Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module SP2

Year B, Term 4

Statistics and Probability:

Probability

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

These teacher notes for the module are to provide background and to overview the module. The notes focus on probability or, in more informal language, chance. The notes cover the big ideas and connections with regard to probability, provide a way to sequence the content, overview teaching ideas, outline the RAMR pedagogy that is the basis of the AIM program, overview the booklet, and relate the module's information to the Australian Mathematics Curriculum. This makes the module unique in that it does not focus on mathematics that determines specific answers (e.g. 2+3=5) but rather on mathematics that provides information to help decision-making in situations where outcomes are not certain, only possible.

Background information for teaching probability

This subsection covers nature of probability and connection to fractions, probability big ideas, and probability teaching (teaching techniques and models and representations).

The nature of probability and its connection to fractions

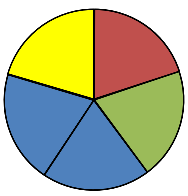
The formal term *probability* is always related to the informal term *chance*. Probability and chance are both concerned with how often we may expect an event to occur. However, chance is a subjective informal estimate of an event whereas probability is an objective formal measurement of an event. Probability describes randomness not haphazardly but as a kind of order different to the deterministic one associated with arithmetic that is able to refine guesses.

Students across all year levels tend to estimate the probability of an event across two or more *sample spaces* by using *ratios* (i.e. the number of favourable outcomes opposed to the number of unfavourable outcomes) rather than *fractions* (number of favourable outcomes out of the total number of outcomes in the entire sample space, *i.e. part of a whole*). This was an unfortunate outcome because **probability is structurally equivalent to fractions**. The implication for teachers is that, until students have an understanding of the part/whole notion of fractions and the connection of this to probability, they will be incapable of true probabilistic reasoning.

Note: A sample space refers to the materials used in a task. Single sample spaces (e.g. 1 die) should be used before multiple sample spaces (e.g. 2 or more dice). A single sample space is much easier for students to determine and compare the probability of an event occurring because it involves fractions with the same name (denominator). Multiple sample spaces may have different numbers of outcomes, thus comparing across sample spaces may involve comparing unlike fractions.

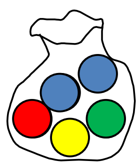
Because probability and fractions share similar concepts and because probability answers are recorded as fractions, teaching probability requires the teacher to use fraction-like teaching. For example, consider the activity below and notice the connection to fractions. The activity shows how probability and fractions are connected pedagogically as well as mathematically. It shows this connection for both area and set models.

Aim: To lead the students to discover the probability of an event occurring (informal language and recording) using an area model and a set model.



What are the chances of spinning blue on this spinner?

[*2 chances out of 5 chances*]



What are the chances of getting, without looking, a blue marble from this bag?

[2 *chances out of 5 chances*]

Stages/Questions that can be asked:

Identify the whole (i.e. the sample space)

- What colours *could* you spin on this spinner? Would it be *possible* to spin red? purple?
- What colours *could* you get from this bag of marbles? Would it be *possible* to get a pink marble?

Examine the parts for equality

- Has the spinner been divided into equal parts? Would the pointer be *just as likely* to stop on one part as on any other part? (OR: Would you have the *same chance* of stopping on any of the parts?)
- Are all the marbles equal, that is, the same size and shape? Would you be *just as likely* to get one marble as any other marble? (OR: Would you have the *same chance* of getting any of the marbles?)

Name the parts (establish the total number of chances, that is, the denominator)

- How many equal parts does this spinner have? How many chances do you have altogether of spinning a colour?
- How many marbles in this bag? How many chances do you have altogether of getting a colour?

Determine the parts to be considered (the outcome preferred, that is, the numerator)

- How many blue parts are there? How many chances do you have of spinning blue?
- How many blue marbles are there? How many chances do you have of getting a blue marble?

Associate the two parts with the fraction name (the probability)

- What chance do you have of spinning blue? (2 chances out of 5 equal chances)
- What chance do you have of getting a blue marble? (2 chances out of 5 equal chances)

Record the probability: 2 fifths (informal); $^{2}/_{5}$ (formal), 0.4, 40%.

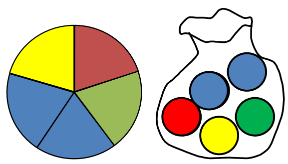
Probability big ideas

To reduce the amount that has to be learnt and build structural schematic understanding, YDM focuses on big ideas. Five major big ideas with special application to probability are as follows.

1. **Probabilistic vs deterministic**. Events are either *probabilistic*, that is, determined by chance (e.g. will it rain?), or *deterministic*, that is, have a certain result (e.g. what is $2 + $5?). The first great understanding is to be able to identify whether a situation is probabilistic or deterministic. The majority of school mathematics is deterministic but the majority of life mathematics is probabilistic. Thus, probability is a crucial teaching area.

1. **Theoretical vs frequentist**. *Theoretical* — derived from making assumptions of equal likelihood within and between the sample spaces (e.g. determining the likelihood of a colour being spun on a spinner). *Frequentist* — calculated from observed frequencies of different outcomes in repeated trials.

2. **Contiguous vs noncontiguous**. Contiguous is where similar parts have a common border or are together; noncontiguous is where parts are separated. For example, the spinner on right is contiguous with regard to blue, and so are the counters.



3. **Interpretation vs construction**. Things can either be interpreted (e.g. what colour will be most likely?) or constructed (set up an experiment to pull coloured balls out of a bag with replacement).

4. **Checking and validation***.* This is a particular problem in probability because a student may look at a simple spinner – two sections red and one section yellow – and say she wants the yellow counter for the race game (one position forward if spinner lands on your colour) because "yellow is my lucky colour". However, if the teacher, trying to correct this misunderstanding, says, "well, let's check it", the check may not contradict the incorrectness of the student's belief. This is because, in all deterministic situations, checking

will show the error, but a game with the spinner may end with yellow winning because of chance. Checking may not be a good technique for convincing students of their probability errors.

Probability activities involving identification of possible outcomes lead to theoretical probability as students are dealing with possible outcomes. Activities which conduct trials to see what happens during random events lead to experimental probability. Because experiments are random, experimental probability may not match theoretical probability. Thus, the teaching of probability requires many experiences — recording the results of those experiences and discussion of these results so that misconceptions can be overcome. The use of games/activities, both fair and unfair, with appropriate teacher questioning will motivate, promote and strengthen students' probabilistic reasoning.

Researchers and practitioners are aware that probabilistic reasoning, particularly in young students, is often generated from personal belief and perceptions. This type of probabilistic reasoning has been classified as *subjective* or *intuitive* probability. Subjective or intuitive probability, however, can be fraught with misconceptions as students may recall situations where a highly desired outcome appeared more difficult to achieve (e.g. rolling a 6 to start in game situations). The random nature of probability makes it very difficult to validate theoretical probability from frequentist or experimental probability. Many trials are needed before experimental probability approaches theoretical probability. As a result experiences are needed with very large data sets. This can be simplified by the use of virtual resources where many spins of spinners, rolls of dice and tosses of coins can be completed very rapidly.

Probability teaching

This subsection looks at effective teaching techniques for probability and the models and representations used.

Teaching techniques

1. **Problem-solving approach**. Statistics and probability appear to be best taught within a problem-solving environment. Students should be encouraged to investigate and experiment as on right. Throughout the development of probabilistic notions and processes, it is recommended that teachers play games and undertake activities that focus on higher-level thinking, go beyond the activities, discuss the situations and reflect on the results.

2. **Metacognition**. It also seems important to develop students' abilities to plan, monitor and evaluate their own work (metacognition). Students should be encouraged to use their "inner voice" (talk to themselves) to keep track of what they are doing, recognise when a subsection has been completed and to identify and correct mistakes. This means that the teacher must ensure that students are aware of the goals of investigations, that is, where they are going and what they will see when they get there.

3. **Open questions**. In particular, open questions can be very important. This is where you turn the question around from what is the answer to what is the question – it is what we are trying to achieve with reversing in the RAMR model. A normal question in probability is: *What is the chance of getting a red from a bag with 3 red and 2 black balls*? An open question is: *What is a situation where the chance is 3/5?*

4. **Probabilistic reasoning**. Probabilistic reasoning grows over time from an initial subjective, non-quantitative reasoning through informal and incomplete quantitative thinking to full numeric reasoning. As chance is essentially a form of measurement, students need to first identify the attribute to be measured. This attribute is *likelihood* which encompasses ideas of randomness, fairness when applied to game experiences, classification of events as impossible, possible and certain, a broad set of descriptive language in everyday and mathematical terms, and symbolic representations.

5. **Concepts and language**. There are some important and unique concepts and accompanying words with regard to probability:
    - Randomness and fairness – this is a major underlying feature – fairness is randomness being true (not biased) and is associated with games.

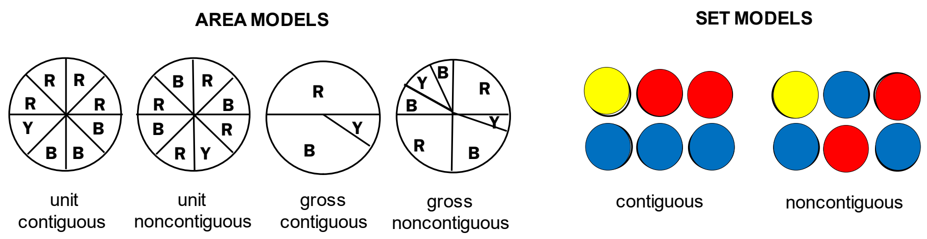
- Likelihood – after *impossible*, *possible* and *certain*, *likely* and *unlikely* need to be fostered through discussion around activities.
- Word bank – possible, impossible, certain, not certain, likely, unlikely, not likely, no doubt, probable, improbable, maybe, fat chance, Buckley's chance, perhaps, dead certain, odds on, sure bet, so-so, 50-50, might happen, more likely, most likely, less likely, least likely.

6. **Inference**. Finally, since probability is so much about life, it is important to ensure that probability enables students to make good decisions about their own lives. This means that teaching how to infer what probability says about life situations, the applications of probability, is an important outcome of probability teaching. As we shall see in the units following, probability can enable people to have some control over daily activity and not have to leave the future to fate or luck.

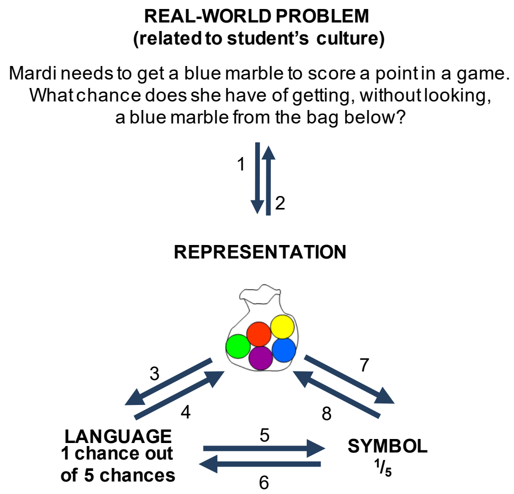
Models and representations

Good models and representations assist learning and problem solving. A sample space can have its components related to the **area model** or the **set model**. As students find area models easier to interpret than set models, spinners should be used before discrete materials such as marbles in developing probabilistic reasoning.

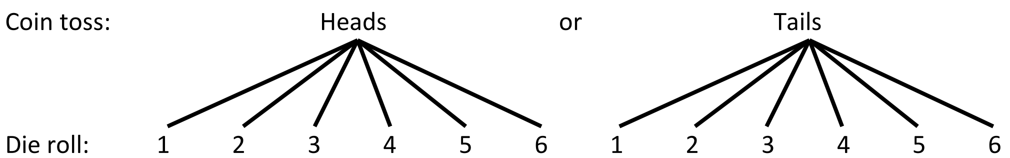
Spinners can be partitioned into equal (*units*) or non-equal (*gross*) segments. As well, like parts can be arranged together (**contiguous**) or split (**noncontiguous**). Unit measurement is easier for students to interpret than gross measurement and contiguous is easier to interpret than noncontiguous.



The representation of probabilistic concepts and processes should proceed from **real models to concrete to pictures to symbols** as for number. Probability is an application of fractions (i.e. part/whole). Therefore similar models (area, set) can be used to represent sample space. The same sequence of models should be kept in mind (area before set). In establishing the representation, language and symbolism of probability, Payne and Rathmell's (1977) model, which underpins RAMR abstraction, should be followed (see on right).



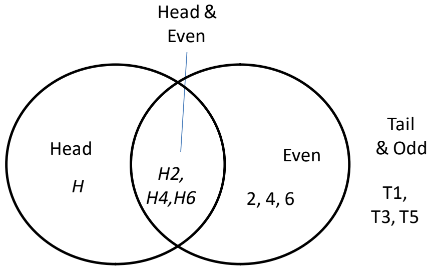
Further ideas to extend students' thinking can involve the combination of events within a trial to provide additional practice in identifying the possible outcomes in consecutive trials (combinatorial counting). **Tree diagrams** can be useful strategies here. For example, list the possible outcomes from tossing a fair coin and a fair die together (see diagram below).



The resulting outcomes and sample space can be more easily identified from the diagram, and enable students to write complete lists of outcomes.

Two-way tables and Venn diagrams enable options to be easily seen. Say, for the example above (coin toss and die roll), we wanted to know the probability of head and even. We can set up a two-way diagram and/or a Venn diagram below; probability is 3/12 or 1/4.

	Odd	Even
Head	3	3
Tail	3	3



Sequencing for probability

This section briefly covers sequencing across probability stages, teaching sequences within stages, and sequencing in this module.

Sequencing across probability stages

The sequence suggested by YDM is to start with classification and to end with applications – this sequence moves through five stages and eight skills as shown in the figure below.

Perceive and identify attribute

- Skill 1: Students can correctly classify an event as impossible, possible or certain

Compare and order likelihood of outcomes

- Skill 2: Students can list all possible outcomes of a chance process
- Skill 3: Students can list all possible outcomes in consecutive trials of a chance process, with and without replacement
- Skill 4: Students can state which outcome is most likely in a single trial of a chance process

Compare and order situations for desired outcome

- Skill 5: Students can correctly choose in what situation an event is more (or most) likely to occur

Measure probabilities

- Skill 6: Students can correctly assign a numerical probability to an event (and use this number for skills 4 and 5)
- Skill 7: Students can estimate the numerical probability of an event using “simulation” methods

Apply probability in inferences

- Skill 8: Students can use probability skills and understanding to make informed decisions and predictions

Teaching sequences

The teaching sequence advocated is to follow the stages/skills above but have students: (a) use area and set models; (b) look at fair and unfair situations; (c) vary the position of the outcomes (contiguous/noncontiguous);

(d) vary the number of outcomes; and (e) undertake both replacement and no replacement activities. While doing this, have students' activities move from one step/one sample space problems to more than one step/more than one sample space problems.

In each of these situations have students: (a) list the outcomes; (b) discuss the likelihood of an event occurring (certain, possible, impossible or yes/maybe/no); and (c) compare two events to determine which is more/less likely to occur. As well, while moving into more than one step situations, it is likewise important to sequence activities so have: (a) two similar sample spaces (one sample space repeated); (b) sample spaces with the same number of outcomes in each space (e.g. 2 spinners); and (c) sample spaces with different numbers of outcomes in each sample space.

It is also important to spend time on events that have consecutive trials and to sequence them so that the events come: (a) from one sample space; (b) from two or more similar sample spaces; and (c) from two or more different sample spaces. Finally, it is also effective to ensure that each stage includes a variety of types of activities such as experiments, simulation trials, and games. However, when undertaking these activities, it is necessary to cover: (a) symbolic representation; (b) relationship to fraction; and (c) comparison and order.

Sequencing in this module

The module has the following sections and units:

Overview: Background information, sequencing and relation to Australian Curriculum

Unit 1: Probabilistic situations (Stage 1 – Skill 1)

Unit 2: Outcomes from probability events (Stage 2 – Skills 2, 3 and 4)

Unit 3: Desired event (Stage 3 – Skill 5)

Unit 4: Probability as a fraction (Stage 4 – Skill 6)

Unit 5: Experimental probability (Stage 4 – Skill 7)

Unit 6: Inference (Stage 5)

Test item types: Test items associated with the six units above which can be used for pre- and post-tests

Appendix A: Connecting probability to fraction

Appendix B: Rich tasks

Appendix C: Game boards

Appendix D: RAMR cycle components and description

Appendix E: AIM scope and sequence showing all modules by year level and term.

Each unit will: (a) define the stages/skills that it covers; (b) discuss approaches to teaching, including RAMR; and (c) provide a variety of activities that build the ideas in the unit.

Note: In probability, it is important to ask many questions because the calculation answer is just a pointer to the probability answer.

Relation to Australian Curriculum: Mathematics

AIM SP2 meets the Australian Curriculum: Mathematics (Foundation to Year 10)

Unit 1: Probabilistic situations	Unit 4: Probability as a fraction
Unit 2: Outcomes from probability events	Unit 5: Experimental probability
Unit 3: Desired event	Unit 6: Inference

Content Description	Year	SP2 Unit					
		1	2	3	4	5	6
Identify outcomes of familiar events involving chance and describe them using everyday language such as 'will happen', 'won't happen' or 'might happen' (ACMSP024)	1	✓	✓	✓	✓	✓	✓
Identify practical activities and everyday events that involve chance. Describe outcomes as 'likely' or 'unlikely' and identify some events as 'certain' or 'impossible' (ACMSP047)	2	✓	✓	✓	✓		✓
Conduct chance experiments, identify and describe possible outcomes and recognise variation in results (ACMSP067)	3	✓	✓				✓
Describe possible everyday events and order their chances of occurring (ACMSP092)	4	✓	✓	✓			
Identify everyday events where one cannot happen if the other happens (ACMSP093)			✓				
Identify events where the chance of one will not be affected by the occurrence of the other (ACMSP094)			✓	✓			
List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions (ACMSP116)	5			✓	✓		
Describe probabilities using fractions, decimals and percentages (ACMSP144)	6			✓	✓		
Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies (ACMSP145)		✓	✓	✓	✓	✓	
Compare observed frequencies across experiments with expected frequencies (ACMSP146)					✓	✓	✓
Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)	7					✓	✓
Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)					✓	✓	✓
Identify complementary events and use the sum of probabilities to solve problems (ACMSP204)	8					✓	✓
Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and' (ACMSP205)						✓	✓
Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)						✓	✓

Unit 1: *Probabilistic Situations*

Skill 1: The student can correctly classify an event as impossible, possible or certain.

Big idea: Probabilistic vs deterministic

This is the first unit and it deals with the first skill, that of classifying events as impossible, possible or certain; that is, recognising when situations involve chance. It is also, obviously, directly focusing on the big idea **probabilistic vs deterministic** because it is being able to recognise between the two. It is the focus of the early years of the Australian Mathematics Curriculum.

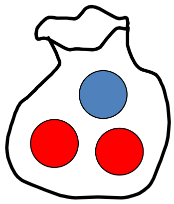
1.1 Defining Skill 1

Materials: Two dice, a bag, three counters (two red and one blue), pen and paper.

Directions: Answer these questions:

1. In a game in which two dice are rolled, is it impossible, possible, certain that I get a total which is:

   (a) 12? (b) 1? (c) less than 15?



2. A bag contains three counters, two red and one blue. I put my hand in and pick out two counters. Is it impossible, possible, certain that one of the counters picked is:

   (a) Blue? (b) Red?

3. Set up both experiments. Try the activities. Do you get what you expected?

1.2 Teaching ideas

The following points are worth making:

1. This is an opportunity to start from the local context and culture of the students. Get students to discuss when events are uncertain and have possible outcomes. Most students have many experiences with chance, e.g. card games, sport, weather. Find out the important ones in your community.

2. Give the students experience of chance or probabilistic situations – here are some examples:

   (a) *Blindperson's Pick*. Blindfold the students and place a "present" in front of them for them to select. Have different coloured and patterned wrapping.

   (b) *Lucky Dip*. Students select counters, balls, blocks from a bag or box, or draw a card from a face-down pack.

   (c) *Come in Spinner*. Students spin spinners with uneven sectors or with numbers repeated (or throw dice with number repeated).

   (d) *Dartboard*. Students throw darts (or blindly stab) at a chequered pattern board.

   (e) *Fishing*. Students fish with a magnet on a line into a box of cardboard fish (with paper clips on their noses) of different sizes and colours.

   (f) *Guessing game*. Students place fewer than 10 objects in a box for other students to guess the amount.

   (g) *Dropsy*. Students drop blocks over their shoulder into a square pattern of numbers.

3. As well as identifying chance situations, get the students to construct chance situations – this is the **interpretation-construction** big idea. Reverse everything as well – here is situation, what is the uncertainty, the area of probability AND here is the area of probability, make up a situation.

4. Discuss characteristics of chance events – look at the focus, e.g. weather, sport, games; discuss whether chance involves activity or a question; consider whether there has to be more than one outcome, look at randomness and whether this is always present.

5. Spend time looking at a chance event – for example, buying a Lotto ticket or a scratchie. The question, *will I win?*, always involves chance. Are there questions that involve Lotto tickets but do not involve probability – what about, *How much is a 'Quickpick'?* So look at changing situations from chance to certainty and back again.

6. Spend time on the **word bank** – see Teacher *Notes* – make sure students understand all the probability words such as certain, possible, impossible, event, outcome and so on. Best to do this by setting fun probability activities and discussing what happens as the activities unfold. Listen to the students, this is your chance to pick up on local words and nuances.

1.3 Classroom activities

Here are a few ideas for classroom activities to help with Skill 1.

1. **Classifying events**. Create three areas (areas on the floor, desks, etc.) and label them *impossible, possible* and *certain.* Have a selection of events for students to place in the relevant area – students could make up events and take them to the position – stand in the position showing their situation. Ask students to write three stories, one for each area. Use local stories.

   Record where the stories go on a table divided into columns by headings impossible, possible and certain.

2. **Creating a language line**. Get two students to stand on left and right in front of room and hold a rope between them, the one on left with *impossible* on a sign and the one on the right with *certain* on a sign. Label the rope *possible*. Students make up situations, summarise them on pieces of paper, tell the other students and then, with other students' help, peg the paper on the rope/line where it best fits – possible ones can be placed halfway or towards one of the ends depending on what they are. For example, *I kicked the football and it flew away* is pegged at impossible, while *I kicked the football and it fell to the ground* is certain. However, *I kicked the football and it went 40 metres* is only possible and depending on the ability of the person can be placed between impossible and certain.

   Students could be asked to form small groups in which they will judge various events as certain, impossible or possible. Events should be localised in the school community. Some examples of situations/events could be: *It will rain tomorrow; drop a rock in water and it will sink; a flower seed planted today will flower tomorrow; the sun will rise tomorrow morning; if you ask someone who was the first Mayor of their town, they will know; you will have two birthdays this year*; and *you will be in bed by 10 pm*. This is a good opportunity to bring in students' identities and cultures. Students could be asked to write events in their families that are certain, possible and impossible. The use of local language could be encouraged.

   Note: This will lead to fractions in probability – impossible will become 0 and certain 1, while possible will be a fraction between 0 and 1.

3. **Continuum**. Using two colours of paper or card stock (say red and blue), cut out circles that are the same size. Cut a radius. Slide one onto the other and rotate one circle around the other to make a spinner base. Different positions of the spinner base could be discussed in terms of fair and unfair (e.g. if you state that red is two steps and blue is one, then fair is 2/3 blue). These spinners can be added to the language line as visual locators between impossible and certain.

You can also use the spinner base to develop the notion of a probability continuum. Have students create a spinner base that is not fair (make your spinner base fair). Then give each student a paper clip and have them come out and place their spinner on a clothes line according to the chances of spinning blue. The completed probability continuum will help students see that chance can be at different places on a continuum between impossible and certain.

4. **Three-dice throw**

Suitability: Year 4–7, 2–4 players. A useful activity to reinforce all basic number facts as well as probability.

Directions: Prepare a large copy of the game board below on cardboard. Obtain three dice and piles of different coloured counters for each student.

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
8	**9**	**10**	**11**	**12**	**13**	**14**	**15**
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**
24	**25**	**26**	**27**	**28**	**29**	**30**	**31**
32	**33**	**34**	**35**	**36**	**37**	**38**	**39**
40	**41**	**42**	**44**	**45**	**48**	**50**	**54**
55	**60**	**64**	**66**	**72**	**75**	**80**	**86**
90	**100**	**108**	**120**	**125**	**144**	**150**	**180**

Rules:

- To begin play, each player in turn rolls or throws all three dice and the player with the smallest sum begins play. Play progresses clockwise around the table.

- Each player in turn throws the three dice. The player then uses the three displayed numbers to make any number which is uncovered on the board. They must use one or two operations on these displayed numbers. For example, suppose they throw a 3, 4 and 5. They could think: (3×4)+5, (3+4)×5, (3×4)-5, or 3×4×5.
- The player is then allowed to cover one resulting number on the board with one of their counters. They are not allowed to cover a number already covered. For example, suppose 60 was uncovered. Since 60=3×4×5, the player could then cover it.
- Scoring: For each number covered, the player scores one point plus an additional point for each box it touches which has been previously covered. For example, suppose a player throws a 5, 2 and 6; and 21 and 14 have been covered. Then the player could cover 22=(5+6)×2 and then score 1 point + 2 points for touching the covered 21 and 14, i.e. 3 points for that turn.
- If a player cannot make an uncovered number on the board from their throw, play moves to the next player, with no scoring.
- The winner is the player with the most points, when no more numbers can be covered.

Variations: Change the scoring – 1 point for each marker (no additional points), or 1 point for each marker touching at least one other (no additional points and no points for isolating cover).

Add a “challenge”: If a player feels that another player did not make the move that scored the most possible points, they may challenge that player. The player does not change the move, but the challenger gets the additional points the better move would have earned. If the challenger is incorrect, they lose one point.

5. **Footy kicks or Netball passes**. Take students out to sports fields and/or courts and attempt to score goals. Discuss the chance of getting a goal from a variety of distances and angles.

Unit 2: Outcomes from Probability Events

Skill 2: The student can list all possible outcomes of a chance process.

Skill 3: The student can list all possible outcomes in consecutive trials of a chance process.

Skill 4: The student can state which outcome is most likely in a single trial of a chance process.

Big idea: Contiguous vs noncontiguous

This unit looks at the three skills that deal with outcomes – determining them all, recognising what happens when they are repeated and picking most likely (and as well, equally likely and least likely). Likelihood of outcomes is one of the major bases of probability. However, for many students, it is tied up with beliefs, feelings of luckiness and misremembered experiences. There is a need to get students to see likelihood in mathematical terms, but the very nature of probability means that, when there are few trials, responses can represent low likelihoods. There needs to be a growing understanding of the need to ensure activity is random, and that there are many trials.

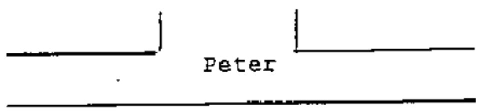
2.1 Defining Skills 2, 3 and 4

Skill 2

Materials: A die, a box of attribute blocks, a bag, pen and paper.

Directions:

1. Answer these questions:
   (a) When I roll a (common) die, what numbers could I get?
   (b) When I pick an attribute block from a bag, what colours and shapes could I get?
   (c) Peter is at a street corner, what are the possible directions for Peter to go?



2. Set up experiments for (a), (b) and (c) above.
   (a) Roll the die and select a block. Do you get what you expected?
   (b) Draw the street corner with chalk on the floor. Walk the street. Which way can Peter go?

Skill 3

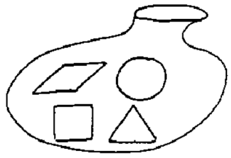
Materials: A die, a pentagon spinner, a bag with four attribute blocks, pen and paper.

Directions:

1. Answer these questions:
   (a) Last time I rolled a die, I got a 6. If I roll it again, what numbers could I get?
   (b) When I spun this spinner last, I got a 2. If I spin it again, what numbers could I get?

   (c) A shop sells three flavours of ice cream: vanilla, lime and chocolate. Yesterday Jane bought a lime ice cream. What flavours could she pick from today?

(d) A bag contains four attribute blocks of different shape. Shane picks out the triangle but does not put it back in the bag. Now Jenny picks a piece. What possible pieces could she pick?



2. Set up and try experiments (a), (b), and (d). Did you get what you expected?

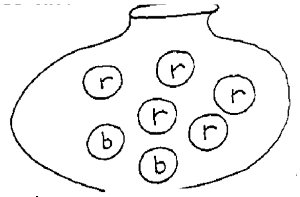
   (a) What changes in (d) would occur if the block was put back in the bag?

   (b) Why do people feel that the previous results will affect their next try?

Skill 4

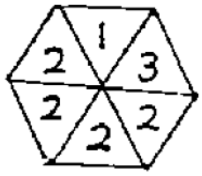
Materials: A bag with five red and two blue counters, a hexagon spinner, pen and paper.

Directions:



1. Answer these questions.

   (a) A bag contains five red and two blue counters. Sally picks one counter from the bag. Are red and blue counters equally likely to be chosen? Which colour counter is more likely to be chosen?

   (b) I spin the spinner at right once. What numbers can I get? Which number is most likely?

   (c) David's mother quickly picks a party hat from the pile containing the following:

   How many sorts of party hats could she pick? Is she more likely to pick one of the two hats on the right?

2. Set up experiments (a) and (b). Trial them 10 times. Did you get what you expected?

2.2 Teaching ideas

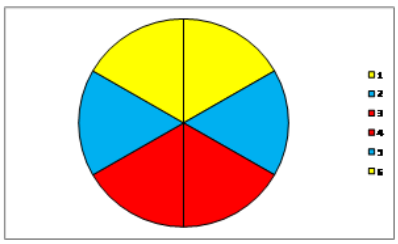
For Skills 2, 3 and 4, we need to follow the sequence:

(a) look first at one-off trials/events and then list all outcomes – check that students are able to do this before moving on;

(b) move on to consecutive trials for the same events as in step (a) above – discuss if this changes the outcomes – for example, if four heads come up from four tosses of a coin, check that students do not believe that a tail is now more likely than a head in the fifth toss;

(c) look at replacement and no replacement (particularly for card activities and selecting things from a bag activities) – check that students know that replacement leaves things the same but no replacement does not;

(d) look at contiguous and noncontiguous models in counters and in spinners – check that students are not making mistakes when things are contiguous (e.g. thinking blue on right is more likely because it has two options);



(e) go through each of these situations above but now look at likelihood of outcomes; and

(f) compare and order different events in terms of the likelihood of the same outcome.

Note: When students are competent, it is possible to progress from qualitative comparisons of more or less likely to numeric comparison and ordering. Students should have experiences comparing two or more sample spaces with the same number of outcomes and two or more sample spaces with different numbers of outcomes but the same number of favourable outcomes to ensure robust understanding.

2.3 Classroom activities

1. **N-counters**

   Materials: 24 counters (12 of each colour), two dice, board as on right, two students or two groups of students to play.

   Directions: Students choose colour. Place all their counters anywhere on the board (can place multiple counters on the same number). In turn, students throw dice and add numbers, both students with a counter on that number remove <u>one</u> counter. Winner is first player to remove all counters.

	2	3	4	
5	6	7	8	9
	10	11	12	

2. **Counter Thief**. A variation of N-counters above. Played by two players. Use a 6×2 grid numbered 1 to 12. Each player has 12 counters and they place them anywhere on the grid. Players roll two dice, add the values and remove the counters from the corresponding grid square. With each roll of the dice, students need to keep a tally record of the dice total. At end of game, provided sufficient dice rolls have occurred, students will notice a bell curve from dice rolls.

3. **Red One**

   Materials: 20 counters: 8 blue, 6 green, 4 yellow, 2 red; group of students

   Directions: The counters are spread out, students can choose a colour.

   Questions:

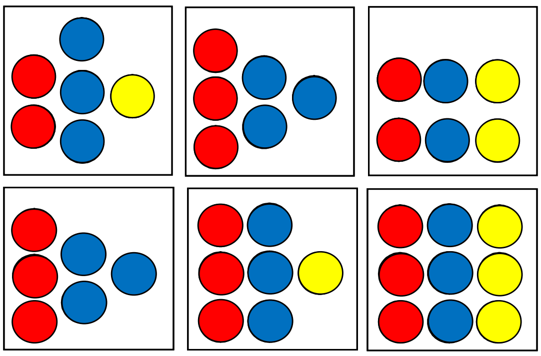
   (a) If you shut eyes and choose, will the red be as likely as the other colours?

   (b) How can you test this? Construct an experiment and test it!

   (c) For what distribution of counters would the red be equally likely?

   (d) Is there a way the counters could be set up so even with the present colours, red is equally likely? [Sort the colours into groups, number groups 1 to 4 – students select a number, then one counter from that group.]

4. **Compare and Order**



   (a) The three containers on right all have the same number of counters. From which container is red most likely to be selected? [Middle container]

   (b) The three containers on the right have different numbers of counters. From which container is red most likely to be selected? [Left container]

   Questions: Is the number of red counters important when the total number of counters is the same? Is the number of red counters important when the total number of counters is different? If not, what is important?

Unit 3: Desired Event

Skill 5: The student can correctly choose in what situation an event is more likely to occur.

Big ideas: Deterministic vs probabilistic, Concepts of a fraction

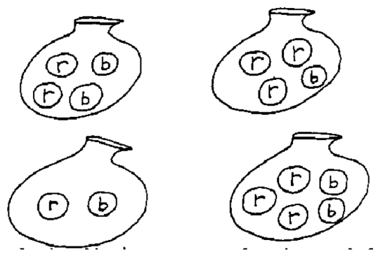
In this unit, we turn around what was in Unit 2. Instead of waiting for the event to happen and predicting its outcome, we choose the outcome we would like and find the event that has most chance of enabling this outcome to happen. It is an act of **reversing**: instead of looking from event to outcome, we look from outcome to event. That is, from *teacher gives event → students choose most likely outcome* to *teacher gives most likely outcome → students choose event.*

This is a very important unit because, in life, we continually have in mind what we would like to happen and we search for ways to achieve it. For example, we want to have a good time, so what should we do? Or we want to obtain a job, so what should we do to maximise our chances?

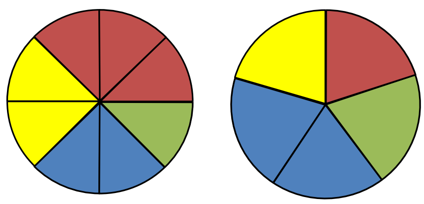
3.1 Defining Skill 5

Materials: A bag with red and blue counters, pen and paper.

Directions:



1. Answer these questions.
   (a) There are four bags of counters on the right. From which bag is Sue more likely to pick a blue counter?
   (b) A lucky dip has two good prizes and three booby prizes, while a second has three good prizes and four booby prizes. Which lucky dip is most likely to give Jack a good prize?
2. Set up experiment.
   (a) Trial each bag 10 times. Did you get what you expected?



3. I have been given these two spinners on right.
   (a) I can pick the spinner and I can pick the colour. Then I spin the spinner and score a point each time my colour comes up.
   (b) Which spinner and which colour should I choose to get to five points in the least number of spins?
4. Set up experiment.
   (a) Trial each spinner 10 times. Did you get what you expected?

3.2 Teaching ideas

The following points should be made.

1. Care should be taken to ensure students see how different this skill is to finding the most likely outcome of an event. This skill requires comparison of events in terms of likelihood of a certain outcome – sometimes when the events are very different.

2. This skill is about life – about what you do next to achieve what you want. So use it as an opportunity for your students to discuss what they want and the best ways they can see to achieve it. However, it is important not to assume things from the background of a teacher. Many students from Indigenous schools do not believe, with good reasons, that working hard at school will enable them to get a good job – they believe that it may work for others but not them. Do not apply your values onto the students – listen to what they say, and let them discuss activities that might best give them what they want.

3. It is also not important to tackle big ticket items. Simple things like what to do that night may be easier. How do they have fun? How do they stay safe?

4. This skill is the beginning of **inference** – using probability data from events to determine the best way to achieve wanted outcomes.

5. This inference is not made from ratio understanding but **fraction understanding** – not from comparing chances of wanted outcomes against other outcomes, but comparing chances of wanted outcomes against all outcomes.

6. In the modern world, Skill 5 can be excellently discussed in relation to computer games – what do you have to do to have the best chance of survival?

3.3 Classroom activities

1. **Lucky June problem**

   (a) June wanted to go on the end of year trip. Her father gave her five white and five black counters and two identical bags. He told her she could put counters in bags however she wanted but he would take the bags, mix them up and then she would have to choose the bag and one counter from it. If the counter was white she could go on the trip. If it was black she could not go on the trip.

   (b) June was clever and so she chose a white and went on the trip. How did she place the counters in the bags to maximise her chance of getting a white?

   (c) This is a simplified version of the famous "lucky Prince" puzzle. To marry the Emperor's daughter and get half his kingdom, the Prince also has to select a white marble. He is given three identical barrels and 50 white and 50 black marbles. He can place the marbles as he wishes in barrels, but in the morning, they will be mixed around and he has to choose a barrel and then a marble from it.

   (d) The Prince was also clever and won. So how did the Prince place his marbles to have the best chance of getting a white?

2. **Card Identities**

   (a) Many students will not need this discussion activity, but use it anyway to determine student knowledge of red and black cards, four suits, number and picture cards, number of cards in pack, 1/52 chance etc.

   (b) Play card games like "21" where each player is dealt two cards. They total the cards and decide if they want another card to be dealt to them. They "bust" if the combined card total is greater than 21.

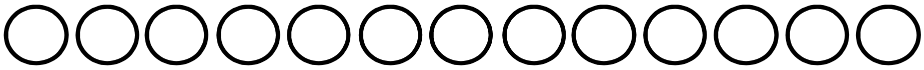
3. **Between the posts**

   (a) Deal two cards for a turn. Player looks at the cards and decides whether to call for another card.

   (b) If the call is made, it is because the player decides he/she has a reasonable chance of getting a third card that falls between the original two cards.

4. **Who wins**

Materials: 13 counters placed in a line; two players or one group of players.



Directions: Choose who goes first. In turn, remove one or two counters. Winner is the person who takes the last counter(s).

Questions:

(a) Who is more likely to win – first player or second player? Assume everyone plays well and no errors.

(b) What if players removed only one counter each time? Who is most likely to win now? What if remove one, two or three counters each time?

(c) Can you construct a game for 9 counters where first player is most likely to win?; construct a game for 10 counters where first player is most likely to lose?

Unit 4: Probability as a Fraction

Skill 6: The student can assign a numerical probability to an event.

Big idea: Theoretical vs frequentist

This unit begins the process of calculation with regard to probabilities and formally connects fraction and probability. Of the two options available, frequentist and theoretical, this unit focuses on the theoretical.

4.1 Defining Skill 6

Materials: A coin, pen and paper.

Directions: Answer these questions.

(a) Ian tosses a coin. What outcomes are likely?

(b) Are these outcomes equally likely?

(c) What is the probability of a tail?

Set up an experiment:

(a) Toss a coin 20 times. Did you get what you expected?

(b) Check you did not fall into a rhythm and keep throwing in the same way.

4.2 Teaching ideas

The basis of this unit is as follows.

1. Theoretical probability is based on equally likely outcomes. In this situation, the probability of what is wanted equals the number of outcomes giving what is wanted divided by the total number of outcomes. For example, to throw a die to get an even number means that the number of outcomes with the desired result is three while the total number of outcomes is six. Thus, the probability is 3/6 or 1/2.

2. Thus, probabilities are fractions and there is a need to follow the sequence given in **Appendix A** which describes the activity and questions needed to connect probability to fraction. It is important to reverse all relationships that emerge and construct events for particular probabilities as well as interpreting given situations in terms of probability. Thus, we need event → probability and probability → event.

3. Ensure that students realise that fraction in probabilities goes from 0 to 1 and not beyond 1. There are not probabilities of, say, 2¾! It is useful to undertake the number-line activity where a string, rope or a line is drawn from 0 to 1 and students peg/place numbers onto it. Here the numbers can be fractions, percents, decimals and probability language such as “some chance”, “possible”, “probable” and so on.

4. To ensure that calculation of fraction probabilities includes all possible outcomes, it is important to build **sample spaces** of all possible outcomes. For example, if two dice are thrown and added, the sample space is as shown below. It means, for example, that the chance of getting a 7 is the number of outcomes giving 7 divided by the total number of outcomes = 6/36 or 1/6.

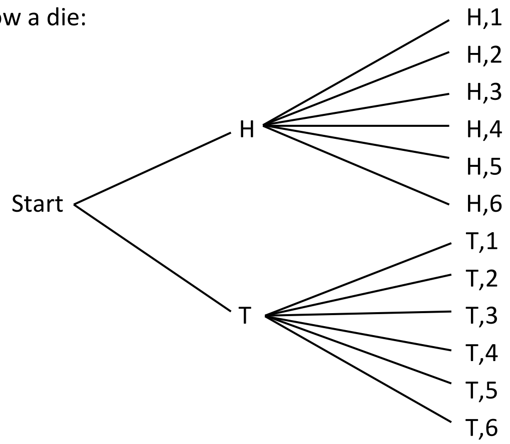
Two-dice sample space:

2	1,1					
3	1,2	2,1				
4	1,3	2,2	3,1			
5	1,4	2,3	3,2	4,1		
6	1,5	2,4	3,3	4,2	5,1	
7	1,6	2,5	3,4	4,3	5,2	6,1
8	2,6	3,5	4,4	5,3	6,2	
9	3,6	4,5	5,4	6,3		
10	4,6	5,5	6,4			
11	5,6	6,5				
12	6,6					

5. In the two-dice sample space above, if we want the probability of getting an odd number sum, we simply add the probabilities for 3, 5, 7, 9, and 11 because these events are disjoint (i.e. no outcomes in common). However, if events are overlapping, this is not so. For example, the probability of getting two numbers the same on the two dice and getting a score over 7 overlap. The results 4,4 , 5,5 , 6,6 are in both sets and so we cannot simply add the probabilities. We need to add the probabilities and then account for the overlap so that these results are not counted twice.

6. In complex sample spaces, there are often two or more stages – e.g. throw two dice, draw three cards, or throw a die and spin a spinner. In these situations a tree diagram such as that below helps with the sample space.

Toss a coin and throw a die:



4.3 Classroom activities

1. **Tug of war**

Materials: die, 1 counter, board as below, 2 players.

X						***						**Y**

Rules: Each player chooses an end (X or Y). The counter is place in position ***. Players in turn throw the die and move the counter the amount shown towards their end. The first player to reach his/her end wins.

Questions (after many games):

(a) Does it take a long time to get a winner? Why?

(b) Which player is more likely to win? Is there an advantage in going first?

(c) If the board was 100 squares long, would there ever be a winner?

2. **Planetfall**

Materials: 1 coin, counters, board as below, 2-6 players.

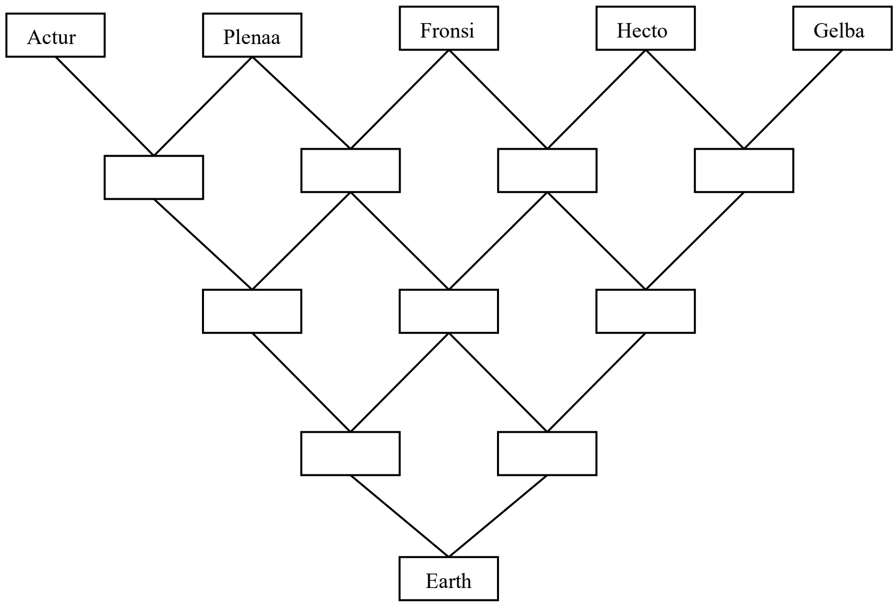
Rules:

(a) Players place counters (spaceships) at start (Earth).

(b) Players in turn toss the coin and move left if heads and right if tails.

(c) Players score 1 point for reaching Fronsi, 2 points for reaching Plenhaa or Hecto and 3 points for reaching Gelbt or Actur.

(d) The first player to make 10 points wins.



Question (after many games): What is the most likely planet to reach? Why?

3. **Feud**

Materials: 2 dice, 2 players.

Rules:

(a) Players in turn throw dice and add the numbers.

(b) If the point sum is 2, 3, 4, 10, 11 or 12, player 1 receives one point. If the sum is 5, 6, 7, 8 or 9, player 2 receives one point.

(c) The first player to 10 points wins.

Questions (after many games):

(a) Is the game fair?

(b) Does it matter whether you are player 1 or 2?

(c) What extra numbers could we give player 1 to even the contest?

4. **Making Fair**

Materials: A page of spinners with different areas for yellow, red, blue. Racetrack game board (see Appendix C) with tracks marked yellow, red, blue.

Activity:

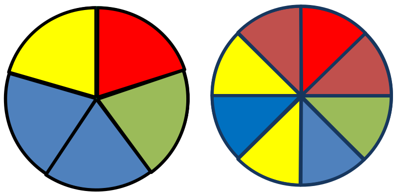
(a) Choose which of the spinners would be fair.

(b) Decide which colour is likely to win with the other (unfair) spinners.

(c) Choose an unfair spinner and make it fair by having different colours or moving different amounts.

Notes:

(a) Make noncontiguous as well as contiguous spinner, e.g. the left spinner is contiguous and the right spinner is not.



(b) The spinner on the left is not a fair spinner as there is twice the chance for blue as for each of the other colours. However, it would be made fair if red, yellow and green meant move two spaces while blue meant move only one space.

(c) If the spinners were given numbers instead of colours then could have two spins and move the sum. Then can look at what is possible in sums and whether sums can be fair.

5. **Internet games**

(a) There are many games of automatic dice rolls, card games, roulette, and so on available to use in the classroom.

(b) Ensure the students have pencil and paper handy to record outcomes, as it is possible learning opportunities will be missed if the students are simply playing the games without reflection.

Unit 5: Experimental Probability

Skill 7: The student can estimate the numerical probability of an event using "simulation" methods.

Big idea: Frequentist vs theoretical

This unit follows the theoretical one of Unit 4. However, now the method to obtain the probability as a fraction is frequentist – found from experiments.

5.1 Defining Skill 7

Materials: Thumbtack, pen and paper.

Directions:

(a) Toss a simple spinning top 50 times and record the number of times that it lands point up and point down in a frequency table.

Outcome	**Tally**	**Frequency**
Point up		
Point down		

(b) Use your results to estimate the probability of a spinning top landing point up, and point down, in a single toss.

5.2 Teaching ideas

Some points to consider are as follows.

1. It is useful to begin with experimental probability by constructing experiments where outcomes are equally likely before moving on to not equally likely examples. In this way, experimental and theoretical can be connected. It also allows the experiments to be validation of the theoretical outcomes.

2. This approach is also useful because it shows that experiments often give different results to theory. This leads to important discussions regarding why experimental (observed) and theoretical (expected) probabilities may differ. One is chance but another is that the experiments become non-random or there are too few trials.

3. This leads to spending time on ensuring that: (a) randomness is retained in experiments that do not have the backing of theory; and (b) sufficient trials are completed to remove extreme probabilities. One needs to check that tosses produce the best possible random results (this often involves bouncing the thrown object back off a wall – see activity 1 in 5.3).

4. Once skill is acquired, students should move from one-step to two or more step experiments. This will lead to the need to undertake complete sample spaces, and to the use of tree diagrams, two-way tables and Venn diagrams.

5. As the number of trials is important to accuracy in probabilities, technological simulations become important tools for detailed investigations.

5.3 Classroom activities

1. **Tossing coin**

   Materials: One coin.

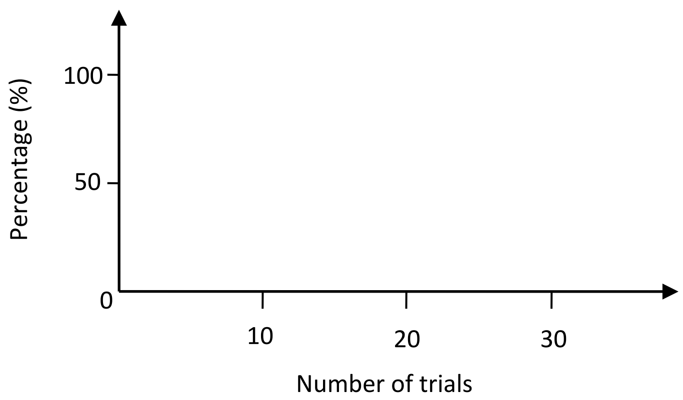
   Directions:

   (a) A trial consists of tossing a coin and recording how it lands. Perform some preliminary flips to determine the best technique for tossing which obtains random landing positions.

   (b) Perform an experiment of 30 trials. Record results after each trial; tally the results, then record the numbers and percentage cumulatively.

Trial	Results		Cumulative record			
	Head	Tail	No. of heads	No. of tails	% of heads	% of tails
1						
2						
and so on						

   (c) Obtain two sheets of graph paper. Draw a line graph using axes as below:



   Questions:

   (a) Which of heads or tails occurred the most in the 30 trials? Would this remain the case if we completed more trials?

   (b) Was there a trend in the graphed percentage? Would this trend give rise to a reasonable estimate of probability? Is this what you expected?

2. **Two-dice difference**

   Materials: Two dice, pad and pencil.

   Experiment: Roll two dice. Calculate the difference between the uppermost faces – take low from high.

   Questions: What difference is likely to occur most frequently? Least frequently? What differences can occur? What is the probability of these differences?

   Procedure:

   (a) Investigate these questions and record your results and findings on pad.

   (b) Devise an experiment to test your conclusions. What did you find? Were your expectations confirmed? How could you improve your experimental procedure?

   (c) If you were the banker in a gambling game of two-dice differences, what odds would you strike for each difference?

3. **Biased Die**

Materials: Cubes with sides as follows: 1, 1, 2, 3, 3, 3.

Directions:

(a) Create a biased die with numbers e.g. 1, 1, 2, 3, 3, 3. One die for each student or in pairs, but students do not get the opportunity to "study" the arrangement of numbers.

(b) Repeatedly roll the die and record the numbers that show up. Students keep rolling until they think they know the number arrangement. A pattern may appear after 10 rolls, where students can make a hypothesis as to what numbers are featured on the die.

(c) After 20 rolls they may be in a position to make a statement as to what the number arrangement is.

(d) Clear frequencies will be seen with over 50 rolls of tally data.

4. **Cube toss**

Materials: Cubes with sides as follows: 1, 1, 2, 3, 3, 3.

Directions:

(a) Ask students what they might get when they roll the cube. What is likely, what is impossible?

(b) Have students roll the cube and record results in bar graph as shown below. Stop when a bar is full. Discuss the findings.

1										
2										
3										

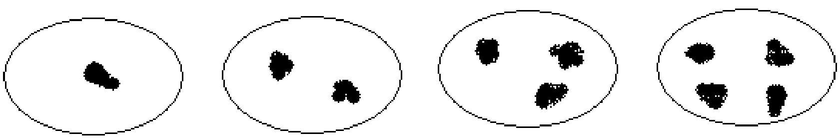
(c) Now introduce the notion of rolling two cubes and recording the sum of the two cubes. Have a short discussion where students predict which bar will fill the fastest or if they will be equal.

(d) Roll the cubes, stop when a bar is full. Discuss the findings.

1										
2										
3										
4										
5										
6										

5. **Spots on rocks**

Materials: Four flat stones or rocks with 1, 2, 3 and 4 in the form of dots on one side and nothing on the other side (see below).



Directions:

(a) The game involves the tossing of four rocks and calculating the resulting score.

(b) Each player tosses the rocks, and keeps a cumulative score (checked by other players if necessary). The first player to reach **exactly** 50 wins.

(c) In a discussion, ask students what sums are possible? What sums were common? What scores are possible in a single turn?

(d) What are all the outcomes (possible combinations of stones)? What is the probability of each score? (*Note*: This game is good for exploring the notion of independent events.)

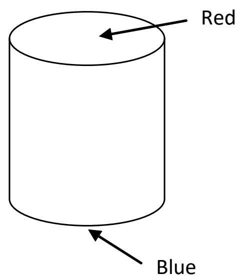
(e) As a variation for scoring, as students toss their way to 50, the computation can be varied once they have achieved at least 40 points. The students can use subtraction, multiplication and division to reach exactly 50.

(f) Again discuss what computations are possible? What computations were common? What scores are possible in a single turn?

(g) What are all the outcomes (possible combinations of stones)? What is the probability of each score?

6. **Cylinder experiment**

Materials: Cylinders of different heights, red and blue coloured marking pens.



Directions:

(a) Mark the ends of the cylinders red and blue, as shown on right, with a marking pen. A trial consists of flipping the cylinder and recording how it lands. Perform some preliminary flips to determine the best technique for flipping which obtains random landing positions. Once you've worked this out, stick to it. (Hint: It is best to throw the cylinder against a wall.)

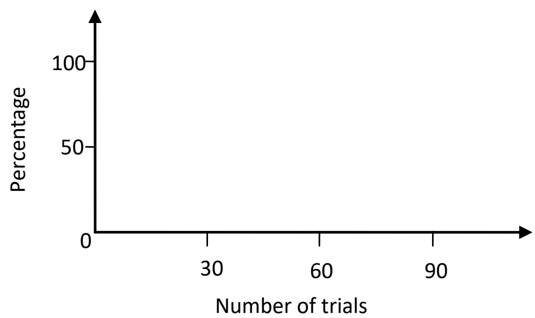
(b) Perform an experiment of 30 trials with one of the cylinders. Record results after each group of three trials. First tally the results and then record the number and percentage of each outcome cumulatively after each group of three trials (use table below).

Trials	Result			Cumulative record					
				Red end		Blue end		Curved Surface	
	Red end	Blue end	Curved Surface	Number	%	Number	%	Number	%
3									
6									
and so on									

(c) Find two other groups using the same size cylinder and copy their results. Record their tallies, then cumulatively add these tallies to your number and percentage columns to gain results of 90 trials.

(d) Repeat directions (b) to (c) with the second cylinders (of differing height), using another version of the table above.

(e) Draw line graphs of the cumulative percentages for both cylinders on graph paper using axes as on right:



Questions:

(a) Which outcome occurred most frequently for the first cylinder? For the second? Would this be the same if we completed more trials? Is there the same chance of getting a blue as a red end? Should there be?

(b) What were the differences in frequency of outcomes for the two cylinders? Is this the difference you would expect taking into account the different height–diameter ratios? Suggest a relationship between the geometry of the cylinder and the frequency of outcomes. How could you test this?

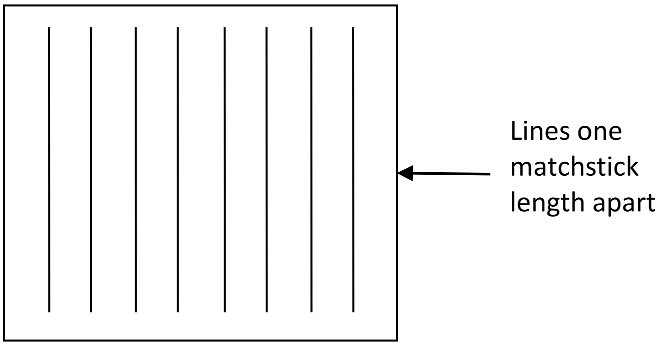
(c) Is there any trend in the graphed results? Will this trend give rise to a reasonable estimate of probability? Do these probabilities accord with your expectations for the cylinders? Why?

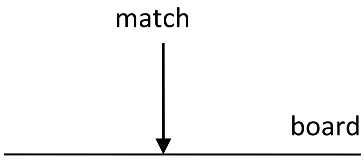
Notes: Using the probabilities worked out above; determine the probability of getting three blue ends in three successive trials. How would you test this? List five other experiments that could be conducted and for which there is no easy theoretical answer.

7. **Match sticks**

Materials: Box of matches, pad and pencil, ruled board as below.

Experiment: Hold the matches vertically about the centre of the board. Drop the matches (shown below).





Question: What is the probability of a match falling across a line?

Procedure:

(a) What would you guess the probability to be?

(b) Design an experiment to test your guess and record your work on your pad.

(c) What factors influence the result of your experiment? Can they be controlled?

(d) What effect would doubling, halving, and so on, the distance between the lines have on probability? Test and record your results.

Notes: Here is some "Pi in the Sky". If $\frac{2n}{d \times f} =$ ____, where n = number of matches dropped, d = distance in matches between lines, f = number of matches on line, what is _____? Use your results to find _____.

Unit 6: Inference

Inference is the last section of this module. It is the ability to use probability data to make decisions and predictions regarding probabilistic situations and frequencies of different outcomes.

6.1 Teaching ideas

1. The focus in inference is on making decisions from experiments. This means that time is needed for the exploration and thinking. It also means that a wide variety of methods should be used to explore the data (e.g. tables and graphs) and care should be taken in gathering the data (care with ensuring randomness).
2. Ensure that all activity is reversed. In other words, teaching that is the teacher providing materials for the students to find probabilities should be, in part, turned around so that some of the time the teacher provides the probabilities and the students come up with the materials and activity that reflects that probability.
3. Such inferential investigations have the ability to handle large data sets and so this adds in the opportunity for the mathematics to start to involve analysis of a wide range of data, analysis that is necessary to solve today's problems.
4. Inferential activity has a large aspect of problem solving and requires confidence, self-efficacy and resilience.

6.2 Classroom activities

1. **Guess what is in the bag**

   Materials: One bag, 10 blocks of different colours, groups of students.

   Directions: Place blocks in bag, draw one out at a time and record. Replace and shake bag. Repeat this. Use record to predict what is in the bag.

   Notes:

   (a) If students find this difficult, have fewer blocks and fewer colours.

   (b) Talk to students about how many times they should repeat the drawing of the block. Why would multiples of 10 be good?

   (c) If student gets answer wrong, ask to do more draws, discuss making draws random.

2. **Inference from experimental probability**

   Materials*:* Bag, small counters or pegs of two colours, another bag with 14 counters of three colours (these bags need to be prepared ahead of time and labelled with the total number of counters so that students do not know ahead of time what colours are in the bags), notebook, pen, graph paper, hand calculator.

   Directions:

   (a) Place one counter of one colour and two counters of a second colour in the bag.

   (b) A trial consists of shaking the bag, selecting a counter (without looking) and replacing the counter in the bag (without looking). Ensure the bag is well shaken.

   (c) Perform an experiment of 30 trials. Record results cumulatively after each trial.

(d) Complete a table with headings as below:

Trial	Result		Cumulative Record			
	1st colour	2nd colour	1st colour		2nd colour	
			Number	%	Number	%
1						
2						
3						

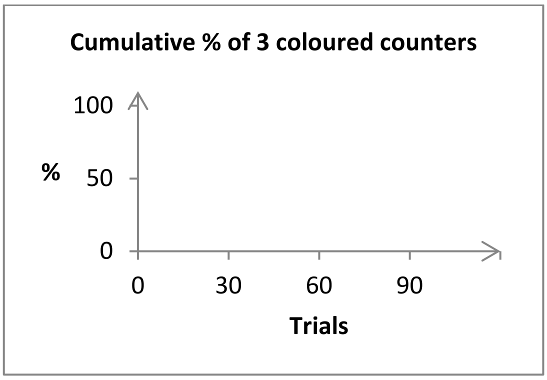
(e) Perform a second experiment of 20 trials with the second bag containing 14 counters. Record results cumulatively after each group of 5 trials.

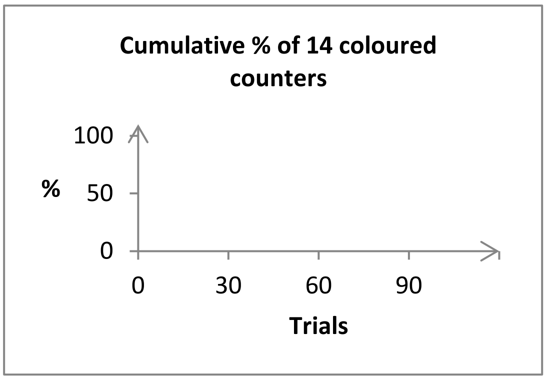
(f) Find three other groups who have finished this experiment and record their tallies and then work out your own cumulative results (making 80 trials in all).

(g) Complete a table with headings as below (use your calculator).

Trial	Result			Cumulative Record					
	1st colour	2nd colour	3rd colour	1st colour		2nd colour		3rd colour	
				Number	%	Number	%	Number	%
1									
2									
3									

(h) Draw line graphs for all colours in both experiments using axes similar to below (use graph paper).





Questions:

(a) From your results, what is your estimate of the number of counters of each colour – in the first experiment (3 counters)? In the second experiment (14 counters)? How could you improve this estimate?

(b) Do any trends evident in the graphs enable you to make an even more accurate determination of the number of blue counters? Why?

(c) Open the second bag. Check its contents. Was your estimate right? Why or why not? Calculate the theoretical probabilities for each colour in the first and second bags. Were these close to the experimental probabilities? Why or why not?

3. **Peggie's Pick**

Materials: Bag containing pegs (2 red, 4 blue, 8 green), pad and pencil.

Experiment: Shake the bag well. Select a peg without looking. Record its colour. Replace the peg.

Questions: What colour pegs are in the bag? If there are 14 pegs, how many of each colour are there?

Procedure:

(a) Design an experiment to investigate the above questions and record all work in your pad.

(b) How could you improve your estimate of the number and colour of pegs? Would more trials help?

(c) Open the bag. Check its contents. Were you right or wrong? If you were wrong, what factors could have influenced the results of your experiment?

Activities: List three other experiments to give, as above, experiences of inference.

4. **How many beads in the container?**

Materials: Plastic or glass container full of beads (with lid taped down), pad and pen. Any measuring device you need.

Procedure:

(a) Guess the number of beads in the sealed container. How accurately can you guess? Within 100, 50, 20, 10, ... of the actual number?

(b) Devise a method which will give a better estimation than guessing. Record your work in your pad. The sealed container must not be opened. You may use another container and beads.

(c) What factors influence your calculation? Can you think of other better methods?

Activities: List other situations in which estimation using the above method is required.

5. **Footy kicks or Netball passes**

This activity is a repeat of goal shooting games played earlier, however at the conclusion of the module, it is expected that the language and representations relating to the goals would contain accurate probability description.

(a) Determine the chance of getting a goal from a variety of distances and angles.

(b) Have students keep a record of shooting positions and draw diagrams.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the probability item types

The probability item types have been divided into six subtests, to match the six units in this module. The six units are in sequence from easy to hard or Prep to Year 9. Thus, the pre-test should start with Subtest 1 item types and continue through the subtests in sequence until it reaches where students can no longer do the items and there is continuous failure. The post-test should include all subtests.

Subtest item types

Subtest 1 items (Unit 1: Probabilistic situations)

1. Your teacher says: It will be a nice sunny day for our sports day next Friday.

   Tick the correct box to tell whether:

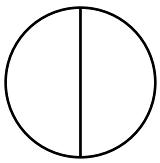
   Fridays are always sunny. ☐

   Fridays are sometimes sunny. ☐

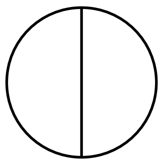
   Fridays are never sunny. ☐

2.

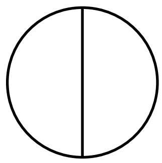
(a) Colour the spinner below so that you will be **certain** to spin red.



(b) Colour the spinner below so that it is **possible** but not certain that you will spin red.

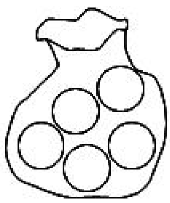


(c) Colour the spinner below so that it will be **impossible** to spin red.

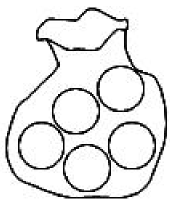


3.

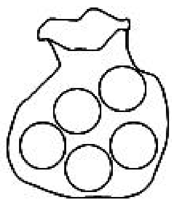
(a) Colour the marbles so that your friend would have **no chance** of picking a blue marble without looking.



(b) Colour the marbles so that your friend would have **every chance** of picking a blue marble without looking.

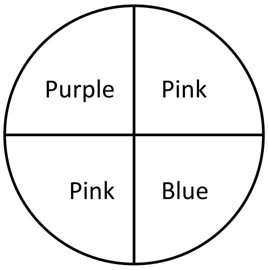


(c) Colour the marbles so that your friend would have **some chance** of picking a blue marble without looking.



Subtest 2 items (Unit 2: Outcomes from probability events)

1. (a) If you were playing a game with this spinner, what colours could you spin?



Circle the colours:

Green Purple Pink Blue Red Orange

(b) Would you always spin pink? Yes / No

(c) Is this a fair spinner to use in a game? Yes / No

Why or Why not? ____

____

2. (a) What numbers could you get if you were playing a game with this normal die?

____

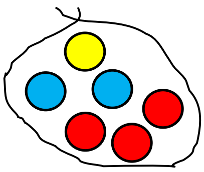
(b) Is it harder to get a 1 than a 6? Yes / No

(c) Could you **sometimes** get a 7 on a normal die? Yes / No

(d) If you rolled a die 600 times, circle how often you would expect to get a 6:

10 times 50 times 100 times 300 times

3. If you were to close your eyes and select a coloured marble from the bag, circle the colour you think you'd **most likely** get and write why.

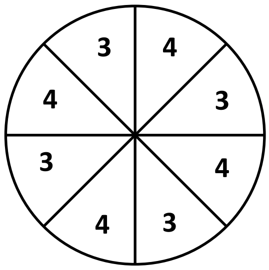


Blue Red Yellow

Because ____

Subtest 3 items (Unit 3: Desired event)

1. Using the spinner shown here, circle TRUE or FALSE for each question:



   (a) You have an equal chance of getting a 3 or a 4.

   TRUE FALSE

   (b) Getting a 3 is **less likely** than getting a 4.

   TRUE FALSE

   (c) Getting a 3 is **more likely** than getting a 4.

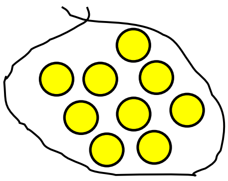
   TRUE FALSE

   (d) You will **always** get a 4 after you get a 3.

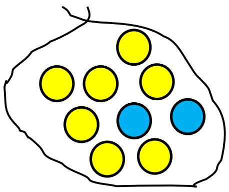
   TRUE FALSE

2. (a) Is there a bag of marbles (shown below) from which you could be certain to pick (without looking) a yellow marble? Yes / No

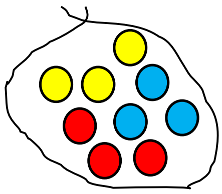
   (b) If Yes, which bag/s (A, B, C)? _____________



Bag A



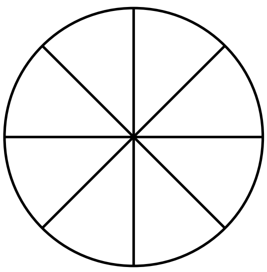
Bag B



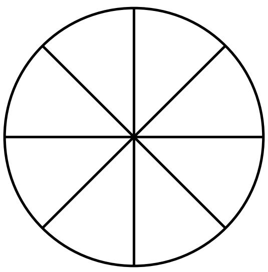
Bag C

3.

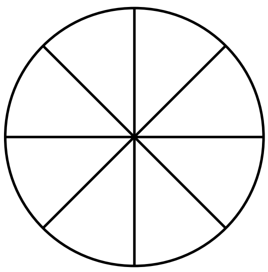
(a) Colour the spinner below so that you will be **certain** to spin red.



(b) Colour the spinner below so that it is **possible** but not certain that you will spin red.

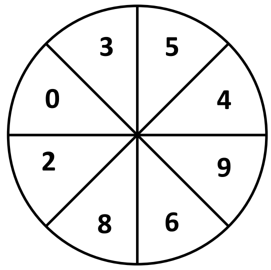


(c) Colour the spinner below so that it will be **impossible** to spin red.



Subtest 4 items (Unit 4: Probability as a fraction)

1. (a) If you were playing a game with this spinner, what numbers **could** you spin? ______________________________



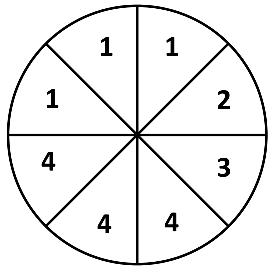
   (b) Is this a **fair** spinner to use in a game? Yes / No

   Why or Why not?

   ____

   ____

2. (a) If you were playing a game with this spinner, what numbers **could** you spin? ______________________________



   (b) Would you **always** spin 4? Yes / No

   (c) Is this a **fair** spinner to use in a game? Yes/ No

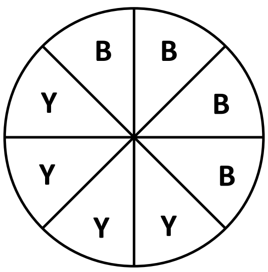
   Why or Why not?

   ____

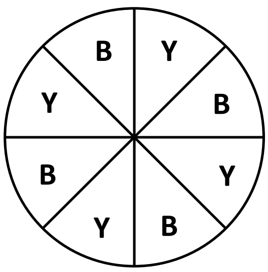
   ____

3. Look at the spinners below and answer the questions.

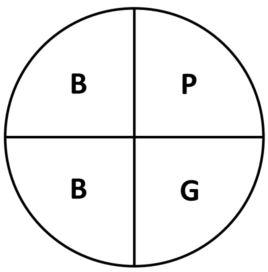
Spinner A



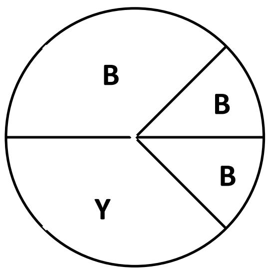
Spinner B



Spinner C



Spinner D



   (a) On which spinner(s) is it impossible to spin purple (P)? __________________________

   On which spinner(s) would you be:

   (b) more likely to spin blue (B) than yellow (Y)? __________________________________

   (c) less likely to spin green (G) than blue (B)? __________________________________

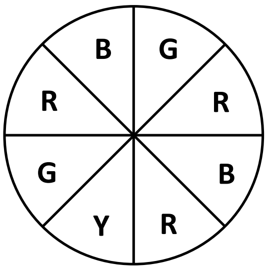
   (d) just as likely to spin yellow (Y) as blue (B)? __________________________________

4. Draw lines to join the picture and the correct statement.

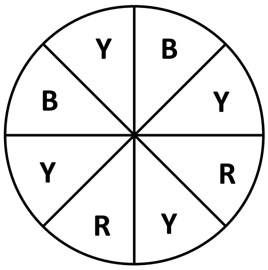
(a) On this spinner, you will be just as likely to spin red (R) as blue (B).

(b) On this spinner, you will be more likely to spin red (R) than blue (B).

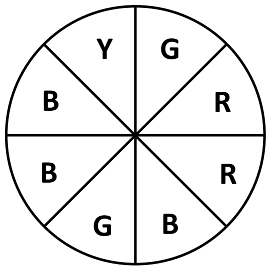
(c) On this spinner, you will be less likely to spin red than blue



Spinner 1



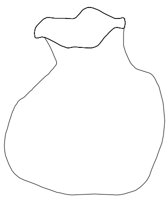
Spinner 2



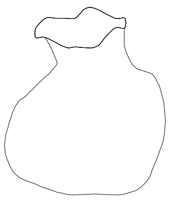
Spinner 3

5. Draw marbles in the bags so that the probability of getting a red marble is:

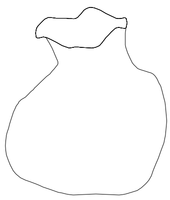
(a) $^{2}/_{5}$



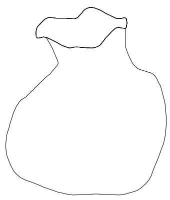
(b) 1



(c) 0

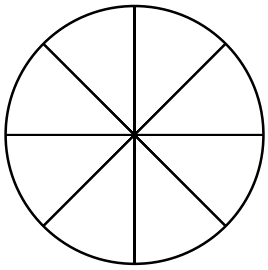


(d) more than $^{1}/_{2}$



Subtest 5 items (Unit 5: Experimental probability)

1. Using purple, pink and orange, colour the spinner below so that you would be most likely to get pink and just as likely to get orange as purple.



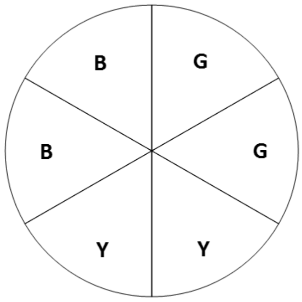
2. Tick the spinner that will give you a better chance of getting blue (B).

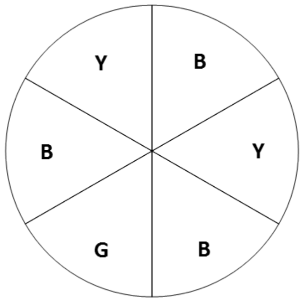
   Explain your choice:

   ____

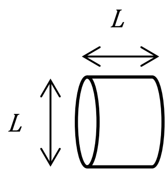
   ____

   ____





3. Cut a piece of dowel so that the length is the same as the diameter. as shown.

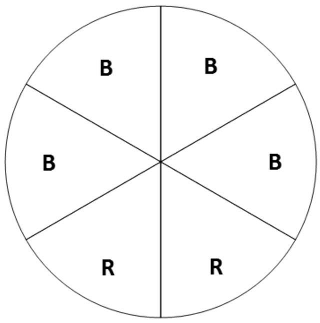


   Colour one end red and the other end blue. Throw dowel at wall and see how it lands on the floor. Record red (R), blue (B), and curved (C) as the landing positions. Find the experimental probability for this dowel.

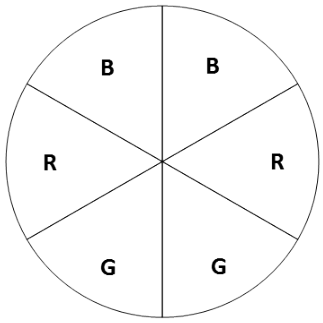
Subtest 6 items (Unit 6: Inference)

1. Draw a spinner and write the numbers 1, 3, 6 and 9 so that you would be most likely to get a 9, just as likely to get a 3 as a 6, and least likely to get a 1.

2. Does Spinner A give you the same chance of getting red (R) as Spinner B? Yes / No



Spinner A



Spinner B

Explain your choice: ____

____

3. If you were to flip a coin 100 times, write how many heads and how many tails you would be likely to get:

(a) Heads? _______________

(b) Tails? _______________

Explain your thinking: ____

____

4. If you are playing a game with two dice, would you be more likely to roll a 7 or 11? _________________

Explain your thinking: ____

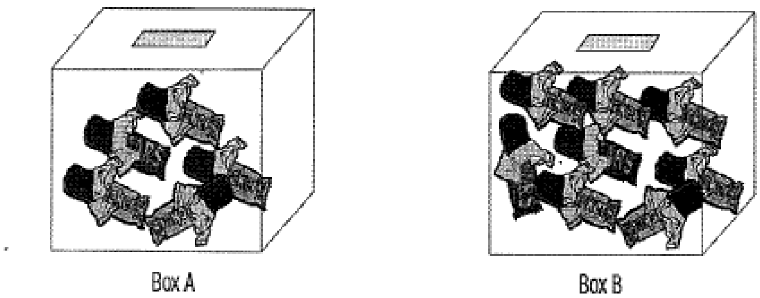
____

5. Draw lines to connect these words to an appropriate place on the number line below.

Impossible certain unlikely likely even chance more chance 50-50

0 ____ **1**

6. Box A below has 1 Mars Bar and 4 Snickers; Box B has 2 Mars Bars and 6 Snickers. (You cannot see through the boxes).



If you want to get a Mars Bars when you take one out, which box is best to choose from, or do they both give you the same chance of getting a Mars Bar?

Tick your answer: Either box ☐ Box A ☐ Box B ☐

Explain your thinking: ____

____

____

7. Rhonda is thinking of a number between 1 and 10 and challenges Tom to guess it. She said, “I’ll give you a clue. The number is greater than 6.”

Has this clue given Tom a better chance of guessing the number or would it not make any difference?

Tick your answer. Has helped ☐ Hasn’t helped ☐

Explain your thinking: ____

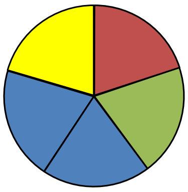
____

____

Appendix A: Connecting Probability to Fraction

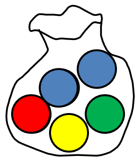
The major connection is between **probability and fraction** (common, decimal, percent). Because probability and fractions share similar concepts and because probability answers are recorded as fractions, teaching probability requires the teacher to help students make the connection between fractions and probability. The following example shows how these two mathematical domains can be connected pedagogically as well as mathematically.

Aim: To lead the students to discover the probability of an event occurring (informal language and recording) using an area model and a set model.



What are the chances of spinning blue on this spinner?

[*2 chances out of 5 chances*]



What are the chances of getting, without looking, a blue marble from this bag?

[2 *chances out of 5 chances*]

Stages/Questions that can be asked:

Identify the whole (i.e. the sample space)

- What colours *could* you spin on this spinner? Would it be *possible* to spin red? purple?
- What colours *could* you get from this bag of marbles? Would it be *possible* to get a pink marble?

Examine the parts for equality

- Has the spinner been divided into equal parts? Would the pointer be *just as likely* to stop on one part as on any other part? (OR: Would you have the *same chance* of stopping on any of the parts?)
- Are all the marbles equal, that is, the same size and shape? Would you be *just as likely* to get one marble as any other marble? (OR: Would you have the *same chance* of getting any of the marbles?)

Name the parts (establish the total number of chances, that is, the denominator)

- How many equal parts does this spinner have? How many chances do you have altogether of spinning a colour?
- How many marbles in this bag? How many chances do you have altogether of getting a colour?

Determine the parts to be considered (the outcome preferred, that is, the numerator)

- How many blue parts are there? How many chances do you have of spinning blue?
- How many blue marbles are there? How many chances do you have of getting a blue marble?

Associate the two parts with the fraction name (the probability)

- What chance do you have of spinning blue? (2 chances out of 5 equal chances)
- What chance do you have of getting a blue marble? (2 chances out of 5 equal chances)

Record the probability: 2 fifths (informal); $^2/_5$ (formal), 0.4, 40%

Appendix B: Rich Tasks

Rich Task 1: Fair Game

Question: Is this game fair? If it is not fair, how would you change it to make it fair?

Directions: Play this game with a friend. You need a coin.

Rules:

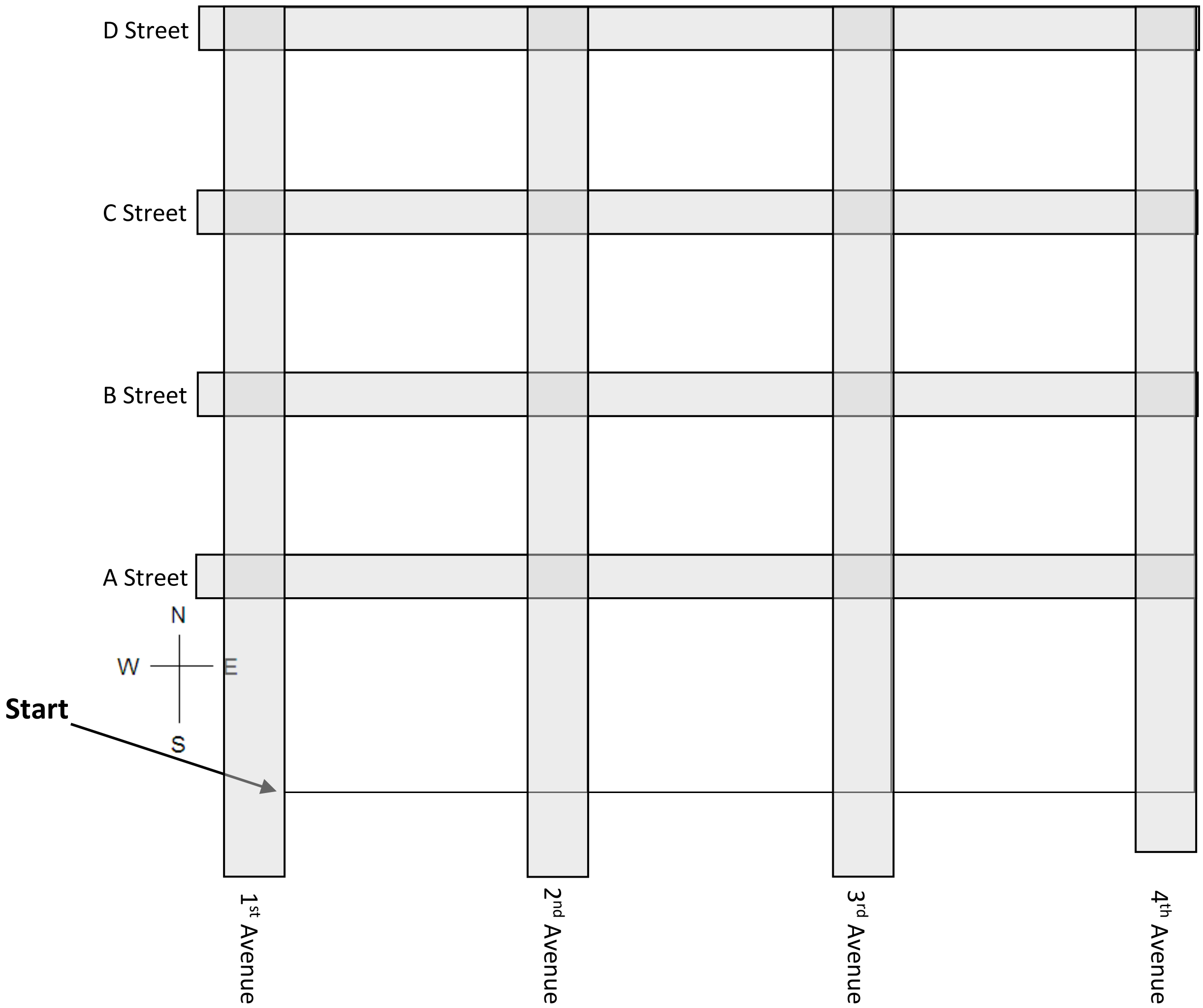
(a) To decide who goes first, each player tosses the coin. The first player to toss two heads in a row goes first.

(b) To take a turn, toss the coin. From start, move 1 block north for heads, 1 block east for tails. Toss and move two more times.

(c) Player 1 gets one point by ending up on either 4th Avenue or D Street. Player 2 gets one point by ending up on only C Street.

(d) The player who gets more points wins the game.

(e) Write a report. Give proof to support your conclusion about the fairness of the game.



Notes: In this task, students explore the concept of chance events in a game setting. They need sufficient playing time to decide if the game is fair or not, and then further time to change the rules if they need to make it fair. Although students will know how to take turns and play a board game, they could need assistance in analysing the probable outcomes of the game. The task can be given at any time of the module. Students have the opportunity to use a tree diagram to show their working – possible outcomes.

Presenting the problem

(a) Demonstrate the rules where the players start at 1st Avenue and A Street. The games piece is played on the lines or roads, not on a rectangle as this is a housing block.

(b) Students need to explain their thinking fully. Encourage students to make sketches, charts, tables or lists to make their reports clear.

Assessment criteria

(a) The reasoning used to decide if the game is fair.

(b) How clearly they present their thinking.

(c) If they decide the game is unfair, whether they are able to change the rules to make the game fair.

Prompts to get students started

(a) Who is more likely to win? Can you explain why?

(b) Which player would you prefer to be? Why?

(c) What makes the game fair or unfair?

(Rich Task modified from: Westley, J. (1994). *Puddle questions: Assessing mathematical thinking (Grade 7).* Creative Publications, California, USA)

Rich Task 2: Baby Boys

In 1998, the news that 22 boys were born in a row at King Edward Memorial Hospital hit the headlines. Doctors worked frantically from 3:30 am on August 20 to 10:53 pm on August 22, to deliver the same gender one after the other. This was a great surprise, so many boys and not a single girl delivered during this short period.

Problem: Discuss the probability of such an event happening. Design a simulation, which looks at a similar problem of 50 consecutive births, and determine how many times a run of three boys would occur.

Planning:

(a) What do you need to know before you begin?

(b) What do you need to do to solve this problem?

(c) What assumptions do you need to make?

(d) What do you predict you will find?

Solving: Follow your plan to solve the original problem.

Results:

(a) Accurately summarise your results. What did you discover?

(b) Was your prediction correct? Why? Why not?

(c) How do the girls' results compare with the boys' results?

(d) How could you extend this investigation further?

Appendix C: Game Boards

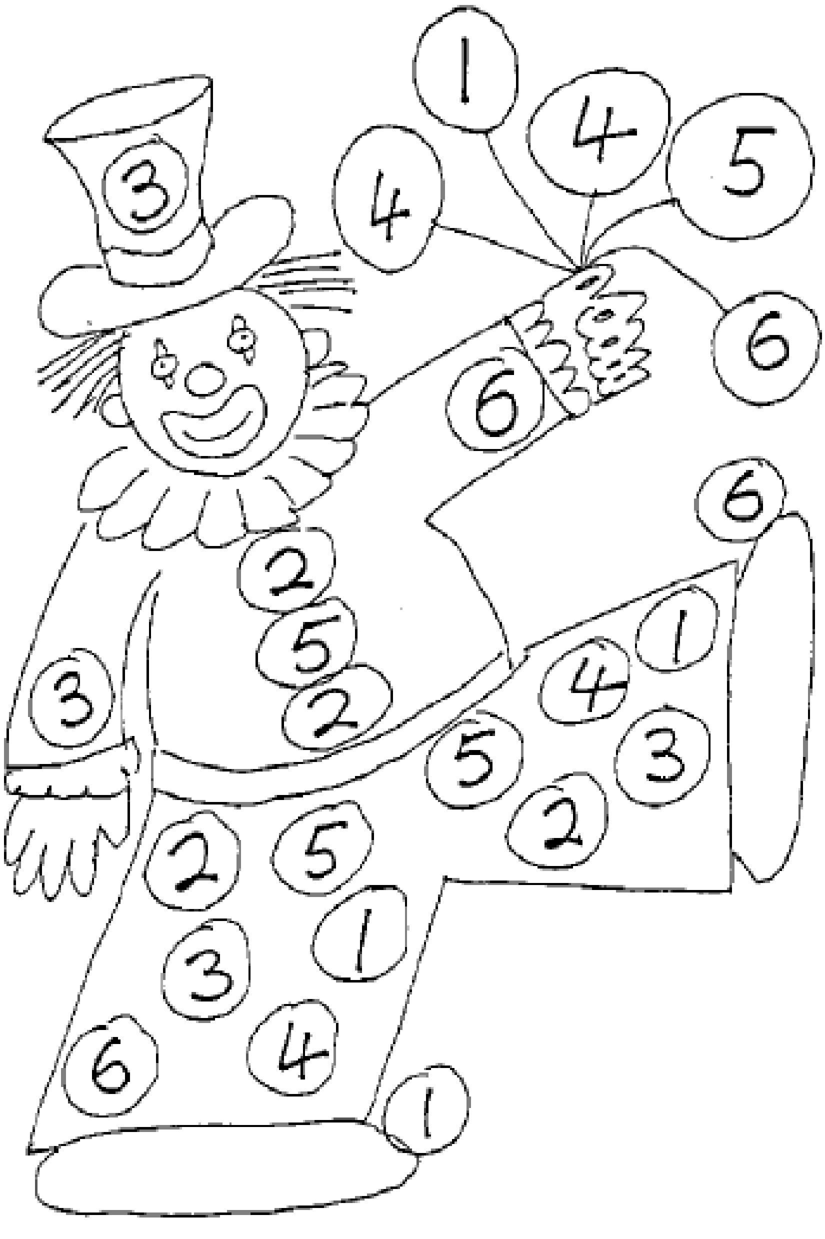
Decorate the clown game

Materials: Playing sheet for each player, counters, die.

Number of players: 2, 3, 4

Rules:

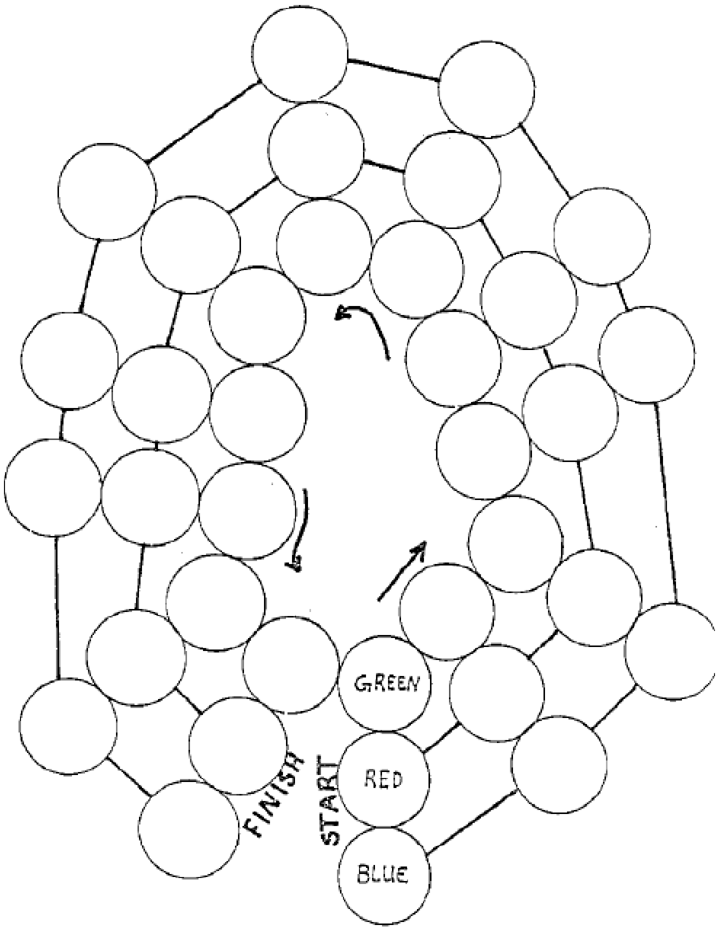
- Each player takes a playing sheet showing a clown and small counters.
- Players take turns to roll the die and then cover the matching numeral on the clown with a counter. For example, if a 3 is rolled you may cover a balloon, one of the 3 spots on the pants or the pompom on the shoe.
- The winner is the first person to decorate the clown completely.



Race game boards

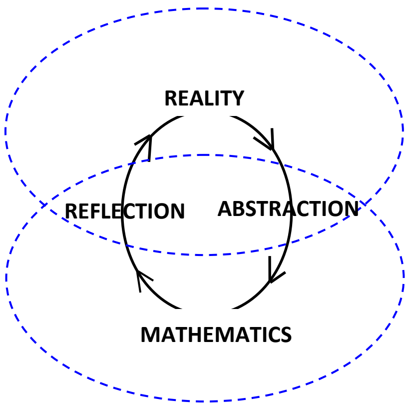
Can be used with fair spinner/die or unfair spinner/die.

FINISH	FINISH	FINISH
START	START	START



Appendix D: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix E: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system **N2: Decimal Number Numeration** Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches **SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae **N3: Common Fractions** Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships **O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject **N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference **G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs **N5: Directed Number, Indices and Systems** Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio **G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation **SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics **O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

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