QUT
YuMiDeadly
*Growing community
through education*

YuMi Deadly Maths

# AIM Module SP3

## Year C, Term 3

# Statistics and Probability:

## Statistical Inference

## ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

## CONDITIONS OF USE AND RESTRICTED WAIVER OF COPYRIGHT

## DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.
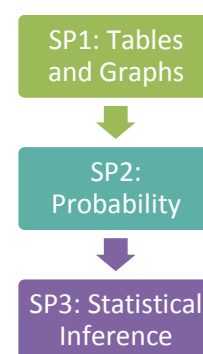
# Contents

# *Module Overview*

Statistics and probability are crucial everyday life skills and important mathematical topics. Situations in real life can be of a predictable or random nature. Most decision-making of modern society is based on graphs, probability and statistical inference. In advertising, politics and economics, samples are organised, survey questions developed, answers sought, results tabulated and organised, and predictions displayed with averages and graphs to show distributions, relationships and trends before decisions are made. *What do people want in a car? Should Queensland have daylight saving*? Many computer banks are filled with the raw data on which such decisions will be made.

AIM looks at the area of statistics and probability through three lenses. As large amounts of raw data are incomprehensible, the first module, SP1 *Tables and Graphs*, covers data gathering and representation and supplies a visual way of presenting the range of alternatives available and of indicating the density of interest (e.g. most popular/likely). It builds these representations around an understanding of their purpose.

Because nearly all life decisions involve uncertainty, requiring decisions to be made of possibilities and probabilities, the second module, SP2 *Probability*, provides teaching material pertinent to uncertainty, involving the measurement of the likelihood of events in chance processes. At the point of experimentation, probability activities merge with the activities of the third module.

This third module, SP3 *Statistical Inference*, moves on to analysing and interpreting data, providing an indispensable tool for comprehending the raw data on which decision-making is based, and involving measures of central tendency and distribution such as *means*, *medians* and *deviation*, as a framework with which to describe what happens. It also introduces some more complex representations (e.g. box and whisker graphs).

## Background information for teaching statistical inference

This section covers the nature of statistical inference, teaching ideas, connections and big ideas.

### Nature of statistical inference

There is a body of opinion that statistics is a strand different to the other strands of mathematics. This is because the other strands of mathematics are seen to focus on invariant or unchanging properties and processes that generalise across contexts; for example, "turnarounds" or the commutative law for addition (i.e. $a + b = b + a$) is a law that holds for all numbers – it is a general or generic law. Against this, statistics is seen as understanding variation (or difference) within a particular context, that is, the data that has been gathered on a particular situation. It certainly appears to be a reasonable argument that statistics is applied to particular and often complex contexts (e.g. what is the height of a typical Year 7 student?) which have error and uncertainty at their basis.

However, YDM is unsure about this difference: (a) statistics has invariant properties (e.g. formula for mean), processes (e.g. box and whisker graphs) and concepts (e.g. the idea of error or uncertainty); (b) other strands of mathematics (e.g. probability and measurement) appear to have uncertainty and error in their applications; and (c) even operations have to take account of error when applied to the real world (e.g. bridge building). It seems that the particularity of statistics is in terms of the data and questions that it is designed to handle. However, statistics reflects traditional Aboriginal and Torres Strait Islander thinking in its ability to handle large data sets and

complex interactions in particular contexts. In rich tasks that focus on complex but particular situations, Indigenous students have been found to excel (e.g. the tasks in the New Basics program of the 2000s).
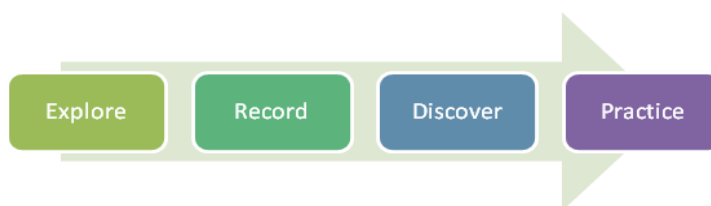
Of course, it is important to realise that statistics requires students to **infer from data,** not just describe data. This difference between description and inference is shown below.

| DESCRIPTION | INFERENCE |
|---|---|
| • Describes data | • Goes beyond data |
| • Expressed with certainty | • Uncertain (how sure are we) |
| • Often based on personal experience | • Evidence-based arguments |

Because of its nature, statistics and statistical inference are now **very important** in mathematics. This is because they reflect modern life and modern problems in terms of the information revolution that is occurring. Statistics enables complexity to be solved within a context. Statistics is part of normal development – people make inferences everyday (e.g. what film shall we go to?).

## Teaching approaches

Statistics involves quantifying and analysing sets of responses to surveys, questions and chance events. As a result, number ideas and concepts are naturally reinforced in statistics activities. Sequences of ideas, materials, language of quantity and representations used for building number ideas and concepts are also appropriate for use in statistics. Specific number sequences may be found in the Number modules but are also listed here. Statistics and probability are best taught within a problem-solving environment. Students should be encouraged to investigate and experiment as in the figure below.



Explore → Record → Discover → Practice

Students need to develop skills to investigate problems. A commonly used format for investigations is below.



Recognise and formulate problem → Collect relevant data → Organise data → Analyse and interpret data → Draw and defend inferences → Relate to original problem

Statistics also benefits from active teaching approaches, particularly inquiry-based approaches that give precedence to experiences that enable students to come to terms with variation and uncertainty. Thus, like problem solving, statistics is best learnt by doing because only in this way can students appreciate uncertainty and variation, and the undetermined nature of much of this, that is, appreciate and understand the untypicality of typical.

Statistics is an opportunity for integration with other maths topics (particularly measurement). It benefits from finding contexts and problems that interest students. Its focus, and most important component, is learning to make decisions with data – to make inferences. Also importantly, this decision-making or inferring is part of a process or sequence that has components as below, and can be seen in the cycle on below right:

Posing a question/problem and predicting possible answers

Devising a plan to tackle the question

Collecting and analysing data

Making decisions acknowledging uncertainty (inferring)

Defending the inferences with argument



PROBLEM → PLAN → DATA → ANALYSIS → CONCLUSION → PROBLEM

In the early years, this is an informal process involving understanding variation, predicting, hypothesising, and criticising. In the later years, it involves: (a) analysis and interpretation of data; (b) investigation and comment on different forms/representations of data; (c) relationship of data to questions/issues and evaluation of these issues/questions in terms of data, particularly relationship between purpose and choice of data; (d) box plots and relation to distributions; and (e) discussion of distribution of data, using terms including *skewed*, *symmetric* and *bi-modal*.

Teaching statistics also provides the opportunity for enhancing the Proficiencies in the Australian Mathematics Curriculum: (a) **understanding** – covering terms such as *sample*, *population*, *random*; (b) **fluency** – covering working with calculations; (c) **problem solving** – devising a strategy for analysing data to answer a question; and (d) **reasoning** – generalising from data and analysis to a conclusion.
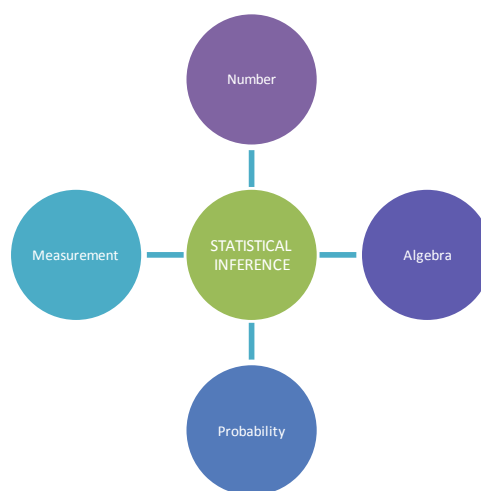
## Connections

Statistical inference activities: (a) use number and operations in the data and its manipulation; (b) commonly involve measurement in the collection of data; (c) involve algebraic forms of thinking in terms of constructing inferences because of the uncertainty of the numbers; and (d) use probability as rich sources of activities. Consequently, number concepts (e.g. percentages), operations (e.g. adding and dividing for means), algebra (e.g. thinking operationally when the numbers are not given), and probability (e.g. investigations of uncertainty) are linked to statistical inference (as on right). Forms of data representation (e.g. bar and circle graphs) obviously connect to geometry, but as they are accounted for in Module SP1 *Tables and Graphs*, they are not connected here. The **connections** important to statistical inference are as follows.



1. **Probability**. It is argued that **statistical literacy** is a meeting point of statistics and probability in the everyday world, whereby statistical tools need to be developed in relation to general contextual knowledge and critical literacy for everyday decision-making. Thus, the major connection between statistics and other mathematics topics is with probability, particularly at the level of experimental probability.

1. **Number and operations**. Other important connections are with number and operations, particularly with respect to forms of data and formulae for centre and deviance. In particular, different forms of number use (*categorical* – simply using number to denote something like a town or a response with no measurement involved; *ordinal* – numbers only give order and the additive and multiplicative difference between 2 and 4 may be very different to that between 3 and 5 and 3 and 6 respectively; and *interval* – additive and multiplicative differences hold).

2. **Measurement**. This is also an important connection because measurement is the common basis for the data and both statistics and measurement share a similar relationship with respect to error and uncertainty.

3. **Algebra**. Finally, although not necessarily using letters, much of the inferential thinking in statistics is algebraic in form in that it deals with uncertainty with respect to number. It is similar to that required for budgets.

It should be noted that some relationships can be complex as data can be in: (a) a category form such as colour or home city; (b) an ordinal form such as short, normal and tall or 1, 2 and 3; or (c) an interval form such as height in centimetres. The data can also be in: (a) discrete and discontinuous form like cost to nearest dollar; or (b) continuous form like mass in kg to decimal places.

Weaknesses or lack of conceptual understanding in number, algebra, measurement and probability will hamper students' progress in statistical inference. Where students are demonstrating lack of success in the

statistics, it is necessary to ascertain whether the difficultly lies with the statistical concepts and processes or with the numerical, algebraic, measurement and probability concepts and processes.

## Big ideas

Big ideas are mathematical ideas that underlie topics and recur across the years of schooling. Looking at statistics, it seems evident that there are some big ideas underlying statistical inference. A few are given here.

### Global big ideas

1. **Chance vs certainty**. In arithmetic, problems have **certain** answers, that is, $4 + 7 = 11$. However, in statistics, decisions can be made in terms of **chance** or **uncertainty**. The data is not absolute; it shows that there are more options and thus predicts the best chance for an outcome. It is important that students know when they are in certain and when they are in chance situations.

2. **Accuracy vs exactness**. This goes hand-in-hand with the first big idea. In arithmetic, answers can be calculated exactly. However, in measures and in drawing inferences from data, there is sometimes no exactness; there is only being as accurate as possible or as required.

3. **Interpretation vs construction**. It is essential to be able to interpret data but this can be assisted by learning how to construct data.

### Statistical inference big ideas

1. **Variation and uncertainty**. This includes the global understandings that are the basis of beliefs about the nature of mathematics and thinking about mathematics, particularly in number and operations:

   - experimental vs theoretical,
   - absolutist vs probabilistic,
   - accuracy vs exactness, and
   - continuous vs discrete.

2. **Centrality of context**. Although this could be considered with the big ideas above, it is separate because of its applicability to inference. The focus of statistical inference is to make decisions about the particular, using thinking and processes that are more generic.

3. **Integration of information**. The complexity of statistical questions often requires the integration of other mathematics topics and other disciplines/subjects (e.g. science).

4. **The relation between sample and population**. This is unique to statistics where a small subset is used to determine findings about the total population – and the ways in which uncertainty can be decreased by appropriate relationships between sample and population.

5. **The efficacy of models and simulations**. This is not unique to statistics but is particularly important because of the role that models and simulations can play in reducing uncertainty and understanding variation.

6. **Formulae for concepts of centre and deviation**. This covers mean (average score), median (middlemost score) and mode (score with highest frequency); and range (largest score – lowest score), quartiles (breaking range into quarters), mean deviation (average of differences between scores and mean), and standard deviation (square root of average of squares of differences between scores and mean).
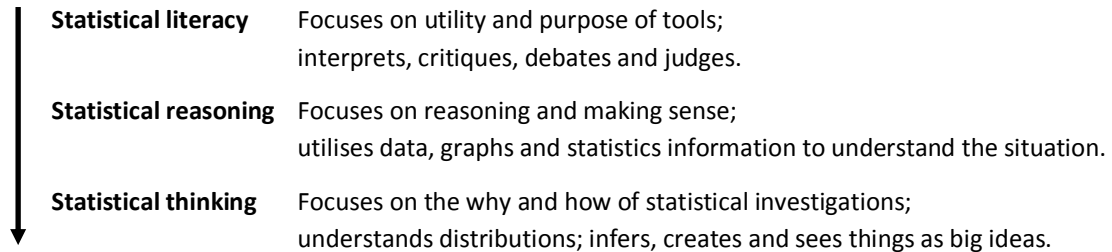
### Strategy big ideas

1. **Data-driven/Complex thinking**. Statistics problem solving and investigations should be based on data and, therefore, are often examples of complex thinking.

2. **Evidence-based/Inferential thinking**. Statistics problem solving and investigations should be driven by evidence from the data and, therefore, are often examples of inferential thinking.

# Sequencing for statistical inference

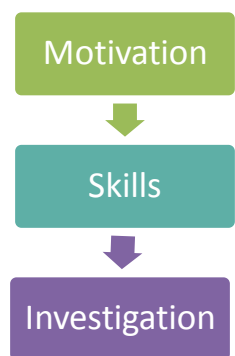This section briefly looks at the role of sequencing in statistics in general and in this module in particular.

## Sequencing in statistics

The development of statistical inferences goes through three steps as below:

**Statistical literacy**    Focuses on utility and purpose of tools; interprets, critiques, debates and judges.

**Statistical reasoning**    Focuses on reasoning and making sense; utilises data, graphs and statistics information to understand the situation.

**Statistical thinking**    Focuses on the why and how of statistical investigations; understands distributions; infers, creates and sees things as big ideas.

Because of its nature, statistical inference is suited to inquiry and rich task approaches to teaching. Therefore, it is suited to an approach to teaching based upon the notion of Renzulli (1977) that mathematics ideas should be developed through three stages:

- **Stage 1**. Motivate the students – pick an idea that will interest the students and will assist them to engage with mathematics.

- **Stage 2**. Provide prerequisite skills – list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.

- **Stage 3**. Provide integrating tasks – end the teaching sequence by setting students an open-ended investigation to explore.

Motivation → Skills → Investigation

Finally under sequencing, we look at the types of questions, tasks or projects that can be set across the years of this module (Years 3 to 9). This we break into four levels of **problem types**:

**Level A: Simple** – one uncertainty, e.g. *Do most students have brown eyes?*

**Level B: Multiple** – two or more uncertainties, e.g. *Do tall children run faster?*

**Level C: Extended** – two or more uncertainties plus need for other maths/science knowledge, e.g. *What year level has the healthiest lunch?, What is the best design for a loopy aeroplane?*

**Level D: Complex** – all of Level C plus differences between types, e.g. *Do typical Year 7 students eat healthy cereals?*

## Sequencing in this module

For the purposes of this module, we have divided the focus of the inferential statistics into a **five-step sequence** from early years to later years:

Early inference

Development of inferential thinking

Data and central tendency

Data distribution

Inferential misrepresentation

The reason for these five stages is to look at statistical literacy development in Unit 1, begin the movement to statistical reasoning in Unit 2, and then start to build towards statistical thinking and full inferencing in Units 3 and 4. We end with Unit 5, looking at the role of misrepresentation in statistics (i.e. "how to lie with statistics") that has led to the statement "there are lies, damned lies, and statistics".

This sequence will also enable us to cover the development of the important process of making decisions with respect to data, and also the development of meaning and formulae for a series of concepts that assist with describing data and its distribution, for example, mean, median, mode, range, deviation, quartiles, and outliers. As these concepts with respect to statistics are late in mathematics development, it means that early learning should focus on the development of statistical literacy leading to reasoning, inferential thinking and the strategies and approaches that go with this. Then, later, the new ideas will be added for a more sophisticated language for inference which will build onto the early thinking.

Unit 1 looks at Level A problem types, then Unit 2 moves on to Level B problem types. Units 3 and 4 cover Levels C and D problem types.

The sections of this module are, therefore, as follows.

**Overview**: Background information, sequencing, and relation to Australian Curriculum

**Unit 1**: Early inference

**Unit 2**: Development of inferential reasoning

**Unit 3**: Central tendency

**Unit 4**: Data distribution

**Unit 5**: Inferential misrepresentation

**Test item types**: Test items associated with the five units above which can be used for pre- and post-tests

**Appendix A**: Statistics activity

**Appendix B**: Cultural implications of statistical inference

**Appendix C**: RAMR cycle components and description

**Appendix D**: AIM scope and sequence showing all modules by year level and term.

Each unit has three parts:

- an *overview of unit* that delineates the important foci of the unit and provides information to help in teaching the specific ideas of the unit;

- *activities* that introduce, teach and consolidate the ideas in the unit (and can form the basis of RAMR lessons, see **Appendix C**) – starting from motivating situations in the students' world and then building necessary concepts and skills; and

- *investigations* that integrate the ideas and connect to other units in an enquiry and problem-solving approach.

Thus, the module combines two approaches to teaching:

- *structural/RAMR teaching* of activities that lead to the discovery and abstraction of mathematical concepts and skills (processes, strategies and procedures) starting from the world of the students; and

- *integrative rich-style tasks* which allow students an opportunity to solve problems and build their own personal solution, and which give opportunities to combine knowledge across the units.

## Relation to Australian Curriculum: Mathematics

| AIM SP3 meets the Australian Curriculum: Mathematics (Foundation to Year 10) | | | | | | |
|---|---|---|---|---|---|---|
| Unit 1: Early inference<br>Unit 2: Development of inferential reasoning<br>Unit 3: Central tendency | | Unit 4: Data distribution<br>Unit 5: Inferential misrepresentation | | | | |

| Content Descriptions | Year | SP3 Unit | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169) | 6 | ✓ | ✓ | ✓ | ✓ | |
| Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171) | 7 | | | ✓ | ✓ | ✓ |
| Describe and interpret data displays using median, mean and range (ACMSP172) | | | | ✓ | ✓ | ✓ |
| Investigate techniques for collecting data, including census, sampling and observation (ACMSP284) | | | ✓ | ✓ | ✓ | ✓ |
| Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206) | 8 | | | | ✓ | ✓ |
| Investigate the effect of individual data values , including outliers, on the mean and median (ACMSP207) | | | | | ✓ | ✓ |
| Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293) | | | | | ✓ | |
| Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources (ACMSP228) | 9 | ✓ | ✓ | ✓ | ✓ | ✓ |
| Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283) | | | | ✓ | ✓ | ✓ |
| Determine quartiles and interquartile range (ACMSP248) | 10 | | | | ✓ | |
| Construct and interpret box plots and use them to compare data sets (ACMSP249) | | | | | ✓ | |
| Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250) | | | | | ✓ | |
| Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253) | | | | | ✓ | ✓ |

# Unit 1: Early Inference

This section covers early activities in statistical inference – it looks at the middle primary years. The focus is on understanding statistical literacy as the first step in inference. This means a focus on utility and purpose of tools, and on building abilities to interpret, critique, debate and judge.

## 1.1    Overview of unit

As stated above, this beginning unit focuses on statistical literacy as a meeting point of probability and statistics in the everyday world. It begins the development of statistical tools in relation to general contextual knowledge and critical literacy. The tools are primarily those relating to the collection and organisation of data and using that data to make decisions. The activities tend to include: (a) posing questions and collecting categorical/numerical data; (b) describing and interpreting data in context; and (c) carrying out surveys and recording data accurately. They begin to build an appreciation of informal inferences and some informal understanding of variation and uncertainty.

The outcomes aimed for include:

(a)  building proficiencies – understanding, fluency, problem solving and reasoning;

(b)  building critical and creative thinking – developing inquiry (identifying, exploring, organising), generating ideas, reflecting on thinking and process, and analysing, synthesising and evaluating; and

(c)  building the heart of statistics – formulating and testing of hypotheses, justifying conjecture with evidence, and inferring with convincing argument.

In particular, integrate measurement with statistics and challenge students' idea of certainty; for example, how certain are they that their measure is correct?

The practicalities for this include shifting focus from data points (e.g. "Kym watches 10 hours of TV") to holism or characterising groups (e.g. "most of my class watch between 10 and 15 hours of TV a week"). This has been characterised as having the ability to "distinguish signal from noise" (e.g. tuning a small radio). It is best undertaken through investigating questions characterised as level A (see 1.3 Investigations) – by allowing students to draw inferences from data they have gathered.

It is important to remember that students' capabilities in drawing informal inferences need to be recognised, with increased exposure to a range of statistical representations that require interpretation and explanation beyond basic descriptions. If students are not exposed to informal inference in the primary school, the introduction of formal statistical tests in the late secondary school can become a meaningless experience because students will not have developed an intuition about the story conveyed by data.
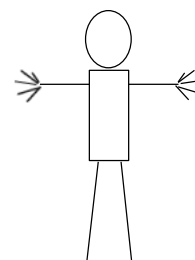
## 1.2    Activities

### Introductory activity

**Materials**: Measuring devices, methods to represent bar graphs (rectangular sheets of paper, maths mat, graph paper, pen and paper, and so on)
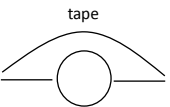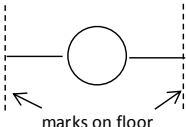
**Instructions**:

- Choose a student to stand with arms outstretched and organise all students to measure the student's arm span from fingertip to fingertip.

- Record all the students' data on the board to nearest cm. Organise the students to graph this data in terms of individual lines or frequencies.
- Use the data and the graphs to discuss the following informally: range, centre, outliers, certainty, and typical.
- Ask the students to make decisions from the data as to: What is John's arm span? What is the variation and why is it occurring? What is uncertain?
- In particular, ask what the students would do if all they had was the data and no way to measure the student's arm span. What do they think is the "correct answer" or the "best answer" and why?

**Points for discussion:** In most examples of this activity, there is a wide variation in measures. This gives an opportunity to discuss errors in measurement and the way in which different ways of measuring may lead to different measures. For example, the following can lead to error:

(a)

Measure too long because of bend in measuring tape.

(b)

This can lead to discussion of different ways of measuring such as laying the students on the ground and marking lines on ground and measuring between these. Would this be more accurate?

(c) Students not accurate when measuring with a tape – particularly if have a 1 m tape and it has to be moved for the overall measure.

(d) The student moves between different students measuring – the arms could move backwards and forwards, after a time tiredness may make the arms sag so not at right angles to the body, the fingers may curl, and so on.

This activity can lead to good discussion on data such as what is the middle, what is the average and are there outliers, what are these and why would they occur? Finally, asking the students to come to a conclusion or consensus just from the data can lead to great discussion justifying different outcomes. This then is an opportunity to discuss the idea of error but, even more importantly, the idea of uncertainty.

## Foci of activities

There are four foci here. Teachers need to choose which of these is/are appropriate for their students:

1. **Ensuring basics**. It is important to run activities that build the abilities that underlie inference. The first of these includes gathering data, recording data, graphing data and describing data. For example, it is possible to gather all students' shoe sizes by using shoes to make the graph. This allows for recording (tallies and tables), graphing (bar graphs and frequency bar graphs) and informally discussing what "most students' size is" (e.g. centre, average or typical) and what are unusual sizes (e.g. range, outliers). The second of these is to return to measuring. This leads to discussion on errors and how they can be made and how to take account of them.

2. **Making decisions**. It is important, after or during basics, that activities like measuring arm span above are undertaken to (a) extend discussion to **error and uncertainty**; (b) build ability to **make decisions** from the data; and (c) **defend decisions** from the data in relation to the specifics of the measuring.

3. **Posing problems and devising data gathering**. As well as (2) above, it is important to build the students' ability to work from the problem only. So we need to build ability to pose questions and devise ways to gather data for their answers. We need to reduce our support for the students – just ask a question like "how far do we jump?" and allow the students to work out ways to gather data for this. Then, justification for the inference is not just in terms of data but in terms of relevance of the data.

4. **Building complexity**. The above introductory activity on measuring arm span is so specific yet filled with uncertainty. The next step that could be undertaken is to begin to add in extra uncertainty by asking not for a specific arm span but for a "typical" arm span. This starts to extend the ideas in Unit 1 to Unit 2.

## 1.3 Investigations

### Investigation 1

1. Choose an investigation like one of those in the level A examples below. Try to make it relevant and motivating for your students.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the outcomes for this unit and the four foci in the activities as appropriate to the students.

| **Level A** | How tall is John? |
|---|---|
| **investigations** | How far do we jump? |
| | What is the best recipe for play dough? |
| | What kind of books do we like? |
| | What is the best design for an obstacle course? |
| | What makes a toy car go further – a steep or a low ramp? |
| | How long does it take to tie a shoelace? |
| | Do most kids in class have brown eyes? |
| | Are we getting better at skipping? or Can we get better at skipping? |

### Investigation 2

Choose something that has more than one way of getting an answer, for example:

(a) How many advertisements do they have on TV? Does it change for different programs?

(b) What is the most popular car colour?

(c) Who is the best player on a football team? [Class chooses team]

(d) Which is the best class from the maths test?

Always predict to start and then follow Polya's 4 stages (SEE, PLAN, DO, CHECK).

(a) To see, discuss the question so everyone is clear what has to be done and what has to be known or assumed to be able to tackle the question.

(b) To make a plan, discuss what should be done, and what you have to collect and what you have to look up to follow the plan. Work out the sequence/order in which you will do things. Check if you've missed something, or that there is not another way.

(c) To do, follow the plan and write a report, giving inferences.

(d) To check, go back over what you have done, see if there is another way to solve it or another solution, try to highlight what you have learnt, and try to generalise what you have done to an extension of the question.
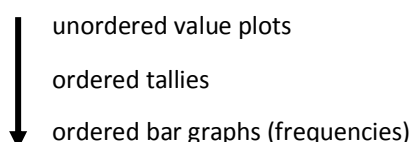
# *Unit 2: Development of Inferential Reasoning*

This unit covers the move from statistical literacy to statistical reasoning – it looks at the upper primary years. Statistical reasoning focuses on reasoning and making sense of data in context – it utilises data, graphs and statistics information to understand problems and situations.

## 2.1    Overview of unit

As stated above, this unit covers the move from literacy to reasoning. Its aim is to:

(a) reinforce students' understanding of statistical literacy and begin their movement to statistical reasoning;

(b) enable students to experience statistical processes before they learn more formal rules and formulae (this is considered crucial to lead to better statistical understandings in secondary school);

(c) facilitate students to begin to focus more on inference ("beginning inference") – covering variation, prediction, hypothesising and criticising; generalising beyond data, using data as evidence, and continuing to acknowledge uncertainty; and

(d) prepare students to move their understanding of variation and uncertainty from informal/intuitive to formal.

This unit also aims to ensure students understand and can use a variety of representations of data. This is because statistical reasoning benefits from seeing connections between different representations of data, particularly when students move to new representations to better infer findings. In particular, understanding of inference improves as students move through the graphical forms of data used in inference as below:

unordered value plots

ordered tallies

ordered bar graphs (frequencies)

This unit also covers building appropriate language, more formally introducing terms such as: outliers, error, most likely, centre, and so on; and introducing sampling and the relation of samples to population. In terms of practicalities, it is important to provide experiences such as: (a) carrying out surveys and recording data accurately; (b) working with frequency data and converting it into percentages; (c) appreciating and employing the process of making "informal inferences"; (d) learning and using the language of the process (pose, predict, sample, random, population, infer, conclude, decide, certainty); and (e) using random samples in collecting data and using the results from the samples to make decisions about a statistical question. The Australian Bureau of Statistics has websites with many useful data sets that students can use, sample and contribute to.

In particular, this unit advocates the following.

1. **Challenge that randomness is determined**. Teachers need to **challenge** the common misconception that randomness is determined (e.g. "I never throw sixes"), especially with respect to life. Students need to be challenged to understand that throwing a die can give random numbers from 1 to 6; but they also need a stronger challenge to ensure that random events in life give all possibilities.

2. **Integrate contextual knowledge**. Give a variety of experiences of data and making inferences in particular contexts and show how the context affects decision-making.

3. **Give intuitive experiences first**. Teachers should always give experiences that build intuitive understanding before moving to formulae and to procedures (e.g. look at data intuitively for average before teaching the formula for mean).

4. **Teach purpose and aggregation**. Teach that graphs are a purposeful tool before stressing the features of good graphs, and aggregate data in the mind; that is, stress seeing data globally as a distribution not as a collection of points.

It is important to go beyond routine questions that relate directly to data and get the students to interpret information from data sets and graphs. Inference includes taking account of variation, predicting, hypothesising, and criticising. It has three components: generalising beyond the data, using data as evidence, and acknowledging uncertainty in the conclusion. There are three types of questions:

(a) from the data (answers can be read directly from the graph);

(b) between the data (involve comparing categories on the graph); and

(c) beyond the data (students infer reasons why or predict from the graph).

As a reversing activity students can be asked to suggest what questions may have been asked to generate the data in the graph. Sharing questions and responses can lead to significant engagement with the data and may suggest further avenues that students can explore in generating their own data collections.

## 2.2 Activities

1. **Softball throwing**

(a) A class had to choose a representative for a softball throwing contest. Three children volunteered. Each volunteer was asked to make five throws which were measured with a trundle wheel to the nearest metre. The results were as shown on the table below.

| Volunteers | Their 5 throws (to the nearest metre) |
|------------|----------------------------------------|
| Rachel | 28, 23, 22, 24, 27 |
| Betty | 24, 23, 27, 24, 27 |
| Tony | 23, 27, 29, 18, 26 |

(b) Who would be the best representative? Who is the most consistent? Who has the longest throw?

(c) What should our criteria be for selecting the best representative? Who has the best typical throw? How do we define typical? Is consistency important? Should we have measured more or less than five throws? Should bad throws be excluded? Is anything important lost in rounding to the nearest metre?

(d) Develop an argument for your choice.

*Note 1*: Students can be allowed to put information on tables, or to tally throws into sections, say 15–19, 20–24, 25–29, etc. if this helps them. Students can also graph the results and work out averages if this also helps. Encourage students to take into account the context (one big throw wins) and their analysis of data in their arguments for their representative.

*Note 2*: If students refine the way to represent the data in arriving at their inference (e.g. from a simple plot of points to a frequency bar graph), this is said to be an example of "transnumeration" (Wild & Pfannkuch, 1999) – "changing representations to engender understanding" (p. 227). This shows strong knowledge growth and increased understanding. Observe students to see if this happens.

2.  **Canned food**

    (a) There are 23 brands of baked sausage in plum sauce. Sixteen use microwaves to cook the sausages and sauce, and 7 use steam. The cans always display how they are cooked but stores always seem to have different brands or unbranded cans. To enable consumers to determine which type of can to buy, a consumer group tests all brands and marks them out of 20 on quality. The marks for each of the 23 brands are given in the table below.

    | Type of can | Marks for all brands of that type |
    |---|---|
    | Microwaved | 7 4 18 5 4 12 9 3 4 3 16 9 5 13 7 18 |
    | Steamed | 10 14 5 11 15 12 9 |

    (b) Which type, microwaved or steamed, is best to buy when there is no brand? Which type has the best mark? Which type has the most consistent?

    (c) Determine criteria for making a judgement. Develop an argument for your choice.

    *Note*: Again let students use any form of table, graph or determination of average that they think will help them. Check for the misconception that the type with the highest score is best even if other scores are low – in this context, it could be argued that one needs consistency as do not know what is being bought.

## 2.3    Investigations

### Investigation 1

1.  Choose an investigation like one of those in the level B examples below. Try to make it relevant and motivating for your students.

2.  Let the students work out their own way to tackle the question – discuss and reflect.

3.  Use every opportunity to direct attention to and reinforce the movement from statistical literacy to statistical reasoning, and take every opportunity to meet the four challenges and use the three types of questions.

    **Level B investigations**
    What year level has the healthiest lunch?
    What is the shortest shadow?
    How much do we spend at the fete?
    What is a typical hand span?
    Is rolling your tongue hereditary?
    How many commercials do we watch?

### Investigation 2

1.  Choose a topic that has rich online data, for example, something from statistics that can be found on the Internet such as crime statistics, road safety statistics, weather statistics, sport statistics, TV ratings.

2.  Teacher poses a question, or better still have a discussion and see what the students come up with (problem posing).

3.  Put students into groups to investigate the question and infer and defend a conclusion.

4.  Use the process from 1.3 – PREDICT, SEE, PLAN, DO, CHECK – the plan now must focus on data to select.

# Unit 3: Central Tendency

This unit looks at statistical inference at the end of the primary years and the beginning of the secondary years. As such, it covers the move from statistical reasoning to the beginning of statistical thinking – beginning to focus on the why and how of statistical investigations and to understand distributions, to infer and create, and to see things as big ideas. The unit does this within the task of introducing mean, median and mode, the measures of central tendency. These will enable inferences to be better determined and defended with argument.

## 3.1 Overview of unit

This unit continues to focus on statistical reasoning and introduces the mean, median and mode as another tool to facilitate the comparison of data sets and the process of inferring and predicting from data. This period is the time when students' understandings should be changed from informal to formal and from intuitive to formulae. However, this move should follow understanding. This means spending time building intuitive understandings for mean as average, median as the middle one, and mode as the most common one. YDC would recommend allowing students to develop their own formulae before giving them the precise statistics formulae (this is the creative component of Abstraction in the RAMR cycle).

It is recommended that: (a) students be encouraged to predict before finding mean, median or mode and to reverse from mean-median-mode to data (as well as data to mean-median-mode); (b) teaching integrates statistical inference with probability (e.g. using experimental probability investigations as a basis for inferences); and (c) questioning keeps attempting to move students' understandings from literacy to reasoning to thinking. [*Note*: An activity that can be used here is the Monte Carlo method to find area of irregular shapes. To do this: (a) copy the shape onto a rectangular grid of Cartesian coordinates; (b) use random numbers to generate points and plot them; (c) count points in and out of the shape; and (d) use the percentage in the shape × total area of grid to give area of the shape.]

The formal definitions of mean, median and mode are introduced along with formulae and examples. Consider the following data set containing results in a mathematics test out of 20 for two groups of students (the Angels and the Aces).

| Angels | 7 | 8 | 10 | 9 | 5 | 7 | 11 | 5 | 10 | 7 | 9 |
|--------|---|---|----|---|---|---|----|---|----|---|---|
| Aces | 8 | 7 | 14 | 10 | 8 | 9 | 7 | 16 | 8 | 15 | 8 |

1. **Mean**. The mean is the average of the scores. To calculate it, the scores are added and the result divided by the number of scores (i.e. mean = Σ score/number of scores). In the Angels and Aces example, the following means are evident:

   Angels     7+8+10+9+5+7+11+5+10+7+9 = 88      88÷11 = 8

   Aces      8+7+14+10+8+9+7+16+8+15+8 = 110      110÷11 = 10

2. **Median**. The median is the middle number when numbers are placed in order from lowest to highest. For an odd number of scores like here with 11 scores, the median is the 6th score. For an even number of scores (say 14), the median is the average of the middle two scores (for 14 numbers, this would be the 7th and 8th scores).

   Angels     5, 5, 7, 7, 7, 8, 9, 9, 10, 10, 11 – the 6th term is 8 – the median

   Aces      7, 7, 8, 8, 8, 8, 9, 10, 14, 15, 16 – the 6th term is 8 – the median

3. **Mode**. The mode is the most commonly occurring score. In the example above, the mode for the Angels is 7, while the mode for the Aces is 8.

Once calculated, the measures of central tendency (mean-median-mode) and the data sets are applied back to reality. The questions are:

- What do these measures tell us? How might they be useful?
- What can be inferred from these measures? What relationships can be seen?
- How representative of the class are the measures of central tendency?

## 3.2 Intuitive and process activities

### Intuitive introductory activities

The best introductory activities are those that enable the students to intuitively understand mean, median and mode. Here are some examples. Choose the one or more appropriate to your students. It is not necessary to do them all – only those that are needed.

1. **Straws or strips**. Cut 6–10 straws or strips of paper in various lengths. Ensure that there is more than one of some lengths. Give strips to another group. They put the strips side by side (shortest to longest) and:

   (a) find median (middle one) and mode (most common); and

   (b) predict where they think mean will be, and work out an informal way to find mean through making lengths the same (cutting bits off the longest and adding to the shortest).

2. **Unifix**. Repeat the above using coloured unfix cubes – have sets of 5–7 colours and students make bar graphs by putting colours together. Then they move cubes around for mean (best to prepare Unifix so mean is a whole number).

3. **Packets of M&Ms or Smarties**. Give out packets, students sort into colours and make bar graph shortest to longest. Students find mode and median – then predict and move M&Ms/Smarties to get all same height bars for mean.

   Can move bars so in a line with a marker between bars – this shows that the mean is found by adding and dividing by number of colours (leads to formulae) – see below.

4. **Birthdays**. The above can be repeated with A4 sheets with names of students placed on a bar graph under months of their birthdays. This allows the paper sheets to be cut into fractions to get common length for the mean.

5. **Picture**. A picture of a row of people of different heights can give a visual image of average – it is the line in which tall can be added to short to get common height. Heights made with streamers is another introductory activity for mean.

### Process activities

This section more formally introduces the three measures of central tendency, moving from data to formulae. It also look at the importance of allowing students to make up their own formulae before they are given the formal ones of mathematics.

1. **Data and centres**

   (a) Pick a topic to gather data on, e.g. shoe sizes. Construct a physical graph from everybody's shoe sizes like the one following (the 0 represent students).

```
                                    0
                                    0
                        0           0
                        0           0           0
            0           0           0           0
            0           0           0           0
            0           0           0           0                       0
0                       0           0           0           0           0           0           0
```

| Shoe size: | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

(b) List the shoe sizes above in order, e.g. 3, 5, 5, 5, 5, 6, 6, ... and so on. Calculate mean, median and mode. Repeat this for your data.

(c) Record data on a frequency table as below for the data in (a). Add cumulative frequencies. Repeat this for your data.

| Cumulative frequency (Cf) | 1 | 1 | 5 | 11 | 19 | 24 | 25 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency (F) | 1 | 0 | 4 | 6 | 8 | 5 | 1 | 2 | 1 |
| Number (n) | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

(d) Graph the data from the frequency table as a bar graph and the cumulative frequency as a line graph. Repeat this for your data.

(e) Compare your data with data from (a). What is the difference in the mean, median and mode? If there is a difference, why? If there is not, why not?

(f) Describe and justify from your class's data only (as the other is made up) what you would think a typical shoe size for a class at the same year level as yours would be. Do you need more data for this? Why?

2. **Building formulae**

(a) Repeat (a) to (d) from Investigation 1 for another set of data, e.g. hand length to nearest centimetre.

(b) If you had to calculate the mean, median and mode from the frequency and cumulative frequency table, how would you do it? Try some ways to see if you arrive at the same mean, median and mode.

(c) Here are some data in a table:

| Cf | 2 | 6 | 11 | 12 | 18 | 20 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|
| F | 2 | 4 | 5 | 1 | 6 | 2 | 3 | 1 |
| n | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Calculate mean, median and mode from the table.

(d) **Challenge**: In your group construct a formula for finding the following measures (if a formula is possible): (a) mean, (b) mode, and (c) median.

(e) Compare your formulae with the formal formulae from your teacher. What is different? What is the same?

## 3.3    RAMR lesson: Measures of central tendency

This is an example of RAMR lesson for mean, mode and median.

### Reality

Determine from students their initial understanding of the term *average*. Discuss populations (Australian Bureau of Statistics website has much useful statistical data) as a basis of statistical information or find a relevant topic with statistical information for students to start with – perhaps cricket run rates, bowling statistics of players, swimming times if near Olympic or Commonwealth Games. Discuss the uses of these statistics, how they are used, what information they convey. This leads into need for calculations and interpretation of measures. Gather own personal statistics – height, mass, foot length, arm length … make three paper streamer replicas of these measures (not crepe streamers as these will stretch). Ensure students have their names on their streamers and a label for what measure it represents.

### Abstraction

#### Body

Students line up in order of height. Discuss what the average height might be. Have students consider what the middle value is. Find middle value by counting in from the ends. Create a chart of students' heights by securing streamers to a large flat surface (wall, floor, maths mat). Clearly label the middle value as **median**. (If there are even numbers of students the median will be halfway between the two central students' heights.)

Looking at the chart, have students identify any sections where students are the same height. Find the height that occurs most often. Discuss this measure as the **mode**. Label clearly.

Consider the tallest and shortest values, run a line of string across the chart from left to right across all streamers at the shortest and tallest values, discuss the difference between the two measures as the **range**.

Ask students what could be done to make everyone the same height. How could this be achieved? Some suggestions may be to take height off all taller people or add height to all shorter people. Have students consider how to make all the streamers the same length without having any streamer left over. This is time consuming but worthwhile to build a kinaesthetic understanding of the process behind calculating the **mean.** Students shorten longer streamers and add to shorter streamers to make them all the same height.

Make a new chart with an additional set of streamers. Mark across the mean with string or a drawn line. Discuss the differences between the mean and the actual measurements of height. This is the **mean deviation.**

*Note*: This activity can also be linked to graphing activities to discuss scaling and truncation of axes. Students can be challenged in groups to make their graph of heights fit on an A3 page while still representing their heights. Students usually tend to fold their heights in half or quarters – scaling. It is also useful to discuss the possibility of choosing a base height to work from and remove the section of streamer that is this long. What this shows is that scaling on a graph indicates the overall relationship between values in proportion; truncation displays the difference or the range more effectively but loses the overall relationship.

#### Hand

List students' heights on board. Link heights to scale on graph. Students make own graphs to represent heights. Mark in mean, median, mode as discovered. Record median and mode as numeric values. Discuss the actions taken in determining the mean. Explore ways of working with the values to make them all the same.

#### Mind

Look for strategies to simply calculate the mean. Consider patterns in students' explorations which lead to adding all values and dividing by the number of values to determine the mean.

## Mathematics

Discuss measures of central tendency discovered. Where might they be most useful and/or most appropriate? Explore other measures taken to further practise calculation of mean, median, mode, range, mean deviation. Explore relationships between mean and median. Investigate what happens to the mean and median if further values are added. This can be done by working through a second abstraction cycle using foot lengths and calculating the mean and median as lengths are added to the list (i.e. start with two values and calculate the mean and median [will be the same], add a value and recalculate, record changes, …). Discuss the effects of adding values to the mean and median.

## Reflection

Generalise the effects of changes in the data set on the mean and median.

Explore ways of making the mean and median of a set of figures the same and different.

Explore the effect of skewed data on mean values. Reverse from a given set of values to indicate what the data set might look like.

## 3.4    Further mean, mode and median activities

Once again, it is not necessary to do all these activities. They give a variety of ideas – choose the activities that your students need. The first four reinforce a particular central tendency – the last one practises all three measures of central tendency (mean, mode and median).

### Mean activity – Drawing instruments

**Materials**: Box or tray of writing instruments (students can use their own); strips of scrap paper, or students can tear these themselves; sticky tape

**Instructions**:

1.   Discuss **Mean –** simply the average of all the items in a sample. To compute a mean add up all the values and divide by the total number of items in the data set.

2.   Do a hands-on activity with drawing instruments. Each student selects out two things that can be used to draw with (pens, pencils, coloured pencils, crayons, charcoal, etc.). Have the students be creative as to what can be used to draw.

3.   Students work with a partner to cut a strip of paper to the length of each drawing instrument. Then, they tape the strips together and fold into four equal pieces.

4.   Discuss why four? (4 drawing instruments used so divide by 4).

5.   Students measure the length of these pieces and that is their mean.

6.   Discuss how this is the same as what we do mathematically. Finally, apply it by finding the mean of the same data used in previous lessons.

### Mode activity – Measuring smiles

**Materials**: String, tape measures, paper strips or streamers, calculators

**Instructions**:

1.   Discuss **Mode –** the most frequently occurring value in the data set.

2.   Working with a partner, students use a piece of string and a measuring tape to measure their own smile – rounded to the nearest centimetre.

3. Each student then cuts out a strip of paper streamer the length of their smile and writes their name on their strip as well as the length of their smile.

4. Create a column graph of smile lengths; the column with the most smiles in it is the mode.

5. As a class, manipulate the smiles to discuss mean and median.

6. Introduce the notion of outlier data if you have any worthy smiles.

7. You can investigate metric conversions to millimetres and metres.

8. Students can then work together in small groups then as a class to tape their smiles together. Does your class smile stretch across the room?

## Median activity – Finding what is typical

**Materials**: Unifix cubes

**Instructions**:

For finding median, use Unifix cubes (but you could use stacks of non-interlocking as well) to show how many of something we have.

1. Discuss **Median –** the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.

2. Have class make a stack of cubes to represent:

   (a) how many people in your family

   (b) how many pencils in your desk

   (c) how many pets you have.

3. To find the median, line up in order from least to greatest (left to right) and find the exact middle person.

4. Discuss with the class how the middle person gives us information about what is typical for whichever numerical data we are investigating. Be sure to demonstrate examples using an even number of pieces of data and an odd number of pieces of data so students can see how that works.

5. After doing several of these types of lining up, counting to the middle, discussing that median shows us what's typical, then you can move to a paper and pencil method for finding median, always emphasising the importance of putting the data in order from least to greatest before finding the exact middle.

6. As far as connecting median to the real-world reality, think of some things that we make decisions about based on what we've come to believe is typical for certain situations. For example:

   (a) holiday plans may be based on typical weather patterns, temperature or rainfall;

   (b) expectations for sporting event outcomes may determine whether fans turn up to watch;

   (c) a coach looks at what is typical about an athlete, and then makes decisions on whether to use the player, when to send a player in or take one out.

One thing to remember is that median is just one kind of average – just one way to look at what's typical.

## Median activity – Hat sizes

**Materials**: Tape measures, pencil, post-it notes or scrap paper

**Instructions**:

1. Discuss **Median –** the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.

2. To create a **reality link**, use hat sizes. Pose a problem about a shop owner not knowing how many hats to order in each size. The students, in pairs, estimate and then measure each other's head size with tape measures.

3. The students agree on their head measures, and each student writes theirs on a post-it note or piece of paper.

4. Have the students line up from greatest to least. One from each side of the line can sit down until you reach the median.

5. You can then use the data to make a variety of graphs to show the information. Mean and mode can be calculated from the data.

6. As an independent activity, have students repeat hat activity but have the retailer selling sneakers. The students will need to understand the need to look for shoe size, order these from largest to smallest, eliminate from each end until median is found, graph etc. to find mean and mode.

## Mean, mode and median card games

**Materials**: Deck of cards (1 [Aces] to 9 cards only), scrap paper, pencil, calculator (optional)

**Number of players**: 5

**Instructions**:

Review the definitions of these key terms with the class:

- *Mean* is simply the average of all the items in a sample. To compute a mean add up all the values and divide by the total number of items in the data set.
- *Median* is the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.
- *Mode* is the most frequently occurring value in the data set.

Deal out 7 cards to each player. Ask each player to arrange their cards in sequential order. Aces count as the number 1. Then, depending upon which game you want to play, follow the directions below:

1. **Finding the Mean game**. Each player finds the total value of the digits on their cards, then divides the total by 7 (the total number of cards) to find the mean. For example, if the cards in your hand are Ace, 2, 4, 6, 8, 8, 9, then the sum of those digits is 38. Dividing the sum by 7 yields 5 (rounding to the nearest whole number). If this was your hand, you'd have scored 5 points in this round. Because computation can be tricky without paper at this age, feel free to give your students a pencil and paper to find the mean. Or, to keep the game moving at a faster pace, you may allow use of a calculator.

2. **Finding the Median game**. Each player finds the median card in their hand and that number is their point value for that round. Thus, using the hand above, the median of the cards is 6, since it's the value of the middle card.

3. **Finding the Mode game**. Each player finds the mode in their hand of cards, which represents their point value for that round. If there is no mode, then they don't score any points in that round. However, if there are two modes (two numbers occur the same number of times), then the player snags the point values for both modes! In the example above, the mode would be 8, since it occurs most often.

The winner of each game is the first person who scores 21 points.

## 3.5    Investigations

Any group of people vary in their data, and many distributions are possible. What does this say for the numbers (mean, mode, and median) that we use to describe them? The following investigations involve constructing different distributions so we can look at this. They look at the question – what does mean, mode, and median tell us about a distribution?

*Note:* It is not necessary to do all these investigations – **simply do the one or the ones that are appropriate to your students**.

### Investigation 1: Answering a question

1.  Choose an investigation like one of those in the level C/D examples below. Try to make it relevant and motivating for your students. But make sure that the investigations move onto two or more uncertainties and other knowledges required.

2.  Let the students work out their own way to tackle the question – discuss and reflect.

3.  Use every opportunity to direct attention to and reinforce the outcomes for this unit and utilise central tendencies as appropriate for your students.

| **Levels C and D Investigations** | Do we eat healthy cereal? |
|---|---|
| | What is the best design for a loopy aeroplane? |
| | How long is 10,000 steps? |
| | Is it better to buy or make Chinese food? |
| | Does Barbie have human dimensions? |
| | What is reaction time? |
| | How far does an origami frog jump? |

### Investigation 2: Constructing distributions

1.  Suppose a class had small and large students and no in-between sizes.

    (a) Construct a frequency table for shoe size for this class which gives the same mean as in 3.2 *Process activities 1: Data and centres* but with no 6, 7, or 8 shoe sizes.

    (b) What happens to mean, median and mode?

    (c) Draw a frequency graph. How is it different to your original graph from 3.2 *Process activities 1: Data and centres*?

    (d) Can you keep the median the same?

2.  Suppose two giants joined the class with shoe sizes of 25 and 27.

    (a) Redo mean, median and mode for your data. What is the difference?

    (b) Draw a frequency graph. How is it different to your original graph?

    (c) How could you change the shoe sizes for the rest of the class so that the mean was reduced to the same number as in 3.2 *Process activities 1: Data and centres*?

    (d) Can you make the median the same as in the process activity?

3.  What if we had a lot of students with size 2 shoes?

    (a) Let's add five size 2 students and remove two from each of 6, 7, and 8. What happens to the mean, median and mode?

(b) Draw a frequency graph. How is it different to your original graph from the 3.2 *Process activities 1: Data and centres*?

(c) Can you increase the sizes of some of the other students so you get the same mean? What happens to mode and median?

4. The temperatures for a week were:

| S | M | Tu | W | Th | F | S |
|---|---|----|---|----|---|---|
| 23 | 24 | 23 | 22 | 23 | 24 | 22 |

(a) What was mean, median and mode? Did you notice that all three are the same?

(b) Can you get the mean to differentiate from the median and mode by changing some temperatures? Which ones and by how much? Can it be done with a simple change?

(c) Modify the above to make a distribution with a low mean.

(d) Modify the above to make a distribution with a high mean.

(e) Draw the frequency graphs for (b), (c) and (d). How are they different?

## Investigation 3: Understanding what mean-median-mode mean for a distribution

1. Prepare/find data of the following types:

    (a) symmetrical data (like normal curve) with low and high numbers matched and most numbers in the centre;

    (b) symmetrical data but with a hole in the middle – very little data in middle;

    (c) skewed low data – most data in low end;

    (d) skewed high – most data in high end; and

    (e) data with a few high outliers.

2. Find mean, median and mode. Compare and contrast for different data sets. Draw conclusions regarding the effect of different distributions on mean-median-mode.

3. What kind of data would have mean, median and mode the same? What would have mean and median the same but two modes? What would have mean higher than median, lower than median?

# Unit 4: Data Distribution

This unit continues the move from literacy → reasoning → thinking in statistical inference. It does this by looking at range, quartiles, mean deviation and standard deviation. It also looks at two new graphing techniques of complex bar graphs and box plots which use range, mean/median and quartiles in their design. It continues discussion of distribution of data, using terms including *skewed*, *symmetric* and *bi-modal*. Finally, it looks at sampling.

## 4.1 Overview of unit

The first focus of this unit is distributions, particularly in frequency graphs. It continues to look at the relationships of mean, median and mode with data but also introduces new distribution concepts of range, quartiles, mean and standard deviation, and their relationships to data. It also focuses on reversing, that is, constructing data to achieve given ranges, centres and deviations. Finally, it introduces new graphs based on distributions, that is, complex bar graphs showing range, and box and whisker graphs, introducing new language such as skewed and normal.

The second focus of this unit is to begin to introduce sampling. To do this we look at gathering data from large populations and discuss informally how it would be done. We also introduce terms such as *sample* and *represents population*. Thus we discuss ways to have a sample more closely represent the population, introducing such ideas as random sampling, stratified random sampling, cluster sampling and multi-stage sampling (strata and clusters).

*Note*: This is where we need to go back and look at the sections on challenging misconceptions and building understanding in Unit 2 (see sections 2.1 and 2.2).

The new ideas raised in this unit include the following.

1.  **Range**. In a list of data this is the difference between the greatest and least value. Consider the data for the Angels and Aces mathematics test shown in Unit 3 (section 3.1):

    The range for the Angels is 11−5 = 6

    The range for the Aces is 16−7 = 9

2.  **Quartiles**. The four quartiles show the scores at which data is divided into four parts. For the Angels and Aces data, this is as follows (put data from smallest to largest):

    **Angels**    5  5 ⑦ 7  7 ⑧ 9  9 ⑩  10  11 – quartiles are at 7, 8 and 10

    **Aces**      7  7 ⑧ 8  8 ⑧ 9  10 ⑭  15  16 – quartiles are at 8, 8 and 14

    *Note:* For continuous data, median is found by finding/calculating score that is half way (halving the number of scores) and the 1$^{st}$ and 3$^{rd}$ quartiles by finding/calculating the scores that are ¼ and ¾ of number of scores.

3.  **Deviation**. This is the measure of difference between the scores and the mean. Early deviation looks at the average of the differences between score and mean (where differences are always positive) – the mean deviation. Deviation is a concept that is tied to the idea of centre; it measures the extent to which your data deviates from the centre or mean. As an example, the Angels and Aces data gives the following:

    **Angels**    1, 0, 2, 1, 3, 1, 3, 3, 2, 1, 1    sum is 18    Mean deviation is 1.6

    **Aces**      2, 3, 4, 0, 2, 1, 3, 6, 2, 5, 2    sum is 30    Mean deviation is 2.7

*Note*: There are two matters that should be especially noted. The first is that statistical measures are only possible if data can be added, subtracted and divided. Scores from counting (interval data) are acceptable but scores denoting only the order (ordinal data) or simply the membership (category data) of categories are not acceptable. The second is the reason for calculating these statistical measures in the first place. This is that they enable a mass of data to be succinctly described. Reducing data to sets of representative figures enables comparison, inference and prediction to be more easily deduced from the data set.

*Standard deviation* is a measure of difference between score and mean which gives more weighting to large differences. This is because it averages the square root of the square of the differences between mean and scores. In the Angels and Aces example it is as follows:

**Angels**  squares of differences:   1, 0, 4, 1, 9, 1, 9, 9, 4, 1, 1
sum = 40   standard deviation $= \sqrt{40} = 6.3$

**Aces**   squares of differences:   4, 9, 16, 0, 4, 1, 9, 36, 4, 25, 4
sum = 112   standard deviation $= \sqrt{112} = 10.6$

Compare the standard deviation to the mean deviation. What are the differences? Why?

Now we have a complete set of centres and deviations, they can be used to compare data. Central tendency may appear to be a straightforward way of comparing two data sets to make inferences and predictions. However, the incidence of misrepresentation and misconceptions with statistics (see Unit 5) means that it is important that students are encouraged to explore the use of misinformation in statistics for robust understanding and critical literacy in statistical situations in the real world. Rather than just learning these as a set of ideas and features to watch for it is more useful to challenge students to understand misrepresentation sufficiently that they can produce deliberately misleading statistics. This is much more engaging and will result in a stronger learning experience.

In the Angels/Aces example we can tabulate the measures as below for comparison.

|  | Range | Mean | Median | Mode | Mean Deviation | Standard Deviation |
|---|---|---|---|---|---|---|
| **Angels** | 6 | 8 | 8 | 7 | 1.6 | 6.3 |
| **Aces** | 9 | 10 | 8 | 8 | 2.7 | 10.6 |

As can be seen, the scores of the Aces are more spread out than the scores of the Angels (this can be seen in both the range and the deviation) and are higher (this can be seen in the mean and mode), yet this is due to a few high scores as the median shows there is little difference in the bulk of the scores.
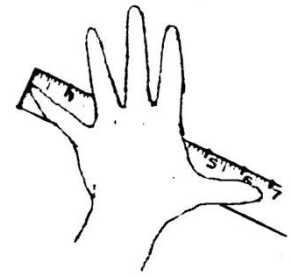
Thus, it is important to build intuitive understanding of measures such as mean or deviation so they can be applied back to the original question to analyse the trends shown in the data. This is applying the statistics back to reality and is the culminating activity for all data handling and analysis activities. Inferring from the measures and predicting outcomes are important skills for decision-making and should involve discussion and debate. Written justifications for decisions using the measures calculated are also appropriate activities here.

## 4.2    Activity

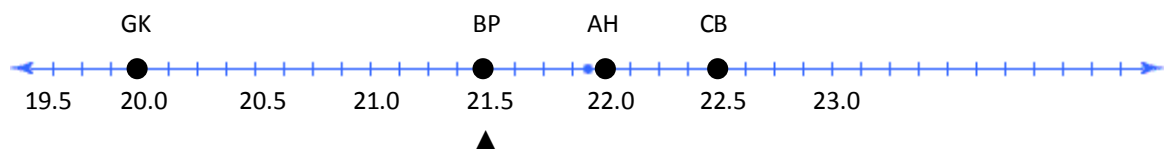### Are you a deviant? Comparing hand spans

**Materials**: Pencil, tape measures or rulers, paper

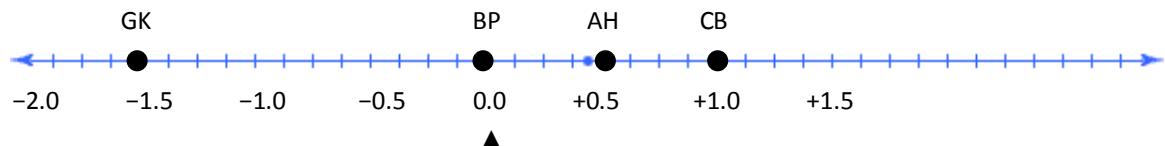**Instructions**: Have students work in groups of four.

1.  Record the hand span for every person in your group. (Let the students decide if it matters if the left or right hand is used.)

2.  Find the mean hand span for your group.

3.  Make a dot plot of the results for your group. Write names or initials above the dots to identify each case. Mark the mean with a wedge ( ▲ ) below the number line.

    For example:

    

4.  Discuss two reasons why the measurements are not all the same.

5.  Measure (either with ruler or calculate numerically) how far your hand span is from the mean of your group. How far from the mean are the hand spans of the others in your group?

6.  Make a second dot plot. This time, plot the differences (deviations) from the mean. (Compute these by subtracting the mean from each observation, e.g. GK has hand span of 20.0, mean = 21.5. So 20.0 − 21.5 = −1.5.) Again, label each dot with names or initials.
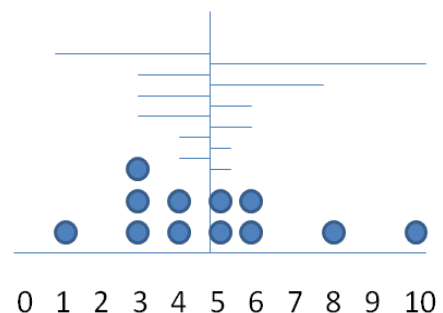
    For example:

    

7.  Explain how to get the second plot from the first without computing any differences.

    *We can just shift the number line (think of the line being able to slide to the right) so that the wedge now indicates the zero point on the number line.*

8.  Using the idea of deviations from the mean, can you come up with a "typical" distance (deviation) from the mean? Explain.

    *A "typical" distance from the mean seems to be around 0.75. We can get this by taking the average distances: (1.5 + 0.0 + 0.5 + 1.0) / 4 = 0.75. We do this by adding everything as if it is positive – this is the mean deviation.*

**Extension activity**: Create some data in your class that can be plotted in a dot plot where multiple occurrences are found for a given value (e.g. number of hours of TV watched on Saturday). Find the mean and plot as per graph on right. By finding the mean distances of the horizontal lines from the mean, an approximate standard deviation can be found.
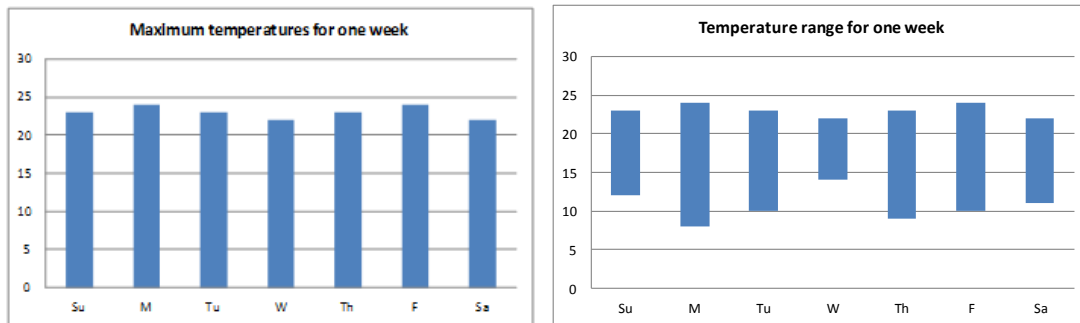
## 4.3    Distribution graphs
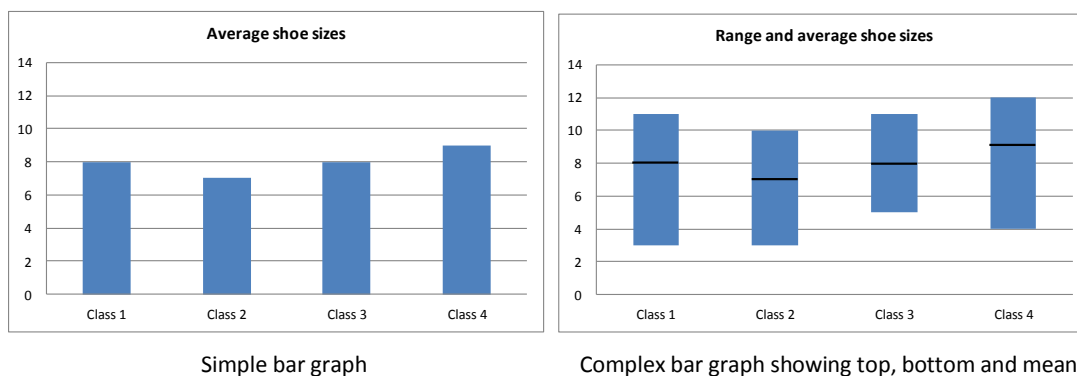
There are two types of graphs.

### Complex bar graph showing range

Take data on temperatures for a week. One way to look at these would be to graph the maximum temperature on a bar graph. However, if we graph bars from minimum to maximum temperature (this will look like the graph on the right below) this shows maximum and minimum temperatures and the range of the temperature across the day.



*Activity*

1. Collect shoe size data for four classes.

2. Average the data for each class and calculate low/high and range for each class.

3. Represent the average shoe size data on a bar graph for the four classes.

4. Represent the data with bars showing low to high and a line for average on a graph. The two graphs should be different as follows:



Simple bar graph

Complex bar graph showing top, bottom and mean
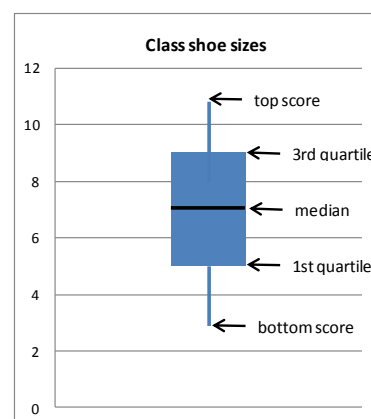
5. Which gives the best results in terms of simplicity? Which is best for inferences?
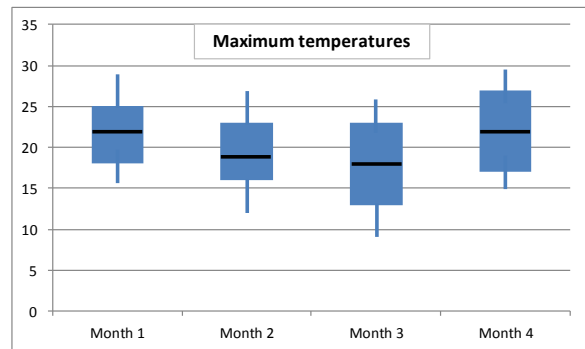
### Box and whisker

Take data on shoe sizes for a class. Find median and divide into quartiles. Graph this as on the right:



*Activity*

1. Find data on temperatures for four months (find data from the internet).

2. Calculate lowest and highest temperature and median and quartiles for each month.

3. Draw a box and whisker graph for each month, side by side (called stacked) – you should get a graph that looks like that on the right.

4. Draw a box and whisker graph for the three months combined.

5. Which is better? The stacked or the combined? Which is easier to make inferences from?



## 4.4 Sampling

In the real world, and sometimes in class, there are too many sources of data. Therefore, to get an idea of the data distribution, we take a sample. The important thing for accuracy is that the sample reflects the population. Thus, the sample that is considered most likely to do this is the random – or formed by chance. However, as it is chance, it still may not represent the population – but it is considered to have more chance of doing this than other ways.

*Activity*

1. Gather data from all the students in the class – say height.

2. Put names of students on pieces of paper, put pieces of paper in a bag, and select 10 at random.

3. Compare mean, mode, median, range and standard deviation of the 10 with the same measures for the whole class – were the 10 representative of the class?

There are four types of sampling:

1. Random sampling – sample chosen by chance.

2. Stratified random sampling – population broken into different characteristics which are given a percent representation, then random within each strata (e.g. may wish men and women to be the same percent – 50% for each – so randomly sample the same number from each strata).

3. Cluster sampling – population divided into clusters that are the same in some way, clusters are chosen randomly, but all people in the clusters chosen are surveyed.

4. Multi-stage sampling (strata and clusters) – combination of 2 and 3.

*Activity*

1. Choose something on which to survey all the students in the school.

2. Choose a sampling technique and implement it.

3. Analyse the data (mean, mode, median, range, deviation, and so on).

4. Infer findings from the population from your sample data. Do you think that the sample was accurate?

5. In elections, the whole population votes but to predict the election, polls use samples. Find out how they do this. Are they accurate?

## 4.5    Investigations

As stated before, any group of people vary in their data and many distributions are possible. What does this say for the numbers (mean, median, mode, range and deviation) that we use to describe them? The following tasks involve constructing different distributions so we can look at range and deviation. It looks at the question – what do mean deviation and standard deviation tell us about a distribution?

### Investigation 1: Answering a question

1. Choose an investigation like one of those in the level C/D examples below. Try to make it relevant and motivating for your students. But make sure that the investigations move onto two or more uncertainties, other knowledges required, and differences between types.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the outcomes for this unit (moving from reasoning to thinking, using measures of deviation, using complex bar or box and whisker graphs, and using sampling) as appropriate for your students.

| **Levels C and D Investigations** | Do we eat healthy cereal? |
| --- | --- |
| | What is the best design for a loopy aeroplane? |
| | How long is 10,000 steps? |
| | Is it better to buy or make Chinese food? |
| | Does Barbie have human dimensions? |
| | What is reaction time? |
| | How far does an origami frog jump? |

### Investigation 2: Why we need standard deviation

1. Take all the data sets from section 3.2 *Process activities 1: Data and centres* and section 3.5 *Investigation 2: Constructing distributions, directions 1–3*. This is five sets of data. For all five distributions, calculate:

    (a) range;

    (b) mean deviation; and

    (c) standard deviation.

2. What do you notice about the mean deviation and the standard deviation? Was one distribution's deviation higher than the rest? Which distribution had this higher deviation? Why? Any interesting results in the other deviations? How did the mean and standard deviations relate to each other?

3. In a new study, you gathered data on shoe size and found that the mean was 13.

    (a) What does the distribution look like if the standard deviation is high?

    (b) What does the distribution look like if the standard deviation is low?

    (c) What does the distribution look like if the standard deviation is high and the mean deviation is low?

### Investigation 3: Reversing

Provide students with means, medians, modes, ranges, mean and standard deviations and see if they can develop data for such measures.

# Unit 5: Inferential Misrepresentation

This section is different to the others in that it looks at how statistics can be used to misrepresent or "lie". This is a powerful way of reinforcing statistical inferential thinking, and also a powerful way to understand how persuasion may be given the credibility of mathematical correctness. The unit looks at methods for misrepresentation and then at activities and investigations.

## 5.1    Overview of unit

### Ways of misrepresentation

Darrell Huff in his book *How to lie with statistics* (Gollancz, London, 1969 – now out in Penguin) describes a series of ways in which statistics can misrepresent.

1.    **Sample bias**

Statistics showing central tendency for a large population are nearly always based on a small sample. Such samples may not represent the population as a whole and so bias the statistic. For example, Oxendorf University may trumpet that its graduates of 10 years standing earn on average $145 165 per year. Is this really correct? A closer look at how this statistic was arrived at may provide insight.

It is likely that the information on salaries was collected by replies to an emailed survey. In this case, only those graduates whose email addresses were known and who bothered to reply were included in the result. Furthermore, the statistic was calculated not on their actual salary but on what they said their salary was. These factors mean that the statistics are open to problems of lying and bias towards more successful graduates, whose addresses are known and who have support staff to reply to the email (and who may inflate their salaries).

The statistics of average salary being $145 165 may also conceal large differences. What about deviation? The statement seems to imply that such a salary is what **every** graduate can expect!

We should always realise that all samples have a bias – towards people with more money, more education, more information and alertness, better clothing and more conventional and settled appearance – because these are the people who most interviewers feel more at ease with.

So, when faced with a survey result, say "how was the information collected?".

2.    **Wrong average**

In a factory or enterprise, there may be, for example:

*   a manager/owner earning $900 000 per year;
*   a partner earning $300 000 per year;
*   two assistant managers earning $200 000 per year;
*   a sales manager earning $114 000 per year;
*   three sales people earning $100 000 per year;
*   four information technology staff earning $74 000 per year;
*   a foreperson earning $60 000 per year; and
*   12 workers/clerical staff earning $40 000 per year.

In this case, the **mode** (the wage/salary occurring most frequently) is $40 000 (the workers at the bottom of the range). The **median** (the wage/salary in the middle – 12 people earn more, 12 people earn less) is $60 000 per

year (the foreperson), while the **mean** (the average) is \$114 000 per year, but only 4 of the 25 people earn more than this. Depending on the data, mean, median and mode may be the same or differ widely. Where there is a large range of values which contains a few very large values and many close together low values (which is typical of income statistics), the mean is high and the mode low. The median is the best measure of centre.

So, when faced with an average, say "which average?".

3.  **Missing information**

Statistically inadequate samples (small ones) can produce just about any result. Therefore if we ignore unfavourable samples, we can end up with an "independent laboratory test" certified by a "public accountant" proving just about anything. Four out of five people liking "Exo teeth licorice" can be just that – groups of five people were asked if they liked "Exo" until one group was found where four out of five did. The 30 previous groups in which fewer than four liked "Exo" need not be considered.

The average alone can be misleading. A town with cold nights and hot days can end up with a delightful average temperature. We need information on range and deviation as well as average.

Words have different meanings to different people. What do statisticians mean when they say that "Tuffo" cleans twice as bright? Is this twice as bright as other cleaners or twice as bright as before cleaning?

4.  **Irrelevant statistics**

Darryl Huff gives the old adage: "if you cannot prove what you want, demonstrate something else and pretend it is the same". Statistics about related matters are often used to support arguments for which there is no direct support. The statistic quoted may well be true but not for the situation to which it is directed.

For example, "laboratory controlled tests" may indeed show that "Basho" destroys 9 out of 10 germs when used in high concentrations in a test tube, but will it do anything in your mouth in dilute concentrations? Young people from 16 to 21 may indeed have more car accidents than the 50 to 55 age range, but this may be due to driving more. Accidents per person **per kilometres driven** may show that it is safer, for a 100 kilometre drive, to be with the young person!

5.  **Direct misrepresentation**

Statistical data can be directly misrepresented. For example, juvenile delinquency figures can take a large jump when the courts change their recording procedures to count charges for group activities to each individual. Five youths stealing from a house can change from one offence to five break and enter offences, five being unlawfully on premises and five stealing offences (15 offences in all).

Percentages can make increases smaller or larger, depending on what you want, by choosing the appropriate base. Percentage increases can also look different to absolute increases. For example, someone taking a 50% pay cut from \$800 to \$400 per week is not going to be happy when that 50% is returned, but on the \$400, i.e. to \$600. We would not think it right if our 50% rabbit burger was made by mixing one rabbit with one bullock.

## Talking back to statistics

Darrell Huff recommends that we form a general impression of the argument and then ask the following five questions. Make these the basis of teaching.

1.  **Who says so?** Look for bias, missing information, wrong measures, ambiguous statements and value-laden names (the prestigious university). What are the interests of the claimants?

2.  **How do you know?** Look at how the statistic was calculated (the gathering of the data, sample size, type of data, calculations, and so on).

3. **Is there anything missing?** Is there range and deviation as well as average? What average? Try to look past percentages to raw scores. Look to see if key words are properly defined. Be wary of value-laden labels. Look at the groupings and categories. How were these selected?

4. **Did someone change the subject?** Watch for the switch from data to conclusion. More reports of cases do not mean more cases! What people say they do may not be what they do. Watch comparisons. Are they between different things?

5. **Does it make sense?** Many a statistic is false on its face. The magic of numbers suspends belief. Be wary of the decimal for the poverty line for a family of four. This is near impossible to directly measure. It has to be calculated from estimates.

## 5.2   Activities

### Introductory activities

Complete the following activities. Discuss Huff's misrepresentation types after the questions have been attempted and discussed.

1. Suppose a university wanted to show that its students earned more by going to university.

   (a)  What data would they need to gather?

   (b)  How would this gathering be done?

   (c)  Does this lead to bias? How?

2. We have the following data:

   - a manager/owner earning $900 000 per year;
   - a partner earning $300 000 per year;
   - two assistant managers earning $200 000 per year;
   - a sales manager earning $114 000 per year;
   - three sales people earning $100 000 per year;
   - four information technology staff earning $74 000 per year;
   - a foreperson earning $60 000 per year; and
   - 12 workers/clerical staff earning $40 000 per year.

   (a)  Calculate the mean, median and mode.

   (b)  What do you notice?

   (c)  How does this come about?

   (d)  What is the best of the three measures (mean, mode, median) for giving the centre of the data?

   (e)  What is wrong with the average or mean as the number giving the middle?

3. Answer the following statistical questions.

   (a)  What does an advertisement mean when it says there is evidence that Tuffo cleans twice as bright? [Twice as bright as what?]

   (b)  What does the Tuffo advertisement mean when it says 4 out of 5 housewives recommend Tuffo? [Does it mean 80% of all housewives?]

   (c)  The disinfectant ingredient in Basho cleaner has been shown in laboratories to kill 99% of all bacteria. What does this mean for Basho coming out of the spray can onto the kitchen table? [Does the spray can Basho have the same concentrations as Basho's ingredients in the laboratory?]

(d) Do statistics showing drivers aged 22–30 have more accidents mean that they are worse drivers than drivers aged 52–60? Why or why not?

(e) Is it a 50% rabbit burger if you mix the meat of one rabbit with the meat of one cow? Is there any way this could be looked at as true?

## Recognising misrepresentations

Refer to the five misrepresentations of Huff. Classify each of the following as one of the misrepresentation types. Then look at them in detail and answer the questions supplied.

1. Johnny threw a coin five times. He got a head only once. "That convinces me," said Johnny, "from now on, I'm always going to call tails!"

   (a) What is wrong with Johnny's conclusion?

   (b) What could a teacher do to show Johnny the incorrectness of his conclusion?

   (c) What does this say about the teacher who says that MAB are useless because when she tried them on some students last year, they did not work?

2. Bread rose from 80 cents to $1.20, milk dropped from $1.00 to 60 cents. "This is straightforward," said June, "bread has increased 50% and milk has decreased 40%, we have had a rise in the cost of living!" "No!" said Janette, "bread has increased $33\frac{1}{3}$% and milk dropped $66\frac{2}{3}$%, we have had a reduction in the cost of living."

   (a) Both are right in their percentages but how?

   (b) What has really happened to the cost of living, based on the cost of these two items?

   (c) How is this possible?

3. "The average family of four requires $673.86 per week to survive", states the report.

   (a) How is such a statistic calculated – from what information?

   (b) Is the decimal justified?

   (c) Why is it nearly always used?

   (d) Does this mean that all families of four can survive on this amount?

4. The principal says that the class has a mathematics average of 6 out of 10.

   (a) What could this mean?

   (b) Is it useful information on its own?

   (c) Does it mean that over half the students passed?

5. Before she used "Mucho" hair lotion, the lady is pictured dowdy, lank and sad. Afterwards, she is pictured smiling with bouncy and glistening hair.

   (a) What do these pictures tell us about "Mucho"? Anything of value?

   (b) Why do such pictures continue to be used?

6. The factory has an increased wages bill of 10%, an increased advertising bill of 10% and increased material costs of 10%.

   (a) Are they justified in raising their prices by 30%?

   (b) Why/why not?

## 5.3    Investigations

1.  **"Against the assertion" poster**. The world is full of assertions and sayings. For example, "there is more crime now than in the past"; "a stitch in time saves nine"; "the unemployed are dole bludgers"; "many hands make like work"; "people are less friendly than in the past".

    (a)  Choose an assertion that most people agree with. Gather data regarding the assertion, represent the data and draw inferences from the data, to make a poster that disagrees with the assertion.

    (b)  Look at Huff's five ways of misrepresenting. For example – gather data from a sample which is biased by choice of sample or choice of interviewer or statements in the interview; reinterpret data so that there is a different result (e.g. look at accidents per km driven not total accidents, or look at crimes per person not crimes on their own); gather data on something that looks like it is the same but is not, but can act as if it is; or use percentages of centres differently.

    (c)  Construct the poster. Try to be convincing in data and arguments. Make it really attractive and appealing.

    (d)  Did you find that there some data that cannot be misrepresented easily? Why?

2.  **"Both sides" posters**. Prepare a double poster display for an assertion as below.

    (a)  Choose an assertion (e.g. Young drivers are more dangerous than old drivers) that is suitable for (b) below.

    (b)  Gather data and prepare two side-by-side posters, one presenting data and drawing inferences in a way that supports the assertion, and the other presenting data and drawing inferences in a way that rejects the assertion (make the posters attractive and appealing on both sides, with graphs and headings and so on).

    (c)  Think of opposing ideas and how you could be able to find opposing data – remember that there may need to be a different way of looking at the data. Think of different ways your data could be biased and your graphs look better in their support of bias. Try not to simply repeat the idea in (1) above.

    (d)  Display your posters side by side.

3.  **"Paradox" poster**. There is a paradox in percent in that two sets of data can show a reduction yet the combined data show an increase. For example, there could be two groups – in the first, most prefer A and in the second most also prefer A; however, when put together, the combined data shows most prefer B.

    (a)  Find this paradox. Describe it.

    (b)  Why is it possible? Represent it on a poster.

# *Test Item Types*

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

## Instructions

### Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

### Information on the statistical inference item types

The statistical inference item types are divided into five subtests, one for each of the five units in this module. The five units are in sequence. Units 1 and 2 cover early inference and inferential reasoning, Unit 3 covers central tendency (e.g. mean, mode, standard deviation, and so on), while Units 4 and 5 cover data distribution and inferential misrepresentation. Thus, Units 3, 4 and 5 are at junior secondary level.

As stated in other modules in Year C, this means that the pre-test should focus on the items in the early subtests, moving up the subtests until students can no longer do the items. It also means that the post-test must cover all the later subtest items. As with many of the Year C modules, Statistical Inference is predominantly at junior secondary level and this may mean that many of the subtests will be difficult for students. However, Statistical Inference is also a module that deals with students' ability to use data sets to make inferences in situations of variation and uncertainty, thus it can only be really tested by an investigation. So the final question in Subtest 5 is set as an investigation (or rich task) and can be graded as follows:
A completely done [4], B reasonably done [3], C borderline [2], D fail but some information [1] and E nothing correct/did not attempt [0].

## Subtest item types

### Subtest 1 items (Unit 1: Early inference)

1.  The class measured John's height with a measuring tape.

    (a)  Would you expect all measures to be the same? Why or why not?

    _____

    _____

    (b)  How different would you expect the measures to be at most? 1 cm? 10 cm? Why?

    _____

    _____

    (c)  Why would different students get different measures?

    _____

    _____

2.  Ten (10) of your classmates got these measures for John's height:

    140 cm    142 cm    144 cm    142 cm    147 cm    141 cm    142 cm    143 cm    141 cm    143 cm

    (a)  Tally the results into a table (the first two rows have been done for you; fill in the empty spaces in the rest of the table).

| Height (cm) | Tally | No. of students with measure |
|:---:|:---|:---:|
| 140 | \| | 1 |
| 141 | \| \| | 2 |
| 142 | | |
| | | |
| | | |
| 147 | | |

(b)    Draw a graph of the frequencies.



(c)    What would you pick as the most likely height? Why? _____

_____

_____

(d)    What happened to the measurer who got 147 cm? _____

_____

## Subtest 2 items (Unit 2: Development of inferential reasoning)

1.  Suppose you have to answer this question: "Do typical Year 8 students have arm lengths that are half their height?"

    (a)  What does "typical" mean? _____

    _____

    (b)  Briefly describe how you would go about answering this question. _____

    _____

    _____

    (c)  How would you gather data? _____

    _____

    (d)  What data would make you answer the question with YES? What data for NO?

    _____

    _____

2.  For the question in 1 above, the following data is collected:

| Person | Height | Arm length |
|--------|--------|------------|
| John   | 135 cm | 68 cm      |
| Mary   | 121 cm | 55 cm      |
| Frank  | 142 cm | 70 cm      |
| Bruce  | 149 cm | 75 cm      |
| Sue    | 118 cm | 52 cm      |
| Tom    | 139 cm | 70 cm      |
| Ben    | 144 cm | 71 cm      |

    (a)  What does this data say about height and arm length? Why? _____

    _____

    (b)  Does this data reflect typical students? _____

    _____

    (c)  What could the data be saying about girls? _____

    _____

## Subtest 3 items (Unit 3: Central tendency)

**Materials**: Calculator, pen and paper

1. Here is data gathered by the boys in the class for their shoe sizes:

| Boys | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | B11 | B12 | B13 | B14 | B15 |
|------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| Shoe size | 4 | 9 | 11 | 6 | 9 | 7 | 7 | 10 | 12 | 8 | 5 | 6 | 7 | 8 | 7 |

Calculate the following (show working):

(a) Mean

(b) Mode

(c) Median

2. (a) If you had to use this data to answer the question, "What is a typical boy's shoe size for this class?", would you use the **mean**, the **mode**, or the **median**? Why?

_____

_____

(b) For what data on shoe size could the **mean** not be good at giving the "middle" for shoe sizes? _____

_____

(c) For what data on shoe size could the **mode** not be good at giving the "middle" for shoe sizes? _____

_____

3. (a) Write a set of 5 boys' shoe sizes so that the **median** and **mean** are higher than the **mode**.

(b) Write a set of 5 boys' shoe sizes so that the **median** is less than the **mean**.

## Subtest 4 items (Unit 4: Data distribution)

**Materials**: Calculator, pen and paper

1.    Here is data gathered by the girls in the class for width of their hand spans:

| Girls | G1 | G2 | G3 | G4 | G5 | G6 | G7 | G8 | G9 | G10 | G11 | G12 | G13 | G14 | G15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Handspan size in cm | 14 | 19 | 18 | 21 | 19 | 17 | 17 | 20 | 15 | 18 | 20 | 20 | 17 | 18 | 17 |

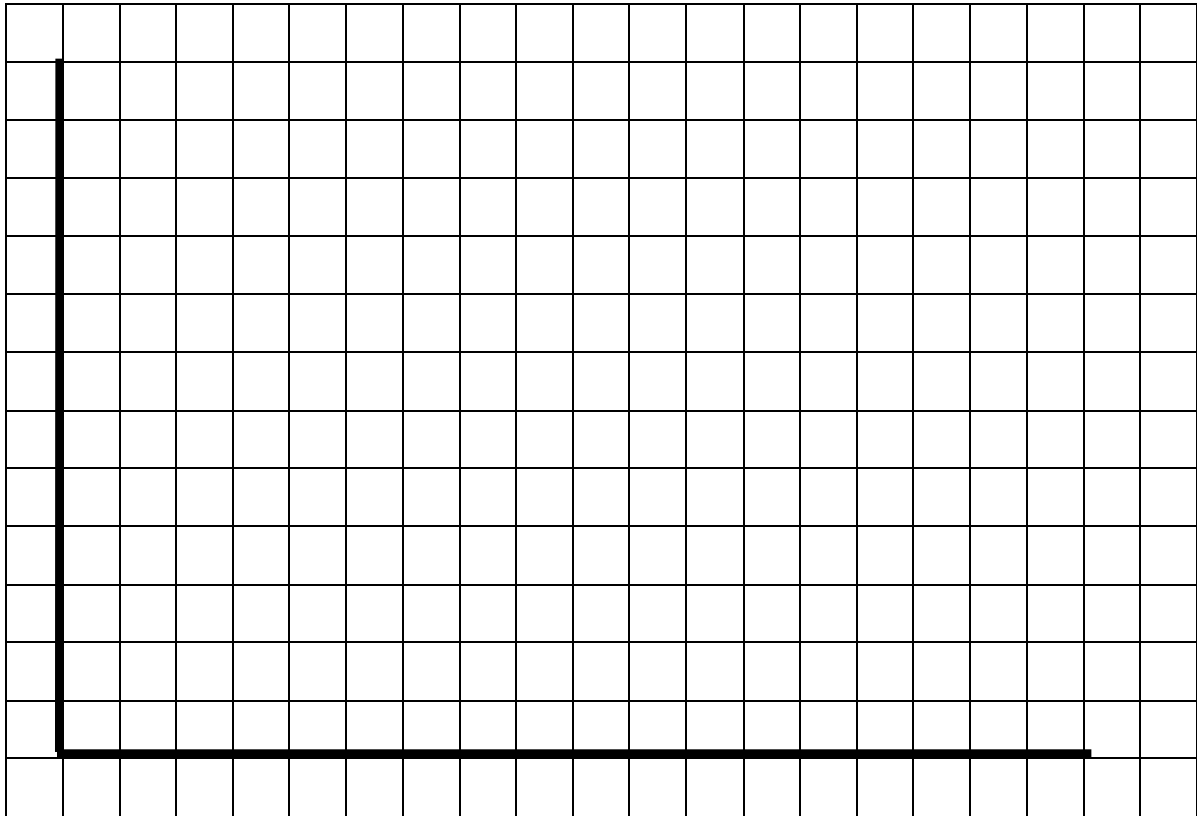Calculate the following (show working):

(a)    Range

(b)    Mean deviation

(c)    Standard deviation

(d)    $1^{st}$ and $3^{rd}$ quartiles

2.    Construct a single box and whisker graph for the girls' data on hand spans.



3.    (a)    What is the difference between calculating deviation by **mean deviation** and
             calculating it by **standard deviation**? _____

      _____

      _____

      (b)    Why is standard deviation often the chosen way to measure deviation?

      _____

      _____

4.    Write a set of 9 girls' hand span measurements so that the standard deviation would be
      much larger than the mean deviation.

## Subtest 5 items (Unit 5: Inferential misrepresentation)

1.  Describe the following ways of misrepresenting statistics.

    (a) Sample bias: _____

    _____

    (b) Missing information: _____

    _____

    (c) Irrelevant statistics: _____

    _____

2.  Joe said, "For the last 3 weeks it has been colder than I can ever remember. There is no such thing as global warming."

    (a) What is wrong with Joe's argument? _____

    _____

    _____

    (b) Can you think of more than one problem with it? _____

    _____

    (c) Does this mean he is right or wrong with his conclusion? _____

    _____

3.  **Investigation**: Gather data for a "both sides" poster as shown below. [See end of Unit 5 for information on this type of poster.]

| Young drivers are bad drivers | Young drivers are good drivers |
|---|---|
| | |

# *Appendix A: Statistics Activity*

**Materials**: Graph paper, calculator, pen and paper, protractor

1.  Here is data gathered by the boys in the class for shoe sizes:

| Boys | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | B11 | B12 | B13 | B14 | B15 |
|------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| Shoe size | 4 | 9 | 11 | 6 | 9 | 7 | 7 | 10 | 12 | 8 | 5 | 6 | 7 | 8 | 7 |

    (a) Draw a frequency table of shoe sizes for the boys.

    (b) Draw a bar graph (columns) for the frequency of the shoe sizes for the boys.

    (c) Draw a cumulative frequency table for shoe sizes for the boys.

    (d) Draw a line graph of cumulative frequency for the boys' shoe sizes.

2.  Here is data gathered by the girls for shoe size, height and favourite colour:
    (R-Red, B-Blue, Y-Yellow, G-Green)

| Girls | G1 | G2 | G3 | G4 | G5 | G6 | G7 | G8 | G9 | G10 | G11 | G12 |
|-------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| Shoe size | 7 | 3 | 5 | 4 | 8 | 5 | 6 | 4 | 7 | 4 | 6 | 5 |
| Arm length cm | 65 | 34 | 38 | 38 | 63 | 38 | 54 | 34 | 57 | 40 | 50 | 42 |
| Favourite colour | R | Y | Y | Y | B | R | G | G | B | Y | R | G |

    (a) Draw a scattergram or scatter plot of the girls' shoe size against arm length.

    (b) Draw a circle graph of the girls' favourite colours.

    (c) Draw a stem and leaf graph of the girls' arm lengths.

3.  Looking at the data in 1 and 2, what inferences can you draw?

# *Appendix B: Cultural Implications of Statistical Inference*

There are four sides to statistics and probability and culture.

First, there are arguments for statistics and probability causing difficulties similar to number for Indigenous cultures. Statistics and probability are inherently linked to understanding of number and fractions. Essentially statistics serve to enumerate data and present it in a form for analysis which may be summarised in measures of central tendency or presented graphically. Probability assigns a numeric value to the likelihood of an event. As such, cultural implications for number can be extended to the mathematics strands of statistics and probability. However, there are also specific cultural implications for each of these domains of mathematics.
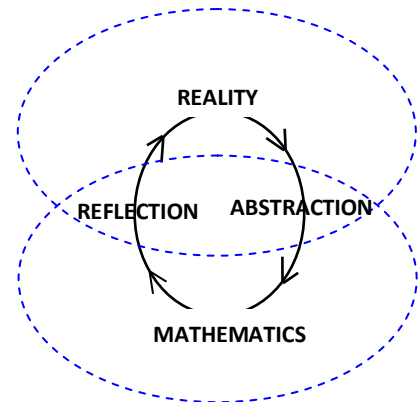
Second, there are arguments that statistics has been used for the oppression of Indigenous cultures. Statistics are used to summarise, describe and represent data for analysis. Data is frequently gathered by research. Students may be exposed to statistics that have been collated and reported that are not representative of their cultural group but instead based on extreme cases that exist in any cultural group. Where any given group has been widely researched, or has experienced the effects of colonisation, such as Aboriginal and Torres Strait Islander groups, statistics may be in existence that paint extremely negative pictures of the group. Where statistics have been reported, represented and disseminated in negative contexts, use of these figures as examples can be detrimental or distressing to these students. While it is important for students to realise and be able to interpret cases of misrepresentation of statistics, this should be done sensitively. Smith (1999) has excellent arguments showing how research and its findings, particularly where statistics is used, have always been detrimental and have assisted the oppression of Indigenous peoples.

Third, probability is linked to the cultural life of Indigenous communities through cards and gambling. Probability describes likelihood of chance events and is frequently linked to gambling. While this makes a very real-world link for some students and can be an area of mathematics where students excel, teachers need to be sensitive to initiatives and feeling within their local community. In some communities cards are outlawed in an attempt to deal with local issues. As a result, activities within these communities should be conducted in such a way that playing cards are not needed and gambling games minimised.

Fourth, in opposition to the first argument in this sub-section, statistics covers thinking, particularly in Investigations, that enables complex situations to be understood. In fact, it focuses on complexity in specific contexts. This is similar to the powerful form of thinking that Indigenous elders have with respect to their land. Therefore, Indigenous students should be strong in statistical reasoning and thinking because it reflects their cultural learning approaches. There is anecdotal information that this was the case when "New Basics" was trialled in Indigenous community schools.

# *Appendix C: RAMR Cycle*

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

---

REALITY
- **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea.
- **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known.
- **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.

---

ABSTRACTION
- **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea.
- **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities.
- **Creativity**: Allow opportunities to create own representations, including language and symbols.

---

MATHEMATICS
- **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea.
- **Practice**: Facilitate students' practice to become familiar with all aspects of the idea.
- **Connections**: Construct activities to connect the idea to other mathematical ideas.

---

REFLECTION
- **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge.
- **Applications/problems**: Set problems that apply the idea back to reality.
- **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

---

# *Appendix D: AIM Scope and Sequence*

| Yr | Term 1 | Term 2 | Term 3 | Term 4 |
|---|---|---|---|---|
| **A** | **N1: Whole Number Numeration**<br>Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system<br><br>**N2: Decimal Number Numeration**<br>Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system | **O1: Addition and Subtraction for Whole Numbers**<br>Concepts; strategies; basic facts; computation; problem solving; extension to algebra<br><br>**M1: Basic Measurement (Length, Mass and Capacity)**<br>Attribute; direct and indirect comparison; non-standard units; standard units; applications | **O2: Multiplication and Division for Whole Numbers**<br>Concepts; strategies; basic facts; computation; problem solving; extension to algebra<br><br>**M2: Relationship Measurement (Perimeter, Area and Volume)**<br>Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae | **G1: Shape (3D, 2D, Line and Angle)**<br>3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches<br><br>**SP1: Tables and Graphs**<br>Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction |
| **B** | **M3: Extension Measurement (Time, Money, Angle and Temperature)**<br>Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae<br><br>**N3: Common Fractions**<br>Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability | **G2: Euclidean Transformations (Flips, Slides and Turns)**<br>Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships<br><br>**O3: Common and Decimal Fraction Operations**<br>Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation | **A1: Equivalence and Equations**<br>Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject<br><br>**N4: Percent, Rate and Ratio**<br>Concepts and models for percent, rate and ratio; proportion; applications, models and problems | **SP2: Probability**<br>Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference<br><br>**G3: Coordinates and Graphing**<br>Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs |
| **C** | **A2: Patterns and Linear Relationships**<br>Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs<br><br>**N5: Directed Number, Indices and Systems**<br>Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems | **A3: Change and Functions**<br>Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio<br><br>**G4: Projective and Topology**<br>Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks | **O4: Arithmetic and Algebra Principles**<br>Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation<br><br>**SP3: Statistical Inference**<br>Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences | **A4: Algebraic Computation**<br>Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics<br><br>**O5: Financial Mathematics**<br>Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities |

*Key:* N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

**YuMiDeadly**

*Growing community
through education*