Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module M3

Year B, Term 1

Extension Measurement:

Time, Angle, Money and Temperature

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

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DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

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Module Overview

This is the final measurement module and it covers all the stages for teaching the measurement topics of **time, angle, money** and **temperature**. Except for money, these are measurement topics that do not have a base 10 numbering system – instead they use base 60, 360, and 100. All these four areas also have basic things that have to be learnt early, almost independent of the five stages below. For example, “it’s 8 o’clock and it’s Tuesday morning”, “we’re travelling North”, “have you got a $5 note?”, and “what is the temperature today?” In particular, money and time are multifaceted. Time has point of time (e.g. time-telling or reading a clock), sequence of time (hours in a day, days in the week, months in the year, and so on) as well as duration of time that follows the five stages. Money has knowing the coins and notes as well as measuring money as value. For the reasons above, time, angle, money and temperature are called extension measures.

Similar to modules M1 and M2, the major big idea for M3 is **continuous vs discrete**. All measurement topics are continuous and cannot naturally be counted or represented by numbers. However, the invention of unit has allowed the continuous to be “discretified” or partitioned into units and these units can be counted. This application of unit changes learners’ perception of the units, and perception of their world. Thus, the sequence for teaching these measurement topics has three parts: (a) understanding the measurement topic in its natural continuous state (Stages 1 and 2); (b) the introduction of unit and number to measurement topics (Stage 3); and (c) understanding the standard units adopted by Australia and applications of, and relationships between, these units (Stages 4 and 5).

(*Note*: In measurement, number can never be alone; measures are given by **number and unit**).

Background information for teaching extension measurement

Measurement is a topic whose teaching can clash with non-numerical cultures. This is discussed in Module M1 *Basic Measurement.* This section for this module focuses on big ideas and connections.

Big ideas

The big ideas for measurement are:

1. **Continuous vs discrete**. Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted.

1. **Interpretation vs construction**. Things can either be interpreted (e.g. a m is 100 cm) or constructed (construct a m out of 10 cm lengths of straws).

2. **Notion of unit**. Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts (e.g. 1000 mm makes a m and 1000 m makes a km).

3. **Multiplicative structure.** Standard units are designed so that they reflect place value in that adjacent positions are related by moving left (× base) and moving right (÷ base), where the base is 10 for metrics but 60 for time and angle and, in practical terms, 100 for dollars-cents and temperature.

4. **Common units**. We must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60) and, if we do so, the object with the biggest number has the most attribute.

5. **Inverse relation**. Same as Extension of field properties principle (i.e. the bigger the unit, the smaller the number – e.g. 2 m = 200 cm).

6. **Accuracy vs exactness.** Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100) or exactly (5 275 + 3 873 = 9 148).

7. **Attribute leads to instrumentation**. The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring).

8. **Triadic relationships**. When three things are related, there are three problem types (e.g. measuring length has three components, the object, the number and the unit – thus we can set problems where the object is unknown, the number is unknown or the unit is unknown).

9. **Balance rule**. Whatever you do to one side of an equation you have to do to the other to keep things equal.

Other than *continuous vs discrete* whose major importance is highlighted above, the big ideas with particular importance are *common units*, *inverse relation* and *accuracy vs exactness* which have a powerful role in nonstandard units, and *triadic relationships* which has a strong role in the application and formulae stage.

Important connections involving measurement

The three connections discussed in Modules M1 and M2 are different for the time, angle, money and temperature in M3 than they were for the measures length, mass, capacity, perimeter, area and volume in M1 and M2. They have been changed as below.

1. **Connection between unit, fractions and division**. This connection gains in importance. The connection remains between fractions that divide a whole into equal pieces and division that divides a time measure (e.g. hours) into equal sub-measures (e.g. minutes). However, with non-metric relationships, the fraction understanding that a minute is a 60^{th} of an hour gains in importance as there is no similar decimal relationship. And we still have the other consequence that division, fractions and units of measure all have inverse relations – that is, the fewer to share among (divide by) the more each sharer gets; the smaller the denominator, the larger the fraction; and the smaller the unit the larger the number of units (e.g. the activity measures 2 hours or 120 minutes).

2. **Connection between fractions, decimals and unit conversion**. For the measures in M1 and M2, this was only decimals, and the metric names related to place values and conversions (e.g. if mm were ones, m were thousands and km millions making conversions ×1000 and ÷1000). This remains for money and, to a lesser extent, temperature. But the conversions for time are ×60 and ÷60 for hours to minutes and ×3600 and ÷3600 for hours and seconds, while the conversions for angle are ×90 and ÷90 for right angles to degrees, and so on. There are also ×12 and ÷12 for years and months, and ×360 and ÷360 for full turns to degrees, and so on. Only in decades and centuries do decimals re-emerge.

3. **Connection between PV, whole-part, and algebra computation**. For money, temperature and time (in years, decades and centuries), the same separation strategy (see Modules O1 and O2) for addition, subtraction, multiplication and division holds as for whole numbers. For time and angle (for time up to a year), the separation strategy holds as for mixed numbers and part-whole charts. Here, a similar process holds for 2h 34m + 5h 42m, for example, as it does for 58+34, but carrying minutes to hours happens when equal to or above 60, not 10.

Note: Unlike Module M2, there is no connection between all four measures. The only similarity is between angle and time, where angle does break into minutes and seconds similar to time for plotting directions on boats. This is because units for time and angle are Babylonian in background, where 60 was the base. This explains the relation based on 60 between seconds, minutes and hours. However in shape, the Babylonians were attracted to the fact that 6 equilateral triangles connect to make a full turn and they assigned the 60 to each triangle making the full turn 360 degrees (but still keeping the minutes and seconds for each degree). Babylonian angle still plays a wide part in our measures because the sea distance unit of nautical mile is the distance at the equator of one minute of angle at the centre of the earth (and knot is one nautical mile per hour).

Sequencing for extension measurement

This section covers the five stages for sequencing the teaching of any measure, and the sequence in this module.

Stages of measurement teaching

The teaching of the measurement topics time, angle, money and temperature follow the same stages below as in the previous measurement modules.

Stage 1 – Attribute identification

This stage focuses on students understanding the attribute (or concept) of the measure. Activities to identify attributes: should follow **rich experiences** with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, and so on. They should also involve **developing meaning** for all the specialist attribute language that accompanies measurement topics.

The central idea in learning about an attribute is to experience it. However, if students have difficulty identifying the attribute from other characteristics of the experience, there are two general ways to introduce any attribute by providing examples where: (a) the only thing that is the **same** is the attribute, and (b) the only thing that **varies** is the attribute.

Stage 2 – Comparing and ordering

This stage focuses on comparing (i.e. two examples) and ordering (i.e. three or more examples) the amounts of attribute in examples. The process of ordering is based on comparison; the ability to compare two examples is extended to ordering three examples by identifying the one that is **between** the other two. Stage 2 activities are learnt in two parts: (a) **direct comparison** and order where examples are compared directly to each other; and (b) **indirect comparison** and order through an intermediary. Stage 2 activities involve **no units** and **no numbers**; the total amounts of the attribute present are **compared or ordered holistically**.

Stage 3 – Non-standard units

This stage has two foci: (a) introducing the **notion of unit**; and (b) the development of measurement **processes** (using instruments with which to measure) and measurement **principles** (i.e. big measurement ideas that hold across all measurement topics). To prevent too much new information being given at once, the units are non-standard or class/learner chosen so that the learner is familiar with them. The measurement processes differ for different topics; they are related to the proper use of the measuring instruments. The **measurement principles** are the techniques for using units. These are: (a) *common units* – common units must be used in measuring, comparing/ordering and calculating amount of attribute and, in this case, the example with the most attribute has the larger number of units (this leads to the need for a standard); (b) *inverse relation* – the bigger the unit, the smaller the number and vice versa; and (c) *accuracy vs exactness* – all units give rise to error, with smaller units being more accurate but more difficult to apply, so there is a need to choose the level of accuracy required for the job. This principle requires a tolerance for error and also leads to the skill of choosing appropriate units and to developing skill in estimating as well as accurate measuring.

Stage 4 – Standard units

This stage focuses on the introduction of the standard units accepted by Australia. It should be remembered that these units should only be introduced **after** the need for a standard has been determined by recognising the limitations of non-standard units. It is recommended that they be preceded by the use of a class-chosen **common unit** (if appropriate). The recommended sequence for introducing standard units is: (a) **identifying** the unit through experiencing it or constructing it; (b) **internalising** the unit through relating it to body or everyday activities; and (c) **estimating** with the unit before measuring. Stage 4 activities should also relate to the **decimal number system** to build understanding of conversion between units and should continue to develop the measurement **processes and principles** begun in Stage 3.

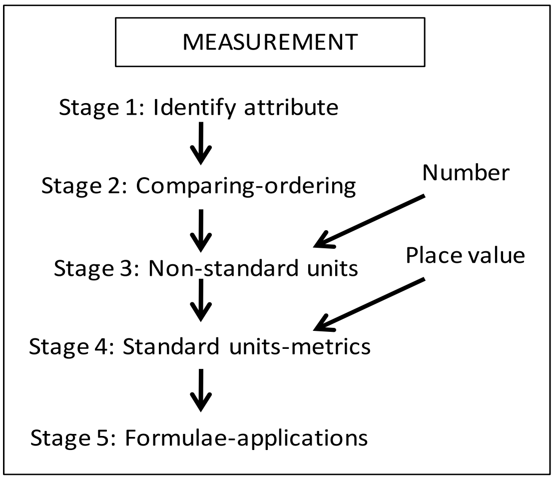
Stage 5 – Applications and formulae

This section focuses on: (a) **applications** of measures to the real world (i.e. calculating the measure of things); and (b) any **formulae** for determining measures (tends to be restricted to angle and money).

Sequencing in the module

The teaching of the measurement topics time, angle, money and temperature follow the sequence as on the diagram on right. The first two stages have **no number**, the third introduces unit and number, the fourth introduces standard units, while the fifth looks at applications and introduces formulae.

The module covers all stages for duration of time, angle, money as measure, and temperature, and also provides some ideas for point of time (reading a clock), sequence of time (days, months, decades, centuries) and for money-handling skills (notes and coins for dollars and cents).



The **six sections** in the module are:

Overview: Background information, sequencing, and relation to Australian Curriculum

Unit 1: Time – point of time, sequence of time and duration of time through the five stages (attribute, comparing, non-standard units, standard units, applications and formulae)

Unit 2: Angle – attribute, comparing, non-standard units, standard units, applications and formulae

Unit 3: Money – money-handling skills and money as value through the five stages (attribute, comparing, non-standard units, standard units, applications and formulae)

Unit 4: Temperature – attribute, comparing, non-standard units, standard units, applications and formulae

Test item types – test items associated with the three units and built around the five stages which can be used for pre- and post-tests

Appendix A: RAMR cycle components and description

Appendix B: AIM scope and sequence showing all modules by year level and term.

This sequence, like that in the previous measurement modules, has two components: (a) a sequence **across** the four units from time to temperature, and (b) a sequence **within** each unit that follows the five stages. The teaching ideas within each stage within each cycle follow the RAMR cycle which is described in **Appendix A**.

However, it is not necessary to complete all units in the sequence as they are in this module. In fact, you may find it more useful to do all the Stage 1 activities together, then all the Stage 2, all the Stage 3 and all the Stage 4 activities together, up to all the Stage 5 activities together (for time, angle, money and temperature). There are relationships between metric units and formulae for time, angle, money and temperature that can be used to make this an efficient way to teach in Stages 4 and 5.

Relation to Australian Curriculum: Mathematics

AIM M3 meets the Australian Curriculum: Mathematics (Foundation to Year 10)					
Unit 1: Time Unit 2: Angle	Unit 3: Money Unit 4: Temperature				
Content Description	**Year**	**M3 Unit**			
		1	**2**	**3**	**4**
Compare and order the duration of events using the everyday language of time (ACMMG007) Connect days of the week to familiar events and actions (ACMMG008)	**P to 2**	✓			
Tell time to the half-hour (ACMMG020) Describe duration using months, weeks, days and hours (ACMMG021)		✓			
Tell time to the quarter-hour, using the language of 'past' and 'to' (ACMMG039) Name and order months and seasons (ACMMG040)		✓			
Use a calendar to identify the date and determine the number of days in each month (ACMMG041)		✓			
Tell time to the minute and investigate the relationship between units of time (ACMMG062)	**3**	✓			
Convert between units of time (ACMMG085) Use am and pm notation and solve simple time problems (ACMMG086)	**4**	✓			
Compare 12- and 24-hour time systems and convert between them (ACMMG110)	**5**	✓			
Interpret and use timetables (ACMMG139)	**6**	✓			
Solve problems involving duration, including using 12- and 24-hour time within a single time zone (ACMMG199)	**8 to 9**	✓			
Investigate very small and very large time scales and intervals (ACMMG219)		✓			
Identify and describe half and quarter turns (ACMMG046) Identify angles as measures of turn and compare angle sizes in everyday situations (ACMMG064)	**P to 3**		✓		
Compare angles and classify them as equal to, greater than or less than a right angle (ACMMG089)	**4**		✓		
Estimate, measure and compare angles using degrees. Construct angles using a protractor (ACMMG112)	**5**		✓		
Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles (ACMMG141)	**6**		✓		
Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)	**7**		✓		
Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)			✓		
Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)			✓		
Recognise, describe and order Australian coins according to their value (ACMNA017) Count and order small collections of Australian coins and notes according to their value (ACMNA034)	**1 to 2**			✓	
Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents (ACMNA059)	**3**			✓	
Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies (ACMNA080)	**4**			✓	
Create simple financial plans (ACMNA106)	**5**			✓	
Use scaled instruments to measure and compare lengths, masses, capacities and temperature s(ACMMG084)	**4**				✓

Unit 1: Time

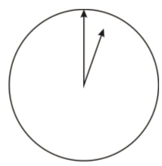
The first important aspect of time is that there are three perspectives: (a) **point of time** (e.g. reading a clock); (b) **sequence of time** (e.g. knowing the days of a week); and (c) **duration of time** (e.g. how long is a minute). As a measurement, the most appropriate perspective is duration of time. This section looks at the three perspectives and gives the five stages for duration.

The second important aspect of time is that it is not a base 10 attribute – it has a base of 60 – being a construction of the Persian or Babylonian Empire which had 60 as an important number.

1.1 Point of time

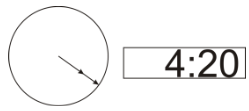
Point of time is crucial for employment as it enables the worker to tell time, keep time logs and calculate the amount of time on a job. Three aspects are examined.

Sequence for time-telling skill



Begin with full turns, part turns, quarter and half turns. Study the clock face and the two hands, working with non-geared and geared clocks, looking at clockwise and anti-clockwise turns. Relate full turn of large hand to movement of small hand. Relate full turn to an hour and o'clock; relate part turns to half-past, quarter past and quarter to. Use geared clock activities to discover relation between small-hand and large-hand movements.

Relate part turns to fractions. Use angle wheels, rotagrams, and Rotascan clock (see below). Look at:



- counting by fives, telling time in five-minute intervals (5 past, 10 past, etc.);
- introducing sixtieths in relation to clock face;
- introducing notion of minute;
- reading time on digital clocks;
- reading 24-hour clocks; and
- relating, e.g. 5:43 to 17 minutes to 6.

Use worksheets with clock faces, relating drawings of time on face with language and digital time. Relate different ways to use language to tell time. Introduce the second, using stopwatches, and apply timing to sport and navigation.

Look at point of time and sequence of time. Discuss when things happen in each day. Ask students, aides, parents, elders for local ways of telling the time. Introduce the calendar – days, weeks, months, years. Look at the Torres Strait calendar – how does it differ? Look at movement of stars and moon.

Kinaesthetic activities for time-telling

Turn the classroom into a clock – number 12 at front and 1 to 11 around the walls. Turn bodies counting o'clocks as point at numbers; turn bodies counting minutes by 5. Act out times with students' hands as hours and minutes – go both directions (reverse) teacher calls a time → students act as the hands of a clock, and teacher acts as hands of a clock → students call a time (both directions also for o'clock and minutes by 5s).

Relate time to movement of the sun. Act out pointing at the sun as it moves across the sky while calling out hours of the day and times when things are done (e.g. start of school, lunch, and so on). Connect point of time to these well-known times of the day (also TV programs).

Lesson plan for time-telling

Materials: Two ropes and a post. A timer. A5 numbers 1-12 (or 24). Two paper plates, two hairpins, scissors and glue. An analogue clock.

Reality

Students discuss and list events that take small and large amounts of time, e.g. washing up, getting to school, sleeping, time between breaths, eating morning tea, running 100 m, becoming a teenager.

Students describe the different units we use to record time especially second, minute, hour, day, week, fortnight, month, year.

- Discuss the mechanics of an analogue clock.
- Discuss the mechanics of a digital clock. Discuss as many different locations digital clocks are found in the home, and in the school.
- Discuss where students might have seen a timetable (bus stops, train station, TV guide, classroom wall for break times, etc.).

Abstraction

Students go outside and form a human clock by making a circle and using a post and two ropes/ribbons.

Students discuss and help their peers with the minute and hour hands to move to different times of the day, keeping track of both hour and minute hand movement and that it is co-ordinated.

Teacher calls out new times e.g. “Go ahead in time 1 hour and 20 minutes”; “Go backwards in time 75 minutes”.

Back in the classroom students create a giant analogue clock around the walls of the classroom. Print off large A4 numbers 1-12 (or 13 – 24 if your students are ready for 24-hour time). Stick the numbers around the walls starting with 12 in the centre of wall above the whiteboard (or centre of wall where most students see it from their desk). Place the 6 opposite the 12 on the back wall. Have students place the remaining numbers. By standing a student in the centre of the room, you can have the student model any particular time, or change in time. This can be made into a game.

A second abstraction classroom activity is where you have the students make a Peek-a-boo clock using two paper plates and hairpins (see right).

Mathematics

Practice/Symbols. Students come back to the classroom (if outside) or return to their desks (if inside) and record some of the times they saw demonstrated as well as a changed time and explain how far each hand moved to arrive at the new time. They record how many hours and minutes and the correct pronunciation of each time.

It is essential students have an understanding of counting in fives as they explore changes in time around the analogue clock face. This is difficult to do with digital clocks.

Students work in pairs with their partner and make two different times on their peek-a-boo clocks and then each calculates the difference between the clocks. Students draw and record several examples.

Students can make up an analogue clock with their “own numbers” for a little maths problem e.g. for the 1 o’clock, they might like to write “½ of 2”; at 2 o’clock they might put 5 – 3, and so on. When completed, students can set a time on their clock, and swap clocks with another student. The class then has to solve each little math problem and tell the time.

Connection activities would focus on relation between fractions and movement of the hands of a clock.

Reflection

Validation/Application. Students create a story using six clock times and illustrate their story with representations of the clock times. The six clock times should describe events in the story.

An extension of this might be to use students “own number” clock at the front of the room. Set a time on it, and have the class create a time story using the little maths problems and the time as shown on the clock.

Extension. Activities would be to extend the 12-hour clock to 24-hour time. The peek-a-boo clock would help here. Also acting out two turns of the classroom clock – 1 o’clock, 2 o’clock, ... , 12 o’clock, 13 o’clock, ... , 24 o’clock.

1.2 Sequence of time

This is a crucial area – knowing day, date, month and year. It requires familiarity with hours across a day, am and pm, days in a week, days (and weeks) in a month, months in a year, decades and centuries, AD and BC, and so on. The history of many of the names is interesting – the names of the days and the names of the Norse gods (Saturn, Sun, Moon, Tuor, Wodin or Odin, Thor and Fria). The source of the names of the months – the god Janus, the Caesars Julius and Augustus, and the Latin names for 7, 8, 9 and 10.

An activity to use kinaesthetic activity is to relate seasons to actions that are typical of those seasons – e.g. summer and swimming for the south-east.

Calendars are also interesting in this area. There are many interesting ideas using calendars – e.g. the difference between the sum of opposite corners of days forming a square or rectangle on a calendar within one month is always zero. (Why?)

Tuesday	Wednesday
1	2
8	9
15	16

(16 + 1) – (15 + 2) = 17 – 17 = 0

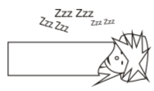
Look at point of time and sequence of time – integrate the two. Discuss when things happen in each day. Ask students, aides, parents, local leaders and elders for local ways of telling the time. Introduce the calendar – days, weeks, months, years. Look at local calendars – Aboriginal and Torres Strait Islander calendars – how do they differ? Look at the movement of stars and moon. Connect this work to astronomy. Look at movements of the moon (full, new, etc.).

1.3 Duration of time – Identifying the attribute of time

The perspective of duration of time is outlined in full using the five measurement stages.

Time is often called the fourth dimension. It is the duration we spend undertaking an activity. We are fixed within the flow of time. We are unable to move through time at our own volition or at our own speed and direction. Students are continually experiencing time. We do not have to set up special “changes” for them to experience. They only need to be directed to reflect on their own experience. But, of course, we cannot set up any contrasting experience (such as no change in time or twice as fast a change in time) with which to compare. Time is also a very subjective experience. Duration is the component that relates to the learning sequence as it is the measurement of an amount of time.

Activities



Arrange pictures of events in the order in which they happened, hang pictures with pegs on a clothes line in this order (with the size of the pictures related to the length of the event and the closeness of the pictures related to how closely they occur next to each other in time).

Associate events with the time of the day, e.g. when do we come to school? Look at how cycles repeat themselves (e.g. out of bed, get dressed, eat, clean teeth, etc.).

Experience activities that take a short or a long time. Wait while others do things (e.g. hop 10 times, run around oval, etc.). Note when things start and finish. Experience a variety of activities which all take the same time. Relate the passing of time to another activity (e.g. shading in a column or row on a blackboard).

1.4 Duration of time – Comparing and ordering time

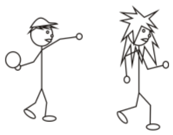
Once students understand the attribute of time they can explore ways to compare and order the time of different activities and events. The direct comparison of time is quite simple. It aligns with the concept explored in relation to length where a direct comparison can only be done effectively with a common baseline. When directly comparing the duration of activities it is important to start them at the same time. Once two or more activities have been started at the same time the duration can be compared and described. The activity that continues when the others finish is the activity that took the longest time. The activity that finished first is the one that took the smallest or shortest time. The others are in between.

The indirect comparison of time, without using units, requires an understanding of the transitivity principle. This principle was described in Module M1 in relation to length. The transitivity principle holds that:

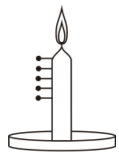
- if event A takes a longer amount of time than event B, and
- if event B takes a longer amount of time than event C; then
- event A will take a longer amount of time than event C.

One event is used as an intermediary (in this case event B). The intermediary event will need to be something familiar to the student. For example you could compare the amount of time taken to read a paragraph out loud with how long it takes to brush your teeth (a familiar activity that you have a good feeling for how long it takes), then compare the reading of another paragraph also with the event of brushing your teeth. Then the time taken for reading the two paragraphs can be compared after consideration of the intermediary event (cleaning your teeth).

Activities



Directly compare which activity takes longer by “having a race” (e.g. time for a ball to stop bouncing and walking across a room, running around building and writing name and address, etc.). Ensure children start both activities at the same time.



Indirectly compare activities by **timing both events with the same technique** (i.e. the amount of sand to run out of a small hole, the length of a taper candle that is burnt down, the length of the column on a blackboard that is shaded, the distance a ball rolls down an incline, the depth a holed container sinks to, etc.).

Relate common events (e.g. eating breakfast is longer than a short cartoon on TV but dressing takes less time, etc.). Label strips of paper with events; the length of the strips of paper is determined by the length of time the event took.

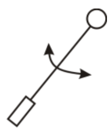
Warm up: Guess how long? The teacher sets a timer for 20 seconds and the girl students sit with eyes closed and raise their hands when they think 20 seconds is up and the boys judge. Swap roles and the boys try to guess. Teacher secretly sets the timer for a particular time, e.g. 28 seconds, and students listen until buzzer rings, write down and try to guess how long it was. Check to see who was the closest.

1.5 Duration of time – Non-standard units for time

Non-standard units of time need to be activities that have duration. Generally units will be of a short duration but longer activities can be used to time longer events. The focus with these units is that the measure of time will be the count of the number of times the unit activity is repeated to time the event in focus. For example the time it takes one student to run around the school oval could be measured in star jumps done by another student. Keeping these non-standard units uniform is likely to be difficult so these sorts of activities are good to introduce the idea of a count of units as a measure but also to introduce the need for standard units.

The duration of activities measured in non-standard units is a good way for young students to gain an experience of the length of time in each standard unit. For example, counting the number of times a student can write their first name in one minute helps the students develop an understanding of the length of a minute. Students can create non-standard clocks that can be used to measure time by marking equal spaces on a candle or by pouring sand from one container into another that has marked graduations. Students can also observe the passage of time related to the movement of the sun by observing shadows or making a sundial.

Activities



Use a regular action to time events (e.g. counting, pulse, a pendulum’s swing, hopping, the roll of a ball bearing in a curved groove, etc.). Look at history for interesting methods.

Calibrate the regular change in something to time events (e.g. a candle burning down is marked with pins at regular distances - thin taper candles are best), water or sand pouring out a small hole into another container marked into height intervals, a container marked at intervals down its side sinking in water, a ball rolling slowly down an incline (with obstructions) marked into distance intervals, etc.).

Relate time to the movement of the sun. Construct a sundial. Discuss historical time measurers. Discuss terms such as midday, morning, afternoon, evening (use worksheets and relate times to drawings). Develop the measurement principles.

Use angle activities to develop the **big ideas** of measurement as they pertain to angle.

1.6 Duration of time – Standard units for time

The standard units of time are not metric. Time is base 60. The relationships between units of time need careful consideration and exploration to be understood. While the smallest unit of time is the second, when times less than one second need to be considered it is fractions of a second that are the focus. The fractions used are decimal fractions. So time is measured in seconds, tenths of seconds, hundredths of seconds etc. The metric prefixes can be used to refer to fractions of a second just as they are with other metric measurement units, e.g. 1/1000 of a gram is a milligram. However there is a tendency for any fraction of a second to be referred to as a millisecond which is incorrect.

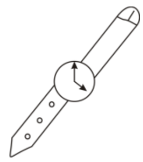
The standard units of time are:

1 minute (min) = 60 seconds (secs)
1 hour (hr) = 60 mins = 3 600 secs
1 day = 24 hours = 1 440 mins = 86 400 secs
1 week = 7 days = 168 hours = 10 080 mins = 604 800 secs
1 fortnight = 14 days = 336 hours = 20 160 mins = 1 209 600 secs
1 month = 28/29/30/31 days (depending on which month)
1 year = 365.242 days
1 decade = 10 years
1 century = 100 years

Activities

Introduce minutes, hours, seconds, clock faces, digital time, 24-hour clock using the four steps of common unit, identification, internalisation and estimation. Determine what children can do in a minute. Work out the length of a favourite TV program.

Experience a minute and other time intervals by doing something for this time. Relate these times to everyday events like brushing teeth or eating, or school lessons, etc. Turn back and call out when 20 seconds is up. Do this while doing something else (e.g. hopping, holding breath, etc.). Choose actions that may change perception of time.



Use stop watches and clocks, etc. to time events. Calibrate a sun dial in hours (e.g. put a stick in the ground and mark where shadow points when each hour is up).

Relate daily events to the times they commonly occur (use worksheets and join pictures to times). Work out a roster (daily or for a special activity). Relate digital time to clock faces. Introduce 24-hour time.

Measuring devices for time using standard units

Clocks and stopwatches are devices used to measure the passage of time and the duration of events. As time is constantly moving on and it can't be stopped the measurement of the amount of time an activity or event takes requires noting of the start time and the finish time and then a calculation if standard clocks are used as the measuring devices. Students can be cued to start their event when the second hand of the clock is on the 12 (indicating the start of a minute) and then noting where on the clock this hand is when the activity is finished. Then a calculation will be needed to determine the duration. If the activity takes longer than one minute the number of minutes will need to be remembered or recorded as well as where the second hand was when the activity ended. This provides an added complication to the use of standard clocks for measuring duration.

The other option for measuring duration is to use a stopwatch. Stopwatches can be either digital or analogue. Stopwatches allow duration to be measured by being started at the same time as the event. These devices keep time in seconds, minutes and fractions of seconds and when stopped record the length of time of the event. Analogue stopwatches generally record seconds and minutes only. Digital stopwatches record minutes, seconds and fractions of a second. Digital stopwatches are quite complex as they record more than just minutes and seconds but also the tenths and hundredths of seconds. For young students all the numbers on a digital stopwatch can be confusing. The figure below shows an analogue and a digital stopwatch.

An analogue and a digital stopwatch for measuring duration of time

1.7 Duration of time – Applications and formulae

Because time does not stand still it is necessary to complete calculations to measure the duration of an activity or event. The start time and finish time are noted and the duration is calculated as the difference between these two times. As described above the use of stopwatches to measure duration requires the reading and interpreting of a scale (analogue) or digital total rather than a calculation.

There are no formulaic applications of time except through the conversion of units. Time can be measured using clocks or timing devices and once known the duration of an activity or event can be described using any of the related units of time. The length of time taken to run a marathon can be described in hours; hours and minutes; hours, minutes and seconds; hours, minutes, seconds and fractions of a second depending on the accuracy required. All these conversions rely on calculation.

Providing students with opportunities to solve real-life problems that require the measurement of time helps them to apply their understandings of this attribute in ways that relate to life beyond school. Lot of these are related to **rate** (e.g. water wastage per day, speed – km/hr, and so on).

The following outlines four different time applications students may have experienced – especially if they have travelled.

24-hour time

Discuss where students might have seen 24-hour time used. Responses might include on digital clocks, on some TV guides, train or airline ticketing information. Some students may have heard it on TV shows from America, or in use by the military.

Discuss why 12-hour time might have been invented. What are the advantages? What are the disadvantages?

Present a clock face that shows both 12- and 24-hour time. Have the students create a table of corresponding times (e.g. 1pm = 1300 hrs, 2pm = 1400 hrs and so on). Discuss strategies (other than memorising) for the relationships between 12- and 24-hour time. Get students to create a story using 24-hour times and illustrate their story with representations of the clock times. The clock times should describe events in the story.

Consider Indigenous time and tidal change. Have students research the significance of the tides to coastal Indigenous peoples in the past, and in the present.

Timetables (not to be confused with times tables)

Discuss the purpose of a timetable in everyday life. Consider this from multiple points of view – the students', the adults in their community, and perhaps people living in a very busy city.

Have a number of timetables available for the students to practise reading. Make up some stories telling the student they need to be at a sport game by 2pm, what bus or ferry should they catch from the bus stop or jetty near their house? Try to use authentic timetables that are specific to the local environment. When students are comfortable with these, use a timetable for a local but larger town or city (like Townsville or Brisbane). As an extension activity, consider a tube timetable for London, having the student pretend they are travelling to see an event at the Olympics! Let the student select the event and locate the timetable on the Internet.

Time zones

Print out a copy of Australia and its time zones map. Choose a map that clearly illustrates the time zones, as well as the different states. Photocopy the map – one for each pair of students. Discuss the concept of time zones with students. Explain that time zones exist because the sun rises and sets at different times in different parts of Australia and the world. Discuss the implications of this in terms of TV coverage of big sporting events like the Olympics (events are held in the daytime in London, but viewed 'LIVE' during the night time in Australia).

Shine a torch and hold it over a globe. Rotate the globe and explain that the torch represents the sun. As you rotate the globe, children will see how the sun illuminates different parts of the earth at different times. It is important to rotate the globe correctly, from left to right. (Australia has time generally 10 hours ahead of London, so we need to get the torchlight before London). Inform students that you will be discussing different time zones in Australia. Have them look at the map and explain the regions of the different time zones:

> Australia has three time zones: Eastern Standard Time (EST) for the eastern states, Central Standard Time (CST) for the Northern Territory and South Australia and Western Standard Time (WST) for Western Australia. CST is half an hour behind EST and WST is two hours behind EST.
>
> Daylight Saving Time: Most Australian states wind their clocks forward an hour during the Daylight Saving period. New South Wales, Australian Capital Territory, Victoria, Tasmania and South Australia do this from the beginning of October to the beginning of April. Western Australia, the Northern Territory and Queensland don't observe the practice of Daylight Saving.

Explain that there is a time difference between the time zones. For example, when it is noon in the Eastern time zone, it is 11.30 am in the Central time zone, and 10.00 am in the Western time zone.

Ask students to determine the times in different time zones. State different times in different time zones and ask them what time it would be in another time zone. For instance, you could ask, "If it's 3 o'clock in the Western time zone, what time is it in the Eastern time zone?" or "If it's midnight in Brisbane, what time is it in Perth?"

Time logs

Get students to fill in time logs – time in and time out – ask them to calculate the time spent on each job. Try to get examples of actual logs so students can see relevance in filling them in.

If students have trouble working out how long spent on a job, try the following:

1. **Number line**. Develop a number line for 7 am to 4 pm – mark in hours, ¼ hours. Get students to mark in starting and ending times on number line. Use the line to work out how long was taken. Look at the hours and the minutes between start and finish. To assist getting over the 12 noon restart of the numbers – do the difference in 2 steps – up to the 12 noon and after the 12 noon, or translate the number line to 24-hour time.

Additive subtraction. Do subtractions using the "shopkeeper's algorithm", that is, starting from smaller and building to larger. For the example start 8:35 am and finish 1:20 pm, make a series of jumps from 8:35 to 1:20 – first go to the next hour, then to 12, then to finish hour and finally to finish minutes. Then add up all the jumps – this is the subtraction (the difference).

8:35 am	
	0 h 25 m
9:00 am	
	3 h
12:00 noon	
	1 h
1: 00 pm	
	0 h 20 m
1:20 pm	
	4 h 45 m

Unit 2: Angle

Angle is the amount of turn between two directions (arms) represented as two arrows from a common starting point. The amount of angle is the amount of turn, not the length of the arrows. Angle is also the corner of 2D or 3D shapes hence the link between this topic and geometry.

The measurement attribute of angle is measured in terms of full turn and this is 360 degrees. It is 360 degrees because the Babylonians visualised a full turn as 6 equilateral triangles meeting at a point and gave each corner of the equilateral triangles an angle of 60 degrees (thus angle, like time, has a base 60). Small angles are also measured in minutes (1/60th of a degree) and seconds (1/60th of a minute).

2.1 Identifying the attribute of angle

The attribute of angle relates very closely to the mathematics topic of geometry. When two lines radiate from a common point they form an angle. Angles are the corners of 2D and 3D shapes and the study of trigonometry combines measurement of angles and sides in right angle triangles. For young students the attribute of angle is best experienced as a measure of the amount of turn. They can turn their bodies, observe the movement of the hands of clocks and investigate the properties of shapes by comparing and measuring the corners. The measurement of the amount of turn or size of angles follows the same teaching sequence as other measurement attributes.

Angles in real life will be either static or dynamic. A static angle does not change and a dynamic one does. The corner of a room or corner of a window is an example of a static angle. The hands of a clock are an example of a dynamic angle which changes during the day. Discuss with students other examples of static and dynamic angles. If you have swinging doors in your classroom, explore different sized dynamic angles created by opening the door wider or narrower. Have students create static angles and dynamic angles using their bodies.

There are a range of words used to describe angles in particular size ranges. An **acute** angle is an angle between 0° and 90°. An **obtuse** angle is an angle between 90° and 180°. A **reflex** angle is an angle between 180° and 360°. A **revolution** angle is a full 360° circle.

Activities

Discuss things that turn, experience turning with the children’s bodies, follow the turns of another (e.g. “follow the leader”, “Simon says”, etc.). Relate the turns to the directions to which the body points at the start and end of the turns. Notice that a turn begins and ends with a direction, so define angle as the amount of turn from one direction to another. Use the turn activities in the Geometry module G1. Use **rotagram** and **angle wheel** activities. Turn the pages of a book, turn geostrips in relation to each other. Give directions to a blindfolded partner (e.g. “turn right, turn left, turn further, ...”, etc.).

Try to use activities that students find real, for example, skate boards doing 360 degree jumps. Try to connect these to the formal work, for example, once the skateboard is experienced in reality, a small toy skateboard could be used to translate the movements on the paper and then be replaced with a drawing of an angle – joining reality to mathematics.

2.2 Comparing and ordering angles

Once students understand the attribute of angle they can explore ways to compare and order the size of different angles.

Direct and indirect comparison of angles

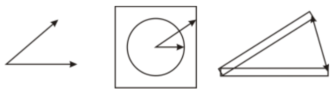
Representations of different angles can be place directly on top of each other and the size of the angles can be compared. Students need to focus on the angle and not be distracted by the length of the lines or sides of the angle. Many students mistakenly believe that changing the length of the arms of an angle will change the size of the angle. These students are focussing on the area or space between the arms rather than the amount of turn between the two arms. The two angles at right are the same size but the far right angle has longer arms.

To indirectly compare angles requires the use of an intermediary angle. Often when angles are being compared students use the size of known angles, e.g. right angle, as the intermediary. Each angle to be compared is compared to a right angle and this knowledge can be used to compare the initial angles.

Activities

Continue turn with body from one direction to another. Experience small and large turns.

Compare angles directly by taking a copy of one angle as a sector of a circle and placing this copy over the other angle. The copy can be made on tracing paper and cut out.



Compare angles indirectly by using an angle wheel or rotagram (see above) or by using **two hinged strips** (see above) – these can be joined by string a set distance from the hinge to hold the angle. In this angle copier, the length of the string determines the size of the angle (see above). A compass and a ruler can also copy an angle by measuring the distance between the two rays (or directions) with the ruler along an arc a fixed radius from the vertex (arc is drawn by compass held at a fixed radius).

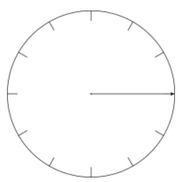
Construct a **right angle measurer** by folding paper and comparing with angles to classify them as acute or obtuse (see on right for fold). Open out right angle measurer to get a straight-line angle. Use this to show when an angle is reflex. Show four of the angles is a full turn – relate to N, E, S and W.

2.3 Non-standard units for angle

Non-standard units for angle can be made from sectors and by making own non-standard "protractor". The use of language to describe whole and part turns, particularly in the giving and following of directions, can be considered as a non-standard unit for angle. These units are uniform non-standard as they are likely to be the same each time as they relate to a fraction of a full turn or full circle. Students can be instructed to make a quarter or half or full turn to the left or right and the result will be a change in their direction. (Do not use degrees.) Use angle activities to develop the **big ideas** of measurement as they pertain to angle.

Activities

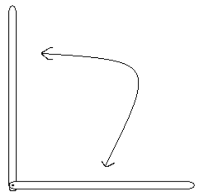
Make non-standard units by cutting circle into sectors (or pie pieces). Measure angles by filling in space between two directions with small sectors. Measure extent of turn by how many small **sectors** will fit in the angle. Construct a non-standard protractor by cutting out a plastic circle with marks on edge (as on right). This can be done on computer by using Excel and constructing a pie chart of, say, 12, 40, 60 or 100 equal numbers, and printing this on plastic



transparency. A calibrated homemade protractor can also be constructed from a rotagram or an angle wheel. The angle is read from the intervals marked along the arc (it is good to have a starting point).

Approximate angle measures can be found by counting how many of an object can be fitted into the angle at a fixed distance from the vertex (e.g. fingers, pencils, etc.). Once again, these can be used to develop the measurement principles.

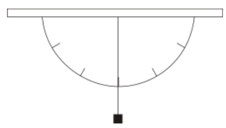
2.4 Standard units for angle



The standard units for angle are base 60 and relate to the 360 degrees that are in a circle or one full turn. When a finer measurement than one degree is needed fractions of a degree are referred to but they are not decimal fractions. The fractions of a degree are 60ths of an angle and are called minutes and seconds (which are other base 60 measures). With angle, particular sized angles are commonly used as referents and so are like standard units for this attribute, e.g. a right angle is an angle measuring 90° or ¼ of a circle (or of a full turn). The **standard units** for angle are:

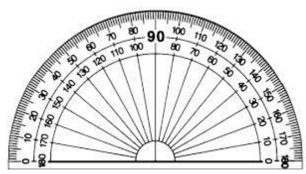
- 1 degree (°) = 1/360th of a full turn or of a circle
- A right angle (∟) = 90°
- A full turn = 360°

Activities



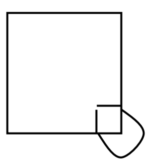
Construct a sector equal to 1 or 2 degrees. Experience the protractor. Use the 4 steps - common unit, identification, internalisation and estimation. Measure things on your body (e.g. angle between thumb and forefinger).

Measuring devices for measuring angle in standard units



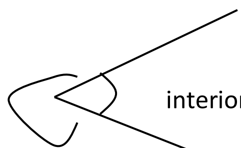
The measuring device most commonly used to measure angle is the protractor. Protractors are generally a full or half circle and have individual degrees marked usually in multiples of 10. They measure in degrees. Measuring angles with a protractor is quite complex. Generally the full circle protractors are easier for students to use and understand. It also helps if the protractors are see-through so they can be aligned with the arms of the angle being measured.

Every pair of lines that share a common point can represent two different angles that will add to 360°. When these angles form the corner of a 2D shape these angles are referred to as interior and exterior angles. The figure below shows the interior and exterior angles of one corner of a square as well as the same idea for an angle that is not the corner of a 2D shape. This causes difficulty when using a protractor for many students as they need to identify which of the two angles they are measuring.



interior angle = 90°

exterior angle = 270°



interior angle = 30°

exterior angle = 330°

It is important for your students to recognise the names of the parts of an angle – arms and vertex – and that the size of an angle is the amount of turn required by one arm in relation to the other arm.

- Have students use "angle strips" and protractors to measure angles around the room. Have students create angle strips from pieces of cardboard, and use these to replicate angles in classroom, then measure the angle using a protractor. The angle strip is held together with a split pin.

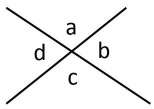
- Have students draw a variety of angles and estimate their size, then check using the protractor. Once students are familiar with the use of a protractor, have them explore a flower garden and find examples of where angles appear in nature in fixed patterns (e.g. a daisy with five petals has each petal at an angle of approximately 72°). Students can be sent out into the garden to find an example of a 35° angle in a bush (many leaves grow at 35° from the stem).
- Most of the angles found in nature are the result of rotation and symmetry. So it is not surprising that many diagrammatic representations of the Central and Western Desert people represent such symmetry in their descriptions of kinship. Have students explore such representations on the Internet, and perhaps create their own kinship representations.

2.5 Applications for angle

There are many applications for angle in the sense of using formulae to calculate other measures once the size of particular angles is known. This aspect of mathematics is usually described and taught in relation to geometry. These aspects of angle geometry are based on there being 360 degrees in a full circle or full turn and that parallel lines and lines that cross parallel lines create angles with particular equivalences.

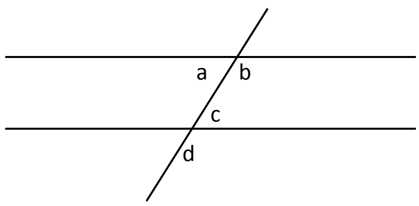
Angle properties

Vertically opposite angles are angles formed by any two intersecting lines. The two pairs of opposite angles will be equal in measure.



Angles a and c are vertically opposite and equal in measure
Angles b and d are vertically opposite and equal in measure

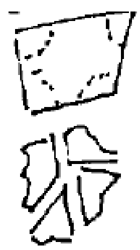
Parallel lines angle properties. *Alternate angles* are formed when a straight line traverses (crosses) a pair of parallel lines. Alternate angles are on opposite sides of the transversal and will be equal in measure. Alternate angles form a Z shape. *Co-interior angles* are formed when a straight line traverses a pair of parallel lines. Co-interior angles are on the same side of the transversal and add to 180° (are supplementary). Co-interior angles form a C shape. *Corresponding angles* are formed when a straight line traverses a pair of parallel lines. Corresponding angles are on the same side of the traversal. Corresponding angles form an F shape.



Angles **a** and **c** are alternate angles and equal in measure
Angles **b** and **c** are co-interior angles and add to 180°
Angles **b** and **d** are corresponding angles and equal in measure

Angle sums of polygons are as follows: (a) the interior angle sum of an n-sided polygon is (n-2) × 180 degrees; and (b) the exterior angle sum of an n-sided polygon is always 360 degrees. **Other shape-angle properties** are as follows. The angle within a circle with the diameter as its base is always 90 degrees. The diagonals of a square and rhombus meet at 90 degrees, the diagonals of a rectangle and parallelogram do not. Shapes that tessellate have angles that are factors of 360 degrees or add to 360 degrees or 180 degrees.

Teaching activities



Tear off the corners of polygons to determine the interior angle sum rules for these polygons (as on right). Triangulate polygons into triangles for the same purpose. "Walk" around regular polygons to find relation of number of sides to the exterior angle (the amount that has to be turned).

Use the activities in the Geometry Module G1. Relate angles in regular polygons, relate interior and exterior angles in triangles (and determine the number of triangles in any polygon), relate angles in circles (particularly those from diameters), relate angles in parallel line situations, relate angles between diagonals to the various quadrilaterals, relate angles to tessellation, relate angles in line and rotational symmetry, and so on.

Use angles on a sphere to introduce nautical mile as the distance along the equator of one minute of angle at the centre of the earth.

2.6 Time and angle rich task

In this task, students create a machine that measures a single minute. There is clearly no correct design for the machine. The creativity is limitless. What is interesting is how students tackle the problem, how they organise their thinking, and what levels of complexity they can imagine and construct. The results can be revealing in a number of ways. Students will need to troubleshoot their machine designs as they strive to reach the desired 1-minute run time for the marble. Students may make designs that are more sophisticated than you expected.

The mathematical ideas are:

- Understanding measurement
- Understanding time periods
- Using angles to increase or decrease speed.

Presenting the problem

- You may need to discuss what a ball run is. Consider showing the students video examples from the Internet
- Help students recognise that they will have to work in a team of three for best results (many hands to hold and tape joints etc.).

Assessment criteria

The major emphasis of the task is recognising the single minute as a unit of time. You may find 1 minute is too long a period of time for your students, so suggest the combination of machines (joining of two groups – will the time be a simple addition of the individual times?). To a lesser degree, you are interested in their representation and understanding of angles and gradients in the real world.

Presentation

One way to present investigations is with posters. One for this investigation is on the next page.

“I’ll do it in a minute!” machine

Ever been asked to do something, and you say “I’ll do it in a minute”?

Here is your chance to build yourself a machine that will let you know when your minute is up so you have to go and do something.

Build a minute machine made from junk paper and cardboard and masking tape.
The contraption needs to send a rolling marble through tubes and funnels, across tracks and bumpers, and to be caught EXACTLY 1 minute later in a disposable cup.

In the design of your minute machine, make sure you have at least one reflex angle and one acute angle between the tracks.

Unit 3: Money/Value

Money as a measurement is the measure of the value or cost of something. There are two components – money-handling skill, and money as measuring value.

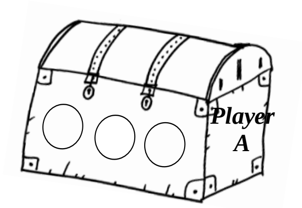
3.1 Money-handling skill

This involves being able to recognise and use money for everyday activities such as shopping, buying, selling, getting change, calculating amounts, and so on. Some activities are as follows.

1. Recognise coins through coin rubbings, drawing coins, coin posters, etc. Understand the value of coins and relate coins to each other. Recognise what can be bought for certain coins, add and subtract coins, and make change.
2. Setting up a "shop" – have things to buy and sell, use play money, etc. Look at computation and word problems in money situations.
3. Organising a party or a trip somewhere – working out costs of things (travel, accommodation, drinks, food, entertainment and so on) and making a plan and keeping to a budget.
4. Setting up a family budget – identifying all costs for a family and then relating these to income and working out how much in each area (rent, petrol, food, entertainment, and so on) can be spent.

Game: Change the treasure

Instructions: Use play money if real money is not advisable.



(a) Student throws two dice, and adds the numbers.

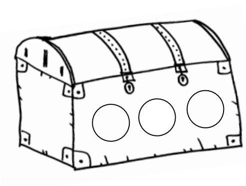
(b) Round to the nearest 5c (does it come closer to 5c or 10c?)

(c) Pick up that coin and add it to your treasure stash.

(d) If you can combine the value to make a different coin, then exchange it. For example, 5c + 5c → 10c or 10c + 10 c → 20c.

(e) The winner is the person who first makes a total of 60c (3 × 20c coins or 50c + 10c).

Extension: Vary the dice and coins. For example, use 10c and 20c coins to make 50c (using dice with numbers 1-10).



(a) Have students read an amount of money on a price tag, and then calculate the exact amount using coins and/or notes. This can be played as a shop keeper's game.

(b) Discuss the notion of rounding (Australia does not use 1c or 2c coins, there is a discussion about whether to discontinue the 5c piece – what impact would that have on small items like lollies still sold "2 for 5c" in some corner stores?).

(c) Practise rounding prices up and down for the sale of single items. Practise rounding prices up or down for multiple items purchased at once.

(d) Take the students to the local shop and have the shop keeper show students how a till works. Explore what maths the register person needs to help a shopper make a purchase. If possible, visit a shop with a very old cash register that requires the worker to calculate the correct change for the customer.

(e) Have students check their change after making a purchase from a pretend shop.

(f) Have students explore money transactions in different currencies. Pretend the students are buying a "McDonald's Happy Meal" in three different countries (Australia, Japan and America). What currency would they use? What money (notes and coins) would they need to pay an exact amount? What change would they need if they had to overpay? An interesting discussion point is what do they actually purchase as a Happy Meal in Asian countries? (cabbage is often used instead of a pickle on a cheese burger, the sauce is often fish sauce, and sometimes seaweed is used).

Extra activities. Introduce the coins and notes used in Australia, using where possible identifying, internalising and estimating activities. Play "shop", determine everyday items that can be bought for the coin/note under discussion or multiples of that coin/note. If possible, involve children in actual buying and selling, actually having to use or plan to use money (e.g. organising lunches and lunch money, buying, cooking and making and selling food on days the tuckshop is closed; organising an outing and determining the money needed, etc.).

3.2 Money as value – Identifying the attribute of money (value)

As a measure, money is not as absolute as length but varies according to demand. It may seem strange to place money within the measurement context but measurement teaching approaches can be useful in instructing students about money. The measure of value is subjective in that an object could hold great value to one person but not to another. While this value might not be monetary, discussions about what students value, may give them an idea of the concept of trying to measure this value.

The cost or value of an object or experience is not tangible and needs to be represented either using symbols (e.g. $) or by using coins and notes that can represent the equivalence in value or cost. Therefore before students can work on the measurement aspects of this attribute, they need experiences with number and quantity and they need to have experienced the recording of quantities using numbers and other symbols. Students will not be able to compare or order the cost or value of items without the associated number concepts to compare coins and notes or symbolic representations of amounts of money including the use of a decimal point to separate the whole dollars from the part dollars (cents).

Activities

Discuss what things are the most valuable for students (e.g. "what one thing would you take to be marooned with, ...", etc.). Discuss why they value it, and how they would show it had value. Choose amongst alternatives. Discuss things that have little value. Make a collage of pictures of expensive and cheap things.

3.3 Money as value – Comparing and ordering money

Once students understand the attribute of money, value and cost they can explore ways to compare and order the cost and value of different objects. The difficulty with comparing money is that the measure is not visible or tangible. The comparison must be done to representations of the value or cost.

Activities

Subjectively compare objects to see which one is valued more highly. Indirectly compare by relating to a common object (e.g. I like my toy more than this pen but the trip to get hamburgers is better than the toy, etc.). Discuss how many of one object is a fair trade for another. Organise the class into groups all with a different product and discuss how the groups would barter one thing for another (introduces supply and demand). Look at the history of the students to whom you are teaching – was bartering common?

Students can be encouraged to barter and swap items or activities for other things. To make the swap or barter they will need to compare the value of their items. This comparison could be done directly or indirectly. A novel way to discuss the "value" of items, or a barter system, would be to explore the swapping of food that may occur at morning tea or lunch time, e.g. a meat and salad sandwich (pretty tasty and filling) swapped with a small cup cake or biscuit (junk food).

Direct and indirect comparison of money/value

Students can **directly compare** their valuations of particular objects to see which object is valued more highly or which student values the object more highly. These comparisons are not absolute and are subjective but serve to focus on the attribute of value and the differing amounts of the attribute that different objects possess.

Students can **indirectly compare** the value of items or activities by comparing them to other intermediary objects or activities by considering how much each is liked comparatively. To enable comparison of different activities according to which is liked more or less the liking is equated with having greater value for that student. A student could say that they like playing soccer better than watching television and they like going to the movies better than playing soccer. The transitive relationship between these activities means that it can be deduced that this student likes going to the movies better than watching television.

3.4 Money as value – Non-standard units for money

When students are encouraged to barter or swap skills or items for other activities or objects, they consider the value of what they have and what they would accept in return for this. By introducing a count of particular items that are considered to equal the value of an object or activity students are working with non-standard units of value. If the objects being used as non-standard units are all the same they can be described as being uniform non-standard units. The benefit of using uniform non-standard units is that the count of these units for one item can be compared to the count of the same units for another item. Through the use of non-standard units students develop an understanding of the need for a standard unit.

An interesting topic to discuss with students is how early civilisations worked before they had money as we know it. Bartering was one method but in many cultures a particular object was valued and was used as a non-standard unit of currency. For example, cowrie shells or pigs were highly valued in some island cultures and particular tasks were considered to be equivalent to a particular number of these items. Discuss the use of a common or valued object as a medium of exchange (e.g. Cowrie shells, oxen or pigs or horses or other animals, marbles, etc.). Discuss the historical development of money. Again look at history of local people. Discuss the **talent**, an iron ring that had the mass of the gold needed to buy an ox in biblical times– became a basis for money and mass.

Use angle activities to develop the **big ideas** of measurement as they pertain to money.

3.5 Money as value – Standard units for money

At this point, activities here are similar to those under money-handling skill. One activity is to look at different currencies. The standard units of money in different countries vary. In Australia our monetary system is decimal and the units are related by multiples of 10. This means that money provides a close link to the study of our number system which is also decimal.

By using money for actual purchases and shopping experiences including the giving of change, students can be helped to understand the value and relative value of the money in our society. Young students should start with $1 and $2 coins and combinations of these rather than 5c and 10c coins as they can be counted relatively easily. Activities involving students making particular values with coins are quite complex. Counting money to find the total value when there are a number of different coins is difficult due to the need to change the count.

The standard units of money in Australia are:

1 dollar ($) = 100 cents

The standard coins and notes in our currency are:

Coins: 5c, 10c, 20c, 50c, $1 and $2
Notes: $5, $10, $20, $50 and $100

- Have students explore the different countries that use the term Dollars and Cents as their currency. What do our Asian neighbours use? Is it always base 100?
- Have students identify equivalent values for coins or notes (two 10c pieces = 20c). This can be done virtually with pictures on Word or PowerPoint.

Comparison of standard units

Comparison can be made of the actual coins in our system of money with the focus being the comparison of their monetary value. Due to the nature of the coins and notes in our money, students can focus their comparisons not on the associated value but on the physical characteristics of the coins and notes themselves. This can lead to misunderstandings. For example, the 50c coin is larger in size than both the $1 and $2 coins although these coins are of greater value.

- Compare this with the New Zealand monetary system. Have the students find out why the sizes of the Australian and New Zealand coins do not follow the same pattern.
- Have students compare coins from neighbouring Asian countries and investigate the relationship between size of coin and its value.

Comparison of amounts of money written symbolically requires a good understanding of number concepts including place value so that students know to compare the place with the greatest value when comparing written amounts of money. For example, $34.10 is of greater value than $29.65 even though each number in the second price is larger than those in the first price, except the first number.

3.6 Money as value – Applications for money

There are no formulaic applications of money except through the conversion of units. Providing students with opportunities to solve real-life problems that require the use of money to measure the value or cost of items or activities helps them to apply their understandings of this attribute in ways that relate to life beyond school. As students progress through school the financial aspects of mathematics focussing on shopping, giving change, designing budgets and working with percentage discounts and interest provide many opportunities for problem solving in real-life contexts.

- GST is a recent phenomenon in Australia's financial history, but most students do not know it exists as it is automatically included in the regular price of an item. Compare this with the added tax applied in America. If you were to hand over the exact amount printed on a price tag in Target in America, they would ask you for the additional sales tax of between 1% and 10% (depending upon which US State you are in!). The current average US sales tax is 9.6%.
- Assist students to set up a business in the classroom with a purpose (e.g. to raise money for a class excursion). The business is to do a sausage sizzle one lunch time. The class needs to establish a budget, purchase items, and calculate what GST was required, what items were exempt, how much to charge per sausage on bread and so on. Provide students with information on which shops are giving discounts on bread, onions, sausages, tomato sauce, etc. Calculate the percentage discount and overall savings. If you can, take the students to the supermarket and look at the price tags to determine best value for money (cost per 100g).
- Explore the methods various shops use to express discounts (sales tags: "From $10", "Nothing over $5.00", "30% off lowest marked price").
- Note that other more advanced aspects of money will be included in Module O5 *Financial Mathematics*.

Unit 4: Temperature

Temperature is a measure of how hot or cold objects are – a very subjective experience without measuring instruments. The most important reference points are the freezing point and boiling point of water.

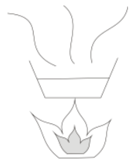
4.1 Identifying the attribute of temperature

This stage builds meaning for the attribute temperature.

Materials. Pen, paper, heat sources (e.g. sun, shade, ice, and so on), a variety of clothing for hot and cold situations, and other materials as mentioned in activities.

Abstraction

Body



Experience different temperatures by touching warm and cold objects. Use the environment to experience these temperature changes. Go outside and walk in a sunny area and a shady area. Discuss differences experienced and reasons why the temperature feels different. If able, organise for students to experience cold rooms, freezers, hot houses, and rooms that are very hot.

Use clothes to experience the difference between feeling warmer and cooler. Have students describe the temperature. Use this as an opportunity to build a rich vocabulary for describing temperature (e.g. hot, cold, freezing, warm, and so on). Organise students to walk (in bare feet) along surfaces with different temperatures (e.g. concrete, grass, sand, mud, water, and so on). Talk to students how each day feels and relate to temperature chart.

Hand

Experience many different temperatures. Examples of activities include: (a) moving different objects to experience varying degrees of warmth from sunlight and coolness from shade; (b) touching ice and warm water; (c) feeling containers where the contents affect the temperature (e.g. water tanks, coffee cups, and so on). Develop the notion that sometimes senses can’t be relied upon by heating up hand then placing quickly on a warm surface and vice versa (i.e. cool hand and place on a warm surface)!

Mind

Shut eyes and think of different temperatures. Draw situations and use different colours to represent different temperatures. Act out how it feels on a hot day, cold day, and so on (e.g. acting out shivering and perspiring).

Mathematics

Practice

Continue to experience, describe and visually represent different temperatures. Even worksheets with pictures can represent different temperatures. Use a daily weather chart to record students’ observations about the weather. Discuss daily temperature and the variation that can occur within a day.

Connections

Construct a homemade temperature gauge made up of a long thin tube connected to a reservoir or just a long thin tube. As it heats up, the liquid should expand and rise up the tube. In this way, there is a connection between temperature and length.

Reflection

Application

Look at temperatures in local environment, and places where the temperature differs (e.g. fireplace, refrigerator).

Extension

Flexibility. Think of all the situations that are hot and cold in the world (e.g. ice and snow, active volcanoes, and so on). Look for places (e.g. deserts) that are both hot and cold.

Reversing. Make sure that teaching goes from: (a) teacher provides a situation which has a certain temperature (e.g. a refrigerator) → student says whether hot or cold; and (b) teacher chooses whether hot, cold, warm, and so on → students think up situation.

Generalising. Spend time on activities that reinforce three generalities: (a) feeling temperature is objective (i.e. relates to the person and their previous experience – a cold hand chilled from a refrigerator will find an object is hotter to touch than a hand that has been in hot water which will find the object cold); (b) big idea: attribute leads to instrumentation (e.g. look at what happens to things as they get hotter and colder and try to work out something that will provide a measure – in particular, the activity in "Connections" will lead to the thermometer); and (c) the connection between length and temperature.

Changing parameters. Instead of experiencing temperature, look at how change of temperature affects us.

4.2 Comparing and ordering temperature

This stage compares and orders temperature without use of numbers.

Materials. Pen, paper, heat sources (e.g. sun, shade, ice, and so on), a variety pictures showing various temperatures (e.g. hot, freezing, roasting, and so on), and other materials as mentioned in activities.

Abstraction

Body

It is not possible to directly compare temperature – we have to use indirect methods like touching or feeling both objects to see which is hotter. These activities involve comparing temperatures directly with the hand, moving hand from one instance to another: the hand will feel the change. There is also relating temperatures by comparison with or to known things (e.g. that one is hotter than my hand feels yet this one is not, and so on).

Comparison has to move on to ordering by temperature. Activities include filling cups with water at different temperatures and letting children order the cups from warmest to coldest. Need also to introduce comparison temperature language (e.g. warmer, colder, hotter, warmest, hottest, coldest, and so on).

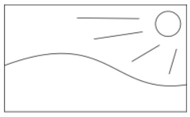
Hand

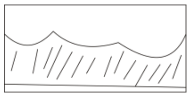
Students should experience a variety of temperature comparisons and orderings. The following are some examples. Use touch to compare and order a variety of things at different temperatures, using out in the sun, in a cold place, refrigerators, freezers, ovens, and so on, as sources of these objects and their different temperatures.

Use materials and their melting and boiling points to signify temperature changes. For example, provide students with a bowl of ice cubes to take outside on a sunny day. Encourage them to decide where they want to put the ice cubes. Have student predict what will happen to the ice cubes that are left in the different places.

Students should observe that ice melts at different rates in the shade and in the sun. They will also learn that solid ice will turn to liquid when heated. Discuss how the change in temperature from freezer to sunlight outside causes the ice to melt. Thus we have freezing temperatures and melting temperatures. Other materials can also be used to allow students to indirectly relate temperatures by what they do to other things (e.g. at this temperature, the paraffin is runny but at this colder temperature, it remains solid here, and so on). Construct a simple temperature measurer with a thin tube and coloured water in a small container, discovering how the coloured water goes up the tube as the temperature rises.

Use experiences and drawings to consider comparison and order of temperatures. Discuss hot days and cold days, order pictures that show hot and cold. Discuss summer and winter, what is common to wear, what is commonly eaten and where is the common vacation area? Keep a weather chart, drawing and describing the weather on each day. Relate this work to other subjects (e.g. Science). Look at how heat and cold affects humans, animals, food and other materials. Look at the environment and consider how changes of temperature affect things such as butter, care of pets, local vegetation, wild animals, fish, and so on. Discuss the effect of hot and cold. Use language to talk about feelings and observations concerning different types of weather. Discuss health effects (e.g. survival in hot and cold, influenza, and so on).





SPECIAL ACTIVITIES: "HOW DOES IT FEEL?"

Have students place their hands on their cheeks to feel how warm or cool their hands are, then have the students rub their hands together briskly for about 30 seconds, and put them against their cheeks again. Ask the students if their hands are warmer or cooler after they rubbed them together.

Arbitrarily label several spots in the room which have different temperatures for the students to touch; for example, window glass, metal shelf, spot in the sunlight, spot in the shade, and so on. Let the students try to determine which spot has more heat and which spot has less heat. Let the students make statements such as, "The metal shelf has more heat than the window glass." Let the students attempt to order the spots by the amount of heat they feel. Point out to the students that the things that have more heat than their hands feel warm or hot to them and vice versa.

Obtain two glasses of water from the tap, and let the students feel them to see that they are both cool. Set one in the shade and one in the sun. Thirty minutes later, let the students feel them again. Discuss why one glass now feels warm, whereas the other is still cool.

Let two student "judges" stand in front of the room. Have six or so students file past and lay their hands on the "judges'" cheeks. The "judges" try to decide who has the coldest/warmest hands.

Ensure students are introduced to comparing and ordering language. Use virtual situations and pictures to also compare and order by temperature.

Note: If in an Indigenous community, get an elder to describe the changes in temperature across a year and how this affects country.

Mind

Imagine hot and cold and all temperatures in between and either side.

Mathematics

Practice

Give students the opportunity to continue to compare temperatures through activities, virtual materials and pictures in worksheets. Ensure all language relating to comparing and ordering temperature is known. Keep a daily temperature chart with visual representations of temperature over extended time periods, say one term month. Make comparisons of temperature based on daily, weekly, and seasonal differences.

Connections

Use the homemade temperature measurer to draw connections between temperature and length.

Reflection

Validation/Application

Discuss real-world situations for hotter/colder. Find things that are between the two ends of a comparison (e.g. hot water and an oven).

Extension

Flexibility. Brainstorm hot-cold situations in the world. Mix things up – where is it very cold and very hot at the same time? Where is it same temperature all the time?

Reversing. Make sure teaching goes from: (a) teacher provides two temperature situations→ students use feel and give temperature comparison word, and (b) teachers give temperature comparison word → students provide examples that meet that word. Also remember from section 1.2 that comparison can be considered as a *triad* – with three parts, first situation, comparison word, and second situation – thus there are three "directions", or three problem types: (a) give first situation and word (e.g. hotter) and students find a cooler situation; (b) give second situation and a word (e.g. hotter) and students give a hotter situation; and (c) give two situations and ask for word(s) to relate them (e.g. "the freezer is cooler than the refrigerator").

Generalising. Generalise that (a) need a better measuring system than feel to accurately order situations by temperature; and (b) there are three problem types in comparison as in reversing.

Changing parameters. Look beyond temperature to how different temperatures affect life (ours and the world around us).

4.3 Non-standard units for temperature

This section introduces the notion of a unit and the measurement processes and principles. Because measuring instruments for temperature (other than by feel) are highly technical, there is not much scope for non-standard units.

Materials. Pen, paper, heat sources (e.g. sun, shade, ice, and so on), large thermometers without scale, a variety of pictures showing various temperatures (e.g. hot, freezing, roasting, and so on), and other materials as mentioned in activities.

Abstraction

Body

Use your feelings to make your body a rough non-standard instrument, as follows: freezing-1, cold-2, cool-warm-3, body temperature-4, warm-5, hot-6, and hotter than I can touch-7. Use this to give a number to temperatures.

Hand

There are only a few examples here as follows: (a) Make up a checklist for temperature based on effect on materials – freezes ice, ice melts but paraffin does not, and so on. This will require scientific knowledge and special materials. (b) Construct a homemade thermometer with a thin tube and red coloured water and attach to cardboard with regular lines on it, numbered from 0 to whatever. (c) Obtain a large thermometer and stick this on cardboard with made up regular lines on it, numbered from 0 to whatever. (d) Make up a series of pictures or virtual films showing different temperatures (e.g. ice blocks, sunny day, and so on) and give a number to each from cold to hot – use number to measure temperature roughly. These can be used to give a number to temperature.

Mind

Imagine a thermometer with red line going up and down with temperature.

Mathematics

Formality/Practice

Give students the opportunity to continue to compare temperatures through one of the above methods. Continue with the daily temperature chart with visual representations of temperature over extended time periods, say one term month, making comparisons of temperature based on daily, weekly, and seasonal differences.

Connections

The important one is relationship between temperature, land, flora, fauna and our behaviour.

Reflection

Validation/Application

Look at measuring temperature in real-world situations. Set up temperature problems based on non-standard units in everyday life situations.

Extension

Flexibility. Find use of non-standard temperature units in local community activity. Discuss with elders.

Reversing. Consider the non-standard triad for temperature: situation, temperature as a number, and temperature unit. Give the three types of activities in lessons – where situation, number and unit are unknown.

Generalising. Here the objective is to extend the understanding to teach the following: (a) *continuous vs discrete* big idea – how units make continuous temperature discrete steps; (b) *common units* big idea – how steps in scales must be common/same for measuring and comparison of temperatures, how must set a common unit after a while in sequence of activities, and the bigger number is hotter when units are common/same. All this should lead to the need for a standard; (c) *inverse relation* big idea – comparing non-standard units (particularly the distance between lines on thermometer) will lead to the larger the unit, the less the number; and (d) *accuracy vs exactness* big idea – discuss units to see that smaller gaps between lines is more accurate, that accuracy is not always needed and so skills in choosing appropriate units and estimating are useful. Because there is only one unit (^{o}C), there is not much need for this.

Changing parameters. Once again, the main extension here is to relate temperature and environment.

4.4 Standard units for temperature

This section focuses on introducing the degrees centigrade or Celsius (^{o}C). Temperature is a measure of how hot or cold objects are – a very subjective experience without measuring instruments. The most important reference points are the freezing point and boiling point of water. There is only one unit here as there is only one temperature unit (^{o}C).

Materials. Thermometer, pen, paper, ice, water for boiling.

Reality

Find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Common unit

Although difficult for temperature, after the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring temperature against a class chosen unit – e.g. temperature of body.

Identification

Use a thermometer to measure temperature of ice and boiling water and some common things in between.

Internalisation

Use a thermometer to measure body temperature, fridge temperature, tap water temperature, and so on. Look up recipes and find common cooking temperatures.

Mathematics

Estimation

Estimate first and then measure the temperature of a variety of objects (find some interesting things) inside, in the sun, slightly heated, etc. Complete estimates and measures of object before moving onto the next.

OBJECT	Estimate	Measure	Difference

Research the effect of altitude on boiling and freezing points of water. Estimate first. Research other interesting temperature – estimate first – e.g. magma from a volcano, temperature at which iron turns to liquid, and so on.

4.5 Applications for temperature

There are no formulae for temperature. And, since there is only one unit, applications can only be built around two of the three triad types. The two types left are:

- **Number unknown** – *What is the temperature of this material?*
- **Object unknown** – *Find a material with temperature 62° Celsius*.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the extension measurement item types

Similar to Modules M1 and M2, the extension measurement item types are in four subtests to match the module units. The subtests are not divided into the five measurement stages because most of the items relate to the applications of these attributes in the real world; however the items in the Time subtest are separated into point of time, sequence of time and duration of time.

As always, the selection of item types should be determined by the teacher's knowledge of their students. For maximum effect, it is best to mix items/sub-items from different attributes/subtests so that students have to pick what attribute is covered by particular items and sub-items – particularly in the post-test. Pre-test and post-test items need to come from each subtest.

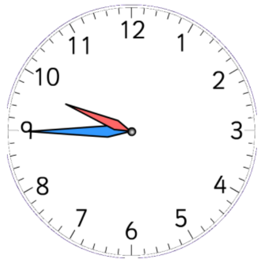
As with the other Measurement modules, it is possible to consider this module as four parts and have pre- and post-tests for each part. However, it is important that long-term performance in all four of time, angle, money/value and temperature is measured at the end of the module.

Subtest item types

Subtest 1 items (Unit 1: Time)

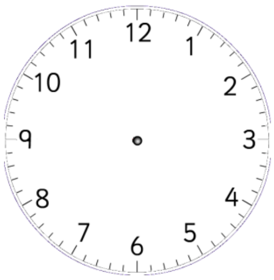
Point of time

1. What time is shown on the clock?

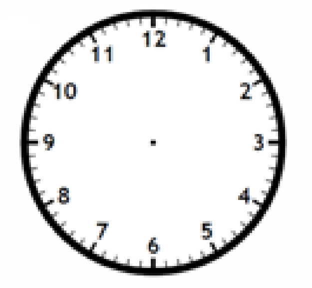


   ____

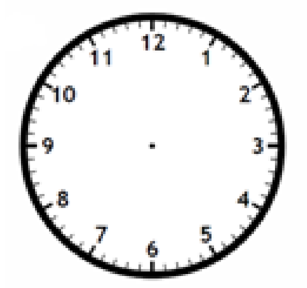
2. Draw the hands on the clock to show 7.30.



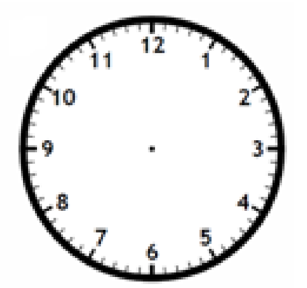
3. Draw the hands on the clocks to show the time:



You arrived at school today at 8:30 am



You will leave school today at 3:17 pm



You will finish dinner tonight at 7:05 pm

4. On the timetable below, draw a circle around the time I would need to <u>catch</u> a bus from the depot to the shop if I wanted to be at the shop between 4:15 and 4:25 pm.

Departs bus depot	Stops at shop
3:30pm	3:50pm
3:40pm	4:00pm
3:50pm	4:10pm
4:00pm	4:20pm
4:10pm	4:30pm

5. If it is 1 am on Saturday morning in Queensland, and Western Australia is 2 hours behind Eastern Standard Time, what time and day is it in Perth? ___________________________

Sequence of time

6. Fill in the missing months in the correct order in the table below:

	February		
May		July	
			December

Duration of time

7. How much time has passed between 9:25 pm and 10:40 pm? _____________________

8. Michael caught a bus at 9:25 am and arrived at the shops at 10:40 am. How long did the trip take? _______________________

9. A boat trip across to an island took 2 hours 25 minutes, and coming back took 2 hours. How much time altogether was spent travelling in the boat? ________________________

10. What is 2 hours 20 minutes after 3:50 pm? _______________

11. The electrician worked for 2 hours yesterday, and then today she worked for 3 hours 25 minutes. How long did she work all together? ______________________________________

12. My bus timetable says the school bus is due at my bus stop at 3:20pm. It will take my school bus 15 minutes to get me from school to my stop. What time will my bus leave school?

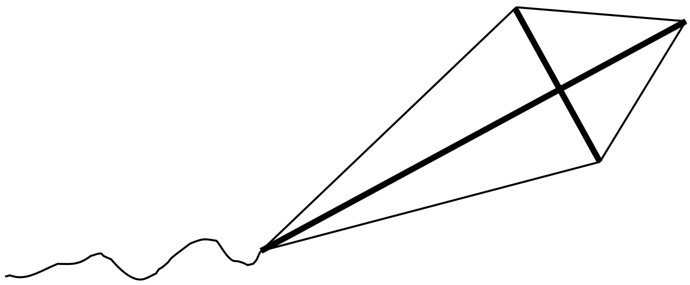
____

Subtest 2 items (Unit 2: Angle)

1. On the square, tick (✓) a right angle

2. How many degrees in a right angle? _______

3. On this kite there are a number of angles:

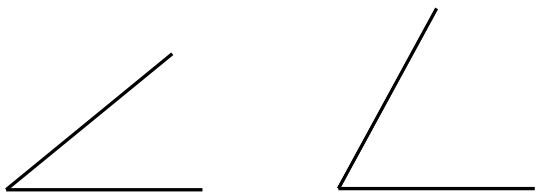


   (a) Draw two ticks (✓) to show two v*ertically opposite angles*.

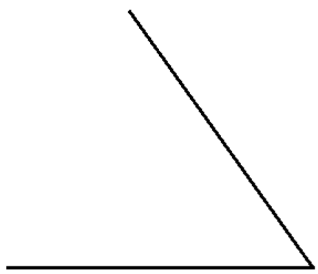
   (b) What do you know about the size of vertically opposite angles? _______________

   (c) Shade in an acute angle.

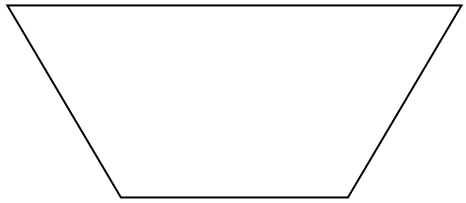
4. Draw a circle around the larger of the two angles.



5. Use your protractor to measure the size of the angle:



6. On the picture of a trapezoidal desk:



(a) shade in an obtuse angle

(b) draw two ticks (✓) to show two co-interior angles

(c) mark an external angle

(d) Measure the size of each of the angles and write them in on the diagram.

Subtest 3 items (Unit 3: Money/Value)

1. I need to pay the shop keeper 45 cents for a packet of lollies. Write down two different coin sets I could use to pay the exact amount.
   (Hint: 1 coin set is 20c + 20c + 5c. Show me 2 others).

   (a) ____

   (b) ____

2. My mum sent me to the shop to buy a $3.50 carton of eggs. She gave me a $10.00 note. How much change do I need to bring home? _____________________

3. You are asked to help a friend sell sausages on bread to get money for the local footy team. If a sausage on bread is $1.50, and someone gives you $2.00, how much change do you give back? _______________________

4. How many cents in $2.00? ____________

5. I want to buy 2 cans of Fanta for $1.50 each, and a pie for $2.50. I give the shopkeeper $5.

   (a) Have I given the shopkeeper enough money? ______________

   (b) If not, how much more money will the shop keeper ask me for? (show your working).

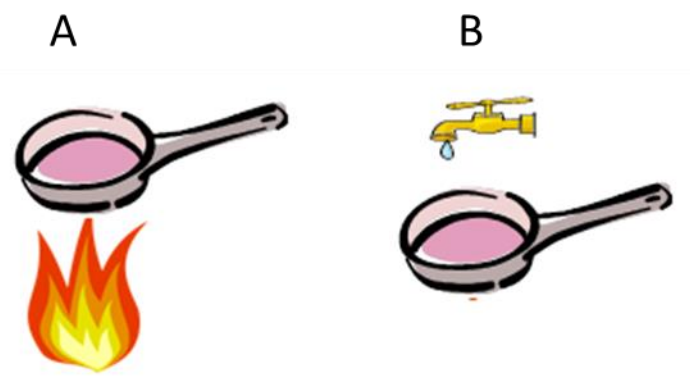
Challenge questions

6. I have three Australian notes in my wallet; how much money might I have?

7. Try to come up with five or more different solutions.

Subtest 4 items: (Unit 4: Temperature)

1. Which frypan is hotter? A or B ? (circle your answer and say why)



Why? ____

2. How can we measure which of two objects is colder without a thermometer?

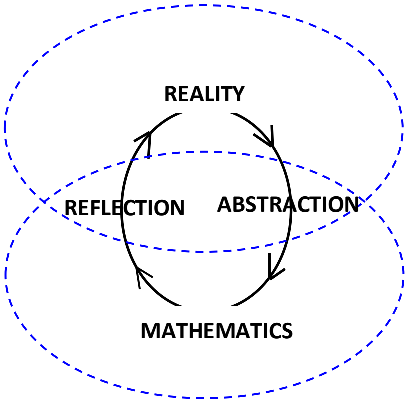
____

____

3. (a) Water forms ice at __________°C

(b) Water boils at __________°C

Appendix A: RAMR Cycle



AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).

The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix B: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

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