Accelerated Inclusive Mathematics Early Understandings Project

YuMi Deadly Maths

AIM EU
Module N4

Number: Early Fractions

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The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning. The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life. YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

DEVELOPMENT OF THE AIM EARLY UNDERSTANDINGS MODULES

In 2009, the YuMi Deadly Centre (YDC) was funded by the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This resulted in a Year 7 to 9 program of 24 half-term mathematics modules designed to accelerate learning of very underperforming Indigenous students to enable access to mathematics subjects in the senior secondary years and therefore enhance employment and life chances. This program was called Accelerated Indigenous Mathematics or AIM and was based on YDC's pedagogy for teaching mathematics titled YuMi Deadly Maths (YDM). As low income schools became interested in using the program, it was modified to be suitable for all students and its title was changed to Accelerated Inclusive Mathematics (leaving the acronym unchanged as AIM).

In response to a request for AIM-type materials for Early Childhood years, YDC is developing an Early Understandings version of AIM for underperforming Years F to 2 students titled Accelerated Inclusive Mathematics Early Understandings or AIM EU. This module is part of this new program. It uses the original AIM acceleration pedagogy developed for Years 7 to 9 students and focuses on developing teaching and learning modules which show the vertical sequence for developing key Years F to 2 mathematics ideas in a manner that enables students to accelerate learning from their ability level to their age level if they fall behind in mathematics.

YDC acknowledges the role of the Federal Department of Education in the development of the original AIM modules and sees AIM EU as a continuation of, and a statement of respect for, the *Closing the Gap* funding.

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Module Overview

This module follows on from ideas in Number modules N1, N2, and N3 but applied to fractional understandings.

These modules are designed to provide support in Years F to 2 to improve Year 3 mathematics performance. The AIM EU modules are based on the AIM Years 7 to 9 modules, which are designed to accelerate mathematics teaching and learning to where underperforming mathematics students (at around Year 3–4 level in Year 7) can learn six years of mathematics in three years and thus access Year 10 mathematics as mainstream students. The AIM EU modules are designed to accelerate learning in the early years so that students with little schooling cultural capital at the start of their Foundation year can learn the school mathematics understandings normally taught in home, plus those taught in Years F–2, in three years and reach Year 3 with strong Year 3 mathematics knowledge. The nine AIM EU modules covering Number and Algebra Years F to 2, plus background on the modules, are described in **Appendix A**.

AIM EU uses the YuMi Deadly Mathematics (YDM) pedagogy, which is based around the structure of mathematics (sequencing, connections and big ideas) and a Reality–Abstraction–Mathematics–Reflection (RAMR) teaching cycle that is described in **Appendix B**. The YDM pedagogy endeavours to achieve three goals: (a) to reveal the structure of mathematics, (b) to show how the symbols of mathematics tell stories about our everyday world, and (c) to provide students with knowledge they can access in real-world situations to help solve problems. The YuMi Deadly Centre (YDC) argues that the power of mathematics is based on how the structure of connections, big ideas and sequences relates descriptively (with language) and logically (through problem-solving) to the world we live in.

This module is the fourth in a sequence of four, *N1 Number: Counting*, *N2 Number: Place Value, N3 Number: Quantity* and *N4 Number: Fractions*, which cover Number from before Year F to Year 2. This module is designed to ensure teachers cover before Year F work as well as the F–2 work.

This chapter introduces and overviews the module by: (a) discussing what is involved in pre-fractional thinking and early fractional understanding, (b) identifying the connections and big ideas that enable deeper understanding, (c) exploring how the content in N4 is best sequenced, (d) discussing teaching and cultural implications, and (e) summarising the structure of the module.

Early fractional understanding and pre-fractional thinking

Similarly to what was done in Module O1, this module has to focus on ideas that precede fractions that are pre-fractional.

Pre-fractional thinking

The following provides a list of some of the important pre-operational and, therefore, pre-fractional thinking abilities.

1. **Pre-counting**. Play-based activities and pre-counting activities such as sorting and classifying according to one or more criteria (see *Module N1 Counting*) and seeing patterns for one or more attributes (see *Module A1 Patterning)*.
2. **Language**. Activities such as telling stories, creating ideas for dialogue, using thinking language (involving words such as think, know, guess, remember), giving oral explanations of thinking and reasoning (not written explanations), developing ability to ask questions, and, most importantly, building listening skills.

3. **Thinking outside oneself**. Understanding others' minds, particularly that the beliefs of other people may be different, and that other people may have only partial knowledge not necessarily all of it (and that because they have partial knowledge they will not necessarily have all of it).
4. **Creativity**. Using a creative approach to activity such as predicting what might happen in relation to future events, suggesting alternative actions (which could have been taken in the past), and using counterfactual reasoning.
5. **Planning**. Showing "planful, thoughtful, purposeful" behaviour, constructing their own rules (particularly for the purpose of problem-solving), and reasoning logically from given precepts.

Early Fractional Understanding

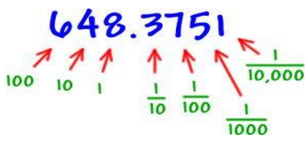
The teaching sequence for early fractional understandings needs to provide experiences that will help children to understand the following mathematics ideas with regard to fractions:

- that when we split something into two equal-sized parts, we say we have halved it and that each part is half the original thing
- that we can partition objects and collections into two or more equal-sized parts and the partitioning can be done in different ways.
- that we use fraction words and symbols to describe parts of the whole. (NB the whole can be an object, a collection or a quantity) and that the same fractional quantity can be represented with a lot of different fractions. We say fractions are equivalent when they represent the same number or quantity.
- that we can compare and order fractional numbers and place them on a number line.

Core fractional understandings

The early years should lay the foundations for the core understandings of fractions that are developed across primary and into early secondary. Thus the early teaching sequence has to prepare students for the following meanings of fractions:

- knowing that a fractional number can be written and understood as a division, a decimal or as an operator
- knowing that a fraction symbol may show a ratio relationship between two quantities



Future general fraction understandings

The important thing is that early teaching lays strong foundations and prepares students for later knowledge. This does not mean that this later knowledge is taught in F–2, but that the F–2 teaching of fractions does not work against this later knowledge – it is in harmony with it.

In very general terms, not to be used in F–2 or even later students, some important later-knowledge ideas are as follows.

- that fractions such as p/q are constructed by taking a whole **dividing it into q equal parts**, naming the parts as qths (and 1/q), taking the p parts and naming this as p/qths (in general, there are some special names, halves for twoths, thirds for threeths), noting that p/p and p pths is one.
- that fractions such as p/q are p equal parts out of q equal parts where the **q equal parts are seen as one whole**, and that the symbol represents a single amount (quantity) but has been designed so that it allows the fraction to also represent multiply by p and divide by q.

- that have to move **between wholes and fractions and fractions and wholes** (reversing, if given something as p/q, need to break this into p equal parts and make q of these parts to get the whole).

- that wholes can be combined with fractions to make mixed numbers which can be renamed as improper fractions, and can be represented on whole part charts and number lines – 4 and $^2/_3$ is 4 wholes and 2 thirds, between 4 and 5 on the number line, and is 14 thirds if we change everything to thirds.

- that equivalent fractions are based on multiplying by one in its many forms – e.g. $^2/_3 \times {}^2/_2 = {}^4/_6$.

Connections and big ideas

The starting point for all YDC AIM modules is the connections between mathematics topics and using these connections to accelerate learning, in particular in the formation of big ideas whose learning will provide understanding across mathematics topics and across year levels.

Importance of connections in early years

YDC believes mathematics is best understood and applied in a schematic structured form which contains knowledge of when and why as well as how. Schema has knowledge as connected nodes, which facilitates recall and problem-solving. The basis of the YDM philosophy is that knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning.

In particular situations, ideas can be connected until they form a schema that is a big idea. A big idea is one that applies in more than one mathematics topic area and more than one year level. This means

Understanding schematic structure enables teachers to:

(a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;

(b) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem-solving;

(c) *choose effective instructional materials, models and strategies* – mathematics that is connected to other ideas or based around a big idea can be taught with similar materials, models and strategies; and

(d) *teach mathematics in a manner that enables later teachers to more easily teach more advanced mathematics* – by pre-empting the knowledge that will be needed later, preparing linkages to other ideas, and building foundations for big ideas the later teachers will use.

Models, symbolic representations and connectivity

Models show the mathematical similarity or equivalence of whole numbers, decimal numbers, common fractions, and percent, rate and ratio, while symbolic representations tend to show difference. In fact, to see the similarities that assist in learning, differences in notation have to be discounted.

Models common to representations of fractions are the usual ones of *area* (e.g. one-third is represented by one-third of a block of chocolate), set or collection (one-third is represented by six of the nine children in the group) and; *number line* (one-third is represented as a point one-third of the distance along zero to one on a line. These models are also used with whole numbers and decimal numbers so they connect different forms of number.

However, **symbolic and language representations** differentiate and **act against connections** – making students think that the different number types represent different things. For example, a whole partitioned into 5 pieces and 3 being taken is 3 ÷ 5 ("three divided by five"), $^3/_5$ ("three fifths"), 0.6 ("zero point six" or "six tenths"), 60%

("sixty percent"), and 3:2 ("ratio three to two"), all different notations and languages. Because of history, we have all these different number types to describe the same things, all looking syntactically different.

Major big ideas

There are five major big ideas for all numbers that apply in a similar way to whole numbers, decimal numbers, common fractions, and percent, rate and ratio. These are as follows.

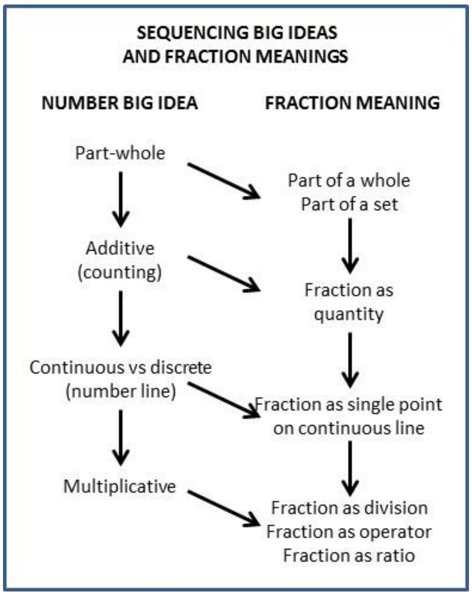
1. **Part-whole (includes Notion of a unit)**. The basis of number is the unit which is grouped to make large numbers and partitioned to make fractions. Thus everything can be seen as part and whole – a ten is a tenth of a hundred and a whole of 10 ones. The concept here is that the number of equal partitions in the whole **name the fraction** and the processes are **reading and writing**. The whole can be a collection of objects.
2. **Additive structure**. A number's value is the sum of its parts, e.g. $\frac{5}{6}$ is five one-sixths. Fractions count forwards and backwards following an odometer type pattern that increases/decreases the number of wholes at $\frac{6}{6}$ ths. The concept for this idea is counting and the processes are **counting patterns**. It enables fractions to be seen as a single **quantity** on a line and sets up **mixed numbers**.
3. **Continuous vs Discrete/Number line**. Regardless of its symbols every fraction is a single quantity represented by a point on a number line, has rank and can be compared to other numbers. The number-line representation changes perspective of number – for example, the 0 is no longer nothing, it is the starting position of positive whole numbers (and of rulers and other measuring devices). The concepts for this idea are **comparison/order**, **rank** and **density** and the processes are **comparing/ordering**, **rounding**, and **estimating**.
4. **Multiplicative structure**. There is a multiplicative relationship between fractions and wholes. The concept for this idea is **multiplicativity** and processes are **renaming** (giving improper fractions). It relates fraction like ¾ to 3 **divided** by 4, and that which multiplies by 3 and divides by 4.
5. **Equivalence**. Sometimes a single quantity can be represented by more than one number. For example, $\frac{2}{3}$ is the same as $\frac{4}{6}$, and $\frac{3}{5}$ is the same as $\frac{6}{10}$. Equivalence often reflects adding zero (the additive identity) or multiplying by one (the multiplicative identity and the method of equivalence used in fractions).

Sequencing

One outcome of seeing mathematics as a structure is that concepts are not singular – there is more than one for every mathematics idea. For fractions, there are seven meanings based on the big ideas:

- **part of a whole** and **part of a set or collection** – part-whole big idea;
- **a quantity and a number that can be counted** – additive big idea;
- a **single point on a continuous number line** – continuous vs discrete big idea;
- **division**, **ratio** and **operator** – multiplicative big idea.

Because this module is for Years F to 2, this module will only focus on the first four of the above meanings.



The diagram on right shows the relationship between number big ideas and fraction meanings.

Sequence of meanings

To develop a rich meaning for fractions, it is essential to cover all the meanings from real-world situations and later mathematics. This means introducing the following concepts in the following order:

FRACTION AS PART OF A WHOLE (part of a cake, part of a liquorice strip)

FRACTION AS PART OF A COLLECTION (part of a class)

FRACTION AS A NUMBER OR QUANTITY ($^{3}/_{4}$ is halfway between $^{1}/_{2}$ and 1)

FRACTION AS A CONTINUOUS QUANTITY ON A NUMBER LINE ($^{2}/_{3}$ is smaller than $^{3}/_{4}$)

FRACTION AS DIVISION ($^{3}/_{4}$ is 3 divided by 4 – puts fractions on calculators)

FRACTION AS RATIO ($^{3}/_{4}$ can be considered as 3:4 even though, for fractions, the 4 is a whole)

FRACTION AS OPERATOR ($^{3}/_{4}$ is multiplication by 3 and division by 4)

For this module, we will consider only those topics that are part of the curriculum for Years F–2. Thus we will not focus on fractions as division, ratio and operator. This leaves part of a whole, fraction as part of a collection, fraction as number and quantity, and fraction as a continuous entity.

Sequence across units in module

The decision not to include division, ratio and operator meanings of fraction in the module, leaves the content of the module fairly straightforward.

(a) **Fraction as part of a whole**. This unit begins the work on fraction by looking at parts and wholes. It introduces the need for equal parts and reverses the activities, whole to parts AND parts to wholes. More time is spent on equal and unequal before we look at halves and quarters.

(b) **Fraction as part of a collection**. This repeats unit 1 but for a set not an area. Part of a collection is more difficult than part of a whole because the students need to identify the whole as all the collection together and then break it into equal parts. Equal and unequal and halves and quarters are again the end focus. We experience different ways to partition

(c) **Fraction as a number or quantity**. A fraction such as ¾ is a symbol with two numbers and a diagonal line. Students have to realise that it is one integrated symbol and represents one number. This unit is therefore important in focusing on counting fractions and reading and writing fractions. Early equivalence is also focused on, along with the relationship between folding strips and fractions on the number line

(d) **Fraction on a continuous number line**. This moves on from 3 above to placing fractions on number lines that are continuous so that any fraction can be placed. The line can also be larger than one and so mixed numbers can be placed on it. This enables comparison and order, rank and approximations to be undertaken

Sequences within units

The following summarises the activities with each of the five units in this module O2 *Meanings and Solving*

Fraction as part of a whole

- Partitioning the whole - breaking wholes into parts
- Equal parts - connecting partitioning the whole with equal parts to introduce fraction
- Forming whole - reversing wholes to parts and looking at parts to wholes
- Equal and unequal - ensuring students understand the nature of equals for the area focus of part of a whole
- Halves and quarters - applying above to 1/2, 1/4, 2/4 and 3/4, plus others

Fraction as part of a collection

- Partitioning groups - breaking groups into parts (seeing group as a whole)
- Equal parts of a group - connecting partitioning to equal to introduce fraction
- Forming wholes - reversing wholes to parts and looking at parts to wholes for groups
- Equal and unequal - ensuring students understand the nature of equals for groups
- Halves and quarters - applying above to 1/2, 1/4, 2/4 and 3/4, plus others.

Fraction as number or quantity

- Counting in fractions - count in fractions using 1/2's and 1/4's & other known fractions, using improper fractions and wholes and parts as counts go past the whole
- Reading fractions - names based on number of equal parts, develop pattern
- Writing fractions - language and symbols based on equal parts/naming unit fractions
- Same fraction-different numerals - improper fractions, mixed numbers and below
- Early equivalence - looking at 2/4 and 1/2 for example

Fraction on a continuous number line

- Partitioning lines - breaking 0-1 length into parts, relation to parts of a whole
- Equal parts - ensuring students understand equal re number lines
- Forming wholes - reversing and going from part of a line to o-1 of a line
- Mixed numbers - use continuous lines to build wholes and parts and mixed numbers
- Counting, comparing ordering - using lines to do these concepts/processes and looking at patterns that emerge

Teaching and culture

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework and the impact of Western number teaching on Indigenous and low-SES students.

Teaching implications

Because the teaching in this module moves from before-school knowledge through to Year 2 knowledge, the teaching implications are as follows.

1. **Language teaching ideas**. The module will require you to do play and language activities that will build operations. Some ideas for this are: (a) use storytelling time to develop thinking and to model verbalised thinking; (b) create ideas for dialogue using thinking language, particularly opportunities for structured dialogue; (c) develop questioning, focusing on students' ability to ask questions; and (d) wonder together with students.
2. **Thinking–solving teaching ideas**. There will also be a large focus on starting students on thinking and solving. Some ideas for this are: (a) develop spatial, logical, creative and flexible thinking skills and skills in decision-making, plus metacognition; (b) provide opportunities to explore identifying and describing attributes, matching and sorting, and comparing and ordering; (c) create thinking times and develop the

ability to ask questions that stimulate children's thinking and encourage children to elaborate on their ideas; (d) recognise creativity in approach by students and need for solitary as well as social play; and (e) provide opportunities to plan and reflect/evaluate thinking with students and to solve problems.

3. **RAMR cycle**. The module will be based on teaching following the RAMR cycle. Each unit will have at least one RAMR exemplar lesson. The lessons will: (a) start with something students know and in which they are interested; (b) move on to creatively representing the new knowledge through the sequence body → hand → mind; (c) develop language and symbols, and practice and connect components; and (d) finally, reflect the new knowledge back into the lives of the children using problems and applications, and focus on ensuring flexibility, reversing and generalising (by changing parameters if needed). The RAMR cycle is in **Appendix B**.

4. **Models**. It is important that students connect symbols, language, real world situations and models in many and varied contexts and forms. For models, it is important that there is a balance of set and number line (or length) models. Set models are discrete items like money, fingers, counters and other objects, while the number line model is a ruler or steps or jumps along a line (this means that 2+3 can be two books joining 3 books, set model, or two steps and then three more steps, number line model). With new meanings, it is also important to begin to use array models and combinations models.

Cultural implications

In this section, we move on from just looking at teaching to the cultural implications in this teaching, because students who need AIM EU modules include Indigenous and low-SES students.

1. **Teaching Indigenous students**. Aboriginal and Torres Strait Islander students tend to be high context – their mathematics has always been built around pattern and relationships. Their learning style is best met by teaching patterning that presents mathematics structurally as relationships, without the trappings of Western culture. As Ezeife (2002) and Grant (1998) argued, Indigenous students should flourish in situations where teaching is holistic (from the whole to the parts). Thus, problems and investigations where there is opportunity for creativity and patterning should have positive outcomes for Indigenous students as long as the problems are realistic, make sense within the Indigenous students' context and matter to the students. In general, this means a lesser focus on algorithms and rules, and a greater focus on patterning, generalisations and applications to everyday life. It also means a strong language focus to translate the students' abilities to the world of standard English.

2. **Teaching low-SES students**. Interestingly, holistic teaching is also positive for low-SES students. Three reasons are worth noting. First, low-SES students tend to have strengths with intuitive–holistic and visual–spatial teaching approaches rather than verbal–logical approaches. Thus, a focus on solving problems that make contextual sense and for which the answers matter and with a strong language component should be positive for low-SES students. Second, many low-SES students in Australia are immigrants and refugees from cultures not dissimilar to Aboriginal or Torres Strait Islander cultures. They are also advantaged by holistic algebraic and patterning approaches to teaching mathematics. Third, many low-SES students and their families have long-term experience of failure in traditional mathematics teaching, resulting in learned helplessness. This can be overcome with a focus on investigations along with a strong language focus. Holistic-based problem-oriented teaching of mathematics through patterns is sufficiently different that students can escape their helplessness – particularly if taught actively and from reality as in the RAMR model.

3. **Prior-to-school knowledge**. Both Indigenous and low SES students can come to school with lots of knowledge from their culture and background, but little knowledge that helps with schoolwork. The modules in these books are designed to provide this "prior-to-school" knowledge but it is important that their cultural and context knowledge is also equally appreciated and maintained. Pride in heritage and connection to heritage is important in learning "school" knowledge.

Structure of module

Components

Based on the ideas above, this module is divided into this overview section, four units, a review section, test item types, and appendices, as follows.

Overview: This section covers a description the module's focus, connections and big ideas, sequencing, teaching and culture, and summary of the module structure.

Units: Each unit includes examples of teaching ideas that could be provided to the students, some in the form of RAMR lessons, and all as complete and well sequenced as is possible within this structure.

Unit 1: Fraction as part of a whole

Unit 2: Fraction as part of a collection

Unit 3: Fraction as a number or quantity

Unit 4: Fraction on a continuous number line

Module review: This section reviews the module, looking at important components across units. This includes the teaching approaches, models and representations, competencies and later activity (where the activity in this module leads to in Years 3 to 9).

Test item types: This section provides examples of items that could be used in pre- and post-tests for each unit.

Appendices: This comprises three appendices covering the AIM EU modules, the RAMR pedagogy, and proposed teaching frameworks for operations.

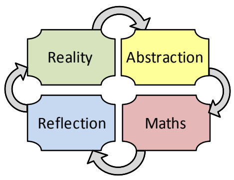
Further information

Sequencing the teaching of the units. The four units are in sequence and could be completed one at a time. However, each of the units is divided into sub-ideas (concepts and processes) that are also in sequence within the unit. Therefore, schools may find it advantageous to: (a) teach earlier sub-ideas in a later unit before completing all later sub-ideas in an earlier unit; (b) teach sub-ideas across units, teaching a sub-idea in a way that covers that sub-idea in all the units together; or (c) a combination of the above.

The AIM EU modules are designed to show sequences within and across units. However, it is always YDC's policy that schools should be **free to adapt the modules to suit the needs of the school** and the students. This should also be true of the materials for teaching provided in the units in the modules. These are exemplars of lessons and test items and schools should feel free to use them as they are or to modify them. The RAMR framework itself (see Appendix B) is also flexible and should be used that way.

Together, the units and the RAMR framework are designed to ensure that all important information is covered in teaching. Therefore, if changing and modifying the order, try to ensure the modification does not miss something important (see **Appendix C** for detailed teaching frameworks).

RAMR lessons. We have included RAMR lessons as exemplars wherever possible in the units of the module. Activities that are given in RAMR framework form are identified with the symbol on the right.



Suggestions for improvement. We are always open to suggestions for improvement and modification of our resources. If you have any suggestions for this module, please contact YDC.

Unit 1: Fraction as Part of a Whole

To understand fractional numbers we need students to learn to read, write, say and interpret them in everyday use. We need them to be able to order them and understand the relevance of the order, and recognise equivalent forms. We need to examine the early understandings students have to have to begin their journey to understanding fractional numbers.

The first understanding is fractions as parts of a whole. There are important ideas that lie behind fractions as parts of a whole that need to be understood. The five subsections 1.1 to 1.5 that make up this unit provide focus on these important ideas.

Background information

Number understanding is built around the one or the unit as in the diagram below:

←	**ONE**	→
Parts of one Fractions and decimals		Groups of ones Tens, hundreds, and so on

Common fractions are parts of a whole. They are formed by breaking a whole into equal parts and use the name of the number of equal parts to give the name of the fractions (except for the first few).

Thus, learning of fractions involves wholes and equal parts, going from wholes to equal arts and reverse, and beginning to learn the well-known fractions.

1.1 Partitioning the whole

1. Early understanding prerequisite to fraction parts is the capacity to break things/wholes up. Children need a lot of experience breaking things up into parts/partitioning.
2. They also need to learn that:
   (a) There are many ways to break things/the whole up

   (b) When splitting the whole into parts the whole should be completely used up

   (c) Regardless of how we split the whole into parts the whole remains the same amount
3. Activities to do the above include:
   - Box buildings and block towers
   - Playdough rolling
   - Banana sharing
   - Paper collage and card making
   - Body parts
   - Playing cards and bead threading

1.2 Equal parts of a whole

1. When working with the ideas below always ensure students have a **whole, the same as the one they are partitioning, to compare the parts they partition**. This matching and comparing continues to build their understanding of: (a) when splitting a whole into equal parts the whole should be completely used up; and regardless of how we partition, the whole remains the same.

2. Start to build the basic fractions through half. Remember, when we split something into two equal sized parts we say we have halved it and that each part is half the original thing. We should note that the language and social meanings of halving: halving will often simply mean to split into shares; the word half is often associated with fairness and sharing; will refer to it as any number of shares, e.g. we all got half; or associate the word half with two and use it whenever there are two parts even if they are not equal in size. Finally, students must be able to produce the partition for themselves and name each part as 'one half'. It is not sufficient for them to be able to just colour in half on a pre-drawn and partitioned shape.

3. Students must be able to attempt (they might not get it quite right) to achieve equal parts because they know they must be equal. They need to understand that:

   (a) equal parts need not look alike but they must have the same size or amount of the relevant thing, i.e. the equality of two halves refers to the relevant quantity not appearance;

   (b) the partitioning can be done in different ways (they may think there can only be two pieces) a half may be one part of two or two parts of four, four parts of eight and so on; and

   (c) the parts can be in any and different arrangements.

4. Students should be encouraged to use a variety of strategies to partition quantities into equal shares; folding; symmetry; dealing out or measuring/matching.

5. Activities include the following:

   - Invite children to explain the meaning of a half in everyday situations e.g. provide a range of things such as sandwich, apple, half a packet of biscuits, and piece of paper. Ask students how they know it is a half in each case?
   - Have students focus on the act of halving in everyday activities for example half an apple, halve some playdough, halve a sheet of paper etc. Discuss with students what they need to think about in each case. Encourage them to find strategies to make sure the portions are equal. Have them decide what to do if something is left over. Ask: *Is this the only way you can halve it?*
   - Repeat activities from **1.1** and focusing on equal parts of the whole.
   - Have students fold several identical rectangular pieces of paper to represent dividing/sharing chocolate bars into halves in different ways. Ask children to find ways to check that each "chocolate bar" is in halves.
   - Making and cutting sandwiches.
   - Creating columns.
   - Folding quadrants for Thinkboard templates.
   - Making Pattern block composite shapes.
   - Playdough constructions.

1.3 Forming wholes

1. This is reversing 1.1 and 1.2. And is important. It is done by taking the equal parts and reforming the wholes with familiar objects and things i.e. reversing the splitting/partitioning into parts and putting them back together. Also have to ensure that no part of the whole is over looked/left out/lost, i.e. the whole always stays the same no matter how it is partitioned.

2. When forming a whole from a part, form different wholes as follows:

   (a) Forming a whole from an equal part means that there can be different wholes made up of different numbers of equivalent parts.

   (b) Forming a whole from one of two equal parts means that there can be equivalent wholes of two equal parts but they may look different

   (c) Forming a whole from one of four equal parts means equivalent wholes of four equal parts but they may look different

   (d) Forming a whole from one of eight equal parts means equivalent wholes of eight equal parts but may look different

3. Activities are as follows:

   - Segment creations- e.g. ask students to segment a piece of citrus fruit and decide if the segments are equal size. Have them re-form the fruit.
   - Same equal parts different creations e.g. sharing sandwiches cut in four quarter squares and four quarter triangle.
   - Numbers of parts e.g. ask students to compare fraction parts using 3 paper circles that are equal in size. Have them fold cut and cut one circle into halves; one into quarters; one into eighths. Re-compose on a whole circle and compare the parts

1.4 Equal and unequal

1. Experience partitioning into equal and unequal parts. Note that when making equal parts, they need not look alike but they must have the same size.

2. Encourage students to see that:

   (a) When splitting a whole into equal parts the whole should be completely used up

   (b) No matter how we partition the whole remains the same

   (c) The more parts something is split into the smaller each part is (important)

3. Activities:

   - Challenge students to think about fraction equivalences as they play with models partitioned into equal parts e.g. fraction cakes say: *You have two quarters of the cake I have one half a cake. Do we have the same amount of cake?*
   - Shading grids – **four colours.** Use 3 × 4 grids. Shade in 4 colours. Make sure each colour covers an equal area of the grid. *How do you know each arrangement has equal amounts of each colour?* Repeat with other grids and link to this to the appropriate fraction. Use grids with triangles.
   - Pattern blocks – have students use geometrically designed materials, e.g. pattern blocks, to partition larger shapes into equal parts. Ask: *How many different equal parts can a hexagon be partitioned into?*
   - Divide shapes by one dotted line and ask students to tick those that are equal.

1.5 Halves and quarters

1. Experience activities that break a whole into 2 and 4 equal parts. Name the parts and the fractions that emerge. In particular, students need extensive experience with partitioning and re-partitioning things to understand that one half is the same as two quarters in every situation.

2. Other activities include students physically re-partitioning objects e.g. paper, cutting and folding and visualising partitioning and repartitioning objects

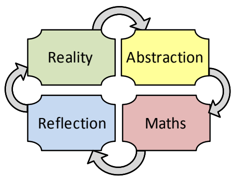
3. Repeat 1 and 2 above when extending to understand that one half is the same as two quarters is the same as four eighths.

4. In this sub-section we are building the foundations for students to be able to construct and describe easily modelled or visualised fractional equivalences in words for example they may say that a half of a pizza is the same as two quarters or that a quarter of a pizza is the same as two eighths and so on.

5. Other activities:

    - Group students in threes. Give them each a strip of paper that is equivalent in length. Ask them to fold one in half, one in four or quarters, one in 8 or eighths. Ask them to compare their strips. What is different? What is the same? Introduce fractional language to show that one whole strip can be two halves, four quarters, 8 eighths
    - Ask students to compare fractions using 3 paper circles that are equal in size. Have them fold and cut one circle into halves, one into quarters and one into eighths. Have students match the parts to determine how many quarters and eighths fit exactly on half a circle.. Later repeat this with paper rectangles. Ask: what stays the same? What is different? Ask: In what context do we need to share objects that are circles?
    - Symmetry games e.g. shading grids; mirror reflections; symmetry art; patterns and symmetry; pattern blocks
    - Making sandwiches: give students a plate of 'sandwiches' cut in halves and quarters by using squares, triangles and rectangles. Ask students to take a half a sandwich each. Ask them to re-form the whole sandwich.

1.6 RAMR lesson: What is a Half?



Learning goal: A half is one of two equal parts of a whole

Big Ideas: Part-whole; equal parts of the whole

Resources: Magic maths box of stuff (paper plates, plastic cups, clocks, paper, string, straws, coins, boxes, blocks, seashells, beads, balls, bags, and anything else you have collected,

Reality

Local knowledge: Ask your students to think about what is a half? Where do you see them /use them /seen them today?

Prior experience: N1, A1, O1

Kinaesthetic: Place students in groups. At each group provide a magic box of maths stuff. Ask them to individually choose and make a half, and then work as a group to write/come up with/say a definition. Have each student describe their half and the action they did. Ask: what makes it a half? Or how do you know it is a half? Have each group share their definition. Have a discussion about what is a fraction? (an equal part of a whole)

Abstraction

Body: Have students work in groups to make body fractions identifying the equal parts and the whole

Hand: Repeat the kinaesthetic activity with each student choosing a different whole from previously and ask them to create another half and/or fraction

Mind: Have students work in pairs and give instruction to partner to construct an equal part that they are thinking of in their head/mind's eye. Play I am thinking …….

Creativity: Have students create a collage poster to explain a half

Mathematics

Language/symbols: split, partition, share, equal, fraction part, equal-sized, quantity, half, halving, whole, fairness, sharing, number of parts

Practice: Have a range of activities from 1.1 and 1.2 for students to practice partitioning the whole into two equal sized parts. Ensure each student has the opportunity to describe what they have done using fraction words

Connections: doubling; counting; sharing, grouping

Reflection

Validate: Have students complete the posters they created in the abstraction stage and create a presentation to share their knowledge with parents/another class/ a class visitor

Applications/Problem solving.

Challenge students to think about fraction equivalences as they play with models partitioned into equal parts e.g. fraction cakes say: You have two quarters of the cake I have one half a cake. Do we have the same amount of cake?

Pattern blocks -have students use geometrically designed materials e.g. pattern blocks, to partition larger shapes into equal parts. E.g. ask: Show me which shapes can partitioned into half? How many different equal parts can a hexagon be partitioned into?

Extension:

- *Flexibility*: *Think* about what is a half of a half?
- *Reversing*: Present the students with 'a half' and ask them to create the whole.
- *Generalising/Changing parameter:* I was listening to the radio and I heard the announcer say half. What might she have been referring to?

Unit 2: Fraction as a Part of a Collection

The next understanding is fraction as part of a collection or set. Partitioning collections into any number of parts is significant in linking multiplication, division and fractions. Again, there are important ideas that lie behind fraction as parts of a collection that need to be understood. The subsections in this unit provide these ideas.

Background information

Simplistically, there are two ones in the world – the first in one thing, the second is one collection or group of things. The difficulty with groups and collections is that they look more than one and the mind has to visualise them as one thing (this is called **unitising**).

Therefore, we have to repeat the activities from part of a whole for part of a collection. However, there is added difficulty as part of a whole is

whole → equal parts → fraction

while part of a collection is

collection → unitise to whole → equal parts → fraction.

2.1 Partitioning groups

1. Children need extensive experience in splitting a wide range of collections into equal sized parts. The greater range of materials and contexts explored the richer the student's experience.
2. There are many ways to break things/the collection up. They can be shared into equal parts by dealing out or distributing.
3. Activities:
   - sharing strategies- count out collections
   - Box buildings- partition building structures
   - Block play- share out blocks for a joint construction; group students in pairs or fours and have them work as a team on each equal part.
   - Playdough – endless activities for partitioning e.g. making balls; worms; snakes;
   - Paper collage- use egg cartons, cardboard coloured paper etc. for children to create their own equal partitions posters
   - Playing cards- games, dealing out, making pairs
   - Bead threading- partition by number, colour, shape; vary the whole
   - Counting collections- get out the Magic maths box to sort and partition collections how many groups? How many in the group?

2.2 Equal parts of a group

1. The ideas in part of a whole are repeated here:

   (a) When splitting the collection into equal parts the collection should be completely used up

   (b) Regardless of how we split the collection into parts the collection remains the same amount

   (c) The more shares/equal groups the collection is split into the smaller each share is

2. Students need to be able to construct the partitions without being given pre-drawn diagrams. Students also must be able to produce the partition for themselves and name each part. As 'one half'. It is not sufficient for them to be able to just colour in half on a pre-drawn and partitioned collection. They must be able to attempt (they might not get it quite right) to achieve equal parts because they know they must be equal.

3. Students need to understand that equal parts need not look alike but they must have the same size or amount of the relevant thing i.e. the equality of two halves refers to the relevant quantity not appearance. As well, they should know that the partitioning can be done in different ways (they may think there can only be two pieces) a half may be one part of two or two parts of four, four parts of eight and so on, and parts can be in any and different arrangements.

4. Students should be encouraged to use a variety of strategies to partition quantities into equal shares; symmetry; dealing out or measuring/matching.

5. Activities:

   - What is a fair share? Brainstorm everyday sharing situations where equality of quantity is important.

   - Segments. Use mandarins and have children peel and segment the piece of fruit themselves. Have students decide if the segments are equal. If they are equal, then invite students to explain how two, three and then four people could share the whole mandarin.

2.3 Forming wholes

1. This is taking the equal parts and reforming the collection/set with familiar objects and things, i.e. it is reversing the splitting/partitioning into parts and putting them back together. It is important because we frequently ask children to partition the whole but rarely the reverse, "here is a part, make me the whole"?

2. Ensure no part of the collection/set is over looked/left out/lost, i.e. the collection/set always stays the same no matter how it is partitioned

3. Get the students to experience enough activities that they realise:

   (a) Forming a collection/set from a part can give different wholes,

   (b) Forming a collection/set from an equal part can give different collections/sets made up of different numbers of equivalent parts,

   (c) Forming a collection/set from one of two equal parts can give equivalent collection/set of two equal parts but may look different,

   (d) Forming a collection/set from one of four equal parts can give equivalent collection/set of four equal parts but may look different, and

(e) Forming a collection/set from one of eight equal parts can give equivalent collection/set of eight equal parts but may look different

4. Activities:

   - **Collection creations**: give students a **part** of a collection and ask them to make the whole collection. Can you make a different collection? Is this possible? Why can these collections be different?
   - **What is the whole**? Same equal groups, different creations: give the students a part of collection and say this is an **equal part** of the collection, make the collection. Have students compare their collections. What is the same? What is different?
   - Numbers of parts numbers in each part. Give students a part of a collection and say this **one of two equal parts** of a collection. Make the collection. Ask: what do you notice? Should everyone have the same collection?
   - Give the students 3 blocks and say this is one quarter of the collection. Make the whole collection. How many in the collection? And so on.

2.4 Equal and unequal

1. Look at equal and unequal partitions of a set. Get students to experience sufficient examples that they realise that:

   (a) No matter how we partition the collection/set it remains the same, and

   (b) The more groups the collection is split into the smaller number in each group.

2. Activities;

   - Sharing collections: have students share collections that can be easily subdivided. For example, 12 crackers with peanut butter between two, three, four plates so that each plate has the same amount of crackers. Have students try 5 plates. Ask: why is it difficult to share12 crackers between 5 plates? When is sharing collections easy? When is it difficult?
   - Equal shares: Read the class familiar stories, e.g. *The Doorbell Rang* by Pat Hutchins. Then ask students to say how many equal shares the collection is subdivided into each time. Ask: why would it be difficult to share 24 between 5 people? When is making portions easy? When is it difficult?
   - Sharing Equality: have students in groups of 4 share a pack of cards. Ask them to share it equally. Ask: how can they be sure it is equal? Ask: what fraction of the cards have you got? Have you really got a quarter of the cards? How can you check?
   - Party baskets: have students investigate ways of sharing different numbers of sweets among different numbers of party baskets without left overs or cutting up the sweets.
   - Grouping: Give students collections of discrete items that cannot be cut up e.g. marbles, shells, rocks, counters, blocks. Have them sort them into two groups: those that can be shared into equal groups and those that cannot. Ask: which collections can be halved and which collections cannot? Introduce add and even numbers to describe the two categories.
   - Give students counters and ask them to decide how many different ways a packet of 24 sweets could be split into equal portions. Have students record these.

2.5 Halves and quarters

1. Students need extensive experience with partitioning and re-partitioning collections to understand that one half is the same as two quarters in every situation.

2. This should involve students physically re-partitioning collections and also visualising partitioning and repartitioning collections.

3. Repeat 1 and 2 above when extending to understand that one half is the same as two quarters is the same as four eighths.

4. We are building the foundations for students to be able to construct and describe easily modelled or visualised fractional equivalences in words for example they may say that a half of a group is the same as two quarters or that a quarter of a group is the same as two eighths and so on.

5. Activities:

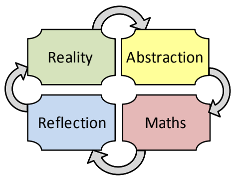
   - What is a half of a half? Have students solve and explain using as many contexts and materials as they can think of to prove their solution.

   - Repeat the problem asking them to use pattern blocks to show possible solutions

   - Grouping: Give students collections of discrete items that cannot be cut up e.g. marbles, shells, rocks, counters, blocks. Have them sort them into two groups: those that can be shared into equal groups and those that cannot. Ask: which collections can be halved and which collections cannot? Introduce add and even numbers to describe the two categories. Extend to other partitions such as can you find quarters, thirds why? Why not? Can you sort them into 8 groups?

   - Make pizza and play parts of a circle exploring a half; a half of a half; and a half of a half of a half of a circle. Give students pizza paper circles and have them explore the half of a half for as far as they can.

2.6 RAMR lesson for fraction as part of a collection



Learning goal:

- Provide extensive experience in splitting a wide range of collections into equal sized parts.
- There are many ways to break things/the collection up.

Big Ideas: Part-whole; equal parts of the whole

Resources: The greater range of materials and contexts explored the richer the student's experience. Magic maths box of stuff (paper plates, plastic cups, clocks, paper, string, straws, coins, boxes, blocks, seashells, beads, balls, bags, and anything else you have collected,

Reality

Local knowledge: what collections need to be shared in your neighbourhood/community/family? Ensure every child has the opportunity to share a sharing story from their life.

Prior experience: N1 A1 O1; this module 1.1

Kinaesthetic: Read *The Doorbell rang* by Pat Hutchins. Have students construct the groups and act out the story with materials to share the 'cookies'. Then ask students to say how many equal shares the collection is subdivided into each time. Ask: why would it be difficult to share 24 between 5 people? When is making portions easy? When is it difficult?

Abstraction

Body: Have students work in groups of 2 or four to Construct Box buildings- partition building structures

Hand: Block play- share out blocks for a joint construction; group students in pairs or fours and have them work as a team on each equal part.

Playdough – endless activities for partitioning e.g. making balls; worms; snakes;

Mind: In pairs, have one student describe what they can see in their mind's eye to describe a collection partitioned. The other student has to construct it. Ask: how many groups? How many in each group?

Creativity: Have students visualise, tell then construct groups using Paper collage - use egg cartons, cardboard coloured paper etc. for children to create their own equal partitions of a set posters

Mathematics

Language/symbols: collections, sets, split, partition, share, equal, fraction part, equal-sized, quantity, half, halving, whole, fairness, sharing, number of parts, group, number of groups, number in group

Practice: What is a fair share? Brainstorm everyday sharing situations where equality of quantity is important.

Segments. Use mandarins and have children peel and segment the piece of fruit themselves. Have students decide if the segments are equal. If they are equal, then invite students to explain how two, three and then four people could share the whole mandarin.

Connections: Counting; operating: multiplication and division

Reflection

Validate: Give students counters and ask them to decide how many different ways a packet of 24 sweets could be split into equal portions. Have students record these.

Have students in groups of 4 share a pack of cards. Ask them to share it equally. Ask: how can they be sure it is equal? Ask: what fraction of the cards have you got? Have you really got a quarter of the cards? How can you check?

Party baskets: have students investigate ways of sharing different numbers of sweets among different numbers of party baskets without left overs or cutting up the sweets.

Applications/Problem solving. Half of the people in a family are males. What might a drawing of the family look like? (Do the students understand that there must be the same number of people in each group? Make sure a range of solutions are viewed)

Extension:

- *Flexibility:* Grouping: Give students collections of discrete items that cannot be cut up e.g. marbles, shells, rocks, counters, and blocks. Have them sort them into two groups: those that can be shared into equal groups and those that cannot. Ask: which collections can be halved and which collections cannot? Introduce add and even numbers to describe the two categories. Extend to other partitions such as can you find quarters, thirds why? Why not? Can you sort them into 8 groups?
- *Reversing*: Collection creations: give students a part of a collection and ask them to make the whole collection. Can you make a different collection? Is this possible? Why can these collections be different?
- *Generalising/Changing parameters:* 1/4 of the class order lunched from the tuckshop each day. How many students might there be in the class and how many of them order lunch?

Unit 3: Fraction as a Number or Quantity

As has been stated before, to understand fractional numbers we need students to learn to read, write, say and interpret them in everyday use. We need them to be able to order them and understand the relevance of the order, and recognise equivalent forms. We need to examine the early understandings students have to have to begin their journey to understanding fractional numbers.

In this unit we look at helping children to develop an understanding of the vocabulary and notation of fractions. During this early understandings stage the emphasis is on the meaning of the fraction words rather than the symbolic form.

Background information

The following ideas have been part of Unit 1 and Unit 2. We list them as a summary:

1. The most common uses of fractions relates to the part-whole idea.
2. The whole can be an object (Unit 1), a collection (Unit 2) or a quantity (Unit 4)
3. Students need to link the action of sharing into a number of equal parts with the language of unit fractions: one half, one third, one quarter and so on.
4. Students should be able to say that there are two equal parts and so each part is one half; four equal parts and so each part is one fourth or one quarter; eight equal parts and so each part is one eighth.
5. Students should learn to count forwards in simple fractional amounts, relating the count to equal quantities.
6. Only after they are comfortable with using fraction words should students be expected to learn to use the symbolic conventions for reading and writing fractional amounts. E.g. if we partition something into four equal parts: each part is called one fourth or one quarter and is symbolically written as $^1/_4$; three of the parts are called three fourths or three quarters and are written $^3/_4$.
7. Students need to understand that to find three quarters of ‘a whole’ one must separate the whole into equal parts and take three out of each four parts.
8. The equal parts may not look alike but they must have the same measure of number, mass, length etc.
9. The equal part maybe a single thing, many things or part of a thing.
10. Through comparison activities students should be able to demonstrate to themselves relationships such as a quarter of a pie is larger than an eighth of it. Gradually they will develop the complete idea that one eighth of a whole is one in each eight parts of the whole. This understanding leads to equivalent fractions.

3.1 Counting in fractions

1. Counting in fractions and talking about the count is a way to have students practice **saying** the fraction words and building confidence to say and use them.
2. Students need to link the action of sharing into a number of equal parts with the language of unit fractions: **one** half, **one** third, **one** quarter and so on.
3. Students should be able to say that there are two equal parts and so each part is one half; four equal parts and so each part is one fourth or one quarter; eight equal parts and so each part is one eighth.

4. Students should learn to count forwards in simple fractional amounts, relating the count to equal quantities.

5. Activities:

   - Counting out portions e.g. cakes, pizzas, small tarts, sandwiches in halves, fourths or quarters, thirds, fifths, sixths, eighths
   - Counting across more than one whole for example saying ‘three thirds or one whole’, four thirds or one and one third and so on for halves, quarters, fifths, sixths, eighths , and sevenths (use the days of the week) Always use contexts that match the fractional numbers so learning about counting fractions has a purpose for the students. For example counting orange halves
   - Counting backwards in fractions once the students are proficient at forward counting.
   - Fraction machine like a function machine (Module A2) pass in objects/collections/ quantities and ask it to make ¾ etc.

3.2 Reading fractions

1. Only after students are comfortable with using fraction words should students be expected to learn to use the symbolic conventions for reading fractional amounts. For example, if we partition something into four equal parts: each part is called one fourth or one quarter and is symbolically written as $^1/_4$ and has to be read as one-fourth (there is no recognition of the line in reading. Similarly; $^3/_4$ is read as three fourths.

2. There are many representations of fractions that young students may get asked to read, objects, shapes, quantities in containers, sets/collections of all manner of items, lengths and so on. Students need to be exposed to as wide a variety of contexts as possible to learn’ to read’ the situation as a fractional quantity.

3. Introduce reading the fraction words and the symbolic representation. Be aware that the symbols are difficult and can lead to misconceptions. For example the symbols have two numbers and a line that suddenly appears; this is a number-a fractional number (as against a whole number). It is a ‘part of a whole’ number. How is it made up? Reading ¼ as one part of four, etc.

4. Activities:

   - Focus on language and labels to reinforce reading
   - Use partner games that require reading fractions.
   - Also be aware that halves and thirds, and fourths as quarters, do not reinforce the pattern that, e.g. $^3/_7$ is three sevenths and $^4/_{11}$ is four elevenths. The early fractions should be twoths, threeths, fourths and fiveths

3.3 Writing fractions

1. The same problems exist in writing as for reading. Thus, as stated in 3.2, only after students are comfortable with using fraction words should students be expected to learn to use the symbolic conventions for writing fractional amounts. For example, if we partition something into four equal parts: each part is called one fourth or one quarter and is symbolically written as $^1/_4$; three of the parts are called three fourths or three quarters and are written $^3/_4$.

2. In the early stages students draw picture representations of fractions of things, collections and quantities. Drawing equal partitions is difficult for young children to learn and needs a lot of practice. However, drawing pictures is an important part of developing meaning for such symbols as $^3/_7$.

3. Drawing partitions should follow on extensive experience with constructing partitions with objects, things, collections and quantities.

4. Finally writing and matching the symbols to the drawing and the collection/object/quantity needs reinforcement

5. Activities:
   - Use labelling parts, posters and counting parts
   - Try to get pattern across that number on top of line is the number of parts being considered, while number under the line is the number of equal parts and names the fraction.

3.4 Same fraction different numerals

1. Students need to understand that to find three quarters of 'a whole' one must separate the whole into equal parts and take three out of each four parts. The equal parts may not look alike but they must have the same measure of number, mass, length etc.

2. The equal part maybe a single thing, many things or part of a thing.

3. Students will to understand that one half, two fourths and four eighths are different ways of representing the same quantity (e.g. 1/2, 2/4 and 4/8 are the same fraction, as is 3/4 and 6/8).

4. Activities:
   - Real world contexts
   - Finding fractions: have students work in pairs. Give each pair a range of 'wholes', such as three straws, a semicircle, one cup of water, a piece of string and a bag of rice. Ask students to find a unit fraction such as a quarter of each of the wholes. Invite them to record what they have done in pictures and words, how they did this and the result for each. Discuss and compare.

3.5 Early equivalence

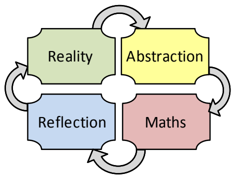
1. Through comparison activities students should be able to demonstrate to themselves relationships such as a quarter of a pie is larger than an eighth of it. Gradually they will develop the complete idea that one eighth of a whole is one in each eight parts of the whole. This understanding leads to equivalent fractions.

2. Students will to understand that one half, two fourths and four eighths are different ways of representing the same quantity (leading onto later understanding that every fraction has an infinite number of equivalent forms).

3. Students should find equivalent fractions by physically or mentally repartitioning materials, e.g. 1/3, 2/6 and 4/12.

4. Activities:
   - The following can represent same size fractions, e.g. Pattern blocks, Bead strings, Pizza circles
   - Finding fractions: Have students work in pairs. Give each pair a range of 'wholes', such as three straws, a semicircle, one cup of water, a piece of string and a bag of rice. Ask students to find a unit fraction such as a quarter of each of the wholes. Invite them to record what they have done in pictures and words, how they did this and the result for each. Discuss and compare.

3.6 RAMR lesson for fraction as a number or quantity



Learning goal**: to say, read and write fractions**

Big Ideas: •The most common uses of fractions relates to the part-whole idea.

•The whole can be an object (Unit 1), a collection (Unit 2) or a quantity (Unit 4)

•Students need to link the action of sharing into a number of equal parts with the language of unit fractions: one half, one third, one quarter and so on.

Resources: Pens, paper. The greater range of materials and contexts explored the richer the student's experience. Magic maths box of stuff (paper plates, plastic cups, clocks, paper, string, straws, coins, boxes, blocks, seashells, beads, balls, bags, and anything else you have collected,

Reality

Local knowledge: what collections need to be counted in your neighbourhood/community/family? Ensure every child has the opportunity to share a counting in fractions story from their life. Where do you see fractions written? What do they look like? Half way home, half a glass of water, portions of pizza, half a minute, six and a quarter years old, sports distances, measurements

Prior experience: N1 A1 O1; this module 1.1 &1.2 Check students can count, know numbers, can partition whole into equal parts, construct and use number line model (length model), can identify the whole(area, set or length model)

Kinaesthetic: Apple parts activity. Have a bag of apples. Have one student hold one whole apple. Cut a second apple into two halves of the whole. Ask: are these the same size? Why? Cut another apple into half. Give each of the four halves to a student. Ask the group to count how many halves. One half, two halves or one whole, three halves or one whole and one half, four halves or two wholes. Continue to build the count until every student is holding a half or a whole. Ask: How many whole apples do I have if I have 4 halves? Have the student reconstruct the apple wholes.

Abstraction

Body: Counting out portions e.g. cakes, pizzas, small tarts, sandwiches in halves, fourths or quarters, thirds, fifths, sixths, eighths

Counting across more than one whole for example saying 'three thirds or one whole', four thirds or one and one third and so on for halves, quarters, fifths, sixths, eighths , and sevenths (use the days of the week) Always use contexts that match the fractional numbers so learning about counting fractions has a purpose for the students.

Hand: Paper folding, cutting, rectangles, circles, strips; grid models; pattern blocks; portioning food, whole and whole groups; measuring, to allow students to create their own counting sequences

Mind: Have students close eyes and imagine objects cut into fractions

Creativity: Students make/draw their own construction of fraction part counting. Encourage students to create their own representations, language and symbols

Mathematics

Language/symbols: collections, sets, split, partition, share, equal, fraction part, equal-sized, quantity, half, halving, whole, fairness, sharing, number of parts, group, number of groups, number in group, unit fraction names, words, symbols (to match and read and copy to label)

Practice:

- Language, fraction labels using partner games;

- Writing and labelling own fraction representations
- Make fractions: have students work in pairs. Give each pair a range of materials. Ask students to find a unit fraction such as a quarter of a whole. Invite them to record what they have done in pictures and words, how they did this. Discuss and compare.
- Play mix and match bingo with picture, symbol, word

Connections: Counting, operating

Reflection

Validate: Play Finding fractions: have students work in pairs. Give each pair a range of 'wholes', such as three straws, a semicircle, one cup of water, a piece of string and a bag of rice. Ask students to find a unit fraction such as a quarter of each of the wholes. Invite them to record what they have done in pictures and words, how they did this and the result for each. Discuss and compare.

Applications/Problem solving. Pattern block problem

Extension:

- *Flexibility* Look for all the places 1 ¼ might be used. Make a list
- *Reversing*: Erica ate ¾ of a cake, show what she ate.

Unit 4: Fraction as a Continuous Quantity on a Line

In this unit we look at introducing children to the idea that we can count compare and order fractions on a number line. During this early understandings stage the emphasis is on recognizing a fraction is a number and placing it on the line, later students will learn to develop a repertoire of strategies for comparing and ordering fractions.

Once again, the sub-sections of this unit will provide the major ideas of the unit.

Background information

Fractions are often used to describe quantities (three quarters of an orange) but they also represent numbers (the number three quarters) that have their own properties and their own position on a number line. For example, a number between 1 and 2 and closer to 2 is an approximate description of the position of the number '1 and three quarters'.

Thus, we can compare and order fractions on a number line just as we do with whole numbers. However, identifying each whole is important at this early stage and assists in placing the fractions.

Finally, as students count in fractional amounts they develop a sense of the relative magnitude and position of easily visualised fractions, such as half, quarters and one and two thirds.

4.1 Partitioning

1. Children need extensive experience in splitting/partitioning a length into equal sized parts. The greater range of materials and contexts explored, the richer the student's experience.
2. Children need careful introduction to representing this partitioning as a continuous quantity as a number line (see Module N3)
3. Activities:
    - Standing Ropes: Stretch a skipping rope across the room/floor/wall. Mark one end of the rope '0'; and the other end '1'. Invite students to stand on or next to the rope to indicate fraction positions, such as a half or a quarter. Add a second rope to extend the line to 2(3) so the students can indicate a position for one and a half lengths of rope. Ask students to explain how they make their decision about where to stand.
    - String: Have students halve a piece of string. Ask them to label the parts. Extend to quarters and so on.
    - Bead string: Have students segment the bead string and name the parts.
    - Paper tapes: Have students fold and segment a paper strip. Label zero, one half, one whole and so on. Ask: how is this half different from half an apple? Draw out the idea that the fractions on the tape show a position on the tape.
    - Chalk and cement: have students draw and construct their own fraction lines on cement. Have them step out their fraction lines.

4.2 Equal parts

1. Partitioning the whole or collection into equal parts is important. Students need to be able to construct the partitions without being given pre-drawn diagrams. This is also particularly so when the partitioning is for part of a number line.

2. Students must be able to produce the partition of a section of a number line for themselves and name each part. For example, For 1/2, It is not sufficient for students to be able to just colour in or mark half on a pre-drawn and partitioned line. They need to construct the two halves or the point where 1/2 is placed on the number line as well.

3. They must be able to attempt (they might not get it quite right) to achieve equal parts, because they know they must be equal.

4. Activities:

    - Rope folding and labelling: Use a large rope outdoors and have groups of students fold and label partitions. Have teams toss bean bags from end of line to positions on the line which they then have to correctly identify.

    - Bead string tens

4.3 Forming wholes

1. This remains an important activity, being able to identify the whole and the parts in the number line. It presupposing common wholes along the line.

2. It requires deconstructing the number line and reconstructing the number line, identifying the whole and the parts for each fraction partition set individually e.g. halves; fourths; thirds; fifths; eighths and so on

3. Activities:

    - Rope and lids

    - Bead strings

    - Hands measure

4.4 Mixed numbers

1. When the number line is divided into whole distances, and the fraction parts have been marked onto the line between each whole number, students need to be able to count in fractional amounts e.g. one, one and a half, two, two and a half, and so on.

2. As students count in fractional amounts they develop a sense of the relative magnitude and position of easily visualised fractions, such as half, quarters and one and two thirds.

3. Students can justify the order of the numbers using materials, diagrams or words.

4. Activities:

    - Apples

    - Metres

4.5 Counting, comparing, ordering

1. Students can compare and order fractions on a number line just as they do with whole numbers. However, it is important to identify each whole at this early stage.

2. Counting fractional amounts along the number line is important is getting students to see the line as the same as the whole number ones – just with fractions marked in between the wholes..

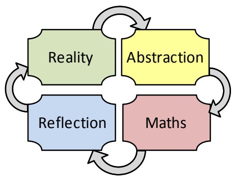
3. As students count in fractional amounts they develop a sense of the relative magnitude and position of easily visualised fractions, such as half, quarters, and one and two thirds.

4. Comparing is in terms of which number is closest to the zero – obviously 11/8 is larger than 5/8. This is straightforward if there is only one fraction (eighths). We can therefore order like denominators and we can use benchmarking to help us order other fractions .For example, 2 and 2/5 is larger than 2 and 4/7 because 2/5 is less than 1/2 and 4/7 is larger than 1/2.

5. Activities:

   - Rope standing
   - Bead strings
   - Chalk and cement

4.6 RAMR lesson for fractions on a number line



Learning goal: Represent fractions using linear materials and reading, counting and comparing fractions and mixed numbers on the number line.

Big ideas: Part-whole; whole-part; representing fractions as a number on a model number line.

Resources: Rope, fraction labels; fraction pegs,

Reality

Local knowledge: Where do students use/see fraction partitions in daily life? Fruit, sport competition, food, using resources and so on.

Prior experience: Check students' vocabulary of fractional parts and their experience in finding halves, fourths and eighths of shapes/objects and collections.

Kinaesthetic: Lay the rope out marking the starting point (always zero) and the whole (1 rope):

Have students walk the whole length of the rope.

Ask a student/s to predict where ½ the rope would be and lay markers at estimate.

Check accuracy of estimation by asking a student to take the end of the rope back to the start. Hammer in the marker. Discuss the accuracy of student estimations/approximations. When do you need to be exact? When is it okay to approximate? Ask: what is this one half of? What if we lay another half to the end of this whole rope? How many rope halves would there be? How many whole ropes? Ensure all students have the opportunity to walk along the rope counting the halves out loud i.e. one half, two halve or one whole, three halves or one whole and one half. Extend and practice counting for as long as is needed for every student to experience this and be confident in their understanding. Use Bean bag toss as a team game to practice. Students take it in turns to toss bean bag from zero line along the stretched out rope then read how far along it is. Is it one half or one whole or 11/2 or is it somewhere in between? Which is it nearer to?

At a later stage or for another lesson repeat this activity marking ¼, remembering to walk the distances.

Divide the rope into ½, walk the distance.

Repeat for ¼ and ⅛, walk each distance.

At each division, place a marker showing ½, ¼, ⅛.

And so on. Eventually use different ropes and build each alongside to construct a 'picture' or diagram' of fractional parts of the rope. Have students read and compare.

Abstraction

Body: Game: Relay Race to Rope Fractions (flags left beside rope):

Four equal teams with students behind their leaders at the starting line.

Teacher shows a random fraction card.

First student runs to that fraction and back; taps the next student's hand and goes to the end.

Another random fraction card is shown. Repeat process. First team with all its members back is the winner.

Hand: Using equal paper strips of different colours, students glue a whole strip onto a blank sheet of paper and write the name/numeral beside it (1 whole, 1/1).

Fold another strip (different colour) into halves, draw a black line over the fold and paste it under the whole and write 2 halves = one whole beside it.

Extend to two wholes.

As each fraction is introduced repeat activity and add strips.

Mind: ask students to visualise running a race? How far have the contestants travelled when they are halfway there? And so on

Creativity: Students draw/tell stories of fraction lengths they have seen ½, ¼, ⅛

Mathematics

Language/symbols: Fraction, partition, equal, not equal, parts, half, halves, halving, quarter, quarters, fourths, eights, whole, same, shape, object, benchmark, compare, share, solve, equivalent.

Practice Record drawings of the rope activity, naming each part.

Find and show equivalent fractions by comparison. Repeat apple activity from RAMR in Unit 3, placing apple parts along the number line.

Explore virtual linear activities to represent, compare and share halves, quarters and eighths:

Connections: whole number to fraction number symbols; relationship to operating and division; measurement

Reflection

Validate: Have students create a picture representing all the ways they know about fractions and representing fractions now.

Have students draw a chalk number line outside and partition it in halves/quarters/ eighths etc. Then have them jump along it saying/naming the fraction parts.

Applications/Problem solving. I am less than one but more than zero. I am bigger than one half. What number am I? Have students guess and discuss strategies they use to work it out. Ask students to create their own fraction clue problem to share with class.

Extension:

- *Flexibility*: Give students various fraction tapes (folded in haves, quarters, eighths, etc.) Ask is 2 halves the same size as four quarters? And so on clarify the whole. Have students label the fraction parts on the tapes. Ask: How is the half marked on the tape different from say, half an apple? How is it the same?
- *Reversing*: Here is a half/quarter/eighth, make the whole.
- *Generalising/Changing parameters:* What might we call fractions for line folded into 3 parts? 6 parts? And so on. What happens when we break the whole into more equal pieces?

Module Review

This section looks back across the units in this module, looking for commonalities in terms of teaching approaches, models and representations and critical teaching points (or competencies). It also discusses where the fraction ideas in this module go in terms of later fraction understandings.

Teaching approaches

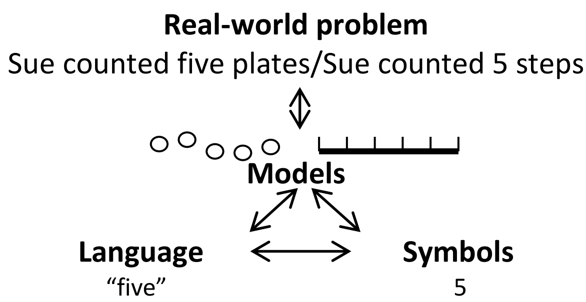
This subsection looks at the ways of teaching ideas common within the units and across most of mathematics. In doing so, we need to consider the ideas in the Review section of AIM EU Module O3, specifically:

- developing young children's thinking skills in relation to creativity as well as reasoning and enquiry;
- drawing upon **explicit strategies** to help very young students develop their skills in counterfactual reasoning; and
- focusing on pedagogical approaches to questioning specifically to improve students' understanding of the role of questions.

In particular, Module O3 recommends that teachers to explore the extent to which:

- their questions can focus specifically on stimulating children's thinking;
- they can create timetabled opportunities for "thinking times" which signal to the children that a non-ordinary (and possibly counterfactual) kind of thinking is being encouraged;
- more opportunities can be created in the classroom for structured dialogue;
- children can be invited to construct written opinions and arguments;
- "story-time" can become an opportunity to develop children's thinking;
- traditional sorting and sequencing tasks can be an opportunity for children to verbalise their thinking;
- play equipment can present children with possibilities for developing their imagination;
- children can be given opportunities for solitary as well as social play;
- children can be asked to evaluate their work critically;
- additional adults in the classroom can be used to develop children's thinking; and
- creative activities can encourage creative "possibility thinking", as well as creative skills.

As well as the above, fractions are numbers and therefore should respond to teaching based on pedagogies that are effective for numbers. In particular this would mean adapting the Payne and Rathmell pedagogical framework below to fractions:



The Payne and Rathmell (1977) triangle for early number

Models and representations

Models and representations are common ways of providing students with thinking images that support learning and applying mathematics. They show the mathematical similarity or equivalence of whole numbers, decimal numbers, common fractions, and percent, rate and ratio, while symbolic representations tend to show difference. In fact, to see the similarities that assist in learning, differences in notation have to be discounted.

Models common to representations of fractions are the usual ones of **area** (e.g. one-third is represented by one-third of a block of chocolate); **set or collection** (one-third is represented by six of the nine children in the group); and **number line** (one-third is represented as a point one-third of the distance along zero to one on a line). These models are also used with whole numbers and decimal numbers so they connect different forms of number. (*Note*: Volume can also be another model but it is usually not as helpful as area, set and line.)

However, symbolic and language representations differentiate and act against connections – making students think that the different number types represent different things. For example, a whole partitioned into five pieces and three pieces being taken can be any of the following:

3 ÷ 5 ("three divided by five"), $^3\!/_5$ ("three fifths")

0.6 ("zero point six" or "six tenths") 60% ("sixty percent")

3:2 ("ratio three to two")

all different notations and languages. Because of history, we have all these different number types to describe the same things, all looking syntactically different.

Critical teaching points

Critical teaching points (sometimes called competencies) represent important steps in the teaching process. For fractions these are as follows:

- understanding equals and unequals in the attributes normally used to show fractions, for example, common attributes such as areas, objects in collections, and length along a line, to less common attributes used for fraction such as time, volume and mass;
- determining whether parts are equal for the attributes used to represent fractions;
- partitioning the whole into equal parts where the whole can be an area, a collection of objects, a volume, or a number line from 0 to 1; and
- reversing the above and forming a whole of which the fraction is part.

In the earliest years, these critical teaching points focus on the simplest fractions and give rise to sequences of ideas to be mastered, such as: forming the whole, equal and unequal, halves and halving, and halves and quarters. Furthermore, the notion of fraction is not just about parts. A fraction is a result of a partitioning into parts where the fraction is the parts being considered in relation to **all the parts being seen as one whole**. This means $^3\!/_5$ is 3 out of 5 parts where the 5 parts are one whole. Thus, understanding that $^3\!/_5$ is not in ratio form (3:5 or 3 to 5) involves **partitioning** a whole into parts and visualising the parts as a whole (called **unitising**).

Overall, we have the following sequence:

- partitioning objects <u>and</u> collections <u>and</u> continuous quantities;
- unitising the partitioned components back into a whole for objects, collections and continuous quantities;
- partitioning (and unitising) in different ways;
- introducing and using fraction words and symbols;
- comparing and ordering fractions; and
- knowing that fractions can be written in different ways and the fraction symbol can show different meanings.

Later fraction understandings

Fractions have a powerful role in showing that numbers can become larger (through grouping) and can become smaller (through partitioning). It is part of a sequence of mathematics ideas focusing on less than one (and larger than −1). This sequence of connected ideas includes division, fraction, decimals, percent, probability, rate and ratio.

All these ideas relate to other numbers in terms of multiplicative relationships, that is, $^3/_5$ of 20, 57% of 89, cement and sand in ratio 3:5, and chance of winning is 1 in 3. Most of them also relate to numbers larger than one (or less than −1) such as $1^3/_4$ (only probability is purely between 0 and 1).

The sequence of ideas that fractions move through in later primary and early secondary years is therefore as follows.

1. **Meanings of fraction**:
   (a) fraction as part of a whole – divide a cake into four equal pieces, one piece is $^1/_4$;
   (b) fraction as part of a collection – 12 lollies partitioned into 3 equal groups is 1 part out of 4 or $^1/_4$;
   (c) fraction as quantity on a number line – 0 to 1 is divided into 5 lengths, so the first jump is $^1/_5$;
   (d) fraction as division – two cakes shared equally among three people is $^2/_3$ each so $^2/_3$ is 2 divided by 3; and
   (e) fraction as operator – $^2/_5$ of 10 is 2 groups of 2 objects or 4 using fraction as part of a collection, while multiplying 10 by 2 and dividing by 5 is 10 × 2 ÷ 5 = 4, thus $^2/_5$ is that which multiplies by 2 and divides by 5.
2. **Inverse relations**: the bigger the denominator the smaller the fraction if numerator stays the same.
3. **Mixed numbers and improper fractions**:
   (a) Fraction numbers can be greater than 1 – for example, $^7/_5$ is $^5/_5$ plus $^2/_5$ – which equals $1^2/_5$.
   (b) Mixed numbers and improper fractions are related – 1 whole is $^7/_7$, 2 wholes is $^{14}/_7$, so 2 $^3/_7$ is 14 + 3 = $^{17}/_7$ – this relation is similar to renaming in whole numbers.
   (c) Numbers can be counted – $^1/_7$, $^2/_7$, $^3/_7$, $^4/_7$, $^5/_7$, $^6/_7$, one whole, 1 $^1/_7$, 1 $^2/_7$, 1 $^3/_7$, and so on. This counting can be forward and backward.
4. **Equivalent fractions, reciprocals and operations**:
   (a) $^3/_5$ is the same value as $^5/_{10}$ and as $^9/_{15}$ and $^{63}/_{105}$. This is because they all cancel down to $^3/_5$ or you get from one to another by multiplying by 1 (e.g. $^3/_5$ × $^{21}/_{21}$ = $^{63}/_{105}$ and $^{21}/_{21}$ is 1) .
   (b) Equivalent fractions enable unlike denominator fractions to be ordered, added and subtracted.
   (c) $^2/_3$ × $^4/_5$ = $^8/_{15}$ by using area model.
   (d) $^2/_3$ ÷ $^4/_5$ = $^{10}/_{12}$ = $^5/_6$ by multiplying by reciprocal (6 ÷ 2 = 6 × $^1/_2$ = 3; thus $^2/_3$ ÷ $^4/_5$ = $^2/_3$ × $^5/_4$).
5. **Structural connections**:
   (a) The placing of fractions into sequences of fractions equivalent to each other is an equivalence class.
   (b) Positive and negative whole numbers and fractions form a rational number system.
   (c) Fractions are represented by different symbol forms and relate to different topics (e.g. percent), meaning that we have to be able to convert between these (e.g. 25% = $^1/_4$).
   (d) The topics in the above are all examples of multiplicative structure and thus their problems can be solved by common methods (e.g. P-P-W diagram, double number line, and change diagrams).

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different to pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "not known" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the Module N4: Early Fractions test item types

This section includes:

1. Pre-test instructions;
2. Observation Checklist; and
3. Test item types.

Pre-test instructions

When preparing for assessment ensure the following:

- Students have a strong sense of identity; feel safe, secure and supported; develop their emerging autonomy, interdependence, resilience and sense of agency; and develop knowledgeable and confident identities.
- Students are confident and involved learners, and develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity.

When conducting assessment, take the following into consideration:

- Student interview for diagnostic assessment in the early learning stages is of paramount importance.
- Use materials and graphics familiar to students' context in and out of school.
- Use manipulatives rather than pictures wherever possible.
- Acknowledge the role of using stories in this early number learning, enabling students to tell stories and act out understandings to illustrate what they know.
- Playdough and sand trays are useful for early interview assessment situations.

Ways to prepare students for assessment processes include the following:

- In **individual teaching times**, challenge students' thinking. "Challenging my thinking helps me to learn by encouraging me to ask questions about what I do and learn. I learn and am encouraged to take risks, try new things and explore my ideas."
- In **group time**, model and scaffold question-and-answer skills by using sentence stems to clarify understandings and think about actions. Encourage students to think of answers to questions where there is no one correct answer, and to understand that there can be more than one correct answer (e.g. *How can we sort the objects?*).
- In **active learning centres**, use activities such as imaginative play, sand play, playdough, painting, ICTs and construction to think and talk about different ways of using materials, technologies or toys. Ask questions and take risks with new ideas.

Other considerations:

- Preferred/most productive assessment techniques for early understandings are observations, interviews, checklists, diary entries, and folios of student work.
- Diagnostic assessment items can be used as both pre-test and post-test instruments.

Remember:

Testing the knowledge can imply memory of stuff; asking the students **what they can do with knowledge** requires construction and demonstration of their understanding at this early understandings level.

N4 Early Fractions: Observation Checklist				
Unit	**Concept**	**Knows**	**Can construct/do/tell/solve**	**Tripping points**
1. Fraction as part of a whole	Partitioning	How to break whole into parts	Build, cut, fold, draw partitions of wholes	Using all the whole No matter how we split the whole always remains the same
	Equal parts	Fraction parts of the whole must be equal size	Make, fold ,draw, count, equal parts	Equal parts need not look alike Partitioning can be done in different ways Parts can be in any arrangement
	Forming wholes	Reversing the above two concepts to reform the whole	Form a whole from a part, an equal part; one of a number of equal parts.	Ensure no part of the whole is overlooked.
	Equal and unequal	Equal parts need not look alike more parts split into the smaller the parts	Partition shapes into equal and unequal l parts	Use up all whole The whole remains the same
	Halves and quarters	½ is one of 2 equal parts of a whole	Repartition objects cutting, folding, drawing visualising	One half is the same as two quarters
2. Fraction as part of a collection	Partitioning groups	How to split collections into parts	Uses sharing strategies to count out , dealing, distributing	The whole stays the same
	Equal parts of a group	Fraction parts of the whole must be equal size	More shares the collection is split into the smaller each share is	Equal parts need not look alike Partitioning can be done in different ways Parts can be in any arrangement
	Forming wholes	Reversing the above two concepts to reform the whole	Form a whole from a part, an equal part; one of a number of equal parts.	Ensure no part of the whole is overlooked.
	Equal and unequal	No matter how we partition the collection/set it remains the same	Partition collections into equal and unequal parts	When is sharing collections easy/hard?
	Halves and quarters	Partition and repartition	Repartition groups of ½, ¼, 1/8	Half of a half. What is the whole?
3. Fraction as a number or quantity	Counting in fractions	You can count in fractions	Can say and use fraction words	Unit fractions
	Reading fractions	symbols	Read One part out of two	'Twoths' ' threeths' and fourths, fiveths
	Writing fractions	Attempts writing symbolic conventions	Identify and talk about written symbols	Matching symbols, words, drawings, models
	Same fraction different numeral	One whole can be 4 parts, take out 3 parts for 3/4	Can say: 1is 2/2is 4/4; 1 ½ is 3/2	4/4 is the same quantity as 1 2/2 is the same quantity as 1
	Early equivalence	½ is larger than ¼; ½=2/4=4/8	Build, cut, fold, draw, and distribute to show.	In a range of contexts: set, object, length, quantity
4. Fraction as a continuous quantity on a number line	Partitioning	Split lengths into parts	Fold paper strips/ string; rope segments; cut straws	The whole stays the same
	Equal parts	Equal Parts linear whole	Equal and unequal partitions	When is sharing easy/hard?
	Forming wholes	Identify the whole and parts on the number line	Deconstruct reconstruct number line	Ensure no part of the whole is overlooked.
	Mixed numbers	Identify each whole on the number line	See the line as the same as the whole number one	Visualise 1/2s 1/4s 1/8s
	Counting comparing and ordering	Identify and compare different fractional amounts	Justify order of fractions	Closeness to zero

Subtest item types

Subtest 1 items (Unit 1: Fraction as Part of a Whole)

1. AIM Subtest Questions style:

    (a) AIM Subtest Question indent style

    (b) AIM Subtest Question indent style

2. AIM Subtest Questions style

Subtest 2 items (Unit 2: Fraction as Part of a Collection)

1. AIM Subtest Questions style:

   (a) AIM Subtest Question indent style

Subtest 3 items (Unit 3: Fraction as a Number or Quantity)

1. AIM Subtest Questions style:

   (a) AIM Subtest Question indent style

Subtest 4 items (Unit 4: Fraction as a Continuous Quantity on a Number Line)

1. AIM Subtest Questions style:

   (a) AIM Subtest Question indent style

Appendices

Appendix A: AIM Early Understandings Modules

Module content

1st module **Number N1: Counting** *Sorting/correspondence *Subitising *Rote *Rational *Symbol recognition *Models *Counting competencies	**2nd module** **Algebra A1: Patterning** *Repeating *Growing *Visuals/tables *Number patterns	**3rd module** **Algebra A2: Functions and Equations** _Functions_ *Change *Function machine *Inverse/backtracking _Equations_ *Equals *Balance
4th module **Number N2: Place Value** _Concepts_ *Place value *Additive structure, odometer *Multiplicative structure *Equivalence _Processes_ *Role of zero *Reading/writing *Counting sequences *Seriation *Renaming	**5th module** **Number N3: Quantity** _Concepts_ *Number line *Rank _Processes_ *Comparing/ordering *Rounding/estimating _Relationship to place value_	**6th module** **Operations O1: Thinking and Solving** *Early thinking skills *Planning *Strategies *Problem types *Metacognition
7th module **Operations O2: Meaning and Operating** *Addition and subtraction; multiplication and division *Word problems *Models	**8th module** **Operations O3: Calculating** *Computation/calculating *Recording *Estimating	**9th module** **Number N4: Early Fractions** _Concepts_ *Fractions as part of a whole *Fractions as part of a group/set *Fractions as a number or quantity *Fraction as a continuous quantity/number line _Processes_ *Representing *Reading and writing *Comparing and ordering *Renaming

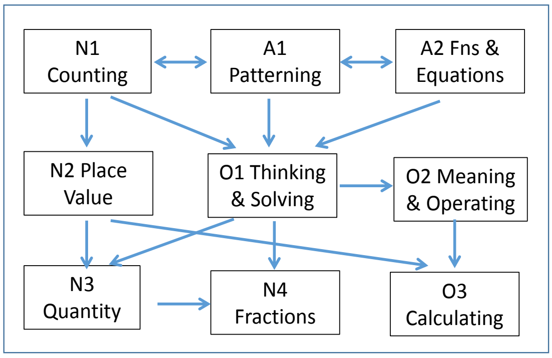
Module background, components and sequence

Background. In many schools, there are students who come to Prep/Foundation with intelligence and local knowledge but little cultural capital to be successful in schooling. In particular, they are missing basic knowledge to do with number that is normally acquired in the years before coming to school. This includes counting and numerals to 10 but also consists of such ideas as attribute recognition, sorting by attributes, making patterns and 1-1 correspondence between objects. Even more difficult, it includes behaviours such as paying attention, listening, completing tasks, not interfering with activity of other students, and so on.

Teachers can sometimes assume this knowledge and teach as if it is known and thus exacerbate this lack of cultural capital. Even when the lack is identified, building this knowledge can be time consuming in classrooms where students are at different levels. It can lead to situations where Prep/Foundation teachers say at the end of the year that some of their students are now just ready to start school and they wish they could have another year with them. These situations can lead to a gap between some students and the rest that is already at least one year by the beginning of Year 1. For many students, this gap becomes at least two years by Year 3 and is not closed and sometimes widens across the primary years unless schools can provide major intervention programs. It also leads to problems with truancy, behaviour and low expectations.

Components. The AIM EU project was developed to provide Years F–2 teachers with a program that can accelerate early understandings and enable children with low cultural capital to be ready for Year 3 at the end of Year 2. It is based on nine modules which are built around three components. The mathematics ideas are designed to be in sequence but also to be connected and related to a common development. The modules are based on the AIM Years 7–9 program where modules are designed to teach six years of mathematics (start of Year 4 to end of Year 9) in three years (start of Year 7 to end of Year 9). The three components are: (a) Basics – A1 *Patterning* and A2 *Functions and Equations*; (b) Number – N1 *Counting* (also a basic), N2 *Place Value*, N3 *Quantity* (number line), and N4 *Fractions*; and (c) Operations – O1 *Thinking and Solving*, O2 *Meaning and Operating*, and O3 *Calculating*. These nine modules cover early Number and Algebra understandings from before school (pre-foundational) to Year 2.

Sequence. Each module is a sequence of ideas from F–2. For some ideas, this means that the module covers activities in Prep/Foundation, Year 1 and Year 2. Other modules are more constrained and may only have activities for one or two year levels. For example, Counting would predominantly be the Prep/Foundation year and Fractions would be Year 2. Thus, the modules overlap across the three years F to 2. For example, Place Value shares ideas with Counting and with Quantity for two-digit numbers in Year1 and three-digit numbers in Year 2. It is therefore difficult, and inexact, to sequence the modules. However, it is worth attempting a sequence because, although inexact, the attempt provides insight into the modules and their teaching. One such attempt is on the right. It shows the following:



1. The foundation ideas are within *Counting*, *Patterning* and *Functions and Equations* – these deal with the manipulation of material for the basis of mathematics, seeing patterns, the start of number, and the idea of inverse (undoing) and the meaning of equals (same and different).

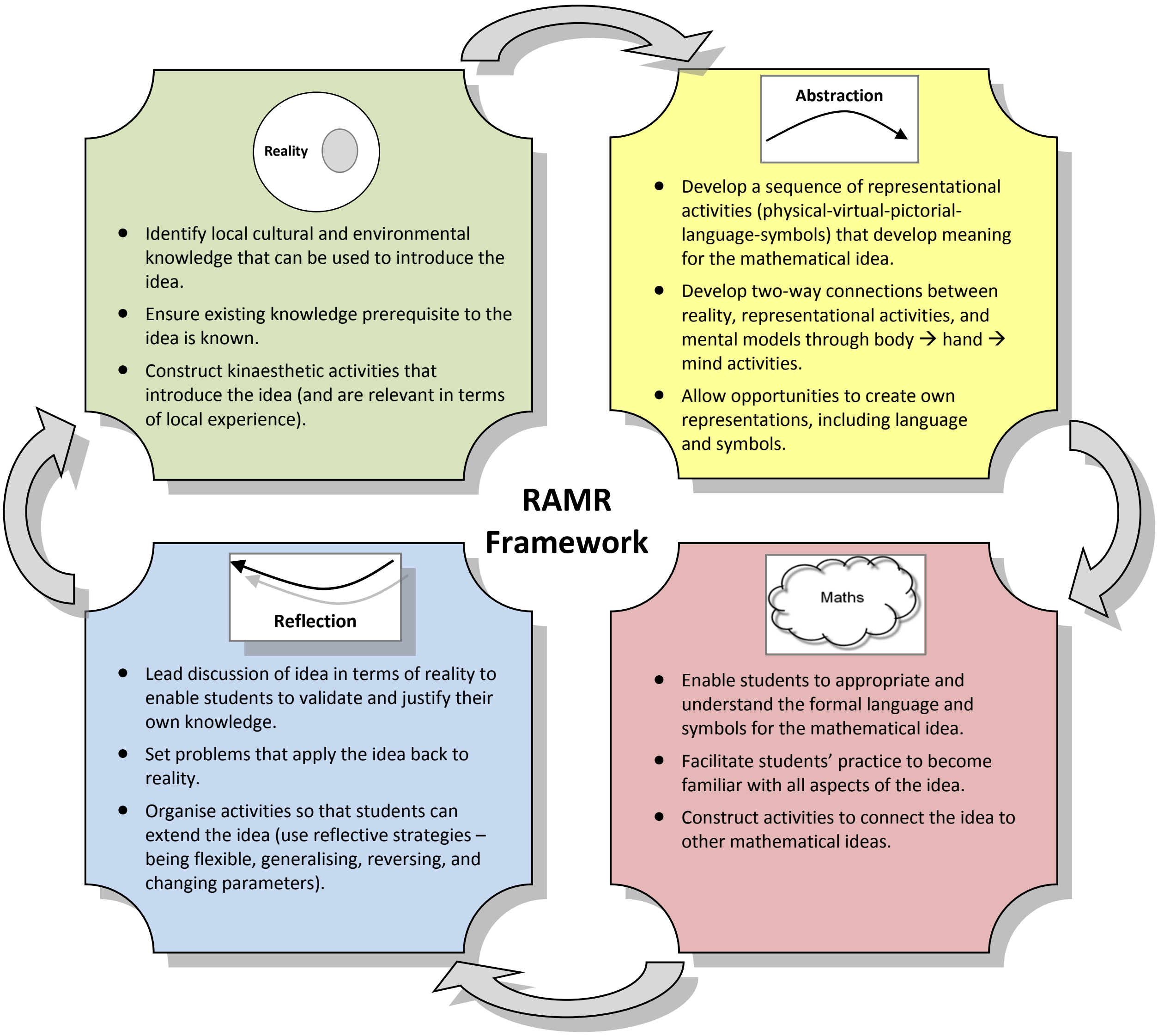
2. The central components of the sequence are *Thinking and Solving* along with *Place Value* and M*eaning and Operating* – these lead into the less important *Calculating* and prepare for *Quantity*, *Fractions* and later general problem-solving and algebra.

3. The *Quantity*, *Fractions* and *Calculating* modules are the end product of the sequence and rely on the earlier ideas, except that *Quantity* restructures the idea of number from discrete to continuous to prepare for measures.

Appendix B: RAMR Cycle

AIM advocates using the four components in the figure below, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem-solving, flexibility, reversing and generalising (see figure on right). The innovative aspect of RAMR is that Reality to Abstraction to Mathematics develops the mathematics idea while Mathematics to Reflection to Reality reconnects it to the world and extends it.

Planning the teaching of mathematics is based around the four components of the RAMR cycle. They are applied to the mathematical idea to be taught. By breaking instruction down into the four parts, the cycle can lead to a structured instructional sequence for teaching the idea. The figure below shows how this can be done.



The YuMi Deadly Maths RAMR Framework

Appendix C: Teaching Framework

Teaching scope and sequence for early fractions

TOPIC	SUB-TOPICS	DESCRIPTIONS AND CONCEPTS/STRATEGIES/WAYS
Early Fractions	Fraction as part of a whole	
	Fraction as part of a collection	
	Fraction as a number or quantity	
	Fraction as a continuous quantity on a number line	

Proposed year-level framework

YEAR LEVEL	NUMBER – EARLY FRACTIONS	
	Semester 1	**Semester 2**
Prep		
1		
2		
Focus		

The numbers in here will need to be decided by teachers.

Accelerated Inclusive Mathematics Project

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