Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module O5

Year C, Term 4

Operations:

Financial Mathematics

Prepared by the YuMi Deadly Centre
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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

This module covers the mathematics of financial literacy. As such, it uses number, operations, algebra, measurement and statistics/probability knowledge from previous modules and applies it to finance and consumer contexts. Much of what is being covered in this module has been covered in the number, operations and algebra modules, and also in the measurement modules (particularly Module M3 which covered money as a measure), and statistics and probability modules (particularly SP2 *Probability*).

However, the module integrates these mathematics ideas and directs them to one context and this is a worthwhile mathematical activity. Further, the context of finance affects all Australian people as it relates to work and pay, buying and selling, housing and cars, credit cards and debt, and so on.

This Overview section of the module covers the background information needed for financial mathematics, the sequencing that will be used in this module and its relationship to the *Australian Curriculum: Mathematics*.

Background information for teaching financial mathematics

This subsection looks at the mathematical basis of finance and the big ideas from number, operations and algebra that affect this mathematics.

Finance in mathematics

Finance deals with numbers, operations and algebra. In terms of number, finance uses whole numbers, decimal numbers, fractions, and percent, rate and ratio, including directed numbers (assets and liabilities). With respect to operations, it covers all four operations (addition, subtraction, multiplication and division) plus percent, rate and ratio applications. With respect to algebra, it covers formulae, variables, expressions and equations, change and functions, and all the principles and strategies of arithmetic and algebra.

This means that financial mathematics has connections as follows:

1. **Number –** whole numbers, directed numbers and decimals (tenths and hundredths) when discussing dollars and cents and how much money you have/level of debt, and percent in decimal and fraction form when considering interest rates (and also rate and ratio when considering differences between prices, lenders, packages and plans).

1. **Operations** – calculations of money and comparisons of the effect of rates, and so on; development of budgets, planning trips and events, financial planning, and so on; best buys in supermarkets and so on; and checking statements, determining change rates and understanding effect of stocks and shares.

2. **Measurement** – relation to value as an attribute for money; history of finance (e.g. bartering, use of non-standard units, money lending and rise of gold standard); familiarity with present money (coins and bills); and formulae.

3. **Algebra** – development of formulae for interest (simple and compound), profit and loss, and discounts and taxes, particularly for rates that are on final amount (such as GST).

4. **Statistics and probability** – determination of the effect of probabilities on likely financial outcomes (risk, potential debt) and making inferences with respect to investments; interpreting tabular and graphical information; determining best plans for phones and health; determining whether it is best to buy, lease or mortgage; and identifying scams and rip offs.

Big ideas for financial mathematics

The big ideas for financial mathematics include those for number, operations, algebra, measurement, and statistics and probability. Here in the table below is a list of major ones taken from the AIM *Overview* booklet (pp. 19–20). More detail of the big ideas can be found in modules on the same mathematics strand.

Global big ideas	• *Symbols tell stories.* The symbols of mathematics enable the world to be described succinctly and in a generalised way (e.g. 2 + 3 = 5 means caught 2 fish and then caught another 3 fish, or bought a $2 chocolate and $3 drink, or joined a 2 m length of wood to a 3 m length, and so on). • *Change vs relationship.* Everything can be seen as a change (e.g. 2 goes to 5 by +3) or as a relationship (e.g. 2 and 3 relate to 5 by addition). • *Probabilistic vs absolutist.* Things are either determined by chance (e.g. will it rain?) or are exact (e.g. what is $2 + $5?). • *Accuracy vs exactness.* Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100) or exactly (5 275 + 3 873 = 9 148). • *Continuous vs discrete.* Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted. • *Part-part-total/whole.* Two parts make a total or whole, and a total or whole can be separated to form two parts (e.g. fraction is part-whole, ratio is part to part; addition is knowing parts, wanting total).
Numeration big ideas	• *Part-whole/Notion of unit.* Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts (e.g. if there are six counters, each counter can be a unit, making six units, or the set of six can be one unit.) • *Concept of place value.* Value is determined by position of digits in relation to ones place. • *Additive/Odometer.* All positions change forward from 0 to base, then restart at 0 with position on left increasing by 1, and the opposite for counting back (e.g. $2^{3}/_{5}$, $2^{4}/_{5}$, 3, $3^{1}/_{5}$, and so on). • *Multiplicative structure.* Adjacent positions are related by moving left (× base); moving right (÷ base). Base is normally 10 or a multiple of 10 in Hindu-Arabic system and metrics. • *Number line.* Quantity on a line, rank, order, rounding, and density.
Equals, operations and algebra big ideas	• *Concepts of the operations.* Meanings of addition, subtraction, multiplication and division. • *Equals and order.* Reflexivity/non-reflexivity – A=A but A is not > A; Symmetry/antisymmetry – A=B → B=A while A>B → B<A and A<B → B>A; Transitivity – A=B and B=C → A=C and A>B and B>C → A>C. • *Balance.* Whatever is done to one side of the equation is done to the other. • *Identity.* 0 and 1 do not change things (+/− and ×/÷ respectively). • *Inverse.* A change that undoes another change (e.g. +2/−2; ×3/÷3). • *Commutativity.* Order does not matter for +/× (e.g. 8+6 / 6+8; 4×3 / 3×4). • *Associativity*. What is done first does not matter for +/× (e.g. (8+4)+2 = 8+(4+2), but (8÷4)÷2 ≠ 8÷(4÷2)). • *Distributivity*. ×/÷ act on everything (e.g. 2×(3+4) = 6+8; (6+8)÷2 = 3+4). • *Compensation.* Ensuring that a change is compensated for so answer remains the same – related to inverse (e.g. 5+5 = 7+3; 48+25 = 50+23; 61−29 = 62−30). • *Equivalence*. Two expressions are equivalent if they relate by +0 and ×1 – also related to inverse, number, fractions, proportion and algebra (e.g. 48+25 = 48+2+25−2 = 73; 50+23 = 73; ${}^{2}/_{3} = {}^{2}/_{3} \times {}^{2}/_{2} = {}^{4}/_{6}$). • *Inverse relation for −, ÷ / direct relationship +, ×.* The higher the number the smaller the result (e.g. 12÷2 = 6 > 12÷3 = 4; 1/2 > 1/3); the higher the number the higher the result (e.g. 4+3 < 4+7). • *Backtracking.* Using inverse to reverse and solve problems (e.g. 2*y*+3 = 11 means *y*×2+3, so answer is 11−3÷2 = 4). • *Basic fact strategies.* Counting, doubles, near 10, patterns, connections, think addition, think multiplication. • *Operation strategies.* Separation, sequencing and compensation. • *Estimation strategies*. Front end, rounding, straddling and getting closer.

Measurement big ideas	• *Concepts of measure*. Length, perimeter, area, volume, capacity, mass, temperature, time, money/value, angle. • *Notion of unit.* Understanding of the role of unit in turning continuous into discrete. • *Common units.* Must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60). • *Inverse relation.* The bigger the unit, the smaller the number (e.g. 200 cm = 2 m). • *Accuracy vs exactness.* Same as Global principle (e.g. cutting a 20 cm strip usually does not give a length of exactly 20 cm). • *Attribute leads to instrumentation.* The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring). • *Formulae.* Perimeter, area, volume formulae. • *Using an intermediary.* Using string to compare length of a pencil with distance around a can.
Statistics and probability big ideas	• *Tables and graphs.* Types of charts and tables, comparison graphs, trend graphs and distribution graphs. • *Concept of probability.* Chance (possible, impossible and certain), outcome, event, likelihood. • *Inference concepts.* Variation, error, uncertainty, distribution, sample, and inference itself. • *Experimental vs theoretical.* Knowing when something can be calculated or determined by trials. • *Equally likely outcome.* Outcomes as a fraction by number giving result ÷ total number. • *Formulae.* Mean, mode, median, range, deviation, standard deviation, quartiles. • *Integration of different knowledges.* For example, question *Do typical Year 7 students eat healthily?* requires some form of data gathering, determining typical, and determining healthy eating.
Pedagogy big ideas	• *Interpretation vs construction/Generation vs illustration.* Things can either be interpreted (e.g. what operation for this problem, what properties for this shape) or constructed (write a problem for 2+3=5; construct a shape of 4 sides with 2 sides parallel) – activities should generate students' knowledge not illustrate teachers'. • *Connections lead to instruction/Seamless sequencing.* Two connected ideas are taught similarly and progress from one to the other should not involve changing rules. • *Pre-empting and peel back/Compromise and reteaching.* Look forward and back – teach for tomorrow and rebuild from known – be aware what ends and what lasts forever and rebuild ideas not lasting. • *Gestalt leaps and superstructures.* Look out for ways of accelerating knowledge. • *Language as labels/Construction before explanation.* New ideas to be constructed not told. • *Unnumbered before numbered.* Big ideas are best started in situations without number. • *Creativity.* Let students create own language and symbols (particularly to support pattern). • *Triadic relationships.* When three things are related, there are three problem types (e.g. 2+3=5 can have a problem for: ?+3=5, 2+?=5, 2+3=?). • *Problem solving.* Metacognition, thinking skills, plans of attack, strategies, affects, and domain knowledge. • *RAMR cycle.* All components of RAMR cycle are big pedagogy ideas.

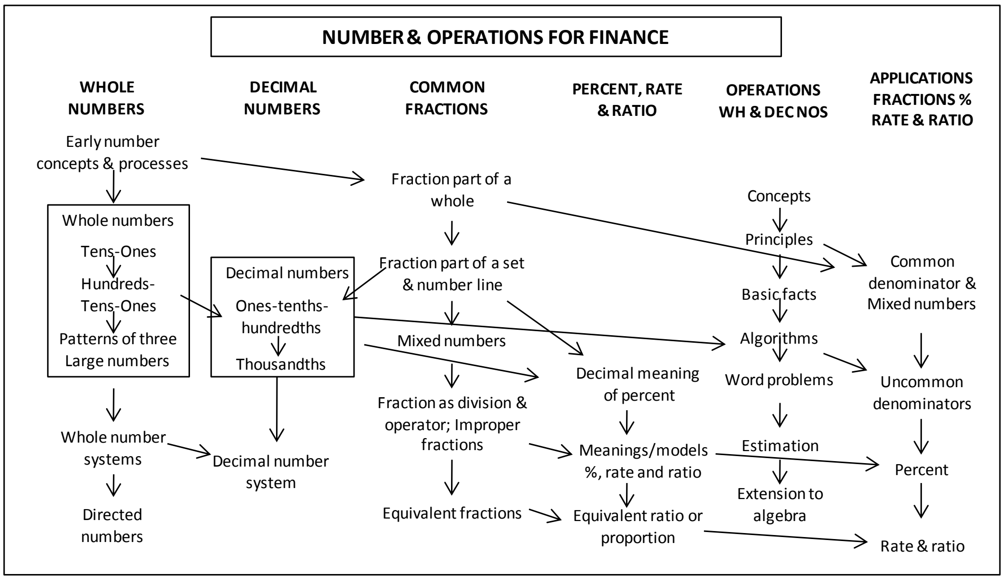
Sequencing for financial mathematics

This subsection looks at overall sequencing for financial mathematics in terms of number and operations and then at the particular sequencing for finance that is the basis of this module.

Overall sequencing

The sequencing for financial mathematics follows the sequencing for number and operations, since financial mathematics, in terms of mathematics, is the application of number and operations. The overall sequences for these two areas are shown below in the diagram titled "number and operations for finance".

The diagram shows that number for finance develops from early whole numbers (e.g. recognition of coins and bills, buying and giving change, in terms of whole numbers) and fractions to decimal numbers (e.g. money in terms of decimals such as $24.56) until it gets to applications in percent (e.g. 45% of $120) and rate and ratio (e.g. petrol costs $1.65 per litre, how many litres for $70?). In particular, percent is important as it leads to understanding of profit, loss, discount, interest, taxes such as GST, and so on.



Sequencing in this module

To cover a range of activities to do with finance, this module has the following sections.

Overview: Background information, sequencing and relation to Australian Curriculum.

Unit 1: Early financial mathematics activities – money activities such as buying and giving change that have to do with a whole number understanding and a focus on notes and coins.

Unit 2: Percentage and ratio activities – moving on to simpler uses of percent such as profit, loss and discount, application of tax such as GST, and rate/ratio activities such as best buys in a supermarket.

Unit 3: Simple and compound interest – moving on to understanding the finance of borrowing and lending money, from simple to compound interest.

Unit 4: Extension to algebra – although a lot of the activities in earlier units use algebraic thinking (e.g. budgets where spending more on one area means spending less in another area), this looks at the formal outcomes of having an algebraic perspective on finance, particularly through formulae.

Unit 5: Plans, budgets and other financial investigations – this unit looks briefly at the projects that can be set for students or groups of students that are based on finance (e.g. plans for events such as a class trip; preparing budgets for one person or a family; determining which plan for a mobile phone is best).

Test item types: Test items associated with the five units above which can be used for pre- and post-tests.

Appendix A: RAMR cycle components and description.

Appendix B: AIM scope and sequence showing all modules by year level and term.

Each of the units is based on sets of ideas for classroom activities and, where possible, full RAMR lessons (see **Appendix A**) are also provided to enrich the ideas and to provide a framework for teaching.

Relation to Australian Curriculum: Mathematics

AIM O5 meets the Australian Curriculum: Mathematics (Foundation to Year 10)						
Unit 1: Early finance and mathematics activities Unit 2: Percentage and rate/ratio activities Unit 3: Simple and compound interest Unit 4: Extension to algebra Unit 5: Plans, budgets and other financial investigations						
Content Description	**Year**	**O5 Unit**				
		1	**2**	**3**	**4**	**5**
Recognise, describe and order Australian coins according to their value (ACMNA017)	1 to 3	✓				
Count and order small collections of Australian coins and notes according to their value (ACMNA034)		✓				
Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents (ACMNA059)		✓				
Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies (ACMNA080)	4	✓				
Create simple financial plans (ACMNA106)	5					✓
Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (ACMNA127)	6		✓			✓
Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (ACMNA128)			✓			✓
Make connections between equivalent fractions, decimals and percentages (ACMNA131)			✓		✓	✓
Investigate and calculate percentage discounts of 10%, 25%, and 50% on sale items, with and without digital technologies (ACMNA132)			✓		✓	
Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)	7	✓	✓		✓	
Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)		✓			✓	
Recognise and solve problems involving simple ratios (ACMNA173)			✓		✓	
Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)			✓			✓
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)	8	✓	✓		✓	
Solve a range of problems involving rates and ratios , with and without digital technologies (ACMNA188)					✓	✓
Solve problems involving profit and loss, with and without digital technologies (ACMNA189)			✓			
Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)	9		✓		✓	✓
Solve problems involving simple interest (ACMNA211)				✓		

Unit 1: Early Financial Mathematics Activities

Financial mathematics begins with money. This unit looks at early activities that focus on money, predominantly using whole numbers. It begins with a list of ideas, then describes a RAMR lesson, and finally gives an investigation.

1.1 Money activities

The following is a list of starting activities – textbooks and the Internet will uncover many more.

1. **Getting to know coins and notes**. Obtain coins and notes or copies of them (play money) and play games and activities to enable the students to instantly recognise them:
   (a) draw copies of coins and notes (both sides) – do rubbings of the coins;
   (b) play matching games of note/coin and name/value.
2. **Relating coins and notes**. Look at different combinations of coins and notes to make other coins and notes (e.g. *How many ways can you make 50 cents?*).
3. **Memory games**. Make cards with coins/notes, names, values and pictures of other notes/coins which add to the same amount; turn them over, students have to make pairs by remembering where the other card is when turned over. This activity is highly recommended as it really gets the mind to focus on what coins/notes look like.
4. **Games**. Set up games to be played with play money:
   (a) commercial games such as Monopoly;
   (b) matching games such as snap, bingo and cover-the-board (have to match picture of coin/note with name, amount, and, if wanted, mixtures of other coins/notes that give the same amount);
   (c) made-up games such as throwing a die and giving each number a value (e.g. 5 cents through to $2); the players take turns throwing die and collect money shown by die – first one over $5 or $10 wins – or can be first one who can make exactly $5 or $10 wins;
   (d) race games where you land on squares which give or remove money (students keep records of how much lost or given) – when someone finishes, the person with the most money wins; or the other way around – players all start with a certain amount of money and you can miss a turn or pay a fine if you land on a square with a negative move (students choose whether to use their money to pay the fine or hold for a better time to use their money and miss a turn instead) – first one to the end wins.
5. **Using coins and notes in community settings**. Set up shops where students can buy, sell and give change using play money. Students can use adverts in papers to make shop fronts with pictures and amounts.
6. **Classroom money**. Classes with behaviour and attendance money have used the following:
   (a) Students are given an amount of money per day that they attend and are also given extra as rewards for good behaviour and are fined for bad behaviour.
   (b) Once a week, the class sets up as a town with bank and shops; students go to the bank and take out money to spend in the shops and students can exchange school play money for lollies, pens, rubbers, rulers, early leaving, time on computers, or whatever the school and teacher allow (students have a passbook in which to record their attendance money, rewards and fines).

(c) It is useful to have major items (e.g. a football) that require months of saving, good behaviour and good attendance to get.

(d) The teacher has to buy some things but a lot can be got from handouts from groups (e.g. AFL) coming around to encourage students to play their sport.

This sets up activities that, when completed, improve attendance and behaviour as well as improve number, operations and financial maths.

Note: In one school, the teacher even got students to pay electricity bills, council rates, and so on, each week so they realised there were outlays as well as inputs in everyday life (the amount per day for attendance had to rise) – this teacher even required those with low attendance and high fines to borrow to pay these outlays, and to pay interest on the loans.

7. **Eating out**. Students are given money and have to buy a meal from a restaurant guide (the Internet is good to get these). This can also be the start of a worksheet of exercises (e.g. *How much for the soup and the steak? How much change from $50 if you buy the chicken and the ice-cream? If you have $100, can you buy the oysters and the lobster?*).

8. **Buying online**. Similar to the above but use online shopping guides to buy things.

9. **Worksheets**. Made up of amounts and pictures of notes/coins – students add, subtract, multiply and divide – problems given in pictorial and word form (pictures of notes/coins that buy one coat – *how much for one coat?*)

10. **Computers**. All the above on virtual worksheets, activities and games – see YuMi Deadly Centre website for virtual activities with money (Student Learning resources, Number and operations – Finance and virtual manipulatives http://ydc.qut.edu.au/projects/project-resources/student-learning-projects-resources/).

1.2 RAMR money calculation lesson: "Tuckshop orders"

Learning goal: Students will make calculations with money.

Content description: Number and algebra – Money and financial mathematics. Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies [ACMNA080].

Big idea: Money calculations, part-part-whole.

Resources: Play money Australian notes and coins, tuckshop menu, shop catalogues.

Reality

Local knowledge

Ask students to tell their money stories, for example, how much they get for pocket money, items they have seen increase in cost, items that are cheaper if you buy more in number or in quantity, how they like to spend their pocket money, what are essential goods to buy.

Prior experience

Check that students know the Australian coins/notes and their value. Check that they understand the multiplicative relationship that exists between them.

Kinaesthetic

Display the tuckshop menu and prices. Have the students act out the roles of student buying and person selling. After one student buys from the tuckshop giving money and receiving change, reverse so that the buyer becomes the seller and the seller goes to the back of the line to become a buyer. Students and teacher check each transaction for accuracy.

Abstraction

Body

Tuckshop orders. Each student is given a purse containing different amounts of money and a tuckshop menu. They select items from the menu and calculate how much their order will cost and how much change they will receive. One by one they go to the student tuckshop convenor, make their purchase/s and give the total cost. The convenor counts out the change and the student checks the amount given. All students participate in calculating total cost and amount of change to be given at each transaction. Use the "think addition" strategy for giving change, e.g. $5 − $3.45: + 5c (up to $3.50), + 50c (up to $4), + $1 (up to $5). Change is 5c + 50c + $1 = $1.55.

Hand

Whole to parts – given $10 note. In groups, make as many different combinations as possible to show other ways of giving $10 change when a $10 note is not available. Share with the class.

Parts to whole. Students count the money in their purses, record the parts and tally the whole. Share the results.

Mind

Students visualise tuckshop items and their cost and in their mind see the notes and/or coins that would be needed to buy the item. Reverse: Given $5 to spend – *What could be bought? How much change would there be? How could that be given to you?*

Creativity

Students write their own tuckshop menu and cost per item. Buy 3 items, calculate cost and change required.

Mathematics

Language/symbols

Dollars, cents, coins, notes, equivalent amounts, change, tender, subtract, estimate, round, change, budget.

Practice

1. Use the thinkboard to create practice examples for students to complete the missing parts, for example:

Story With my birthday money, I am buying an iPad worth $469.95. How much change will I get from $500?	**Materials**
Picture	
Language	**Symbols**

2. Use catalogues from various stores. Have examples that involve all four operations.

3. Include worksheet on equivalence in money using different notes and coins. Also have virtual activities.

Connections

Place value, strategies for the four operations.

Reflection

Validation

Students discuss where money exchange occurs in their world, e.g. getting/spending pocket money, shopping, banking.

Application/problems

Provide applications and problems for students to apply to different contexts independently, e.g. *Calculate how much you will save in a year if you bank $1.50 each week out of your $10 weekly pocket money. If you had done that since you were 4 years old, how much would you have now?*

Extension

Flexibility. Students are able to give many ways to represent an amount of money.

Reversing. Students should work from stories through to symbols on the board, and then reverse from symbols back to stories: stories ↔ acting out/modelling with materials ↔ pictures (drawing) ↔ language ↔ symbols.

Generalising. Students understand that large coins/notes are multiples of smaller coins/notes so that there are many ways to represent the same amount. The four operations that are applied to integers are applied in the same way to money.

Changing parameters. Students are given examples using money that extend to ten thousands. Students explore currencies of other countries to investigate the relationship between the money denominations of the particular countries.

Teacher's notes:

- Teachers using thinkboards often photograph them when students have finished filling them out (but not removing the materials) so that a permanent record of the students' work can be displayed (the materials section on the board is not permanent).
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section.
- The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point. Reflection should not be undertaken until students have mastered the mathematical concept.

1.3 Investigation: Shopping

This financial investigation requires students to engage in shop activities.

Directions

1. **Setting up a shop**. Decide on a focus of the shop. Collect pictures of a variety of things that could be sold in the shop (use newspapers, advertising materials, Internet). Prepare a "shop front" display with name of shop and pictures/objects and prices of at least 20 things you have for sale.

2. **Running your shop**. Obtain play money and sell the objects in your shop. Give the correct change. Calculate the price of two things for the customers. Keep a record of sales and change given.

3. **Inspecting other shops**. Become a shop inspector. Go to other shops. Buy things. Pay in large bills. Check the shopkeeper's calculations for total price and providing correct change. Keep a record of your purchases and change give.

4. **Reporting on shopping**. Write a report describing your shop, how you calculated the totals and change, your recording method as a shopkeeper and an inspector, and what you think is the best shop.

Teacher hints

- The idea here is for students to practise calculating cost of items and giving of change using any method they wish. Give number lines or allow students to use money on PVC if this will help.
- Organise so that all prepare a shop (groups may be necessary). Get half to buy from others and vice versa. Need to be organised here.
- Encourage students to be creative and artistic in the presentation of the shop but also to be well structured.
- Check students' method of recording sales/change before start of activity.

Assessment rubric

Activity	Excellent (A)	Good (B)	Satisfactory (C)	Effort shown (D)	No effort shown – unsatisfactory (E)
Shop	Excellent structure, creativity and presentation	Good structure, creativity and presentation	Adequate structure, creativity and presentation in most areas	Does not meet adequate standards but effort shown	Inadequate and no effort shown
Recording form	Well thought out recording forms	Useable recording forms	Forms able to be used in most situations	Inadequate forms but effort shown	Inadequate and no effort shown
Shop-keeping	All correct calculations	Nearly all calculations are correct	Mostly correct calculations	Mostly incorrect calculations but effort shown	Incorrect and no effort shown
Inspections	All correct checking	Nearly all correct checking	Mostly correct checking	Inadequate checking but effort shown	Inadequate and no effort shown
Report	Explains clearly with much detail	Explains clearly with some detail	Readable report	Does not meet requirements but effort shown	Does not meet requirements and no effort shown

Unit 2: Percentage and Rate/Ratio Activities

The major applications of number and operations into financial mathematics lie with percentage and rate/ratio. In this unit, we go through the mathematics of the above and follow this with two RAMR lessons, one on percent and taxes, and one on rate/ratio and best buys.

2.1 Percentage and rate/ratio activities

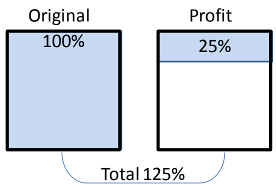
Percentage and profit and loss

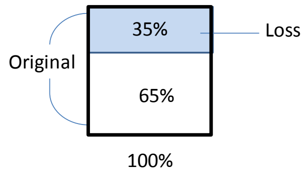
Profit and loss require the percent to be calculated by adding profit to 100% and taking loss from 100%.

Thus, using size diagrams:

(a) **Profit** of 25% means have original 100% plus the 25% = 125%. **As a decimal this is 1.25**.

(b) **Loss** of 35% means reducing original by 35% = 65%. **As a decimal this is 0.65**.





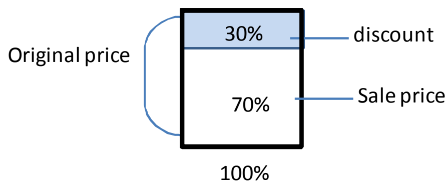
The usual practice is to look at profit and loss across a collection of cases. For example, *If I had $5000 in shares and made a loss of 8% in a year, had $3000 in property and made of profit of 12% in a year, and had $3000 cash and made a profit of 5% in a year, what was the loss/profit of the whole amount?* Taking each part separately, the table below gives the answer.

Starting Amount	Profit/Loss	Finishing Amount
$5 000	Loss 8% = $400	$4 600 (0.92 × 5000)
$3 000	Profit 12% = $360	$3 360 (1.12 × 3000)
$3 000	Profit 5% = $150	$3 150 (1.05 × 3000)
$11 000 TOTAL		**$11 110 TOTAL**

The totals mean that a profit of $110 was made out of the $11 000. This is a $\frac{110}{11000}$ = $\frac{1}{100}$ = 1% profit.

The use of tables becomes very important in problems of the above type as the strategies of "break the problem into parts" and "exhaust all possibilities systematically" are used.

Percent and discount



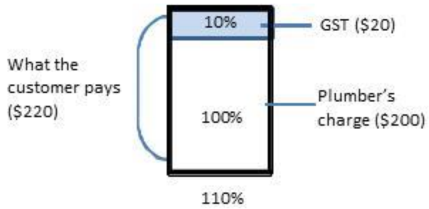
Discount is similar to loss in that it is a reduction from 100% and normally we have to work out 100% less (subtract) that reduction. For example, if the original price was $140 and there was a discount of 30%, this would mean that the sale price would be 70% (100 − 30) of the $140 or 0.7 × 140 = $98. This, of course, is the same as calculating 30% of $140 = $42 and subtracting this from the original price.

However, we have to be careful about what is the 100%. If we had the same discount and the sale price was $140, then the original price would be 100% and the sale price of $140 would be 70%. This would mean that 1% would be 140/70 = $2 and 100%, the original price, would be $200. However, **it should be noted that this is not the same as adding 30% of $140 to the $140** (which would equal $140 + $42 or $182). The reason for this is that the $140 is not what we are taking the percent discount of, it is not the 100% as it is in the problem above – it is 70%.

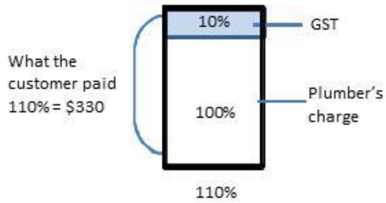
Percentage and taxes

Taxes are another way in which % is used. If a tax on importing clothes is 17.5%, then, if we want to import \$2000 worth of clothes, we have to pay an extra 0.175 × 2000 = \$350.

The Goods and Services Tax (GST) is another example of this. If a plumber, John, charges a customer \$200, he also has to add on 10% GST which makes the total bill \$220. Thus we have the diagram on the right.



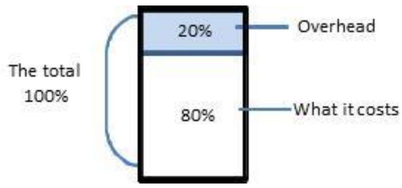
What if John the plumber does not record the separate amounts and only knows the total amount he has been paid by his customers – what does he pay the government in GST? Say John is paid \$330, what GST does he have to pay?



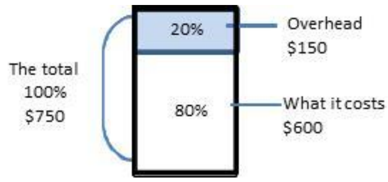
The problem here is that the GST is not 10% of \$330 or \$33. The amount of GST is $\frac{10}{110} = \frac{1}{11}$ of \$330, that is \$30.

The plumber has to know that he has to charge the amount he wants to get plus 10% to cover GST. However, if he is looking at what he gets from the customer, he has to divide this by 11 to get the GST he has to pay to the government. The important point is to understand where the 100% is.

Sometimes the percent is charged on the final amount, as in the case of some overheads. A company may decide that it charges what it costs + 20% but the 20% is not on the cost but on the total. Here the diagram is as shown on right.



How do we work out the total and the overhead for a \$600 cost? We know that \$600 is 80%, so 1% is $\frac{600}{80}$ = \$7.50, therefore 100% is \$750. This means that the overhead is \$150, which is 25% of the \$600 but only 20% of the total of \$750, as in the diagram on the right.



This means that the customer has to pay 25% above what it costs so the company can make 20% on the total. This is different to GST and most taxes.

Rates/ratios and best buy in supermarket

A good application of rates and ratio is to visit a supermarket, look at costs and calculate the best buy. Modern supermarkets now have already done a lot of the work for us by stating cost/weight on their displays.

There are **two rates** that can be used to determine best buy in a supermarket as the example of two cans of beans (A and B) in the table below shows.

A **Mass: 440g, Cost: \$2.10**	B **Mass: 300g, Cost: \$1.55**
Rate 1: Cost/weight (\$/g)	Rate 2: Weight/cost (g/\$)
A gives \$2.10 ÷ 440g = 0.0048 \$/g	A gives 440g ÷ \$2.10 = 210g/\$
B gives \$1.55 ÷ 300g = 0.0052 \$/g	B gives 300g ÷ \$1.55 = 194g/\$
Thus B is more expensive because each gram costs more.	Thus B is more expensive because you get less grams for each dollar.

(*Note*: Lower is better for Rate 1, i.e. \$/g; while higher is better for Rate 2, i.e. g/\$.)

One could look at a lot of other situations and use rates to work out best value (although value is more than money – something may be more expensive but tastes better). For example, what are the costs of events (e.g. football games, concerts) in relation to how long they go for? This gives possibilities of working out cost/time (lower the better) and time/cost (higher the better). Of course often there is more than one thing involved in determining value. For instance, a mobile phone might have higher text costs and lower call costs or vice versa. This leads to more complex investigations.

Often, the things being compared are given in **ratios**, that is, 3 hours of play for $7 or 3:7, while 5 hours is $11 or 5:11. Sometimes we can directly compare the ratios but usually we have to change to a rate (e.g. dollars/hour is 7/3 or $2.33 for the first and 11/5 or $2.20 for the second which is better value in terms of cost per hour).

2.2 RAMR lesson on percentage: “Bright budgeting”

Learning goal: Students will make calculations with money amounts and make financial decisions.

Content description: Number and algebra – Money and financial mathematics.

Big idea: Part-whole.

Resources: Envelopes with receipts and invoices; Receipts and Invoices worksheet – table with two columns: Total Spent and GST; Maths Mat and elastics; brochures, e.g. JB Hi-Fi, Dick Smith, Big W; calculators; envelopes with $100; float for each shop; pads, pencils.

Reality

Local knowledge

Students describe the ways they get money of their own – e.g. pocket money, birthday gifts, working for Grandma, interest on their money in the bank – and what they do with the money they have, e.g. spend it, save it, save up for something special, Christmas gifts.

Prior experience

Revise rounding of money to the nearest five cents and percentage, especially 10%.

Kinaesthetic

Maths mat. Students use elastics to show $50 (5 rows of 10). This is how much the article cost the seller to produce so that a profit can be made. The government now charges 10% or $^1/_{10}$ of the $50 as GST. How much is the government getting as GST? [$5] Students use another elastic to show the extra GST of $5. How much must the seller charge the customer now to retain the profit? [$50 + GST $5 = $55 or 100% of the cost price (that includes profit) PLUS 10% of the cost that goes to the government as GST] What fraction of the selling price of $55 is the GST of $5? [$^5/_{55} = {}^1/_{11}$].

Lucky dip. Students take one envelope from a basket with envelopes containing receipts and invoices. They examine the receipts and invoices, and record in a table the Total Spent and GST component.

Students check the GST on their invoices with calculators. To do this, the total on the invoice or receipt must be divided by 11 to find the GST component or $^1/_{11}$ of the total equals the GST. Students compare all their invoices with a partner and discuss the pattern they see in all these amounts. [The GST is $^1/_{11}$ of the total.] How do we know what the seller’s cost was? [Take the GST away from the Total on the invoice.]

Have students research the GST on the Internet.

Note: Some items on a receipt/invoice may be asterisked as GST free, in which case the total GST on the invoice will be smaller than 10% of the base cost and the calculation of GST from the total may not be exactly $^1/_{11}$.

Abstraction

Body

Act it out. Point of sale – set up shops for various outlets with catalogues on the "counter" so that students can match advertised amounts in spending their \$100 cash.

Encourage students to "shop around" and prepare a plan of the items they want to purchase, prioritising them so that their spending does not exceed their budget of \$100. Prompt them to compare prices so that they purchase the item from the shop with the least cost and examine the catalogues closely to buy at sale prices. Always round to the nearest 5 cents and estimate what the change will be.

Reverse. Calculate the change to be received from purchased items, rounding to the nearest 5 cents, estimating their change and continually checking to see they are spending within their budget.

Hand

Search online for the cheapest prices for your wish list of goods. Note that price may determine quality and therefore only goods that are the same model and brand can be equally compared. Write a spending plan for your wish list that is not to exceed \$500. Have two sections: **Income** (sources or where the money has come from) and **Expenditure** (where the money is going). List the name of the goods to be purchased, the shop, the full price, the sale price, and keep a running total. Calculate the sale price where all goods have a reduction of 25%. Calculators may be used.

Mind

Close your eyes and see a receipt for a total of \$33.00. How much GST has been charged? [$^1/_{11} \times \$33 = \$3$]. In your mind, see what the GST on the invoice is if the base amount is \$245 [$10\% \times \$245 = \$24.50$] and add these together to give the total amount on the invoice [$\$245 + \$24.50 = \$269.50$].

Creativity

Create a spending plan for your wish list of goods. The sky's the limit but you must be able to show the goods and the total you have spent and also the sources of this money.

Mathematics

Language/symbols

GST, goods, services, tax, component, spending plan, budget, income, expenditure, purchase, cost, financial plan, savings, balance, overspending, profit

Practice

Calculators may be used in the following examples.

1. You are having a birthday party. Create a list of items to be purchased to help make your party a great celebration. Search the Web to calculate the budget you will need to be able to carry out your plan. Look at your savings account. Are you overspending? How can you cut down the cost? Search: www.colesmyer.com.au and www.homeshop.com.au

2. Create the budget spreadsheet for this information and calculate the balance:

   INCOME: Walk neighbour's dog – \$5, owed from a friend – \$3, pocket money – \$4, money for birthday – \$12

   EXPENDITURE: Pay for school excursion – \$3, buy Mum a birthday gift – \$9.90; buy a pencil case – \$3.90.

3. Worksheet exercises (examples below):

   (a) The cost of a watch is \$410 excluding GST. Find the total cost including GST.

(b) Your dad stops to fill up with petrol at Sam's petrol station. The car takes 80 litres at $1.60 per litre. How much profit does Sam make on the deal if he bought the petrol at $1.30 per litre? (Remember the GST.) [Cost: 80 × $1.30 = $104 + GST: 10% × $104 = $10.40 so Sam's cost = $114.40. Selling price: 80 × $1.60 = $128. Profit: $128 − $114.40 = $13.60]

(c) A crate of 20 two-litre bottles of milk is sold at $2.75 a bottle and a profit of $4 is made on the sale. What did the milk cost to buy per bottle? (There is no GST on milk. It is one of the essentials and so is exempt from GST.)

(d) JB Hi-Fi is having a one-day sale and giving 10% off everything storewide. What will you pay for a laptop originally marked at $1350 if you buy it during the one-day sale?

Connections

Relate to money, percentage, ratio, bank interest.

Reflection

Validation

Students check where budgets and financial plans are necessary in the real world, e.g. family budgets, holiday budgets, wedding budgets, school financial planning, business and government financial planning.

Application/problems

Provide applications and problems for students to apply to different real-world contexts independently. For example, *With $60 you must: select enough food for three days for a family of three. This includes breakfast, lunch and dinner, remember that one family member is 11 years old and the other two are adults. List each grocery item and its price*. Another option is to look at bank interest on savings – see Unit 3.

Extension

Flexibility. Students are able to understand and apply the many factors that need to be considered in financial planning.

Reversing. Students are able to move between planning a budget ↔ modelling it ↔ interpreting financial plans ↔ naming the GST component ↔ calculating the amount of GST, starting from and moving between any given point.

Generalising. Financial planning is essential to ensure all parts of society live within their means. Income determines the amount available for expenditure. The GST is a tax that applies to most goods and services. The GST is 10% of the selling price.

Changing parameters. How much more would you have to pay on a $50 pair of shoes if the GST changed to 12.5% and the same profit margin was kept as when the GST was 10%?

Teacher's notes

- Students need a thorough understanding of division by 10 to be able to calculate GST on base amounts without a calculator. Calculators can be used for division by 11 when calculating GST from the total invoice.
- Explicit teaching **that aligns with students' understanding** is present in every part of the RAMR cycle and has particular emphasis in the Mathematics section.
- The RAMR cycle is not always linear but may necessitate revisiting the previous stages at any given point. Reflection should not be undertaken until students have mastered the mathematical concept.

2.3 RAMR lesson on ratio: "Smart shopping"

Learning goal: Students will compare and evaluate two shopping options.

Content description: **Number and algebra – Fractions and decimals**

Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies [ACMNA127].

Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check reasonableness of answers [ACMNA128].

Money and financial mathematics

Investigate and calculate percentage discounts of 10%, 25%, and 50% on sale item, with and without digital technologies [ACMNA132].

Big idea: Discrete vs continuous, multiplicative structure, percentage, fractions.

Resources: Student-made shop name, posters, brochures, signs, price tags, vouchers, tick and flick record sheet, comparative table, hundred board, counters, discount cards, worksheets.

Reality

Local knowledge

Discuss the various ways families do their shopping, e.g. bulk buying, supermarket, convenience store, products on sale, online. What are the differences in these? When are these used? What are the benefits and disadvantages in these? Is there one main type used most of the time?

Prior experience

Use shopping examples to revise calculating simple fractions of a quantity ($\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$) and calculating percentage discounts (10%, 25%, 50%).

Kinaesthetic

Act out a shop using stationery materials and calculators.

1. Groups of three to four students set up a shop with five products, e.g. erasers, lead pencils, blue pens, felt pens, rulers.
2. For each shop, students choose a name for their shop, design posters and/or brochures advertising their products and prices (full price and reduced price, amount of discount, etc.), and create a financial plan regarding:
   (a) how much stock to purchase – quality, appeal, cost;
   (b) buying and selling price of goods;
   (c) desired profit margin;
   (d) incentives, e.g. buy one get one free, buy one get 50% off second vouchers;
   (e) if and when to advertise a sale.
3. Students then commence operating their shops, including:
   (a) roles for each partner, e.g. sales person, cashier, recorder – one person for each role **or** everybody undertaking all three roles (completing the sale);
   (b) record keeping of all business transactions (legal requirement for GST and income tax);
   (c) expenditure – buying price of goods, advertising costs (standard $5);
   (d) income – record of all sales of goods and takings; check that change is correct;

(e) profit or loss the business has made;

(f) individual partner's share of the profit/loss (ideally all shops have the same number of partners).

4. **Reverse**: One at a time, each partner from every store is released from duties at the shop to purchase one of each product from the other stores. Partners may NOT buy from their own store. Students shop around to get the best deal for each product. Students keep a record of their purchases, product, price and the shop where the purchase was made.

5. After all the shopping has been completed, the stores compare their profits or losses and discuss the results to determine which shop made the most profit, which had the greatest loss, and which gave the best value. What are the advantages or disadvantages in these scenarios?

Abstraction

Body

Gathering information via an investigation plan to determine store with the better value. In groups of three to four students, conduct an investigation to compare the prices two different stores offer on five products, e.g. toothpaste, muesli bars, cereal, milk, flour to see which store gives the better value. Students may select the two stores to compare, e.g. Westfield chain stores, local supermarket, general corner store.

Students use personal visits, newspaper and online advertisements and catalogues, store brochures to compare prices for the investigation. Visit: www.colesmyer.com.au and www.homeshop.com.au

Include consideration of:

- travel expenses (petrol, parking, taxi, bus, train);
- specials (sale items; buy one get one free; buy two for reduced price; percentage discounts; reduced prices for quick sale – check due date);
- prices against quantities (per 100 g price; 200 g @ $3.50 against 500 g @ $6).

Hand

Make a shopping list and compare prices in a table to determine the store with the most savings (name the stores):

Item	Price: Store 1	Price: Store 2	Savings	
			Store 1	**Store 2**
1.				
2.				
3.				
4.				
5.				
Total savings				

__________ is the store that gives the better overall value. On the five items in our shopping list, __________ gave total savings of ______ .

Mind

Close your eyes and in your mind see the same product in two different stores where one store has a better price than the other store. Where will you make your purchase? See a product costing $12 that is advertised at a 10% discount. What is the discount you will get?

Creativity

Create a brochure showing products at reduced prices.

Mathematics

Language/symbols

Unit fraction, quantity, addition, divide, multiply, sale, percentage, discount.

Practice

1. **Game: Super Savings**. 2 to 4 players, Hundred board, counters, discount cards (multiple examples), for example:

   (a) Clearance sale: 25% off original price. What is the discount on a shirt marked $20?

   (b) Which is the better voucher and by how much? Jumper $120, A. $50 off or B. 50% off

   (c) Which is the better deal and by how much per DVD? 2 DVDs @ $48 or 5 DVDs @ $125.

   Rules: In turn, players select a discount card from the pack, calculate the savings they acquire and move their counter forward one square for every $5 saved. First to 100 squares, wins.

2. Worksheets, for example:

   (a) A six pack of drinks cost $13.50. What is the unit price?

   (b) How much discount do you get if a pair of shoes marked at $85 has 10% discount?

   (c) If you have paid $80 less on a suitcase marked at $320, what was the percentage of discount you received?

   (d) What is the better deal? ½ kg steak @ $19 or 3 kg steak @ $105? How much per kg do you save?

Connections

Relate to GST, mode, interest, overhead costs.

Reflection

Validation

Students check where shopping around occurs in the real world, e.g. One Big Switch (health, electricity, fuel), bank interest rates, car prices, sales in shops, and so on.

Application/problems

Provide applications and problems for students to apply to different real-world contexts independently. For example: *You want a smart phone for your birthday. Compare prices from two or more shops and work out the best plan or agreement to take*.

Extension

Flexibility. Students are able to compare and calculate prices and their discounts using a variety of strategies (simple fractions, percentages, rates, ratios, subtraction) and, given the amount of discount, they can then calculate the percentage of discount.

Reversing. Students are able to move between original price ↔ reduction strategies (buy one get one free, 25% discount, 100 g cost etc.) ↔ reduced price ↔ amount of discount, starting from and moving between any given point.

Generalising. Many factors impact the outcome in business transactions: cost price, selling price, supply and demand, the need for reductions to obtain sales, the discount percentage given, competition among outlets, prudent use of sales to sell over-stocked items or those nearing their use-by date. A book of records must be kept itemising an outlet's income and expenditure for tax purposes and for monitoring products for ordering and checking purposes. Stores use a variety of tools to discount prices.

Changing parameters. Include calculation of GST in the investigations; calculating with different percentages, e.g. 15%, 30%, 40%; track and examine the mode of products for future ordering.

Teacher's notes

- Ensure students have a solid knowledge of calculations with common fractions and percentage before proceeding with the lesson.
- Explicit teaching **that aligns with students' understanding** is present in every part of the RAMR cycle and has particular emphasis in the Mathematics section.
- The RAMR cycle is not always linear but may necessitate revisiting the previous stages at any given point. Reflection should not be undertaken until students have mastered the mathematical concept.

Unit 3: Simple and Compound Interest

Percent, rate and ratio have many applications in financial mathematics (e.g. interest, profit, loss). In this unit, we look at applications of percent in simple and compound interest.

3.1 Percent and simple interest

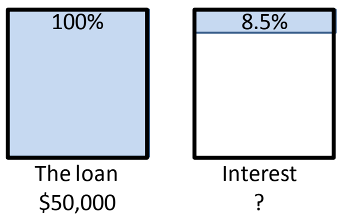
Interest is charged when an amount is borrowed; it is a percent of what is borrowed to be paid over a period of time (in normal finance this is usually a year). For example, $50 000 may be borrowed and the interest rate is 8.5% per year. This means at the end of the year the borrower must pay back $50 000 plus 8.5% of $50 000.

There are two types of interest, simple and compound. Compound interest is when the interest is calculated and charged more than once in the year. For example, the 8.5% per year on the $50 000 may be charged (we say compounded) every 3 months. This means that, for compound interest, the amount on which the interest is charged increases by the interest at each compound (see section 3.2). The interest rate is pro-rata – the monthly interest rate would be 8.5% ÷ 12.

This section looks at simple interest and the next section at compound interest. Some points regarding working out simple interest on the problem *Joan borrowed $50 000 at 8.5% per year, how much did she pay at the end of the year?* are given below.

1. **Size diagram**. We can draw a size diagram as on right – the interest is extra to the $50 000, so we have two squares. The payment in % terms is 108.5. Calculating 1% (the unitary method) gives 50 000/100 = 500. So 108.5% = 500 × 108.5 = $54 250. This is the amount Joan has to pay.

   (*Note:* We should also discuss with students the similarity between profit and interest, that is if profit is 25%, then selling price is 125% and if interest is 8.5%, the repayment is 108.5%.)



2. **Patterns**. If we look at interest problems in order to find a pattern or rule that we can use to replace the need for diagrams, it becomes evident that there are two rules:

   (a) simple interest is amount borrowed × interest rate as a decimal; and

   (b) repayment is amount borrowed + (amount borrowed × interest rate as a decimal).

3. **When time is different to a year**. Since interest is a rate per year, then the interest and the repayment depend on how long the amount is borrowed. For 3 months or ¼ of a year, the rate is ¼ of 8.5% or 8.5% ÷ 4 = 2.125%. Thus the amount paid back is:

   $50 000 + $50 000 × 2.125% = $50 000 + $50 000 × 0.02125

   = $50 000 + $1 062.50

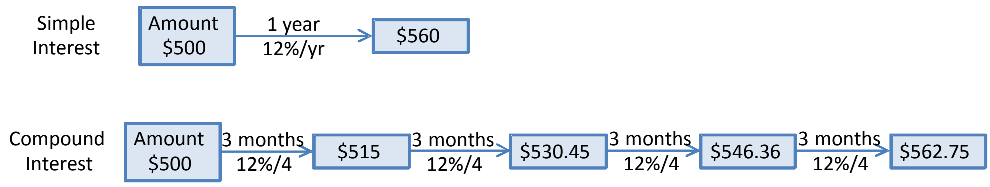
   = $51 062.50

3.2 Percent and compound interest

Compound interest is when the interest from a previous period is added to the amount for the next period and the new interest acts on a larger number. Thus it is possible to have a 12% interest but this interest to be charged each month at 1% and the amount on which the interest acts to continually get larger. Consider the following comparison.

Simple interest 12%/year	Compound interest 12%/year compounded every 3 months
Amount borrowed \$500 After year, amount owing is \$500 + interest = \$500 + 12% of \$500 = \$500 + 0.12 × \$500 = \$500 + \$60 = \$560 *Interest is \$560 – \$500 = \$60*	Amount borrowed \$500 After 3 months, amount owing is \$500 + interest = \$500 + 12%/4 of \$500 = \$500 + 3% of \$500 = \$515 After 6 months amount owing is \$515 + 12%/4 of \$515 = \$515 + 3% of 515 = \$515 + \$15.45 = \$530.45 After 9 months, amount owing is \$530.45 + 12%/4 of \$530.45 = \$530.45 + 3% of \$530.45 = \$530.45 + \$15.91 = \$546.36 After 12 months, amount owing is \$546.36 + 12%/4 of \$546.36 = \$546.36 + 3% of \$546.36 = \$546.36 + \$16.39 = \$562.75 *Interest is \$562.75 – \$500 = \$62.75*

This simple interest – compound interest difference can be seen diagrammatically as follows.

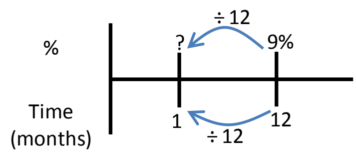


The compounding adds another \$2.75 to the amount to be paid and increases the real interest rate by 2.75/500 = 0.0055 or 0.55%, that is more than an extra ½ %. For credit cards, the interest rate used to be 18% compounded daily which was equivalent to 19.2% simple interest.

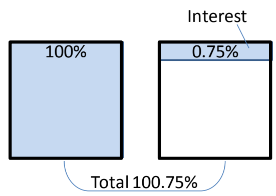
Note: Both simple interest and compound interest lead to formulae. These are discussed in Unit 4 Extension to Algebra.

To teach compound interest, go through the following steps:

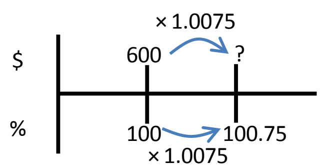
1. **Length less than a year.** Spend time teaching that interest rates are normally per year and have to be reduced when only acting for part of a year. For example, a rate of 9% per year is ÷ 12, 9/12 = 0.75% for one month. The double number line helps: ? = 9 ÷ 12 = 0.75%.



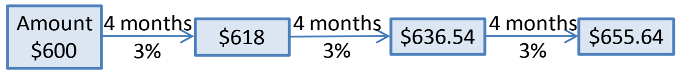
2. **Changing interest to total amount.** Practise that an interest rate of 0.75% means that the repayment is 100% + 0.75% (amount + interest). Practise that this is 100.75% or the same as 1.0075 of the loan. A size diagram helps (as on right).



3. **Make students familiar with calculations of interest for a short length.** Spend time practising calculation of interest for short periods (the double number line helps again). For example, a loan of \$600 for a month at 9%/year is interest rate per month, which is 0.75% and 100.75% as total amount. This gives final amount of \$604.50.



4. **Do series of calculations across short periods.** For example, \$600 at 9% per year compounded every 4 months is 3 calculations of 3% from \$600 to final amount as the figure on right shows:



5. **Patterns.** Look for a pattern from simple interest to get a formula. The formula is:

$$\text{Repayment} = \text{Amount } (1 + \frac{I}{M})^N$$

where I is interest, M is number of compounds per year and N is total number of compounds until repayment.

Unit 4: *Extension to Algebra*

Algebra is the generalisation of arithmetic; thus, this extension unit, in particular, seeks to generalise the material so far in this module. This will be done in two sections, the first on generalisations of models and procedures, and the second on formulae.

4.1 Generalisations

The module contains many options for generalisation. These often do not involve letters. They are a result of discussing examples and asking students for patterns – they are the final R in RAMR (reflection). **It is very important that this is done, and students are encouraged to develop their own "correct" generalisations even if they are informal and idiosyncratic**. Some useful ones are given below.

1. **Percent ↔ Number**. Percent is hundredths, therefore 27.8% becomes 0.278 as a decimal number. This change looks like as follows for PVC:

Percent (%)

H	T	O	t	h	th
	2	7 •	8		

↔

Decimal Number

H	T	O	t	h	th
		0 •	2	7	8

Thus the **general rule** for percent to decimal conversions is (a) numerals shift two places to right (÷100) for percent to decimal number; and (b) numerals shift two places to left (×100) for decimal number to percent. The decimal number form is best for calculating percentage; for example, 27% of \$60 = 0.27 × 60.

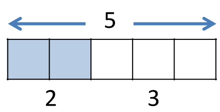
2. **Rate**. Rates are given as "number" "attribute"/"second attribute" (e.g. 24 km/hr, 6 litres/km and \$3/kg). Rate problems, therefore, deal with these attributes. Looking at a lot of problems will show that attributes "cancel" as well as numbers (e.g. 3 hr @ 40 km/hr = 120 km; the hr/hr seems to become 1). This leads to the following general rules:

(a) if rate and first attribute is given, the operation is usually multiply; and

(b) if rate and second attribute is given, the operation is usually divide.

The generality (b) above comes from division, and its relation to multiplication by reciprocal. So, for example 160 km @ 40 km/hr, the calculation is 160 km ÷ 40 km/hr = 160 km × $\frac{\text{hr}}{40\text{ km}}$ = 4 hrs (the km seem to divide away).

3. **Ratio and proportion**. Ratios are part-to-part and fractions are part-to-whole. For example, in diagram on right, the 5 components give a ratio of 2:3 (part to other part) and two fractions $\frac{2}{5}$ and $\frac{3}{5}$.



In general, this means that ratio A:B gives two fractions ($\frac{A}{A+B}$ and $\frac{B}{A+B}$), while fraction $\frac{C}{D}$ gives another fraction $\frac{D-C}{D}$ and a ratio C:D−C.

Proportion divides ratios in sets equivalent to a starting ratio, that is, A:B = 2A:2B = 3A:3B and so on (e.g. 2:3, 4:6, 6:9, and so on). This means that the rule for proportion is that ratios are in proportion (equivalent) if they cancel down to the same starting ratio (e.g. 12:16 cancels down to 3:4 and 51:68 cancels down to 3:4, so 12:16 and 51:68 are in proportion).

4. **Triad and Change**. All percent, rate and ratio examples have three components as follows:

   Percent: Amount, percent and percentage

   Rate: Amount of second attribute, rate, and amount of first attribute

   Ratio: Amount of first component, ratio, and amount of second component

   These translate to change as follows:

	START	**× multiplier** →	**FINISH**
Percent:	Amount	× percent as decimal →	percentage
Rate:	Amount of 2^{nd} attribute	× rate →	amount of 1^{st} attribute
Ratio:	Amount of 1^{st} component	× ratio as fraction →	amount of 2^{nd} component
Result	**Unknown means ÷**	**Unknown means ÷**	**Unknown means ×**

5. **Calculating multiplicative change**. If we have two numbers, say 56 and 32, then how we get from the first (56) to the second (32) by multiplication is × 32/56. In general:

$$\boxed{A} \xrightarrow{\times B/A} \boxed{B}$$

4.2 Formulae

There are two formulae for interest.

1. **Simple interest**. Looking at examples, we can see that the interest is, as a decimal, Interest × Amount. This can become a formula:

$$P = IA$$

   where P is interest to be paid, I is interest rate as a decimal, and A is the amount. The amount to be repaid therefore is Amount + Interest × Amount and the formula is:

$$R = A + IA = A(1 + I)$$

   where R is repayment. If we add in N for the number of years, the formulae become:

$$P = NIA \quad \text{and} \quad R = A(1 + NI)$$

2. **Compound interest**. Compound interest is a series of simple interests. If the interest rate (I) is set for a year and the amount (A) is borrowed and interest is compounded M times a year, then, for the first period, the amount owing is:

$$A(1 + \frac{I}{M})$$

   Now compound interest acts as follows – as a series of I/M interest amounts added to the starting amount:

$$\boxed{\text{Amount}} \xrightarrow{\text{Interest}} \boxed{\text{Amount of } 1^{st} \text{ interest}} \xrightarrow{\text{Interest}} \boxed{\text{Amount of } 1^{st} \text{ \& } 2^{nd} \text{ interest}} \xrightarrow{\text{Interest}}$$

   However, for the second and subsequent compounds, the interest is not on A but on A plus the previous interest. So the compounding goes as follows:

$$\boxed{\text{A}} \xrightarrow{\text{I/M}} \boxed{\text{A(1+I/M)}} \xrightarrow{\text{I/M}} \boxed{\text{(A(1+I/M))×(1+I/M))}} \xrightarrow{\text{I/M}}$$

   and so on. If there are N compounds, then the final step gives:

$$\underbrace{A(1+I/M)\times(1+I/M)\times(1+I/M)\times \ldots\ldots \times(1+I/M)}_{\text{N times}}$$

Thus, the compound interest formula is:

$$R = A \times (1 + \frac{I}{M})^{N}$$

In this formula, R is repayment, A is starting amount, I is yearly interest rate in decimal number form, M is number of compounds per year and N is total number of compounds before repayment.

Unit 5: Plans, Budgets and Other Financial Investigations

One of the important outcomes of a study of financial mathematics is that students can use the knowledge to make better decisions as consumers. This means that they can investigate the best situation in which to place or use their money, plan the use of this money in these situations, and make budgets. This unit looks at plans, budgets and financial investigations. It describes some investigations in detail and then lists some more at the end.

5.1 Investigation 1: Planning a party

The first investigation on finance requires students to plan and organise a party, particularly in terms of cost.

Directions

1. **Type of party**. Decide on a focus for the party, including the theme, number of people, time of day, what is being provided in terms of food and drink, activities and so on.

2. **Plan for party**. Write a plan for your party including a list of everything that will be needed for the party and the people who are coming – decorations, type of food, drink, equipment and the numbers of each needed. Ensure your party provides food as well as refreshments.

3. **Costs of the party**. Use an Excel spreadsheet, or a list and a calculator, to calculate the cost of your party. Obtain realistic prices for the items needed, calculate the costs of the items and then the total cost of the party.

4. **Restricted party costs**. Modify your party so that it can be paid for less than $10 per person.

5. **Report**. Present your plans and costs as a written report.

Teacher hints

It is important that students look at the party plans realistically and cost it appropriately for the people attending. However, it is also crucial to get the arithmetic correct. Encourage the students to be diverse and creative for this party. If needed , provide students with lists of costs of products from local stores.

Assessment rubric

Activity	Excellent (A)	Good (B)	Satisfactory (C)	Effort shown (D)	No effort shown – unsatisfactory (E)
Plan	Complete and creative plan presented very clearly in all areas	Complete and creative plan with good presentation in most areas	Adequate structure, creativity and presentation in most areas	Inadequately presented plan but effort shown	Inadequate and no effort shown
Costs	Correct and well structured list and numbers of costs	Nearly always correct and well structured list and numbers of costs	Mostly correct and adequate list	Only some costs considered but effort shown	Not complete and no effort shown

Restricted costs	Excellent understanding of cost reductions to <$10/person	Adequate cost reduction to <$10/person	Poor modification but does keep the costs <$10/person	Not able to restrict cost but effort shown to have an adequate party	Ineffective in restriction and no effort shown
Report	Explains clearly with much detail	Explains clearly with some detail	Readable report	Does not meet requirements but effort shown	Does not meet requirements and no effort shown

5.2 Investigation 2: Developing a family budget

This second investigation on finance requires students to develop a budget for a typical family in their community.

Directions

1. First, list all of the things a family in the students' community would have to pay for in a week (expenditure), then, find the general cost of things in their community. To start, here is the beginning of a list:
   - Housing – rent or mortgage payments, electricity or gas, and rates etc (if appropriate).
   - Food – meals for the week, drink, etc.
   - Clothing – ongoing purchases to keep the family in clothes.
   - Furniture – ongoing replacement of things broken or worn out.
   - Car/travel – petrol, servicing, and repayments.
   - Entertainment – sports, movies, parties, DVDs, travel, etc.
   - Obligations – gifts, visits, meetings, etc.
   - Other – things that are special to your family.
2. Within a budget, it is necessary to balance the expenditure with the income. The second step in preparing the budget is therefore to focus on income. List the various ways a family gets money (income). To start, here is the beginning of a list:
   - Wages/Salaries – all family members.
   - Income from training.
   - Allowances and social benefits (e.g. CentreLink).
   - Pensions – age, invalid, etc.
   - Other forms of income such as payments, sales, etc.
3. The third part of the investigation requires students to develop a family budget which displays how much money the students' family receives each week (income) and lays out a plan for the money they pay each week (expenditure) in which the expenditure is less than the income. Here is a list of tasks to do. Complete these steps.
   - Family description – describe a typical family (e.g. how many adults, teenagers, children, babies etc.) in the community. Describe their forms of income – how many are working, how many on allowances and pensions. Consider how much they should have to spend.
   - Income – make a list of income and work out average weekly amounts.
   - Expenditure – make a list of expenditure and work out average weekly amounts.
   - Budget – adjust expenditure so it is less than income but represents a balance of expenditure.
   - Report – write a report on your budget, what you think of it and what it might lead to.

Teacher hints

The focus of this investigation is to involve the students with their family working on a budget that has some similarities with their family's budget. However, care must be taken that there are other options when this does not work out as it may be inappropriate to move into this area. It may be that a simple family may have to be substituted for normal extended families. But it would be beneficial if they could consult their family on possible incomes and expenditures. Any way the students could work with their families on this project would be good.

Encourage creativity and chunking (putting things together into more general expenditure items). Encourage students to use a spreadsheet (e.g. Excel).

Note: The focus is on family because the family (not the individual) is the most appropriate focus in many communities.

Assessment rubric

Activity	Excellent (A)	Good (B)	Satisfactory (C)	Effort shown (D)	No effort shown – unsatisfactory (E)
Family description	Complete and detailed description	Good description	Adequate description, covering important points	Effort shown but limited description and missing important aspects	Limited description, missing important aspects and effort not shown
Income	Complete list, good amounts, correctly calculated	Good list and correct calculation	Adequate list, mostly correct calculation	Effort shown but only some income considered	Only some income considered and effort not shown
Expenditure	Complete list, good structure, correctly calculated, good amounts	Reasonably complete list and structure, correct calculations	Adequate lists, mostly correct calculations	Effort shown but only some expenditure considered	Only some expenditure considered and effort not shown
Budget	Creative modifications	Good modifications	Adequate modifications that work	Effort shown but modification does not work	Modification does not work and effort not shown
Report	Insight shown in effect of budgets	Some insight shown in effect of budgets	Readable report	Effort shown but does not meet requirements	Does not meet requirements and effort not shown

5.3 Further investigations

The ideas above and in Units 1 to 4 are just a start with regard to what is possible in financial and vocational mathematics in terms of percent, rate and ratio. They can be the basis of longer investigations or **rich tasks** that explore in an open-ended way. The following are examples of such tasks.

1. **Using the Internet**. Use the Internet to gather information to make decisions about something that is motivating to the students. For example:

   (a) buying a smart phone and working out the best plan/agreement to be on;

   (b) borrowing money to buy a car; or

   (c) the difficulties with using money on a credit card.

2. **Reversing the procedure**. For example, in many applications, we start with a percent problem and end with an answer, so why not start with an answer and end with a problem? To reduce difficulty, we can limit the problem to having a % between 10 and 90 (and to not being a multiple of 10 if we want to make it more difficult, and starting with an amount of 50 or over to make it simpler). A particular activity could be: *Find a problem where the answer is $68 and this represents the total amount or 100% (type 2 problem).*

   We have to start somewhere so we could say "what if $51 was the % paid and then you had to find the total?". As $51 ÷ $68 = 0.75, the problem could be "Jack paid 75% of the cost, which was $51, what was the total cost?"

   Other reversing problems could be to write problems for: (a) Type 1 problem; answer $124; (b) Type 2 problem; answer $245; (c) Type 3 problem; answer 26%.

3. **Everyday activities**. Take an everyday activity for a truck company such as driving. Look at average speeds (km/hr) for truck drivers and average distances driven per hour. Consider how long a driver can drive until they have to rest.

   (a) How long can a driver travel on average before they have to rest?

   (b) How long would it take for a truck to travel to Melbourne and return with one driver?

   (c) If you have to have 4 trucks per week travelling from Brisbane to Melbourne, how many drivers do you need?

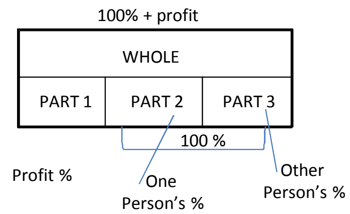
4. **Reducing costs**. Activity *Cheap houses* is an investigation into why square houses are cheaper The reasons for this lie in the area/perimeter ratio for houses – you get the most area for the least perimeter in a square house. This makes it cheaper to build.

Note: It should be noted that, as problems become more complex and have more than two steps, the models need to be extended as follows:

Problem: A painting was sold for a profit of 22%. The profit was kept in an account. The rest of the money was split 35% to Fred and the remainder to John. If the sale price was $2 350, how much did John get?

We would now need to use a P-P-P-W model as on right.

3 PARTS FOR WHOLE



Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the financial mathematics item types

The financial mathematics item types are divided into five subtests, to match the five units in this module. The five units are in sequence: Unit 1 covers early ideas to do with recognition and use of money, Unit 2 covers percent and rate based ideas such as profit and best buys, Unit 3 covers simple and compound interest, Unit 4 looks at extension to algebra and formulae, and Unit 5 looks at investigations and rich tasks which are an important outcome of financial maths activities (e.g. What is the best way to buy a mobile phone?).

Therefore, the pre-test should start at Subtest 1 and work its way up the subtests until students are continuously unable to answer the item types, and the post-test should cover all the subtests and also have an investigation. The investigation can be graded by a rubric, for example: A completely done [4], B reasonably done [3], C borderline [2], D fail but some information [1] and E nothing correct/did not attempt [0].

Subtest item types

Subtest 1 items (Unit 1: Early finance and mathematics activities)

1. Calculate how much in total for the following:

   (a) coat $75, pants $59, belt $27

   (b) laptop $563, laptop cover $87, external hard drive $146

2. How much change would you get from $100 if you bought a DVD for $28.75 and a CD for $27.55?

3. How much change would you get from $200 if you bought a shirt for $56.95, a hat for $34.75 and shoes for $115.95?

4. Complete the thinkboard.

Story June was thinking of spending her $50 on 2 CDs costing $21.50 and $16.45. How much change would she get?	**Materials**
Picture	
Language	**Symbols**

Subtest 2 items (Unit 2: Profit and loss and best buys)

1. (a) John had to pay 30% of the $60 cost. How much did he pay?

   (b) Jill paid $60. This was 30% of the cost. What was the cost?

2. (a) The discount of 40% reduced the dress to $80. How much did the dress originally cost?

   (b) There was a discount of 40% on all jeans. The jeans originally cost $80. How much did Alex pay after the discount?

3. (a) The pasta cost $2.40 for 350 g and $3.30 for 500 g. Which is the better buy?

   (b) The 1 litre bottle costs $4.80. The 600 mL bottle costs $2.80. You need to buy 3 litres, is it a better buy to get five 600 mL bottles or three 1 litre bottles?

Subtest 3 items (Unit 3: Simple and compound interest)

1. The bank gives 4.5% **simple interest** on savings accounts. How much interest will Fred earn on $5000 across 2.5 years?

2. The credit union gives 4% interest on savings accounts, **compounding monthly**. How much interest will Fred earn on $5000 across 2.5 years?

3. Another company gives 4% interest on $5000 across 2.5 years, **compounding weekly**. How much better is this for Fred?

4. The laptop costs $1200. The bank charges 6% per year simple interest for a loan of $1200 to buy the laptop. The shop charges the same rate of interest on a loan to buy the laptop, but compounding monthly. Over 1.5 years, which will cost less (the bank loan or the shop loan) and by how much?

Subtest 4 items (Unit 4: Extension to algebra)

1. If sand and cement are mixed in the ratio A: B and we have 20 tonnes of cement, write the algebraic sentence that would give the tonnes of sand required.

2. If I drive the car at an average speed of C km/hr, then:

   (a) How far would I get in D hours?

   (b) How long would it take to drive E km?

3. What is the formula for repayment R for simple interest when A is the amount, I is the interest and N is the number of years?

4. What is the repayment R for compound interest when A is the amount, I is the interest, M is the number of compounds per year and N is the number of years?

Subtest 5 items (Unit 5: Plans, budgets and other financial investigations)

Investigation

Consider an outing for your class of 25 students to travel to a nearby town or city to see the local show. Prepare a plan for how much this will cost for each student.

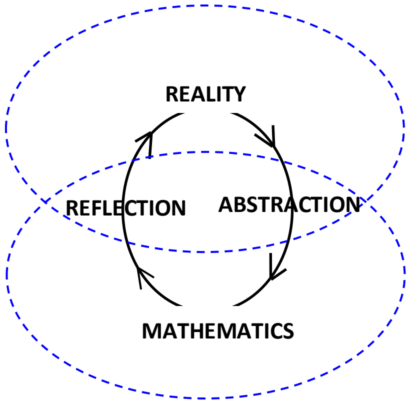
To do this:

- decide how long you will attend and what you will do
- prepare a list of all costs (e.g. travel, entry cost to the show, food, drink, rides, and so on)
- use the Internet to get these costs
- calculate total cost and divide by the number of students.

Write a report to justify this cost.

Appendix A: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematical ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix B: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

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