Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module M1

Year A, Term 2

Basic Measurement:

Length, Mass and Capacity

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

This module, Module M1 *Basic Measurement*, is the first of the measurement modules and covers all the stages for teaching the measurement topics of **length, mass** and **capacity**. These are measurement topics that do not lead to formulae and are "basic" in the way they move from concept to units and applications, and in the way place value is related to metrics.

Note: Perimeter is defined as the distance around an object and could easily be included as part of length but is left to Module M2 *Relationship Measurement* which deals with perimeter, area and volume which have related formulae.

Background information for teaching measurement

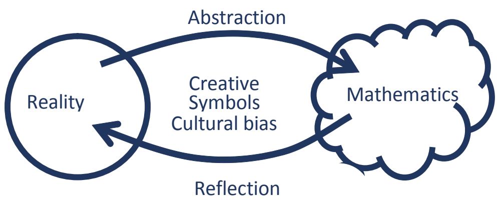
In this section we look at cultural implications and big ideas and connections for length, mass and capacity.

Cultural contextualisation

Cultural bias

The mathematics taught in Queensland schools is based on Western culture, originating in Europe and influenced by America. As presented in schools, Western mathematics is based on number – whole numbers, fractions, money, measures, chance and data. As such, it reflects an imperative of Western culture regarding the primacy of number. In this, mathematics reflects an often unidentified component of its nature, that it is culturally based and biased and that its teaching has a large aspect of enculturation in relation to Western culture. (*Note:* Two other major cultures are also similar in their focus on mathematics and number, Hindu-Arabic and Chinese-Japanese, and Australia has many people from these cultures, but Australian mathematics education is still primarily European based.)

By its nature, mathematics is a creative cultural and contextual *abstraction* (or generalisation) of everyday life that empowers people to solve their problems. However, as shown in the diagram on right describing AIM's view of the philosophy behind mathematics knowledge growth (Matthews, 2006), such abstraction has to take account of *local culture and context* in the abstraction process which gives rise to cultural bias. Unthinking mathematics teaching transfers that bias to students, and can negatively affect cultural beliefs and pride in heritage of students not of the dominant culture.



THE RELATIONSHIP BETWEEN LIFE AND MATHS

Particular effect of measurement

Number applies to discrete objects. However, there are many important attributes that are not discrete, rather they are **continuous**, for example, length and area (in fact all of the measures). A child's height does not grow in jumps of 1 cm – it smoothly increases. Similarly, the ground and the ocean spread in all directions continuously, they are not naturally broken into square metres. Thus, number does not naturally apply to measures such as length, mass, capacity, area, volume, temperature, time, angle, and value.

Western culture invented the unit to apply number to these continuous measures; that is, a set amount of the measure that could be repeated across the object being measured to determine the number of the units that fitted into the object. This finding changed number from general to specific and means that, in measurement, it is necessary to state both the number and the unit, such as 56 cm, 4.5 m^2, 6 kg, and so on. Thus, Western culture found a way to "discretify" the continuous – to break the unbroken into parts that can be counted. It is important that the units which enable this to happen fit together across what is being measured without gaps and overlaps. Therefore the type of unit, particularly its shape, is important, especially in area, volume and angle.

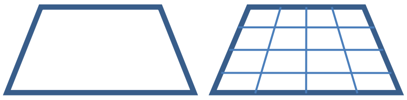


Diagram A Diagram B

It is also important to note that using units can change the way a person perceives the attribute or measure. For example, land can look very different when seen as a continuous area than when seen as broken into units (Diagrams A and B on right). Diagram B tends to focus attention on parts and personal ownership and then on to ideas like "how much", "how can I get more", "what can I buy and sell", "how can I exploit this bit", while Diagram A focuses attention on the whole and on to ideas like "how does it all interact" and "how can we keep it together". Diagram B looks like house blocks while Diagram A looks like a national park. This change in perception can affect cultural values. For example, some Indigenous cultures have no conception of "owning land", and teaching area as square units can change this profoundly and challenge culture.

Questions

Therefore, questions that are basic for this module are as follows.

1. Why would a society give high cultural importance to number? What effect does this have on that society? What strengths and weaknesses of such a society could be attributed to their number orientation? That is, what strengths and weaknesses does number give a society like Australia?

1. How would a number-oriented society interact with other societies, particularly societies that do not have its number orientation? What effect could Western teaching of number have on children from non-Western societies (e.g. Australian Indigenous people)?

2. What role does measurement have in relation to number? Could the teaching of Western measurement have a negative effect on non-Western societies (e.g. Australian Indigenous people)?

3. In particular, how can we teach Western measurement so that Indigenous students understand it yet stay strong in and proud of their culture? (Both outcomes are necessary for a strong future for Indigenous students.)

Some answers

To teach measurement in its natural state and to teach Western "numbered" measurement, YDM recommends a five-stage process. Stages 1 and 2 are pre-number, identifying the attribute and comparing and ordering it without number; Stage 3 introduces the notion of unit and applies number to continuous attributes for the first time – using child-chosen or informal units; Stages 4 and 5 are post introduction of number and deal with standard units (metrics) and applications.

For Indigenous schools where the culture differs from the mainstream Western culture of normal classrooms, it is important to use the stages of measurement teaching to maintain Indigenous culture as well as learning Western measurement. To achieve both these outcomes, the following are important. It is important to spend time on Stages 1 and 2 – identifying the attribute, and comparing and ordering (without units or number). This means spending time getting to know what the attribute means (e.g. what area, mass and angle actually are), and what it means for there to be more or less of the attribute (e.g. what does it mean for an area, mass or angle to be larger than another one).

The crucial part of this stage is to enable the students to understand measures such as length, area, mass and capacity as continuous entities and to relate this to the understanding of these attributes that are in the local culture. This requires an understanding of **associated language** as attribute language is a crucial part of early measurement. If we take the example of length, we find that lengths in different directions and of different types have different English words, that is, long/short, tall/short, wide/narrow, thick/thin, deep/shallow, high/low, and so on. Some differences between words are subtle such as the difference between "tall" and "high"; other words also mean special types of length (e.g. "distance"). All these different words associated with length need to be understood. With students of a different culture and home language, this requires initially associating Western meaning with local cultural understanding and Western mathematics language with home language. And besides the language, students also have to learn **associated symbols** for the standard units, for example, for length, we have mm, cm, m, and km.

Thus, mathematics teaching needs to be from the local world of students to the formal abstract mathematics and back again so that it imitates the nature of mathematics. This means that **mathematics teaching should move between three worlds:** (a) the culture and context of Indigenous culture; (b) the informal understandings based on materials, pictures, informal language and patterns that can bridge from everyday life to abstract mathematics; and (c) formal abstract mathematics (formal language and symbols).

Big ideas and connections

Big ideas for measurement

The big ideas for measurement are an important part of this module and are as follows:

1. **Continuous vs discrete**. Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted.

2. **Interpretation vs construction**. Things can either be interpreted (e.g. a m is 100 cm) or constructed (construct a m out of 10 cm lengths of straws).

3. **Notion of unit**. Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts (e.g. 1000 mm makes a m and 1000 m makes a km).

4. **Multiplicative structure.** Standard units are designed so that they reflect place value in that adjacent positions are related by moving left (× base) and moving right (÷ base), where the base is 10 for metrics but 60 for time and angle and, in practical terms, 100 for dollars-cents and temperature.

5. **Common units**. We must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60) and, if we do so, the object with the biggest number has the most attribute.

6. **Inverse relation**. Same as Extension of field properties principle (i.e. the bigger the unit, the smaller the number – e.g. 2 m = 200 cm).

7. **Accuracy vs exactness.** Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100) or exactly (5 275 + 3 873 = 9 148).

8. **Attribute leads to instrumentation**. The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring).

9. **Triadic relationships**. When three things are related, there are three problem types (e.g. measuring length has three components, the object, the number and the unit – thus we can set problems where the object is unknown, the number is unknown or the unit is unknown).

10. **Balance rule**. Whatever you do to one side of an equation you have to do to the other to keep things equal.

Major big ideas

The major big idea here is 1 above, **continuous vs discrete**. All measurement topics are continuous and cannot naturally be counted or represented by numbers. However, the invention of unit has allowed the continuous to be “discretified” or partitioned into units and these units can be counted. This application of unit changes learners’ perception of the units. Thus, the sequence for teaching measurement topics has three parts: (a) understanding the measurement topic in its natural continuous state (Stages 1 and 2 below); (b) introducing unit and number to measurement topics (Stage 3 below); and (c) understanding the standard units adopted by Australia and applications of formulae for, and relationships between, these units (Stages 4 and 5 below).

Big ideas 5, 6 and 7 have a special role in measurement – they are the **measurement principles**. Big idea 5, *common units*, ensures that, when measurements are compared or used in computation, they are common units; and if there are common units, the bigger the number the more the attribute. Big idea 6, *inverse relation*, gives the important principle that the bigger the unit, the smaller the number and vice versa. This is why, in measurement, number can never be alone: measures are given by **number and unit**. Big idea 7, *accuracy versus exactness*, underlies that, in measurement, instruments are such that there is only accuracy not exactness. This leads to being able to choose appropriate units. These three principles are part of Stage 3, Non-standard units.

As well as the above big ideas, big idea 9, *triadic relationships*, underlies the three types of applications in Stage 5, and big idea 10, *balance rule*, enables the subject of a formula to be changed, also in Stage 5.

Important connections involving measurement

There are three connections that are very important.

1. **Connection between unit, fractions and division**. Fractions divide a whole into equal pieces, division does the same, and so does a unit – it divides a length measure (e.g. length) into equal sub-measures (e.g. m or hand span). Thus, division, fractions and units of measure all have inverse relations – that is the less to share (divide) the more each sharer gets, the smaller the denominator, the larger the fraction, and the smaller the unit the larger the number of units (e.g. a wall measures 7 m or 7 000 mm).

2. **Connection between place value (PV) and metric conversion**. If whole number and decimal place values are built around a pattern of threes (macrostructure) ... billions, millions, thousands, ones, thousandths, ... with a sub-pattern (microstructure) of Hundreds, Tens and Ones, then thousandths can be mm, ones can be m and thousands can be km, or ones can be mm, thousands m and millions km. In this way, place value can be seen as the same structure as metrics. If multiplicative structure is built between the macrostructure, we can see that thousandths → ones → thousands → millions is ×1000 and millions → thousands → ones → thousandths is ÷1000. This leads to metric conversions, for example, mm → m → km is ×1000 (1000 mm = m) and km → m → mm is ÷1000 (m÷1000=mm).

3. **Connection between PV, metric and algebra computation**. For 34 + 52, the PVs are separated so 4 ones is added to 2 ones to give 6 ones and 3 tens is added to 5 tens to give 8 tens. Thus, for 3 m 46 cm + 5 m 21 cm, the answer is 3 m + 5 m and 46 cm + 21 cm which is 8 m 67 cm, and for 3a + 4b add 5a + 2b is 3a + 5a and 4b + 2b which is 8a + 6b.

Sequencing for basic measurement (length, mass and capacity)

This section briefly looks at sequencing for basic measures and within this module.

Sequencing in basic measurement

The sequencing lies in the five stages. It is crucial to work through all these stages in order as follows.

Stage 1	→	**Stage 2**	→	**Stage 3**	→	**Stage 4**	→	**Stage 5**
Identify attribute		Comparison (no numbers)		Notion of unit/ non-standard units		Standard units		Applications and formulae

This is particularly so for Stages 1 and 2 where number is not used and Stage 3 where the notion or idea of measurement **unit** is introduced. Do not be led into believing that the first three stages are unimportant because they do not deal with metrics – they lay the foundation for understanding measurement – particularly the big ideas of measurement.

Stage 1 – Attribute identification

This stage focuses on students understanding the attribute (or concept) of the measure. Activities to identify attributes: should follow **rich experiences** with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, and so on. They should also involve **developing meaning** for all the specialist attribute language that accompanies measurement topics.

The central idea in learning about an attribute is to experience it. However, if students have difficulty identifying the attribute from other characteristics of the experience, there are two general ways to introduce any attribute by providing examples where: (a) the only thing that is the **same** is the attribute, and (b) the only thing that **varies** is the attribute.

Stage 2 – Comparing and ordering

This stage focuses on comparing (i.e. two examples) and ordering (i.e. three or more examples) the amounts of attribute in examples. The process of ordering is based on comparison; the ability to compare two examples is extended to ordering three examples by identifying the one that is **between** the other two. Stage 2 activities are learnt in two parts: (a) **direct comparison** and order where examples are compared directly to each other; and (b) **indirect comparison** and order through an intermediary. Stage 2 activities involve **no units** and **no numbers**; the total amounts of the attribute present are **compared or ordered holistically**.

Stage 3 – Non-standard units

This stage has two foci: (a) introducing the **notion of unit**; and (b) the development of measurement **processes** (using instruments with which to measure) and measurement **principles** (i.e. big measurement ideas that hold across all measurement topics). To prevent too much new information being given at once, the units are non-standard or class/learner chosen so that the learner is familiar with them. The measurement processes differ for different topics; they are related to the proper use of the measuring instruments. The **measurement principles** are the techniques for using units. These are: (a) *common units* – common units must be used in measuring, comparing/ordering and calculating amount of attribute and, in this case, the example with the most attribute has the larger number of units (this leads to the need for a standard); (b) *inverse relation* – the bigger the unit, the smaller the number and vice versa; and (c) *accuracy vs exactness* – all units give rise to error, with smaller units being more accurate but more difficult to apply, so there is a need to choose the level of accuracy required for the job. This principle requires a tolerance for error and also leads to the skill of choosing appropriate units and to developing skill in estimating as well as accurate measuring.

Stage 4 – Standard units

This stage focuses on the introduction of the standard units accepted by Australia. It should be remembered that these units should only be introduced **after** the need for a standard has been determined by recognising the limitations of non-standard units. It is recommended that they be preceded by the use of a class-chosen **common unit** (if appropriate). The recommended sequence for introducing standard units is: (a) **identifying** the unit through experiencing it or constructing it; (b) **internalising** the unit through relating it to body or everyday activities; and (c) **estimating** with the unit before measuring. Stage 4 activities should also relate to the **decimal number system** to build understanding of conversion between units and should continue to develop the measurement **processes and principles** begun in Stage 3.

Stage 5 – Applications and formulae

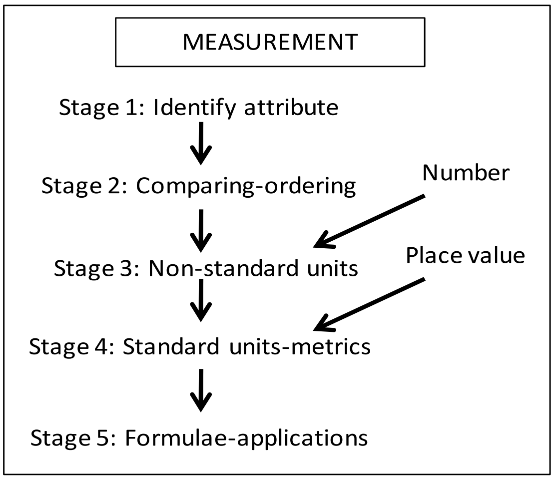
This stage focuses on: (a) applications of measures to the real world (i.e. calculating the measure of things); and (b) any formulae for determining measures (tends to be restricted to some perimeter, area, volume and angle).

Note: There are no formulae for length, mass and capacity.

Sequencing in this module

This module has to cover all stages for length, mass and capacity. The sequence for each measure is the five stages as on right. The first two stages have **no number**, the third introduces unit and number, the fourth introduces metrics, and therefore is based on place value, while the fifth looks at applications and introduces formulae.

The metrics for length, mass and capacity are the same as place value – milli is ones, unit is thousands, and kilo is millions. Because of size, we also utilise the unit tonne which stands for a billion grams, and use grams, kilograms and tonnes for the normal range of measures.



The sections of this module are as follows:

Overview: Background information, sequencing and relation to Australian Curriculum

Unit 1: Length – attribute, comparing, non-standard units, standard units, applications

Unit 2: Mass – attribute, comparing, non-standard units, standard units, applications

Unit 3: Capacity – attribute, comparing, non-standard units, standard units, applications

Test item types: Test items associated with the three units built around the five stages which can be used for pre- and post-tests

Appendix A: Teaching tools to assist with aspects of the units

Appendix B: RAMR cycle components and description

Appendix C: AIM scope and sequence showing all modules by year level and term.

The sequence within this module does not follow the same structure as for the three modules that preceded it. Here, there is a sequence across the three units in that, normally, measurement teaching would follow the sequence length → mass → capacity. However, the strong sequence is **within** each of the three units where the five stages form the subheadings of each unit.

Within each stage in the units, the teaching ideas follow the RAMR cycle which is described in **Appendix B**.

Relation to Australian Curriculum: Mathematics

AIM M1 meets the Australian Curriculum: Mathematics (Foundation to Year 10)				
Unit 1: Length Unit 2: Mass Unit 3: Capacity				
Content Description	**Year**	**M1 Unit**		
		1	**2**	**3**
Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language (ACMMG006)	**P to 2**	✓	✓	✓
Measure and compare the lengths and capacities of pairs of objects using uniform informal units (ACMMG019)		✓	✓	
Compare and order several shapes and objects based on length and capacity using appropriate uniform informal units (ACMMG037)		✓	✓	
Compare masses of objects using balance scales (ACMMG038)				✓
Measure, order and compare objects using familiar metric units of length, mass and capacity (ACMMG061)	**3**	✓	✓	✓
Use scaled instruments to measure and compare lengths, masses and capacities (ACMMG084)	**4**	✓	✓	✓
Choose appropriate units of measurement for length, capacity and mass (ACMMG108)	**5**	✓	✓	✓
Calculate the perimeter of rectangles using familiar metric units (ACMMG109)		✓		
Connect decimal representations to the metric system (ACMMG135)	**6**	✓	✓	✓
Convert between common metric units of length, mass and capacity (ACMMG136)		✓	✓	✓
Find perimeters and areas of parallelograms, rhombuses and kites (ACMMG196)	**8**	✓		
Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)		✓		

Unit 1: Length

Length is a measure of how far – along an object, a line, a road or the side of a building, or around an object. It is the 1-dimensional measure. It has a large vocabulary of words (e.g. short, narrow, shallow, and so on). It is normally integrated with perimeter (the distance around something). However, for AIM, perimeter is covered in Module M2 *Relationship Measurement* so that it can be connected to area and volume.

1.1 Stage 1 for Length: Attribute identification

This stage focuses on identifying length using appropriate vocabulary: short or long, wide or narrow, thick or thin, high or low, deep or shallow, near or far, up or down, distance around, perimeter. Although distance around is used in this unit, the main development of perimeter is in Module M2 *Relationship Measurement*.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations. For example, distance, heights, legal limits for fish, size of trucks or road trains.

Abstraction

Body

Use actions, mimes and dances that represent growing higher or growing longer and shorter (e.g. a tree growing taller, a fish getting longer). Act out with students' bodies all the length words.

Cut strips of ribbon to students' heights and stick on walls or mark heights on walls; draw around a student lying down; draw around a student's foot.

Experience objects in class and home that are long/short, and so on (e.g. the board is wide, the door is high, a pencil is narrow, a book is thick or thin, and so on). It is always comparative.

Check the different sizes of tree trunks by hugging trees. Or trace a finger around the outside of an object.

Hand

Experience a variety of materials. Put out five pencils of different lengths (what is different?) or put out a feather, a pencil, a strip of paper, a can, and a duster that are all the same length (what is the same?).

Hold up an object, look for things that are the same length as it, look for things that are different lengths to it.

Experience different ways to get length – distance around a can, diagonally across a blackboard.

Get students to follow directions to find things – it's far from the desk, beside the bin, and so on.

Mind

Students shut eyes and you tell them a particular object and ask them to think of things that are thinner, wider, taller etc. than that object. Think of things that are the same length.

Mathematics

Practice

Worksheets where students sort objects into long and short, thick and thin, and so on.

Connections

Show non-connections – find long and thin, short and wide, and so on.

Reflection

Application

Take new understanding of length out into the school grounds and identify long/short, wide/narrow, and so on.

Extension

Reversing. Get students to draw their own long and short, or own thin and thick, and so on. (Note: Go from word to drawing or object as well as object or drawing to word.)

Generalising. Look at high and tall. Is a tall student or object on the ground taller than a short student or object on a chair (or just higher)? What does this mean when thinking about length?

Hold up a piece of wood, say is this long or short? Discuss what people might say in different situations (e.g. it's long for a piece of firewood but short for the trunk of a tree). Discuss that long/short depend on perspective or situation or experience of people. Discuss what the attribute of length is – its "longness" or "shortness".

Discuss ways in which we could determine which of two examples is longer.

Changing parameters. Consider length not being a straight line. Is a curly line long or short? What about a spiral line? What happens when a tall student lies on the ground?

1.2 Stage 2 for Length: Comparing and ordering

Comparing activities with two examples should come before ordering activities with three or more examples. The change from comparing to ordering is more difficult as it requires a focus on what is *between* the other two examples.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

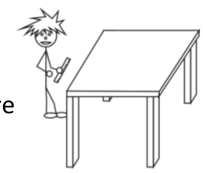
Body

Comparing. Directly compare and order using students' bodies. Begin with comparing length attributes of two bodies (e.g. height). Continue to develop vocabulary for length by creating situations that will allow students to compare which body is taller/shorter, higher/lower, longer/shorter distance around and so on.

Ordering. Compare length of parts of body and extend to ordering experiences by using more than two examples of each. Include situations where students experience comparison and ordering between thickest/thinnest, nearest/farthest, deepest/shallowest, and so on.

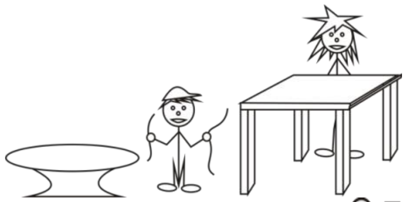
Hand

Comparing. Directly compare lengths where **both** objects can be physically moved together for the comparison. Also should compare thicknesses and widths of objects.



Directly compare lengths where **only one** of the objects can be physically moved. Compare different types of length, including thicknesses, width and depth.

Indirectly compare lengths where **neither object** can be moved by using an **intermediary** (e.g. string, paper strips, part of own body, and so on). Compare different types of length, including thicknesses and depth.



Ordering. Repeat all the cases above but this time **ordering** more than two examples. Focus on finding the "between" examples. Introduce language such as tallest/shortest, longest/shortest, widest, narrowest, and so on. Introduce language and categorise objects by near and far.

Virtual manipulatives can be a very powerful way to learn about ordering and comparing.

Models can be longer/shorter, taller/shorter, wider/narrower, etc. Different versions of things can be moved around and objects, animals, and so on built from parts that are different lengths (e.g. put the shortest tail on the longest dog, put the tallest boy into the shallowest end of the pool, and so on). As well, things can be moved around contexts (e.g. put the ball next to the chair, put the bat away from the chair, and so on). Repeat the above for pictures (e.g. circle the wider, or widest, door in the picture, and so on).

Mind

Shut eyes and imagine someone getting taller/shorter, wider/narrower, and so on. Us this as an opportunity to discuss all the different words for length comparison – shut your eyes, imagine you have jumped into the deepest end of the pool, what will happen; now you jump in the shallowest part, what will happen?

Imagine, then draw and describe a variety of objects, demonstrating comparisons and orderings of length. For example: a taller and shorter tree; a higher or lower book in a bookcase; a wider or narrower fish; a set of fish swimming at different depths.

Special activity. What is longer – the height of a glass or the distance around its rim?

Mathematics

Practice

Give students many opportunities to compare and order the length of objects both directly and through the use of an intermediary.

Introduce notation for comparison (e.g. A>B and B<A). Make sure the rules of comparison are known – non-reflexive: A is not < A; antisymmetric: A<B → B>A; and transitive: A>B and B>C → A>C.

Connections

Connect to line – draw a straight line, take two objects of different length and, in turn, put one end on the start of the line and mark the end points on the line. Discuss what the positions of the end points mean [longer is further along the line].

Reflection

Application

Discuss real-world situations of longer/shorter, and so on (do all the words). Find things that are between (e.g. wider than the door but narrower than the desk, and so on).

Extension

Flexibility. Think of situations where you would use the length words (i.e. longer/shorter, longest/shortest, wider/narrower, widest/shortest, and so on). Mix things up – find the object which is the shortest and widest, and so on; find the object which is far from the chair, near the post, taller than the car, wider than the post box, and so on.

Reversing. Make sure you do all the directions – words ←→ objects (e.g. you give the comparison such as wider or widest, and the students find objects where one is wider or widest; you provide the objects, students determine which is wider/widest, taller/tallest, thinner/thinnest, and so on).

Remember that comparison can be considered as a **triad**, with three parts: first object, comparison word, and second object – thus there are three “directions”:

(a) give first object and word (e.g. thinner) and require a thicker object;

(b) give second object and a word (e.g. thinner) and ask for an object that the second object is thinner than; and

(c) give two objects and ask for word(s) to relate them. This is developing the *triadic relationship* big idea.

Generalising. Try to find things that hold true for all comparisons of length. Put two objects together in a way that shows the shorter is longer [leads to the generalisation that you must have a common starting point].

Generalise the connections activity above [the object whose end point is further along the line is the longer]. Discuss how this could lead to a way of measuring length. This begins understanding of the *attribute leads to instrumentation* big idea. Show that an object can be both longer and shorter [comparison depends on the size of the object to which it is compared].

Generalise the reversing activity above [when three things exist, there are three directions and three problem types]. Comparing involves first object, second object and comparison. Problem types are: (a) give the two objects, ask for the comparison; (b) give first objects and comparison, find second object; and (c) give second object and comparison and find first object (see Stage 5).

Changing parameters. Instead of looking for, for example, longest or shortest, look for a length in between (e.g. find something whose length is in between the board’s height and the door’s width). Look at length which is not straight. Use intermediaries to find which of a wriggly line and a spiral is longest.

1.3 Stage 3 for Length: Non-standard units

This section focuses on introduction of the idea of using a constant unit to measure length; and development of measurement processes and principles for length.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Introduce the idea that a given length, like a pace, can be used to give a number to a larger length. Pick a wall, determine how many teacher paces it takes to “walk” the wall (e.g. 9 paces). Then say that this means that the wall is 9 teacher-paces long – a **number** and a **unit**. Get the students to measure things with their own paces and write down the number and the unit.

Always get students to estimate before measuring and to give answers as numbers and units. Make students aware of the correct use of paces in measuring. Measure a wall by correct pacing. Then, act out various “incorrect” ways to do the pacing, as follows. (*Note*: When we get different numbers for the same wall, ask the students what is unfair about these examples. This method is called “torpedoing”.)

1. **Not starting at the beginning of the wall.** Start 2 or 3 paces along the walls and ask why the number is less. Start well before the wall and ask why the number is larger.

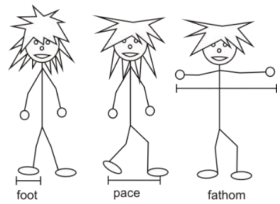
2. **Not ending at the end of the wall.** Try this and ask why the number is larger or smaller depending on when the walking finishes.

3. **Not walking in a straight line.** Try a really wobbly walk and ask why the number of paces is larger.

4. **Not keeping paces the same**. Walk the wall with a variety of different length paces. Ask why the number is larger or smaller depending on whether paces are mostly short or long. This leads to the first aspect of the *common units* big idea – **units in a measure must be the same**.

Repeat the above two steps for different lengths, short and long, fitting the body parts on the right into the lengths (e.g. how many digits along the side of an A4 sheet?)

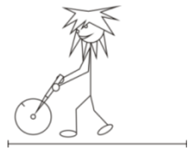
Again use "torpedoing" to get across correct measurement processes: (a) start and end units in line with the start and end of the object being measured; (b) place units beside each other in a straight line; (c) use the same units throughout the measuring; and (d) do not have gaps between the units.



Hand

Use a variety of non-standard units to measure the lengths (e.g. how many dusters long is the board, how many pencils wide is the desk, how many sticks wide is the playground?). Objects that could be used as units to measure length could include pencils, pencil cases, sheets of paper, blackboard dusters, Smarties, Cuisenaire rods, blocks, straws, cardboard strips, pegs, paper clips, lengths of dowel, lengths of string, and so on.

Good units are **multiples of lengths of everyday things**. For example, cut out a copy of your foot and use this as a non-standard unit. Use anything that **rolls** and count the revolutions turned when the object is "wheeled" along the length (e.g. bicycle wheel, cotton reel, can, dish, and so on).



Make up a **non-standard ruler** – string beads or pieces of straw onto string, stick short sections of paper end on end, and so on. Count the number of units which are alongside the object being measured. Use **virtual materials and pictures** to experience non-standard units.

Things that also should be done: (a) repeat the activities from Body to reinforce the measurement processes that give accurate measures; (b) always estimate before measuring; and (c) always give answers as number and unit.

Mind

Have students draw units beside the object being measured. Have students imagine units being used to measure the length of an object. Also imagine accurate measurement processes used in the Hand experience and represent how the length was measured.

Mathematics

Practice

Continue to provide situations for students to measure length – use materials, pictures and worksheets. Estimate first. Reinforce accurate measurement processes while the practice is being undertaken. Allow opportunities for students to identify inaccurate measurement processes.

Connections

Connect non-standard measurement to number-line division. For example, the diagram on the right can be considered as the length of the rod divided by the paper-clip length. For division, the more people there

are to divide the cake among means that each person gets less cake. It is the same here, increasing the length of the paper clip means less paper clips. This leads into the *inverse relation* big idea.

Reflection

Application

Measure with non-standard units in real-world situations. Set up measurement problems based on non-standard units.

Extension

Flexibility. Find non-standard units in local community (e.g. number of cans of fuel to measure distance travelled by boats). Try to get students to think of ways non-standard units are used in the world, local and otherwise. Use history and look at other units used in the past (e.g. the mile which was 2000 paces of the Roman army).

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types, as in Stage 5. Thus, make sure all three directions are taught. This leads to the *triadic relationship* big idea.

Generalising. Here the objective is to extend the understanding from Abstraction and Mathematics to teach continuous vs discrete and the three measurement principles as follows. This is a major part of this unit.

1. **Continuous vs discrete**. Discuss how the act of measurement is the same as counting on a number track, ladder or line. Look at the line **without** regular marks or divided into units. Look at the line **with** these things. Elicit that the line is continuous and cannot be naturally counted. Discuss how the unit breaks it into parts that can be counted. Relate to ruler and measuring with rulers. This leads to the *continuous vs discrete* big idea – that **there are two ways that number is applied**: (a) to discrete objects, and (b) to continuous things such as lines by the use of units to discretify the continuous line.

2. **Measurement principle 1: Common units.** Use torpedoing to show that we cannot know how long something is if we do not know the unit or have the **same unit**. For example, come into the class and say you caught a 24 unit long fish and draw on board. Then say also that a friend caught a 36 unit long fish and ask students to mark beside the drawing of the 24 unit fish where the 36 unit fish would end. Then put up a smaller fish for the 36 unit fish and ask how can this be? [Most students will say that using smaller units.]

   Another example is to set a tall student and a short student to pace a wall and write their answers up in number of paces. The numbers should be different and you can ask why is this so? [Once again most students will say because pace length is different – the bigger number is caused by the smaller pace.] Repeat activities like this as much as needed. This leads to the second aspect of the *common units* big idea – that **units must be the same length when comparing objects**.

   Then set a **common class unit** – let the students choose it – with which all can measure the lengths of objects. Discuss what a bigger number will mean in this case when all are using the same length unit. This leads to the third aspect of the *common units* big idea – that **when units are the same, the larger number specifies the longer object.**

   Move discussion onto situations where something, say for a house, is being made in one town or country and wanted in another town or country. What is needed to ensure that the thing being made is the right length? Discuss buying long and selling short – using a tall person to buy fabric by the fathom and a small person to sell fabric by the fathom – discuss how this would lead to anger and difficulty in buying the right amount of fabric. This leads to the fourth aspect of the *common units* big idea – that **there is a need for a standard**.

3. **Measurement principle 2: Inverse relation.** Measure things with large and small units, with units that are $^1/_2$ and $^1/_3$ the length of other units. Record the results as follows.

UNIT	OBJECT	NUMBER
Small stick	Desk	13
Medium stick	Desk	9
Large stick	Desk	6
$^1/_2$ large stick	Desk	12
$^1/_3$ large stick	Desk	18
Double large stick	Desk	3

   The pattern is easy to follow – the larger the unit the smaller the number and this is in inverse relation to the unit length. It is like division, decrease or halve the divisor is to increase or double the quotient and vice versa. (Activities like this lead to the *inverse relation* big idea – **the larger the unit, the smaller the number and vice versa.**)

4. **Measurement principle 3: Accuracy vs exactness**. Get students to cut ribbons to 12 small units (e.g. 12 fingers). Get them to compare ribbons and they will find slight differences in length. Ask them why? [Most students realise that in measurement you cannot be exact, you can only be as accurate as you want or are able.] Get students to measure a length in **complete** larger units and smaller units and then cut ribbon to the length shown with both units. Compare the ribbons with the object. Students should see that the smaller units are more accurate. This leads to the first consequence of the *accuracy vs exactness* big idea – that **smaller units give greater accuracy**.

   Discuss what smaller units being more accurate means. Discuss whether this means that we should always measure in small units. Argue about whether there are times when such accuracy is not needed. Provide students with measurement tasks for contexts (e.g. measure the window for a replacement window sill, measure the side of the classroom so we can work out if it is longer than other classrooms, and so on). Discuss what would be the best units for each task. Continue this line of thought by giving students a variety of objects of differing length and a variety of measurement instruments. Have them select and record the appropriate instrument to measure the length of each object. Practise these concepts with more tasks where students devise their own measurement plans, involving them selecting the unit of measurement and estimating and checking: Students must reflect on the quality of their measurement instrument and accuracy of estimation. This leads to the second consequence of the *accuracy vs exactness* big idea – students require **skill in being able to choose appropriate units.** Discuss whether estimation is ever good enough. Find situations in which it is. This leads to the third consequence of the *accuracy vs exactness* big idea – students require **skill in estimating**.

Changing parameters. What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? What if the units were not length – say they were mass or area? Would they need similar study of measurement processes and principles? Would this study have common points?

1.4 Stage 4 for Length: Standard units and metric conversions

This stage introduces the standard metric units of length and their relationships:

1 centimetre (cm) = 10 millimetres (mm)
1 decimetre (dm) = 10 cm
1 metre (m) = 10 dm = 100 cm = 1 000 mm
1 kilometre (km) = 1 000 m = 100 000 cm = 1 000 000 mm

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring length with a class-chosen unit – e.g. a piece of dowel, a pencil. This can be used by students to show that with a common unit, a higher number really means a longer length.

Identification

Cut 1 cm pieces from different coloured drinking straws. Thread these pieces along a string in groups of 10 of one colour followed by 10 of another colour. This can be used to identify centimetres and groups of 10 centimetres to make up a metre.

Using 1 cm grid paper, cut 10 strips that are 10 cm in length. Tape these together to form a folding 1 m measuring strip. This should be placed on cardboard to make it more durable. This again identifies a cm and a m.

Attempt to cut a 1 cm piece of paper into 10 equal slices. Discuss how small they are. Find a long distance (along a road or around an oval), get students to estimate how far to make a kilometre, then use a trundle wheel to have everyone walk an actual kilometre.

Internalisation

Use a measuring tape to measure and record your personal body measures:

Height	Arm span	Head circumference	Leg length
Around the wrist	Foot	Length of hand	Ankle to knee
Wrist to elbow	Left hand	Thumb	Index finger

Find a reference length in your body which is approximately:

1 cm ……………………………………………………………………………………………........

10 cm ……………………………………………………………………………………….........

1 m ……………………………………………………………………………………….............

1 mm ....

Use this measure to estimate different items around the room – measure to check how close.

Mark out a 10 m distance using a measuring tape. Determine how many of your paces equal this 10 m. Pace the following distances, make some up, and use this value to convert paces to metres:

DISTANCE	PACES	Estimated conversion to METRES
Length of room		
Width of board		
Length of veranda		
Distance around the classroom		
Distance around the building		

Mark out 100 m. Use a stop watch to time how long it takes you to walk this distance. Use this time to determine how long it would take you to walk a kilometre.

Find a local well known distance that is around 1 km – check if your time matches.

Estimation

Estimate, using a variety of techniques – internal measures, time, paces, etc., a variety of items or lengths Then measure the same lengths and see how close you get.

Estimate larger distances that students may walk every day – classroom to canteen, school to shop or significant landmarks in the community and then measure these distances. Try to develop the distance of a kilometre.

Place-value connections

Set up the place-value cards:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
				←	3	7	→		

1. Move numbers left and right and use a calculator to determine relationships in moves (×10 to left and ÷10 to right, and so on).

2. Look at names of units – research what centi, milli and kilo mean.

3. Place metric units under place-value positions as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo metre	Hecto metre	Deka metre	metre	deci metre	centi metre	milli metre

4. Use relationships from (1.) above to show relationships between metric units.

5. Use a marker on the unit you are looking at to show the conversions.

Example: Place 37 m on the place-value chart. This is m, so put an orange (or other marker) on the metre place. This shows 37 m. If we wanted to convert to millimetres, leave the 3 and 7 where they are and move the marker to the millimetre position. Fill the 3 "empty" places with zeroes. This shows 37 000 mm which is the same as 37 m.

Mathematics

Metric expanders

Construct a larger copy of **Expander A** (kilometres, metres and millimetres) located in **Appendix A1** and cut it out. Fold the expander like number expanders. Use them to relate km, m and mm as for place-value cards.

Metric slide rule

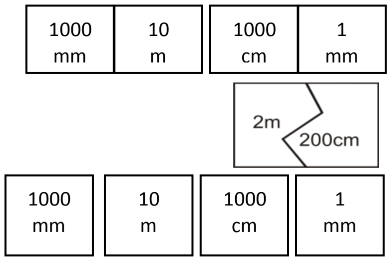
Copy the metric slide rule in **Appendix A2**. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale. Use the slide rule to relate metrics and decimal numeration.

Practice

Give students experience with specialist measuring instruments such as callipers, verniers, etc. Undertake outdoor activities such as height measurement, orienteering and scale drawings/surveying.

Consolidate the metric conversions through drill – some examples:

- Dominoes
- Bingo
- Mix and Match cards
- Card decks (for Concentration, Gin Rummy, Snap etc.)



Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (m and mm), mass (kg and g, t and kg) and volume (L and mL).

1.5 Stage 5 for Length: Applications and formulae

As there are no formulae for length (except with respect to perimeter and circumference which are covered in Module M2 *Relationship Measurement*), applications for length should be built around the idea of a triad – there is an object, a unit of measure and the number of units.

Thus, applications for length are built around three types of problems:

- **Number unknown** – *measure this book in grams*.
- **Object unknown** – *find an object that is 2 kg in mass*.
- **Unit unknown** – *this object is 350 units, are these units g or kg?*

Unit 2: Mass

Mass is the measure of the inertia of an object – how much force it takes to move the object and how much force it takes to stop the object from moving. The important points are that mass:

- may not be proportional to the volume of the object; and
- can be understood as the amount of pressure the object pushes down on something holding it up.

2.1 Stage 1 for Mass: Attribute identification

The stage focuses on building the meaning of mass as heft and resistance to push; and identifying mass using appropriate vocabulary.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

Give students many experiences using their bodies with **hefting** objects, trying to lift different objects (e.g. "how much can I lift?" Be careful here). Look at and feel how hard things press down on the hand.

Make the students' bodies a beam balance by hanging plastic bands off each hand. Put weights on each side and let students feel the heft of each side and which is heavier. Display two bags, for example, one filled with dried leaves and the other with sand, for the students to identify by hefting the bags and determining which bag is heavier and/or lighter. Use a seesaw as a balance beam with students on each side.

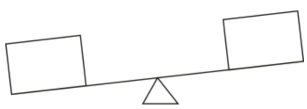
Make sure all mass language is introduced and experienced – mass, weight, heavy/light, heavier/lighter, heaviest/lightest, heft, and so on.

Hand

Provide a variety of experiences showing the effect of mass on pushing down on things. For example: (a) placing things on the end (or middle) of a stick and seeing how much the stick bends (or bows); (b) using a spring or a rubber band to see how long different objects stretch out the spring or rubber bands when lifted with it, or how much the objects compress a spring when the object is placed on top of it.

Use the idea of pushing and pulling as well as hefting. Show students two large cartons and ask which one is heavier. Tell students that the cartons are too large to be lifted and ask if there is another way to compare them. Let students take turns to push or pull the cartons. Discuss objects that have to be pushed or pulled rather than lifted (e.g. beds, couches, and so on). Discuss how effort in pushing and pulling also is an experience of mass.

Experience the balance beam. Try different objects on each side. Develop the notion of balance and how we can balance and unbalance. Look at questions such as "how can we get this side to go up?".



Experience hefting masses that are large yet light, small yet heavy, and so on. Make decisions regarding heft and pushing effort for virtual examples and pictures.

Mind

Students close eyes and imagine lifting a light and then a heavy object. Find a way to visually represent the difference in effort displaced to show the difference in mass. Repeat this activity for pushing and pulling two objects of significant mass difference.

Mathematics

Practice

Continue experiencing hefting and pushing masses through activities, virtual materials, pictures and worksheets. Use rubber band measurers and beam balances.

Connections

Similar to the "measuring container" idea in capacity, the rubber band or bending stick mass measurers can be shown as a connection between mass and length. The further they stretch down in length, the heavier the mass.

Reflection

Application

Explore examples of mass in the everyday world of the students. Set activities to investigate what kinds of masses exist in students' lives and whether they are heavy or light.

Extension

Flexibility. Think of many things that are heavy or light. Relate this to size. Does this relationship always hold? What could make a larger thing lighter – even if it is a larger version of the other one?

Reversing. Get students to experience mass (a) when teacher gives object and students experience its mass, and (b) when teacher describes a mass and students find or construct an object with that mass.

Generalising

1. **Relativity**. Look at the relativity of heavy and light as words. Discuss/show how, in different situations, the same object can be heavy or light. For example, a motor bike is light compared to cars but heavy compared to backpacks.
2. **Big idea: Attribute leads to instrumentation.** Discuss the attribute of mass. Think of it as heft. Discuss how we might measure this heft [stretching a spring]. Try to elicit that the length the spring is stretched is a way of measuring mass.

2.2 Stage 2 for Mass: Comparing and ordering

This stage focuses on comparisons without reference to number; it begins with comparing then moves to ordering.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

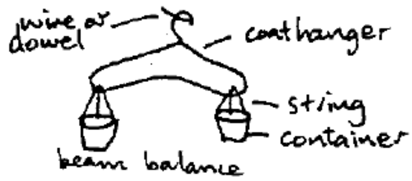
Body

Have students make their body into a beam balance (e.g. a plastic bag on each hand). Experience weights on each side and determining which is heavier (or lighter). As people have different strengths on each side, always transfer the weights to the opposite sides before making decision – heavier weights should be heavier regardless of which arm they are on. Move on from comparing to ordering by finding the "between" weight/mass. Introduce comparing words – heavier/lighter, heaviest/lightest, and so on.

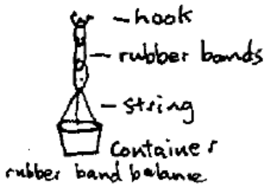
Have students directly compare their masses with each other using a seesaw-type beam balance – a plank across a low brick will do but be careful.

Hand

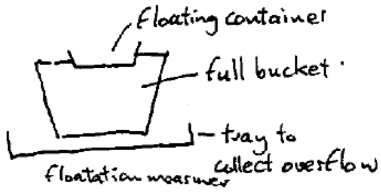
Compare two objects/masses by a variety of ways – beam balance, rubber band mass measurer, flotation measurer, and stocking measurer. Compare **directly** with the beam balance. Compare **indirectly** with the other balances as the objects have to be placed in turn and the length of the stretch gives the comparison.



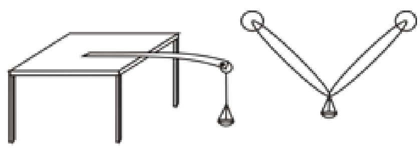
1. A homemade beam balance can be constructed from a wire coat hanger, string and two margarine containers (see right).



2. A homemade spring balance or rubber band balance can be constructed from a string, a wire hook, a margarine container and rubber bands (see left). This balance can be used to compare mass by seeing which object stretches out the rubber band the furthest.



3. A flotation measurer can also be constructed to compare masses (see right). This is a tall container floating in water with sufficient plasticine in the bottom to prevent it tipping. Objects placed in the container cause it to sink deeper into the water. The object causing the furthest sinking is the one with the most mass.
4. A stocking measurer is easily made – a mass in the leg of the stocking will stretch like a rubber band with the heavier mass stretching further.



Other homemade mass measurers can be made from pieces of wood or plastics or metal as on right (rubber bands can also be used). How much the material bends or stretches determines the mass of the object.

Extend to ordering three or more objects using the mass measurers.

Spend time ensuring accuracy in measuring with spring/rubber band measurer: (a) focus on the increase in stretch due to the mass (not the lowest point) unless you make sure that the un-stretched length remains in one place; (b) put a small mass in tray or bottom of stocking so there is tautness in the un-stretched length; and (c) check that rubber bands, springs, and stockings have not been overstretched. Also spend time on beam balance measuring – the balance arms must be the same length.

Use virtual and pictorial means to experience mass comparisons – drawings of beam balances can easily show weight comparisons.

Mind

Imagine, then draw and describe a variety of objects, demonstrating comparisons and orderings of mass using all the measurers.

Mathematics

Practice

Continue to give many opportunities for students to compare and order mass, estimating before calculating. For example, use a seesaw balance and have students make a seesaw using soft drink cans/bottles and shoe box or plastic container lids. Ask students to balance the empty lid on the can first. Then put one object such as a toy car on one side and ask students to find things that make the lid balance. Make shapes out of play dough or clay that have equal mass but a different shape.

Make sure that students understand comparison notation (e.g. > and <) and the rules of comparison (i.e. non-reflexive, antisymmetrical, and transitive), as in section 1.2.

Connections

Continue the link between mass and length for spring balances, rubber band measurers and other mass measurers which go up and down depending on mass.

Reflection

Application

Relate understanding back to everyday life – what things in the world have large and small mass (e.g. a truck has greater mass than a car, a pencil has less mass than a stapler, and so on). Set problems to do with ordering mass that make sense back in the students' world.

Extension

Flexibility. Think of as many pairs of things as possible that have more/less mass than each other. Think of situations where mass has to be taken into account (e.g. small planes or boats) and different lifts – elevators, goods on a truck, a forklift.

Reversing. Make sure teaching goes from: (a) teacher provides masses→ students use measurers to give comparison word, and (b) teachers give comparison word → students provide containers that meet that word.

Also remember that comparison can be considered as a **triad**, with three parts: first object, comparison word, and second object – thus there are three "directions", or three problem types: (a) give first mass and word (e.g. heavier) and students find a lighter mass; (b) give second mass and a word (e.g. greater) and students find a mass that the second mass is greater than; and (c) give two masses and ask for word(s) to relate them (e.g. "the second mass is lighter"). This is the *triadic relationship* big idea.

Generalising. Generalise the accuracy rules that have already been started: (a) more/less mass means a greater/less increase in stretch or the side of the balance going down/up (note that more mass means side goes down); (b) don't overstretch the rubber bands, springs, etc.; (c) make sure the beam balance's arms are the same length; and (d) for comparison, like all triadic relationships, there are three problem types as in reversing. This is developing the *triadic relationship* big idea.

Changing parameters. Discuss what happens in a beam balance if the arms are different lengths or in a rubber band measurer if the bands are overstretched.

2.3 Stage 3 for Mass: Non-standard units

This section introduces the notion of unit, and develops the measurement processes and principles for mass.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

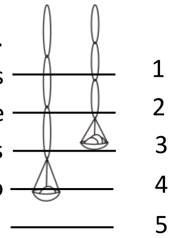
Use seesaw large beam balance to work out how many bricks balance a student – be careful. Use plastic bag beam balance to determine the number of small masses that balance one larger mass. Small objects that can be used include dusters, ball bearings, marbles, books, MAB ones, stones, pencils, small cans of baked beans and spaghetti, and so on. Always **estimate** first.

Hand

Use a variety of measurers to find the mass of an object in terms of smaller masses (e.g. the litre bottle of water is five spaghetti cans). Again, always **estimate** or **predict** first.

(a) The beam balance is easy – how many of the masses will balance the object?

(b) The homemade spring balance can be placed against a blackboard or a sheet of paper. The container can be weighted with plasticine to make the rubber bands taut. This position can be marked and called 0. Then regular intervals can be marked on the blackboard/sheet going down. When objects are placed in the container, their mass can be read off the position that the rubber band stretching allows the container to move down to.



(c) The flotation measurer can also be used for non-standard units by marking regular intervals on the side of the flotation measurer. Similar for other stretching measurers.

Both the spring balance and the flotation measurer can also be used similarly to the beam balance. An object is placed in the container and the position where the container ends up is marked. The object is removed and smaller objects added until the same position is reached.

Undertake a variety of activities including:

(a) find an item that is bigger than a particular object but lighter than it;

(b) find the mass of different objects and order them;

(c) find objects that are of different volumes but same mass and different mass but same volume;

(d) compare very different objects (e.g. does object X have the same mass as object Y?; how many object X might have the same mass as object Y?; how can you find out?);

(e) find an object that has the same mass as a given object but is a different size; and

(f) use a beam balance to order five identical containers filled with different types of objects (place in order from lightest to heaviest) – because the containers are not perceptually different, this task requires students to make multiple comparisons on the beam balance.

Look at what is needed for accurate measurement, for example:

(a) arms of balance the same length; and

(b) lines behind rubber band balance starting at taut position, and being equally spaced apart.

Utilise virtual and pictorial activities (again a beam balance can show mass in a picture by the picture of the masses on one side). Ensure students know how to be accurate with the scales (see section 3.2).

Mind

Imagine larger and smaller mass being compared on a measurer in the mind with eyes shut. Draw examples of what is imagined.

Mathematics

Practice

Continue to provide situations for students to experience measuring mass with non-standard units using all the measurers, and also use virtual materials and worksheets with pictures. Estimate first.

Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.

Make sure that students understand comparison notation (e.g. > and <) and the rules of comparison (i.e. non-reflexive, antisymmetrical, and transitive), as in section 1.2.

Connections

Connect non-standard measurement of mass to division. For example, working out how many small masses balance an object is the same as dividing the object's mass by the small masses. This means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units or masses mean less of them to balance the object and vice versa. This leads into the *inverse relation* big idea.

Reflection

Application

Measure mass with non-standard units in real-world situations. Set up mass problems based on non-standard units in everyday life situations.

Extension

Flexibility. Find use of non-standard mass units in local community activity. Try to get students to think of all the ways non-standard mass units are used in the world, local and otherwise. Use history (e.g. the talent was the weight of gold to buy an ox). Look at old stories – like the one where the prince discovers how to weigh the elephant.

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types:

- **Number unknown** – *how many dusters to balance a book?*
- **Object unknown** – *find an object which is 7 dusters in mass.*
- **Unit unknown** – *the mass of the book is 8, what is the unit?*

Make sure all three directions are taught. This leads to understanding the *triadic relationship* big idea.

Generalising. Here the objective is to extend the understanding from Abstraction and Mathematics to teach continuous vs discrete and the three measurement principles as follows.

1. **Continuous vs discrete**. Discuss how mass non-standard units have broken up continuous mass into small parts that allow mass to be counted. This leads to the *continuous vs discrete* big idea – that **there are two ways that number is applied**: (a) to discrete objects, and (b) to continuous things such as the use of units like spaghetti cans to discretify the continuous mass.

2. **Measurement principle 1: Common units.** Use torpedoing to show that: (a) we cannot measure accurately if we vary the masses being balanced; and (b) we cannot know if an object is heavier/lighter than another unless we use the **same unit**. Do activities like balancing an object with a variety of units of different mass and ask, what is the problem here? Why can't we count these units? Do activities where you say that a heavy object is 4 units and a light object is 16 units and ask why this could be. This leads to the first and second aspect of the *common units* big idea – that **units must be the same size when measuring and comparing objects**.

Then set a **common class unit** – a common mass that all will be measured against and measure using this. Elicit what bigger numbers mean now. This leads to the third aspect of the *common units* big idea – that **when units are the same, the larger number specifies the heavier object.**

Discuss if different mass units were used in different towns and countries, what would this mean? We could be paying a lot of money and end up with, say, a small load of concrete because the company you buy from has small units. Set up spring balances with different spaced lines and different rubber bands and show that the same object is a different number in each measurer. Ask why? This leads to the fourth aspect of the *common units* big idea – that **there is a need for a standard**.

3. **Measurement principle 2: Inverse relation**. Measure things with large and small masses and record results on a table as in section 1.3. Study the results for patterns. This is best done with the students working in pairs. Activities like this lead to the *inverse relation* big idea – **the larger the unit, the smaller the number and vice versa** – and to the understanding that **measuring in units is like dividing**.

4. **Measurement principle 3: Accuracy vs exactness.** Three activities:

   (a) Get students to measure an object twice with a rubber band measurer. Was it the same answer both times? There should be a difference in the second measure because of error. Discuss this error and whether we always want to be exact. Discuss how to make the measuring more accurate.

   (b) Measure the mass of an object with large and small units. Discuss which is more accurate if giving answers in whole numbers of units.

   (c) Give students a variety of masses as non-standard units (e.g. books, dusters, cans, and so on) to use in a variety of measuring activities. Have the students select the appropriate mass to measure with and record the mass as number and object.

   Discuss situations where estimation would work. This leads to the three consequences of the *accuracy vs exactness* big idea – **smaller units give greater accuracy**, students require **skill in being able to choose appropriate units**, and students require **skill in estimating**.

Changing parameters. What if the spring cannot stop from becoming stretched and therefore change how far it stretches. How do we handle this?

What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

2.4 Stage 4 for Mass: Standard units

This stage introduces the gram, kilogram and tonne; and metric conversions for mass units:

1 gram (g) = 1 000 milligrams (mg) = 1 000 000 micrograms (μ)
1 kilogram (kg) = 1 000 g = 1 000 000 mg
1 tonne (t) = 1 000 kg = 1 000 000 g

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

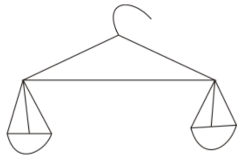
Common unit

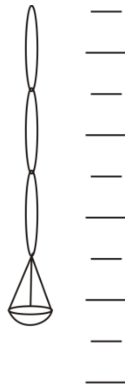
After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring mass with a class-chosen unit – e.g. a stapler, a particular rock. This can be used by students to show that with a common unit, a higher number really means a greater mass.

Identification

Construct mass measurers as follows:

> <u>Method 1</u> – a wire coat hanger, string and 2 margarine containers (plus masses).
>
> <u>Method 2</u> – a long piece of paper, 3 rubber bands, string and a margarine container (calibrate the "spring balance" with masses – mark lengths on the paper).





Use these measurers to make up plastic bags, or other containers, containing 100 g, 250 g, 500 g and 1 kg of various materials (pasta, sand, marbles, rice, etc.). Feel the weight of a centicube – this weighs 1 gram (if no access to these, feel the weight of a paper clip – close to 1 gram).

Find examples of utilities and trucks that hold 1, 2, 5 and 10 tonnes. A normal home trailer will hold ½ tonne. A metre cube of water is 1 tonne.

Internalisation

Use a bathroom scale to measure your own mass in kg.

Measure 1 L of water.

Find objects in the environment that measure approximately 1 kg, 500 g, 250 g, 100 g, 50 g and 1 g. Make up lumps of plasticine to these measures.

Find out the mass of a car and a 4-wheel drive, an elephant etc.

Estimation

Estimate first and then measure the masses of the following objects/people (find some more to measure). Complete estimates and measures of object before moving onto the next.

OBJECT	Estimate	Measure	Difference
Duster			
Case or port			
Shoe			
Another student			
Teacher			

Look up pictures of utilities and trucks, estimate how many tonnes they can carry, and then check by research.

Place-value connections

Set up the place-value cards with place-metric cards as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo gram	Hecto gram	Deka gram	gram	deci gram	centi gram	milli gram

Again analyse the meaning of kilo and use relationships between place-value positions to reinforce relationships between metric units.

Mathematics

Metric expanders

Construct a larger copy of **Expander C** (tonnes, kilograms and grams) in **Appendix A1** and cut it out. Fold the expander like number expanders. Use them to relate t, kg and g as for place-value cards.

Metric slide rule

Copy the metric slide rule in **Appendix A2**. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale.

Use the slide rule to relate metrics and decimal numeration.

Practice

Consolidate the metric conversions through drill – some examples:

- Dominoes
- Bingo
- Mix and Match cards
- Card decks (for Concentration, Gin Rummy, Snap, etc.)

1000 g	1000 kg	1 t	1 g

1000 g	10 kg	1000 kg	1 t

Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (km, m, cm and mm), mass (kg, g and t) and volume (kL, L and mL).

2.5 Stage 5 for Mass: Applications and formulae

Applications for mass should be built around the idea of a triad – there is an object, a unit of measure and the number of units – as there are no formulae for mass.

Thus applications for mass are built around three types of problems:

- **Number unknown** – *measure this book in grams.*
- **Object unknown** – *find an object that is 2 kg in mass*.
- **Unit unknown** – *this object is 350 units, are these units g or kg?*

Unit 3: Capacity

This unit covers capacity which is the measure of **liquid volume** and has litres as its standard units. In terms of definition, capacity is the amount of space enclosed by a solid shape (e.g. a jug or container). Solid volume with its cubic units (e.g. m^3) is in Module M2 *Relationship Measurement* (although it is common to integrate and important to connect liquid and solid volume).

3.1 Stage 1 for Capacity: Attribute identification

This stage focuses on identifying capacity using appropriate vocabulary.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

Experience a lot of situations in which parts of the body hold water or rice (e.g. one or two cupped hands). Discuss the difference.

Try to hold a lot and hold a little. Introduce the term **capacity** (i.e. the amount a container holds), and large and small capacity. Also introduce capacity as a special word for **volume** of pourable materials. Also introduce pouring words like full and empty.

Hand

Allow students to play with water, rice, sand or other material, pouring from one container to another, filling and emptying. Experience capacity in different ways – long thin containers, short fat containers, large containers, small containers, interesting containers, and so on.

Experience finding containers that hold the same amount – "have the same capacity". Experience what happens when the container you are pouring from has more (capacity) than the container you are pouring into, and vice versa. Estimate how much a container will fill another before pouring.

Experience capacity without pouring – decide which container has small or large capacity by looking at it – experience containers fitting inside each other and discuss what this means for capacity.

Mind

Encourage students to imagine containers, imagine the containers being filled and emptied, and imagine the containers being made bigger and smaller to change capacity.

Mathematics

Practice

Use materials, computers and pictures in worksheets to experience capacity. Differentiate it in drawings from length. Experience having one container (e.g. a cup of water) and a larger container (e.g. an empty ice-cream container) and guessing how high up the water will go; repeat this activity for a variety of containers.

Connections

Obtain a tall thin container and start pouring from other containers into the tall thin one. Mark on the tall thin container how high each container fills the tall thin container. Discuss how capacity here relates to length (or height).

Reflection

Application

Relate understanding of capacity back to everyday life – what things in the world have capacity (e.g. cups, glasses, petrol tankers, and so on) – what has big/small capacity?

Extension

Flexibility. Think of as many things as you can that have capacity. Visit a supermarket and find different containers which have the same capacity.

Reversing. Up to now the students have gone from container → experiencing capacity. Reverse this – go from capacity as an experience → container. Thus, do activities where students construct containers for particular purposes.

Generalising. Discuss what makes a container have large or small capacity. Discuss how we can measure this. Think back over connections – is there an idea here? Try to elicit that capacity could be measured by how high up the side you can fill a given container. This is the beginning of the *attribute leads to instrumentation* big idea.

Changing parameters. What if material being poured could contract or expand? What happens if the material becomes a gas as it is poured (e.g. it is only a liquid because it is under pressure and pouring releases the pressure). What does capacity mean here? What if it freezes as it is being poured – what if a liquid turns to ice? (e.g. what happens when a full water bottle is put in the freezer?)

3.2 Stage 2 for Capacity: Comparing and ordering

In this stage capacity is compared and ordered without reference to number, and vocabulary is developed for these comparisons and orderings (e.g. holds more/less, it is larger/smaller, it is the largest/smallest).

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

Compare and order capacity of students' hands. Compare how much rice can be held in one or two cupped hands of students. Begin with comparing two students and then introduce a third and so on to order capacity and determine who is able to hold the greatest/smallest amount. Focus on finding the capacity between the other two.

Reinforce pouring words like full and empty and introduce comparing words such as more/less, larger/smaller capacity, greater/lesser capacity, and so on.

Hand

Use sets of containers and compare them directly by placing one inside another (works best for similar shaped containers but different shapes will also work as long as inside each other) or beside each other. Extend to ordering three or more different containers.

Indirectly compare and order by pouring water, rice or sand (and other liquids) from one container to another and seeing which container holds more. Elicit from the students that if the first container still has material after the second is filled, the first has larger capacity; and if the first container is emptied before the second is filled, then the second has larger capacity. Extend this to order. Ensure students understand that there must be no spillage, the first container must be full, and the second container empty for the pouring to be accurate.

Continue this even more indirectly by choosing a container (tall and thin is best) that the other containers can pour into and on the side of which marks can be made to show the height of the poured material. Elicit from the students that the container whose poured material is higher up the side has the greater/greatest capacity and the mark lower up the side has the smaller/smallest capacity. This continues to develop the big idea that *attribute leads to instrumentation*.

Remind students that capacity is a special form of volume so that this word can also be used. Experience greater/smaller and greatest/smallest capacity using virtual means and using pictures.

Mind

Imagine, then draw and describe, a variety of objects, demonstrating comparisons and orderings of capacity. For example: an empty and full container, two identical containers with the same capacity, two different containers with the same capacity. Imagine them in the mind, and imagine them getting larger and smaller and holding more and less.

Mathematics

Practice

Use comparison and ordering experiences from Abstraction to practise being able to compare and order capacities. Continue to discuss comparative language for capacity such as large, larger, largest, small, smaller, smallest, full, empty, less, more, and so on. Present students with a variety of comparing and ordering problems, for example:

- Find a container that will hold more water than this one.
- Find a container that will hold less water than this one.
- Find a container that will hold the same amount of water as this one.
- Find three containers and put them in order based on the size of their capacity.
- Order pictures of objects or pictures of partly filled glasses or containers.

Make sure that students understand comparison notation (e.g. > and <) and the rules of comparison (i.e. non-reflexive, antisymmetrical, and transitive), as in section 1.2.

Connections

Continue to draw the connection between length and capacity through "measuring containers" as in section 3.1.

Reflection

Application

Relate understanding of comparing and ordering capacity back to everyday life – what things in the world have large and small capacity (e.g. a jug has greater capacity than a glass, a bucket has more capacity than a bottle, a petrol truck has less capacity than a swimming pool, and so on). Investigate (e.g. using the Internet) to find out things that you are unsure of.

Extension

Flexibility. Think of as many pairs of things as you can that have more/less capacity than each other. Visit a supermarket and find different containers which can be ordered in terms of capacity.

Reversing. Make sure teaching goes from: (a) teacher provides containers → students give comparison word; and (b) teachers give comparison word → students provide containers.

Also remember from section 1.2 that comparison can be considered as a **triad**, with three parts: first container, comparison word, and second container– thus there are three "directions", or three problem types: (a) give first container and a word (e.g. larger) and require a larger capacity container; (b) give second container and a word (e.g. greater) and ask for a container that the second container is greater than; and (c) give two containers and ask for word(s) to relate them (e.g. the second container has less capacity). This is developing the *triadic relationship* big idea.

Generalising. Generalise the three things that have already been stated: (a) more and less capacity for a container depends on container it is being compared to; (b) comparison by pouring is only accurate if first container is full and second container is empty; (c) when pouting into a "measuring container", the container that fills the measuring container to the highest point is the largest and vice versa; and (d) for comparison, like all triadic relationships, there are three problem types. This is developing the *triadic relationship* big idea.

Changing parameters. Again discuss ordering capacities for when the material changes as it is being poured.

3.3 Stage 3 for Capacity: Non-standard units

The section introduces the notion of unit, and develops the measurement processes and principles.

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

Fill containers with handfuls of materials (e.g. water, sand, rice, and so on). Count how many handfuls to fill the container (say it is 7). Say that the capacity of the container is 7 handfuls.

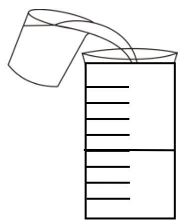
Hand

Pour material into containers from smaller containers and count how many small containers full are needed to fill the lager container. Use spoons, thimbles, lids, small glasses, egg cups, buckets, and so on. Ensure students are measuring with accuracy which means having the following accurate measurement processes:

- ensuring that the unit container (e.g. the glass) is always full;
- ensuring that the container to be measured (e.g. the jug) starts empty; and
- ensuring there is no spillage of material during pouring.

Always **estimate** before pouring and calculating capacity. Always say the capacity of containers in terms of number and unit (which is the container that is being counted), for example, the jug has a capacity of 9 glasses.

Make a homemade "measuring container". Calibrate by regularly spaced lines, from a glass or jar which has a constant diameter. Use this to measure capacity/volume of containers, by pouring material into the measuring container from various containers.



Use virtual materials and pictures to further experience measuring capacity of containers in terms of non-standard units.

Mind

Have the students imagine containers being filled from smaller containers and determining how many of the smaller container fit into the larger. Also imagine a "measuring container" being filled from other containers and the capacity being determined by how many lines up the side it fills the measuring container to. Make drawings of what is imagined.

Mathematics

Practice

Continue to provide situations for students to experience measuring capacity with non-standard units – use containers, pouring materials and also use virtual materials and worksheets with pictures. Estimate first.

Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.

Connections

Connect non-standard measurement of capacity to division. For example, working out how many glassfuls are in a jug is the same as dividing the jug's volume by the glass's volume. As stated in section 1.3, this means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units or glasses mean fewer glassfuls to fill the jug and vice versa. This leads into the *inverse relation* big idea.

Reflection

Application

Measure capacity with non-standard units in real-world situations. Set up capacity problems based on non-standard units, e.g. cupfuls, bottlefuls, etc.

Extension

Flexibility. Find capacity non-standard units in local community (e.g. measuring the capacity of a fuel tank in terms of cans of fuel, making cakes using tablespoons and cups as measuring units). Try to get students to think of all ways non-standard capacity units are used in the world, local and otherwise.

Use history and look at other units used in the past (e.g. the gallon which was the amount of wheat in a standard barrel).

Reversing. The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types:

- **Number unknown** – *how many jugs to fill the bucket?*
- **Object unknown** – *find an object which is 5 jugs in capacity.*
- **Unit unknown** – *the container has a capacity of 7, what is the unit?*

Make sure all three directions are taught. This leads to understanding the *triadic relationship* big idea.

Generalising. Here the objective is to extend the understanding from Abstraction and Mathematics to teach continuous vs discrete and the three measurement principles as follows.

1. **Continuous vs discrete**. Discuss how the capacity non-standard units have broken up a continuous body of water into small parts that allow capacity to be counted. This leads to the *continuous vs discrete* big idea – that *there are two ways that number is applied*: (a) to discrete objects, and (b) to continuous things such as capacity by the use of units like glassfuls to discretify the continuous volume.

2. **Measurement principle 1: Common units.** Use torpedoing to show that: (a) we cannot measure accurately if we vary the container as we count containers full; and (b) we cannot know if a container is larger/smaller than another unless we use the **same unit**. Measure a container sloppily, at times only half filling the container you are pouring from. Say a large container is 5 units and a small container is 15 units and ask why this could be. This leads to the first and second aspect of the *common units* big idea – that **units must be the same size when measuring and comparing containers**.

Then set a **common class unit** – a common size container, and measure using this. Elicit what bigger numbers mean now. This leads to the third aspect of the *common units* big idea – that **when units are the same, the larger number specifies the longer object.**

Discuss if different units were used in different towns and countries, what would this mean? We could be paying more for a smaller bottle? This leads to the fourth aspect of the *common units* big idea – that **there is a need for a standard**.

3. **Measurement principle 2: Inverse relation.** Measure things with large and small containers and record results on a table as in section 1.3. Study the results for patterns. This is best done with the students working in pairs. Activities like this lead to the *inverse relation* big idea – **the larger the unit, the smaller the number and vice versa** – and to the understanding that **measuring in units is like dividing**.

4. **Measurement principle 3: Accuracy vs exactness.** Three activity sets:

   (a) Get students to put 5 cups in a container and then measure it out in the same cups. There should be a difference in the second measure because of error. Discuss this error and whether we always want to be exact. Discuss how to make the measuring more accurate.

   (b) Measure the capacity of a container with large and small units. Discuss which is more accurate if giving whole numbers of units.

   (c) Give students a variety of containers as non-standard units (e.g. buckets, glasses, thimbles, and so on) and a variety of measuring activities. Have the students select the appropriate measuring container and record the capacity as number and container.

   Discuss situations where estimation would work. This leads to the three consequences of the *accuracy vs exactness* big idea – **smaller units give greater accuracy**, students require **skill in being able to choose appropriate units**, and students require **skill in estimating**.

ACTIVITY: GUESS WHAT HOLDS MORE

1. Take two pieces of A4 paper.
2. Make the round part of a cylinder longwise from one A4 sheet and the round part of a cylinder shortwise from the other.
3. Use more tape and paper to give each cylinder a base.
4. Pour rice from one cylinder to the other. Which is larger? Why?

Changing parameters. What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

3.4 Stage 4 for Capacity: Standard units

This stage introduces the formal measurements for capacity – millilitre, Litre and kilolitre (Litre as an abbreviation is a capital L so it is not easily confused with a 1 or any other symbol). The relationships between the standard units are:

1 litre (L) = 1 000 millilitres (mL)
1 kilolitre (kL) = 1 000 L = 1 000 000 mL

Reality

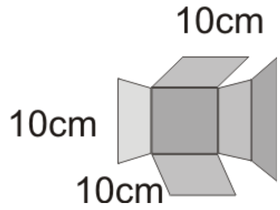
Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring capacity with a class-chosen unit – e.g. a common object like a glass or cup. This can be used by students to show that with a common unit, a higher number really means a greater capacity.

Identification



Using 1 cm grid paper, draw and cut out a net for a cube of side 1 cm. Fold and tape to make the cube. Fill the 1 cm cube with rice or sand. Pour this into another container. This represents 1 mL.

Use cardboard to make a net for a cube of side 10 cm as on right. Tape and fold this cardboard to make the cube. This cube is 1 L. Check this by pouring 1 L of water or sand into it. Check that this cube holds the same as a 1 L soft drink bottle. This is the same size as a MAB thousand block. (Similarly, a cubic box of side 1 m is a kilolitre.)

Calibrate a container into 100 mL levels using either of the following methods:

> Method 1 – take a glass jar and pour 100 mL amounts into it, marking the levels with tape as you go.
>
> Method 2 – take a 1 L milk carton, cut off the top and use a ruler to divide the height into 10 equal intervals.

Make a cubic metre with m length dowelling – put a MAB thousand cube in it. Note that there are 1000 MAB blocks in the cubic metre – so cubic metre is a kilolitre.

Internalisation

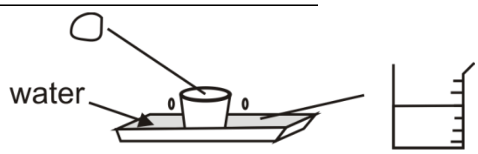
Obtain a collection of jars and jugs and pour 250 mL and 500 mL into them and note levels. Try to estimate where the levels will be before pouring.

Estimation

First estimate and then measure the capacity of objects as below (find some of your own). Estimate and measure each object before moving onto the next. **Estimate the capacity – do not estimate length, breadth, height**. Use measuring cylinders for checking.

OBJECT	Estimate	Measure	Difference
Capacity			
Cup			
Glass			
Bottle			
Plastic container			

Use an overflow tray and a measuring cylinder, to find the volume of objects by immersion and overflow. Estimate first.



OBJECT	Estimate	Measure	Difference
Lump of plasticine			
Rock			
Your fist			

Place-value connections

Set up the place-value cards with place-metric cards as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo Litre	Hecto Litre	Deka Litre	Litre	deci Litre	centi Litre	milli Litre

Analyse the meaning of milli and kilo and use relationships between place-value positions to reinforce relationships between metric units.

Mathematics

Metric expanders

Construct a larger copy of **Expander B** (kilolitres, litres and millilitres) in **Appendix A1** and cut it out. Fold the expander like number expanders. Use them to relate kL, L and mL as for place-value cards.

Metric slide rule

Copy the metric slide rule in **Appendix A2**. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale.

Use the slide rule to relate metrics and decimal numeration.

Practice

Consolidate the metric conversions through drill – some examples:

- Dominoes
- Bingo
- Mix and Match cards
- Card decks (for Concentration, Gin Rummy, Snap, etc.)

1000 mL	1000 L	1 KL	1 mL
1000 mL	10 L	1000 L	1 mL

Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (km, m, cm and mm), mass (T, kg, and g) and volume (kL, L and mL).

3.5 Stage 5 for Capacity: Applications and formulae

Again, as there are no formulae in capacity (that are not part of solid volume), applications for capacity should be built around the idea of a triad – there is an object, a unit of measure and the number of units.

Applications for capacity can be built around three types of problems:

- **Number unknown** – *measure the capacity of this bowl in millilitres.*
- **Object unknown** – *find an object that is 4L.*
- **Unit unknown** – *this object is 500 units, are these units mL or L?*

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the basic measurement item types

The basic measurement item types are in three subtests to match the module units. Each of the subtests is divided into five parts – attribute, comparison, non-standard units, standard units and applications/formulae. The major parts of the tests are non-standard and standard units as non-standard has the principles and processes, and standard has the formal units, their relation to place value and their conversions.

Thus the sequencing in the subtests is across the five stages and pre-tests should focus on early stages (i.e. attributes and comparison as well as non-standard units), while the post-tests should be constructed so that they ensure the final stages are covered (i.e. standard units and applications). However, as always, the selection of item types should be determined by the teacher's knowledge of their students.

For **maximum effect**, it is best to **mix items** from different attributes/subtests so that students have to pick what attribute is covered by particular items and sub-items – **particularly in the post-test**.

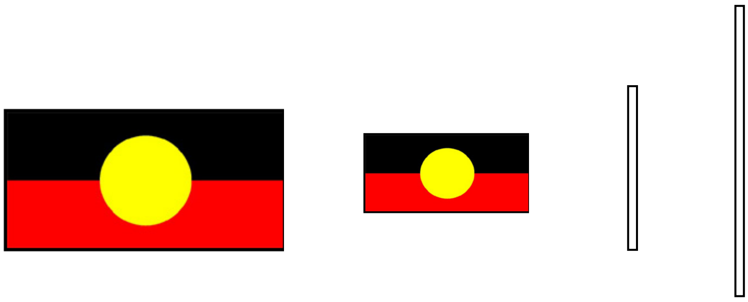
There are two final points to be made. First, regardless of knowledge of the latter stages, a crucial thing in understanding measurement is knowing what the measure is as an attribute, and this is Stages 1 and 2 (i.e. attribute and comparisons with no numbers). Second, it is possible to consider this module as three parts and have pre- and post-tests for each part. However, it is important that long-term performance in all three of length, mass and capacity is measured at the end of the module.

Subtest item types

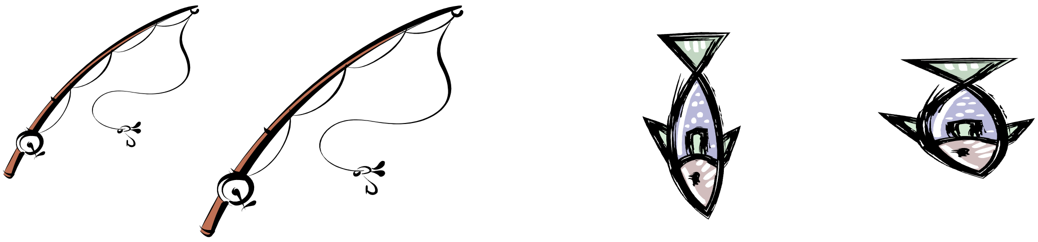
Subtest 1 items (Unit 1: Length)

Stage 1 – attribute

1. (a) Draw a line from the wider flag to the shorter pole.



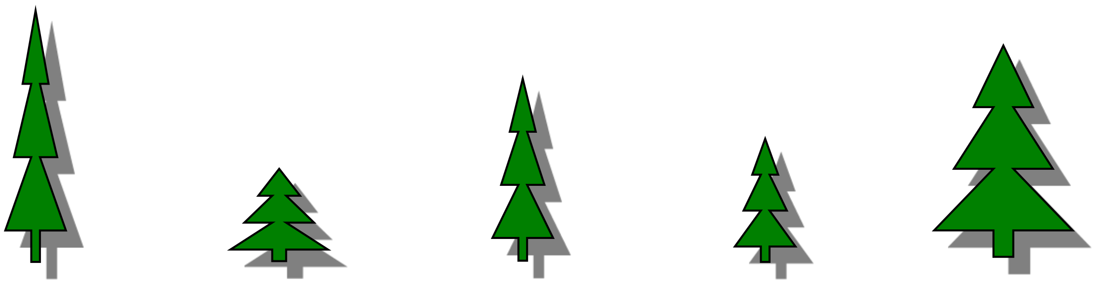
(b) Draw a line from the longest fishing rod to the widest fish.



Stage 2 – comparison

2. (a) Order these trees from <u>shortest</u> to <u>tallest</u>: ___________________________________

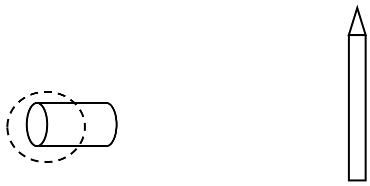
A B C D E



(b) Order these kangaroos from <u>tallest</u> to <u>shortest</u>: ______________________________

A B C D E

3. Without counting anything, how could you work out if the length of the pencil is longer than the distance around the can?

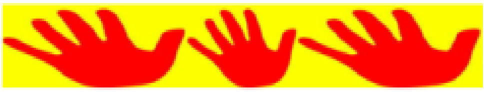


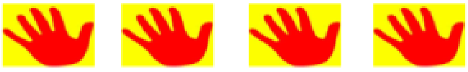
____

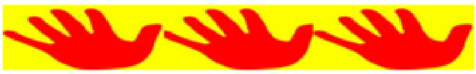
____

Stage 3 – non-standard units

4. Three students measured the width of the computer screen using handspans. They got different answers.

Donna  3 handspans

Joseph  4 handspans

Eli  3 handspans

(a) Whose answer is measured correctly? ____

(b) Why?____

____

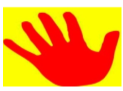
(c) Paul also measures the width of the computer and he finds that his measurement is 2 handspans. He has measured it correctly. Could he still be correct if Eli measured it with 3 handspans? Why?

____

____

(d) Gary measured the width of the whiteboard with his hands and got 15 handspans.

Gary’s hand: 

Paula’s hand: 

How many handspans would you expect Paula to get if she measured the same whiteboard with her hands?

Circle the answer: 12 handspans 30 handspans 40 handspans 65 handspans

5. Measure the width of your desk using your hands:

The desk is

(how many) (what unit)

Stage 4 – standard units

6. What do these measurements refer to?

cm ____

mm ____

km ____

m ____

Stage 5 – applications/formulae

7. Fill in the following conversions:

(a) 15 cm = ________ mm

(b) 3 m = ________ cm

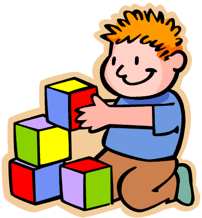
(c) 4600 m = ________ km

(d) 60 mm = ________ cm

Subtest 2 items (Unit 2: Mass)

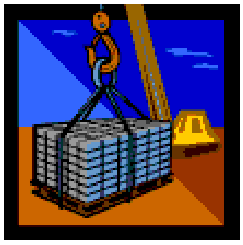
Stage 1 – attribute

1. Draw a line to match the picture to the word that it best illustrates.



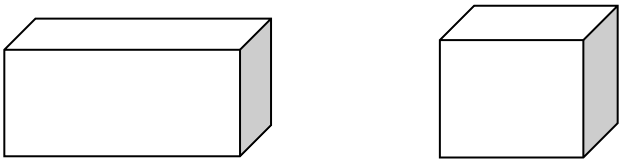
Heavy

Light



Stage 2 – comparison

2. Could these two containers have the same mass? Why?



A B

____

____

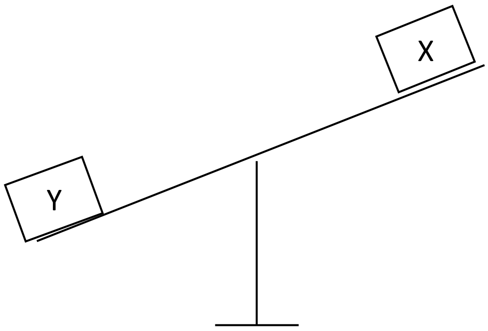
3. Which box weighs more, X or Y? Why?

____

____

____

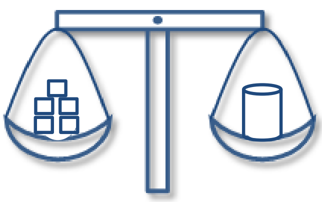
____



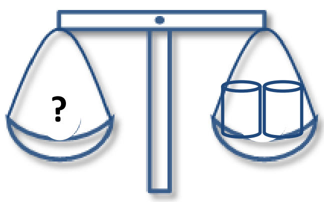
Stage 3 – non-standard units

4. In diagram A, five (5) blocks have been placed on one side of the scales to balance the jar on the other side. How many blocks might be needed to balance the two jars in diagram B?

A



B



Stage 4 – standard units

5. What do these measurements refer to?

   t ____

   g ____

   kg ____

Stage 5 – applications/formulae

6. Fill in the following conversion:

   (a) 5000 g = ________ kg

   (b) 6.5 t = ________ kg

   (c) 2.7 kg = ________ g

Challenge question

7. Using only a 1 kilogram, a 2 kilogram and a 5 kilogram measures and scales, how many different combinations can you make? e.g. 7 kilograms with the 2 and the 5 kilogram measure.

Subtest 3 items (Unit 3: Capacity)

Stage 1 – attribute

1. Which of these has the greatest capacity, A or B? ________________

A

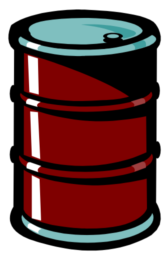
1 L

B

600 mL

Stage 2 – comparison

2. Albert did not have any measuring equipment. How could he determine which one of these two containers has a greater capacity?



____

____

Stage 3 – non-standard units

3. Bill and Ted are measuring the capacity of a barrel.

   Bill measures by counting buckets full.

   Ted measures by counting cans full.

   Who will have the largest number to fill a barrel, Ted or Bill?______________________

   Why? ____

   ____

Stage 4 – standard units

4. What do these measurements refer to?

L ____

kL ____

mL ____

Stage 5 – applications/formulae

5. Fill in the following conversions:

(a) 13 000 mL = _______ L

(b) 2 kL = ___________ L

(c) 8.5 L = ________ mL

(d) 6000 L = ________ kL

Appendix A: Teaching Tools

A1 Metric Expanders

Fold shaded part so it is only shown as expanders are opened.

Expander A

			Kilometres					Metres					Millimetres	

Expander B

			Kilolitres					Litres					Millilitres	

Expander C

			Tonnes					Kilograms					Grams	

A2 Metric Slide Rule

✂

Whole Numbers				Decimal Fractions		
TH	H	T	O	t	h	th
1000	100	10	1	0.1	0.01	0.001

✂ slit

✂ slit

How does it work?

		3	0	0				What is my new number?

✄ slit

Whole Numbers				Decimal Fractions		
TH	H	T	O	t	h	th
	3	0	0			
1000	100	10	1	0.1	0.01	0.001

✄ slit

Pull the slider one place to the left to multiply by 10; Pull the slider two places to the left to multiply by 100 etc.
Pull the slider one place to the right to divide by 10; Pull the slider two places to the right to divide by 100 etc.
3 in the Hundreds place – how many ones is that? etc.

A3 Place Value (PV) Chart

Whole number PVs				Decimal PVs		
TH	H	T	O	t	h	th
1000	100	10	1	0.1	0.01	0.001

Cut out PV chart and slides

Cut along dotted lines
and insert slides

Slides

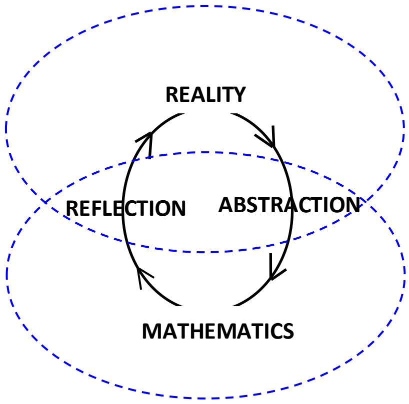
			km			m		cm	mm				

						L			mL				

			t			kg			g				

Appendix B: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix C: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

Accelerated Inclusive Mathematics Project

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