Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module N2

Year A, Term 1

Number:

Decimal Number Numeration

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

As described in Module N1 *Whole Number Numeration*, mathematics is based on numerical and algebraic thinking with numbers, variables and operations integrated with geometrical and visual thinking. As a product of human problem solving, albeit mainly from European, Hindu-Arabic and Japanese-Chinese cultures, mathematics has, historically and presently, two aspects at the basis of its structure: number and geometry. Of these two, number has the largest share of mathematics development as it is the major component of the Australian Curriculum strands: Number and Algebra, Geometry and Measurement, and Statistics and Probability.

This Module N2 *Decimal Number Numeration* focuses on decimal numbers and is the second module in Level A. It follows on from whole numbers and leads into operations and measurement. It is dependent on "part of a whole" fraction understanding of the new place-value positions it adds to the whole number place-value positions. Its purpose is to provide a vertically structured module of units that teaches the core of decimal number numeration for Years 2–9. Similar to whole numbers, it is built around the five big ideas from the *Whole Number Numeration* module but applied to the particular needs of decimal numbers.

Background information for teaching decimal number numeration

This section looks at how decimal numbers, whole numbers and fractions are connected and what this means for teaching decimal numbers.

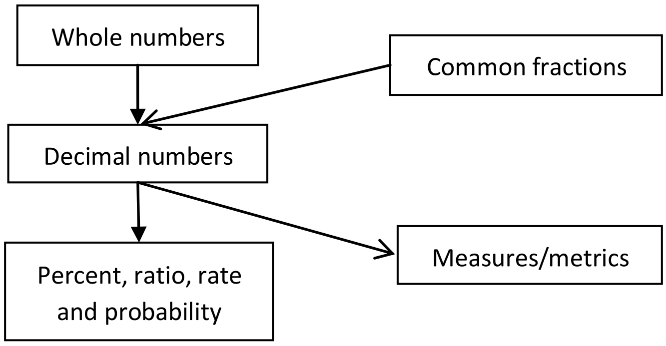
Connecting whole numbers and fractions to decimals

The importance of this module is that it starts to provide information on the second half of the role of the unit or one in numbers. The basis of number is the one – it is grouped (e.g. tens, hundreds) to give whole numbers and then it is partitioned to give parts of one, that is, common fractions and decimal numbers. Thus, the module is designed to provide information on how the decimal number component of partitioning can be taught. Thus, the module:

- is a pivotal component of number – leading on from whole numbers to measures (metrics), percent, ratio and rate, and covers both rational and irrational numbers;
- extends the whole number system by bringing in fraction place-value (PV) positions – extending ones, tens, hundreds, and so on, to tenths, hundredths, and so on;
- enables the one/unit to go in both directions – grouping to make large numbers, partitioning to make small numbers; and
- builds a number system that is symmetrical about ones, bi-directional in PV position relationships, and exponential in structure.

The inverse relationship between grouping (building whole numbers) and partitioning (building decimal numbers) means that decimal place values work in both directions or are bi-directional (e.g. hundreds partition into tens and tens group into hundreds; tenths partition into hundredths and hundredths group into tenths). It also means that decimals have connections with whole numbers, common fractions and topics that relate to these two ideas, that is, metrics, percent, ratio, rate and probability, as the figure on the next page shows.

To efficiently develop decimal numbers, there is a need to have **seamless development** along the connections in this figure. This is done by basing instruction for both whole numbers and decimal numbers on the similarities between common fractions/decimal numbers and whole numbers and taking account of differences. The only similarity between decimal numbers and common fractions is that decimal numbers use common-fraction place-value positions. The differences between them are in the way each is represented by symbols (e.g. a half is 1/2 in fraction symbols and 0.5 in decimal symbols).



The similarities between whole and decimal numbers are: (a) place-value positions are determined from relation to ones; (b) numbers are compared/ordered by looking for larger/largest digit in largest PV position; (c) both use a place-value symbol system with numerals referring to place values; (d) both are based on the same big ideas as listed below; (e) numbers are added and subtracted by aligning PVs; and (f) numbers are multiplied and divided by adding and subtracting PV position to the ones (e.g. 10×1000 = 10 000; 10÷1000 = 0.01; 0.01×0.001 = 0.0001; 0.01÷0.001 = 10). Shortcuts such as "multiplying by 10 is adding a zero" should not be taught.

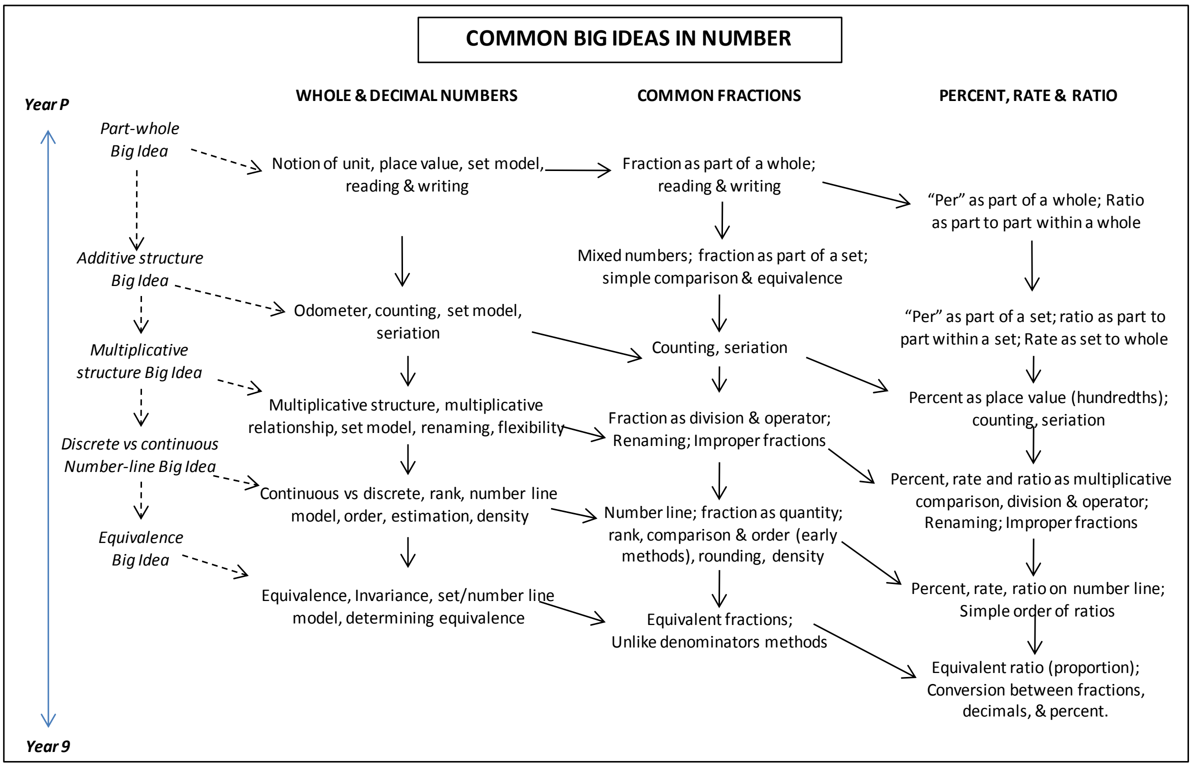
The differences between decimal numbers and whole numbers have to be taken into account. The major ones are: (a) new place-value positions that are fractions; (b) a different convention for determining ones position (decimal point directly after the ones); (c) symmetry about the ones (tens-tenths; hundreds-hundredths, and so on); and (d) greater density of numbers (i.e. whole numbers are spaced – there are no whole numbers between 24 and 25, while decimal numbers are very dense – there are infinite decimal numbers between 24 and 25).

However, there is a difference that is hard to take into account – that is, decimal numbers use whole-number like notation for symbols but use fraction language for saying the decimal. For example, 2.38 is 2 ones, 3 tenths and 8 hundredths but is said "two and thirty-eight hundredths". Thus, there need to be activities where decimal and fraction representations are integrated.

Big ideas

AIM believes that mathematics should be taught so that it is accessible as well as available (see *AIM Overview* booklet), that is, learnt as a rich schema containing knowledge of when and why as well as how. Rich schema has knowledge as connected nodes which facilitates recall (it is easier to remember a structure than a collection of individual pieces of information) and problem solving (content that solves problems is usually peripheral, along a connection from the content on which the problem is based). As a consequence, YuMi Deadly Maths AIM argues that knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning.

The big ideas for decimals are mainly the same as for whole numbers but there are a few extra because of the extension to fraction place values. The big ideas that are the same as whole numbers are: (a) **part-whole** (or notion of a unit); (b) **additive structure** (counting/odometer); (c) **multiplicative structure**; (d) **continuous–discrete/number line**; and (e) **equivalence** (we apply these big ideas to all numbers as in the figure on the next page). The extra big ideas are as follows: (a) **unitising and reunitising**, that is, making a collection into one whole and redoing this for different partitions (e.g. 12 can form into a one as 12 objects – unitising, and re-form as 4 rows of 3 objects – reunitising); (b) **density** being a component of continuous–discrete for decimal numbers where there are many numbers between any two numbers; (c) **flexibility** being a component of multiplicative structure in allowing the unit to be any PV position, for example, 24 ones is 2.4 tens and is also 0.24 hundreds; and (d) the **rational-irrational** dichotomy in real numbers where some decimals numbers are common fractions and some are not.



Models and representations

There are two other aspects that need to be discussed when looking at decimal numbers: physical, virtual and visual models; and symbolic representations.

Models are common across whole numbers, decimal numbers and common fractions. They are: (a) **set and area** models (e.g. counters, sticks, circles, rectangles); (b) **position** models such as place value charts (PVCs), abaci, chip trading and digit cards; (c) **number line** models (or strips); and (d) **patterning** models such as number expanders.

1. **Whole numbers**. The common models used are set and position models where numbers are joined by using materials such as bundling sticks, MAB, or money (anything where individual objects are counted) or counters or number cards on PVCs (chip trading) or abaci. The area model is used to a small extent in the relationship between flats, longs and units to provide the idea that a hundred is 100× a unit. Numbers are placed on number lines to assist with order, rank and other processes. Finally, numbers can be expanded on the number expander (and there are special expanders for large numbers).

2. **Decimal numbers**. Sets and PVCs can still be used through money, chip trading or an abacus (and digit cards on a PVC can be very useful). The area model is more widely used with, for example, a 10×10 square being one, a strip being a tenth and a small square being a hundredth. Placing numbers on number lines is useful for rank, order, rounding and density. Finally, numbers can still be expanded on a number expander and patterns seen with digits on large PVCs.

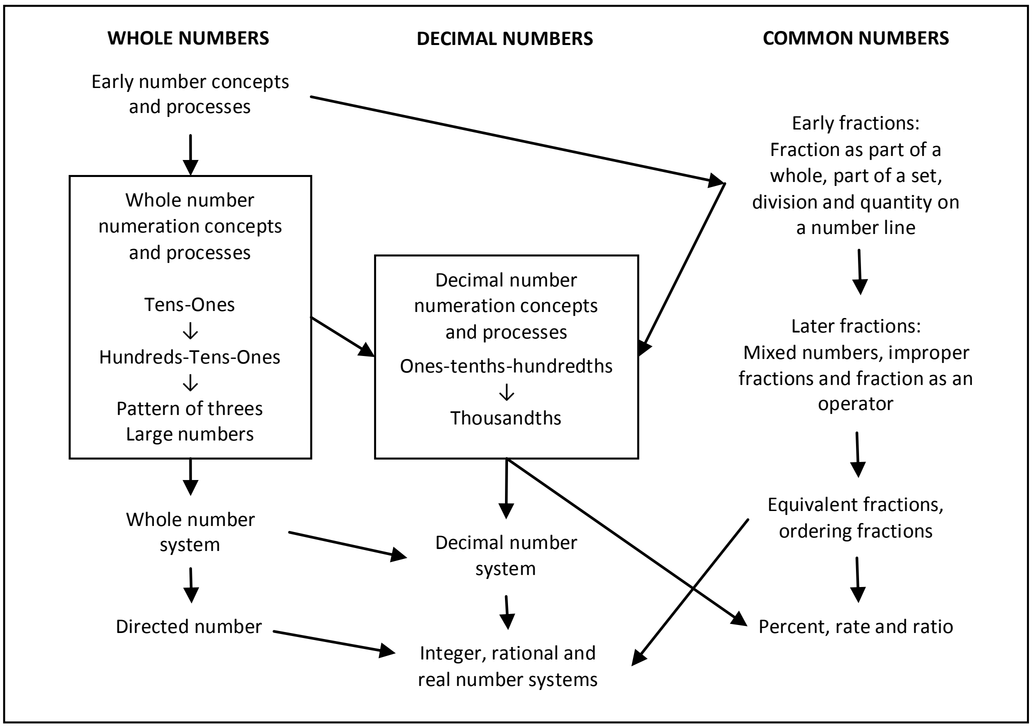
Symbolic and language representations. As stated above, symbols and language are different to models because they separate and make connections difficult. At their basis, decimal numbers and common fractions are similar but have very different notation and language and this difference in symbols and language makes the mathematics look different when it is in fact the same.

Sequencing for decimal number numeration

This section briefly looks at the role of sequencing in teaching decimal numbers and how the units are sequenced in this module.

Sequencing wholes, fractions and decimals

These sequencing relationships for common fractions and decimal numbers are given in the figure below. It shows how whole numbers and common fractions give rise to decimal numbers and then, in turn, how the growth of numbers is affected by decimal numbers.



Because this module is for Year 8 students and, therefore, focuses on **reteaching**, it is possible to **integrate ideas and use shortcuts to gain efficiencies to allow for acceleration**. Therefore, the module does the following:

- uses the part-of-a-whole meaning of common fraction to introduce tenths and hundredths as fractions, and uses reunitising to show how tenths and hundredths relate to decimal notation;
- introduces decimal number numeration by extending multiplicative structure in whole numbers back past the one to tenths and hundredths so that decimal numbers are an extension of whole numbers;
- consolidates decimal numbers by moving from tenths and hundredths to thousandths in two sections, one based on positional understandings (place value) and the other on number-line understandings (both ways teaching decimal numeration big ideas as an extension of whole number big ideas); and
- begins the process of understanding decimals as a real number system encompassing whole numbers, rationals and irrationals.

There is one particular part of the sequence that needs special attention – this is at unitising/reunitising, when teaching moves from tenths to hundredths and is recorded in decimal notation. For fractions, 43 hundredths is just 43 hundredths but for decimal notation, it needs to be understood as 4 tenths and 3 hundredths so it can be written as 0.43. Thus 40 hundredths must be seen as 4 tenths. This is not as simple as it seems – it means seeing 40 hundredths as 4 ten-hundredths out of 10 ten-hundredths – this requires the unit/whole to be initially seen as a hundred parts and then changed, in the mind, and seen as 10 lots of 10 parts. This is called **reunitising**. Students need a lot of practice in it, and this practice must involve going both ways, 40 hundredths to 4 tenths and 4 tenths to 40 hundredths.

Sequencing within this module

In this module the sections are as follows. The units follow the sequence in the dot points above.

Overview: Introduction, background information, sequencing and relation to Australian Curriculum

Unit 1: Tenths and hundredths – fractions to decimals

Unit 2: Tenths and hundredths – whole numbers to decimals

Unit 3: Numeration to thousandths – positional activities (big ideas 1, 2, 3 and 5 – notion of unit, additive structure, multiplicative structure, and equivalence)

Unit 4: Numeration to thousandths – number-line activities (big idea 4 – continuous vs discrete, number line, rank, order and rounding)

Unit 5: Numeration past thousandths – pattern and symmetry

Test item types: Test items associated with the five units above which can be used for pre- and post-tests

Appendix A: Activities and templates for activities in the units

Appendix B: RAMR cycle components and description

Appendix C: AIM scope and sequence showing all modules by year level and term.

The sequence of units follows the structured sequence that is the basis for this module. Each unit is itself divided into one or more RAMR lessons and the stages of RAMR are used as subheadings within each unit (see **Appendix B**).

Most of the units are based around a big idea and accompanying concepts and processes (see Module N1 *Whole Number Numeration*). This is to ensure that all the concepts and processes associated with each of the five big ideas are covered.

Relation to Australian Curriculum: Mathematics

AIM N2 meets the Australian Curriculum: Mathematics (Foundation to Year 10)						
Unit 1: Tenths and hundredths – fractions to decimals Unit 2: Tenths and hundredths – whole numbers to decimals Unit 3: Numeration to thousandths – positional activities Unit 4: Numeration to thousandths – number-line activities Unit 5: Numeration past thousandths – pattern and symmetry						
Content Description	**Year**	**N2 Unit**				
		1	**2**	**3**	**4**	**5**
Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033)	**2**	✓	✓			
Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation (ACMNA079)	**4**	✓	✓	✓		
Recognise that the place value system can be extended beyond hundredths (ACMNA104)	**5**		✓	✓	✓	✓
Compare, order and represent decimals (ACMNA105)				✓	✓	
Use estimation and rounding to check the reasonableness of answers (ACMNA128)	**6**				✓	
Multiply and divide decimals by powers of 10 (ACMNA130)			✓	✓	✓	✓
Round decimals to a specified number of decimal places (ACMNA156)	**7**				✓	

Unit 1: Tenths and Hundredths – Fractions to Decimals

This unit covers the common fraction component of decimal numbers – looking at what tenths and fractions mean as part of a whole and how reunitising enables 34 hundredths to be seen as 3 tenths and 4 hundredths and as 0.34, building the relationship between language and symbols for decimals.

1.1 Tenths/hundredths – fractions as part of a whole

Teaching decimal numbers involves introducing new place-value positions. These positions (i.e. tenths, hundredths, and so on) are fraction place values and thus, we need to introduce fraction as part of a whole for these particular fractions (see **Appendix A1** for examples of paper folding for fractions). This is a starting point for the learning of decimals.

A. Tenths as part of a whole (area/number-line models)

The following teaching sequence should be used:

Identify the whole

Partition into equal parts (e.g. 10 equal parts)

Name the parts – tenths (may have to acknowledge other fractions here but we are only working with tenths)

Consider a certain number of parts (e.g. five parts)

Name the fraction (five tenths)

Reality

Discuss what it is that makes a whole. Can we use the same criteria every time we are looking at breaking something into parts? It is important that it is understood that a whole is not necessarily ONE, for example:

On Monday I bring in 1 cake to share with 10 students – 1 cake is the whole that will be shared.

On Tuesday I bring in 2 cakes to share with 10 students – 2 cakes is the whole that will be shared.

On Wednesday I bring in half a cake to share with 10 students – half a cake is the whole that will be shared.

On Thursday I bring in 1 cake that is the size of the table to share with 10 students – 1 large cake is the whole.

Each day/situation is different but at each point we are identifying the whole as what is available to be shared.

Abstraction

Body

This can be done using the strip mat. Use a square card to cover the "parts" to indicate one tenth. Three parts covered with square cards is three tenths. *We are only using the language here without introducing the symbols.*

Strip mat (one whole) *Square card (one tenth)*

Hand

Use different shapes and quantities that are or can be broken into 10 equal pieces. For example, use a smaller version of the strip map and colour or cover squares to give different fractions, e.g. 3 tenths, 8 tenths (can also use the 10×1 grids in **Appendix A4)**.

Mind

Do activities as follows: *I am running a race and I run two tenths of the race – am I closer to the start or the finish? I cut a cake into 10 pieces and kept 4 tenths and I gave the rest to my friend. Who got more cake?*

Mathematics

Introduce the symbols for one tenth – 1/10 means one part out of 10. 5/10 means 5 parts out of 10.

Use practice activities such as: (a) shade these fractions – 3 tenths, 9 tenths, 14 tenths (and so on) on shapes – rectangles, squares, L shapes (and so on); (b) shade 10 tenths – *What is 10/10?, 20 tenths?*; and (c) activities such as in **Appendix A2**.

Note: We are not teaching fractions as such here; just the idea of breaking a whole into 10 equal parts and one part being one tenth. Two parts is two tenths, 20 tenths is two wholes and so on. We are relying here that there is some basic knowledge of fractions to be able to label this as tenths. If not, you need to spend a little bit of time building up one half, one quarter, one twentieth and so on, just a couple of fractions to get the idea.

Reflection

Reverse working from the whole to part and **work part to whole** (identifying the whole from the part).

Identify fraction (e.g. 2/10); ask to make whole.

Find one part (e.g. construct 1/10); observe construction (e.g. to find 1/10 when given 2/10 means halving what has been identified as 2/10).

Find whole by putting together the necessary copies of the one part (e.g. make 10 copies to give 10 tenths or make 8 tenths to put together with 2 tenths).

Construct more than one differently shaped whole (e.g. L shape, P shape, steps, etc.)

B. Hundredths as part of a whole (area/number-line models)

Reality

Identify situations where a part or a whole is separated into further parts, for example:

One school separated into 3 parts (grades), these parts are separated into further parts (classes) – there are now 3 parts separated into 3 parts again – 9 parts altogether.

I cut the cake into 10 pieces for my 10 students. Each piece is 1 part – 1 tenth. Each of the 10 people must share their piece of cake with their brother – each part is separated into 2 parts – 10 parts separated into 2 more parts – now 20 parts altogether.

Abstraction

Body

Again using the strip mat – take one of the one-tenth squares and cut into 10 pieces or strips. Say: *How many of these strips will fit into one square?* [10]. *How many strips do we need to cover each of the 10 squares?* [100]. *If we need 100 of these strips to cover the mat, what do you think we call this strip?* [One Hundredth]. Using the hundredth strips, put out 7. Say: *How much of the mat is covered?* [Seven hundredths]. Put out 18 on the mat covering 18 hundredths. Say: *How much of the mat is covered?* [Eighteen hundredths].

Strip mat (one whole)

Square cards (one tenths)

Strips (one hundredths)

Hand and Mind

Have students make the tenths and hundredths step by step and recognising the significance of each cut as they make it, labelling each part as they go. They can use small copies of the strip mat or the **10×10 grids in Appendix A4**. Begin with a whole – they can construct a square (easy if it is a multiple of 10 cm, e.g. 20×20 cm) – label this one whole. Accurately cut this shape into 10 equal-sized pieces (strips) – label each of these one tenth. Accurately cut these tenths into 10 equal pieces each (small squares) – label each of these one hundredth.

This can be done with different shapes, not just squares. These pieces can be kept in envelopes for later in the module wherever students need to use concrete materials.

Mathematics

Introduce the symbols for one hundredth – 1/100 means one part out of 100. 5/100 means 5 parts out of 100.

Use a hundreds grid to shade 3 tenths, shade 18 hundredths, shade 1 whole, 8 tenths and 4 hundredths etc. – see **Appendix A2**.

Reflection

Reverse the activity of making one hundredth from a whole and give the students the "one hundredth" and they must construct the whole by constructing the tenth, labelling it and then constructing the whole. (One small box – they need 10 lots of 10 boxes; half a circle – they need 100 half circles.)

1.2 Tenths/hundredths – reunitising for decimals

Reunitising is when you have to make a whole, for example, not 12 squares but 3 rows of 4 squares – then 4 squares is 4/12 in terms of squares but also 1/3 in terms of rows of 4. This is important for tenths/hundredths because we have to show 34 hundredths is 3 tenths and 4 hundredths by looking at rows of 10 so we can relate to place value.

Reality

Think of situations where you may need to reunitise. In the example of the school and classes above there may be situations where you need to walk about the whole school or the 3 grades or the 9 classes.

A cake that is pre-cut in 10 pieces that then each have to be divided into two – the person serving the cake will want to know that it is 20 pieces.

Abstraction

Body

Using the **Maths Mat** (6 × 10), put the elastics show one whole as one row of 10. Surrounding each of the parts with elastic, count the squares to determine:

One whole row = ? tenths; Two whole rows could also be ? tenths.

One row and 8 tenths = ? tenths; Two rows and 3 tenths = ? tenths; Five rows and 8 tenths = ? tenths.

8 tenths = ? hundredths; 4 tenths and 2 hundredths = ? hundredths.

One row, 2 tenths and 4 hundredths = ? tenths and 4 hundredths = ? hundredths, etc.

Hand

Using **MAB** or hundreds charts, begin with the initial number. Use the flat as the one. Then 8 wholes, 2 tenths and 3 hundredths would be 8 flats, 2 longs and 3 units. Ask questions like: *How many tenths? How many hundredths?* Use MAB on ones-tenths-hundredths PVC. (*Note:* Ensure students can use the blocks this way and are not still seeing the flat as 100, long as 10 and unit as one – if they are go to the 10×10 grids as below.)

Another way is to use 10×10 grids (see **Appendix A4**). Ensure students see a whole square as a one, a column or row as a tenth and a small square as a hundredth. Then shade ones, tenths and hundredths and ones and hundredths – always reverse everything: (a) ones-tenths-hundredths to ones and hundredths and just hundredths and reverse; and (b) name to shading and reverse (shading to name).

A recommended activity is to use the Fractions to Decimals worksheet in **Appendix A5**. This has 10×10 grids to shade, a ones-tenths-hundredths PVC with Unifix, a small PVC on which to write in the numerals, and a place to write the decimal number (can replace this with a calculator). Use the materials all together to move from ones-tenths-hundredths as fractions to decimal numbers.

- Step 1: Shade, say, 2 wholes and 34 hundredths.
- Step 2: Discuss the shading – show it is 2 wholes, 3 tenths and 4 hundredths.
- Step 3: Translate this to PVC by putting 2 counters in ones, 3 counters in tenths and 4 counters in hundredths.
- Step 4: Transfer this to a small PVC with 2 in ones, 3 in tenths and 4 in hundredths.
- Step 5: Write the number as 2.3 4.
- Step 6: Do the reverse, starting with, say, 1.6 2. And continue on back and forth.

Mind

Students to picture the body and hand activities in their heads – whether they pick the mat or the shading, they answer the same types of questions but they only do it in their minds.

Mathematics

Activities that involve the formal recognition that there are 10 tenths in one whole and 100 hundredths in one whole and that allow students the flexibility to convert between the three units. For example, how many hundredths in 2 ones, 4 tenths and 7 hundredths and reverse; what is 4 ones and 35 hundredths in ones, tenths and hundredths and reverse.

Students need to practise with multiple wholes, e.g. 4 wholes 3 tenths – to learn that the same rules apply for all numbers, not just between 0 and 1.

Reflection

Give problems and applications with decimals. Extend thinking by such activities as: *Find two other ways to express 4.56*; and so on. Be flexible in presentation, reverse everything and state generalisations.

Unit 2: Tenths and Hundredths – Whole Numbers to Decimals

This unit looks at how there can be a seamless transition from whole numbers to decimal numbers. It covers whole number ideas in the form that they can extend to decimals, uses multiplicative structure in whole numbers to develop decimal number PV positions of tenths and hundredths, and connects common fractions, whole numbers and decimal numbers.

2.1 Whole number ideas

The idea of this unit is to recap the whole number work so that it is remembered in a form that makes decimals easy (and removes all differences that cause difficulty). This means seeing whole numbers and decimals as the **same** in as much as possible as follows.

1. **PV positions are determined from the ones –** in whole numbers, the ones is the right-most position; in decimal numbers, the ones is the position with a dot after it.

   In whole numbers, the PV positions move left from the ones following tens, hundreds, thousands, and so on. In decimal numbers, the PV positions move left and right symmetrically about the ones position as below:

   Hundreds Tens **Ones** tenths hundredths

2. **Larger numbers have larger digits in the largest PV positions –** it is only because of the placement of the ones position in whole numbers that more digits means larger numbers. This latter idea does not translate but the former idea is the same for decimal and whole numbers.

   Numbers to be compared should be placed into their PV positions and then compared.

3. **Addition and subtraction is by aligning PVs –** it is recommended not to use aligning by decimal points. (This is because the shortcut of "align decimal points" can lead to the big idea not easily extending to time, mixed numbers and variables.)

4. **Multiplication and division is by adding/subtracting number of PV positions to ones** (include ones in number) – for example:

   10×1000 = 1+3 positions = 4 positions = 10 000;

   10÷1000= 1−3 positions = −2 positions = 0.01

5. **Multiplying and dividing by 10 moves PV positions to left and to right** – not to be remembered in terms of zeros and movement of decimal point as these are only consequences of the PV movement.

At the same time we must remember the **differences**; decimal numbers have the following which whole numbers do not have: (a) fraction PV positions; (b) different convention for determining ones position; (c) high density (no whole numbers between 24 and 25, while infinite decimal numbers between 24 and 25, 24.2 and 24.3, 24.37 and 24.38, and so on).

Note: It is important to go back and check students' understandings of whole numbers and ensure there are no shortcut understandings which will not extend to decimals. For example: (a) more digits, larger number; and (b) to ×10, add a zero.

2.2 Multiplicative structure to develop tenths/hundredths

Use the multiplicative structure big idea that was developed during the whole number module to continue into the multiplicative structure of decimals (see **Appendix A3** for Wholes to Decimals worksheet). This is a way to build decimals directly from whole numbers.

Reality

The concept of decimal numbers is familiar to students as they use money in the everyday world. Discuss other ways money is referred to.

Introductory activity

Materials: Line with 0 cents at one end (LHS) and $2 on the other (RHS), yellow arrows with numbers written in dollars and cents between 0 cents and $2 on one side (e.g. 1 dollar and 34 cents; 27 cents) and same number written in decimals and dollar signs only on the other side (e.g. $1.34, $0.27), and green arrows with the same numbers on them as decimals only (e.g. 1.34, 0.27).

Instructions:

1. Set up the line with 0 cents at one end and $2 at the other. Have students place yellow arrows at the appropriate point on the line with the side up that is in dollars and cents.
2. Discuss as they place them, what the students notice. Different representations should be offered, and reasons for those.
3. Flip the arrows over (in the same position) to show the decimal dollar format. Discussion around the difference should highlight the second decimal place (hundredths).
4. Next have the students place the green arrows (decimal side up) beside the matching yellow arrow and notice the similarities and differences.

Abstraction

Body

Use number bibs or cards with students standing in front of labelled place values, to experience this concept.

> Start with a number, e.g. 23 (two students in front of tens label and three students in front of ones label). At this point, only have the ones, tens, hundreds, thousands etc. labels displayed.
>
> ×10, place values change to reflect this change (record the changing number at each multiple) – first change, tens student moves to the left (their right if facing class) to the hundreds and ones student moves to the tens etc.
>
> Repeat three more times – ask students for ×10 rule (×10 means digits move left – **not** add a zero).
>
> Now work backwards (e.g. starting with 23 000), ÷10 on calculator three times, changing PVC numbers and recording – again ask students for ÷10 rule (÷10 means digits move right – **not** remove a zero).

Students should now have the number 23 on their calculator. Ask the question: *What would happen if we ÷10 again?* Discuss possibilities, directing the discussion toward the idea that the ones digit will now be ten times smaller therefore shift to the right. *This position is called TENTHS* – draw on the earlier experience of dividing a whole into ten equal parts being one tenth.

We need a way to record the ones position – how do we record 2 ones and 3 tenths? – it can't be 23 as this means something different – encourage students to understand that we need a new way of showing the ones. Let students use their own new way (e.g. hat on top of ones) to record the ones position.

Then ask question: *What would happen if we ÷10 again?* Discuss possibilities, directing the discussion toward the idea that the tenths digit will now be ten times smaller therefore shift to the right. *This position is called HUNDREDTHS* – draw on the earlier experience of physically dividing a tenth into ten equal parts being one hundredth.

Hand

Students start with calculators, PVC/digit cards or multiplicative sliders on PVCs and pen/paper and can follow the same process as they did with body but here they are working the calculator and recording all themselves. Students can use the concrete materials they had developed earlier to show that each next PV was a multiple of 10 of the previous one. Each time the "unit" was cut into 10, there were 10 times more units of the new PV.

Mind

Picture the number 54 in your head. If you multiply this number by 10, what number do you have? If you divide it by 10 and then 10 again, what number do you have? and so on.

Mathematics

Spend a brief time with recording method chosen by students – then introduce decimal point. Stress that the point has no position – it only exists to show the ones. Always write decimal numbers with no position for point (i.e. 2.3 4 not 2 . 3 4 or 2.34 – *this is difficult to show with typed numbers*). YDC staff have found it useful to always circle the ones and the point to drive home that it is the ones about which decimals pivot (not the point), for example:

2 (5.) 3 6

Discuss continually ÷10 and what this means for new decimal place-value positions – stress symmetry around the ones, for example:

4 9 2 (5.) 3 6 7

Tens Ones tenths
Hundreds Ones hundredths
Thousands Ones thousandths

H	T	O	H	T	O	H	T	O	tenths	hundredths	thousandths
Millions			Thousands			Ones			Parts of One		

Reflection

Now that students have built the PV structure around the decimal point, return to the reality of this concept.

Look at examples in a newspaper, on the internet or on TV where decimal numbers are used. What are they used for? What do they mean in each example?

Is it reasonable that any measure could be expressed with a decimal? A metre, the number of laps of the oval, the number of people? Often a whole number is sufficient – especially when counting.

Unit 3: Numeration to Thousandths – Positional Activities

This unit looks at the number big ideas for ones, tenths, hundredths and thousandths that relate to position. The following points need to be made.

- Part-whole, additive structure, multiplicative structure and equivalence activities involve PV charts and materials on these charts – these materials can relate to size (e.g. tongue depressors whole, cut into tenths and cut into hundredths – each tenth cut into further tenths) but this is difficult. Good learning can also come from counters and digit cards being placed on PVCs.
- Use the decimal point **only** as the way to designate the ones position. Extend **seamlessly** from whole to decimal numbers. Stress the difference between writing and saying decimal numbers – decimal numbers should be read using the place values rather than reading each digit. For example: (a) 1.3 7 is one and thirty seven hundredths, rather than one point three seven; and (b) 9.5 3 4 is nine and five hundred and thirty-four thousandths, rather than nine point five three four.
- When writing decimal numbers, try not to give the decimal point a position by making extra space for the dot (sometimes it is useful to circle number and dot).

3.1 Part-whole, place value, reading and writing

Early activities showing size

Tenths

Cut bundling sticks into 10 equal pieces. Then have three types of materials – single sticks, bundles of 10 sticks, and sticks cut into 10 equal pieces.

Place material on a tens-ones-tenths PVC. Have students also writing numbers on small versions of the PVC, and placing numbers on calculators, and writing with pen/paper.

Relate stories ↔ materials ↔ pictures ↔ language ↔ symbols for tenths. Go both ways: stories and materials → symbols AND symbols → stories and materials.

Hundredths

Obtain 10×10 grids (see **Appendix A4**), ones-tenths-hundredths PVC, small pictures of PVC, Unifix, calculators and pen/paper.

Relate stories ↔ materials ↔ pictures ↔ language ↔ symbols for tenths and hundredths – recap that 50 hundredths = 5 tenths and 34 hundredths is 3 tenths and 4 hundredths. Go both ways: stories and materials → symbols AND symbols → stories and materials.

An effective way is to use 10×10 grids to direct students to shade 3 wholes, 4 rows and 6 little squares – transfer this to a ones-tenths-hundredths PVC using coloured counters for ones, tenths and hundredths – translate this to a small PVC using digits, and write as a decimal (or put on a calculator).

Then reverse this process by giving the decimal and asking for small PVC, large PVC, and shading to be completed. Move forwards and backwards like this. Emphasise the change from shading 40 squares to seeing this as 4 in the tenths place and vice versa.

Extension activities

Use digit cards on PVCs – focus on the position.

Give students PV cards and get them to line up in correct PV positions. Have other students stand in front of them with digits to make numbers. Go both ways: numbers → students in position AND students in position → numbers.

Extend the ideas from earlier to include thousandths by extending tenths/hundredths by seeing the pattern that relates PVC to language and symbol – go both ways.

You could also relate tenths, hundredths and thousandths to Hundreds, Tens, Ones of thousandths to continue the pattern of three as below in the example RAMR activity.

Relate back to whole numbers.

Example RAMR activity

Below is an example of an activity using the PVC below.

H	T	O	H	T	O	H	T	O	tenths	hundredths	thousandths
Millions			Thousands			Ones			Parts of One		

Abstraction

Body

Use the "body" place cards for students to "be" the digit in different place values. Have the class read out the name of the number they see. *Remember that this exercise is more for the students being the numbers and locating their positions etc. than it is for the remainder of the class, so change the students so they all have a turn in both roles.

Hand

Place PV cards on mat and use this as a basis to discuss what is happening. Practise reading and writing decimal numbers to thousandths – play games such as "Wipe-out". Use digit cards to represent numbers on PVCs to help students see how many zeros in wiping 6 from 1.268.

Mind

Play Wipe-out without the calculator:

- Say the number "one and forty-nine hundredths" and students must picture the number in their heads.
- Say – "a number that has 8 hundredths, 9 tenths and 4 ones" and students must only write down the number (4.98).

Mathematics

Allow students opportunities to read and write the numbers as well as just write the numbers for the word descriptions. The place values do not need to be given in order. In fact they should be given out of order to really determine if students understand place value.

Reflection

The Activity Sheet, "Colour in Decimats" (from *APMC magazine, 2010*) could be used here to extend the reading and writing of decimals as it changes the unit and parameters that you are working with throughout the activity. This activity can be modified as a game with different outcomes.

3.2 Additive structure, counting, odometer and seriation

Recap the role of counting and seriation in whole number PV positions – ensure students can work over the bridging numbers, for example, if counting in tenths: 2.8, 2.9 to 3.0, NOT 2.91 or 2.10 etc.

Use a variety of activities to get students to count in decimal place-value positions:

- Place digit cards on PVC or use an abacus. Pick a PV position. Change the numbers in this position as students count. Discuss what happens after we get to 9 in a PV position.
- Repeat counting back – discuss what happens when get to 0 in a PV position.
- Use calculators to count in decimal PV positions (e.g. enter 3.258, + .01 and keep pressing = to count forwards in hundredths position – subtract to count backward).
- Extend counting to show examples of odometer pattern for counting on and counting back – ask students to identify pattern.
- Look at examples of seriation – looking at making one decimal PV position larger and smaller.
- Practise counting patterns – e.g. 4.7, 4.8, 4.9, ___, ___, ___; 4.6 6, 4.6 8, ___,___,___.
- Discuss how odometer is a recurring pattern that affects whole numbers, decimals and metrics (base 10) and time, angle, common fractions for other bases (e.g. $2\,^{4}/_{6}$, $2\,^{5}/_{6}$, $3\,^{0}/_{6}$, $3\,^{1}/_{6}$, and so on).

3.3 Multiplicative structure, renaming and flexibility

This concept was covered earlier in the module using the area model. Students now have a more formal understanding of decimals and their relation to the number line and can use more sophisticated techniques.

Some examples of what to do are:

- Use students as PV positions and have another student moving up and down in front of them – discuss what operation changes numbers as students move. Reverse this – ask what ×10 does for the position of the number. Do the same for division.
- Place digits on a PVC going from thousands to thousandths, place on calculator and record, ×10 and ÷10 and note how digits move. Do this as doing body work above. Go both ways:
- operation $\rightarrow$ move AND move $\rightarrow$ operation
- Repeat this for ×100, ÷100, ×1000, and ÷1000. Ask students to state rules for all these activities.
- Record all the changes (e.g. 2.54 × 10 is 25.4) and discuss what moving digits a number of PV positions means for the positioning of the ones position (as determined by the decimal point – never give any rules in terms of decimal points) – see below:

  The ones position moves to right as digits move to left.

$$2\ \textcircled{5.}\ 3\ \ 6\ \times 10 = 2\ \ 5\ \textcircled{3.}\ 6$$

- Practise examples such as 7.23 × 10 = __________; 26.5 ÷ 100 = _________. Discuss how the ×10/÷10 structure applies in metrics and, in other bases, to time, angle and so on.
- Place digits on PVC from thousands to thousandths, place an object on PVC to act as decimal point – discuss role of decimal point.
- Place digit cards to make a number (e.g. 256.78); discuss this as 25 tens, 6 ones and 78 tenths – move decimal point to tens – discuss how this makes 25.678 tens from 256.78 ones. Set up examples where

decimal point is moved and discuss what the number is (e.g. 34.6 becomes 0.346 hundreds, 0.457 becomes 45.7 hundredths – discuss how this is also 45.6%. Relate to metrics.

- Discuss how flexibility of decimal point means renaming and vice versa. Relate to $4.65 being 46.5 ten cent pieces.

3.4 Equivalence

Recap equivalence from whole numbers. Note that it is not extensive and relates to placement of zeros. For example, which placement of zero changes the number: 3 to 30 or 3 to 03?

Extend this to decimals by looking at adding zeros into decimal numbers – which of these change the value of the number?

- 2.46 → 02.46
- 2.46 → 2.406
- 2.46 → 20.46 or 2.046
- 2.46 → 02.46

Do many of these and see if students can build a **pattern** – where can zeros be added without changing the value of the decimal number?

Unit 4: Numeration to Thousandths – Number-Line Activities

This unit looks at decimal numeration to thousandths for number lines. It covers the continuous–discrete/number line big idea, the concepts of rank and density, and the processes of comparison and order. Number lines do not show PV positions separately but place the number as a whole in one position. This is useful for rank and order and also estimation and density. It is important to use number lines for number as jobs in mines and construction use number this way. The unit covers these ideas, concepts and processes and ends by looking at two of the concepts/processes from both positional and line perspectives.

4.1 Continuous–discrete/Number lines, rank, order and rounding

A. Rank and order

Ranking and ordering decimal numbers is an important big idea that will transfer to the measurement units. Students must have an appropriate method to do this.

Reality

Relate examples of ordering decimal numbers to the examples that students have uncovered during activities for whole numbers and other topics (e.g. 5 023 and 5 203, or 1 metre 45 cm and 1.5 metres). The actual unit or measure of the examples need not be emphasised as it is just the numbers we will be examining.

Abstraction

Body

Use rope, pegs and pieces of paper to place decimal numbers on number lines. Begin with basic tenths – one tenth, two tenths, three tenths, ..., ten tenths (one) to give students a reference to be able to place numbers on the number line. Add the numbers 0.1, 0.2, 0.3, ..., 1 to the number names on the number line.

On an empty rope number line place your beginning and end numbers, i.e. 0 and 2, then have students peg numbers on the number line – first tenths, then tenths and hundredths. Allow students time, or encourage them, to self-correct – ask *Do all the numbers appear to be in the correct places? Are there any changes that anybody would like to make?*

Using masking tape on the floor the same activity can be completed using digit cards and word names. The tape on the floor makes it easier to extend to thousandths. Compare decimal numbers by their PV positions and identify larger number by the digit in the largest PV position. Show that decimal numbers with more digits can be smaller, e.g. 3.45 is smaller than 4.2.

Mind

By picturing the position on the number line of numbers, or the numbers' distance from zero, students can tell you which number is larger – 7.8 or 7.9;   5.678 or 5.7?

Mathematics

Complete activities where students must order and rank and compare three or more decimal numbers. They should be able to do this without necessarily using a number line every time.

Reflection

Challenge students to rank numbers in the thousands or millions, where there is more information than is necessarily important to the decimal ranking, e.g. rank 154 639.3268 and 154 639.6587, and so on.

B. Density

Discuss how looking at decimals as a number line (or length) changes discrete to continuous – that there are an infinite number of numbers between 7 and 8 and there are an infinite number between 7.4 and 7.5 etc. It is always possible to get a number between two others, e.g. *Give me a number between 4.56 and 4.57*.

Abstraction

Body

Two students stand on either side of the classroom. They are the ends of the number line, say 11 and 14 and they write their number on a card that they hold on their point. Then the next student picks a number between these, say 12, writes a card and stands on the number line and holds it in place. The next student must choose a number between 11 and 12, say 11.6. The next student chooses a number between 11 and 11.6, say 11.5. The next must choose between 11.5 and 11.6, say 11.54. The next between 11.5 and 11.54. The activity continues to numbers with up to 6 or so decimal places.

Students may need some reminding of the multiplicative nature of the number system, that each new "smaller" PV position is a division of ten from the previous position.

Hand

Students can make the place cards that begin with two consecutive whole numbers and are broken down until there are 6 decimal place values, in the same way as in "Body" above. They should test these on their peers.

Mind

Students should be able to answer the activities in their minds.

Mathematics

Students should be formally recording numbers that are in the ten thousandths, hundred thousandths. These names are worth familiarising students with.

Reflection

Choose a number, e.g. 2.5468, and students reverse the concept and choose the previous two "place value" numbers that it would have been between, i.e. 2.5468 is between 2.546 and 2.547.

Choose a number, e.g. 5.2145, and students choose two other numbers that could have been chosen, i.e. 5.2145 is between 5.214 and 5.215 – other numbers could have been 5.2148 or 5.2143.

C. Rounding and estimation

The ability to estimate with decimals is important when doing money and measurement.

Reality

Discuss the importance of being able to estimate and where it may be used. For example, if a measured distance from the airstrip to the school is 4.678 km and a visitor needed a rough idea how far to go, you would give an estimate of "about 5 km" rather than 4 km as 5 km is more accurate.

Abstraction

Body

Students place numbers on a number line, e.g. start at 3 and put place cards down in tenths until 5. Examine each of the tenths values and decide which whole number they are closest to. Discuss the rule about the midpoint (5) being rounded up – that if the number 5.678 is being rounded to the nearest whole number then the hundredths and thousandths are insignificant. If the number is being rounded to the nearest tenth then the thousandth place value is insignificant.

Place digit cards on the number lines and decide what they should be rounded to.

Have students "be" the digit decimal numbers using the digit bibs. For example, if the number is 4.621, and you are rounding to the nearest tenth, the digits that are not considered (thousandths) should sit down on the spot. The number is then rounded.

Hand

In pairs or singularly, students use a number line at their desks (e.g. between 1.7 and 1.9) and have number cards (e.g. 1.75, 1.84, 1.8995) that they place on the number line then slide to the correctly rounded tenths position. Have a number of packs ready so students can swap when they are finished.

Mind

Ask students to mentally determine the appropriate rounding for decimal numbers.

Mathematics

Place decimal numbers on number lines so that students can determine rounding to various decimal PVs (e.g. 2.457 is between 2.4 and 2.5 and is closer to 2.5 than 2.4 so 2.457 is 2.5 (rounded to the nearest tenth).

Practise these skills – use the game "Target" (e.g. 17 × ? = 500.6 so use calculator to determine by trial and error what the ? is).

Reflection

Reverse the questions – *What are two numbers that if rounded to tenths could be rounded to 5.6?*

4.2 Using number lines as well as place-value charts

In this section, we look at how taking two perspectives (positional as well as number line) from two big ideas (part-whole and continuous vs discrete) deepens understanding.

A. Counting, odometer, seriation and number line

Recap the role of counting and seriation in whole number PV positions – ensure students can work over the bridging numbers, for example, if counting in tenths: 2.8, 2.9 to 3.0, NOT 2.91 or 2.10 etc.

Abstraction

Body

Step forward counting out loud with each increasing or decreasing step. Count around the room with each person saying the next number.

Hand

Use calculators to count in decimal PV positions. Clap once – students must say what is next – clap three times and students must say the number, counted three tenths up or down etc.

Mind

Say a number – e.g. 1.34 and you are counting in tenths – clap once – students must say what is next – clap three times and students must say the number, counted three tenths up or down etc.

Mathematics

Extend counting to show examples of odometer pattern for counting on and counting back – ask students to identify the pattern.

- Practise patterns – e.g. 4.7, 4.8, 4.9, ___, ___, ___; 3.6 6, 3.6 8, ___,___,___.
- Look at examples of seriation – looking at one decimal PV position larger and smaller.
- Discuss how odometer is a recurring pattern that affects whole numbers and decimals.

B. Multiplicative structure, renaming and flexibility on number line

This concept was covered earlier in the module using the area model. Students now have a more formal understanding of decimals and their relation to the number line and can use more sophisticated techniques.

Abstraction

Body

On a number line from 0 to 5 mark a point – e.g. two and forty-five hundredths. Ask students to walk from 0 to 2.45 in PV steps – e.g. two big steps for whole numbers, 4 smaller steps for tenths and tiptoes for hundredths.

How else could we walk the same distance?

- 2 big steps for whole numbers and 45 tiptoe steps
- 2 big steps, 3 smaller steps for tenths and 15 tiptoe steps
- 1 big step, 14 smaller steps and 5 tiptoes, etc.

Ask: *If I started with 15 tiptoes and two whole steps, how many smaller steps would I need? If I started with one whole step and 12 smaller steps, how many tiptoes would I need?* and so on.

Hand

The same activity can be completed with materials at student desks using cut up matchsticks, straws, toothpicks, skewers or lengths of beads etc. that can be arranged in a line to represent a number line.

Mind

Ask questions like: *2.37 is ? many hundredths?, ? many tenths*. *456 hundredths can be written as ?*

Mathematics

Students should complete exercises to rename numbers to the hundredths and thousandths.

C. Comparison and order

The power gained for this process of integrating two perspectives has been canvassed in N1. The number line shows that 3.48 is not as long as 4.2. However, many students, when seeing only the symbols, think that 3.42 is larger than 4.2 because it has more digits. This error can occur through developing a shortcut of "more digits means larger number" that some teachers use when ordering whole numbers. To avoid this problem it is necessary to look at order from a place value as well as number line perspective, as follows.

- Step 1 – start from number line and it shows that 4.2 is larger than 3.48.
- Step 2 – record the numbers being compared in two columns – aligning PV positions:

  3. 4 8
  4. 2

- Step 3 – study the columns for the pattern that always works. [The largest digit in the larger PV position always signifies the larger number – for example, for 3.48 and 4.2 , the larger PV is the ones – in the ones, 3.48 has a 3 and 4.2 has a 4. Thus 4.2 is the larger number.]

Unit 5: Numeration Past Thousandths – Pattern and Symmetry

The decimal numbers form a system that rolls on forever in both directions – getting as large as we want or as small as we want. This unit enables students to study possibilities and to see pattern and symmetry.

Step 1. Study the place-value positions – display them as below:

___________ _____________

Thousands thousandths

Hundreds hundredths

Tens tenths

Ones

Encourage students to see the symmetry in the PV positions about the ones. Discuss what would be in the left underline position – so what would be in the right underline position – get students to continue the pattern.

Step 2. Look at tenths, hundredths and thousandths:

- We know that 30 hundredths is 3 tenths – thus, 1 tenth is 10 hundredths;
- Construct a thousandth – a large square cut into 10×10 smaller squares with each square cut into 5×2 small rectangles – the large square is one, a row is a tenth, each small square is a hundredth and each small rectangle is a thousandth.
- Use this to show that a hundred thousandths are 1 tenth and 10 thousandths are 1 hundredth.
- This means that tenths-hundredths-thousandths can be seen as H-T-O thousandths.

Step 3. Look at the whole numbers in the pattern of threes as below:

and so on ← H-T-O Billions H-T-O Millions H-T-O Thousands H-T-O Ones

Add in the extra PV positions from Step 2:

__________ ___________

H-T-O Millions H-T-O ______

H-T-O Thousands H-T-O thousandths

H-T-O Ones

Encourage students to see the symmetry and to extend this pattern – discuss how similar it is to the Step 1 pattern.

Step 4. Can students now see this as a system? That is, patterns of three that go on in both left and right directions? Get students to make a large drawing of this symmetry pattern.

Step 5. This will be left to Year C when we do directed numbers and indices in Module N5 – we will write the system in scientific notation – we will also look at reals, rationals and irrationals. Then we will see that the decimal number system and the real number system are continuous, bi-directional and symmetrical.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the decimal number item types

There are four subtests related to Units 1 to 4 of this module. There is no subtest for Unit 5 as this unit just builds on the first four. Subtest 1 covers fraction understanding of decimal place-value positions for tenths and hundredths. Subtest 2 covers understanding of tenths and hundredths as an extension of whole numbers, looking at fraction and decimal notation. These two are smaller than Subtests 3 and 4.

Subtests 3 and 4 extend understanding to tenths, hundredths and thousandths. Subtest 3 covers the big ideas of notion of unit (place value, reading and writing), additive structure (seriation, counting), multiplicative structure (×10/÷10, renaming, flexibility) and equivalence (role of zero). Subtest 4 covers number-line work (order, rank, rounding, density) and relates these back to place value.

To ensure there is a diagnostic sequence, Subtests 3 and 4 need to have items covering tenths and tenths and hundredths as well as items covering tenths, hundredths and thousandths. Items at all three levels are given. However, it is expected that the pre-test may be restricted to tenths or to tenths and hundredths items depending on performance level of students. The post-test should include thousandths examples.

Subtest item types

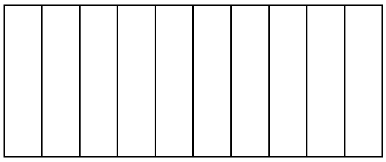
Subtest 1 items (Unit 1: Tenths and hundredths – fractions to decimals)

1. Write the decimal number for each name:

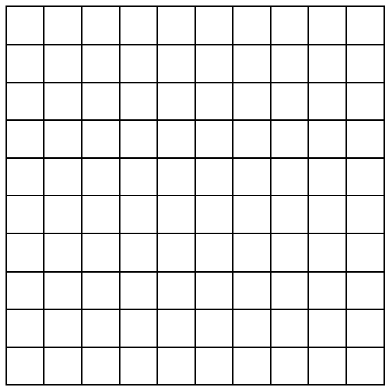
   (a) One and eight tenths __________

   (b) Two and sixteen hundredths __________

2. (a) Shade 0.4 of the shape below.



   (b) Shade 0.75 of the shape below.



3. Write the decimal number that is:

   (a) five hundredths __________

   (b) four ones, seven tenths, eight hundredths __________

4 Circle the number where 7 is worth the most:

   8.67 7.21 4.71 70.3

5. Write the number that is:

   (a) 0.1 more than 8.57 __________

   (b) 0.1 less than 6.99 __________

   (c) 0.01 more than 4.32 __________

   (d) 0.01 less than 2.60 __________

Subtest 2 items (Unit 2: Tenths and hundredths – whole numbers to decimals)

1. Write the decimal number that is:

   (a) seven hundredths __________

   (b) five ones, two tenths, nine hundredths __________

2. Write any number that comes between 3.4 and 3.5 __________

3. Write the decimal number for each name:

   (a) Five and three tenths __________

   (b) Four and nineteen hundredths __________

Challenge Question

4. A child told me that she was six and a half. How many different ways can you write her age with numbers?

Subtest 3 items (Unit 3: Numeration to thousandths – positional activities)

Tenths

1. Continue these counting patterns:

   (a) 4.6, 4.7, 4.8, _______ , _______ , _______

   (b) 7.4, 7.3, 7.2, _______ , _______ , _______

2. Round to the nearest whole number:

   (a) 3.7 ________

   (b) 1.2 ________

3. Write the number that is:

   (a) 0.1 more than 3.6 __________

   (b) 0.1 less than 7.9 __________

Tenths and hundredths

4. Continue these counting patterns:

   (a) 4.54, 4.53, 4.52, _______ , ________ , ________

   (b) 4.06, 4.07, 4.08, ________, _______ , _______

5. Round to the nearest whole number:

   (a) 6.78 ________

   (b) 1.08 ________

6. Circle the number where 5 is worth the most:

   8.65   5.21   4.51   50.3

7. Write the number that is:

(a) 0.01 more than 6.72 __________

(b) 0.01 less than 3.70 __________

Tenths, hundredths and thousandths

8. Write the answers :

(a) $3.4 \div 10 =$ __________

(b) $0.097 \times 10 =$ __________

(c) $0.05 \times 100 =$ __________

(d) $4 \div 10 =$ ____________

(e) $0.067 \times 10 =$ __________

(f) $0.08 \times 100 =$ __________

9. Write the missing operations (e.g. × 10 or ÷ 100 and so on)

(a) 4.3 __________ = 430

(b) 14.5 __________ = 145

(c) 53 _________ = 5.3

(d) 0.125 _________ = 125

Subtest 4 items (Unit 4: Numeration to thousandths – number-line activities)

Tenths

1. Write any number that comes between 4 and 5 __________

2. In each box below, circle the number with the larger value:

(a) | 12 | 2.9 | (b) | 0.7 | 0.09 |

Tenths and hundredths

3. Round to the nearest whole number:

(a) 9.28 __________

(b) 3.78 __________

4. In each box below, circle the number with the larger value:

(a) | 1.5 | 1.32 | (b) | 0.56 | 5.2 |

5. Write in order from smallest to largest value:

6.05 , 6.93 , 6.1   __________ , __________ , __________

Tenths, hundredths and thousandths

6. Show where 0.8 and 0.25 would be on the number line below:

7. Write a number that could be at each of the arrows:

__________   __________

8. Challenge questions

(a) The difference between two numbers is 0.3. What might the two numbers be?

(b) Write down five (5) numbers between 3.01 and 3.1.

(c) What numbers could be rounded off to 5.8?

Appendix A: Activities and Templates

A1 Paper folding for fractions

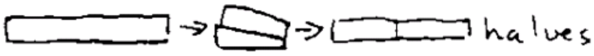
Some examples of materials that are useful for partitioning, and directions for folding paper to make different fractions, are given below. These folds are part of the technical knowledge needed by teachers to successfully work with students in classrooms on practical work on fraction as part of a whole.

1. **Real-world materials:** Everyday materials that can be cut into fractions using area or length models (e.g. cakes, pies, pizzas, liquorice, and so on).

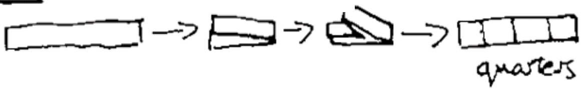
2. **Paper rectangles:**

Halves	–	fold in half
Quarters	–	fold in half and half again
Eighths	–	fold in half three times
Thirds	–	fold so that fold goes halfway back along rectangle
Fifths	–	roll rectangle 2½ times, and flatten where rolls end
Sixths	–	roll 3 times for sixths, 3½ times for sevenths and so on

3. **Paper strips:**



Halves: Fold in half.



Fourths/quarters: Fold in half and half again.

Eighths: Fold in half, fold in half, fold in half again.

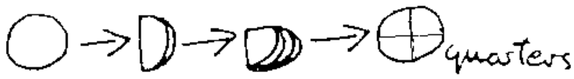
Thirds: Fold so that the fold is halfway back along the strip.

Fifths: Roll strip 2½ times and flatten.

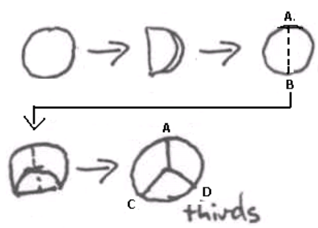
Sixths/sevenths: Roll 3 times for 6ths and 3½ times for 7ths.

4. **Paper circles**:

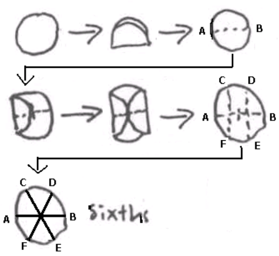
Halves: Fold in half.



Fourths/quarters: Fold in half top to bottom, then fold in half left to right.



Thirds: Fold in half downwards, crease lightly and open (see AB). Fold from B to the centre. Open out and you should see two points (see CD). Draw lines from A to the centre, and from C and D to the centre.



Sixths: Fold in half, crease, open and draw a line across the crease (see AB). Fold from A to the centre; fold from B to the centre; open out and you should see four points (see CDEF); draw lines from C to E and from D to F.

A2 Tenths/hundredths activities

Game: Growing by tenths

Materials:

Make students individual copies of an activity sheet that is composed of five 10×1 grids (see Appendix A4); a coloured pencil; a die.

Directions:

- This activity requires counting by tenths (so no regrouping will occur on the first throw).
- The game should be played first without recording and then with informal recording in conjunction with a small whole/part chart.
- Each student takes turns in throwing a die and then colouring that number of tenths on their activity sheet. The object of the game is to be the first to colour the whole sheet of five grids.

Questioning:

Before the game

Show me one whole. Has the whole shape been divided into equal parts? How many equal parts? So what fraction of the whole shape is each part? How many tenths equal the whole? Show me 1 tenth, 3 tenths, 8 tenths.

During the game

How many tenths can you colour? Will you be able to colour the whole square? How many more tenths will you need before the whole square can be coloured? How many whole ones have you coloured? What fraction of this one have you coloured? How much of the 5 ones have you coloured so far?

At some stage during the game, stop the students and ask each student to say how many squares they have coloured (**identifying** mixed numbers or improper fractions).

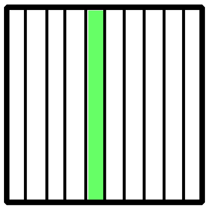
Then ask the class, who has coloured the largest amount, the smallest amount (**comparing**), and how they can tell.

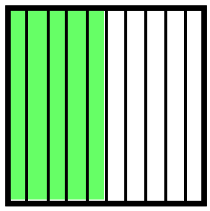
After the game

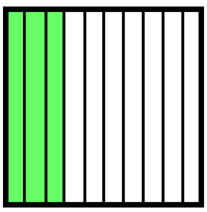
Who came second? How can you tell? (**comparing**) Who came third? How can you tell?

Mix-and-match cards

1. Use 10×10 grids to make up more of these.
2. Copy onto card – one colour for all materials.
3. Cut out each set of three and then cut into squares.
4. Students have to sort into rows that represent the same number.
5. Repeat for hundredths.

1 tenth		0.1

0.5	5 tenths	

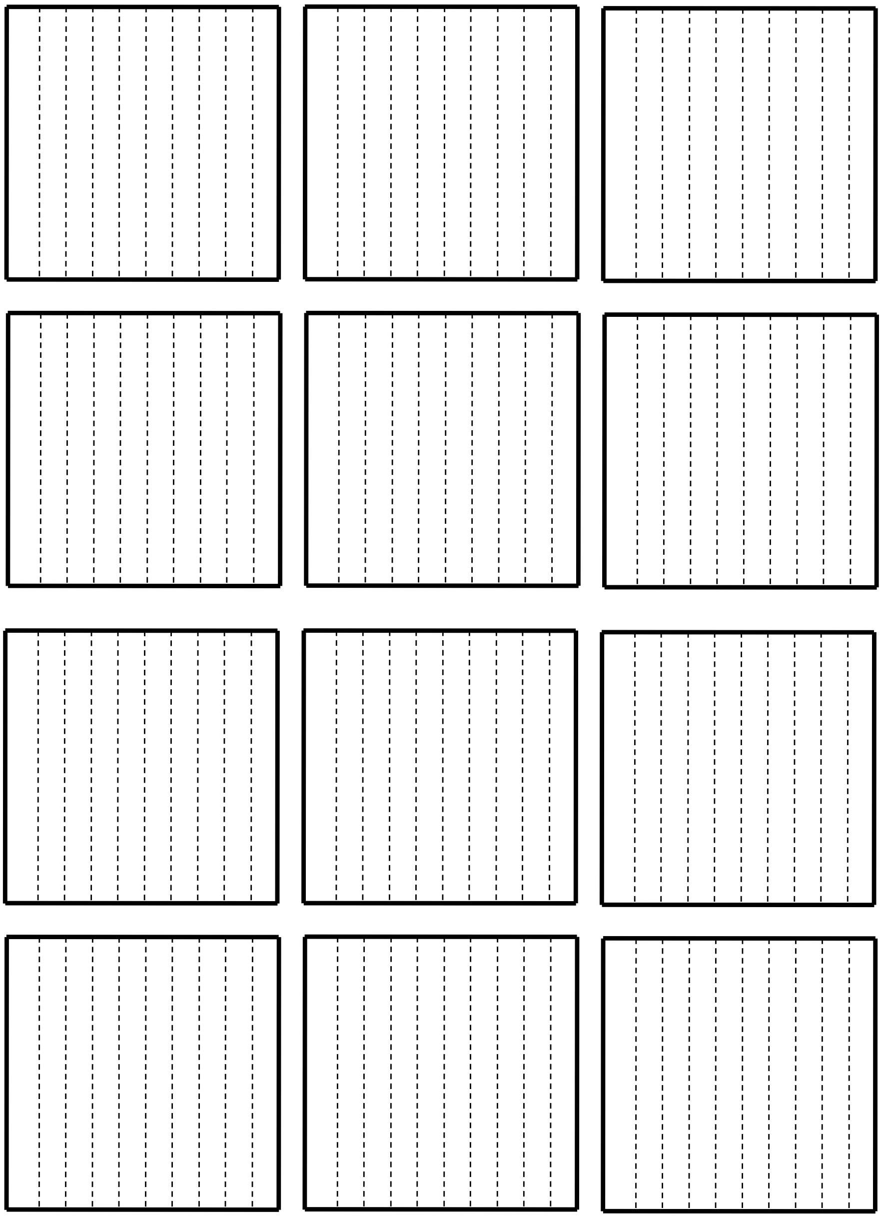
	0.3	3 tenths

A3 Wholes to Decimals worksheet

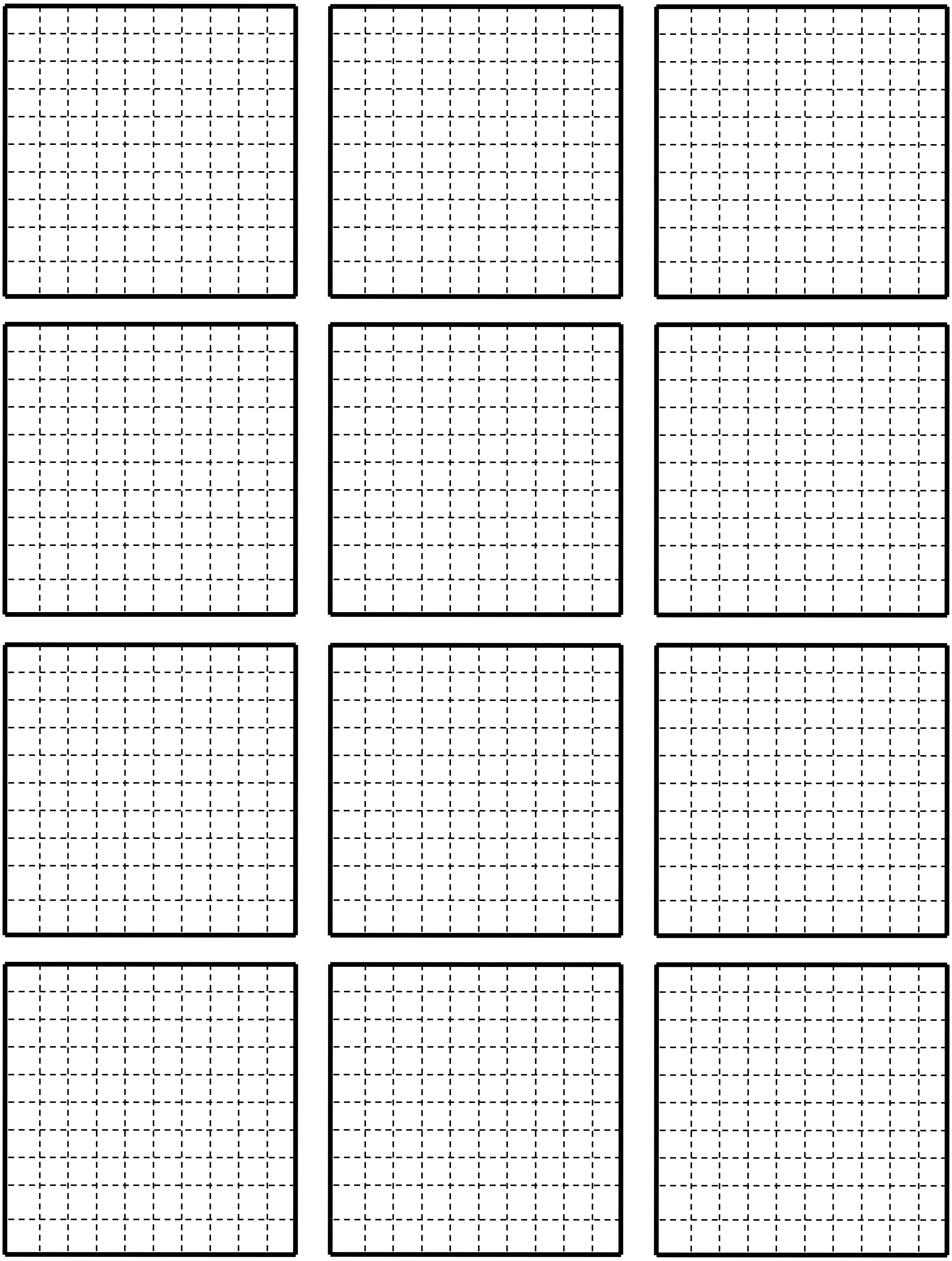
	H	T	O	H	T	O		
Operation	Thousands			Ones				Number

A4 10×1 and 10×10 grids for tenths/hundredths

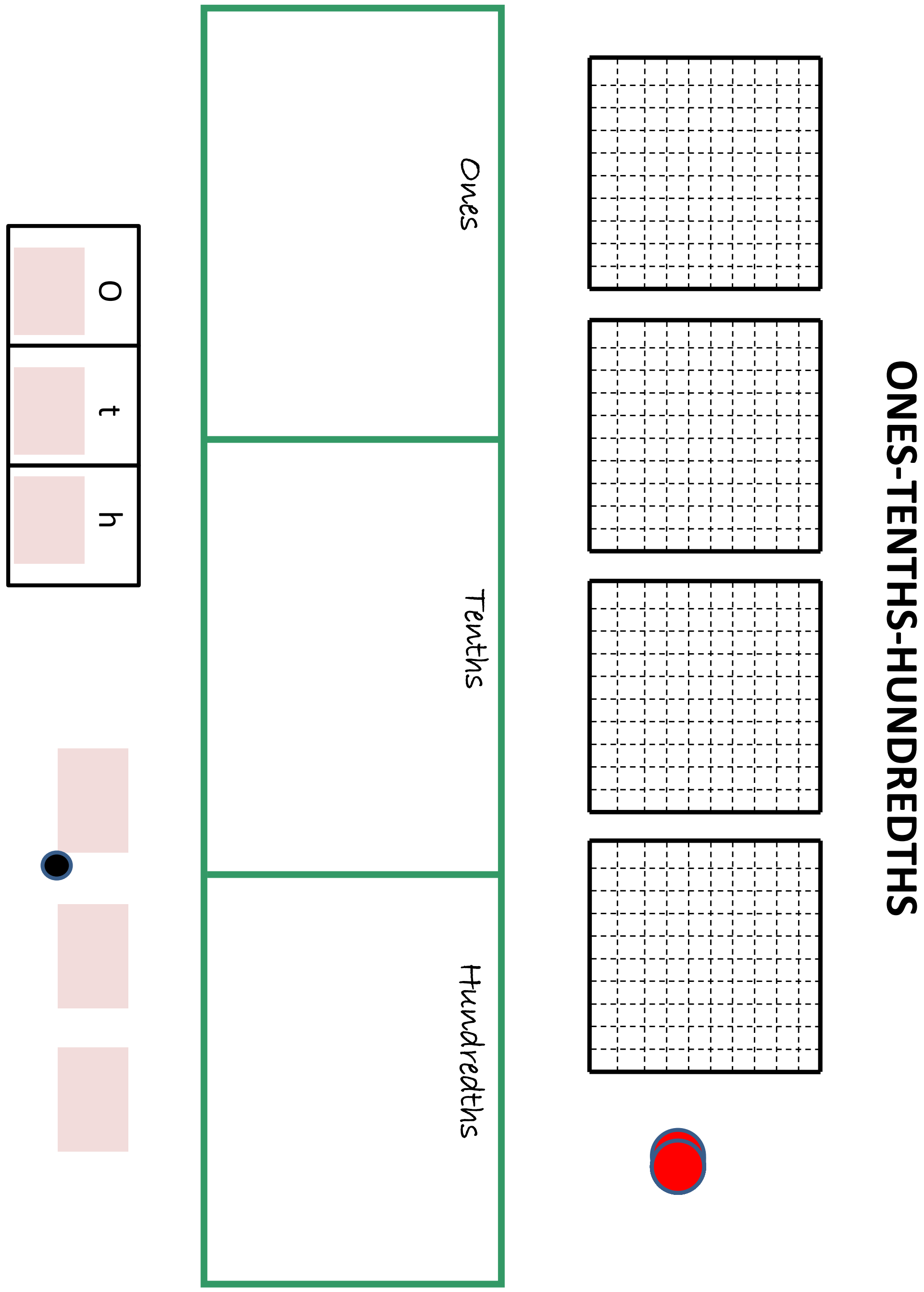
10×1 grids



10×10 grids

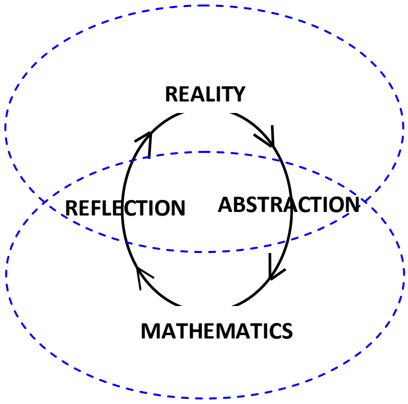


A5 Fractions to Decimals worksheet



Appendix B: RAMR Cycle

AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).



The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix C: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system **N2: Decimal Number Numeration** Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra **M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches **SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae **N3: Common Fractions** Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships **O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject **N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference **G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs **N5: Directed Number, Indices and Systems** Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio **G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation **SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics **O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

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