YuMi Deadly Maths

AIM Module M2

Year A, Term 3

Relationship Measurement:
Perimeter, Area and Solid Volume

Prepared by the YuMi Deadly Centre
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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: Growing community through education.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

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DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government’s Closing the Gap: Expansion of Intensive Literacy and Numeracy program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students’ learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

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Module Overview

This is the second of the Measurement modules and it covers all the stages of the sequence for teaching the measurement topics: perimeter, area and volume (solid). These are measurement topics that lead to formulae and relate to length. They follow on from the basic measurement topics, length, mass and capacity (liquid volume), that do not have formulae. In particular they all follow on from and are related to length. We call them the relationship measurement topics. Thus this is Module M2 Relationship Measurement.

Background information for teaching relationship measurement

In this section we list again the big ideas of measurement and highlight the four which have most effect, and look at the special connection between perimeter, area and volume. They relate, in symbols and formulae, to length as follows: perimeter is length (m); area is the square of length (m²); volume is the cube of length (m³).

Big ideas for measurement

The big ideas for measurement are an important part of this module, so we repeat them from Module M1 Basic Measurement.

1. Continuous vs discrete. Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted.

1. Interpretation vs construction. Things can either be interpreted (e.g. a m is 100 cm) or constructed (construct a m out of 10 cm lengths of straws).

2. Notion of unit. Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts (e.g. 1000 mm makes a m and 1000 m makes a km).

3. Multiplicative structure. Standard units are designed so that they reflect place value in that adjacent positions are related by moving left (× base) and moving right (÷ base), where the base is 10 for metrics but 60 for time and angle and, in practical terms, 100 for dollars-cents and temperature.

4. Common units. We must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60) and, if we do so, the object with the biggest number has the most attribute.

5. Inverse relation. Same as Extension of field properties principle (i.e. the bigger the unit, the smaller the number – e.g. 2 m = 200 cm).

6. Accuracy vs exactness. Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100) or exactly (5 275 + 3 873 = 9 148).

7. Attribute leads to instrumentation. The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring).

8. Triadic relationships. When three things are related, there are three problem types (e.g. measuring length has three components, the object, the number and the unit – thus we can set problems where the object is unknown, the number is unknown or the unit is unknown).

9. Balance rule. Whatever you do to one side of an equation you have to do to the other to keep things equal.
Major big ideas

As for the previous measurement module M1, the major big idea here is 1 above, **continuous vs discrete**. All measurement topics are continuous and cannot naturally be counted or represented by numbers. However, the invention of unit has allowed the continuous to be “discretified” or partitioned into units and these units can be counted. This application of unit changes learners’ perception of the units. Thus, the sequence for teaching measurement topics has three parts: (a) understanding the measurement topic in its natural continuous state (i.e. Stages 1 and 2 below); (b) introducing unit and number to measurement topics (i.e. Stage 3 below); and (c) understanding the standard units adopted by Australia and applications of formulae for, and relationships between, these units (i.e. Stages 4 and 5 below).

As in Module M1, big ideas 5, 6 and 7 have a special role in measurement – they are the **measurement principles**. Big idea 5, **common units**, ensures that, when measurements are compared or used in computation, they are common units; and if there are common units, the bigger the number the more the attribute. Big idea 6, **inverse relation**, gives the important principle that the bigger the unit, the smaller the number and vice versa. This is why, in measurement, number can never be alone: measures are given by **number and unit**. Big idea, **accuracy vs exactness**, underlies that, in measurement, instruments are such that there is only accuracy not exactness. This leads to being able to choose appropriate units. These three principles are part of Stage 3, **Non-standard units**.

As well as the above big ideas, big idea 9, **triadic relationships**, underlies the three types of applications in Stage 5, and big idea 10, **balance rule**, enables the subject of a formula to be changed, also in Stage 5.

Connections between attributes

The connections described in Module M1 *Basic Measurement* still hold for this module. These are:

- connection between unit, fractions and division – in the way they all divide a length into equal parts and have the inverse relation principle;
- connection between fractions, decimals and unit conversion – in that, being metrics, conversions are powers of 10 (as they are between place values) but with the milli ε→ one conversion being ×1000 and ÷1000 for perimeter, ×1 000 000 and ÷1 000 000 for area, and ×1 000 000 000 and ÷1 000 000 000 for volume; and
- connection between PV, whole-part, and algebra computation – in that the separation strategy (see Module O1) applies to operating on perimeter, area and volume as it does on whole and decimal numbers.

However, a more powerful connection is evident across perimeter, area and volume, and this is the reason they have a module to themselves; they relate to each other, to length and to shape by their dimensions. Perimeter, like length, is one dimensional (1D) and acts on lines; area is two dimensional (2D) and acts on plane or 2D shapes; whole volume is three dimensional (3D) and acts on solid or 3D shapes.

This relation is most evident in formulae and notation as follows:

<table>
<thead>
<tr>
<th>SHAPE</th>
<th>FORMULA</th>
<th>NOTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>Rectangle</td>
<td>Perimeter = 2(L+W)</td>
</tr>
<tr>
<td>2D</td>
<td>Rectangular surface</td>
<td>Area = L×W</td>
</tr>
<tr>
<td>3D</td>
<td>Rectangular prism</td>
<td>Volume = L×W×H</td>
</tr>
</tbody>
</table>
Sequencing for relationship measurement

This section covers the five stages for sequencing the teaching of any measure, and the sequence in this module.

Stages of measurement teaching

The teaching of the measurement topics perimeter, area and volume (solid) follow the same stages below as in the previous measurement module.

Stage 1 – Attribute identification

This stage focuses on students understanding the attribute (or concept) of the measure. Activities to identify attributes: should follow rich experiences with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, and so on. They should also involve developing meaning for all the specialist attribute language that accompanies measurement topics.

The central idea in learning about an attribute is to experience it. However, if students have difficulty identifying the attribute from other characteristics of the experience, there are two general ways to introduce any attribute by providing examples where: (a) the only thing that is the same is the attribute, and (b) the only thing that varies is the attribute.

Stage 2 – Comparing and ordering

This stage focuses on comparing (i.e. two examples) and ordering (i.e. three or more examples) the amounts of attribute in examples. The process of ordering is based on comparison; the ability to compare two examples is extended to ordering three examples by identifying the one that is between the other two. Stage 2 activities are learnt in two parts: (a) direct comparison and order where examples are compared directly to each other; and (b) indirect comparison and order through an intermediary. Stage 2 activities involve no units and no numbers; the total amounts of the attribute present are compared or ordered holistically.

Stage 3 – Non-standard units

This section has two foci: (a) introducing the notion of unit; and (b) the development of measurement processes (using instruments with which to measure) and measurement principles (i.e. big measurement ideas that hold across all measurement topics). To prevent too much new information being given at once, the units are non-standard or class/learner chosen so that the learner is familiar with them. The measurement processes differ for different topics; they are related to the proper use of the measuring instruments. The measurement principles are the techniques for using units. These are: (a) common units – common units must be used in measuring, comparing/ordering and calculating amount of attribute and, in this case, the example with the most attribute has the larger number of units (this leads to the need for a standard); (b) inverse relation – the bigger the unit, the smaller the number and vice versa; and (c) accuracy vs exactness – all units give rise to error, with smaller units being more accurate but more difficult to apply, so there is a need to choose the level of accuracy required for the job. This principle requires a tolerance for error and also leads to the skill of choosing appropriate units and to developing skill in estimating as well as accurate measuring.

Stage 4 – Standard units

This stage focuses on the introduction of the standard units accepted by Australia. It should be remembered that these units should only be introduced after the need for a standard has been determined by recognising the limitations of non-standard units. It is recommended that they be preceded by the use of a class-chosen common unit (if appropriate). The recommended sequence for introducing standard units is: (a) identifying the unit through experiencing it or constructing it; (b) internalising the unit through relating it to body or everyday activities; and (c) estimating with the unit before measuring. Stage 4 activities should also relate to the decimal number system to build understanding of conversion between units and should continue to develop the measurement processes and principles begun in Stage 3.
**Stage 5 – Applications and formulae**

This stage focuses on: (a) **applications** of measures to the real world (i.e. calculating the measure of things); and (b) any **formulae** for determining measures (tends to be restricted to perimeter, area, volume and angle).

**Sequencing in this module**

The teaching of the measurement topics perimeter, area and volume follows the sequence as on right. They start from length, and use number from Stage 3 and place value from Stage 4.

The module covers all stages for perimeter, area and solid volume (following on from length, capacity and mass in Module M1). All activities should relate to the real world of the students as far as possible.

The **five sections** in the module are:

- **Overview**: Background information, sequencing and relation to Australian Curriculum
- **Unit 1**: Perimeter – attribute, comparing, non-standard units, standard units, applications and formulae
- **Unit 2**: Area – attribute, comparing, non-standard units, standard units, applications and formulae
- **Unit 3**: Solid volume – attribute, comparing, non-standard units, standard units, applications and formulae
- **Test item types**: Test items associated with the three units built around the five stages which can be used for pre- and post-tests
- **Appendix A**: Teaching tools to assist with aspects of the units
- **Appendix B**: RAMR cycle components and description
- **Appendix C**: AIM scope and sequence showing all modules by year level and term.

This sequence, like that in module M1, has two components: (a) a sequence **across** the three units from perimeter to volume, and (b) a sequence **within** each unit that follows the five stages. The teaching ideas within each stage within each cycle follow the RAMR cycle which is described in Appendix B.

However, it is not necessary to complete all units in the sequence as they are in this module. In fact, you may find it more useful to do all the Stage 1 activities together, then all the Stage 2, all the Stage 3 and all the Stage 4 activities together, up to all the Stage 5 activities together (for perimeter, area and volume). There are relationships between metric units and formulae for perimeter, area and volume that can be used to make this an efficient way to teach in Stages 4 and 5.
### Relation to Australian Curriculum: Mathematics

**AIM M2 meets the Australian Curriculum: Mathematics (Foundation to Year 10)**

Unit 1: Perimeter  
Unit 2: Area  
Unit 3: Solid volume

<table>
<thead>
<tr>
<th>Content Description</th>
<th>Year</th>
<th>M2 Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare and order several shapes and objects based on area and volume using</td>
<td>P to 3</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>appropriate uniform informal units (<a href="#">ACMMG037</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare objects using familiar metric units of area and volume (<a href="#">ACMMG290</a>)</td>
<td>4</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>Choose appropriate units of measurement for area, volume (<a href="#">ACMMG108</a>)</td>
<td>5</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>Calculate the perimeter and area of rectangles using familiar metric units ([</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>ACMMG109](#))</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Connect decimal representations to the metric system (<a href="#">ACMMG135</a>)</td>
<td>7</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>Solve problems involving the comparison of lengths and areas using appropriate</td>
<td>8</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>units (<a href="#">ACMMG137</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connect volume and capacity and their units of measurement (<a href="#">ACMMG138</a>)</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Establish the formulas for areas of rectangles, triangles and parallelograms and</td>
<td>10</td>
<td>✓  ✓</td>
</tr>
<tr>
<td>use these in problem solving (<a href="#">ACMMG159</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate volumes of rectangular prisms (<a href="#">ACMMG160</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Choose appropriate units of measurement for area and volume and convert from one</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unit to another (<a href="#">ACMMG195</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(<a href="#">ACMMG196</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigate the relationship between features of circles such as circumference,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>area, radius and diameter. Use formulas to solve problems involving circumference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and area (<a href="#">ACMMG197</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develop the formulas for volumes of rectangular and triangular prisms and prisms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in general. Use formulas to solve problems involving volume (<a href="#">ACMMG198</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the areas of composite shapes (<a href="#">ACMMG216</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the surface area and volume of cylinders and solve related problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(<a href="#">ACMMG217</a>)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve problems involving the surface area and volume of right prisms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(<a href="#">ACMMG218</a>)</td>
<td></td>
<td>✓  ✓</td>
</tr>
<tr>
<td>Solve problems involving surface area and volume for a range of prisms, cylinders</td>
<td></td>
<td>✓  ✓</td>
</tr>
<tr>
<td>and composite solids (<a href="#">ACMMG242</a>)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 1: Perimeter

Perimeter is a special name for length when it measures the distance around something like a shape, a tree or a house. Like length, it is a measure of one dimension. As such, it is related to height or strips of various types (e.g. rulers), but is best associated with distance or walking. Of course, there are other ways than the obvious to measure perimeter (e.g. trundle wheels) and it often requires an intermediary such as string. It is hoped that a rich and varied mixture is given below. (Note: Perimeter for circular shapes, buildings, and so on, is given a special name – circumference.)

Because of its close relation to length, perimeter should simply be seen as an extension of the length activities of Module M1. Thus, this section is smaller than area and volume – it should be read and undertaken with work on length.

1.1 Stage 1 for Perimeter: Attribute identification

This stage builds meaning for the attribute. It translates the attribute length to the special case of perimeter.

Reality
Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body
Use actions, mimes and dances to reinforce the notion of length: act out growing higher or growing longer and shorter, etc. (e.g. a tree growing, a fish getting longer). Act out with students’ bodies all the length words — long/short, tall/short, wide/narrow, thick/thin, high/low, deep/shallow, near/far, up/down, and so on.

Experience, “distance around” things to introduce perimeter — e.g. walking around a building or a shape marked on the ground, running around the oval — note that this restricts language to longer/shorter and smaller/larger. Experience distance around round things such as a roundabout — introduce the special word of “circumference”. Use words “distance around” before introducing “perimeter” or “circumference”.

Hand
Experience length with a variety of materials — discuss whether long/short, thick/thin, and so on. Put out pencils of different lengths (what is different?) or put out a feather, a pencil, a strip of paper, a can, and a duster that are all the same length (what is the same?). Hold up an object, look for things that are the same length as it, look for things that are different lengths to it. Experience length in terms of distance around or perimeter — run finger around shapes and objects, cut strips of material to fir around head or waist, tie string around objects (how much do we need?), put tape around objects, and so on.

Mind
Students shut eyes and imagine walking around something that takes a long time or short time, or putting string around objects that are large and small. They describe the things they imagine — in terms of “distance around”. Students shut eyes and think of a perimeter — then make it longer/shorter, and so on. Students draw the original. Students think of things that are the same perimeter.
Mathematics

Practice
Worksheets where students sort objects into long and short perimeters.

Connections
Show non-connections – find long and square and short and round, and so on.

Reflection

Application
Take new understanding of perimeter/circumference out into the playground and identify long/short.

Extension

Flexibility. Think of many perimeters that are long/short, and so on. Find a stone with the longest perimeter, and so on.

Reversing. Get students to draw shapes that have long/short perimeters.

Generalising. Look at different perimeters – what makes them long and short? Discuss what people might say in different situations – is there only one view of long and sort for all people? (e.g. it’s long for a piece of firewood but short for the trunk of a tree). Discuss that long/short depend on perspective or situation or experience of people. Discuss ways in which we could determine which of two perimeters is longer. Try to elicit the beginnings of getting the attribute leads to instrumentation big idea.

Changing parameters. Consider perimeter not being a straight line. Could we have a squiggly perimeter?

1.2 Stage 2 for Perimeter: Comparing and ordering

Before introducing numbers in measuring situations, it is useful to experience the concepts of perimeter and circumference, by comparing and ordering different examples. Students should undertake comparing activities between two examples, before ordering three or more examples. The change from comparing two to ordering three or more is difficult. It requires a focus on between – i.e. identifying the example which is between the other two. (Note: Direct comparison is difficult for perimeters and thus we mainly compare and order perimeters through an intermediary – e.g. string.)

Reality
Where possible, find real-life contexts or use relevant objects or situations to embed the activities in.

Abstraction

Body
Indirectly compare and order distances around parts of students’ bodies – use string or strips of paper or any other flexible length – use these intermediaries to compare and order – do not use numbers. Mark how long they are or cut them to length, and then directly compare and order. State which “distance around” or perimeter is longer-larger or shorter-smaller.

Hand
Indirectly compare and order distance around objects using intermediaries – use string, rope, and so on. Either mark two or more intermediaries and compare, or use one intermediary and see if more or less length is used for distance around or perimeter.
Also experience comparing and ordering distance around (circumference) circular objects and circles.

For ordering, focus on finding the “between” examples. Introduce language such as longest/shortest and largest/smallest. Note: Can compare distances around large things by running at same speed and seeing who finishes first. Complete both types of activities:

(a) perimeter/circumference to length activities (e.g. rolling bicycle wheel one revolution along a wooden plank, wrapping a paper strip around a cylinder/can, opening out a wire rectangle to compare it with a stick, etc.);

(b) perimeter/circumference to perimeter/circumference activities (e.g. using string or paper as an intermediary, opening out wire or geostrip shapes and comparing the total lengths of their sides, and rolling two cans one revolution each to see which can rolls further, etc.).

**Drying glasses activity (an idea from the ‘I hate mathematics’ book)**

*When you dry a glass with a towel, you dry up the side and around the top rim. Which is longer, the distance up the side (the height) or the distance around the top (the circumference of the circle)? Use your towel. Mark off the height of the glass on the towel. See if this distance will wrap around the top. Is this always true? Are there glasses that do not do this?*

**Mind**

Shut eyes and imagine different distances around – which is longer/shorter? How would you determine this? Draw different examples.

**Mathematics**

**Practice**

Give students many opportunities to compare and order “distances around” (both perimeters and circumferences) indirectly – even using string on worksheets.

**Connections**

Connect to principles for working out the longer “distance around” – must make sure to go all the way around, and line – draw a straight line, take two objects of different length and, in turn, put one end on the start of the line and mark the end points on the line. Discuss what the positions of the end points mean [longer is further along the line].

**Reflection**

**Application**

Discuss real-world situations of longer/shorter distances around. Find things that are in between two perimeters or circumferences.

**Extension**

**Flexibility.** Think of all the situations where you would use the length words (i.e. longer/shorter, longest/shortest) with regard to distance around, perimeter or circumference.

**Reversing.** Make sure you do all the directions – objects → perimeter length, and perimeter length → objects. That is: “Which of these two objects has the longer perimeter?” and “Find me an object with a longer perimeter than this object”.

Remember that comparison can be considered as a **triad** – with three parts, first object, comparison word, and second object – thus there are three “directions” (this is developing the **triadic relationship** big idea):
(a) give first object and words (e.g. longer perimeter) and require a shorter perimeter object;

(b) give second object and words (e.g. shorter perimeter) and ask for an object with a perimeter that the perimeter of the second object is shorter than; and

(c) give two objects and ask for which has the longer/shorter perimeter.

**Generalising.** Try to find things that hold true for all comparisons of perimeter and circumference. Obviously the circle with the smaller circumference fits inside the circle with the larger circumference. Is there something similar for perimeters? The ideas in connections above can be generalisations – e.g. do not overlap. Show that a perimeter can be both longer and shorter (comparison depends on the size of the object to which it is compared). Generalise the reversing activity above.

**Changing parameters.** Instead of looking for, for example, longest or shortest, look for a length in between (e.g. find something whose length is in between the board’s height and the door’s width). Look at length which is not straight. Use intermediaries to find which of the distance around a wriggly shoe is longest.

### 1.3 Stage 3 for Perimeter: Non-standard units

This section introduces the notion of unit using non-standard units and uses these units to develop measurement processes and principles for perimeter. It uses ideas from length and applies them to perimeter and circumference.

**Reality**

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

**Abstraction**

**Body**

Experience pacing around objects and counting paces. Get the students to measure things with their own paces and write down the number and the unit. Always get students to estimate before measuring and to give answers as numbers and units. *(Note: Make students aware of the correct use of paces in measuring distance around – e.g. start and finish at the same place – use the method of “torpedoing” described in M1 Length subsection 1.3.)* Discuss the answers different students get for distance around – discuss whether paces are mostly long or short. This leads to the first aspect of the common units big idea – units in a measure must be the same.

Repeat the above for different body parts as the non-standard unit (see on right). Discuss how errors can be made – overlapping units, separating units, not ensuring units form a straight line, not going around in the shortest way (see Module M1 Length section 1.3).

**Hand**

Use a variety of non-standard units to measure perimeters and circumferences (e.g. how many dusters around the edge of the board, how many pencils wide around the desk, how many sticks wide around the playground?). Objects that could be used as units to measure length could include pencils, pencil cases, sheets or paper, blackboard dusters, Smarties, Cuisenaire rods, blocks, straws, cardboard strips, pegs, paper clips, lengths or dowel, length of string, and so on. Good units are multiples of lengths of everyday things. For example, cut out a copy of your foot and use this as a non-standard unit.
Use anything that rolls and count the revolutions turned when the object is “wheeled” along the length (e.g. bicycle wheel, cotton reel, can, dish, and so on). The rolling method is particularly good when the shape is irregular. So is putting string around the shape and then measuring the string. Make up a non-standard ruler – string beads or pieces of straw onto string, stick short sections of paper end on end, and so on. Count the number of units which are alongside the object being measured. Use an object to mark a long piece of paper so it acts like a home-made measuring tape. Things that also should be done: (a) repeat the activities from body to reinforce the measurement processes that give accurate measures; (b) always estimate before measuring; and (c) always give answers as number and unit.

Mind

Have students draw units beside distance around being measured. Have the students imagine units being used to measure perimeter and circumference. Also imagine accurate measurement processes used in the hand experience and represent how the perimeters and circumferences were measured.

Mathematics

Practice

Continue to provide situations for students to measure perimeter and circumference – use materials, pictures and worksheets. Estimate first. Reinforce accurate measurement processes while the practice is being undertaken. Even run worksheets where students have to identify inaccurate measurement processes for perimeters and circumferences.

Connections

Connect non-standard measurement to number to number line division for perimeters and circumferences. For example, the diagram on the right can be considered as a perimeter divided by the paper-clip lengths.

For division, the more people there are to divide the cake among means that each person gets less cake. It is the same here, increasing the length of the paper clip means less paper clips. This leads into the inverse relation big idea.

Reflection

Application

Measure perimeters and circumferences with non-standard units in real-world situations. Set up measurement problems based on non-standard units.

Extension

Flexibility. Find non-standard units in local community (e.g. number of cans of fuel to measure distance around an island by boats). Try to get students to think of all ways non-standard units are used in the world, local and otherwise. Use history and look at other units used in the past (e.g. the mile which was 2000 paces of the Roman army).

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types, as in Stage 5. Thus, must make sure all three directions are taught. This leads to the triadic relationship big idea.

Generalising. As for length, perimeter and circumference can continue to extend the understanding from Abstraction and Mathematics to teach continuous vs discrete and the three measurement principles: common units, inverse relation, and accuracy vs exactness (see Module M1 Length section 1.3.) This is a major part of this unit. There is a particular importance in focusing on a need for a standard.
Changing parameters. What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? What if the units were not length – say they were mass or area? Would they need similar study of measurement processes and principles? Would this study have common points?

1.4 Stage 4 for Perimeter: Standard units and metric conversions

The standard units of mm, cm, m and km and conversions for these units (10 mm = 1 cm, 1000 mm = 1 m, 100 cm = 1 m, 1000 m = 1 km) have been introduced in Module M1 Length section 1.4. Perimeter and circumference can only extend this. These units are the formal basis of measurement.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations. The accompanying test is also available to measure prerequisite knowledge.

Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3 (see Module M1 Length for more on this), time can be spent measuring distance around with a class chosen unit – e.g. a piece of dowelling. This can be used by students to show that with a common unit, a higher number really means a greater length.

Identification

Cut 1 cm pieces from different coloured drinking straws. Thread these pieces along a string in groups of 10 of one colour followed by 10 of another colour. This identifies a centimetre and a metre. Use these, and other constructions from Module M1 Length (like putting 10 cm strips together – see on right), to measure perimeter and circumference.

Internalisation

Use a measuring tape to measure and record your personal body measures if they are perimeters/circumferences:

<table>
<thead>
<tr>
<th>BODY PART</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance around head</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance around foot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance around waist</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance around wrist</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find a distance around which is approximately: 1 cm, 10 cm, 1 mm and 1 m.

Mark out a 10 m distance using a measuring tape. Determine how many of your paces equal this 10 m. Pace the following distances, make some up, and use this value to convert paces to metres:

<table>
<thead>
<tr>
<th>DISTANCE AROUND</th>
<th>PACES</th>
<th>METRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Building</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Playground</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Estimation

First estimate, and then measure the length of the objects (find some interesting ones), to complete the table below. Estimate and measure each distance before starting the next.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance around blackboard</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance around window</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance around door</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Look at local maps, estimate differences on the map and then measure how many km around things (parks, city centre, and so on).

Place-value connections/Metric conversions

Do the same as in Module M1 Length section 1.4:

(a) set up the place-value cards from Module N2, Units 1 and 4 at the front of the room, or use mat;
(b) research what centi, milli and kilo mean;
(c) place metric units under place-value positions; and
(d) use multiplicative relationship between place-value (PV) positions to show relationships between metric units.

Mathematics

Practice and connections

As in Module M1 Length, use metric expanders and metric slide rules to relate/connect metrics to PV and to each other (conversions). Practice with instruments (rulers, tapes) and undertake orienteering and then draw maps to scale. Consolidate the metric conversions through drill.

Reflection

Repeat the reflections for non-standard units but using standard units, i.e. look at applications, flexibility, reversing, generalising and changing parameters. Show how non-standard principles such as inverse relation extend to metrics.

1.5 Stage 5 for Perimeter: Applications and formulae

This is a major extension of length.

Applications

The approach is built around the big idea of triad – there being object, number and unit in any measure means that there are three problem types for each to be unknown:

- **Number unknown** – *measure around this room in metres*.
- **Object unknown** – *find an object with a perimeter of 16 cm*.
- **Unit unknown** – *this object has a perimeter of 35 units, are these units cm or mm?*
Formulae

There are three major formulae that can be discovered.

**Perimeter of rectangle**

Draw rectangular shapes on graph paper and record lengths, widths, \( L + W \), and perimeters on a table as follows. Discussion will lead students to discover that the perimeter of a rectangle is \( 2(L + W) \).

<table>
<thead>
<tr>
<th>Shape</th>
<th>( L )</th>
<th>( W )</th>
<th>( L + W )</th>
<th>Perimeter</th>
</tr>
</thead>
</table>

**Perimeter of regular polygons**

In a similar way, the students can discover that the perimeter of an \( n \)-sided regular polygon of length of side \( L \) is \( n \times L \). This makes the perimeter of an equilateral triangle \( 3 \times L \) and the perimeter of a square \( 4 \times L \).

**Circumference of a circle**

1. Find a circle such as a wheel, hoop or a cylinder, and mark it on edge.
2. Roll it so that it does one revolution and draw a line that represents this roll (this is circumference).
3. Draw three copies of wheel/circle along this line at diameter. Ask students before they place copies of the wheel along the line to guess how many diameters in the revolution.

Do this with more than one size wheel. Students will see that there are “three and a bit” diameters in the revolution. Use this for examples such as: For a 2-metre diameter garden, how many metres of bricks around the edge? – 6 and a reasonable bit.

When students have this, introduce \( \pi \). This can be done by measuring and dividing circumference by diameter. It is best if this is done by cylinders:

1. Put 10 cylinders side by side and measure across all 10 and divide by 10 for diameter.
2. Wrap string 10 times around cylinder, measure string and divide by 10 for circumference. Then put on a table as below for different circles/cylinders to show that there is a common ratio called \( \pi \) (which is 3-and-a-bit or more accurately 3.14) which relates diameter and circumference.

<table>
<thead>
<tr>
<th>Circle</th>
<th>( C )</th>
<th>( D )</th>
<th>( C \div D )</th>
</tr>
</thead>
</table>

This will show that circumference of a circle \( (C) \) is \( \pi \times D \) where \( D \) is diameter and circumference of a circle \( (C) \) is \( 2 \times \pi \times r \) where \( r \) is radius and half the diameter.

Two activities for finding circumference of a circle are given on the following pages.
Activity A – Circumference of a circle

To find the formula for calculating the circumference of a circle, follow the procedure below:

(a) Find a sheet of paper, three different-sized lids, a ruler, and a pen. Label each lid small, medium, and large, according to their relative sizes.

(b) Use a ruler to draw three straight lines across a sheet of paper.

(c) For each of the three lids in turn, mark a line on it in one place on its side (as shown in the figure at right). Turn the lid on its side, and position the lid at one end of one of the lines you drew on the paper so that the line you drew on the lid touches the paper. Roll the lid one time along the paper (as shown in figure on right below). Mark the position where it stopped on the paper.

(d) For each of the three lids, lay the lid flat, and see how many fit on the line (as in figure immediately below). Note there are 3-and-a-bit circles in the line.

(e) Is it the same 3-and-a-bit for all the lids? What does this mean?

(f) For each lid in turn, measure this distance called \( C \) on the line (see figure above right) and measure the distance called \( D \) directly across the centre of the lid from edge to edge (as shown in the figure at right). Make sure you label the lid for which \( C \) and \( D \) are calculated.

(g) Write the data you collected in step (f) on the table below (remember to include the units of length that you are using) and then divide \( C \) by \( D \) for the third column:

<table>
<thead>
<tr>
<th>Lid</th>
<th>( C )</th>
<th>( D )</th>
<th>( C \div D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(h) What did you get in the third column for each of the lids? Was it close to 3-and-a-bit? What does this mean?
Activity B – Circumference of a circle

(a) From Activity A, you should have observed that the value for \( C \div D \) is equal to 3-and-a-bit. This value represents the ratio of the circumference \( (C) \) of a circle to the diameter \( (D) \) of the circle. A better approximation of this ratio is known to be 3.14 and is more formally referred to as pi and is symbolised using the Greek letter \( \pi \).

(b) So, we have developed the formula \( C \div D = \pi \). Rearranging the formula, we obtain \( C = \pi \times D \). Hence, the formula for calculating the circumference of a circle is \( C = \pi \times D \). That is, to find the circumference of a circle, we multiply the diameter of the circle by \( \pi \). By the way, your calculator may have a \( \pi \) button. If so, then you can use this button to calculate the circumference of a circle. If not, then you can just use 3.14 in place of \( \pi \).

(c) Match each of the expressions on the left with its approximated calculation on the right. Draw a line to connect the expression with its calculation.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 2 \pi )</td>
<td>0.52</td>
</tr>
<tr>
<td>2. ( \pi + 2 )</td>
<td>1.27</td>
</tr>
<tr>
<td>3. ( \pi \div 6 )</td>
<td>1.86</td>
</tr>
<tr>
<td>4. ( 5 - \pi )</td>
<td>2.37</td>
</tr>
<tr>
<td>5. ( 3 \pi \div 4 )</td>
<td>3.57</td>
</tr>
<tr>
<td>6. ( 4 \pi - 9 )</td>
<td>5.14</td>
</tr>
<tr>
<td>7. ( \pi^2 )</td>
<td>6.28</td>
</tr>
<tr>
<td>8. ( 4 \div \pi )</td>
<td>9.87</td>
</tr>
</tbody>
</table>

(d) Use the formulae \( D = C \div \pi \) and \( C = \pi \times D \) to complete the tables below. Round your answers to the same number of places as given.

<table>
<thead>
<tr>
<th>( C )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>48 cm</td>
<td></td>
</tr>
<tr>
<td>3.5 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>718 mm</td>
</tr>
<tr>
<td>86 cm</td>
<td>0.19 km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.75 m</td>
</tr>
<tr>
<td></td>
<td>41 cm</td>
</tr>
<tr>
<td>777 mm</td>
<td></td>
</tr>
<tr>
<td>34.8 cm</td>
<td>21 m</td>
</tr>
</tbody>
</table>

(e) What will the formulae be for the radius \( r \)?
Area is a measure of the coverage of two-dimensional space enclosed within a two-dimensional shape or surface. Area relates to length in that the standard units are square units whose sides are the lengths of standard units (squares with side of km, m, cm and mm). It is measured by these squares, directly and through the use of formulae – \( \text{mm}^2 \), \( \text{cm}^2 \), \( \text{m}^2 \) and \( \text{km}^2 \) – with a new unit, hectare (h), added in (to fill the gap between \( \text{m}^2 \) and \( \text{km}^2 \)).

2.1 Stage 1 for Area: Attribute identification

This stage building understanding of what area really is – “coverage” of a surface.

Reality

Teachers need to have a discussion relating to real-life examples of the area attribute e.g. painting a wall, tiling a floor, paving a path.

Abstraction

Body

Start with experiencing area – wrapping packages, wrapping strangely shaped (or hard to wrap) parcels, painting and colouring in. Introduce the term area as “coverage” (and large and small area). Use bodies to “cover” the floor, or hands to cover the desk.

Hand

Repeat body activities with hands – covering desktops with paper, covering play areas with paper, cutting and pasting one shape to cover another, and using stamps or paint on hands to cover a space with colour.

Experience area in different ways – long thin shapes, short fat shapes, large shapes, small shapes, interesting shapes. Discuss that area requires things to be closed. Discuss that area does not have to be flat and provide experiences covering non-flat things.

Mind

Encourage students to imagine surfaces and the covering of surfaces.

Mathematics

Practice

Use materials, computers and pictures in worksheets to experience area. Get students to colour shapes that have area and do not have area – hexagon yes, hexagon with part of a side missing no, rolled up long thin rectangle yes, spiral no (see below). Use pictures of surfaces as well.

Connections

Discuss how area relates to length – look at a rectangle. What is it about the lengths of the sides that will affect area? Make length and width longer – discuss how they may relate.
Reflection

Application

Relate understanding of area back to everyday life – what things in the world have area (e.g. cups, tables, walls, footballs, and so on) – what has big/small area? Introduce surface area – the term means “area of the surface”.

Extension

Flexibility. Think of as many things as you can that have an area. Walk around the school. Find things with the same or similar area.

Reversing. Up to now the students have gone from shape/surface → experiencing area. Reverse this – go from area as an experience → shape/surface. Thus, do activities where students construct areas for particular purposes e.g. wrapping a present, covering a book.

Generalising. Discuss what makes a surface/shape have large or small area. Discuss how we can measure this. This is the beginning of the attribute leads to instrumentation big idea.

Changing parameters. How can we change area – what kind of materials will do this – blow up a balloon?

2.2 Stage 2 for Area: Comparing and ordering

In this stage, area is compared and ordered without reference to number.

Reality

Where possible, find real-life contexts for area to embed the activities in; for example, you could compare the area of different sporting fields – football, netball, tennis.

Abstraction

Body

Compare and order things related to body – e.g. compare and order area of students’ hands, feet, etc. One idea is to lie down and trace around the body and compare with other bodies. Begin with comparing two and then introduce a third and so on to order area and determine who has greater/smaller and then greatest/smallest amount. Focus on finding the area between the other two. Reinforce words associated with area.

Hand

Directly compare and order areas by placing one area on top of another area e.g. book on top of a seat, pencil case on top of sheet of paper, etc. Draw around something and compare this with the same thing of different size but same shape – e.g. pencil cases or bags. First compare two, then order more than two.

Indirectly compare and order area by covering one area with paper and then transferring this paper to another area. This is very good for areas of differing shape where the paper has to be cut and rejoined to fit on the other area. You can use other material to do the same thing, e.g. cloth, plastic, etc.

If there is not enough material, then first surface is smaller – if too much material, then second surface is smaller.

Practise with dissections – jigsaws and shape puzzles – before doing this to get idea of cutting and re-forming. Use tangrams (and other shape puzzles) to cover shapes/spaces/surfaces, and then break the dissections apart and rejoin differently to cover other shapes.

Experience greater/smaller and greatest/smallest area using objects, virtual means and pictures.
Mind

Imagine, then draw and describe a variety of surfaces, demonstrating comparisons and orderings of area. For example: different desk tops or pin boards. Imagine them in the mind, and imagine them getting larger and smaller and having more and less area.

Mathematics

Practice

Use comparison and ordering experiences from the examples developed above to practise being able to compare and order areas. Continue to discuss comparative language for area such as large, larger, largest, small, smaller, smallest, and so on. Present students with a variety of comparing and ordering problems, for example: find a surface larger/smaller than this one. Order pictures of objects or pictures of shapes by area.

Make sure that students understand comparison notation (e.g. \( > \) and \( < \), \( = \) and \( ≠ \)) and the rules of comparison (i.e. non-reflexive, antisymmetrical, and transitive).

Connections

Continue to draw the connection between length and area.

Reflection

Application

Relate understanding of comparing and ordering area back to everyday life – what things in the world have large and small area (e.g. playgrounds, ovals, desks, etc.) For example, what is the largest park in your local area? – investigate (e.g. Google Maps and Internet) to find out. Discuss where we need area – e.g. painting a house, building a wall, paving a floor or courtyard.

Extension

Flexibility. Think of as many pairs of things as you can that have more/less area than each other. Visit a hardware shop or look around your school to find different pavers which can be ordered in terms of area.

Reversing. Make sure teaching goes from the teacher providing surfaces to students giving comparison words, as well as teachers giving comparison words to students providing surfaces. Also remember that comparison can be considered as a triad – with three parts, first surface, comparison word, and second surface– thus there are three “directions”, or three problem types (this is developing the triadic relationship big idea):

(a) give the first surface and word (e.g. larger) and require a larger area surface;

(b) give the second surface and a word (e.g. greater) and ask for a surface that is larger in area than the second surface; and

(c) give two surfaces and ask for word(s) to relate them (e.g. the second surface has less area).

Generalising. Generalise the three things that have already been started:

(a) more or less area of a surface depends on the surface it is being compared to;

(b) comparison by covering is only accurate if surfaces are fully covered; and

(c) for comparison, like all triadic relationships, there are three problem types as in reversing.

Changing parameters. Again discuss ordering areas for when the material can be stretched.
2.3 Stage 3 for Area: Non-standard units

Introduces the notion of unit and develops the measurement processes and principles in terms of number. Area is measured in terms of number and non-standard units (e.g. the board is covered by 15 A4 sheets of paper).

Reality

Where possible, find real-life contexts for area to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Use hands to cover surfaces – put paint or water on hands so can see coverage – say that the area is 7 hands (i.e. give number and unit).

Hand

Using a variety of materials as units, e.g. dusters, blocks, cubes, tiles, sheets of paper, hands, etc. Particularly interesting are chalk-filled dusters on paper, stamps on paper, wet hands on blackboard, etc. Use geoboards, both square and isometric (triangles). Ensure students are measuring with accuracy which means having the following accurate measurement processes:

- ensuring that the shapes cover without gaps and overlaps (or with regular gaps and overlaps);
- counting part units or counting those over ½ a shape and not counting those less than ½ a shape; and
- ensuring that the surface is covered to the edges (in some cases, this means choosing the unit to match the shape of the surface).

Always estimate before covering and calculating area. Always say the area of surfaces in terms of number and unit.

Use tessellations as non-standard units, e.g. triangles, quadrilaterals, hexagons, Escher-type drawings, etc. Make plastic overlays of these tessellations (e.g. O/H plastic) so they can place easily on the shapes. Alternatively, cut out shapes and place on top of tessellating grids. (Note: Do not be afraid to use non-tessellating shapes to cover and count.) Use Tangram pieces to cover shapes and assign unit value to one of the smallest Tangram pieces – then use this to assign a number to the other pieces. Use virtual materials and pictures to further experience measuring area in terms of non-standard units (e.g. containers).

Discuss the following:

(a) how different units give different numbers for area (small units give large numbers and vice versa);
(b) which size units are most accurate (small units) but discuss why we cannot use these all the time (it would take too long);
(c) what type/size of unit would be better for large/small surfaces and why (depends on how accurate you want to be and how quickly you have to do the job);
(d) what shape of unit is the most effective (squares as these form arrays and arrays can be quickly calculated by multiplication); and
(e) what is needed if giving an area over the phone (need a common or standard unit).

Mind

Have the students imagine surfaces being covered with smaller shapes and determining how many of the smaller shapes fit onto the larger. Make drawings of what is imagined.
Mathematics

Practice

Continue to provide situations for students to experience measuring area with non-standard units – use grids, overlays, and tessellating shapes to measure the area of shapes on paper/worksheets. Always estimate first.

Reinforce accurate measurement processes while the practice is being undertaken. Even have worksheets where students have to identify inaccurate measurement processes.

Connections

Connect non-standard measurement of area to division. For example, realising that working out how many hexagons cover a surface is the same as dividing the surface into equal parts (i.e. area is related to fractions and division). This means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units mean less number of units and vice versa. This leads into the inverse relation big idea.

Reflection

Application

Measure area with non-standard units in real-world situations. Set up area problems based on non-standard units.

Extension

Flexibility. Find area non-standard units in local community (e.g. measuring the area of paving in terms of large pavers). Try to get students to think of a variety of ways non-standard units are used in the world, local and otherwise, for area. Use history and look at other units used in the past (e.g. acres).

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types.

- Number unknown – how many hexagons cover the surface?
- Object unknown – find a surface which is 12 hexagons in area.
- Unit unknown – the surface has an area of 7, what is the unit/tessellating shape?

Thus, you must make sure all three directions are taught.

Generalising. Here the objective is to extend the understanding from Abstraction to Mathematics and to teach continuous vs discrete and the three measurement principles as follows. This is a major part of this unit.

1. Continuous vs discrete. Discuss how the area non-standard units have broken up a continuous surface into small parts that allows that area to be counted. This leads to the continuous vs discrete big idea – that there are two ways that number is applied: (a) to discrete objects, and (b) to continuous things such as area by the use of units like hexagons to “discretify” the continuous area.

2. Measurement principle 1: Common units. We need to show that: (a) we cannot measure accurately if we vary the unit, and (b) we cannot know if a surface is larger/smaller than another unless we use the same unit. Measure a surface sloppily; allowing units to have gaps/overlaps, and ask what is wrong? Say John measured a large surface as 6 units and Jack measured the same surface as 15 units and ask why this could be. This leads to the first and second aspect of the common units big idea – that units must be the same size when measuring/ comparing containers.

Then set a common class unit – select a common sized tessellating shape and measure using this. Elicit what bigger numbers mean now. This leads to the third aspect of the common units big idea – that when units are the same, the larger number specifies the larger object.
Discuss if different units were used in different towns and countries, what would this mean? We could be paying more for paving. This leads to the fourth aspect of the common units big idea – that **there is a need for a standard**.

Look particularly at the best area unit in terms of shape – why square? (use of arrays and multiplication?)

3. **Measurement principle 2: Inverse relation.** Measure things with large and small units and record results in a table. Study the results for patterns. This is best done with the students working in pairs. Activities like this lead to the inverse relation big idea – the larger the unit, the smaller the number and vice versa – and to the understanding that **measuring in units is like dividing**.

4. **Measurement principle 3: Accuracy vs exactness.** Get students to measure a surface with three different sized and shaped tessellating shapes. Which is most exact/least exact? Repeat for other surfaces. Which unit is best for the surface? Why do we want to be accurate? Are there cases where this is not needed? Discuss situations where estimation would work. This leads to the three consequences of the accuracy vs exactness big idea – smaller units give greater accuracy, students require **skill in being able to choose appropriate units**, and students require **skill in estimating**.

**Changing parameters.** What if the units were the standard ones, would they act the same with regard to big ideas as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

### 2.4 Stage 4 for Area: Standard units

This section focuses on introducing the square millimetre, square centimetre, square metre, and square kilometre. There are two parts – introduction of units and metric conversions.

#### A. Introduction of metric units

**Reality**

Where possible, find real-life contexts for area to embed the activities in; for example, using relevant objects or situations.

**Common unit**

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring area with a class chosen unit – e.g. a square piece of paper. This can be used by students to show that with a common unit, a higher number really means a greater length.

**Abstraction**

**Identification**

Use paper and cardboard to construct a square mm, cm and a square m (square m can also be made by connecting 4 one-metre lengths of dowel or metre rulers into a square). Place the square mm inside the square cm inside the square m – discuss the differences in size and how many of the smaller fit into the larger (100 mm² in 1 cm², 10 000 cm² in 1 m², 10 000 m² in 1 hectare, and 100 hectares in 1 km² – there is also 1 000 000 mm² in 1 m² and 1 000 000 m² in 1 km²).

Attempt to experience a hectare (a square 100 m by 100 m) and a square km, by walking around such spaces.

**Internalisation**

Relate the units to mm², cm², m², h and km² to something in students’ world. Find something on their body which is 1 mm² and 1 cm², find something in their everyday world that is 1 mm² and 1 cm² (e.g. end of a paper clip and end of a pencil). Draw around student’s body – cut out a square metre from newspaper taped together
and then cut this up to cover the drawing around the body – how much is covered by a square metre? Look for square metres in the classroom – part or all of a door, a window, a blackboard, the teacher’s desk top, and so on. Get a map of the local area – draw around areas that are a hectare or a square km. How many house blocks fit into the hectare? How many road blocks fit into a square km? Google Map is a good resource for exploring area in the physical landscape.

Make up plastic grids for 1 cm squares and a grid with 2 mm squares (4 mm²) – overlay these on shapes and determine the area by counting. Make up m², h and km² grids for various maps and overlay these to work out areas for different maps (this activity will require a discussion of scale). Use the scale on the map to work out the grid size. Use cm graph paper (preferably in the form of plastic overlays) to measure area of hand, A4 paper, bag, etcetera (anything that is an everyday item for the student). Use centicubes or MAB units and pack into spaces (these are 1 cm²). Use geoboards (1 cm squares) and rubber bands to make and calculate areas. Use MAB flats – these are $\frac{1}{100}$ of a m²).

**Realisation**

Some realisation questions to explore with the students are:

1. Would you use square metres or square centimetres to measure:
   (a) the cover of a book?
   (b) a sports oval?
   (c) a CD cover?
   (d) the classroom floor?
   (e) a floor rug?

2. Name four things that are:
   (a) smaller than 1 m²
   (b) about 1 m²
   (c) larger than 1 m².

Learn the two techniques, “breaking into parts” and “enclosing with a rectangle”, plus the technique of finding the area of a triangle by halving the appropriate rectangle.

Use these techniques to calculate the areas of polygons (any type). Calculate the area inside curved shapes by counting the squares that are all or more than half way inside the shape.

**Technique 1: Breaking into parts**

This relies on techniques as follows:
Technique 2: Enclosing with a square or rectangle

Area is $16 - 4 - 4 - 1 - 1 = 6 \text{ cm}^2$

Mathematics

Estimation

First estimate and then measure the area of objects, and record the information in a table like the one below. Estimate and measure each object before moving onto the next. **Ensure the students are estimating the area – and not estimating the length and width.**

Use 2 mm and 1 cm grid overlays. Use MAB units or centicubes. Use paper copies of a square metre with paper copies of 10 cm $\times$ 10 cm squares or MAB flats (1/100 of a square metre) and 10 cm $\times$ 100 cm rectangles (1/10 of a square metre).

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paper clip box</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pencil case</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student desk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Door</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whiteboard</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat the above on maps for hectares and square km using grid overlays from map scales.

Repeat the above for very small things and 2 mm grid paper.
Practice

Continue to provide situations for students to experience measuring area with standard units – use **grids and overlays** to measure area of shapes on paper/worksheets. **Always estimate first.** Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.

Connections

Connect standard measurement of area to division as for non-standard units – connect to the **inverse relation** big idea.

Students can make $1m^2$ squares of paper.

Use the $1m^2$ pieces of paper to measure the area of a basketball court (if you have a small class, perhaps select a handball court or something smaller – students might get bored with making a large number of $1m^2$). Let students devise their own method for working out the area coverage.

Most students will begin to lay down squares in a grid of rows and columns, and count them all individually. Others may lay them around the perimeter and fill in the middle, then count them individually. Large spaces are particularly good to find area. Students need to be made aware that the area of rectangles and squares is related to the units along the rows and columns of the shape/area.

Students will see that the area is in fact an array, and work out how many in the first row, and then how many columns, and then either multiply them or use repeated addition. Some students may not need to use the “squares” and simply want to measure the sides and multiply the measurements. This is the beginning stage of developing the formula for the area of a rectangle.

Reflection

Application

Measure area with standard units in real-world situations. Set up area problems based on standard units.

Extension

**Flexibility.** Find standard units of area in the local community (e.g. measuring the area of a house – how many square metres?) Try to get students to think of ways standard area units are used in the world, local and otherwise, for area.

**Reversing.** The components of a standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types: (a) **Number unknown** – for example, how many $m^2$ cover the surface?; (b) **Object unknown** – for example, find a surface which is $12m^2$ in area; and (c) **Unit unknown** – for example, the surface has an area of 70, what is the unit? Thus, must make sure all three directions are taught. This leads to understanding the **triadic relationship** big idea.

**Generalising.** Here the objective is to extend the understanding from Abstraction to Mathematics to teach **continuous vs discrete** and the three measurement principles as follows. This is a **major part** of this unit.

1. **Continuous vs discrete.** Discuss how the area standard units have broken up a continuous surface into small parts that allows the area to be counted. This leads to the **continuous vs discrete** big idea.

2. **Measurement principle 1: Common units.** Show that you must use same unit for the square units and the length units that give the shape – (i.e. cannot have 40 cm by 2 m rectangle to work out area without changing all length units to the same unit – e.g. $40 \times 200$ or $0.4 \times 2$). This leads to the first and second aspect of the **common units** big idea – that **units must be the same size when measuring/comparing containers.** Then set a **common class unit** to lead to when units are the same, the larger number specifies
the longer object, and discuss what happens if different units are used to lead to the fourth aspect of the common units big idea — that there is a need for a standard.

3. **Measurement principle 2: Inverse relation.** Measure things with large and small units to lead to the inverse relation big idea — the larger the unit, the smaller the number and vice versa.

4. **Measurement principle 3: Accuracy vs exactness.** Further discuss the use of large and small units, the reasons why we want to measure area, and when we can use estimates, to introduce the consequences of the accuracy vs exactness big idea — smaller units give greater accuracy, students require skills in being able to choose appropriate units, and students require skills in estimating.

**B. Conversion of metric units for area**

**Reality/Abstraction**

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

**Abstraction**

Revise length conversions:

- 1 cm = 10 mm
- 1 m = 100 cm
- 1 km = 1000 m

Make a 1 cm square. How long is each side? Use 1 mm grid paper.

How many 1 mm² fit inside 1 cm²?

Students can count, multiply rows or devise their own method to determine 1 cm² = 100 mm². For some students you will need to emphasise the 10 rows of 10 – mm squares.

Repeat this process for 1 m² = 10 000 cm².

**Mathematics**

Use patterns to develop ways to calculate:

1 m² = 10 000 cm² (students must know this conversion)
2 m² = _?_ (20 000 cm²)
3 m² = _?_ (30 000 cm²)

11 m² = _?_ (110 000 cm²) ← Some students will see the pattern. How do you do it?

Try saying the conversion aloud with the students — For every one square metre there are ten thousand square centimetres.

2 m² = 2 × 10 000 = 20 000 cm²
8 m² = 8 × 10 000 = 80 000 cm²
2.3 m² = 2.3 × 10 000 = 23 000 cm²

Repeat for cm² to mm².

The emphasis is on the student knowing and understanding the relationship between the units.

**Place-value connections**

Set up the place-value cards as follows:
Use multiplicative relationships between PV positions to look at the conversion rate between the area units, mm$^2$, cm$^2$, m$^2$, h, and km$^2$: 100 mm$^2$ = 1 cm$^2$; 10 000 cm$^2$ = 1 m$^2$; 10 000 m$^2$ = 1 h; 100 h = 1 km$^2$. Be flexible with the decimal point to show, e.g. 2.45 m$^2$ = 24 500 cm$^2$ (place decimal point after m$^2$, place numerals in PV positions for 2.45, move decimal point to after cm$^2$).

**Mathematics**

**Practice/Formality**

Use metric expanders and metric slide rules to relate/connect metrics to PV and to each other (conversions). Metric expander is PV set up above with blanks for spaces between metrics. It folds like ordinary number expanders with the pleat fold at the units.

Metric slide rules can be two types:

1. PV chart going from millions to millionths and the metric units on the slide as follows. The slide can move across and back placing any metric area unit at the ones (to see where the other units are). Note – some educators have the PV on the slide.

2. PV chart having metric area units where the number PVs normally go, and numbers on slides to move across and back.

Give students experience with specialist measuring techniques such as plastic overlays of cm grid paper. Undertake outdoor activities using maps and scales. Consolidate the metric conversions through drill – dominoes, bingo, mix and match cards, card decks for snap and concentration.

**Connections**

Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. Area can be seen to have very large conversions, therefore we have to use a large number of PV positions to get from mm$^3$ to km$^3$.

**Reflection**

Look at metric conversions in the world. Think of where they are needed. Look at applications, flexibility, reversing, generalising and changing parameters. Show how non-standard principles such as inverse relation extend to metrics.
2.5 Stage 5 for Area: Applications and formulae

This section covers: (a) applications, (b) formulae, and (c) relationships between perimeter and area. The development of formulae is a major component of area.

A. Applications

The approach is built around the big idea of triad – there being object, number and unit in any measure means that there are three problem types for each to be unknown: (a) Number unknown – measure the area of the wall in m\(^2\); (b) Object unknown – find an object with an area of 16 m\(^2\); and (c) Unit unknown – this object has an area of 35 square units, are these units cm\(^2\) or mm\(^2\)?

B. Formulae

The formula for area of rectangle can be discovered. Three other formulae can be introduced by relating them to this area.

Note: It is crucial to maintain the same symbols. That is not to have A rectangle = L×W and A triangle = (B×H)/2. Choose one set of symbols: we have chosen L and W, then A rectangle = L×W and A triangle = (L×W)/2. This strengthens students’ abilities to see connections between formulae and to remember formulae through these connections (the best way to do this).

Abstraction

Students may have missed the link from grids to formulae. Have students calculate the area of a classroom or basketball court etcetera using square metres. Lay out the first “row”. How many squares fit along this row?

Formucae building

Verbalise the row and column array – There are 14 rows of 16 square metres in each row. The area is 14 rows of 16 square metres.

Area = 14 rows of 16 m\(^2\)
Area = measure of width × measure of length
Area = Width × Length
Area = W × L

Area of a rectangle by discovery

Draw rectangular shapes on graph paper and record lengths, widths, and area as number of squares on a table as follows. Discussion will lead students to discover that the area of a rectangle is L × W.

<table>
<thead>
<tr>
<th>Shape</th>
<th>L</th>
<th>W</th>
<th>Area</th>
</tr>
</thead>
</table>

Area of a parallelogram by connection

Cutting and re-forming shows that a parallelogram is a rectangle with the same length (L) and perpendicular width (W), thus the area of a parallelogram is also L × W. Also, you might need to have students draw parallelograms and cut off the triangular ends to turn and attach to the other end of the parallelogram to create a rectangle.
Area of a triangle by connection

Cutting and re-forming shows that two triangles are a rectangle with the same base length (L) and perpendicular width (W). Thus a triangle is half a rectangle and the area of a triangle is \( \frac{1}{2} (L \times W) \).

Area of a circle by connection

Cutting and re-forming shows that a circle can be considered as a rectangle of length \( \pi \times r \) and width \( r \) and thus the area of a circle is \( \pi \times r \times r \) or \( \pi r^2 \). To do this, we have to remember that the circumference is \( 2 \times \pi \times r \). The cutting and re-forming is shown in the following activity.

Activity – Area of circle

1. Find a piece of paper, a large lid (at least 12 cm in diameter), a ruler, and scissors.
2. Measure the diameter of the lid, and calculate the radius (we shall call this \( r \)).
3. Place the lid firmly on the piece of the paper, and draw a circle around it. Calculate the circumference (\( C = 2 \times \pi \times r \)) and the area (\( A = \pi \times r^2 \)).
4. Draw lines across the circle to form 16 sectors that are approximately equal in area, as shown in the figure at right. (Hint: keep halving.)
5. Cut out the 16 sectors, as shown at right. Arrange the sectors in the configuration shown below. Note that this configuration looks very much like a rectangle. Cutting one of the sectors in half and putting one half at the start and end of the “rectangle” makes it even more like a rectangle.

6. What is the height? What is the length? Can you see how these relate to radius and circumference?
7. Measure the height and length of the rectangle. Calculate the area of the rectangle. Divide the length of the rectangle by the radius. Did you get an answer that is close to \( \pi \)? Remember that \( \pi \) is approximately equal to 3.14.
8. Did you recognise that the height of the rectangle is the same as the radius of the lid? What does this mean for length?

Mathematics

Through the Abstraction phase of finding and calculating area, students will be moving to the formal mathematical recording and calculating. Continue practising the calculations of area, using the correct mathematical terms and representations.
C. Relationships between length/perimeter and area

The idea here is to investigate and discover relationships between length, perimeter and area. For length and area, the relationship is based on squaring, that is, if the lengths of side of a rectangle or the radius of a circle increase by 2 or 3 times, the area increases by 4 or 9 times (the square of the increase in length). This is important because it means a pipe of double width will carry 4 times the water. The relationship between perimeter and area is more varied — the perimeter can vary for the same area, however there are some relationships: (a) the largest area for a given perimeter is the circle, and (b) if restricting shapes to rectangles, the largest area for a given perimeter is a square. Overall, the circle for any shape and square for rectangles give the best area for the least perimeter.

Length $\leftrightarrow$ area

Make rectangles and circles where lengths and radii are doubled — find areas — see that area is 4 times as large.

E.g. rectangle $2\, \text{cm} \times 3\, \text{cm} = 6\, \text{cm}^2 \rightarrow$ rectangle $4\, \text{cm} \times 6\, \text{cm} = 24\, \text{cm}^2$

Perimeter $\leftrightarrow$ area

Find ways to show that perimeter can be very long for a small area, and to find what shapes give the largest area for a given perimeter (see below for some activities).

1. Cut up “fat” rectangles of paper to cover long thin rectangles as on right — we can see that the long thin rectangle has same area but a greater perimeter, and we can see that, if we keep slicing, we can end up with any length of side we want for the same area.

2. Start with cm graph paper, make a circle of string which is, say 16 cm long and then make a variety of shapes with this string (circles, ellipses, rectangles, triangles, etc.) on the cm grid paper and count squares for area. Make a table with headings shape and area, and see which shape has the largest area — it will be the circle. This activity can also be done using geo boards and “the mat”.

3. Start with cm graph paper and draw a variety of rectangles to show that the square has greatest area where a square $(4 \times 4)$ is $16\, \text{cm}^2$ and a rectangle $(2 \times 6)$ is $12\, \text{cm}^2$.

4. Another way is to start with an area, and use a fixed number of blocks to make a variety of shapes of the same area and compare their perimeters.

5. Finally, this leads to an investigation of why square houses are the cheapest. Get students to make a square, rectangular (ranch style) and L-shaped house of same area on graph paper — ask students to put in walls to make three bedrooms, kitchen, lounge, bathroom, etcetera. Then measure lengths of walls and the square will have the shortest length of walls and so cheapest to make — concrete base and roof are same for all houses in terms of area.
Volume is a measure of the amount of space enclosed or taken up by a three-dimensional (3D) shape. It is measured by filling the interior of 3D shapes by units (e.g. cubes). Volume refers to solid volume (e.g. cubic metres) and is different to capacity (e.g. litres). Solid volume is determined by the amount of space enclosed or displaced, that is, displacement as well as filling. It is measured by formulae and is in cubic units (e.g. mm$^3$, cm$^3$, m$^3$, and km$^3$) because it relates to length but in three directions.

### 3.1 Stage 1 for Volume: Attribute identification

This stage builds meaning for the attribute volume. It has similarities to the capacity section of Module M1 *Basic Measurement: Length, Mass and Capacity*. It should be seen as different to the mass section of M1. Volume is an attribute of a 3D object. For liquids, it is called capacity. We are looking here at solid volume. You need to ensure students see the difference between mass and volume – mass is heft, how hard the object pushes down on the hand, while volume is the amount of space occupied by the object. Experience small heavy and large light objects so that students see that mass (heaviness) is not necessarily directed related to volume (size).

#### Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

#### Abstraction

**Body**

Start with re-experiencing length and area – how tall people are, the size of the area of their hands or body. Now look at how big/small people are. Act out being big (standing tall and spreading arms and legs) and small (bend over, crouch and pull in arms and legs so become a “ball”). Say big objects have large volume and small objects have small volume. Discuss the difference. In this way, introduce volume as a special word for size.

**Hand**

Allow students to play with different-sized objects or to make different-sized constructions. Do the following: building sandcastles, building with blocks and other material, making things with plasticine, and so on. Continue with a variety of size activities, e.g. packing things away, filling a box or carton with material, enclosing space, building a house or a fort, stacking materials, blowing up a balloon, making cakes (or any type of cooking), acting out or miming big and small with students’ bodies, enclosing space with students’ bodies.

Experience volume as the amount of space an object takes up → immerse objects in water and watch the level rise for larger objects. Experience volume situations (as the amount held in a container) with virtual materials and pictures. For virtual materials, use “click and drag” to change volume.

**Mind**

Encourage students to imagine containers and objects. Imagine the containers being filled and emptied, and the objects displacing space. Imagine containers/objects being made bigger and smaller (to change volume) – draw big and small things (e.g. mouse and elephant).
Mathematics

Practice

Use materials, computers and pictures in worksheets to experience volume. Differentiate it in drawings from length and area. Experience different size objects – small and large cars, etc. – be varied. Experience drawings of things that have volume and things that do not (or at least cannot be filled or cannot displace volume).

Connections

Look at long, narrow and thin objects and short, wide and thick objects. Does volume relate to length and area of surface? Discuss this!

Reflection

Application

Relate understanding of volume back to everyday life – what things in the world have volume, what do not (e.g. boxes, cars, houses, boats, petrol tankers, and so on) – what has large/small volume?

Extension

Flexibility. Think of as many things as you can that have volume. Visit a supermarket and find different containers which have the same volume.

Reversing. Up to now the students have gone from container $\rightarrow$ experiencing volume. Reverse this – go from volume as an experience $\rightarrow$ container. Thus, do activities where students construct containers for particular purposes or build things to certain sizes.

Generalising. Discuss what makes an object have large or small volume. Discuss how we can measure this. Think back over connections – is there an idea here? Try to elicit how volume could be measured. This is the beginning of the attribute leads to instrumentation big idea.

Changing parameters. What if material made into a 3D shape could contract and expand? What does volume mean here?

3.2 Stage 2 for Volume: Comparing and ordering

In this stage, volume is compared and ordered without reference to number.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Hand

Direct comparison and order can be done by placing objects inside each other – works best for similar shaped containers but different shapes will also work as long as inside each other. Can also compare/order directly when objects placed beside each other if difference is obvious. Extend to ordering three or more different containers. Discuss large and small, and also empty and full.

Construct something and then make it larger in volume by adding more, or smaller in volume by taking things away. Blow up balloons to make thing larger in volume – use plasticine to increase or decrease volume. Compare and order pictures of objects – like houses and cars – need for differences to be obvious.
Indirectly compare/order by: (a) pouring water, rice or sand (and other liquids) from one container to another and seeing which container holds more; and (b) packing objects inside a container/box and then seeing if more or less objects can be packed into a second container/box. Do the same thing by immersing objects in water – determining the larger object either by the height the water rises or the amount that is spilled over (in both cases the water level must be the same to start). Two interesting ways to do this are to mark the increase for one object and to see if the other makes the water go higher or lower, or to fill a bucket to the brim with the first object in it and then to remove this object and to place in a second to see if there is any spill. Discuss large/larger/largest and small/smaller/smallest volume.

Mind

Imagine, then draw and describe a variety of objects, demonstrating comparisons and orderings of volume. For example: small and large men, small and large buildings, small and large dogs, and so on. Imagine objects getting larger and smaller in volume.

Mathematics

Practice

Use comparison and ordering experiences from above to practice being able to compare and order volumes. Continue to discuss comparative language for volume such as large, larger, largest; small, smaller, smallest; full, empty, less, more, and so on. Present students with a variety of comparing and ordering problems, for example: (a) make something bigger in volume than this; (b) make something smaller in volume than this; (c) find objects that are the same volume, and (d) order three or more objects by volume. Order pictures of objects by volume. Make sure that students understand comparison notation (e.g. > and <) and the rules of comparison (i.e. non-reflexive, antisymmetrical, and transitive).

Connections

Continue to draw the connections between length, area of surface and volume.

Reflection

Application

Relate understanding of comparing and ordering volume back to everyday life – what things in the world have large and small volume. Think about different rooms in a house – why are some bigger than others?

Extension

Flexibility. Think of as many pairs of things as you can that have more/less volume than each other. Visit a supermarket or the council lockup or the car park, and so on, and find different things that are large and small.

Reversing. Make sure teaching goes from: (a) teacher provides objects \( \rightarrow \) students give comparison word, and (b) teachers give comparison word \( \rightarrow \) students provide objects. Also remember from section 1.2 that comparison can be considered as a triad – with three parts, first container, comparison word, and second container– thus there are three “directions”, or three problem types: (a) give first container and word (e.g. larger) and require a larger volume object to be found; (b) give second object and a word (e.g. greater) and ask for an object that the second object has more volume than; and (c) give two containers and ask for word(s) to relate them (e.g. the second container has less volume). This is developing the triadic relationship big idea.

Generalising. Generalise the three things that have already been stated: (a) more and less volume for an object depends on the object it is being compared to; (b) comparison by filling with objects is only accurate if you fill the objects the same way; (c) the object that takes more fill or displaces more water has the larger/largest volume, vice versa; and (d) for comparison, like all triadic relationships, there are three problem types as in reversing.

Changing parameters. Again discuss ordering volume when the material changes in size.
3.3 Stage 3 for Volume: Non-standard units

This stage introduces the notion of unit and develops the measurement processes and principles. Volume is measured in terms of number and non-standard unit (e.g. the box holds 14 blocks).

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Hand

Use solid volume ideas. For example, pack small containers with same objects (e.g. cubes or other small 3D shapes). Measure the volume of a tissue box by stacking Unifix cubes inside. Discuss how to ensure there are no gaps or overlaps or that there is regularity in the gaps and overlaps. Move on to larger containers if you have access to large, same size and shape, objects to pack with. For example, collect supermarket containers – match boxes, 1 litre milk cartons, Weet-Bix boxes, and so on and use these for packing inside larger boxes. You can even pack with tin cans as long as you do it regularly.

Find the volume of objects that are prisms – that is, they have a uniform shape throughout. Get students to stack the Unifix cubes to “make” the shape \( \rightarrow \) count the cubes.

Repeat the process. Construct the first layer: The first layer – the base – has 16 cubes, how many layers do we need to make the shape? Answer might be 12 layers, so....

Develop the formula:

- The first, or base layer has 16 cubes on the base; it is 12 layers high.
- The volume is 12 layers (height) with 16 on the base layer.
- Volume is 12 height layers on the base area of 16.
- Volume is 12 height layers \( \times \) area of base (16).
- Volume is height \( \times \) area of base.

This understanding of formula will build over time; the final statement is where students should be by Stage 5.

Always estimate before measuring and calculating volume. Use virtual materials and pictures to further experience the measuring of volume in terms of non-standard units (e.g. containers).

Mind

Have the students imagine containers being filled with collections of the same objects and determining how many of the smaller objects fit into the larger. Make drawings of what is imagined. Think about the best objects to use for the measure – how best to ensure they pack well?

Mathematics

Practice

Continue to provide situations for students to experience measuring volume with non-standard units – use containers, and also virtual materials and worksheets with pictures. Estimate first. Reinforce accurate measurement processes while the practice is being undertaken. Even have worksheets where students have to identify inaccurate measurement processes.

Continue with finding volume; encourage students to record as accurately as possible.
Connections

Connect non-standard measurement of volume to division. This leads into the inverse relation big idea.

Reflection

Application

Measure volume with non-standard units in real-world situations. Set up volume problems based on non-standard units.

Extension

Flexibility. Find volume non-standard units in local community (e.g. measuring boots of cars in terms of travel cases or bags of cement/mulch). Try to get students to think of ways non-standard volume units are used in the world, local and otherwise. Use history and look at other units used in the past (e.g. the gallon which was the amount of wheat in a standard barrel).

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types: (a) Number unknown – for example, how many blocks to fill the box?; (b) Object unknown – for example, find a box that will hold 10 blocks; and (c) Unit unknown – for example, the container has a volume of 7, what is the unit – this block or that block?

Thus, make sure all three directions are taught. This leads to understanding the triadic relationship big idea.

Generalising. Here the objective is to extend the understanding from Abstraction to Mathematics to teach continuous vs discrete and the three collections of measurement principles: (a) to compare, units have to be common and larger number of units means more volume if common; (b) inverse relation for volume units/units act like division/need for a standard unit; and (c) have to look for accuracy not exactness/smaller units are more accurate but take longer/need to be able to select appropriate units. This is a major part of this unit.

Changing parameters. What if the units were the standard ones, would they act the same with regard to measurement principles as non-standard units? Would this study have common points regardless of whether unit is non-standard or standard?

3.4 Stage 4 for Volume: Standard units and metric conversions

This section focuses on introducing mm$^3$, cm$^3$, m$^3$, and km$^3$ – the cubic units – and on relating the conversions between them to PV positions. The relationships here are: 1000 mm$^3$ is equal to 1 cm$^3$ $(10 \times 10 \times 10)$; 1 000 000 cm$^3$ is equal to 1 m$^3$ $(100 \times 100 \times 100)$; and 1 000 000 000 m$^3$ is equal to 1 km$^3$ $(1000 \times 1000 \times 1000)$.

A. Introducing metrics

Reality/Abstraction

Context

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring volume with a class chosen unit.

Discuss “What is the most efficient volume unit?” – The cube can be argued to be the best because it packs well, is based on square area units, and leads to multiplication.
Identification

Construct and/or experience mm$^3$, cm$^3$, m$^3$, and km$^3$. For 1 mm$^3$, try to find something that is 1 mm long, 1 mm wide and 1 mm high. For 1 cm$^3$, use 1 cm grid paper, draw and cut out a net for a cube of side 1 cm, fold and tape to make the cube, and compare with a centicube and a MAB unit. For 1 m$^3$, make a cubic metre out of 12 pieces of 1 m long dowelling or metre rules (or use a commercial construction kit). For km$^3$, look on internet for something that is as large as this – the capacity of a large dam may be large enough.

Work out How many 1 cm$^3$ fit into 1 m$^3$?

The MAB blocks can help here. The small cubes (the ones) $\Rightarrow$ 1000 of them fit in the largest MAB cube (the hundreds block).

How many of the large MAB cubes will fit in a metre cube?

Construct a metre cube out of dowel or metre rulers, and place the large MAB in one corner. Allow students multiple experiences to explore the relationships:

- $1,000$ mm$^3 = 1$ cm$^3$
- $1,000,000$ cm$^3 = 1$ m$^3$

Realisation

Explore the following realisation question with the students:

Name four objects that could be measured using m$^3$, and then another four objects that could be measured using cm$^3$.

Internalisation

Find things in everyday life that are 1 mm$^3$, 1 cm$^3$, 1 m$^3$, and 1 km$^3$ – so that the students can refer back to them. Some ideas are: (a) obtain a collection of small rectangular prisms (boxes – e.g. matchbox) and pack these with MAB units to find their volume; (b) look at classroom and work out how many cubic metres of air in it; and (c) find out how many cubic km of water in a nearby dam.

Mathematics

The formal conversions, as outlined above for area, can be repeated here. It is important that students have some grasp of the actual size of the mm$^3$, cm$^3$, and m$^3$ as conversions are difficult without this.

Estimation and recognition

First estimate and then measure the volume of objects as below (find some of your own). Measure L, W and H with tape and use calculator to find volume to check the estimates. Estimate and measure each object before moving onto the next. Estimate the volume – do not estimate length, width, height.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>Estimate</th>
<th>Measure</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoe box</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cupboard</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under the table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classroom</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Connections

Build the connection between metric units for volume and PV – see next section.
Reflection

Look at metrics for volume in the world. Think of where needed. Look at applications, flexibility, reversing, generalising and changing parameters. Show how measurement principles such as inverse relation extend from non-standard to standard units.

B. Metric conversions

Reality/Abstraction

Place-value connections

Set up the place-value cards using metric cards as follows:

<table>
<thead>
<tr>
<th>Quintillions</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>km³</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quadrillions</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>m³</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trillions</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm³</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Billions</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm³</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use multiplicative relationships between PV positions to look at the conversion rate between the area units, mm³, cm³, m³, and km³ – 1000 mm³ = 1 cm³; 1 000 000 cm³ = 1 m³; 1 000 000 000 m³ = 1 km³; and 1 000 000 000 000 mm³ = 1 km³. Be flexible with the decimal point to show, e.g. 2.45 m³ = 2 450 000 cm³ (place decimal point after m³, place numerals in PV positions for 2.45, move decimal point to after cm³).

Mathematics

Practice

Use metric expanders and metric slide rules to relate/connect metrics to PV and to each other (conversions). Metric expander is PV set up above with blanks for spaces between metrics: It folds like ordinary number expanders with the pleat fold at the units.

Metric slide rules can be two types:

1. PV chart going from millions to millionths and the metric units on the slide as follows. The slide can move across and back placing any metric area unit at the ones (to see where the other units are). Note: Some educators have the PV on the slide.

2. PV chart having metric area units where the number PVs go, and numbers on slides to move across and back.

Give students experience with specialist measuring techniques associated with measuring length, height and width (e.g. vernier callipers). Undertake outdoor activities using maps and scales. Consolidate the metric conversions through drill – dominoes, bingo, mix and match cards, card decks for Snap and Concentration.
Connections

Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. Volume can be seen to have very large conversions, therefore we have to use a large number of PV positions to get from mm$^3$ to km$^3$.

Reflection

Look at metric conversions in the world. Think of where they are needed. Look at applications, flexibility, reversing, generalising and changing parameters. Show how non-standard principles such as inverse relation extend to metrics.

3.5 Stage 5 for Volume: Applications and formulae

This section covers: (a) applications; (b) formulae; and (c) relationships between length, area and volume, and between volume, capacity and mass. The development of formulae is a major component of volume.

A. Applications

The approach is built around the big idea of triad – there being object, number and unit in any measure means that there are three problem types for each to be unknown: (a) Number unknown – measure the volume of the box in cm$^3$; (b) Object unknown – find an object with a volume of 350 000 cm$^3$; and (c) Unit unknown – this object has a volume of 450 000 cubic units, are these units cm$^3$ or mm$^3$?

The interesting problems are type (c) – unit unknown – as these are not prevalent in schools; these problems require the students to determine the unit used on an object given to them when also given the number of units. They are more difficult if the object is given a number which could be the number of units in any of perimeter, area and volume; and even more difficult if could be length, mass or capacity as well.

B. Formulae

The formulae for volume can be mostly found by connection to area and to other already known volumes and some discovery activities as below. Continue with activities, as in Stage 3 → moving students towards formula.

Reality/Abstraction

Volume of rectangular prism

Construct a rectangle of length 6 cm and width 4 cm as on the right. Use formula for area of a rectangle to work out its area (24 cm$^2$). Use this 6 cm × 4 cm rectangle as a base for a prism. Construct a prism out of 24 cm cubes that has a 6 cm × 4 cm base and height of 1 as on the right – find volume by counting cubes – note that volume is 24 cm$^3$, the same number as its base area. Put another layer of 24 cubes on top – now made a prism of base 24 and height 2 as on the right – find its volume by counting cubes – note that its volume is 48 cm$^3$ or double 24 cm$^3$ or $2 \times 24$ cm$^3$ or $2 \times$ base in cm$^3$. Repeat this for height 3 and height 4. Put these on a table as below. Look at pattern – what is relation of area of base and height to volume – repeat this for other rectangular prisms – discover that volume of a rectangular prism equals area of base × height or $L \times W \times H$. (Note: This can be done virtually – use a drawing like on the right and keep adding copies of it onto the top of the rectangular prism – so you can see that the height multiplies the base for volume.)

<table>
<thead>
<tr>
<th>Height</th>
<th>Area base</th>
<th>Base × height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

Volume of any prism

Repeat the above for other shaped prisms. You may not be able to go back to counting cubes, but you can see that any prism of height 1 has a volume equal to its base area as a number with the units changing from square
units to cube units. Repeat this for height 2, 3 and 4. Use thick examples of the shape and stack them. Use virtual shapes as for the rectangle example above and keep adding them to the prisms. Discuss pattern – students should be able to discover that **Volume of any prism = area of base × H.**

**Volume of cylinder**

Repeat above argument/activity for cylinders of height 1, 2, 3, 4, and so on. Discover that **Volume of a cylinder = area of circular base × H = π × r² × H.**

**Volume of any pyramid**

Construct pyramids and prisms with a corresponding (same) base – use nets and cardboard – leave one base of the pyramid open and the base of the prism open. Fill the pyramid with something like rice, and pour the pyramid contents into the prism (corresponding or same base) – you will see that three pyramids fill each of the prisms of the corresponding or same base. This means that **Volume of any pyramid is 1/3 of the prism of same base = 1/3 × base area × H.**

**Volume of cone**

Repeat above with cylinders and cones of corresponding or same bases. This will show that **Volume of cone = 1/3 of volume of cylinder of same base = 1/3 × area of circular base = 1/3 × π × r² × H.**

**Note:** Area and volume of spheres are difficult to relate to anything else – **Volume of a sphere is 4/3 × π × r³; Area of surface of sphere is 4 × π × R².**

C. **Relationships between attributes**

As we did for the relationships between length, perimeter and area in section 2.5, we need to investigate and discover the relationship(s) between length, area and volume, and look at the particular relationship between volume, capacity and mass.

**Length → volume**

Here are three activities that can show the “8 times for 2 times” relationship.

1. Construct rectangular prisms from cubes – make the second prism double the third in L, W and H – what happens to the volume? It will be found that the volume increases 8 times when length increases 2 times. Repeat for 3 × L, W and H. Does volume increase 9 times? (Challenge – what happens when only L and W change and H stays the same?)

2. Use plasticine or play dough to make two spheres – make them as spherical as possible – keep adding plasticine or play dough to the second until its diameter is 2 times the first. Weigh both spheres. The weight increases 8 times for the larger sphere, thus the volume will also be 8 times. This shows the 2× → 8× or, more generally, the cubic relationship between radius/diameter and volume.

3. Repeat (1) above for cylinders – doubling the radius and H – this will also show the cubic relationship. (Challenge – what happens to volume of cylinder when H is doubled? What happens when R is doubled?)

**Surface → volume**

The surface → volume relationship is similar to the perimeter → area relationship – for a fixed volume, surface area can vary widely, for a fixed surface area – spheres and cubes give the best volume. However, the volume to surface-area ratio increases as things grow in size. Test this with cubes of side 1, 2, 3 and so on, using the table below:

<table>
<thead>
<tr>
<th>Side of cube</th>
<th>Surface area</th>
<th>Volume</th>
<th>Volume ÷ surface area</th>
</tr>
</thead>
</table>
Small things therefore have a large volume to surface-area ratio, while large things have smaller volume to surface-area ratios. Here are two activities that enable students to investigate volume to surface-area ratio.

1. **Roll ups** (an idea from the *I hate mathematics* book). Obtain two pieces of A4 paper, tape, extra paper and some rice or dried beans. Roll the two pieces of paper into cylinders, one “longways” and one “shortways” (see right). Add a bottom to each container and stick down. Use the beans to determine which container holds the most.

Which container holds the most / least – the short fat one or the tall skinny one?

2. **Look after the baby.** Make a shape from 5 square counters. Note the area and perimeter. Make the shape again but double original scale, and again, but triple. Note the areas and perimeters for all three. Record this information on a table. What is the pattern in the relationship between area and perimeter as the size increases? Repeat the above for a 9-counter square, doubling and tripling the length and width. What pattern is emerging in the perimeter ↔ area relationship? Start with 3 cubes and make any 3D shape. Double and triple this in scale. Calculate volume and surface area. What is the effect on surface area of doubling and tripling lengths of sides? What is the effect on volume?

Make cubes of side 1, 2, 3, 4, 5 and 6. Calculate area of base, volume and surface area for these cubes. Put these on a table. What pattern emerges?

What happens to the surface-area to volume ratio? What important consequence does this have for babies? Think about their ability or inability to stay cool or warm.

**Volume, capacity and mass**

There is a particular relationship between the attributes of volume, capacity and mass. This relationship is unique to metric measures and does not apply in a similar way to imperial measures. A cube that is 1 cm × 1 cm × 1 cm has a volume of 1 cm³. If this cube was filled with water (at 4° C) it would weigh 1 gram (g) and the capacity would be 1 millilitre (mL). So the relationship is 1 cm³ = 1 mL = 1 g.

This relationship is seen in car engine sizes where the volume of the cylinders in the engine is sometimes described as cc (cubic centimetres) or as litres (L). A vehicle could have a 4000 cc motor which could also be describe as a 4 L engine.

The relationships between units are:

1 cubic cm (cm³) = 1 cm × 1 cm × 1 cm = 1 millilitre (mL) = 1 gram (g)
1 cubic decimetre (dm³) = 10 cm × 10 cm × 10 cm = 1 litre (L) = 1000 mL = 1 kilogram (kg) = 1000 g
1 cubic metre (m³) = 100 cm × 100 cm × 100 cm = 1 kilolitre (kL) = 1000 L = 1 tonne (t) = 1000 kg.
Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students’ performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students’ existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students’ effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put “not known” beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that any pre-test is a series of questions to find out what they know before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the post-test, the students should be told that this is their opportunity to show how they have improved.

For all tests, teachers should continually check to see how the students are going. Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the relationship measurement item types

The relationship measurement item types are in three subtests – one for perimeter (Unit 1), one for area (Unit 2) and one for volume (Unit 3). Each of the subtests is divided into five parts – attribute, comparison, non-standard units, standard units and applications and formulae. For perimeter, area and volume, there are more item types in Stage 5, applications and formulae, than there were for length, mass and capacity because of the presence of formulae. Stages 3 (non-standard units) and 4 (standard units) will still be important, and the form of standard units and conversions between these units will be complicated. However, regardless of knowledge of the latter stages, a crucial thing in understanding measurement is knowing what the measure is as an attribute, and this is Stages 1 and 2 (i.e. attribute and comparisons with no numbers).

The sequencing in the subtests is across the five stages and pre-tests should focus on early stages (i.e. attributes and comparison as well as non-standard units), while the post-tests should be constructed so that they ensure the final stages are covered (i.e. standard units and applications). As always, the selection of item types should be determined by the teacher’s knowledge of their students. It can be advantageous to mix items from different attributes/subtests so that students have to pick the attribute as well as the stage (particularly in the post-test). However, it is also possible to consider this module as three parts and have pre- and post-tests for each part (with a combined test at the end).
Subtest item types

Subtest 1 items (Unit 1: Perimeter)

Stage 1 – attribute

1. (a) The diagram below is of a basketball court. Mark the perimeter.

(b) The circles show where John walked. Put a cross on the shapes where he walked a perimeter:

(c) The diagram below is the floor plan of a house. Use arrows and lines to show the perimeter of the house and garage.
Stage 2 – comparison

2. Describe a way to compare the **perimeter** around the following shapes, without using numbers.

___________________________________________________________________________
___________________________________________________________________________

Stage 3 – non-standard units

3. Two students measured around the same shape with paperclips. Amy used 12 paperclips (picture A) and Ben used 8 paperclips (picture B).

(a) Whose measurement is more accurate? _________________________________

(b) Why? __________________________________________________________________
___________________________________________________________________________

Stage 4 – standard units

4. (a) mm is a shortened form for __________________

(b) 6 m = _______ cm

(c) 2 km = _______ m
Stage 5 – applications/formulae

5. This hexagon shape has 6 sides; it is not drawn to scale. Each of the sides has a length of 3 cm and the diagonals have a length of 8 cm.

(a) Describe in words how to find the perimeter of the hexagon.
___________________________________________________________________________
___________________________________________________________________________

(b) Calculate the perimeter of the hexagon.

6. This kite shape has 4 sides; it is not drawn to scale. Two of the sides have lengths of 3 cm and the other two sides have lengths of 8 cm. The horizontal from one side to the other is 5 cm.

(a) Describe in words how to find the perimeter of the kite shape.
___________________________________________________________________________
___________________________________________________________________________

(b) How do you calculate the circumference of this circle?
Subtest 2 items (Unit 2: Area)

Stage 1 – attribute

1. (a) Describe how to compare the areas of these two shapes without units.

___________________________________________________________________________

___________________________________________________________________________

(b) Which is bigger and why?

___________________________________________________________________________

Stage 2 – comparison

2. There are six shapes shown on the grid below:

A

B

C

D

E

F

Use the grid to answer the following questions:

(a) Name another shape that has the same area as A __________

(b) Name another shape that has the same area as E __________

(c) Order the shapes D, E, F from smallest to largest __________
3. There are four shapes shown on the grid below:

Use the grid to answer the following questions:

(a) Name another shape that has the same area as A __________

(b) Name another shape that has a smaller area than A __________

(c) Order the shapes A, B, C from smallest to largest _______________

Stage 3 – non-standard units

4. Jack used 12 carpet tiles to cover a table. Jill used 20 books to cover the same table.

(a) Why are the measurements different? _______________________________________

__________________________________________________________________________

(b) Which has the larger area – the carpet tile or the book? _________________

Stage 4 – standard units

5. (a) Write down something that you would measure in square centimetres.

__________________________________________________________________________

(b) Write down something that you would measure in square metres.

__________________________________________________________________________

(c) How many square centimetres in a square metre? __________________________
Stage 5 – applications/formulae

6. What is the area of the shape on right if each of the graduation marks shows 1 m?

7. Using a ruler, draw a shape that is 12 cm$^2$ (12 sq. cm).

8. Find the area of these shapes (the measurements are in centimetres).

   (a)

   (b)
Subtest 3 items (Unit 3 Volume)

Stage 1 – attribute

1. Think about things in your classroom that you can see:

   (a) Name two things that have volume ____________________________________________

   (b) Name one thing that does not have volume _____________________________________

   (c) Explain the difference _____________________________________________________

   ___________________________________________________________________________

Stage 2 – comparison

2. How could you find out (without a ruler) which of the two boxes of chocolates has more
   volume (holds more)?

   ___________________________________________________________________________

   ___________________________________________________________________________

Stage 3 – non-standard units

3. Lots of small cubes (shown below) were used to measure the volume of Boxes A and B.

   Measuring cubes Box A Box B

   When they were counted, Box B had more cubes than Box A.

   (a) Does this mean that Box B has a larger volume than Box A? ____________________

   (b) Why or why not? ___________________________________________________________
Stage 4 – standard units

4. Think of something in the classroom, school or local area that you could measure in:
   (a) Square centimetres: __________________________________________________
   (b) Square metres: ______________________________________________________
   (c) Square kilometres: ____________________________________________________

Stage 5 – applications/formulae

5. Find the volume of the following shapes:
   (a) A box with height 3 cm, length 4 cm and width 2 cm

   (b) A can with height 10 cm and diameter 4 cm

6. Find the volume of the following 3D objects.
   (a) The area of the top of the box is 32 cm$^2$. The height is 8 cm.

   (b) The height of this cylinder is 2 m and the diameter of the base is 4 m.

Challenge question

7. Describe how to calculate the volume of a tent.

   __________________________________________________
   __________________________________________________
   __________________________________________________
Appendix A: Teaching Tools

A1 Metric Expanders

Fold shaded parts so they are only shown as expanders are opened.

Expander A

Expander B

Expander C

Expander D
### A2 Place Value (PV) Chart

<table>
<thead>
<tr>
<th>Whole number PVs</th>
<th>Decimal PVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH</td>
<td>t</td>
</tr>
<tr>
<td>H</td>
<td>h</td>
</tr>
<tr>
<td>T</td>
<td>th</td>
</tr>
<tr>
<td>O</td>
<td></td>
</tr>
</tbody>
</table>

| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |

Cut out PV chart and slides
Cut along dotted lines
and insert slides

### Slides

km | m | cm | mm |
---|---|----|----|
L  | mL|

<table>
<thead>
<tr>
<th>t</th>
<th>kg</th>
<th>g</th>
</tr>
</thead>
</table>
AIM advocates using the four components in the figure on right, reality—abstraction—mathematics—reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).

The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the pattern of threes where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

<table>
<thead>
<tr>
<th>REALITY</th>
<th>ABSTRACTION</th>
<th>MATHEMATICS</th>
<th>REFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <strong>Local knowledge</strong>: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea.</td>
<td>• <strong>Representation</strong>: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea.</td>
<td>• <strong>Language/symbols</strong>: Enable students to appropriate and understand the formal language and symbols for the mathematical idea.</td>
<td>• <strong>Validation</strong>: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge.</td>
</tr>
<tr>
<td>• <strong>Prior experience</strong>: Ensure existing knowledge and experience prerequisite to the idea is known.</td>
<td>• <strong>Body-hand-mind</strong>: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities.</td>
<td>• <strong>Practice</strong>: Facilitate students’ practice to become familiar with all aspects of the idea.</td>
<td>• <strong>Applications/problems</strong>: Set problems that apply the idea back to reality.</td>
</tr>
<tr>
<td>• <strong>Kinaesthetic</strong>: Construct kinaesthetic activities, based on local context, that introduce the idea.</td>
<td>• <strong>Creativity</strong>: Allow opportunities to create own representations, including language and symbols.</td>
<td>• <strong>Connections</strong>: Construct activities to connect the idea to other mathematical ideas.</td>
<td>• <strong>Extension</strong>: Organise activities so that students can extend the idea (use reflective strategies – flexibility, reversing, generalising, and changing parameters).</td>
</tr>
</tbody>
</table>
### Appendix C: AIM Scope and Sequence

<table>
<thead>
<tr>
<th>Year</th>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>N1: Whole Number Numeration</td>
<td>O1: Addition and Subtraction for Whole Numbers</td>
<td>O2: Multiplication and Division for Whole Numbers</td>
<td>G1: Shape (3D, 2D, Line and Angle)</td>
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<tr>
<td></td>
<td>Early grouping, big ideas for H-T-D pattern of threes; extension to large numbers and number system</td>
<td>Concepts; strategies; basic facts; computation; problem solving; extension to algebra</td>
<td>Concepts; strategies; basic facts; computation; problem solving; extension to algebra</td>
<td>3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches</td>
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<tr>
<td></td>
<td>N2: Decimal Number Numeration</td>
<td>M1: Basic Measurement (Length, Mass and Capacity)</td>
<td>M2: Relationship Measurement (Perimeter, Area and Volume)</td>
<td>SP1: Tables and Graphs</td>
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<tr>
<td></td>
<td>Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system</td>
<td>Attribute; direct and indirect comparison; non-standard units; standard units; applications</td>
<td>Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae</td>
<td>Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction</td>
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<td></td>
<td>Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability</td>
<td>Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships</td>
<td>Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject</td>
<td>Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference</td>
</tr>
<tr>
<td></td>
<td>O3: Common and Decimal Fraction Operations</td>
<td>O4: Arithmetic and Algebra Principles</td>
<td>N4: Percent, Rate and Ratio</td>
<td>G3: Coordinates and Graphing</td>
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<tr>
<td></td>
<td>Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation</td>
<td>Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification; expansion and factorisation</td>
<td>Concepts and models for percent, rate and ratio; proportion; applications, models and problems</td>
<td>Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs</td>
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<td></td>
<td>Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs</td>
<td>Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio</td>
<td>Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities</td>
<td>Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics</td>
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<td>N5: Directed Number, Indices and Systems</td>
<td>G4: Projective and Topology</td>
<td>SP3: Statistical Inference</td>
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<td></td>
<td>Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems</td>
<td>Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks</td>
<td>Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences</td>
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<td></td>
<td>O4: Arithmetic and Algebra Principles</td>
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</tbody>
</table>

**Key:** N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.