Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module SP1

Year A, Term 4

Statistics and Probability:

Tables and Graphs

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

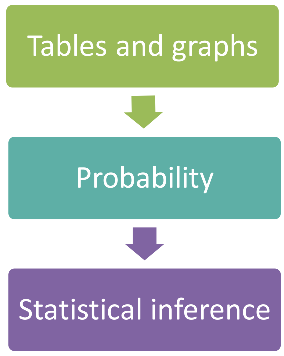
Contents

Module Overview

Statistics and Probability are crucial everyday life skills and important mathematical topics. Most decision-making of modern society is based on statistics, graphs and probability. In advertising, politics and economics, samples are organised, survey questions developed, answers sought, results tabulated and organised, and predictions displayed with averages and graphs to show distributions, relationships and trends before decisions are made. *What do people want in a car? Should Queensland have daylight saving*? Many computer banks are filled with the raw data on which such decisions will be made.

Large amounts of raw data are incomprehensible; *Statistics* is an indispensable tool for comprehending the raw data on which decision-making is based. Measures of central tendency such as *means* and *averages* supply a framework with which to describe what happens. *Graphs* supply a visual way of presenting the range of alternatives available and of indicating the density of interest (e.g. most popular/likely).

Probability is equally important because nearly all life-decisions involve uncertainty, and thus are made on possibilities and probabilities. Situations in real life can be of a predictable or random nature. Probability involves the measurement of the likelihood of events in chance processes.



Thus, with this module, we begin the series of modules that cover Statistics and Probability up to Year 10. There are three modules in the series: SP1 *Tables and Graphs* – looking at gathering and representing data in tables, charts and graphs; SP2 *Probability* – looking at teaching of probability as a concept and a fraction; and SP3 *Statistical Inference* – looking at inferring conclusions from data.

Background information for teaching tables and graphs

This section covers the knowledge that can allow a teacher to assist students to see mathematics knowledge as a structure of recurring ideas.

Connections

Table and graph statistics activities involve counting responses to questions and assigning numeric values to chance events for analysis. Subsequently, number concepts of counting, fractions and percent are inherently linked to statistics and probability activities. Table and graph statistics are also highly visual and rely on classifying, organising and summarising data into rows and columns and using this data to draw rectangles, lines and dots within a Cartesian framework or to divide a circle into sectors.

The data for the tables and graphs can be in: (a) a category form such as colour; (b) an ordinal form such as short, normal and tall; or (c) an interval form such as height in centimetres. The data can also be in: (a) discrete and discontinuous form like cost to nearest dollar; or (b) continuous form like mass in kg to decimal places. It is also the case that the gathering of data, particularly interval and continuous data, involves some act of measuring attributes such as distance and temperature. Thus, statistics is connected to the other mathematical strands **as on right** and in terms of the detail following this statement.

1. **Pre-number and Number** – predominantly classifying and patterning, counting, fractions and percent but also including operations;

1. **Algebra/Number sense** – basic ability to notice relationships and comparisons; to recognise the relative magnitudes of the comparisons and relationships; and to understand continuous, discrete and linear relationships;

2. **Geometry** – rectangular and circular shapes, Cartesian coordinates, square tessellation (e.g. patterns of rows and columns), angle, and visualisation; and

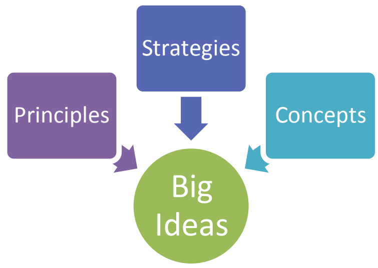
3. **Measurement** – understanding of attributes, order and measurement with respect to distance, mass, capacity, area, volume, time and temperature.

Weaknesses or lack of conceptual understanding in one of these connected topics, particularly counting, fractions and percent, will hamper students' progress in tables and graphs. Where students are demonstrating lack of success in the statistics of tables and graphs, it is necessary to ascertain whether the difficultly lies with the statistical concepts and processes or with the numerical, algebraic, geometrical and measurement processes attached to their representations.

Please note that although *Probability* is a separate module from the two *Statistics* modules, probability activities provide rich sources of activities for statistics and these topics should be used together to provide students with clear ideas of the links between the topics.

Big ideas

Big ideas are mathematical ideas that underlie topics and recur across the years of schooling. They can be principles, strategies and concepts. Looking at tables and graphs, it seems evident that there are some big ideas underlying tables and graphs. A few are given here.



Global principles

1. **Chance vs certainty**. In arithmetic, problems have **certain** answers, that is, 4+7=11. However, in statistics, decisions can be made in terms of **chance**. The data is not absolute: it shows that there are more options and thus predicts the best chance for an outcome. It is important that students know when they are in a certain and when they are in a chance situation.

2. **Accuracy vs exactness**. This goes hand-in-hand with the first big idea. In arithmetic, answers can be calculated exactly. However, in measures and in drawing inferences from data, there is sometimes no exactness; there is only being as accurate as possible or as required.

3. **Interpretation vs construction**. It is essential to be able to construct tables, charts and graphs (and understand how they are constructed) as well as to read or interpret them (and understand how they represent data). Interestingly, one of the most powerful ways to learn to interpret tables, charts and graphs is to have experience constructing them.

Concepts

1. **Matrix structure**. In tables and graphs, data is gathered and combined into rows and columns, and mostly drawn on paper which has a Cartesian structure (the rows and columns of a matrix).

2. **Notion of unit** (and commonality of unit). Units must be selected that effectively measure the concept under focus and the same size units from the same starting point must be used in comparisons and trends.

3. **Scale**. Components of representations must have commonality in scale and not be stretched or squashed or truncated.

4. **Multiplicative structure**. Comparisons should be understood in a multiplicative sense (such as percentages) and the significance of their comparisons and trends should be understood in a multiplicative sense.

Strategies

1. **Data driven/Complex thinking**. Statistics problem solving and investigations should be based on data and, therefore, are often examples of complex thinking.

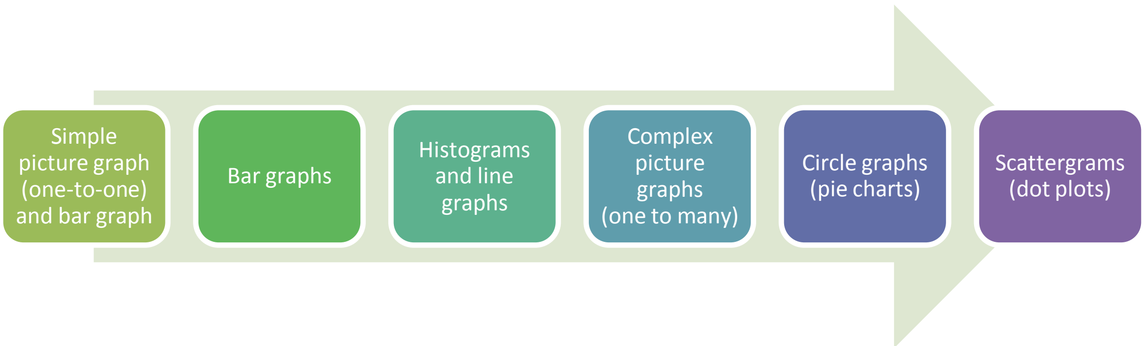
2. **Evidence based/Inferential thinking**. Statistics problem solving and investigations should be driven by evidence from the data and, therefore, are often examples of inferential thinking.

Sequencing

This section briefly looks at the role of sequencing for tables and graphs, sequencing of teaching, and then looks at the role of sequencing in this particular module.

Sequencing of tables and graphs

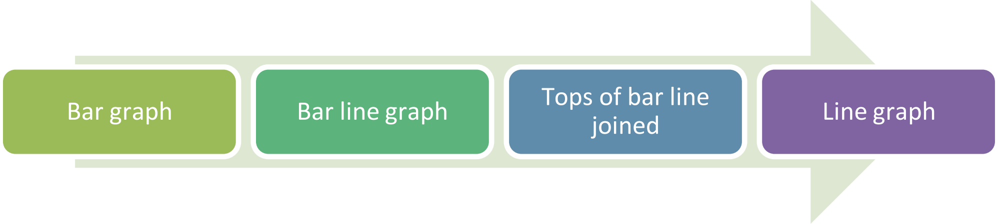
The various forms of graphs need to be introduced to students in a sequence whereby complexity and abstraction move from low to high as in the figure below. To connect graphs to the imperatives (i.e. comparison, frequencies, trends–relationships), this module has slightly changed this order for some graph types (notably complex picture graphs). This sequence is the basis of the overall sequencing in the diagram above. Following this path means working across the sections, and this is expedited by investigations that involve:

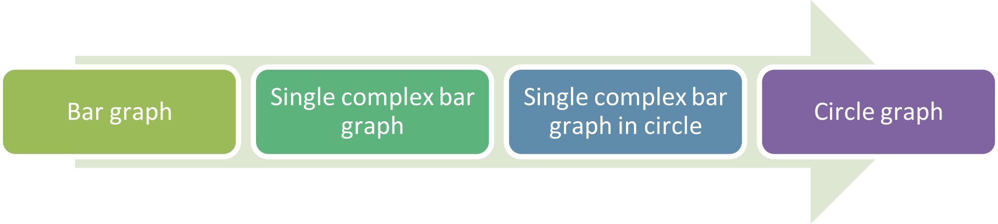


The development of understanding of the most common form of graph, the bar graph, should proceed through the stages described below. These stages are within Unit 2 that focuses on comparison and picture–bar graphs.

Stage 1: one-to-one correspondence	• Using physical materials to compare two rows or columns (children themselves, then concrete materials)
Stage 2: more columns	• Using physical materials to compare more than two rows or columns • Transition to more permanent recording form with pictures/drawings
Stage 3: stick on	• Discontinuing to use physical materials, gradually replacing drawings with stuck on squares and allowing more than one drawing/square per child
Stage 4: squared paper	• Moving from sticking on squares to shading or colouring squares on graph paper
Stage 5: abstract representation	• Replacing shading squares with strips, rectangles and lines (bar line graphs, histograms, line graphs)

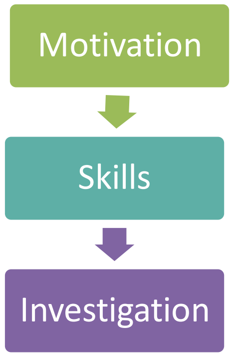
The development of understanding across other graphs may be facilitated with the following sequences. This is part of Unit 4 and the development of line graphs. The sequences for developing understanding of line and circle graphs show the power that comes from integrating some statistic ideas across units.





Sequencing of teaching approaches

This module uses the RAMR cycle but also uses an approach to teaching based upon the notion of Renzulli (1977) that mathematics ideas should be developed through three stages.



- Stage 1: Motivate the students – pick an idea that will interest the students and will assist them to engage with mathematics.
- Stage 2: Provide pre-requisite skills – list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.
- Stage 3: Provide integrating tasks – end the teaching sequence by setting students an open-ended investigation to explore.

This means that each unit will have two foci: (a) activities that form the basis of RAMR lessons – starting from motivating situations in the students' world and then building necessary concepts and skills; and (b) an investigation that integrates the ideas and connects to other units in a problem-solving approach. Thus, the modules combine two approaches to teaching: (a) structural/RAMR teaching of activities that lead to the discovery and abstraction of mathematical concepts and skills (processes, strategies and procedures) starting from the world of the students; and (b) integrative rich-style tasks which allow students an opportunity to solve problems and build their own personal solution, and which give opportunities to combine knowledge across the units.

Sequencing in this module

This module looks at the various forms of tables, charts and graphs. The focus of this teaching is to:

(a) organise and represent data efficiently in ways that enable outcomes such as comparisons, trends and relationships to be easily seen;

(b) construct and read a variety of tables/charts and graphs (e.g. bar graphs, picture graphs, line graphs, circle graphs, stem and leaf graphs, and scattergrams);

(c) recognise forms of tables, charts and graphs most effective for different outcomes (e.g. comparison, trends, relationships to whole); and

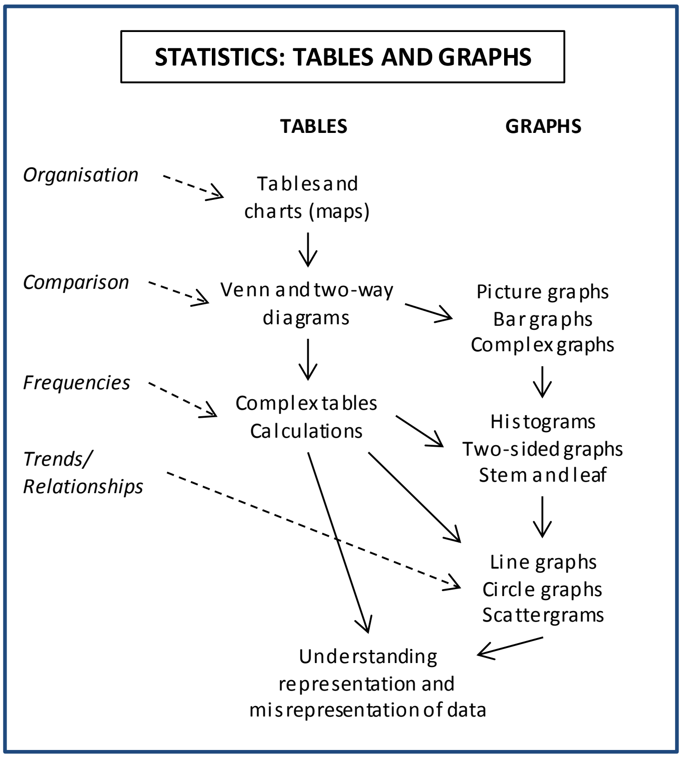
(d) understand how different representations can be used to misrepresent data.

The module has been divided into five imperatives in sequence:

- Organisation
- Comparison
- Frequencies
- Trends/relationships
- Misrepresentation

Forms of charts, tables and graphs have been assigned to each of these imperatives. These components have been ordered. This order is the basis of the sequencing in the diagram on right.

Tables and charts become more complex as the graphs become more complex and vice versa. However, there are ways of connecting more complex tables and graphs to their simpler forms (see the diagrams in subsection on sequencing tables and graphs).

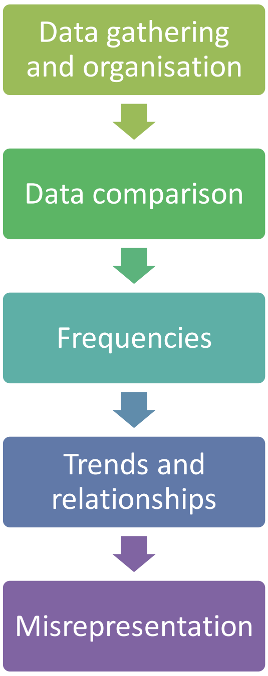


The module is divided into sections and units as follows and as in diagram below right.

Overview: Background information, sequencing and relation to Australian Curriculum.

Unit 1: *Data gathering and organisation – Tables and charts*. This unit covers a variety of tables (e.g. simple, regular, irregular) and charts (e.g. strip maps, branch maps).

Unit 2: *Data comparison – Picture and bar graphs*. This unit covers simple comparison graphs (e.g. picture/block graphs, bar or column graphs, bar line graphs), complex comparison graphs (e.g. one-to-many picture graphs, pictograms, complex bar graphs) and charts (Venn and Carroll or two-way diagrams). Comparison is based on length (or height). The complex picture graphs involve scale.



Unit 3: *Frequencies – Tables, calculations, histograms, two-sided and stem-leaf graphs*. This unit moves from gathering individual pieces of data to counting the number of times the same numerical data is present on its own or in groups. This leads to tables with many columns and tables which are used to calculate costs, and graphs which are based on frequencies. In the graphs, the length or height determines frequency, even in examples which appear to be area based.

Unit 4: *Trends and relationships – Lines, circles, and scattergrams*. This unit covers line graphs, circle graphs and scattergrams which are graphs designed to represent trends across data and relationships between some data and all the data. The trends are shown by length or height and the relationships by either length/height or angle (even in cases where they appear to be shown by area).

Unit 5: *Misrepresentation – How to lie with statistics*. This unit provides an overview of how different tables and graphs show different representations and uses this understanding to explore how statistics can misrepresent data through how data is gathered and how it is presented in a graph.

Test item types: Test items associated with the five units above which can be used for pre- and post-tests.

Appendix A: Extra material for teaching tables and graphs, including definitions and examples.

Appendix B: RAMR cycle components and description.

Appendix C: AIM scope and sequence showing all modules by year level and term.

The relationship between outcomes and graphs is as follows:

- Bar graphs – comparisons (e.g. which is larger or smaller?)
- Line graphs – trends and comparisons
- Histograms, two-sided and stem-leaf graphs – comparisons (of frequencies)
- Circle graphs – comparisons and relationships (part-part and part-whole)
- Scattergrams – trends and relationships.

Note: The drawing and reading of box plots is not part of this module. This method of graphing will be covered in Module SP3.

Activity plans

In the units, the teaching is described by general descriptions, activities and reflections. These do not follow the Reality–Abstraction–Mathematics–Reflection (RAMR) model in Appendix B. Brief descriptions of classroom activities are provided without giving detail of what would be included in a RAMR designed lesson. These ideas have to be translated by teachers into the RAMR cycle by using your knowledge of your students. Interestingly, this means that sometimes less detailed activity plans can be the most effective in providing teaching ideas to teachers. Also, statistics appears to be a mathematics area that is global in perspective. Everyday real activities (reading tables, understanding graphs, following football statistics) are its focus and, therefore, this focus **appears** to be omnipresent, rendering reality a part of all four stages in RAMR.

Finally, although this module is divided into units, it is important to take every opportunity **to teach across units**. For example, if practising tallying for Unit 1, this would mean immediately using the tally material to construct tables and charts and, where appropriate, construct picture and bar graphs for Unit 2 or line graphs and scattergrams for Unit 4. In other words, don't stay within a unit when there are opportunities to use material from one unit in another unit. This is because statistics appears to be less sequential and more integrative than arithmetic, and to reflect an investigatory approach to teaching. This lower emphasis on sequencing often means that it is often **better to teach by rich tasks that cut across the units** and not step-by-step through the units. *Note:* Show how graphs are constructed and read but ensure that **electronic means of constructing graphs** is a major part of this module.

Relation to Australian Curriculum: Mathematics

The table on the following page shows how the content of this module aligns with the ACARA *Australian Curriculum: Mathematics*. It should be noted that, although some work is done in this module, the following will be the main focus of the third Statistics and Probability module, Module SP3 *Statistical Inference*:

- analysis and interpretation of data;
- investigation and comment on different forms/representations of data;
- relationship of data to questions/issues and evaluation of these issues/questions in terms of data, particularly relationship between purpose and choice of data;
- box plots and relation to distributions; and
- discussion of distribution of data, using terms including "skewed", "symmetric" and "bi modal".

AIM13 SP1 meets the Australian Curriculum: Mathematics (Foundation to Year 10)

Unit 1: Data gathering and organisation
Unit 2: Data comparison
Unit 3: Frequencies
Unit 4: Trends and relationships
Unit 5: Misrepresentation

Content Description	Year	SP1 Unit				
		1	2	3	4	5
Choose simple questions and gather responses (ACMSP262)	1	✓				
Represent data with objects and drawings where one object or drawing represents one data value. Describe the displays (ACMSP263)		✓	✓			
Identify a question of interest based on one categorical variable. Gather data relevant to the question (ACMSP048)	2	✓	✓	✓		
Collect, check and classify data (ACMSP049)		✓	✓			
Create displays of data using lists, table and picture graphs and interpret them (ACMSP050		✓	✓	✓		
Identify questions or issues for categorical variables. Identify data sources and plan methods of data collection and recording (ACMSP068)	3	✓	✓	✓		
Collect data, organise into categories and create displays using lists, tables, picture graphs and simple column graphs, with and without the use of digital technologies (ACMSP069)		✓	✓	✓	✓	
Interpret and compare data displays (ACMSP070)		✓	✓	✓	✓	
Select and trial methods for data collection, including survey questions and recording sheets (ACMSP095)	4	✓	✓	✓	✓	
Construct suitable data displays, with and without the use of digital technologies, from given or collected data. Include tables, column graphs and picture graphs where one picture can represent many data values (ACMSP096)		✓	✓	✓	✓	
Evaluate the effectiveness of different displays in illustrating data features including variability (ACMSP097)			✓	✓	✓	✓
Pose questions and collect categorical or numerical data by observation or survey (ACMSP118)	5	✓		✓		
Construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies (ACMSP119)			✓	✓		
Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)	7		✓	✓	✓	✓
Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)			✓	✓		
Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)	8	✓	✓		✓	
Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)	9	✓	✓	✓		✓
Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi modal' (ACMSP282)		✓	✓	✓	✓	✓

Unit 1: Data Gathering and Organisation – Tables and Charts

This unit covers the basis of Module SP1 *Tables and Graphs* – gathering data and recording it in an organised way in tables and charts. It will be the first of two units that look at this gathering and organising. When we get to frequency distributions (Unit 3), we come back and look again at tables.

1.1 Gathering data

It is very important to give students experience with a variety of data forms. In Module SP2 *Analysing and interpreting data*, we look at the crucial relationship between objective or purpose and data choice. For example, when looking at whether under 25s are more dangerous drivers than over 50s, one has to take into account the number of kilometres driven each day. It may be possible that, in total, more under 25s are in accidents than over 50s but that this reverses when looking at accidents per km driven.

It is also important that students understand the pre-number ideas of sorting and classifying. Often this can be a language problem. With Indigenous students, this can be overcome by using local language such as **"which mob belongs"** – putting things with their right "mob" can be a useful way of introducing sorting.

Activities

A. Data sorting

Materials: Scissors, Data set in **Appendix A2.1**

Instructions:

1. Cut out the squares. Keep the frame.
2. Group them any way you like (be prepared to explain your grouping).
3. Count the number in each group.
4. Repeat above, gathering your own data using blanks next page – compare the two sets of data.

Note: This is rich data that can be used in the next unit. So, if the students are interested, jump to the next unit with this data, then return to this unit for more data gathering.

B. Data selection

Instructions:

1. Consider the following contexts. Choose three types of data that you would like to find for each context.
2. Choose one context and collect data – put data in squares like activity 1.
3. Give data to another student/group of students to sort.

CONTEXT	THREE TYPES OF DATA
1. Teachers	
2. Cars	
3. Football players	
4. Mobile phones	
5. Local animals	
Your own context	

Reflection

- What kinds of data are there?
- Is there data you can just see? What data do you need to measure?
- What if data is in terms of descriptions – how do we reduce it to one thing?

1.2 Constructing tables and charts

Tables and charts are often not explicitly taught – students are left to pick up what they are, and how they work, by chance. In this section, we provide some ideas for directly teaching charts and tables. However, we also provide ideas for experiences – to ensure that students experience a variety of them, from “Googled” directions to bus routes to timetables and so on. Experience can also be a good teacher.

Activities

A. Gathering data through clustering or grouping circles

Instructions:

1. Organise students to group in circles on floor so that people can cluster around characteristics such as eye colour, gender, and left or right handedness, and so on. A lot of different interpretations and comparisons can be made.
2. This data is recorded and the numbers of students in the clusters can be made into simple tables.
3. Students can be organised into columns with the same spacing and starting line so that this data leads to bar graphs. Clustering gives rich data to make into tables, graphs and comparisons.
4. This activity can be repeated for “hand” material – that is, data on cards or collections of logic blocks to be sorted by colour, size, shape, and so on – where the cards/blocks are placed into groups on a desk.

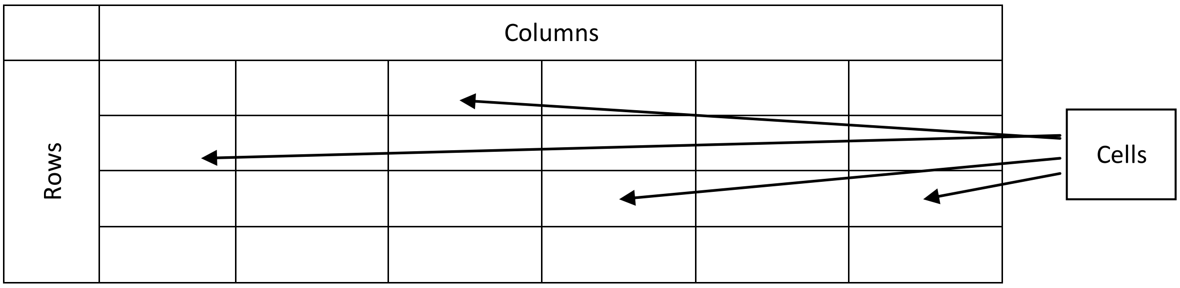
B. Constructing a table

A table is based on an array or matrix – columns and rows. You need to become familiar with reading tables across and down. Tables require the following: (a) working out what will be in the columns – their names/titles, (b) working out what will be in the rows – their names/titles, and (c) placing or reading the information correctly in the cells.

Instructions:

1. Choose a context and a topic: e.g. hair colour. Make a table using the following steps.
2. Step 1 – choose the rows – these are the focus of the table, for example, each student or the different hair colours (it depends on what you want the table to do, count hair colours or give you the hair colour of each person) – these are down the side of the table.
3. Step 2 – choose the columns – this is where different things occur for the rows, for example, you might have a column for girls, boys, and total, or you might list the hair colours if these are not the rows – these are across the top of the table.
4. Step 3 – choose the cells – these are what are in the squares of the table, for example, the number of girls/boys/total with that hair colour or a tick if the student in that row has that hair colour.

Thus a table is as follows:



C. Tallying and tables

Tallying is a method of putting down one stroke | for each item and putting a cross line when moving from 4 to 5, 9 to 10 and any time we make a new 5, that is, 1 |, 2 ||. 3 |||, 4 ||||, 5 ~~||||~~, 6 ~~||||~~ |, 7 ~~||||~~ || and so on. This means that 10 is ~~||||~~ ~~||||~~ and 17 is ~~||||~~ ~~||||~~ ~~||||~~ ||.

Materials: Magazine, book or newspaper; pen; tally sheets as below, two dice; clothes pegs and line; 12 index cards with numbers 2 through 12 on them, arranged as in (c).

Instructions:

1. Construct a table and use it and tallying to complete the activities below.

   (a) Different hair colours in our class.

Hair colour	Blonde	Brown	Black	Red
Tally				
Total				

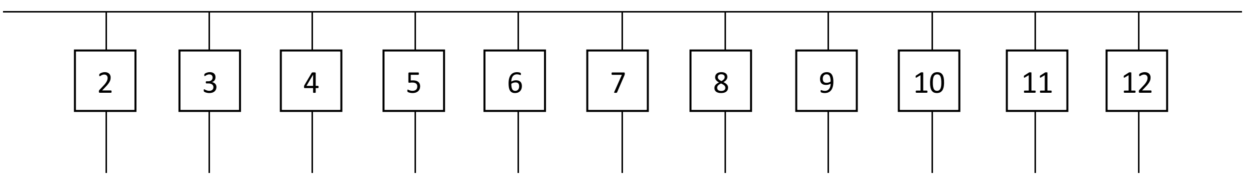
   (b) Different cars in the car park.

Type of car	Holden	Ford	Japanese origin	European origin
Tally				
Total				

2. Open the paper, book or magazine at any page, tally each letter on that page (or in an article on that page), and combine results into a total class tally. Questions – What are the most common and least common letters? What is the use of this information?

Letter	Tally	Total	Letter	Tally	Total	Letter	Tally	Total
A			J			S		
B			K			T		
C			L			U		
Continuing on to I			Continuing on to R			Continuing on to Z		

3. Toss the two dice 50 times, adding the two numbers for each throw. Place a clothes peg on the line hanging from the index card with the total on it for each throw. Questions – Which number has the most pegs? The least pegs? If we tossed the dice 1000 times, what would you expect to happen?



D. Charts and tables

Materials: Packets, newspapers, magazines, manila folders, sticky tape/glue/scissors, logic blocks.

Instructions:

1. **Tables.** Collect everyday examples of tables (e.g. packets of cereal, papers, timetables, etc.). Get the students to experience tables with them. To do this – stick the best inside manila folders, place suitable titles on the outside of the folders, and make up work cards to go with the folders, which require the students to find information from the tables.
2. **Tables**. Construct tables which display: (a) the timetables for Year 1, 2 and 3 classes; and (b) the seating arrangements for your class.
3. **Strip maps**. Draw a strip map diagram of how you get from home to school – mark in major landmarks. Use a road map to draw a strip map from Brisbane to the Gold Coast (or other locations in your local area). Draw a strip map of a train line in your area.
4. **Strip map (timeline).** Draw a timeline of what you did yesterday. Compare this with a friend.
5. **Strip map.** Use the following data to construct a strip map for Bus 2:

Stoodysville's bus timetable

	Bus 1	Bus 2	Bus 3
Monteysville	9:00	9:13	9:27
Rubber Road	9:11	9:35	9:51
Factory	9:21	9:55	
Fredsville	9:35	10:23	10:29
King Bridge		10:52	
Road's End		11:14	11:02

6. **Venn diagram**. Draw up a large Venn diagram in terms of red and large logic blocks. Place the blocks in their appropriate sections. How many blocks in each section after a complete set of logic blocks has been placed?
7. **Carroll or 2-way diagram**. Draw up a large Carroll or 2-way diagram in terms of triangular blocks and small blocks. Place the blocks in their appropriate sections. How many blocks in each section after a complete set of logic blocks has been placed?

Reflection

- What is data? What kind of data should we find? [Data can be ticks, words, tallies, numbers. We have to find the data that makes sense in terms of what we want the table to do.]
- Can data be used in more than one way? Interestingly, data collected in one way can often be used in many ways in different tables.
- What kind of data is best shown on tables? What kind of data is best shown in strip maps? Why do you sometimes see both at a bus, train or boat stop?
- Where would you use Venn or Carroll diagrams?

1.3 Reading tables and charts

The best way to learn to read tables and charts is to construct them – this is why this is the third sub-unit and not the first.

Activities

A. Reading tables

Instructions:

1. Consider the following table for trains – columns are stations, rows are trains, and cells are times. In **Appendix A2.2** Timetables, there is a description of four ways to read a table like that below. Read about these ways and then use them in examples below. At the end, there is an example that involves more than just reading from the table.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1	9:16 am	9:47 am	10:23 am	11:26 am	11:58 am	12:22 pm
2	9:45 am		10:43 am		12:06 pm	12:42 pm
3	10:07 am	10:39 am		12:09 pm		1:35 pm
4	10:54 am	11:28 am	12:03 pm	1:01 pm	1:42 pm	2:06 pm

2. Way 1: When does train 1 arrive at Karlin? Train 3 arrive at Ooptan? Train 4 arrive at Mont?
3. Way 2: What train arrives at Karlin at 10.43 am? What train arrives at Ooptan at 1.35 pm?
4. Way 3: At what station does train 4 arrive at 11.28 am? At what station does train 1 arrive at 11.58 am?
5. Way 4: What train misses two stations and what are the stations?
6. If you miss train 1 and have to wait for the next train, how late will you be at Nanty? What about Mont?
7. Can you make up more examples like (6) above?

Reflection

- What things can make tables hard to read? [Strange shapes, lack of titles and clear names for rows and columns, huge amounts of data.]
- What do you have to be good at to read a table? [Understanding vertical/horizontal and rows/columns, and being able to move eyes along rows/columns without jumping a row or column.]
- What should you look at before you start to find answers on a table? [Names of rows and columns, type of data in the cells, and missing cells.]

1.4 Investigation

Take the data from Activity 1 in sub-section 1.1 – the squares of data. Create as many forms of charts and tables with this data as you can. Make a poster called "The Power of Data" and put all your ideas in it in a creative and attractive way.

What graphs could you make from these different charts and tables? Add these to your poster.

Are there some charts and tables which cannot be made into graphs?

Unit 2: Data Comparison – Picture and Bar Graphs

This unit covers another of the bases of this module, Module SP1 *Tables and Graphs*, drawing and reading picture and bar graphs which are designed to show comparison. Line graphs and circle graphs can be developed as an extension of bar graphs, so picture and bar graphs are an important part of the development of tables and graphs.

In Unit 1, we showed that charts and tables could be built around words and symbols like ticks as well as numbers. However, because of their nature, picture and bar graphs have to represent number because their diagrammatic representation as a graph shows difference as a comparison between lengths. Thus for comparison, we are restricted to data that are numbers of measures (i.e. numbers and attributes – e.g. 24 m or $31).

2.1 Introducing simple picture and bar graphs

As shown in the Sequencing sub-section of the Overview section, bar graphs are developed from picture graphs which are in turn developed in a sequence from simple two-way graphs. The sequence is as follows:

Stage 1: One-to-one correspondence – two columns – students, physical objects or pictures;

Stage 2: More columns – students, physical objects or pictures;

Stage 3: Stick on – using paper squares instead of students, physical objects and pictures;

Stage 4: Squared paper stage – shading squares on graph paper;

Stage 5: Abstract representation stage – drawings of rectangles on axes; and

Stage 6: Using lines for bars – replacing rectangles with a line.

Activities

A. People graphs

Instructions:

1. Use the Maths Mat to act out with students' bodies, bar graphs, 1-1 correspondence, and labels (e.g. more, less). A good example is to label columns on the Mat with numbers 0, 1, 2 and so on. People line up, for example, based on the *number of siblings in family when growing up*.
2. Students place an object in the square they are in (e.g. recycled envelopes with names written on). Participants can step back to view the graph on the mat, but they still can identify their own position and view as a whole to bring out the language (e.g. bars, 1-1 correspondence).
3. Students then go back to graph paper and represent what is on the Mat by colouring squares. Names are no longer the central focus – the focus is on the number in each column as the bar graph is completed.
4. The bar graph can be condensed to form a table. Note that this will require new labelling.

B. Teaching principles of bar graphs

For bar graphs to be legitimate and provide easy observational comparison, they must have the following:

1. **the appropriate labelling and scaling –** (a) two axes, (b) title for the graph, (c) titles for each axes, and (d) scales for each axes. The scales can be categorical (e.g. list of pets) for the horizontal axis but are normally interval (i.e. numbers) and discrete (only the counting numbers, 1, 2, 3, and so on, no 2.5 or 3.68); and

2. **the correct positioning of the pictures or squares** – (a) the same size pictures and squares, (b) the same spacing for the pictures and squares, (c) 1-1 correspondence, and (d) pictures and squares starting at the same point (0).

These principles of simple picture and bar graphs can be (and should be) taught.

Instructions:

1. Use the Maths Mat to act the following out with students' bodies: (a) same spacing, (b) 1-1 correspondence, (c) labelling, and (d) starting from the same point (0).
2. Use the Maths Mat to act out simple bar graphs.
3. Use the method called "torpedoing" (developed in the 60s and 70s by Bruner) to teach the graph principles. In this method, the teacher deliberately makes errors so that the students can argue that the teacher is wrong, and give the correct answers. **Appendix A4** has an example of a torpedoing activity to teach same size show equal spacing, 1-1 correspondence, and starting from zero.
4. Get to know how to **use computers** to draw all the types of bar graphs.

Reflection

- What makes an effective bar graph?
- What are the principles of a good bar graph?
- What are easy ways to mislead with a picture or bar graph?

2.2 More complex graphs

The sequence at the start of section 2.1 takes bar graphs from materials via pictures to abstract rectangles. We now look at these abstract graphs.

Activities

A. Moving from pictorial to abstract

Instructions:

1. **Appendix A5** shows the sequence from pictures to abstract bar graph. Read this sequence.
2. What questioning would you use at each of the steps in this sequencing – e.g. how do you move from pictures to squares, and squares to shading and shading to rectangles? What stays the same and what changes?
3. Gather some data from a group of people/students – have more than two columns (e.g. hair colour). Draw/construct four graphs showing the transition from pictures to abstraction as in **Appendix A5**.

B. Recognising and fixing errors in bar graph presentation from pictures to abstract

Once we get to abstract bar graphs, there is another principle to recognise, that the width of the graphs should be common, and that bars should not touch unless they represent touching intervals.

Instructions:

1. Look at the students' drawings of bar graphs in **Appendix A5**. What is wrong with the graphs?
2. Redraw the graphs in **Appendix A5** so that they are more appropriate/correct.
3. If they are not already, turn each graph into its abstract form.

Reflection

- What is the sequence from start to abstraction for bar graphs?
- Are sequences like this useful for Year 8 students with little mathematics knowledge? Or can you go directly to the abstract form?

2.3 Reading comparison graphs

Graphs like picture and bar graphs are important because they visually show what/who has the most or least, and whether differences are large or small. Interestingly, although bar graphs are constructed by drawing rectangles of different heights, most people see the difference between heights multiplicatively. It is easier to see that two bars are such that one is about double the other than to read off the numbers and see that one is 23 more than the other.

Activities

A. Construct and read

Instructions:

1. Organise the students to gather data that could be made into a bar graph from other students or from Internet sources. Place this data into a table.
2. Convert the table data into a bar graph. Then answer questions with regard to the graph. The students' experience in making the graph should assist them in answering questions.
3. **Appendix A3** contains a bar graph (it is the first graph) and questions. Get the students to gather data on rainfall for 2011 in their local area as for this graph. Ask the same questions. (*Note:* The percentage question should only be done if the students understand percent.)
4. **Appendix A3** also has a simple picture graph about rainy days (no. 4). Answer the questions for this graph. Look for other situations where a picture graph may be better than a bar graph.
5. Ask the students if there is anything about where they live that they would like to find out. Organise them to gather data and construct bar graphs. Get the students to answer questions about the graph. (*Note*: It is important that the students are able to answer direct questions. Questions requiring interpretation will have another module, SP3, in which the skill will be taught – however, I would ask these questions.)

B. Reading

Instructions:

1. Obtain examples of bar graphs from magazines, newspapers, promotion material, advertisements, Australian Bureau of Statistics, Bureau of Meteorology, and so on. Get the students to help find such graphs on the Internet.
2. Set questions for these graphs – help the students to be able to find information from the graphs and the corresponding tables.
3. Get the students to make up their own questions for other students to answer – question posing is an excellent way to teach question answering.

Reflection

- Are bar graphs relevant to students? How can we make them relevant?
- In other situations, students such as those in AIM schools have been engaged by gathering and graphing data that concerns their history and their present situation. What data interests your students? Data with regard to family? Data with regard to major historical events in community?
- In other countries, teachers have looked at data that emerges from the country's colonial past – even that to do with injustice and invasion (e.g. Uncle Ernie Grant has found much statistical data from studying punitive expeditions against Indigenous people). They have also looked at the present situation (e.g. average wages and unemployment rates as compared to non-Indigenous people). For example, the statistics associated with local community councils may be interesting. These have been very successful with regard to literacy and numeracy but have stirred up violence in some cases. Are they worth the risk?
- Could we look at controversial things in our schools? What safeguards would we need (e.g. permission and support of Elders)? Chris Sarra had students looking at substance abuse, violence in the home and life expectancy when at Cherbourg. Is this something only he could do?
- What about involving the students in AIM's pre-post testing? Each student could do their own data.

2.4 Investigation

Organise the students to undertake a study of their lives that sets up bar graphs. Make a poster on this. Ideas could be:

- A poster on Indigenous people in Australia – the rates of Indigenous people playing NRL and AFL, the rates of Indigenous people in various jobs, looking at a variety of jobs or a few particular vocations.
- A poster on mathematics improvement – give weekly tests on basic facts and so on and students graph each week their results (a teacher did this very successfully at Shalom Christian College in 2007, another did it in Dajarra a year earlier).
- A poster on the local council – expenditure on various things across the last few years.
- A poster on native animals in the area – how many of each? – or of trees and grasses.
- A "care for country" poster looking at land degradation in the local area, or land usage.

Unit 3: Frequencies – Tables, Calculations, Histograms, Two-Sided and Stem-Leaf Graphs

This unit moves on to more complicated/advanced tables and graphs – looking at how tables are used for solving problems and calculations (e.g. timetables and budgets), and looking at histograms and stem and leaf graphs. The focus is still comparison but the tables and graphs are more complicated.

There are three complications.

1. By their nature, picture and bar graphs are frequencies – they show how many students have blue eyes, or the mm of rainfall on Friday. However, things become more complicated when the x-axis is not in terms of category data (e.g. pets, or names of students) or ordinal data (e.g. months of the year) but rather in terms of interval data like height. It does not make sense to have the number of people of height 155 cm as one will be 155.1 cm and another 154.6 cm. The answer lies in finding the number of people whose heights are between 154.5 cm and 155.5 cm. If this is done, then the rectangles touch each other and all the area is covered. These are called **histograms**.

2. The second complication is to look at bar graphs that go negative. An example of this is on election nights when looking at swings in terms of percentage votes from one party to the other. For the political parties, the rectangles/bars for some parties will be above the line showing a *swing to* while others will be below the line showing a *swing away*. Such graphs are called **two-sided complex bar or column graphs**. They can also be continuous like histograms.

3. The third complication is to use the graph to show the distribution of the data alongside the comparison. For example, data from heights could be placed on a vertical line, marked in 10 cm intervals, by writing the actual height for each student in each 10 cm interval, in a list small to large beside the interval. One can compare distributions by having different data on each side (e.g. girls vs boys). Such graphs are called **stem and leaf graphs**.

3.1 Constructing and using frequency tables/distributions

Here we look at timetables and budgets. Timetables were started in Unit 1 – we now look again at them and the ideas that were in **Appendix A2.2.**

Activities

A. Table activities

Instructions:

1. Look again at Appendix A2.2. Examine the bus timetable below:

Major stops	**AM**			**PM**		
Goodna Station	-	9.40	11.40	1.45	3.39	5.39
Redbank Station	-	9.46	11.46	1.51	3.45	5.45
Redbank Plaza	7.50	9.51	11.51	1.56	3.50	5.50
Riverview Primary Sch	7.55	9.56	11.56	2.01	3.55	5.55
Riverview Gardens	8.00	10.01	12.01	2.06	4.00	6.00
Dinmore Station	8.05	10.06	12.06	2.11	4.05	6.05
Bundamba Primary Sch	8.09	10.10	12.10	2.15	4.09	6.09
Booval Fair	8.12	10.13	12.13	2.18	4.12	6.12
Ipswich (Bell St)	8.19	10.20	12.20	2.25	4.19	6.19

2. Answer these questions:

    - What time would you need to catch the bus from Redbank Plaza in order to get to Bundamba Primary before school starts?
    - If the bus leaves Dinmore Station at 2.11 pm, what time would it arrive in Ipswich? If I was travelling from Redbank to Ipswich to arrive at the same time, what time would I have to leave?

3. Obtain actual train and bus timetables and set questions to be answered.

B. Calculations and budgets

Instructions:

1. Look at **Appendix A2.3**. Examine how to set up the table to budget for the 4-day trip. Repeat this for a woman doing the trip. Make up your own 4-day trip costs.

2. Develop a list of items for the following situations:

    - things other than clothing that you need to take on a holiday; and
    - things that you might need to put in a new house/car to make it more livable/drivable.

3. Translate the two lists from (2) into budget tables as follows:

    - a table that determines the cost for these other items for a holiday of two weeks; and
    - a table that determines the cost of the extras across 6 months.

    Use the Internet to find values of items and work out the budget for each of the two situations.

Reflection

- What is important in setting up and interpreting timetables?
- What is important in working out a budget?
- How can you be sure that you have exhausted all possibilities for the budget?

3.2 Constructing and using complex picture graphs and histograms

Activities

A. Picture graphs and scales

Instructions:

1. **Appendix A3** has a complex picture graph. It is the second graph under activity (4). Answer the questions. What is the scale in this exercise?

2. Answer, in particular, part (c) of this activity (4) in **Appendix A3**. The thing to watch is that these scale picture graphs are useful when numbers are large.

3. What are the difficulties, the mistakes that might be made, with scale picture graphs?

4. Find examples of complex picture graphs with scales. Are they effective? Are they easy to follow? What are their scales?

B. Histograms

Instructions:

1. **Appendix A6.2** describes how a histogram is developed. Read this Appendix. Answer the questions.

2. Gather your own data on heights and repeat the process.

3. If class weights were between 44 kg and 91 kg – what intervals would you have for the weights if you only wanted around six intervals?

   [All weights are in range found by 91−44=47; six intervals means that interval length is 47÷6=nearly 8, so length of 8 – have to choose interval length of either 7 or 9 if need odd number for centralising which is seven intervals, as 7×7=49>47 and 7×6=42<47, or six intervals, as 9×5=45<47 and 9×6=54>47. Of course, it can tolerate even length, so interval length of 8 will do.]

4. Gather data for continuous things like height or weight and develop other histograms. Follow the steps of **Appendix A6.2**.

5. Search Internet and magazines, etc., for examples of histograms and answer questions about them.

6. Use computers to construct histograms.

Reflection

- What is the problem of scale in complex picture graphs?
- As scale is a problem, why use scaled picture graphs? Are there situations where they are effective?
- Why do we need histograms when we have bar graphs? Do we need this special name? Are they just bar graphs with bars touching?
- Do we need odd length intervals? And if we do, when do we need them? Do we also need the zeroes at each end (other than to make the polygon)?

3.3 Constructing and using two-sided complex bar graphs

These graphs are like bar–column graphs that go in either direction – positive or negative or comparing two sets of data.

Activities

A. Using people

Instructions:

1. Gather data on eye colour. Record the number of students for each colour separately for girls and boys. (*Note*: Data could also be gathered around *preferences* – favourite pet, favourite colour, politics; *places* – suburb where they live, where they were born; *what they have or own* – type of computer, type of phone, number of siblings; and so on.) It is normal for this graph that the data is not continuous. This data would be similar to that for a simple bar or column graph.

2. Using a strip of masking tape for the axis (usually vertical), mark sections by eye colour. Students stand to the side of the sections going out from the stem – girls on right and boys on left. This is a two-sided bar graph.

3. Get students to place a label where they are standing, step back and view what they have made, and then make a pen-and-paper copy of the graph. Discuss the language.

B. Constructing two-sided graphs

Instructions:

1. Choose a topic by which things can be characterised using positive and negative numbers and graph this. Normally this would be a horizontal axis with columns going off from either side. It should be noted that these graphs tend to have x-axis as discontinuous and the data tends to be categorical.

   - One way to do this is to pick something where there are positive and negative numbers and graph students' scores or graph class averages. One possible example of this is NAPLAN scores above and below the state average.
   - A second way to do this is to choose something for data gathering where there is comparison with an average. For example, choose something for which there is census or other Australia-wide data, gather data from class, compare class averages with Australian averages, and graph positive and negative differences.
   - A third way to do this is to compare data from one time to another. For example, the values of shares could be compared from start to end of the year, as could house prices (with increases showing as positives and decreases as negatives). Also, could have a series of devices to randomly generate numbers. These could be trialled twice and the first trials compared with the second. Another good example is pre-post data.

2. Choose a topic where you are comparing two sets of data on the same things (e.g. test results for boys and girls). These would usually be a vertical axis with horizontal bars for comparing two groups and a horizontal axis with vertical bars for positive and negative data. For example, the data for selecting cards (1 to 6), throwing a die, and spinning a 1-6 spinner could be as follows for 5 girls and 8 boys (the first row of data is the first trial and the second row of data is the second trial):

Cards		Die		Spinner	
Girls	Boys	Girls	Boys	Girls	Boys
1, 5, 4, 3, 2	5, 3, 5, 2, 1, 3, 4, 1	1, 5, 4, 5, 1	6, 4, 2, 5, 1, 1, 3, 2	3, 6, 3, 1, 1	4, 3, 4, 1, 5, 4, 2, 4
4, 6, 3, 4, 2	6, 2, 6, 3, 1, 4, 6, 2	5, 3, 6, 4, 4	5, 4, 3, 6, 1, 5, 4, 4	3, 1, 5, 6, 4	3, 2, 2, 6, 2, 2, 1, 2

   - Average all students' first and second trials and draw a two-sided graph with horizontal axis and vertical bars of differences in the three averages ($2^{nd} – 1^{st}$). Repeat for six differences with boys and girls separately.
   - Average boys' and girls' results separately for 1^{st} trials and 2^{nd} trials, and draw a two-sided graph with vertical axis and horizontal bars with girls on right and boys on left for all six trials.

3. One common use of these graphs is the graphs shown in election-result coverage looking at voting patterns which show swings (positive and negative) for and against parties. Look these up and discuss them.

4. Search Internet and other sources of information for two-sided graphs. Describe how they are made. Pose questions about them.

Reflection

- Where are two-sided graphs useful?
- How are they different from bar graphs? What is the use of this difference?

3.4 Constructing and using stem and leaf graphs

These are similar to the two-sided graphs above but show distribution within an interval. The data tends to be continuous.

Activities

A. Using people

Instructions:

1. Choose a topic for which people have a measure. This could be age, street number of house, height, mass, and so on. For this example, we will choose height.
2. Measure the height of all students.
3. Using a strip of masking tape for the "stem" (normally vertical), mark sections as 0–9, 10–19, 20–29 and so on. Students stand to the side of the section in which their height belongs, going out from the stem – in order with lowest beside the stem. This is a one-sided, made-with-bodies stem and leaf graph. To make it two-sided, put females on one side and males on other.
4. Get students to place a label where they are standing, step back and view what they have made, and then make a pen-and-paper copy of the stem and leaf graph. Discuss the language.

B. Constructing two-sided graphs

Instructions:

1. The following is data from students' masses:

   Girls: 37, 35, 56, 38, 41, 46, 62, 51, 43, 46, 31, 37, 60

   Boys: 48, 51, 83, 76, 63, 64, 49, 58, 63, 72, 65, 62, 58, 70, 49

   - Use the data to draw a one-sided stem and leaf graph with all data together.
   - Use the data of boys and girls separately to draw a two-sided stem and leaf graph with boys on right and girls on left.
   - Which graph is best for what? Why?

   Note: You can use a very abstract form of stem and leaf graph where the stem just has the tens and the leaves have the ones. For example:

   The number circled means that there is a 34 on the right-hand side. The number in a square means that there is a second 33 on the left-hand side.

			1	6	4	5		
			8	5	2	3	4	6
8	7	4	1	4	8	9		
	8	[3]	3	3	1	(4)	8	
	5	2	1	2	6	6	4	
			2	1	4			
				0				

2. Search the Internet and other sources of information to find stem and leaf graphs.
3. Describe how they are made. Pose questions about them.

Reflection

- Where are stem and leaf graphs useful?
- How are they different from bar graphs? Is this difference useful?

3.5 Investigation

There are multiple opportunities here:

1. **Get students to interpret timetables**. Obtain a collection of timetables for buses, trains (from bus/train operators, or off the Internet) or something different (e.g. planes) and see if students can read them. Set questions from the timetables for students to answer. For example, questions could go from simple (e.g. "What time does train 3 get to whatever?") to complex (e.g. "How long for train 3 is it from whatever to somewhere else? Is this the same for all trains? Why would these times be different?").

2. **Set a rich timetable task**. Get the free Prevocational Maths materials from the YuMi Deadly Centre website (Vocational Learning resources). Two that come to mind are: *11.6 The Man from Hungary*, and *12.6 Rocking Around the World.*

3. **Set up budget situations**. Provide students with a variety of information for a variety of situations (e.g. different costs for mobile phones), and work with them to develop a table. Discuss how tables can be different (e.g. they can be square or rectangle; they can have missing sections), and perform different tasks (e.g. to compare different options, to describe different options, to ensure all options are covered, to calculate the costs involved).

4. **Set a budget rich task.** Once again the Prevocational Maths material from the YDC website has some good ideas. Two are: *11.2 The Big Day Out* and *11.4 Exchange Student*.

5. Finally, **planning a party** and **developing a family budget** are really worthwhile tasks – these are described in the free financial mathematics activities that can be found on the YDC website (Student Learning resources).

Unit 4: Trends and Relationships – Lines, Circles and Scattergrams

This unit extends the focus of comparison to using comparisons to see trends and relationships – between parts and between a part and the whole. Trends are best seen in line graphs and relationships in circle graphs and scattergrams (or scatter plots), though looking at more than one line graph on the same axes also leads to relationships.

4.1 Constructing and using line graphs

Activities

A. Tables to bar to line graphs

Instructions:

1. **Appendix A5** describes how a bar graph can become a line graphs. Read this appendix. Copy the bar graph and make the line graph.
2. Obtain some data and turn it into bar graphs, and then turn them into line graphs the same way as in Appendix A5.
3. Discuss how this process of finding the x axis positions and going up the heights of the bar gives (x,y) positions that can lead to line graphs – that a table gives a series of two values that are the point in the line graph.
4. Do the activities (1) and (2) in **Appendix A6.1**. This shows how to shortcut from table to line graph.

B. Creating and using line graphs

Instructions:

1. Obtain data that needs to show trends (e.g. rainfall, payments) and construct line graphs.
2. Look up line graphs and determine questions for students to answer to show they can read them.
3. An example is in **Appendix A3**, second graph– answer the questions with the graph in the Appendix.
4. Use computers to construct line graphs.

C. Relating line graphs to stories

Instructions:

1. Consider a line graph of the height of water in a bath – the water is turned on –and the height of the water increases – someone gets in, gets out and pulls the plug – what happens to the height of the water? – what does a line graph look like of height against time?
2. Make up stories about the bath and then construct the graph. Take graphs and turn them into bath stories. See handouts.
3. Try other stories, e.g. a car leaves home, meets someone, stops, then drives on – what does its distance from home against time graph look like?
4. These graphs can be complicated, with many lines, but there is always a story – see handouts.

D. Relating line graphs to other line graphs

Instructions:

1. If there is more than one line graph, the two graphs can be compared. Find examples of this and compare – get two sets of data and put on same graph and compare.
2. Comparing across graphs adds an extra dimension – graphs of average monthly maximums and minimums can be made for a year for all cities – can we tell the city from the graph?

Reflection

- What is the difference between trends and comparisons?
- Why are lines important in trends?
- How many different ways can we see relationships in lines – within the lines, across lines and across graphs?

4.2 Constructing and using circle graphs

Activities

A. From bars to compound bars to circle graphs

Materials: pictures of eyes to colour in, 2 cm graph paper, pole and streamers, sticky tape, scissors, paper circles, 100 beads – 10 blue, 10 red and so on.

Instructions:

Make up a simple set of data for a picture or bar graph. For example, get 20+ students to group themselves around eye colour.

1. Step 1 – give each student a picture of an eye to colour the same as their eyes – get to line up as a bar graph on the mat – place eyes where they are standing, move off mat to see the graph.
2. Step 2 – colour in 2 cm graph paper squares with pencils to make a copy of the bar graph.
3. Step 3 – re-form the human bar graph and then get bars to join end-on-end to make a single complex bar graph – place down coloured pictures of eyes and step back to see the whole complex bar.
4. Step 4 – cut strips of 2 cm squares and join and colour in to make a copy of the complex bar.
5. Step 5 – re-form the human complex bar, and move into an equally spaced circle around pole – people on ends of bars for each colour take a streamer and pull out to make a "maypole" – place pictures and streamers on ground and step back to see the circle graph.
6. Step 6 – put paper copy of complex bar as a circle around the circle paper and draw lines to centre from end of bars of one colour to make a copy of circle graph – work out the fraction for each sector.
7. Step 7 – put 100 beads around the circle graph and calculate percent of each sector. (*Note:* You can show how to go from fraction to percent by this method.)

This method of data → bar graph → compound bar or strip → circle graph → percent is an excellent activity – involves going from activity with body to paper work and back again. A complete plan for it will soon be on the YDC website.

B. Reading and constructing circle graphs

Instructions:

1. Complete the circle graph activity from **Appendix A3** activity (3).
2. Look for circle graphs on the Internet and in magazines, etc., and set questions to see if students can read them.
3. Complete the circle graph activities from **Appendix A6.1** – activities (3) to (5). Spend time turning percent into angle out of 360 and vice versa.
4. Gain data by setting up an activity to gather data from people, make a table and tally, draw a bar graph and then construct own circle graphs. (*Note:* Circle graphs are hard to construct unless using a computer.)
5. Do many examples of circle graphs on the computer.

Reflection

- Circle graphs use similar data to bar graphs; what is the difference in the relationships they show?
- Is it still important to do circle graphs by hand with protractors, or should we just use the computer?

4.3 Constructing and using scattergrams

Activities

A. Reading and constructing scattergrams

Instructions:

1. Complete the scattergram activities from **Appendix A6.1** – activities (6) to (10). Answer the questions.
2. Complete the scattergram activities from **Appendix A3** – activity (5). Answer the questions.
3. Try to find scattergrams on the Internet and from reports and so on. Try to argue for relationships.
4. Gather data for scattergrams – look at things that may relate (e.g. hand span and height, arm span and foot length, and so on) and may inversely relate (e.g. hours watching TV and hours doing homework). Do not miss out on some data that does not relate.
5. Discuss outliers and special cases and their effect on relationship.

Reflection

- How close together do the dots have to be to show a relationship?
- Relate scattergram relationships to estimates of line of best fit.

4.4 Comparing graph types

The crucial thing for this module is to understand what types of representations (tables and graphs) most suit which data and which purpose.

Activities

A. Comparing different graphs

Instructions:

1. Take one set of data and show how it can be represented by different graphs.
2. Discuss which representation is most effective.

B. Relating graphs to data

Instructions:

1. **Appendix A7** has five sets of data. Look at this data.
2. Complete the tasks at the top of the data set.
3. Determine which graph type is best/better for each data set (if possible).

C. Features of graphs – what are they good for?

Instructions:

1. Copy the table from **Appendix A9.3 – Reflection activity**.
2. Complete the table for each type of graph listed.

Reflection

- Can all graphs apply to all data?
- Is purpose more important than data type in determining what graph to use?
- If the above is the case, does this make graphs circular – the purpose selects them because they show the purpose in the best light?

4.5 Investigation

There are two investigations:

1. The first is a group activity involving cooperative problem solving. Groups of four receive four cards which have different representations of the same data – stem and leaf, box and whisker, raw unsorted data and bar graph. What do we get from each? (Mostly for discussion and language, to see what can be figured out.)
2. The second is individual (though it could be group) and requires each student to develop a graph from a question or position. The task is described in **Appendix A10.**

Unit 5: Misrepresentation – How to Lie with Statistics

This unit covers correct representation and misrepresentation. Ever since the statement "lies, damned lies and statistics", there has been an awareness that statistics can be manipulated to misrepresent. This is also true of graphs as we will now show.

5.1 Misrepresentation in gathering data

Activities

A. Methods of data misrepresentation

Instructions:

1. The gathering of data can lead to misrepresentation in three ways: (a) the way questions are asked (leading questions); (b) the person who asks the questions (interviewer bias); and (c) who is chosen to answer the questions (sample bias).
2. Look up these ideas and see what they mean. They can be powerful – a group once sent out an attractive blonde and an attractive brunette to ask 20 young men if gentlemen preferred blondes – the results were diametrically opposite between both interviewers.
3. We will look at this more in Module SP3 – until then read Darrel Huff's book, *How to lie with statistics*.

5.2 Misrepresentation in the construction of graphs

Activities

A. Methods of graph misrepresentation

Instructions:

1. Read **Appendix A8** (first page and top of the second page on line graphs)**.** Complete activity 4.
2. Read activity 5. See how pictures show a stronger presentation than bars.
3. Bar graphs can also be made to misrepresent in the same way as line graphs – truncation, stretching vertically and narrowing the graph make differences look bigger, while not truncating, squashing down and widening make differences less. Try this out for a bar graph.

B. Fixing up misrepresentations

Instructions:

1. Read **Appendix A9.1**.
2. Complete activities 1 to 7.
3. List the inaccuracies; and redraw the graphs correctly.
4. Redraw the graphs in **Appendix A9.2**.

Reflection

- Have you seen this type of thing in graphs? Search advertisements and reports (and the Internet) for examples.
- How much of misrepresentation is use of visuals and how much is manipulation of data?

5.3 Investigation

There are three possibilities:

1. Look at something from the news. For example, it is commonly stated that China's economy is strong. Look this up on the Internet. Are there graphs that support this? Are there other graphs that show other possibilities? Look at the graphs and discuss what they show. Discuss also how the graphs could misrepresent the real situation.
2. Look on the Internet for graphs that show change and how to prevent this change. For example, graphs of the concentration of members of audiences within a lecture show that concentration declines in the middle of a lecture. There are also graphs that show that an activity in the middle of the lecture causes concentration to rise. Find such graphs. What are they telling us? That taking breaks increases the amount learnt?
3. Pick a statement like "cars are safer now than years ago" or "there is more crime now than 50 years ago". Gather data on these statements and present two posters that do their best to purport opposite views. Use your "how to lie" knowledge to make them both convincing!

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the tables and graphs item types

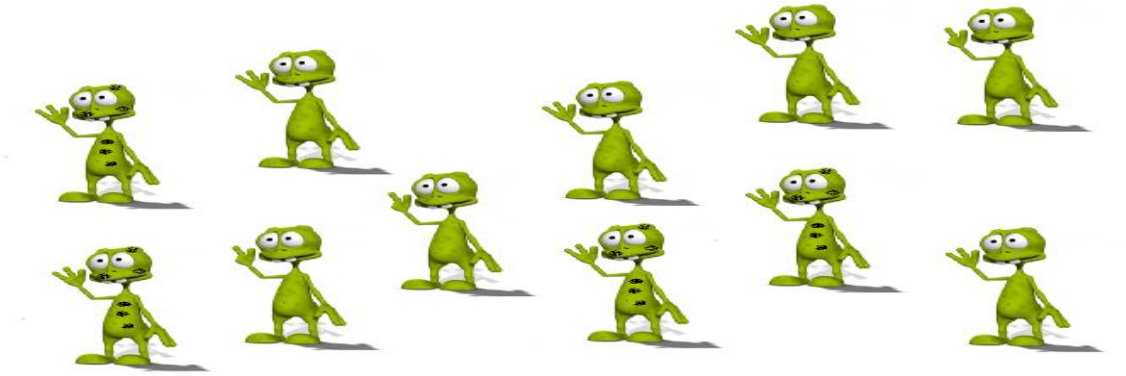
There are five subtests to match the five units. The units show a strong vertical sequence, therefore the pre-test should begin with Subtest 1 and work its way up the subtests to where the teachers' knowledge of their students means that the students have never studied this area. The post-test will consist of all the subtests.

Because of what charts and graphs are, the test items could be somewhat large and be composed of situations where data has to be organised into tables and graphs have to be constructed. They could also be items that do the reverse, provide the tables and graphs and ask questions about them to test interpretation. Therefore, it is possible for items to have many parts and to cover concepts from different units in one item.

Subtest item types

Subtest 1 items (Unit 1: Data gathering and organisation – tables and charts)

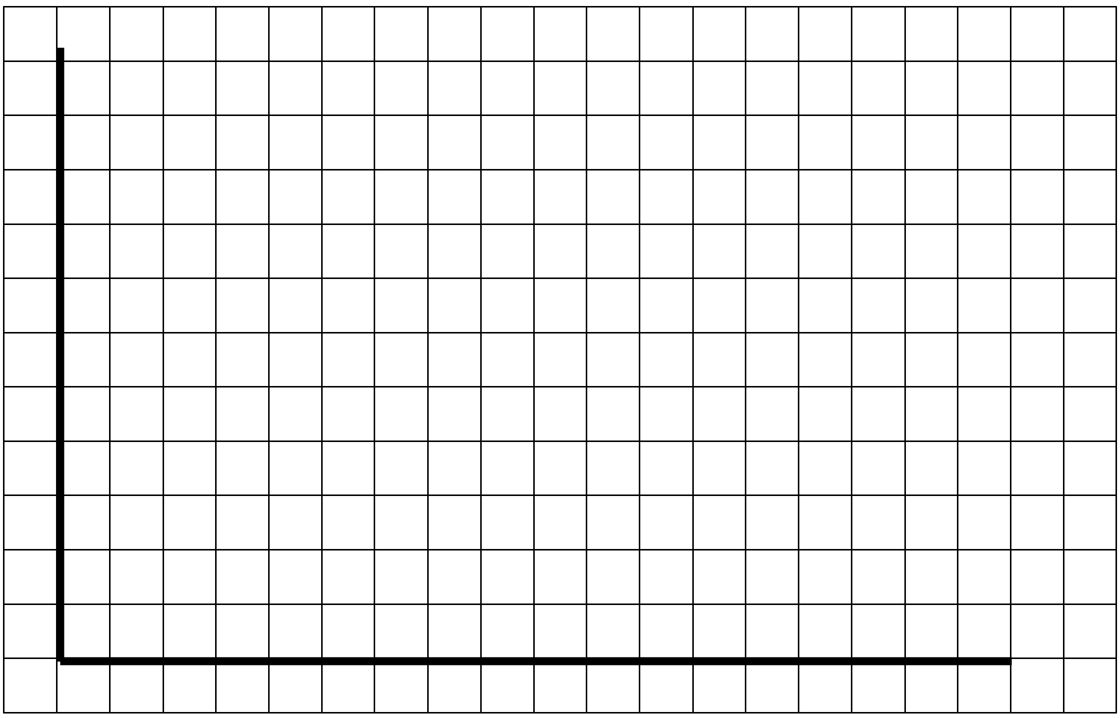
1. (a) There are three types of aliens. Look at which are the same and **put them into three groups**. (HINT: draw circles around those that are the same.)



(b) The aliens have been flying to earth each month. Their travel times are in the table below:

	January	February	March
Red space ship	4 days	3 days	4 days
Blue space ship	7 days	4 days	5 days
Green space ship	6 days	4 days	3 days

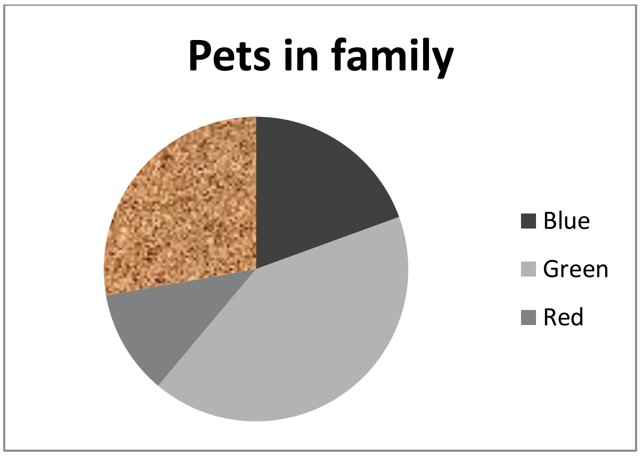
Choose 2 coloured space ships and draw a graph.



Subtest 2 items (Unit 2: Data comparison – picture and bar graphs)

1. What's wrong? Find the mistakes on the graphs.

   (a) Find two mistakes on the graph and write why they are wrong



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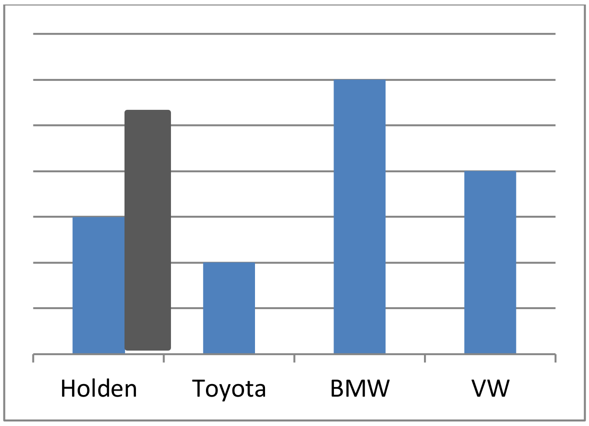
   (b) Find three mistakes on the graph and write why they are wrong



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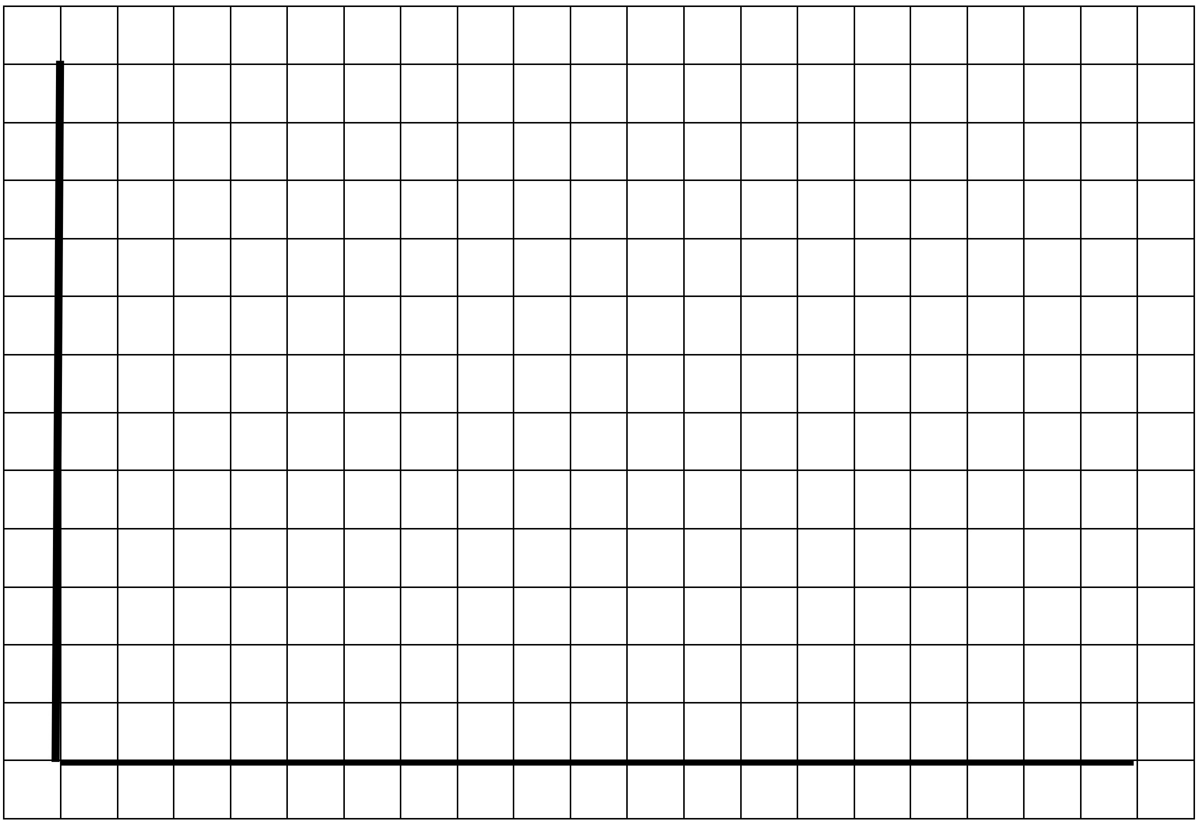
Subtest 3 items (Unit 3: Frequencies – tables, calculations, histograms, two-sided and stem-leaf graphs)

1. Football stats about tackles, passes, tries, kicks and conversions.

   (Teacher reads attached play commentary leading to a try and conversion. Teacher can read as many times as necessary, or can provide a written copy to students.)

   (a) Draw a table and record the number of tackles, passes, tries, kicks and conversions.

   (b) Draw a graph of the football data from your table.



Subtest 3: Football commentary

The ball is kicked off by Todd Carney and received by Billy Slater for Queensland, Billy passes the ball to Matthew Scott and he charges up and is tackled on the 20m line. Scott plays the ball and Cooper Cronk runs from dummy half and passes the ball to Jonathan Thurston who dummies and then passes to Cooper Cronk who is tackled on the 30m line. Cronk plays the ball and Thurston passes it to Nate Myles who charges up and is tackled at the 40m line. Myles plays the ball and Cronk at dummy half passes to Brent Tate who passes to Thurston who puts in an early kick down towards the New South Wales fullback Brett Stewart. Stewart runs forward and is tackled in the 30m line. He plays the ball and Jarrod Hayne runs off and passes the ball to Josh Morris who passes to James Tamau who charges forward to the 40m line and is tackled. James plays the ball and Carney swoops it up at dummy half and passes the ball to Robbie Farrah who passes to Greg Bird who passes to Luke Lewis who charges up to the halfway line and is tackled. A quick play of the ball and Carney kicks the ball down to Billy Slater who catches it on the full and manages to make his way to the 40m line where he is tackled. Cameron Smith picks up the play-the-ball and passes to Sam Thaiday who makes an explosive run up to the 60m line. He plays the ball and Cooper Cronk passes to Thurston who puts in a short kick which is picked up by Billy Slater who sprints off and just misses the tackle attempt by Brett Stewart to score a try! Thurston lines up the conversion from the right-hand side and kicks it through and over the post – successful.

2. **Materials**: Graph paper, calculator, pen and paper, protractor

Here is data gathered by the boys in the class for shoe sizes:

Boys	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15
Shoe size	4	9	11	6	9	7	7	10	12	8	5	6	7	8	7

(a) Draw a frequency table of shoe sizes for the boys

(b) Draw a bar graph (columns) for the frequency of the shoe sizes for the boys

(c) Draw a cumulative frequency table for shoe sizes for the boys

(d) Draw a line graph of cumulative frequency for boys' shoe sizes

Subtest 4 items (Unit 4: Trends and relationships – lines, circles and scattergrams)

1. Here is data gathered by the girls for shoe size, height and favourite colour (R-Red, B-Blue, Y-Yellow, G-Green)

Girls	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12
Shoe size	7	3	5	4	8	5	6	4	7	4	6	5
Arm length cm	65	34	38	38	63	38	54	34	57	40	50	42
Favourite colour	R	Y	Y	Y	B	R	G	G	B	Y	R	G

(a) Draw a scattergram or scatter plot of the girls' shoe size against arm length

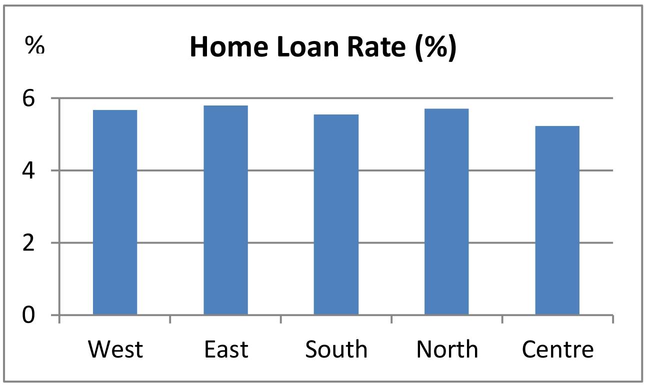
(b) Draw a circle graph of the girls' favourite colours

(c) Draw a stem and leaf of the girls' arm lengths

Subtest 5 items (Unit 5: Misrepresentation – how to lie with statistics)

1. The five banks all offered different rates for home loans:

West Bank	5.67%
East Bank	5.81%
South Bank	5.55%
North Bank	5.72%
Centre Bank	5.24%



Centre Bank wanted to prepare a brochure that emphasised that they had the lowest rates.

(a) Change the graph so that Centre Bank's rate looks much less than the others – state what you did to enable this to happen.

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(b) Is there another way the data above could be graphically presented to make Centre Bank look a lot better than the other banks? Describe it.

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Appendix A: Extra Material for Tables and Graphs

A1 Definitions, descriptions and examples of tables and graphs

Tables

The simple table (list)

This is represented by the table of contents on a cereal packet or the list of prices at a shop. It consists of words and figures in two columns. An example is:

Table 1. *Ruffy's Corn Cereal*

Average contents per serving:	
Vitamin C	25 mg
Iron	17 mg
Niacin	11 mg
Riboflavin	38 mg

Another simple table which is useful to link to probability activities is the frequency table. Frequency tables list the sample space (possible outcomes) of an event in a table with the number of combinations that lead to the outcome listed underneath. Frequency tables can be converted to a single continuous strip of boxes with each outcome coloured suitably, similar to a stacked bar graph. This can be curved in a circle and used to create early circle graphs or combined with hundreds beads to convert to percentages.

The regular table

This is the matrix style table where there are two or more columns of data. For example:

Table 2. *Materials collected by the students in 3Z*

	Dan	Joe	Fred	Sue	Anne
Milk bottle tops	5	7	4	11	2
Cotton reels	2	9	12	4	3
Orange juice bottles	5	2	6	2	11
Egg cartons	15	5	4	9	10

The irregular table

This is similar to the matrix table above but not all rows are the same (e.g. a class seating plan).

Table 3. *Seating plan of our classroom*

	Table 1	**Table 2**	**Table 3**	**Table 4**	**Table 5**	**Table 6**
A	Joe	Fred	Sue	Joan	Bill	Alex
B	Abel	____	Jill	Judy	Bob	____
C	Anita	____	Greg	Tom	Betty	Brigit
D	Derek	Ted	John			
E	Nick	Chris	Peter			
F	Frank	Fran	Greta			

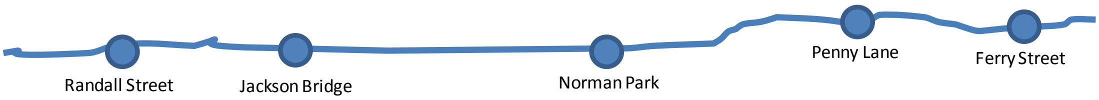
Charts

Charts attempt to display information more visually, to relate the display to what actually occurs (e.g. road maps, bus routes, timelines of history).

The strip map

This could be the bus route of an area, the major highway route of a journey or the timeline of a history topic. A line is drawn and on this line is marked references to major features (e.g. bridges, towns or happenings). Two examples are:

Bus route



My Saturday

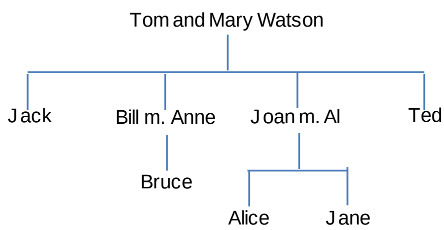
|-------------------|--------------------|-------------------|--------------------|---------

Breakfast *Shopping with Mum* *Lunch* *Play with friends* *Dinner*

The branch map

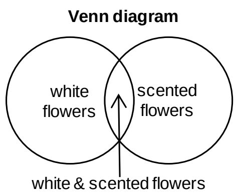
This is a combination of strip maps, involving branching as in a tree. The most straightforward examples are genealogy diagrams (family tree of parents, grandparents, etc.). The skill of following directions from a map is an important life skill in our society. An example of a branch map is:

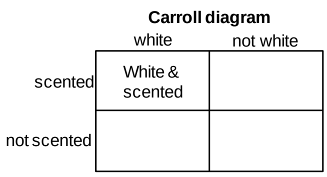
Mary Watson's family tree



Venn and Carroll or 2-way diagrams

These are visual ways to represent membership in different sets and subsets. An example is:

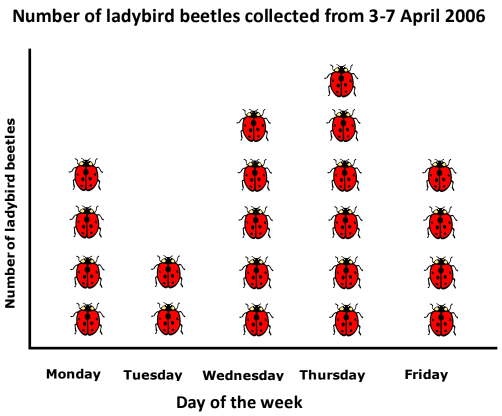


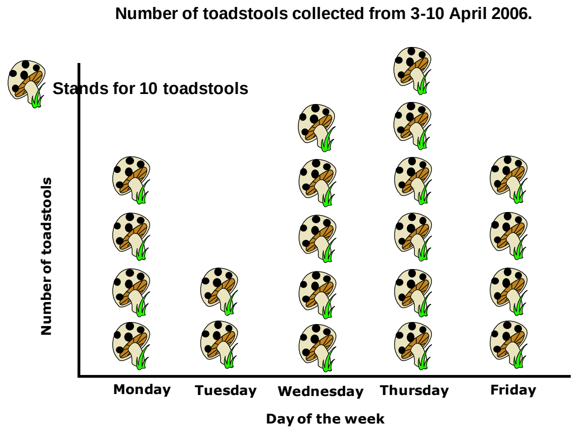


Graphs

Picture graphs (pictographs)

Picture graphs facilitate comparisons of quantities. They can represent the data with one-to-one or one-to-many correspondence. It is important to ensure that students understand the need for picture graphs to use the same size images to represent each piece of data and that pictures must be consistently spaced. These are the beginning understandings for scale and proportion and are most easily accomplished by giving students same-sized pieces of paper to create their initial graphs. Picture graphs are easily converted to bar graphs.

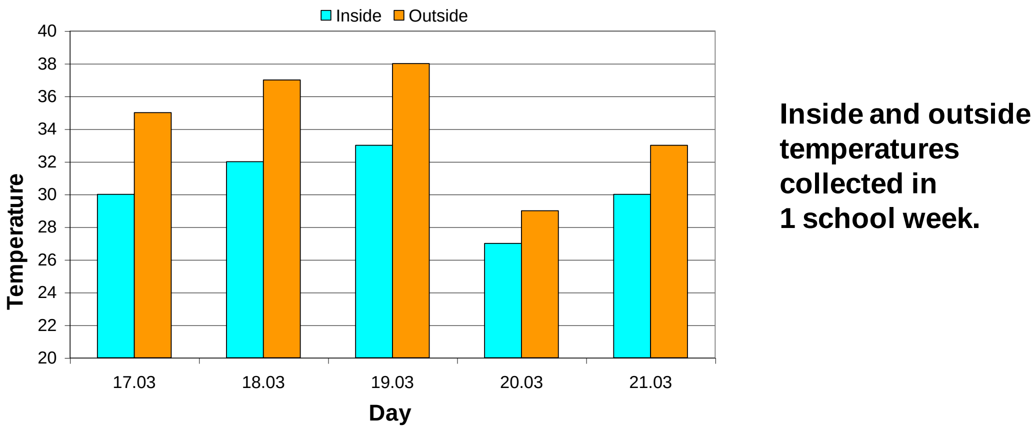




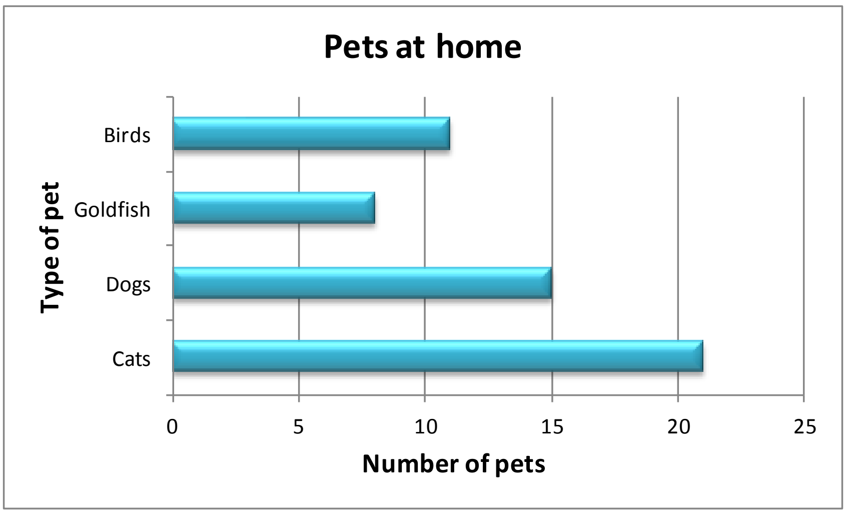
Picture graphs showing one-to-one and one-to-many correspondence

Bar graphs

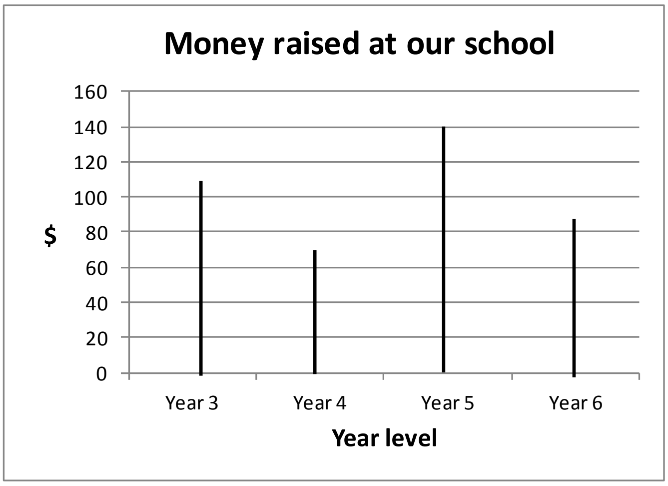
Bar graphs are easily created initially with blocks and sticky notes which can then be transferred to squared paper before working with abstractly ruled scales. Bar graphs facilitate comparisons of quantities and can be displayed with vertical bars (also called columns), horizontal bars, or bar lines. Below is an example of each.



Example 1: Vertical bar or column graph



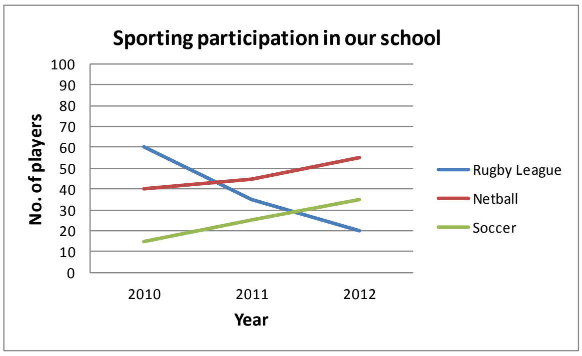
Example 2: Horizontal bar graph



Example 3: Bar line graph

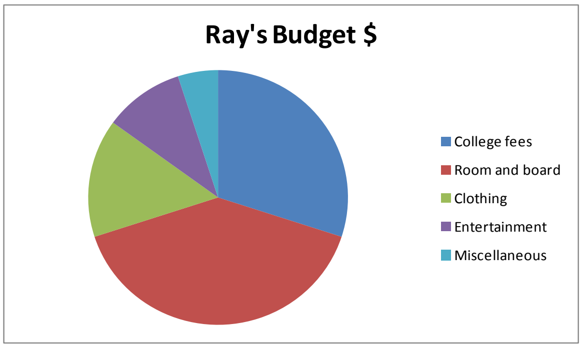
Line graphs

Line graphs can be used for comparison and for expressing allocations of resources, but they are particularly useful for communicating trends. Line graphs are also useful for representing continuous data.

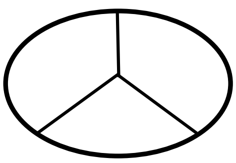


Circle graphs (pie charts)

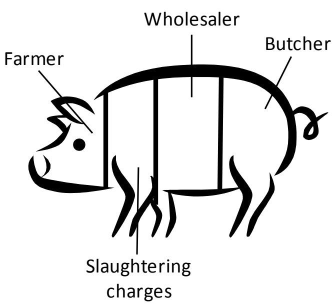
Circle graphs are used to picture the totality of a quantity and to indicate how portions of that totality are allocated. The example on right is a circle graph indicating how one college student spent his budget.



Circle graphs are in some instances made on shapes other than circles. For example:



Where the egg money goes

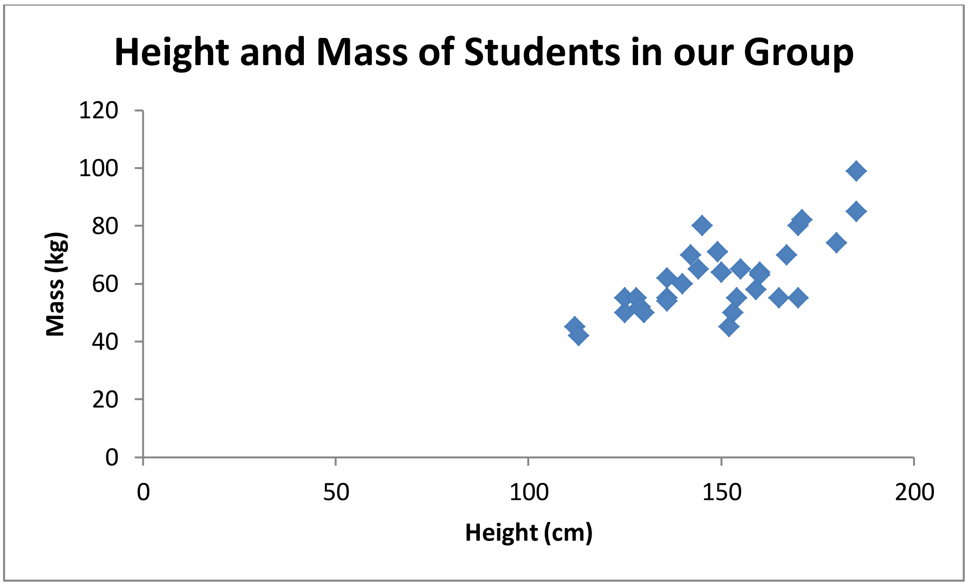


Where the pork money goes

To successfully engage with circle graphs students need to understand Part-Part-Whole relationships and that the complete shape represents the whole while the segments represent the parts of the whole. Circle graphs link very closely with fraction understandings and percents.

Scattergrams

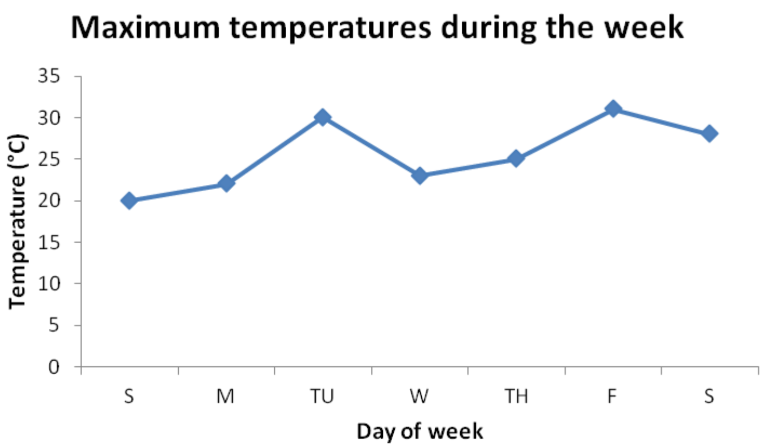
Scattergrams show relationships between two different sets of data. The scattergram is made for data which is not in sequence (in terms of the horizontal axis) and is unsuitable for a line graph. Here is a scattergram which shows that mass is related to height.



Overall features of graphs

A graph should have the following components:

1. Title
2. Vertical
   (a) Axis
   (b) Scale
   (c) Title
   (d) Units
3. Horizontal
   (a) Axis
   (b) Scale
   (c) Title
   (d) Units (if relevant)



A2 Data grouping, timetables and using tables to work out costs

A2.1 Data grouping activity

Cut out the squares. Keep the frame. Group them any way you like (be prepared to explain your grouping). Count the number in each group. Repeat above, gathering your own data using the blanks on the next page – compare the two sets of data.

Data for activity

Person 1 Gender: F Eye colour: Blue L/R handed: R	Person 2 Gender: F Eye colour: Brown L/R handed: R	Person 3 Gender: M Eye colour: Brown L/R handed: R	Person 4 Gender: F Eye colour: Green L/R handed: R
Person 5 Gender: M Eye colour: Blue L/R handed: R	Person 6 Gender: F Eye colour: Brown L/R handed: R	Person 7 Gender: F Eye colour: Green L/R handed: R	Person 8 Gender: F Eye colour: Blue L/R handed: R
Person 9 Gender: F Eye colour: Brown L/R handed: R	Person 10 Gender: M Eye colour: Brown L/R handed: R	Person 11 Gender: F Eye colour: Green L/R handed: R	Person 12 Gender: M Eye colour: Green L/R handed: R
Person 13 Gender: F Eye colour: Blue L/R handed: L	Person 14 Gender: F Eye colour: Brown L/R handed: R	Person 15 Gender: F Eye colour: Green L/R handed: R	Person 16 Gender: F Eye colour: Blue L/R handed: L
Person 17 Gender: F Eye colour: Brown L/R handed: L	Person 18 Gender: F Eye colour: Blue L/R handed: R	Person 19 Gender: M Eye colour: Blue L/R handed: R	Person 20 Gender: M Eye colour: Brown L/R handed: L

Blank cards for data gathering

Person 1 Gender: Eye colour: L/R Handed:	Person 2 Gender: Eye colour: L/R Handed:	Person 3 Gender: Eye colour: L/R Handed:	Person 4 Gender: Eye colour: L/R Handed:
Person 5 Gender: Eye colour: L/R Handed:	Person 6 Gender: Eye colour: L/R Handed:	Person 7 Gender: Eye colour: L/R Handed:	Person 8 Gender: Eye colour: L/R Handed:
Person 9 Gender: Eye colour: L/R Handed:	Person 10 Gender: Eye colour: L/R Handed:	Person 11 Gender: Eye colour: L/R Handed:	Person 12 Gender: Eye colour: L/R Handed:
Person 13 Gender: Eye colour: L/R Handed:	Person 14 Gender: Eye colour: L/R Handed:	Person 15 Gender: Eye colour: L/R Handed:	Person 16 Gender: Eye colour: L/R Handed:
Person 17 Gender: Eye colour: L/R Handed:	Person 18 Gender: Eye colour: L/R Handed:	Person 19 Gender: Eye colour: L/R Handed:	Person 20 Gender: Eye colour: L/R Handed:
Person 21 Gender: Eye colour: L/R Handed:	Person 22 Gender: Eye colour: L/R Handed:	Person 23 Gender: Eye colour: L/R Handed:	Person 24 Gender: Eye colour: L/R Handed:

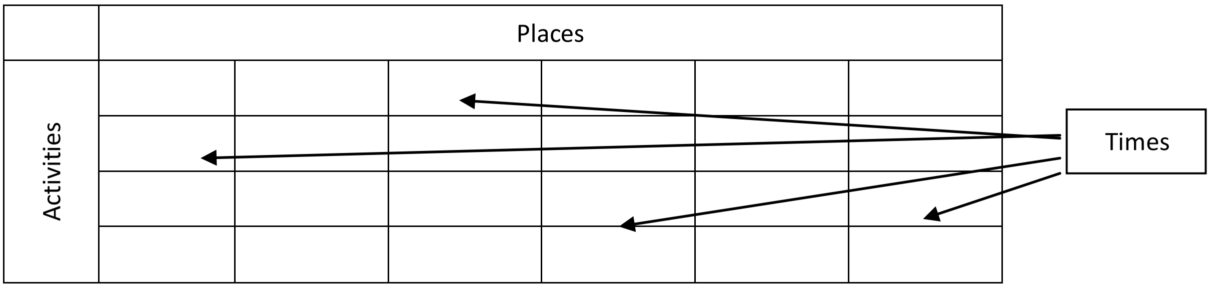
A2.2 Timetable activity

Constructing a timetable

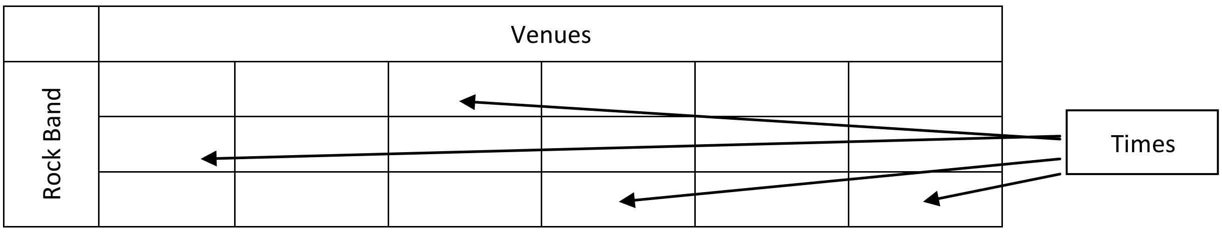
The best way to understand timetables is to construct one. For this, you have to consider:

1. **Activities** – this is the focus of the timetable, for example, trains, buses, classes in a school, appointments. These are usually down the side of the table.
2. **Places** – this is where the activities occur or may stop, for example, trains stop at stations; appointments occur in numbered surgery rooms. These are usually across the top of the table (but may not be, sometimes timetables mix things up).
3. **Times** – these are what are in the squares of the table – the times at which the activities occur at a certain place.

Thus a timetable is as follows:



To teach how to construct a timetable, think of a timetable situation. Try to choose something different. For example, when bands are playing at different venues, get students to create their own personal weekly or monthly "gig guide".



Four ways to read a timetable

Consider the following timetable for trains – columns are stations, rows are trains, and cells are times.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1	9:16 am	9:47 am	10:23 am	11:26 am	11:58 am	12:22 pm
2	9:45		10:43 am		12:06 pm	12:42 pm
3	10:07	10:39 am		12:09 pm		1:35 pm
4	10:54	11:28 am	12:03 pm	1:01 pm	1:42 pm	2:06 pm

Way 1: Finding information in a cell (here, finding a time) – to do this, look across → and down ↓. For example: to find when train 2 gets to Karlin, we start at train 2 and move across to Karlin's column. Then circle what time appears in that box – 10:43 am.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2			10:43 am			
3						
4						

Way 2: Finding which row to use (here, finding a train) – to do this, look for the station and then go down for the time and across for the train. For example: if you need to get to Nanty just before 12:30 pm, you start at Nanty, and look down until you get to 12:06 pm, then across to the left for the train. Then you can circle which train number is needed.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2					12:06 pm	
3						
4						

Way 3: Finding which column to use (here, finding a station) – to do this, find the train required, go across to the time and look up for the station. For example, to find which station you would have to get off from train 3 that was just after 12.00 pm, we start at train 3 and move across to a train near 12:00 pm, then move up to the station. Then circle the station you need to get off at.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2						
3				12:09 pm		
4						

Way 4: Finding things that have no answers: (here some cells are blank) – to do this, you have to understand that sometimes tables have blank spaces because, here, the train does not travel to a certain station. For example, train 2 does not stop at Jackin. To find these things out, look for gaps and missing spaces.

A2.3 Using tables to work out costs

Setting up tables

To show how lists can become tables, we will look at an example, *What would be the cost of clothing a man for a 4-day trip?* There are four steps as follows.

1. **Develop a list**. One way to do this is to consider the clothing needed in terms of parts of the body and then different activities; for example, a list could be:

   Shoes, Socks, Underwear, Pants, Shirts, Coat/jumper, Hat, Pyjamas

2. **Consider all needed for table**. One way to do this is to think of one item and what is needed to work out cost (e.g. the number of items, where to buy it, the cost and so on). Because of the nature of the task of buying clothes, there is also a need to consider any discounts.

   Item, Place to buy it (shop), Number of items, Cost of each item, Any reductions, Totals

3. **Translate to a table**. An effective way to do this is to put the needs (Step 2) across the top and the list of all items (Step 1) down the left-hand side of the table.

Shop	Item	Number	Cost/item	Any reduction	Totals
Athlete's Foot	Shoes				
Target	Socks				
Target	Underwear				
Jeans West	Pants				
Chaps Menswear	Shirts				
Chaps Menswear	Coat/jumper				
Target	Hat				
Target	Pyjamas				
OVERALL TOTAL					

4. **Determine numbers**. The next step is to complete the table by filling in the gaps – determining the number of items, the shop, and the cost/item (plus any reductions).

Shop	Item	Number	Cost/item	Any reduction	Totals
Athlete's Foot	Shoes	1			
Target	Socks	4			
Target	Underwear	4			
Jeans West	Pants	2			
Chaps Menswear	Shirts	4			
Chaps Menswear	Coat/jumper	1			
Target	Hat	1			
Target	Pyjamas	1			
OVERALL TOTAL					

Calculating with tables (budgets)

Once the table has been set up, then operations can be used to determine totals (budgets) as follows.

1. **Select appropriate operations**. Selecting the right operations within a table is easier than in real-life situations, but still needs to be accurate. In general, the rules are: (a) addition for totals and overall totals, (b) subtraction for any reductions (discounts), and (c) multiplication of number × cost or number × reduced cost for total in each row. Of course, if we have to work backwards (e.g. find the cost per item when we know the cost of four items), the operations can invert (e.g. from multiplication to division).
2. **Determine form of totalling**. In a table, you can find the overall total at the end or you can total cumulatively – keep a running total that advances at each row.
3. **Complete the operations**. Here are examples for overall total and cumulative total based on the cost of clothing for a 4-day trip for a man.

Overall total:

Shop	Item	Number	Cost/item	Any reduction	Total
Athlete's foot	Shoes	1 pair	$160.00	$32.00	$128.00
Target	Socks	4 pairs	$6.90		$27.60
Target	Underwear	4	$3.50	$0.30	$12.80
OVERALL TOTAL					$168.40

Subtraction $160.00–$32.00

Multiplication 4 × $6.90

Subtraction $3.50–$0.30=$3.20 Multiplication 4 × $3.20

Addition of the column

Cumulative total:

Shop	Item	Number	Cost/item	Any reduction	Cumulative total
Athlete's foot	Shoes	1 pair	$160.00	$32.00	$128.00
Target	Socks	4 pairs	$6.90		$155.60
Target	Underwear	4	$3.50	$0.30	$168.40

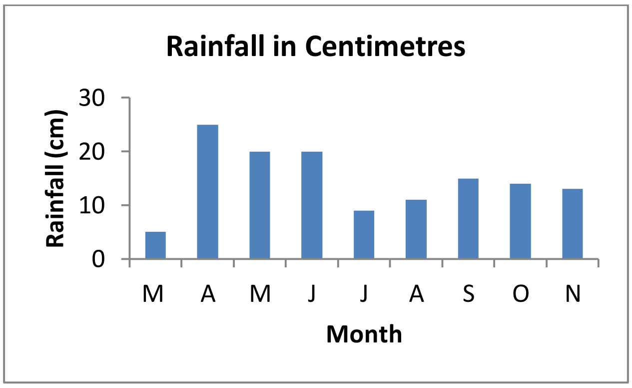
Adding $128 to total of row ($27.60)

Adding $155.60 (the amount in the line above) to the total of the row ($12.80)

A3 Reading and interpreting graphs

For each of the following graphs, answer the questions listed below them.

1. **Bar Graph:** Students in a Year 4 class constructed the following rainfall graph for one school year.



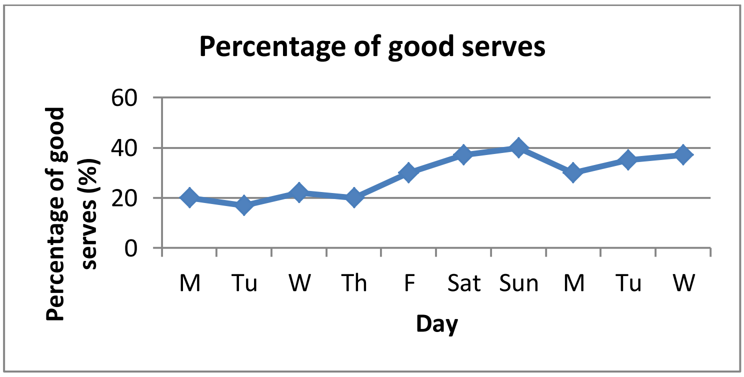
   (a) Which month had the most rain?

   (b) What were the approximate amounts of rainfall in March, April and May?

   (c) What part of the school year would you call the rainy season?

   (d) About what percentage of the total rainfall fell in November?

2. **Line Graph**: As part of a program to improve her serve in tennis (where she had a tendency to double fault), Mary decided to make a graph of her daily serving success. Below are the results for 10 days.



   (a) Is Mary improving?

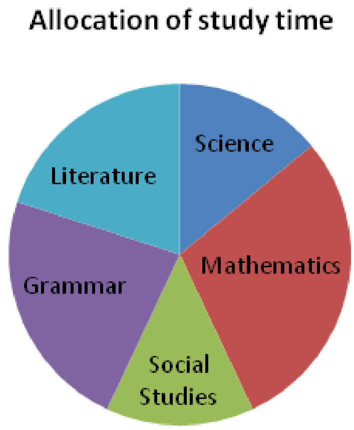
   (b) Which was Mary's best serving day? Her worst?

   (c) Approximately what percentage of her total serves over the 10 days were good (Be careful!)

   (d) Complete:

Day	M	T	W	Th	F	Sat	Sun	M	Tu	W
% of good serves										

3. **Circle graph**: A Year 8 class had lessons in science, social studies, mathematics and English (split by the teacher into literature and grammar). The students were upset about the length of the mathematics assignments and presented the following graph to their mathematics teacher to convince her she was using an unfair share of their study time.



(a) The alert mathematics teacher told the class that they were complaining to the wrong teacher. Who did she have in mind?

(b) About what percentage of study time did the class feel they spent on each of their four subjects?

(c) What special skills are involved in reading a circle graph that are not required for the previous types of graphs?

4. **Picture graph**: A Year 2 class wanted to keep track of the number of dry and rainy days during a week, so they used the following method of representing the data, using cut out faces.

Rainy Days

RAINY	☹ ☹ ☹ ☹
DRY	☺ ☺ ☺

(a) What advantages can you see in using a picture graph in this situation instead of a bar graph?

(b) An upper primary class may use a picture graph for rainy and dry days over half a year as follows:

Rainy Days

RAINY	✗✗✗✗✗✗✗✗✗
DRY	✓✓✓✓✓✓✓✓✓✓✓✓✓✓✓

(each ✗ and ✓ represents 5 days)

What is the extra difficulty here for students reading the graph? For what questions concerning the graph will errors mostly occur?

(c) Which of the following kinds of data lend themselves to picture graphs?

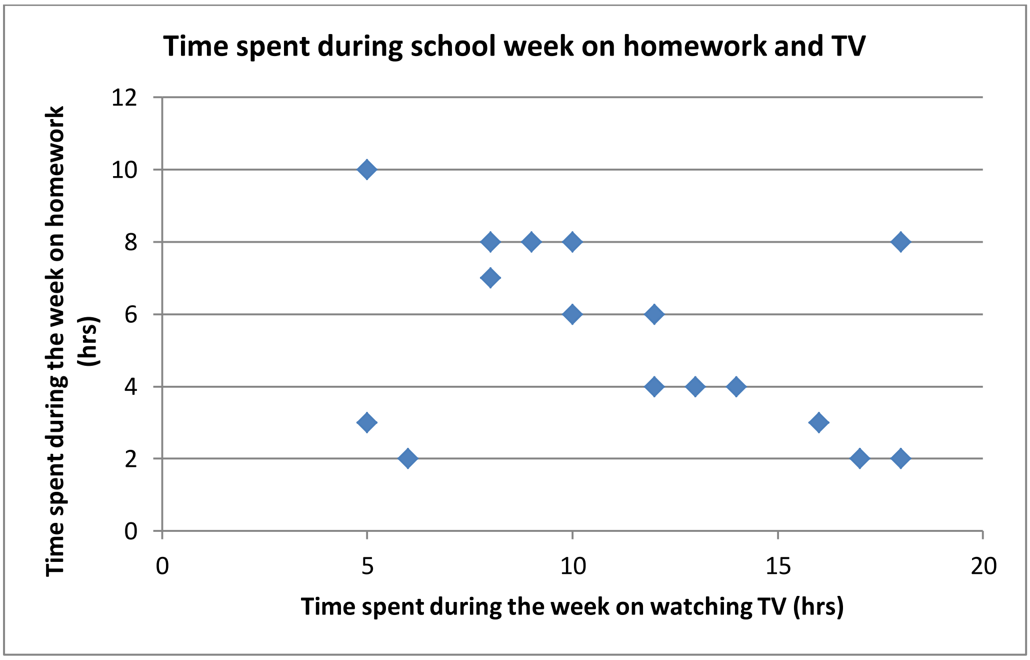
i) Maximum temperature on each day of the week

ii) The number of students who ride bikes, walk or are driven to school

iii) The relative numbers of science, mathematics and social studies books in the library

iv) The number of recyclable cans collected by each child in the class.

5. **Scattergram**: To show the relation between time they spent on homework and time they spent watching TV, a class made a scattergram as follows:



(a) Is it true that students who watch a lot of TV do little homework (and vice versa)? That is, is TV watching inversely proportional to homework time or is there no relation?

(b) Are there any students who do not fit the pattern? Describe them.

(c) Which of the following kinds of data lend themselves to scattergrams?

i) Length of bean plants for each day of the week

ii) Length of foot and circumference of head

iii) The sunset hours of each day of the week

iv) Circumference and diameter of circles.

What is particular about data that is amenable to scattergrams?

A4 Teaching the principles of good picture graphs through making it unfair

1. Construct a stage 2 / stage 3 bar graph as described below.

Type of Pet	Number of students with that pet
Dog	9
Cat	7
Fish	3
Bird	5

2. Construct a poster framework as below:

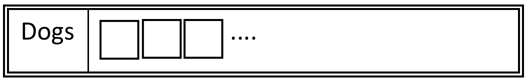
PETS WE HAVE	
Dogs	
Cats	
Fish	
Birds	

3. Cut out 24 rectangles of paper of the same height (a height to fit in the rows of your poster) but of the length so that 17 are squares and 7 are rectangles where the length is 1.5 times this height.

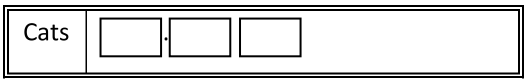
4. Represent the data on the pets using these squares/rectangles of paper (by placing a "D" on 9 squares for the dogs, an "F" on 3 squares for the fish, a "B" on 5 squares for the birds and a "C" on the 7 rectangles for the cats).

5. Paste the squares/rectangles appropriately on the poster as follows:

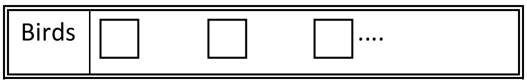
   (a) Place the dogs end to end neatly, starting on the left of the poster, e.g.



   (b) Place the cats also end to end neatly, starting on the left of the poster, e.g.



   (c) Place the birds end to end with a gap between (still starting from the left) so that they extend to the right further than the dogs, e.g.



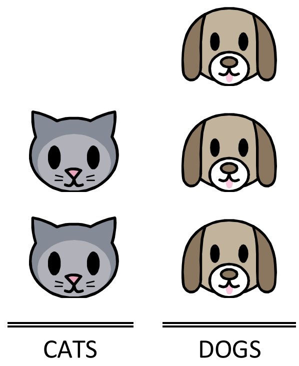
   (d) Place the fish end to end neatly but starting sufficiently to the right that it extends longer than the dogs, e.g.

6. Discuss the fairness of what has happened. What rules should these types of graphs follow to be fair? [Same size squares, arranged evenly, starting from LHS – all like the Dogs]

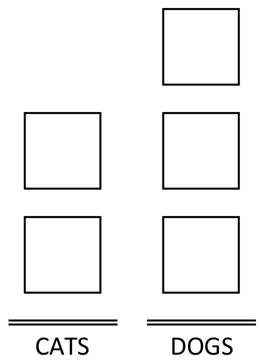
A5 Transitions: Picture → bar graphs and Bar → line graphs

A5.1 Picture graphs to bar graphs

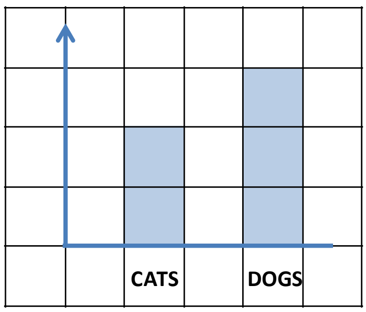
Simple picture graphs where one symbol represents one item, e.g.



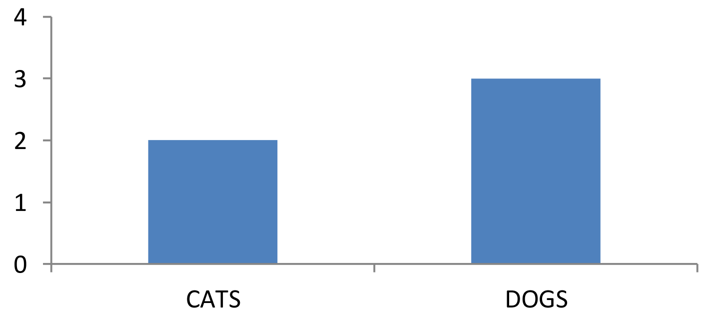
Can have their symbols replaced by squares, e.g.



Then we can replace constructing rows or columns of such squares with the shading in of squares on graph paper, e.g.

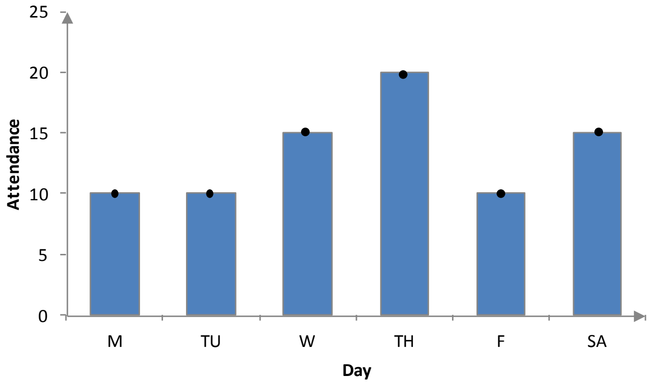


And, finally, remove the graph paper and just use bars, e.g.

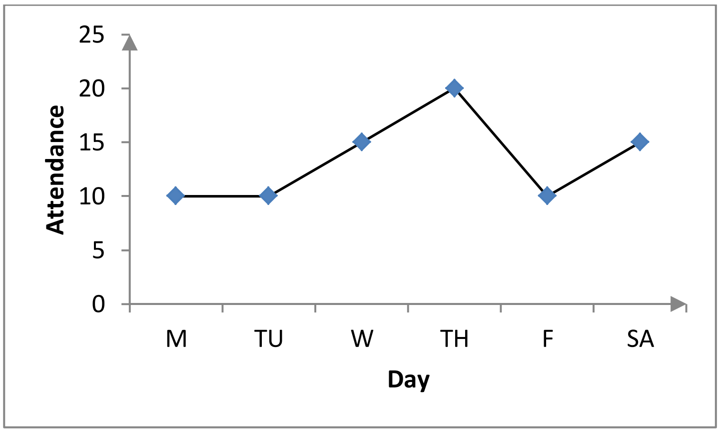


A5.2 Bar graphs to line graphs

For bar graphs with an appropriate horizontal axis, the bars may be replaced by dots on the top centre of each bar, e.g.



And the dots can then be joined by a line to make a line graph, e.g.



Once ability in plotting points has been developed for line graphs, it can be transferred to scattergrams.

A6 Constructing line, circle and scattergrams, and constructing histograms

A6.1 Line and circle graphs and scattergrams

1. Complete the following:

   Step 1 – using graph paper make a bar graph out of the following data:

Basic Facts Drill	Number correct
Monday	29
Tuesday	27
Wednesday	31
Thursday	38
Friday	49

   Step 2 – place a dot at the top centre of each bar and join with a line.

   Step 3 – repeat step 2 on a new piece of graph paper without drawing the bar graph.

2. Construct a series of questions that you would ask students to achieve steps 2 and 3 above.

3. Construct a circle graph or pie chart for the following data:

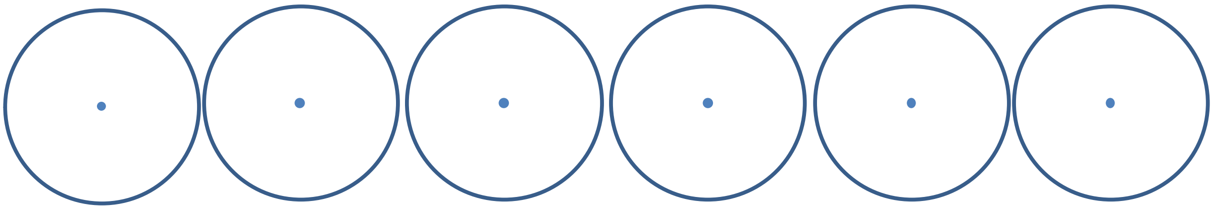
Economic Sector	Percentage of Export
Farming	43
Mining	35
Manufacturing	22

   Draw a circle. Convert the percentages to an angle out of 360 – multiply by 360 and divide by 100. Use a protractor to mark in the sectors.

4. Complete the following table:

Percentage	Fraction (common)	Sector angle
25	1/4	90
45		
	3/8	
		120
	2/5	
62		
		80

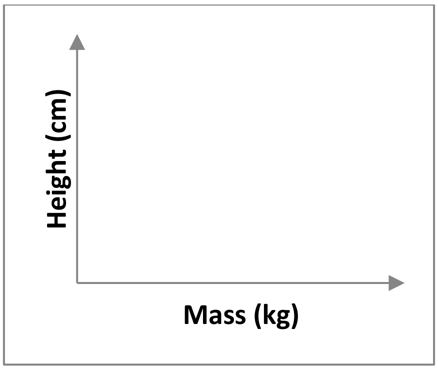
   Draw each sector angle (except the 90°) on the circles below:



5. List the prerequisite knowledge of mathematical concepts and processes needed by a student before they could undertake the construction of a circle graph.

6. Measure the height and mass of each student in the class.

7. Draw up axes on graph paper.



8. Take the mass of the first student and draw a pencil line with a ruler vertically through the point where this mass is on the horizontal axis. Take the height of this first student and draw a pencil line horizontally through the point where this height is on the vertical axis. Mark where the two lines cross with a bold point.

9. Mark a point, using the technique in (8) above for the mass and height of every student.

10. Study the resulting scattergram. Is it true that, as a general rule, taller students tend to be heavier and vice versa?

A6.2 Constructing histograms

1. Obtain the height information from measuring heights to make a bar graph. List the highest and lowest height. Subtract these two values to get the **range** of the heights.

2. Using this range, determine an interval size that divides the heights into approximately seven sections or intervals. For example, a list of heights from 121 cm to 165 cm (a range of 44 cm) could be divided into seven 7 cm intervals (it is important to have the interval length an **odd number** so that there is a central number to define each bar), as follows:

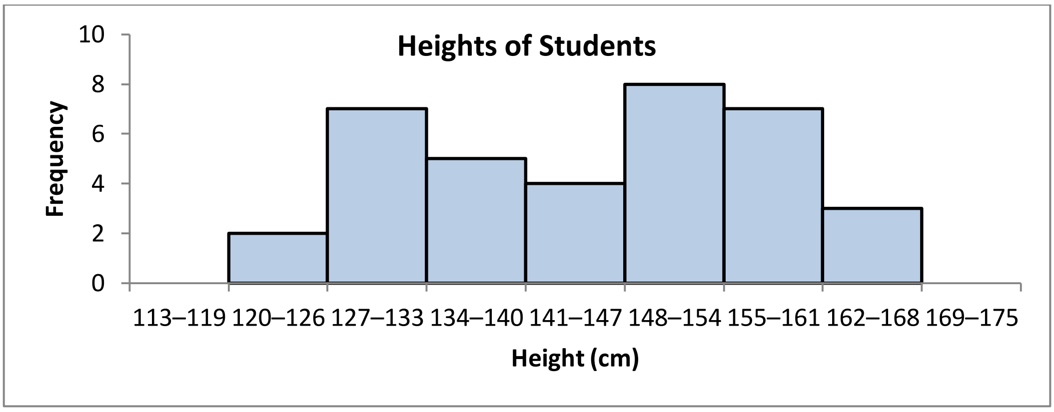
   120 – 126
   127 – 133
   134 – 140
   141 – 147
   148 – 154
   155 – 161
   162 – 168

3. Add the previous and following intervals to this list. For example 113 – 119 and 169 – 175 would be added to the above list. This is so that the ends of the range of data can be shown by the zero values.

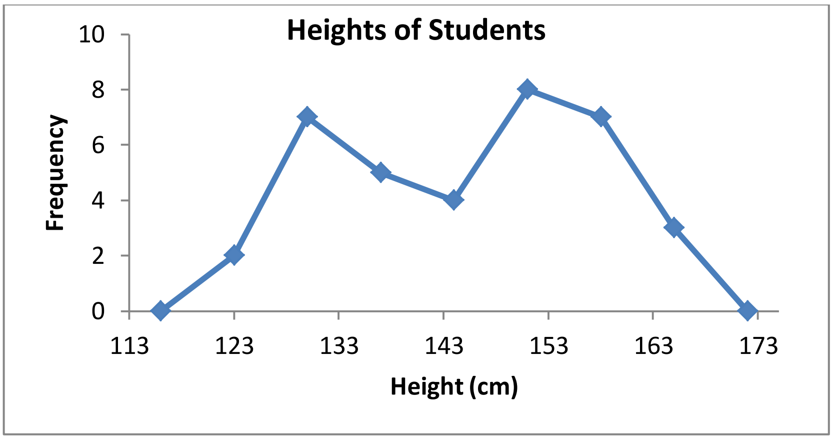
4. Make up a table as below and tally into each section the heights that are present in that interval. For example, see the possible tally below:

INTERVAL	TALLY	FREQUENCY
113 – 119		0
120 – 126	II	2
127 – 133	~~IIII~~ II	7
134 – 140	~~IIII~~	5
141 – 147	IIII	4
148 – 154	~~IIII~~ III	8
155 – 161	~~IIII~~ II	7
162 – 168	III	3
169 – 175		0

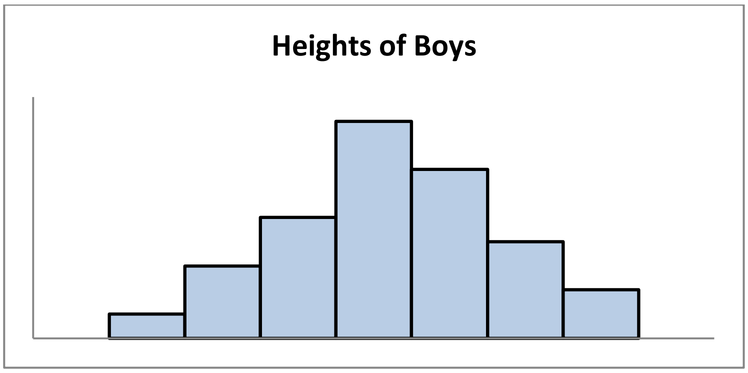
5. Use this data to draw a bar graph, as the example below shows.



6. Convert the bar graph to a line graph (remember the zeros at the end) as the example shows. This graph is called a polygon.



7. The example above is a common bar graph for coeducational classes. A bar graph of heights for boys only or girls only is as below.



Why is there a difference?

A7 Determining best graphs for five data sets

Below are five sets of data. For each set of data construct a bar graph, line graph, circle graph, picture graph or scattergram to represent the data. Choose a different type of graph for each set of data (basing your choice on the appropriateness of the type of graph for the data).

1.

Student	**Books read so far this year**
Sally	1
Mary	7
Bill	4
Don	3
Joe	2

2.

Place of residence	**Fraction of year spent**
Home	2/3
Hotels	1/2
Cottage	1/6
Grandmother's	1/12

3.

Age in months	**Height in cms**
26	71
47	93
71	92
78	120
34	79
54	97
63	105

4.

Spelling test	**Johnny's score**
1	50
2	90
3	33
4	79
5	97

5.

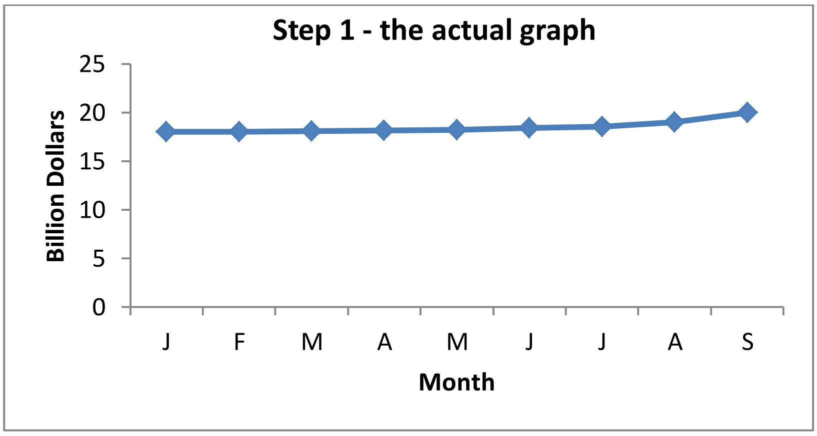
Diameter of circle (cm)	**Circumference of circle (cm)**
1	3.1
1.5	4.7
3.3	10.4
4	12.6

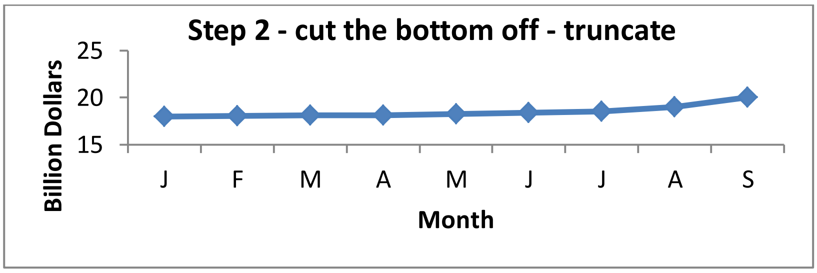
A8 How to lie with graphs

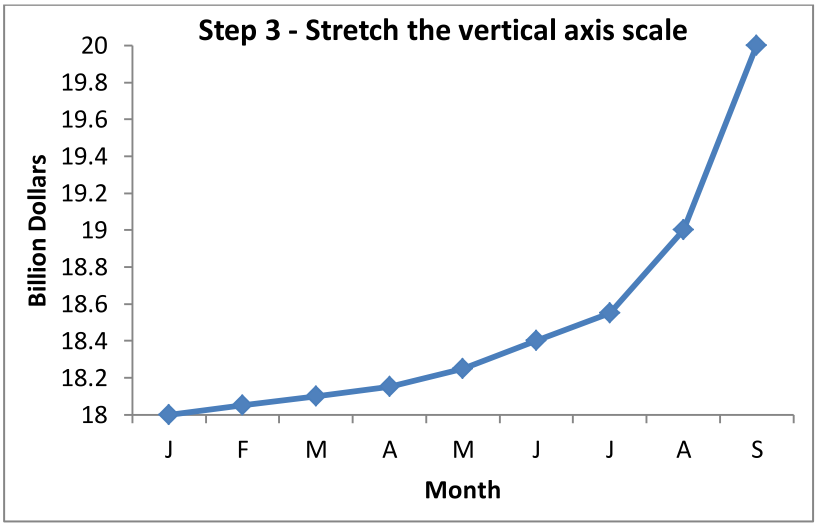
One of the best ways to "lie" or "misrepresent" with data is with graphs. The vertical axis scale can be reduced to increase the slope of the graph. Graphs can be truncated (this suppression of the zero is a powerful tool in misinformation). Scales need not be given. The graph can only show the top of the scale to accentuate differences which are small in comparison to the numbers being considered.

"Trend" lines can be drawn though the scattered sets of points in the scattergram. Over-confident extrapolation which extends the graph well beyond factual content can be full of errors and very misleading if no indication (e.g. a dotted line) is given that it is being done.

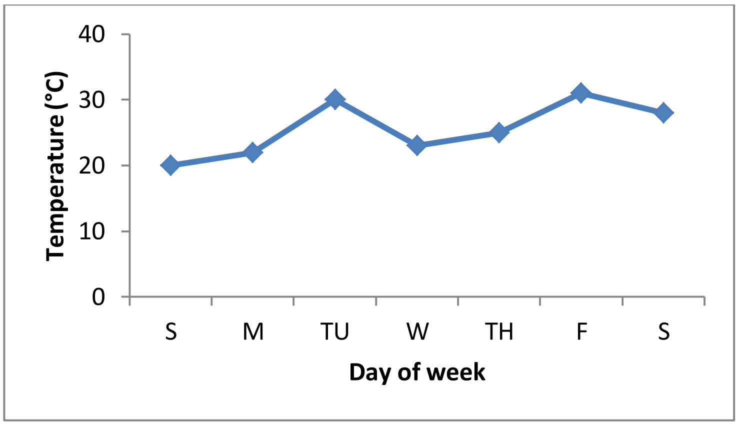
Here is an example of how to make a 10% increase in the gross national product more impressive – in three steps (idea from Darrell Huff, *How to Lie with Statistics,* Penguin).

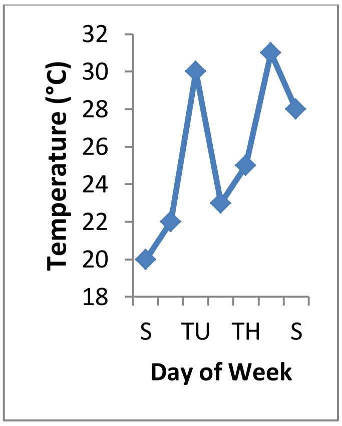






1. Look at these two graphs representing the same data. Do they convey the same message?





2. If you wanted to emphasise the variability of the weather, which graph would you use? What does the other graph emphasise?

3. What changes have occurred in the vertical and horizontal axes and scales to bring about these changes in impression?

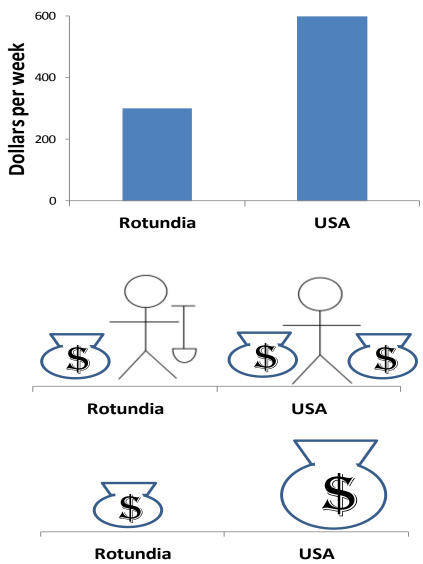
4. Use the following data on government spending in the second half of the year to draw line graphs which emphasis that:

   (a) Government spending has increased; and

   (b) Government spending has remained steady.

Month	Spending ($m)
July	195
August	193
September	196
October	194
November	197
December	202

5. Picture graphs and drawing-type bar graphs (pictograms) can be very misleading. For example, Darrell Huff (*How to Lie with Statistics* (Penguin)) has the following three ways to show that the workers of Rotundia earn half as much as the workers of USA.



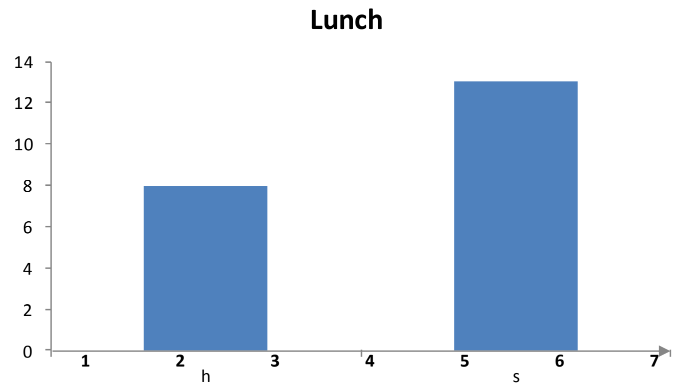
The drawings in the last picture have the USA bag of money being twice as high as the Rotundia bag of money. But the USA bag is also twice as wide and appears twice as thick. Hence the impression is left with the reader that the USA wages are eight times more than the Rotundia wages.

A9 Finding better graphical representations that do not misrepresent

A9.1 Listing and redrawing to removing inaccuracies

Information

The following graph, purporting to show students who eat lunch at home and at school, has many inaccuracies.

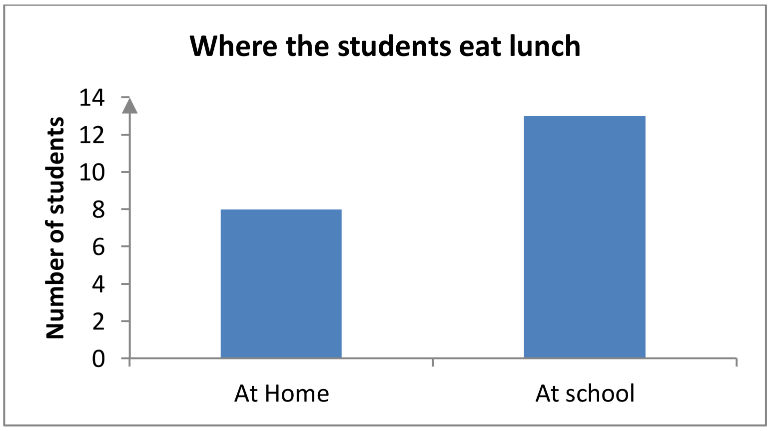


(a) There are no titles giving meaning to the horizontal and vertical axes. There is no indication of what the “h” and “s” stand for.

(b) There should be an arrow on the vertical axis (showing that the numbers continue) but not on the horizontal axis.

(c) The bars should be evenly spaced.

This would be a better presentation:



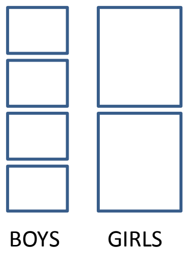
Activity

For the following graphs:

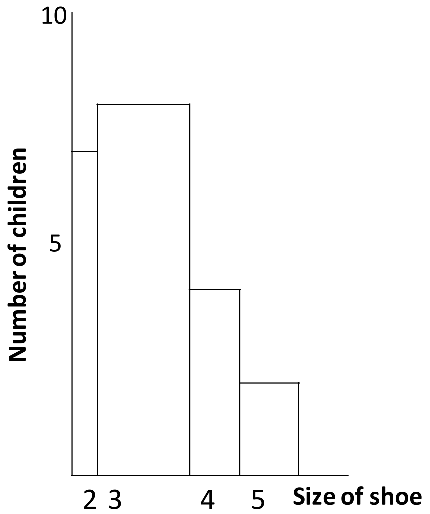
(a) list the inaccuracies; and

(b) redraw the graph "correctly" or in a better presentation.

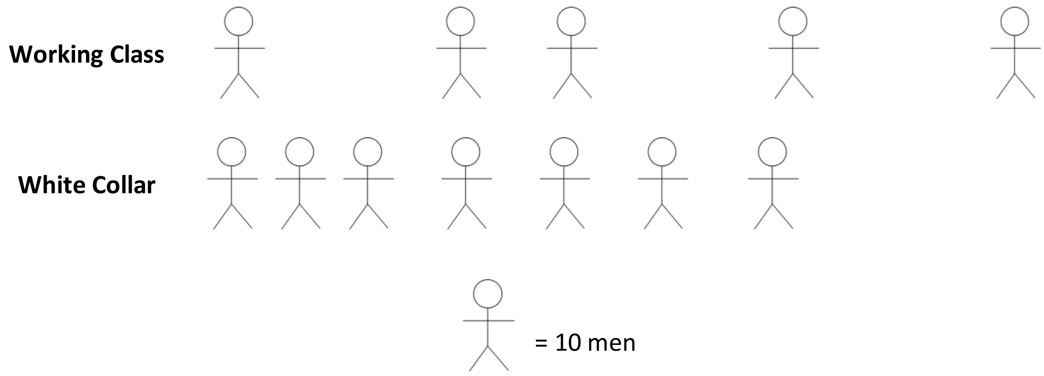
1. Students going to camp



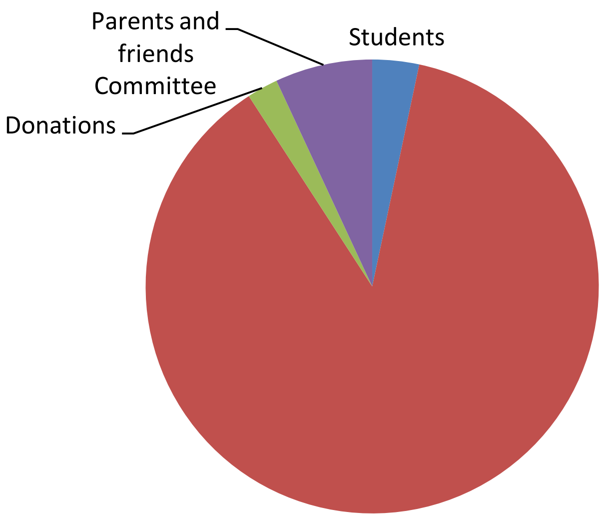
2. The shoe size students have



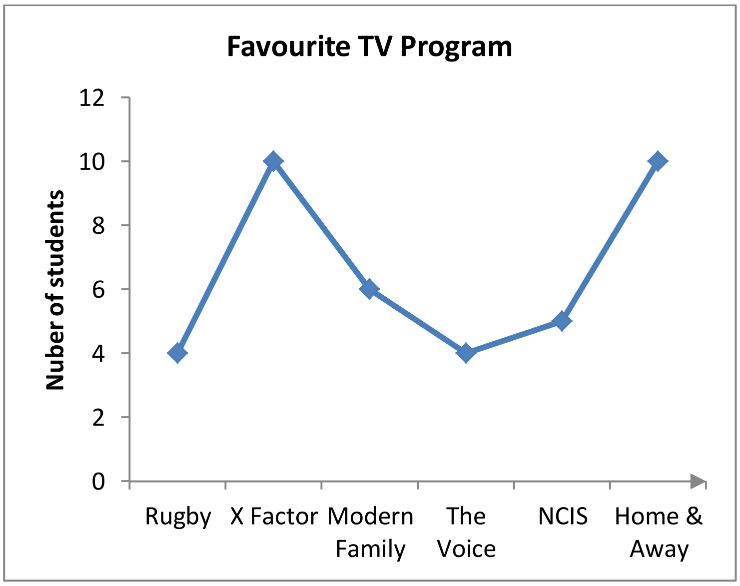
3. Number of workers



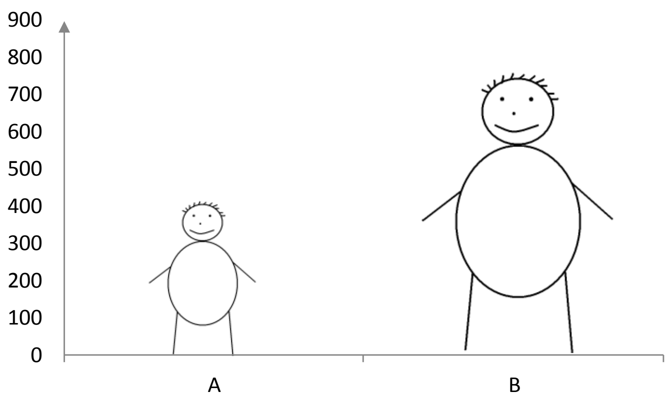
4. School revenue



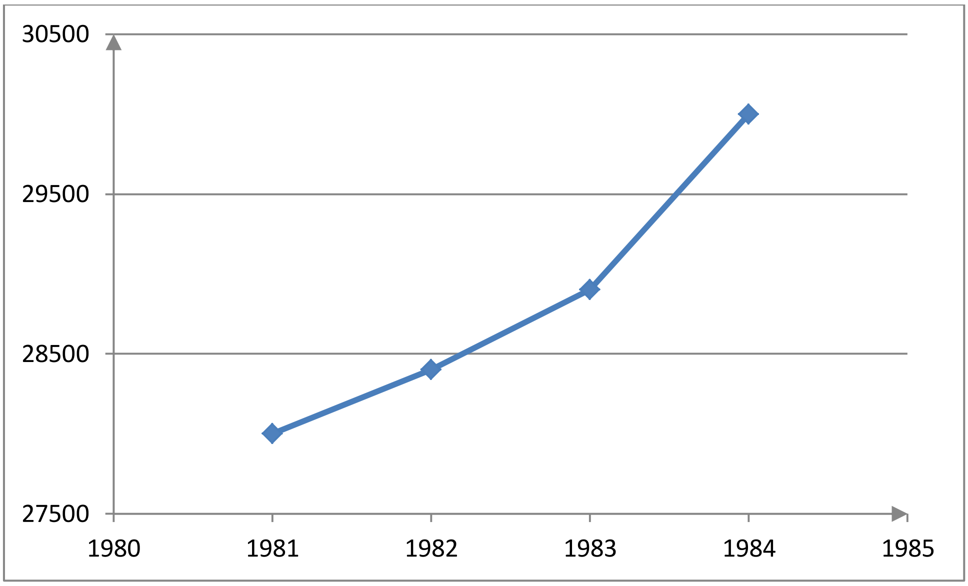
5. Students' favourite TV programs



6. Comparing size of schools

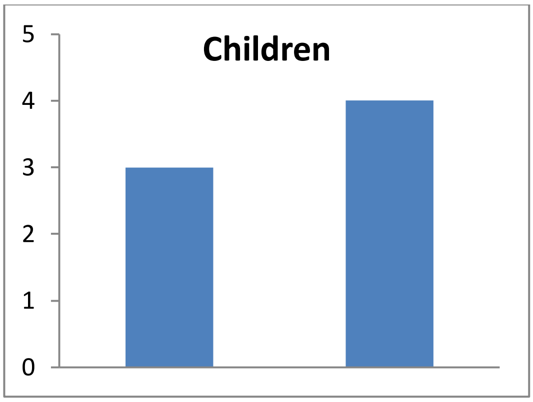


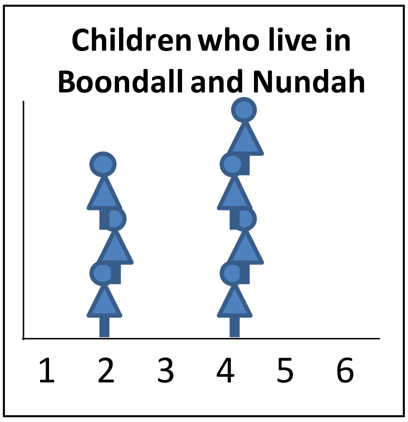
7. Salary changes

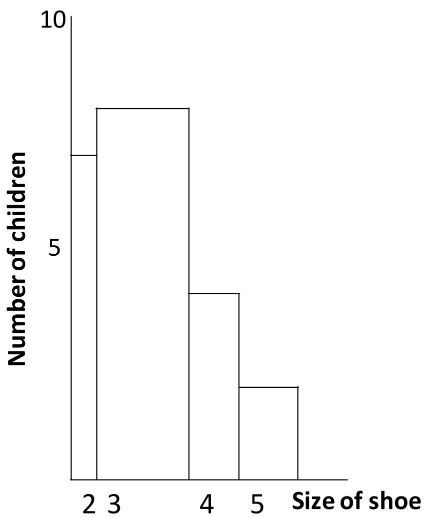


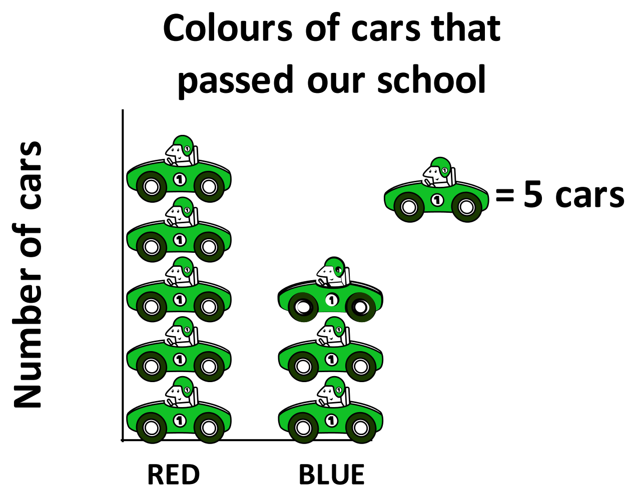
A9.2 Redraw graphs so they are clearer to understand!

Please redo the following so they are clearer to understand.









A9.3 Reflection activity

Look at graphs that have been redone and fill in the following table:

	Features of Graphs	
	Particularly good for:	**Not particularly good for:**
Bar graph		
Line graph		
Picture graph		
Circle graph		
Scattergram		

A10 Investigation involving graphing

1. **Choose a question or position**

   Think of a question or position and devise a data collection experiment to shed light on the question or to support or refute the position. Use your imagination: almost any data collection experiment would be fine – check with your teacher.

   Examples:

   - Take a poll of attitudes towards a political issue
   - Time a particular activity
   - Analyse writing or speech for frequency of use of various parts.

2. **Collect data to answer the question or support the position**

   Set up experiments, surveys, etc. from which you can collect your data (you may have to devise a convenient table or chart for collecting it).

3. **Decide on a message contained in the data**

   Organise your data and look for patterns and trends that support a point of view, position or message.

4. **Choose a type of graph and a scale that will easily and clearly communicate the message**

   Decide what types of graphs (bar, line, circle, picture graph or scattergram) are most suitable to present your data. You may use several types of graphs depending on the type of data and the message you wish to convey. For bar, line and picture graphs, choose a scale that will accurately communicate your required message.

5. **Construct your graph(s)**

   Draw up your graph(s) and prepare a report consisting of a description of your experiment, how you collected your data, the data, your graph(s), and a description of any answers, messages or conclusions that are warranted by the data.

6. **Present the graph to others for evaluation**

   Place this report on a poster and display it for discussion by other members of your class.

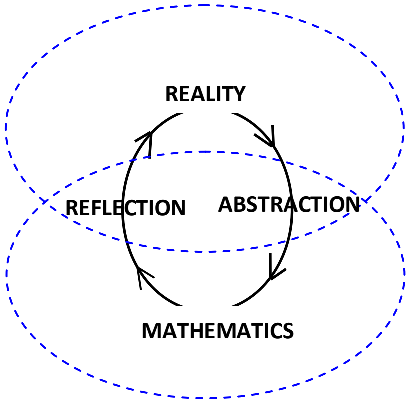
   In this discussion, you should first focus on how the data was collected, how it was organised and why particular graphs were chosen in preference to others. Then the class should address itself to the questions:

   - What was the message?
   - Did the data support the message?
   - Could another kind of graph or a different scale have been used to better convey the message?

   The class should pay particular attention to the general question of which type of graph seems most appropriate for which kind of data.

Appendix B: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix C: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system **N2: Decimal Number Numeration** Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches **SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae **N3: Common Fractions** Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships **O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject **N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference **G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs **N5: Directed Number, Indices and Systems** Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio **G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation **SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics **O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

YuMiDeadly
Growing community through education

Faculty of Education
School of Curriculum
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059

CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
Email: ydc@qut.edu.au
Website: ydc.qut.edu.au