Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module A2

Year C, Term 1

Algebra:

Patterns and Linear Relationships

Prepared by the YuMi Deadly Centre
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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

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DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

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Module Overview

This is the second module on algebra, Module A2 *Patterns and Linear Relationships*. It follows Module A1 *Equivalence and Equations* and precedes Modules A3 *Change and Functions* and A4 *Algebraic Computation* (however, Module O4 *Arithmetic and Algebra Principles* is in the algebra sequence as much as the operations sequence – see **Appendix B**). This module focuses on patterns and uses them to introduce variable and graphing. There are two pattern types to explore in algebra to promote early algebraic thinking and introduce the notion of variable, namely repeating and growing patterns. Repeating patterns are simplest for introducing activities that engage students in noticing and identifying patterns, whereas growing patterns introduce more complex relationships between terms. There are also two types of growing patterns, linear and nonlinear. We will be focusing on linear growing patterns but there is a unit on nonlinear at the end of the module.

Background information for teaching patterns and linear relationships

This section looks at the various types of patterns (repeating, linear growing and nonlinear growing) and what can be done with them, lists the big ideas associated with patterns, and briefly discusses the cultural implications of algebra teaching (which are positive for Indigenous people).

Repeating patterns

Repeating patterns are linear sequences of objects, pictures or numbers that form a pattern because a section of them repeats; for example:

0 x 0 x 0 x 0 x 0 x 0	repeating part: 0 x
o l l o l l o l l o l l o l	repeating part: o l l

The crucial skill is to be able to go from pattern to repeating part and repeating part to pattern. The major ideas to be developed from repeating patterns are: (a) generalisation, (b) representing generalisations with variables (introduction to algebra), (c) fractions and ratio, and (d) equivalent fractions and ratio (proportion).

The common materials to be used consist of objects of different colour, size, shape, and so on, with any one or more attributes determining the basis of the pattern, plus small numbered cards (1 to 5), tables, and calculators. It is useful to have magnetic copies of these objects/cards that can be placed on whiteboards.

Note: Repeating patterns which are broken into repeating parts can have the parts numbered. This numbering is useful in generalisation, although there is a problem that, for example, two repeats usually means an original and one repeat if we start from 1; it is more sensible to do this than start from 0:

O | |	O | |	O | |	O | |	O | |
1	2	3	4	5

Linear growing patterns

Growing patterns are series of terms where there is a fixed part and a growing part as on right. If the growing amount is always the same, then it is linear. In the pattern on the right, 0 is fixed and X is growing by one each time.

			X
		X	X
	X	X	X
0X	0X	0X	0X

When the growing part does not grow (grows by zero), you have a repeating pattern as on right.

0X 0X 0X 0X

It is possible for the fixed part to not exist (to be zero) as on right.

			x
		x	x
	x	x	x
x	x	x	x

Numbering of growing pattern terms

There is debate as to whether the numbering of linear growing pattern terms should start from 0 or 1. Starting from zero has advantages in that it: (a) makes the connection between patterns and graphs more obvious (the zero term is important as it is the y-intercept of the graph); and (b) relates to real-world situations where there is always a starting measure before the first change is measured. However, when dealing with unnumbered patterns made from materials, it is reasonable and usual to start the numbering from 1.

YDM's answer to this dilemma is to start repeating patterns when they use objects from 1 but to move to starting from 0 when beginning to use numbers in patterns and when translating patterns to graphs. In this module, **starting with 0 begins with Unit 4**.

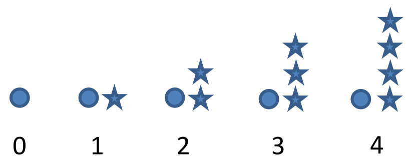
Of course, numbering from 0 or 1 is not that important for ideas but does change the constant part of the pattern rule:

Growing pattern	ox	oxx	oxxx	oxxxx	
Numbering from zero	0	1	2	3	Pattern rule: *n*+2
Numbering from one	1	2	3	4	Pattern rule: *n*+1

In fact, we could number the position of the terms for any number. It is just easier for graphing if we have a zero term.

Pattern rules and graphing

The focus on growing patterns is to identify what is called the *pattern rule* which describes the growth. For patterns like that on the right, there are two types of rules.

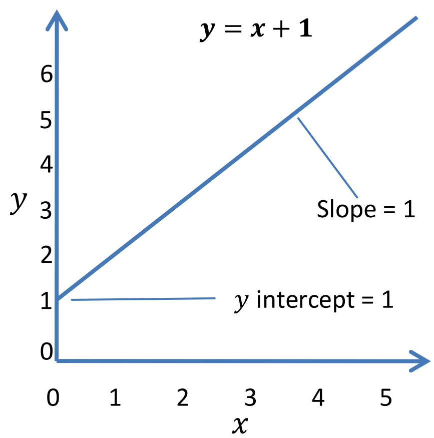


(a) *Sequential*: the nth term is the previous term + 1.

(b) *Position*: the nth term is 1 + n or is n +1 since:

0 term is	1 O and 0 X	(1 + 0) = 1
1 term is	1 O and 1 X	(1 + 1) = 2
2 term is	1 O and 2 X	(1 + 2) = 3
and so on.		

Position rules enable linear growing patterns to be used to introduce the notion of variable. They also can be used to plot graphs as straight lines. When this is done, a relation exists between the graph, the growing part and the fixed part – the growing part is the slope and the fixed part is the y intercept (for $y = x + 1$, slope is 1 and y intercept is 1) as shown on right.



Note: In previous times, sequential pattern rules (e.g. "1 more") were considered to be trivial. With the growth of computers this has changed. In the example above, the position pattern rule gives the function $y = x + 1$ but the sequential pattern rule gives the function $y(1) = 2$, $y(k + 1) = y(k) + 1$. This is now how functions are represented in programming.

Note: For all activities, it is crucial to go from **pattern to pattern rule** and **pattern rule to pattern**.

Activities, materials and major ideas

Students gain a better understanding of patterning if given experience finding number pattern rules without using a table. An example of this is below. (*Note*: Here, we are starting the pattern from 0 so as to accommodate the y-intercept in later years, but see the discussion about this above.)

Example

Consider the following:

	X	XX	XXX	XXXX	XXXXX
X	XX	XXX	XXXX	XXXXX	XXXXXX
0	1	2	3	4	5

The fixed part is the one X from term 0, the growing part is two extra Xs each new term. In a table it is easy to see that term 0 is 1, term 1 is 3, term 2 is 5, term 3 is 7 and so on, leading to a pattern of $2n + 1$ for the nth term. However, if we stay with visuals, then more is possible. The visuals can be interpreted as

- *two rows, the top is n and the bottom is $n + 1$, making the pattern $n + n + 1$;*
- *a double row of length n and an extra X, making the pattern $2n + 1$; and*
- *a double row of length $n + 1$ with a missing X, making the pattern $2(n + 1) - 1$.*

The focus on visuals gives the students an understanding that there are different equivalent algebraic expressions for a pattern rule. The different interpretations of the visuals also provide arguments to support that the pattern rule holds for all items: they provide justification.

The common materials to be used consist of numbers or objects of different colour, size, shape, and so on, with the number of objects (or the numbers themselves) determining the basis of the pattern, plus small numbered cards (1 to 5), tables, and calculators. Again it is useful to have magnetic copies of objects, cards and numbers.

The major ideas to be developed are

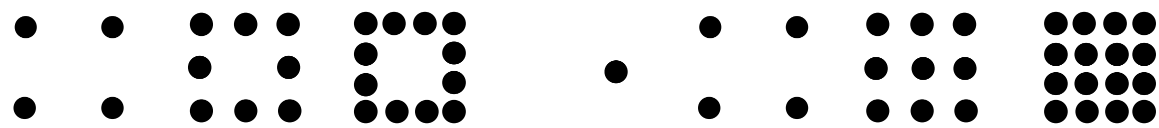
- generalisation and justification of generalisation;
- visual manipulation and numerical tabulation;
- representation of generalisations with variables (introduction to algebra); and
- introduction of line graphs and relation of slope and x-intercept to growing and fixed parts of growing patterns.

As well, the difference between use and non-use of number tables can be identified; that is, number tables make identification of position pattern rule easier but determining the rule from visuals alone enhances students' ability to justify their rule and to find more than one version of the rule (which helps develop equivalence of expressions).

Nonlinear growing patterns and patterns in other strands

Nonlinear growing patterns can be constructed in such a way that they do not grow in a constant manner (i.e. by the same amount each time). For example, the pattern on the left below (open square) grows by 4 each time while the pattern on the right (filled square) grows by increasing amounts:

0 1 2 3 (and so on) 0 1 2 3 (and so on)



The left-hand pattern is 0, 4, 8, 12, 16, and so on (the multiples of 4) which is the rule $4n$ (a linear equation) and the right-hand pattern is 1, 4, 9, 16, 25, and so on (the squares starting at 1^2) which is the rule $(n + 1)^2$ (a quadratic, thus a nonlinear equation).

There is a **relationship** that gives whether the pattern is a quadratic or a cubic and so on. Consider the patterns following:

Pattern A 1, 3, 5, 7, 9, 11, and so on
subtracting consecutive terms gives 2, 2, 2, 2, and so on (all the same)
so the rule has n in it (it is linear).

Pattern B 2, 3, 5, 8, 12, 17, and so on
subtracting consecutive terms gives 1, 2, 3, 4, 5, and so on
subtracting consecutive terms again gives 1, 1, 1, 1, and so on (all the same)
so the rule has n^2 in it (it is a quadratic and nonlinear).

Pattern C 2, 5, 10, 18, 30, and so on
subtracting consecutive terms gives 3, 5, 8, 12, and so on
subtracting consecutive terms again gives 2, 3, 4, 5, and so on
subtracting consecutive terms again gives 1, 1, 1, 1, and so on (all the same)
so the rule has n^3 in it (it is a cubic and nonlinear).

Note: If we had to go through the subtraction four times, we would have n^4 in the rule, and so on.

Patterns in other mathematics strands can be used to obtain understanding of ideas and recall of facts in these strands. For example, the order of place-value positions, the relationship between adjacent place-value positions, counting patterns and the odometer principle, multiplication basic facts, higher decade facts (3+4=7 → 30+40=70) and repeated addition are all examples of understandings and facts that can be obtained by seeing patterns. These patterns are considered in modules associated with number (N) and operations (O).

Big ideas for patterns

The major **big mathematics ideas** that patterns will enable to be taught are:

(a) *the act of generalisation* (i.e. skill in generalising – being able to develop a rule that holds for all numbers) and justification of generalisation;

(b) *visual manipulation and numerical tabulation* – number tables make identification of position pattern rule easier but determining the rule from visuals alone enhances students' ability to justify their rule and to find more than one version of the rule (which helps develop equivalence of expressions);

(c) *the meaning of variable* (i.e. being able to state a rule with a letter, e.g. the nth position has 2n+1 objects);

(d) *equivalence* (when two things are equal such as $^2/_3 = {}^4/_6$) – for fractions and ratio; and

(e) *linear graphs* and relation between slope and y intercept and fixed and growing parts.

There are also **big teaching ideas** for patterns as follows:

(a) *unnumbered → numbered* – start with unnumbered activities before moving onto numbered activities – unnumbered activities enable big ideas to be more easily seen;

(b) *development* – one attribute → two or more attributes, one operation → more than one operation, and addition and subtraction → multiplication and division → all four operations; and

(c) *exploration/reality* – act out real-world situations, enquire, discuss and allow students to come up with their rules – that is, think of real situations in which the patterns could exist (even if only evaluations).

Cultural implications

Traditionally, in Queensland schools, mathematics and its teaching both reflect Western culture. Therefore, differences in mathematics performance can stem from a different cultural view of what it means to be good at mathematics. Commonly, in most school environments, this is determined by gauging students' performance levels from test items that reflect non-Indigenous learning styles, namely solving meaningless problems by pen-and-paper means. In those problems, there are often marked differences in errors between Indigenous and non-Indigenous students. One case study of Indigenous students' errors found that underperformance tended to reflect mistakes in procedures rather than understanding (reflecting the position of Grant (1998) that Indigenous students see the whole rather than the parts).

Therefore, it is important to teach mathematics on an equitable basis with Western mathematics reflecting **"both ways"** approaches (Ezeife, 2002). Western teaching is traditionally compartmentalised, resulting in an education system in schools (whether oral or written) focusing on the details of the individual parts rather than the whole and relationships within the whole. By contrast, Indigenous students tend to be holistic learners, appreciating overviews of subjects and conscious linking of ideas (Grant, 1998). In fact, Indigenous people have been characterised as belonging to "high-context culture groups" (Ezeife, 2002) which are characterised by: a holistic (top-down) approach to information processing in which meaning is "extracted" from the environment and the situation. Low-context cultures use a linear, sequential building block (bottom-up) approach to information processing in which meaning is constructed (Ezeife, 2002).

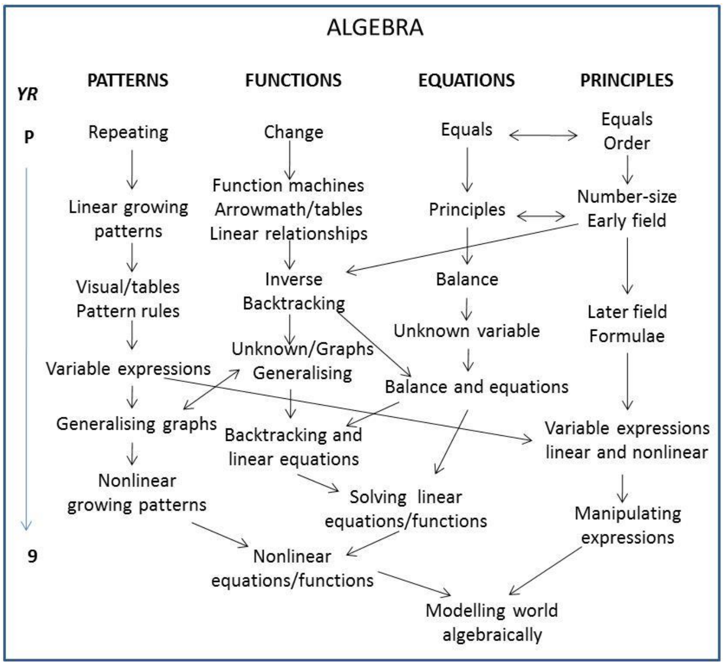
What this means is that students who use **holistic thought-processing** are more likely to be disadvantaged in mainstream mathematics classrooms. This is because Westernised mathematics is largely presented as hierarchical and broken into parts with minimal connections made between concepts and with the children's culture and community. It potentially conflicts with how they learn. If this is to change, curriculum and assessment need to be made more culturally sensitive and community orientated.

Thus, we have a **confluence of results**. Indigenous students are high context and learn best with holistic teaching (Grant, 1998; Ezeife, 2002). Mathematics in its most powerful form is based on structural understanding that is learnt best by holistic teaching. Algebra is the component of mathematics that is based on mathematical structure, and is capable of presenting mathematics holistically.

As a consequence, it seems that algebra is the form of mathematics that is most in harmony with Indigenous culture and learning style. Because of this, **algebra understanding should be a strength of Indigenous students** if it is taught through pattern and structure (rather than through sequential teaching of rules and algorithms). It seems likely that algebra is a subject in which Aboriginal and Torres Strait Islander students should excel. Finally, because of its relationship with arithmetic, this understanding of algebra should enable enhanced understanding of and proficiency with arithmetic. As well as this, the RAMR framework itself is a product of an Indigenous approach to teaching mathematics.

Sequencing for patterns and linear relationships

This section briefly looks at the role of sequencing in algebra and in this particular module.



Sequencing in algebra

The overall sequence for algebra is given in the figure on right. It relates four components – patterns (patterns and linear relationships), functions (change and functions), equations (equivalence and equations), and principles (arithmetic and algebra principles). In AIM, these four components are covered in Modules A1, A2, A3, O4, and A4 (see **Appendix B**).

The sequence begins with **patterns** as training in the act of generalisation by finding pattern rules and relating to graphs. It then moves on to **functions**, starting from change rules in transformations, using real situations, tables and arrowmath notation before equations and graphs, solving for unknowns by the use of the balance rule. After this it moves to relationships that in arithmetic and algebra are represented predominantly by **equations**, solving them by the use of the balance rule. The sequence is completed by focusing on arithmetic and algebraic **principles** and extending these to methods such as substitution, expansion and factorisation (algebraic computation).

There are three important aspects of sequencing for algebra because of its generalised nature and reliance on big ideas, particularly principle big ideas. These principles hold generally across mathematics in the building of big ideas.

1. **Unnumbered work before numbered**. Within each module, the sequencing begins with unnumbered activities, as these enable the big ideas to develop, moves on to numbers and arithmetic situations and then moves to generalised situations. YDM follows the view of the Russian mathematics educator Davydov that to build big ideas like the **balance rule** requires **initially working in unnumbered situations** as numbers tend to result in students looking for answers not generalisations of concepts, processes (relationships/changes) and strategies.
2. **Processes not answers**. The basis of algebra is things that hold for all numbers not particular answers. For 2+3 the processing is 2+3 (i.e. joining 2 things and 3 things) which gives answer 5. For x+3, the process and the answer are the same, that is, x+3. Thus, algebra has to be built around big ideas not computation. It should be noted that this has a consequence, that arithmetic does not teach two-operation processes well because the students simply do each process as it happens, for example, 2×5+3 becomes 2×5=10 and 10+3=13, two single steps not one double step. This means that time needs to be spent on teaching the processes of two-step operations (e.g. the inverse of 2×5+3 is −3 and ÷2 not the other way around).
3. **Separation to integration**. The sequence begins simply, in a separated manner, but by the time junior secondary is reached, the components are more integrated and connected to allow patterns, functions and equivalence all to be expressed in the same way (by equations), and for results to cover nonlinear as well as linear relationships and changes.
4. **Modelling as end point**. The overall end point of algebra is modelling as well as manipulation of symbolics. Computers and special calculators can do the manipulations to simplify and solve for unknowns – what is important, like in arithmetic, is to apply the knowledge to the world and solve problems – to model the world algebraically. This is important because most students cannot see the relevance of, say, $x + y = 7$ to their everyday world. Yet, with understanding it is very relevant. It could mean that you bought two things at a shop for $7. Then the cost of the first thing (x) plus the cost of the second thing (y) is equal to $7. This gives parameters in which thinking can be used. Suppose we were working in whole dollars. Then the first thing could cost $1 and the second cost $6, or $2 and $5, or $3 and $4, and so on.

Sequencing in this module

The sequence for this module begins with repeating patterns and then shows how these can be extended to growing patterns. We move on to linear growing patterns where we focus on pattern rules (sequential and position) and on the techniques for identifying generalisation (using visuals and tables). After this, linear growing patterns are used to introduce variable, equations and graphs, with particular emphasis on the relationship between growing part and constant part and coefficient of variable/slope and constant term/y-intercept. However, as we also show, repeating patterns can be used to introduce variable as well as equivalent fractions and ratio. Finally, there is some time spent on nonlinear patterns. Sequencing details for repeating and linear growing patterns are given below, followed by the module structure.

Repeating patterns. The normal sequence of activities is as follows:

(a) copying, continuing and completing repeating patterns and identifying repeating part, constructing repeating patterns (without direction and when given a repeating part);

(b) finding what object is at a position or finding positions for objects (e.g. what object is the 13th term? what terms are square red counters?);

(c) breaking pattern into repeats and connecting to growing patterns, for example:
0 x 0 x 0 x 0 x … → 0x 0x 0x 0x … → 0x 0xx 0xxx 0xxxx … ; and

(d) representing repeats on tables and generalising tabled numbers to variables, and using tables of repeats to introduce fractions, equivalent fractions, ratio and equivalent ratio (proportion).

Linear growing patterns. The normal sequence of activities is as follows:

(a) copying, continuing and completing growing patterns, and constructing growing patterns (without direction and when given a pattern rule);

(b) finding what objects are at a position and what position has certain objects (e.g. what term has the 20th red circle?);

(c) identifying growing and fixed parts of visual patterns, using this to identify pattern rules (sequential and position) without use of number tables, and different versions of pattern rules (leads to equivalence of expressions – number sentences with no equals), and justifying why it works for all terms;

(d) identifying growing and fixed parts and pattern rule from number tables, and using pattern rules to introduce variable and algebraic expression; and

(e) representing patterns with graphs and relating growing and fixed parts to slope and y-intercept of graph respectively.

Module structure. This module is divided into the following sections and units.

Overview: Background information, sequencing and relation to Australian Curriculum

Unit 1: Early repeating pattern activities – repeats and rules for position

Unit 2: Initial linear growing patterns activities – changing repeating to linear growing patterns and exploring sequential and position pattern rules through growing and fixed parts

Unit 3: Visuals and tables – comparing tabular and visual methods for growing patterns and tables with repeating patterns

Unit 4: Building algebraic generalisations – introducing generalisations, variable and line graphs from linear growing patterns, introducing 0 as first term, and building generalisation from repeating patterns

Unit 5: Extensions – using repeating patterns for equivalence and relating linear growing pattern characteristics (growing and fixed parts) to slope and y-intercept, and line graph functions

Unit 6: Nonlinear patterning activities – generalisation and variable and relating fixed and growing parts to functions (quadratics, cubics and so on)

Test item types: Test items associated with the six units above which can be used for pre- and post-tests

Appendix A: RAMR cycle components and description

Appendix B: AIM scope and sequence showing all modules by year level and term.

The teaching information in the units is a combination of lists of activities and RAMR cycle ideas. Each unit is divided into one or more subsections and the stages of RAMR (see **Appendix A**) are used as subheadings within each unit. However, not all stages are shown – there tends to be a focus on reality and abstraction.

The last two units will be a challenge to the students and so only central ideas are included in testing.

Relation to Australian Curriculum: Mathematics

AIM A2 meets the Australian Curriculum: Mathematics (Foundation to Year 10)

Unit 1: Early repeating pattern activities
Unit 2: Initial linear growing pattern activities
Unit 3: Visuals and tables
Unit 4: Building algebraic generalisations
Unit 5: Extensions
Unit 6: Nonlinear patterning activities

Content Description	Year	A2 Unit					
		1	2	3	4	5	6
Sort and classify familiar objects and explain the basis for these classifications. Copy, continue and create patterns with objects and drawings (ACMNA005)	F	✓	✓				
Investigate and describe number patterns formed by skip counting and patterns with objects (ACMNA018)	1	✓	✓				
Describe patterns with numbers and identify missing elements (ACMNA035)	2	✓	✓				
Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)	4		✓	✓			
Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107)	5		✓	✓	✓		
Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133)	6		✓	✓	✓	✓	
Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)	7		✓	✓	✓	✓	
Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)				✓	✓	✓	
Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)					✓	✓	✓
Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)	8				✓	✓	✓
Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)	9						✓
Solve problems involving linear equations, including those derived from formulas (ACMNA235)	10						✓

Unit 1: Early Repeating Pattern Activities

Patterns are an excellent way to teach students the act of generalisation and to introduce variables, algebraic expressions, and graphs. There are two types of patterns, repeating and growing; and three types of pattern rule to identify – the repeating part for repeating patterns, and the sequential and position rules for growing patterns. Materials used are objects with different attributes (e.g. size, shape, colour) and cards with numbers 1, 2, 3, 4 and 5 on them; and a teacher set of these materials magnetised for placing on magnetic whiteboard. This unit looks at early repeating pattern activities and determining the repeating part.

1.1 Copy, continue and create repeating patterns (unnumbered)

The steps here are as follows. Start with real things (e.g. real songs/dances).

1. Use movements and sound (e.g. body movements, clapping, music and dance steps) to follow patterns of activity.
2. Extend these activities to where students have to copy and continue a pattern that repeats.
3. Encourage students to create their own patterns of movements and sound and lead others in copying their patterns.

Abstraction

Body

Use movements and sound to follow patterns of activity. Students can copy patterns, and then begin to continue them. Encourage the students to find the repeating part within a pattern. Students often identify only part of the repeating part (e.g. X X X not O X X X), so language must direct students to find all of the repeating part.

Activity: Feel the Pattern

Students create a pattern using their bodies. This could be sitting and standing, or clapping and clicking, or using music and dance.

1. Begin with a pattern: sit, stand, sit, stand, and so on. Ask students:

   What is another way we can show this same pattern? Clap, click, clap, click, ...
2. Ask students to continue the pattern (e.g. If the last student is sitting, what do you think comes next? What about if the last student was standing?).
3. Teacher begins a pattern (e.g. sit, stand, sit, stand, ...) and asks students if they can identify the part that repeats. Allow them to show teacher using actions and words.
4. Next allow the students to complete a pattern, that is to fill in the empty spaces (e.g. what action will fill the gap in sit, stand, stand, sit, stand, stand, sit, ____, stand, ______ ...).
5. Encourage the students to create their own patterns of movement and sound, then identify the repeating part. Here students sometimes construct a symmetric design which does not go linearly on forever (e.g. X X X O O X X O X X O O X X X) not a continuing repeating pattern (e.g. X X X O O X X X O O ...).
6. Reverse the idea and give the students a repeating part and ask them to create the pattern such as O O X X X (e.g. O O X X X O O X X X O O X X X ...).

Variation: Use a repeat where the part starts and finishes with the same object (e.g. X O O X); introduce a third object (e.g. X O R X O R...). Younger students may find it easier to work with repeating patterns if the objects are very different. This often means two attributes different – both colour and shape (e.g. red O, blue X).

Hand

Translate the pattern used within the body activity into a hand activity using resources such as counters, blocks, pictures, etc.

Activity: Make the Pattern

Materials: Concrete manipulatives.

1. Using a variety of concrete manipulatives allow students to create a repeating pattern and identify the repeating part.
2. Give students a pattern and allow them to translate it into their concrete manipulative. Ask students to continue the pattern.
3. Give students an incomplete pattern and allow them to create the complete pattern. Ask the students to create an incomplete pattern and ask a friend to complete it.
4. Give the students a repeating part and ask for it to be repeated three times.

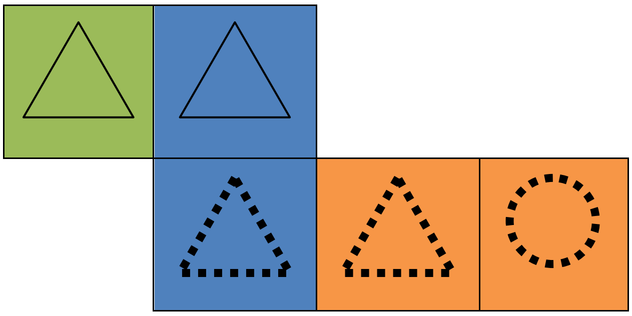
Variation: Add a third object to repeat, have only one attribute different (e.g. red spot and blue spot, not red spot and blue square).

The following activity gives students practice at changing only one attribute.

Activity: Only Change One

Materials: Only Change One cards.

1. Students are dealt out 7 cards each; one card is placed right side up in the centre and the remainder are placed right side down forming a pick-up pile.
2. Moving in a clockwise direction students place one card. To play a card, it must be placed next to a card with one attribute different (background colour, shape or outline) as below:



3. If a student cannot play they pick up from the pile. The first student to use all their cards is the winner.

The commercial card game "Blink" is a reverse of this activity: all attributes but one can be changed.

Mind

Have students think about a pattern with only one attribute changing. Ask students to find the repeating part. Share these with the class. Now ask the students to think of a pattern with more than one attribute. Ask students to find the repeating part.

1.2 Determining what object is in a position in a repeating pattern (numbered)

The following activities allow students to represent the positions of objects within patterns as numbers.

Activity: Where in the Pattern?

1. The teacher begins by providing a repeating pattern (e.g. X O O X O O ...), then questions students*: How can we determine the position of one particular object? For example I want to talk about this one X O* ***O*** *X O O.* Form the idea of numbering the objects as below:

X	O	O	X	O	O	X	O	...
1	2	3	4	5	6	7	8	...

2. The teacher then asks the students to identify the object (X or O) that is in a particular position, e.g. the 8th position as in the example above. The sequences involved in this are:
   (a) initially allow the students to draw extra objects until the position is reached before asking them to work it out in their head;
   (b) initially give the students positions to find that are 5 or less ahead of the last item placed before going to positions further out (e.g. finding the 10th position is easier than the 13th position); and
   (c) initially allow students to copy the pattern when they are asked to determine the object in positions before asking them to find a position in a pattern that the teacher has put out.

Variation: Give students an object and ask them to find the positional number. Can one object have more than one positional number?

Techniques students may use

1. It seems that determining what object is in a position requires students to coordinate two things in their minds – the pattern and the position number, or, more difficult, synchronise these two things as moving, in their mind, along the pattern of objects and along the number for the position of the objects.

2. It also requires the students to identify the whole repeat and recognise its components (e.g. two Xs and one O). This is easier to do if the students can put out the objects as they go, the number for the position past the last object placed is within the students' subitisation range (normally less than or equal to 5), or the students have familiarised themselves with the pattern by placing it out themselves.

3. Sometimes, the students use a wrapping technique as below – here the students appear to be using multiples of 6 not 3 and seeing 27 as 3 more than 24 and counting halfway along the 6 objects.

   X O O X O O
   X O O X O O
   _ _ _ _ _ _

4. As students gain skip counting proficiency or improved understanding of multiplication, they can use the pattern to find the object and so larger numbers can be set as below, e.g. find the 27th term. The students can determine that, for example, the 27th term is an O because they see that X O O ... is a pattern of three and 27 as a multiple of three must be the last object in the pattern of three (i.e. the students can jump in threes). As a beginning to this, students start to manipulate the objects emphasising features as below – they start to think of the pattern in terms of its last object.

X	O	O	X	O	O	_	_	…	_	_	O	_	_
1	2	3	4	5	6						27		

↓

X	O	O	X	O	O	_	_	…	_	_	O	_	_
		3			6						27		

Because skip counting by five and the five times tables are more familiar to students, then five-object patterns are easier than three- or four-object patterns, e.g. students find that a pattern like O O X X X ... is easier than a pattern like O X X X ... .

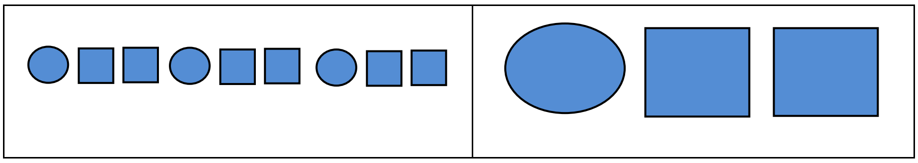
5. Finally, students seem to find patterns like O O X ... easier to find objects in positions than patterns like X O O ... because the students can tag the third or repeat ending object.

The following activity allows the students to practise their understanding of repeating patterns and identifying the repeating part.

Activity: Match It!

Materials: Match It cards.

1. Within the set of cards there are pattern cards and there are repeating part cards. These cards are all shuffled together to form one deck.
2. Each player is dealt 5 cards.
3. The aim is to match the pattern card with the repeating part card as below:



4. The play is similar to that of go fish. In turn, a player can ask another player if they have a certain card. If the asked player has that card it must be given. The first player continues to ask until the asked player does not have the card. The player then picks up from the deck and it is the next player's turn.
5. When a player matches their card with another they place their cards on the table in front of them. Other players must confirm the match is correct.

Reflection

Validation

Have students think about a repeating pattern that they see in reality. Think about what part repeats and identify it. Think about how many times it repeats within reality. Is it continuous? Does it have an end?

Application

Does this work if the pattern has more than three objects? If given a part of the pattern (e.g. from object 4 to 9) can you find the pattern?

Extension

What is the pattern if each object has numerous attributes? Can there be two patterns within one series of objects (e.g. by colour and by shape)?

Unit 2: *Initial Linear Growing Pattern Activities*

This unit looks at early growing pattern activities and determining the sequential and position pattern rules – with emphasis on the position rule. It covers how to sequence seamlessly from repeating to growing patterns.

2.1 Moving from repeating to linear growing patterns (unnumbered)

Reality

Where are growing patterns used in everyday life? Question the students giving examples such as: *When a seedling was planted it was 3 cm tall. Every day it grew 2 cm taller. How tall was it at the end of day 7?*

Abstraction

Body. The following activity allows the students to use their bodies to create a pattern.

Activity: Let's Grow!

Materials: A3 digit cards.

1. Use body movements to create a repeating pattern (e.g. stamp, stamp, clap, stamp, stamp, clap, ...). Teacher questions the students to show different aspects of the pattern: *Translate the pattern into a position (e.g. sit, sit, stand, sit, sit, stand, ...). What is the repeating part?*
2. Ask half the students to form the pattern around the room. Break the pattern into repeats by grouping the students in each repeat.

				O	O	O
XOO	XOO	XOO	OR	XO	XO	XO

3. Ask some of the remaining students to be the label for each repeat, holding a digit card. At this point there should be four students within each group: three as the pattern and one as the label. Introduce the word "term" as the position of the repeat.

				O	O	O
XOO	XOO	XOO	OR	XO	XO	XO

XOO	XOO	XOO	...and so on
1	2	3	

4. Tell the students that the pattern is going to grow; choose which object will grow (i.e. stand or sit). Apply this to the pattern with the remaining students.

						O
		X			O	O
	X	X		O	O	O
XOO	XOO	XOO	OR	XO	XO	XO
1	2	3		1	2	3

Variations: Grow the other object, use three objects instead of two, grow both objects, grow both objects at different rates (as shown below), change the way the objects are presented and so on.

		X
		XO
	XO	XO
O	XO	XO
XO	XO	XO
1	2	3

This presents repeating patterns as a precursor to growing patterns and links the two together.

2.2 Copying, continuing, completing and constructing linear growing patterns (unnumbered)

Abstraction

Hand

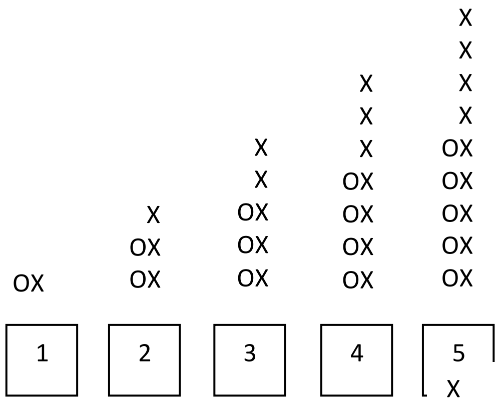
Once growing patterns are introduced, students can be asked to copy, continue, and complete growing patterns set up by the teacher. This can be done through the use of concrete manipulatives.

Repeat the "Let's Grow" activity using concrete manipulatives and A7 digit cards. This way each student is able to create their own pattern and explain how it is growing. Ensure students build patterns using a variety of attributes (e.g. colour, size, shape).

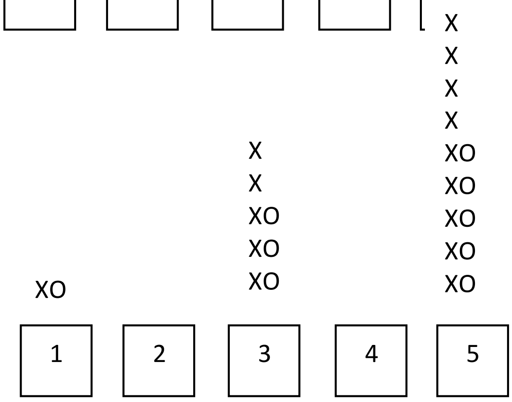
Activity: Growing Patterns

Materials: Concrete manipulative, term cards.

1. Teacher shows a growing pattern, see right. [This example has X increasing by two and O increasing by one in each new term.]
2. Students make their own copy of this pattern using a concrete manipulative.
3. Students extend the pattern finding the next few terms (e.g. this could involve making the 6th and 7th terms in the pattern on right). Then use term cards to place under the concrete resource.



4. Ask the students to make some further terms (or tell what is involved in these terms) such as the 10th or 20th term. This requires some understanding of the pattern rule but can be completed by considering what happens term by term.
5. Ask the students to make the first five terms of their own growing pattern. Ask them to explain what is involved in the patterns and how the terms are growing.
6. The sixth step is to have students complete a pattern where there are gaps in the example given, as on right. It should be noted that it is more difficult to complete than continue a pattern. It is also harder if the terms given are not regular, e.g. when you are given the 1st, 2nd and 5th terms.



Mind

Students watch as the teacher creates a pattern using actions, concrete manipulatives, images, etc. Students each mentally figure out the number of terms and the positional rule (i.e. what happens to make it grow) that the teacher has shown the students.

2.3 Determining growing and fixed parts and finding/using linear pattern rules (unnumbered)

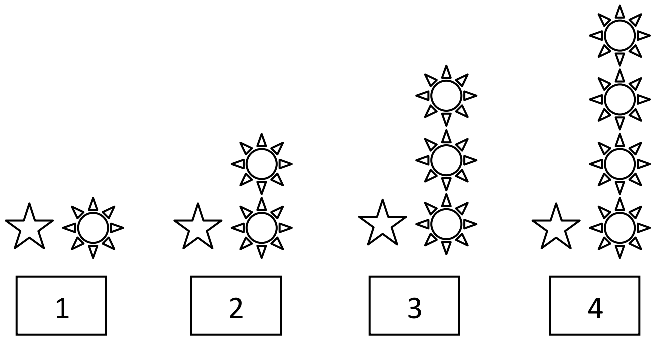
The important part of growing patterns is to identify the general rule for the pattern that enables any term to be determined; this is called the pattern rule. Determining pattern rules is assisted by identifying in the pattern what grows and what stays fixed – the growing part and the fixed part rule.

Activity: What Grows and What Doesn't?

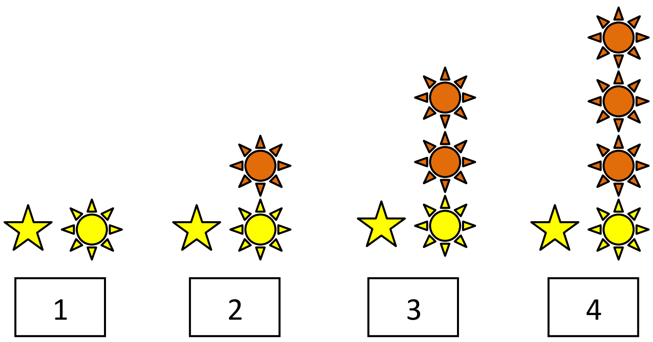
1. Teacher begins with a simple growing pattern question:

   Two people are on a train (a man and a woman); at each stop an extra man gets on, how many women and men?
   What part is there every time? This is the fixed part.
   What part changes every time? This is the growing part.

2. Teacher shows students a pattern, for example:



3. Students and teacher identify the part that is fixed for every term and shade it a colour. Then determine the part that grows and shade it a different colour.



4. Use the above information to work out what each term will look like (e.g. the 10th term is 1 star and 10 suns, the 20th term is 1 star and 20 suns).

5. Allow students to create their own growing pattern and identify the fixed and growing parts.

Using pattern rules

Pattern rules are identified to allow the onlooker to determine what is in any term. There are two types of pattern rule – the *sequential* pattern rule which gives the difference between sequential terms, and the *position* rule which relates number of objects to the term position.

Activity: Determining the Pattern Rules

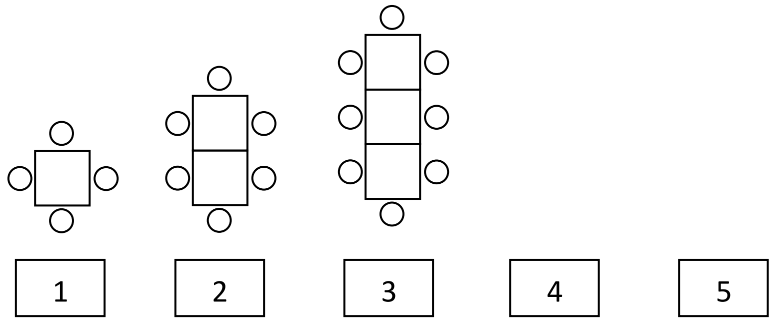
1. Teacher introduces the two rules: positional and sequential and relates them to the above pattern:

   The sequential rule for the pattern above is "add one sun", and the position rule is 1 + position number.

   We should accept messy language at this stage; later we can give the rule in terms of n.

2. Apply this using examples of real-world problems:

   A table in a restaurant sits 4, two tables pushed together sit 6, and so on. How many will 15 tables pushed together into a row sit?



It is important to reverse the process and find the position of the term with a given number of objects. For example, in the "What grows and what doesn't" pattern above, what term has 62 objects? In the second (real-world) pattern, how many tables in a row to sit 18 people?

Reflection

Validation

Review the reality to ensure students can answer the question:

When a seedling was planted it was 3 cm tall. Every day it grew 2 cm taller. How tall was it at the end of day 7?

Application

Ask students to create their own real-world problems, and then give them to a friend to solve.

Extension

Is there a way to find a generalisation with number for each of the patterns? (e.g. $n + 1$)

Unit 3: Visuals and Tables

There is a debate about whether to solve patterns from visuals or tables. For some students, tables can be quicker but visuals supply more information, explain why the rule holds for all terms and enable more than one pattern to be found (although all patterns are equivalent). This unit looks at both methods – covering repeating patterns and applications as well.

3.1 Using tables in growing linear patterns (unnumbered)

Reality

Revisit the reality used in Unit 2: *When a seedling was planted it was 3 cm tall. Every day it grew 2 cm taller. How tall was it at the end of day 7?* Is there another way of displaying this information?

Introduce the idea of using a table to display the information as below:

Day	Height in cm
1	3
2	5
3	7
4	9
and so on	

Abstraction

Body. The following activity allows the students to see initial number growing patterns.

Activity: Make a Growing Pattern

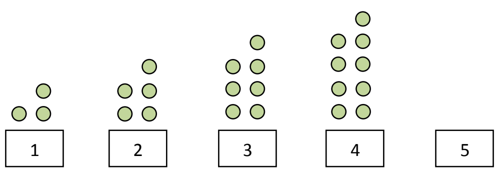
1. Ask the students to create a growing pattern. Begin with asking for two objects. Have students give physical examples of what you could do.
2. Ask the students to think of a fixed part and create four terms.
3. Ask students to add a growing part for the four terms created.
4. What is the sequential rule and what is the positional rule?
5. Term by term, ask students to count the number of objects within each term. Can you see a pattern with the numbers?

Hand. The following activity is an example of how pattern rules can be found within unnumbered growing patterns.

Activity: Pattern Rules

Materials: 20 counters per student, term cards per student.

1. Students copy, using counters and term cards, a pattern that teacher displays, for example:



2. Look for the sequential rule. This does not change regardless of different representation. Here the sequential rule is "add 2".

   To assist with generation of rules from visual cues, it is important to teach students to visualise objects in different ways. Consider the pattern above: this could be viewed in three ways:

   - **Person A** sees one double tower and an extra (for position 3, a double tower of height 3 with an extra object);
   - **Person B** sees two single towers (for position 3, a tower of height 3 and a tower of height 4); and
   - **Person C** sees a double tower with one missing (for position 3, a double tower of height 4 with one object missing).

3. Find the positional rule. Each of the above visual interpretations leads to a different but equivalent position rule:

   - Person A leads to the position pattern rule of "2 times the position plus 1";
   - Person B leads to the position pattern rule of "position plus position plus 1";
   - Position C leads to the position pattern rule of "2 times (position plus 1) minus 1".

4. Ask students to justify their visual interpretation and their rule.

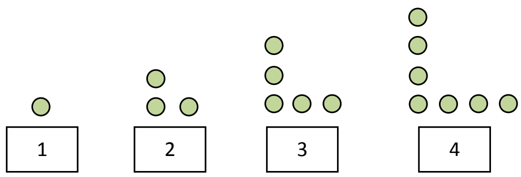
Variation: Ask students to find the number of objects for any position using n to give the rule in algebraic form (e.g. Person A: $2n + 1$, Person B: $n + n + 1$, Person C: $2(n + 1) - 1$); reverse the process working from the rule to the pattern.

3.2 Using tables in growing patterns (numbered)

The following activity is one way students could use tables to represent a numbered growing pattern.

Activity: Building a Table

1. Teacher provides a growing pattern as on right:



2. Students construct and complete a table as below:

No. of term	No. of objects
1	1
2	3
3	5
4	7

3. Students find the sequential pattern rule in the visual and in the table. In the visual students find what is constant and what changes (e.g. the first circle is the constant and the change is the extra two circles added at each new term). In the table this is done by looking down the right-hand side column of the table.

4. Students then find the positional rule; this is done by looking across the table. The position rule is found by checking if there is some multiplication and/or addition/subtraction that works for all numbers. For example, *1×2 = 2 which is 1 more than 1, 2×2 = 4 which is 1 more than 3, so try this on next (yes, 3×2−1 = 5, 4×2−1 = 7 and so on). Thus, the position rule is that, for a number like 256, the number of objects is 2×156−1, and in everyday language, as a generalisation, it is "2 × term number − 1".*

   It is a good idea to ask the students what it would be for any position number n but not expect all to get the answer as an algebraic expression.

5. Reverse the direction by giving a positional rule such as "3 times the position plus 2" and ask students to construct a pattern for this rule. This can be done for a sequential rule as well.

The table is simpler to use to find the position rule. However, the visual method gives the reason for the rule and gives more than one rule.

In discovering and representing the pattern rule, students go through the steps below, giving the rule:

(a) for small numbers;

(b) for large numbers (called quasi-generalisation);

(c) in language; and

(d) in algebra using letters.

3.3 Tables in repeating patterns

It is also possible to use repeating patterns to develop generalisation. To do this, a repeating pattern is displayed, the students are asked to break it into repeats, and then complete a table for these repeats from which generalisations can be found. An example of this is below.

Pattern: XXOXXOXXO …

Repeats: XXO XXO XXO

Table: as on right

No. of repeats	No. of X's	No. of O's	Total
1	2	1	3
2	4	2	6
3	6	3	9

Generalisations: Look for "patterns" or "rules" **down and across** the table. The down generalisations include: repeats go up by 1, X's go up by 2, O's go up by 1, and total goes up by 3. The across generalisations include: number of X's is twice the number of repeats, number of O's is equal to the number of repeats, and total is three times the number of repeats.

3.4 Patterning applications

YDM is based on relating mathematics to reality – in particular, the RAMR cycle argues that mathematics should come from reality (through abstraction) and return to reality (through reflection). In this module, we have focused on the sequence in building algebraic ideas. This is particularly so with patterns where we have given emphasis to how we build from repeating to growing patterns to finding the pattern rule with little focus on real-life examples. This small subsection is to provide balance and to discuss how we can ensure that we start and end with reality.

Repeating pattern examples

The major sources of reality for repeating patterns are as follows.

1. **The built environment**. Many components of the built environment will repeat; this could be as simple as window, window, balcony repeated, or window, window, door, window, garage, repeated. Paving is often a repeating pattern as is tiling – colours and shapes. Garden beds, trees and lawn can follow repeating patterns – particularly with respect to edges.

2. **Art and design**. Many forms of art repeat – for example, weaving can produce a colour pattern for a scarf, knitting and crocheting can follow complex repeating patterns of different stitches. All fabric designs are repeats as are many of the artistic designs on buildings (e.g. frieze patterns). This area is a real opportunity to involve different cultures and their art and design.

3. **Dance and music**. Drumming and clapping rhythms are repeated patterns as are rhythms using double basses and guitars. Playing music is a great way to introduce repeating patterns. YDC has developed with a music group called JAM (Join Australian Music) a set of mathematics lessons that use repeating patterns to

enable students to develop mathematics knowledge based on drumming and clapping rhythms, pitch, loudness, and notes/beat (for further information, contact ydc@qut.edu.au).

4. **Poetry**. Some poetry and song lyrics follow repeating patterns (e.g. limericks). The mathematics of the repeating patterns can be introduced as a way to analyse the writing.

Growing pattern examples

It is harder to find good reality examples for growing patterns. However, some examples are as follows.

1. **Growing shapes**. There are some fun ones in this area, like using counters to grow shapes (e.g. squares, triangles, pentagons, L's, X's, Y's, N's, W's, and so on). Be careful to ensure these are linear if focusing on linear relationships.

2. **Real-world situations**. The real world can also provide examples like the relationship between chairs and tables in a restaurant. However, be careful, because many of these such as the number of ways one could go up a number of stairs are nonlinear examples that lead to square or triangular numbers (1, 3, 6, 10, 15, ...) and Fibonacci sequences (1, 1, 2, 3, 5, 8, 13, ...). Nonlinear patterns are covered in Unit 6.

3. **Mathematics itself**. Many of the relationships in mathematics start off as patterns. For example, interior angle sums for n-sided polygons, the number of diagonals in an n-sided polygon, the rule for place-value positions when multiplying/dividing decimal numbers, and so on. This is a good opportunity to integrate aspects of mathematics – use discovering of a formula as a chance to solve growing patterns.

4. **Scientific experiments**. One way to have something with reality for any growing pattern is to argue that you are testing a chemical on tree growth. The tree starts with the fixed part, say two leaves, then after each day it has grown three extra leaves, and so on (this is pattern $3n + 2$; pattern $4n + 3$ would start with three leaves and grow four each day). Talking about days and leaves helps students work out the pattern. For this experimental approach, it is appropriate to start from 0 because the extra leaves grow after 1 day, 2 days and so on. The tree with its two starting leaves is day 0.

Unit 4: Building Algebraic Generalisations

A major step is to use the patterns to build generalisations and algebraic representations. It is at this point that: (a) starting from 0 can be useful, and (b) we move from patterns with objects to patterns with numbers. We also start to add graphs, and we build generalisations from repeating patterns.

4.1 Building algebraic generalisations from linear growing patterns

It is important to ensure that students can find and express generalisations from patterns. There are three stages to this development: (a) in quasi-generalisation form (for large numbers – e.g. the 674th position is 3 × 674 + 2); (b) in language (informally – e.g. "three times the position plus two"); and (c) in algebraic form (e.g. $3n + 2$). To do this, follow the steps below.

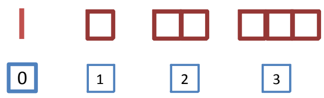
Reality

Bring in reality by, for example, considering someone building a set of small yards enclosed with fences, starting with one fence:

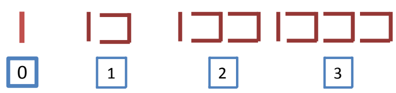
Abstraction

Body

Construct a pattern of yards as on right and determine the number of fences and yards. This could be stated as a construction problem (e.g. *How many fences to make 1, 2, 3 yards?*).



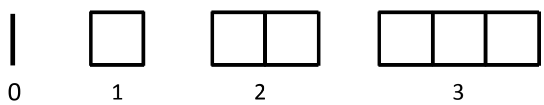
Identify the fixed and growing parts of the pattern (the first fence is fixed and then it grows by three fences each term – see diagram on right); these relate to the pattern rule.



Activity: Masking Tape Pattern

Materials: Masking tape, 2 felt tip pens (different colours).

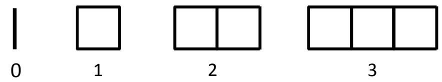
1. Use masking tape to create a pattern on the floor, for example, as on right:



2. Allow students to add the terms.
3. Using a coloured felt tip pen highlight the fixed part. Using another colour, highlight the growing part.
4. Without using a table, students form a sequential and a positional rule.
5. Students then create a table of the pattern on the board and see if the rules work for the numbers.
6. Question the students: *What about the 10th term? What about the 382nd term?* This is done verbally. *Can you write it on the board?* This is done symbolically (e.g. 3 × 382 + 1). *Is there a way it could be written to make it easy to find the value of any term?* Teacher introduces the concept of n and displays the rule in algebraic form: $3n + 1$.

Hand

Replace the yards with drawings of lines, as on right:



Mind

Move on to developing a pattern in the mind or in a table:

(a) Mind – each new yard has 3 fences so yard 1 = 1 + 3 fences; yard 2 = 1 + 3 + 3 fences, and so on. Thus 10 yards is 1 + 10 × 3 and 100 yards is 1 + 100 × 3.

(b) Table

Fences	1	4	7	10	13	and so on
No.	0	1	2	3	4	

Mathematics

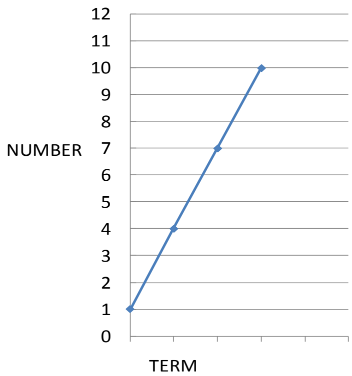
Find the pattern rule and reverse the pattern rule: the n^{th} yard is 1 + 3n fences; 82 fences is 81 + 1 or 3 × 27 + 1 fences, so the number of yards is $\frac{82-1}{3} = 27$.

4.2 Using patterns to introduce linear graphs

The Masking Tape Pattern activity in 4.1 can be extended to graphing. Once the position pattern rule has been determined as an algebraic expression, a graph can be constructed. To do this, continue as follows.

1. Construct a **table** and place early values in it:

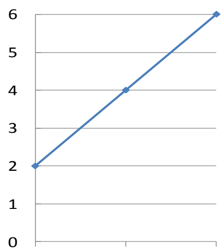
Number of sticks	4	7	10	13	16	
Position	1	2	3	4	5	and so on

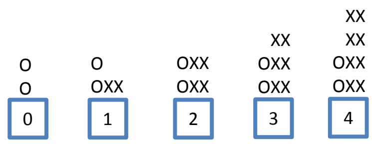


2. Use the table to draw the **graph** as on the right. To do this graph you need to add in the 0 term as below:

Number of sticks	1	4	7	10	13	16	
Position	0	1	2	3	4	5	and so on

3. **Reverse everything** – provide a graph as below and make up a pattern to match this graph (use the fact that term 0 = 2, term 1 = 4, term 2 = 6, etc. to construct a pattern – see below right).



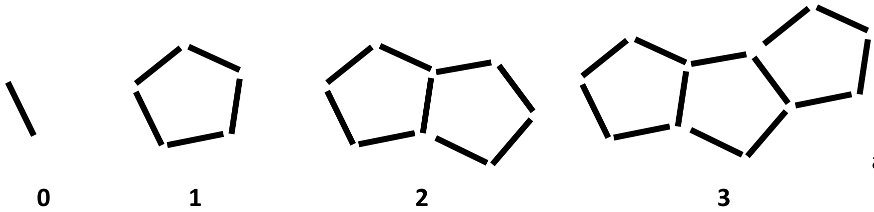


There is a relationship between graphs and patterns that holds generally for growing parts and, in a way, for fixed parts. The slope of the graph is the growing part, and the y intercept is the fixed part of a growing pattern. It works in the two examples above. The pattern at the start of subsection 4.1 grows by 3 and has a fixed part of 1, and gives a graph with slope of 3 and a y intercept of 1. The graph in step 3 above has a slope of 2 and a y intercept of 2 and the pattern it comes from grows by 2 and has a fixed part of 2.

Thus, we can build the generalisation of graphs of straight lines, $y = mx + c$, where m is slope and c is y-intercept, by graphing patterns that grow by m and have a fixed part of c.

4.3 Moving from physical to mathematical patterns (and reversing)

Up to now, we have focused on patterns that emerge from using counters or sticks, for example:



and so on

However, physical objects cannot show negatives and these are perfectly acceptable in patterns. Thus, we now move from physical to mathematical patterns, for example:

Number	7	5	3	1	−1	−3	?
Position	0	1	2	3	4	5	6

and so on

It is now useful to think of first terms as position zero.

We can even give the pattern in the middle, for example:

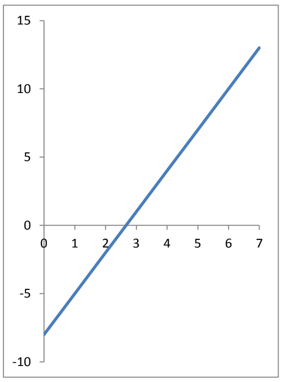
Number	?	?	?	1	4	7	?	?
Position	0	1	2	3	4	5	6	7

The steps are the same, as follows.

1. Construct a table:

Number	−8	−5	−2	1	4	7	10	13
Position	0	1	2	3	4	5	6	7

and so on



   (a) Look for a pattern: growing part is +3, starting is −8; pattern is ×3 −8

   (b) Give the pattern rule: n^{th} term is $3n - 8$.

   (c) Reverse the pattern rule: position for amount k is $(k + 8) \div 3$.

2. Draw graph: slope = 3, y-intercept = −8.

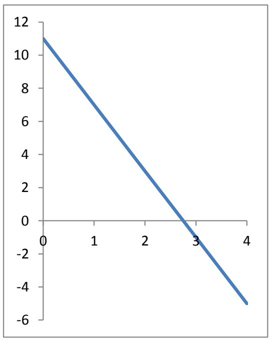
3. Reverse everything: go from graph to pattern.

It is also possible for the graph and growing part to be negative, as in the following example.

1. Table:

Number	11	7	3	−1	−5
Position	0	1	2	3	4

and so on



   (a) Pattern: growing part is −4, start is 11; pattern is ×−4 +11.

   (b) Pattern rule: n^{th} term is $11 - 4n$.

   (c) Reverse: position for amount k is $(11 - k) \div 4$.

2. Draw graph: slope = −4, y-intercept = 11.

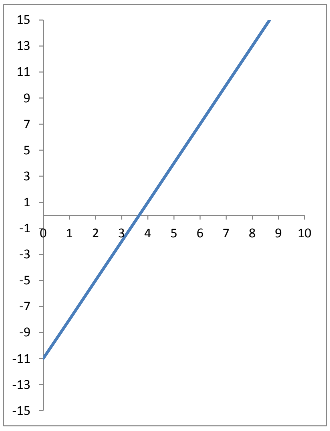
3. Reverse everything.

We can also begin the table from any position.

1. Table:

Number	7	10	13	16	19
Position	6	7	8	9	10

and so on



   (a) Pattern: growing part is 3, constant part is at the zero position where the number will be −11; pattern is ×3 −11.

   (b) Pattern rule: n^{th} term is $3n - 11$.

   (c) Reverse: position for amount k is $(k + 11) \div 3$.

2. Draw graph: slope = 3, y-intercept = −11.
3. Reverse everything.

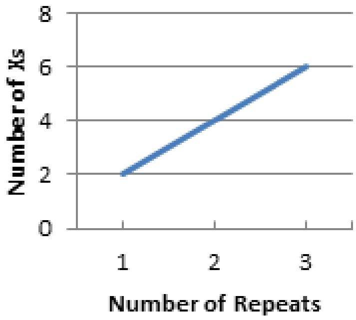
4.4 Building generalisations from repeating patterns

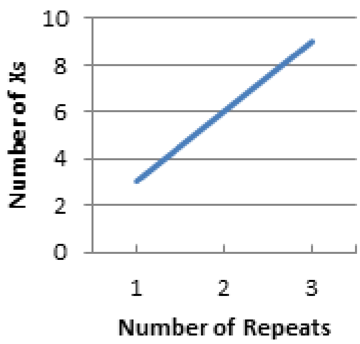
The early activities can be extended to use repeating patterns as a means of teaching generalisation to algebraic expressions (as is possible in growing patterns).

1. Teacher presents a **repeating pattern** (e.g. X X O X X O X X O ...).
2. Complete the **table** below and find the down and across pattern rules.

No. of repeats	No. of X's	No. of O's	Total
1	2	1	3
2	4	2	6
3	6	3	9

3. **Generalise** how to find the numbers for n repeats. Do this in language first – O's are the same as number of repeats, X's are double the number of repeats, and so on.
4. State generalisations as **algebraic expressions** – if n represents the number of repeats, the number of X's is $2n$, the number of O's is n, and the total number is $3n$.
5. **Reverse** the **repeats-to-number** activity to **number-to-repeats** – how many repeats are needed for 26 X's? [$26 = 2n$ so it is 13 repeats).
6. **Reverse** the activity **algebraically** (e.g. if there are n X's, then there are $\frac{n}{2}$ repeats).
7. **Graph** the relationship between number of X's and number of repeats, see top right.
8. **Reverse overall** by constructing a repeating pattern of X's and O's for a given graph, e.g. the graph on right. This graph has 3 X's for 1 repeat, 6 X's for 2 repeats, and so on, but does not show number of O's. So, if there is one O, the repeating pattern is OXXX, OXXX and so on or, if there are two O's, it is OOXXXOOXXX and so on. (*Note:* the y-intercept is 0. This is because for no repeats this is always zero.)





Unit 5: Extensions

This unit looks at extensions: (a) how physical materials can be used to extend understanding of the algebraic forms of the pattern rule; (b) how repeating patterns can be used to extend mathematics ideas such as equivalent fractions and proportion; and (c) how growing patterns can help understanding of slope and y-intercept for linear graphs. Finally, it looks at relationships between patterns and graphs for linear equations.

5.1 Reinforcing variables (cups and counters)

Pattern rules are an effective way to introduce the notion of variable and the notion of expression as a way of stating a generalisation, for example, finding the pattern rule for the growing pattern below:

	@@	@@@	@@@@	@@@@@
@	@	@@	@@@	@@@@
0	1	2	3	4

The pattern is 2 × position + 1 or double anything + 1 or $2n$ + 1 for the n^{th} position. However, if some students have some difficulty understanding $2n$ + 1, an effective method for reinforcing understanding of variable is with physical materials such as cups and counters – the cup represents any number and each counter represents one. (*Note:* There are other materials that do this as well, e.g. envelopes and small squares).

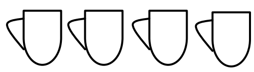
Activity: Cups and Counters

Materials: 10 cups and 10 counters per student.

Multiplication of a variable: Consider a pattern as follows:

	@	@@	@@@	@@@@	
	@	@@	@@@	@@@@	
	@	@@	@@@	@@@@	
	@	@@	@@@	@@@@	
0	1	2	3	4	and so on

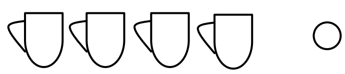
The pattern is four times the position or $4n$. In terms of cups and counters, this is 4 cups as the cups represent an unknown or variable amount of counters depicted by letters. Thus, $4n$ is 4 unknowns or 4 cups, as on right.



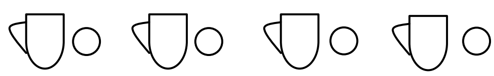
Multiplication of a variable and addition/subtraction of a number: This is a pattern whose rule has two operations. Consider the pattern below:

@	@@	@@@	@@@@	@@@@@	
	@	@@	@@@	@@@@	
	@	@@	@@@	@@@@	
	@	@@	@@@	@@@@	
0	1	2	3	4	and so on

Here, the pattern is four times the position plus one, that is, $4n + 1$. This is 4 cups and one counter as on right.



Cups and counters allow different meanings to be explored, for example, the difference between $4n + 1$ above and $4(n + 1)$ on right.



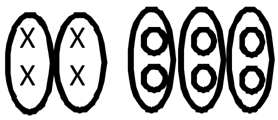
5.2 Repeating patterns to equivalence

1. **Pattern.** Start with a repeating pattern, e.g. X X O O O X X O O O X X O O O
2. **Repeats.** Change it to repeats, e.g. XXOOO   XXOOO   XXOOO
3. **Table.** Complete a table that contains fractions and ratios.

No. of repeats	No. of X's	No. of O's	Total	Fraction X's	Ratio X:O
1	2	3	5	$^{2}/_{5}$	2:3
2	4	6	10	$^{4}/_{10}$	4:6
3	6	9	15	$^{6}/_{15}$	6:9

4. **Generalisation.** Look at what n repeats will give (repeats to O's and X's).
5. **Reversing.** Go from numbers of O's and X's to number of repeats (e.g. 72 O's means $^{72}/_{3}$ repeats = 24 repeats).
6. **Equivalence.** Discuss whether fractions and ratios are equivalent (or in ratio terms, in proportion). For example, is 2:3 = 4:6?

   Note: A student explained it to class as shown on right. She stated that 2:3 is XXOOO and 4:6 is XXXXOOOOOO. However, she rearranged the objects as on right and argued that this showed that XXXXOOOOOO is the same as XXOOO.



Same as XX 000

7. **Reversing overall.** Make up a repeating pattern where the ratio is 2:5 (e.g. O O X X X X X O O X X X X X O O X X X X X and so on).

Note: (a) The teaching approach is to have class discussion. Let students propose generalisations. Don't say if generalisations are right or wrong. Focus on down and across generalisations. Encourage students to justify their point of view. (b) Repeating patterns can be done with number, e.g. 1 2 2 1 2 2 1 2 2, ..., but numbers here act like objects.

5.3 Growing patterns to slope and y-intercept of linear equation/graph

Growing patterns can be related to line graphs and linear equations, as described below.

Step 1: Relating patterns to graphs

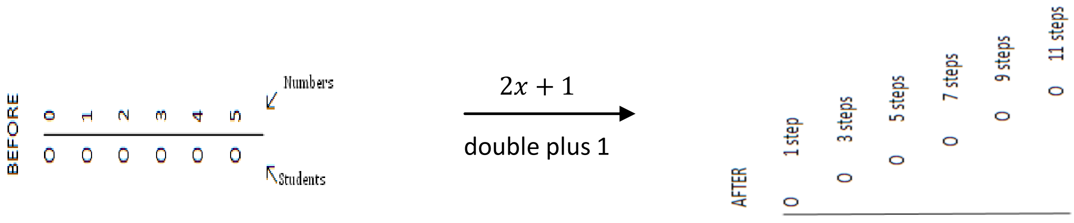
1. **Relating patterns to linear equations**. Start with a pattern, e.g. oo x, oo xx, oo xxx, oo xxxx, and so on. Determine fixed and growing parts, determine position rule $[n + 2]$ and plot graph. Rename position as x and pattern value as y, redraw graph and rewrite the rule ($y = x + 2$). This is the linear equation or function that the graph now represents. Repeat this activity if necessary.
2. **Reverse the situation**. Start with a linear equation e.g. $y = 3x - 2$. Draw a table of x and y values as on right and fill in the table for x= 0, 1, 2, 3 and so on. Draw the graph.

y				
x	0	1	2	3

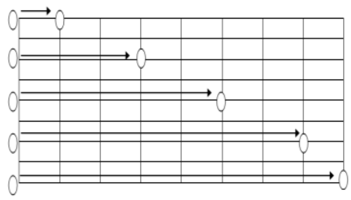
   Go backwards and forwards with other examples – graph → linear equation (interpreting) and linear equation → graph (plotting).

3. **Special kinaesthetic activity**. Take students out to the school yard and line them up on a line. Number them in order from 0 to whatever. Take a linear equation (say $2x + 1$) and say to each student that they have to double the number given to them, add one, and take that many steps (see diagram below). When

they have finished, discuss that they have made the graph of linear equation $y = 2x + 1$ with their bodies. *Note:* (a) Train students to take the same-size steps. (b) They can also do negative steps – stepping back.



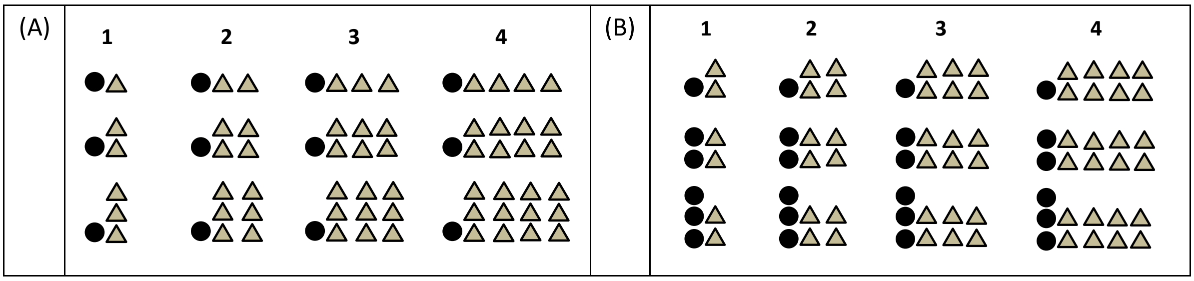
It is best if you can do this with actual students on a grid as shown on right (using a mat or a grid painted on the school yard). Students need to stand on intersections of lines so they imitate a graph.



Get students to hold a rope so they can see that they have made a straight line. Then students can replace themselves with a token and draw a copy of the graph to see what they have acted out. Finally, students should be encouraged to imagine the change in their mind (so completing body → hand → mind).

Step 2: Relating collections of patterns to graphs

1. Gather **collections of patterns** – the first (A) where the growing part varies but not the constant part, the second (B) where the constant part varies but not the growing part – try to make them real to students:



2. Look at **properties of the patterns** – ask the students to:
   - find the growing part and the constant part for each of the patterns;
   - find the position pattern rule for each of the patterns; and
   - draw a graph for each of the patterns.

3. Look at **changes in the graphs and relate to patterns** – discuss the graphs. How do the graphs for (A) change? How does this relate to the change in the patterns? How do the graphs for (B) change? How does this relate to the change in the patterns?

4. **Table the data/find the rule** – put the following on a table as on right. Discuss – let students come up with rules/relationships between patterns and linear graphs

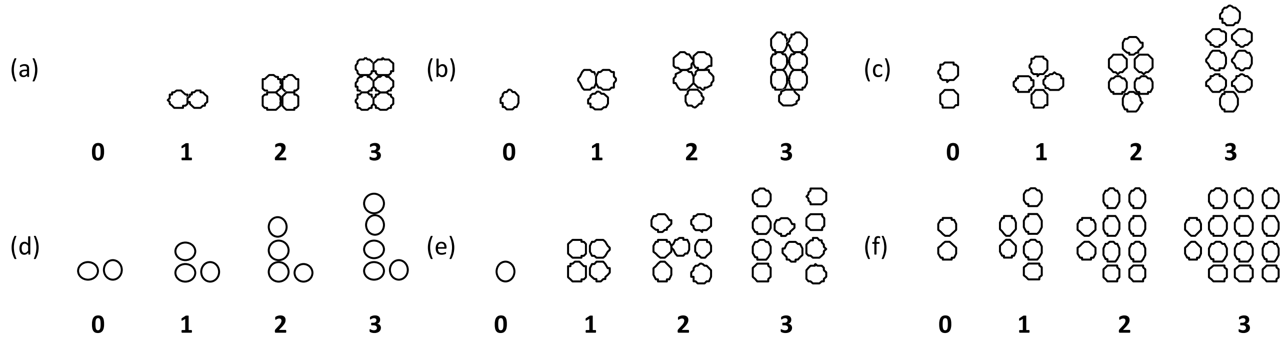
Growing part	Constant part	Slope of graph	y-intercept of graph

5. **Reverse** – discuss how this would help us if we started from the graph and finished with the pattern – finding a pattern for a linear graph? Give a graph of y-intercept 3 and slope 4 and ask for a pattern.

5.4 Relationships between patterns and graphs for linear equations

Relationships between patterns and graphs of linear equations can be found by relating solutions to different patterns looking for similarities and differences.

1. Provide examples of patterns that have fixed growing parts (pattern rule is a linear equation) and are related in a way that enables relationships with regard to graphs and functions to be seen. For example:



Note: The composition of, and position rules for, the patterns and their graphs are as follows. Examples have to be chosen so that the linear relationships that are wanted are available from the patterns.

PATTERN	COMPOSITION	POSITION RULE	GRAPH CHARACTERISTICS
(a)	Grows by 2, no fixed part, starts with 0	$2n$	Slope 2, y-intercept 0
(b)	Grows by 2, fixed part of 1, starts with 1	$2n + 1$	Slope 2, y-intercept 1
(c)	Grows by 2, fixed part of 2, starts with 2	$2n + 2$	Slope 2, y-intercept 2
(d)	Grows by 1, fixed part of 2, starts with 2	$n + 2$	Slope 1, y-intercept 2
(e)	Grows by 3, fixed part of 1, starts with 1	$3n + 1$	Slope 3, y-intercept 1
(f)	Grows by 4, fixed part of 2, starts with 2	$4n + 2$	Slope 4, y-intercept 2

2. For each pattern, analyse the values visually and identify the fixed and growing parts.
3. For each pattern, find the value of some positions (e.g. 10, 100, 39, 64), find the position of certain values (e.g. 17, 71, 149 for pattern b), write the pattern rule in English, find the value for position n, and find the position for value k.
4. Draw the graph of each pattern and identify the slope and y-intercept.
5. Look for patterns between characteristics of pattern, position rules, and characteristics of graph (can be useful to complete a table like that above). First compare (a), (b) and (c) noting similarities and differences – what changes (fixed part) and what does not change (growing part). Then compare (d) with (c) – now growing part changes and fixed part stays the same. After this compare (e) and (f) – both growing part and fixed part are different (but how does this difference relate to graphs, its slope and y-intercept?).

What follows is the **general rule for slope**: if the growing part is p, the pattern rule starts with pn and the graph has slope p. This relationship is even more obvious when we restrict ourselves to numbers. For example:

Number	7	5	3	1	−1	−3
Position	0	1	2	3	4	5

and so on

The growing part is −2, the fixed part is 7, so the pattern is $7–2n$. This makes the slope −2 and the y-intercept 7. This means that the function for a linear equation is $f(x) = mx + c$ where m is the slope (and the growing part if from a linear growing pattern) and c is the y-intercept and the constant part for the zero term ($n = 0$).

Note: It is useful to compare the patterns above to triangular numbers; a pattern which does not have fixed linear growth.

Unit 6: Nonlinear Patterning Activities

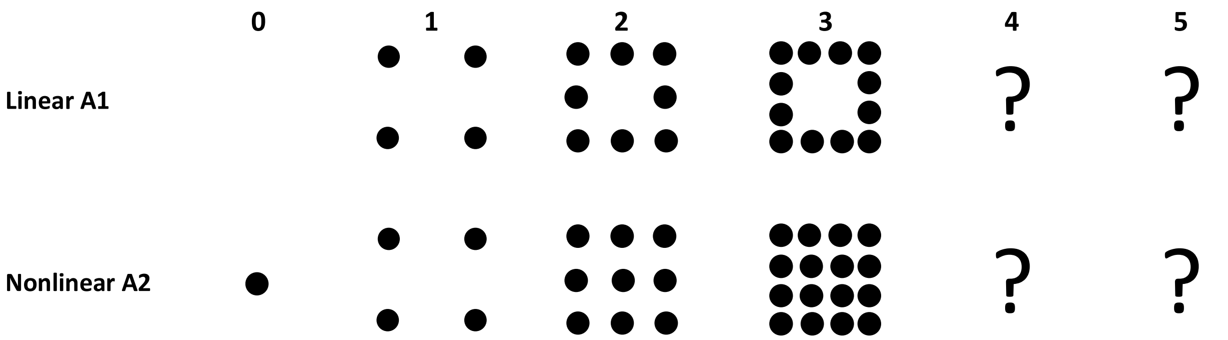
Linear patterns are the important patterns for up to Year 9 for two reasons. First, they are the simplest form of algebra and so allow learning of forms such as $2n$, $n+3$ and $2n+3$. Second, they lead to understanding of graphs of lines and linear equations which are the major focus up to Year 9. However, there are also opportunities for nonlinear pattern work to pre-empt quadratics and other nonlinear forms in Year 10 onwards.

6.1 Nonlinear patterns and graphs

Here are some steps for investigating linearity and nonlinearity.

Comparing linear and nonlinear

1. Consider the two patterns below:

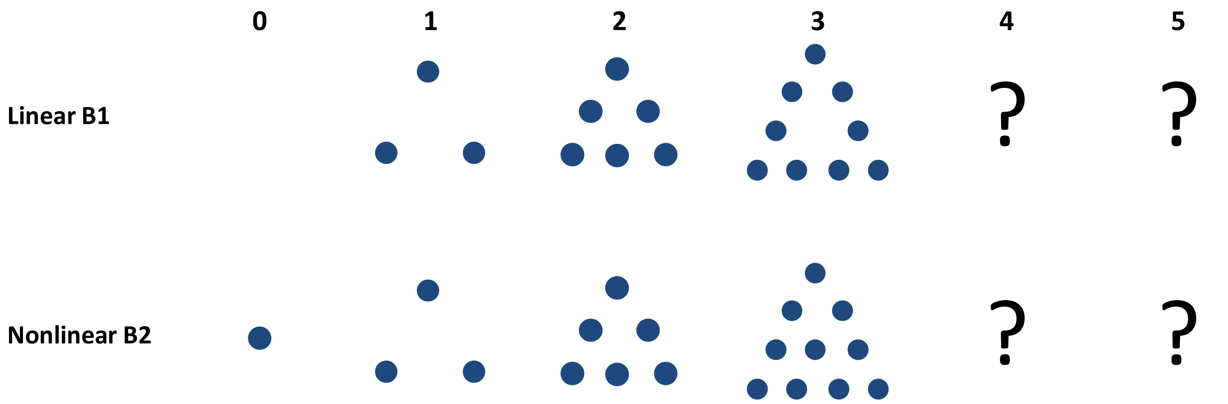


   (a) Complete terms 4 and 5 by drawing or with counters.

   (b) Look at term 5 visually and use this pattern to determine the 10^{th} term, 100^{th} term and 156^{th} term for A1 and A2.

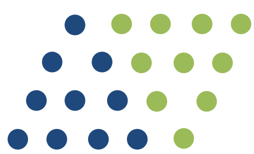
   (c) Describe the positional rule for A1 and A2 (in language and as an algebraic expression).

2. Consider the two patterns below:



   Repeat questions (a) to (c) above.

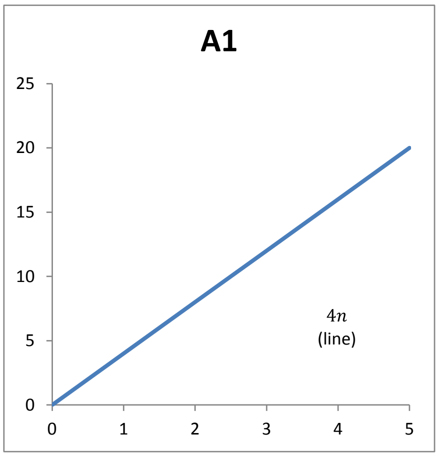
   Hint: To find the rule for B2, put two triangles together, see that they form a parallelogram as on right – and so the triangle is half of this.

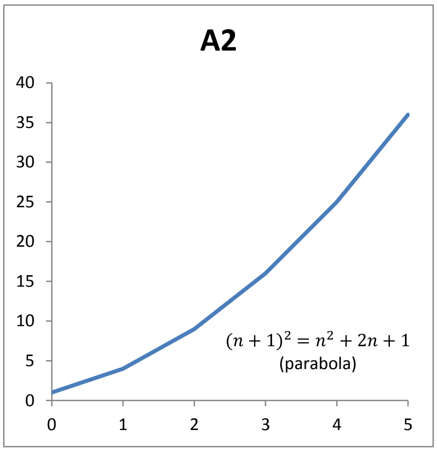


3. Comparing graphs of linear and nonlinear patterns:

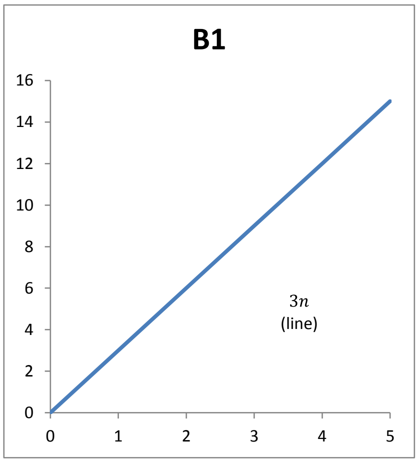
(a) Plot points on a table and then transfer them to graphs.

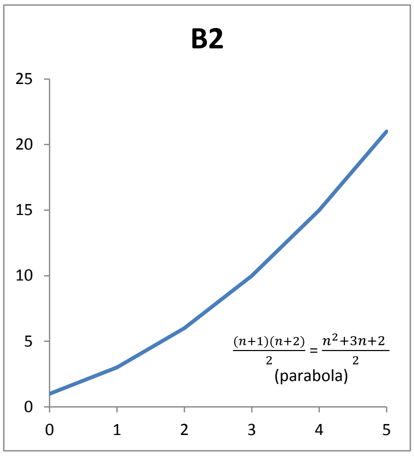
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
A1	0	4	8	12	16	20	24	28
A2	1	4	9	16	25	36	49	64





	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
B1	0	3	6	9	12	15	18	21
B2	1	3	6	10	15	21	28	36



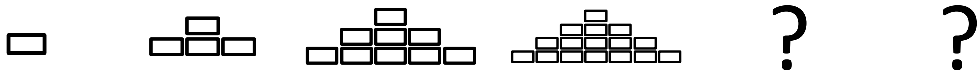
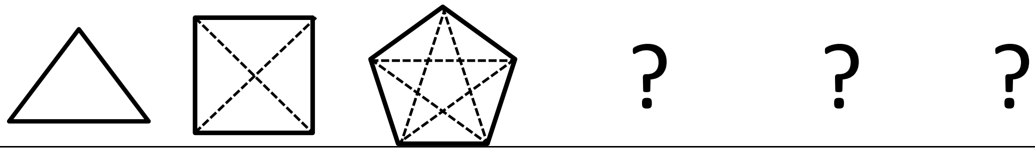
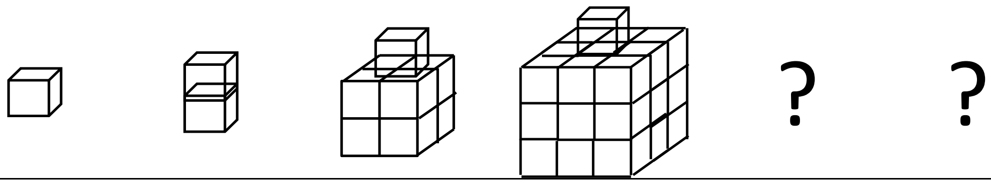
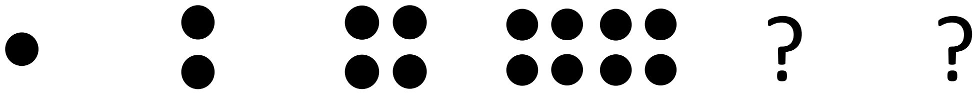
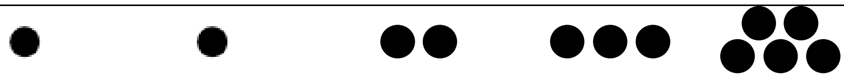


(b) What pattern is in these results?

4. Finding the position rule (challenge):

(a) Find the position rule for the following nonlinear patterns (some of them are difficult).

(b) Draw graphs for patterns C, E and F and then D and G. What do you notice?

	0	1	2	3	4	5
C						
D						
E						
F	 *Note:* Drawing F is showing a pattern which doubles every time.					
G	 *Note:* Drawing G is showing a pattern which adds the previous number to the one before that.					

6.2 Reversing relationships to look at graph → pattern

1. Start with a nonlinear pattern, for example $n^2 + 3n$.

   (a) Construct a table:

Term (n)	**0**	**1**	**2**	**3**	**4**	**5**
Number ($n^2 + 3n$)	0	4	10	18	28	40

   (b) Construct a pattern to fit this (call this pattern **H**). *Hint:* Terms 3 and 4 are shown below:

```
O O O      O O O O
O O O      O O O O
O O O      O O O O
X X X      X X X X
X X X      X X X X
X X X      X X X X
           X X X X

  3           4
```

2. Repeat step 1 for:

   (a) Pattern rule $2n^2 + n$ (call this pattern **I**).

   (b) Pattern rule $3n^2 - n + 2$ (call this pattern **J**).

   (c) Pattern rule $n^3 - n$ (call this pattern **K**).

3. Draw the graphs of patterns H to K.

4. Looking at all the graphs A to K, what relationships can you see between graph type and pattern rule?

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6.3 Growing and fixed parts (challenge)

In linear patterns, we found that the growing and fixed parts of a pattern related directly to the graph. For example:

O O	O O	O O	O O	O O
	X	X X	X X X	X X X X
	X	X X	X X X	X X X X
	X	X X	X X X	X X X X
0	**1**	**2**	**3**	**4**

The above pattern has a fixed part of 2 and a growing part of 3. The growing part of 3 can be seen in the table by looking at differences between consecutive terms – these are all 3. Therefore the equation for the position rule for this pattern has a $3n$ in it and the graph has a slope of 3. The y-intercept is related to the fixed part which is 2, but it depends on what happens when n is zero. Here it is 2.

We can now say that the pattern rule is $3n + 2$, that the coefficient or power of n is 1, and that the relationship and graph is linear, with slope of 3 and y-intercept of 2.

Position	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Number of objects	2	5	8	11	14	17	20	23
Difference		3	3	3	3	3	3	3

The question is whether something similar to this holds for nonlinear. To explore this we look at an example of a quadratic (e.g. example A2 which is $n^2 + 2n + 1$).

1. Take quadratic A2 and put the information into a table.

Position	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number	1	4	9	16	25	36	49	64	81

2. Now subtract differences and continue until differences are the same or constant.

Position	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number	1	4	9	16	25	36	49	64	81
1st difference		3	5	7	9	11	13	15	17
2nd difference			2	2	2	2	2	2	2

3. It took two differences to get the constant. Is there a pattern here – one difference to constant for linear like $3n + 2$; two differences to constant for quadratic like $n^2 + 2n + 1$? Is it possible that all quadratics take two differences to get a constant?

4. The constant is 2, this is the coefficient of n in $n^2 + 2n + 1$. Is this always the case for quadratics?

5. Repeat the above for the quadratic patterns in examples B2, C, I and J. Does it hold?

6. Try this difference technique for cubics E and K. Is there a similar rule for cubics?

7. Is there a rule that relates difference to coefficient of n?

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the pattern and linear relationship item types

The pattern and linear relationship item types are divided into six subtests, one for each unit in this module. The six units represent a sequence of pattern knowledge from early childhood through to Year 9 – there are no units at the same level looking at different concepts – all units are integrated and in sequence. They begin with repeating patterns, move on to linear growing patterns and pattern rules, and end with generalisation, extensions into graphs and equivalence, and a small final unit on non-linear patterns.

This means that the pre-test should focus on the items in the early subtests, moving up the subtests until students can no longer do the items. It also means that the post-test must cover all the later subtest items. However, it is important that sufficient content is included in the pre-test to ensure that: (a) teaching begins where students are at; (b) what is missed out is because the students cannot answer the questions; and (c) the pre-test provides both achievement level and diagnostic information. It is also important that sufficient content is included in the post-test to ensure that: (a) what is not included is because students can do this; (b) what is included will give the level of achievement at the end of the module; and (c) legitimate comparisons can be made between pre- and post-tests in terms of effect. Finally, always read the questions to the students and explain any contextual information as long as this does not direct to the answers.

Subtest item types

Subtest 1 items (Unit 1: Early repeating patterns)

1. The following is a repeating pattern:

□ ○ ○ ○ □ ○ ○ ○ □ ○ ○ ○

(a) Circle the repeating part.

(b) Fill in the gaps:

□ ○ ○ ○ □ ○ ____ ____ ____ ____ ○ ○ □ ○ ○ ○

(c) Continue the pattern to the left for one repeat and to the right for one repeat.

□ ○ ○ ○ □ ○ ○ ○ □ ○ ○ ○

2. Looking at the following pattern:

□ ○ ○ □ ○ ○ □ ○ ○

(a) What is the shape in position 3 within the pattern? ________________________

(b) If the pattern was continued to the right, what would the shape in position 15 be?

____

3. Draw your own pattern, using circles and squares, that has three repeats. Circle the repeating part.

4. Draw □ ○ ○ □ ○ ○ □ ○ ○ □ ○ ○ as repeats.

1	2	3	4

Subtest 2 items (Unit 2: Initial linear growing patterns)

1. This pattern is a repeating pattern:

□ □ ○ □ □ ○ □ □ ○

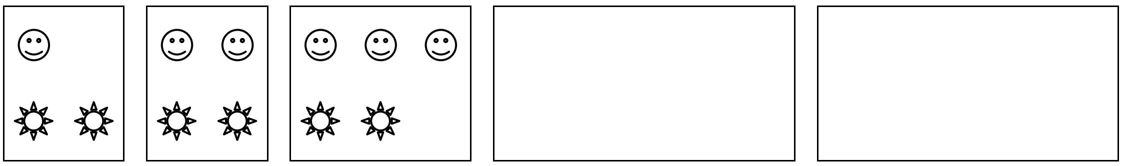
(a) Draw it as repeats:

1	2	3	4

(b) Turn it into a growing pattern by growing one of the parts:

1	2	3	4

2. This pattern is a growing pattern:



1 2 3 4 5

(a) How can you tell it is a growing pattern? ______________________________________

____

(b) Draw the next two terms (4 and 5) in the pattern:

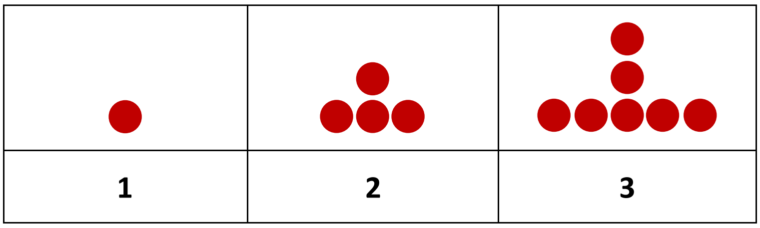
(c) What is the part that stays the same? ______________________________________

(d) What is the part that is different (the growing part)?

____

Subtest 3 items (Unit 3: Visuals and tables)

1. This is a growing pattern:



(a) Draw the 5^{th} and the 10^{th} terms below:

(b) Describe the pattern visually: ____

____

(c) Describe the 376^{th} term: ____

____

(d) Describe the n^{th} term (“any number”): _____________________________________

____

2. This table shows the numbers in the pattern from Question 1:

Term number	1	2	3	4	5		10
Number of objects	1	4	7			16	
Difference							

(a) Fill in the gaps in the table.

(b) What is the sequential rule? ____

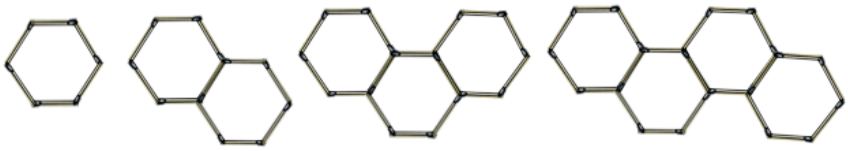
(c) What is the position rule for this pattern? ____________________________________

(d) How does this position rule relate to the “difference” row? ______________________

____

Subtest 4 items (Unit 4: Building algebraic generalisations)

1. In this pattern, each new term is created by joining more matchsticks.



1 2 3 4 5

(a) Draw the 5th term. Think of how the hexagons grow visually.

(b) In words, describe how the pattern is growing (the sequential rule): ________________

____

(c) Can you use this to make a rule for the pattern? _________________________________

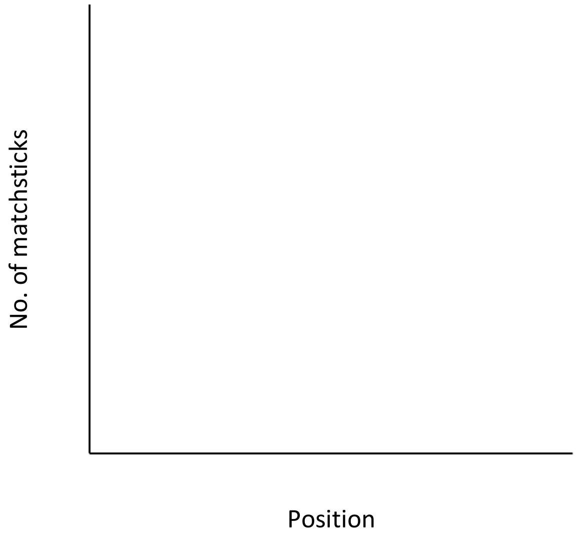
2. (a) The hexagon pattern is displayed in the following table. Fill in the empty spaces.

Position	Number of matchsticks	Position	Number of matchsticks
1	6		36
2	11	73	
3			241
4		121	
10		n	
100			k

(b) Write the position rule (position → number of matchsticks): ______________________

(c) Write the rule for number of matchsticks → position: _____________________________

3. (a) Draw a graph of these values on the axes below. Label the axes.



(b) What is the growing part and what is the constant part of the pattern?

Growing part: ____________________

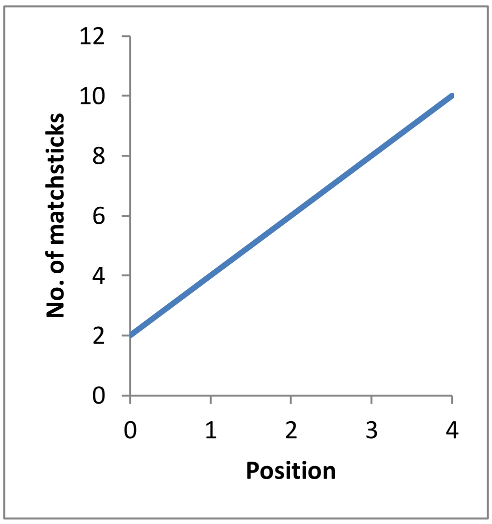
Constant part: ____________________

(c) What is the relation between (b) above and the slope and y-intercept of the graph?

____

____

4. Draw a pattern that gives this graph:



Subtest 5 items (Unit 5: Extensions)

1. This is a repeating pattern: X X O O O X X O O O X X O O O

(a) Complete the table for the repeating pattern.

No. of repeats	No. of X's	No. of O's	Total no. of X's and O's	Fraction X/Total	Ratio X:O
1	2	3	5	2/5	2:3
2	4	6	10	4/10	4:6
3					
4					
10					
31					
n					

(b) How are the fractions in the "Fraction X/Total" column related to each other?

____

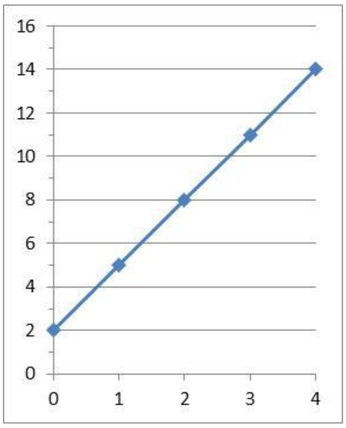
(c) How are the ratios in the "Ratio X:O" column related to each other?

____

2. What is the relationship between the slope and y-intercept in a linear graph and the growing amount and constant amount in a linear pattern?

____

3. This graph has a slope of 3 and a y-intercept of 2.



(a) What is its equation? ____________________

(b) What is the 0 term and the growing part of the pattern?

____

(c) Draw a pattern that would give this graph.

4. Here is a growing linear pattern:

X **O** **O**	**X** **O O** **O O**	**X** **O O O** **O O O**	**X** **O O O O** **O O O O**
0	**1**	**2**	**3**

(a) What is the constant part and what is the growing part of this pattern?

Constant part: ____________________

Growing part: ____________________

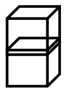
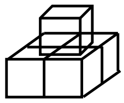
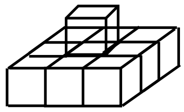
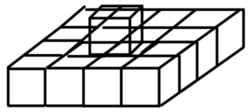
(b) Complete the table for this pattern.

Position	0	1	2	3	31	n	
No. of objects	3	5	7	9			k
Difference		2	2	2			

(c) What is the equation of the graph for this pattern? ______________________________

Subtest 6 items (Unit 6: Nonlinear patterning)

This is a nonlinear pattern:

				
0	**1**	**2**	**3**	

1. Describe the pattern and how it grows. Use the example of the 10th position.

   ____

   ____

2. Complete the table for this pattern.

Position	**0**	**1**	**2**	**3**	**4**	**5**	**6**
No. of objects	2	5	10	17			
First difference		3	5				
Second difference			2				

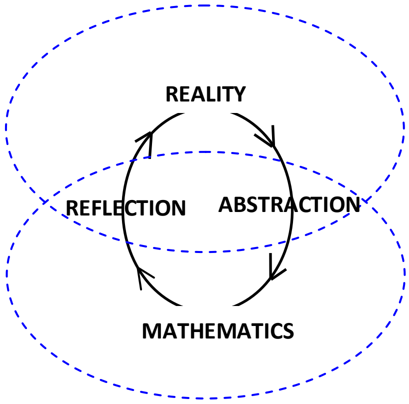
3. What type of graph comes from this pattern? ________________________________

   ____

4. Can you use the 0 position and the changes to write the equation of the graph?

   Equation:

Appendix A: RAMR Cycle



AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).

The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix B: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

YuMiDeadly
Growing community
through education

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through the YuMi Deadly Centre

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