



YuMi Deadly Maths

AIM Project Overview:

**Objectives, Acceleration
framework, Pedagogy,
Culture and school change,
Implementation options**

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The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is <http://ydc.qut.edu.au>.

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DEVELOPMENT OF THE AIM MODULES

The Accelerated Inclusive Mathematics (AIM) modules were originally developed from 2010 to 2013 as part of the Accelerated Indigenous Mathematics project funded under the Commonwealth Government’s *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This project aimed to assist secondary schools with beginning junior-secondary Indigenous students who were at Year 2/3 level in mathematics by developing a junior-secondary mathematics program that accelerates the students’ learning to enable access to mathematics subjects in the senior-secondary years and therefore enhance employment and life chances. The project developed three years of modules (Years A to C) that are vertical sequences of learning to take students from their ability level to their age level in mathematics. The YuMi Deadly Centre acknowledges the role of the Commonwealth Department of Education, Employment and Workplace Relations (DEEWR) in the development of the Accelerated Indigenous Mathematics project and these modules.

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Contents

	Page
1. Background and Purpose	1
1.1 History	1
1.2 Development	2
1.3 Reconceptualisation	3
1.4 Objectives	4
1.5 Imperatives	5
2. AIM Acceleration Framework.....	7
2.1 Teaching–learning imperatives	7
2.2 Acceleration structure	8
2.3 Module framework.....	9
2.4 Testing framework	12
3. YDM Pedagogical Approach	15
3.1 Sequencing and connections	15
3.2 Big ideas.....	17
3.3 RAMR framework.....	20
3.4 RAMR cycle	23
3.5 Related or prior pedagogies.....	24
3.6 Associated pedagogies	26
4. Cultural and School Change Implications	29
4.1 Cultural implications.....	29
4.2 PD and school change.....	31
4.3 Teacher knowledge	33
5. Implementation Options.....	35
5.1 Forms of implementation	35
5.2 Training provisions.....	37
References.....	39

Figures and Tables

	Page
<i>Figure 1</i> Years 8–10 students' enhanced mathematics learning as a result of accelerated intervention	2
<i>Figure 2</i> Normal/horizontal and unit-based/vertical mathematics growth	9
<i>Figure 3</i> Different module types	10
<i>Figure 4</i> Sequenced structure of a module.....	13
<i>Figure 5</i> Sequences/connections between number, operations and algebra	16
<i>Figure 6</i> Sequence/connections between mathematics strands	16
<i>Figure 7</i> Relationship between perceived reality and invented mathematics (adapted from Matthews, 2009)	21
<i>Figure 8</i> Abstraction sequence from reality to mathematics.....	22
<i>Figure 9</i> RAMR cycle.....	23
<i>Figure 10</i> The Wilson cycle	24
<i>Figure 11</i> Payne and Rathmell triangle framework for teaching fraction $\frac{1}{3}$	24
<i>Figure 12</i> Two perspectives of progress	30
<i>Figure 13</i> The YDC effective PD cycle (adapted from Clarke & Peter, 1993).....	31
<i>Figure 14</i> Cycle of school change and leadership.....	33
<i>Figure 15</i> Shulman's knowledge types	33
<i>Figure 16</i> Student evaluation process	34
<i>Table 1</i> AIM module scope and sequence	11
<i>Table 2</i> Major big ideas by topic area or strand.....	19
<i>Table 3</i> Using the RAMR cycle to plan mathematics instruction for a mathematical idea	24

1. Background and Purpose

The Accelerated Inclusive Mathematics project has been developed from the DEEWR-funded *Closing the Gap* Accelerated Indigenous Mathematics project. The name has only changed because the project now runs in schools where it is used with non-Indigenous as well as Indigenous students and because it has been **reconceptualised** for a wider role in these schools. Therefore, to prevent confusion, in this Overview booklet and all associated modules and materials, **the acronym AIM will only be used for the new conception Accelerated Inclusive Mathematics**. If we are referring to Accelerated Indigenous Mathematics, we will write the name in full.

This Overview booklet describes the components of the AIM project. These components have much in common with the components of the Accelerated Indigenous Mathematics project but include new material developed: (a) as a result of the four years (2010–13) the Accelerated Indigenous Mathematics project was trialled in Indigenous Years 8–10 classrooms; and (b) as part of the reconceptualisation.

This section of the Overview looks at the background to AIM and its purpose. It covers the historical development of the project, its reconceptualisation to the present version, and its objectives and imperatives.

1.1 History

The genesis for the project was Indigenous secondary schools where significant numbers of Year 8 students exhibited mathematics performance levels at Year 3 and below. If taught mathematics at Year 8 level, these students were likely to learn little mathematics, disengage from school and enter post-compulsory years with mathematics understanding inadequate to access meaningful employment or tertiary education, thus becoming dependent on social welfare (Baker, Street, & Tomlin, 2006; Cooper, Baturo, Warren, & Grant, 2006; De Bortoli & Thomson, 2010; Ewing, Baturo, Cooper, Duus, & Moore, 2007).

In the early to middle 2000s, YuMi Deadly Centre (YDC) staff worked in projects with many of these Indigenous schools with Year 8 students as described above. The projects were attempting to improve mathematics learning outcomes so that these students could have improved employment and life chances. The only way that YDC staff could see to do this was to try to develop teaching that would remediate students' understandings and enable them to catch up to their grade level. This meant teaching seven to eight years of mathematics in three years, that is, **accelerating** the learning of the students.

Initial acceleration projects were not the success that was hoped because YDC staff could not control all the aspects of teaching. The projects were small and trials tended to be class by class and covering one topic. The teaching was overwhelmed by: (a) problems across the school, such as attendance, behaviour, and community support; (b) difficulties with the topic being due to weaknesses in other topics; and (c) the lack of involvement by other teachers leading to lack of support across the school.

However, there were positive results and a major success. First, YDC staff became aware of a successful project by a principal to improve teaching and learning in a primary school using the *Stronger Smarter* philosophy of C. Sarra (2003) which argued that change had to be whole school, focus on behaviour and attendance as well as curriculum, be in harmony with the culture of the students, and based on high expectations. Second, staff became involved in a successful longitudinal study following non-Indigenous students' learning of algebra ideas across five years (Years 2 to 6). It was based on learning through big ideas (mathematics ideas that can be used across topics and year levels) and was the basis of the structured sequence theory described in section 2.2. Third, staff became involved in successful projects to teach mathematics to vocational students by focusing on the context of the vocation.

Finally, there was a school-wide project which was very successful in acceleration. In 2006–08, YDC staff worked with teachers in an Indigenous P–12 school to improve mathematics learning. After two years of disjointed activity in individual classes, the school adopted a common approach to teaching and behaviour that was in line with that being advocated by YDC and decided to use this approach to teach the Number strand in every class (covering whole numbers, decimal numbers and fractions), using pre- and post-tests developed by YDC to determine instruction and evaluate effectiveness.

The school comprised nearly 100% Aboriginal and Torres Strait Islander students with nearly all students in Years 8–10 exhibiting about Year 3 level knowledge of mathematics. The pre- and post-tests contained items from Years 2, 3, 4 and 5 in such a manner that grades of 25%, 50% and 75% showed understanding at Year 2, Year 3 and Year 4 levels respectively. YDC staff collaborated with the teachers to develop a plan for the mathematics instruction that suited the school. This intervention was very successful as can be seen in Figure 1. The pre-post test results showed that Years 8–10 students in this very underperforming low SES Indigenous school gained, on average, 1.5 to 2 years in mathematics understanding after less than two terms of intervention (a 3-to-4× acceleration).

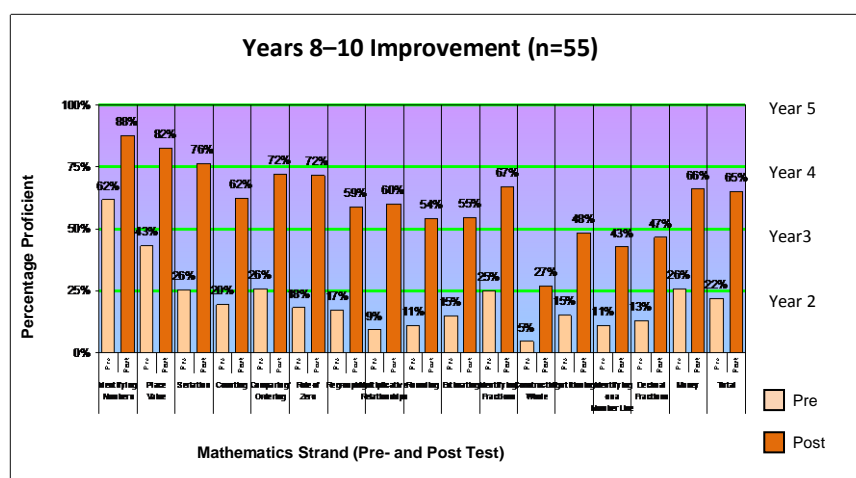


Figure 1 Years 8–10 students' enhanced mathematics learning as a result of accelerated intervention

This success, combined with YDC staff's other existing and past research projects, showed that acceleration is possible in low SES Indigenous schools with underachieving students if interventions are designed with the following three imperatives:

1. Mathematics is taught in units or modules that develop **structural understanding of big ideas** through sequences of activities that show how the ideas grow across Years 3 to 9 (these ideas are described in Chapter 2 of this Overview booklet).
2. Mathematics activities are **contextualised** into local culture and language and tailored to the needs of the school (these ideas are described in Chapter 3 of this Overview booklet).
3. Mathematics teaching is integrated with **whole-school change** to improve: (a) student attitude, behaviour and confidence; (b) teacher knowledge, confidence, pedagogy and expectations; and (c) local community involvement and support (these ideas are described in Chapter 4 of this Overview booklet).

1.2 Development

As a result of the above success, YDC staff designed the Accelerated Indigenous Mathematics project as a longitudinal study using design experiment and action-research case studies to investigate how the mathematics progression of Years 8–10 students with mathematics achievement levels at Year 3 or below could be accelerated so that normal mathematics options could be accessed in Year 11. The proposal was built for schools where the Indigenous students' underperformance did not appear to be related to their ability but

rather their attendance and engagement. It was based on contextualisation, an active teaching pedagogy and a structural approach to mathematics that focused on crucial big ideas and revealed the sequences, connections and relationships that are part of the structure of mathematics. The question was could such a pedagogy make it possible to accelerate the mathematics learning of underperforming Indigenous Years 8, 9 and 10 students?

Because they could not be in schools all the time, YDC staff could not teach the students directly in the project and needed to work through the teachers, developing resources and using professional development (PD) to improve the capacity of teachers to teach mathematics effectively. This led to the following four outcomes in YDC's projects to accelerate mathematics learning of Indigenous students in schools:

- (a) to develop effective **instructional materials** to be the basis of teaching and learning that accelerates mathematical performance;
- (b) to develop **PD structures and approaches** that are effective in providing teachers with the motivation, confidence, knowledge and skills to accelerate mathematics learning;
- (c) to construct **theory** with regard to how mathematics **learning can be accelerated** from initial performance ability level to appropriate year level; and
- (d) to construct **theory** with regard to how mathematics **PD can enable teachers to use instructional materials to accelerate** the mathematics learning of their students.

YDC successfully applied to the Federal Department of Education, Employment and Workplace Relations (DEEWR) for a *Closing the Gap: Expansion of Intensive Literacy and Numeracy* grant for the Accelerated Indigenous Mathematics project in 2008 and received the grant for the project at the end of 2009.

However, with the first trial of AIM in 2010, it became evident that attempting to cover all topics each year would not assist acceleration as much as dividing the curriculum into components, called modules, and building a **vertical curriculum** around these modules. This meant that only some topics could be covered in any one year, and the remaining topics had to be left to later years, to fill the gaps. It also resulted in half-term modules that covered Year 3 to Year 9. This vertical framework is still the basis of the new AIM and is described in Chapter 2 of this Overview booklet.

Across the four years it was trialled with DEEWR funding, the success of the project was dependent on school support and teachers adopting the pedagogy. Where this was done, there was success, with underperforming Year 8 Indigenous students successfully transitioning from AIM into Year 11 mathematics subjects at the end of Year 10. A description of these results can be found on DEEWR's [Teach Learn Share website](#).

Historical note. During the period of the DEEWR *Closing the Gap* Accelerated Indigenous Mathematics project, 2010–13, the Queensland curriculum became the Australian Curriculum and changed from Years 1–12 to P–12, with primary shifting from Years 1–7 to P–6 and secondary from Years 8–12 to 7–9 (junior secondary) and 10–12 (senior secondary), and compulsory years ending at Year 9 not 10. However, with Year 7 not physically moving to secondary until 2015, the DEEWR Accelerated Indigenous Mathematics project was trialled in Years 8–10 but with the mathematics ending at Year 9. The new AIM project is designed for Years 7–9.

1.3 Reconceptualisation

The original idea for AIM, which was also the basis of the four-year DEEWR-funded Accelerated Indigenous Mathematics project, was to build the AIM pedagogy around the 24 modules as a replacement for normal mathematics curriculum activity. This meant that the PD sessions were built around going through how to teach the modules that were the focus of the next school term, and these PD sessions were restricted to teachers who were assigned to classes where AIM was to be the mathematics curriculum.

Thus, AIM only affected a few classes containing very underperforming students. However, most AIM schools had many other classes of students who, although not at Year 3 level at the beginning of secondary school, were below age level in mathematics. The question was how to use the immense resource that is AIM to help

the teachers of these classes to work within the Australian Curriculum in a diagnostic and remedial manner – to use the AIM modules to assist with students who are one or two years behind on particular topics.

It has been evident for some time that teachers of mathematics often have a weakness with sequencing. When faced with students who do not have the prerequisites for a topic, teachers can be unsure of what mathematics knowledge precedes the area under consideration, how they can determine the particular understanding that is yet to be learnt, and what is the most effective sequence of steps to teach from what the students know to what they do not.

The 24 AIM modules provide teaching ideas for mathematics topics in sequence from Year 3 to Year 9. They can be used by teachers as a resource:

- (a) to determine prerequisite knowledge (and the big ideas that underlie the topic under consideration, and connections of this topic to other topics);
- (b) to determine ways to check what prerequisite knowledge is known (and not known);
- (c) to provide instructional strategies to teach the not-known prerequisites in an effective sequence; and
- (d) to provide instructional strategies to teach the topic at age-appropriate level.

The final point above is because the 24 AIM modules have units of work up to Year 9, so their final units are also a resource of age-appropriate activities to teach Years 7–9 mathematics, even for students with no difficulties.

Thus, in line with YuMi Deadly Maths Teacher Development Training projects, **AIM has been reconceptualised to focus on training teachers in a remedial mathematics pedagogy that will enable them to develop diagnostic assessments, remedial sequences, and effective instructional strategies.**

The vertical instructional sequences of units in the 24 modules can be used by a mathematics teacher to determine what goes before and after a topic to be taught – the modules can work with standard classes by giving information on how to remediate prerequisites and how to teach new material. Training in this pedagogy, as well as knowledge of the sequences in the 24 modules, is now the focus of the AIM project. The objective is to use the AIM materials to make teaching more effective across a range of class types in Years 7–9.

The 24 AIM modules support the pedagogy, rather than being the pedagogy. This enables the AIM modules to have much wider use in secondary schools by supporting all classroom mathematics teaching in Years 7–9.

Thus, there are two options in using AIM (see Chapter 5):

- (a) **replacement** – replacing the Australian Curriculum for Years 7–9 with the 24 AIM modules (in the order given in Table 1, section 2.3); and
- (b) **support** – using the 24 modules to enable the Australian Curriculum to be implemented with diagnosis and remediation.

1.4 Objectives

The overall objective for AIM is to develop **theory and practice** on how the mathematics learning of underperforming Years 7, 8 and 9 students can be accelerated to where the students can access later mathematics subjects. The expectations of AIM are to:

- (a) improve the mathematics learning in Years 7–9 students, particularly for underperforming students, by apportioning Years 3–9 Australian Curriculum mathematics content into three years and providing a teaching approach that will accelerate mathematics learning; and
- (b) write, trial, develop and refine a three-year mathematics program for accelerating this content that provides a resource to teachers to enable underperforming students to progress to post-compulsory mathematics courses, gain employment, and improve life chances.

Therefore, the outcomes of AIM are:

- (a) student learning – motivation, confidence and understanding to progress to post-compulsory mathematics;
- (b) materials – culturally and contextually appropriate teaching modules (sequences of units) and tests designed so that Years 3–9 mathematics can be taught in three years and so that they can be used by teachers as replacement for the Years 7–9 curriculum or as support for diagnosing and remediating mathematics difficulties and teaching mathematics topics;
- (c) services – PD workshops to train teachers in remedial pedagogy and in using the 24 modules, and an online support framework covering email communication, discussion forum and help desk; and
- (d) research – involving all teachers in action research on their practices, analysing data provided by teachers on researcher actions \leftrightarrow teacher practices \leftrightarrow student learning, and drawing implications for modules and PD workshops and for a theory of mathematics acceleration.

As stated in sections 1.2 and 1.3 above, the design of AIM was influenced by the successes in earlier YDC projects, in terms of philosophy and pedagogy and by Sarra (2003) and VET experiences. As a consequence, the AIM project tends to reflect a convergence of cognitive, cultural and affective research, and to be built around a framework for acceleration, as follows.

1. *Cognitively*, the project focuses on **structural learning** of mathematics; on abstraction or generalisation (Cooper & Warren, 2008), on vertical sequences, and on application to the world; aiming to reveal the big ideas of mathematics (the abstract schema of Ohlsson, 1993) and to develop a “mathematical eye” through which to view and interpret the world. The pedagogy is based on social constructivism (English & Halford, 1995) but built around a cycle of instruction with its roots in Wilson (1976, 1982; adapted by Ashlock, Johnson, Wilson, & Jones, 1983), and Baturo, Cooper, Doyle, and Grant (2007).
2. *Affectively*, the project is based on integrating mathematics instruction with **whole-school change** programs, similar to those advocated in the Stronger Smarter Institute (Sarra, 2003), namely, to challenge behaviour, build pride, make identity positive with respect to learning, ensure high expectations, enable local leadership and integrate school and community.
3. *Culturally*, the project is based on **contextualising** mathematics to students’ culture and language. The research of YDC staff in communities where language, activity and culture is different to that of mainstream schools has shown that: (a) local language is a major part of mathematics instruction (Schäfer, 2010; Young, van der Vlugt, & Qanya, 2005); (b) local knowledge must be made legitimate within the classroom; and (c) local culture must be celebrated and not negated (Baker, et al., 2006; Ewing, Cooper, Baturo, Matthews, & Sun, 2010). In this way, students’ mathematics learning begins with **what is already known**.

1.5 Imperatives

The basis of AIM is to improve outcomes for underperforming students by improving mathematics learning. The imperative behind AIM is to find theory that begins the process of enabling the underperforming students to access later years mathematics subjects that enable tertiary education and/or apprenticeships and traineeships, enhancing employment and life chances (see Nguyen, 2010). Thus AIM represents a win-win solution for government, community and individuals in relation to “closing the gap” (cf. Department of FaHCSIA, 2009; SCRGSP, 2009).

AIM is particularly set up to improve employment and life chances of Aboriginal, Torres Strait Islander, and low SES students. Many of these students perform in mathematics at early to middle primary years or below and disengage from school. Their trajectory is often unemployment, welfare dependence and insufficient literacy and numeracy to gain employment without significant intervention. For YDC staff, the project represents a convergence of social, economic and educational benefit, because it is designed to:

- (a) give underperforming students the opportunity to change their future from welfare dependency (with all its concomitant problems of violence, substance abuse and poor health) to productive employment;
- (b) increase the pool of potential skilled workers for industry, particularly in rural and remote areas, the lack of which is putting at risk Australia's economic development; and
- (c) illuminate learning theory with regard to mathematics development to prevent future under-performance.

YDC staff realise that AIM is a challenging project. Many teachers of underperforming Years 7–9 students are not secondary mathematics trained; they are teaching out of field. They have to learn mathematics and mathematics education, and to implement these ideas in challenging classroom situations, often to students of different cultural backgrounds. However, YDC always follows the imperatives below, and these are also the basis of AIM.

1. All people deserve the deepest mathematics that empowers them to understand their world and solve their problems, and this is possible if mathematics is taught as a conceptual structure, life-describing language, and problem-solving tool.
2. All people can excel in mathematics and remain strong and proud in their culture and heritage if taught actively, contextually, with respect and high expectations and in a culturally safe manner.
3. All teachers can be empowered to teach mathematics with the outcomes above if they have the support of their school and system and the knowledge, resources and expectations to deliver effective pedagogy.
4. All communities can benefit from strong, empowering mathematics programs that profoundly and positively affect students' future employment and life chances if school and community are connected through high expectations in an education program of which mathematics is a part.

2. AIM Acceleration Framework

The AIM project is based on over 20 years of teaching by YDC staff in low SES and Indigenous classrooms. It has two aspects. The first is the overall framework of the AIM teaching program, the series of modules that cover all the mathematics from Years 3 to 9. The second is the approach to teaching each module, the pedagogy of YuMi Deadly Maths (YDM) that enables effective teaching and acceleration through year levels in each module to enable underperforming students to catch up in mathematics. This chapter describes the first aspect, the AIM framework for acceleration that is the basis for implementing AIM, covering teaching–learning imperatives, acceleration structure, module framework, testing framework, and module implementation. The second aspect, the YDM pedagogy, is the focus of the next chapter.

2.1 Teaching–learning imperatives

At the beginning of designing the original Accelerated Indigenous Mathematics project, the focus was on how to do more in less time. As a result, the plan for acceleration was, and still is for AIM, based on the following four *imperatives*.

Imperative 1: Chunking. Mathematics topics are to be taught in large chunks, not as small pieces spread over years. For example, place value from Years 3–8 has been planned to be taught, not as individual places (ones, then tens, then hundreds, and so on) but in terms of place-value periods (ones, tens, hundreds of *ones*; ones, tens, hundreds of *thousands*; ones, tens, hundreds of *millions*; and so on). Thus, the focus is on building a holistic understanding around whole numbers as a complete multiplicative system – the approach to teaching that YDC has found to be effective with Indigenous students.

Imperative 2: Structure. Mathematics topics are to be taught as a structure (i.e. a connected set of ideas) both *within domains* – as per place value of number; and *across domains* (e.g. place value and metric measures). For example, conversion of metric units is connected to place value (same multiplicative base, 1000); multiplication and division are related to measuring, fractions and chance. With respect to the last example, students are led to recognise the **big ideas** of mathematics (see Chapter 3) and to understand that the following mathematics ideas are not different; they are all examples of the *inverse relation principle* (big idea): large divisors give small quotients in division, large units give small numbers in measurement, large denominators give small fractions, and many possibilities gives small chances of winning in probability.

Imperative 3: Active pedagogy. Mathematics topics are to be taught actively, with actions and material leading to ideas in the mind (i.e. mental models); teaching is to follow the sequence body→hand→mind. Mathematics topics are to relate to real-world problems (from everyday experiences) and relate back to real-world problems, integrating real-world situations, physical, virtual and pictorial models, language, and symbols. Lessons are to entail moving between these representations that require knowledge of teaching methods at sequenced levels: the *technical* (knowing how to use appropriate materials to model mathematical ideas), *domain* (knowing the specific methods and materials for the mathematical idea) and *generic* level (knowing the pedagogies of *generalising*, *reversing* and *flexibility* that can be used for all mathematics teaching).

Imperative 4: Culture, community and school focus. The cultural implications of Western mathematics are to be made visible, frameworks for learning are to be used that relate to the local community, and mathematics is to be contextualised into culture and related to home language. As well, the local community is to actively participate in the school and local knowledge is to be made legitimate in the classroom. Finally, schools are to adopt change processes similar to those advocated by the Stronger Smarter Institute, so that school processes build pride in heritage, change identity to where there is a belief that students can learn, confront behaviour, ensure cultural safety and high expectations and are based on local leadership.

2.2 Acceleration structure

The framework for acceleration used in AIM, and in the original Accelerated Indigenous Mathematics project, is based on a theory (Warren & Cooper, 2009) for building big mathematical ideas. This theory is built around understanding of mathematical ideas being the ability to move between four representations of these ideas: (a) symbols; (b) language; (c) physical, virtual and pictorial; and (d) graphical representations (if they are appropriate). It also involves being able to move between various models of these representations such as set, number line, array, and so on. Warren and Cooper's theory argued that mathematics knowledge growth for big ideas is through **structured sequences across** models/representations not within a model/representation.

Structured sequence theory

The theory argues that these structured sequences have the following properties.

1. **Isomorphism.** Effective models and representations have strong isomorphism to desired internal mental models, few distracters, and many options for extension. In other words, these models/representations grow with the idea – for example, 3×4 is an array which can grow into the area model which is the basis of multiplication of larger numbers (7×23), fractions ($\frac{4}{5} \times \frac{2}{3}$), and algebra ($x(x + 2)$).
2. **Sequence.** Sequences of models/representations develop so there is increased flexibility, decreased overt structure, increased coverage and continuous connectedness to reality. In other words, the balance model for equations moves from a physical balance to a pictorial balance to an abstract balance that can handle division and negatives.
3. **Nestedness.** Ideas behind consecutive steps are nested wherever possible. That is, later thinking is a subset of earlier. For example, the first understanding of equals should be “same value as”, and the second understanding should be the result of the calculation which is a particular form of “same value as”. That is, the meaning of equals in $4 + 2 = 6$ is nested within $4 + 2 = 5 + 1$, so $4 + 2 = 5 + 1$ comes first.
4. **Integration.** More complex and advanced mathematical ideas can be facilitated by integrating models. For example, solving algebraic equations is a combination of number-line understanding of inverse operation and balance-beam understanding of the balance rule. However, some complex and advanced ideas may require the development of *superstructures* if complexity leads to *compound* difficulties. For example, the compensation principle for addition is to do the inverse to the other number (e.g. $8 + 5 = 10 + 3$ by adding and subtracting 2), while the compensation principle for subtraction is, simplistically, the opposite (e.g. $14 - 6 = 18 - 10$ by adding 4 to both numbers). This opposite difference between them can cause confusion and, thus, is called a compound difficulty. However, if a superstructure of understanding is built around subtraction as the inverse of addition and division as the inverse of multiplication, it is almost self-evident that there has to be an opposite process for subtraction in relation to addition.
5. **Comparison.** Abstraction is facilitated by comparison of models/representations to show commonalities that represent the kernel of desired internal mental model. In simple terms, $2 + 3 = 5$ makes more sense when seen in joining counters (set model) and steps along a number track (number-line model.)

The implication of this theory is that acceleration is enhanced if instruction is divided into vertical units of work (the AIM project refers to these vertical units as **modules**) that structurally sequence mathematical ideas from early childhood to junior secondary. In this way, the modules can build big ideas as well as concepts, strategies and processes. Early trials of this in AIM and other YDC mathematics projects have shown that:

- (a) the first stages of a module (that cover the early years of the sequence) have to be completed slowly and carefully to build the connections that frame out the big mathematical ideas; and then
- (b) the later stages (that cover the later years) can be covered quickly in gestalt-like leaps of understanding (i.e. leaps to holistic ideas or superstructures which enable deeper understandings of the original parts).

Overall, this implies that mathematics learning of underperforming Year 7 students can be accelerated if: (a) instruction, models, representations and language follow a nested line of abstraction from ability level to age level; and (b) instruction has its basis in carefully built, contextually and culturally appropriate foundational ideas enabling rapid and gestalt advances across higher level and more generalised topics (Holmes & Tait-McCutcheon, 2009; Thomas & Tagg, 2007).

Curriculum implications

Accelerating mathematics learning by modules which develop a big idea from early childhood to junior secondary has implications for curriculum if the project is to cover, as it must, all the mathematics from Year 3 to Year 9 in three years (which we will call Years A, B and C).

First, although it allows teachers to teach to ability but still spend some time on age-appropriate activities, it means that only part of the mathematics curriculum can be covered in each year. This is because modules change the way the mathematics is accelerated (see Figure 2). Instead of building all topics in three stages as in the “normal” growth diagram on the left in Figure 2, one-third of the topics would be completed in each year, with the Years B and C filling in the gaps left from Year A as in the “vertical” growth diagram on the right in Figure 2. Thus, the mathematics curriculum moves **from a horizontal to a vertical structure**.

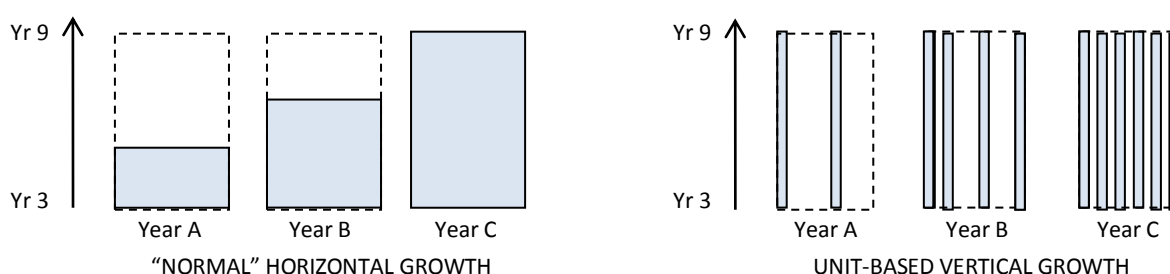


Figure 2 Normal/horizontal and unit-based/vertical mathematics growth

This idea of modules and unit-based growth has resulted in success with trials in previous years in other projects. In particular, acceleration of whole numbers has been successful using this vertical approach. However, as stated above, the use of modules means that, although only some mathematics topics would be taught in Year 7, they would be taught to junior secondary level. To stress what this means a second time, this means that **growth in mathematics would change** from enhancing learning of **all** topics across each of the three years to **adding new topics each year** to fill in the gaps (as in Figure 2). Both structures would reach the same outcome by the end of Year 9 but in different ways.

2.3 Module framework

With the theoretical structure of the modules accepted, the task was then to determine the number of modules and what they would cover. In this section, we look at this in terms of internal module structure and scope, sequence and coverage across the modules.

Internal module structure

The modules were designed, in part initially and in part as a result of trials, to have the following components.

1. An **introduction** section that describes the focus of the module, the connections and big ideas that underlie the module, and the vertical sequence that the units will move through.
2. The **vertical sequence of units** that take the big ideas from Year 3 to Year 9, but modified to ensure that (a) a proper foundation for the sequence is first built (i.e. sometimes the first unit includes work from Years P–2, e.g. equivalence and equations starts with Prep physical balance work); and (b) the last units can be covered from the earlier units (i.e. sometimes the sequence ends early and later work is left to the

end of other modules whose knowledge is needed for the later work to be understood, e.g. whole number systems with scientific notation is left for the module on indices).

3. **Pre-post test items** for the teachers to be able to (a) select the appropriate starting unit for their students, and (b) check that students understand all the units when the module is finished.

Further to this, early trials of the modules showed that some modules were less effective than others because there was so much work in the modules that was new to the teachers. For example, for many teachers operations are synonymous with algorithms, procedures and worksheets, but in the AIM modules the focus was on concepts (meanings and models), principles (properties and laws) and strategies (general approaches that show direction to answer). This was too much new knowledge for most teachers so two changes were made:

- (a) the section on principles was moved later in the sequence of modules – to where it became part of the movement from arithmetic to algebra; and
- (b) the change or transformation meaning of operations was moved to a later unit on functions – to where its development through function machines was applied to a variety of situations.

Where possible, modules are built around a central mathematics big idea, or collection of ideas, or a theme or topic in mathematics. In many cases, this allows for the modules to have a single sequence of units. These units may contain more than one idea but each idea is integrated into a single sequence with the other ideas. However, there are differences (see Figure 3 below). In some cases, the modules are divided into a number of different, but related, topics, and vertical sequences, often very similar, have been developed for each topic (e.g. basic measurement is a series of similarly sequenced units in three large sections). As well, there are further modules that combine these two approaches (e.g. whole number is five units giving different ideas for three-digit numbers, and then back to a central sequence leading to large numbers and number systems).

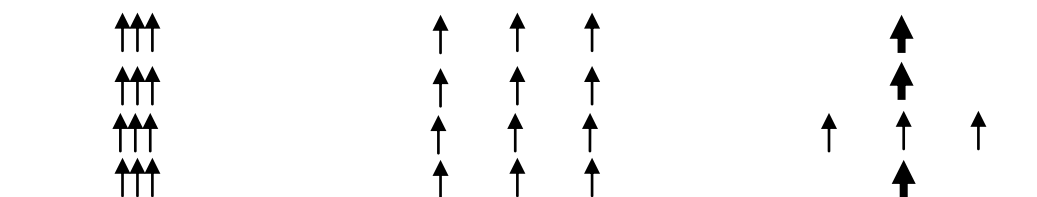


Figure 3 Different module types

Another factor that affected module structure was the need for strong foundations. Often there is a preponderance of activities in the early units with later units covering more years of work in fewer activities. This is because, when foundations are set, acceleration occurs through the later units.

Interestingly, the module structure was found in practice, during classroom trials, to have real advantages:

- (a) it gave teachers the freedom and confidence to drop down and teach at ability level because they saw that such instruction would accelerate the students' knowledge up to age level;
- (b) it gave the students a sense of progress and confidence that it was possible to reach age-appropriate standards;
- (c) it convinced teachers that it was appropriate to spend time on one topic area as long as learning was in connected and integrated chunks to enable students to make progress;
- (d) it enabled students to experience big ideas across year levels and facilitated strong generalisations of these big ideas;
- (e) it enabled teachers to see growth of students' ideas as the students moved through the units and to make on-the-run, in-class assessments of progress (to know where their students were); and
- (f) it allowed students to have the most powerful form of acceleration, where strong foundations enable gestalt-type leaps in later knowledge to superstructures.

Structure across modules

To determine the focus of the modules required looking at the mathematics content for Years 3 to 9 and breaking it into parts. The result is the scope and sequence in Table 1 below. The process by which this module scope and sequence was developed is described after the table.

Table 1 *AIM module scope and sequence*

Yr	Term 1	Term 2	Term 3	Term 4
A	N1: Whole Number Numeration Early grouping, big ideas for H-T-O; pattern of threes; extension to large numbers and number system	O1: Addition and Subtraction for Whole Numbers Concepts; strategies; basic facts; computation; problem solving; extension to algebra	O2: Multiplication and Division for Whole Numbers Concepts; strategies; basic facts; computation; problem solving; extension to algebra	G1: Shape (3D, 2D, Line and Angle) 3D and 2D shapes; lines, angles, diagonals, rigidity and properties; Pythagoras; teaching approaches
	N2: Decimal Number Numeration Fraction to decimal; whole number to decimal; big ideas for decimals; tenths, hundredths and thousandths; extension to decimal number system	M1: Basic Measurement (Length, Mass and Capacity) Attribute; direct and indirect comparison; non-standard units; standard units; applications	M2: Relationship Measurement (Perimeter, Area and Volume) Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	SP1: Tables and Graphs Different tables and charts; bar, line, circle, stem and leaf, and scatter graphs; use and construction
B	M3: Extension Measurement (Time, Money, Angle and Temperature) Attribute; direct and indirect comparison; non-standard units; standard units; applications and formulae	G2: Euclidean Transformations (Flips, Slides and Turns) Line-rotation symmetry; flip-slides-turns; tessellations; dissections; congruence; properties and relationships	A1: Equivalence and Equations Definition of equals; equivalence principles; equations; balance rule; solutions for unknowns; changing subject	SP2: Probability Definition and language; listing outcomes; likely outcomes; desired outcomes; calculating (fractions); experiments; relation to inference
	N3: Common Fractions Concepts and models of common fractions; mixed numbers; equivalent fractions; relationship to percent, ratio and probability	O3: Common and Decimal Fraction Operations Addition, subtraction, multiplication and division of common and decimal fractions; models, concepts and computation	N4: Percent, Rate and Ratio Concepts and models for percent, rate and ratio; proportion; applications, models and problems	G3: Coordinates and Graphing Polar and Cartesian coordinates; line graphs; slope and y-intercept; distance and midpoints; graphical solutions; nonlinear graphs
C	A2: Patterns and Linear Relationships Repeating and growing patterns; position rules, visual and table methods; application to linear and nonlinear relations and graphs	A3: Change and Functions Function machine; input-output tables; arrowmath notation, inverse and backtracking; solutions for unknowns; model for applications to percent, rate and ratio	O4: Arithmetic and Algebra Principles Number-size, field and equivalence principles for arithmetic; application to estimation; extension to algebra; simplification, expansion and factorisation	A4: Algebraic Computation Arithmetic to algebra computation; modelling-solving for unknowns; simultaneous equations, quadratics
	N5: Directed Number, Indices and Systems Concept and operations for negative numbers; concept, patterns and operations for indices; scientific notation and number systems	G4: Projective and Topology Visualisation; divergent and affine projections; perspective; similarity and trigonometry; topology and networks	SP3: Statistical Inference Gathering and analysing data; mean, mode, median, range and deviation; box and whisker graphs; large data sets, investigations and inferences	O5: Financial Mathematics Applications of percent, rate and ratio to money; simple and compound interest; best buys; budgeting and planning activities

Key: N = Number; O = Operations; M = Measurement; G = Geometry, SP = Statistics and Probability; A = Algebra.

The initial process for forming the modules was in three parts. First, the modules were divided using the strands from the Australian Mathematics Curriculum but with Operations separated from Number and Algebra, and Statistics and Probability considered as one strand. This gave six topic areas of Number, Operations, Algebra, Geometry, Measurement, and Statistics and Probability. Second, the length of time for a module was set at half a school term (i.e. normally 5 weeks) which, in three years, means at most 24 modules. Each of the topic areas was therefore divided into modules that could be completed in half a term, based on a sub-topic or a collection of connected sub-topics that cover the same mathematics big ideas. Third, the modules were collected into three year levels (called Years A, B and C) so that (a) each year covers an important cross-section of mathematics, (b) there is a sequence across the years, and (c) there are connections made within each year (to get chunking across as well as within modules).

For the original Accelerated Indigenous Mathematics project, and in the new AIM being used as a replacement, the years were based on the uses of mathematics and how mathematics changes from Years 3 to 9:

1. **Year A.** Basic mathematics necessary for employment in a trade (whole and decimal numbers, measurement, operations, shape, and tables and graphs) – it enables connections between metrics and place value, and operations to be applied in measurement situations.
2. **Year B.** Multiplicative approaches to mathematics (fractions; percent, rate and ratio; probability; and equations and equivalence), and completion of Year A ideas (measurement and flips-slides-turns geometry) – also completes the coverage of all mathematics necessary for an apprenticeship.
3. **Year C.** Generalisation to algebra (patterns, functions, principles, and algebraic computation) and advanced topics (negatives and indices, projections-topology, statistical inference, and financial mathematics) – extends understandings to the topics that underlie the Years 10–12 mathematics subjects that lead to university and high-status mathematics careers (e.g. engineering, accountancy).

The divisions and the year level foci above led to a scope and sequence for AIM (see Table 1) of 24 modules that teach mathematics content from Year 3 to Year 9 and cover the three years with two modules each term. These 24 modules were written and, where possible, trialled in classrooms with low-performing students. The modules in Table 1 were refined as a result of these trials. As can be seen in the table, each module is signified by a letter (giving topic area) and a number which gives order.

2.4 Testing framework

The characteristics of the modules, with the vertically sequenced units, provide the basis of the acceleration needed for the low-performing students to catch up, and allow teachers to teach both at the ability level of their students at the start of the module and at the age level of their students at the end of the module. To allow teachers to know where to start a module and to determine how far up the module the students have gone, testing also needs to be associated with each unit in the module, and this testing has to include diagnostic pre-testing to know where to start and diagnostic post-testing to know where the students have finished and what still has to be learnt.

Each module grows knowledge in particular mathematics ideas. When all modules are combined, they grow knowledge across all ideas. The growth within modules can be shown by the difference in achievement between the pre- and post-test results and, unless an overall summative test is added, the growth across the modules needs to be determined by some aggregation of the post-tests in relation to the pre-tests. This will be discussed later in this section, but one conclusion is evident, that to maximise implementation, the pre-tests and the post-tests have to be both **diagnostic** (formative) and **achievement** (summative).

Relation of units and tests

Because modules are designed with vertical sequences, then each unit could, depending on the quality of the design, represent a different level of mathematics knowledge, and this could be related to year levels. Then

knowing the highest unit at which a student can perform in a module is a measure of achievement (in terms of year level knowledge) as well as a diagnosis of what a student has learnt and needs to learn.

Within modules

Thus, an effective answer to pre- and post-tests is to design the modules so that there are diagnostic-achievement tests (we shall call them *subtests*) for each of the vertical units; and the highest level subtest is normally associated with the highest level unit. This enables the subtests to be used by teachers to determine where their students start, to check that their students complete the module, and to gain information on how to teach to get their students to the end of the module. As well, teachers can also use them together for the full pre- and post-tests.

Figure 4 illustrates this relationship between units and subtests visually. It should be noted that numbers of units could be more or less than five but that the number of subtests is the same as the number of units and the final test is normally the highest level one. Thus Figure 4 is the final structure required for each module. **This structure has been designed into the modules but not yet trialled in classrooms to see if it is effective.** This will be the next move in terms of research with any trials of these modules. It is hoped that this research will provide information to enable each module to be refined to this structure.

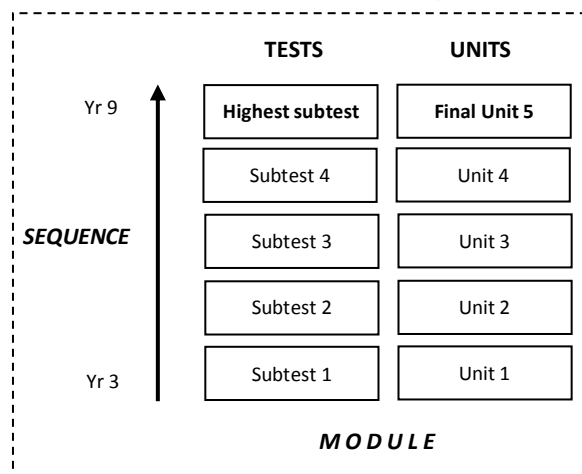


Figure 4 Sequenced structure of a module

Across modules

As stated above, knowing where students start and finish a module in terms of vertical units can be used to show overall growth of knowledge. To do this, the units have to be designed so that students' responses to year levels can be translated in terms of Year 3 to Year 9 knowledge with respect to the Australian Mathematics Curriculum. When this is done, it is possible for the discontinuous growth in knowledge across modules, with later years filling in gaps from earlier years, to be translated to a general across-modules growth in terms of mathematics year levels. For such an overall testing regime, the relation of subtests to vertical units as shown in Figure 4 could be aggregated to enable an overall grade in terms of year level of mathematics understanding. Such a grade would be based on rapid changes in some mathematics ideas (as modules covering these ideas are completed) and no change in other ideas (where modules covering these other ideas have not yet been taught). It would be best recorded in terms of levels within the six topic areas (Number, Operations, Algebra, Geometry, Measurement, and Statistics and Probability) as well as an overall level.

Interestingly, such an aggregated grade could enable diagnosis to relate to year levels and provide information on students' general progress through year levels, as well as through the individual modules, which could be used to indicate extra teaching needed in terms of modules as well as that needed for different levels of understanding within the modules.

This aggregation of post-test responses to get an overall grade has yet to be trialled and is another next move in future trials of AIM materials. Because it is based on discontinuous learning movements (as only one module covering a few ideas is being taught at any one time), it may be effectively done by weighted averaging of module final post-test results (which could range from no knowledge of any unit to full knowledge of all units depending on whether the module has been covered).

Preparation and administration of tests

Giving modules as per Figure 4 means that the subtests, and the item types in them, are given unit by unit. They can therefore be used unit by unit, or all together. The subtest items can be used to construct tests for the start and end of each module (the pre- and post-tests). This gives the testing a double edge: (a) the

subtests can be used unit by unit as a way of checking knowledge before and after each unit, and (b) the subtests can be integrated to make overall pre-tests and post-tests.

Because each school's teachers and students are different, the relationships between subtests and the pre- and post-tests vary and the following parameters need to be followed:

- (a) the pre-tests should normally not include all items in the subtests otherwise the pre-test would result in continuous failure because of the low performance of the students with regard to junior secondary curricula (particularly the highest level in a module, Year 9) – teachers need to choose the level of subtest at which to end the pre-test, or select subtest items appropriately for their students;
- (b) the post-tests would need to include all the higher levels of subtests but may not consist of all the early subtests depending on whether students can do the higher subtests or not – once more, the teacher can make the appropriate selection;
- (c) it is important that the two tests (pre and post) can be used for comparisons, thus the higher subtests that are not put into the pre-test have to be assumed as zero for the students, and similarly the lower subtests not put in the post-test have to be assumed as correct; and
- (d) if the above assumptions cannot be made or are uncertain, all subtests, or a reasonable number of subtests, need to be included in both the pre-test and the post-test.

For flexibility, the design for assessment items for modules that has emerged from the AIM trials is that the items given in subtests relate to the units, with the assessment items in order. This is to enable teachers to have options: (a) they can pre-test and post-test students at the start and end of each module (or simply use the item types in a worksheet at start and finish); (b) they can combine the subtests to form an overall pre-test and/or post-test for the module as a whole (to see if the module has improved performance); and (c) they can allow for differences in students by using the sequence of item types to select test items appropriate for their students yet maintain sequence integrity.

Overall, the subtest items are given flexibly in sequenced lists so that teachers can do all the following:

1. Use their knowledge of their students to ensure that pre-tests are based on item types that students might reasonably know but do not become a continuous stream of “don't knows” – this means (a) not putting subtest material in the pre-tests that it is known that students cannot do; and (b) ensuring students know that it is OK in pre-tests to not answer items because the pre-test is given before instruction (it is to find out if they know anything before it is taught).
2. Reduce the number of item types to make the pre-tests as short as possible without removing important questions – subtest items attempt to cover all possibilities and it is not necessary to give students all possibilities when they have shown they cannot answer any possibility.
3. Ensure that there are sufficient common assessment item types in both the pre- and post-tests to allow for pre-post comparisons and to record what was in the pre- and post-tests and what were the common items – for example, all item types in pre-tests should be given in post-tests to see how far up the year levels students have come through being taught the modules.
4. Encourage students to try all items in post-tests and do as well as they can in the post-tests, because these final tests are based on the highest units in the sequences to determine how much they have learnt.

Finally, it is recommended that pre-tests and post-tests be compared by Effect Size (Google this for further information). To do this, calculate the difference between the pre-test and post-test means and divide this by the average of the standard deviations of the pre- and post-tests. If the resulting calculation is higher than 0.4 (the higher the better), this is an indication that teaching the module was more effective than normal teaching.

3. YDM Pedagogical Approach

As stated at the beginning of Chapter 2, this chapter looks at the pedagogical approach to teaching each module. This teaching is based on the mathematics pedagogy that underlies AIM, the YuMi Deadly Maths (YDM) pedagogy. This YDM pedagogical approach enables effective teaching and acceleration through year levels in each module to enable underperforming students to catch up in mathematics. This pedagogy has two parts: the first deals with the way the mathematics ideas are developed **across the units** in a module; while the second deals with the ways the ideas **within each unit** are to be taught. The first part is described in the first two sections of this chapter, sequencing and connections (3.1), and big ideas (3.2). The second component is described in the middle sections of this chapter, RAMR framework (3.3) and RAMR cycle (3.4). The ideas on which this RAMR framework and cycle are based are described in the last two sections, related or prior pedagogies (3.5) and associated pedagogies (3.6).

3.1 Sequencing and connections

YDM and AIM believe that mathematics should be taught so that it is accessible as well as available, that is, learnt as a rich schema containing knowledge of when as well as how. Rich schema has knowledge as connected nodes which facilitates recall (it is easier to remember a structure than a collection of individual pieces of information) and problem solving (content that solves problems is usually peripheral, along a connection from the content on which the problem is based).

As a consequence, YDM and AIM argue that knowledge of the structure of mathematics, particularly of sequences and connections (and big ideas), can assist teachers to be effective and efficient in teaching mathematics. This is because it enables teachers to:

- (a) **determine what mathematics is important to teach** – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;
- (b) **link new mathematics ideas to existing known mathematics** – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem solving;
- (c) **choose effective instructional materials, models and strategies** – mathematics that is connected to other mathematics or based around a big idea commonly can be taught with similar materials, models and strategies; and
- (d) **teach mathematics in a manner that makes it easier for later teachers to teach more advanced mathematics** – by preparing the linkages to other ideas and the foundations for the big ideas the later teacher will use.

Thus it is essential that teachers know the mathematics that precedes and follows what they are teaching, because they are then able to build on the past and prepare for the future.

Major sequences/connections

European mathematics grew out of two views of reality: the first was number, the amount of discrete objects present; and the second was the world around us, the shapes and the spaces we live in. The basis of number was the unit, the one. Large numbers were formed by grouping these ones, and small numbers (e.g. fractions) by partitioning these ones into equal parts. The operations of addition and multiplication, and the inverse operations of subtraction and division, were actions on these ones which joined and separated sets of numbers.

Algebra was constructed by generalising number and arithmetic, and representing general results with letters. Figure 5 illustrates this relationship diagrammatically.

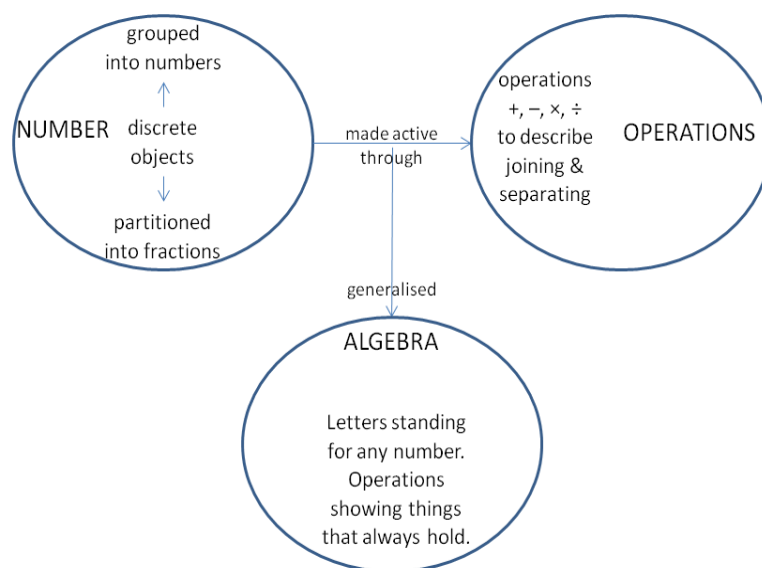


Figure 5 Sequences/connections between number, operations and algebra

Number, operations and algebra, with input from geometry, gave rise to applications within measurement, and statistics and probability. This relationship is diagrammatically represented by Figure 6. This gives a framework for Years P to 9 that enables mathematics as a whole to be considered. It provides an overview and sequence for the connections upon which teaching should be built (e.g. number and geometry before measurement, fractions before numerical probability).

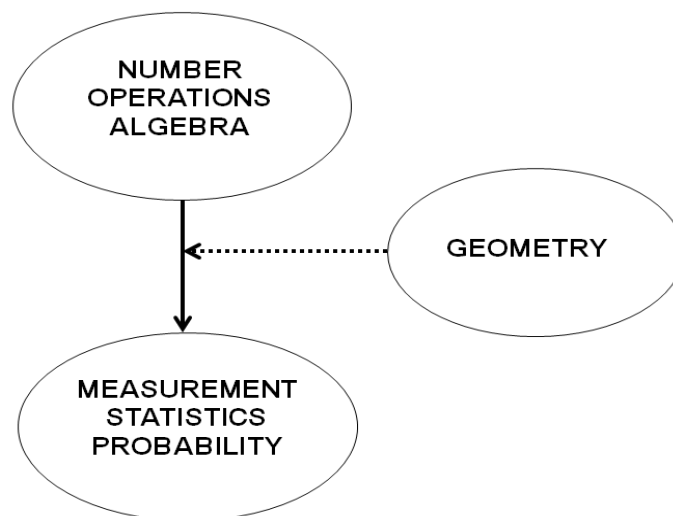


Figure 6 Sequence/connections between mathematics strands

Seamless sequencing between connected ideas

One of the crucial things is that sequencing between connected ideas should be seamless, that is the movement from one idea to the next should not be inhibited by things being taught for the first idea that do not transfer, or worse still do not work, for the second idea. Such seamless sequencing is a major feature in the modules because the units in a module are connected and they form a vertical sequence. If students are to move through the work in a module with acceleration, then the sequence from one unit to the next needs to be straightforward. Similarly, modules from the same topic area are connected and often the work in the second module is based on the first. Once again, there is benefit for acceleration when ideas from the first module also apply in the second. Some examples should assist:

1. **Whole to decimal numbers.** Decimal numbers extend whole numbers by adding in another side of place values based on fractions (i.e. tenths, hundredths, and so on) – whole number ideas should always apply in decimal numbers – that is, we need to teach that: (a) multiplication by 10 is a movement of one place to the left, not adding a zero to the right; and (b) the bigger number has the largest digit in the largest place value, not the number with the most digits.
2. **Whole number algorithms to algebra calculation.** If we teach that whole numbers are added by adding like place values (and renaming if needed) this can translate to algebraic addition being adding like variables – this is even stronger if we use vertical addition for algebra as below.

$$\begin{array}{r}
 462 \\
 + 235 \\
 \hline
 697
 \end{array}
 \longrightarrow
 \begin{array}{r}
 3a + 7b \\
 + 5a + 2b \\
 \hline
 8a + 9b
 \end{array}$$

3. **Decimal numbers to percent.** If we teach flexibility of decimal notation (e.g. $24 = 2.4 \text{ tens} = 0.24 \text{ hundreds}$), we can teach that percent being hundredths is place value with the decimal point after the hundredths instead of after the ones – this means that $0.075 \text{ ones} = 7.5 \text{ hundredths} = 7.5\%$.

Mathematics is replete with examples like this, and this is one of the strengths of the YDM and AIM approach, that more difficult mathematics is made easier by ensuring prerequisite mathematics is taught so that it is an easy translation from the simpler to the more difficult. This is one of the bases of YDM itself, that the earlier years are taught in a way that makes it easier for the later years.

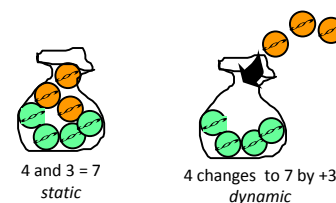
3.2 Big ideas

Big ideas are mathematics ideas that can be used in many year levels and across different topic areas. Knowing mathematics in terms of big ideas is a powerful way to accelerate mathematics learning. A big idea covers so much of mathematics and there are not that many of them.

Nature and justification of big ideas

The nature of big ideas is that they have a wide effect. For YDM, big ideas have some or all of the following properties:

1. **They provide generic approaches to a wide range of ideas** – they encompass viewpoints that cross boundaries. For example, many mathematical actions can be considered as relationship (*static*) and transformation (*dynamic*), as in the addition example on right.
2. **They apply across topic areas** – they have some generic capabilities that are not restricted to a particular domain (e.g. the inverse relation in division between divisor and quotient also applies to measurement, fractions and probability).
3. **They apply across year levels** – they have the capacity to remain meaningful and useful as a learner moves up the grades (e.g. the concept of addition holds for early work in whole numbers, work in decimals, measures, common fractions, and algebraic variables).
4. **Their meaning is independent of context and content** – it is encapsulated in what they are and how they relate, not to the particular context in which they operate. For example, the commutative law says that *first number + second number = second number + first number* irrespective of content type (e.g. whole numbers, decimal and common fractions, algebra or functions).
5. **They are teaching approaches that apply across ideas** – they have the capacity to apply to many situations. For example, the teaching approach of reversing (reversing the order of activity in a lesson) applies everywhere (including going whole to part and part to whole, shape to symmetry and symmetry to shape, algorithm to answer and answer to algorithm).



The justification for focusing on big ideas is that they are very effective ways to accelerate mathematics learning. For YDM, this is because of the following reasons.

1. **One big idea can apply to a lot of mathematics** – this makes them powerful ways to teach and understand mathematics. For example, part-part-whole, multiplicative comparison (double number line) and start-change-end diagrams can solve most fraction, percent, rate, and ratio problems (i.e. they reduce cognitive load).
2. **One big idea can cover work that would need many procedures and rules to be rote learnt.** For example, the distributive law and area diagrams can be used to understand and solve 24×37 , $\frac{2}{5} \times \frac{4}{5}$ and $(x - 1)(x + 2)$ problems.
3. **Big ideas are organic in that later learning can fit within them** – they build structural connectivity across domains of mathematics, thus developing rich schema that can easily accommodate new ideas. For example, building the notion of inverse as “undoing things” and teaching the inverse relationships between $+2$ and -2 ; $\times 5$ and $\div 5$; x^2 and \sqrt{x} ; p^3 and p^{-3} ; $(p)^n$ and $(p)^{1/n}$; $f(x) = 2x + 1$ and $f(x) = (x - 1) \div 2$ can make it really easy to understand integration as the inverse of differentiation in calculus.

Types of big ideas

YDM identifies five types of big ideas:

1. **Global big ideas** – these relate to nearly all mathematical ideas and all year levels. For example, the *commutative principle* is not a global big idea because it only refers to addition and multiplication situations but on the other hand, *transformation and relationship* is global because it refers to all mathematics, saying that every idea can be considered both as a change and as a relationship.
2. **Concept big ideas** – these are the meanings of ideas that are common across mathematics. For example, the meanings behind equals and multiplication – such meanings have large impact and can help in many topic areas, from operations to algebra and measurement to statistics.
3. **Principle big ideas** – these are relationships where meaning is encoded in the relation of the parts, rather than in their content. For example, the commutative principle (turnarounds – e.g. $1^{\text{st}} + 2^{\text{nd}} = 2^{\text{nd}} + 1^{\text{st}}$) is an example of a principle big idea because it also holds for many contexts (e.g. whole numbers, decimals, fractions, variables, functions), while $2 + 3 = 5$ is contentful (holds for 2, 3 and 5).
4. **Strategy big ideas** – these are ways of solving exercises and problems that apply to a range of mathematics across year levels. For example, the part-part-total (PPT) structure underpins all operations and is a powerful strategy in solving word problems and fraction, percent and ratio problems.
5. **Pedagogy big ideas** – these are ideas for teaching that are generic in their application – that can apply to the teaching of many mathematics ideas. For example, the teaching approach of *reversing* where the teaching direction between teacher and student is reversed (e.g. from “what is $5 + 8$?” to “what addition facts give answer 13?”) can apply in many situations other than addition.

Major big ideas

Table 2 on the following two pages lists some of the more important big ideas. A complete list is available on the YDM online learning Blackboard site as a supplementary resource for schools involved in YDM projects. Each module also contains the particular list for the topics in that module.

Table 2 Major big ideas by topic area or strand

Global big ideas	<ul style="list-style-type: none"> • <i>Symbols tell stories.</i> The symbols of mathematics enable the world to be described succinctly and in a generalised way (e.g. $2 + 3 = 5$ means caught 2 fish and then caught another 3 fish, or bought a \$2 chocolate and \$3 drink, or joined a 2 m length of wood to a 3 m length, and so on). • <i>Change vs relationship.</i> Everything can be seen as a change (e.g. 2 goes to 5 by +3) or as a relationship (e.g. 2 and 3 relate to 5 by addition). • <i>Probabilistic vs absolutist.</i> Things are either determined by chance (e.g. will it rain?) or are exact (e.g. what is \$2 + \$5?). • <i>Accuracy vs exactness.</i> Problems can be solved accurately (e.g. find $5\,275 + 3\,873$ to the nearest 100) or exactly ($5\,275 + 3\,873 = 9\,148$). • <i>Continuous vs discrete.</i> Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted. • <i>Part-part-total/whole.</i> Two parts make a total or whole, and a total or whole can be separated to form two parts (e.g. fraction is part-whole, ratio is part to part; addition is knowing parts, wanting total).
Numeration big ideas	<ul style="list-style-type: none"> • <i>Part-whole/Notion of unit.</i> Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts (e.g. if there are six counters, each counter can be a unit, making six units, or the set of six can be one unit.) • <i>Concept of place value.</i> Value is determined by position of digits in relation to ones place. • <i>Additive/Odometer.</i> All positions change forward from 0 to base, then restart at 0 with position on left increasing by 1, and the opposite for counting back (e.g. $2^3/5$, $2^4/5$, 3, $3^1/5$, and so on). • <i>Multiplicative structure.</i> Adjacent positions are related by moving left (\times base); moving right (\div base). Base is normally 10 or a multiple of 10 in Hindu-Arabic system and metrics. • <i>Number line.</i> Quantity on a line, rank, order, rounding, and density.
Equals, operations and algebra big ideas	<ul style="list-style-type: none"> • <i>Concepts of the operations.</i> Meanings of addition, subtraction, multiplication and division. • <i>Equals and order.</i> Reflexivity/non-reflexivity – $A=A$ but A is not $> A$; Symmetry/antisymmetry – $A=B \rightarrow B=A$ while $A>B \rightarrow B<A$ and $A<B \rightarrow B>A$; Transitivity – $A=B$ and $B=C \rightarrow A=C$ and $A>B$ and $B>C \rightarrow A>C$. • <i>Balance.</i> Whatever is done to one side of the equation is done to the other. • <i>Identity.</i> 0 and 1 do not change things ($+/-$ and \times/\div respectively). • <i>Inverse.</i> A change that undoes another change (e.g. $+2/-2$; $\times 3/\div 3$). • <i>Commutativity.</i> Order does not matter for $+/\times$ (e.g. $8+6$ / $6+8$; 4×3 / 3×4). • <i>Associativity.</i> What is done first does not matter for $+/\times$ (e.g. $(8+4)+2 = 8+(4+2)$, but $(8\div 4)\div 2 \neq 8\div (4\div 2)$). • <i>Distributivity.</i> \times/\div act on everything (e.g. $2\times(3+4) = 6+8$; $(6+8)\div 2 = 3+4$). • <i>Compensation.</i> Ensuring that a change is compensated for so answer remains the same – related to inverse (e.g. $5+5 = 7+3$; $48+25 = 50+23$; $61-29 = 62-30$). • <i>Equivalence.</i> Two expressions are equivalent if they relate by $+0$ and $\times 1$ – also related to inverse, number, fractions, proportion and algebra (e.g. $48+25 = 48+2+25-2 = 73$; $50+23 = 73$; $2/3 = 2/3 \times 2/2 = 4/6$). • <i>Inverse relation for $-$, \div / direct relationship $+$, \times.</i> The higher the number the smaller the result (e.g. $12\div 2 = 6 > 12\div 3 = 4$; $1/2 > 1/3$); the higher the number the higher the result (e.g. $4+3 < 4+7$). • <i>Backtracking.</i> Using inverse to reverse and solve problems (e.g. $2y+3 = 11$ means $y\times 2+3$, so answer is $11-3\div 2 = 4$). • <i>Basic fact strategies.</i> Counting, doubles, near 10, patterns, connections, think addition, think multiplication. • <i>Operation strategies.</i> Separation, sequencing and compensation. • <i>Estimation strategies.</i> Front end, rounding, straddling and getting closer.
Measurement big ideas	<ul style="list-style-type: none"> • <i>Concepts of measure.</i> Length, perimeter, area, volume, capacity, mass, temperature, time, money/value, angle. • <i>Notion of unit.</i> Understanding of the role of unit in turning continuous into discrete. • <i>Common units.</i> Must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60). • <i>Inverse relation.</i> The bigger the unit, the smaller the number (e.g. 200 cm = 2 m). • <i>Accuracy vs exactness.</i> Same as Global principle (e.g. cutting a 20 cm strip usually does not give a length of exactly 20 cm). • <i>Attribute leads to instrumentation.</i> The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring). • <i>Formulae.</i> Perimeter, area, volume formulae. • <i>Using an intermediary.</i> Using string to compare length of a pencil with distance around a can.

Geometry big ideas	<ul style="list-style-type: none"> • <i>Concepts.</i> All types of angles, lines, 2D shapes and 3D shapes, flips-slides-turns, symmetries, tessellations, dissections, congruence, coordinates (Cartesian, polar), plotting graphs (slope, y-intercept, distance, midpoint), types of projections, similarity, trigonometry, topology, networks. • <i>Formulae.</i> Angle, length, diagonal and rigidity formulae and relationships – interior angle sums, Pythagoras, trigonometry (sine, cosine and tangent), number of diagonals, number of lines to make rigid. • <i>Reflection and rotational relationship.</i> Number of rotations equals number of reflections; rotation angle double reflection angle (holds for symmetry and Euclidean transformations). • <i>Euler's formula.</i> Nodes/corners plus regions/surfaces equals lines/edges plus 2 (holds for 3D shapes and maps). • <i>Transformational invariance.</i> Topological transformations change straightness and length, projective change length but not straightness, and Euclidean change neither. • <i>Visualising.</i> Mental rotation, choosing starting piece.
Statistics and probability big ideas	<ul style="list-style-type: none"> • <i>Tables and graphs.</i> Types of charts and tables, comparison graphs, trend graphs and distribution graphs. • <i>Concept of probability.</i> Chance (possible, impossible and certain), outcome, event, likelihood. • <i>Inference concepts.</i> Variation, error, uncertainty, distribution, sample, and inference itself. • <i>Experimental vs theoretical.</i> Knowing when something can be calculated or determined by trials. • <i>Equally likely outcome.</i> Outcomes as a fraction by number giving result ÷ total number. • <i>Formulae.</i> Mean, mode, median, range, deviation, standard deviation, quartiles. • <i>Integration of different knowledges.</i> For example, question <i>Do typical Year 7 students eat healthily?</i> requires some form of data gathering, determining typical, and determining healthy eating.
Pedagogy big ideas	<ul style="list-style-type: none"> • <i>Interpretation vs construction/Generation vs illustration.</i> Things can either be interpreted (e.g. what operation for this problem, what properties for this shape) or constructed (write a problem for $2+3=5$; construct a shape of 4 sides with 2 sides parallel) – activities should generate students' knowledge not illustrate teachers'. • <i>Connections lead to instruction/Seamless sequencing.</i> Two connected ideas are taught similarly and progress from one to the other should not involve changing rules. • <i>Pre-empting and peel back/Compromise and reteaching.</i> Look forward and back – teach for tomorrow and rebuild from known – be aware what ends and what lasts forever and rebuild ideas not lasting. • <i>Gestalt leaps and superstructures.</i> Look out for ways of accelerating knowledge. • <i>Language as labels/Construction before explanation.</i> New ideas to be constructed not told. • <i>Unnumbered before numbered.</i> Big ideas are best started in situations without number. • <i>Creativity.</i> Let students create own language and symbols (particularly to support pattern). • <i>Triadic relationships.</i> When three things are related, there are three problem types (e.g. $2+3=5$ can have a problem for: $?+3=5$, $2+?=5$, $2+3=?$). • <i>Problem solving.</i> Metacognition, thinking skills, plans of attack, strategies, affects, and domain knowledge. • <i>RAMR cycle.</i> All components of RAMR cycle are big pedagogy ideas.

3.3 RAMR framework

The AIM project is based on a philosophical model that has emerged from an analysis of the nature of mathematics. This model provides a framework for a pedagogic cycle by using the relationships between reality and mathematics as a cycle for planning. This section describes the model, its features and its components.

Philosophical model

To have an approach to teaching mathematics that takes account of the cultural capital students bring to the classroom and negates the traditional Eurocentric nature of school mathematics, it is necessary to consider the nature of mathematics. Mathematics starts from observations in a perceived *reality*. An aspect of a real-life situation is selected and *abstracted* using a range of mathematical *symbols*. The resulting *mathematics* is used to explain reality and solve problems. It is validated and extended by being *critically reflected* back to reality. The cycle from reality to mathematics and back means that abstraction and reflection are *creative* acts; the invented mathematics as a structure, language and problem-solving tool is built around *symbols*; and the mathematics and how it is used in reality is framed by the *cultural bias* of the person creating the abstraction and reflection. The act of abstraction requires learners to move from reality to symbols, and the act of

reflection requires learners to extend this knowledge by relating symbols back to reality. This cyclic process is encapsulated in Figure 7.

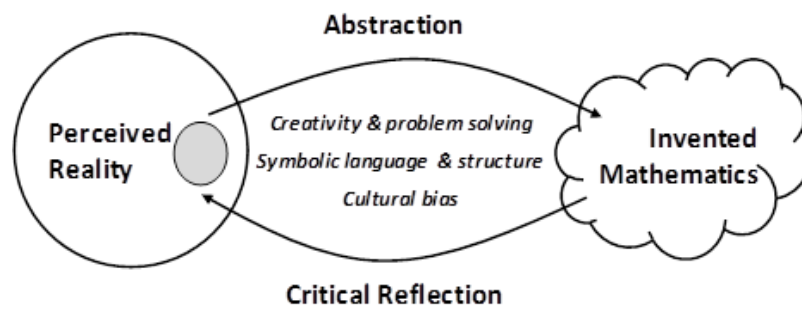


Figure 7 Relationship between perceived reality and invented mathematics (adapted from Matthews, 2009)

Features

Creativity, symbols and cultural bias are features of the model in Figure 7. The first, *creativity*, is particularly evident in the abstraction and critical reflection cycle. It is important to note that this cycle is similar to other artistic pursuits such as dance, music, painting and language as different forms of abstractions. Therefore, mathematics can be considered as another art form and, in theory, relates to these other forms of abstractions. In essence, it is possible to develop empowering pedagogy that allows students to be creative and express themselves in the mathematics classroom. This would allow students to learn mathematics from their current knowledge (i.e. from the students' social and cultural background), thereby providing agency through creativity and ownership over their learning.

As a product of the abstraction process, *symbols* and their meanings are important features of the model since they connect the abstract representation with reality. However, it is common that students do not make these connections easily and view mathematics as just sums with no real meaning. This is further exacerbated for students when they first learn algebra, and letters are suddenly introduced into mathematics without any obvious reason except that we are now learning algebra. Interestingly, focusing on creativity within mathematics provides the opportunity for students to generate their own symbols to represent their understanding of the mathematical process. These symbol systems can then be compared to and assist in understanding the meanings of current symbols, symbolic language and their connection to reality. In addition, this can also lead to the teaching and learning of the underlying structure of mathematics, providing students with a holistic view of mathematics.

The third feature, *cultural bias*, exists in all aspects of the abstraction and critical reflection cycle. The observer expresses their cultural bias in the way they perceive reality and decide on which aspect of reality they wish to focus. In the abstraction process, the form a symbol takes and the meanings that are attached to this symbol or group of symbols is biased by a cultural perspective. Finally, the critical reflection processes are underpinned by the cultural bias within the abstraction process and the observer's perception of reality. If we have an understanding and appreciation of the cultural bias within mathematics, new innovative pedagogy can be developed that moves beyond some cultural biases so that students can relate to mathematics but also gain a deep understanding for the current form of mathematics and how mathematics is used.

Components

The philosophical relationship of Figure 7 can be deconstructed into four components: reality, abstraction, mathematics, and reflection. The nature of each of these components is as follows.

Reality

The reality component of the cycle is where students: (a) access knowledge of their environment and culture; (b) utilise existing mathematics knowledge prerequisite to the new mathematical idea; and (c) experience real-world activities that act out the idea. The focus in this component is to connect the new idea to existing ideas

and everyday experiences. Among the kinaesthetic, physical and visualisation activities that predominate in this component, it is vital that students be provided with opportunities to generate their own experiences and verbalise their own actions. This generation and verbalisation provides the students with ownership over their understanding of the mathematical idea.

Abstraction

The abstraction process is where students experience a variety of representations, actions and language that enable meaning to be developed that carries mathematical ideas from reality to abstraction. Representations, actions and language will predominantly be as in Figure 8 below, however students should also be provided with opportunities to create their own representations, including language and symbols, of the mathematical idea that has been initially experienced through physical activity. This allows students to have a creative experience that will, firstly, develop meaning and, secondly, attach it to language and symbols. The sharing of other students' representations provides students with alternative views of the same idea attached to varied symbolic representations. Discussions on the use of different symbols enables students to: (a) critically reflect on their journey (enabling them to justify and “prove” their ideas); (b) understand the role of symbols in mathematics (enabling them to understand the relation between symbol, meaning and reality); and (c) be ready to appropriate (Ernest, 2005) the commonly accepted symbols of Eurocentric mathematics.

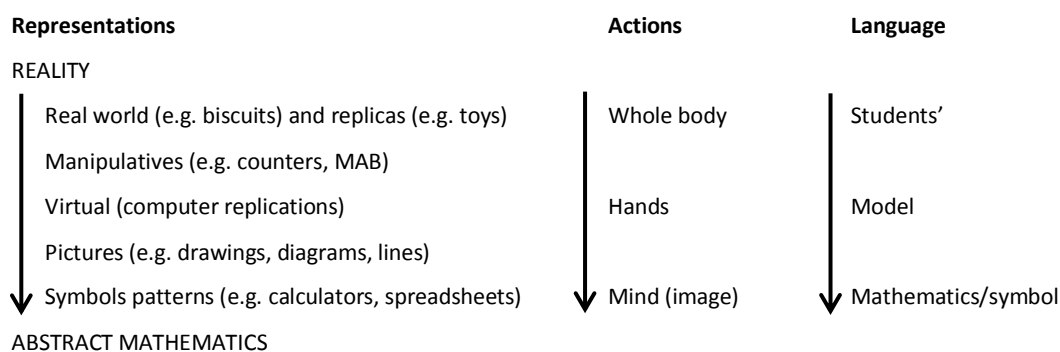


Figure 8 Abstraction sequence from reality to mathematics

The act of abstraction requires the learner to generalise a mathematical idea from examples in the world to symbols in the artificial world of mathematics. It means that the learner has to move from reality to symbols, for example, connecting the real-life situation of three children joining two children to make five children with the symbols $2 + 3 = 5$. The recommended way to do this is to move through a sequence of representations of the mathematical idea from reality to abstract (as in Figure 8).

The representations can be external (real-world activities, materials, images, pictures, language and symbols) or internal (mental images of external representations), with learning occurring when structural connections are made between the two (Halford, 1993). The external representations facilitate the internal representations while accompanying language and actions become increasingly abstract (as in Figure 8).

Mathematics

The mathematics component of the cycle is where students: (a) appropriate the formal language and symbols of Eurocentric mathematics; (b) reinforce the knowledge they have gained during the abstraction; and (c) build connections with other related mathematical ideas. The focus is to assist students to construct their own set of tools (filling their “mathematical toolbox”) that will enable them to recognise and recall mathematical ideas from the language and symbols associated with the ideas, thus adding to their bank of accessible knowledge. The connections between new and existing ideas enable better recall of mathematical ideas and improve problem solving. It is easier to remember ideas in terms of how they are related to each other (structural understanding) than as many disconnected pieces of information. The ideas that help in problem solving are often connected peripherally to the central idea to which the problem refers.

Reflection

The critical reflection process is where the new mathematical ideas are: (a) considered in relation to reality in order to validate/justify understandings; (b) applied back into reality in order to solve everyday life problems; and (c) extended to new and deeper mathematical ideas through the use of reflective strategies, namely, flexibility, generalising, reversing and changing parameters. As well as reflecting on the mathematics they have learnt in relation to the world they live in, this process involves students' consideration of the journey from reality to mathematics via abstraction that they took in developing the mathematical ideas. It requires reflection on what they learnt, how they learnt it, and why they learnt it. It also requires them to justify their outcome.

Reflection is more powerful than it seems at first glance. It requires the learner to validate their mathematics learning against their everyday life, thus generating ownership of the knowledge. However, it is also a method of extending learning as the reflection acts on the abstracted mathematics in relation to reality. For example, students can reflect on $3 + 4 = 7$ and see that if one addend, say the 3, was reduced by 2, then the sum, 7, has to be reduced by 2 to keep the equation equal, the beginning of the balance rule. The extension of knowledge through critical reflection can be assisted by the use of four strategies: flexibility, generalising, reversing and changing parameters.

Along with abstraction, reflection forms an important cycle (thesis-antithesis-synthesis) with perceived reality and mathematics. It is through this cycle that mathematics knowledge is *created*, *developed* and *refined*. Mathematical knowledge is created (the thesis) by abstraction from perceived reality. This knowledge is trialled within itself for consistency (proof) and against reality for effectiveness (application). Problems that emerge in proof or application (the antithesis) are used to amend the mathematics (the synthesis) and the cycle continues.

3.4 RAMR cycle

AIM advocates using the four components of Figure 7, Reality–Abstraction–Mathematics–Reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language, and symbolic representations, i.e. body \rightarrow hand \rightarrow mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem solving, flexibility, reversing and generalising (see Figure 9 on right). The innovative aspect of RAMR is that the right half develops the mathematics idea while the left half reconnects it to the world and extends it. For example, whole-number place value built around the **pattern of threes** where hundreds-tens-ones applies to ones, thousands, millions, and so on, can be easily extended to metrics by considering the ones to be millimetres, the thousands to be metres and the millions to be kilometres.

Planning the teaching of mathematics is based around the RAMR cycle if it is deconstructed into components that are applied to a mathematical idea. By breaking instruction down into the four parts and taking account of the pedagogical approaches described earlier, the cycle can lead to a structured instructional sequence for teaching the idea. Table 3 briefly outlines how this can be done. Prerequisite mathematical ideas are considered in the reality and mathematics components of the cycle, while extensions and follow-up ideas are considered in the reflection component.

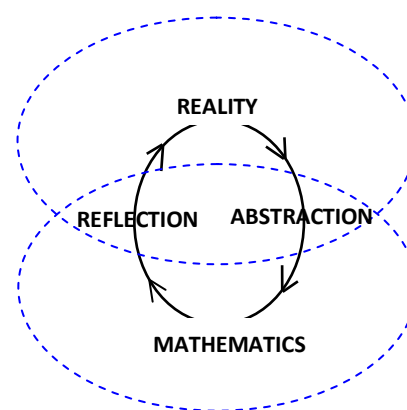


Figure 9 RAMR cycle

Table 3 Using the RAMR cycle to plan mathematics instruction for a mathematical idea

REALITY <ul style="list-style-type: none"> • Local knowledge: Identify local student cultural-environmental knowledge and interests that can be used to introduce the idea. • Prior experience: Ensure existing knowledge prerequisite to the idea is known. • Kinaesthetic: Construct kinaesthetic activities, based on local context, that introduce the idea.
ABSTRACTION <ul style="list-style-type: none"> • Representation: Develop a sequence of representational activities (physical to virtual to pictorial materials to language to symbols) that develop meaning for the mathematical idea. • Body-hand-mind: Develop two-way connections between reality, representational activities, and mental models through body → hand → mind activities. • Creativity: Allow opportunities to create own representations, including language and symbols.
MATHEMATICS <ul style="list-style-type: none"> • Language/symbols: Enable students to appropriate and understand the formal language and symbols for the mathematical idea. • Practice: Facilitate students' practice to become familiar with all aspects of the idea. • Connections: Construct activities to connect the idea to other mathematical ideas.
REFLECTION <ul style="list-style-type: none"> • Validation: Facilitate reflection of new idea in terms of reality to enable students to validate and justify their new knowledge. • Applications/problems: Set problems that apply the idea back to reality. • Extension: Organise activities so that students can extend the idea (use reflective strategies – <i>flexibility, reversing, generalising, and changing parameters</i>).

3.5 Related or prior pedagogies

The RAMR cycle is related to and has been constructed from mathematics pedagogies that have been commonly and successfully used in mathematics for many years. Five of these are briefly described as follows.

Wilson cycle

The Wilson cycle (see Ashlock et al., 1983) is a pedagogy that has been used by YDC staff for many years with success. The RAMR cycle was based on ideas from the Wilson model, particularly in abstraction, mathematics and reflection. However, RAMR has the added perspective of emphasising building from reality back to reality. The Wilson cycle specifies six steps (Figure 10 on right): (a) teach the idea informally (real-world situations, representations and informal language); (b) introduce the formal mathematics language and symbols; (c) undertake activities specifically to connect the new knowledge to existing knowledge; (d) practice (games, practice activities and worksheets); (e) apply knowledge to solve problems; and (f) see if students can undertake activities that can extend knowledge to new knowledge without having to go through all the steps. The cycle also advocates continuous *checking and diagnosis* of students' understandings to ensure no errors become habituated.

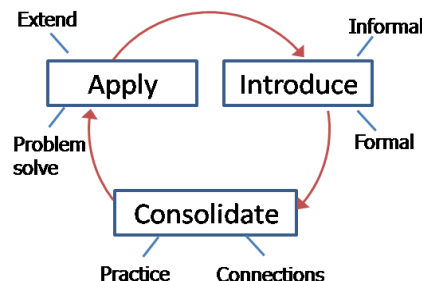


Figure 10 The Wilson cycle

Payne and Rathmell triangle

As a framework, Payne and Rathmell's (1977) triangle model relates real-world situations, representations, language and symbols (see Figure 11 on right). Real-world situations are identified and modelled with body, hand and mind. The physical, pictorial and virtual materials, and accompanying mental-visual models, are connected to language and symbols, studied and reinforced as two-way connections. The abstraction process in RAMR is designed to cover the ideas in this framework.

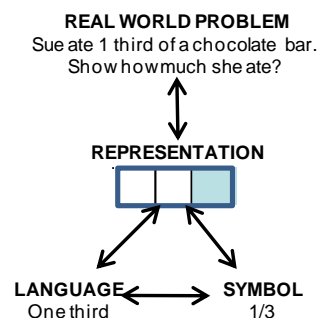


Figure 11 Payne and Rathmell triangle framework for teaching fraction $\frac{1}{3}$

Multi-representational instruction

This approach involves using many representations in teaching episodes (e.g. MAB, place-value charts, calculators and pen/paper) to relate model with language and symbol. It involves continuously linking representations. As argued by Duval (1999), mathematics comprehension results from the coordination of at least two representational forms or registers; the multifunctional registers of natural language and figures/diagrams, and the mono-functional registers of notation systems (symbols) and graphs. Learning is deepest when students can integrate registers. Multi-representations work well with the Payne and Rathmell triangle and are the basis of the abstraction component of the RAMR cycle.

Knowledge types

The four components of the RAMR cycle were also a product of Baturu's (1998) modification of Leinhardt's (1990) four knowledge types: (a) entry (knowledge of mathematical ideas before instruction – from experience); (b) representational (knowledge of physical materials, virtual materials and pictures used to develop the ideas); (c) procedural (knowledge of definitions, rules and algorithms); and (d) structural (knowledge of relationships and concepts). All four types have to be developed in a unit of work with the final goal being structural.

It is easy to see that these four knowledge types are similar to the RAMR components – entry is reality, representational is abstraction, procedural is in the first part of mathematics, and structural is in the last part of mathematics joined with reflection.

Available and accessible knowledge

There are two ways to make errors: (a) not having the knowledge available (i.e. not having learnt the knowledge); and (b) having the knowledge available but not accessing it (i.e. having learnt the knowledge but not realising that it can be applied in the task situation). Thus, mathematics teaching needs to build strong available **and** accessible mathematics knowledge. In particular, accessible knowledge is very productive as it is a form of deep learning and enables performance to continue into higher mathematics.

Teaching for accessible knowledge requires more than teaching for available knowledge. Accessing mathematics knowledge means knowing it on its own merits and knowing when, where and how it can be applied to task situations. Thus, accessible knowledge is based on mathematical ideas being held in *rich schemas*. Such schemas have four characteristics:

- (a) they **define** – completely describe all the ways the mathematical idea can be thought of;
- (b) they **connect** – store knowledge so that all relationships are evident and all related knowledge is connected;
- (c) they **apply** – contain all the different ways the mathematical idea can be used in reality; and
- (d) they **remember** – organise and store all experiences students have had with the mathematical idea.

The RAMR cycle has been designed to be effective in developing rich schema and, therefore, accessible knowledge. It involves abstraction and reflection and begins and ends with reality thus ensuring mathematical ideas are well defined, connected, applied and experienced. It identifies with techniques particularly useful for accessibility such as explicitly relating the mathematical ideas to as wide a collection of out-of-school experiences as possible and teaching that mathematics and its symbols form a highly connected structure, a language to describe the world and a set of tools for solving the world's problems.

Overall, the secret to accessible knowledge is to do **both sides** of the RAMR cycle: *abstract* the mathematical idea and *practise* the idea but not to stop there, to draw a breath and move on to *connections* and *reflections*. In particular, the **reflective strategies** are useful; *flexibility* gives the students a rich set of applications (particularly non-prototypic activities), *generalising* integrates knowledge which means there are fewer things

to apply, *reversing* often means that real-world instances are constructed (the best way to teach interpreting the world), and *changing parameters* builds connections.

As well as improving applications and problem solving, improving accessible knowledge has the potential to improve results in testing which is more than just repeating known procedures. One of these is the NAPLAN tests.

3.6 Associated pedagogies

There are other pedagogies that are important to an effective RAMR lesson. These add to the components to build stronger learning outcomes. Four of these are described below.

Levels of instruction

Baturo et al. (2007) identified three levels of instruction that need to be taken into account in lesson planning: (a) *technical* – becoming familiar and proficient with the use of materials; (b) *domain* – knowing what materials and what activities will provide experiences effective for learning the topic being taught; and (c) *generic* – knowing instructional strategies that hold for all topics. These three levels of instruction should be taken into account for all activities within the RAMR cycle.

Four of the most important generic strategies have particular application to the reflection component of RAMR:

- (a) **flexibility** (experiencing the mathematical idea many ways);
- (b) **reversing** (teaching in the opposite direction – whole to a fraction and then fraction to a whole);
- (c) **generalising** (developing the idea into a generality); and
- (d) **changing parameters** (considering what would happen if something changed).

As well, most other components of RAMR are also important; for example, starting from what the students know and are interested in, and making connections. In fact, the RAMR model is itself a generic strategy as it can be used with all topics.

However, these are not the only such strategies. For example, continuously diagnosing students' knowledge, and asking students to reflect on what they have learnt are also important generic strategies.

Learner-centred principles

Overall, effective mathematics teaching means taking account of good principles of teaching and learning in mathematics. A good set of principles has been developed from a review of the literature by Alexander and Murphy (1998) as follows. These should always be taken into account when using the RAMR model.

1. **Knowledge base** – one's existing knowledge serves as the foundation of all future learning by guiding organisation and representations, by serving as a basis of association with new information, and by colouring and filtering all new experiences.
2. **Situation/context** – learning is as much a socially shared knowledge as it is an individually constructed enterprise.
3. **Development and individual differences** – learning, while ultimately a unique adventure for all, progresses through various common stages of development influenced by both inherited and experiential/environmental factors.
4. **Strategic processing** – the ability to reflect upon and regulate one's thoughts and behaviour is essential to learning and development.
5. **Motivation and affect** – motivational or affective factors, such as intrinsic motivation, attributions for learning, and personal goals, along with motivational characteristics of learning tasks, play a significant role in the learning process.

Inquiry approaches

Allowing students to explore topics, discuss and arrive at their own conclusions is also important, as the following three examples show. First, inquiry is important in mathematics lessons because it is a good way of enabling students to construct their own knowledge in relation to discussion with peers and teachers (Cobb, 1994; English & Halford, 1995). This *social constructivism* is a highly recommended way to run lessons.

Second, inquiry can be enhanced by the inclusion of instruction in what some educators call *cognitive metaphors*. This is something that is done before the mathematics is taught. It involves teaching students about how their minds work and learn, and how they can keep track of their learning. With this knowledge, students can approach mathematics instruction with an understanding of themselves as learners and an ability to monitor and measure their own learning.

Third, at its best, the inquiry approach eschews the teacher as “sage on the stage” and, to a lesser extent, as “guide on the side” and advocates a new role of “meddler in the middle” (McWilliam, 2005). It co-opts students to be **complicit** in their own learning by making the students *co-constructors* of their knowledge (Claxton, 1999).

Inquiry approaches that use social constructivism, cognitive metaphor and/or co-construction, are useful pedagogies to integrate with RAMR for strong learning. Of course, it is important that inquiry is undertaken in classrooms where debate and discussion is open to all students. This usually involves: (a) not making common conclusions or right answers the necessary end point of the activity, (b) ensuring all students are culturally safe, and (c) having the highest expectations that the students can handle the work.

Metacognition

Many of the pedagogies above require students to know **when** to use a mathematics idea as well as what idea to use and how to use the idea. For example, the particular difference between available and accessible knowledge is that accessible understanding knows when to use knowledge as well as how to use it. The basis of this “when” is *metacognition*, that is, **awareness and control** of own thinking particularly with regard to the ability to plan, monitor, evaluate and make decisions with regard to progress on an application of a mathematical idea.

Metacognition for application of knowledge can be enabled with two teaching strategies, namely, *match-mismatch* (Charles & Lester, 1982) and *metacognitive training* (Brown, 1995). Match-mismatch involves three explicit teaching steps: (a) teach the knowledge; (b) teach when to use the knowledge (i.e. for what problem types); and (c) provide experiences that match and mismatch situations (first providing tasks on all of which the idea can be used, and then tasks on only some of which the idea can be used to give the students experience of “when”).

Metacognitive training involves directly teaching students models of how learners think and to use these models to keep a record of their thinking (with the view of enabling them to be aware of and control their thinking). The training can be supported by activities, namely “random reporting”, “think-pair-share”, social interaction roles, and reciprocal teaching as follows.

1. **Random reporting.** Problems are given to groups to solve; the group is told that, when they are finished, any member of the group could be chosen to report, so they have to ensure all members understand the problem and solution.
2. **Think-pair-share.** Give students a problem which has more than one possible way to solve it and ask them to work alone; after a time, put into pairs and ask them to listen to each other’s answers and then come to a common conclusion; a little further along, put students into groups of four and ask each group to come up with a common best answer – students’ arguments about who is right make students aware of their thinking.
3. **Social interaction roles.** Place students in groups and assign a role to each member – one recommended way is groups of three with one student designated as leader (they make the decision if there is dissension), one as recorder (they write down and report the group’s findings – only they can use a pen);

and one as checker (they check the others' thinking); change the roles for different problems and ensure students stay in their roles; the delineation of the roles causes all to consider the other students' thinking as well as their own.

4. **Reciprocal teaching.** Students take turns to be the teacher of their group; these "teacher" students are taken out and given the task they have to teach to other students (along with teaching hints); students have to prepare a plan to ensure all students in their group learn; the role of teacher requires students to consider their own and other students' thinking.

4. *Cultural and School Change Implications*

The previous chapters have stated that an important component of AIM is to take account of the cultural differences between the underperforming students and the middle-class Western culture of the school and to ensure there is a focus on school change and community involvement in relation to the PD and other support sessions. This chapter focuses on cultural implications, PD and school change, and teacher knowledge for positive change.

4.1 Cultural implications

The AIM project is to improve mathematics performance of students who have underachieved and who may not reflect the dominant culture of both school and Australian society. This may include students who are Indigenous, as well as low SES students from single and unemployed parent homes.

Indigenous students

The underachievement of Aboriginal and Torres Strait Islander students is, in part, a consequence of being part of a dispossessed people who have been considered by the dominant culture as primitive with no value for a modern society. This has implications for the way mathematics is taught to Aboriginal and Torres Strait Islander students. The devaluing of Indigenous cultures still continues today; the notion perpetuated by the education system that humankind evolved from hunter-gathers to technologically advanced societies does not provide a sense of pride for Indigenous students about their culture. It ignores the reality that Aboriginal and Torres Strait Islander people have powerful and sophisticated forms of mathematical knowledge that enable the complexity evident in total ecosystems to be understood. It leads to disengagement and non-attendance.

Aboriginal and Torres Strait Islander students predominantly come to school with a home language which is not standard English and with knowledge, skills, and patterns of interaction that are not appreciated by schools and do not match what facilitates success in school. This mismatch is particularly evident in the way mathematics is taught in schools. Aboriginal and Torres Strait Islander students tend to be active holistic learners, appreciating overviews of subjects and conscious linking of ideas (Grant, 1998). In fact, Indigenous people have been characterised as belonging to *high-context* culture groups using a holistic (top-down) approach to information processing in which meaning is extracted from the environment and the situation. In contrast, mainstream Australian culture is characterised as a *low-context* culture and uses a linear, sequential building block (bottom-up) approach to information processing in which meaning is constructed (Ezeife, 2002). School mathematics is traditionally presented in a compartmentalised form where the focus is on the details of the individual parts rather than the whole and relationships within the whole, a form of presentation that disadvantages Aboriginal and Torres Strait Islander students.

Low SES students

Historically, educational institutions have favoured higher to middle-class backgrounds, beliefs and practices. This is due to a number of factors including the history of the purpose of schooling across its development and the socio-economic backgrounds of the majority of teachers and curriculum developers (Meadmore, 1999). As such, there are pre-existing patterns of communication and interactions (or discourses) endemic to education in Australia which are not favourable to lower SES students (Meadmore, 1999). Thus, the middle-class Eurocentric culture of Australian schools and implicitly understood patterns of communication and interactions serve to further marginalise students from low socio-economic backgrounds from school mathematics. The nature of discourses within school practices do not always successfully link to nor validate mathematical practices that may be part of low SES students' out-of-school experiences (Baker et al., 2006) leading to insufficient links being made between students' existing mathematical knowledge and practices and school

mathematics. In these cases, students may disassociate from school mathematics and feel they cannot succeed, particularly if their home skills and knowledge are not valued nor actively sought (Thomson, 2002).

Expectations may also pose difficulties for low SES students as for Indigenous students. Low SES parents may perceive mathematics as alienating and unnecessary or too difficult for their children to learn; this can lead to students not expecting to succeed in mathematics, having low expectations of themselves and their future roles in society, and thus not participating in mathematics classrooms. Teachers may also have low expectations of low SES students and often believe that lower level or *life* numeracy is all that is needed for these students (Baker et al., 2006). The resulting emphasis of mathematics for these students becomes utilitarian, rote and procedural mathematics tasks that are not explicitly related to overarching mathematical structures.

Strengths and weaknesses of Eurocentric mathematics

Interestingly, the Eurocentric mathematics that is taught in Australian schools has weaknesses due to its cultural bias. Because of the way their culture was developing, European societies developed mathematics to help them explain their world and solve their problems, particularly to explain space, time, and eventually, number. When trading became a way of life, a need developed to be more precise in representing and quantifying value to have a shared agreement of how values could be compared (“is mine worth more than yours”), a more sophisticated process than quantification as it involves rate (e.g. 3 cows = 1 boat). Over time, the European mathematics’ quantification and comparison system grew to encompass a variety of numbers (common and decimal fractions, percentages, rates and ratios), measures of time and shape (length, area, volume, mass and angle), and two operations (addition and multiplication) and their inverses (subtraction and division). The system was also generalised to findings that hold for all numbers and measures, and the resulting mathematics area of algebra has grown in importance as science and technology has expanded.

The weakness of European mathematics lies in its strength, the success of its quantifying and comparing systems in underpinning the growth of science and technology. This has resulted in longer and healthier lives and the devices that support work, home life and leisure. Western society now has the tools to change our environments to make living better; we can cool the hot, warm the cold, clear the land, bring in new plants and animals, and clothe, shelter, and feed large populations. However this triumph has affected European culture and society. Success has come to mean increasing numbers and continued growth; smaller numbers and negative growth are to be avoided (and are given names that signify failure, such as “recession”). The culture appears to have little ability to understand harmony and act sustainably; it tries to dominate the land, the sea, the weather, and the animals, birds and fish, with little understanding of how things interact to allow human life, resulting in poverty, hunger, war, pestilence and climate change. Mathematics can be developed which would reinforce planetary equity and sustainability (see Figure 12); however, such mathematics requires less dominance by number, less need for growth, and an emphasis on living in harmony with land and sea; a non-Eurocentric form of mathematics.

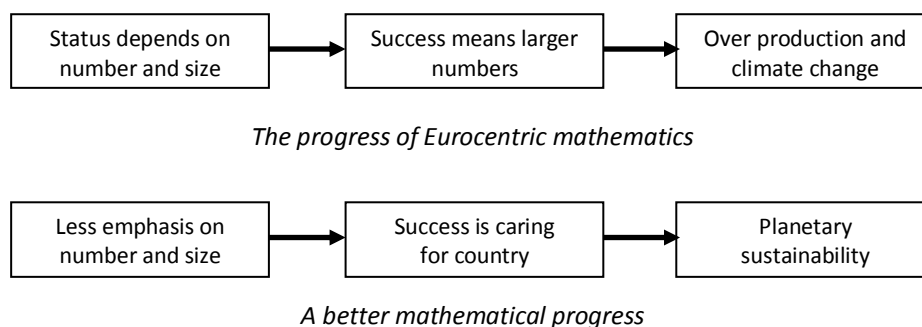


Figure 12 Two perspectives of progress

Implications for teaching culturally diverse students

Traditional teaching of mathematics can confront Indigenous and low SES students' cultures and perpetuate the belief that success in mathematics classrooms requires the rejection of culture (i.e. that one has to become "white" or middle class). To be effective, mathematics teaching has to enhance mathematics outcomes but retain pride in culture and heritage. Approaches that seem to be effective are as follows.

1. **Discuss the role of mathematics.** Confront the Eurocentric nature of traditional school mathematics by making students aware of the cultural implications of mathematics teaching, and draw attention to which mathematics ideas change perceptions of reality. Discuss the role of mathematics in European culture and draw attention to its strengths and weaknesses. Make mathematics available to all students.
2. **Legitimise and contextualise the mathematics.** Legitimise local Indigenous or other cultural knowledge and integrate students' culture and mathematics instruction, to match the mathematics classroom to the cultural capital brought by the students (Bourdieu, 1973). Contextualise mathematics into the life and culture of students, use models and activities from the everyday lives of the students. Use a local cultural framework for learning. Take account of English not being the first or home language of students; develop language and be aware of different meanings for mathematics words.
3. **Modify teaching pedagogies.** Present mathematics as a holistic structure that can empower the learner by focusing on big ideas and using instructional strategies that relate acting, creating, modelling and imagining. Modify teaching pedagogies to include cultural perspectives.
4. **Integrate with whole-school changes.** Realise that approaches to improve mathematics learning cannot stand alone, and they need to be allied with whole-school activities that include the local community, challenge attendance and behaviour, have high expectations, develop local leadership and give a strong role to local teacher aides.

4.2 PD and school change

Changing the teaching in Year 7 requires a change at teacher and school level. The AIM project requires Year 7 teachers to teach below year level and to only teach certain topics. It also requires effective and pertinent PD and leadership for change.

PD and teachers' practices

AIM sees PD and changing teachers' classroom practices as being a cycle of affective readiness for change, pertinent external input, effective classroom trials, positive student outcomes, and supportive reflective sharing that leads to further readiness and so on (see Figure 13).

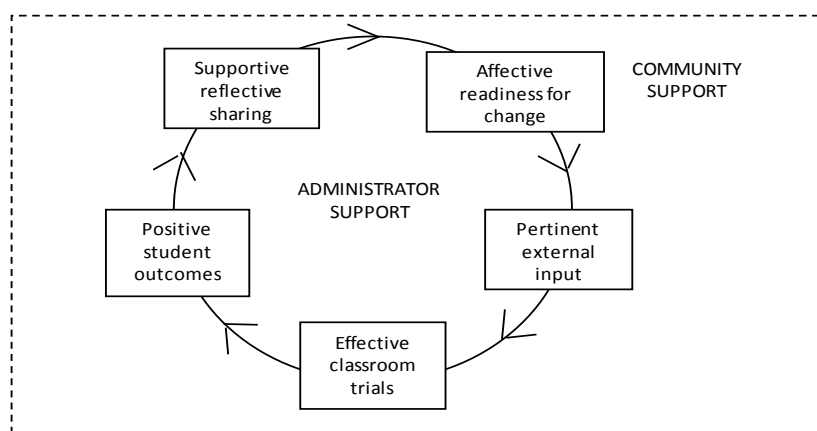


Figure 13 The YDC effective PD cycle (adapted from Clarke & Peter, 1993)

AIM acknowledges the importance of the **interaction** between researcher input and teacher need and readiness for this input, and the **role of success** (in terms of student outcomes) when trialling new ideas. It recognises that

positive student responses along with initial readiness are crucial to successful change and that these are facilitated by: (a) pertinent, relevant and innovative ideas and resources at input (the AIM resources and PD workshops); (b) just-in-time support before and during classroom trials (assistance with planning, visits to model teaching approaches and provide feedback on teaching); (c) data gathering in an action-research process during classroom trials that can be seen as positive in terms of student outcomes; and (d) opportunities for teachers to share their successes and to attribute them to their ability. It also recognises the important role of the principal (and other administrative staff) and the local community. Without the support of the principal and other senior administrators, few if any interventions succeed in changing school practices. The same is true without the support of the local community.

AIM recognises that input through resources and PD workshops is a long way from student outcomes. Thus, for the project to be transformational within schools and improve student outcomes in mathematics, it requires six tiers of interaction, namely: (a) the community; (b) external researchers (YDC staff); (c) systems; (d) school administrators (principals and senior administrators such as heads of department and/or heads of curriculum – HODs and HOCs); (e) classroom teachers; and (f) students. YDC staff have control over resources and PD and, through visits, some involvement in school policies and classroom teaching. However, much of the teaching that affects student outcomes will be undertaken by teachers without involvement of YDC staff. And it is this teaching that determines the central outcome of student success that determines effectiveness of intervention. This distance between YDC staff and student outcomes gives importance to other people; thus meaning that we have to consider: (a) key people in delivery (particularly principals, administrators and teachers); (b) parents, carers and other community members; and (c) students.

School change and leadership

The provision of new mathematics teaching ideas is often insufficient for sustainable improvement in underachieving students' learning of mathematics. The new ideas have struggled to have positive effects when low attendance and negative behaviour are endemic across a school, when school practices and learning spaces disengage students, when positive partnerships are not formed between teachers and their teacher aides, when classrooms do not involve community leaders or acknowledge local knowledge, and when teachers do not believe that the students are capable of the work. The ideas have been successful when they have been integrated into whole-school changes that challenge attendance and behaviour, integrate and legitimise local community knowledge, build in practices to support the culture of the students, and change teacher attitudes towards and relationships with the students. Thus, the AIM project is much more than a set of new teaching ideas; it integrates: (a) a particular teaching–learning approach to mathematics (i.e. RAMR) that is designed to empower students within their culture; and (b) an approach to PD and school change that is designed to facilitate change to support community involvement and student engagement.

The AIM approach to school change and leadership is based on the belief that schools with students underachieving in mathematics can only enhance mathematics learning with a program that focuses on mathematics and on school change together. In simple terms, AIM believes that schools should see themselves as part of, not apart from, the communities in which their students live, and should see their role in terms of YDC's vision, as *growing community through education*. AIM believes that school change can have a profound effect in creating emancipatory environments that actively seek to improve the educational outcomes and life chances for Aboriginal, Torres Strait Islander, low SES, and indeed **all** students, and strong school leadership plays a critical role in acknowledging the existence of this student excellence.

AIM has been influenced by the philosophy and success of the Stronger Smarter Institute which argues that school change and leadership cycles through four requirements (see Figure 14): community–school partnerships; local leadership; positive student identity; and high expectations. It aims to develop not just new capabilities but also shifts in thinking individually and collectively. Maintaining the cycle of the four requirements ensures sustained growth towards enhanced learning. It creates and sustains emancipatory environments that enhance the opportunities of the students who attend, and challenges mechanisms and processes that continue to reproduce disengagement among many students within the schooling system.

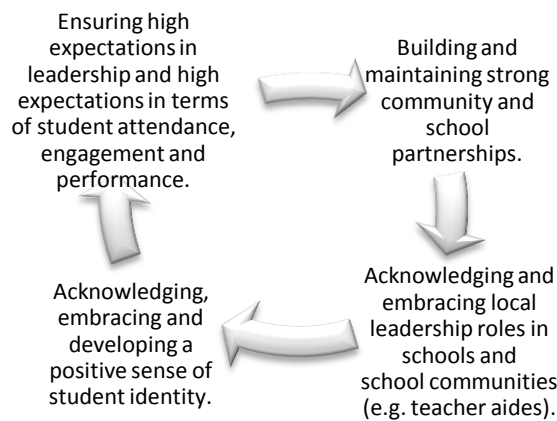


Figure 14 Cycle of school change and leadership

4.3 Teacher knowledge

Low SES students and, particularly, Indigenous students, tend to be holistic in learning style, moving from whole to parts, and not aligned with traditional procedural/algorithmic teaching which moves part to whole. To take advantage of this, the approach to mathematics teaching advocated in AIM (and in YDM that underlies it) focuses on big ideas (concepts, strategies and principles), vertically sequenced modules, and the RAMR cycle. However, this is a teaching approach that requires a lot from teachers – understanding of mathematics structure, active pedagogy and classroom control of behaviour. The AIM pedagogy also relies on teachers making decisions as to instruction themselves, based on their understanding of mathematics and their knowledge of the individual students. AIM provides teaching ideas and activities but not in the form of scripts or “recipes”.

Shulman’s knowledge types

Thus, AIM pedagogy requires teachers to know all three of Shulman’s knowledge types for effective mathematics teaching: *subject matter* (knowledge of mathematics content in terms of how it is structured, sequenced and connected), *pedagogic content* (knowledge of how to teach mathematics) and *lesson planning* (general knowledge of how to organise and run a lesson – includes behaviour management). Figure 15 illustrates these three knowledge types. To facilitate this, the activities of AIM are designed to build the capacity of teachers. However, in the past, it has taken time for teachers to come to terms with what is required to effectively implement an AIM-type mathematics program.

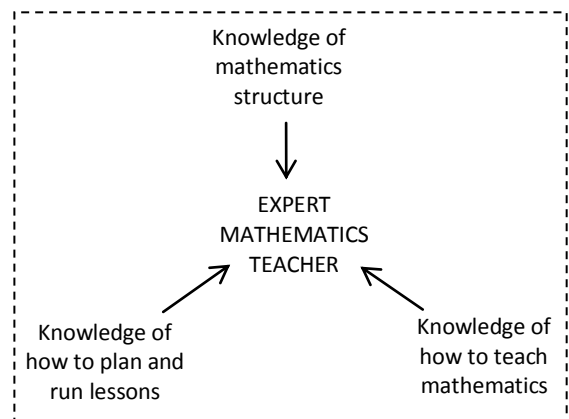


Figure 15 Shulman’s knowledge types

Similar to teachers, teacher aides need subject-matter, pedagogic-content and lesson-planning knowledge. AIM believes in providing teacher aides with the same PD as the teachers; however, the knowledge is usually placed more within the framework of the particular modules and within the framework of their role in the classroom. This does not mean simplification because the highest quality knowledge is required for one-to-one or one-to-many tutoring.

Building capacity

To build the capacity of AIM project schools to teach mathematics effectively, the plan consists of four steps (as in Figure 16 on right):

- (a) development of *mathematics resources* (with appropriate tests),
- (b) provision of *PD and school support*,
- (c) *classroom trials* of resources and *teacher feedback* on effectiveness, and
- (d) *student testing at the start and end* of the trials.

The teacher feedback and pre-post test results are used by AIM to modify materials. Thus in all versions of AIM, YDC staff always plan to: (a) write or refine mathematics resources, (b) provide PD, and (c) visit each school to meet teachers and provide school-specific PD.

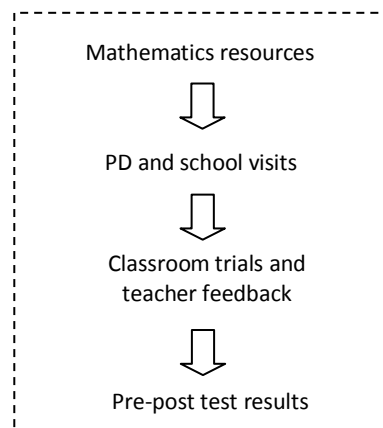


Figure 16 Student evaluation process

As described in section 2.4, pre-post tests have two purposes. The first is to indicate the starting point for instruction (the pre-test) and areas in need of further remediation (the post-test). The second is to provide baseline data for any improvements shown by comparing pre- and post-test results.

Teacher feedback

For quality control, to make the resources more effective and to enhance theory, YDC encourages all teachers trialling the AIM material to keep a record of what they do and their students' responses. We also ask teachers to keep an annotated plan of what they do. Both the plan and the record of the students' responses will help the teachers and the researchers.

Teachers who trial new ideas with a view to recording how things go will get much more out of the trials. This is because they will have systematic information on whether the new ideas are worthwhile, and information on how their teaching is going and how their practices could be improved. It takes a while to become expert with a new approach and an action-research view of what is happening really helps. It is effective in improving teacher knowledge.

However, it is also important to do this data gathering and reflection in a group, so YDC recommends that schools set up communities of practice, groups where teachers can get support for their trials and can reflect on what happened together. For example, when physical materials were introduced in primary classes in the 1970s and 80s, there were often difficulties because students were not used to working other than in rows with worksheets or textbooks; they did not know how to behave, became excited with the material and out of control. However, when given a chance to play with the material and having been taught how to act in groups with material, the new approach was seen to work.

5. Implementation Options

The reconceptualisation of the original Accelerated Indigenous Mathematics project into the new Accelerated Inclusive Mathematics project was to enable use of the AIM modules with most Years 7–9 students and not just those whose performance level was Year 3 or below. To do this, AIM has been reconceived as a project focusing on training teachers in a remedial pedagogy that is an extension of the YuMi Deadly Maths pedagogy to take account of students who are missing prerequisite knowledge to that being taught. The modules have been reconceived so that they can be both a resource for planning diagnosis and remediation and providing instructional strategies in sequence, and a complete three-year course that can replace the normal Mathematics curriculum for very underperforming students.

This chapter looks at these two forms of implementation and the training provisions by which the AIM project operates.

5.1 Forms of implementation

As described in section 1.3, there are two options for using the AIM modules and two options for the AIM project: replacement and support.

Replacement

This is the option that was used in the original DEEWR-funded Accelerated Indigenous Mathematics project. Students whose performance level is at Year 3 or below are placed in AIM classes and given the 24 modules, as in section 2.2 and Table 1. The teachers implement the 24 modules in the 12 terms from Year 7 to Year 9, approximately half a term per module, giving pre-tests and post-tests.

Characteristics

Replacement AIM does not follow the Australian Curriculum across each year. Students only study 8 topics in each year. Therefore, it may be difficult for them to leave the AIM class and go to a regular maths class. However, the program is such that they will complete all work in the Australian Curriculum up to Year 9 across the three years (see section 2.2 and Figure 2). At the end of Year 9, they will have had the same experiences as a student from a regular maths class. The program is also flexible enough that some schools using the original Accelerated Indigenous Mathematics materials have been able to place more able students out of AIM classes into mainstream classes.

The modules are based on vertical sequences of units with the first and foundation units given in detail. This allows acceleration up the module and across the units from Year 3 to Year 9 knowledge because the carefully and slowly developed foundations of the module can be used as a basis for rapid growth in later units. For example, in whole numbers, the module spends a large amount of time looking at two- and three-digit numbers in terms of the five big ideas of place value, counting, multiplicative structure, number line and equivalence. This is done slowly and completely across five units. Then, by focusing on the pattern of threes in the place-value structure of whole numbers, the understanding of whole numbers can be easily extended to millions and to a number system structure. This is because whole numbers can be seen as hundreds, tens and ones (H-T-O) of ones, H-T-O of thousands, H-T-O of millions, and so on. In this way, the properties of a three-digit number such as 256 (2 hundreds 5 tens 2 ones or two hundred and fifty-six ones) can be easily transferred to 256 thousands (2 hundred thousands 5 ten thousands and 6 thousands or two hundred and fifty-six thousands) and 256 millions (2 hundred millions 5 ten millions 6 one millions or two hundred and fifty-six millions).

Thus it is the nature of the modules (i.e. their being based on one topic and a few big ideas, using the same teaching methods and teaching materials, and sequencing from Year 3 to Year 9) that enables acceleration. In

particular, restricting each module to one topic enables the work of one week to be used the next and does not require students to remember many topics as when there is continual changing of topics, materials and ideas.

Implementation

To implement this option, schools assess their students and place those with low levels of mathematics into AIM classes. There will be pressure for these classes to stay together across the next three years due to having a different, and restricted within each year, curriculum to regular maths classrooms.

Once started, Year 7 AIM students move on to Year 8 AIM classes and Year 8 AIM students move to Year 9 AIM classes, after each year. In this way, in three years, students can cover all the work in Years 7–9. The three years of AIM modules are denoted A (Year 7), B (Year 8) and C (Year 9). This is because some schools use the materials in classes other than in Years 7, 8 and 9 or out of year level. For example, some schools use Year A in Year 10 prevocational transition classes because the modules cover much of what is needed for a trade, and some schools use Year A in Year 8 classes when they get new students with low performance.

The PD provided to teachers using AIM as replacement is straightforward. The teachers are trained in how to teach the modules that they will use in the next term at the end of the previous term or very early in the new term and then they repeat this training in their schools in teaching the module to students. The modules are designed so that there are two for each term. Trainers should also regularly recap and practise earlier ideas.

Support

This is a new option. It has a more general focus and does not require the existing mathematics curriculum to be changed or replaced. It supports the present curriculum to meet a variety of performance levels in students.

Characteristics

As with replacement, this option uses the characteristic that modules show the vertical development of topics to make them a resource for teaching. If students do not have the prerequisites to undertake a lesson, teachers can look up the module covering that topic as a sequence. This enables teachers to see the topic as a sequence of ideas and lessons, to determine where to start on this topic, and to be given lesson plans for prerequisites and plans for teaching at year level.

Over the years, YDC staff have found that teachers are often unable to state what teaching comes before and after a lesson. The modules are a resource of such information and so can be used, if there are prerequisite difficulties, by teachers to assist them to remediate the difficulties that their students have. So the modules become a large database of teaching ideas for the teachers.

Finally, because they are used as a database, the modules may be better presented in topic clusters than in three years. This means presenting all the Number modules first, and then all the Operations modules and so on. On the other hand, the year level sequence may still be useful.

Implementation

The big difference between the options is in their implementation. The replacement option involves teachers being taught each module that they will then teach in the future, by going through the module. The trainers and teachers know which modules will be taught in each term. They can focus on these few modules and learn how to effectively teach them. Furthermore, they need only learn eight modules each year.

In contrast, the modules to be used for the support option could be any module for which the students show weaknesses. Teachers need to study their class and determine the extent of class difficulties before they can identify the modules that may be useful. Then the modules need to be checked for useful activities.

Thus, teachers need to have a working knowledge of all of the 24 modules as any topic area could be a problem for their students. As all topics are taught in the support option, there could be quite a number of modules to check, some of which may not match with the students' year level. Once teachers have identified the relevant

modules, the mathematics performance level of the students in relation to the units in all of the selected modules is determined and teaching activities selected to remediate the difficulties and to teach the set work.

Implementing this option of AIM is best undertaken if integrated with YDM. Teachers using the option can be trained in the general YDM Teacher Development PD sessions. These teach the teachers how to use effective pedagogies in all topic areas. Then, the general YDM books can be replaced by the more specific AIM modules as a way to implement AIM; that is, the AIM modules become a resource for the general overall teaching of mathematics. Implementation of support AIM also means that teachers need to acquire a working understanding of using assessment items and approaches to determine what mathematics needs to be remedied, and to determine when students understand what has just been taught and can move on to further work.

So instead of just repeating what has been shown in the PD, teachers need a wider and more flexible preparation to use AIM in the support option. However, this form of AIM can be used in most lessons in a secondary school and, therefore, has much wider application, making the training worthwhile.

5.2 Training provisions

The focus of the AIM project is to evaluate and refine the remedial pedagogy, the modules, and the PD workshops by studying teachers' and students' reactions to them in terms of achieving acceleration of maths understanding. This requires trials in classrooms by teachers who know the modules and the pedagogy on which they are based. The model of PD and teacher change followed by AIM (see Figure 13 in section 4.2, and also Baturo, Warren, & Cooper, 2004; Lamb, Cooper, & Warren, 2007) is that a new approach to teaching is successful if:

- (a) teachers believe there is a need to change their pedagogy;
- (b) motivating PD is provided showing a new approach;
- (c) teachers trial something of this new approach and find it successful; and
- (d) teachers believe, on reflection, that this success is due to the effectiveness of the new pedagogy.

Thus, implementing AIM in schools requires that teachers be interested in changing their present pedagogy to AIM; that they receive enthusing PD on AIM that convinces them they need to, and can, change; and that they are supported in their first trials to be successful.

Implementation structure

In view of the above, to set up the implementation of AIM in schools, YDC does the following:

1. **Resources.** AIM provides all schools with this Overview booklet and the 24 modules.
2. **PD support.** AIM provides four teachers (called trainers), and principals in certain sessions, with sufficient PD to be able to use the modules. These teachers then train the other teachers in their school.
3. **Online support.** AIM provides all schools with online support in the form of a website with extra resources, a discussion forum, and a helpdesk to answer questions sent by teachers.
4. **Action research.** AIM provides PD on how to use an action-research approach to trialling AIM modules so that teachers can learn from the trials and provide feedback to YDC on quality of resources, PD and online support.

This would normally be six or more days of PD per year (and up to nine days if AIM is the only training being provided). The PD covers the ideas in this Overview booklet and how to use the modules, and provides PD sessions on each module. The pedagogy advocated is very active, and the PD emulates this as it models the teaching advocated within the Overview and the modules. In particular, the PD:

- (a) acts out the teaching of the vertical sequence of units by going through each unit in sequence and acting out with materials the suggested teaching activities;

- (b) allows the teachers and aides time to experience using movements (acting out with bodies) and physical materials that are advocated in the units;
- (c) focuses on integrating mathematics, mathematics education, and classroom management, and stresses the need to work with community and to take account of culture;
- (d) uses the activities in (a) and (b) above to provide teachers with the knowledge of, and skills to use, the theories of structure and pedagogy that underlie AIM; and
- (e) gives time for reflection and discussion so that teachers can discuss problems, share insights, and understand their own teaching in relation to the ideas presented in the PD.

Additional services

There is more certainty of success with AIM if there are also in-school sharing experiences and in-school support, particularly to support the trainers. AIM training focuses on mathematics, mathematics education, and lesson planning. This requires a lot from the teachers and is difficult for trainers to provide in a rushed school situation.

Therefore, AIM can provide, at extra cost, the following additional services.

1. **Additional trainers.** Schools can negotiate for more than four teachers to become trainers.
2. **Expert practitioners.** Schools can negotiate for expert practitioners to visit their school, support the trainers, and work with the teachers in classrooms (modelling teaching, planning, observing, and providing after-school special PD).
3. **Teacher aides.** Schools can negotiate for special training for their teacher aides so that they can assist in the instruction.
4. **Other projects.** Schools can negotiate for teachers to be trained in YDC's basic training (YDM Teacher Development) and YDC's enrichment and extension training (MITI). It is an advantage to be generally trained in YDM before starting AIM. And MITI provides a rounded program by supplying training and resources to extend able students.

There can be cost reductions in integrating basic, remedial and extension materials and pedagogies as well as learning advantages.

Integrating assessment

Across the four years of the Accelerated Indigenous Mathematics trials, the module booklets were reduced in complexity. As stated in section 2.3 on the module framework, ideas were moved around to ensure that no module has so many ideas new to teachers that it prevents teachers believing that they can undertake the units successfully. The final step of this was to reduce the introduction sections of the modules by removing material that is in this Overview booklet.

In the Accelerated Indigenous Mathematics trials across the four years, teachers were provided with full pre- and post-tests. This was possible because of YDC staff's familiarity with the teachers' classrooms. However, in the final modules, only lists of subtest item types for each unit are provided. This means that, in the future, teachers may need to: (a) use their knowledge of assessment and of their students to select items to construct appropriate pre-tests; and (b) use their general knowledge of test items to modify pre-test items and select other items to construct appropriate post-tests.

This means also, in terms of implementation of modules, that PD with the teachers will cover **two extra areas** to the list above:

- (a) sufficient theory of assessment, particularly the relation between assessment construction and validity of responses, to enable teachers to determine appropriate diagnostic and achievement testing for their students; and
- (b) skills and experience for teachers to be able to modify and use the assessment types to construct appropriate pre- and post-tests.

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