Accelerated Inclusive Mathematics Project

YuMi Deadly Maths

AIM Module N1

Year A, Term 1

Number:

Whole Number Numeration

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. The project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students' learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Department of Education, Employment and Workplace Relations in the development of the AIM project and these modules.

Contents

Module Overview

This is the first module in Year A of the AIM scope and sequence – it covers **whole numbers** from early grouping to hundreds, tens and ones, and to large numbers by the pattern of threes. Its purpose is to teach whole number numeration from Year 2 level to Year 9 level in a series of vertically sequenced units as deeply as prerequisites allow (e.g. the whole number system is not presented in powers of 10 as indices are yet to be covered). It is the basic module of the AIM program, is constructed around five big ideas, and is followed by the module on decimal numbers (which also uses the same five big ideas).

Note: This module does not provide ideas already covered in the *AIM Overview* booklet.

Background information for teaching whole number numeration

This section looks at some important aspects of whole numbers related to what they are and how this affects effective teaching and acceleration. It then focuses on the way the topic can be taught from five big ideas.

Historical context

Human thinking has two aspects: verbal logical and visual spatial. Verbal logical thinking, associated in some literature with the left hemisphere of the brain, is the conscious processing of which we are always aware. It tends to operate sequentially and logically and to be language and symbol (e.g. number) oriented. On the other hand, visual spatial thinking, associated in some literature with the right hemisphere of the brain, can occur unconsciously without us being aware of it. It tends to operate holistically and intuitively, to be oriented towards the use of pictures, and seems capable of processing more than one thing at a time – as such it can be associated with what some literature calls simultaneous processing.

Our senses and the world around us have enabled both these forms of thinking to evolve and develop. To understand and to modify our environment has required the use of logic and the development of language and number. It has also required an understanding of the space that the environment exists in and of shape, size and position that enables these to be visualised (what we call geometry). **Mathematics** is therefore a product of human thinking that has emerged from solving problems in the world around us and **has two aspects at the basis of its structure: number and geometry**. We should always keep this dichotomy in mind, but realise that good thinking integrates both and that mathematics can be built from either side.

The unit or the one is the starting point for number. It is the basis for all numbers. Units can be grouped to produce whole numbers (ones, tens, hundreds and so on), or they can be partitioned to produce decimal numbers (tenths, hundredths, and so on) or common fractions (e.g. sixths, ninths, and so on). Through this relationship, whole numbers are connected to decimal numbers and common fractions, and to the other versions of fraction, that is, percent, rate and ratio.

One of the bases of the RAMR pedagogy is the Payne and Rathmell model (see *Overview* booklet) which advocates starting from the real world, modelling number with sets and number lines (materials, computers and pictures) and then introducing the language and symbols. After this, activities and questions should be constructed that encourage students to connect and move flexibly, in all directions, between real world, model, language and symbols.

The model highlights why number is difficult to teach in the English language. This is because our number system is based upon a Hindu-Arabic invention. In Hindu-Arabic languages, text is written right to left (the opposite of the English language) and counting uses a base-10 number system represented using 10 symbols. However, the English language system is written left to right and counting was based upon a base-20 system (built around scores). Consequently we have the problem of the teens (seventeen should really be onety-seven). On top of this, we do not usually say zeros in a number, e.g. 405 is said as 4 hundred and five, not four hundred and zeroty-five. This gives a particular problem in teaching number between language and symbols – the language does not fully reflect the pattern in the numbers. For some students, YDM believes that it is necessary to initially give language that reflects structure (e.g. use "onety-seven" instead of "seventeen" and "four hundred and zeroty-five" instead of "four hundred and five").

Big ideas, concepts and processes

There are five major big ideas that apply to all numbers (whole numbers and decimal numbers, common fractions, and percent, rate and ratio), namely, part-whole (notion of unit), additive structure (counting), multiplicative structure, continuous vs discrete (number line), and equivalence.

1. **Part-whole (includes Notion of a unit)**. The basis of number is the unit which is grouped to make large numbers and partitioned to make fractions. Thus everything can be seen as part and whole – a ten is a tenth of a hundred and a whole of 10 ones. The value of each digit in the number system is determined by its position relative to the ones position, by its place value. The value of each digit is given by that digit number of place values and the value of a number is found by adding these (i.e. 204 is 200+4). The pattern also holds for fractions (e.g. sixths) and measures whose base is not 10 (e.g. time).

   There is also a sub-big idea in that the pattern for the groupings and partitioning in numbers is a **pattern of threes**, that is, ones-tens-hundreds ones, ones-tens-hundreds thousands, ones-tens-hundreds millions and so on. This means that knowledge can be transferred from ones to thousands to millions and so on. The concept for this big idea is **place value**, and the processes are **reading and writing**.

2. **Additive structure**. A number's value is the sum of its parts, e.g. 234 = 200+30+4. Thus, place values increase and decrease by adding and subtracting. The consequence of this is that each place-value position counts forwards and backwards and values of place-value positions increase and decrease.

   There is also a sub-big idea in that counting follows a pattern, the **odometer pattern** – forwards from 0 to 9 and back to 0 as the number on left increases by 1 (e.g. 274, 284, 294, 304, 314, ...); backwards from 9 to 0 and back to 9 as the number on left decreases by 1 (e.g. 234, 224, 214, 204, 194, 184, ...). For fractions, this counting pattern differs, as it does for measures (e.g. time) that are not base 10.

   The concept for this big idea is **counting** and the processes are **seriation** and **counting patterns**.

3. **Multiplicative structure**. A number's value is a sum of the products of value and place, e.g. 234 = 2×100 + 3×10 + 4×1. This means that there is a multiplicative relationship between adjacent place-value positions – values increase ×10 when moving to the left and decrease ÷10 when moving to the right. Thus, the value of a number is found by adding the multiples of digit × place value (e.g. 204 = 2×100 + 0×10 + 4×1).

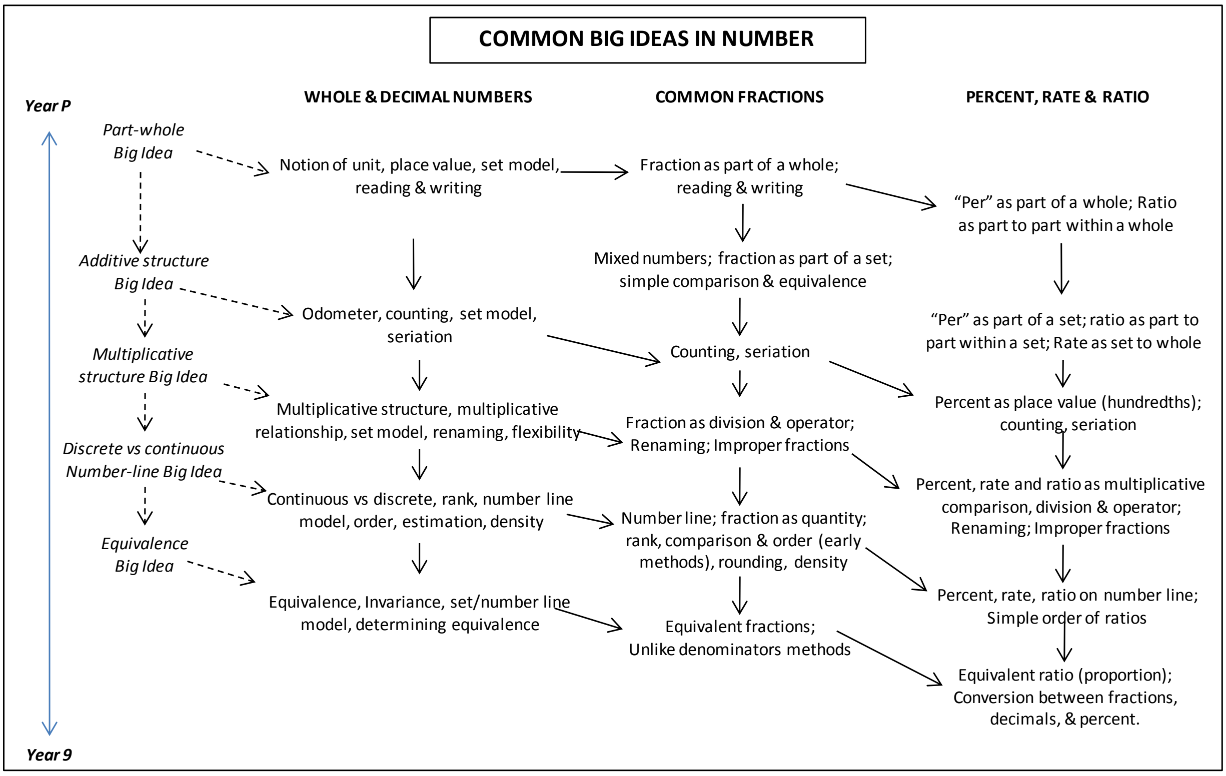
   The concept for this big idea is **multiplicativity** and the processes are **renaming** and **flexibility**. Flexibility is thinking of things more than one way by changing the unit (e.g. 300 is 30 tens or 0.3 thousands and so on).

4. **Continuous vs discrete/Number line**. Regardless of place values and digits, each number is a single quantity represented by a point on a number line, has rank and can be compared to other numbers. The number-line representation changes perspective of number – for example, the 0 is no longer nothing, it is the starting position of positive whole numbers (and of rulers and other measuring devices).

   The concepts for this idea are **comparison/order**, **rank** and **density** and the processes are **comparing/ordering**, **rounding**, and **estimating**. Order just means to work out the larger/largest but rank means to place on line where they should be proportionally. For example, 91 being larger than 32 can be shown by students in a row with 91 after 32, but rank is shown by the 32 being on a line one third of the way between 0 and 100 and the 91 being near the 100. Density is how many numbers between adjacent numbers.

5. **Equivalence**. Sometimes a single quantity can be represented by more than one number. For example, 04 is the same as 4, 2.40 is the same as 2.4, $^{2}/_{3}$ is the same as $^{4}/_{6}$, and 3:5 is the same as 6:10. Equivalence often reflects adding zero (the additive identity) or multiplying by one (the multiplicative identity).

The application of the five big ideas to number topics is shown graphically in the figure below.



This figure has been designed as follows: (a) whole and decimal numbers are combined for brevity (and because they are similar); (b) the topics in whole and decimal numbers, common fractions, and percent, rate and ratio have been placed, as far as possible, to approximately align with year levels shown on left; (c) the big ideas are in italics and the dotted arrows show their sequence and the topics which they underlie; and (d) the down and angled solid line arrows show sequences within topics and the sequence across topics but within a big idea.

Note: To give prominence to big ideas and ensure that students recognise and use them in later modules, we will base the units in this module around big ideas by making them part of the headings of units or sections. However, many concepts and processes are a combination of big ideas and so to recognise and implement this we will sometimes return to a previous big idea in the middle of showing instruction for a later big idea.

Sequencing for whole number numeration

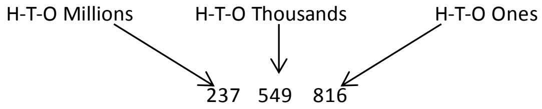
This section looks at the how whole numbers are sequenced and how we have applied sequencing in this module.

Sequencing in whole numbers

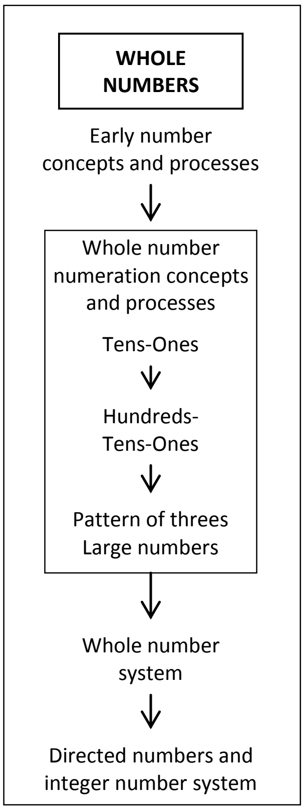
The secret of sequencing for acceleration in number is to build early number (hundreds, tens and ones) around big ideas that also apply to larger whole numbers and then to other forms of number. The five big ideas listed above apply to whole numbers of any size and to common fractions, decimal numbers, and percent, rate and ratio. This means that once these ideas are covered in early whole numbers, they can be quickly transferred to larger whole numbers and then to other number areas. Thus, the development is as follows:

H-T-O → Thousands → Millions → Number system → Other numbers

To enable this acceleration across the whole-number area, the secret is to build ideas through the **pattern of threes** – that ones, thousands and millions (and so on) all have the same pattern built around H-T-O (Hundreds, Tens and Ones) as below.



This means that whole number numeration should be seen as having two components: (a) a **substructure of three** , that is, Hundreds-Tens-Ones; and (b) a **superstructure of a sequence of powers of 10 that go up by three** each time, that is, ones, thousands, millions, billions, trillions, quadrillions, and so on. It should be noted that in modern parlance, the names for each set of three are often replaced by **ones, kilo, mega, giga, tera**, and so on – the language of computer memory. Therefore, the sequence for whole numbers is as shown in the figure on right and below:



- Early number
- Understanding everything about H-T-O ones (all five big ideas)
- Extending this understanding to H-T-O thousands and H-T-O millions (and so on)
- Whole number systems
- Directed numbers and integer system

For this module, we will focus on the second and third steps of this sequence. We will leave whole number systems, directed numbers and integer systems to Module N2 *Decimal Number Numeration* (where we will look at number systems), and to Module N5 *Directed Number, Indices and Systems* where we will look at Year 9 understandings of number.

Note: If for some reason the "pattern-of-threes" extension from H-T-O to larger numbers does not work, then big ideas will need to be revisited for larger numbers. Therefore, do not treat this sequence as automatic – it will vary depending on student learning. For this reason, **we provide teaching ideas for large numbers for the big ideas in Appendix A at the end of this module**.

Sequencing in this module

In this module the sections are as follows:

Overview: Background information, sequencing and relation to Australian Curriculum

Unit 1: Notion of unit/Part-whole, place value, and reading and writing 3-digit numbers

Unit 2: Additive structure, seriation, counting and odometer for 3-digit numbers

Unit 3: Multiplicative structure and renaming for 3-digit numbers

Unit 4: Continuous vs discrete, number line, rank, rounding and ordering for 3-digit numbers

Unit 5: Equivalence for 3-digit numbers

Unit 6: Extension to large numbers using pattern of threes

Test item types: Test items associated with the six units above which can be used for pre- and post-tests

Appendix A: Teaching ideas for large numbers based on the five big ideas

Appendix B: RAMR cycle components and description

Appendix C: AIM scope and sequence showing all modules by year level and term.

The sequence of units follows the structured sequence that is the basis for this module. Each unit is itself divided into one or more RAMR lessons (see *Overview* booklet), and the stages of RAMR are used as subheadings within each unit. This is so that this module can act as an exemplar of the RAMR pedagogy. However, not all sub-stages are given in each RAMR lesson.

Most of the units are based around big ideas and concepts and processes to ensure that all are covered. However, this structure may not be instructionally efficient unless there is integration. Therefore, we have used the RAMR stages to gain instructional efficiencies by having more than one idea in a RAMR cycle. For example, in Unit 1, the part-whole idea and its associated place-value concept and reading-writing process are taught together in one cycle; while in Unit 4, the number line is integrated with place value in one cycle to ensure that the order of numbers that is evident on the number line is considered in place-value terms to enable the generalisation that the digits in the highest place values determine the largest number.

Relation to Australian Curriculum: Mathematics

AIM N1 meets the Australian Curriculum: Mathematics (Foundation to Year 10)							
Unit 1: Place value, reading and writing, 3-digit numbers Unit 2: Seriation, counting and odometer Unit 3: Multiplicative structure and renaming	Unit 4: Rank, ordering and rounding Unit 5: Equivalence Unit 6: Extension to large numbers						
Content Description	**Year**	**N1 Unit**					
		1	**2**	**3**	**4**	**5**	**6**
Recognise, model, read, write and order numbers to at least 100. Locate these numbers on a number line (ACMNA013)	**1**	✓	✓	✓	✓		
Count collections to 100 by partitioning numbers using place value (ACMNA014)			✓	✓	✓		
Recognise, model, represent and order numbers to at least 1000 (ACMNA027)	**2**		✓	✓	✓	✓	✓
Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting (ACMNA028)			✓	✓	✓	✓	✓
Recognise, model, represent and order numbers to at least 10 000 (ACMNA052)	**3**						✓
Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053)						✓	✓
Recognise, represent and order numbers to at least tens of thousands (ACMNA072)	**4**						✓
Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems (ACMNA073)							✓
Use estimation and rounding to check the reasonableness of answers to calculations (ACMNA099)	**5**				✓		✓
Recognise that the place value system can be extended beyond hundredths (ACMNA104)							✓
Investigate everyday situations that use integers. Locate and represent these numbers on a number line (ACMNA124)	**6**				✓		

Unit 1: Notion of Unit/Part-Whole, Place Value and Reading and Writing 3-Digit Numbers

This unit covers the big idea of **notion of unit/part-whole** and then moves on to the **place value** concept and the **reading-writing** process. These big idea, concept and process are integrated into one full RAMR cycle – the RAMR stages are given as major headings with the big idea, concept and process that are covered in them – not all of the big idea, concept and process is covered in each section. It is important to note that this integration is designed to bring about efficiencies without losing effectiveness.

Big idea

The **part-whole** big idea here is the foundation of the one as the starting point to build a number system by giving position to one/unit, group, group of groups, and so on, as follows.

1. Perceiving number requires flexibility in changing unit from a single to a group to a group of groups where anything can be the unit with other things becoming part of that unit or groups of that unit. For example: *If tens are the unit, then ones are part of that unit and hundreds are groups of 10 of that unit.*
2. We switch the unit of count depending on what we want to count, for example: *We can count songs, CDs (each with 12 songs) and packs (each containing 5 CDs with 12 songs each).*
3. To understand (and read and write) numbers in terms of place value (PV), students must continuously change their perception of unit (and gain a multifaceted perception of unit), for example: *324 requires students to realise that the 2 is two tens (ten is the unit) and also 20 ones (one is the unit).*
4. The part-whole big idea builds reading and writing by using PV to read numbers (symbol to language) and to write numbers (language to symbols). This **requires knowledge** of the role of zeros; and **understanding** that digits in numbers follow strict language and symbol patterns (excepting teens and zeros).

Introductory activity: Mystery number on your back

With sticky labels form a 3-digit number to stick on someone's back. The person may ask questions, and the class can answer only *Yes* or *No*. Broad questions are best, e.g. *Is it bigger than 400? Is there a five in the hundreds place? Is there a zero in it?* Continue until the correct number is guessed.

It is best to model this with an adult first, and then note the language that students use. They will use language that they understand, and are confident in using. The activity can be used with more people, with the rule that each person may answer only one question, to each other person, so they have to mingle to keep asking. You may choose to begin using 2-digit numbers before 3 digits.

1.1 Reality – notion of unit, place value and reading-writing

Local contexts

Find real-life contexts to embed the activities in: *Where in the world do we find numbers?*

Discuss different uses of numbers – house numbers, numbers on the back of footballers, and so on. Discuss what the different parts of the numbers mean. Identify particular uses of number in the life of your students. Encourage a wide variety of contexts, e.g. sports, cooking, time, distance, fishing, travel. Watch for opportunities for one/unit, group, group of groups, and so on.

Prerequisites

Using the chosen context, check students' understanding of numerals 1 to 10, reading and writing, seriation and ordering.

Check students' understanding of grouping – forming groups, counting individual objects and counting groups.

Kinaesthetic

Pick out two or three of the contexts – play games and sing songs to act out numbers in local contexts. For example, metres on a Rugby League field, using people as objects on a large racetrack or 10×10 board (a large version of a game), adding up numbers on really large playing cards with people as dots, number rhymes about something locals do, counting as people walk past (what happens if there is a large number of people), or walking along a large ruler, and so on.

1.2 Abstraction – notion of unit, place value and reading-writing

Early grouping: Body

Musical groups. Get students to group themselves into groups of a number less than 10, say 4 or 6, and then get them to describe how many groups and how many were left over. Try this for tens and ones.

Early grouping: Hand

Counting boxes. Place material (e.g. Unifix or straws) on a simple two-sided PV chart, "groups" on left in shaded area and "ones" on right. Make up nonsense names for the groups. Start with groups under 5. Students state number of groups and number of ones as they move left hand from left-hand side (LHS) of the chart to right-hand side (RHS) of the chart, e.g. *3 groups and 4 ones*. Always stop and allow students to **predict** what will happen when next unit is added or removed. Repeat this for larger numbers until you reach tens and ones.

Discuss how these unit groups could be considered two ways in terms of unit. For example, a dozen eggs could be one dozen or 12 single eggs. Discuss how different ways of thinking about things gives options.

- A tournament can be seen as one tournament or 20 teams or 160 players. Another example is months, weeks, days, hours, minutes etc.
- Discuss possibilities – is it possible for a unit to be a group of groups? For example, a box of 12 packets each with 20 pencils – the unit can be a pencil, a packet, or the box.
- Ask students to give examples for ones, groups, groups of groups (and even groups of groups of groups), and so on. Get them to **invent** their own objects to use as models for ones, groups and groups of groups.
- Note the language that the students use and model correct mathematical language.

Tens/Ones: Body

Tens	Ones

Mat activity. Use the mat for establishing number sequence and how the tens numbers label the rows. When you finish a row (0–9), you move to the next row. Mark the second row as "1 ten" then "1 ten and 1"; "1 ten and 2", etc. Identify where numbers are on the grid, giving the number name; and reverse by standing on a square and asking for the number name. Students point to row name (left hand) and column name (right hand).

People material. Count tens and ones by putting students into circles of tens and left overs. Laminate photographs of groups of ten and single students and use these, instead of bundling sticks/MAB, on Ten-One PV charts.

Tens/Ones: Hand

PV materials. To develop tens and ones: (a) use materials (e.g. **straws or bundling sticks,** Multi-base Arithmetic Blocks [MAB], money); (b) record on PV chart; (c) enter on calculator; and (d) write on paper. For normal numbers, introduce specific language: *4 tens and 2 ones* before *forty-two*; for zeros start with *four tens zero ones* and *forty-zero* before just *forty*; for fourteen use *one ten four ones* before *onety-four* before *fourteen*. Do not relate *4 tens* to 40 otherwise students may see *4 tens and 3 ones* as 403. **Move hands** from tens to ones as you say the numbers.

Play games such as throwing dice and adding/subtracting the number of ones shown, using bundling sticks, making a new ten from 10 ones or trading a ten for 10 ones when required, first to or from four tens wins (or throw two dice and stop at 100 or go to 0 from 100). Always ask *How many tens? How many ones left over?* and *How many more to next ten?*. **Stress** that the number of tens goes in LH place-value position and the number of leftover ones goes in RH place-value position. Have calculators available – use materials, written numbers, language and typing into calculators all happening at the same time.

Have pen and paper and calculators available in conjunction with these activities. Always record and say numbers as you use materials, moving left hand from tens to ones, to begin connections between symbols, language and materials. Speak initially in terms of tens and ones before giving formal names, and record on small PV charts before simply writing the numbers.

Note that the strength of size differentiating material like MAB on a PV chart is that manipulation of materials can kinaesthetically do two things at once – build position (larger place values on left) and build size (LHS digit is worth 10 of the RHS digit). However, the weakness is that material for 4 tens has 40 ones in it, so 4 tens and 3 ones can appear to be 40 on the left and 3 on the right (i.e. 403). **The way to prevent this** is to never say "40 is 4 tens" and to always stress that the students write **the number of tens** (not the number of ones in the tens) on the LHS and **the number of left over or loose ones** on the RHS.

Montessori numbers. Show numbers as individual parts on a PV board, and place them one over the other to see the numerals, e.g. 300 and 60 and 5 becoming 365**.**

$\boxed{300}\ \boxed{60}\ \boxed{5} \longrightarrow \boxed{3}\boxed{6}\boxed{5}$

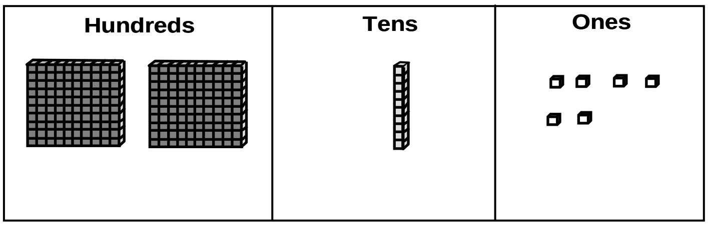
Hundreds/Tens/Ones: Hand

Hundreds	Tens	Ones

PV materials and hand movements. Repeat the above but for three digits. Do 456 before 450 before 406 before 400 before 416 type numbers. *Note:* 400 is *4 hundreds 0 tens and 0 ones* – it is not just *4 hundreds* (otherwise *4 hundreds 2 tens and 3 ones* might be written as 40023). Stress that LHS is **number of hundreds**, middle is **number of left-over tens** and RHS is **number of left-over ones**.

When placing material on PVC, put left hand on LH position and, as you move left hand to the right, say *4 tens and 5 ones*, or *6 hundreds, 7 tens and 2 ones*, **repeat hand movement** saying *forty-five*, or *six hundred and seventy-two*. Do hand movements **over and over** as you change hundreds, tens and ones by adding or taking away numbers in place-value positions (add 3 ones, take 2 ones, add 5 tens, take 2 hundreds, and so on).

Zeros and teens. Initially do activities with 3-digit numbers that have no zeros or teens, then move on to zeros and teens. Make students aware of how zeros and teens make numbers harder to follow. For reading, give a real-world story: *Malcolm counted this many cars*. Show the number with MAB, for example, as below. Students say the number: *Two hundred and sixteen*. Say: *Show me the two hundred part; show me the ten part; show me the six part*. Discuss what the number should/could have been called: *two hundred and onety-six*.



For writing with zeros and teens, use strategies as given for reading numbers but have students write their responses. State the numbers as they should be (e.g. *onety-seven* or *twoty-zero*) before giving proper names. **Compare** the number with two hundred and sixty and two hundred and sixty-one. Put out MAB for both these numbers. Say: *How many ones in two hundred and sixteen, how many in two hundred and sixty, and how many in two hundred and sixty one?* **Repeat** for tens and hundreds. **Repeat** for other numbers. For example: 311; 518; 470; 407; 200. **Swap roles** – the teacher says the number and the students show it with MAB. Don't forget to ask the students to show the hundreds part of the number, the tens and so on.

Hundreds/Tens/Ones: Mind

Shut eyes. Have students shut eyes and imagine the MAB and PVC and say it aloud as they move their hands.

Use calculators. Say the number out loud, record the number on small PVCs, on paper and enter on a calculator (get students to add the relevant number on the calculator when adding material to PVC). (*Note:* Give special attention to saying the teen numbers and numbers that have zero in them.)

Creativity

Let students make up their own number symbols and ways to group numbers.

1.3 Mathematics – notion of unit, place value and reading-writing

Language and symbols/Practice

Games. Spend time giving digits for students to read. Use flash cards of MAB, bundling sticks, words, symbols. Start with 2-digit numbers and then 3 digits. Use dice and cards to randomly select digits – have digit cards (small cards with numbers on them) and PV charts. Spend time giving varying numbers as digits for students to say as numbers; reverse and say number and students write as numerals or enter on calculator Have two teams of students who compete to make numbers as they are called, e.g. 23.

Matching representations. **Mix and Match cards** – match words and numerals on cards on the floor – students are to find matching pairs. Also play **Cover the board** and **Bingo** games where one of the three representations is shown (numeral, language, and drawing of PVC with materials) and students have to find one of the two other representations. Use **thinkboard** with different sections for stories, materials, pictures, symbols, language. **Column worksheets** – prepare column worksheets with one column filled in (students fill in others) as follows:

Picture of material (MAB on PVC)	Language	Symbol
	Three hundred and seven	

Read-write calculator. Call out three digits, students enter on to their calculator and say the number; say a number, students enter on calculator and then they say digits they used. Play **Wipe-out** – students enter a 3-digit number, e.g. 278, into their calculator. Ask: *How can you make the 7 a zero? Are there different ways of doing this? What number have you got now?* Try other numbers.

Give the place values in wrong order. Ask: *What number is 6 tens, 8 hundreds and 7 ones?* **Do a lot of these** – it is important that students see that a number is determined by hundreds, tens and ones and it does not matter in what order the place-value positions are given. Reverse this – get students to give all the different ways that 587 could be given – 5 hundreds, 8 tens, 7 ones; or 8 tens, 7 ones, 5 hundreds; or 7 ones, 8 tens, 5 hundreds; and so on.

Number expander. Use number expanders to show hundreds, tens and ones and then fold along dotted lines (pleat fold) to show numeral and vice versa.

5	Hundreds	7	Tens	2	Ones

⟷

5	7	2

Connections

Relate H-T-O to other forms of ones, groups, groups of groups, then to other topics where numbers are used (e.g. measures).

1.4 Reflection – notion of unit, place value and reading-writing

Validation/Application

Validate by applying ideas into the world of the students. Also apply the notion of unit back to the real world. Where in the world do we have groups and ones interchanging depending on how we think of them? Get students to read and write numbers as they appear in their world. Make posters on where they use numbers.

Extension

Flexibility. Brainstorm situations where numbers are used. An example is to ask how many ways can we make 61 (e.g. 9 ones less than 7 tens), looking out for the student who says 1 hour and 1 minute. Do this continuously as you move through numbers (e.g. 75 is ¾ in metres and centimetres, 125 is 1/8 of 1000).

Reversing. Go in both directions – numerals to name and name to numerals, MAB to name and name to MAB, numerals to MAB and MAB to numerals, and so on. This is a crucial component of building mathematics knowledge.

Generalising. Ask: *These are not numbers, but what if I have Fred hundreds, Frank tens and Fonzy ones, what would I call the number?* Answer – *Fred hundred and Frankty-Fonzy!*

Changing parameters. See if students can extend to thousands using pattern of threes. *What if I had a 6-digit number – 452 781 – can you say it using hundreds, tens and ones?* Answer – *Four hundred and fifty-two thousand, seven hundred and eighty-one!*

Unit 2: Additive Structure, Seriation, Counting and Odometer for 3-Digit Numbers

This unit covers the big idea of **additive structure/counting,** the concept of **seriation**, the process of **counting** forwards/backwards (the pattern in counting), and the principle of **odometer.** It is structured around the RAMR stages and related to which of the above is involved in that stage.

Big idea

Any number is the sum of its components, that is, 243 = 200+40+3. Thus any place value adds/subtracts by increasing and decreasing. This is called additive structure. The basis of **additive structure** is as follows.

1. **Seriation** – the ability to determine one more or less in each place value (e.g. 10 more than 53, 1 less than 419, 100 more than 345). It is closely related to counting and odometer and is also the basis for counting patterns. For example, a seriation activity like 300 more than 247 is just counting on from the 2 in the hundred for 3 counts. That is, 247 → 347 → 447 → 547.

2. **Counting** – that any place value position counts like the ones position (e.g. 50, 60, 70, 80, ...; 338, 328, 318, 308, 298, ...). This relates to counting patterns (and, as we see below, to odometer).

3. **Odometer** – the strict counting forward and backward patterns that are similar to a car's odometer: (a) counting forwards – 0, 1, 2, ..., 8, 9 and then returning to 0 with the digit on the left increasing by 1; and (b) counting backwards – 9, 8, 7, ..., 1, 0 and returning to 9 with the digit on the left reducing by 1.
   (*Note:* This means that counting, seriation and counting patterns can all be integrated around this odometer principle.)

2.1 Reality – seriation, counting patterns and odometer

Where possible, find real-life contexts to embed the activities in, using relevant objects or situations. For example:

- seriation situations – 10 years since 2003, 10 years older/younger, and so on;
- counting situations, e.g. count downs; and
- odometer situations, e.g. digital clock, petrol pump, electricity meter.

2.2 Abstraction – seriation, counting patterns and odometer

Body

Mat. Always use the mat before the paper grid (99 board) – the mat enables activities to be acted out using students' bodies. Use the 6 × 10 mat as a shortened 99 board by placing numbers 1, 2, 3, and so on, in the top row and 10, 20, 30, and so on, on the side (that is, extend to the second row beginning at 10, third row beginning at 20 and fourth row beginning at 30, etc.). Walk around this mat using the activities that you would do with finger on the grid paper.

Flip chart. Students with number bibs stand in rows of 10 and act out flip chart to show odometer rules. Repeat using smaller key ring numbers and/or flip charts on mat.

Montessori numbers. Can act these out on the mat. Montessori numbers are separate cards of different length which when aligned on the left edge, form a number (as below). Of course, we will do only 3-digit numbers at this stage.

500 000 | 40 000 | 7000 | 300 | 80 | 2 ⟶ | 5 | 4 | 7 | 3 | 8 | 2 |

Hand

99 board. Do the following starting activities.

1. **Getting to know the patterns of numbers.** Have students read columns and rows. For example, 4, 14, 24, ...; and 60, 61, 62, ..., and notice the patterns.

2. **Reading a column.** It can be useful to have students read the column as "four, onety-four, twoty-four", and so on. Then the pattern in the column can be seen – it is that "four" is said each time with the tens going up, that is, the ones stay the same and the tens increase.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

3. **Reading a row.** The row is read as "sixty, sixty-one, sixty-two", and so on. Then the pattern in the row becomes apparent – it is that the "sixty" is said each time with the ones increasing, that is the tens stay the same and the ones increase.

4. **Jigsaws.** Cut the 99 boards into jigsaw puzzles and get students to re-form them. Students can make puzzles for each other. Hand out 99 boards with parts missing and students have to complete the numbers.

5. **Knowing where numbers are – placing numbers by tens and ones.** Always start at zero. For *position to number* – get students to start at zero and move down and across, encouraging students to see pattern, e.g. that 3 down and 7 across is 37. For *number to position* – move the tens down and ones across, e.g. 54 is 5 down and 4 across (starting at zero).

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

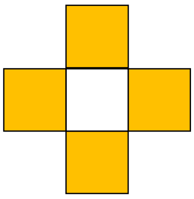
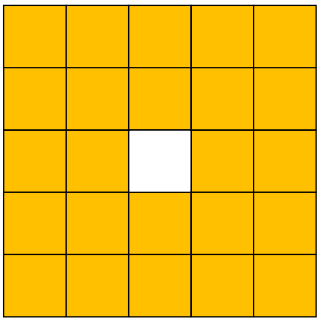
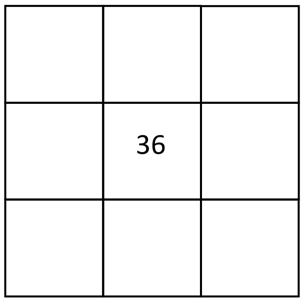
In the first teaching direction, 3 down and 7 across is given and students find they reach 37. In the second teaching direction, 54 is given and students find the movements (5 down and 4 across) that reach this number.

Do the following activities to practise finding, placing and relating numbers on a 99 board.

1. **Seriation for two-digit numbers.** Use the board to identify the numbers on the left, right, above and below a chosen number – show how left and right is 1 less and 1 more, above and below is 10 less and 10 more. For example, look at 78: 1 less is 77, 1 more is 79, 10 less is 68 and 10 more is 88. Start with the number and determine (use a calculator) 1 less, 1 more, 10 less and 10 more and then find these numbers on the board and their relation to the starting number (circle the number as for 46 on right).

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

2. **Windows and squares.** Construct a 99 board window with a hole in middle, place over a number so you can only see that number and write numbers 1 less and 1 more, above and below. Examples of windows are as below on left:

   Construct 3×3 squares with a number in the middle (as above right) and ask for surrounding numbers.

3. **Three-digit numbers**. Extend these ideas to 99 boards that start with 100, 200, etc. – that have three-digit numbers – even ones that start at 43 or 327.

4. **Patterns**. Construct patterns of the type 356, 366, 376, ___ , ___ , and so on, for different place values (also have decreasing patterns). After constructing, get students to decipher the patterns – have to find the changing place value. Make sure students know what happens as place-value position goes up to 9 or down to 0. Get students to construct "hard" patterns for each other.

MAB. Put MAB on PVCs, select a place value (ones, tens or hundreds) and add MABs one at a time to this value. Start with the ones (1, 2, 3, 4, ...) then tens (10, 20, 30, 40, ...) then hundreds. At each addition state the number, then write the number. Students can add on a calculator as you go along – enter a number, say 254 and +10, clap hands, students press =, state the number. Start at a number other than zero. Also count backwards, removing one MAB piece at a time (remember to write and record on calculator).

Repeat above with flip cards, counting and writing as they go. Also count backwards, flipping cards backwards (remember to write and record on calculator).

Cup odometer. Construct an odometer out of three foam cups. Turn cups horizontally and write numbers 0 to 9 evenly spaced on lines on outside top/rim of each foam cup. Put foam cups inside of each other, turn right-hand cup 0, 1, 2, ... 7, 8. When you pass 9 back to zero, turn middle cup one position forward. If middle cup passes 9, turn left-hand cup one position forward. Reverse this and count backwards: turn right-hand cup 9, 8, 7, …, 2, 1. When it passes zero back to 9, turn middle cup one position backwards. If middle cup passes zero, turn left-hand cup back one position.

Mind

Imaginary board. Ask students to close eyes and imagine the 99 board. Get them, with eyes shut, to find numbers, to state numbers that are 1 more and less or 10 more and less. Ensure they have a picture of the board they can use for seriation and patterns.

Mental odometer. Shut eyes and imagine the odometer turning – count as it goes. Get students to imagine a three-digit number, choose a place value, and imagine counting on or back by that place value, rolling the numbers up and down.

Calculator patterning. Use calculators to count in any place value and to show odometer pattern. The activity can be called – "teaching your calculator to count".

1. **Counting patterns/Any PV position counts.** This can be done as a regular activity at the beginning of a lesson, using paper, pen and a calculator. Choose a starting number (e.g. 376) and the number to be added (e.g. 10). Firstly predict what comes next (e.g. 386) secondly add 10 on the calculator and if guess is right put a tick beside it. If guess is incorrect, cross out and write correct one (from the calculator) beside it, and continue. Continue with this pattern until past 430.

   Another common way to use calculators is to give the class a starting number and a number to add, get them to put on calculator, and then, each time you ring a bell, press equals and call out the number. This can be used for any starting point/any added number (e.g. 0/1, 0/5, 87/1, 345/10, 1386/100, and so on), including decimals, and is good for bridging the tens or hundreds. It teaches counting patterns.

2. **Odometer pattern.** Repeat the activities from Counting patterns but this time get the students to state only the PV position that is being counted, e.g. 324 – "two", 314 – "one", 304- "zero", 294 – "nine", and so on – going forward and backward. **This activity is particularly strong for the calculator**. Do it for all three PV positions – hundreds, tens and ones. It shows that the 8, 9, 0, 1 and the 1, 0, 9, 8 odometer patterns hold for all place values and positions.

2.3 Mathematics – seriation, counting patterns and odometer

Language and symbols/Practice

Construct 3×3 squares with number in middle and ask for other numbers. This can be extended to 3-digit numbers.

	36	

Build more complex shapes on which to fill in the numbers. Extend this by building jigsaw pieces from 99 boards with only one number written in one square and ask students to fill in other squares (see examples F and G below).

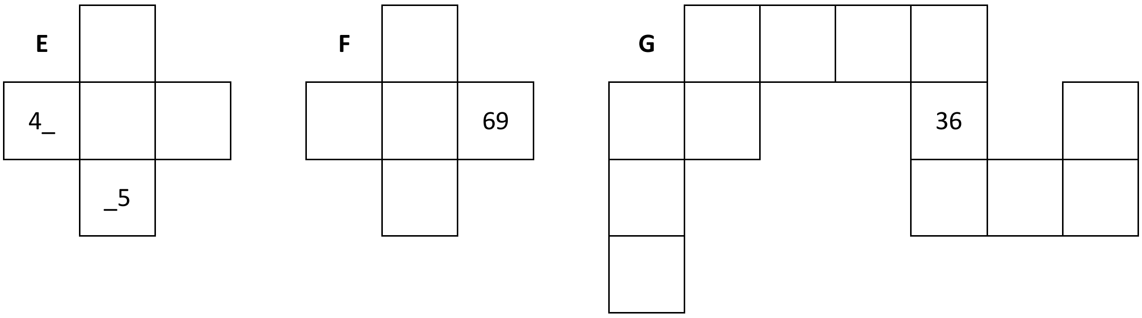
Reverse the process and provide the numbers on the sides and ask for the number in the middle. Here are some examples (easy → hard).

A	62	
71		73
	82	

B	16	
25		27
	36	

C	78	
87		8_
	9_	

D		
		43
	52	



(*Note:* The materials and ideas above were taken from Numeration materials developed under the leadership of Associate Professor Annette Baturo for training Indigenous teacher aides.)

Practise counting in any place value with calculator and flip cards.

Use materials, calculator and worksheets to practise examples like: *What is 10 more than 367?* Also complete counting patterns such as: continue 485, 495, ___, ___, ___ or continue 627, 617, 607, ___, ___. Complete worksheets of patterns such as 324, 344, 364, ____, ____, ____.

Connections

Ask question: *Where else do we count?* Look at counting in other mathematics topics, for example, fractions, metric measures (e.g. cm) and non-metric measure (e.g. time).

2.4 Reflection – seriation, counting patterns and odometer

Validation/Application

Look at situations when you read/count numbers – post boxes, sporting scores, distance, road signs, phone numbers, codes and passwords, etc.

Extension

Flexibility. Count weeks and days, years and months, metres and centimetres, hours and minutes, and so on.

Reversing. Do both directions – teacher gives start and adding/subtracting PV – students show pattern; AND teacher gives pattern – students give start and adding/subtracting PV.

Generalising. Look to make general rules about everything, for example:

1. **Part-whole**. Relate to place value – look at ones, 10 ones forms 1 ten, 10 tens forms 1 hundred – what is the unit? The answer, of course, is that it can be all three – it depends on what you are looking at. Ask students for other things that act like this – e.g. kilometres, metres and centimetres. (This is why it is a big idea – the notion of unit occurs right through mathematics from Prep to senior secondary school.)

2. **All PVs count**. Show that any place value counts. Use this to work out what it means, for example, to be within 30 of 654 – roll the 5 up 3 tens to 684 and roll back 3 tens to 624, so you have to be between 624 and 684. Generalise this idea.

3. **Odometer rules.** These are 8, 9, 0, 1 with digit on LHS increasing by 1; and 1, 0, 9, 8 with digit on LHS decreasing by 1.

Changing parameters. Do these generalities work for numbers of more than 3 digits – how do we do this/show this? What if we had 4 digits – could we still count? Does odometer work for 5, 6, 7 digit numbers? How does it work for days and weeks, years and months, sixths?

Unit 3: Multiplicative Structure and Renaming for 3-Digit Numbers

This unit focuses on the big idea of **multiplicative structure** and the process of **renaming**. The integration of these two into one common set of RAMR stages is more straightforward than in Units 1 and 2. Renaming can be mainly covered in connections (Mathematics stage) and extension (Reflection stage).

Big idea

1. The **multiplicative structure big idea** is the understanding that adjacent PV positions **relate to each other multiplicatively**, that is, one place to the left is multiplication by 10 and one place to the right is division by 10. Extending this, two places to the left is ×100 and two places to the right is ÷100, three places is ×1000 and ÷1000, and so on.

2. It should be noted that many place-value materials (e.g. bundling sticks, MAB) are additive and cannot be easily used to teach multiplicativity. Materials such as digit cards on a PVC and a calculator for the multiplication are more useful.

3. Multiplicativity in place value enables understanding (and calculating) that reveals that numbers can have more than one meaning in terms of PV positions. For example, 362 can be 3 hundreds, 6 tens and 2 ones or 36 tens and 2 ones, and so on. This is **renaming** and it is aligned with multiplicative structure because students must understand that ones increase 10 times as they move to become tens which means that $10 = 10 \times 1$ and, thus, 6 tens is $6 \times 10 = 60$ ones.

4. There are two types of renaming: (a) simple renaming of PV position – 362 can be 36 tens and 2 ones; and (b) more complex renaming which renames only part of each place value – 362 is 2 hundreds, 14 tens and 22 ones.

3.1 Reality – multiplicative structure and renaming

Where possible, find real-life contexts to embed the multiplicative nature of this unit into. Use relevant objects or situations. For example, 10 tickets at $2 cost $20; 10 tables of 6 people makes 60 people; 1 hour of exercise for 10 days makes 10 hours or 1 hour of exercise per day for 1 week makes 7 hours. Certain foods in our society can help – e.g. a packet of lifesavers is 10 lifesavers, so 2 packets of lifesavers × 10 = 20 lifesavers and 60 lifesavers ÷ 10 = 6 packets of lifesavers.

For renaming, discuss that things being multiples of other things leads to being able to do things in many ways (e.g. 10 $1 coins in a $10 note enables $20 to be paid with two $10 notes, one $10 note and 10 $1 coins or 20 $1 coins – and more ways if we bring in the $2 coin and the $5 note).

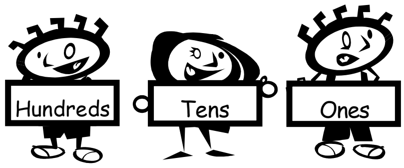
It is useful to look for contexts where there are different ways to make amounts. One example is the “float” that shopkeepers have to have from which to give change. This relates to the different ways that money can form given amounts, e.g. $60 is $10 and $50 or three $20s. This can end up as classical renaming, e.g. $32 is 12 $1 coins and two $10 notes.

There are also particular situations, e.g. needing $1 coins for meters. This can show that three $10 notes and two $1 coins can become 32 $1 coins, that is 3 tens and 2 ones = 32 ones.

3.2 Abstraction – multiplicative structure and renaming

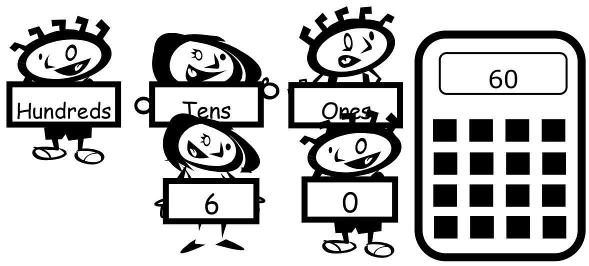
Body

Number/PV bibs. Give three students PV cards and organise them to sit/stand in correct position. Give another student a digit card, say 6, and get them to stand in front of each position.

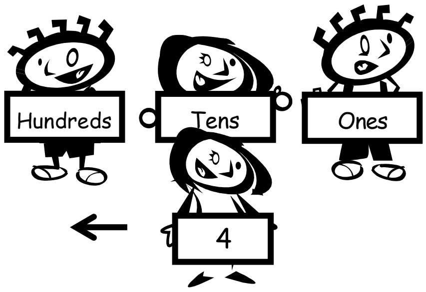


Add zero cards to show what each number means. Press buttons to place numbers on calculator, e.g. 60.

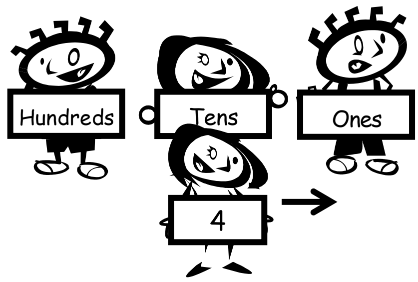
Repeat this for 2 and 3-digit numbers on cards in front of PV cards, e.g. 230, 604, 14, 824, 615. Move from cards to calculator and calculator to cards. Say numbers in terms of hundreds, tens and ones and properly.



Multiplicativity. Put a digit card in front of PV cards, move card left and right, use calculator × and ÷ buttons to show relationship in moves, e.g. 6 tens → 6 ones is ÷10 and 6 ones → 6 hundreds is ×100.



Put a number in calculator, e.g. 40 and multiply or divide by 10, move cards to show these multiplications and divisions. The move to the left shown by the arrow is ×10. The move to the right shown by the second arrow is ÷10.



Mat. Repeat this with the mat with PV cards above a 3×10 mat and digit cards on mat – can move cards or slide mat relative to the PV positions.

Hand

Give the students not involved with the cards a small set of digit cards and a PVC (or a Slide Rule which is the PVC slit on both sides to enable a strip of paper to slide along under the PV positions), pen and paper, and a calculator. As the students make numbers in the front of the classroom and move along, the other students copy the movements with their digit cards, write down the changes on paper and make the change on their calculator with an appropriate × or ÷. This is an excellent example of a multi-representation lesson – a powerful way to teach mathematics.

Reverse – as teacher, give the ×10 or ÷100 change and ask students to move digits.

Mind

Encourage the students to find and write down patterns in movements and their relation to ×10 and ÷10. Ask students for a pattern (i.e. move left one place is ×10 and move right one place is ÷10). Encourage students to study changes that move two places and three places.

3.3 Mathematics – multiplicative structure and renaming

Language and symbols/Practice

Worksheets – these should operate in both directions, giving the multiplication or division and asking for the movement, and giving the movement and asking for the × and ÷ 10 or 100.

Connections

There are two strong connections to make. The first is with metrics and converting from one unit to another. The second is to use the multiplicative structure on PV positions to show renaming. The metrics will be done in the Measurement modules, but the renaming is here.

First type of renaming

1. Put out MAB for a 3-digit number like 273. Take the hundreds and trade them for tens – this shows that 2 hundreds, 7 tens and 3 ones is the same as 27 tens and 3 ones. Can also use the same materials to show that 273 = 2 hundreds, 7 tens, 3 ones which, if you trade all the tens, is 2 hundreds, 0 tens and 73 ones.

2. Use **number expanders** to show, e.g. that 273 is 2 hundreds 7 tens 3 ones but also 27 tens and 3 ones. Also show that 45 tens 9 ones is the same as 4 hundreds 5 tens 9 ones or 459. Fold only at the hundreds in the example below.

2	Hundreds	7	Tens	3	Ones	⟺	2	7	Tens	3	Ones

3. Have students visualise in the mind these changes happening.

4. Use number expanders and worksheets, and then worksheets alone, to practise trading in all of one place-value position (e.g. 4 tens 5 ones = 45 ones). Make sure examples go both ways: from number to expanded form, and from expanded form to number.

Second type of renaming

1. Use MAB and trading to show, e.g. that 273 is 2 hundreds, 7 tens, 3 ones which can be 1 hundred, 17 tens, 3 ones or 1 hundred, 15 tens, 23 ones, or many other combinations, and vice versa.

2. Use PVC to show that, e.g. 1 hundred, 15 tens, 23 ones is three layers – 1 hundred, 15 tens = 2 hundreds 5 tens, and 23 ones = 2 tens 3 ones which adds to 273.

3. Brainstorm as many renamings as possible for a 3-digit number.

4. Practise renaming. Use worksheets to go from renaming back to number. Use examples like:

   3 hundreds 2 tens 7 ones = 2 hundreds ___ tens 17 ones: Work out how many tens [answer = 11].

5. Get students to work out as many ways as they can for renaming a 3-digit number.

3.4 Reflection – multiplicative structure and renaming

Validation/Application

Students communicate their understandings of patterns discovered above. Get students to construct their own renaming or to find renaming in their home culture/neighbourhood.

Particularly apply to renaming – look at examples such as “nineteen ninety four” for 1 994 or one thousand, nine hundred and ninety-four. Take up the money shopping idea from Reality and look at different ways to pay and give change. Use a thinkboard.

Extension

Flexibility. Where in the world do we have changes such as this other than whole numbers and metrics? What about time, money?

Reversing. Must do both directions – change to operation AND operation to change.

Generalising. Make up a generality for any grouping (e.g. days/weeks is ×7). Students should see that other number areas are **related in a similar way to whole numbers but possibly with different × and ÷.** For example, weeks and days relate ×7 and ÷7, hours and minutes relate ×60 and ÷60. Metrics relate via 10 but, of course, the mm – m, the g – Kg and mL – L relate ×1000 and ÷1000.

Investigate where else renaming is used in the world of the students. Generalise the results of this type of renaming. Discuss if renaming is still important when numbers have 5, 6 and 7 digits.

Changing parameters. How does multiplicative structure extend to 5-digit numbers? The understanding that a move left is ×10 and a move right is ÷10 has to be generalised to where the structure of the number system is understood in terms of any adjacent place-value positions relating ×10 and ÷10, with this relationship being **continuous across all place values and bi-directional in application**. Will both types of renaming still work if number has 5, 6 or 7 digits?

There is also a need to build flexibility in seeing that 247 is really 247 ones. It can also be 24.7 tens and 2.47 hundreds and 0.247 thousands. However, we will leave this to Module N2 on decimal numbers.

When looking at groups and groups of groups, renaming is an important understanding of the part-whole big idea. If we have matches, boxes of matches and cartons of match boxes – then the numbers of matches we have depends on how these are opened – e.g. open the carton and we are one less carton but many more match boxes.

Unit 4: Continuous vs Discrete, Number Line, Rank, Rounding and Ordering for 3-Digit Numbers

This is an important unit because it shows that even continuous things such as length, area and volume can have number applied to them when we use units. It also reflects that this application of number changes both the meaning of numbers (particularly the meaning of zero) and how we view the world.

The unit covers the big idea of **continuous vs discrete/number line**, the concept of **rank** and the processes of **rounding/estimation** and **comparing/ordering**. This is done in **two cycles** of RAMR – the **continuous vs discrete, rank and rounding cycle** and the **comparing and ordering cycle**. The first cycle integrates the big idea of continuous vs discrete, with the concept of rank and the process of rounding. Mostly, continuous vs discrete is covered in the Reality and Abstraction stages, while rounding is covered in the Mathematics and Reflection stages, as an application of rank. The second cycle is more straightforward but comparison and order has to be connected to place value, so this is part of the connections subsection. (*Note:* Where there is only a small amount of information, stages could be amalgamated – see 4.5.)

Big idea

Lines are continuous and are not divided into discrete sections that can be counted. However, number-oriented cultures developed the notion of measuring unit to divide continuous lines into equal length discrete parts that could be counted. This **discretifying of the continuous** had three effects:

(a) it changed the role of numbers (0 was no longer nothing, it was the starting point; numbers were no longer assigned to actions/things, they were the ends of things happening);

(b) it changed our perception of the world (integrated wholes became compartmentalised parts); and

(c) it allowed negatives (why not keep going past the zero with the units, we will get negatives).

Thus, the placement of the number becomes based on this equal partitioning of the line. However, since many measures are associated with lines it is important that all numbers are related to positions on lines as well as sets of objects. It gives rise to the following.

1. **Rank**. Regardless of the number of digits that make up a number, each number is a single point on a number line, for example, 657 is a point close to and just below 660. This form of representation ranks numbers in terms of distance from end points of number lines.

2. **Rounding**. Knowing position on a number line enables values to be determined to which numbers under consideration are nearest. Placement of numbers on number lines allows ease of estimation.

3. **Order**. Number representations on a number line provide an indication of size. Numbers can be compared and ordered in terms of position along the line. The higher place values are the starting point for comparing and ordering numbers – comparison is aligned with rank. The number 400 is a hundred more than 300 so 368 cannot be larger than anything over 400. (*Note:* This idea has to be related to PV so that the rule for ordering from PV can be constructed.)

4.1 Reality – continuous vs discrete, rank and rounding

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations. In this unit, this would be length situations – you might look at the construction of a building or a garden and discuss when counting and number are obvious (e.g. number of plants, number of lengths of wood) and not obvious (e.g. length of the path, length of wood to finish the job). Discuss how we measure and why it is needed – so we can go to a shop and ask for the right length of wood.

For rounding, think about instructions on travelling – "How far to the next town?" "About 50 km!" Is this 50 accurate or is it just close enough? Is there a problem if it is 20 km out (could run out of petrol or time)? How accurate would you like it to be? To the nearest 10 km – so it means that the distance is between 40km and 60km? Or more accurate (what if you are walking)?

4.2 Abstraction – continuous vs discrete, rank and rounding

Body

Continuous vs discrete. Count discrete objects in the environment, e.g. people, cars, desks. Compare this with finding numbers through pacing or rulers/measuring tapes. Experience how a unit breaks up a line and makes it countable.

Preparing for rank. Use pacing, number tracks, rulers, and racetrack game boards to find the position along a line or a track of numbers by counting **– notice the positions of numbers (larger numbers are further along the line or track).** Pick a distance at the school (e.g. length of a long building or a playground) – say it starts at 0 and ends at 100. Get students to walk to 50, 25, and other numbers. Discuss where each number will be. Repeat for other starting and ending numbers (e.g. 0 to 999, 50 to 150).

Rank. Put a start and end number on two students (say 0 on one student and 100 on the other) and put them in front of the class (students will see 0 on the left) and have the students hold a rope between them. Give other students numbers on paper between 0 and 100 and pegs and they have to peg on rope where they think this number would be. Get other students to help more accurately place the number. Discuss where numbers would be (e.g. in the middle, near an end). Repeat for other starting and ending numbers. Go both ways (reverse) – give number → students place; give position → students guess number. Extend this to 1000 with ropes or pegs or a masking tape line on the ground. Again get students to place numbers, get the class to help place the numbers, and discuss how we could show placement is reasonable. Use examples like 487 (near half), 990 near one end, and so on. Make sure to reverse the activity – peg cards with A, B, C, etc. on them on to line and get students to write down/guess what A, B, etc., are.

Hand

Reinforce rank. Repeat the body activities but on lines drawn on paper or virtual lines.

Introduce rounding. Use rank to break number lines into equal parts – every 10 for 0 to 100, every 100 for 0 to 1000, every 10 for 300 to 500. Place numbers on these lines – discuss which 10 or 100 is closest – introduce this as rounding to the nearest 10 or 100. Introduce **rounding up** and **rounding down.** Discuss what one can round to – what if it was to nearest 5 instead of 10? Act out placing numbers on a line and then rounding.

Go both ways. Give number, they place and round to nearest 10; give rounding, they give an example of a number that would round like that.

Halfway. Discuss what happens when the number is halfway – state that the convention is to round up.

Mind

Reflect on things that can be counted and what cannot. Point out that, normally, items have to be discrete (individual and separated) to be counted. Point out that the world is full of discrete things (chairs, people, animals, days, grains of sand, etc.) but that some things **have had to be "changed"** to become countable.

Discuss length – it is not countable unless a unit of measure is used. Length is continuous (as is area, volume, mass, time, and so on) but units make it discrete. Ask why we want to turn the continuous into the discrete?

(So we can apply number to it.) Look at a line broken into equal parts – discuss how the numbers are different in relation to counting blocks – particularly the zero (show zero for blocks and then for line).

Have students imagine a line in their mind and then use the imagined line to place numbers. Repeat this for different lines. Show numbers on lines which are wrongly placed – get students to change the placement.

Have students imagine a ranked number line and round. Have them shut their eyes and point out numbers on imaginary line, then show with fingers to where they would round.

4.3 Mathematics – continuous vs discrete, rank and rounding

Language and symbols/Practice

Familiarity with number lines. Continue to engage in counting and measuring activities that demonstrate the continuous vs discrete big idea and the role of rank in evenly partitioning the line.

Experience lines. Relate to a measuring tape (mm-m) – measure things and write down length in numbers, and then mark these lengths on a single line (recording what was measured under the line). Find number lines in the everyday world. Explore different uses of number lines. Point out that 0 is now starting point, not nothing – rulers start at zero and even a calculator when cleared shows a zero. Get students to construct their own lines; for example, construct a number line from 600 to 800 – mark in sufficient points to show where things are but not too many – too many points and it's too crowded; not enough points and it's too hard to read.

Experience a variety of lines, e.g. can lines be circular? What about speedometers, clocks, and so on?

Position and quantity worksheets. Continue with worksheets that relate position to value on a line with two end numbers (e.g. place 929 on a line starting from 700 and getting to 1000). Go both ways and have worksheets that give numbers to be placed on a line and other worksheets where points are marked on the line and students have to write down numbers.

Rounding worksheets. Give numbers, students place on ranked line and then round. Progress so that students round directly off the number line. Do worksheets where they go from number → round and round → number.

Connections

Relate to comparison and order – use rank on a number line to introduce bigger/longer, smaller/shorter, and between. Relate to measuring devices. What about scales on maps?

4.4 Reflection – continuous vs discrete, rank and rounding

Validation/Application

Continue the Experience activity above – get students to find number lines in their world. Get students to construct their own measuring devices that use a number line. Use a variety of starting and finishing numbers.

Extension

Flexibility. Make a record of all types of number lines (e.g. semicircular and so on).

Reversing. Go both ways: (a) give numbers and let students place on line **and** give positions on line and ask students to guess numbers (with reasons); and (b) get students to construct number lines **and** to interpret and work on number lines supplied to them.

Changing parameters. Do number lines still work for more than 3 digits? Think about a number line from 0 to 1 million – what would you mark on the line?

Discuss how area would have number applied to it – how would we work out what the number was?

4.5 Reality/Abstraction – comparing and ordering

Where possible, find real-life contexts to embed the activities in, using relevant objects or situations. For example – comparing sport scores or heights, and so on.

Body

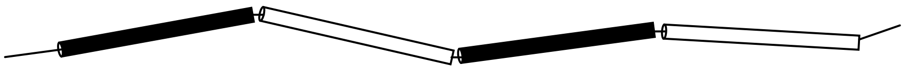
Use pacing, number tracks, rulers, and racetrack game boards to find the position along a line or a track of numbers by counting. Compare two positions – longer is further along the line.

Hand

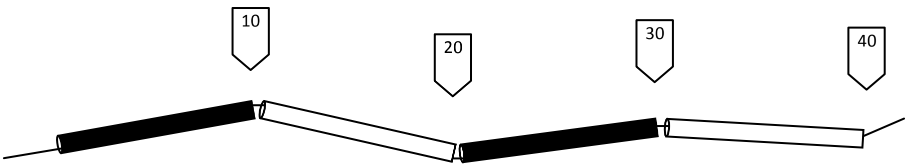
Repeat rank activities. Repeat the activities in section 4.2 (e.g. with pegs and rope, or on a line on board or virtually) but, this time, **use two numbers and state which is larger**. Repeat this for three or more numbers – putting in order from smallest to largest or vice versa (remember to identify the **"between"** numbers). Relate the results of number line to MAB/PVC represented numbers – develop generalisation that the left-most place value is the most important in ordering.

Redirect thinking. Discuss with students how with rank we place the number correctly but now with comparison we are trying to understand the rules (PV rules) by which one number is larger than the other. Direct students to look at a number line as a regular placement of numbers between two points. Then say that, with comparison and order, you want them to use the number line to work out what makes one number larger than the other, particularly in relation to PV positions.

Develop activity to focus attention on PV – Tens/Ones. Make a **straw number line:** Cut 10 cm lengths of straws (five each of two different colours). String straws alternately on string to make a straw number line or "measuring tape" as below. (*Note:* Can compare this with a metre to see how accurate it is; but it need not be a metre. In fact the straws could be any length, we simply need a 10-straw number line/measuring tape.)



Label the end of each straw as follows.



Using the straw number line: Find where the following would be on the "straw tape": 30, 80, 28, 64. State the numbers that are where the teacher is pointing. (Teachers point to positions on one of the "straw tapes".) **Crucial** – focus on teaching the importance of the tens position in determining order of 2-digit numbers.

Find pairs of objects/lengths to measure where one is longer than the other. Measure the pairs and place results on a recording sheet as follows. (Alternative – teacher records on board and ticks the longer.)

Pair of objects	Greater/Longer	Less/Shorter
A1/A2	45	38
B1/B2	82	69
C1/C2	68	64

Discuss results – encourage students to discover a pattern. Test the pattern by providing sets of two numbers for students to pick the larger/longer. Ensure discussion and discovery focuses first on tens and, when they are the same, on ones. Start with examples where the tens are always different and then add in examples where the tens are the same later. The measuring with the straw number line which only has tens marked focuses attention on tens – measures are in terms of tens and the ones have to be guessed. (*Note:* Some educators would argue that the number line should not have the tens marked placing even more focus on how many of these tens as they are counted for each measure.)

Hundreds/Tens/Ones. Repeat this for a 0 to 1000 number line divided into hundreds – can make it out of two colours of cardboard strips repeated five times. Once again gather information and find the rule that the largest digit in the hundreds is the greater number and if hundreds are the same then tens and ones are looked at.

Mind

Imagine using the straw number line – what number would be greater out of 56 and 73 or 29 and 31? Imagine using the cardboard strip number line – what number would be greater, 402 or 385?

4.6 Mathematics – comparing and ordering

Language and symbols/Practice

Using straw tape or drawings of tape measures. Use the straw tape to work out examples as follows:

(a) Tick the larger:

26 or 46   42 or 39   68 or 64   48 or 51

(b) Place the following in order from largest to smallest: 54, 61, 16, 59, 65

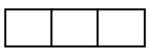
(c) Repeat this for 3-digit numbers.

Introduce symbols. Introduce the symbols – **> greater than** and **< less than**. Use the fact that the symbol itself is wider at the greater number to help memory. Do this for 2 and 3-digit numbers.

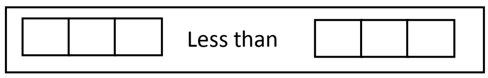
Worksheets. Complete worksheets that practise the ideas in the above material. Do recall activities – place out two sets of two playing cards and use > and < signs between the two sets of cards. Do both of these for 3-digit numbers (three cards on each side).

Games. Play games to become independent of material – the games below focus on getting students to see that the largest digit – in the hundreds – is the most important.

3-digit "Chance-number games". Materials – digit cards, boards as below, digit cards to fit into board, card deck (0–9 or 1–9 only):



Chance-number board



Chance-order board

1. **Chance-number**. 3 cards are dealt one at a time, use first number to place a digit card on board (have to choose hundreds, tens or ones), second and third numbers fill other positions. Player who makes higher number scores 1 point, 0 otherwise. Winner is largest score after 5 games. *Variations*: (1) play game where object is to make lower number; (2) when number is complete, can give up a digit and take the value of a fourth dealt card.

2. **Chance-order together**. 6 cards are dealt, use the numbers to make the left-hand 3-digit number smaller than the right-hand number with digit cards on game board. Score 1 point if the left-hand 3-digit number is correctly smaller than the right-hand 3-digit number. Score 2 points if the smaller 3-digit number is the largest possible. The winner is who has highest score after 5 games. *Variation*: object is to make larger 3-digit number – score 2 points if the larger number is the smallest possible.

3. **Chance-order one-at-a-time**. 6 cards are dealt one at a time, use first number to place a digit card on board (have to choose hundreds, tens or ones in either the left-hand or right-hand 3-digit number),

continue making choices and placing digits on board before next card called. Score 1 if correct and 0 if not. The winner is who has highest score after 5 games.

4. **Chance-order high scoring**. 6 cards are dealt one at a time, use first number to place a digit card on board (have to choose hundreds, tens or ones in either the left-hand or right-hand number), continue making choices and placing digits on board before next card called. Score 0 if not correct but score the value in the hundreds place of the smaller 3-digit number if correct. The winner is who has highest score after 5 games.

Have students work in pairs to design their own game based on innovations of the models just played.

Connections

Relate the games above to place value – ensure all students discover the importance of the highest PV position. Relate to measures. Then ensure they know the rule for when the largest PV positions are the same – e.g. 436 and 452, or 436 and 431.

Note: The sequence of ideas above is assisted by (some would say requires) the numbers being looked at in terms of place value (Unit 1) and place values related to order – students have to arrive at the place-value generality that it is the largest place values that are important in determining order.

4.7 Reflection – comparing and ordering

Validation/Application

Apply comparison and order to the world of the students. Do activities that involve ordering the students' world.

Extension

Flexibility. Find all situations where comparison is important – money, height, mass, and so on.

Reversing. Give numbers → put in order AND give one number and an order → have to find other numbers For example, teacher gives 467, 406 and 470 and students find greatest; teacher states that 621 is the greatest and students have to find another two numbers to make this so (but not easy numbers).

Generalising. This is the most crucial aspect – have to get students to generalise the importance of the largest PV position as the starting point of comparing. This comes from the connections above and integrating Unit 1 and Unit 4.

Changing parameters. Extend above to 4 and more digits. Is the largest PV position still the starting point? Do the students think this rule always applies? What would they look for in thousands/4-digit numbers, or millions/6-digit numbers?

Unit 5: Equivalence for 3-Digit Numbers

For whole numbers, this is a small unit. It focuses on the big idea of equivalence, that is, where two different representations have the same value, e.g. 8 + 5 = 10 + 3; 27 − 25 = 4 − 2; $^2/_3 = {^4/_6}$. For whole numbers, the only possibility for equivalence is a 0 before the numbers, e.g. 004 = 4. However, for some students, this is a difficulty as they believe zeros have no value allowing 204 to be the same as 24.

Thus, this unit has only one small RAMR cycle and it is given in two parts.

Big idea

The big idea here is that it is possible for the number to be the same and the numeral to be different. Do this only for whole numbers, but obviously it can be extended to decimal numbers. It is related to the role of zeros. It is somewhat trivial for whole numbers, but there are new perspectives that make it not so trivial. There are really two ways of looking at equivalence.

1. **Whole numbers as value**. For whole numbers, the rules for equivalence when looking at value are as follows: (a) zeros placed before the other numerals do not change its value, for example, 245 = 00245; (b) placing a zero elsewhere does change the value of the number, for example, 87 ≠ 807, 870, and this is true if more than one zero is involved; and (c) removing a zero does not change a number's value if the zero was before the other numerals, otherwise it changes the value, for example, 065 = 65 but 650 ≠ 65.

2. **Other purposes of number**. The first point about purposes for number other than value is that adding and removing zeros anywhere else than before the other numerals does change the number (the same rule as for value). We are only looking here at zeros **before** the other numerals. What is being proposed is that there are some situations where zeros before the other numerals does change the number. These situations are as follows.

   (a) *Identity numbers*. The first situation is when numbers are used to identify things (and people). An identity card with a number 000568 is often different to one with 568. In fact the difference is one of belonging (or validity). The 000568 identity card probably has to have 6 digits (3 digits is not acceptable). So for small numbers, we add the zeros so that the card is valid.

   (b) *Phone numbers*. A second example is phone numbers. Phoning 0467 234 431 is different to phoning 467 234 431.

   (c) *Order numbers*. A third example is using numbers to order files in computer systems that use alphabetical systems to order. Since AB is before B then 10 is before 2. This necessitates ensuring that zeros are placed in front of numbers because ordering files labelled 4587 and 43311 gives a different result to ordering numbers 04587 and 43311.

   Thus in the modern digital age, there are many situations where a number such as 045876 is not equivalent to 45876.

5.1 Reality/Abstraction – equivalence

Look at the world of the students to find examples in their lives of where a different numeral is given for the same number. Some of these are: How are dates entered on many forms? How are dates entered on credit cards? How do we now write today's date? Are there any dates where you would write something different to normal numerals for numbers? Also, computers, if given files with numbers 1 to 12 will put 11 before 2 – the numbers have to be given in the form 01, 02 to 12.

Body

Give students bibs with numbers on them and get them to make numbers on a mat with H-T-O place values marked on it. See what numbers they make. If one has a zero, ask what the zero does to the number. Is the zero necessary to be there? What value does the number have with the zero? Without the zero?

Hand

Use the numbers in A below as names for files on a computer. Save these files into a folder. Ask Word to order the files (click on "Refresh"). Repeat for numbers in B. What happened? Is there a problem? What do we do to get B into numerical order? Is there a difference if B was changed to C?

A:	7	3	8	2	4	6
B:	7	3	18	2	14	26
C:	7	3	18	2	114	26

Discuss the role of zero in numbers. Look at where zeros change and zeros don't change the value of a number. Look at examples like this: Circle YES or NO

83 → 830 number changed in value – YES/NO?

056 → 56 number changed in value – YES?NO

Then get students to construct these: (a) give students a number to start with (say, 304) and ask them to do something to that number that changes it and something that does not change it; and (b) give students a number (say 407) and ask them for a starting number and what was done to it that changes value and does not change value. Ask students for more than one answer.

Sort the following numbers into groups where the number is the same value.

24 024 240 0024 204 2004 0204 00240

Mind

Imagine a number with no zeros – imagine putting in zeros – when/where does the zero make a difference?

5.2 Mathematics/Reflection – equivalence

Symbols/Practice

Practise situations where zeros change and don't change numbers:

- Give examples like 56 → 506 45 → 045 708 → 78 and so on; ask students to circle change and tick no change.
- Give a set of examples based on 0 and three other digits; ask class to sort into groups where all numerals are the same number; and those which are different.
- Give a 3-column table with starting number, change and no change as headings; fill in one of the columns for each row; ask students to make up the other rows.

Connections

Where else in mathematics do we have this situation where two different numerals give the same number/value?

- In measures?
- Time? (what is 5 past – "oh five?") Dates? (what does Year 95 or 04 mean?)
- Identification numbers? (Fred's student number was 00843 – why is the 00 there?)
- Invoice numbers and numbers on digital machines.

Validation/Application

Ask students to explore the world and find everywhere that numbers are expressed in different numerals to the normal 6, 11, 25, 327, etc. For example, 24-hour time for plane timetables, and military operations when time and angles (bearings) are written in a specific pattern.

Extension

Flexibility. When numerals are different – why? What is the generalisation? Why is it done? (For example, if numbers are to be ordered, and they go to three digits, then often there are three places to fill in and 4 has to be written as 004, 57 has to be written as 057 and 307 has to be written as 307.)

Reversing. Reverse everything. If 64 has to be written as 0064, then what number is 0206?

Generalising. Generalise the ways numerals can be changed by zeros without changing the number (e.g. adding zeros at the start of a whole number) and changing the number (e.g. zeros in between digits or at end of digits of a whole number).

Changing parameters. What if we had six digits, does the rule change? Is this a change or not: 781 → 007801? What starting number is a no change if the end number is 070081?

Unit 6: Extension to Large Numbers Using Pattern of Threes

This unit looks at how the 3-digit numeration knowledge developed for hundreds, tens and ones can be extended to large whole numbers. This will be done in two RAMR cycles:

(a) the first on position understanding (covering the big ideas of notion of unit/part-whole, additive structure/counting, multiplicative structure, and equivalence) which will be based on the pattern of threes in place value; and

(b) the second on number-line understanding (covering continuous vs discrete/number-line big idea, and also covering rank, order and rounding).

The major focus of this unit will be on the positional understanding (as there is much more involved), but the number-line understanding will also be important because, in the Reflection stage, it will have ideas to assist the teaching of very large numbers.

Big idea

The big idea here is that it is possible for large whole numbers (thousand and millions and as a system) to be learnt by a gestalt jump from the H-T-O knowledge (to be an extension of 3-digit knowledge) because the large numbers can be seen as a pattern of threes. This means that we can extend 3-digit knowledge to 6 and 9 and larger digit knowledge by looking at numbers as sets of three PVs as follows. Thus, we do not need to repeat all the steps that students went through to learn the H-T-O knowledge to learn the larger numbers.

H–T–O	H–T–O	H–T–O	H–T–O
Billions	Millions	Thousands	Ones

This is based on seeing PV as two parts:

(a) a macrostructure of a sequence of larger numbers built around multiples of 1000 – ones, thousands, millions, billions, and so on; and

(b) a microstructure within each of these PVs where there are hundreds, tens and ones (H-T-O).

That is, the 9-digit knowledge of millions is seen as three repeats of the H-T-O knowledge – through ones, then thousands and onto millions.

Note: AIM advocates extension as the most efficient way to accelerate understanding of low-performing students in Year 7/8. If it succeeds, it extends the five big ideas and their concepts and processes from 3 digits to 9 digits, that is from H-T-O ones to H-T-O thousands and millions. However, if there is a difficulty with any of the extensions, **Appendix A** has detailed activities for thousands and millions similar to those in Units 1 to 5.

Whole-number system

←	H	T	O	H	T	O	H	T	O	H	T	O
	BILLIONS			MILLIONS			THOUSANDS			ONES		

Understanding the pattern of threes enables students to see the pattern for very large numbers as a system – the place values increase in groups of three with a new name for each group of three.

This means that whole numbers form a system that can handle all numbers no matter how large. After billions, the system moves on to trillions and then uses Latin words – quadrillions, quintillions, sextillions, septillions, and so on. This sequence can be understood in terms of powers of 10, ones being 10^0, thousands 10^3, millions 10^6, billions 10^9, and so on. This will be covered in Year C Module N5 *Directed Number, Indices and Systems.*

6.1 Reality/Abstraction – position extension

Reality of large numbers

Discuss with students where you find large numbers – numbers with many digits. Examples of ideas are: How much does a person earn working in the mines? How much is their favourite singer worth? What does a new car or house cost? How much money does the richest person in Australia have? Bring in people from the community to describe situations which have large numbers or go on visits to places where large numbers are used (e.g. visit the council and find out how much they spend in a year). Find out if students themselves have situations where large numbers are used.

Note that large number situations may have new names. Megabytes and gigabytes are an instance. What does it mean for your phone to have a memory of 16 gigabytes? How many "megs" on a memory stick? What is a "meg"? Some excellent large number activities are virtual – one enables student to travel across the solar system as numbers show distance from earth; another shows the history of earth with the number of years since the earth was formed shown on the screen.

Body

Use the bodies of students. Put One Ones, Ten Ones, Hundred Ones, One Thousands, Ten Thousands, and so on, as labels or bibs on students. Organise them into place-value order and into threes. Alternatively do this on a 6×10 mat – students can walk the place values as they say them.

Hand

PV materials. Use fold out PV charts, Montessori cards and number expanders to show PVs as a pattern of threes. As can be seen in the number expander, these materials focus on how numbers can be seen in terms of the microstructure. For example, the number expander shows that 462 381 759 806 is 462 billions, 381 millions, 759 thousands and 806 ones.

4	6	2	Billions	3	8	1	Millions	7	5	9	Thousands	8	0	6	Ones

Reverse this process – break up large numbers into threes (e.g. 348 651; 832 526 851) – name each of the groups of three – then name the digits in each group of three as if they were three-digit numbers, for example, 832 526 851:

832	526	851
8H 3T 2 Ones of Millions	5H 2T 6 Ones of Thousands	8H 5T 1 Ones of Ones

"8 hundred and thirty-two million, 5 hundred and twenty-six thousand, 8 hundred and fifty-one"

Growing the numbers. An excellent way to show how H-T-O ones moves to large numbers is to teach one microstructure at a time, that is, teach H-T-O ones and then bring in H-T-O thousands and then H-T-O millions, and so on. One way to do this is to use materials where PV charts for ones, thousands and millions **are separate** but can be brought together. One such material to do this is the **Pattern-of-threes PVCs** on the next page. There are three PVCs and digit cards – this material should be cut out and laminated. (*Note:* If bigger numbers are wanted a fourth PVC for Billions can be made.)

The technique for teaching is to start with the ones PVC, then bring in the thousands PVC when ready and place it on the left, then do the same with the millions PVC. Activities are as follows.

1. Give students numbers to place in the pattern-of-threes PVC using the digit cards – get students to read the numbers, moving their left hands as they read across the digits and the ones, thousands, etc. It is important to **move the hand** to ensure that kinaesthetic activity is present to build the PV system as a

mental image. Repeat this with other numbers, with hand movements, to drive home the macrostructure and microstructure of large numbers.

2. Reverse the above and this time read out the number – ask students to make the number with the digit cards. State numerals and PV positions out of order and see if students can correctly put out the digit cards and then read the numbers (moving their hands).

Mind

Shut eyes and imagine large numbers and PV positions and say and write numbers in the head.

Pattern-of-threes PVCs

HUNDREDS	TENS	ONES
MILLIONS		

HUNDREDS	TENS	ONES
THOUSANDS		

HUNDREDS	TENS	ONES
ONES		

Digit cards

0	1	2	3	4	5	6
7	8	9	0	0	0	0
0	1	2	3	4	5	6
7	8	9	0	0	0	0

6.2 Mathematics/Reflection – position extension

Language and symbols/Practice

Continue the reading and writing of numbers using the pattern-of-threes PVCs/digit cards, followed by just reading and writing without PVCs. Use worksheets that relate PVCs, language and numbers. Particular activities are as follows.

1. **Unit-PV-reading-writing**. Once the macrostructure and microstructure of the pattern of threes is known, activities like those in Abstraction (section 6.1) should build good place value and reading and writing of large numbers, particularly if saying numbers, writing numbers, recording on calculators, and using materials are integrated. In particular, the calculator is an excellent method of reinforcing the large number work:

   - give number as digits, students enter on calculator, and read out number AND read out number as it should be, students enter on calculator and say digits;
   - play the game Wipe-out – much better with more numbers – wipe one digit at a time (e.g. 45 786 – wipe the 7) or have more than one repeat of a digit and have to wipe both (e.g. 48 786 – wipe both 8s).

   Note: There can be difficulty with zeros in Wipe-out, e.g. students will subtract 500 from the number 45 786 when asked to wipe 5. If this is the case, (a) play the game with digit cards on PVCs so students can see PVs; (b) spend a lot of time making numbers like 5 000 on the PVCs with the digit cards so students get used to zeros; and/or (c) relate 5 000 to the 5 in 45 786.

2. **Counting-seriation-odometer**. Count by tens and hundreds in 3-digit numbers and reinforce patterns in counting for 3 digits. Move the H-T-O up to the thousands and repeat – is there any difference? Could use bodies (with bibs to make 368) on a mat and then move them from the ones to the thousands. Could use calculators and count in tens for 368 and count in ten thousands for 368 000.

   Use calculators to reinforce. Enter, say, 34 672 and add 1 000 – keep pressing the = sign and calling out the thousands position – note the odometer pattern remains. Repeat for, say, 459 302 and subtract 10 000, pressing equals and calling out the ten thousands position – again note the odometer pattern.

   Practise counting patterns, e.g. 34 568 901, 34 578 901, 34 588 901, 34 598 901, ______, ______, ______, and so on.

3. **Multiplicativity-renaming**. Reinforce the ×10/100 and ÷10/100 movements in 3 digits (use bodies or PVCs/digit cards). Move the 3 digits to thousands, add in ones so 6 digits and again do ×10/100 and ÷10/100 movements. Is there any change in what happens? Remember to go both ways, that is, teacher says movement, students give operation AND teacher gives operation, students give movement. Introduce ×1000 and ÷1000 movements.

   Note how important ×1000 and ÷1000 movements and operations are to metric conversions.

4. **Equivalence**. Recap finding for 3 digits that zeros only make difference when put at end or in middle of digits in a whole number. Zeros at start of a number make no difference. Apply this to larger numbers and see it is the same.

Connections

Large numbers have become commonplace with money and have always been a part of measurement (e.g. solar system) and time (e.g. age of earth). The important thing is to make use of such examples at this point.

Validation/Application

Organise the students to look at large numbers in their lives (e.g. population figures, odometer on cars, gigabytes, hits on YouTube, and so on) and how these affect the students. The idea is to get them to look at these numbers with new understandings.

Extension

Flexibility. Do a poster on large numbers – *Where do we find numbers like this (e.g. 27 465 000)?*

Reversing. Always ensure activities that relate representations of large numbers (e.g. PVC, language, symbols) are reversed, that is, go symbol to language AND language to symbol.

Generalising. Ask the students if they can see a pattern in how numbers get bigger and bigger (e.g. *How do we understand a number that says how far it is to Saturn?*).

Changing parameters. Discuss what words are used for very large numbers – thousands, millions, billions trillions – what is next? Encourage students to look up names of larger numbers on the internet (based on Latin – quadrillions, quintillions, and so on). Use language and position, for example:

...	trillions	billions	millions	thousands	ones
...	terabytes	gigabytes	megabytes	kilobytes	bytes

Note: This RAMR cycle covers four big ideas and their concepts and processes: (a) notion of unit/part-whole, place value, and reading/writing; (b) counting, seriation and odometer; (c) multiplicative structure and renaming; and (d) equivalence. If teachers find that this extension does not work and more teaching has to be done with large numbers to get understandings in these four areas, **Appendix A** at the end of this module has more detailed teaching instructions for teaching thousands and millions and sections A1 to A3 and A5 cover the ideas in this RAMR lesson.

6.3 Reality/Abstraction – number-line extension

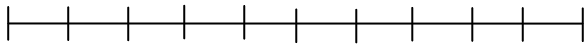
Try to find a way that a number line showing large numbers has meaning – timelines and distances may offer opportunities.

Body/Hand

Use pegs and rope, or a drawing of a number line, to reinforce placement of numbers on a 0–1000 line and rules for order (the biggest place-value position is the important position – the biggest digits there mean the biggest number).

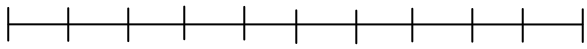
Extend pegs and rope or drawing of number line to 0 to 1 000 000. Think of this as 0 thousands to 1000 thousands. In this way the 0 to 1 000 000 number line will be similar to the 0 to 1000 number line – the numbers are placed in the same way. Extend to 0 to 1 000 000 000 number line – this time, the number line can be thought of as a 0 to 1000 million number line – structurally the same as a 0 to 1 000 000 or a 0 to 1000 number line.

Get students to make the three number lines, on the floor with tape and numbers, or on paper: one for 0 to 1000, one for 0 to 1 000 000, and one for 0 to 1 000 000 000. Drawings of these lines below show their similarity, which seems easier to see if using language. Think of two numbers on the line – which is bigger? Think of the rule for 0 to 1000 – the number with the more hundreds. So what is the rule for 0 to 1 000 000 000 – the number with the more hundred millions.



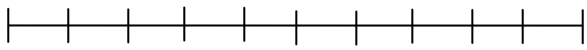
0 200 400 600 800 1000

0 2H 4H 6H 8H 10H H – hundred



0 200 000 400 000 600 000 800 000 1 000 000

0 2HTH 4HTH 6HTH 8HTH 10HTH HTH – hundred thousand



0 400 000 000 800 000 000

200 000 000 600 000 000 1 000 000 000

0 2HM 4HM 6HM 8HM 10HM HM – hundred million

Mind

Shut eyes and imagine a line, say from 0 to a billion or 1000 million – think of this as similar to the 0 to 1000 line – now place numbers like 435 678 121 on it.

6.4 Mathematics/Reflection – number-line extension

Language and symbols/Practice

Aligning. Compare large numbers by putting both on the same PVC and seeing which has the larger digit in the highest PV position, as below:

			H T O H T O		
A is 345 678			thousands ones		largest PV is 100 thousands
B is 96 723	→	**align**	3 4 5 6 7 8	→	A has a 3 in it, B has a 0
			9 6 7 2 3		A is the larger number

Worksheets. Provide pairs of numbers and PVCs to put them in – align PV positions and determine larger/smaller.

Connections

The main connections here are to measurement (e.g. large distances, volumes, masses and so on) and to data and probability (e.g. hits on websites, Gross National Products, census data, chances of winning Lotto).

Validation/Application

Check students can discuss large number lines in their world.

Extension

Flexibility/Reversing. Be flexible with numbers and reverse relationships (line and placement → guess number, and number and line → placement).

Generalising/Changing parameters. Look at PV as a number line as follows.

H	T	O	H	T	O	H	T	O	H	T	O
BILLIONS			MILLIONS			THOUSANDS			ONES		

Where do we go from here? Does this help below?

H	T	O	H	T	O	H	T	O	H	T	O
GIGABYTES			MEGABYTES			KILOBYTES			BYTES		

Motivating activity. Both the activities below can involve a class in very large numbers as they explore possibilities.

1. **Chessboard and rice**. Look up the famous puzzle about wanting one grain of rice for the first square of a chessboard, two grains for the second, four for the third, doubling all the time until you get to the last square. How many grains in the last square? How many on the board in total? How much does this weigh? If you had 100 people each putting a grain on every second, how long would it take to finish all the squares?

2. **Tower of Hanoi** (look it up on the Internet). How many moves for tower of height 2? Tower of height 3? At a move per second, how long to move a 50 tower?

Note: This RAMR lesson covers the continuous vs discrete big idea and all the concepts and processes of Unit 4. The AIM project believes that acceleration-by-extension works for extending whole numbers from 3 digits to 9 digits, and it should not be necessary to reteach all the detail of Units 1 to 5. However, if this is a problem for your students, **Appendix A** has detailed teaching information covering the ideas associated with continuous vs discrete, number line, rank, rounding and ordering for large numbers.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the whole number item types

The whole number numeration item types are divided into two parts: Subtests 1 to 5 which focus on three-digit whole number numeration (and cover the ideas in Units 1 to 5); and Subtest 6 which focuses on numeration for thousands and millions (and covers the ideas in Unit 6 and Appendix A). Thus, items in Subtests 1 to 5 relate to Units 1 to 5, while items in Subtest 6 relate to Unit 6. However, to provide more detail, Subtest 6 items are divided into sets that relate to sections A1 to A5 of Appendix A.

Subtests 1 to 5 should be considered separately to Subtest 6 because they cover two levels of number. Thus, the following seems evident: (a) errors in Subtests 1 to 5 mean that the student is having difficulties with 2-digit and 3-digit whole numbers and should be taught all the module; (b) errors in Subtest 6, and very few errors in Subtests 1 to 5, mean that the student should do the Appendix A material; and (c) few errors in Subtests 1 to 5 and Subtest 6 mean that the student could be in mainstream Year 8 for whole numbers.

Note: It should be recognised that success in whole number tests may not mean success in other more difficult mathematics topics. Thus success in this module may not mean that students do not need AIM modules for other mathematics topics. However, difficulties in this module likely indicate major difficulties in the rest of mathematics.

Subtest item types

Subtest 1 items (Unit 1: Notion of unit/Part-whole)

1. Write the number for each number name:

   (a) Three hundred and forty-nine ________________________

   (b) Five hundred and forty ______________________________

   (c) Two hundred and twelve ____________________________

   (d) Eight hundred and six _______________________________

2. Write in words:

   (a) 268 ____

   (b) 303 ____

   (c) 814 ____

   (d) 550 ____

3. Write the number that has:

   (a) 6 tens 5 ones 4 hundreds ___________________________

   (b) 3 hundreds 2 ones 7 tens ___________________________

   (c) 1 ten 3 hundreds 5 ones ____________________________

   (d) 3 ones 4 hundreds _________________________________

4. (a) Circle the number in which the 4 is worth the most:

   346 401 274

   (b) Circle the number in which the 6 is worth the least:

   601 896 462

Subtest 2 items (Unit 2: Additive structure)

1. What number is:

   (a) 10 more than 728? ______________________________

   (b) 100 less than 685? ______________________________

   (c) 100 more than 310? ______________________________

   (d) 10 less than 500? ______________________________

   (e) 1 less than 140? ______________________________

   (f) 1 more than 299? ______________________________

2. Complete the counting sequences:

   (a) 286, 287, 288, ____________, ____________, _____________

   (b) 596, 696, 796, ____________, ____________, _____________

   (c) 433, 432, 431, ____________, ____________, _____________

   (d) 830, 820, 810, ____________, ____________, _____________

   (e) 364, 374, 384, ____________, ____________, _____________

   (f) 504, 404, 304, ____________, ____________, _____________

3. **Challenge** – Circle the pattern in which the digit 3 changes if the pattern continues for 10 numbers (note: there might be more than one correct answer):

   (a) 234, 235, 236, ….

   (b) 563, 553, 543, ….

   (c) 344, 345, 346, ….

   (d) 371, 361, 351, ….

Subtest 3 items (Unit 3: Multiplicative structure)

1. Write in the missing place value names:

   (a) 8 ones × 10 = 8___________________

   (b) 4 ones × 100 = 4__________________

   (c) 62 ones × 10 = 62_________________

   (d) 15 tens × 10 = 15_________________

   (e) 9 hundreds ÷ 10 = 9_______________

   (f) 6 tens ÷ 10 = 6 ___________________

   (g) 7 hundreds ÷ 100 = 7______________

   (h) 23 tens ÷ 10 = 23 _________________

2. Write the missing numbers:

   (a) 928 = 9 hundreds, _______tens, _______ones

   (b) 684 = 6 hundreds, 7 tens, _______ones

   (c) _______ = 4 hundreds, 19 tens, 6 ones

   (d) 547 = _______hundreds, 14 tens, 7 ones

   (e) 723 = 6 hundreds, _______ tens, 13 ones

   (f) _______ = 3 hundreds, 26 tens, 31 ones

3. **Challenge** – Write the missing numbers following the rules:

   (a) 632= _______hundreds, _______tens, _______ones

   [The rule is that you cannot write a 6]

   (b) 632= _______hundreds, _______tens, _______ones

   [The rule is that you cannot write a 3]

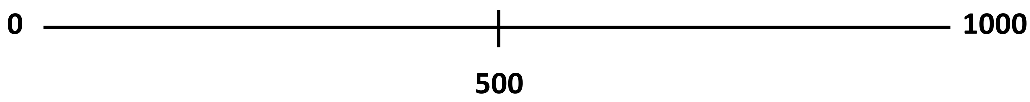
   (c) 632= _______hundreds, _______tens, _______ones

   [The rule is that you cannot write a 6 or a 3]

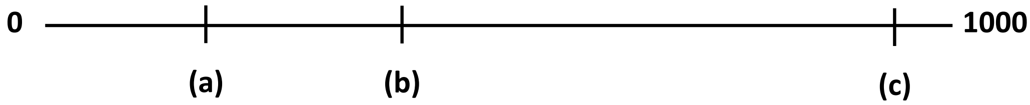
Subtest 4 items (Unit 4: Continuous vs discrete/Number line)

1. Place the following numbers on the number line:

   (a) 601 (b) 87 (c) 756



2. What number could be at (a) __________; (b) __________; (c) __________



3. For each set of numbers, circle the larger value:

   (a) 387, 404 (b) 556, 552 (c) 674, 92 (d) 804, 840

4. Put these numbers in order from the largest to the smallest in value:

   652, 625, 650, 605, 615

   _________, _________, _________, _________, _________

5. Round to the nearest 10:

   (a) 686 ________

   (b) 725 ________

   (c) 303 ________

6. Round to the nearest 100:

   (a) 456 _______

   (b) 238 _______

   (c) 850 _______

7. Circle the numbers that could be rounded to 300:

   (a) 713 (b) 267 (c) 325 (d) 380 (e) 250

Subtest 5 items (Unit 5: Equivalence)

1. (a) Write a zero in 368 without changing its value _________________________

   (b) Write a zero in 368 so that its value does change _______________________

   (c) Write a zero in 368 so that its value changes in a different way ___________

2. (a) Remove a zero from 06040 so that its value does not change _____________

   (b) Remove a zero from 06040 so that its value does change _________________

Challenge items

3. What numbers can you make using 1, 0, 2, 7, 8 and 4? See how many you can find (using each numeral only once in each number). Write them here:

4. Write the largest number you can find using all these numerals: _______________

5. Write the smallest number you can find using all these numerals: ______________

Subtest 6 items (Unit 6: Extension to 9 digits)

A1 Part-whole items

1. Write the number for each number name:

   (a) Six thousand five hundred and nine ________________________

   (b) Fifty-seven thousand two hundred and fourteen __________________________

   (c) Two hundred and eighty-eight thousand ___________________________

   (d) Eight hundred and six thousand ______________________________

2. Write in words:

   (a) 268 767 ____

   ____

   (b) 30 335 ____

   ____

   (c) 8149 ____

   ____

   (d) 5 502 100 ____

   ____

3. Write the number that has:

   (a) 3 millions, 8 hundred-thousands, 2 ten-thousands, 2 thousands, 4 hundreds, 5 ones

   ____

   (b) 6 thousands, 4 hundreds, 3 millions, 7 hundred-thousands, 5 ten-thousands, 6 tens, 2 ones ___________________________

   (c) 9 tens, 9 hundred-thousands , 5 ones, 4 thousands, 2 hundreds, 6 ten-thousands

   ____

   (d) 3 hundred-thousands, 8 tens, 7 thousands, 2 ten-thousands, 5 hundreds

   ____

4. (a) Circle the number in which the 7 is worth the most :

946 175 271 003 497 991 8 965 794

(b) Circle the number in which the 6 is worth the least:

46 175 271 690 600 000 9 995 764

A2 Additive structure items

1. What number is:

(a) 10 more than 3097? ______________________________

(b) 100 less than 24 507?______________________________

(c) 1000 more than 372 514? ______________________________

(d) 10 less than 20 517? ______________________________

(e) 100 less than 5 000 000? ______________________________

(f) 1000 more than 408 753? ______________________________

2. Complete the counting sequences

(a) 7286, 7287, 7288, ____________, ____________, _____________

(b) 53 596, 53 696, 53 796, ____________, ____________, _____________

(c) 16 433, 16 432, 16 431, ____________, ____________, _____________

(d) 29 830, 29 820, 29 810, ____________, ____________, _____________

(e) 103 364, 103 374, 103 384, ____________, ____________, _____________

(f) 1504, 1404, 1304, ____________, ____________, _____________

A3 Multiplicative structure items

1. Write in the missing place value names:

(a) 8 ones × 100 = 8___________________

(b) 4 ones × 1000 = 4__________________

(c) 62 ones × 100 = 62________________

(d) 15 tens × 1000 = 15________________

(e) 9 hundred thousands ÷ 10 = 9______________

(f) 6 thousands ÷ 10 = 6 __________________

(g) 7 hundreds ÷ 100 = 7_____________

(h) 23 tens ÷ 10 = 23 ________________

2. Write the missing numbers:

(a) 928 = 9 hundreds _______tens _______ones

(b) 684 = 6 hundreds 7 tens _______ones

(c) _______ = 4 hundreds 19 tens 6 ones

(d) 547 = _______hundreds 14 tens 7 ones

(e) 57 320 = ________ thousands ________ hundreds _______ ones

(f) _______ = 4 millions 36 thousands 186 ones

3. **Challenge** – Write the missing numbers following the rules:

(a) 874= _______hundreds _______tens _______ones

[The rule is that you cannot write a 7]

(b) 874= _______hundreds _______tens _______ones

[The rule is that you cannot write an 8]

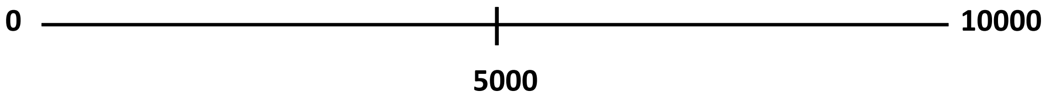
(c) 874= _______hundreds _______tens _______ones

[The rule is that you cannot write a 7 or an 8]

A4 Number-line items

1. Place the following numbers on the number line:

   (a) 4300 (b) 870 (c) 7500



2. What number could be at (a) __________; (b) __________; (c) __________?

3. For each set of numbers, circle the larger value:

   (a) 6799, 7006 (b) 4730, 4729 (c) 13 705, 9780 (d) 5920, 5902

4. Put these numbers in order from the largest to the smallest in value:

   16 057, 16 507, 15 607, 17 056, 15 706

   _________, _________, _________, _________, _________

5. Round to the nearest 1000:

   (a) 1686 ________

   (b) 5725 ________

   (c) 3303 ________

6. Round to the nearest 10 000:

   (a) 83 456 _______

   (b) 47 238 _______

   (c) 155 850 _______

7. Circle the numbers that could be rounded to 3000:

   (a) 2713 (b) 2267 (c) 3825 (d) 3280 (e) 3250

A5 Equivalence items

1. (a) Write a zero in 1368 without changing its value ________________________

   (b) Write a zero in 1368 so that its value does change ______________________

   (c) Write a zero in 1368 so that its value changes in a different way ___________

2. (a) Remove a zero from 06040 so that the value does not change _________________

   (b) Remove a zero from 06040 so that its value does change _________________

Challenge items

3. What numbers can you make using 1, 0, 2, 7, 8 and 4? See how many you can find. Write them here:

4. Write the largest number you can find using all these numerals: _______________

5. Write the smallest number you can find using all these numerals: ______________

Appendix A: Additional Material for Building the Big Ideas for Large Numbers

Hopefully you will not need to use this additional material, but if your students have a problem with a big idea, concept or process with numbers up to a billion, this additional material repeats the activities of Units 1 to 5 for large numbers and may have an activity that helps you teach to remediate this difficulty.

Big idea

Large numbers such as millions are an extension of the hundreds-tens-ones understanding because of the pattern of threes. The same big ideas, concepts and processes can be applied to them as for 0 to 999.

A1 Notion of unit, PV and reading and writing for large numbers

A1.1 Notion of unit

Big idea

Changing the perception of a unit as thousands and millions up to trillions – the 2 in 423 is two tens where tens is the unit or 20 ones, must be applied to the understanding that the 2 in 423 000 is 2 ten thousands where ten thousand is the unit, or 20 thousands or 20 000 ones.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Think of some way to get the body to experience a million. For example, act like the Roman army. They counted each time their left foot was placed down – they called this a passus. Walk 100 passuses around an oval – how far did you get? Think of a thousand passuses, a million passuses. Is it possible to count a long march in millions of passuses – what about 1000 passuses? (*Note:* 1000 passuses was called by the Romans a mile passus or a mile – the old British unit of length.)

Think of other ways to consider a million – how many plane trips across Australia for a million kilometres? – how long will this take?

Hand

Find ways to construct a million. For example, construct a metre cube with a kit or with metre rulers. Place a MAB cube inside it – 1000 cubes will fit in the cubic metre. Place a MAB unit inside the cubic metre – 1 000 000 units will fit in the cubic metre.

Mind

Ask students to shut eyes and imagine a million in some way. For example, imagine $10 notes. The $10 notes are packed into packets of 1000. Think of how many these would be. Pack the packets into a box that is 10 packets wides (10 shorter sides of the $10 bills) and 10 packets long (10 longer sides of the $10 bills). How high will this be?

Find other ways to imagine a million, for example, consider the MAB unit inside the MAB cube and/or the MAB cube inside the cubic metre.

Mathematics

Practice

Find ways to consider a million. For example, use the internet to spend $1 million dollars without buying anything over $100 000. Was it more difficult than you thought?

Connections

Take a large number with the macrostructure showing – for example, 47 million. Discuss what we would do if putting this number into boxes labelled millions, thousands and ones. Would we write "47" in the millions box or would we write "47 000 000"? Discuss why we write "47" – try to elicit it is because millions are our unit in this case.

Reflection

Applications

Apply the notion of considering very large groupings as single units back to the real world. Where in the world do we have groups and ones interchanging depending on how we think of them? Consider particularly how to involve kilo, mega, giga, and tera as they refer to bytes. For example, what does and 135 gigabyte drive mean? Or in everyday language, what does a 135 gig drive mean? Or, how many megs is that memory stick?

Extension

Flexibility. Brainstorm examples of large numbers being used as units. For example, what is a millionaire, a billionaire? What does a $21 trillion stimulus package mean? How many dollars?

Reversing. Go both ways – for example, give $3 trillion and ask for how many dollars, and give $3 000 000 and ask for how we can say this with less length in the numeral.

Generalising. Relate the making of large groups of numbers into units such as billions or gigs to other situations and try to get students to see that making new "units" is a generalisation that we always do when numbers get large. For example: (a) time – look at "centuries", "millennia", and names given to periods of earth's history such as "Jurassic"; (b) speed – look at "mach", and "speed of light" and names from science fiction such as that used in Star Trek for speed; and (c) distance – look at "light years" and names from science fiction. There must be other names – for example "par secs" – what is this? Use the internet.

Changing parameters. What if we looked at very small instead of very large? There are special names for very small things (e.g. "pico", "nano") – look at these too – extend the generalisation to making units out of very small as well as very large.

A1.2 Place-value concept

Big idea

Large numbers are an extension of place value through a pattern of threes to large numbers. Place value is a visual image.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Get six and then nine students to each take a digit from 1 to 9 and arrange themselves right to left to form a large number. Ask them to split into two (6 students) and then three groups (9 students) of three. Start from right discussing the place value of each student – get students to note the sequence from right:

hundred million, ten million, one million – hundred thousand, ten thousand, one thousand – hundred, ten, one

Discuss what each group of three could be called – thousands-ones for 6 students, and millions-thousands-ones for 9 students. Place a sign in front of each group with the macrostructure. Remind students that within the group of three there is the microstructure of hundreds-tens-ones (could give each student a hundred, ten or one along with their number). Do activities where teacher points to a position and students say the PV (e.g. 3 ten thousands) and the reverse, where teacher gives a PV name (e.g. hundred million) and students point to it.

Construct following cards (can also have millions-thousands-ones under the H, T and O):

Millions	Millions	Millions	Thousands	Thousands	Thousands	Ones	Ones	Ones
H	T	O	H	T	O	H	T	O

Organise students to stand in order with PV cards. Get another student (or students) to move in front of PV cards with a digit card and ask remaining students to state PV position. Also reverse – state the PV position and get the remaining students to direct student with digit card to move to this position

Hand

Look at 6 and 9-digit numbers and do the following (which is described for 9-digit numbers).

1. Consider a 9-digit number such as 356 872 913, break digits into threes, i.e. 356 / 872 / 913, introduce and/or recognise that right-hand three digits are ones, middle three digits are thousands and left-hand three digits are millions. Cover the 872 913 with hand and say the remaining three digits as a 3-digit number (i.e. *three hundred and fifty-six*) then add the *millions* – repeat covering 356 and the 913 and saying *eight hundred and seventy-two thousands* – repeat covering 356 872 and saying *nine hundred and thirteen ones* (remind that convention in ordinary speech is not to say the ones). Check students can see the PVs – teacher gives digit → students give PV, and (reverse) teacher gives PV → students give digit. Repeat this for other numbers from 5 to 9 digits.

2. Construct and place the following cards on the wall, get a student to place a digit, say 4, in front of PV cards and to place that card in front of different PV cards.

Millions	Millions	Millions	Thousands	Thousands	Thousands	Ones	Ones	Ones
H	T	O	H	T	O	H	T	O

   ← 4 →

   Get the students to say the place value. For example, where the 4 is above is the ten-thousands PV position, it is "forty thousand". Ensure the activity is undertaken all ways – teacher places digit in front of PV card and asks students to say the PV, and teacher gives PV and students place digit.

3. Provide a copy of the PVCs and digit cards on the next page. These PVCs have been designed to represent the macro and micro PV structure of large numbers. Organise the students to place their PVCs side by side (ones on right, thousands in middle and millions on left). Use the digit cards to represent numbers from 4 to 9 digits on the PVCs (choosing cards from a deck without K, Q, J or Joker, and with the Ace being a 1 and

the 10 being a zero at random is a way to determine what the digits will be in each place). When the digits are placed, ask students to find ones, thousands or millions groups of three on PVCs and reverse by pointing to groups of three and asking students to give name. Repeat this for digits – teacher says digit → students give PV, teacher gives PV → students say digit. It is important to place digits on PVCs and to use the kinaesthetic sense to embed an understanding of the patterns of three.

Pattern-of-threes PVCs

HUNDREDS	TENS	ONES
MILLIONS		

HUNDREDS	TENS	ONES
THOUSANDS		

HUNDREDS	TENS	ONES
ONES		

Digit Cards

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

4. In the situations in Steps 2 and 3, provide digits and place values out of order and construct the number. Start with macrostructure, for example, what number is 876 thousands, 5 millions, 234 ones? [5 876 234] Then do both structures together, for example, what number would be constructed from 4 ten thousands, 3 tens, 5 one millions, 7 hundred thousands, 8 hundreds, 4 ones and 0 one thousands? [5 740 834] Repeat this many times.

   Reverse this – get students to give all the different ways that 5 876 234 could be written.

Mind

Shut eyes and imagine PVCs to help embed patterns from kinaesthetic experience to the mind. Assist this by, when placing material on PVC, always putting left hand on digits as the PV is said. Finally, write large numbers, look at them, break them into threes in the mind, assign macro-PV structure and say the PVs of some of the digits. Reversing, say PV positions and assemble this to a number in groups of three. Imagine numbers as being in a structure:

H T O	H T O	H T O
millions	thousands	ones

Mathematics

Practice

Use charts and worksheets with charts to practise chart → PVs and PVs → chart. Then use worksheets to practice numerals → PVs and PVs → numerals. Also practise giving PVs out of order and determining numerals' positions. Reinforce with the game "Wipe-out" – see below.

Connections

Connect to metrics. The macro/microstructure in the PV charts above works excellently for km, m and mm; kL, L and mL; tonne, kg and g, and so on. For example, for length, the ones can be replaced with millimetres, the thousands with metres, and the millions with kilometres. More complex relationships exist for area and solid volume. These connections are an excellent way to build understanding of metric conversions.

Connect to indices – ones are 10^0, tens are 10^1, hundreds are 10^2, thousands are 10^3 and millions are 10^6. Thus the PVs of large numbers (millions) are H-T-O 10^6 H-T-O 10^3 H-T-O 10^0.

Connect to other names – bytes, kilobytes, and megabytes.

Game: Wipe-out (place value)

Materials: Calculator, worksheet (if wanted). **Number of players**: 2.

Directions: One student calls out a number, e.g. 673, 56 782, 24 875. Other students put in calculator then first student calls out a digit. Other students have to change number on calculator with a single subtraction, e.g. for "wipe the 7", answers would be 603, 56 082, 24 805. Do examples with two digit positions to wipe (e.g. 347 642 – wipe out both 4s). Can be done with a worksheet as below.

Number	Digit	Subtraction	Result
63 284	8	−80	63 204
7 452 892	5	−50 000	7 402 892

Reflection

Application

Use internet, papers and magazines to find examples of large numbers and get students to determine PVs of their digits.

Extension

Flexibility. Discuss and list different ways to get, for example, 7 536 500 (6 thousands, 5 hundreds, 7 millions, 3 ten-thousands, and 5 hundred-thousands; 75 365 hundreds; 7 536-and-a-half thousands; approximately 7½ millions; and so on). Extend applications and look for other examples of large numbers.

Reversing. This has been done throughout Abstraction and Mathematics but check at this point that students can go from numbers to PV positions, and PV positions (even out of order) to numbers.

Generalising. Look at the pattern of threes structure (the macro and microstructure together as below.

H T O	H T O	H T O
millions	thousands	ones

Discuss how this could be extended – what is to the left of millions? Discuss other names, e.g. kilo, mega, giga, tera, and so on. Look at the numbers in terms of indices – how could this be extended?

Changing parameters. What happens if the base is not 10? For example, days, hours, minutes and seconds – how do these relate to "place value"? For example:

DAYS		HOURS		MINS		SECS
17	← x24 / ÷24 →	19	← x60 / ÷60 →	35	← x60 / ÷60 →	43

A1.3 Reading and writing process

Big idea

Building number as a visual image.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Same as the PV activities for smaller numbers, get six and then nine students to each take a digit from 1 to 9 and arrange themselves right to left to form a large number, splitting into groups of three. Again discuss what each group of three could be called – thousands-ones for 6 students, and millions-thousands-ones for 9 students. Place a sign in front of each group with the macrostructure. Remind students that within the group of three there is the microstructure of hundreds-tens-ones (could give each student a H, T or O along with their number). Get rest of class to read out number, pointing to the position as they say the digit – each group of three students steps forward as their three numbers are read (can get them to hold up number and H, T or O as their digit is read and to highlight the millions-thousands-ones). For example (9 students):

782	164	395
seven hundred and eighty-two million	one hundred and sixty-four thousand	three hundred and ninety-five

Again as for PV activities, construct the following cards (can also have millions-thousands-ones under the H, T and O), get students to hold cards in order and other students to hold digit cards. Go both directions, teacher gives number → students form this number, and teacher directs students to form a number → other students read the number.

Millions	Millions	Millions	Thousands	Thousands	Thousands	Ones	Ones	Ones
H	T	O	H	T	O	H	T	O

Hand

Do the following as in the A1.2 PV concept activities (described for numbers up to 9 digits).

1. Consider a 9-digit number such as 356 872 913, break digits into threes, i.e. 356 / 872 / 913, introduce and/or recognise that right-hand three digit are ones, middle three digits are thousands and left-hand three digits are millions. After looking at each part separately, combine the three parts and say the total number **pointing at group of three as you say the group** – *three hundred and fifty-six millions, eight hundred and seventy-two thousands and nine hundred and thirteen*. Repeat this for other numbers from 5 to 9 digits:

2. Construct and place the following cards on the wall, get students to place digits in front of PV cards as below. Get the students to say the number shown, and then to write it on paper and/or enter it on a calculator. Allow students to experience place value in the following ways (reversing): (a) teacher places digits in front of PV cards and asks students to say and write/enter the number; (b) teacher says number and asks students to write/enter number, say PV and place the digits; and (c) teacher writes number and asks students to say number and place the digits:

Millions H	Millions T	Millions O	Thousands H	Thousands T	Thousands O	Ones H	Ones T	Ones O
	3	4	5	0	7	3	7	2

3. Provide a copy of the **Pattern-of-threes PVCs** material (place value charts and digit cards) from section A1.2. Organise the students to place their PVCs side by side (ones on right, thousands in middle and millions on left). Use the digit cards to represent numbers from 4 to 9 digits. When the digits are placed, ensure the students do this: (a) say the number **moving hands to each PV position** as the digits are said, and **pointing to millions, thousand and ones** as they are said (or not said as the situation is for ones); and (b) write the number on paper and enter the number in their calculator.

   It is important to place digits on PVCs and to use the kinaesthetic sense to embed an understanding of the patterns of three. It is also important to repeat this experience of PV in two ways (**reversing**): (a) teacher gives digits to be placed on the PVCs and asks students to read the number, write the numerals and enter the number on calculator; and (b) teacher says number or writes numerals and asks students to place digits on the chart and write/enter the number or say the number.

4. Reinforce reading and writing large whole numbers with number expanders. There are two special number expanders for large numbers – the millions and the mega number expander (for megabytes) – see below. These expanders have three digit positions and folds on ones, thousands and millions – showing the pattern of threes. The number expander pleat folds at the coloured sections leaving 9 boxes for numbers. It then expands out to show the pattern of threes (e.g. 204 761 894 is 204 millions 761 thousands 894 ones). Obviously, these number expanders **are also useful for place value**.

Millions number expander

			millions				**thousands**				**ones**

Mega number expander

			mega				**kilo**				**ones**

Mind

Have students shut eyes and imagine PVCs to help embed patterns from kinaesthetic experience to the mind. Assist this by, when placing material on PVC, always putting left hand on digits as they are said and moving left to right, saying the number as you go (e.g. three hundred and fifty-six millions, eight hundred and seventy-two thousands and nine hundred and thirteen as you move hand L→ R across the digits and PV positions). Finally, write large numbers, look at them, break them into threes in the mind, assign macro-PV structure and say the number. Reversing, say the number and then write it down as groups of three numbers.

Give a real-world story: *Jim spent $1 356 500 on his new mansion overlooking the ocean*. Show the number on a PV chart or millions number expander. Have students read the number. Say: *Show me the ones part; show me the thousands part; show me the millions part*. Say: *How many are there in the ones section, how many are there in the thousands and million section?* **Swap roles** – the teacher says the number and the students show it on the PVC or number expander. Don't forget to ask the students to show the millions part of the number, the thousands part of the number and so on.

Mathematics

Practice

Use charts and worksheets with charts to practise representing, reading and writing numbers (chart → language → numerals AND numerals → language → chart). Then use worksheets and calculators to practise reading and writing (numerals ←→ language). For example, teacher calls digits → students enter on calculator and say number, and teacher says number → students enter on calculator and say digits pressed.

Spend time giving numbers for students to read and write. Use dice and cards to randomly select digits. Spend time giving varying numbers as digits for students to say as numbers; reverse and say number and students write as numerals or enter on calculator (e.g. the "Let your calculator do your talking" type activity). Give **special attention to teens and zeros**. Reinforce with the game "Target" – see below.

Game: Target (reading and writing numbers, plus order and estimation)

Materials: Calculator, worksheet if necessary. **Number of players**: 2.

Directions: Give students a starting number and a target number, e.g. 37 and 9176. Students enter $\boxed{37}$ $\boxed{\times}$ in calculator. Then press $\boxed{\text{guess}}$ $\boxed{=}$, $\boxed{\text{guess}}$ $\boxed{=}$, until they get the target. (No pressing of "clear all".) Students take turns being the starting number provider. After 5 goes each, the winner is the student with the lowest number of guesses. Can be done with worksheet as below.

	Number	Target	Too high	Too low	Correct guess	Number of guesses
(a)						
(b)						

Connections

Connect to metrics (e.g. km, m and mm; kL, L and mL; tonne, kg and g, and so on) similarly to PV activities, but this time say the numbers when, for example, the ones are replaced with millimetre, the thousands with metres, and the millions with kilometres (e.g. three hundred and fifty-six million, eight hundred and seventy-two thousand and nine hundred and thirteen millimetres is three hundred and fifty-six kilometres, eight hundred and seventy-two metres and nine hundred and thirteen millimetres). More complex relationships exist for area and solid volume. These connections are an excellent way to build understanding of metric conversions.

Reflection

Applications

Use internet, papers and magazines to find examples of large numbers and get students to read and write these digits. Also set problems and investigations with large numbers that build understanding of whole numbers as quantity. For example:

- "Spend $100 000 000" – give catalogues of expensive items (such as cars, real estate, etc.) and give students a budget of up to $100 million. May have to put restrictions (e.g. cannot by two of the same thing).
- Shop for data storage items that detail storage space in bytes. (There are plenty of excellent online sites to visit.) Have students write numbers onto expanders or PVC. Extend to ordering greatest to least amounts.
- Look for examples of where big numbers would be used in the students' environment (e.g. bricks to build the school, leaves on a tree).

Note: Reverse these ideas – let the students make up their own prices for things and use to create their own catalogues.

Extension

Flexibility. Discuss and list different situations in which large numbers are used. For example, company profits, government budgets, Australia's deficit, expensive house prices, and so on.

Reversing. This has been done throughout the cycle but check at this point that students can go from PVs ←→ number names ←→ numerals AND numerals ←→ number names ←→ PVs.

Generalising. Look at large numbers and the pattern-of-threes structure that underlies them.

H T O	H T O	H T O
millions	thousands	ones

Discuss how we get bigger numbers – elicit that the pattern of threes continues but with new names (e.g. billions, trillions, etc.). Discuss other names (e.g. kilo, mega, giga, tera, and so on). Look at the numbers in terms of indices – how could this be extended?

Changing parameters. What happens if the base is not 10? For example, days, hours, minutes and seconds – how do these relate to "place value"? For example, how many seconds in: 17 days, 19 hours, 35 minutes and 43 seconds?

A2 Counting, seriation and odometer for large numbers

A2.1 Counting

Big idea

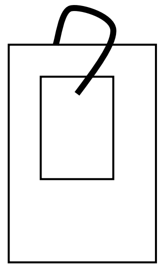
Each large place-value position counts like the ones.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body



Make single place flip cards as on right, ten cards 0 to 9 that can be flipped over, a larger card behind the ten cards with place-value position written on the bottom part of the larger card (e.g. Ten Thousands). Five to nine students take a flip card each and set up in PV order L → R. Teacher gives each position a starting digit. Students flip to this. Then a PV is chosen to count. The student in this position flips as students count. For forwards this means 5, 6, 7, 8, 9, 0 (with digit on LHS going up by 1), 1, 2, 3, 4, and so on. For backwards this means 2, 1, 0, 9 (with digit on LHS going back by 1)

Hand

Imitate the Body work above but with digit cards on a small PVC.

Hand out calculators, enter a large whole number (up to 8 digits), pick a PV (say ten thousands), add a 1 to that PV (i.e. +10 000), and keep entering =, =, =, and so on. Read the number at each equal or just the digit in the PV position. Repeat for counting backwards situations (i.e. −10 000). Keeping class together, clapping for = press, and together stating the number can be a good starting approach.

Write down each number as they press =. See examples of counting forward for ten-thousands position and counting backwards for hundred-thousands position:

376 581	4 276 401
386 581	4 176 401
396 581	4 076 401
406 581	3 976 401
416 581	3 876 401
426 581	3 776 401

Discuss that the counting in any PV position is like counting in the ones position. Show this by covering all of the number except the chosen PV position and the PV position on its left – this would make the left-hand side count forwards example above: 37, 38, 39, 40, 41, 42 – same as counting by ones.

Mind

Shut eyes and imagine a number, pick a PV position, and count in that position forwards and backwards.

Mathematics

Practice

Repeatedly practice with and without calculator to count in any position forward and backward. Also use worksheets to continue patterns where a PV is counted forward and backward as below:

78 690 237, 78 790 237, 78 890 237, ________, ________, ________

Connections

Relate to counting in any situation, time, money, measures (km, m and mm), fractions, and so on.

Reflection

Applications

Look at where large numbers have counting in high PV positions – use internet and talk to people to find examples (e.g. population). Think of problems with counting in high PV positions.

Extension

Flexibility. Try to brainstorm all situations where counting in high PV positions is used – often in estimation situations. For example, estimating budgets and populations.

Reversing. Make sure students go from stating PV position → counting forward and backward in that position, and counting forward and backward in a position → identifying PV position.

Generalising. Continue work from Abstraction to ensure students see the counting generalisation for any place value.

Changing parameters. Extend the generalising of whole numbers above to counting in measures, fractions, and so on. Build a meaning for counting that applies to anything.

A2.2 Seriation process

Big idea

The ability to determine one more or less in each place value (e.g. 10 000 more than 53 000, 100 000 less than 409 000).

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Use the single place flip cards as in A2.1 but simply flip forward and back one flip to find the one PV forward and backward.

Hand

Use digit cards on a small PVC and calculators as in A2.1 but flip only once and press only one = so as to get one PV forward and backward. Try to get the students to think seriation for 10 000 PV for 345 672 is the same as seriation for the digit in the PV position. Here the digit is 4 – so 1 before and after for 4 is 3 and 5; this means 10 000 before and after for 345 672 is 335 672 and 355 672. It is also possible to develop "99 boards" for, say, 300 000 to 399 000 and to use the 99 board technique from Unit 2 (section 2.2) of this module.

Spend extra time looking at one PV more and less for examples that are next to points where numbers change from 0 to 9 and vice versa. In other words, for example:

one 10 000 more and less	for 3 597 320, for 2 608 234, for 5 991 405, for 6 008 431, for 69 992 430 and for 70 002 765

Mind

Again shut eyes and imagine as in A2.1 but again only one PV number forward and backward.

Mathematics

Practice

Give numbers and PV positions and ask for the one before and after in that PV position. Look especially at the 0, 00, 9 and 99 type numbers within the relevant PV position.

Special activities to reinforce seriation are as follows (need to specify which PV to seriate first; in these examples it is the 10 000):

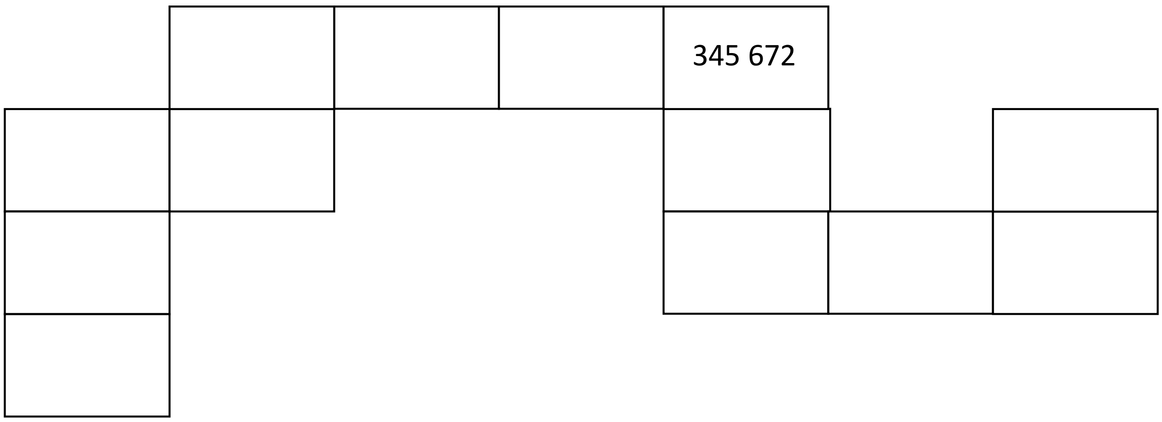
- Give 3×3 squares with number in middle and one other number and ask for the other numbers:

	448 275	
	458 275	

- Give two numbers outside the middle one and ask for the other numbers:

	528 104	
537 104		

- Complete the "jigsaw" piece



Connections

Relate to one before and one after in many other situations (e.g. measures).

Reflection

Validation/Applications

Look for application of seriation for high PVs in large numbers. Often used in estimation.

Extension

Flexibility. Try to find a wide variety of situations where one more/less large PV are used (e.g. estimation examples).

Reversing. Go both ways – PV → one more/less, and one more/less → PV.

Generalising. Develop a generalisation for seriation – this has four parts:

- one PV more and digit in PV between 0 and 8 → same number but PV position increases by one
- one PV more and digit in PV at 9 → same number but PV position reduces to 0 and PV position on left increases by one
- one PV less and digit in PV between 1 and 9 → same number but PV position decreases by one
- one PV less and digit in PV at 0 → same number but PV position increases to 9 and PV position on left decreases by one.

Changing parameters. As for A2.1, looking at measures, fractions, etc.

A2.3 Odometer principle

Big idea

The ability to determine one more or less in each place value (e.g. 10 000 more than 53 000, 100 000 less than 409 000).

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Use same material as in A2.1, but as you state numbers give only the position digit and focus on the change at 9 (forward) and 0 (backward). For example: (a) counting forward by 10 000s: 384 761, 394 761, 404 761, 414 761, and so on, becomes 8, 9, 0, 1, and so on; and (b) counting backward by 100 000s: 4 281 034, 4 181 034, 4 081 034, 3 981 034, and so on, becomes 2, 1, 0, 9, and so on.

Hand

Repeat the activities from A2.1, but again focus on the change to the PV digits at 9 and 0. Add in an extra activity of making an odometer – take foam cups, turn horizontal and put ten digits 0 to 9 and lines regularly around the top of each cup (so the cup is divided into tenths) – put each cup inside the other and then you can turn each cup like an odometer.

Mind

Shut eyes and imagine but focus on the counting passing 9 forward or 0 backward.

Mathematics

Practice

Follow ideas from A2.1 and A2.2 – but always get students to see that the digits increase 7, 8, 9 then drop to 0 while PV on left increases by one; and digits decrease 2, 1, 0 then increases to 9 while the PV on left decreases by one.

Connections

Once again show how this odometer idea works for fractions, measures, and so on, as well as whole numbers.

Reflection

Validation/Applications

Look at problems that apply to odometer situations in everyday life – for example, the odometer in a car.

Extensions

Flexibility. Try to find as many odometer situations as you can.

Reversing. Try always to have lessons: PV and starting number → counting pattern; and counting pattern → PV and starting number.

Generalising. Develop a generalisation for odometer – has two parts: (a) PV, starting number and counting forward → 7, 8, 9, 0, 1, and so on, PV to left increasing by one; and (b) PV, starting number and counting backward → 2, 1, 0, 9, 8, and so on, PV to left decreasing by one.

Changing parameters. Look at action of odometer in measures such as time (e.g. years, months, days, hours, minutes, seconds), fractions, and other situations.

A3 Multiplicativity and renaming for large numbers

A3.1 Multiplicative relationship concept

Big idea

The principle of multiplicativity applies no matter how large the number is.

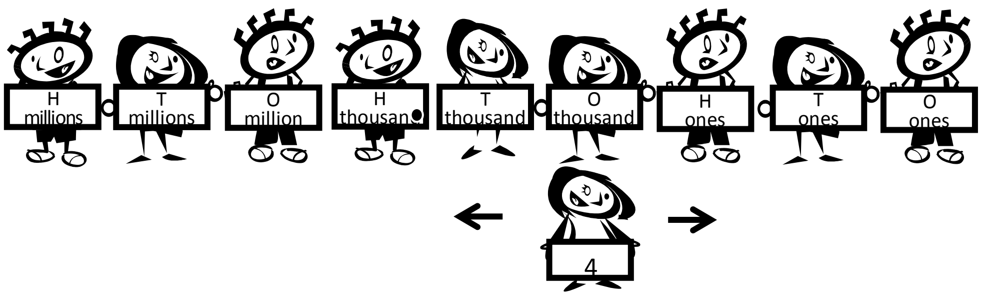
Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Give students cards and organise to stand in line. Get other students with number cards (one, two and more digits) to stand in front of PV cards and to move left and right. Other students use their calculators to determine what multiplication or division (×10, ÷10, ×100, ÷100, ×1000, ÷1000) is the same as one, two, and three places to left or right.



Hand

Students follow what happens at the front with their small PVCs and digit cards, and relate changes to multiplication and division on the calculator. Then, activity can just be with the small PVCs, digit cards and calculators. Then it can move to slide rule – see end of this section.

Mind

Encourage the students to find and write down patterns in movements and their relation to ×10, ÷10, ×100, ÷100, ×1000, and ÷1000. Ask students for a pattern (i.e. move left one place is ×10 and move right one place is ÷10). Spend time on the three-PV position movements (×1000 and ÷1000) – that is, movements across the macrostructure (from ones to thousands to millions).

Mathematics

Worksheets – these should operate in both directions, giving the multiplication or division and asking for the movement, and giving the movement and asking for the × and ÷ 10 or 100 or 1000.

Reflection

Students communicate their understandings of patterns discovered in the steps above.

The understanding that moving left is ×10 and moving right is ÷10 has to be generalised to where the structure of the number system is understood in terms of any adjacent place-value positions relating ×10 and ÷10, with this relationship being continuous across all place values and bi-directional in application. There also has to be a similar generalisation across the macrostructure, that is, that one move left across macrostructure is ×1000 and one move right is ÷1000.

Further, students should also see that other number ideas are also related in a similar way but possibly with different × and ÷. For example, weeks and days relate ×7 and ÷7, hours and minutes relate ×60 and ÷60. Metrics relate via 10 but, of course, the mm–m, the g–Kg and mL–L relate ×1000 and ÷1000. The relationship between metric units is highly connected to movements across the macrostructure (and this is one of the reasons it is very important to teach this).

A3.2 Renaming process

Big idea

Students must be comfortable working with the larger place values.

Abstraction

Use M-TH-O number expanders to show, e.g. that 273 365 478 is 273 millions (M), 365 thousands (TH) and 478 ones (O) but also 273 M and 365 478 O as well as 273 365 478 O – that is, renaming across the macrostructure. Number expanders are below. Use the full number expander to relate changes from one PV position to the next, e.g. 3 hundred thousands (Hun TH) and 14 ten thousands (Ten TH) is the same as 4 Hun TH and 4 Ten TH. Use the M-TH-O number expander to show renaming within the macrostructure – make sure to go both ways, e.g. 273 thousand is 1 hundred thousand, 16 ten thousands and 13 one thousands; 2 hundred thousands, 25 ten thousands and 34 one thousands is 484 thousand.

Number expanders

Make large copies of the following number expanders.

Full number expander:

	Hun M		Ten M		One M		Hun TH		Ten TH		One TH		Hun O		Ten O		One O

M-TH-O number expander:

			M				TH				O

H-T-O number expander for Thousands (similar ones can be made for Ones and Millions):

	H TH		T TH		O TH

A4 Continuous–discrete, rank, ordering and rounding for large numbers

A4.1 Continuous–discrete

Big idea

The principle of continuous and discrete applies no matter how large the number is.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Look at large whole numbers with respect to discrete and continuous entities. Undertake some investigations involving the students and large numbers. For an example that can represent both, look at the investigation to find out if students were to hold hands across Australia, how many students would you need? In terms of discrete, the investigation will give a number of students. In terms of continuous, the investigation will provide a measure of distance across Australia in terms of student fathoms. Discuss the differences in both numbers (e.g. 0 means nothing in discrete but the start of the measuring in continuous; number can be naturally applied to students as objects because they are discrete but distance needs a unit to be repeated across the length to enable number to be applied).

Hand

Investigate large numbers in discrete situations (e.g. population, money, animals in the wild, TV set production, and so on). Investigate large numbers in continuous situations (e.g. length/distance, area, volume, mass, and so on). Discuss differences.

Mind

Shut eyes and imagine number being applied to discrete things (i.e. forming groups, and groups of groups, and so on) and to continuous things (i.e. using units and conversions between units).

Discuss what things can be counted and what cannot. Point out that, normally, items have to be discrete (individual and separated) to be counted. Point out that the world is full of discrete things (chairs, people, animals, days, grains of sand, etc.) but that some things have had to be "changed" to be countable. For example, the beach cannot be counted but grains of sand can; the length of a building cannot be counted but the number of bricks long can. Discuss length – it is not countable unless a unit of measure is used. Length is continuous (as is area, volume, mass, time, and so on) but units make it discrete. Ask why we want to turn the continuous into the discrete? (So we can apply number to it.)

Mathematics

Practice

Do activities where students look at situations (can be represented by pictures in a worksheet) and classify them as discrete or continuous applications of number.

Connections

Gather all discrete and continuous examples into two separate groups. Connect the various examples in each classification.

Reflection

Applications

Set problems with regard to number in discrete and continuous situations.

Extension

Flexibility. Try to brainstorm all the major discrete and continuous situations. In particular, look for number situations that can be both continuous and discrete depending on how they are perceived.

Reversing. Go both ways: starting with a situation → determining if discrete or continuous; and starting with one of discrete or continuous → constructing a situation.

Generalising. Try to identify characteristics of each of the classifications (discrete and continuous) that can be generalised across all examples of each classification. Get students to generalise that discrete can be counted but continuous needs a way (e.g. a unit of measure) to change it into discrete for a number to be used on it.

Changing parameters. Look for situations that are both discrete and continuous. Generalise the characteristics of such dual situations. Is it true that all discrete situations can be perceived as continuous and vice versa?

A4.2 Rank and number-line concept

Big idea

The concept that the further along the number line a number is, the larger it is. This unit is about comprehending the size of these large numbers and working with them. We CANNOT use the rule that the more numerals in a number, the larger it is as this concept cannot be transferred to decimal numbers.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

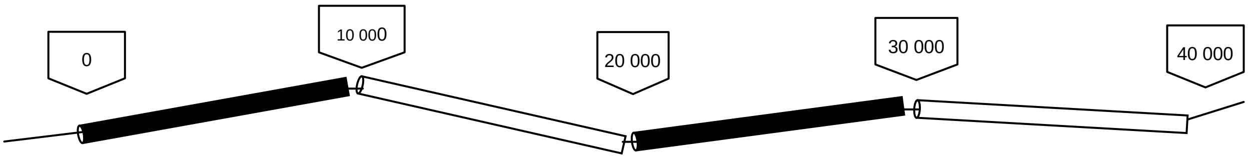
Put a start and end number on two students (say 0 on one student and 100 000 000 on the other) and put them at the front (0 on left looking towards the front). Get these two students to hold a rope between them (reasonably taut). Give other students numbers on paper between 0 and 100 000 000 and pegs and they have to peg on rope where they think this number would be. Get other students to help more accurately place the numbers. Discuss where numbers would be (e.g. in middle, near an end). Repeat for other starting and ending numbers. Go both ways (i.e. reverse) – give number → students place; give position → students guess number.

If students are having difficulties, either (a) divide line into 10 parts and stick paper at 10 000 000, 20 000 000, and so on, so students can see in which ten million interval to place number; or (b) round numbers to nearest thousand or million so students can use knowledge from hundreds-tens-ones (e.g. 54 678 203 can be rounded to 55 million and this is a little over halfway between 0 and 100 million, so 54 678 203 is a little over halfway between 0 and 100 000 000).

Hand

Repeat the above for number lines with various ending numbers, e.g. 0 to 100 000. Give students numbers to place on the line. Point to a position on the line and get students to estimate the number.

If students are having difficulties determining points, divide the line into 10 sections (e.g. mark 10 000, 20 000, and so on) so students have references from which to place and estimate numbers. Teachers can even construct a 10-straw line with each straw representing 10 000 units (see below). Stick numbers on ends of straws as shown.



Mind

Have students imagine the line in their mind and then use the imagined line to place numbers.

Mathematics

Practice

Have students practise placing numbers on number lines and determining what number would be in a position on a number line using worksheets.

Connections

Connect number to measures, particularly of distance. Have students use mm rulers or tape measures to measure things. Discuss how to measure – align 0 with start, read answer off tape. Notice numbers are at the end of the spaces not in the middle of the spaces. Reverse the activity – ask students to find a measure on lines; ask students to measure an object.

Reflection

Applications

Set problems of numbers on number lines – use measurement situations.

Extension

Flexibility. Brainstorm all the number-line situations for large numbers – how about plans for a building in millimetres?

Reversing. Ensure you always go both ways: number → position on line; and position on line → number. Ensure that students **construct number lines as well as interpret them.**

Generalising. Relate placing large whole numbers on number lines to placing rounded numbers on number lines (see *Body*). Enable students to see, for example, that placing 27 608 112 on a 0 to 100 000 000 number line is the same as putting 28 on a 0 to 100 number line, and placing 456 789 on a 0 to 100 000 000 number line is the same as putting 5 on a 0 to 1000 number line.

Changing parameters. If students are capable, expand number lines to include non-decimal measures – for example, nautical miles which are in whole numbers, minutes and tenths of minutes.

A4.3 Comparison and order process

Big idea

Numbers can be ordered by their distance along the number line. The concept that the further along the number line a number is, the larger it is. This unit is about comprehending the size of these large numbers and working with them.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Repeat the *Body* idea from A4.2 but, this time, use two numbers and state which is larger or three numbers and state order smallest to largest. If comparing/ordering numbers like 567 891 and 602 349, use a 0 to 1 000 000 rope. Place the two numbers; the one further from 0 is the larger (the other is the smaller). If necessary, stick 100 000, 200 000, 300 000, and so on, on the rope and then use these to place the two numbers. Spend time on numbers that cross macrostructures – for example 2 090 452 and 897 650. For these numbers use a 0 to 10 000 000 rope (divided into 1 million intervals if necessary). It can then be seen that 2 090 452 is between 2 and 3 million while 897 650 is between 0 and 1 million.

Hand

Repeat the *Hand* ideas from A4.2 but, again, place two or more numbers on a number line and use the one further from 0 to find the larger of two numbers and the one furthest from 0 to find the largest of three numbers. The straw construction is probably not necessary. Ensure that students do all of these: (a) place numbers and compare/order; (b) place one number and find a number that is larger/smaller; and (c) place two or more numbers and find a number that is largest, smallest or between.

Elicit understanding of what makes a number larger by recording sets of two numbers aligned underneath each other by PV, using placement on number line to find larger, ticking larger, then looking at examples to find a pattern. For example:

478 302 ✓	3 456 721	45 706	262 897 ✓	34 502 761 ✓
456 892	4 034 561 ✓	67 329 ✓	95 092	9 678 891

These examples give the pattern that the number with the biggest number in the highest PV position is the larger (taking into account that 95 092 is the same as 095 092 and therefore has a 0 in the 100 000 PV position). Also look at the 478 302 and 456 892 example to see that if the left-most digit is the same, you have to look at the second-left digits and so on.

Mind

Repeat the *Mind* ideas from A4.2, but imagine two or more numbers and then compare or put in order.

Mathematics

Practice

Practice comparing/ordering numbers by using number lines. Discuss the pattern from *Hand* above, and use this to compare/order numbers. Do worksheets where students circle larger or smaller; do worksheets where students put numbers in order from largest to smallest and vice versa; do worksheets where students find numbers between, bigger or smaller than given numbers. Play large-number versions of games "Target" (see A1.3), and "Chance number" and "Chance order" (see below).

Game: "Chance number" (comparison) – 6-digit version

Materials: Digit cards, 6-digit "Chance number" board (as on right), digit cards to fit into board, card deck (0–9 only).

Directions: Nine versions. (1) Teacher (or another student) deals six cards, students use numbers to make smaller/larger number with digit cards on game board (specify whether they have to try to make smallest or largest number for each game). Winner is student with smallest/largest number, as applicable. (2) Eight cards are dealt from which to choose six. Winner is same as (1). (3) Teacher (or another student) deals six cards **one at a time**, students use first number to place a digit card on board in PV of own choice, other numbers fill the other positions. Student who makes higher/lower number (as applicable) scores 1 point, 0 otherwise. Winner is largest score after five games. (4) Same as (3) but, at end, students can give up a number and take the value of a seventh dealt card. (5) Same as (3) but students can give up three numbers and another three cards dealt (one at a time) – can set rule that numbers cannot be risked from the same place value. (6) Same as (3) but score if closest to 500 000. (7) Same as (3) but score if student beats the teacher who is also playing. (8) Same as (3) but score only the LH digit on the board. (9) Any mix of the above.

Game: "Chance order" (comparison) – 6-digit version

Materials: Digit cards, 6-digit "Chance order" board – greater than or less than version (as on right), card deck (0–9 only), digit cards.

Directions: Six versions. (1) Teacher (or student) deals 12 cards, students use numbers to make the left-hand 6-digit number less than the right-hand number with digit cards on game board. Score 1 point if left-hand 6-digit number is correctly less than right-hand 6-digit number. Score 2 points if smaller 6-digit number is the largest possible (while still being less than the RH number). The winner has highest score after five games. (2) Teacher (or student) deals 12 cards **one at a time**, students use first number to place a digit card on any PV on LH or RH side of board, students continue making choices and placing digits on board before next card called. Score 1 if LH number less than RH number and 0 if not. The winner is highest score after five games. (3) Teacher (or another student) deals 12 cards **one at a time**, students use first number to place a digit card on board as in (2), students continue making choices and placing digits on board before next card called. Score 0 if LH not less than RH but score the value of the LH digit on LH number if correct. The winner is highest score after five games. (4) Same as (3), but score sum of two LH digits. (5) Same as (1) to (4) but students have to make the LH 6-digit number greater than the RH 6-digit number.

Connections

Reinforce the relation between order being furthest along a line with order being the number with the largest digit in the highest PV position. Relate this to ordering in measures.

Reflection

Applications

Apply order to real-world situations where numbers have to be larger.

Extension

Flexibility. Try to brainstorm all situations where order of large numbers matters in the world.

Reversing. Go both ways: (a) numbers → order; and (b) order (and some numbers) → all numbers.

However, teachers should direct students to also spend time developing/constructing number lines for different types of numbers (e.g. 7-digit numbers); as well as determining/interpreting where numbers are on lines that have been given to them.

This is the *construction-interpretation* big idea and is very important. This is because construction leads to better interpretation and a lot of interesting learnings. One of these is to discuss how best to do number lines. So when constructing number lines, discuss what works best (too many points and it's too crowded; not enough points and it is too hard to read). And do, for example, 70 000–90 000 lines as well as 0–100 000 lines.

Generalising. Spend time eliciting the pattern for order – three steps:

- align place values and add zeros if necessary so that there is a digit in each PV;
- look at left-most or highest PV, larger digit gives larger number; and
- if left-most digits are the same, move to second left, third left, and so on, until you have a larger digit (and thus a larger number).

Overall, build the idea that, for comparison and order, the larger PVs matter. Note that the games "Chance number" and "Chance order" reinforce this.

Changing parameters. Does the above generalisation still work when not in a base of 10 (e.g. hours, minutes, seconds)?

A4.4 Rounding and estimation process

Big idea

Number lines enable values to be determined to which numbers under consideration are nearest. This unit gives practice using the larger place values.

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

Use the same rope materials but with the rope divided into 10 parts with end points marked in (as below for 5-digit numbers). Numbers are placed on the rope and then decisions made as to which ten thousand they are closest to. For example, 37 352 is closer to 40 000 than 30 000, so we say it is 40 000 rounded to the nearest ten thousand. Can also determine to which 5 000 it is closest – here, 37 352 is closer to 35 000 than 40 000, so an estimate to nearest 5 000 is 35 000.

0 10 000 20 000 30 000 40 000 and so on 90 000 100 000

Hand

Prepare materials as we have earlier but with parts marked in initially. This way students can not only place the number but can also see to what other numbers the original number is closest. Try to discover a pattern from the place values for rounding and estimation. Take a number, say 368 921, compare it with PVs aligned to nearest 100 000s and 10 000s as below and star the correct rounding:

400 000 *	370 000 *
368 921	368 921
300 000	360 000

Two things have to be discovered by looking at a lot of examples:

- to find PV above and below, go to that position in the number and increase digit by one for above, and leave digit as is for below (e.g. for 6 540 872: 7 000 000 > 6 540 872 > 6 000 000); and
- to find closest rounded number with respect to given PV, look at digit to right of PV digit and go to lower if below 5 or go to higher if 5 and above (e.g. for 6 540 872, the 540 872 means that 7 million is the nearest million, but the 40 872 means that 65 hundred thousand is the nearest hundred thousand).

Mind

Imagine these things in the mind – develop pictures of being nearer other numbers due to digits in the largest place values.

Mathematics

Practice

Ensure students practise rounding and estimating – first with number lines and then without, as the pattern from the *Hand* subsection of this unit becomes more evident. Encourage students with difficulties to place things in place-value alignment, and to use seriation to obtain the above and below numbers for different PVs.

5 0 0 0 0 0	nearest 100 000	4 8 0 0 0 0	nearest 10 000
4 7 5 9 8 2		4 7 5 9 8 2	
4 0 0 0 0 0		4 7 0 0 0 0	

Connections

Connect this to measures and even to fractions.

Reflection

Applications

Apply these ideas to problems in everyday life.

Extension

Flexibility. Brainstorm all situations where we would want to round large numbers. Also look at when rounding is better than accuracy and how we would choose the level of accuracy required. This is part of the *Accuracy vs exactness* big idea.

Reversing. **Don't forget this** – make sure teaching goes: number → estimate and estimate → number (e.g. have activities where students select numbers to which the estimate is applicable). For example:

- Round 347 229 to the nearest 10 000
- Which of the following numbers could be rounded to 560 000 to the nearest 10 000
  547 821, 564 929, 59 876, 555 009, and so on.

Generalising. Ensure that the pattern from *Practice* above is well known. It has two sections; (a) choosing PV above and below, and (b) deciding to which of these two is the number closer. It uses seriation with respect to the PV to find the PV above. It uses the PV one position to the right to determine whether the number goes down or up (i.e. 0, 1, 2, 3 and 4 is down, and 5, 6, 7, 8, and 9 is up).

Changing parameters. Again, does this work when not in a base 10 situation (e.g. nautical miles)?

A5 Equivalence for large numbers

Equivalence in large numbers is a fairly simple extension of equivalence in small numbers. As we discussed in Unit 5, equivalence of whole numbers depends on the purpose of the numbers. If the purpose is to provide a measure of value, then the normal rules for zeros follow (e.g. 043 = 43). However, if the purpose is to identify, then the normal rules do not apply (e.g. for the numerical part of a car's registration, 043 is very different to 43 – in fact, 43 would be seen as a partial plate). We will now briefly look at large whole numbers from the point of view of numbers as value and other purposes of number.

A5.1 Equivalence of whole numbers depends on point of view

1. **Whole numbers as value**. For large numbers, the rules for equivalence when looking at value are as follows.
   - zeros placed before the other numerals do not change value, for example: 3 245 678 = 003 245 678;
   - placing a zero elsewhere does change the value of the number, and this is true if more than one zero is involved, for example: 87 432 ≠ 807 432, 870 432, 874 032, 874 302 or 874 320; and
   - removing a zero does not change a number's value if the zero was before the other numerals, otherwise it changes the value, for example: 065 289 = 65 289 but 650 289 ≠ 65 289.
2. **Other purposes of number**. The first point about purposes for number other than value is that adding and removing zeros anywhere else than before the other numerals **does change** the number (the same rule as for value). Thus, we are only looking here at zeros **before** the other numerals.

   What is being proposed is that there are some situations where zeros before the other numerals does change the number. These situations are as follows.
   - *Identity numbers*. The first situation is when numbers are used to identify things (and people). An identity card with a number 000568 is often different to one with 568. In fact the difference is one of belonging (or validity). The 000568 identity card probably has to have 6 digits (3 digits is not acceptable). So for small numbers, we add the zeros so that the card is valid.
   - *Phone numbers*. A second example is phone numbers. Phoning 0467 234 431 is different to phoning 467 234 431.
   - *Order numbers*. A third example is using numbers to order files in computer systems that use alphabetic systems to order. Since AB is before B then 10 is before 2. This necessitates ensuring that zeros are placed in front of numbers because ordering files labelled 4587 and 43311 gives a different result to ordering numbers 04587 and 43311.

   Thus in the modern digital age, there are many situations where a number such as 045876 is not equivalent to 45876.

A5.2 RAMR lesson

Reality

Local knowledge

Try to find something in life where students are labelled with a number and how we can change those labels so they are different (e.g. student ID numbers).

Prior experience

Check that students understand equivalence for 2- and 3-digit numbers. Check that students have the understanding that different reasons for numbers can change when numbers are equivalent.

Kinaesthetic

Act out these numbers – look for where zero affects them.

Abstraction

Body

Set up the Maths Mat as a PVC for less than 9 digits. Get students with bibs to stand in positions and read the numbers. Have a zero pushing in between numbers – read the numbers, are they different because of the zero? Where can the zero stand that does not make the number different?

Set up a 9-digit number with zeros using bibs. Ask students with zeros what the zero does to the number. Is it necessary to be there? What value does the number have without it?

Hand

Redo the steps in the Unit 5 RAMR lesson but with larger numbers – thousand and millions – use the computer to order 08 874, 09 874, 10 874 and so on. Use digit cards on a PVC to show how adding zeros within the numerals moves some of the numerals on the left of the extra zeros to higher place-value positions, changing the value of the numbers. Then show that this does not happen if the zero is on the left of all the numerals, that is, 007 891 is equivalent to 7 891.

Spend time looking for situations where a zero added to the left of the numerals does change things, such as phone numbers, identity numbers, computer ordering, and so on.

Discuss the role of zero in numbers. Look at where zeros change and zeros don't change the numbers. Look at examples like this: Circle YES or NO

83 654 → 830 654 number changed in value? – YES/NO

056 218 → 56 218 number changed in value? – YES/NO

Then get students to construct these on PVC with digit cards: (a) give students a number to start with (say, 304 295) and ask them to do something to that number that changes it and something that does not change it; and (b) give students a number (say, 6 407 801) and ask them for a starting number and what was done to it that changed its value (using zeros only) and what could have been done so as not to change its value. Ask students for more than one answer for the "changing value" option.

Sort the following numbers into groups where the number is the same:

024 361 000 0 240 361 0 024 361 2 004 361 24 361 024 361

Mind

Imagine a number with no zeros – imagine putting in zeros – when/where does the zero make a difference? Imagine a number with zeros – start removing them, when does it make a difference?

Mathematics

Language and symbols/Practice

Practise situations where zeros change and don't change numbers; examples can be: (a) circle change – tick no change questions; (b)sorting numbers with some zeros into same-number groups; or (c) students use zeros to make change or no change. Examples of these are in Unit 5.

Connections

Look where we have situations like this in mathematics where zeros are important; for example, length, mass, time, identification numbers, invoice numbers, and so on.

Reflection

Validation/Applications

Ask students to explore the world and find everywhere that numbers are expressed in different numerals to the normal 6, 11, 25, 327, etc. For example 24-hour time for plane timetables, and military operations when time and angles (bearings) are written in a specific pattern. Make up problems about these.

Extension

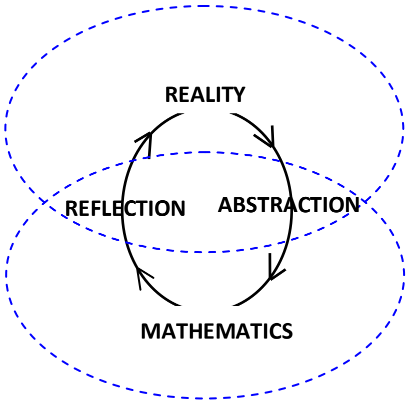
Flexibility. When numerals are different – why? What is the generalisation? Why is it done? [If numbers are to be ordered, and they go to a certain number of digits, then must fill all digits with zeros on left even though one is an early number, e.g. 00006732 is the 6732nd person but needs the eight digits as this is what computer uses.]

Reversing. Reverse everything. If 1642 has to be written as 0001642, then what number is 0002060?

Generalising. Generalise the ways numerals can be changed with a zero without changing the number (e.g. adding zeros at the start of a whole number) and changing the number (e.g. zeros in between digits or at end of digits of a whole number).

Changing parameters. What if we have decimal numbers – does this change the rules? How?

Appendix B: RAMR Cycle



AIM advocates using the four components in the figure on right, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see figure on right).

The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described above, the cycle can lead to a structured instructional sequence for teaching the idea. The table below briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the Reality and Mathematics components of the cycle, while extensions and follow-up ideas are considered in the Reflection component.

REALITY • **Local knowledge**: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • **Prior experience**: Ensure existing knowledge and experience prerequisite to the idea is known. • **Kinaesthetic**: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION • **Representation**: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • **Body-hand-mind**: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • **Creativity**: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS • **Language/symbols**: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • **Practice**: Facilitate students' practice to become familiar with all aspects of the idea. • **Connections**: Construct activities to connect the idea to other mathematical ideas.
REFLECTION • **Validation**: Facilitate reflection of the new idea in terms of reality to enable students to validate and justify their new knowledge. • **Applications/problems**: Set problems that apply the idea back to reality. • **Extension**: Organise activities so that students can extend the idea (use reflective strategies – *flexibility*, *reversing*, *generalising*, and *changing parameters*).

Appendix C: AIM Scope and Sequence

Yr	Term 1	Term 2	Term 3	Term 4
A	**N1: Whole Number Numeration** Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	**O1: Addition and Subtraction for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**O2: Multiplication and Division for Whole Numbers** Concepts; strategies; basic facts; computation; problem solving; extension to algebra	**G1: Shape (3D, 2D, Line and Angle)** 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	**M1: Basic Measurement (Length, Mass and Capacity)** Attribute; direct and indirect comparison; non-standard units; standard units; applications	**M2: Relationship Measurement (Perimeter, Area and Volume)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**SP1: Tables and Graphs** Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	**M3: Extension Measurement (Time, Money, Angle and Temperature)** Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	**G2: Euclidean Transformations (Flips, Slides and Turns)** Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	**A1: Equivalence and Equations** Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	**SP2: Probability** Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	**O3: Common and Decimal Fraction Operations** Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	**N4: Percent, Rate and Ratio** Concepts and models for percent, rate and ratio; proportion; applications, models and problems	**G3: Coordinates and Graphing** Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	**A2: Patterns and Linear Relationships** Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	**A3: Change and Functions** Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	**O4: Arithmetic and Algebra Principles** Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	**A4: Algebraic Computation** Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	**G4: Projective and Topology** Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	**SP3: Statistical Inference** Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	**O5: Financial Mathematics** Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry; SP = Statistics and Probability; A = Algebra.

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