “shifts” as below – what happens to perimeter? Are there any “shifts” that lead to different outcomes?

3. What would happen if the “shift” was not aligned, e.g.

   What changes?

4. Are there other changes to a shape that increase perimeter without affecting area?
   
   (a) What about these two?

   C

   D

   (b) Can you make up more?

   (c) Can you make up different patterns/rules for different shape changes?

5. Can the pattern and area relationships hold if the angles are not right angles? What if you were just adding and taking bits from a perimeter any odd way but the area was staying the same?

6. How does this relate to:
   
   (a) fractals (look it up on the internet);

   (b) perimeter of islands like Australia.

7. Is there a different way to look at area and perimeter? Are there ways area could change without perimeter changing? What are the limits?
**Activity 4: Third option – tessellations**

The shape can tessellate as on right. It can also make frieze patterns.

1. Investigate what types of shapes, like the one we started with, can tessellate or look good in frieze patterns.
   
   (a) Do the “shifts” or angles have to be aligned?
   
   (b) Do the angles have to be right angles?
   
   (c) What about changes to sides as well as top and bottom?

2. What relationship has to exist in position and type of angles for the pattern to tessellate? When will it not tessellate?

3. Are there any other flip, slide and turn activities we could do with the shape? Do these activities enable you to explore and investigate anything?

4. In some modern suburbs, houses are shaped like on the right. They are then placed side by side like below:

   ![Diagram of house shapes]

   Why would this be done? What other shapes could be used to reduce land use?

**Activity 5: Anything else**

Is there another way we could investigate the starting shape in Activity 1?

Please send ideas to ydc@qut.edu.au
**Links to the Australian Curriculum**

The links to the Australian Curriculum cover all areas of mathematics and depend on what avenues students choose to explore. The task also links to the general capability of *Critical and creative thinking*.

### Critical and creative thinking

<table>
<thead>
<tr>
<th>Critical and creative thinking</th>
<th>By the end of Year 8</th>
<th>By the end of Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inquiring – identifying, exploring and organising information and ideas</strong></td>
<td>Pose questions: pose questions to probe assumptions and investigate complex issues. Identify and clarify information and ideas: clarify information and ideas from texts or images when exploring challenging issues. Organise and process information: critically analyse information and evidence according to criteria such as validity and relevance.</td>
<td>Pose questions: pose questions to critically analyse complex issues and abstract ideas. Identify and clarify information and ideas: clarify complex information and ideas drawn from a range of sources. Organise and process information: critically analyse independently sourced information to determine bias and reliability.</td>
</tr>
<tr>
<td><strong>Generating ideas, possibilities and actions</strong></td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals. Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting. Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action.</td>
<td>Imagine possibilities and connect ideas: draw parallels between known and new ideas to create new ways of achieving goals. Consider alternatives: generate alternatives and innovative solutions, and adapt ideas, including when information is limited or conflicting. Seek solutions and put ideas into action: predict possibilities, and identify and test consequences when seeking solutions and putting ideas into action.</td>
</tr>
<tr>
<td><strong>Reflecting on thinking and processes</strong></td>
<td>Think about thinking (metacognition): assess assumptions in their thinking and invite alternative opinions. Reflect on processes: evaluate and justify the reasons behind choosing a particular problem-solving strategy. Transfer knowledge into new contexts: justify reasons for decisions when transferring information to similar and different contexts.</td>
<td>Think about thinking (metacognition): give reasons to support their thinking, and address opposing viewpoints and possible weaknesses in their own positions. Reflect on processes: balance rational and irrational components of a complex or ambiguous problem to evaluate evidence. Transfer knowledge into new contexts: identify, plan and justify transference of knowledge to new contexts.</td>
</tr>
<tr>
<td><strong>Analysing, synthesising and evaluating reasoning and procedures</strong></td>
<td>Apply logic and reasoning: identify gaps in reasoning and missing elements in information. Draw conclusions and design a course of action: differentiate the components of a designed course of action and tolerate ambiguities when drawing conclusions. Evaluate procedures and outcomes: explain intentions and justify ideas, methods and courses of action, and account for expected and unexpected outcomes against criteria they have identified.</td>
<td>Apply logic and reasoning: analyse reasoning used in finding and applying solutions, and in choice of resources. Draw conclusions and design a course of action: use logical and abstract thinking to analyse and synthesise complex information to inform a course of action. Evaluate procedures and outcomes: evaluate the effectiveness of ideas, products and performances and implement courses of action to achieve desired outcomes against criteria they have identified.</td>
</tr>
</tbody>
</table>
Appendices

This section includes the following four appendices:

- Appendix A: Achievement standards
- Appendix B: Affective criteria
- Appendix C: Australian Curriculum mapped to MITI tasks
- Appendix D: MITI tasks mapped to Australian Curriculum
## Appendix A: Achievement standards

<table>
<thead>
<tr>
<th>Conceptual understanding</th>
<th>Procedural fluency</th>
<th>Mathematical language and symbols</th>
<th>Problem-solving approaches</th>
<th>Mathematical modelling</th>
<th>Reasoning and justification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar</td>
<td>Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar</td>
<td>Effective and clear use of appropriate mathematical terminology, diagrams, conventions, symbols and texts, in written and spoken forms</td>
<td>Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar</td>
<td>Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar</td>
<td>Clear explanation of mathematical thinking and reasoning, including logical justification of choices made, evaluation of strategies used and conclusions reached</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations</td>
<td>Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations</td>
<td>Consistent use of appropriate mathematical terminology, diagrams, conventions, symbols and texts, in written and spoken forms</td>
<td>Application of problem-solving approaches to investigate complex familiar or simple unfamiliar situations</td>
<td>Development of mathematical models and representations in complex familiar or simple unfamiliar situations</td>
<td>Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognition and identification of mathematical concepts and relationships in simple familiar situations</td>
<td>Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations</td>
<td>Satisfactory use of appropriate mathematical terminology, diagrams, conventions, symbols and texts, in written and spoken forms</td>
<td>Application of problem-solving approaches to investigate simple familiar situations</td>
<td>Development of mathematical models and representations in simple familiar situations</td>
<td>Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Some identification of simple mathematical concepts</td>
<td>Some recall and use of facts, definitions, technologies and simple procedures</td>
<td>Use of aspects of mathematical terminology, diagrams, symbols and texts</td>
<td>Some selection and application of problem-solving approaches in simple familiar situations</td>
<td>Statements about simple mathematical models and representations</td>
<td>Statements about choices made, strategies used and conclusions reached</td>
</tr>
<tr>
<td><strong>E</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statements about obvious mathematical concepts</td>
<td>Partial recall of facts, definitions or simple procedures</td>
<td>Use of everyday language</td>
<td>Partial selection of problem-solving approaches</td>
<td>Isolated statements about given mathematical models and representations</td>
<td>Isolated statements about given strategies or conclusions</td>
</tr>
</tbody>
</table>
## Appendix B: Affective criteria

<table>
<thead>
<tr>
<th>Following instructions</th>
<th>Working with others</th>
<th>Use of class time</th>
<th>Teacher assistance</th>
<th>Response to feedback</th>
<th>Milestones and deadlines</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>All instructions in class and on the task sheet accurately followed</td>
<td>A committed, cooperative, supportive and inclusive member of the group</td>
<td>Effective and productive use of class time allocated to the task</td>
<td>Interactions with the teacher relate to challenge activities and ideas for extension</td>
<td>All milestones and deadlines met</td>
</tr>
<tr>
<td>B</td>
<td>Minor errors in following instructions given in class and on task sheets</td>
<td>Usually a cooperative and supportive member of the group</td>
<td>Required occasional prompting to stay on task</td>
<td>Some teacher advice needed to undertake the tasks</td>
<td>Task completed without obtaining feedback</td>
</tr>
<tr>
<td>C</td>
<td>Little contribution to the group’s product</td>
<td>Diminished the group’s activities</td>
<td>Unable or unwilling to work productively in the classroom environment</td>
<td>Some adult assistance required to undertake the tasks</td>
<td>Task submitted on time, although some milestones not met</td>
</tr>
<tr>
<td>D</td>
<td>Many instructions ignored</td>
<td>Diminished the group’s activities</td>
<td>Unable or unwilling to work productively in the classroom environment</td>
<td>Some adult assistance required to undertake the tasks</td>
<td>Many milestone missed and task submitted late</td>
</tr>
<tr>
<td>E</td>
<td>Many instructions ignored</td>
<td>Diminished the group’s activities</td>
<td>Unable or unwilling to work productively in the classroom environment</td>
<td>Did not respond to the teacher’s advice</td>
<td>Did not respond to the teacher’s advice</td>
</tr>
</tbody>
</table>

© QUT YuMi Deadly Centre 2017

MITI Investigations Years 7–9  Page 309
### Appendix C: Australian Curriculum mapped to MITI tasks

#### Year 7 Content Descriptions

<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and place value</strong></td>
<td></td>
</tr>
<tr>
<td>Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)</td>
<td>What Are You Worth? (7N3)</td>
</tr>
<tr>
<td>Investigate and use square roots of perfect square numbers (ACMNA150)</td>
<td>Challenging Fractions (8N2)</td>
</tr>
<tr>
<td>Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)</td>
<td>Building a Mathematics Structure (8A3)</td>
</tr>
<tr>
<td>Compare, order, add and subtract integers (ACMNA280)</td>
<td>Directing Numbers (7N2)</td>
</tr>
<tr>
<td></td>
<td>What Are You Worth? (7N3)</td>
</tr>
<tr>
<td></td>
<td>Connected Expressions (7A2)</td>
</tr>
<tr>
<td><strong>Real numbers</strong></td>
<td></td>
</tr>
<tr>
<td>Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)</td>
<td>Challenging Fractions (8N2)</td>
</tr>
<tr>
<td>Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)</td>
<td>Tangled Fractions (7N1) (Challenge)</td>
</tr>
<tr>
<td></td>
<td>Diminishing Fractions (8N1)</td>
</tr>
<tr>
<td>Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)</td>
<td>Tangled Fractions (7N1)</td>
</tr>
<tr>
<td>Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)</td>
<td>Challenging Fractions (8N2)</td>
</tr>
<tr>
<td>Round decimals to a specified number of decimal places (ACMNA156)</td>
<td>Accuracy and Precision (8M1)</td>
</tr>
<tr>
<td>Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)</td>
<td>Square Percentages (8N3)</td>
</tr>
<tr>
<td></td>
<td>Accuracy and Precision (8M1)</td>
</tr>
<tr>
<td>Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)</td>
<td>Square Percentages (8N3)</td>
</tr>
<tr>
<td></td>
<td>Accuracy and Precision (8M1)</td>
</tr>
<tr>
<td>Recognise and solve problems involving simple ratios (ACMNA173)</td>
<td>How Tall is the Criminal? (7N4)</td>
</tr>
<tr>
<td></td>
<td>Rating Our World (7N5)</td>
</tr>
<tr>
<td></td>
<td>How High is that Tree? (9M4)</td>
</tr>
<tr>
<td><strong>Money and financial mathematics</strong></td>
<td></td>
</tr>
<tr>
<td>Investigate and calculate ‘best buys’, with and without digital technologies (ACMNA174)</td>
<td>Rating Our World (7N5)</td>
</tr>
<tr>
<td></td>
<td>Which Card? (9N3)</td>
</tr>
<tr>
<td>Patterns and algebra</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td>Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)</td>
<td>Consecutive Sums (8N4) Growing Patterns and Growing Graphs (8A1) Maths in a Box (8G1) Dividing Diagonals (9A1) (Challenge)</td>
</tr>
<tr>
<td>Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)</td>
<td>Connected Expressions (7A2) Growing Patterns and Growing Graphs (8A1)</td>
</tr>
<tr>
<td>Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)</td>
<td>Building a Mathematics Structure (8A3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Linear and non-linear relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)</td>
</tr>
<tr>
<td>Solve simple linear equations (ACMNA179)</td>
</tr>
<tr>
<td>Investigate, interpret and analyse graphs from authentic data (ACMNA180)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
</tbody>
</table>
| **Using units of measurement** | Rating Our World (7N5)  
Cheap Houses (7M2)  
Diminishing Fractions (8N1)  
Square Percentages (8N3)  
Designing a Kitchen (8M2)  
Crazy Bird Boxes (8M3)  
Maths in a Box (8G1)  
Look Out for the Baby! (9M1) |
| Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159) | |
| Calculate volumes of rectangular prisms (ACMMG160) | Rating Our World (7N5)  
Crazy Bird Boxes (8M3)  
Maths in a Box (8G1)  
Look Out for the Baby! (9M1) |
| **Shape** | |
| Draw different views of prisms and solids formed from combinations of prisms (ACMMG161) | Crazy Bird Boxes (8M3)  
Maths in a Box (8G1)  
Look Out for the Baby! (9M1) |
<p>| <strong>Location and transformation</strong> | Reflecting on Rotations (7A1) |
| Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181) | |
| <strong>Geometric reasoning</strong> | |
| Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163) | Constructive Constructions (9G1) |
| Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164) | Constructive Constructions (9G1) |
| Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166) | Constructive Constructions (9G1) |
| Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165) | Constructive Constructions (9G1) |</p>
<table>
<thead>
<tr>
<th>Statistics and Probability</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chance</strong></td>
<td></td>
</tr>
<tr>
<td>Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)</td>
<td>Dice Doubles (7P1)</td>
</tr>
</tbody>
</table>
| Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168) | Dice Doubles (7P1)  
Fair Game (7P2)  
The Lucky Prince (8P1)  
And Not Or (8P2)  
Is Greed Good? (9P1)  
Monopolising Monopoly (9P2) |
| **Data representation and interpretation** |           |
| Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169) | Are Older Actors Better? (7S1)  
How to Lie with Statistics (9S1) |
| Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170) | Are Older Actors Better? (7S1)  
Distributing Distributions (8S1)  
How to Lie with Statistics (9S1)  
A vs B (9S2) |
| Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171) | Are Older Actors Better? (7S1)  
Distributing Distributions (8S1)  
How to Lie with Statistics (9S1)  
A vs B (9S2) |
| Describe and interpret data displays using median, mean and range (ACMSP172) | Are Older Actors Better? (7S1)  
Distributing Distributions (8S1)  
How to Lie with Statistics (9S1)  
A vs B (9S2) |
## Year 8 Content Descriptions

<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and place value</strong></td>
<td></td>
</tr>
<tr>
<td>Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA182)</td>
<td>Power-ful Mathematics (9N1)</td>
</tr>
<tr>
<td>Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)</td>
<td>Tangled Fractions (7N1) Directing Numbers (7N2) Diminishing Fractions (8N1) Maths in a Box (8G1) Accuracy and Precision (8M1)</td>
</tr>
<tr>
<td><strong>Real numbers</strong></td>
<td></td>
</tr>
<tr>
<td>Investigate terminating and recurring decimals (ACMNA184)</td>
<td>Challenging Fractions (8N2) (Challenge) Square Percentages (8N3) (Challenge)</td>
</tr>
<tr>
<td>Investigate the concept of irrational numbers, including π (ACMNA186)</td>
<td>Diminishing Fractions (8N1) (Challenge 1) It’s All Greek to Me (9N2)</td>
</tr>
<tr>
<td>Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)</td>
<td>Cheap Houses (7M2) Square Percentages (8N3) Accuracy and Precision (8M1) Designing a Kitchen (8M2)</td>
</tr>
<tr>
<td>Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)</td>
<td>How Tall is the Criminal? (7N4) Rating Our World (7N5) Diminishing Fractions (8N1) (Challenges) Maths in a Box (8G1) Square Angles (9M2) How High is that Tree? (9M4)</td>
</tr>
<tr>
<td><strong>Money and financial mathematics</strong></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving profit and loss, with and without digital technologies (ACMNA189)</td>
<td>What Are You Worth? (7N3) Which Card? (9N3)</td>
</tr>
<tr>
<td><strong>Patterns and algebra</strong></td>
<td></td>
</tr>
<tr>
<td>Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)</td>
<td>Algebraic Caterpillars (8A2) Building a Mathematics Structure (8A3)</td>
</tr>
<tr>
<td>Factorise algebraic expressions by identifying numerical factors (ACMNA191)</td>
<td>Algebraic Caterpillars (8A2) (Challenge)</td>
</tr>
<tr>
<td>Simplify algebraic expressions involving the four operations (ACMNA192)</td>
<td>Connected Expressions (7A2) Consecutive Sums (8N4)</td>
</tr>
<tr>
<td><strong>Linear and non-linear relationships</strong></td>
<td></td>
</tr>
<tr>
<td>Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)</td>
<td>Growing Patterns and Growing Graphs (8A1)</td>
</tr>
<tr>
<td>Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)</td>
<td>Connected Expressions (7A2) Challenging Fractions (8N2) (Challenge) Dividing Diagonals (9A1) Taking the Guesswork Out of Maths (9A2)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>MITI Task</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td><strong>Using units of measurement</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195) | Cheap Houses (7M2)  
Designing a Kitchen (8M2)  
Crazy Bird Boxes (8M3)  
Look Out for the Baby! (9M1) |
| Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196) | Crazy Bird Boxes (8M3) |
| Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197) | Look Out for the Baby! (9M1) |
| Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198) | Rating Our World (7N5)  
Maths in a Box (8G1)  
Designing a Kitchen (8M2)  
Crazy Bird Boxes (8M3) |
| Solve problems involving duration, including using 12- and 24-hour time within a single time zone (ACMMG199) | Rocking Around the World (7M1) |
| **Geometric reasoning** | |
| Define congruence of plane shapes using transformations (ACMMG200) | Flippin’ Congruence (7G1)  
Topological Oddities (PS2) |
| Develop the conditions for congruence of triangles (ACMMG201) | Three-Fact Triangles (9M3)  
Constructive Constructions (9G1) |
| Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202) | Flippin’ Congruence (7G1) (Challenge)  
Constructive Constructions (9G1) |
<table>
<thead>
<tr>
<th>Statistics and Probability</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chance</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Identify complementary events and use the sum of probabilities to solve problems (ACMSP204) | The Lucky Prince (8P1)  
And Not Or (8P2)  
Is Greed Good? (9P1)  
Monopolising Monopoly (9P2) |
| Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’. (ACMSP205) | Fair Game (7P2)  
And Not Or (8P2)  
Is Greed Good? (9P1)  
Monopolising Monopoly (9P2) |
| Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292) | And Not Or (8P2) |
| **Data representation and interpretation** |           |
| Investigate techniques for collecting data, including census, sampling and observation (ACMSP284) | Distributing Distributions (8S1) |
| Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206) | How to Lie with Statistics (9S1) |
| Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293) | Distributing Distributions (8S1)  
How to Lie with Statistics (9S1) |
| Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207) | Distributing Distributions (8S1)  
How to Lie with Statistics (9S1)  
A vs B (9S2) |
## Year 9 Content Descriptions

<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real numbers</strong></td>
<td><strong>Cheap Houses (7M2)</strong></td>
</tr>
<tr>
<td>Solve problems involving direct proportion.</td>
<td>Designing a Kitchen (8M2)</td>
</tr>
<tr>
<td>Explore the relationship between graphs and equations</td>
<td>Maths in a Box (8G1)</td>
</tr>
<tr>
<td>corresponding to simple rate problems (ACMNA208)</td>
<td><strong>Powerful Mathematics (9N1)</strong></td>
</tr>
<tr>
<td>Apply index laws to numerical expressions with integer</td>
<td><strong>What Are You Worth? (7N3) (Challenge)</strong></td>
</tr>
<tr>
<td>indices (ACMNA209)</td>
<td>Powerful Mathematics (9N1)</td>
</tr>
<tr>
<td>Express numbers in scientific notation (ACMNA210)</td>
<td><strong>Which Card? (9N3)</strong></td>
</tr>
<tr>
<td><strong>Money and financial mathematics</strong></td>
<td><strong>Powerful Mathematics (9N1)</strong></td>
</tr>
<tr>
<td>Solve problems involving simple interest (ACMNA211)</td>
<td><strong>Powerful Mathematics (9N1)</strong></td>
</tr>
<tr>
<td><strong>Patterns and algebra</strong></td>
<td><strong>Powerful Mathematics (9N1)</strong></td>
</tr>
<tr>
<td>Extend and apply the index laws to variables,</td>
<td><strong>Powerful Mathematics (9N1)</strong></td>
</tr>
<tr>
<td>using positive integer indices and the zero index</td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td>(ACMNA212)</td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td>Apply the distributive law to the expansion of</td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td>algebraic expressions, including binomials, and collect</td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td>like terms where appropriate (ACMNA213)</td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td><strong>Linear and non-linear relationships</strong></td>
<td><strong>Algebraic Caterpillars (8A2)</strong></td>
</tr>
<tr>
<td>Find the distance between two points located on a</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>Cartesian plane using a range of strategies, including</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>graphing software (ACMNA214)</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>Find the midpoint and gradient of a line segment (</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>interval) on the Cartesian plane using a range of</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>strategies, including graphing software (ACMNA294)</td>
<td><strong>Dividing Diagonals (9A1)</strong></td>
</tr>
<tr>
<td>Sketch linear graphs using the coordinates of two</td>
<td><strong>Taking the Guesswork Out of Maths (9A2)</strong></td>
</tr>
<tr>
<td>points and solve linear equations (ACMNA215)</td>
<td><strong>Taking the Guesswork Out of Maths (9A2)</strong></td>
</tr>
<tr>
<td>Graph simple non-linear relations with and without the</td>
<td>**Growing Patterns and Growing Graphs (8A1)</td>
</tr>
<tr>
<td>use of digital technologies and solve simple related</td>
<td>(Challenge)</td>
</tr>
<tr>
<td>equations (ACMNA296)</td>
<td>**Growing Patterns and Growing Graphs (8A1)</td>
</tr>
<tr>
<td></td>
<td>(Challenge)</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>MITI Task</td>
</tr>
<tr>
<td>--------------------------</td>
<td>----------</td>
</tr>
<tr>
<td><strong>Using units of measurement</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Calculate the areas of composite shapes (ACMMG216) | Cheap Houses (7M2)  
Square Percentages (8N3)  
Designing a Kitchen (8M2)  
Crazy Bird Boxes (8M3)  
Maths in a Box (8G1)  
Look Out for the Baby! (9M1) |
| Calculate the surface area and volume of cylinders and solve related problems (ACMMG217) | Rating Our World (7N5)  
Look Out for the Baby! (9M1) |
| Solve problems involving the surface area and volume of right prisms (ACMMG218) | Maths in a Box (8G1) |
| Investigate very small and very large time scales and intervals (ACMMG219) | Power-ful Mathematics (9N1) |
| **Geometric reasoning** |  |
| Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220) | How High is that Tree? (9M4) |
| Solve problems using ratio and scale factors in similar figures (ACMMG221) | Crazy Bird Boxes (8M3)  
How High is that Tree? (9M4) |
| **Pythagoras and trigonometry** |  |
| Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles (ACMMG222) | Diminishing Fractions (8N1) (Challenge 2)  
Maths in a Box (8G1)  
Square Angles (9M2)  
Three-Fact Triangles (9M3)  
It’s All Greek to Me (9N2) |
| Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223) | Square Angles (9M2) |
| Apply trigonometry to solve right-angled triangle problems (ACMMG224) | Square Angles (9M2)  
Three-Fact Triangles (9M3)  
How High is that Tree? (9M4) (Challenge) |
<table>
<thead>
<tr>
<th>Statistics and Probability</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chance</strong></td>
<td></td>
</tr>
<tr>
<td>List all outcomes for two-step chance</td>
<td>The Lucky Prince (8P1)</td>
</tr>
<tr>
<td>experiments, both with and without replacement</td>
<td>And Not Or (8P2)</td>
</tr>
<tr>
<td>using tree diagrams or arrays. Assign</td>
<td>Is Greed Good? (9P1)</td>
</tr>
<tr>
<td>probabilities to outcomes and determine</td>
<td>Monopolising Monopoly (9P2)</td>
</tr>
<tr>
<td>probabilities for events (ACMSP225)</td>
<td></td>
</tr>
<tr>
<td>Calculate relative frequencies from given or</td>
<td>Is Greed Good? (9P1)</td>
</tr>
<tr>
<td>collected data to estimate probabilities of</td>
<td></td>
</tr>
<tr>
<td>events involving ‘and’ or ‘or’ (ACMSP226)</td>
<td></td>
</tr>
<tr>
<td>Investigate reports of surveys in digital</td>
<td>How to Lie with Statistics (9S1)</td>
</tr>
<tr>
<td>media and elsewhere for information on how data</td>
<td></td>
</tr>
<tr>
<td>were obtained to estimate population means and</td>
<td></td>
</tr>
<tr>
<td>medians (ACMSP227)</td>
<td></td>
</tr>
<tr>
<td><strong>Data representation and interpretation</strong></td>
<td></td>
</tr>
<tr>
<td>Identify everyday questions and issues involving</td>
<td>Distributing Distributions (8S1)</td>
</tr>
<tr>
<td>at least one numerical and at least one</td>
<td>How to Lie with Statistics (9S1)</td>
</tr>
<tr>
<td>categorical variable, and collect data directly</td>
<td>A vs B (9S2)</td>
</tr>
<tr>
<td>and from secondary sources (ACMSP228)</td>
<td></td>
</tr>
<tr>
<td>Construct back-to-back stem-and-leaf plots and</td>
<td>Distributing Distributions (8S1)</td>
</tr>
<tr>
<td>histograms and describe data, using terms</td>
<td>How to Lie with Statistics (9S1)</td>
</tr>
<tr>
<td>including ‘skewed’, ‘symmetric’ and ‘bi modal’</td>
<td>A vs B (9S2)</td>
</tr>
<tr>
<td>(ACMSP282)</td>
<td></td>
</tr>
<tr>
<td>Compare data displays using mean, median and</td>
<td>Distributing Distributions (8S1)</td>
</tr>
<tr>
<td>range to describe and interpret numerical data</td>
<td></td>
</tr>
<tr>
<td>sets in terms of location (centre) and spread</td>
<td></td>
</tr>
<tr>
<td>(ACMSP283)</td>
<td></td>
</tr>
</tbody>
</table>
### Year 10 Content Descriptions

<table>
<thead>
<tr>
<th>Number and Algebra</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money and financial mathematics</td>
<td></td>
</tr>
<tr>
<td>Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)</td>
<td>Which Card? (9N3)</td>
</tr>
<tr>
<td><strong>Patterns and algebra</strong></td>
<td></td>
</tr>
<tr>
<td>Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)</td>
<td>Algebraic Caterpillars (8A2) (Challenge)</td>
</tr>
<tr>
<td>Simplify algebraic products and quotients using index laws (ACMNA231)</td>
<td>Power-ful Mathematics (9N1)</td>
</tr>
<tr>
<td>Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)</td>
<td></td>
</tr>
<tr>
<td>Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)</td>
<td></td>
</tr>
<tr>
<td>Substitute values into formulas to determine an unknown (ACMNA234)</td>
<td></td>
</tr>
<tr>
<td><strong>Linear and non-linear relationships</strong></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving linear equations, including those derived from formulas (ACMNA235)</td>
<td>Taking the Guesswork Out of Maths (9A2)</td>
</tr>
<tr>
<td>Solve linear inequalities and graph their solutions on a number line (ACMNA236)</td>
<td></td>
</tr>
<tr>
<td>Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (ACMNA237)</td>
<td>Taking the Guesswork Out of Maths (9A2) (including the Challenge)</td>
</tr>
<tr>
<td>Solve problems involving parallel and perpendicular lines (ACMNA238)</td>
<td></td>
</tr>
<tr>
<td>Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)</td>
<td></td>
</tr>
<tr>
<td>Solve linear equations involving simple algebraic fractions (ACMNA240)</td>
<td></td>
</tr>
<tr>
<td>Solve simple quadratic equations using a range of strategies (ACMNA241)</td>
<td></td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>MITI Task</td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td><strong>Using units of measurement</strong></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving surface area and volume for</td>
<td>Crazy Bird Boxes (8M3)</td>
</tr>
<tr>
<td>a range of prisms, cylinders and composite solids</td>
<td>Look Out for the Baby! (9M1)</td>
</tr>
<tr>
<td>(ACMMG242)</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric reasoning</strong></td>
<td></td>
</tr>
<tr>
<td>Formulate proofs involving congruent triangles and</td>
<td>Constructive Constructions (9G1)</td>
</tr>
<tr>
<td>angle properties (ACMMG243)</td>
<td></td>
</tr>
<tr>
<td>Apply logical reasoning, including the use of</td>
<td>Constructive Constructions (9G1)</td>
</tr>
<tr>
<td>congruence and similarity, to proofs and numerical</td>
<td></td>
</tr>
<tr>
<td>exercises involving plane shapes (ACMMG244)</td>
<td></td>
</tr>
<tr>
<td><strong>Pythagoras and trigonometry</strong></td>
<td></td>
</tr>
<tr>
<td>Solve right-angled triangle problems including those</td>
<td>How High is that Tree? (9M4) (Challenge)</td>
</tr>
<tr>
<td>involving direction and angles of elevation and</td>
<td></td>
</tr>
<tr>
<td>depression (ACMMG245)</td>
<td></td>
</tr>
<tr>
<td><strong>Statistics and Probability</strong></td>
<td><strong>MITI Task</strong></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td><strong>Chance</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP246) | And Not Or (8P2)  
Is Greed Good? (9P1)  
Monopolising Monopoly (9P2) |
| Use the language of ‘if ... then’, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language (ACMSP247) | |
| **Data representation and interpretation** |   |
| Determine quartiles and interquartile range (ACMSP248) |   |
| Construct and interpret box plots and use them to compare data sets (ACMSP249) | Distributing Distributions (8S1)  
A vs B (9S2) |
| Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250) | A vs B (9S2) |
| Use scatter plots to investigate and comment on relationships between two numerical variables (ACMSP251) |   |
| Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252) |   |
| Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253) |   |
## General Capabilities

<table>
<thead>
<tr>
<th>Description</th>
<th>MITI Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical and Creative Thinking</td>
<td>Finding the Winning Strategy (7G2)</td>
</tr>
<tr>
<td></td>
<td>Assuming Too Much (PS1)</td>
</tr>
<tr>
<td></td>
<td>Topological Oddities (PS2)</td>
</tr>
<tr>
<td></td>
<td>Investigating Investigations (PS3)</td>
</tr>
<tr>
<td>Literacy (report writing)</td>
<td>Dice Doubles (7P1)</td>
</tr>
<tr>
<td>Literacy (using Google to find websites with suitable reading levels)</td>
<td>Distributing Distributions (8S1)</td>
</tr>
<tr>
<td>Literacy (converting from one representational form to another)</td>
<td>Dividing Diagonals (9A1)</td>
</tr>
<tr>
<td>Literacy (modality in vocabulary)</td>
<td>Are Older Actors Better? (7S1)</td>
</tr>
<tr>
<td>ITC Capability (spreadsheets)</td>
<td>Are Older Actors Better? (7S1)</td>
</tr>
<tr>
<td></td>
<td>Fair Game (7P2)</td>
</tr>
<tr>
<td></td>
<td>Distributing Distributions (8S1)</td>
</tr>
<tr>
<td></td>
<td>Square Angles (9M2)</td>
</tr>
<tr>
<td></td>
<td>A vs B (9S2)</td>
</tr>
<tr>
<td>ITC Capability (programmable calculators)</td>
<td>Dividing Diagonals (9A1)</td>
</tr>
<tr>
<td></td>
<td>Square Angles (9M2)</td>
</tr>
<tr>
<td>ITC Capability (dynamic geometry software)</td>
<td>Dividing Diagonals (9A1)</td>
</tr>
<tr>
<td>Personal and Social Competence</td>
<td>All tasks can be undertaken in groups</td>
</tr>
</tbody>
</table>
## Appendix D: MITI tasks mapped to Australian Curriculum

<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
</table>
| 7A1  | Reflecting on Rotations      | Representation of transformations on the Cartesian plane, drawing on existing understandings of transformations. | - Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178, Yr 7)  
- Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181, Yr 7) |
| 7A2  | Connected Expressions        | Application of the solutions to sets of six connected linear equations to identify patterns. As a challenge, investigation of the connections between the equations that result in these patterns. | - Compare, order, add and subtract integers (ACMNA150, Yr 7)  
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176, Yr 7)  
- Solve simple linear equations (ACMNA179, Yr 7)  
- Simplify algebraic expressions involving the four operations (ACMNA192, Yr 8)  
- Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194, Yr 8) |
| 7G1  | Flippin’ Congruence          | Investigation of the common transformations (translations, rotations and reflections), leading to the exploration of congruence and patterns. | - Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries (ACMG114, Yr 5)  
- Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (ACMG142, Yr 6)  
- Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMG202, Yr 8) |
| 7G2  | Finding the Winning Strategy | Development of spatial awareness, logic and planning through playing a variety of games of strategy. | - Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries (ACMG114, Yr 5)  
- Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (ACMG142, Yr 6)  
- General Capability of critical and creative thinking |
| 7M1  | Rocking Around the World     | Development of an itinerary for a ten-day world tour by a rock band, taking into account time zones, travelling time, and the time taken for set up and performance. | - Compare 12- and 24-hour time systems and convert between them (ACMMG110, Yr 5)  
- Interpret and use timetables (ACMMG139, Yr 6)  
- Solve problems involving duration, including using 12- and 24-hour time within a single time zone (ACMMG100, Yr 8) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>7M2</td>
<td>Cheap Houses</td>
<td>Investigation of how to build a functional house at lowest cost.</td>
<td>• Create simple financial plans (ACMNA106, Yr 5)&lt;br&gt;• Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109, Yr 5)&lt;br&gt;• Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies (ACMNA132, Yr 6)&lt;br&gt;• Convert between common metric units of length, mass and capacity (ACMMG136, Yr 6)&lt;br&gt;• Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)&lt;br&gt;• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)&lt;br&gt;• Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187, Yr 8)&lt;br&gt;• Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195, Yr 8)&lt;br&gt;• Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208, Yr 9)&lt;br&gt;• Calculate the areas of composite shapes (ACMMG216, Yr 9)</td>
</tr>
<tr>
<td>7N1</td>
<td>Tangled Fractions</td>
<td>Application of calculations with fractions to a practical activity involving four students and two skipping ropes.</td>
<td>• Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (ACMMG142, Yr 6)&lt;br&gt;• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA153, Yr 7)&lt;br&gt;• Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154, Yr 7)&lt;br&gt;• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183, Yr 8)</td>
</tr>
<tr>
<td>7N2</td>
<td>Directing Numbers</td>
<td>Investigation of directed number, including situations where negative numbers arise, arithmetic operations with directed number, and the appropriate use of calculators.</td>
<td>• Compare, order, add and subtract integers (ACMNA280, Yr 7)&lt;br&gt;• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183, Yr 8)&lt;br&gt;• General Capability of ICT capability (using calculators)</td>
</tr>
<tr>
<td>7N3</td>
<td>What Are You Worth?</td>
<td>Investigation of personal net worth, relating it to the addition of positive and negative numbers. Exploration of different ways of representing very large numbers.</td>
<td>• Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149, Yr 7)&lt;br&gt;• Compare, order, add and subtract integers (ACMNA280, Yr 7)&lt;br&gt;• Solve problems involving profit and loss, with and without digital technologies (ACMNA189, Yr 8)&lt;br&gt;• Express numbers in scientific notation (ACMNA209, Yr 9)</td>
</tr>
<tr>
<td>Code</td>
<td>Title</td>
<td>Summary of Task</td>
<td>Australian Curriculum Links</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td>-----------------</td>
<td>----------------------------</td>
</tr>
</tbody>
</table>
| 7N4  | How Tall is the Criminal? | Investigation of ratios, how they are expressed and some practical applications, including to develop a description of a wanted criminal based on clues left at the crime scene. | • Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108, Yr 5)  
• Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)  
• Recognise and solve problems involving simple ratios (ACMNA173, Yr 7)  
• Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8) |
| 7N5  | Rating Our World | Application of ratios and rates to find the costs of household water and electricity and the impact of installing solar panels and/or rainwater tanks. | • Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108, Yr 5)  
• Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)  
• Recognise and solve problems involving simple ratios (ACMNA173, Yr 7)  
• Investigate and calculate 'best buys', with and without digital technologies (ACMNA174, Yr 7)  
• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)  
• Calculate volumes of rectangular prisms (ACMMG160, Yr 7)  
• Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8)  
• Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198, Yr 8)  
• Calculate the surface area and volume of cylinders and solve related problems (ACMMG217, Yr 9) |
| 7P1  | Dice Doubles | Application of probability to identify and report on the unfairness in a game based on simulated throws of two dice. | • Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167, Yr 7)  
• Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)  
• General Capabilities of literacy (report writing) |
| 7P2  | Fair Game | Application of probability to identify the unfairness in a game based on spinners, and to propose rule changes to make the game fair. | • Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)  
• Identify complementary events and use the sum of probabilities to solve problems (ACMSP204, Yr 8) |
| 7S1  | Are Older Actors Better? | Comparison of the ages of the best actor/actress Academy Award recipients to argue whether male or female actors get better as they age. | • Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169, Yr 7)  
• Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170, Yr 7)  
• Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171, Yr 7)  
• Describe and interpret data displays using median, mean and range (ACMSP172, Yr 7)  
• General Capability of literacy (modality in vocabulary)  
• General Capability of ICT capability (using spreadsheets) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
</table>
| 8A1  | Growing Patterns and Growing Graphs | Exploration of linear and non-linear patterns (relations) and the different methods of representing those relations: visually; tables; pattern rules; equations; and graphs. | • Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107, Yr 5)  
• Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA166, Yr 6)  
• Introduce the concept of variables as a way of representing numbers using letters (ACMNA175, Yr 7)  
• Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176, Yr 7)  
• Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193, Yr 8)  
• Graph simple nonlinear relations with and without the use of digital technologies and solve simple related equations (ACMNA296, Yr 9) |
| 8A2  | Algebraic Caterpillars | Introduction to the distributive law by generalising from the calculation of areas using arithmetic methods to an algebraic approach. | • Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA175, Yr 8)  
• Factorise algebraic expressions by identifying numerical factors (ACMNA176, Yr 8)  
• Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA175, Yr 9)  
• Factorise algebraic expressions by taking out a common algebraic factor (ACMNA296, Yr 10) |
| 8A3  | Building a Mathematics Structure | Introduction to the fundamental properties of our number system (associativity; commutativity; closure; identity; inverse; distributive law) and to some key structures such as groups and fields. | • Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151, Yr 7)  
• Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177, Yr 7)  
• Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190, Yr 8) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
</table>
| 8G1  | Maths in a Box         | Investigation of the relations between the size of a sheet of paper used and the height and volume of a box made from that sheet of paper. | • Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107, Yr 5)  
• Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109, Yr 5)  
• Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111, Yr 5)  
• Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123, Yr 6)  
• Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133, Yr 6)  
• Connect volume and capacity and their units of measurement (ACMMG138, Yr 6)  
• Introduce the concept of variables as a way of representing numbers using letters (ACMNA175, Yr 7)  
• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)  
• Calculate volumes of rectangular prisms (ACMMG160, Yr 7)  
• Draw different views of prisms and solids formed from combinations of prisms (ACMMG161, Yr 7)  
• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183, Yr 8)  
• Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8)  
• Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198 Yr 8)  
• Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208, Yr 9)  
• Calculate the areas of composite shapes (ACMMG216, Yr 9)  
• Solve problems involving the surface area and volume of right prisms (ACMMG218, Yr 9)  
• Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles (ACMMG222, Yr 9) |
| 8M1  | Accuracy and Precision | Investigation of the concepts of accuracy and precision in measurements. Exploration of the various representations of measurement error, and the impact of that error on the appropriate number of significant figures in a measurement. | • Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084, Yr 4)  
• Connect decimal representations to the metric system (ACMMG135, Yr 6)  
• Round decimals to a specified number of decimal places (ACMMG156, Yr 7)  
• Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157, Yr 7)  
• Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158, Yr 7)  
• Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183, Yr 8)  
• Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187, Yr 8) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>8M2</td>
<td>Designing a Kitchen</td>
<td>Development of a design and quotation of a kitchen in a new home, including details of the total area of bench space and volume of cupboards.</td>
<td>- Create simple financial plans (ACMNA106, Yr 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109, Yr 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies (ACMNA132, Yr 6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Convert between common metric units of length, mass and capacity (ACMMG136, Yr 6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208, Yr 9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Calculate the areas of composite shapes (ACMMG216, Yr 9)</td>
</tr>
<tr>
<td>8M3</td>
<td>Crazy Bird Boxes</td>
<td>Development of a design and costing of a bird box that does not contain any right angles, including details of the floor area and volume.</td>
<td>- Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108, Yr 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Calculate volumes of rectangular prisms (ACMMG160, Yr 7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Draw different views of prisms and solids formed from combinations of prisms (ACMMG161, Yr 7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198, Yr 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Calculate the areas of composite shapes (ACMMG216, Yr 9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Solve problems using ratio and scale factors in similar figures (ACMMG221, Yr 9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242, Yr 10)</td>
</tr>
<tr>
<td>Code</td>
<td>Title</td>
<td>Summary of Task</td>
<td>Australian Curriculum Links</td>
</tr>
<tr>
<td>------</td>
<td>---------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| 8N1  | Diminishing Fractions | Practical application of fractions, ratios and patterns to the “A” series of paper sizes and other diminishing sequences. | ● Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107, Yr 5)  
● Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109, Yr 5)  
● Solve problems involving addition and subtraction of fractions with the same or related denominators (ACMNA126, Yr 6)  
● Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133, Yr 6)  
● Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153, Yr 7)  
● Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)  
● Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183, Yr 8)  
● Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8)  
● Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles (ACMMG222, Yr 9) |
| 8N2  | Challenging Fractions | Through completion of several activities, development of deep thinking about fractions and enhanced understandings of common and decimal fractions; and identification of the strengths and limitations of each type of fraction. | ● Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152, Yr 7)  
● Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153, Yr 7)  
● Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154, Yr 7)  
● Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155, Yr 7)  
● Investigate terminating and recurring decimals (ACMNA184, Yr 8)  
● Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194, Yr 8) |
| 8N3  | Square Percentages   | Application of geometry to analyse the areas of pieces resulting from the dissection of squares into up to ten different pieces. Calculation of the percentage and/or fraction of the square represented by each piece. | ● Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies (ACMMG088, Yr 4)  
● Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109, Yr 5)  
● Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123, Yr 6)  
● Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157, Yr 7)  
● Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158, Yr 7)  
● Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr7)  
● Investigate terminating and recurring decimals (ACMNA184, Yr 8)  
● Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187, Yr 8)  
● Calculate the areas of composite shapes (ACMMG216, Yr 9) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>8N4</td>
<td>Consecutive Sums</td>
<td>Working with natural numbers, an investigation of the ways that each number can be expressed as the sum of consecutive numbers.</td>
<td>• Investigate and use the properties of odd and even numbers (ACMNA071, Yr 4)&lt;br&gt;• Identify and describe factors and multiples of whole numbers and use them to solve problems (ACMNA098, Yr 5)&lt;br&gt;• Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107, Yr 5)&lt;br&gt;• Identify and describe properties of prime, composite, square and triangular numbers (ACMNA122, Yr 6)&lt;br&gt;• Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123, Yr 6)&lt;br&gt;• Introduce the concept of variables as a way of representing numbers using letters (ACMNA175, Yr 7)&lt;br&gt;• Simplify algebraic expressions involving the four operations (ACMNA192, Yr 8)</td>
</tr>
<tr>
<td>8P1</td>
<td>The Lucky Prince</td>
<td>Application of probability to find the optimum strategy in two puzzles.</td>
<td>• Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)&lt;br&gt;• Identify complementary events and use the sum of probabilities to solve problems (ACMSP204, Yr 8)&lt;br&gt;• List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225, Yr 9)</td>
</tr>
<tr>
<td>8P2</td>
<td>And Not Or</td>
<td>Development of an understanding of attributes, Venn diagrams, the logical connectives (and, or, not), and their applications to probability.</td>
<td>• Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)&lt;br&gt;• Identify complementary events and use the sum of probabilities to solve problems (ACMSP204, Yr 8)&lt;br&gt;• Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'. (ACMSP205, Yr 8)&lt;br&gt;• Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292, Yr 8)&lt;br&gt;• List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225, Yr 9)&lt;br&gt;• Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP249, Yr 10)</td>
</tr>
<tr>
<td>Code</td>
<td>Title</td>
<td>Summary of Task</td>
<td>Australian Curriculum Links</td>
</tr>
<tr>
<td>------</td>
<td>------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| 8S1  | Distributing Distributions  | Introduction to the collection of data, the nature of a statistical distribution, the preparation of frequency distribution tables, and the use of those tables to calculate measures of central tendency and dispersion. | • Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170, Yr 7)  
• Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171, Yr 7)  
• Describe and interpret data displays using median, mean and range (ACMSP172, Yr 7)  
• Investigate techniques for collecting data, including census, sampling and observation (ACMSP284, Yr 8)  
• Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293, Yr 8)  
• Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207, Yr 8)  
• Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources (ACMSP228, Yr 9)  
• Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282, Yr 9)  
• Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283, Yr 9)  
• Construct and interpret box plots and use them to compare data sets (ACMSP249, Yr 10)  
• Calculate and interpret the mean and standard deviation of data and use these to compare data sets (ACMSP278, Yr 10A)                                                                                                                                                                                                                                                                                                                                 |
| 9A1  | Dividing Diagonals          | Application of the midpoint formula to construct a figure on the Cartesian plane. Investigation of the properties of that figure by calculating the distance and gradient of key line segments. | • Introduce the concept of variables as a way of representing numbers using letters (ACMNA175, Yr 7)  
• Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178, Yr 7)  
• Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194, Yr 8)  
• Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214, Yr 9)  
• Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294, Yr 9)  
• General capability of literacy (converting from one form of representation to another)  
• General capability of ITC capability (dynamic geometry software)                                                                                                                                                                                                                                                                                                                                 |
| 9A2  | Taking the Guesswork Out of Maths | Introduction to algebraic methods of solving simultaneous equations. | • Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194, Yr 8)  
• Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215, Yr 9)  
• Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (ACMNA230, Yr 10)  
• Solve problems involving linear equations, including those derived from formulas (ACMNA235, Yr 10)                                                                                                                                                                                                                                                                                                                                                                                                 |

© QUT YuMi Deadly Centre 2017
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>9G1</td>
<td>Constructive Constructions</td>
<td>Investigation of geometric construction techniques, leading to an exploration of: triangle congruence; angle theorems; deductive geometry; and geometric proofs.</td>
<td>• Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles (ACMMG141, Yr 6)&lt;br&gt;• Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163, Yr 7)&lt;br&gt;• Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164, Yr 7)&lt;br&gt;• Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165, Yr 7)&lt;br&gt;• Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166, Yr 7)&lt;br&gt;• Develop the conditions for congruence of triangles (ACMMG201, Yr 8)&lt;br&gt;• Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202, Yr 8)&lt;br&gt;• Formulate proofs involving congruent triangles and angle properties (ACMMG243, Yr 10)&lt;br&gt;• Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244, Yr 10)&lt;br&gt;• Prove and apply angle and chord properties of circles (ACMMG272, Yr 10A)</td>
</tr>
<tr>
<td>9M1</td>
<td>Look Out for the Baby!</td>
<td>Investigation of the relationship between surface area and volume in order to understand why body temperature regulation can be more difficult for babies.</td>
<td>• Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111, Yr 5)&lt;br&gt;• Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)&lt;br&gt;• Connect volume and capacity and their units of measurement (ACMMG138, Yr 6)&lt;br&gt;• Construct simple prisms and pyramids (ACMMG140, Yr 6)&lt;br&gt;• Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159, Yr 7)&lt;br&gt;• Calculate volumes of rectangular prisms (ACMMG160, Yr 7)&lt;br&gt;• Draw different views of prisms and solids formed from combinations of prisms (ACMMG161, Yr 7)&lt;br&gt;• Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195, Yr 8)&lt;br&gt;• Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197, Yr 8)&lt;br&gt;• Calculate the areas and volume of cylinders and solve related problems (ACMMG217, Yr 9)&lt;br&gt;• Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242, Yr 10)</td>
</tr>
<tr>
<td>9M2</td>
<td>Square Angles</td>
<td>Investigation of how carpenters can use a framing square to construct a range of angles.</td>
<td>• Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8)&lt;br&gt;• Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles (ACMMG222, Yr 9)&lt;br&gt;• Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223, Yr 9)&lt;br&gt;• Apply trigonometry to solve right-angled triangle problems (ACMMG224, Yr 9)&lt;br&gt;• General Capability of ICT capability (using spreadsheets and CAS calculators)</td>
</tr>
<tr>
<td>Code</td>
<td>Title</td>
<td>Summary of Task</td>
<td>Australian Curriculum Links</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| 9M3  | Three-Fact Triangles       | Investigation of various combinations of side and angle facts in triangles, leading to the development of tests of congruence. Application of trigonometry to solve problems and develop generalisations about the area of a triangle. | • Describe and draw two-dimensional shapes, with and without digital technologies (ACMMG042, Yr 2)  
• Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies (ACMMG088, Yr 4)  
• Develop the conditions for congruence of triangles (ACMMG201, Yr 8)  
• Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles (ACMMG222, Yr 9)  
• Apply trigonometry to solve right-angled triangle problems (ACMMG224, Yr 9) |
| 9M4  | How High is that Tree?     | Application of a variety of methods, including shadow sticks, to estimate the height of a tree. | • Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084, Yr 4)  
• Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (ACMMA100, Yr 5)  
• Solve problems involving the comparison of lengths and areas using appropriate units (ACMMG137, Yr 6)  
• Recognise and solve problems involving simple ratios (ACMMG173, Yr 7)  
• Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188, Yr 8)  
• Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220, Yr 9)  
• Solve problems using ratio and scale factors in similar figures (ACMMG221, Yr 9)  
• Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245, Yr 9) |
| 9N1  | Power-ful Mathematics      | Investigation of powers with integer exponents, including their properties and laws. Application of powers to scientific notation. | • Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA182, Yr 8)  
• Apply index laws to numerical expressions with integer indices (ACMNA209, Yr 9)  
• Express numbers in scientific notation (ACMNA210, Yr 9)  
• Extend and apply the index laws to variables, using positive integer indices and the zero index (ACMNA212, Yr 9)  
• Investigate very small and very large time scales and intervals (ACMMG219, Yr 9) |
| 9N2  | It’s All Greek to Me       | Investigation of irrational numbers by the completion of several activities.          | • Investigate the concept of irrational numbers, including π (ACMNA186, Yr 8)  
• Investigate Pythagoras’ Theorem and its application to solving simple problems involving right angled triangles (ACMMG222, Yr 9) |
| 9N3  | Which Card?                | Investigation of the different debit and credit cards available. Comparison of the cards with each other and with cash transactions. Determination of the best card for various spending patterns and other circumstances. | • Investigate and calculate ‘best buys’, with and without digital technologies (ACMNA174, Yr 7)  
• Solve problems involving profit and loss, with and without digital technologies (ACMNA189, Yr 8)  
• Solve problems involving simple interest (ACMNA211, Yr 9)  
• Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229, Yr 10) |
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
</table>
| 9P1  | Is Greed Good?            | Application of probability to investigate a game based on throwing a die and to  | • Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)  
• Identify complementary events and use the sum of probabilities to solve problems (ACMSP204, Yr 8)  
• Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’. (ACMSP205, Yr 8)  
• List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225, Yr 9)  
• Calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’ or ‘or’ (ACMSP226, Yr 9)  
• Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP249, Yr 10) |
| 9P2  | Monopolising Monopoly     | Exploration of some of the probabilities associated with the game of Monopoly to  | • Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168, Yr 7)  
• Identify complementary events and use the sum of probabilities to solve problems (ACMSP204, Yr 8)  
• Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’. (ACMSP205, Yr 8)  
• List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225, Yr 9)  
• Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP249, Yr 10) |
| 9S1  | How to Lie with Statistics| Through the preparation of arguments from the same set of data that support and     | • Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169, Yr 7)  
• Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170, Yr 7)  
• Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171, Yr 7)  
• Describe and interpret data displays using median, mean and range (ACMSP172, Yr 7)  
• Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206, Yr 8)  
• Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293, Yr 8)  
• Investigate the effect of individual data values , including outliers, on the mean and median (ACMSP207, Yr 8)  
• Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (ACMSP227, Yr 9)  
• Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources (ACMSP228, Yr 9)  
• Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282, Yr 9) |

Page 336  Appendices  YuMi Deadly Maths
<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Summary of Task</th>
<th>Australian Curriculum Links</th>
</tr>
</thead>
</table>
| 9S2  | A vs B                 | Collection of data about the careers of two celebrities and the presentation of this data to support a conclusion about who has the more successful career.                                                                 | • Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170, Yr 7)  
• Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171, Yr 7)  
• Describe and interpret data displays using median, mean and range (ACMSP172, Yr 7)  
• Investigate the effect of individual data values , including outliers, on the mean and median (ACMSP207, Yr 8)  
• Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources (ACMSP228, Yr 9)  
• Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282, Yr 9)  
• Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207, Yr 8)  
• Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250, Yr 10)  
| PS1  | Assuming Too Much      | A series of problems that challenge students to question the assumptions that they make when solving problems.                                                                 | • General Capability of critical and creative thinking                                                                                                                                                                                                                                                                                                   |  
| PS2  | Topological Oddities   | Investigation of different aspects of topology: topological change and homeomorphism; the Mobius strip and Klein bottle; and map colouring.                                                                 | • Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries (ACMMG114, Yr 5)  
• Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (ACMMG142, Yr 6)  
• Define congruence of plane shapes using transformations (ACMMG200, Yr 8)  
• General Capabilities of literacy (locating websites of a specified reading level)  
• General Capability of critical and creative thinking                                                                                                                                                                                                 |  
| PS3  | Investigating         | An open-ended investigation of a given geometric figure, providing opportunities to pose and respond to questions in the areas of angles, length, perimeter, area, patterns, etc.                                                                 | • General Capability of critical and creative thinking                                                                                                                                                                                                                                                                                                   |  

Key: A = Algebra; G = Geometry; M = Measurement; N = Number; P = Probability; S = Statistics; PS = Problem-Solving