Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module N3

Year B, Term 1

Number:

Common Fractions

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

In Year A, modules were presented that covered N1 *Whole Number Numeration* and N2 *Decimal Number Numeration*. This module extends these two modules by covering numeration for common fractions. This module is based on the five numeration big ideas and associated concepts and processes that held for whole and decimal numbers.

This overview covers background information and sequencing for common fractions and alignment with the *Australian Curriculum: Mathematics*. Background is an important section because of the central place that the idea of common fraction holds in mathematics. It underlies decimals in the way it enables the second movement from the one – from one to parts.

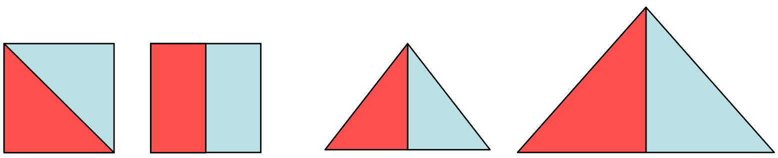
Background information for teaching common fractions

This section covers the partitioning and unitising basis of mathematics, the way that common fractions are connected to other topics and strands, and the big ideas, concepts and processes at the basis of their teaching.

Partitioning and unitising

The basis of fractions is partitioning wholes into parts. The identification of the whole is a vitally important aspect of working with fractions and lack of attention to what the whole is or was can lead to misunderstandings about fractions. Students need a wide range of experiences splitting both collections of discrete objects and continuous wholes into equal parts. Collections can be partitioned into fractions by sharing or equal dealing out. Continuous quantities can be partitioned into fractions by cutting, folding, pouring or weighing. It is also valuable for students to work in reverse and to take a number of equal-size parts and use them to re-form wholes. This skill of being able to see parts as a whole is called *unitising*. Unitising is essential for part of a group because the set of objects has to first be unitised (seen as one whole) before it can be partitioned.

As well as partitioning and unitising, it is crucial that parts being partitioned to and unitised from are equal – in area as well as length. However the parts do not need to be the same shape. It is the quantity or size that needs to be the focus. When fractions of different wholes are to be compared the original whole needs to be the same. In the figure below the two squares are the same whole so the halves of each shape will be the same. However, as the two triangles are not the same size, halves of these two shapes are not the same size, even though they are the same shape.



Comparing fractions of different wholes

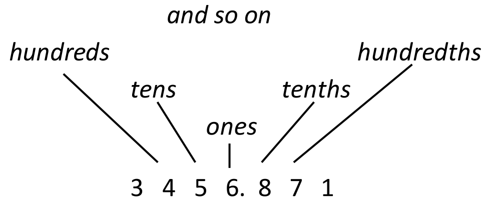
Fractions involve partitioning wholes into parts. When this is done, it is important, for fractions, that **the whole is maintained as the unit** and that **the part does not become the whole or the unit**. This is part of the whole-part principle. Thus, for a fraction such as ¾ to be understood, ¾ must be seen as **three parts out of four parts as one whole**. Three parts out of four parts, without seeing the parts as the whole, is the ratio 3:1.

Note: Many cultures do not have fair shares being equal in a mathematical way. If using “sharing” always speak to an elder or a community leader as to cultural meanings.

Connections between fractions and other topics

There are major connections between fractions and other numbers and topics as follows:

1. **Fraction and whole and decimal number**. Decimal numbers are whole numbers with extra fraction place values. Thus whole numbers and common fraction understandings combine to develop decimal numbers. The symbol structure of decimal numbers is an extension of the whole-number place value structure as follows. However, the language for the decimals follows fraction names, for example, 2.3 7 is "two and thirty-seven hundredths".



1. **Fraction and division**. The great similarity or connection is that fractions are division. This enables many difficult ideas to be made easy. For example, dividing money between a lot of people means each gets little, measuring with a large unit gives a small number, and fractions with large denominators are small.

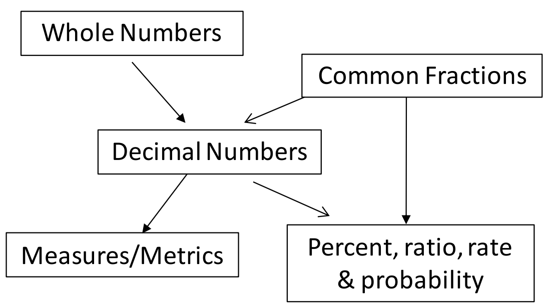
2. **Fraction and measurement**. As well, measuring fixed amounts with units is similar to partitioning a whole into equal parts, so there are similarities between measurement and fractions (see 2 above).

3. **Fraction and percent**. Percent is a fraction where the whole is partitioned into hundredths, so 25% is $^{25}/_{100}$. This relationship also connects percent to the second fractional place value – hundredths.

4. **Fraction and percent, rate and ratio applications**. These three are connected because they are all forms of multiplicative comparison. For example, Jack has $24 and Bill has ¾ of this, June has 30% of this, and Ann's money is in ratio 3:4 with Jack's. Meanwhile, Alan changes $24 for 16 litres of fuel at $1.50/litre. However, it is also useful to remember differences. Fractions have a different language and symbol structure from percent and ratio. Further, there is an important difference between fraction and ratio – **fraction is part to whole while ratio is part to part.** For example, ratio 2:3 is fraction $^{2}/_{5}$.

5. **Fraction and probability**. Fraction is highly connected to probability – probability is a fraction. It is also a pure fraction – has to be equal to or between 0 and 1 (there are no mixed numbers in probability).

6. **Conversions**. That many topics are related to fractions (as on right) raises a final issue with regard to connections. Since there are so many forms of number related to fraction, there is a need to be able to convert between the different forms of fraction – common fraction, decimal, probability, percent and ratio (e.g. the following are equivalent – the common fraction $^{2}/_{5}$ or $^{40}/_{100}$, decimal number 0.4, probability 4 out of 10, percent 40%, and ratio 2:3 or 40:60).



Big ideas in fractions

There are five big ideas that were the basis of numeration for whole and decimal numbers. These remain the basis of fractions and are listed below along with the concepts and processes that are associated with them.

The five big ideas for all numbers

1. **Notion of whole/Part-unit-group**. The basis of number is the whole or unit (or one). It is grouped for large numbers and partitioned for small numbers. However, any part of this can be perceived as a whole. For example, for $3^{2}/_{5}$, there are 3 ones and 2 fifths, and it is possible for the fifth or the one to be considered

as a whole. This enables, in particular, mixed numbers to be considered in terms of place values (wholes and parts) similar to 2-digit numbers being tens and ones.

2. **Additive structure**. Common fractions/mixed numbers count in patterns (e.g. $3^3/_5$, $3^4/_5$, 4, $4\,^1/_5$, and so on) and have seriation (e.g. what is $^1/_5$ more than $3^3/_5$?). They have "odometer" like patterns.

3. **Multiplicative structure**. Common fractions/mixed numbers relate multiplicatively, for example, for $3^2/_5$, fifths to wholes is ×5 and wholes to fifths is ÷5.

4. **Continuous vs discrete/Quantity on a number line**. Common fractions/mixed numbers can be represented on number lines. These representations show rank, comparison and order, rounding and approximation, and density (i.e. there is always another fraction between any two given fractions – $^{15}/_{22}$ is between $^7/_{11}$ and $^8/_{11}$).

5. **Equivalence**. Common fractions have numbers with different notation that are the same amount, for example, $^2/_3 = {}^4/_6$. This is a major part of fractions.

Concepts

1. **Common fractions**. This covers the five concepts or meanings of fraction: (a) *part of a whole* – dividing a whole into equal parts (e.g. $^3/_4$ is 3 parts out of 4 parts equalling 1 whole), an approach that easily leads to common fraction notation; (b) *part of a set* – dividing a group into equal parts (e.g. $^3/_4$ of 8 is dividing 8 as a whole into 4 equal parts and taking 3 of the parts, gives 6); (c) q*uantity on a number line* – a fraction is a single point on the line (e.g. $^3/_4$ is a single point halfway between ½ and 1), an approach which is good for ordering fractions; (d) *division* – fractions are given by numerator divided by denominator (e.g. 3 cakes shared amongst 4 people is $^3/_4$ of a cake to each person, so $^3/_4$ is equivalent to 3÷4); and (e) *multiplier or operator* – fraction $^3/_4$ is that which multiplies by 3 and divides by 4 (i.e. acts as ×3 ÷4).

2. **Mixed numbers/Improper fractions**. This is the way of describing fractions larger than 1 as wholes and parts (mixed numbers), and as fraction notation where the numerator is larger than the denominator (improper fractions). Improper fractions are where the wholes in the mixed numbers are renamed as parts (e.g. $3^1/_3$ is a mixed number composed of three wholes and one third; 3 wholes is 9 thirds so, after renaming wholes as thirds, $3^1/_3$ is improper fraction $^{10}/_3$). Mixed numbers such as $3^1/_3$ can be placed on whole-part charts with 3 in the wholes and $^1/_3$ in the parts.

3. **Equivalent fractions**. Because of their nature, more than one fraction name applies to each fraction. For example, $^2/_3$ of a cake is the same size as $^4/_6$ of the cake. This makes equivalent fractions an example of the equivalence principle (equivalence is equality by adding 0 or, in this case, multiplying by 1, so $^2/_3\times1={}^2/_3$; $^2/_3\times{}^2/_2={}^4/_6$; $^2/_3\times{}^3/_3={}^6/_9$; and so on).

   Note: Equivalent fractions form a structure in mathematics called an *equivalence class*. Equivalence classes divide things into disjoint sets where members of a set form a sequence starting with a base number. Thus, in teaching equivalent fractions, we have to move through **four steps**:

   (a) *Step 1* – showing that different fractions can be equivalent in value;
   (b) *Step 2* – constructing sequences of fractions equivalent to a starting fraction;
   (c) *Step 3* –using these sequences to determine the pattern that relates the fractions; and
   (d) *Step 4* –reversing this pattern to get a rule for determining whether fractions are equivalent.

Fraction processes

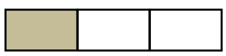
1. **Reading/Writing**. This is the ability to read and write the various forms of fraction – common fractions, mixed numbers, and improper fractions. The naming of the fractions is similar to the naming for the order of numbers (e.g. thirds, fifths, sixths, twenty-firsts) except for the early ones (second – half, fourth – quarter). There are other places where language and numeral do not follow the same pattern (e.g. twenty-one – twenty-first). The names go as follows:

2	TWO	HALF (TWOTH)	3	THREE	THIRD (THREETH)
4	FOUR	FOURTH or QUARTER	5	FIVE	FIFTH (FIVETH)
6	SIX	SIXTH	7	SEVEN	SEVENTH and so on

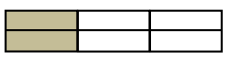
It should be noted that language for fractions comes from the number of equal parts. We recommend that to get the pattern, names be called “twoths”, “threeths”, “fourths” and “fiveths” until the pattern of “fred” equal parts being “fredths” is seen. Once this language is gained, full fraction language and symbols can be seen to emerge from the parts being considered, that is, three out of four equal parts as one whole is “three-fourths”, then “3 fourths”, and “3 line 4” or $^3/_4$. It is useful to remember that $^3/_4$ is a single symbol – its parts are not separate.

For mixed numbers, the language is almost place value. It is given in terms of wholes and parts, for example, $4^2/_7$. A whole-part chart can be used to show the mixed numbers. After this, it is important to maintain counting, for example, $^4/_5$, 1, $1^1/_5$, $1^2/_5$, and so on.

2. **Comparing/Ordering**. This is the ability to compare the various types of fraction. For fractions and mixed numbers, there are the four ways below:
   (a) same denominator – the larger numerator is bigger (e.g. $^4/_5 > {}^2/_5$);
   (b) same numerator – the larger denominator is smaller (e.g. $^3/_7 < {}^3/_5$);
   (c) common denominator – the larger equivalent fraction is bigger (e.g. $^2/_3 = {}^8/_{12}$, $^3/_4 = {}^9/_{12}$, $^9/_{12} > {}^8/_{12}$ means $^3/_4 > {}^2/_3$); and
   (d) mixed numbers – the one with more wholes is larger (e.g. 4 is bigger than 3, then 4¼ > 3½).
3. **Renaming**. This is the ability to change mixed numbers to improper fractions and vice versa by renaming (e.g. $3^1/_3={}^9/_3+{}^1/_3={}^{10}/_3$, while $^{14}/_5$ is $2^4/_5$ because $^{10}/_5$ is 2).
4. **Rounding to nearest whole**. This is ability to determine nearest whole to a fraction. For example, $2^4/_5$ is 3 rounded to nearest whole.
5. **Maintaining the whole**. Fractions involve partitioning wholes into parts. When this is done, it is important, for fractions, that the whole is maintained as the unit and that the part does not become the whole or the unit. This is a component of the whole-part principle. Thus, for a fraction such as $^3/_4$ to be understood, $^3/_4$ must be seen as three parts out of four parts as one whole. Three parts out of four parts, without seeing the parts as the whole, is ratio 3:1 or 3:4 (depending on how it is perceived).
6. **Partitioning and unitising**. The basis of fractions is partitioning wholes into parts. It is also the reverse of this which is being able to see parts as a whole (which is called unitising). Unitising is essential for part of a group because the set of objects has to first be unitised (seen as one whole) before it can be partitioned. As well as partitioning and unitising, it is crucial that parts being partitioned to and unitised from are equal in size/value.
7. **Reunitising**. When dealing with equivalence, a second understanding is necessary. If $^1/_3$ of a whole is compared with $^2/_6$, then to see them as the same requires students understanding $^2/_6$ as one group of two out of three groups of two seen as one whole. This ability to see groups of groups as a whole is called reunitising (as it requires unitising for the group of two to be seen as a unit and then again when the three groups of two is the whole).



$^1/_3$



$^2/_6$

Sequencing in common fractions

This section briefly looks at sequencing for common fractions and sequencing in this module.

Sequencing to develop meaning

The first goal in teaching fractions is to develop **meaning**. The recommendation is that this is first taught with bodies, materials and language only and then notation (symbols) – it is important that the common-fraction notation be not confused with decimal notation, so do not develop them at the same time and focus on differences between them.

To develop a rich meaning for fractions, it is essential to cover all the meanings from real-world situations and later mathematics. This means introducing the following concepts in the following **order**:

fraction as part of a whole (e.g. part of a cake, part of a liquorice strip)

fraction as part of a group (e.g. part of a set)

fraction as a number or quantity (e.g. $^3/_4$ is half way between $^1/_2$ and 1)

fraction as division (e.g. $^3/_4$ is 3 divided by 4 – put fractions on calculators)

fraction as operator (e.g. $^3/_4$ is that which multiplies by 3 and divides by 4 – this enables cancellation of fractions in that, for example, $^3/_4$ is the same as ×3÷4).

In teaching fraction meanings, there are **three models**: (a) *area* – cakes, pizzas, chocolate blocks, paper folding, circles and rectangles (fraction discs), diagrams of 2D shapes; (b) *set* (discrete) – Unifix cubes, counters, logic attribute blocks, students, chairs, and so on; and (c) *length* – liquorice strips, paper strips, fraction mats. There is a fourth model, volume, but in most examples it is equivalent to length.

The **steps** to be followed in teaching meaning of fractions are as follows:

identify whole (focus on the whole)

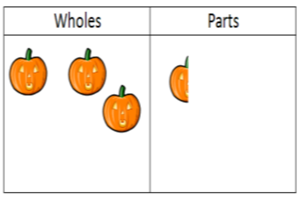
partition into equal parts (break the model into equal parts)

name the equal-sized parts (the total number of parts gives the name)

determine number of parts (the number of parts shaded or taken)

associate fraction name (the number of parts shaded gives the first name and the number of parts in total gives the second).

Once we have the meaning of fractions developed, it is important to extend fractions to quantities large than one. This then introduces mixed numbers and equivalent fractions. Mixed numbers are numbers made up as wholes and parts. They are best introduced on a whole-part chart as on right using materials – wholes and parts. The example on right is 3½. Improper fractions are renaming of mixed numbers. If the example above right was renamed as halves, there would be seven halves. Written as $^7/_2$ this is an improper fraction.



Equivalent fractions need to be developed and this proceeds as follows:

introduce idea of equivalence (e.g. examples showing $^2/_3 = {}^4/_6$)

construct sequences of equivalent fractions (e.g. $^2/_3 = {}^4/_6 = {}^6/_9 = {}^8/_{12}$ = … and so on)

pattern relating equivalent fractions in sequences (e.g. $^2/_3 \times 1 = {}^2/_3 \times {}^2/_2 = {}^4/_6$, and so on)

reversing pattern for rule for equivalent fractions (e.g. cancel down to same fraction).

Finally, the equivalent fraction rule can be applied to order any fraction – this gives the sequence for ordering fractions as follows:

for common denominators, the fraction with the biggest numerator is larger (e.g. $^{4}/_{7} > ^{3}/_{7}$)

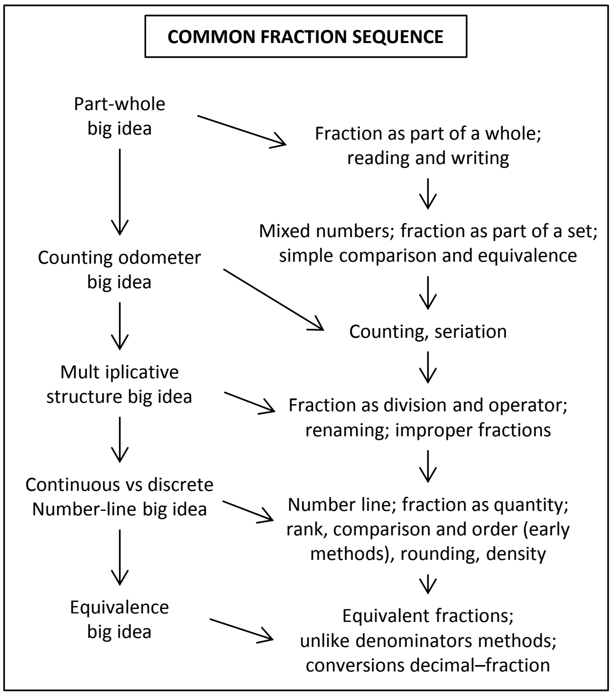
for common numerators, the fraction with the largest denominator is smaller (e.g. $^{4}/_{7} < ^{4}/_{5}$)

for some fractions, finding a fraction between the two to act as a benchmark (e.g. $^{3}/_{7}$ is less than $^{5}/_{9}$ because $^{3}/_{7} < ^{1}/_{2}$ and $^{5}/_{9} > ^{1}/_{2}$)

for any fractions, applying the equivalence rule to change the fractions to common denominators.

Sequencing in this module

The five big ideas used in whole and decimal numbers are as follows: (1) notion of unit or part-whole, (2) additive structure or counting odometer, (3) multiplicative structure, (4) continuous vs discrete or number line, and (5) equivalence. These apply to fractions as well, as can be seen on the right. This means that we begin with the part-whole meanings of fractions (fraction as part of a whole, fraction as part of a set) along with the introduction of mixed numbers. At this point, students can undertake simple comparison and equivalence. From here we go on to additive structure (counting, seriation) and multiplicative structure (fractions as division and operator). Multiplicative structure also leads to improper fractions.



With the focus on number lines, we look at fractions as quantity on a number line, and then go on to rank, comparison and order, rounding and density. Finally, we move on to equivalence and its applications with unlike denominators. We finish by looking at conversions between decimals and fraction, that is, equivalence between these two forms of number.

Therefore, the sections and units of this module are as below:

Overview: Background information, sequencing, and relation to Australian Curriculum

Unit 1: Part-whole fraction concepts and processes – fractions as part of groups and part of sets; reading and writing

Unit 2: Mixed numbers and additive structure – part-whole charts; counting patterns, seriation, and odometer

Unit 3: Multiplicative fraction concepts and processes – fractions as division and as operator; renaming and improper fractions

Unit 4: Number-line fraction concepts and processes – fractions as quantities on number lines; rank, comparison and order, and rounding and approximation

Unit 5: Equivalence – equivalence rule; common denominators; and application to comparison of fractions with unlike denominators

Test item types: Test items associated with the five units above which can be used for pre- and post-tests

Appendix A: RAMR cycle components and description

Appendix B: AIM scope and sequence showing all modules by year level and term.

Most of the units are based around a big idea and accompanying concepts and processes. This is to ensure that all the concepts and processes associated with each of the five big ideas are covered. However, this structure may not be instructionally efficient unless there is integration. Therefore, we have used the RAMR stages to gain instructional efficiencies by having more than one idea in a RAMR cycle. RAMR is described in **Appendix A**.

Relation to Australian Curriculum: Mathematics

AIM N3 meets the Australian Curriculum: Mathematics (Foundation to Year 10)						
Unit 1: Part-whole fraction concepts Unit 2: Mixed numbers and additive structure Unit 3: Multiplicative structure Unit 4: Continuous–discrete/number line Unit 5: Equivalence and comparison						
Content Description	**Year**	**N3 Unit**				
		1	**2**	**3**	**4**	**5**
Recognise and describe one-half as one of two equal parts of a whole (ACMNA016)	2	✓				
Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033)		✓				
Model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole (ACMNA058)	3	✓				
Investigate equivalent fractions used in contexts (ACMNA077)	4	✓	✓	✓		
Count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line (ACMNA078)		✓	✓	✓		
Compare and order common unit fractions and locate and represent them on a number line (ACMNA102)	5	✓		✓	✓	
Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (ACMNA103)			✓	✓	✓	✓
Compare fractions with related denominators and locate and represent them on a number line (ACMNA125)	6	✓	✓	✓	✓	
Solve problems involving addition and subtraction of fractions with the same or related denominators (ACMNA126)			✓	✓	✓	
Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (ACMNA127)					✓	
Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (ACMNA128)						✓
Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies (ACMNA129)						✓
Multiply and divide decimals by powers of 10 (ACMNA130)				✓		
Make connections between equivalent fractions, decimals and percentages (ACMNA131)		✓				
Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)	7		✓	✓	✓	
Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)				✓	✓	✓
Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)		✓	✓	✓	✓	
Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)			✓	✓	✓	✓

Unit 1: Part-Whole Fraction Concepts

This unit introduces the part-whole fraction concepts. It bases this on the notion of unit or part-whole-group big ideas that give rise to *fraction as part of a whole* and *fraction a part of a set* meanings of common fractions. The area and set model are used in these introductions. For this unit, we will initially provide teaching ideas by giving all stages of the RAMR model. It is possible to combine understandings and to do lessons in 1.1 and 1.2 in a more concise and integrated way. However, we have not done so because these two concepts (part of a whole, part of a set) are crucial to mathematics understanding of fraction, and there is often confusion in that students believe that $^1/_3$ of a set of 12 is done by looking at groups of 3, not 3 groups.

1.1 Fraction as part of a whole – area model

This section is based on part-whole-group big idea and focuses on dividing a whole into equal parts. It reflects two notions that underlie the teaching of fraction: (a) unitising – making a whole out of parts (even if only in the mind); and (b) partitioning – making parts out of a whole. It leads to understanding of formal fraction notation but also includes technical knowledge on folding.

Reality

Using local culture and environment. Look for things in local environment that use fractions (e.g. half a glass of water, halfway home).

Existing knowledge. Check that students can count, know numbers, can partition in equal parts and know basic shapes.

Kinaesthetic. Find things in the environment that students can find a fraction of or cut into fractions (e.g. their bodies, fruit).

Abstraction

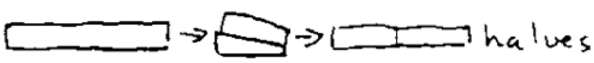
Body

Use food (e.g. cakes, pikelets, pizzas, liquorice strips, and so on) to represent fractions.

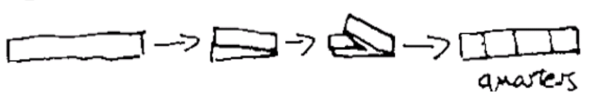
Hand

1. **Paper rectangles/strip**s

Halves: Fold in half.

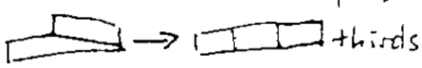


Fourths/quarters: Fold in half and half again.



Eighths: Fold in half, fold in half, fold in half again.

Thirds: Fold so that the fold is halfway back along the strip.



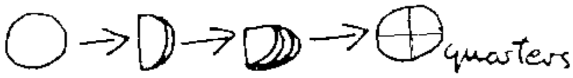
Fifths: Roll strip $2^1/_2$ times and flatten.

Sixths/sevenths: Roll 3 times for 6ths and $3^1/_2$ times for 7ths.

It is important to develop the concept and language of fractions as a whole, being "divided into equal parts", sevenths, thirds etc. It is not necessary, at this point, to start introducing formal symbols.

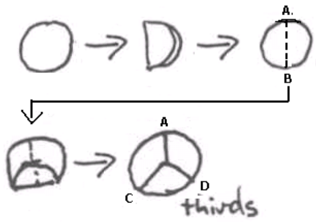
2. Paper circles

Halves: Fold in half.

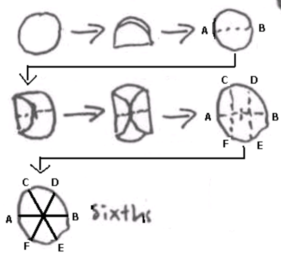


Fourths/quarters: Fold in half top to bottom, then fold in half left to right.

Thirds: Fold in half downwards, crease lightly and open (see AB). Fold from B to the centre. Open out and you should see 2 points (see CD). Draw lines from A to the centre, and from C and D to the centre.



Sixths: Fold in half, crease, open and draw a line across the crease (see AB). Fold from A to the centre; fold from B to the centre; open out and you should see 4 points (see CDEF); draw lines from C to E and from D to F.



3. Models and concepts

Ask the students to fold the paper rectangle, square and circle to make halves in as many ways as they can, and then to make fourths/quarters in as many ways as they can.

Halves. Hold up a paper rectangle: Say: *This is a whole*. Ask the student to say what it is. Say: *Fold the whole to show halves*. [Check and discuss how students did this – some may have folded down and some may have folded across.] *How many equal parts have you made*? [2] *I wonder why we don't call them "twoths"*? Say: *Can you fold it from one corner to the opposite corner to show halves*? [No] *Cut the rectangles from corner to opposite corner. Now put one on top of the other. Are the two parts equal*? [Yes] *So you can't fold a rectangle to show halves but you can cut it to show halves*. Repeat this with a paper square. The students should be able to fold along each diagonal.

Fourths/quarters. Say: *Take a new piece of paper.* Hold it up and say: *This is a whole. Now show me fourths/quarters. How many parts have you got? Is each part the same size? So one whole equals how many fourths/quarters?* Ask the students to say how they folded their paper to show fourths. *Thirds/sixths*. Say: *Take a new piece of paper.* Hold it up and say: *This is a whole*. Ask the students to fold it to show thirds. Say: *How many parts have you got? I wonder why we don't call them "threeths"? Are the parts the same size? Can you make this into sixths or will you need a new piece of paper?*

Repeat for fourths and eighths, then for fifths and tenths.

4. Materials and language

For all materials, e.g. paper rectangles

- Identify whole (hold up your coloured rectangle, say: "this is one whole", hold up the white rectangle, say "this is also one whole").
- Partition into equal parts (fold the white rectangle into four equal parts by folding in half and half again).
- Name the equal-sized parts (ask "how many parts has the whole been divided into?", count the parts "one, two, three, four", state "each part is a fourth or quarter").
- Determine number of parts (shade three of the parts, or cut out three of the parts, place this against the coloured whole).
- Associate fraction name (ask "how many parts are shaded – one, two, three", state "three out of four parts of one whole, so fraction is three-fourths or three-quarters").

Mind

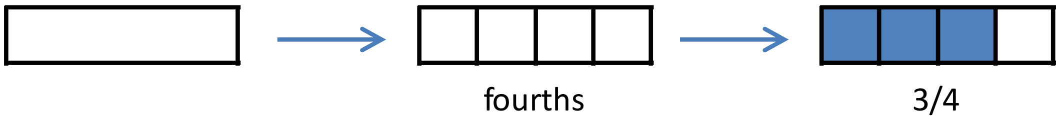
- Get students to shut eyes and imagine shapes cut into fractions.

Mathematics

Formal symbols/language

Repeat the Abstraction activities but go another step, write out the fraction symbol. First be informal (e.g. "3 fourths"), and then formal (e.g. ¾). Ensure students have appropriated the correct symbol.

Explain that common fractions are where the denominator gives the number of parts that the whole has been partitioned into and the numerator gives the number of these parts that are being considered.



Use practice activities like worksheets with four columns (rectangle/strip shaded, circle shaded, language, symbol) and fill in only one column (different for each example) – students fill in other columns, or the **think** board (see 1.2 mathematics stage for an example of this).

Connections

Relate common fraction to division – show how $^1/_3$ is dividing one whole into three equal parts, the way division divides 12 lollies into 3 equal groups. This is more powerful after 1.2.

Reflection

Validation/Application

Apply fraction to local environment – search for places/objects that fractions could be used. Ask for anything that is ¾. For example, 75%, 0.75, 75 cm, 45 minutes, 270 degrees, and so on. Use knowledge to solve simple examples – Andy ate ¾ of the cake – show what he ate.

Extension

Flexibility – see above.

Generalising. Look at volume models and extend area ideas to these. Then, try to get across the generalisation that if you take a whole and break it into *q* equal parts and shade *p* of them that the fraction is *p q*ths or $^p/_q$. Reverse. One of the very important aspects of teaching the fraction meaning is to reverse the process – to ensure that teaching covers <u>all</u> the following:

- WHOLE → PART (give students a paper square, say it is one whole, and ask them to fold it to get ¾; give students 12 Unifix, say it is one whole, and ask them to construct $^2/_3$);
- PART → WHOLE (give students a paper square, say it is ¾, and ask them to make one whole; give students 12 Unifix, say this is $^2/_3$, and ask them to make one whole); and
- WHOLE/PART → WHOLE/PART (give students a paper square, say it is 1 $^1/_4$, and ask them to construct ½; give students 20 Unifix, say this is 1 $^2/_3$, and ask them to make 2 ½).

Note: It is crucial to ensure that students **maintain the whole throughout**. When a paper rectangle is folded into four, some students see four wholes not one whole. Thus, we spend time at the start stressing what the whole is and keeping a coloured whole to compare the part with. Similarly, for Unifix, we spend time at the start ensuring students see the Unifix as one whole group. Other methods to do this are running a finger around the whole while saying "this is one whole" or putting the Unifix on a coloured piece of paper or drawing a circle around the Unifix. The idea is to act out the formation of the whole, so that the kinaesthetic sense is in action as well as sight, hearing and touch.

1.2 Fraction as part of a group – set model

This section focuses on dividing a group into equal parts, and involves sharing sets. It is more difficult because it involves unitising before partitioning. It is also based on the part-whole-group big idea.

Reality

Using local culture and environment. Look for things in local environment that use fractions (e.g. half a class of students, a quarter of a bag of lollies, half the fish caught). Try to find unique things.

Existing knowledge. Check that students can count, know numbers, can partition in equal parts and know part of a group meaning of fraction.

Kinaesthetic. Make fractions with students' bodies, hands and fingers.

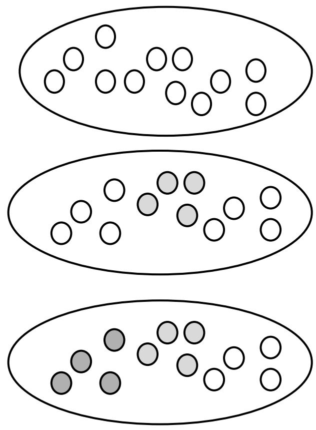
Abstraction

Body

Use a group of students as the whole – for example, six students if you want to work on thirds, halves and sixths. Ask students, *What fraction of our classmates have long hair, have blond hair, have long socks,* etc.? Change the number of students over time.

Using the same strategies and instructions as with the *Hand* activities, use students to represent the scenarios related to their reality.

Hand



- *Identify whole*. Take 12 Unifix. Cover them with hands and say "this is one whole."
- *Partition into equal parts*. Partition the Unifix into three parts – do this by sharing among three.
- *Name the equal-sized parts*. Ask "how many equal parts have we divided our whole into?" Count the parts "one, two, three", state "each part is one third."
- *Determine the number of parts*. Choose two of the groups.
- *Associate fraction name*. Ask "how many groups chosen?" – "one", "two". Say the name "two thirds, we have chosen two thirds of the whole".

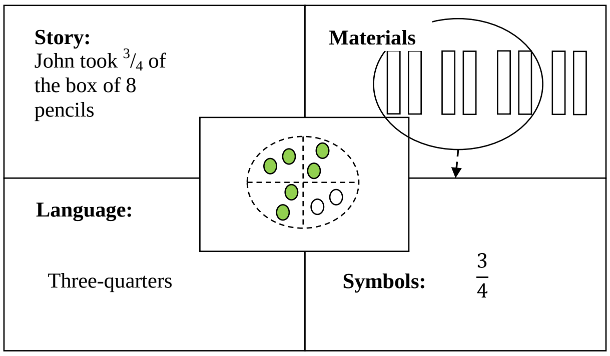
Mind

Students can picture 12 counters in their heads. Get them to "dot" these counters in the air then circle one third of them.

Mathematics

Formal language and symbols. Consolidate– this includes words like denominator and numerator, and so on.

Practice. Relate representations – story, materials, drawings, language and symbols. Use the thinkboard as below. Use worksheets where there are different columns for the representations and only one column is filled in each row. Practise finding fractions of sets: Provide numeral [$^2/_3$] – find $^2/_3$ of 12.



Connections. Relate fractions to division: (a) do examples that relate division to fraction – e.g. 12 lollies to be put into groups of 3 is the same as finding $^{1}/_{3}$ of a group of 12 lollies; and (b) look at similarities between division and fractions (e.g. more people to share with means less to each person – larger denominator means smaller fraction).

Reflection

- Apply fraction to local environment – search for examples, be flexible and look for places/objects that fractions of a set could be used.
- Use knowledge to solve simple examples – e.g. Four people went out fishing and caught 20 fish between them. They each took home an equal share for their families. How many fish did they each take home?
- Reversing – Part to whole – e.g. Eric took home six fish which was a third of all the fish caught that day. How many fish were caught altogether?
- Generalising – what is $^{p}/_{q}$ of a set of n objects?

1.3 Reading and writing fractions

The language for fractions comes from the number of equal parts. We recommend that to get the pattern, names be called “twoths”, “threeths”, “fourths” and “fiveths” until the pattern of “fred” equal parts being “fredths” is seen.

Once this language is gained, full fraction language and symbols can be seen to emerge from the parts being considered, that is, three out of four equal parts as one whole is “three fourths”, then “3 fourths”, and “3 line 4” or ¾. It is useful to remember that ¾ is a single symbol – its parts are not separate.

The naming of the fractions is similar to the naming for the order of numbers (e.g. thirds, fifths, sixths, twenty-firsts) except for the early ones (second – half, fourth – quarter). This is another place where language and numeral do not follow the same pattern (e.g. four – quarter, seven – seventh, eleven – eleventh, twenty-one – twenty-first). The names go as follows:

2	TWO	HALF (TWOTH)
3	THREE	THIRD (THREETH)
4	FOUR	FOURTH or QUARTER
5	FIVE	FIFTH (FIVETH)
6	SIX	SIXTH
7	SEVEN	SEVENTH

and so on

For mixed numbers, the language is almost place value. It is given in terms of wholes and parts, that is, 4 $^{2}/_{7}$. A Whole/Part chart can be used to show the mixed numbers. After this, it is important to maintain counting, that is, $^{4}/_{5}$, 1, 1 $^{1}/_{5}$, 1 $^{2}/_{5}$, and so on.

Unit 2: Mixed Numbers and Additive Structure

This section looks again at fractions but extends ideas to encompass additive structure/counting so we can extend from proper fractions to mixed numbers. It also looks at fraction topics where there is a similarity to place value, that is, the use of the part-whole chart, like a tens-ones chart, for reading and counting fractions and renaming.

2.1 Mixed numbers

This allows us to look at fractions larger than one – composed of whole numbers and proper fractions (e.g. two and three-fourths, three and five-sevenths).

Reality

Using local culture and environment. Look for examples – one and a half sandwiches, two and a quarter cans of fuel, etc.

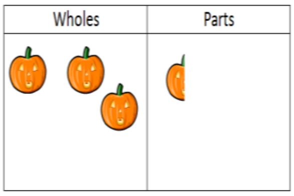
Existing knowledge. Check that students understand the concept of a whole, fractions as part of a whole, and recognising a unit.

Get them to act out more than one (e.g. eat 1 and a ½ cupcakes, walk 2 and a ¼ times around the oval).

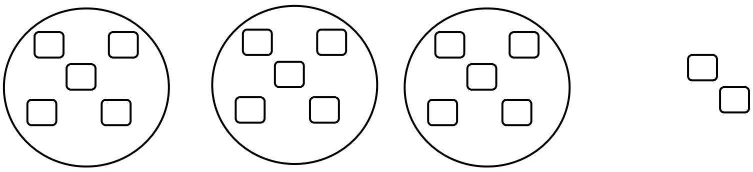
Abstraction

Use a whole-part chart to introduce mixed numbers – wholes are put on the left and parts on the right.

- *Materials*. Place materials for, say, 3 and ½ on the whole-part chart.
- *Language*. Ask how many wholes? [Three] Cover wholes, ask how many parts left over (loose parts)? [half]
- Extend this to larger denominators (e.g. sixths) – place material on whole-part chart.

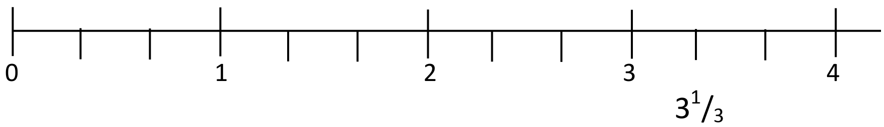


Extend this to groups – use students or physical material to have groups and left overs as for $3^2/_5$ below:



Extend to division (e.g. $3^2/_5$ = $^{17}/_5$ is the same as 17÷5) and to operator (e.g. $\times 3^2/_5 = \times 17 \div 5$).

Extend to number line – a very good way of showing mixed numbers.



Mathematics

Symbols – Ask, *How many wholes?* [Three]. Say, *Write the number 3.* Cover wholes, ask how many parts left over (loose parts)? [Half]. Say, *Write this fraction on the right of the whole number.* 3 ½

Practise working between the symbol ↔ language ↔ picture using the thinkboard.

Reflection

Extend the division principle of fractions and complete activities, e.g. divide seven cupcakes between two people – students could use the set model to represent this and now mathematically say and write the answer as 3½.

2.2 Additive structure (counting, seriation, odometer)

Additive structure was described in relation to whole number and decimals as part of their place value. Fractions do not have place value as whole numbers and decimals do. However the denominator effectively names a fraction and can be used for counting as the place values of whole and decimal numbers can be. For example, it is possible to count in whatever the denominator of a fraction is, e.g. $^1/_3$, $^2/_3$, $^3/_3$, $^4/_3$ etc. Each new fraction is one more part of the same equal parts, that is, *a unit fraction more or less.* For example $^1/_6$ more than $3^2/_6$ is $3^3/_6$. This leads to counting and odometer – $3^3/_6$, $3^4/_6$, $3^5/_6$, $4^0/_6$, $4^1/_6$, and so on.

Counting activities

WHOLES	PARTS

Set up a Whole/Part chart as on right. Obtain some physical materials in the form of wholes and parts – say circle wholes and circles cut into sixths. These then can be placed on the Whole/Part chart to make mixed numbers – e.g. three wholes and four sixths forms $3^4/_6$. So mixed numbers can be made. The activity should go as follows:

- Show materials → Say and write fraction AND Say and write fraction → Put out materials.
- When saying fraction $3^4/_6$, put out materials, place hand over wholes place and say "three wholes" and then move hand to RHS over parts place and say "four sixths".
- Count out wholes and sixths from a starting point, say $3^2/_6$, adding a sixth piece of a circle each time, and saying 3 and 2 sixths, 3 and 3 sixths, 3 and 4 sixths, and so on past 3 and 5 sixths, 4 and 0 sixths and so on.
- Count backwards removing sixths as you go.
- Place race games – for example, for sixths, spin a 1-2-3-4 spinner, in turn, and add the number of sixths that is shown, rename as necessary to have a mixed number (no improper fractions allowed), first player to six wholes wins. (*Note:* You can reverse game and start with six wholes and remove sixths.)

Odometer activities

The odometer pattern is a generalisation and so teaching needs a lot of examples as follows.

- Repeat the counting activities above for many different types of fractions (e.g. fifths, eighths, thirds, and so on).
- Write out the resulting counting sequences for each type of fraction.
- Look through these sequences for a pattern for each sequence (e.g. for sixths, 5 is crucial – after 5 parts the count goes to the next whole, at 0 parts the count goes to the previous whole and 5 parts).
- Look through these sequences for a pattern for when the counting forwards reaches the next whole number and the counting backwards reaches the previous whole number – when does this occur? [**Counting forwards** – go to the next whole number (increases by 1) and zero parts after you count to where the number of parts is one less than fraction denominator; **counting backwards** – after you count down to zero parts, go to the previous whole number (decreases by 1) and the number of parts that is one less than fraction denominator.]
- Get students to write their generalisations in their own way.
- Get students to extend this work – where else do we see different odometer numbers (other than 9 which is the odometer number for whole and decimal numbers)? [What about weeks and days, hours and minutes, metres and centimetres?]

Note: Get students to notice odometer pattern in each fraction type before looking for common pattern across all pattern types.

Unit 3: Multiplicative Structure

There is a multiplicative relationship between the number of equal parts and the whole. For example, for 3 1/6, the wholes are 6 times the $^{1}/_{6}$ths and vice versa. This unit looks at this multiplicative structure and how it is the basis of the division and operator meanings of fractions and how renaming leads to improper fractions.

As well as partitioning and unitising, it is crucial that parts being partitioned to and unitised from are equal. This need for equality of parts is related to the multiplicative structure of fractions. One of the concepts of a fraction is that a fraction is a division e.g. ¼ is 1÷4 (one whole divided into 4 equal parts). 4 × ¼ = 1.

Repeated dividing of wholes into smaller parts, e.g. repeated halving, is also multiplicative and each time the whole is halved the size of the parts is proportionally smaller and the denominator is proportionally larger. For example ½ of ½ is ¼; ½ of ¼ is $^{1}/_{8}$, ½ of $^{1}/_{8}$ is $^{1}/_{16}$ and so on. This multiplicative relationship between fractions is the basis for equivalence understandings and operating with fractions.

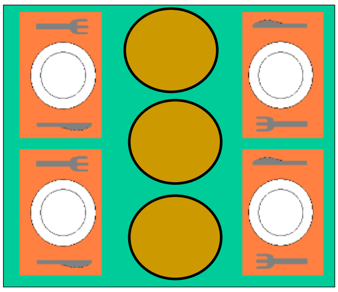
3.1 Fractions as division and operator

This section covers the two meanings of fractions that emerge from the big idea of multiplicative structure: fractions as division, and fractions as operators.

Discussion

Division comes from a multiplicative structure viewpoint, e.g. sixths are one whole partitioned/divided into 6 equal parts. Thus, the same result comes from dividing a whole equally among six people – each will get a sixth. This leads to a collection of activities, where two cakes are shared among three people – each gets $^{2}/_{3}$, four pizzas among five people – each gets $^{4}/_{5}$, and so on. In this way division is connected to fraction to show the mathematical equivalence between division and fraction. It leads to the meaning that **fractions are division**.

These dividing activities are great fun and require problem solving. Put out, say, three circles of paper as cakes, choose four students, and set the task to divide the cakes equally among the four students. This will be done differently depending on how students go about doing it – for example, they might divide each cake among the four people, giving three single quarters; or they might give a half to each person, leaving two halves left which are divided into quarters so that each person gets a half and a quarter. *Note:* The use of plates, table and settings, can be a great way to set up the sharing, as shown on right.



Operator or multiplier is also a part-whole and multiplicative approach. For example, if we look at eight objects, we can consider the eight objects as one unit (as we do for part of a set) and then a single object is 1/8 of the whole unit. Alternatively, the eight objects can be seen as eight which makes a single object the unit (we can also consider the 8 as 100% which makes the unit 12.5%.) If we think of the 8 as eight, we can multiply by 3 and divide by 4 and get 8×3÷4 = 6. If we now think of the 8 as one, we can divide into 4 parts and take 3 of them, and we see that ¾ = 6 also. In this way, we arrive at the meaning **fraction as operator, i.e. fractions ($^{3}/_{4}$) are that which multiplies by numerator (3) and divides by denominator (4)**. This allows cancellation of fractions in operation situations.

RAMR lesson ideas

Reality

Using local culture and environment. Two cakes of the same size are brought in to share for a birthday. Divide the cakes amongst the class.

Existing knowledge. Check that students understand the concept of sharing equal parts – that half does not just mean making two pieces, but two **equal** pieces.

Abstraction

Cut paper strips. Say they are liquorice. Share 3 strips amongst 5 people (i.e. 3÷5). This will take time but students will see that the only way to do this is to cut the strips into 5 equal pieces and distribute – each of the 5 people will get 3 pieces. This means that 3 shared amongst 5 is $^3/_5$ and **fractions are division**.

Repeat this for 3 cakes shared amongst 4 people (see context shown previously). This shows that ¾ is 3÷4.

Mathematics

Ensure formal language and symbols are known. When developing symbols, the best idea is to move as follows:

↓ TEACHER RECORDS STUDENTS' MODELLING
STUDENTS RECORD OWN MODELLING

The meaning "fraction as division" is used with the symbols to provide a way to put fractions on the calculator. For example, $^3/_4$ is 3 divided by 4 (3 cakes shared among 4 people) – on a calculator it is 3 ÷ 4 and is, therefore, 0.75.

Reflection

Application. Calculate three tenths on the calculator and help students join the link where $^3/_{10}$ is 0.3, and $^6/_{10}$ is 0.6. What will $^7/_{100}$ be as a decimal? Check it on the calculator.

Extension. Extend fraction as division to fraction as operator:

Step 1: Take, for example, 8 counters, and find ¾ of them [answer is 6 counters – partition 8 into 4 groups of 2 and take 3 groups]

Step 2: Take, for example, 3 lots of 8 counters (e.g. 8×3) and then divide this into four groups (8×3÷4) [answer is 8×3÷4=6]

Step 3: Relate the two and we have that ¾ acts the same as ×3÷4. This means that fractions can be cancelled and this sets up for equivalent fractions (e.g. $^{24}/_{32}$ = ¾ cancelling by 8).

Step 4: Apply to problems such as find $^3/_5$ of 25 – this is 25×3÷5 = 15.

Thus, ¾ is 3÷4 and also, in applications, the same as ×3÷4.

3.2 Improper fractions

This introduces fraction notation where the numerator is larger than the denominator.

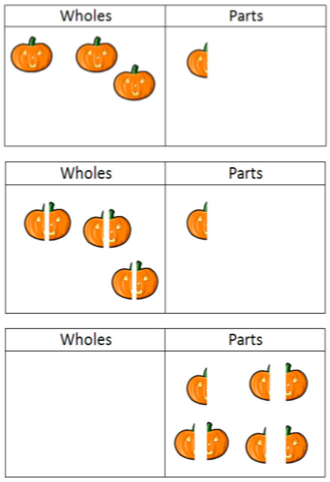
Discussion

The basis of fractions is partitioning wholes in parts. It is also the reverse of this which is being able to see parts as a whole (which is called unitising). Unitising is essential for part of a group because the set of objects has to first be unitised (seen as one whole) before it can be partitioned.

As well as partitioning and unitising, it is crucial that parts being partitioned to and unitised from are equal – in area as well as length. This need for equality of parts is related to the multiplicative structure of fractions. There is a multiplicative relationship between the number of equal parts and the whole. One of the concepts of a fraction is that a fraction is a division e.g. ¼ is 1÷4 (one whole divided into 4 equal parts). 4 × ¼ = 1.

Repeated dividing of wholes into smaller parts, e.g. repeated halving, is also multiplicative and each time the whole is halved the size of the parts is proportionally smaller and the denominator is proportionally larger. For example ½ of ½ is ¼; ½ of ¼ is $^1/_8$, ½ of $^1/_8$ is $^1/_{16}$ and so on. This multiplicative relationship between fractions is the basis for equivalence understandings and operating with fractions.

RAMR Lesson ideas



Abstraction

Regrouping of material on the whole-part chart relates mixed numbers to improper fractions.

Begin with 3½ (as on right). Determine the unit – halves will be our unit.

Represent the wholes as the new unit, i.e. split the wholes into halves.

Change the 3 wholes into halves and move the halves across to the part section of the chart. How many halves? [6 + 1 = 7 halves]

Mathematics

Ensure students know formal names/symbols. Introduce the proper and improper fraction terms – proper fractions are between 0 and 1, NOT "when the numerator is bigger than the denominator". Provide numeral: 3½ is seven halves – $^7/_2$.

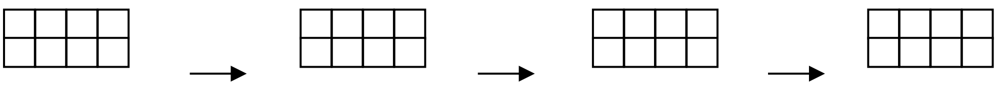
Use the thinkboard to practise representations to and from diagrams ↔ word descriptions ↔ fraction notation. Play improper fraction games below.

Improper fraction games

Formation of improper fractions through renaming of wholes to parts can be practised with shading and trading games.

Sandwich relish

Each player has a game board. A suitable dice is thrown in turn by each player (here dice will have six of $^1/_8$, $^2/_8$, $^3/_8$, $^4/_8$, $^5/_8$, $^6/_8$ and $^7/_8$ on the six sides) and the same number of parts are shaded on the board as the number on the dice. Player states how many wholes and parts they have shaded in total (e.g. gives the fraction shaded across the board as a mixed number). If stated incorrectly, the player misses a turn. The first player with all 4 wholes shaded wins.



Card games

Use the fraction discs, whole-part charts and cards with appropriate fraction names (e.g. 3 tenths, 2 eighths) for the following games.

Lose 5 ones (an adaptation of the whole-number game "Lose 5 tens"). Students start with 5 wholes on the whole-part chart, take turns in drawing a card and taking away that number of tenths, eighths, etc. Students have three turns each, compare numbers when finished and winner is the one who has the smallest number.

Win 5 ones. The opposite to the game above – start with zero and build to 5 ones.

Connect renaming for fractions to renaming in PV.

Reflection

Reversing. Give a representation of 2½ (e.g. a strip of paper, 20 counters) and ask students to find the whole.

Unit 4: Continuous–Discrete/Number Line

As described before, whole numbers and decimals can be applied to both discrete and continuous attributes. This holds true for common fractions and leads to the meaning of **fraction as quantity or a single point on a line**. In the same way the whole numbers can be represented with sets and on lines, fractions represented using continuous models like a piece of ribbon or rope. The use of number lines allows fractions to be visualised and counted. The placement along a number line allows us to investigate other aspects of fractions.

This unit looks at the quantity meaning of fraction, rank, density and rounding, and early comparing and ordering.

4.1 Fraction as quantity on a line – number-line model

In this section we are reminded that a fraction is a single point on the number line. It is an important concept for ordering and rounding fractions. This is part of the continuous-discrete big idea.

Reality

Using local culture and environment. Look for things in local environment that use fractions (e.g. walking halfway home, running two thirds of the football field/netball court).

Existing knowledge. Check that students understand a number line – beginning at zero and that numbers on the number line are partitioned equally.

Kinaesthetic. Choose a distance – walk it – call it one whole. Ask students to estimate and walk half/quarter etc. Walk whole way counting off 1/4s and end at 4/4s. Mark off parts of body – feet 0/4, knees 1/4, hips/waist 2/4, chest 3/4, head 4/4 – get students to point as teacher says fraction – get students to say as you point at own body (reversing).

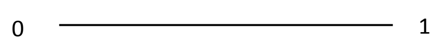
Abstraction

Body

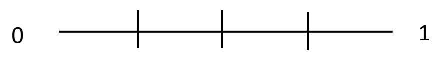
Make a number line on the ground in the classroom or a larger one outside. Follow the processes of the *Hand* activities and step it through the number line.

Hand

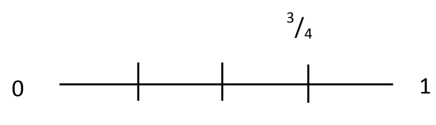
- *Identify whole*. Draw a line and mark it 0 and 1 at ends. Move fingers along it – say “this is one whole.”



- *Partition into equal parts*. Partition the line into four equal lengths – the rectangle paper strips that were made earlier can be used again to represent the number line.



- *Name the equal sized parts*. Ask “how many has the whole been divided into?” Count the parts, sliding your finger along the number line and only saying “one, two, three, four” as you reach each point on the number line. State “each part is a fourth.”
- *Determine the number of parts*. Choose three of the lengths.
- *Associate fraction name*. Ask “how many lengths chosen?” – “three”, say the name – “three fourths”.



Mind

Draw a number line in your mind. Put your finger on 0 in the air and another finger on 1. Keep the picture of the whole number line but now move one of the fingers to show half/third/quarter.

Mathematics

- Provide numeral [$^3/_4$] – mark $^3/_4$ on number line.
- Practise representations to and from diagrams ↔ word descriptions ↔ fraction notation.
- Use the thinkboard.

Reflection

- Apply fraction to local environment – search for examples, be flexible and look for places/objects that fractions of a set could be used.
- Use knowledge to solve simple examples – e.g. Four people shared a race that was 100 m long. They ran a quarter of the distance each. How far did they each run?
- Part to whole – e.g. Simone had one third of a piece of ribbon. She had 30 cm. How long was the whole piece?

4.2 Rank, density and rounding

This section looks at rank on a line and density in terms of always being able to find a fraction between any two given fractions.

Rank

Rank is the concept that the further along the number line a number is, the larger it is. The focus is on the distance of numbers from the end of a number line. Fractions are numbers that each have a place on a number line and so rank applies to fractions. The complexity with rank and whole numbers and decimals is that it is not the number of numerals in a number that determines the size of a number but the relative distance along the line. This requires understanding of density and place value. With fractions there is a different complexity. The ability to place fractions on a number line requires comparison of the fractions. If the denominators of the fractions are different, equivalent versions of the fractions need to be considered to allow for the placement along a number line.

So the size of a fraction in comparison to another cannot be determined by considering the denominator or numerator in isolation. If the denominators are the same the numerators can be compared and the fractions can be ranked. If the denominators are different, equivalent versions of the fractions need to be found by converting them to the same denominator. Therefore "the larger the denominator, the smaller the fraction" does not necessarily apply as it will depend on the numerator as well. For example, $^9/_{12}$ is larger than $^2/_3$ even though twelfths are smaller parts than thirds – it is the number of parts that need to be considered as well.

To **teach rank**, use the techniques of whole and decimal numbers:

- Get a rope and pegs or put a line on the ground – place starting and ending numbers at ends of line – have students holding the rope or placed at ends of the line – could be 0 and 1 for proper fractions, could be 0 and 5 for improper fractions or mixed numbers.

- Give students fraction numbers to peg on line or place on line on floor – discuss the placements in terms of order or guesses and in relation to start and end numbers.

Density

The concept of density relates to the number of numbers that exist between two given numbers. Fractions, like decimals, are dense. There are always fractions (in fact, a countably infinite number of fractions) between any two given numbers. For example, numbers $2^1/_3$, $2^2/_3$ are between 2 and 3; $2^1/_{24}$, $2^1/_{12}$, $2^1/_6$ are between 2 and $2^1/_3$ and so on. This means that fractions are dense and whole numbers are not – in theory, for fractions it is possible to continue partitioning between whole numbers so that the next whole number is never actually reached.

- Show how fractions can be put between any two fractions, and that there are many fractions that could go in between any other two fractions.

0			1
	1/3	9/15	

1/3				9/15
	13/30	1/2	61/120	

- Expand previous number lines as below showing how the equivalent fractions grow.

0																1
0/2								1/2								2/2
0/4				1/4				2/4				3/4				4/4
0/8		1/8		2/8		3/8		4/8		5/8		6/8		7/8		8/8
0/16	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16	9/16	10/16	11/16	12/16	13/16	14/16	15/16	16/16

and so on.

Rounding

This is the ability to round a fraction to the nearest whole number, or 10 or ½. For example, $2^3/_8$ rounded to nearest whole number is 2. The ability to round and estimate fractions is related to the idea of benchmarking. For this, students need to recognise the relationship between the denominator, numerator and related whole.

To **teach rounding** (and estimating), set up number lines marked at the "roundings" place in fractions and see which rounding is closest. For example round $3^3/_5$ to the nearest half; the following steps hold:

(a) construct a number line 0 to 5 and mark in the halves – $^1/_2$, 1, $1^1/_2$, 2, $2^1/_2$, 3, $3^1/_2$, 4, $4^1/_2$, 5;

(b) place on the number line the example $3^3/_5$ – this will be between $3^1/_2$ and 4; and

(c) note that $3^3/_5$ is closer to $3^1/_2$ than to 4 – so $3^3/_5$ rounded to the nearest $^1/_2$ is $3^1/_2$.

4.3 Early comparing and ordering

The ability to compare and order fractions is an important aspect of learning about fractions and is a complex activity. For fractions and mixed numbers, there are four ways to compare and order them:

(a) If the fractions have the **same denominator** – the larger numerator is bigger (e.g. $^4/_5 > {}^2/_5$).

(b) If the fractions have the **same numerator** – the larger denominator is smaller (e.g. $^3/_7 < {}^3/_5$).

(c) If the fractions can be translated to a **common denominator** – the larger equivalent fraction is bigger (e.g. $^2/_3 = {}^8/_{12}$, $^3/_4 = {}^9/_{12}$, $^9/_{12} > {}^8/_{12}$ means $^3/_4 > {}^2/_3$) – see 4.4 below.

(d) If the fractions are mixed numbers – the one with **more wholes** is larger (e.g. 4 is bigger than 3, therefore 4¼ > 3½).

Benchmarking is another strategy that can assist with the comparing and ordering of fractions. Here, the fractions are related to benchmarks (e.g. whole numbers, halves, quarters) to determine size. For example, $^4/_{11}$ is less than $^1/_2$ and $^4/_7$ is greater than $^1/_2$, thus $^4/_{11}$ is less than $^4/_7$.

There is a progression of difficulty with these different types of comparisons. When students are introduced to the idea of comparing fractions it makes sense to follow this sequence. For example, the following pairs of fractions should be compared in the following order:

$^{2}/_{9}$	$^{4}/_{9}$	(like denominators)
$^{2}/_{3}$	$^{2}/_{5}$	(like numerators)
$^{3}/_{4}$	$^{5}/_{6}$	(near 1)
$^{3}/_{7}$	$^{5}/_{9}$	(benchmarking around a $^{1}/_{2}$)
$^{5}/_{8}$	$^{7}/_{11}$	(equivalence)

The list of techniques for comparison provides a repertoire of strategies that teachers should know. They should be used with students in diagnostic situations (i.e. used with students for whom they appear to be suitable), **not necessarily taught to every student regardless of need**. Appropriate and inappropriate rules for the various strategies are as follows.

Same denominators, different numerators (e.g. $^{4}/_{9}$ and $^{2}/_{9}$):

- *Appropriate strategies:* Use materials/pictures to compare the two fractions, get pattern that have to compare the numerators as though they were whole numbers.
- *Inappropriate strategy:* Confusing the numerator with the size factor of the denominator, then using the fraction rule "the larger the denominator, the smaller the fraction" so that 4 ninths is smaller than 2 ninths.

Same numerators, different denominators (e.g. $^{2}/_{3}$, $^{2}/_{5}$):

- *Appropriate strategies:* Using a common whole, construct both fractions and compare. Get a pattern that shows that the smaller denominator is the largest. Using a half as a benchmark so that 2 thirds is more than a half but 2 fifths is less than a half and therefore 2 thirds > 2 fifths.
- *Inappropriate strategy:* Whole-number comparison – because 5 > 3, 2 fifths is larger than 2 thirds.

All different (e.g. $^{3}/_{4}$, $^{5}/_{6}$):

- *Appropriate strategy:* Use 1 as a benchmark so that 3 quarters has 1 quarter left which is larger than the 1 sixth left from 5 sixths.
- *Inappropriate strategy:* Finding the differences, that is, 3 is 1 less than 4 and 5 is 1 less than 6 so the fractions are equivalent.

All different (e.g. $^{3}/_{7}$, $^{5}/_{9}$):

- *Appropriate strategy:* Using a half as a benchmark.
- *Inappropriate strategy:* Finding the differences (in which case these fractions would be equivalent) or using the fraction rule.

From this activity, teachers should understand the importance of providing the students with plenty of approximation activities and the importance of having their students explain their answers because correct answers can be based on inappropriate cognitive strategies.

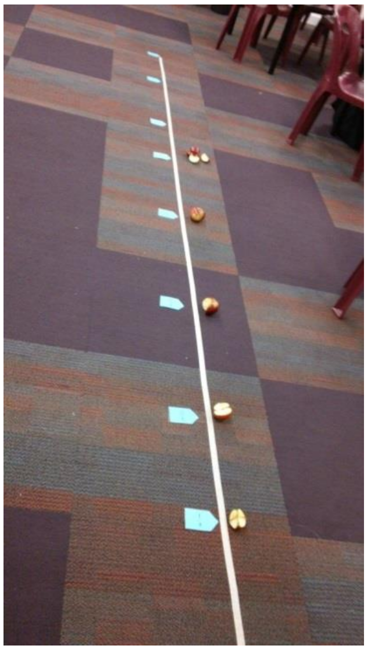
4.4 Integrating representations

Undertake activities that **integrate more than one model/representation** – the activity below is an excellent one for this.

Activity: Apples for the Teacher!

Materials:

- Tape on the floor
- 6–8 small apples each almost cut into quarters but not separated out
- Cards for each of the quarter fraction names up to 2 whole apples



Directions:

1. Begin with 0 apples, putting the card at the base of the line.
2. Ask for students to find one quarter and place both the quarter apple and the fraction numeral a step along the line.
3. When over one, mixed numbers are required and can be discussed.
4. Continue like this to place all the pieces of apple until you run out.
5. When the supply of apples runs out, encourage the students to work out what they need to take from previous collections.

Unit 5: Equivalence and Comparison

This unit looks at equivalence, comparison and conversions. The equivalence activities take up the major part of the unit with the equivalence techniques providing the method that is used for comparing unlike denominator and unlike numerator fractions. Comparison builds on the work already started in Unit 4. Conversion enables change from decimals to fractions.

5.1 Determining equivalence and the equivalence rule

The sequence for teaching equivalence is based on a variety of techniques and follows this sequence:

> **Step 1**: Showing equivalence is possible: real-world situations of equivalence (e.g. showing that $^1/_2 = {}^2/_4$ for a cake) → 3D and virtual equivalence situations (e.g. showing that $^2/_3 = {}^4/_6$ with fraction discs).
>
> **Step 2**: Developing sequences of equivalent fractions with materials, virtual and pictures (e.g. using paper folding or discs and strips to find a sequence of fractions); followed by more sequence work with 2D (e.g. using fraction mats to write down sequences).
>
> **Step 3**: Looking for a pattern (e.g. using existing sequences and/or fraction sticks to find the pattern that determines equivalence).
>
> **Step 4**: Reversing the pattern to obtain a rule for determining equivalence; and practising the pattern.

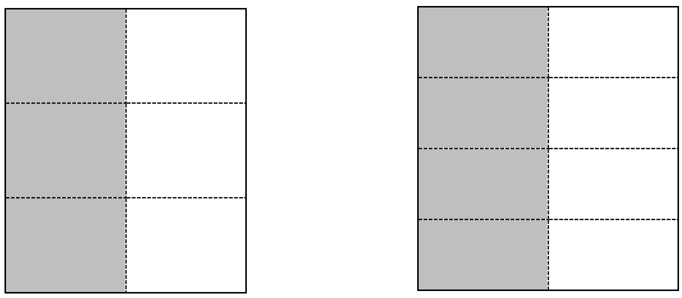
Techniques

There is a series of techniques that can be used to sow equivalence and then to find the equivalence rule that *two fractions are equivalent if they cancel down to the same "starting" fraction*. These methods work excellently as **virtual materials**.

1. **Paper folding**. If an A4 sheet is folded longways and half is shaded, it forms a foundation for equivalence.

   If this A4 sheet is then folded:

   - in half shortways – it shows $^1/_2 = {}^2/_4$
   - in thirds shortways – it shows $^1/_2 = {}^3/_6$
   - in fourths shortways – it shows $^1/_2 = {}^4/_8$
   - in fifths shortways – it shows $^1/_2 = {}^5/_{10}$ and so on.

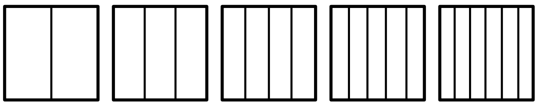
   

   The basis of the folding is that $^2/_4$ has to be seen as "1 lot of 2" in "2 lots of 2" (as a whole) to enable $^2/_4$ to be seen as the same as $^1/_2$. This is an example of reunitising and is difficult for students to do. However, when $^1/_2$ is folded the other way into three, both the 1 part and the whole of 2 parts are broken into three – showing $^3/_6 = {}^1/_2$. This provides the starting point for the equivalence pattern.

2. **Fraction discs and strips**. To compare $^2/_3$ with $^4/_6$, the disc material for each is taken out, one material is placed on a whole and the other *on top* of the first material to see whether they cover the same area. For fraction strips, the two materials are compared by placing the material beside each other and looking to see if they have the same length.

3. **Bases**. One technique for equivalence is to have a series of squares divided into a variety of fractions in a variety of ways. Cut out coloured rectangles to fit over the bases and colour the fractions (could do this by shading). Find a base board for a different partitioning that can be coloured or shaded to show the same area as in the first square).

4. **Overlays**. Start again with squares partitioned into fractions by vertical lines only (see on right). Shade various fractions. Construct a number of plastic see-through overlays made from the same square shape divided into fractions (halves, thirds, quarters, fifths, etc.) by horizontal lines. Then if the overlays are put, in turn, on top of any of the shaded fractions, a sequence of equivalent fractions is determined by how the overlays cut up the fraction (e.g. 2/3=4/6=6/9, and so on). *Note:* Using computers and virtual bases and overlays is very effective for this activity.



5. **Fraction mat**. This is a mat with length examples of ones, halves, thirds, and so on that can be visually compared (see below for a mat with only symbols – other mats can have language only or both).

1 whole									
$\frac{1}{2}$	$\frac{1}{2}$								
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$							
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$						
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$					
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$				
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$			
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

6. **Fraction sticks**. These are strips of paper that show equivalence. An example is given on the next page.

RAMR lesson

Reality

Real-world material and meaning. A round cake can be used to show simple equivalence. Cut the cake into two halves. Cut one half into two quarters. Show that $^{2}/_{4}$ is equal to $^{1}/_{2}$ by comparing the amount of cake in the half and the two quarters. The following could also be used to show equivalence: (a) 12×8 chocolate block – show $^{1}/_{3}$ = $^{2}/_{6}$; (b) long piece of liquorice – show $^{3}/_{4}$ = $^{6}/_{8}$.

Abstraction

Step 1: 2D/virtual/3D material and meaning

- *Area (fraction discs).* Fraction circles or rectangles can be used to compare the following, by placing material on top of each other, and determine which pair is equivalent: (a) $^{1}/_{2}$ and $^{2}/_{4}$; (b) $^{4}/_{6}$ and $^{2}/_{3}$. Ensure examples are also used that do not show equivalence: e.g. $^{3}/_{4}$ and $^{2}/_{3}$.

- *Length (fraction strips).* Fraction strips can be used to compare the following, by placing material beside each other, and determine which pair is equivalent: (a) $^{1}/_{3}$ and $^{2}/_{6}$; (b) $^{1}/_{2}$ and $^{3}/_{6}$. Ensure examples are also used that do not show equivalence: e.g. $^{3}/_{4}$ and $^{4}/_{5}$.

- *Set (Unifix).* Use sets of Unifix to compare the following, by partitioning two sets of Unifix material and comparing the result, and determine which pair is equivalent: (a) 12 Unifix – $^1/_3$ and $^2/_6$; (b) 20 Unifix – $^3/_5$ and $^6/_{10}$. Ensure examples are also used that do not show equivalence: e.g. 28 Unifix – $^5/_8$ and $^4/_7$.

- *Bases and overlays.* Use the bases to find fractions equivalent to a starting fraction. Use overlays on a starting fraction to show other fractions equivalent to it.

Step 2: 2D/virtual/3D material and sequences of equivalent fractions

- Fold two rectangles of paper longways. Shade one half of each. Take one of the shadings and fold it the other way – into half, thirds, fourths, etc. In this way, we can develop a sequence of equivalent fractions equal to $^1/_2$: $^1/_2 = {}^2/_4 = {}^3/_6 = {}^4/_8$ = ... and so on. Use the same method to develop a sequence of fractions equivalent to $^2/_3$.

- Use a ruler and a fraction mat to do the following: (a) find if the following fractions are equivalent: $^3/_4$ and $^9/_{12}$; $^2/_5$ and $^4/_9$; $^6/_8$ and $^9/_{12}$; and (b) develop a sequence of fractions equivalent to $^1/_3$.

Step 3: Patterning material and pattern for equivalent fractions

- Construct a set of fraction sticks (an interesting material – a set is shown below). Make $^2/_5$ out of the 2 and the 5 stick.

- Look for the patterns that determine if two fractions are equivalent.

Fraction sticks

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

A fraction is made by putting two rows together, for example, 2/5:

2	4	6	8	10	12	14	16	18	20
5	10	15	20	25	30	35	40	45	50

There are many patterns that can be seen in the sequence, say, $^2/_3$, $^4/_6$, $^6/_9$, $^8/_{12}$, and so on. *Note:* Students tend to see this pattern additively – that is, they see pattern $^2/_3 = {}^4/_6 = {}^6/_9$ = ... in terms of adding 2 on the "top" and adding 3 to the "bottom". Students need to see this pattern multiplicatively – that is, to see $^2/_3 = {}^4/_6 = {}^6/_9$ =... as multiplying "top" and "bottom" by the same amount. This is the required equivalence pattern: the numerator and denominator both change by the same multiplier or divisor – this relates to multiplication by 1 which equals $^2/_2 = {}^3/_3 = {}^4/_4 = {}^5/_5$, and so on.

Step 4: Rule

Reversing the above pattern gives that **equivalent fractions are fractions that can be cancelled down to the same starting fraction.** This allows cancellation to be used as the way of determining if fractions are equivalent. That is, $^{8}/_{20} = {}^{14}/_{35}$ because both cancel down to 2/5.

Mathematics/Reflection

Practise with sticks and then without sticks to: (a) develop a sequence of equivalent fractions; (b) identify the ×1 instances; (c) cancel down later fractions to the first one; and (d) check equivalence by cancelling down to see if they reduce to the same fraction. For example, fraction A $^{14}/_{21}$ cancels down to $^{2}/_{3}$, fraction B $^{15}/_{20}$ cancels down to ¾ and fraction C $^{36}/_{54}$ cancels down to $^{2}/_{3}$, so A and C are equivalent.

5.2 Comparison for unlike denominator fractions

Comparison is working out which fraction is larger. If the fractions are on a number line, then this is easily seen. However, if fractions have unlike denominators, and their positions cannot be determined, then we use the equivalent fraction methods based on fraction sticks from the previous unit.

Abstraction

Way 1: For simpler fractions

The list of techniques below for comparison provides a repertoire of strategies that teachers should know (see subsection 4.3). They should be used with students in diagnostic situations (i.e. used with students for whom they appear to be suitable), **not necessarily taught to every student** regardless of need.

These activities should be done with the hands (use area/set/number line activities) and then with the mind:

- *Like denominators*. Here, the size of the numerator determines the larger fraction, for example: $^{4}/_{11}$ is smaller than $^{7}/_{11}$.
- *Unlike denominators, like numerators*. Here, the size of the denominator inversely determines the size of the fraction. This is a consequence of "fraction as division" (the more to share – the less each sharer gets). For example: $^{4}/_{11}$ is smaller than $^{4}/_{7}$.
- *"Near one"*. This method follows from the above and is useful for comparing fractions that are close to 1. For example, $^{7}/_{8}$ is less than $^{10}/_{11}$ because $^{7}/_{8}$ is $^{1}/_{8}$ away from 1 while $^{10}/_{11}$ is $^{1}/_{11}$ away from 1.
- *Benchmarking*. Here, the fractions are related to benchmarks (e.g. whole numbers, halves, quarters) to determine size. For example, $^{4}/_{11}$ is less than half and $^{4}/_{7}$ is greater than half, thus $^{4}/_{11}$ is less than $^{4}/_{7}$.

Way 2: For complex fractions (equivalence – unlike denominators and numerators)

Equivalence. When fractions are complicated, then **the equivalence way works for all fractions every time**. In this way, the fractions are converted to the same denominator using equivalence and then compared (**see the fraction sticks in 5.1**). The steps are as follows:

- Use sticks, for example, to make $^{2}/_{5}$ and $^{3}/_{7.}$
- Put these 2 sets of 2 sticks for these two fractions together as below.
- Find the common denominator [35] and compare the fractions $^{15}/_{35}$ and $^{14}/_{35}$. Which is bigger? [$^{3}/_{7} = {}^{15}/_{35}$]

3	6	9	12	15	18	21	24	27	30
7	14	21	28	35	42	49	56	63	70

2	4	6	8	10	12	14	16	18	20
5	10	15	20	25	30	35	40	45	50

Note: Sticks used in this way can compare, add or subtract fractions by aligning the "common denominator".

Mathematics

The above methods in Way 1 and Way 2 provide the foundation for sequencing the comparison of fractions. For example, the following pairs of fractions should be compared in the following order:

$^{2}/_{9}$ $^{4}/_{9}$ (like denominators)
$^{2}/_{3}$ $^{2}/_{5}$ (like numerators)
$^{3}/_{4}$ $^{5}/_{6}$ (near 1)
$^{3}/_{7}$ $^{5}/_{9}$ (benchmarking around a $^{1}/_{2}$)
$^{5}/_{8}$ $^{7}/_{11}$ (equivalence)

However, Way 2 (reinforced with a few examples from Way 1) is all that is needed for any pair of fractions to be compared.

Note: Teachers should understand the importance of providing students with plenty of approximation activities and the importance of having students explain their answers because correct answers can be based on inappropriate cognitive strategies.

5.3 Conversions – fractions, decimals and metrics

Activity: Number line using common and decimal fractions

Materials: Masking tape or rope on the floor (3–4 metres); cards with different representations (0–2) – Yellow for money and significant decimal only; Green for 2 decimal place and fraction out of 100; Blue for common fractions.

1. Start with ordering decimal representation of money (yellow) along the line. Flip cards to show significant decimal, e.g. 70c becomes 0.7, $1.30 becomes 1.3.
2. Beside this place equivalent 2-decimal place card (green); e.g. 0.7 matches 70c, 1.3 matches $1.30.
3. Place common fractions (blue) where possible, noting that mixed numbers 1 $^{1}/_{4}$ is on reverse of improper fraction $^{5}/_{4.}$

Note: This may be best done over a series of lessons. Order similar fractions, e.g. $^{1}/_{4}$, $^{2}/_{4}$, $^{3}/_{4}$, etc.

Extension: Include a red set for percentage, e.g. 120% is also $^{120}/_{100}$. Discuss mixed numbers, improper fractions, etc.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the common fraction item types

The common fraction item types are divided into five subtests, one for each of the five units in the module. The five units reflect a reasonable sequence of teaching ideas but with there are some more advanced ideas in earlier units and some basic ideas in later units. The meanings of fraction (part of a whole, part of a group, division, operator, quantity on a line) are spread across three units (1, 3 and 4).

Therefore, the pre-test could be taken from Subtest 1 and 2 items and the start of Subtest 3 and 4 items. This covers all fraction meanings, mixed numbers and counting ideas. The post-test needs to cover Subtest 3, 4 and 5 items and probably should repeat Subtest 1 and 2 items.

It is important, in both pre-test and post-test, to cover all the meanings of the fractions and to show this with pictures as well as symbols.

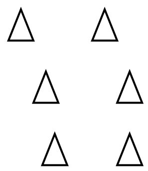
Subtest item types

Subtest 1 items (Unit 1: Part-whole fraction concepts)

1. Represent one half ($\frac{1}{2}$) on each of the following diagrams:

(a) Rectangle

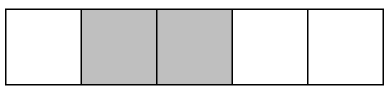
(b) Set: $\frac{1}{2}$ of this group of triangles



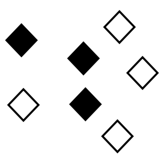
(c) Number line

2. What fraction is represented by each of the following diagrams?

(a) Rectangle

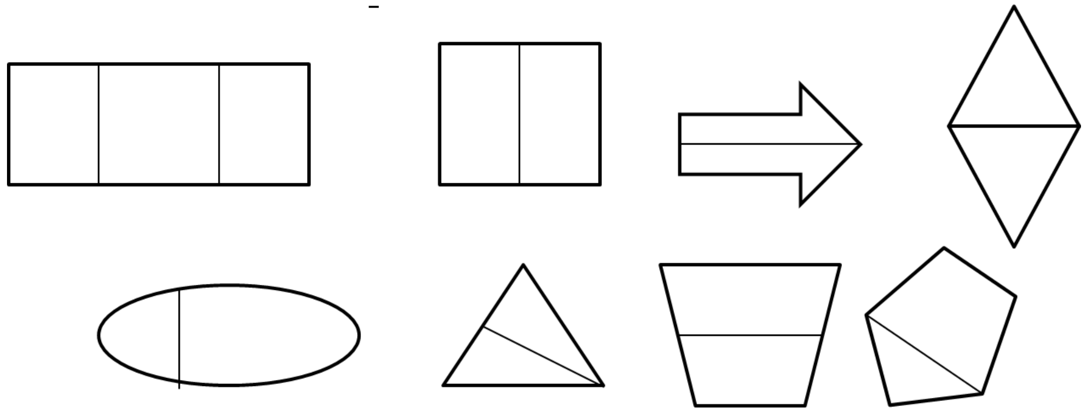


(b) Set

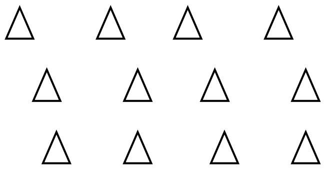


(c) Number line

3. (a) Tick the shapes below that have been divided in half.



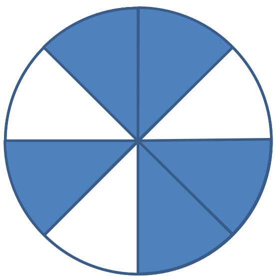
(b) Circle $\frac{1}{2}$ of this group of triangles.



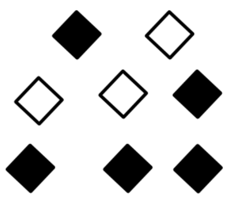
(c) Show the position of $\frac{1}{2}$ on the number line.

4. What fraction is represented by each of the following diagrams?

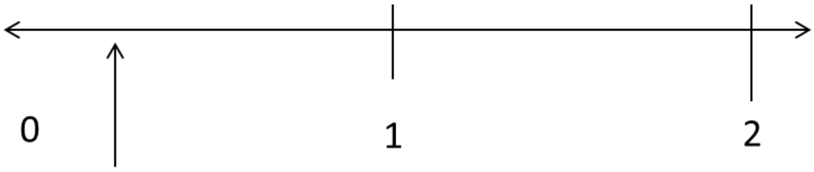
(a) Circle



(b) Set



(c) Number line



5. (a) This shape is part of a whole, it is three quarters ($\frac{3}{4}$). Draw the whole.

(b) This shape is part of a whole, it is one third ($\frac{1}{3}$). Draw the whole.

(c) These pencils are from one and a half ($1\frac{1}{2}$) packets of pencils. Circle how many pencils are in one packet.

Subtest 2 items (Unit 2: Mixed numbers and additive structure)

1. Write the number that is one fifth ($\frac{1}{5}$) more than:

   (a) $\frac{2}{5}$ __________

   (b) $\frac{4}{5}$ __________

2. Write the number that is one eighth ($\frac{1}{8}$) more than:

   (a) $\frac{6}{8}$ __________

   (b) $\frac{7}{8}$ __________

3. Complete the counting sequences:

   (a) $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, ______, ______, ______

   (b) $3\frac{2}{4}$, $3\frac{3}{4}$, 4, ______, ______, ______

   (c) $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, ______, ______, ______

   (d) $4\frac{1}{3}$, $4\frac{2}{3}$, 5, ______, ______, ______

4. (a) Draw a diagram to show $1\frac{2}{5}$

   (b) Draw a diagram to show $1\frac{3}{8}$

5. Write the following as mixed numbers, e.g. 4 thirds = $1\frac{1}{3}$ *or* $\frac{4}{3} = 1\frac{1}{3}$

(a) 7 fifths = ___________

(b) $\frac{8}{7}$ = ___________

(c) 10 quarters = ___________

(d) $\frac{7}{2}$ = ___________

(e) 9 eighths = ___________

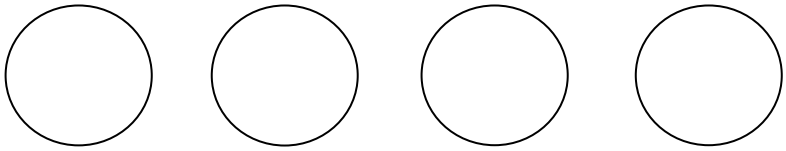
(f) $\frac{6}{5}$ = ___________

(g) 7 thirds = ___________

(h) $\frac{9}{4}$ = ___________

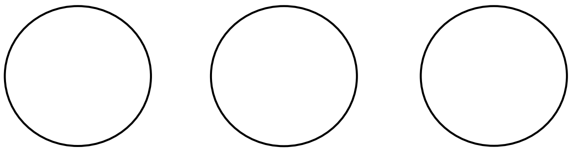
Subtest 3 items (Unit 3: Multiplicative structure)

1. (a) If these four cakes were shared amongst 5 people



What fraction of the cake would each person get? __________

(b) If these three cakes were shared amongst 4 people



What fraction of the cake would each person get? __________

2. How many quarters in:

(a) $1\frac{1}{4}$ __________ quarters

(b) $4\frac{1}{2}$ __________ quarters

(c) $2\frac{3}{4}$ __________ quarters

(d) $3\frac{1}{2}$ __________ quarters

3. (a) One third ($\frac{1}{3}$) of 2 is __________

(b) One fifth ($\frac{1}{5}$) of 4 is __________

Subtest 4 items (Unit 4: Discrete-continuous/Number line)

1. (a) Order these numbers from **smallest to largest.**

$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{5}$

(b) Order these numbers from **largest to smallest.**

$\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{4}$

2. (a) Write a number that is between $\frac{1}{2}$ and 1 __________

(b) Write a number that is between $\frac{3}{5}$ and $\frac{4}{5}$ __________

(c) Write a number that is between 1 and $1\frac{1}{2}$ __________

(d) Write a number that is between $\frac{3}{6}$ and $\frac{4}{6}$ __________

Subtest 5 items (Unit 5: Equivalence and comparison)

1. Using a calculator, convert the following fractions to decimals:

   (a) $\frac{5}{8}$ ___________

   (b) $\frac{13}{18}$ ___________

   (c) $\frac{5}{6}$ ___________

   (d) $\frac{13}{16}$ ___________

2. In each box, circle the fraction that is **not** equivalent (that has a different value) to the others in the box:

   (a)

$\frac{1}{2}$	$\frac{3}{7}$	$\frac{4}{8}$	$\frac{9}{18}$

   (b)

$\frac{2}{3}$	$\frac{6}{9}$	$\frac{5}{12}$	$\frac{10}{15}$

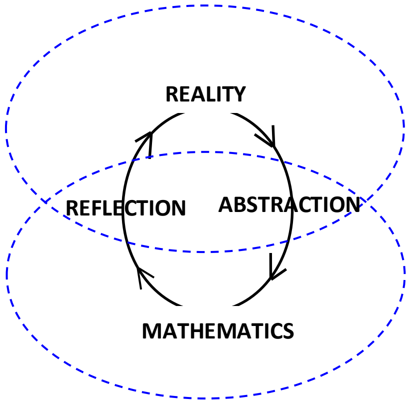
3. I want to sort these fractions into two groups. What might the two groups be and why?

   $\frac{2}{5}$ $\frac{3}{4}$ $\frac{6}{10}$ $\frac{1}{3}$ $\frac{1}{10}$

4. The answer is $\frac{3}{7}$. What might the question be?

Appendix A: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix B: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system **N2: Decimal Number Numeration** Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches **SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae **N3: Common Fractions** Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships **O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject **N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference **G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs **N5: Directed Number, Indices and Systems** Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio **G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation **SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics **O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

YuMiDeadly
Growing community through education

Faculty of Education
School of Curriculum
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059

CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
Email: ydc@qut.edu.au
Website: ydc.qut.edu.au