Accelerated Inclusive Mathematics Early Understandings Project

YuMi Deadly Maths

AIM EU

Module O1

Operations: Thinking and Solving

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, written and refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning. The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life. YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

DEVELOPMENT OF THE AIM EARLY UNDERSTANDINGS MODULES

In 2009, the YuMi Deadly Centre (YDC) was funded by the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This resulted in a Year 7 to 9 program of 24 half-term mathematics modules designed to accelerate the learning of very underperforming Indigenous students to enable access to mathematics subjects in the senior secondary years and therefore enhance employment and life chances. This program was called Accelerated Indigenous Mathematics or AIM and was based on YDC's pedagogy for teaching mathematics titled YuMi Deadly Maths (YDM). As low-income schools became interested in using the program, it was modified to be suitable for all students and its title was changed to Accelerated Inclusive Mathematics (leaving the acronym unchanged as AIM).

In response to a request for AIM-type materials for early childhood years, YDC decide to develop an Early Understandings version of AIM for underperforming Years F to 2 students titled Accelerated Inclusive Mathematics Early Understandings or AIM EU. This module is part of this new program. It uses the original AIM acceleration pedagogy developed for Years 7 to 9 students and focuses on developing teaching and learning modules which show the vertical sequence for developing key Years F to 2 mathematics ideas in a manner that enables students to accelerate learning from their ability level to their age level if they fall behind in mathematics.

YDC acknowledges the role of the Federal Department of Education in the development of the original AIM modules and sees AIM EU as a continuation of, and a statement of respect for, the *Closing the Gap* funding.

Contents

List of Abbreviations

AIM EU	Accelerated Inclusive Mathematics Early Understandings
BAE	begin–act–end
RAMR	Reality–Abstraction–Mathematics–Reflection
SPDC	see–plan–do–check

Module Overview

This module, O1 *Thinking and Solving*, is the fifth of the nine Accelerated Inclusive Mathematics Early Understandings (AIM EU) modules. These modules are designed to provide support in Years F to 2 to improve Year 3 mathematics performance. The AIM EU modules are based on the AIM Years 7 to 9 modules, which are designed to accelerate mathematics teaching and learning to where underperforming mathematics students (at around Year 3–4 level in Year 7) can learn six years of mathematics in three years and thus access Year 10 mathematics as mainstream students. The AIM EU modules are designed to accelerate learning in the early years so that students with little schooling cultural capital at the start of their Foundation year can learn the school mathematics understandings normally taught in home, plus those taught in Years F–2, in three years and reach Year 3 with strong Year 3 mathematics knowledge.

This module is the first in a sequence of three on operations, O1 *Thinking and Solving*, O2 *Meaning and Operating* and O3 *Calculating*, which cover Operations from before Year F to Year 2. This module is designed to ensure that teachers cover the before Year F work in operations as well as the F–2 work. The nine AIM EU modules covering Number and Algebra Years F to 2, plus background on the modules, are described in **Appendix A**.

AIM EU uses the YuMi Deadly Mathematics (YDM) pedagogy, focusing on preoperational ideas, which is based around the structure of mathematics (sequencing, connections and big ideas) and a Reality–Abstraction–Mathematics–Reflection (RAMR) teaching cycle that is described in **Appendix B**. The YDM pedagogy endeavours to achieve three goals: (a) to reveal the structure of mathematics, (b) to show how the symbols of mathematics tell stories about our everyday world, and (c) to provide students with knowledge they can access in real-world situations to help solve problems. The YuMi Deadly Centre (YDC) argues that the power of mathematics is based on how the structure of connections, big ideas and sequences relates descriptively (with language) and logically (through problem-solving) to the world we live in.

This chapter introduces and overviews the module by: (a) discussing what is involved in early thinking and solving with regard to pre-operational ideas; (b) listing connections and big ideas with respect to operations in general; (c) introducing at two teaching big ideas with respect to pre-operational to operational; (d) discussing how operation ideas are sequenced and form two column related to O2 and O3; (e) discussing teaching and cultural implications; and (f) summarising the structure of the module.

Early thinking and solving

As the first module in a sequence of modules on operations in a program (AIM EU) that covers prior to school to Year 2 knowledge, Module O1 *Thinking and Solving* covers the operational thinking that occurs before the Foundation Year. Thus O1 deals with **pre-operational thinking** and its teaching. For students, such thinking consists of ideas before operation symbols and numbers and even before operation meanings themselves (which are in Module O2 *Meaning and Operating*).

Pre-operational thinking

The following provides a list of some of the important pre-operational thinking abilities.

1. **Pre-counting**. Play-based activities and pre-counting activities such as sorting and classifying according to one or more criteria (see Module N1 *Counting*) and seeing patterns for one or more attributes (see Module A1 *Patterning*).

2. **Language**. Activities such as telling stories, creating ideas for dialogue, using thinking language (involving words such as *think*, *know*, *guess*, *remember*), giving oral explanations of thinking and reasoning (not written explanations), developing ability to ask questions, and, most importantly, building **listening skills**.

3. **Thinking outside oneself**. Understanding others' minds, particularly that the beliefs of other people may be different, and that other people may have only partial knowledge not necessarily all of it (and that because they have partial knowledge they will not necessarily have all of it).

4. **Creativity**. Using a creative approach to activity such as predicting what might happen in relation to future events, suggesting alternative actions (which could have been taken in the past), and using counterfactual reasoning.

5. **Planning**. Showing "planful, thoughtful, purposeful" behaviour, constructing their own rules (particularly for the purpose of problem-solving), and reasoning logically from given precepts.

Assessing pre-operational thinking

Teachers need to be able to assess students' pre-operational thinking, in particular, what discrete approaches to developing thinking skills of young minds the students have already been exposed to or are already using. This means that teachers have to know **what critical thinking in young children looks like.** The following is a good indication:

(a) the child has a goal in mind and chooses materials and actions;

(b) the child observes the effects of their actions and can "tell" why something is or is not happening or working as intended; and

(c) the child makes adjustments as needed and continues until they have achieved whatever it is they set out to do to their own satisfaction, which might not necessarily be a teacher's or an adult's idea of a solution.

However, it should be noted that even when a child is showing these indications, the teacher needs to probe further because children:

(a) may not know how they came to know something;

(b) may confuse giving reasons with giving evidence;

(c) may be confused by tasks without a clear context; and

(d) may need guidance in sorting objects by more than one criterion.

Connections and big ideas

The starting point for all YDC AIM modules is the connections between mathematics topics and using these connections to accelerate learning, in particular in the formation of big ideas whose learning will provide understanding across mathematics topics and across year levels. This applies equally (or more so) to pre-operational thinking as to operational thinking.

This subsection overviews the role Operations plays in the structure of mathematics, describing how Operations is **connected to the other strands** within the structure of mathematics and how it is based on a series of **big ideas or principles** that recur across all the years of mathematics.

Importance of connections and big ideas in early years

YDC believes mathematics is best understood and applied in a schematic structured form which contains knowledge of when and why as well as how. Schema has knowledge as connected nodes, which facilitates recall and problem-solving. The basis of the YDM philosophy is that knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning.

In particular situations, ideas can be connected until they form a schema that is a big idea. A big idea is one that applies in more than one mathematics topic area and more than one year level. This means all the operations, their properties and the concept of equals form big ideas.

Understanding schematic structure enables teachers to:

(a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;

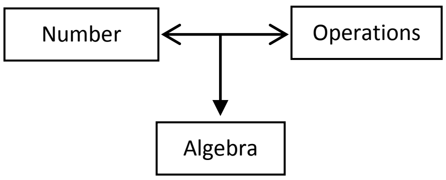
(a) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem-solving;

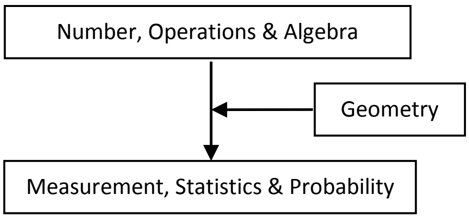
(b) *choose effective instructional materials, models and strategies* – mathematics that is connected to other ideas or based around a big idea can be taught with similar materials, models and strategies; and

(c) *teach mathematics in a manner that enables later teachers to more easily teach more advanced mathematics* – by pre-empting the knowledge that will be needed later, preparing linkages to other ideas, and building foundations for big ideas the later teachers will use.

Connections with respect to operations

Operations concepts are used with Number and lead into Algebra. Finally, through the inclusion of Geometry, Number, Operations and Algebra lead on to Measurement and, more directly, to Statistics and Probability. This can be diagrammatically represented as below:





The major connections between Operations and the other topics are to topics that use number and/or operations as the basis of their mathematics (e.g. Number, Algebra, Measurement, Statistics and Probability). Major connections are as follows.

- *Operations and Number* – an obvious connection as operations need numbers to act on. In particular, the strategies for computation relate to the numeration concepts, that is: (a) separation strategy relies on a place value understanding of 2- to 4-digit numeration; and (b) sequencing and compensation strategies rely on a rank understanding of numeration.
- *Operations and Algebra* – again an obvious relationship as algebra is generalisation of arithmetic activities. In particular, $2x + 3$ relates to an example like $2\times5 + 3$. The difference is that 5 is an actual number while x is a variable.
- *Operations and Measurement* – measurement involves a lot of operations particularly with respect to formulae (e.g. perimeter, area).
- *Operations and Statistics and Probability* – both of these involve operations (e.g. in calculating mean and chance).

As well as between Operations and other topics, there are connections between topics within the two foci of addition and subtraction and multiplication and division; and between topics within Operations. The major connections within Operations are as follows.

- *Inverses* – addition and subtraction are inverses as are multiplication and division.

- *Extensions* – multiplication is repeated addition and division is repeated subtraction.
- *Concepts and problem solving* – the meanings of the operations are the basis of solving problems as they determine which operations relate to which situations.
- *Calculation and estimation* – estimation requires calculation but they also have strategies in common (the calculation strategies help to estimate).

Big ideas with respect to operations

The big ideas for operations are global and come from the principles of a *Field* and *equivalence class* (or extensions of these principles) – a Field is a mathematical structure that is followed by operations on numbers, while an equivalence class is a mathematical structure that is followed by equals. The major big ideas are as follows. Some of the big ideas are considered in more detail in later chapters in this booklet. These sections also consider the properties of operations in terms of number size principles. They also show that subtraction and division are not real operations because they do not obey some of the principles.

1. **Symbols tell stories**. The symbols of mathematics enable the world to be described succinctly and in a generalised way (e.g. 2 + 3 = 5 means caught 2 fish and then caught another 3 fish, or bought a $2 chocolate and $3 drink, or joined a 2m length of wood to a 3m length … and so on).
2. **Relationship vs change**. Mathematics has three components – objects, relationships between objects, and changes/transformations between objects. All relationships can be perceived as changes and vice versa. This is particularly applicable to operations; 2 plus 3 can be perceived as relationship 2 + 3 = 5 or change $2 \xrightarrow{+3} 5$.
3. **Interpretation vs construction**. Things can either be interpreted (e.g. what operation for this problem, what properties for this shape) or constructed (write a problem for 2 + 3 = 5; construct a shape of 4 sides with 2 sides parallel).
4. **Accuracy vs exactness**. Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100 – rounding and estimation) or exactly (e.g. 5 275 + 3 873 = 9 148 – basic facts and algorithms).
5. **Part-part-total/whole**. Two parts make a total or whole, and a total or whole can be separated to form two parts – this is the basis of numbers and operations (e.g. fraction is part-whole, ratio is part to part; addition is knowing parts, wanting total).

Two special pre-operational-to-operational big ideas

The ideas of schema are as crucial in the early years as later, maybe even more so, because the early years lay the foundations for later understandings. Strong foundations build acceleration in learning and powerful mathematics ideas. This is particularly so when big ideas are identified that cover a variety of topics and are useful across more than one year level, and also cover both pre-operational and operational thinking.

For this module, these big ideas should illuminate pre-operational to operational development. There are two big ideas, focusing on thinking and solving which do this that are particularly important for AIM EU.

The beginning–action–end (BAE) big idea.

YDM argues that the basis to understanding operations is what we will call the beginning–action–end, or begin–act–end (BAE) understanding of operations; that is, all operations have a beginning situation, an action that is the operation, and an end situation, as below.

$$\text{BEGINNING} \xrightarrow{\text{ACTION}} \text{END}$$

[addition, subtraction, multiplication, division]

This means that students have to be able to: (a) **recognise** when there is a BAE activity – e.g. *I started with a stack of blocks, John knocked them down and I ended with a pile of blocks*; (b) **identify** differences in beginning and ending – the way things looked, the number of things, and so on; and (c) **discriminate** between different kinds of actions – things that make things messy, things that are tidy, things that remove, things that make larger, and so on. Examples of BAEs are many – in fact nearly everything we do can be considered this way – blowing up a balloon, drawing a picture, walking out of the room, reading a book, painting a block, dressing a doll, and so on.

The see–plan–do–check (SPDC) big idea (from Polya).

We argue that the basis of thinking about and solving problems is Polya's four stages (SPDC) big idea, that powerful thinking when faced with a problem to solve is to work out what the problem is saying (**see**), make a plan to solve it (**plan**), apply that plan to get a solution (**do**) and then check the answer (**check**), as below:

SEE → **P**LAN → **D**O → **C**HECK

Focus of these two big ideas

These two big ideas cover ideas much wider than operations. In fact, they have to be restricted in their actions to move from pre-operational to operational, as the following shows.

BAE can apply to any action, but for operations, the beginning and the end have to relate to items that can be counted (number) and the action that relates to changing number (e.g. joining, separating, removing, comparing, partitioning, and so on). These actions give the concepts of the four operations that are the focus of Module O2 *Meaning and Operating*, showing that the BAE big idea underlies all the four operations. It also underlies word problems because it shows that stories can be interpreted as operations when we determine the beginning, the end and the operation, and work out which is the unknown. Thus the Part-Part-Whole idea of addition can be subsumed under BAE, notably, **beginning** – two parts, **action** – coming together, and **end** – a whole. In this way, addition word problems can be solved by determining which of beginning, action and end are known and unknown (e.g. part unknown is subtraction, whole unknown is addition). It is similar for the other three operations.

SPDC is an approach that can work for any problem but again, for operations, the problem has to act on numbers and the types of things that can be covered have to relate to the actions that give rise to operations. This does not give the direct link to concepts of the four operations as BAE does but it is strong in its application to word problems. SPDC is the framework that has to be applied to the word problems by students when solving them. BAE (begin–act–end) provides the focus to SPDC (see–plan–do–check). It shows that we have to "see" what the beginning, action and end are and then to devise a "plan" based on our knowledge of operations as concepts to solve or "do" the problem (always ensuring we "check" what has happened).

How the two big ideas widen pre-operational thinking

It should be noted that the big ideas have a reflective aspect. They widen what we think of as pre-operational. For example, taking the big ideas into account, it is evident that pre-operational thinking that prepares for, and leads to addition, subtraction, multiplication and division has to include:

(a) students being able to identify actions, beginnings and ends in general, and in everyday life and the classroom, and being able to reverse this process (move from end to beginning);

(b) students being able to see reasonable relationships between the three parts of BAE in general and in relation to the four operations (e.g. given a beginning and an action, what is a reasonable/possible end, given a beginning and an end, what is a reasonable/possible action, and given an end and an action, what is a reasonable possible beginning);

(c) students being able to see beginnings and ends as numbers of objects and actions as changes to these numbers of objects in terms of the actions of the four operations, and to determine whether groups of objects are of different or of same size (needed for the equal groups in multiplication/division); and

(d) students being able to identify and carry out the component parts of SPDC, that is, observe, plan, do and check, and to relate these parts, that is, to make a plan that relates to what they see, to follow that plan when they are doing, and to check in relation to the plan; and to do this in relation to numbers of objects and the actions of the four operations.

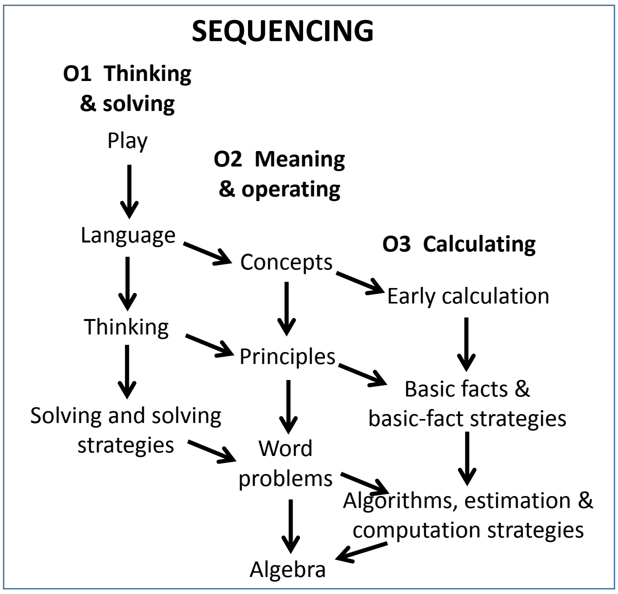
It can also be seen that these ideas need the learning from Module A1 *Patterning* and Module A2 *Functions and Equations*, as well as Module N1 *Counting*. In particular, A1 helps to see the three components of BAE, and A2 helps to see how B and A give E and how to reverse this so A and E give B.

Sequencing

To teach for rich schema, it is essential for teachers to know the mathematics that precedes, relates to and follows what they are teaching, because they are then able to build on the past, relate to the present, and prepare for the future. Thus, YDM presents information in this module as sequences of ideas that relate to and connect with each other. This is particularly important for early understandings as this module is designed to cover prior to school work as well as beginning the Year F to Year 2 sequence.

Sequence across modules

In general, YDM advocates that operations be divided into **two columns or sides**, meanings-operating and computation-calculating. Meanings-operating covers concepts, principles, word problems and extension to algebra, while computation-calculating covers basic facts, algorithms and estimation. This is done to highlight that problem-solving and algebra are based on the meanings-operating side (i.e. concepts and principles), not computation. Of course, expertise with respect to operations is a combination of both sides.



The three AIM EU modules on operations follow this separation in modules O2 and O3. However, to cover pre-operational thinking and prior-to-school experiences), module O1 (*Thinking and Solving*) is placed into the start of the sequence as on right.

Sequence within module O1

To ensure that this module passes through prior-to-school experiences as well as F–2 experiences, this module will be broken into four sections as follows.

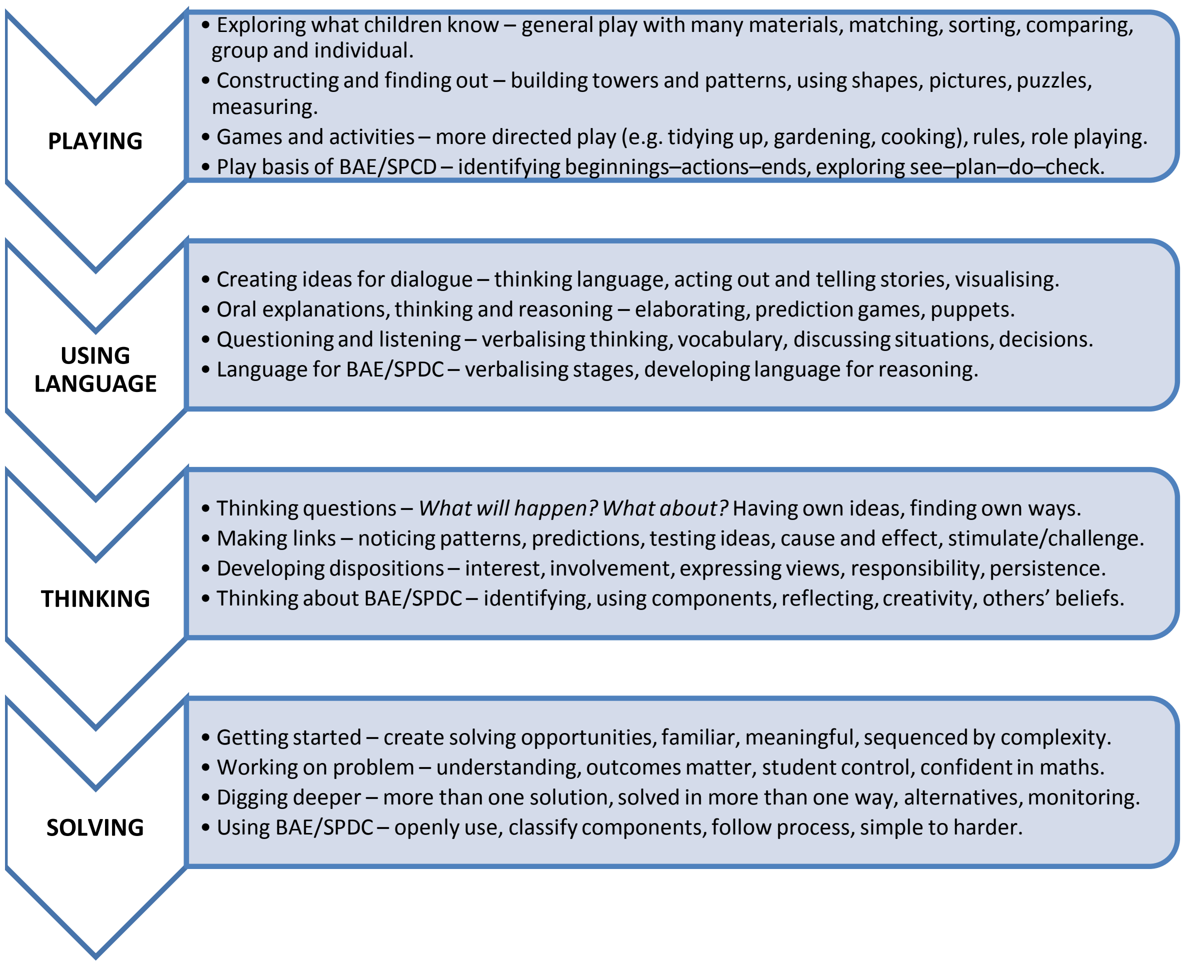
1. **Playing**. Identifying, exploring and doing pre-operational and operational activities – the earliest activities and fundamental ideas.
2. **Using language**. Storytelling, creating ideas, asking questions, listening and developing thinking language – building the language to become a mathematical thinker.
3. **Thinking**. Identifying links, developing thinking skills, choosing directions, identifying alternatives – using BAE (begin–act–end) to understand operations.

4. **Solving**. Working on a problem, digging deeper, making decisions, and reviewing and checking – using SPDC (see–plan–do–check) to solve problems.

Appendix C shows the teaching scope and sequence for O1, O2 and O3 as a table, and provides a proposed year-level teaching framework.

Sequences within units

The diagram below summarises the activities with each of the four units in module O1 *Thinking and Solving*.



Teaching and culture

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework and the impact of Western number teaching on Indigenous and low-SES students.

Teaching implications

Because the teaching in this module moves from before-school knowledge through to Year 2 knowledge, the teaching implications are as follows.

1. **Language teaching ideas**. The module will require you to do play and language activities that will build operations. Some ideas for this are: (a) use storytelling time to develop thinking and to model verbalised thinking; (b) create ideas for dialogue using thinking language, particularly opportunities for structured dialogue; (c) develop questioning, focusing on students' ability to ask questions; and (d) wonder together with students.

2. **Thinking–solving teaching ideas**. There will also be a large focus on starting students on thinking and solving. Some ideas for this are: (a) develop spatial, logical, creative and flexible thinking skills and skills in decision-making, plus metacognition; (b) provide opportunities to explore identifying and describing attributes, matching and sorting, and comparing and ordering; (c) create thinking times and develop the ability to ask questions that stimulate children's thinking and encourage children to elaborate on their ideas; (d) recognise creativity in approach by students and need for solitary as well as social play; and (e) provide opportunities to plan and reflect/evaluate thinking with students and to solve problems.

3. **RAMR cycle**. The module will be based on teaching following the RAMR cycle. Each unit will have at least one RAMR exemplar lesson. The lessons will: (a) start with something students know and in which they are interested; (b) move on to creatively representing the new knowledge through the sequence body → hand → mind; (c) develop language and symbols, and practice and connect components; and (d) finally, reflect the new knowledge back into the lives of the children using problems and applications, and focus on ensuring flexibility, reversing and generalising (by changing parameters if needed). The RAMR cycle is in **Appendix B**.

4. **Models**. It is important that students connect symbols, language, real world situations and models in many and varied contexts and forms. For models, it is important that there is a balance of set and number line (or length) models. Set models are discrete items like money, fingers, counters and other objects, while the number line model is a ruler or steps or jumps along a line. This will be described more fully in the s3ction of the module called Module Review (after the units have been described). This means that 2+3 can be two books joining 3 books (set model) or two steps and then three more steps (number line model).

Cultural implications

In this section, we move on from just looking at teaching to the cultural implications in this teaching, because students who need AIM EU modules include Indigenous and low-SES students.

Teaching Indigenous students. Aboriginal and Torres Strait Islander students tend to be high context – their mathematics has always been built around pattern and relationships. Their learning style is best met by teaching patterning that presents mathematics structurally as relationships, without the trappings of Western culture. As Ezeife (2002) and Grant (1998) argued, Indigenous students should flourish in situations where teaching is holistic (from the whole to the parts). Thus, problems and investigations where there is opportunity for creativity and patterning should have positive outcomes for Indigenous students as long as the problems are realistic, make sense within the Indigenous students' context and matter to the students. In general, this means a lesser focus on algorithms and rules, and a greater focus on patterning, generalisations and applications to everyday life. It also means a strong language focus to translate the students' abilities to the world of standard English.

Teaching low-SES students. Interestingly, holistic teaching is also positive for low-SES students. Three reasons are worth noting. First, low-SES students tend to have strengths with intuitive–holistic and visual–spatial teaching approaches rather than verbal–logical approaches. Thus, a focus on solving problems that make contextual sense and for which the answers matter and with a strong language component should be positive for low-SES students. Second, many low-SES students in Australia are immigrants and refugees from cultures not dissimilar to Aboriginal or Torres Strait Islander cultures. They are also advantaged by holistic algebraic and patterning approaches to teaching mathematics. Third, many low-SES students and their families have long-term experience of failure in traditional mathematics teaching, resulting in learned helplessness. This can be overcome with a focus on investigations along with a strong language focus. Holistic-based problem-oriented teaching of mathematics through patterns is sufficiently different that students can escape their helplessness – particularly if taught actively and from reality as in the RAMR model.

Prior to school knowledge. Both Indigenous and low SES students can come to school with lots of knowledge from their culture and background, but little knowledge that helps with school work. The modules in these books are designed to provide this prior-to-school school knowledge but it is important that their cultural and context knowledge is also equally appreciated and maintained. Pride in heritage and connection to heritage is important in learning ‘school’ knowledge.

Structure of module

Components

Based on the ideas above, this module is divided into this overview section, four units, a review section, test item types, and appendices, as follows.

Overview: This section covers a description the module’s focus, connections and big ideas, sequencing, teaching and culture, and summary of the module structure.

Units: Each unit includes examples of teaching ideas that could be provided to the students, some in the form of RAMR lessons, and all as complete and well sequenced as is possible within this structure.

Unit 1: Playing – exploring and extending what students know. This unit covers the initial teaching of pre-operational and operational ideas, using a play-based approach.

Unit 2: Using language – questioning, listening and creating ideas for dialogue. This unit covers the development of the language needed for thinking and solving problems.

Unit 3: Thinking – links and dispositions. This unit covers the development of questioning, linking and thinking dispositions that begin to enable the BAE and SPDC frameworks to be used.

Unit 4: Solving – getting started and digging deeper. This unit covers the application of BAE and SPDC to identifying operations and solving problems.

Module review: This section reviews the module, looking at important components across units. This includes the teaching approaches, models and representations, competencies and later activity (where the activity in this module leads to in Years 3 to 9).

Test item types: This section provides examples of items that could be used in pre- and post-tests for each unit.

Appendices: This comprises three appendices covering the AIM EU modules, the RAMR pedagogy, and proposed teaching frameworks for operations.

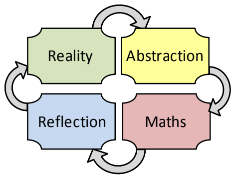
Further information

Sequencing the teaching of the units. The four units are in sequence and could be completed one at a time. However, each of the units is divided into sub-ideas (concepts and processes) that are also in sequence within the unit. Therefore, schools may find it advantageous to: (a) teach earlier sub-ideas in a later unit before completing all later sub-ideas in an earlier unit; (b) teach sub-ideas across units, teaching a sub-idea in a way that covers that sub-idea in all the units together; or (c) a combination of the above.

The AIM EU modules are designed to show sequences within and across units. However, it is always YDC’s policy that schools should be **free to adapt the modules to suit the needs of the school** and the students. This should also be true of the materials for teaching provided in the units in the modules. These are exemplars of lessons and test items and schools should feel free to use them as they are or to modify them. The RAMR framework itself (see Appendix B) is also flexible and should be used that way.

Together, the units and the RAMR framework are designed to ensure that all important information is covered in teaching. Therefore, if changing and modifying the order, try to ensure the modification does not miss something important (see **Appendix C** for detailed teaching frameworks).

RAMR lessons. We have included RAMR lessons as exemplars wherever possible in the units of the module. Activities that are given in RAMR framework form are identified with the symbol on the right.



Suggestions for improvement. We are always open to suggestions for improvement and modification of our resources. If you have any suggestions for this module, please contact YDC.

Unit 1: Playing

AIM EU modules are designed to accelerate the mathematics learning of F-2 students so that pre-operational mathematics knowledge that is acquired prior to school can be covered alongside the Years F-2 curriculum material within the first three years of schooling. The aim of the modules is to reduce/remove the gap in Year 3 between those who come to school knowing the prior to school mathematics knowledge and those that do not. This unit is designed to focus on this prior to school knowledge and too teach the knowledge in an effective manner. Thus, the unit is titled *Playing* as it covers knowledge best learnt in play-based manner.

After providing some background information, the unit covers pre-operational ideas in four steps, focusing on: (a) open activity in exploration; (b) more directed activity in construction; (c) rule based activities in games; and (d) focused activity in looking at the play that underlies the thinking and finding associated with operation, namely the begin-act-end (BAE) and see-plan-do-check (SPCD). It should be noted that this unit: (a) interlinks with Unit 2 as language development is important to the activities; and (b) focuses on ensuring that students understand the operation component of pre-number, pre-counting and early patterning, changing and balancing (the early work in modules O1, A1 and A2). The unit has to have a diagnostic focus on determining what young children know so that can provide support for any gaps with respect to schooling knowledge, and build on any strengths with respect to local cultural knowledge.

Background information

There are four operations in school mathematics: addition, subtraction, multiplication and division. Interestingly, there are only two with respect to mathematical mathematics: addition and multiplication – subtraction is addition of additive inverse (negative) and division is multiplication by multiplicative inverse (reciprocal). However, we shall focus on the operations in terms of school mathematics.

The operations are binary – they act on a beginning number to reach an end number. To find an answer, requires seeing the start, action and end and working out what the operations are, planning and doing the actions of the operations and checking what has been done. This is the BAE and SPDC big ideas.

This means that the pre-operational understandings are composed of all the knowledge, skills an understandings that lie behind:

(a) beginnings and ends (e.g. sequencing activities, pre-number, comparison),

(b) actions (e.g. similarities and differences between beginning and end, change),

(c) identifying a situation/sequence in which there is a begin-act-end (e.g. classifying different situations),

(d) seeing and doing (e.g. observing and acting on observations, identifying same and difference);

(e) planning (e.g. seeing alternatives, making decisions, creative and flexible thinking);

(f) checking (e.g. being systematic, consistent); and

(g) doing more than one thing together for a specific purpose (e.g. linking parts, maintaining sequence).

1.1 Exploring

1. *Playing with what they already know*: General play with many materials – identifying attributes; matching, sorting and classifying; combining groups; separating groups into parts; 1:1 correspondence, rote and rational counting; counting on and skip counting. Doing this in groups and individually. Ensure set materials and line/walking/hopping materials,

2. *Providing opportunities to explore so can find out what the students can do*: For example, identifying, matching, sorting; comparing and ordering – body outdoors → hand materials → mind visualise, and solitary as well social. For example, sort sports equipment.
3. *Playing with a variety of materials to reinforce and support*: For example: sorting, classifying, making designs and patterns, acting out stories, walking and identifying shapes, flowers and so on, forming groups, separation into subsets(different sizes and the same size). Again sort number line material as well.
4. *Focusing on BAE*: Start to look at seeing what is done as three sparts (beginning-action-end). Get students to do things that have the three BAE components. Can they recognise what they are? Can they do things that have the three steps? Note: actions give great opportunities for number line work,

1.2 Constructing and finding out

1. *Open construction*: Building towers with blocks and knocking them over, putting shapes together; balancing activities; selecting shapes to represent something.
2. *Building closed*: Playing with puzzles and jigsaws, working out how they fit together, finding ways to make the overall shape; making patterns and following rules;
3. *Role playing areas*: Shop corner – finding out how much to pay in a shop, act out shopper and shokeeper; nesting, posting and ordering (especially if they are not obvious), and running and hopping along squares.
4. *Using tools*: Using tools in construction, measuring constructions (length and mass)

1.3 Playing games and directed activities

1. *Games*: Playing games the children know → playing new games where students have to follow new rules. Try to build class attitude of "being willing to have a go". Play games where can develop the rules and there are variations in activities and scoring. Play GPS game – children give directions to each other on how to move.
2. *Tools*: Finding out how tools work – measuring tools and other work tools; use these tools with children; do the same for technologies such as tablets.
3. *Being prepared*: Preparing for activities (e.g. scissors, paper, paints for the creative projects); working out how to get the right answer, developing processes for winning.
4. *Directed activities*: sharing equal amounts (e.g. snack time); tidying up and checking nothing is missed or lost; gardening or cooking (e.g. where and what to plant and how many, and measuring ingredients for a recipe with scales, jugs, and so on).

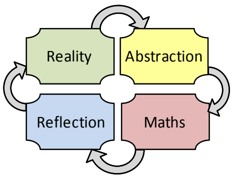
1.4 Play basis of BAE/SPDC frameworks

1. *Identify*: Redo the early activities on functions from Module A1 – focus on input as beginning, change as act, and output as end. Try to identify or interpret everyday activities as BAE (e.g. building a tower, blowing up a balloon, putting toys in a row, sorting big and small into separate piles). Direct children to do BAE activities and identify the parts.
2. *Reverse*: Experience going normal direction (begin-act-end) and then reverse (end-undo action- beginning). Reverse the activities already identified (e.g. letting air out of balloon), then do all new BAE activities as two directions (e.g. *sets* – all colours in one group →reds moved into special group → two groups; two groups → reds put in with other colours → one group; and *number line* – standing at door → walking forward to desk → sitting down; standing up → walking back to door → standing by door)
3. *Changing number*: Start to look at BAE in terms of number and groups. Get children to explore actions that change number and groups (e.g. a group of 3 joins a group of 2 to make a group of 5). Connect with A1

again. Try to do all the four direct types of actions – joining two groups, taking away or separating a group from a bigger group, combining equal groups, and partitioning/sharing into equal groups).

4. *Starting exploration of the SPDC framework*. When talking with students who are doing BAE – ask what they see, what do they plan to do, can they do it, can they check their answer?

1.5 RAMR lesson on playing and operating



Learning goal: Identifying and describing a problem situation

Big ideas: Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity; Beginning-action-end; relationship and change

Resources: A carefully-chosen, inviting set of resources that offer lots of freedom to play, explore, question and try out ideas. Start from something that children might enjoy doing as they play and explore. Such as: construction materials/blocks; sorting/counting materials; pattern making materials/pattern blocks; shape picture materials; puzzles/jigsaws; role playing areas and materials; measuring tools; nesting materials; robots/windup toys for routes.

Reality

Local knowledge: Play items, construction items, and games that are familiar to the children. This initial starting point where ever possible should be at the child's (or children's) own instigation, prompted only by the resources that they are able to interact with in the setting.

Prior experience: N1, A1

Kinaesthetic: Place each different material in a hoop spread around the room. Place students in small groups at each hoop to tell, create, construct and/or pose a story/ problem for others to solve. While children are working approach each group to support, to follow, to question and stimulate thinking. Such as: What happens in the beginning? That is interesting tell me what you are thinking. I am interested to hear your thinking about this. What will happen if…? Have you thought about…? What do you notice about…? What action will you have? I wonder what else /other idea you could try? What happens in the end?

When you approach the first hoop group to identify the story /problem they have posed **model** the questions and statements you want the students to use when they have a turn as the 'solver'. E.g. I am thinking this might be……because I can see….. What other way can I think about this? This is interesting because I could think ……..or ……… Now how could I find out more about this? And so on. After this initial round have other students/ group be the solver in subsequent rounds.

Finally, have each group share back how they came up with their story/proposal/idea/problem? What was their beginning? What action had to be taken? And what had to be found at the end? The problem can be as simple as identifying what has been constructed? Or it might be a quantity problem? Or identifying a counting pattern and so on. It can be useful to construct a list of the problems posed by students for later categorising and further investigation.

Abstraction

Body: Have students use role-play areas to pose and act out problem situations e.g. Working out how much to pay in a shop

Hand: Have students repeat kinaesthetic activity above with own choice of materials e.g. Construction – building, shapes which fit together, balance; Numbers of items in baskets; Puppets in a bus; people in a spaceship; fish in a bucket. Have them use labels to identify the beginning, the action, the end.

Mind: Have students work with puzzles - finding ways of fitting shapes to fit a puzzle and visualise manipulating the puzzle pieces; constructing towers. Have students share with each other what they 'see'. Ask: Can you see how you are beginning? Imagine the ways/actions you might take? What might it look like at the end?

Read a picture book that poses a problem and have children imagine/visualise the problem in the story. E.g. story books: Teddy bears 1-10; 'Who sank the Boat?' 'Changes Changes' by Pat Hutchens.

Creativity: Students make up their own problem situation using own ideas and material choice. Pair students up to swap roles as the 'problem solver' and 'problem poser'.

Again during this time circulate and take the teaching approach to support, to follow, to question and stimulate thinking. Such as: That is interesting tell me what you are thinking. I am interested to hear your thinking about this. What will happen if…? Have you thought about…? What do you notice about…? I wonder what else /other idea you could try? What is the problem? What has to be found? Provide these questions on a small card to other adult classroom helpers. (Are the students demonstrating 'finding out and exploring' behaviour? Have you provided opportunity for the student to 'play with what they already know (maybe have learned in N1 or A1 say?)

Mathematics

Language/symbols: think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember

Practice: Play 'Funny Stories'. There were many papaws growing in my garden last night when I went for a walk. The next evening when I went to pick some for dessert there were less papaws growing in my garden. What could have happened? Students respond and discuss. What is the problem? What action needs to be taken? What needs to be solved/found? Does everyone agree or is there more than one way to look at this? Is there more than one solution? Have children create their own funny story. (Later, make a list/table of the student's stories, the problem, what needs to be found.)

Have pre-prepared picture card problems for students to work on in small groups as they finish the above task. E.g. Bears in space; Unifix stacks; bears in the park; Bears in cars; the mouses tail; pattern block picture cards; What is the problem/ What action can you take / What has to be found? (Are they willing to have a go? Do they understand that the beliefs of others may be different from their own?)

Connections: Sorting, identifying attributes, patterns, matching, ordering, counting Trying to do the same thing in different ways, looking at all the different actions that come out of one situation

Reflection

Validation: Talk with students about actions in their world that change things. Make a collage of changes. Identify B, A and E.

Application/Problem solving: Play what question can you ask? Provide pictures/drawings of problem scenarios and ask students what question could you ask? Have them sort their questions into beginning/action/end.

Extension:

- *Flexibility*: Provide a range of materials/cards that depict three part problems (beginning/action/end). Mix them up and ask students to recreate and tell their own beginning-action-end story problem. Extend. Give them one set of material and ask for all different ways that there could be BAE?
- *Reversing*: Provide an end scenario in models, materials or pictures and ask: What was the beginning of this story problem/ what action was needed/taken? Have students create these.
- *Generalising*: Can children see pattern in the above – things that are the same like moving from one place to another, increasing or decreasing amounts?
- *Change parameters*: Restrict activity to changing number only

Unit 2: Using Language

This unit also covers pre-operational or prior to Year F knowledge, as in the previous unit, but with a focus on language. This means that with respect to the activities and actions that are part of operational and pre-operational knowledge, this unit adds the language to describe and talk about them. It is strongly connected to Unit 1 on playing, repeating a lot of what happens in that unit but adding the language.

The unit is also composed of background information, four steps of activities, namely, *Creating dialogue*, *Oral explanations*, *Questioning and listening*, and *Language for BAE/SPDC*, plus RAMR lessons

Background information

As in Unit 1, we develop the prior to Year F knowledge so that with the acceleration of the learning of the Years F-2 knowledge, students who begin Year F without the expected prior to F understandings can reach Year 3 with no gap between them and other students. This requires a strong focus on language and the relationship between language and action. Therefore, a major part of this unit is to repeat Unit 1 ideas but add in a strong emphasis on language.

For operations and problem solving, there are many new words for young students to learn and to learn how we say these in sentences or maths problems.

First, as seen in Unit 1, students need words to describe their thinking and reasoning such as, think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember beginning, action, end. Too frequently it is assumed that students already know, and know how, and when to use these words.

Second, they meet new maths vocabulary (See 2.3) to use to describe their actions operating and calculating. Using mathematics vocabulary may be confusing because the word means different things in mathematics and non-mathematics contexts or, because two different words sound the same, or because more than one word is used to describe the same concept. This confusion of terms affects the students' ability to operate and solve problems. It is at this point that many students begin to founder and lose confidence as young mathematicians.

Finally, providing classroom structures that encourage conversation, enriching students' capacity to talk about mathematics and partake in the sharing of ideas among students in a maths lesson, including both talking and active listening. To do this we need to consider the following.

2.1 Creating dialogue

1. Creating ideas for dialogue-

    - Arrange furniture so that students can easily turn to each other to see each other. They must be able to speak and listen to classmates. Place an interesting (mathematical) item at each group/pair of desks and invite discussion. Ask: '**What do you notice about this item?**' or 'How do you think this works?' or 'Do you see any patterns?' Or provide an interesting piece of information and ask: 'What can we do with this information?'

    - Encourage students to direct questions and explanations to the class, rather than the teacher.

    - Stand in a variety of spots. This creates the whole room as the forum with each student place as important. As students turn to look at you, their views of the classroom and their position relative to classmates will shift.

- Working in small groups encourages dialogue and ensure the reporting back role is shared around the group to build confidence in tentative students.

- Students need to be taught how to talk to one another in small groups. Some rules for discussion are to do with active listening, thoughtful speaking and respectful collaboration. These might include: everyone in the group is encouraged to contribute; be an attentive listener; each contribution is treated with respect; reasons are asked for; alternatives are discussed before a decision is made and so on. You will want the students to draw up a list in their own words.

2. Developing thinking language-

   - Model verbalising your thinking as you work through a problem using the ‘thinking’ words

   - Play “I think ‘such and such’ because of ‘such and such’. What do you think… (Students name)?” games. This assists students to describe their thinking and to participate in maths conversations. You might use a set of cards depicting scenarios to pass around for each student. Use set and number line instances.

   - When recording student’s ideas, use the students’ words as much as possible. This is a matter of respecting their ideas and enabling them to build confidence in their thinking.

   - Arrange it so that students have a product to share as they explain their thinking e.g. a drawing, chart, or manipulative material. This assists memory and flow of thinking for some children.

   - Have students work together to make a poster of maths thinking words and phrases. Guide the students collaborative talk as they work.

3. Acting out and telling stories-

   - Use storybooks as stimulus to act out problem situations and identify and practice mathematical language in these situations.
   - Provide props and maths rhymes to enact. Enact using set and number line modelling.
   - Counter questions with questions instead of explanations. Students tend to blank out when one student asks a question and the teacher immediately gives an explanation. Asking another question or saying, “What do the rest of you think about that?” or “**Has anyone some thinking they would like to share about that?”**

4. Visualising-

   - Have students visualise an oral story problem as you tell it. Ask students in turn to share what they see. Even when a description is accurate, take time to ask whether anyone saw it a different way? Ask: **did anyone visualise something different but discard that idea**? Help your students to build confidence in their own ideas, knowledge, language and insights by showing that problems can be solved in a variety of ways.

   - Tangents are good- when ideas emerge from the student discussion and captures imaginations, go with it. Keep the big ideas in mind and don’t forget to return to the concept you plan to teach but capture the teachable moment.

2.2 Oral explanations

1. Oral explanations- it is important to give students more time to think before expecting a response.

   - To build confidence to speak, play red group blue group pair activities where red group partners listen to the blue group partner’s explanation and retell what they say. Continue to use ‘talk partners’ whenever possible during lessons.

- Try not to repeat or paraphrase everything students say. This can give the impression that the student is being corrected and may indicate to others that they don't to listen unless the teacher speaks. If needed ask them to repeat what they said for others benefit.
- Listen carefully to student responses. Ask for clarification so that you and others really understand.

2. Thinking and reasoning- students need lots of opportunities to practice joint engagement with one another's ideas to think aloud together, solve problems or make mutual meaning.
    - Honour diverse ideas even if they're incorrect. Do not quickly agree or disagree. Give students time to think and to justify. Ask**: 'Can anyone build on Sally's thinking/ideas**?
    - Respect diverse ideas, methods and examples- these may use stories, diagrams, pictures, technology and so on.
    - Use sentence starters: I think that… I found that…. When I was building I…. today I made… I was thinking ….. I like … because…..
3. Elaborating- how will you encourage children to express their ideas and views?
    - Take time to allow students share different ideas and solutions. Even when a correct solution has been told/shown, ask if there are other ways to do it. This helps to deepen understanding, encourages student to be more willing to work with their own strategies, rather than thinking there is only one solution.
    - Focus students on a problem, puzzle, picture, process or question. Stimulate discussion by asking these types of questions: What do you notice about…? **Do you see any patterns?** What is similar about this and that? What is different about this and that? **How do you think this works?**
    - When students have difficulty thinking about a concept ask: What if? Questions. Encourage them to create the possibilities using 'exploratory' talk. Introduce to students how to speak tentatively using words like 'perhaps', 'if', 'might', 'maybe', 'probably', 'wouldn't it?'
    - Be sure to do the above in set and number line situations and modelling.
4. Prediction games, puppets
    - Use puppets and invite students to use puppets to express their ideas. This can encourage hesitant students to speak and participate in discussions.
    - Play 'I predict' games or other games with scripted 'thinking' language
    - Play board games that offer choices on roll of the dice. This encourages thinking out loud and giving an explanation of choice made.
    - Play maths charades and simple maths Pictionary. These invite opportunity for elaboration by the players.

2.3 Questioning and listening

1. Questioning and listening- we need to ask ourselves what sort of questions will challenge children cognitively?
    - Remind children that conversation is a two way operation requiring both talking and listening
    - Play 'I have, who has?' games which involves a set of cards with clues e.g. I have 16 who has 21? Ensure you have the correct number of cards for the number of students. Or do it with objects. I have the teddy who has the car?

- Remember a student listens harder when a peer speaks than when an adult does. Encourage the children to prepare their own questions.
- Train students to listen to their classmates' observations by asking questions that engage**: "Does this work?" "What do others suggest?"** Or use the single word **"More?"**' to encourage other students to add their ideas.

2. Vocabulary and verbalising thinking- consider how can you model the kind of language and subject vocabulary that you want them to use?
   - Problems can be solved in many ways and we encourage children to explore their own methods of arriving at a solution. Real world problems do not come in a ready set out algorithm or an equation but rather as words and actions. Students need time to practice how to decode these words and actions before turning them into numbers and operations. We need to introduce and model the appropriate use of words in maths many times before students become confident using them themselves. Connecting these words to pictures, actions, materials and stories pertinent to the child's context is essential.
   - Traditional sorting and sequencing tasks can be an opportunity for children to verbalise their thinking.
   - Where ever possible problems need to come from the students' world and be of importance to them to solve. This assists their ability to talk about the problem, verbalise their thinking and associate new maths words into their everyday life.
   - Here is a brief vocabulary:

TOPIC	WORDS
Pre-operational	think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember beginning, action, end thinking language, telling maths stories, storytelling, acting out stories, explaining, predicting, forget, mean, reason plan, planning, finding, testing, checking
Addition	More, add, makes, plus, equals, and, equation, number, joined, altogether, parts, whole, is the same as, equal to, total, increase, sum, another, together, forward-back, above-below and so on
Subtraction	Subtract, take (away), deduct, decrease, difference, less, minus, reduce, remove, leave, left
Multiplication	Multiply, times, of, group, together, left, any, product
Division	Divide, share, remainder
Problem solving	Missing part, known part, un-useful, part- part-whole, how many more, given, needed, parts, whole, missing parts, number line, model, solution

3. Discussion situations- and what ways can you use to extend dialogue?
   - Use pictures to stimulate discussion in the group or groups
   - When observations or questions are brought up by one student, ask "what do the rest of you think about this idea? Does it make sense? Encourage them to consider other examples that would show that the observation is or is not always true.
   - When students first encounter a new concept, encourage them to describe ideas in their own words before introducing the specialised maths terms/words. Then model the use of the terms extensively before asking students to use them.

4. Decision making- consider how you can ask children to explain their thinking when they give the wrong answer.

    - Encourage students to use brain storming ideas to consider alternatives before making decisions Ask: **what else could we consider/do**?
    - Model concept maps to gather as many ideas as possible from students, allow them time to discuss and make choices where to begin.
    - When students are having difficulty doing something or deciding, ask them, "Is there an easier way to do this? Or "In what other ways could this be done?" Rather than telling them what to do.

2.4 Language for BAE/SPDC frameworks

1. Verbalising stages of BAE

    - Mathematics arises from the real world (*reality*) in the form of problems to solve and that we need to *abstract* the essential components of each problem in order to solve the problem. Young students need repeated opportunities to discuss these problem scenarios to identify and state the parts: the beginning, the action, the end.
    - **The beginning–action–end (BAE) big idea**. We argue that the basis to understanding operations is what we will call the beginning–action–end, or begin–act–end (BAE) understanding of operations; that is, all operations have a beginning situation, an action that is the operation, and an end situation, as below.

ACTION

BEGINNING ⟶ END

[addition, subtraction, multiplication, division]

This means that students have to be able to: (a) **recognise** when there is a BAE activity – e.g. *I started with a stack of blocks, John knocked them down and I ended with a pile of blocks*; (b) **identify** differences in beginning and ending – the way things looked, the number of things, and so on; and (c) **discriminate** between different kinds of actions – things that make things messy, things that are tidy, things that remove, things that make larger, and so on. Examples of BAEs are many – in fact nearly everything we do can be considered this way – blowing up a balloon, drawing a picture, walking out of the room, reading a book, painting a block, dressing a doll, and so on.

2. Developing language for reasoning

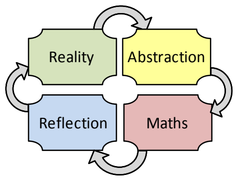
    - As students work with real world problem scenarios they need to develop the language for reasoning and using it to discuss their position. Language of reasoning for young students: think, know, guess, remember, forget, mean, reason, predict.
    - At this stage students are visualising the parts of the story/ problem and making connections to materials and experiences they know might help them to verbalise their thoughts.

3. Options in BAE

    - Visualisation, is the ability of students to imagine in their minds the actions associated with the problem/story.
    - Connecting number story components-story to story question and story to story answer
    - Working out what parts need to be operated on. Is it the beginning or the action or the end part?
    - Can we undo or think backwards? All of these need to be discussed by the student as they act out the story problem.

4. Verbalising stages of SPDC

- Similarly we need to introduce and have young students discuss and enact the **see–plan–do–check (SPDC) big idea** (from Polya). We argue that the basis of thinking about and solving problems is Polya's four stages (SPDC) big idea, that powerful thinking when faced with a problem to solve is to work out what the problem is saying (**see**), make a plan to solve it (**plan**), apply that plan to get a solution (**do**) and then check the answer (**check**),
- SPDC is the framework that has to be applied to the word problems by students when solving them. BAE (begin–act–end) provides the focus to SPDC (see–plan–do–check). It shows that we have to "see" what the beginning, action and end are and then to devise a "plan" based on our knowledge of operations as concepts to solve or "do" the problem (always ensuring we "check" what has happened).
- Investigations and problem solving activities help students develop group collaborative talking skills or, to use talk in order to learn. The classification game is a good activity requiring them to talk to each other and verbalise the stages for problem solving. Give each group plenty of small pieces of paper and a topic each e.g. shapes, patterns, numbers, counting, groups, animals. Each group draws or writes down examples of the category on the separate pieces of paper. The group them sorts and classifies the pieces of paper. You ask: why have they organised their examples in this way? Exchange papers with another group. Will this group classify differently? Can they guess the title you gave the other group? (Containers of items can be used in pre-writing/pre-drawing groups)



2.5 RAMR lesson on using operating language

Learning goal: Telling the problem situation/story

Big ideas: Identifying the parts; Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity, beginning-action-end; relationship and change.

Resources: A carefully-chosen, inviting set of resources that offer lots of freedom to play, explore, question and try out ideas. Start from something that children might enjoy doing as they play and explore. Such as: construction materials/blocks; Unifix cubes; sorting/counting materials; pattern making materials/pattern blocks; shape picture materials; puzzles/jigsaws; role playing areas and materials; measuring tools; nesting materials; robots/windup toys for routes.

Reality

Local knowledge: Play items, construction items, and games that are familiar to the children. This initial starting point where ever possible should be at the child's (or children's) own instigation, prompted only by the resources that they are able to interact with in the setting. (Creating ideas for dialogue)

Prior experience: N1, A1, early ideas for thinking

Kinaesthetic: Place each different material in a hoop spread around the room. Place students in pairs at each hoop to tell, create, construct, act out and/or pose a story/ problem. While children are working approach each group to support, to follow, to question and stimulate thinking. Such as: That is interesting tell me what you are thinking. I am interested to hear your thinking about this.

Have each pair in turn act out and tell their problem situation/story. Ask the non-speaker 'have you some thinking to add to what ' student 1 ' said?

Repeat the activity requesting this time that the other of the pair will be the speaker/reporter/teller.

Finally, have each group share back how they came up with their story/proposal/idea/problem? What was the beginning? And what had to be found? What was the action? What was the end? The problem can be as simple as identifying what has been constructed? Or it might be a quantity problem? Or identifying a counting pattern and so on. It can be useful to construct a list of the problems posed for later categorising and further investigation.

Abstraction

Body: Have students use role-play areas to pose and act out problem situations e.g. Play Dress ups audience have to work out what they are dressing up for?

Hand: Have students repeat kinaesthetic activity above with own choice of materials e.g. Construction – building, shapes which fit together, balance; Numbers of items in baskets; Puppets in a bus; people in a spaceship; fish in a bucket; counters; animals;

Mind: Have students tell a problem story to other students. Ask the students to see in their mind's eye (visualise/picture) what is happening in the story. Ask students' to think of questions to ask the storyteller.

Creativity: Students make up and tell their own unique problem situation story. Pair students up to swap roles as the story teller and the listener/questioner. Encourage children to describe what they are thinking. Ensure plenty of time for students to come up with their solution

Again during this time circulate and take the teaching approach to support, to follow, to question and stimulate thinking. Such as: That is interesting tell me what you are thinking. I am interested to hear your thinking about this. What will happen if…? Have you thought about…? What do you notice about…? I wonder what else /other idea you could try? Tell your problem situation/ story to a friend and ask what the problem is? What has to be found? (Have you provided ideas for each student to create dialogue? Has every student had the opportunity to give an oral explanation of their thinking and reasoning?)

Mathematics

Language/symbols: think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember, thinking language, telling maths stories, storytelling, acting out stories, explaining, predicting, forget, mean, reason

Practice: Play "Making stacks" using Unifix cubes. Have students respond to the following challenges:

Make a stack of 5 blocks. Ask: how can you check this? [Compare, count and so on]

Make a stack of 5 blocks using two colours. Ask: what do you notice?

Make a stack of 5 blocks using only two colours. Ask: What is different here?

Make a stack of 5 blocks. Each block must be a different colour. Ask: how can you be sure?

Make a stack of 5 blocks. The same colour blocks must not touch. Ask: how did you go? (This is more difficult than you might think. Students will have difficulty holding the two rules together as one idea)

Make a stack of 5 blocks. Two colours must have two blocks each.

Make a stack of 5 blocks. You must use at least two colours. Ask: what do you notice?

For this game only go as far as the fourth challenge (or as far as students can manage) on the first occasion. Aim to keep students confidence high.

Listen very carefully to students' responses to your questions. Work carefully to discover what they mean, not what you want them to mean. Respond to the student supportively with replies such as: Did I hear you say…. Or I did not get that. Try again and tell me... Or I like what you said, but what if you looked at it in this way…. Or that is interesting why did you say that?

Have students in pairs and have them play their own 'make stacks' game taking turns to instruct or construct.

(Has every student had the opportunity to ask a question? Has every student had the opportunity to be listened to?)

Make and display a short list of questions for students to use.

Connections: Counting, sorting, patterns,

Reflection

Validation:

- Read a story book, such as A lion in the night by Pamela Allen, allowing children to make their own journey and create their own maps as in the story. Or 'Ten apples up on top' by Theo Le Sieg, allowing children to make their own equations with counters as in the story. This will give the students the opportunity to talk and use the vocabulary already illustrated through the reading and sharing of the text.

Applications/problem solving

- Have students tell a problem story. Have other student retell first student's problem situation. Have a third student choose and ask a question (from the question wall if needed).
- Start focussing on what makes sense – divide situations/stories into three parts as below

B	A	E
Ask a student to set up a story, who was there, what was it like?	Ask a second to describe an action that makes sense for that story	Ask a third to describe the end (an end that makes sense in relation to story)

- Allow students to discuss options (different actions and endings) (*flexibility*)
- Start with A or E and write in the other two parts (*reversing*)
- Provide options and students choose the best to do for any column – state options and get students to discuss if they are sensible – look for patterns and relationships (e.g. things get bigger when our action is to bring more in?) (*generalising*)
- Restrict to number stories (*changing parameters*)

Unit 3: Thinking

This unit starts to focus on operational as well as pre-operational. Its objective is to lay the foundations for O2 which will look at meaning and operating for the four operations. Therefore, its thinking starts to move from play and language to slightly more formal activities. The content is a little more directed and begins to look across the ideas in the four operations, to see them as a basis for thinking. The unit is composed of background information, four sets of activities (thinking questions, making links, developing dispositions and thinking about the BAE and SPDC frameworks) plus RAMR example lessons.

Background information

We have played and developed language. Now is the time to combine the activities in play with the language development and start to have thinking conversations, particularly with respect to operations and the thinking behind them as concepts and the thinking behind their use in problem solving.

There is a tendency for schools to switch too quickly to symbols and calculation when faced with operations. However, the crucial part of operations that pays off in later mathematics is meanings and principle (the general rules of operations). In play activity, two children joining three other children for a game is the addition of two and three and gives five; while two groups of three children joining is multiplication and gives six children. The answers are unimportant long term in relation to the other learnings in these two examples which are that: (a) addition and multiplication are the same in being a joining; (b) addition and multiplication are different in multiplication has equal groups; and (c) addition and multiplication are different in that, in addition, all 2 and 3 (and 5) are numbers of children in the groups while, in multiplication, 3 (and 6) are also the numbers of children in the groups but 2 is the number <u>of groups</u>. This thinking enables problem solving and algebraic work with the operations. Thus, long term, important operation knowledge is much more than calculation.

3.1 Thinking questions

1. Model questioning
    - Provide opportunities to play. Help students develop hypotheses. "If we do this, what do you think will happen?" "Let's predict what we think will happen next."
    - Provide space and time for students to ask questions.
    - Use ‘thinking language’ involving words such as ‘think’, ‘know’, ‘guess’ ,‘remember’
    - Thinking questions to model:

      What will happen if…?

      Have you thought about..?

      What is your problem? Have you thought about…?

      What happens when you test…? Why do you think this will happen?

      How can you fix this? What do you notice about these…(numbers)..?

      Tell me what were you thinking when you were…….?

      Why do you think you thought that?

      How do you think you went?

2. Students make up their own thinking questions. What will happen? What about?

    - Help children view themselves as problem solvers and thinkers by asking open-ended questions. Rather than automatically giving answers to the questions your or whether you view them as correct or not. You could say, "That is interesting. Tell me why you think that." Use phrases like "I am interested to hear your thinking about this." "How would you solve this problem?" "Where do you think we might get more information about this problem?"

    - Gather the questions the students use and display them to refer to and for other students to use as a reference.

3. Creating situations for students to have own ideas

    - Construction play indoors or outdoors with large equipment encourages students to work together. In doing so they need to share and discuss their own ideas. **Encourage thinking in new and different ways**. By allowing children to think differently, you're helping them to use their own creative problem solving ideas. Ask questions like, "What other ideas could we try?" or encourage coming up with other options, "Let’s think of all the possible solutions."

4. Enabling students to find their own way through a problem

    - Don't solve all problems immediately for children. Instead say: “That is interesting, tell me what you are thinking” and provide enough information so children don't get frustrated, but not so much information that you solve the problem for them.

    - Support the student to research further information. You can help students develop critical thinking skills by guiding them towards looking for more information. Say, "Now how could we find out more? Who might know about this? Shall we ask them? Or shall we try searching in a book or on the computer?"

3.2 Making links

This is the point at which can begin to focus more explicitly on the components of BAE – restricting initially to the et model and simple addition and subtraction but being prepared to do comparison and inverse problems

1. Encourage students to notice patterns - restrict to two types of action (joining and separating), and two types of groups (unequal and equal)

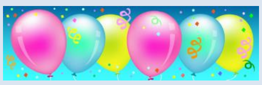
2. Encourage the students to make predictions and then test their ideas and check results,

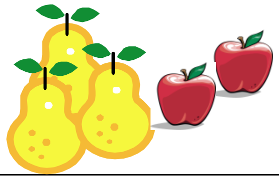
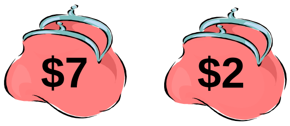
3. Get the students to experience and then discuss cause and effect, Use line as well as set modelling situations.

4. Stimulate the students to do interesting things and trial their ideas – challenge to students with questions that push their boundaries.

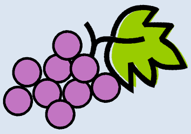
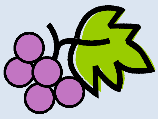
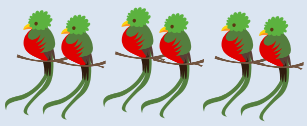
Use the ideas like those below. Note: This is in written form because it is conveying ideas to teachers. In reality it would be done by materials and actions. It also provides practice in visualising (on its own) with actions and materials before using visuals to look at operations. Remember:

BODY → HAND → MIND

	What we know	What it looks like
a.	John has 4 balloons.	 OR 
b.	Millie had \$5. She gave \$1 to Mitch.	 OR

		
	Picture	**Story**
a.		There is I more pear than apples. OR There is one less pear than apples.
b.	 Angela Bronwyn	Angela has more money than Bronwyn. OR Bronwyn has $2 less than Angela.

	Story	**Picture**
a.	2 red cats and 5 yellow cats.	
b.	6 footballs and 3 cricket bats.	
c.	5 cakes; eat 2	

	Begin	**Action**	**End**
a.	 9 grapes	Sam ____________	 5 grapes
b.		Spend $1.	
c.	________________	Lost 3	
d.		______________	
e.	________________	1 burst	

These ideas are taken for Baturo A (2014), *CDAT Operations*, YuMi Deadly Centre, Brisbane QUT. They are set only so make sure you develop **number line instances** as well.

3.3 Developing dispositions (metacognition, group work?)

1. Focus on developing interest and involvement – use engaging and exciting classroom events, both planned and unexpected, to stimulate and challenge children's thinking.
2. Encourage students to express views – use the think, pair, share activity (problem given to children individually, then put into pairs and fours to discuss answers – aim is that students will have the experiences to understand that because someone has partial knowledge of something they will not necessarily have all of it.
3. Give students responsibility – use the technique of social interaction roles where students in groups of three solve problems with one designated as leader, one as recorder/reporter and one as checker – aims is that students will have experiences to understand that the beliefs of others may be different from their own.
4. Facilitate persistence and resilience – promote persistence in problem-solving by encouraging independent play and enquiry, and assist students to realise that they will not get everything correct and to come back from failure.

3.4 Thinking about BAE/SPDC

1. Identifying and using components BAE – move on to activities like those below where the right option is to be determined and ticked – remember again that though they are written, they are better done with activities and discussion.

	Story	**Answer**
a.	Sam has 7 pencils. He would like to have 10 pencils.	*Sam needs 3 more pencils.* *Sam needs to give away 3 pencils.*
b.	Jane gave Ben $3; she had $5 left. How much money did Jane have at the start?	*Jane had $2 at the start.* *Jane had $3 at the start.* *Jane had $8 at the start.*

	Story	*Answer*
a.	Judith had 5 marbles. She won 3 more. Judith had 5 marbles. She lost 3.	*Judith has 8 marbles.*
b.	David had 9 frogs; 2 jumped away. David had 2 pet frogs; he bought 7 more.	*David has 9 pet frogs.*

Answer	**Story**
$12	Paul has $10; Dad gave him $2. *How much money does Paul have now*? Paul has $10; he spent $2. *How much money does he have now*?

8 pets	Lola had 10 pets; she gave1 kitten to her friend. *How many pets now*? Anne has 3 cats; Susie has 5 dogs. *How many pets altogether*?

	Beginning	Action	End
a.	Anne had $10.	She *spent* $4.	More than $10. Exactly $10 Less than $10.
b.	Geraldine had 7 ribbons.	She *bought* 4 new ribbons.	More than 7 ribbons. Exactly 7 ribbons. Less than 7 ribbons.

	Beginning	Action	End
a.	Sam had 9 marbles.	Jack gave him 2 more. He lost 9 marbles.	He has 0 marbles left.
b.	Vinesh has $6.	His brother gave him $4. He gave $4 to his brother.	He now has $10.

	Beginning	Action	End
a.	Bill owns 7 books. Bill owns 9 books.	He read 2 of his books.	7 books left to read.
b.	12 lollies at the start. 9 lollies at the start.	Jill ate 9 lollies.	0 lollies left.

These ideas are taken for Baturo A (2014), *CDAT Operations*, YuMi Deadly Centre, Brisbane QUT. They are set only so make sure you develop **number line instances** as well (e.g. *John walked 6 km and then back 4 km, how much further to home?*)

2. Move on to identifying and using the components of SPDC (see-plan-do-check) – start with see, plan and do. Use simple word problems (e.g. John's family had three cats. They got two more kittens – how many cats does the family have now).

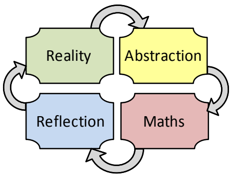
SEE	PLAN	DO	CHECK
Get students to discuss what is said – they could draw the cats – they could draw the extra two cats – they could act it out	Now they have to make a plan – a counter for each cat – three orfiinally and then two other counters for the new cats	This could be to discuss how to get the answer – put out the cats as counters and add, count on, etc.	

Repeat this for a <u>number line activity</u>. Add in the check in all instances.

3. <u>Reflecting and checking</u> – need to discuss what was done and check that it gives a sensible and correct answer. After a few see-plan-do activities, move on to checking. There are a few good ways to do this: (a) one is to follow the plan but this time check each step, (2) a second is to look at the answer in light of the question – is it sensible, and (3) a third is to wok backwards (e.g. take the answer and remove two cats – do you get back to three?)

4. In these activities, try to be <u>creative,</u> discuss a lot, and let students see that other may have different ideas/beliefs than they have. <u>Set up a poster with the four steps</u> on it and write in each of the 4 positions, things students should do. Use the 4 steps to frame your lesson plans. Focus on the strategies of *acting it out* and *making a drawing* along with *given, needed and wanted* and *rewrite the problem.*

3.5 RAMR Lesson on thinking



<u>Learning goal</u>: To be able to identify actions and choose ways to go about solving problems i.e. have their own ideas-make links- choose ways to do it OR beginning-action-end

<u>Big ideas</u>: Beginning-action-end; Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity.

Resources: A carefully-chosen, inviting set of resources that offer lots of freedom to play, explore, question and try out ideas. Start from something that children might enjoy doing as they play and explore. Such as: construction materials/blocks; Unifix cubes; sorting/counting materials; pattern making materials/pattern blocks; shape picture materials; puzzles/jigsaws; role playing areas and materials; measuring tools; nesting materials; robots/windup toys for routes.

Reality

Local knowledge: Play items, construction items, and games that are familiar to the children. This initial starting point where ever possible should be at the child's (or children's) own instigation, prompted only by the resources that they are able to interact with in the setting.

Prior experience: N1, A1, Can recognise a problem situation and/or make up a problem situation; Can tell the problem story.

Kinaesthetic: Have students act out: Problem Dress the monkeys. Here are three monkeys. In the cupboard there are three shirts; a red one, a blue one, and a yellow one. There are also three pairs of shorts; a green pair, and orange pair, and a purple pair. How many different ways can you dress the monkeys?

Ask: what ways can we go about solving this problem? Think of an idea to share? List the ways the students come up with (ideas) as they act out the possibilities. Ask: what do you notice? Keep track of /record any patterns identified by students; or any other ways to do things they suggest. Draw a table with two headings: ideas; ways to do things

Abstraction

Body: Have students use role-play areas to pose and act out a problem situation e.g. Puppies in baskets. The 20 puppies need to go to sleep but there needs to be an even number of puppies in each of the 3 dog baskets. Can you help the puppies to go to sleep? Assist them to make a list of ideas and ways to do things

Hand: Have students repeat kinaesthetic activity above with own choice of props. E.g. dress teddies; ducks in cars; Have them draw their ideas.

Mind: Tell a combinations story and ask students to imagine/visualise it. Ask: What do they 'see'?

Creativity: Students make up their own problem situation using own ideas and material choice. Pair students up to swap roles as the 'problem solver' and 'problem poser'. This time the solver has to identify their idea and the way they chose to go about it. Ask: what other way might you go about it?

Again during this time circulate and take the teaching approach to support, to follow, to question and stimulate thinking. Such as: That is interesting tell me what you are thinking. I am interested to hear your thinking about this. What will happen if…? Have you thought about…? What do you notice about…? I wonder what else /other idea you could try? What is your idea? What ways can you go about it? Provide these questions on a small card to other adult classroom helpers.

[Has each student had the opportunity to express their own ideas? Can they identify an action?]

Mathematics

Language/symbols: think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember, thinking language, telling maths stories, storytelling, acting out stories, explaining, predicting, forget, mean, reason, **plan, planning, finding, testing, checking**

Practice: Play 'Blocks in socks'. Collect a few brightly coloured socks. Focus student's attention on the collection of coloured socks. Explain that you will be placing some blocks in each sock. (Tell the students to close their eyes (no peaking) while you do this. Say: One of these socks has more blocks than any other. Ask: how could we find this sock? Invite a student state their proposal (action) and come (feel the socks; heft the socks; shake the socks) to see which has the most. Invite another student with a different idea. When all ideas have been exhausted have the first student repeat their action and say: now let's see if this sock has the most blocks. Empty each sock and stack the blocks on top. Ask the students whether their class mate was correct? Why? Why not? What action might work better? (Discuss the most blocks and the least blocks concept) Repeat the game as needed.

Extend to 'Pass the socks'. Students sit in a circle with a few blocks in front of them. Every fourth student has a sock. Tell the students with the socks that you will be holding up a number of fingers. The students with the socks in front of them will place that many blocks into the sock. When you call out pass the sock is passed to the person on the left. Repeat for the next two students. On the fourth move, the students take the blocks out and make a stack in front of them. Ask: who has the highest stack? Hopefully they are the same. How many blocks altogether. Make a copy of the stack to stay on show. Instruct students to break up the stacks and put the blocks back in the sock. But keep **one or two** in their hand and not tell anyone what they have. The sock is then passed to the next student. Some blocks will be left in the sock. Ask the students how can we tell how many are in the sock now? Discuss proposed ideas and actions. Have students take the blocks out of the socks and stack them. Compare their proposals. Then compare their stacks. Discuss: which student took the most blocks altogether? Which took the least blocks altogether? How many blocks are in the small stack? How many are in the larger stack?

Or Read Susanna Gretz's Teddy bears go shopping. This book lends itself to the students identifying the problems in the story and choosing topics and actions.

Beginning- having your own idea: own thinking; finding ways;

Action-making links: patterns; predictions; testing; grouping; sequence; cause/effect; drawing; acting out; use materials; make a list; use a table; count; sort;

End-choosing which way/s: planning; checking; changing strategy; reviewing

Ask: Who can add to/ build on 'X's' thinking? Possibility thinking

Are the students able to look for actions to solve the situation? Can they make a choice and come to a decision? Build a chart of actions you might take to solve certain situations. Display where students can refer to it .

Connections: Sorting, identifying attributes, patterns, matching, ordering, counting

Reflection

Validation:

- Provide a range of problem situations. Have students choose an action and share their own idea, make links and explain the way they went about solving it.

- Repeat the monkey problem type as ice-cream cones with two flavours. Use felt board for students to tell the story

Extension:

- *Flexibility*: Have students feel object in feely bag and draw it on the white board for other students to name. Turn what is felt into a BAE situation.
- *Reversing*: Make a collection, ask students to work out and describe how they have made it. Describe attributes have children identify object from a collection. Work backwards from end to beginning.
- *Generalising*: Try to get students to see pattern – to pick out similar ideas – in particular focus on joining and separating actions as important.

Unit 4: Solving

With solving, this unit starts to lay the foundation for both O2 and O3 and also for solving word problems. The unit is also the one in which the BAE and SPDC frameworks become explicit for working with operations. The unit covers background information, four activities (Getting started, Working on a problem, Digging deeper, Using BAE/SCDP frameworks), plus a RAMR lesson. This unit will have a greater focus on SCDO framework.

Background information

The begin-act-end (BAE) and see-plan-do-check (SPDC) frameworks are effective in solving word problems.

BAE framework. For instance, the word problem, *Fred took \$56 from his wallet to pay for the shirt, he had \$121 left in his wallet, how much did he have in his wallet to start with?* can be analysed with BAE framework as follows:

B	A	E
Wallet at start	Spent \$56 on shirt	Have \$121 left
Unknown amount	Subtract \$56	Final amount \$121
\$56 + \$121 = \$177		

This could be solved in many ways: (a)noting that have to invert/go backwards which makes the answer an addition; (b) working out that B has to be the amount that is \$56 bigger than E; or (c) seeing it as the "inverse" meaning of addition (see module O2). We will look further at this understanding and model in this unit.

It should be noted that the word problem above is one that is commonly completed incorrectly by students. This is because the \$56 is taken out of the wallet and this is often seen as subtraction and so the students calculate \$121 - \$56 to get the answer.

It should also be noted that it is set model – make sure there is a balance of number line model problems (*e.g. John was walking from the town, he stopped and started walking back, he met Fred who said he had walked 2 km from the town, John said he had walked 6 km back so far. How far did John walk until he turned back?).*

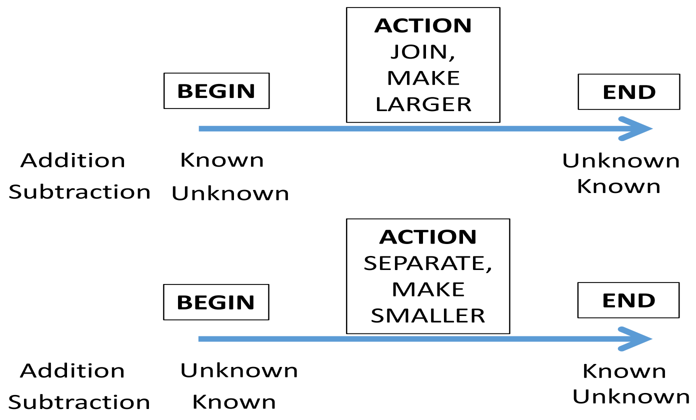
SPDC framework. The SPDC framework also can help solve the word problems as above:

SEE – seeing what B, A and E are (B is to be a amount that when \$56 is subtracted you get \$121
PLAN – finding a way to do this (e.g. see that the unknown is at the start so must work backwards which means adding; think of it as adding \$56 so find \$6 more than \$121 and then \$50 more than \$126)
DO – completing the addition
CHECK – seeing if answer works for subtraction (e.g. calculating \$177 - \$56) and checking it gives \$121.

This means that frameworks like BAE and SCDP are an essential part of finding solutions to operation problems. This is along with knowledge of the meanings of the four operations which cover all the ways addition can appear.

Finally, it is time to begin to look at what will be the focus of operations addition and subtraction in a more formal sense in module O2 *Meaning and operating*. This is looking at actions like joining that make a number of things larger and like separating that make a number of things smaller.

The use of BAE provides a stronger base for consideration of the two operations addition and subtraction because they are inverses of each other. As the diagram below shows, addition and subtraction depend on **where the unknown is as much as what the action is**. As we can see, if we begin with an unknown and take something away make things smaller) and get a known, then to find the unknown we have to add the known and the change



4.1 Getting started

Create opportunities for solving and plan for problem posing. Use familiar contexts, ensure meaningful purposes and monitor mathematical complexity, and use BAE as a structure to find the solution.

1. Create solving opportunities – Set up situations and have discussions that lead to problem situations. Initially do this in everyday familiar contexts for the students. Restrict to joining and separating for this unit. (We will do multiplication, combining equal groups, and division, sharing into equal groups, in O2. Use BAE as follows:

B	A	E
Get student to tell you what the beginning is?	Get students to tell you what the action is?	Get students to tell you what the end is?

   Use the information from the two other parts/columns to help you work out what is unknown?

2. Use SEE and focus on starting – setting up discussions on understanding problem – make these problems interesting and worthwhile solving.

3. Note that there are three ways that an action can increase number:

   - by something joining in (e.g. extra people or money) – e.g. three people walking were joined by two more;
   - by a comparison where one of the sets is larger – e.g. there were three more women than men; and
   - by looking backwards to the beginning when something is removed (e.g. the farmer removed 2 cows, there were three left, how many cows to start with?

4. It is crucial to sequence situations/activities from easy to hard unless you are challenging the students. A problem is more complex when it uses unfamiliar situations, it requires the solver to work backwards, and it has more than one step.

4.2 Working on a problem

Young Children need problems: which they understand which are in familiar contexts; where the outcomes matter to them - even if imaginary; where they have control of the process; and involving mathematics with which they are confident.

1. Provide simple word problems and allow students to have control. Let them try to do it themselves but be ready with the SPDC and/or the BAE frameworks?)

2. Set up discussions on plans, options and possible outcomes when discussing what to do (PLAN?). Begin with problems where outcomes matter. Try to match problems to students so that build confidence. Allow students to work in groups but also to work alone.

3. Use the SPDC framework as in Unit 3: Provide problems and build instruction around four steps. Challenge the students with problems that have more than one step (e.g. buying two things and working out how much change?). Spend time in lessons where working out each step is the focus. Even provide the answer and then spend time working out how to get there. Get students to suggest ways to make errors

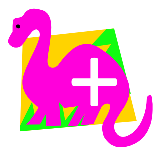
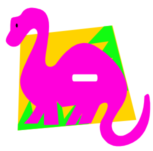
SEE	PLAN	DO	CHECK
What the problem is asking – discuss what it means	How can we tackle the problem Make a plan	Following plan – check no errors	Checking that answer is correct

4. Most importantly – **reverse the process**. Give students a topic and an answer and let them make up problems that will have that answer. Try to get more than one possible problem. Change context (shopping problems, sport problems, and so on). Get then to act out their problem – with other students, with materials, with human/animal figures.

4.3 Digging deeper

Choose rich problems -problems may have more than one solution and can be solved using a range of methods at different levels. Ask solving questions such as: Getting to grips: What are we trying to do; connecting to previous experience: Have we done anything like this before; planning: What do we need; considering alternative methods: Is there another way; monitoring progress: How does it look so far; evaluating solutions: Does it work? How can we check? Could we make it even better?

1. Provide problems with more than one solution. Ask students to give more than one way of solving the problems. Discuss alternatives.
2. Look at ways that drawings or acting out with materials will help find solutions.
3. Set up metacognitive group activities such as think pair share and social interaction roles.
4. Use the BAE activities with operation sign as follows. Tick the story that relates to operation and vice versa.

	Story	Operation
a.	• 6 plums in one bag; 8 in another bag. *How many plums altogether*? OR • Angus had \$10. He bought a kite for \$7. *How much money does he have left*?	
b.	• Jack gave his friend \$3; he had \$5 left. *How much money did he have at the start*? OR • Charlotte had 7 dolls; she gave 2 to her friend. *How many dolls does she have now*?	

	Story	Operation
a.	Amy had 5 Smarties; she ate 3 of them. *How many Smarties did she have left*?	$\begin{array}{r} 5 \\ -\ \underline{3} \\ \underline{2} \end{array}$ $\begin{array}{r} 5 \\ +\ \underline{3} \\ \underline{8} \end{array}$
b.	Peter had 8 marbles; he won 2 more. *How many marbles does he have now*?	$\begin{array}{r} 8 \\ -\ \underline{2} \\ \underline{6} \end{array}$ $\begin{array}{r} 8 \\ +\ \underline{2} \\ \underline{10} \end{array}$

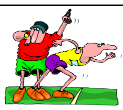
	Story	Action
a.	David had $7. He spent $5. *How much money now*?	7 + 5 or 7 − 5
b.	A burger costs $8; a drink costs $3. *How much altogether*?	8 + 3 or 8 − 3

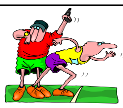
	Action	Story
a.	6 + 3	6 eggs in one nest; 3 eggs in another. *How many eggs altogether*? 9 eggs in one nest; 3 eggs hatched. *How many eggs left to hatch*?
b.	10 − 6	Team A scored 10 points; Team B scored 6 points. *What was the difference in scores*? Team A scored 6 points; Team B scored 4 point. *How many points scored altogether*?

These ideas are taken for Baturo A (2014), *CDAT Operations*, YuMi Deadly Centre, Brisbane QUT. They are set only so make sure you develop **number line instances** as well.

4.4 Using BAE/SPDC frameworks

1. Openly use frameworks with symbols and stories as follows – in the example below, tick the best option. These activities are trying to connect visuals and symbols and language (and visuals).

	Begin 	**Action** 	**End** (Ring) 
a.	Pete had $6.	Earned $3 more.	Pete now has $3. Pete now has $9.
b.	Ned had 5 marbles.	Lost 2 marbles.	Ned now has 7 marbles. Ned now has 3 marbles.

	Begin 	**Action (Ring)** 	**End** 
a.	Tippi had 7 bracelets.	Join 7 and 9. (7 + 9) Take 7 from 9. (9 – 7)	She now has 9 bracelets.
b.	Nigel had 6 stamps.	Join 6 and 4. (6 + 4) Take 4 from 6. (6 – 4)	Nigel now has 4 stamps.

[**These ideas** are taken for Baturo A (2014), *CDAT Operations*, YuMi Deadly Centre, Brisbane QUT. They are set only so make sure you develop **number line instances** as well.]

2. There should be three options – which one is missing? It is havig options at the start – make up some examples of this? Why is start unknown important (leads to inverse being operation?)
3. Make up some activities similar to above for See-Plan-Do-Check – 4 columns, each has options in turn.
4. Make up BAE activities for symbols – here are some?

	Start	**Finish (Ring)**	**Operation**	
a.	Trish had 6 plums.	Trish now has 2 plums	6 + 2 = 8 6 – 4 = 2	
b.	Bill had $7.	Bill now has $6.	$\begin{array}{r} 7 \\ -\ 1 \\ \hline 6 \end{array}$	$\begin{array}{r} 7 \\ +\ 6 \\ \hline 13 \end{array}$

	Start	**Finish (Ring)**	**Operation**
a.	Barbie has 6 balloons.	Barbie now has 2 balloons. Barbie now has 12 balloons.	6 + 6 = 12
b.	Jill had 3 trains.	Jill now has 8 trains. Jill now has 2 trains.	$\begin{array}{r} 3 \\ -\ 1 \\ \hline 2 \end{array}$

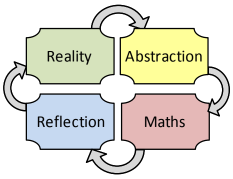
a.	Six plus two equals eight. 6 plus 2 equals 8.	6 + 2 = 8

	6 minus 2 equals 8. 2 plus 8 equals 6.	
b.	10 subtract 3 equals 7. 10 and 7 equal 3. 10 take away 7 equals three. 10 minus 7 is equal to 3	10 – 7 = 3

a.	5 minus 2 equals 3.	5 + 2 = 7 5 – 2 = 3 3 – 5 = 2
b.	5 plus seven equals twelve	5 + 7 = 2 5 – 7 = 12 5 + 7 = 12

These ideas are taken for Baturo A (2014), *CDAT Operations*, YuMi Deadly Centre, Brisbane QUT. They are set only so make sure you develop **number line instances** as well.

4.5 RAMR lesson on Solving



Learning goal: having students identify and try different ways to work it out

Big ideas: construction & Interpretation

Resources: Socks, pegs, clothesline, knitting wool and needles; drawing and collage materials

Reality

Local knowledge: Explore problems pertinent to the student using Play items, construction items, and games that are familiar to them. This initial starting point where ever possible should be at the child's (or children's) own instigation, prompted only by the resources that they are able to interact with in the setting.

Prior experience: N1, A1 Recognising a problem situation; telling a problem story; looking for actions to solve the situation.

Kinaesthetic: **Socks for Spiders** Read/recite ‘A spider’s Bedsocks’ by Phil Mena

A spider knitted bedsocks-

He made them big and bright.

He put them on his little feet

To keep them warm at night.

And when they needed washing

He used a lot of pegs

To hang the socks outside to dry-

Eight pegs, eight socks, eight legs.

Have students act out poem. How do we begin to do this? Allow children time to work out how to represent a spider. Students linking arms back to back while seated on the floor is one solution to creating a spider. How many students for a spider? How many socks? How many spiders could we make with our class? How many socks?

Review the activity

1. Recap how did we **begin** (See) to solve the problem of acting out the poem? What did we try? What worked? What worked best?
2. What did we do **to work on (Plan)** the problem?
3. What else did we discover? (**Digging deeper**, taking further) (Do)
4. What's our **conclusion**? (check)

Create a 4 step Solving poster* for students to refer to while doing the activities that follow.

Abstraction

Body: How many other creatures can you think of for bedsocks? Act out Ladybugs; horses and so on

Hand: use a play tray** to have students repeat kinaesthetic activity above with own choice of materials, counters; provide board games for students to play which involve choice of paths or outcomes.

Mind: Have students visualise the mice in 'Mice' by Rose Fyleman (I think mice Are rather nice Their tails are long Their faces small, They haven't any chins at all. Their ears are pink, their teeth are white, They run around the house at night. They nibble things They shouldn't touch and no-one seems To like them much. But I think mice Are nice! Students can draw their mind pictures and compare them.

Creativity: Now have students create their mice from collage materials. Pose some questions like: I Have 9 tails, 8 eyes, 10 ears, 15 legs. How many complete mice can I make? (Adjust numbers to match your age group)

Mathematics

Language/symbols: think, solve, pose, problem, else, know, guess, predict, maybe, might, perhaps, notice, remember, thinking language, telling maths stories, storytelling, acting out stories, explaining, predicting, forget, mean, reason, **plan, planning, finding, testing, checking**

Practice: Each student or pair of students choose a picture story book (Choose from Books you can count on by Rachel Griffiths). They need to identify the problem and make a plan of how the problem can be solved. Have children present their problem and plans to the rest of the class. (Example story titles: Teddy Bears Go Shopping, Susanna Gretz;A Bag Full of Pups, Dick Gackenbach; Anna's mysterious multiplying jar, Mitsumasa Anno; One, two, three going to sea, Alain; The great Big Enormous Turnip, Alexei Tolstoy; Alexander who used to be rich last Sunday, Judith Viorst; The smallest Turtle, Lynley Dodd;

Connections: Sorting, identifying attributes, patterns, matching, ordering, counting

Reflection

Validation: Play what question can you ask? What plan would you use? Provide pictures/drawings of problem scenarios and ask students what question could you ask?

Extension: Ask students to make a problem from this poem and provide a solving plan. What different problems did they find? How many different ways to solve each problem? How did they check?

A Dreadful Thought by Rose Fyleman

Oh! Mrs. Spider,

How dreadful it would be,

If your children needed stockings,

Like Jennifer and me!

For every little spider,

Never mind how small,

Every little spider

Has eight legs in all!

Four pairs of stockings,

Four for every one,

You never, never, never,

Would get your knitting done!

A 4 step solving poster might look like this: Where ever possible use the student's actual words to create the poster

1. Getting started: How do we begin to solve the problem? (See) Identify the problem Talk about the problem List what we might try	**2.Working on the problem (Plan)** Plan Prepare Predict
4.Concluding (Check) Review what you have done Tell what you have done Compare what you have done	**3.Digging deeper (Do)** Try other ways Monitor what we do Find ways to check what we have done

Module Review

This section reviews the units in this module. It looks across the units and identifies outcomes that go beyond the particularities of the units. The first of these is general **teaching approaches**, that is, ways of teaching ideas common within the units and across most of mathematics. The second is **models and representations**, that is, common ways of providing students with thinking images that support learning and applying mathematics. The third is **competencies**, that is, abilities that are important across the units and into the future.

Teaching approaches

Things we need to consider:

- Whilst classroom approaches to young children's thinking skills aim to develop reasoning, enquiry and creativity, most recent psychological research has focused on children's powers of reasoning and enquiry exclusively (and not **creativity**)
- Current approaches to teaching thinking skills do not draw upon **explicit strategies** to help very young pupils develop their emergent theory of mind or their skills in counterfactual reasoning
- whilst the psychological literature reveals that children find some kinds of question more difficult or confusing than others, few studies relating to pedagogical approaches focus on **questioning** specifically#

Encourage teachers to explore the extent to which:

- their questions can focus specifically on stimulating children's thinking
- they can create time tabled opportunities for 'thinking times' which signal to the children that a non-ordinary (and possibly counterfactual) kind of thinking is being encouraged
- more opportunities can be created in the classroom for structured dialogue
- children can be invited to construct written opinions and arguments
- 'story-time' can become an opportunity to develop children's thinking
- traditional sorting and sequencing tasks can be an opportunity for children to verbalise their thinking
- play equipment can present children with possibilities for developing their imagination
- children can be given opportunities for solitary as well as social play
- children can be asked to evaluate their work critically
- additional adults in the classroom can be used to develop children's thinking
- creative activities can encourage creative 'possibility thinking', as well as creative skills

Finally, because algebra is the generalisation of arithmetic, it will be necessary to focus on the development of the new concept of variable as standing for any number and on the big ideas from arithmetic that carry through into algebra (e.g. concepts of operations and equals, principles associated with operations and equals).

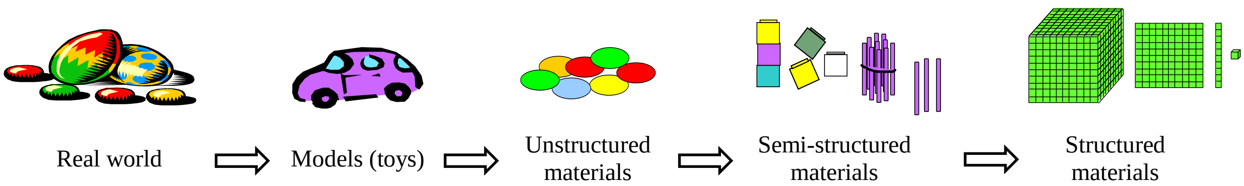
Models and representations

The end product of operations is a **symbol language** (e.g. 2 + 3 and 2 + 3 = 5). There are problems in this symbol language because a focus on answers and not relationships leads many students to believe that 2 + 3 = 5 is acceptable and 5 = 2 + 3 is not; for many students the equals sign has become a symbol for "put the answer here" or "do something" when its real meaning is "same value as". Mathematically, *7 subtract 3 equals 4* is represented with symbols as 7 – 3 = 4; it must be seen as 7 – 3 **is the same value as** 4. This means that it is possible and equally correct to show 7 – 3 = 4 as 4 = 7 – 3. Similarly, students also need to be familiar with relationships that are not closed to a single answer such as 2 + 3 = 4 + 1.

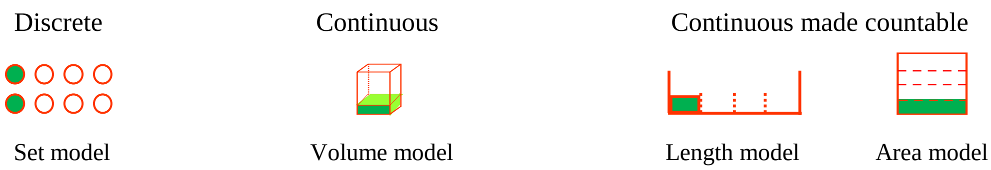
This symbol language has to relate to everyday language and real instances. Many forms of symbols are possible and all relate to stories as the addition stories below show. The interesting point is that treating the symbols like a concise language (so 7 + 4 is 7 things joining 4 things), enables the symbols to tell stories and to describe the world, an outcome more powerful than answers. This is a major part of building the concepts of the operations (see sections 2.1 and 3.1).

One way to represent this symbol language is to use **models**. Models connect and unify mathematics, whereas symbols tend to emphasise difference. For operations, there are three models: **set** (e.g. Unifix and counters), **length** (e.g. Unifix stuck together or number lines), and **array or area** (e.g. counters, Unifix, dot paper or graph paper), which is for multiplication and division (e.g. 3 × 4 is 3 rows of 4 or a 3 by 4 rectangle). These models do much more than just help with meaning – they show structure and apply across many topics. For example, the array model can help with: basic facts (e.g. 4 × 7 is 4 rows of 7 which is 4 rows of 5 plus 4 rows of 2); algorithms (e.g. 24 × 7 is 24 rows of 7 so it is 20 rows of 7 plus 4 rows of 7); fractions/decimals and percent (e.g. 0.2 × 0.4 is a rectangle 2 tenths by 4 tenths which gives 8 squares in 100 or 0.08). Multiplication is the inverse of division and ×1 or $\times\frac{2}{2}$ or $\times\frac{3}{3}$ and so on leaves everything unchanged (e.g. $\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$, and so on).

YDM focuses on knowing **connections between real-world situations, models, language and symbols**, and strategies that lead to meaning and generalisation rather than rote definitions and procedures (or algorithms) that lead to particular answers for particular numbers. In this approach, models can be used in the same sequence as for number and can be tailored to suit the level of number representation students are working at. Kinaesthetic acting out of operations should be completed first which can then be transferred to tangible concrete models and finally abstract number sentences (see figure below).



Models also assist us to link **discrete and continuous**; there are a range of models that cross that divide as the examples below show. The continuous made countable models are particularly important.



As number is taken from discrete objects and applied to continuous attributes, its nature changes. Numbers designate ends of units and zero represents the start of units and not nothing (although it represents no units).

Competencies

The major competencies for this unit are as follows:

Identifying Beginning, action and end through

Play -in computational situations (identify attributes; one to one counting; combine collections; Count on; skip count/grouping ;)

- Recognise a problem situation and or make up a problem situation
- Be willing to have a go at resolving it

Language - tell problem stories

- Use language of reasoning and thinking
- Develop ability to ask questions

Thinking -Look for actions to solve situations

- Think of ideas (creativity)
- Make links
- Choose ways to do things

Solving – being able to get started

- Trying different ways to do things
- predicting

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different from pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "don't know" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the Module O1: Thinking and Solving test item types

This section includes:

1. Pre-test instructions;
2. Observation Checklist

Pre-test instructions

When preparing for assessment ensure the following:

- Children have a strong sense of identity; feel safe, secure and supported; develop their emerging autonomy, interdependence, resilience and sense of agency; and develop knowledgeable and confident identities.
- Children are confident and involved learners; and develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity.

When conducting assessment, take the following into consideration:

- Student interview for diagnostic assessment in the early learning stages is of paramount importance.
- Use materials and graphics familiar to students' context in and out of school.
- Use manipulatives rather than pictures wherever possible.
- Acknowledge the role of using stories in this early number learning, enabling students' telling stories and acting out understandings to illustrate what they know.
- Playdough and sand trays are useful for early interview assessment situations.

Ways to prepare children for assessment processes include the following:

- In **individual teaching times,** challenge students' thinking. "Challenging my thinking helps me to learn by encouraging me to ask questions about what I do and learn. I learn and am encouraged to take risks, try new things and explore my ideas."
- In **group time,** model and scaffold question and answer skills by using sentence stems to clarify understandings and think about actions. Encourage students to think of answers to questions where there is no one correct answer, and to understand that there can be more than one correct answer (e.g. how can we sort the objects?).
- In **active learning centres,** use activities such as imaginative play, sand play, playdough, painting, ICT and construction to think and talk about different ways of using materials, technologies or toys. Ask questions and take risks with new ideas.

Other considerations:

- Preferred/most productive assessment techniques for early understandings are observations, interviews, checklists, diary entries, and folios of student work.
- Diagnostic assessment items can be used as both pre-test and post-test instruments.

Remember:

Testing the knowledge can imply memory of stuff; asking the children **what they can do with knowledge** requires construction and demonstration of their understanding at this early understandings level.

O1 Thinking and Solving: Observation Checklist			
Unit	**Concept**	**Knows**	**Can construct/ do/tell**
1. Playing	Exploring	To observe details, recognise likeness and difference between Recognises a problem situation in play	Sort and classify ideas e.g. 'I could do this or that' Experiments with and creates problem situations in play
	Constructing and finding out	There are constructs /rules/parameters for certain situations	Prepare and collect things Can 'Decide Do and Describe' actions in play
	Playing games and directed activities	There are simple rules To Listen	Develop simple rules Take turns and share in simple scenarios (2 -3 students)
	Play basis of BAE/SPDC frameworks	There is a framework to help solve situations : BAE	Is willing to have a go at solving using simple framework BAE
2. Using Language	Creating Dialogue	Can talk to explain their thinking and answers in math: 1 on 1, in pairs, and in small groups To use thinking words	Ask simple questions e.g. what do you think? What do you see? Listen attentively for a short time Speak thoughtfully and with respect of others ideas
	Oral Explanations	Act out and tell problem situations To visualise problem stories Uses 'might' 'maybe' 'perhaps' 'probably'	Use their own words Use sentence starters if needed Play prediction games Uses 'thinking' words
	Questioning and listening	To ask 'Does this work?' 'I'm not sure, What do you think?' 'What else could we do?' To listen to others responses	Can ask a question Can make their own question Can respond to another's question Use language and some question words
	Language for BAE/SPDC	Beginning- action- end See-Plan Do- Check	Use the BAE scenario to describe what they are doing Enact the SPDC framework
3. Thinking	Thinking questions	thinking language 'think', 'know', 'guess' ,'remember'	Uses model questions e.g. 'What will happen if…?' Makes up own questions
	Making links	To predict; Two types of actions- of join and separate Two types of groups-equal and unequal	Trials ideas Discuss outcome of joining and separating Make equal and unequal groups Joins and separates
	Developing dispositions	What they are interested in can help you understand and develop maths ideas Persist and persevere	Share their interests/experiences in and to maths thinking Take responsibility in thinking groups Continue until they are satisfied with result
	Thinking about BAE/SPDC	Can be creative and use own ideas for BAE There are different ideas/beliefs/ways to SPDC	Use BAE with their thinking Acknowledge others ideas/ beliefs Accept their maybe several ways/solutions to resolve stories/problems using SPDC
4. Solving	Getting started	Identify beginning, action and end	Tell own problem and find beginning, action, end
	Working on a problem	To find ways/ tools to assist e.g. draw, materials, discuss	Show 'planful, thoughtful, purposeful ' behaviour Uses props-act out, draw, visualise, discuss
	Digging deeper	There may be more than one solution and more than one way to get there	Can consider have we done anything like this before? Asks: Could I make it even better? Reverse order of actions
	Using BAE/SPDC	The framework has parts and you need to identify what part is missing.	Identify part missing and tell with pictures/materials the solution.

Appendices

Appendix A: AIM Early Understanding Modules

Module content

1st module **Number N1: Counting** *Sorting/correspondence *Subitising *Rote *Rational *Symbol recognition *Models *Counting competencies	**2nd module** **Algebra A1: Patterning** *Repeating *Growing *Visuals/tables *Number patterns	**3rd module** **Algebra A2: Functions and Equations** *Functions* *Change *Function machine *Inverse/backtracking *Equations* *Equals *Balance
4th module **Number N2: Place Value** *Concepts* *Place value *Additive structure, odometer *Multiplicative structure *Equivalence *Processes* *Role of zero *Reading/writing *Counting sequences *Seriation *Renaming	**5th module** **Number N3: Quantity** *Concepts* *Number line *Rank *Processes* *Comparing/ordering *Rounding/estimating *Relationship to place value*	**6th module** **Operations O1: Thinking and Solving** *Early thinking skills *Planning *Strategies *Problem types *Metacognition
7th module **Operations O2: Meaning and Operating** *Addition and subtraction; multiplication and division *Word problems *Models	**8th module** **Operations O3: Calculating** *Computation/calculating *Recording *Estimating	**9th module** **Number N4: Early Fractions** *Concepts* *Fractions as part of a whole *Fractions as part of a group/set *Fractions as a number or quantity *Fraction as a continuous quantity/number line *Processes* *Representing *Reading and writing *Comparing and ordering *Renaming

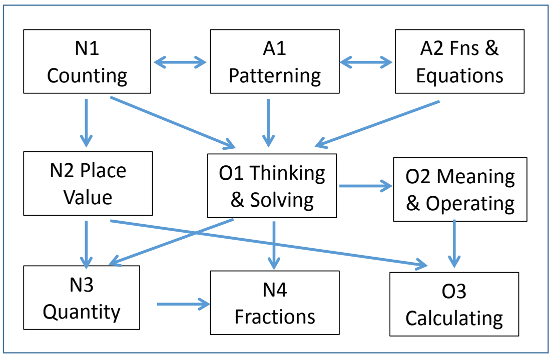
Module background, components and sequence

Background. In many schools, there are students who come to Prep with intelligence and local knowledge but little cultural capital to be successful in schooling. In particular, they are missing basic knowledge to do with number that is normally acquired in the years prior to coming to school. This includes counting and numerals to 10 but also consists of such ideas as attribute recognition, sorting by attributes, making patterns and 1-1 correspondence between objects. Even more difficult, it includes behaviours such as paying attention, listening, completing tasks, not interfering with activity of other students, and so on.

Teachers can sometimes assume this knowledge and teach as if it is known and thus exacerbate this lack of cultural capital. Even when it is identified, it can be time consuming to build this knowledge in classrooms where children are at different levels. Thus, it can lead to situations where Prep teachers say at the end of the year that some of their students are now just ready to start Prep and they wish they could have another year with them. These situations lead to a gap between some students and the rest that is already at least one year at beginning of Year 1. For many students, this gap becomes at least two years by Year 3 and is not closed and sometimes widens across the primary years unless schools can provide major intervention programs. It also leads to problems with truancy, behaviour and low expectations.

Components. The AIM EU project was developed to provide Years P-2 teachers with a program that can accelerate early understandings and enable children with low cultural capital to be ready for Year 3 at the end of Year 2. It is based on nine modules which are built around three components the mathematics ideas are designed to be in sequence but also to be connected and related to a common development. The modules are based on the AIM Years 7-9 program where modules are designed to teach six years of mathematics (end Year 3 to end Year 9) in three years (start Year 7 to end Year 9. The three components are: (a) Basics – A1 *Patterning* and A2 *Functions and equations*; (b) Number – N1 *Counting* (also a basic); N2 *Place value*; N3 *Quantity* (number line); and N4 *Fractions*; and (c) Operations – O1 *Thinking and solving*; O2 *Meaning and operating*; and O3 *Calculating*. These nine modules cover early Number and Algebra understandings from before Prep to Year 2.

Sequence. Each module is a sequence of ideas from P-2. For some ideas, this means that the module covers activities in Prep, Year 1 and Year 2. Other modules are more constrained and may only have activities for one Year or for two Years. For example, Counting would predominantly be Year P and Fractions Year 2. Thus, the modules overlap across the three years P to 2. For example, Place value shares ideas with Counting and with Quantity for two digit numbers in Year1 and three digit numbers in Year 2. It is therefore difficult, and inexact to sequence the modules. However, it is worth attempting a sequence because, although inexact, the attempt provides insight into the modules and their teaching. One such attempt is on the right. It shows the following:

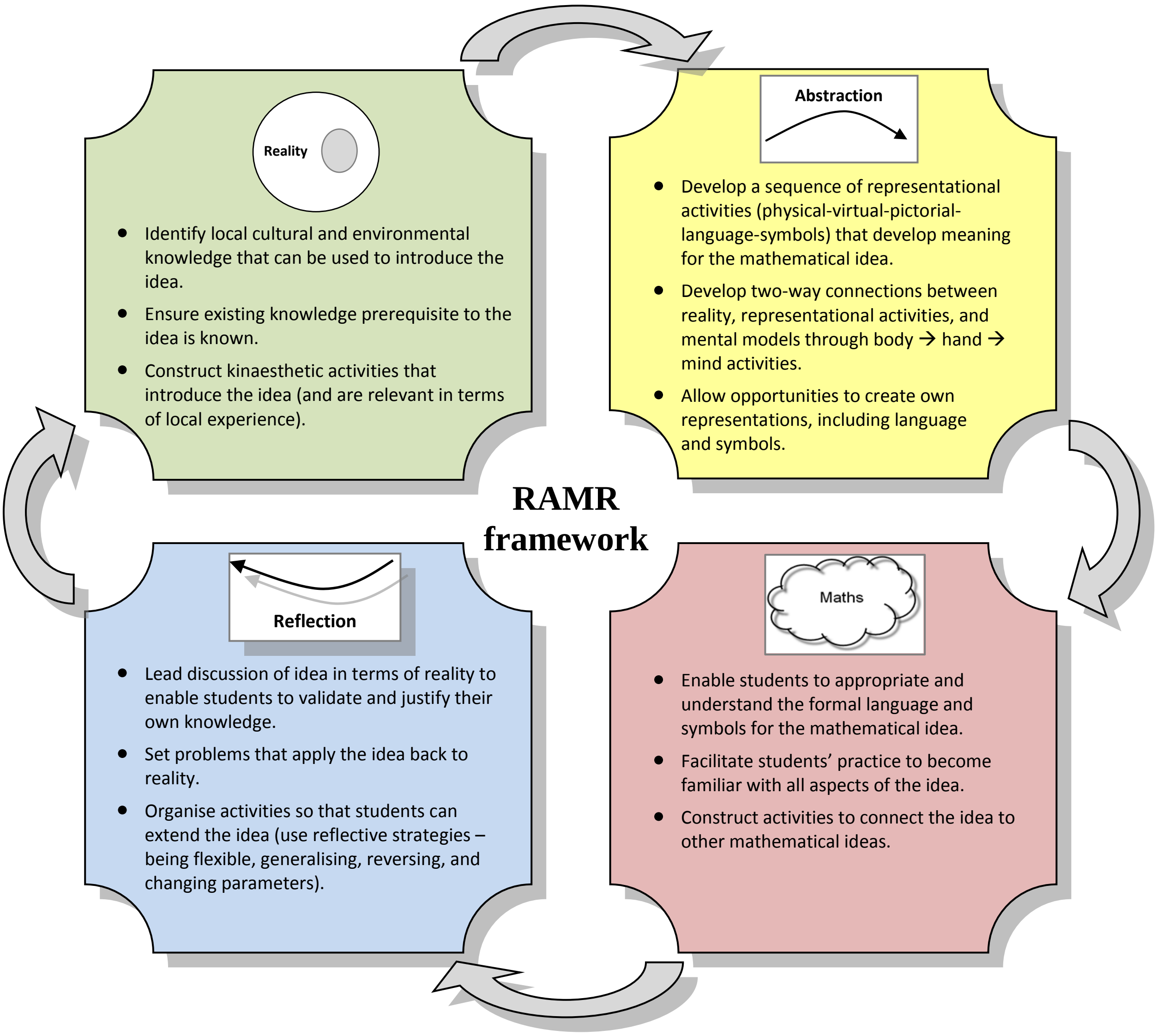


1. that foundation ideas are within *Counting*, *Patterning* and *Functions and Equations* – these deal with the manipulation of material for the basis of mathematics, seeing patterns, the start of number, and the idea of inverse (undoing) and the meaning of equals (same and different);
2. that the central components of the sequence are *Thinking and solving* along with *Place Value* and M*eaning and operating* – these lead into the less important *Calculating* and prepare for *Quantity*, *Fractions* and later general problem-solving and algebra; and
3. that *Quantity*, *Fractions* and *Calculating* are the end product of the sequence and rely on the earlier ideas, except that *Quantity* restructures the idea of number from discrete to continuous to prepare for measures.

Appendix B: RAMR cycle

AIM advocates using the four components in the figure below, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem-solving, flexibility, reversing and generalising. The innovative aspect of RAMR is that Reality to Abstraction to Mathematics develops the mathematics idea while Mathematics to Reflection to Reality reconnects it to the world and extends it.

Planning the teaching of mathematics is based around the four components of the RAMR cycle. They are applied to the mathematical idea to be taught. By breaking instruction down into the four parts, the cycle can lead to a structured instructional sequence for teaching the idea. The figure below shows how this can be done.



The YuMi Deadly Maths RAMR cycle

Appendix C: Teaching framework

Teaching scope and sequence for thinking and solving

TOPICS	SUB-TOPICS	DESCRIPTIONS AND CONCEPTS/STRATEGIES/WAYS
Thinking and Solving	**Playing** – exploring and extending what students know	Recognise a problem situation Make a problem situation Act out a solution
	Using language – questioning, listening, dialogue, vocabulary needed for solving problems	Tell the problem story Question/Listen Use thinking language
	Thinking – links and dispositions	Beginning-Action-End Choosing Persevering
	Solving – getting started and digging deeper	See–Plan–Do–Check Getting started Trying different ways

Accelerated Inclusive Mathematics Early Understandings Project

YuMiDeadly

Growing community through education

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