Accelerated Inclusive Mathematics Early Understandings Project

YuMi Deadly Maths

AIM EU
Module N2

Number:
Place Value

DRAFT 3, January 2017

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ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning. The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life. YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

DEVELOPMENT OF THE AIM EARLY UNDERSTANDINGS MODULES

In 2009, the YuMi Deadly Centre (YDC) was funded by the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This resulted in a Year 7 to 9 program of 24 half-term mathematics modules designed to accelerate learning of very underperforming Indigenous students to enable access to mathematics subjects in the senior secondary years and therefore enhance employment and life chances. This program was called Accelerated Indigenous Mathematics or AIM and was based on YDC's pedagogy for teaching mathematics titled YuMi Deadly Maths (YDM). As low income schools became interested in using the program, it was modified to be suitable for all students and its title was changed to Accelerated Inclusive Mathematics (leaving the acronym unchanged as AIM).

In response to a request for AIM-type materials for Early Childhood years, YDC is developing an Early Understandings version of AIM for underperforming Years F to 2 students titled Accelerated Inclusive Mathematics Early Understandings or AIM EU. This module is part of this new program. It uses the original AIM acceleration pedagogy developed for Years 7 to 9 students and focuses on developing teaching and learning modules which show the vertical sequence for developing key Years F to 2 mathematics ideas in a manner that enables students to accelerate learning from their ability level to their age level if they fall behind in mathematics.

YDC acknowledges the role of the Federal Department of Education in the development of the original AIM modules and sees AIM EU as a continuation of, and a statement of respect for, the *Closing the Gap* funding.

Contents

Module Overview

This Accelerated Inclusive Mathematics Early Understandings (AIM EU) module, N2 Place Value, is the second module dealing with number, being preceded by N1 Counting (see **Appendix A** for full list of modules). AIM EU modules are to support teachers in accelerating mathematics learning in Years F to 2 to assist with closing the gap between Indigenous and low-socioeconomic status (SES) students that is evident from the foundation year of primary schooling with respect to Western school mathematics.

Understanding three-digit numbers is a large component of the number work in Years F to 2 mathematics, and this number work has to cover:

- part-whole (place value);
- how numbers are sequenced (additive structure);
- how adjacent place-value positions are related (multiplicative structure);
- position on a number line (quantity); and
- role of zero (equivalence).

As a consequence, we have divided whole number into two modules following N1: (a) the number work that continues on from counting and relates directly to place value (this module, N2); and (b) the number work associated with the number line and magnitude (N3 – Quantity).

This module looks at grouping and place value related to place value itself and additive and multiplicative structure (leaving position and equivalence to N3). This section overviews the module and looks at the role of place value in number, connections and big ideas, sequencing, pedagogy and teaching, and the structure of the module.

Place value in early childhood and primary

In Years F–2, it is normal to look at numbers from 0 to 999, that is, one-, two- and three-digit numbers. This is a most important development in number for the following reasons:

1. In Foundation/Prep we learn to count 1:1 and to understand that there is 1:many counting. The formal component of 1:many is to count in tens and hundreds as well as ones. The module N1 Counting has focused on this development from 1:1 to 1:many and this is taken forward in N2.
2. Module N2 looks at tens and ones (T-O) and hundreds, tens and ones (H-T-O). As listed in the big ideas section of this overview, there are five big ideas in number (part-whole, additive structure, multiplicative structure, quantity or number line, and equivalence) that hold for all number types. Whole number place value in H-T-O ones begins this process.
3. H-T-O provides the basis for the pattern of threes that is the basis of large whole numbers and decimal numbers. In whole and decimal numbers there is the following pattern of threes (H-T-O) that enables numbers to go on to billions, trillions and so on, or back to thousandths, millionths and so on. If students have a strong understanding of number in H-T-O ones, they can easily extend this understanding to H-T-O thousands and millions and H-T-O thousandths. It is one of the powerful ways we accelerate learning in early number and H-T-Os.

H	T	O	H	T	O	H	T	O	H	T	O
Millions			Thousands			Ones			Thousandths		

The knowledge in this N2 module lays the foundations for large numbers and decimals. It also provides a basis for reteaching H-T-O ones and their properties to students who are having problems with place value in higher grades than F–2. Then it is easy to extend the ideas into larger and smaller numbers.

Connections and big ideas

Connections

The major connections to place value in H-T-O ones are as follows:

(a) all the other H-T-O patterns of threes (e.g. thousands, thousandths, and so on), that is, place value in whole and decimal numbers in general;

(b) metric units in measurement (particularly those with H-T-O type metric conversions) – for example, in length, H-T-O ones becomes millimetres (mm), H-T-O thousands becomes metres (m), and H-T-O millions becomes kilometres (km);

(c) mixed numbers in a simple whole-part table; and

(d) operations in algebra (relate to the traditional algorithms which use H-T-O place value).

Big ideas for place value

1. **Part-whole (includes Notion of a unit)**. The basis of number is the unit which is grouped to make large numbers and partitioned to make fractions. Thus everything can be seen as part and whole – a ten is a tenth of a hundred and a whole of 10 ones. The value of each digit in the number system is determined by its position relative to the ones' position, that is, by its place value. The value of each digit is given by that digit number of place values and the value of a number is found by adding these (i.e. 204 is 200 + 4). The pattern also holds for fractions (e.g. sixths) and measures whose base is not 10 (e.g. time).

   There is also a sub-big idea in that the pattern for the groupings and partitioning in numbers is a **pattern of threes**, that is, ones-tens-hundreds ones, ones-tens-hundreds thousands, ones-tens-hundreds millions and so on. This means that knowledge can be transferred from ones to thousands to millions and so on. The concept for this big idea is **place value**, and the processes are **reading** and **writing**.

2. **Additive structure**. A number's value is the sum of its parts, e.g. 234 = 200 + 30 + 4. Thus, place values increase and decrease by adding and subtracting. The consequence of this is that each place-value position counts forwards and backwards and values of place-value positions increase and decrease.

   There is also a sub-big idea in that counting follows a pattern, the **odometer pattern** – forwards from 0 to 9 and back to 0 as the number on left increases by 1 (e.g. 274, 284, 294, 304, 314, ...); backwards from 9 to 0 and back to 9 as number on left decreases by 1 (e.g. 234, 224, 214, 204, 194, 184, ...). For fractions, this counting pattern differs, as it does for measures (e.g. time) that are not base 10.

   The concept for this idea is **counting** and the processes are **seriation** and **counting patterns**.

3. **Multiplicative structure**. A number's value is a sum of the products of value and place, e.g. 234 = 2 × 100 + 3 × 10 + 4 × 1. This means that there is a multiplicative relationship between adjacent place-value positions – values increase ×10 when moving to the left and decrease ÷10 when moving to the right. Thus, the value of a number is found by adding the multiples of digit × place value (e.g. 204 = 2 × 100 + 0 × 10 + 4 × 1).

   The concept for this idea is **multiplicativity** and the processes are **renaming** and **flexibility**. Flexibility is thinking of things more than one way by changing the unit (e.g. 300 is 30 tens or 0.3 thousands and so on).

4. **Quantity/Number line**. Regardless of place values and digits, each number is a single quantity represented by a point on a number line, has rank, and can be compared to other numbers. The number-line representation changes perspective of number – for example, the 0 is no longer nothing, it is the starting position of positive whole numbers (and of rulers and other measuring devices).

The concepts for this idea are **comparison/order**, **rank** and **density** and the processes are **comparing/ordering**, **rounding**, and **estimating**. Order just means to work out the larger/largest but rank means to place on a line where they should be proportionally. For example, 91 being larger than 32 can be shown by students in a row with 91 after 32, but rank is shown by the 32 being on a line one third of the way between 0 and 100 and the 91 being near the 100. Density is how many numbers between adjacent numbers.

5. **Equivalence**. Sometimes a single quantity can be represented by more than one number. For example, 04 is the same as 4, 2.40 is the same as 2.4, $\frac{2}{3}$ is the same as $\frac{4}{6}$, and 3:5 is the same as 6:10. Equivalence often reflects adding zero (the additive identity) or multiplying by one (the multiplicative identity).

Implications

We believe that mathematics is best understood and applied in a schematic structured form which contains knowledge of **when and why as well as how**. Schema has knowledge as **connected nodes**, which facilitates recall and problem-solving. Knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning. It enables teachers to:

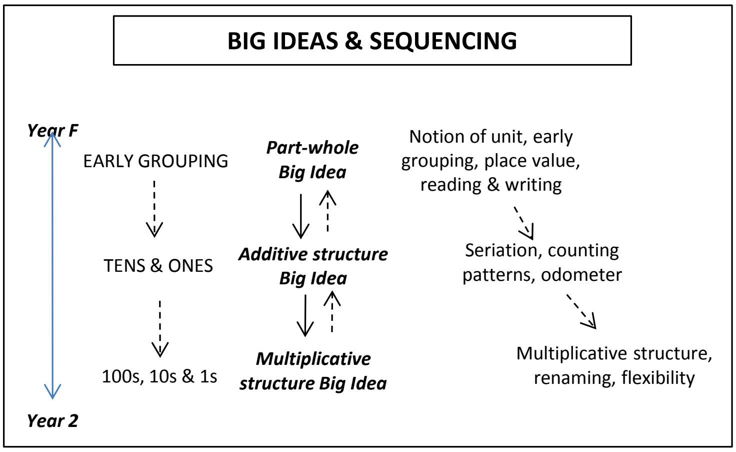
(a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;

(b) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem-solving;

(c) *choose effective instructional materials, models and strategies* – mathematics that is connected to other ideas or based around a big idea can be taught with similar materials, models and strategies; and

(d) *teach mathematics in a manner that enables later teachers to teach more advanced mathematics* – by preparing linkages to other ideas and foundations for big ideas the later teachers will use.

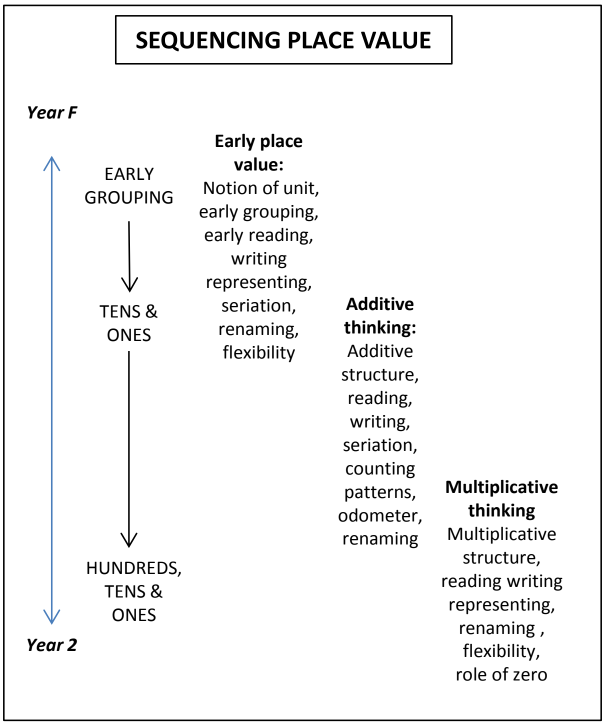
Sequencing



The sequencing of the teaching of place value is complex (as can be seen in the figure on right). First, there are three **big ideas**: part-whole, additive structure, and multiplicative structure. There is argument that these three should be in sequence and there is merit in this: **part-whole** leads to tens and ones, to 24 being 2 tens and 4 ones, and covers notion of unit, place value itself and reading and writing process for two-or-more-digit numbers; (b) **additive** says that 24 is 20 + 4, and covers seriation (1 place value more and 1 place value less), counting patterns and the odometer principle; while (c) **multiplicative** stresses that the 2 tens are ten × 2 ones, looks at the effect of ×10 and ÷10 on place-value positions, and covers renaming and flexibility.

Second, there is the size of the numbers, the number of digits: (a) Year F looks at **single-digit numbers** and **early grouping** leading into two digits, (b) Year 1 covers **tens and ones** (two-digit numbers to 99) for all place-value ideas, and (c) Year2 covers **hundreds, tens and ones** (three-digit numbers to 999) for all place-value ideas.

Thus we have at least two major sequences interacting which can mean that early multiplicative structure ideas may precede the advanced additive ideas and that odometer for two digits may follow place value for three digits. Originally, when we first looked at the possible sequences for this module, and we had decided that we would stop the module after multiplicative structure, we had thought to have three units: early grouping, tens and ones (T-O), and hundreds, tens and ones (H-T-O). However, we have decided to mix things up and so the sequence we are looking at in this module is as on right.



Unit 1 (Early Place Value) focuses on early place value and goes into tens and ones (T-O) – pre place-value activities such as early grouping are also here, and renaming and flexibility are first considered here but revisited in Unit 3.

Unit 2 (Place-Value Additive Thinking) focuses on additive, focusing on tens and ones (T-O) but going into hundreds, tens and ones (H-T-O), looking mainly at the seriation, counting patterns and odometer that make up additive structure.

Unit 3 (Place-Value Multiplicative Thinking) focuses on multiplicative but contains some aspects of T-O as well as its main focus of H-T-O, finishes reading and writing and stresses renaming and flexibility. It also covers the role of zero.

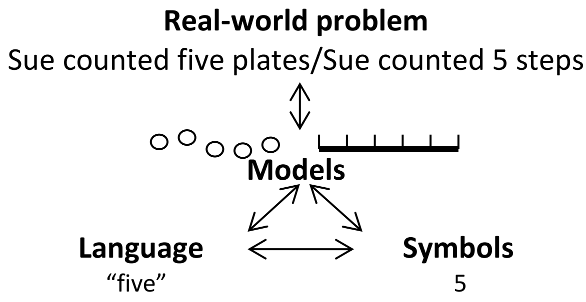
Teaching and culture

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework (see **Appendix B**) and the impact of Western number teaching on Indigenous and low-SES students.

Teaching implications

The first teaching implication is that mathematical ideas that are structurally similar can be taught by similar methods. For example, fractions and division are similar and both are taught by partitioning sets into equal parts – except that the set is seen as one whole for fractions and a collection of objects for division. Similarly, early grouping, place value for whole numbers, place value for decimals and mixed numbers can all be taught with size materials and position charts. The second teaching implication is that number instruction is both diverse and generic. There are particular ways to teach the varied topics in this module and they will be discussed as we meet them.

The **Payne and Rathmell (1977) triangle** is one of the underpinnings of the Abstraction and Mathematics stages of the RAMR framework. It is important in teaching number because of its focus on the relationship between models, language and symbols. Activities and questions should be constructed that encourage students to connect and move flexibly between models, language and symbols in all directions, as shown in the diagram below.



The Payne and Rathmell (1977) triangle for early number

Cultural implications

Aboriginal and Torres Strait Islander students may find the teaching of number confronting because of the differences of the number-oriented culture of the mathematics classroom and their culture, and because many students are from low-SES backgrounds.

Aboriginal and Torres Strait Islander cultures followed a different path from number-oriented cultures (European, Indian-Arabic, and Chinese-Japanese) in the development of mathematics; for Indigenous cultures, people were seen as more important than number so their mathematics specialised in areas other than number. This different focus could be seen as emanating from their cultural beliefs with regard to group rather than individual ownership. Thus, the teaching of number, operations and measurement can bring Australian mainstream Eurocentric school teaching into conflict with Indigenous students; it can be a topic that can, in the terms of Indigenous mathematics and mathematics-education researcher Dr Chris Matthews, designate these cultures as primitive. It must be taught with care as part of a European culture that Indigenous people need to understand. It should not be celebrated as something that raises some people above others.

For low-SES Aboriginal and Torres Strait Islander students in Australia, the outcome is exacerbated. As low-income people, these students are sometimes considered as unsuccessful. The number systems created as part of Eurocentric mathematics have always benefited high-SES people at the expense of low-SES people, and promulgated the idea that bigger numbers (e.g. money, house cost, cars) are better, and mean that the person with the bigger numbers is more successful. The way the numbers function within Eurocentric societies achieves two outcomes simultaneously: (a) it benefits one class of people at the expense of the other, and (b) it puts the blame for their lack of benefit on the actions of the class that is not benefited. The mathematics of number, operations and measurement must be taught with care to low-SES students because its teaching can designate these students as failures. And if the students are both Indigenous and low SES, even greater care must be taken.

Structure of module and sequence of units

Based on the ideas above, the module is divided into five components, providing information on how to teach and assess place value in Years F to 2.

Components

The components of this module consist of the following.

Overview: This section covers place value in early childhood and primary years; connections and big ideas; sequencing; teaching and cultural implications; and structure of module and sequence of units.

Units: This section has three units, Early Place Value, Place-Value Additive Thinking, and Place-Value Multiplicative Thinking. They are described in the Sequencing subsection above.

Module review: This section reviews the module looking at important components across units. For this module, these are the processes that were described in the subsection above on big ideas. It also covers the major ways of modelling and representing the number concepts and processes, the important competencies associated with two- and three-digit numbers, and information on later mathematics (how three-digit numbers are extended into large and small numbers).

Test item types: This section provides examples of types of items for each unit that could be used in pre- and post-tests.

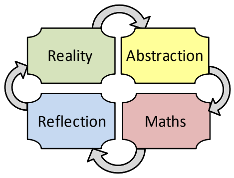
Appendices: This comprises three appendices covering the AIM EU modules, the RAMR pedagogy, and proposed teaching frameworks for place value.

Further information

Sequencing the teaching of this module. The three units cover the teaching of place value as described in the Sequencing subsection above. The three units are in sequence and could be completed one at a time. However, each of the units is divided into sub-ideas (concepts and processes) that are also in sequence within the unit. Therefore, schools may find it advantageous to: (a) teach earlier sub-ideas in a later unit before completing all later sub-ideas in an earlier unit; (b) teach sub-ideas across units, teaching a sub-idea in a way that covers that sub-idea in all the units together; or (c) a combination of the above.

The AIM EU modules are designed to show sequences within and across units. However, it is always YDC's policy that schools should be free to adapt the modules to suit the needs of the school and the students. This should also be true of the materials for teaching provided in the units in the modules. These are exemplars of lessons and test items and schools should feel free to use them as they are or to modify them. The RAMR framework itself (see Appendix B) is also flexible and should be used that way. Together, the units and the RAMR framework are designed to ensure that all important information is covered in teaching. Therefore, if changing and modifying the order, try to ensure the modification does not miss something important (see Appendix C for detailed teaching frameworks).

RAMR lessons. We have included RAMR lessons as exemplars wherever possible in the units of the module. Activities that are given in RAMR framework form are identified with the symbol on the right.



Suggestions for improvement. We are always open to suggestions for improvement and modification of our resources. If you have any suggestions for this module, please contact YDC.

Unit 1: Early Place Value

Early place value is crucial to the development of number. It is like a large building where much time is spent on the foundations but when these are set the building rises quickly. AIM EU modules are to accelerate learning in Years F–2. The secret of this is the foundations – speeding up learning often requires slow and complete learning of the foundations. The later years are where the acceleration occurs.

As a start, there are important ideas, as follows, that lie behind two- and three-digit place value that need to be understood. They are the foundations:

- The most important number in the system is 1 because it is the ***unit*** from which we make numbers larger or smaller.
- Sorting – correspondence and counting require focusing on groups and groups of groups as well as individual objects. Students need to be able to switch between different notions of the units. They need to realise there is 1:1 and 1:many.
- Early grouping sets up the notion of place value by allowing students to experience groups and ones, and grouping and ungrouping.
- Developing a sense of numbers beyond 10 is important; e.g. 11 is 10 and one more, 12 is 10 and two more, and so on.
- The nature of our number system means that the value of a digit in any number is determined by its place and we only need 10 symbols to represent any number from very small to very large numbers. Learning to understand how to read and write and represent numbers can be problematic for young learners.
- Students need to learn the convention for showing where the ones are for whole numbers, that is, on the RHS, then the tens to the left of the ones and so on.
- Students need to develop the concept that one group of 10 equals 10 units within that group and associate it with the numerals involved and represent it with materials and chart.

Particularly important. For place-value concepts to be understood, each student requires a great deal of hands on experience with grouping and regrouping concrete materials. The use of materials should progress very gradually from unstructured, for example, buttons, beads, bears, beans and bundling sticks, to structured materials, for example, Unifix cubes and then MAB. In so doing, students move from using materials that can be separated for regrouping to those that must be "traded" for regrouping. Students should manipulate a wide variety of unstructured and structured materials so that meaning is not attached to just one type of material.

This unit looks at the foundations – it covers notion of a unit, early grouping, early place value and early reading, writing, representing, seriation, renaming and flexibility.

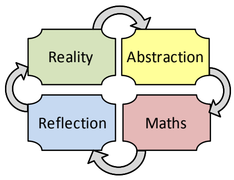
1.1 Notion of unit/one

The most important number in the system is 1 because it is the unit from which we make numbers larger or smaller.

1.1.1 Notion of unit activities

1. Look for contexts: Discuss different uses of numbers; house numbers; sportspersons numbers; numbers on buses; numbers on toys; cooking and numbers.
2. Use personal numbers.
3. Look for examples with one/unit, group, and groups of groups.
4. Count blocks and groups of blocks.

1.1.2 RAMR lesson for notion of unit



Learning goal: Anything can be a unit, a single object or a collection of objects. Units can form groups and units can be partitioned into parts.

Big idea: Part-whole.

Resources: Collections, posters, pictures, story books, boxes, boxed items.

Reality

Local knowledge: Find a local context.

Prior experience: What is students' understanding of sorting, identifying counting, numerals 0–9?

Kinaesthetic: Play games and sing songs to act out numbers that are significant in your local context.

Abstraction

Body: Play musical number groups having students group themselves in groups using numbers less than 10. Have students describe the number in the group and the number left over.

Hand: Use counting boxes with a small number of items in each. Have students group and count. Then use 2 frames, 5 frames, and 10 frames. *Ask: How many different ways can you group things?*

Mind: Have students picture each other's groups in their minds.

Creativity: Have students create their own models for ones, and groups, and groups of groups.

Mathematics

Language/symbols: Note language students are using and introduce appropriate mathematical language – words, phrases, questions. This language will include symbols, digits, pictures, words.

Practice: Unpack and repack boxed items – say, one box, 10 in each box, etc. Identify pictures of things, e.g. one bird – one flock of birds; one pup – one litter of pups; one dozen/box eggs, twelve eggs; one suburb, many houses; one family, 5 people; one class, 15 students; and so on.

Connections: Connect to measures, e.g. of the day.

Reflection

Validation: Have students find something in their day that is a unit or a group of units.

Applications/Problem-solving: I am one student in my class, how many in the class? How did you work it out? Can you show me how you worked it out on paper? With materials? Acting it out?

Extension:

- *Flexibility*: I have 9 objects, how can I group them? How else can I group them?

- *Reversing*: Show a diagram of counters indicating a unit and a grouping. Ask students to describe what it shows. Give a number; ask students to show different ways it could be grouped. Have students create a picture with materials for you to identify the unit and the groups of units.

- *Generalising/Changing parameters*: Ask students to work with nine objects. Identify the unit and the groups. What happens if one/two/three more objects are added?

1.2 Early grouping

This is an important mathematical idea that sets up the notion of place value by allowing students to experience groups and ones (with the groups on the left-hand side), and grouping and ungrouping. It also shows the power of kinaesthetic learning to teach a visual image (like place value) by moving the body to left and right as students put out groups and ones. As well as putting objects into groups, students need practice **saying** how many groups and how many left over. The activities below describe how to teach it.

1.2.1 Early grouping place-value activities

1. Which is more? Ask students to say which is more, 3 or 4. Have the students convince you of this with materials.

2. Play number labels – have students write temporary number labels to show how many in various containers in the classroom, or how many in students' groups at one time to do certain activities. Ask: Which number do you need to write? How can you be sure? Where could you check how?

3. Play number/group bingo. Use 2, 5 and 10 frames.

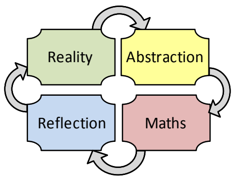
4. Handfuls cover up – students place hands over pile of counters/bugs/bears/cars/buttons, etc. Put three under left hand, the other hand covers the rest. Say: one group of three and how many? Place another three under left hand. Say: two groups of three and how many? Continue until no more groups of three can be made. Repeat with different groupings.

5. Play next number. Roll dice to get a number of objects. Ask: What will happen next? Take special note when approaching 10. Ask: What is different about 9 and 10? Or 7 and 11? etc. Have students use a calculator to model.

6. Roll dice to model "reach my number". For example, the number is 12. Write the number. Ask: How would the groups look if the number was written like this – 21?

7. Introduce groups and ones charts.

1.2.2 RAMR lesson for early grouping



<u>Learning goal</u>: Early grouping.

<u>Big idea</u>: Part-whole.

<u>Resources</u>: Groups/Ones board, counters, Unifix cubes or bundling sticks.

Reality

Local knowledge: Look for examples of groups in local shops, everyday life and culture.

Prior experience: This activity extends one-to-one counting to one-to-many and relies on students knowing left and right (L-R) positions.

Kinaesthetic: Sort students into groups of three, four or five. Students count how many groups and how many individual students are left over.

Abstraction

Body: **Sequences**. Make different groups out of a number of loose ones. For example, make groups of three out of eight loose ones. Say: *Put out 8 ones (counters). Do you have enough ones to make a group of three?* [Students should say, "Yes".] Say: *Show me your group of three. How many groups of three do you have?* [1] *How many loose ones left?* [5] Repeat these questions until no more groups can be made. Repeat with other groups (such as groups of 5, 4, 8, 10).

Hand: **Play the Zurkle game** (Baratta-Lorton, 1981).

- Choose a number between 2 and 9, say, 4. Make up a nonsense name for this number, say "zurkle". Count four things a few times – *one, two, three, zurkle*. Put left hand on Groups on Groups/Ones board and say: *This side is for zurkles*; move left hand to Ones and say: *This side is for ones*. Start by moving left hand from Groups to Ones saying as you go: *zero zurkles and zero*.
- Then ring a bell; students add one Unifix cube to the Ones side, move left hand and say: *zero zurkles and one*. Keep ringing bell, adding another one, moving left hand and saying, until you reach *zero zurkles and three*. Then ask predictive question – *What do we do when the bell rings again?* [Add one, make a zurkle by connecting Unifix, move to left-hand side, move hand, say: *one zurkle and zero*]. Continue counting forward in zurkles and ones, moving hand, saying, and asking predictive questions, until four zurkles (can ask what this could be). Then count back removing a one and asking predictive question when there are zero ones.

Show groups and left-over ones using virtual materials – counters, groups of counters and Groups/Ones boards. Interpret and draw pictures in terms of groups and ones.

Mind: **Mental models**. Get students to shut eyes and imagine a Groups/Ones board. Give a starting number of groups and ones. Then say to add or remove ones and get students to state how many groups and how many left-over ones. Ensure they have a picture in the mind.

Creativity: **Creating own representations**. There is creativity here with nonsense names. You could also have students make up a counting system when only using one hand. It can be a lot of fun to use "fee" as one, "fie" as two, "foe" as three, and "fum" as four and have the giant in Jack and the Beanstalk counting his golden goose eggs.

Mathematics

Language/symbols: group, grouping, ones, ungrouping, Groups/Ones board (graphic organiser), enough, left-over, renaming, numerals, how many groups? How many left-over ones?

Practice: Have students play the Zurkle game in small groups. Then progress to:

- Play race game where students spin a 1-2-3 spinner and add ones shown, always stating how many zurkles and how many left-over ones – first to 4 zurkles wins. Can race backwards too.
- Repeat game for other grouping numbers and nonsense names (2 to 5 before 6 to 9).

Connections: Ensure that these early grouping activities are connected to the first tens and ones activities.

Reflection

Extension: Early grouping activities can lead to many ideas that later will be introduced with tens and ones: (a) position – that groups are on the left of ones; (b) groups of groups – that four zurkles (or zurkle zurkles) needs a new name (like ten tens is one hundred); and (c) renaming – that groups are formed and broken up. One could also introduce numerals for groupings – for zurkles, this would mean that two zurkles and three ones is 23.

1.3 Early place value and reading/writing/representing

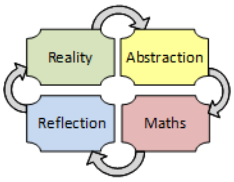
The basis of early place value and, particularly, reading, writing and representing is that the same 0–9 digits can represent all whole numbers. As well, order of the digits makes a difference to the numbers. Students need to know why 10 and 01 are not the same by constructing representations, adding numeral labels, and identifying why numerals change. Also need to look at introducing zero and what it can do.

Get students to construct a variety of representations of a number. Introduce and use charts: groups and ones, then tens and ones.

1.3.1 Early place value, reading/writing/representing activities

- Unifix cube games
- Calculator games
- Bingo boards
- Dice games, e.g. place-value sequence game 0–10
- Card games
- Using numerals with 10 frames
- Placing numerals with ladder grid, stacking ladder grids
- Lily pads
- Bugs
- Grab bag

1.3.2 RAMR lesson for early place value, reading/writing/representing



Learning goal: Develop an understanding of the concept of place value when applied to two-digit numbers — that is, the relative value of digits in the tens and ones places. Read, write, compare and represent numbers to 20 initially, then to 99. Students can also represent these numbers using a variety of materials.

Big ideas: Part-whole-group; reading, writing numbers.

Resources: Beans, paddle-pop sticks, Unifix blocks or similar; laminated place-value mat or paper, spinner/dice.

Reality

Local knowledge: Ask students where they have used or seen numbers, especially larger numbers. Some good examples could include distances on signposts, prices, competitions, media. Ask some students to share some numbers they know in symbols and/or words.

Prior experience: Grouping, one-to-one counting to one-to-many, students knowing left and right (L-R) positions, number names.

Kinaesthetic: Play Build to Ten. Spin or roll, say 3; three students join together and state “we are 3 students”. Have a student record the number on a small whiteboard and hold up. Roll/spin again, say 5, join the other three and state “we are 8 students”. Have student change number on whiteboard. Roll again, say 2, have students join others and state “we are 10 students”. Have student change number on whiteboard. Ask: *What is the 1 representing? What is the zero representing? Is this 10?* Place an elastic around the 10 students and have them state “we are one ten and zero units”. Roll the die/spinner; repeat. Continue until the students have built a second ten. Ask: *How many? How else can this be represented? How can we write it? How can we represent it?* Have different students try the different roles.

Abstraction

Body: Play Build to Ten on the number mat.

Hand: Model with 10 frames, place-value mats, calculators, counters, beans, in pairs.

Mind: Ask students to visualise the tens and ones they have made for a number.

Creativity: Have the students create their own representations, using choices of materials, drawing and writing.

Mathematics

Language/symbols: Ones, tens, digit, numbers with parts, which part of the number does this digit represent? How much does this digit represent? Base ten, number system. Place value – we show numbers greater than nine by using place value. Value of a digit.

Practice: Play race to 20 where they roll a dice, put the number of beans out that correspond to the number on the die, onto the place-value chart and keep doing this until they reach the nominated number. Use charts, units and tens of various materials, dice. When they have enough to make a ten of beans substitute for a ten bean. Ensure students read each number as it is made i.e. I now have 11 beans, that is one ten and one 1 of beans. Be sure to read numbers with zero similarly e.g. I have 20 beans which is two tens and zero ones.

Have students individually record their numbers as they play the game.

Connections: Connect to zero, tens counting patterns, larger and smaller numbers.

Reflection

Validation: Have students choose a personally significant number (e.g. house number, birth date [day/month], age [own or family member]) to read, write and represent with place-value understanding.

Applications/Problem-solving. What numbers can you make that are less than 20 and have a 1 in the tens place? What numbers can you make using the digits 1 and 2? How are they the same? How are they different? I am thinking of a number between 0 and 25 with a 9 in it. What might it be?

1.4 Seriation for early place value

Seriation skills are placing numbers in the correct order. Young children who understand seriation can put numbers in order from lowest to highest, smallest to largest. Eventually, they will come to understand that 6 is higher than 5 or 20 is higher than 10. Thus seriation includes a focus on order, position, amount, and looks specifically at 1/10/100 more/less in relation to understanding place value in the base 10 number system.

1.4.1 Early place-value seriation activities

Early place-value seriation activities are as follows:

1. **Blank 5 frames** (and counters). Use 5 frame to show numbers. For example, use a blank 5 frame, and place two counters in. Ask the student to place counters in the other boxes (to make five) and ask how many more you need to fill the row. How many altogether? Repeat this for other numbers 1–5.

2. **Blank 10 frames** (blank 10 frame and counters). Use 10 frames to show numbers. For example, use a blank 10 frame, and place three counters in one row. Ask the student to place counters in the other boxes (to make 10) and ask how many more do you need to make 10. How many altogether? Repeat this for other numbers.

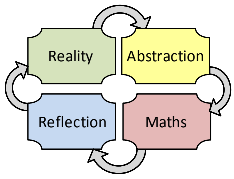
3. **Make 10** (Blocks, straws, counters, objects, or pictures). Put out two blocks, straws, counters, objects, or pictures and ask the student how many more to make 10 (do this with other numbers – up to 10). Roll a large die. Have the student say the number. Ask: *How many more to make 10?* Add using labels for the numbers.

4. **Lily pads**. Have students create lily pads and use cubes for frogs. Pick up a hand full of cubes and place on lily pad. Ask: how many? Place a dot for each one on pad. Repeat. Order lily pads. Extend to adding labels.

5. **Rearranging collections**. Have students rearrange collection of objects so it is easier to count. Have them label and order their collections.

6. **In-between game**. Use three students; 6- or 10-sided die for each student; and 30 counters. 1. Each student rolls a die without the other students seeing the number rolled. 2. Each student, in turn, tries to predict whether they have the lowest, highest or in-between number. 3. After hearing all the predictions, they may change their prediction. 4. Students show their number. 5. Each student who was correct receives a counter. 6. The winner is the first to get 10 counters.

1.4.2 RAMR lesson for seriation for early place value



Learning goal: Develop an understanding of the concept of place value when applied to two-digit numbers — that is, the relative value of digits in the tens and ones places. Read, write, compare, represent and **order** numbers to 20 initially, then to 99. Students can also represent these numbers using a variety of materials.

Big ideas: Place value, seriation, comparison/order.

Resources: Maths Mat, 0–99 grid, counters. Containers of balls/bean bags/cone markers, etc.

Reality

Local knowledge: Ask students where they have used or seen two-digit (or more) numbers, especially larger numbers. Some good examples could include distances on signposts, prices, competitions, and media. Ask some students to share some numbers they know in symbols and/or words.

Prior experience: Grouping, one-to-one counting to one-to-many, students knowing left and right (L-R) positions, number names.

Kinaesthetic: Place a pile of equipment (collections) in the centre of the circled students. (Ensure one collection of objects has 10 fewer items than number of students in class.) Ask them to assist you to sort it out. What do they suggest? Have students in pairs or groups organise then show and explain to the group what they did. Do comparisons arise? What language and orderings are they using? Record these as they feedback. Ask: *Who had more? Who had less?* Record. Choose pre-planned collection and ask a student to hand out one to each student. Discuss what happens. Ask: *How many more? How many less did we have? How could we model and represent this?*

Abstraction

Body: Move to the mat. Have students model the numbers from their collections on the mat. Consider the order. Have them take turns to instruct and model one more, one less, 10 more, 10 less, of each collection. Ask: *What changes? What stays the same?*

Hand: Have students model mat action on 0–99 grids with counters.

Mind: Have students visualise where students will move to for 10 less, 10 more, one less, one more, then watch to check.

Creativity: Ask students how else they could represent numbers on the grid. Use blank grid and wipe-away pens. Or have them create on the mat with number cards. Start with 7 instead of zero in the first square. Where is one more? etc.

Mathematics

Language/symbols: One more, one less, 10 more, 10 less, before, after, between, greater than, less than, larger/smaller numbers, largest/smallest numbers, value of tens, value of ones place, place-value system to represent larger numbers with the same 0–9 digits.

Practice: Make and play grid puzzles, grid patterns, number ladders, model the numbers with base 10 materials.

Connections: counting patterns, large numbers, writing patterns, reading patterns, rounding.

Reflection

Validation: Using base 10 materials, how many ways can you model the number 25? What number is one more? One less? Ten more? Ten less? How can you model these?

Applications/Problem-solving: How many two-digit numbers can you make with a two in the tens place? A two-digit number has one 3 in it. What number could it be? How many different numbers might there be?

1.5 Renaming/flexibility for early place value

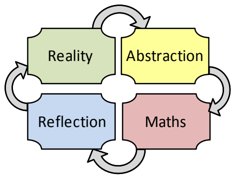
This early renaming and flexibility lays the foundations for the multiplicative structure. It is based on the following.

1. Students do not see 10 as a unit of any kind. They focus on the individual items that make up the 10.
2. Moving students from seeing 10 as ten items to 10 as one ten and zero ones is a big step and needs lots of practise using handling materials, grouping and ungrouping materials, and the introduction and use of the tens and ones board.
3. Students need to develop the concept that one group of 10 equals 10 units within that group and associate it with the numerals involved and represent it with materials and chart.
4. "Ten" is of course the building block of our base 10 numeration system. Young children can usually "read" two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. For example, a 5-year-old might be able to correctly read 62 as sixty-two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values. Ten frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the left of the first frame, and later introducing digit cards, will further assist the development of place-value understanding. Additional frames full of 10 counters can be used to show numbers over 19 so that students develop mental images of numbers based on their experiences with these resources.

1.5.1 Early place value renaming/flexibility activities

1. Calculator count. Use a calculator to explore counting to 10. Start at a number and predict what number comes next. When student gets to nine, predict, and do. Ask: What has changed? Why do you think this might have happened? Continue and discuss what stays the same, what changes, and why.
2. Use 10 frame activities then use two frames. Repeat predict activity. Plenty of activities with 10 frames will enable students to think automatically of numbers less than 10 in terms of their relationship to 10.
3. Using bead strings, say numbers and model what they might look like for the 10 part and the ones part.
4. Using Unifix cubes, play lily pads. Once the students reach 10 have them group into 10 stick and ones.
5. Roll dice and place materials onto chart to build a 10. Say one 10 and zero units. Reverse the process to "unbuild" the 10 into its units.
6. Use bead strings to 'build' a ten. Reverse the process and say One ten no units etc.

1.5.2 RAMR lesson for renaming/flexibility early place value



Learning goal: Renaming and flexibility.

Big idea: Part-whole.

Resources: Collections, posters, pictures, story books, boxes, boxed items.

Reality

Local knowledge and prior experience: Discuss range of numbers in students' lives, naming places, magnitude ideas, counting patterns (forwards and backwards), reading, writing and saying numbers.

Kinaesthetic: Get students to group themselves into groups of a number less than 10, say four or six, and then get them to count how many groups and how many are left over. Try this for tens and ones.

Abstraction

Hand: **Making groups of 10.** Pick up a handful of straws. Say: *We're going to see how many tens we can make out of these ones. Do you have enough ones to make a group of 10? Show me. Put a rubber band around them to hold them together. Where will you put the 10 on the Place Value Chart – the Tens place* [pointing to the place] *or the Ones place* [pointing]*? Do you have enough ones to make another ten?* Repeat until no more tens can be made. Then, say: *What number have you made? Count by tens first* [10, 20, 30, 40, etc.]. Repeat the activity above a couple of times with a different number of ones.

Making numbers. Say: *Show me the number that has 2 tens 5 ones on the Place Value Chart. Show me the 2 tens part. Show me the 5 ones part*. Repeat with these numbers: 1 ten 2 ones, 3 tens 8 ones, 4 ten 5 ones, 2 tens 0 ones. Say: *Show me the number, twenty-five, on the PVC. Show me the 20 part. How many tens make 20? Show me the five part. Which is worth more – the 20 part or the five part?*

Build the number, write the number, say the number using the PVC, tens and ones headings.

1. **Counting and trading ones for tens**. Show 36 with bundling straws on the PVC. Ask the students to write the number. Put out another one straw, saying: *I'm adding 1 more one. Write the number now*. Repeat until you get to 39. Put out 1 more and ask: *Do I have enough ones to trade for a ten?* [Trade] *What number do I have now? Write it.* Keep going for a few more numbers. Ask the students to read the numbers they have written. Swap roles. Let the students make the numbers while you write the numbers. Repeat the above but have the students use the calculator to count as you build the numbers with bundling straws.
2. **Ungrouping tens to make ones.** Put 2 tens 7 ones on the PVC. Ask: *How many tens do you have? How many ones? Roll the dice and take that number of ones away.* [Suppose 4] Ask: *Do you have enough ones to take away 4 ones?* [Yes] *Show me. Roll the dice again*. [Suppose 5] Say: *Do you have enough ones to take away 5 ones?* If No, ask: *How can you get more ones from this number?* [Ungroup a 10 and put the 10 ones in the Ones place.] *How many tens do you have now?* [1] *How many ones?*
3. Repeat until the students have no more straws left. Play "Lose 5 tens 6 ones" if you have two or more players. Each player starts with 5 tens 6 ones of bundling straws on the PVC. Take turns in rolling the dice and taking off that many ones, trading tens for ones when necessary. First player to lose all the straws is the winner.

Mind: When placing material on PVC, put your left hand on left-hand position and, as you move your left hand to the right, say "four tens and five ones", or "six hundreds, seven tens and two ones"; then repeat hand movement, saying "forty-five", or "six hundred and seventy-two". Do hand movements over and over as you change between hundreds, tens and ones (e.g. add 3 ones, take away 2 ones, add 5 tens, take away 2 hundreds, and so on). Use calculators – have students say the number out loud, record the number on small

PVCs, on paper and enter on a calculator (get students to add the relevant number on the calculator when adding material to PVC). Give special attention to zeros and teens.

Creativity: Have students construct their own two-digit number system for a 12-fingered alien race – make up symbols and number names.

Mathematics

Language/symbols: Enable students to discuss the place value and to use the vocabulary, models and draw symbols.

Practice: **Blockbuster**. Play games such as Blockbuster or make a flat/lose a flat. Throw one die and add/subtract number of ones shown, making a new ten from 10 ones or trading a ten for 10 ones when required, first to or from four tens wins; or throw two dice and stop at 100 or go to 0 from 100. Always ask: *How many tens? How many ones left over?* and *How many more to next ten?* Stress the number of tens goes in LH place-value position and the number of left-over ones goes in RH place-value position.

Play trading games.

Connections: Connect to early grouping and larger and smaller numbers.

Reflection

Reversing. You say numbers/write numbers, ask students to show material; you show material, ask students to write/say numbers. Reverse with many materials, for example, lead students through MAB on Hundreds/Tens/Ones PVC → saying hundreds, tens, ones → saying number properly → recording on small PVC → entering on calculator; then reverse – calculator → small PVC → proper language → saying hundreds, tens, ones → MAB on PVC. Then go backwards and forwards from materials ←→ digits, giving examples that have zeros and teens.

Changing parameters. Give the place values in the wrong order. Ask: *What number is 7 ones and one ten?* Do a lot of these – it is important that students see that a number is determined by tens and ones and it does not matter in what order the place-value positions are given. Reverse this – get students to give all the different ways that 37 could be given – 3 tens, 7 ones; 7 ones, 3 tens; 17 ones, 2 tens; and so on.

Unit 2: Place-Value Additive Thinking

Additive thinking is also a foundation of number. So once again, we look at the early work in this area so that we can accelerate ideas later. The foundation ideas are as follows.

1. Place value is the pattern in the way we put digits together that enables us to write an infinite sequence of whole numbers and easily put any two whole numbers in order.
2. The order of the digits makes a difference to the number, so 24 is different from 42.
3. Students associate the place-value meanings of hundreds, tens, and ones more in terms of order in placement initially, than in base-ten groupings (see Unit 3 Place-Value Multiplicative Thinking).
4. Some place-value misunderstandings come about through linguistic complexity of our place-value system in English. We don't name tens as in some other languages e.g. "sixty" vs. six tens; as well, from 13-19 we say the number of units before the tens – seventeen vs. one ten and seven units.
5. Place-value is difficult to teach and learn as it is often masked by successful performance on superficial tasks such as counting by ones on a 0–99 or 1-100 Number Chart.
6. Students need to be able to count to a number, say 42 and see that it is the same as four groups of 10 and two ones, or three groups of 10 and 12 ones, and so on. In essence they need to be able to identify groups and count in groups.
7. Counting continues to play a key role in constructing base 10 ideas about quantity and connecting these concepts to symbols and the oral names for numbers. Activities need to involve concrete materials, while using place-value language orally, charts and symbols.
8. We need to connect place-value concepts with the real world, working with numbers from our lives, homes and community that are meaningful for the students.

After studying the place value of numbers to 99, larger numbers and then decimal fractions are progressively introduced. The same gradual progression of concrete, verbal and symbolic representations is recommended with each extension of numbers e.g. with the introduction of hundreds, thousands, or tens of thousands.

2.1 The additive structure of place value

The additive structure of place value relates to the ability to **count** in each place value. We can count on and back in the Ones, Tens, Hundreds, Thousands etc. places and each count is in units of the place value. It is possible to determine one more or less in each place value (e.g. 10 more than 53, 1 less than 419, 100 more than 345, 10 000 more than 53 000, 100 000 less than 409 000). Each large place-value position counts like the ones. For example, if you are counting on in tens each count is 10, i.e. 10, 20, 30, 40. You have counted 4 tens and arrived at 40. This is closely related to counting and odometer and is also the basis for patterning.

Place value encompasses the understanding that our number system is based on the formation of groups of 10 and that groups of 10 can be counted as individual entities. Students apply their understanding of place value when they model, compare, record, count and classify numbers to 99 and beyond. A great deal of hands-on experience with grouping and regrouping concrete materials is required for place value to be clearly understood.

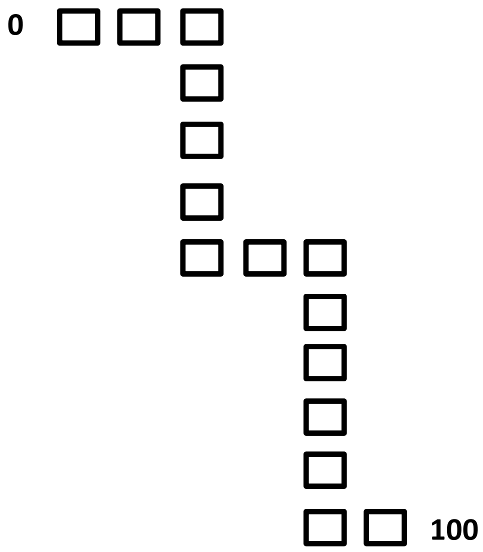
The terms regrouping and renaming are used when referring to the development of place-value concepts. *Regrouping* refers to the process of changing the way in which concrete materials are grouped together and may involve either "unbundling" or "trading" groups of 10. *Renaming* refers to recording the various equivalent forms of numbers using word and non-word symbols. Twenty three can be renamed as 2 tens and 3 ones; 1 ten and 13 ones; 0 tens and 23 ones. Students demonstrate an understanding of place value in two-digit numbers when they can: represent and explain the value of each digit; construct, record and identify numbers which are one more or one less than other numbers and 10 more and 10 less than other numbers; regroup tens and ones using bundling materials; represent and explain the renaming of tens and ones; and compare and classify two-digit numbers.

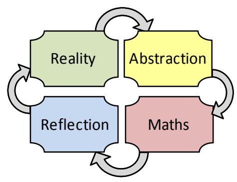
2.1.1 Tens and ones place-value activities

Activities for tens and ones place value are as follows.

1. **Counters in a box**. Have boxes of counting materials (take away food containers are great for this), one box to each group of three or four students. Ask students to estimate/guess how many are in the box. Then work as a team to count how many. Did students use groups to count? What groups did they use? Which were the most efficient? Why? Have students represent their numbers on paper. Ask: Can you write/represent it a different way?
2. **Present or absent**. Have each student write their name in a box on a 10 × 4 grid as they arrive. Students can see at a glance how many are at school by checking the number of tens and ones on the grid.
3. **Drop sticks**. Students can play alone or in groups of two or three; use a set of paddle-pop sticks for each group. Include 10 or fewer sticks all with the same dot dice pattern, 10 or fewer sticks all with the same digit, a combination of dot dice pattern and numerical sticks. Directions: 1. Provide students with a set of appropriately challenging sticks. 2. Determine whether students will find the total of the sticks that land face up or the sticks that land face down. 3. Players drop the sticks and determine the total. Options: students can record their thinking by writing down the numbers to make tens and the unit number left over, etc.
4. **Beans**. Invite students to count a handful of beans and record how many. Point to the digit representing the decade; for example, point to the 3 in 34 and ask students to find that number of beans. Focus on what the 3 in that place means. Repeat for the 4.
5. **Grab bag**. Place MAB in a bag. Students grab a handful then work out what number they have altogether; how many tens and how many ones; how many all in ones. Do this three times. Have students sequence their numbers and make comparisons.
6. **Make 19.** Provide a die and Unifix cubes. Make two teams. Take turns to roll the die. Click together that many cubes. The first team to make 19 wins. Over 19 busts. Teams can sit on a number if they deem they are close enough or teams can play to continue rolling until they get the exact number.
7. **Regrouping**. Have pairs of students decide who has the most matches. Each pair takes two handfuls of matches and groups them to make counting easier. Then, they count the matches before regrouping them in a different way and re-counting. Ask: *Which grouping made it easier to count? Did you get the same number? Why? Why not? Is it easier to see "how many" in the groups of three or groups of 10? Why?*
8. **Mat sequence**. Set up a 0–99 board. Use laminated 0–99 boards and counters to model for individual students while others work on the mat. Use 0–99 board puzzles and jigsaws.
9. **Five Tower game**. Materials: two players, Unifix cubes, two dice, one copy of the Recording Sheet per student. 1. Roll the dice and take that many cubes. 2. Snap them into a tower. 3. Take turns doing this with your partner until you each have five towers. 4. Each player snaps his/her own five towers together into a train. 5. Compare your tower with your partner's. Whose is longest? How much longer? 6. Each partner now breaks his/her own train into tens and ones. 7. Record your results on the recording page.
10. **Place-value game.** Each player has a game sheet and takes it in turns to throw two 10-sided dice. The numbers are used to create two-digit numbers, e.g. a 6 and a 3 could be 63 or 36. Players record their numbers in the most appropriate position between 0 and 100. If numbers cannot be placed, the player misses a turn. The winner is the first to fill all the spaces.

Game sheet





2.1.2 RAMR lesson for place-value tens and ones

Learning goal: Develop understanding of place value in two-digit numbers.

Big ideas: Part-whole; full value of digits is determined by their position as well as their digit.

Resources: Maths Mat, bead strings, pen, paper, pack of cards, pencils.

Reality

Local knowledge: Go for a two-digit number hunt in your local community.

Prior experience: Grouping, counting, number names and digits.

Kinaesthetic: Play tens activities on the mat. Model counting in tens forwards and backwards from given numbers within 0–99. *What is 10 more? What is 10 less?* Play stand on a number, e.g., that has two tens in it. *Is there another? Six ones? Is there another?* etc. Compare two-digit numbers with single-digit numbers.

Abstraction

Body: Use flip cards and place-value bibs for students to act out tens and ones places to model two-digit numbers.

Hand: Play place-value tens. The object of the game is to make the biggest number with two cards that have been randomly turned over. Students have a place-value chart with tens and ones in front of them. The cards are placed in a pile, face down in the middle of the table. Players take turns to pick one card from the top of the pack, turn it over, and place it in either the tens or ones column. Once a card is placed onto the paper its position cannot be changed. Once each player has their two columns filled, players read their numbers and the player with the largest number wins.

Ask: *Can you tell me what each digit within a number represents?* For example, in the number 72, the 7 tells you how many tens are in the number and the 2 tells you how many ones are in the number. Observe: Are students able to read two-digit numbers accurately? Are students able to place two-digit numbers in order, from smallest to largest and vice versa?

Mind: Have students visualise grouping and ungrouping tens, then imagining a two-digit number as what the digit in the tens place might look like in value, etc.

Creativity: Invite students to come up with new ways to represent the places and values of the places of their own chosen numbers.

Mathematics

Language/symbols: Value, place, tens, ones, representing additive place value, place-value charts, number parts.

Practice: These tasks or similar:

Show 23. *What do you see? 23 beads or 10 + 10 + 3 beads?*

Show 13. *What do you see? What has changed?*

Show 10. Show 20. *What happened? What can you now see?* Show 30. Show 40. (*Can you describe?*) *What is happening?* Ten more or building the tens counting pattern. Reverse with 10 less.

Have the students repeat this, moving one bead at a time. Reverse with one less.

Work with your student to read the tasks and move the beads on the bead string to answer the question. The tasks can start off with simple examples and move to more challenging tasks. As your students work, ask them questions about the numbers they are using: *What number is 10 more than this number? What number is 10 less than the number shown? What number is 1 more than this number? 1 less than this number? How many tens in 60? How many ones in 60? How many tens and ones in 60?* Point to the written digit 0 in 60 and ask: *How many does this digit indicate?* Point to the 6 and ask: *How many does this digit indicate?*

Have the students record the value of the numbers in the different ways they identify as they work. Use place-value charts for other representations.

Connections: Bigger and smaller numbers, rounding to the nearest 10, counting on counting back from the nearest 10, big number values, ordering numbers.

Reflection

Validation: Have students write the number that has 2 tens and 7 ones. Write the number that has zero tens and 9 ones.

Ask: *What number is 10 more than 27? What number is 10 less than 27? What number is one more than 27? What number is one less than 27? Which number has the larger value, 23 or 32?* Draw a model to show this.

Provide examples with illustrations of materials to be grouped and ungrouped as tens and ones. Ask students to identify these values.

Applications/Problem-solving. Ask students: *A two-digit number has more tens than ones. What could the number be? How do you know your number is correct?*

2.2 Seriation for hundreds, tens, and ones

With regard to place value, the position of a digit tells us the quantity it represents. Then learning to additively partition is being able to read and write 25 as 2 tens + 5 ones, representing 25 with Unifix cubes then MAB; or bead strings before MAB for larger numbers.

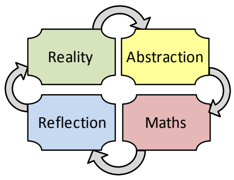
2.2.1 Hundreds, tens, and ones seriation activities

The following are seriation activities. Many are extensions of activities from 2.1.1.

1. **Counters in a box**. See previous activities, adjust contents to suit numbers you are working with.
2. **Drop sticks**. See previous activities, adjust contents to suit numbers you are working with.
3. **Beans**. See previous activities, adjust contents to suit numbers you are working with.
4. **Grab bag**. See previous activities, adjust contents to suit numbers you are working with.

5. **Make 119**. See previous activities, adjust contents to suit numbers you are working with.
6. **Mat sequence 0–99**. Use laminated 0–99 boards and counters to model for individual students while others work on the Maths Mat. Use 0–99 board puzzles and jigsaws.
7. **Reach my number**. Ask students to make their own place-value kits. This kit includes: a sheet of A4 paper which they rule into three columns with headings (from left to right) “Hundreds”, “Tens”, “Ones”, and three sets of the digits 0–9 on card and collections of straws made up as singles and bundles of 10. Have students take turns to roll a 10-sided die or use a spinner to spin a number from 0 to 9 and use their place-value kits to keep score. When students reach the target number (e.g. 145), ask: How would the groups look if the number was 154 or 541? Which parts would be different?
8. **In-between game**. Use three students; 6-, 10-, 12-, 20-, or 30-sided die for each student; and 30 counters. 1. Each student rolls a die without the other students seeing the number rolled. 2. Each student, in turn, tries to predict whether they have the lowest, highest or in-between number. 3. After hearing all the predictions, they may change their prediction. 4. Students show their number. 5. Each student who was correct receives a counter. 6. The winner is the first to get 10 counters.
9. **Guess My Number**. Materials: Hundred Chart pocket chart or an overhead of a Hundred Chart. 1. Pick a secret number from the hundred chart and don’t tell the students what it is. 2. Have students ask you questions to try and guess what the number is. Sample questions include: is it more than/less than, odd/even, end in __, have __ tens/units, etc. 3. Cross off numbers on the overhead chart or flip them on the pocket chart as they are eliminated by student questions. 4. Use a tally to keep track of how many guesses it takes the class before they identify the secret number. Next time you play, try to beat that number of guesses.

2.2.2 RAMR lesson for seriation for hundreds, tens, and ones



<u>Learning goal</u>: Demonstrate seriation and patterning with the 99 board.

<u>Big ideas</u>: Part-whole; full value of digits is determined by their position as well as their digit.

<u>Resources</u>: Maths Mat, 99 boards and 129 boards, counters.

Reality

Local knowledge: Numbers relevant to students in their community.

Prior experience: Grouping, counting, number names and digits, tens and ones.

Kinaesthetic: Play body bingo on the mat.

Abstraction

Sequences. The 99 board represents number in terms of rows down for tens and columns across for ones. Activities focus on building understanding of position of numbers so that students can easily determine one more and less and 10 more and less. A sequence of possible activities follows.

Getting to know the patterns of numbers. Have students read columns and rows. For example, 4, 14, 24, ...; and 60, 61, 62, ..., and notice the patterns.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Reading a column:

It can be useful to have students read the column as “four, onety-four, twoty-four”, and so on. Then the pattern in the column can be seen – it is that “four” is said each time with the tens going up, that is, the ones stay the same and the tens increase.

Reading a row:

The row is read as “sixty, sixty-one, sixty-two”, and so on. Then the pattern in the row becomes apparent – it is that the “sixty” is said each time with the ones going up, that is, the tens stay the same and the ones increase.

Cut 99 boards into jigsaw puzzles and get students to re-form them. Get students to make puzzles for each other. Hand out 99 boards with parts missing and students have to complete the numbers.

Knowing where numbers are – placing numbers by tens and ones. Always start at zero. For position to number – get students to start at zero and move down and across, encouraging students to see the pattern, e.g. that three down and seven across is 37. For number to position – move the tens down and ones across, e.g. 54 is five down and four across (starting at zero).

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

In the first teaching direction, three down and seven across is given and students find they reach 37. In the second teaching direction, 54 is given and students find the movements (five down and four across) that reach this number.

This is an example of the generic strategy of **reversing** – teaching in both directions.

These activities practise finding and placing numbers.

Teaching seriation. Use the board to identify the numbers on left, right, above and below chosen number – show how left and right is one less and one more, above and below is 10 less and 10 more.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Look at 78: one less is 77, one more is 79, 10 less is 68 and 10 more is 88.

Start with the number and look at the left, right, above and below numbers – use a calculator to determine relation of these numbers to the original number (circle the number as for 46 on the left).

Start with the number and determine (use a calculator) one less, one more, 10 less and 10 more and then find these numbers on the board and their relation to the starting number.

Three-digit numbers. Extend these ideas to 99 boards that start with 100, 200, etc. – that have three-digit numbers.

***Patterns**.* Construct patterns of the type 356, 366, 376 and so on for different place values (also have decreasing patterns). After constructing, get students to decipher the patterns – they have to find the changing place value. Make sure students know what happens as place-value position goes up to 9 or down to 0.

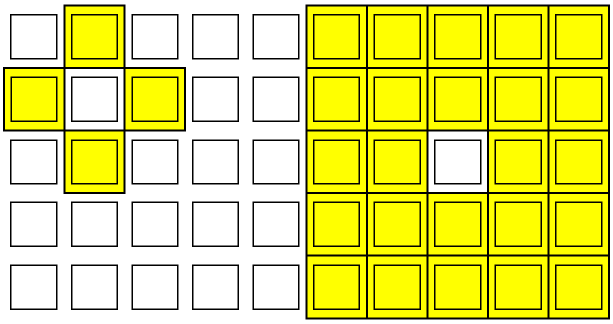
***Mental models**.* Ask students to close their eyes and imagine the 99 board. Get them, with eyes shut, to find numbers, to state numbers that are one more and less or 10 more and less. Ensure they have a picture of the board they can use for seriation and patterns.

Mathematics

Practice. Play "Three in a row". Players in turn take two cards from a pack with 1 to 9 in it (Ace is 1; K, Q, J and 10 removed) and cover any number they can make (e.g. 4 and 3 could be 43 or 34) with a counter (students can remove opponent's counter to place theirs). The first player to get three in a row (row, column or diagonal) wins.

Other practice activities

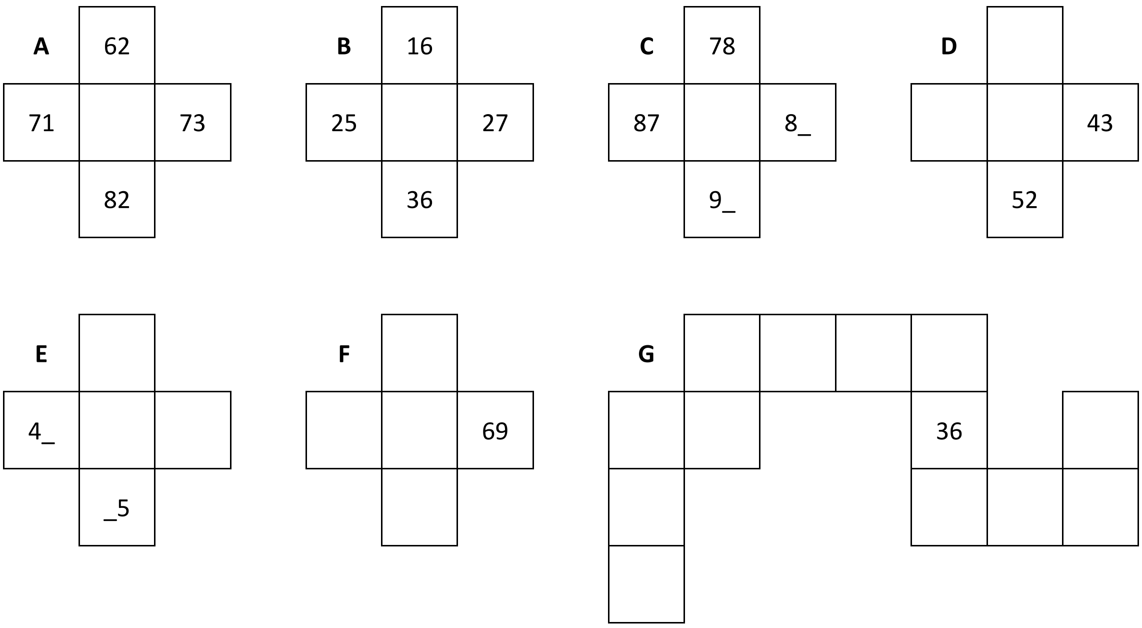
1. Construct a 99 board window with a hole in the middle; place over a number so only that number is visible and ask students to write numbers one less and one more, above and below. Examples of windows are:



2. Give 3 × 3 squares with number in middle and ask for other numbers.
3. Give 3 × 3 squares with numbers on outside and ask for number in middle (reversal of step 2 above).

	36	

4. Give a section of a number board and provide some numbers. Vary the difficulty or number of squares. Also extend to use larger numbers. See examples A to G below:



Reflection

Validation: What numbers below 1000 can you find that have 6 in the tens place? What numbers below 1000 can you find that have 6 in the hundreds place?

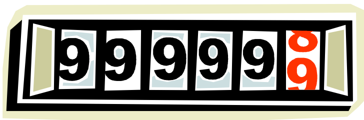
Applications/Problem-solving: I am thinking of a number between 10 and 100 with a single 9 in it. What might my number be?

Extension:

- *Flexibility:* What numbers can you make up using 6, 5, and 8?
- *Reversing*: A two-digit number contains exactly one 4. What might the number be?
- *Generalising/Changing parameters:* I wrote down a number with one zero in it but I cannot remember what it was. I know it was between 600 and 900. What might it have been?

2.3 Counting patterns/odometer principle

The odometer principle states that, in any place-value position, numbers count the same as in the ones place. Counting forwards, they go up to 9 and return to 0 with the digit to the left increased by 1; counting backwards they go down to 0 then return to 9 with the digit on the left reduced by 1. For fractions, the same happens but the number to which the counting goes in counting forward or changes in counting backward is one less than the number of parts in the fraction.



Students need to develop an understanding of the odometer principle in whole-part situations. It includes whole and decimal numbers, where the grouping is 10, and common fractions and mixed numbers, where the grouping may be any whole number. It is particularly important for whole and decimal numbers where each place value (PV) counts forward 7, 8, 9 and then goes to 0 with the PV on the left increasing by 1; and each PV counts backwards 2, 1, 0 and then goes to 9 with the PV on the left decreasing by 1. It should also be noted that odometer, along with place value, rank, and multiplicative structure, is one of the four meanings of whole and decimal number numeration.

We organise or group collections in various ways to make it easier to see how many there are. Groupings based on tens is the standard way to do this because we have chosen to build groupings of 10 into the way we write numbers.

Students should develop the idea that the way we write numbers makes it easy to count forwards and backwards in tens, hundreds, and so on, as well as from any number. For example, counting forwards in tens from 17 (e.g. 17, 27, 37) is easy if you think about what must happen in the tens place rather than trying to add 10 each time. Students can develop a sense of how the tens shift along through activities, such as jumping along a written or imagined number line, or on a 0–99 mat; dropping vertically down levels in a 0 to 109 chart; or adding tens in the form of "longs" to numbers represented in MAB materials.

2.3.1 Hundreds, tens and ones counting patterns/odometer activities

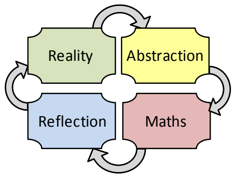
1. **Biggest number**. Select students to write the biggest number they know at the top of a display board. Ask each student: *What is one more? Write the new number beneath the first.* Then, have students add to the sequence each day and say the new number. Ask: *Can this number sequence come to an end?*

2. **Number scrolls**. Ask students to generate number sequences using the constant function on their calculators over the decades and hundreds. Then, have students read, say, predict and verify the numbers from the calculator display.

3. **Numbers as words**. Invite students to select a range of numbers, up to and including tens of thousands, and then write them as words. Select students to read their list of numbers out loud. Have other students listen for the word "and". Ask: *Is "and" used in the same way when we say* 176 *and* 26 076*?*

4. **Odometer rings**. Have students make a bicycle odometer. Ask students to write the numbers 0 to 9 vertically on four strips of paper. Then, have them cut four squares in the piece of card, wide enough to thread the strips of paper through. Thread each strip through a hole in the card. Form loops with the strips and join the ends with sticky tape. Ask students to decide what changes after each 9 in a sequence. Have them use the odometer to read a number sequence. Ask: *What number is one more than 99 (109, 189, 1099)? Which numbers change when one is added to each 9? What is the pattern in these changes?* (Styrofoam coffee cups can be used as an odometer as well.)

2.3.2 RAMR lesson for counting patterns/odometer for hundreds, tens and ones



Learning goal: Develop proficiency with large number place value.

Big ideas: part-whole, full value of digits is determined by their position as well as their digit, additive structure.

Resources: MAB, bundling sticks and rubber bands, Unifix, PVCs, flip cards (numbers 0 to 9 that can be flipped over – four-ring folder will do – can make three- and four-digit numbers then flip over any place value), 99 boards (normal and three-digit versions), calculators, pen and paper.

Reality

Local knowledge: Look for things that follow odometer pattern in local environment and culture – e.g. an odometer from a car, clock or boat.

Prior experience: Check that the students have counting and an understanding of place value for the number of digits being considered.

Kinaesthetic: Get the students to act out an odometer. For example, set up three 0–10 tracks; three students at 0 on each track; ones place student walks along track. When ones-place students get to 9, step off and another student starts at 0, while student in tens place steps forward one space. When tens-place student makes it past 9, the hundreds-place student steps forward one place. Could have a hopping relay race to see who gets to 500 first.

Abstraction

Body/hand: Sequences. Move through bundling sticks on PVC, MAB on PVC to flip cards to calculator. For example: (a) put MAB on PVCs, pick a place value, add MAB one at a time to this value, state number, write number, add on calculator as you go along – also count backwards, removing one MAB piece at a time (remember to write and record on computer); (b) repeat above with flip cards, counting and writing as you go – also count backwards, flipping cards backwards (remember to write and record on computer); and (c) use calculator, enter a number, say 254 and +10, clap hands, students press =, state the number – get them to write the numbers down to see the changes; get them to −10 as well and count backwards.

Repeat the activities from (c) above but this time get the students to state only the place-value position that is being counted, e.g. 324 – "two", 314 – "one", 304 – "zero", 294 – "nine", and so on – going forward and backward. This activity is particularly strong for the calculator. Extend the ideas to normal 99 boards and 99 boards that have three-digit numbers.

Mental models. Get students to shut eyes and imagine the odometer pattern – get students to call out the numbers as you clap hands, without a calculator and with eyes shut. If you can, obtain a virtual sequence of an odometer turning over and get students to imagine this with eyes shut. Do this for an odometer turning backwards. Get students to think of numbers in place-value positions as **being able to turn over or back** – so the 4 in the tens position in 847 turns over to 5, 6, 7 if counting forwards, or to 3, 2, 1 if counting backwards.

Creating own representations. Ask students to make an odometer of another base – *What would a 12-fingered Martian odometer look like?* (they can make up own symbols). Get odometer to work both ways.

Mathematics

Appropriation. Ask students to write down the sequences as they use calculators so that they can see how numeral patterns of odometer counting work.

Practice. Use completing patterns such as 376, 386, 396, ___, ___, ___, ___ and 343, 342, 341, ___, ___, ___, ___ to practise odometer pattern. Construct patterns of this type for different place values (also have decreasing patterns) and get students to decipher the patterns (have to find the changing place value). Make sure students know what happens as place-value positions go up to 9 or down to 0. Show that all place-value positions count.

Connections. Connect odometer work to place value and counting. Also connect to seriation – being 10 or 100 more is the same as one step in odometer counting; for example, 10 more than 348 is the second term in counting sequence 348, 358, 368, ___, and so on.

Connect to comparison and order – get students to use mental model of digits rolling forward or back to understand what it means to be within 30 of 847 – get students to see this as within 3 in the tens position, as below: the 817 and the 877 give the ends of the interval that is within 30 of 847.

___, ___, **817**, 827, 837, 847, 857, 867, **877**, ___, ___

Roll the 5 in 654 up 4 tens to 694 and roll back 5 tens to 604. Discuss what it means to be close. Look at being within 30 of 812.

Reversing. Get students to construct odometer patterns, not just decipher them.

Reflection

Validation. Discuss the action of an odometer – ask students to find where these are in their communities (e.g. reading electricity use). Discuss if it is a sensible way to record change in numbers.

Application. Apply to problems of odometer readings before and after travel – working out how far travelled. Apply odometer to counting situations with large numbers.

Flexibility. Extend the odometer activities above to find all situations where this odometer action occurs in the world – include ones that are not in students' experience.

Generalising. Try to get the students to state what is happening in place-value positions as a generalisation. Get the students to think about the change as they count to 9 or back to 0 (i.e. every position counts forward from 1 to 8, 9, and then back to 0 with digit on left increasing by 1, and backward from 9 to 2, 1, 0, and then back to 9 with digit on the left reducing by 1).

*Changing parameters**.*** Extend to other bases or groupings – count weeks and days, years and months, m and cm, hours and minutes – note what happens as you near the grouping number. Count fractions, e.g. $1\,\frac{3}{5}$, $1\,\frac{4}{5}$, $2\,\frac{0}{5}$, $2\,\frac{1}{5}$, and so on. Try to get students to see that every position counts forward to one less than the grouping, and then back to 0 with the digit on left increasing by 1; and backward to 0, and then back to one less than the grouping with the digit on left reducing by 1.

2.4 Reading, writing and representing hundreds, tens, and ones

For students to read and write whole numbers, they need to be able to distinguish the 0 to 9 digits from other symbols, connect these symbols with their names, and learn how to put these symbols together to represent the whole numbers after 9. Place value is the key to understanding how we say, read, write and calculate with whole numbers. It is the pattern in the way we put the digits together that enables us to write an infinite sequence of whole numbers and to easily put any two whole (or decimal) numbers in order.

Students have to understand the following important characteristics of the place-value system: (a) the order of the digits makes a difference to the number, so 28 is different from 82; (b) the position (or place) of a digit tells us the quantity it represents; for example, in 526, the 2 indicates 2 tens or 20; but in 247, the 2 indicates 2 hundreds or 200; (c). zero is used as a place holder; it indicates there is none of a particular quantity and holds the other digits “in place”; for example, 27 means 2 tens and 7 ones, but 207 means 2 hundreds, 0 tens and 7 ones.

2.4.1 Reading, writing and representing hundreds, tens, and ones activities

Activities in this area are as follows.

1. **What’s your number?** Materials: three dice, recording sheet. 1. Roll the three dice. 2. Arrange all three in order to make six different numbers, e.g. 3 2 6 326, 263, 632, 362, 236, 623. 3. Record your numbers as in the example above. 4. Order your numbers from least to greatest. 5. Show your numbers using pictures, standard notation, expanded form.

2. **Expanded notation**. Invite students to read and record numbers as expanded notation (e.g. 28 is 2 tens and 8 ones). Have them also write numbers from expanded notation shown in place-value order as well as reversed order. Students should know that 8 ones and 2 tens, or 2 tens and 8 ones, are both 28.

3. **Bingo**. Give students practice in recognising number symbols. To begin, students could use Bingo cards that include the numbers 0 to 10. Gradually extend the numbers to include the teens and decades (e.g. 25, 52, 34, 43, 91, 19).

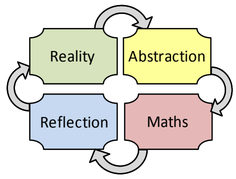
4. **Next number**. Ask students to read out loud the numbers on their calculators as they use the constant function to count. Stop students at 9, then ask: *What number will be next? Check to see if you are correct. What is different about 9 and 10? Has the calculator used these single numbers before?* Use students’ responses to discuss the number of digits and the difference the place makes.

5. **Biggest number**. Select students to write the biggest number they know at the top of a display board. Ask each student: *What is one more?* Write the new number beneath the first. Then, have students add to the sequence each day and say the new number. Ask: *Can this number sequence come to an end?*

6. **Numbers as words**. Invite students to select a range of numbers, up to and including tens of thousands, and then write them as words. Select students to read their list of numbers out loud. Have other students listen for the word “and”. Ask: *Is “and” used in the same way when we say 176 and 1 076?*

7. **Comparing numbers**. Have students work in pairs. Ask one student to write down and then call out a “really big number” for their partner to enter into the calculator. Have them compare the number on the screen with the written version of the number. Ask: *Are they the same? If not, what do you think happened? How was the number said, or how was it heard?*

2.4.2 RAMR lesson for reading, writing and representing hundreds, tens, ones



Learning goal: Develop understanding of place value in three-digit numbers.

Big ideas: Part-whole, full value of digits is determined by their position as well as their digit, additive structure.

Resources: Place Value Charts (PVC), Maths Mat, MABs, pen, paper, calculator, straws, paddle-pop sticks, other bundling materials.

Reality

Local knowledge: Numbers in your world.

Prior experience: Grouping, counting, early place-value experiences.

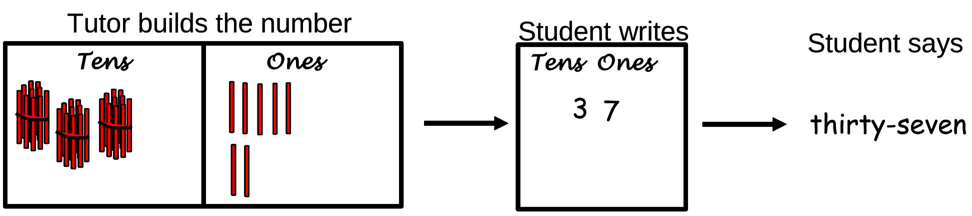
Kinaesthetic: Play guess my number. Place a two- (then later, three-) digit number on each student's back. They need to ask other students yes/no questions to identify their number, e.g. *Has my number a two in it? Is my number even? Is there a number less than five in the tens place?* Begin by giving them no strategies so they can identify them as they go along and eventually identify the most efficient strategies and especially those with which place value assists.

Abstraction

Body: Use mat, PVC, role play.

Hand: Making numbers. Say: *Show me the number that has 1 hundred 2 tens 7 ones on the Place Value Chart. Show me the 2 tens part. Show me the 7 ones part*. Repeat with these numbers: 2 hundreds 4 tens 2 ones, 3 tens 8 ones, 0 hundreds 1 ten 5 ones, 4 hundreds 5 tens 0 ones. Say: *Show me the number one hundred and twenty-seven, on the PVC. Show me the one hundred part. Show me the twenty part. How many tens make twenty? Show me the seven part. Which is worth more – the twenty part or the seven part?* Repeat for the numbers 231, 143, 360, 75.

Writing the numbers. Use the bundling straws and PVC to make numbers like the one shown below. Ask the students to write the number and then read it.



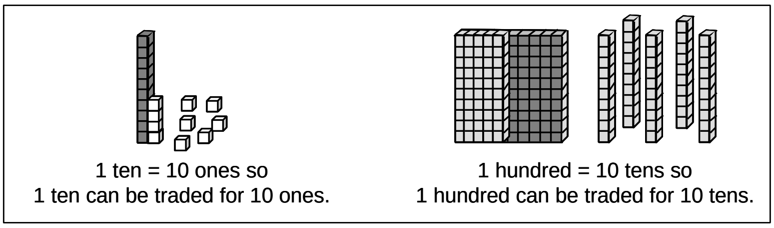
Ungroup hundreds, tens, and ones as in previous reading/writing RAMR for two-digit numbers except using three-digit numbers.

Sequences: Hundreds/Tens/Ones. The sequence below shows activities for H-T-O.

1. Repeat the above but for three digits. Do 456 before 450 before 406 before 400 before 416 type numbers. *Note*: 400 is "4 hundreds 0 tens and 0 ones" – it is not "4 hundreds" (otherwise "4 hundreds 2 tens and 3 ones" may be written as 40023). Stress that the LHS is the number of hundreds, the middle is the number of left-over tens and the RHS is the number of left-over ones. A sequence of activities with MAB is given below.

2. Check whether your students know that 10 MAB ones = 1 MAB ten by having them line up the small blocks beside the long block (see figure below). Then check that they know that 10 MAB tens = 1 MAB hundred. Play "Lose 6 tens 6 ones" with MABs on the PVC. Put out 66 on chart with MAB. Throw two dice; students add the two numbers and remove that number of ones from the chart, regrouping as they go. Students

record the number of MAB with a calculator as they play the game. First to 0 wins. Don't forget to ask the questions: *Do you have enough ones to take away XXX ones?* Make sure students are familiar with the word "trade" and its meaning.

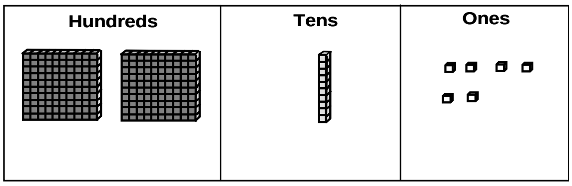


Trading with MAB

3. Introducing hundreds. Ask students to show 99 with MABs on the PVC. Say: *Show me the ninety part. How many tens? Show me the nine part? How many ones? Add 1 more one. How many ones do you have now? Do you have enough ones to trade for a ten?* [Students should trade] *How many tens do you have now? Do you have enough tens to trade for a hundred?* [Students should trade] *What number do you have now? Keep adding ones* [make sure students put them in the ones place] *until you get to 112*. Ask students to read the number now. Ask: *How many hundreds? How many tens? How many ones?* Ask students to show each of these numbers with MABs on the PVC: 574; 832; 333; 740; 200; and 409.

4. Calculator counting (students using PVC, MAB, pen and paper, and calculator). Start with 364 on the PVC. Ask students to read the number [three hundred and sixty-four] and write the number on the paper. Pick a place value, say hundreds, Say: *Add one hundred* – students put out one more hundred, write the new number under the starting number, and add 100 on the calculator. Keep doing this until students have 964. Ask students to read their calculator number and to check whether it's the same as their blocks number. Ask students what place changed first when they were counting by hundreds. Check the written numbers to see if only the hundreds changed. Repeat this for adding 1 in the tens or ones place.

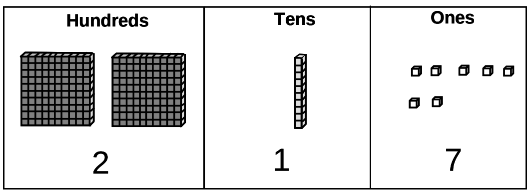
5. Reading numbers with zeros and tens. Give a real-world story: *Malcolm counted 216 cars*. Show the number with MAB (see figure below). Students say the number: "two hundred and sixteen".



Number of cars shown with MAB

Say: *Show me the two hundred part; show me the ten part; show me the six part.* Discuss what the number should/could have been called: "two hundred and onety-six". Compare the number with 260 and 261. Put out MAB for both these numbers. Say: *How many ones in two hundred and sixteen, how many in two hundred and sixty, and how many in two hundred and sixty-one?* Repeat for tens and hundreds. Repeat for other numbers, for example: 311; 518; 470; 407; 200. Swap roles – the teacher says the number and the students show it with MAB. Don't forget to ask the students to show the hundreds part of the number, the tens and the ones.

6. Writing numbers with tens and zeros. Give a real-world story: *Malcolm counted 217 cars*. Build the number with MAB (see figure below). Students say the number: "two hundred and seventeen".



Number of cars shown with MAB and digit cards

Say: *Show me the two hundred part; put a digit card underneath to show this part. Show me the tens left over (the ten part); put a digit card underneath to show this part. Show me the left-over ones (the seven part); put a digit card underneath to show this part.* Ask the student to make a small PVC on paper and then write the number shown on the large PVC with MAB. Compare with 270 and 271. Make with MAB and add digit cards. Discuss the differences in the hundreds, tens and ones and the similarities in the language. Repeat for other numbers, for example: 412; 813; 630; 702; 300. Swap roles – write a number on the small PVC and say the number; the students show it with MAB. Don't forget to ask the students to show the hundreds part of the number, and so on.

Thousands. Repeat the above but for four digits. Once again leave zeros and teens to last. Look up history of numbers and show students other systems – e.g. Roman, Chinese, Mayan.

Mind/Mental models. When placing material on PVC, put your left hand on left-hand position and, as you move your left hand to the right, say "four tens and five ones", or "six hundreds, seven tens and two ones"; then repeat hand movement, saying "forty-five", or "six hundred and seventy-two". Do hand movements over and over as you change between hundreds, tens and ones (e.g. add 3 ones, take away 2 ones, add 5 tens, take away 2 hundreds, and so on). Use calculators – have students say the number out loud, record the number on small PVCs, on paper and enter on a calculator (get students to add the relevant number on the calculator when adding material to PVC). Give special attention to zeros and teens.

Creativity: Creating own representations. Have students construct their own three-digit number system for a 12-fingered alien race – make up symbols and number names.

Mathematics

Language/symbols: Hundreds, odd/even, extended PVC, extended written notation place value.

Practice: **Number expander**. Use a number expander to relate, in both directions, numbers to their expanded form (e.g. 4326 → 4 thousands 3 hundreds 2 tens 6 ones; and 4 thousands 3 hundreds 2 tens 6 ones → 4326). Pleat fold the expander at the coloured section so that it becomes, when folded, just four spaces and, when opened, the four spaces plus the expansion – you can, of course, only fold open some sections at a time.

	Thousands		Hundreds		Tens		Ones

Read-write calculator. Call digits, students enter on calculator and say the number; say a number, students enter on calculator and then say the digits they used. Play "Wipe-out" – give a number and call out a digit – students enter number on calculator and reduce digit to zero with a single subtraction.

Trading games. Play "Win three hundred". Start with 6 tens. Roll the dice and put out that number of tens (make sure students put the tens in the tens place and trade when possible). Ask students to say the number each time (e.g. "one hundred and twenty"). Play "Lose two hundred". Start with two hundred, roll the dice and take away that number of tens, trading when necessary.

Cards/jigsaw games. Bingo: have one student with flash cards with numbers on them, show numbers in turn, other students look on their sheet and cover language or pictures of number, first to get three in a row wins. Cover-the-board has card decks with symbols, language and pictures and a base board with, say, PVCs with numbers – students in turn place cards on top of board numbers or other player's cards which are different representations of their cards; the person with the most number of their cards on top at the end, wins. Mix-and-match is placing different representations of numbers on cards, cutting cards the same way, leaving piles of pieces for student to put back together so all representations are of the same number. (You can also make up dominoes).

Game "Close, closer, closest". Put 200 on PVC with MAB for each player. Each player enters 200 on calculator. Remove 10, J, Q, Ks from a card deck. Shuffle cards. Deal two cards to each player. Players, in turn, form a two-digit number from their cards, add this MAB to PVC (completing all trading) and add number to calculator. They read the number and say how many tens and ones at each play. The game ends when the first player passes 600. The winner is the player closest to 600 at that point. Option: Start at 600 and remove the two-digit number at each turn. Closest player to 200 wins.

Worksheets. Prepare column worksheets with one column filled in (students fill in others) as follows.

Picture (MAB on PVC)	**Language**	**Symbol**
	Three hundred and seven	
		5302

Practise with virtual materials.

Connections: Early grouping and to larger and smaller decimal number work.

Reflection

Validation: Ask students to write the number that has 6 hundreds, 2 tens and 7 ones. Write the number that has 4 hundreds, 0 tens and 9 ones.

> *What number is 10 more than 227? What number is 10 less than 227? What number is one more than 227? What number is one less than 227? Which number has the larger value, 123 or 321? Draw a model to show this.*
>
> Provide examples with illustrations of materials to be grouped and ungrouped as tens and ones. Ask students to identify these values.

Applications/Problem-solving. A two-digit number has more tens than ones. What could the number be? How do you know your number is correct? A three-digit number has more hundreds than tens, what could it be?

Changing parameters. Give the place values in the wrong order: *What number is 6 tens, 8 hundreds and 7 ones?* Do a lot of these – it is important that students see that a number is determined by hundreds, tens and ones and it does not matter in what order the place-value positions are given. Reverse this – get students to give all the different ways that 587 could be given – 5 hundreds, 8 tens, 7 ones; 8 tens, 7 ones, 5 hundreds; 7 ones, 8 tens, 5 hundreds; and so on.

2.5 Renaming in additive place value

Renaming and flexibility are a consequence of different ways of looking at things. Additivity in place value does bring new ways of looking at numbers as in the following examples:

1. Additive thinking in place value makes it easy to split a number into parts. There are standard place-value partitions, such as 582 = 500 + 80 + 2, but often non-standard partitions are more helpful. For example, when operating, thinking of 582 as 382 + 200 helps us subtract 198.
2. The position of a digit tells us the quantity it represents; in 23 the 2 indicates 2 tens or 20 but in 236 the 2 indicates 2 hundreds or 200.
3. This quantity can be represented in different ways: 23 as 2 tens and 3 ones or as 23 ones; 236 as 2 hundreds, 3 tens and 6 ones or as 2 hundreds, 2 tens and 16 ones, etc.

It is useful to remind students of the flexibility and renaming that additive structure brings during study of additive structure. We look at this in more detail in Unit 3.

Unit 3: Place-Value Multiplicative Thinking

Multiplicative structure is a part of place value that is not taught to the same level as place value and additive thinking. It is important, however, particularly in application to metric conversions and understanding of number when the basis is not ones (such as percent).

It is based on there being a constant multiplicative relationship between places. This is something that even young students come in contact with early: the times 10 nature of our number system. It is the basis of connecting counting in tens with place-value understanding and is the reason that zero has a special role in place value. It provides extra strength to the importance of what is the ones in setting up place value. At its basis is that place value determines what the number is by multiplying the digits in place-value position with the value of the place, for example, 365 = 3 × 100 + 6 × 10 + 5 × 1.

This unit looks at the nature of multiplicative structure, the teaching of multiplicative structure, counting patterns and odometer for H-T-O, renaming and flexibility, and the role of zero and 1.

3.1 Nature of multiplicative structure

Multiplicative structure is both a principle and a concept. It is the understanding that adjacent place-value positions relate to each other multiplicatively; that is, one position to the left is 10× larger (multiplication by 10) and one place to the right is 10× smaller (division by 10). It is important for students to understand that the change in numbers is in relation to their place-value positions. These changes to numbers should not be remembered as the adding or removal of zeros.

Ones		
Hundreds	Tens	Ones
		3
	3	0
3	0	0

×10 ×10

Ones		
Hundreds	Tens	Ones
3	0	0
	3	0
		3

÷10 ÷10

Moving the number 3 two places to the left changes the number from 3 to 30 then to 300. 3×100 = 300. Moving the 3 two places to the right changes it from 300 to 30 and then to 3.

Students can be helped to see that in general whenever a number is moved one place to the left it is ×10, two places to the left is ×100, etc. The same works in reverse. A number moved one place to the right is ÷10, two places to the right is ÷100, etc. There is a constant multiplicative relationship between the places, with the values of the positions increasing in powers of ten, from right to left. To find the quantity that a digit represents, the value of the digit is multiplied by the value of the place; for example, in 3264, the 2 represents 200 because it is 2 × 100.

At these beginning stages of multiplicative understanding, focus on the 10× nature of the tens place first. When students are confident with the multiplicative nature of the numbers in the tens place, introduce with 100s and continue to build the pattern for larger numbers.

3.2 Teaching multiplicative structure

Where possible, find real-life contexts into which to embed the multiplicative nature of this unit. Use relevant objects or situations. For example, 10 tickets at $2 costs $20; 10 tables of 6 people makes 60 people; 1 hour of exercise for 10 days makes 10 hours. Certain foods in our society can help – e.g. a packet of lifesavers is 10 lifesavers, so two packets of lifesavers × 10 = 20 lifesavers and 60 lifesavers ÷ 10 = 6 packets of lifesavers.

The structure above is best taught by the body part of RAMR – by getting students to move left and right as the digits are multiplied by 10 and divided by 10 respectively.

3.2.1 Hundreds, tens, and ones activities

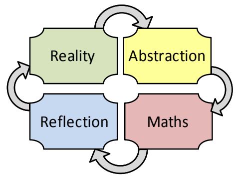
Activities for multiplicative structure involve body activities such as the following, which can be imitated with hand activities by those not involved.

1. **Number cards**. Have students classify numbers by negotiating with each other to decide on the number groups. Give students a card with a number between 0–1001 and ask them to organise themselves into appropriate groups (e.g*. our numbers are in the hundreds/tens/ones*.) Ask: *What does this group of numbers have in common?*
2. **HTO activity**.
3. **Mat sequence with folding cards**.
4. **Bibs and digits** (see 3.2.2).

Support these hand and body activities with mind and practice activities such as the following.

1. **Next number**. Ask the class to say number names in order, say beginning at 81, then each student in turn says the next number up to and over 100. Extend the activity by beginning the count at 201, or 560 to 360 etc. Stop the class count at change over points (e.g. 89, 99, etc.) and ask: *How do you know what number should be next?*
2. **Place-value calculator**. Ask students to enter 1, then 0 into their calculator and say which place the 1 is in (tens). Have them enter another 0 and say where the one is now (hundreds). Continue recording the place names as they appear. Ask: *How many places across does the 1 need to move in order to say 10 000? What is the pattern in the names of the places?* Construct a picture/diagram to record what is happening.
3. **Wipe-out**. Play with the whole class. Enter a number, such as 256, into the calculator. Ask: *How can we make the 5 a zero?* (Subtract 50.) *Why did you do that? What number have we got now?* Make the 2 a zero. Try larger numbers when students are ready. Later, have students play "Wipe-out" in pairs, taking turns to give each other the instructions.

3.2.2 RAMR lesson for place value and multiplicative structure



<u>Learning goal</u>: Some body hand mind tasks to teach multiplicative structure.

<u>Big ideas</u>: Part-whole, full value of digits is determined by their position as well as their digit, additive structure, multiplicative structure.

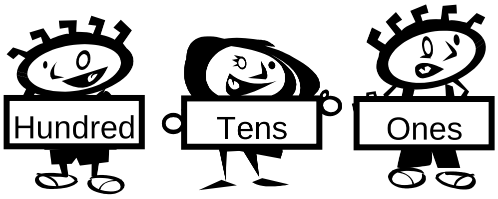
<u>Resources</u>: Place-value names, digit cards, place-value charts, slide rule for place value, calculators, pen and paper.

These materials can be used to study what happens when digits change place-value position – to make students aware of the multiplicative (×10 or ÷10) relationship between place-value positions. The following is a sequence of activities for three-digit numbers that cuts across reality and abstraction. It can also be easily extended to four-digit numbers.

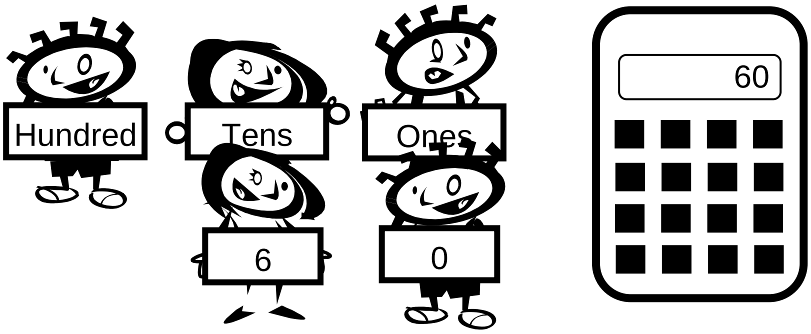
Reality

Kinaesthetic. Make up large copies of digit cards and place-value positions, e.g. three or four place values; give students calculators and undertake the following activities.

Using students' bodies. Give three students PV cards and organise them to stand in correct positions. Students set up their digit cards and PVCs (or their slide rules), pen and paper, and calculator.

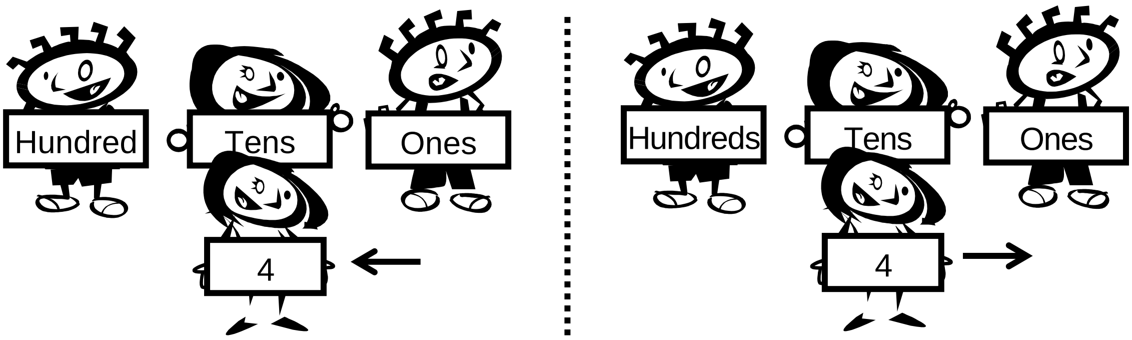


Give another student a digit card, say 6, and get them to stand in front of each position. Add zero cards to show what each number means. Press buttons to place numbers on calculator (see below, e.g. for 6 in tens position); write numbers on paper.



Repeat this for two- and three-digit numbers on cards in front of PV cards, e.g. 230, 604, 14, 824, and 615. Move from cards to calculator and calculator to cards (reversing). Say numbers in terms of hundreds, tens and ones and properly. Reverse everything: movements to ×, ÷; and ×, ÷ to movements.

Exploring the movement. Put a digit card in front of PV cards, move card left and right, use calculator × and ÷ buttons to show relationship in moves (e.g. 6 tens going to 6 ones is ÷10 and 6 ones going to 6 hundreds is ×100). Put a number in calculator, e.g. 40 and multiply or divide by 10; move cards to show these multiplications and divisions (note that the place-value cards could be stuck on wall); write numbers down.



e.g. × 10 ÷ 10

As the students in the front of the classroom move along the PV cards, the other students copy the movements with their digit cards/slide rules, write down the changes on paper, and make the changes on their calculator with an appropriate × or ÷. Make sure that: (a) activities go both ways along cards (to lower and higher PVs); (b) activities are reversed – that you move the student and ask for the × or ÷; and give a × or ÷ and ask for where the student should move to; and (c) moves are one, two and three PV places.

Abstraction

Translation to pictures. After acting out the above in front of the class, all students should focus on their own small digit cards and PVC materials, or their slide rule. They should be directed to move digits across the PV positions left and right (note that this material can also be in virtual form). As they do this, they should write numbers on paper and follow the movements with calculator.

Set up PVC and place a 4 on the ones of the PVC (or use a slide). Ask students: *How many ones, how many tens, how many hundreds?* Ask students to write number on a small PVC and to put on a calculator. Say: *Move the 4 to the tens of the PVC*. Repeat questions and ask students to write number on the small PVC. Ask students how they change 4 ones on calculator to make 4 tens. Show ×10.

Repeat moving 4 from tens to hundreds. Ask: *How do you change 4 tens on calculator to make 4 hundreds?* Show ×10. Ask: *Can you see a pattern? What happens if a number moves from ones to tens to hundreds?* Repeat moving the 4 from hundreds to tens to ones. Repeat using the following starting numbers: 6, 23.

Now start with the × and ÷ and ask where to move the digit card. Switch back and forth, starting with a move and asking for the × and ÷ and starting with a × and ÷ and asking for the move. Type a number on the calculator (one digit, a ten or a hundred) and place digit card on PVC. Multiply or divide by 10 or 100. Ask students to move the digit to match. Ask if the students can see a pattern (move to the left, ×10; move to the right ÷10).

Completeness. Make sure all the following are done:

(a) Use of more than one digit. Move more than one digit along the PV positions.

(b) Extension to moves of two and three PV positions. Encourage students to study changes that move two places and then three places in both directions.

(c) Reversing. Ensure that all activities reverse everything. That is, show or give a movement → ask for the × or ÷; give a × or ÷ → ask for the movement.

Mental models. Ask students to imagine the cards and PVC in their mind – ask them to move finger with eyes shut along their imaginary PVC as questions are asked and to translate movement back and forth to × or ÷.

Mathematics

Language/symbols: Multiplicative, multiplication, division, place-value patterns, sequence.

Practice: Worksheets – these should operate in both directions, giving the multiplication or division and asking for the movement, and giving the movement and asking for the × and ÷ 10 or 100.

Connections: Grouping, additive place value, very large numbers.

Reflection

Validation: Ask students to choose two numbers to compare. Ask them to tell which is greater and how much greater. Ask: *How do you know?*

- *One number is a lot more than another one. Both are greater than 100. What could the two numbers be?*
- *How are the numbers 10 and 100 alike? How are they different?*
- *How are the numbers 350 and 550 alike? How are they different?*

Students should be asked for the patterns in movements and relation to ×10 and ÷10 and to write this down in their own words (generalisation). (*Note:* The pattern is better seen with more place-value positions. The materials can also be in virtual form and displayed with a data projector or smart board.) This should be extended to ×100 and ÷100, and to ×1000 and ÷1000, then reversed and the pattern asked for moving to the left one, two and three PV positions and the same to the right.

3.3 Counting patterns/odometer for hundreds, tens, and ones activities

1. **Number scrolls**. Ask students to generate number sequence using the constant function on their calculators over the decades and hundreds. Then, have students read, say, predict and verify the numbers from the calculator display. Ask: *What changes after each 9 in a sequence? Which numbers change when a 1 is added to each 9? What is the pattern in these changes?*
2. **Construct a simple odometer**. Using three Styrofoam cups write the numerals 0–9 around the lip of each. Fit the three together. Ask students to represent various 3 digit numbers. Play next number modelling with the "odometer". Ask: *What number is 1 more than 99 (109, 189)? What changes after each 9 in a sequence? Which numbers change when a 1 is added to each 9? What is the pattern in these changes?*
3. **Counting in hundreds**. Ask students to use constant addition on their calculator to count in hundreds. Have them predict which number will come next then press = to verify. Ask: *How many hundreds did you put in to make 900? How many hundreds are in 1000 (2000)?*

3.4 Renaming and flexibility

This section is important in terms of flexibility – e.g. 7.5 % is 0.075 as a decimal.

As students' number understanding increases they use more places in the place-value system. They start with whole numbers and move through tens, hundreds, thousands, and so on, and then later also use decimal places, such as tenths, hundredths, thousandths, and so on. Although the curriculum sometimes specifies how many places students should be considering, there are some key principles that apply across all of them.

In our base 10 system, once we have 10 of a particular place (e.g. 10 hundreds) we start using the next place value (e.g. 1 thousand). As we go from one place value to a neighbouring place value the magnitude of the numbers changes by a factor of 10. That is, for example, 7 tenths is 10 times bigger than 7 hundredths and 5 hundreds are 1/10 the size of 5 thousands.

Moreover, the places continue beyond thousands and millions, and beyond hundredths, with each change of place involving a factor of 10. This is part of the "endless" HTO cycle, part of which is shown here.

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

×10 ×10 ×10 ×10 ×10 ×10 ×10 ×10

For renaming, discuss that things being multiples of other things leads to being able to do things in many ways (e.g. 10 × $1 in a $10 note enables $20 to be paid with two $10 notes, one $10 note and 10 $1 coins, or 20 $1 coins).

The teaching strategy here is to move through the material construction models of the number (paddle-pop sticks, bundling straws, MABs, number expanders) from additive place-value understanding, **to link it to multiplicative thinking and the recording symbols for multiplicative thinking.** As in progressing through these models in developing additive understandings, students will take time to progress from one of these models to the next, linking each to the ×10 symbolic representation of multiplicative thinking.

Unbundle and bundle straws/paddle-pop sticks to rename numbers; then use MABs to rename and exchange to rename numbers; next, use number expanders to rename and represent actions with the materials asking students to record using the ×10 multiplicative symbolic representation.

1. Ensure that students are familiar with using paddle-pop sticks and rubber bands to represent three-digit whole numbers, as an extension of work with two-digit numbers. For example:

   53 is made with 5 bundles of ten and 3 singles, or 5 × 10 and 3 × 1.
   153 is made with 1 group of 10 bundles of ten (i.e. a hundred group), 5 bundles of ten and 3 singles.

   Then ask them to make 153 using only bundles of ten and singles (i.e. 15 bundles of ten and 3 singles). Give them practice with other three-digit numbers. Students can make challenges for each other to complete. The advantage of using the paddle-pop sticks is that all the individual units are easily seen and can be bundled and unbundled readily; a disadvantage is that many sticks are required for larger three-digit numbers. Students can prepare bundles of 10 and of 10 tens (100) to keep for use on many occasions. Ask them to record using ×10 multiplicative symbolic representation.

2. Ensure that students are familiar with using MAB to represent numbers. For example:

   53 is made with 5 tens and 3 ones, or 5 × 10 and 3 × 1.
   53 can also be made with 53 ones, 53 × 1.
   153 is made with 1 hundred block, 5 tens and 3 ones: 1 × 100 and 5 × 10 and 3 × 1.

   Note: Emphasise that 53 separate ones is cumbersome compared with the convenience of using 5 tens and 3 ones instead. Highlight that there are still the same number of blocks (really, the total volume is still the same).

   Then ask them to make 153 using only:

   tens and ones (15 tens and 3 ones) or 15 × 10 and 3 × 1.
   hundreds and ones (1 hundred block and 53 ones) or 1 × 100 and 53 × 1.

   Make the point that while we could also use 153 separate ones, it is too cumbersome.

   Give students practice with other three-digit numbers and recording using ×10. Students can also make challenges for each other to complete. MAB are useful because quite large numbers can be represented. A disadvantage is that a ten, for example, has to be exchanged for 10 separate ones, rather than broken up into 10 ones. Teachers will need to highlight that the same number of blocks is present after the exchange.

3. Number expanders are useful tools because they offer a hands-on way of manipulating the symbolic representation (numeral) of a number. They make a bridge from physical to symbolic models for number. Opening and closing the number expander acts as a reminder of the actions with materials (but not as a replacement for this). Because they do not physically model the size of the number, they can be made with any number of place-value columns and so can represent very large or small numbers at more advanced levels.

   Below are some pictures of a number expander with three place-value columns, showing various ways of "expanding" (or renaming) 258.

2	hundreds	**5**	tens	**8**	ones

The number 258 has been written in the blank place-value columns. This shows
258 = 2 hundreds + 5 tens + 8 ones.
Or 2 × 100 and 5 × 10 and 8 × 1

2	hundreds	**5**	**8**	ones

This shows 258 = 2 hundreds + 58 ones
Or 2 × 100 and 58 × 1

2	**5**	tens	**8**	ones

This shows 258 = 25 tens + 8 ones
25 × 10 and 8 × 1

2	**5**	**8**	ones

This shows 258 = 258 ones
Or 258 × 1

- Use a template of 8 number expanders to provide each student with their own copy.
- Ask students to write their own three-digit whole number (e.g. 517) into the three blank rectangles.
- Show students how to fold the expander: the shaded rectangles are folded in half with a "valley fold" and then a "mountain fold" is used to put the shaded rectangle behind the white rectangle on its left.
- Ask them to fold and unfold at various places to make as many different expansions as they can.
- Have students record what their expanders make and show using × 10 symbols.
- Have students represent it with their calculators.

3.5 Role of zero and activities in place value

1. **Exploring 10/100**. Ask: *What does the zero/s tell us?*
2. **Numbers with zeros**. Discuss and explore particularly interesting examples such as numbers with zeros. For example, 410 is 41 tens 0 ones; 507 is 50 tens 7 ones; 700 is 7 hundreds, or 70 tens or 700 ones using expanders.
3. **Ten times greater**. Organise students into pairs. Invite students to use their calculators to find out what is ten times greater than given numbers (e.g. 30, 172, 109, 200, 210, 4550). Ask: *Can you see a pattern? Try to explain to your partner why that happens. What will 10 times 7568 be? Test it and see.*
4. **Multiplying by ten**. Have students predict the effect of multiplying a number by 10. Use the overhead projector calculator and begin with any single digit. Ask: *If we multiply this number by 10, what will the number be? If we multiply by 10 again, what will the number be? How many tens in 100 (1000)?*
5. **Two- and three-digit numbers**. Ask students to use grid paper to draw a diagram that shows the size of each of the digits in a two-digit (e.g. 88) then a three-digit number (e.g. 888). Ask: *How do you know you have the size right for each of the digits? How much bigger is the second 8 than the first?* Later, have students represent the size of the digits in other three-digit numbers (e.g. 256) without using grid paper.
6. **Role of ones digit in setting up place values**. Discuss the role of the ones digit. The RHS number in whole numbers is the ones place.

Module Review

This

Place-value processes

Students learn the place-value processes of reading/writing, seriation, comparing/ordering, renaming, and rounding/estimating whole numbers to understand the following.

1. Whole numbers are in a particular order and the patterns in the way we say them help us to remember the order.
2. There are also patterns in the way we write whole numbers that help us remember the words.
3. The cyclical nature of each set of three places in the hundreds, tens and ones (HTO) model.
4. The role of zero as a place holder.
5. Place value enables us to think of the same whole number in different ways which can be useful as they need these concepts to calculate. Students need to be able to think flexibly of numbers as being made up of other numbers. Groupings based on tens is the standard way to do this because this is the way we write numbers. Often non-standard partitions of numbers are more useful particularly when we calculate.
6. Place value should not be taught in isolation or just as a prerequisite to computation. Rather, students should be challenged to use a variety of mental, diagrammatic (models and representations) and informal writing strategies to work our problems for themselves.

Models and representations

Bundling materials

0–129 board

HTO HTO

Calculator

Odometer

??

Critical teaching points

Critical teaching points represent important steps in the teaching sequence. They are as below.

Students demonstrate their understanding of the principles of base 10 number patterns by:

- sorting and classifying numbers;
- developing the concept of number patterns;
- recognising and using number patterns;
- identifying numbers that are repeated;
- reading the number pattern in as many ways as possible;

- predicting the next number in the number pattern and where it will be placed;
- describing number patterns as relationships;
- renaming and flexibility;
- understanding the role of zero; and
- validating predictions.

Later mathematics

Incomplete

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different from pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "don't know" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the Module N2: Place value test item types

This section includes:

1. Pre-test instructions;
2. Diagnostic Mapping Points;
3. Observation Checklist and Teacher Recording Instrument; and
4. Test item types.

Pre-test instructions

When preparing for assessment ensure the following:

- Students have a strong sense of identity; feel safe, secure and supported; develop their emerging autonomy, interdependence, resilience and sense of agency; and develop knowledgeable and confident identities.
- Students are confident and involved learners, and develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity.

When conducting assessment, take the following into consideration:

- Student interview for diagnostic assessment in the early learning stages is of paramount importance.
- Use materials and graphics familiar to students' context in and out of school.
- Use manipulatives rather than pictures wherever possible.
- Acknowledge the role of using stories in this early number learning, enabling students to tell stories and act out understandings to illustrate what they know.
- Playdough and sand trays are useful for early interview assessment situations.

Ways to prepare students for assessment processes include the following:

- In **individual teaching times**, challenge students' thinking. "Challenging my thinking helps me to learn by encouraging me to ask questions about what I do and learn. I learn and am encouraged to take risks, try new things and explore my ideas."
- In **group time**, model and scaffold question-and-answer skills by using sentence stems to clarify understandings and think about actions. Encourage students to think of answers to questions where there is no one correct answer, and to understand that there can be more than one correct answer (e.g. *How can we sort the objects?*).
- In **active learning centres**, use activities such as imaginative play, sand play, playdough, painting, ICTs and construction to think and talk about different ways of using materials, technologies or toys. Ask questions and take risks with new ideas.

Other considerations:

- Preferred/most productive assessment techniques for early understandings are observations, interviews, checklists, diary entries, and folios of student work.
- Diagnostic assessment items can be used as both pre-test and post-test instruments.

Remember:

Testing the knowledge can imply memory of stuff; asking the students **what they can do with knowledge** requires construction and demonstration of their understanding at this early understandings level.

N2 Place Value: Diagnostic Mapping Points

1. Early place value

Notion of unit/one

- Have egg cartons and objects for eggs; pick up a carton and ask: *How many cartons?* Open carton and ask: *How many eggs?* Ensure students understand number and can apply to groups, to individual items, and to both.

Early groups and ones left over

- Make a group of three counters and a group of one counter. Ask student to show which has "more".
- Make a group of two counters and a group of three counters. Ask student to show which group has "less".
- Give student 11 counters. Ask student to make groups of 3. Ask: *How many groups? How many left over?*
- Have student give you counters and pose a grouping question for you to make.

"Ten-ness"

- Using blank 10 frames, die and counters, ask student to roll the die, place the number of counters on their 10 frame. Roll again: *How many on the 10 frame now?* Continue until the 10 frame is full and there are some left over.

  Ask: *What is your number? How many tens? How many ones left over?* Ask student to choose the correct numeral card (and later, to write the number).
  Give student another 10 frame to place ones left over. Roll again and continue questioning.

Reading, writing, representing

- Except for the teens, does the student say the places in the order in which the digits are written from left to right?
- Have student choose a two-digit number card. Ask them to represent the number with materials. Ask: *Can you represent it differently? Is it the same quantity?*
- Have student represent 36 with counters/bears/sticks or similar. Ask: *How do you know there are 36? Can you represent it a different way? What other groups can you make? Can you make groups of 10? How does this help?*

Renaming/flexibility

- In the above activity, point to the 3 and say, *Show me which part represents this numeral.* Point to the 6. Ask: *Which part represents this numeral? How many altogether? Can 36 be represented as 3 tens and 6 ones and also as 36 ones?* Ask the student to represent 36 on the place-value table both ways.

2. Place-value additive thinking

Additive structure

- Tip a container of 56 beans out and ask student to count them. Do they group them to count? If they count in ones, ask: *Is there another way you could count the beans?* If they group them to count (say in twos) ask them if there is another way they can group them to count the beans. Ask them to write the number down. Point to the 5 and ask: *How many beans does this tell me?* Point to the 6 and ask: *How many beans does this tell me?* If the students have not grouped them in tens already ask them to group the beans to represent 56.

Seriation hundreds/tens/ones

- Give student a blank 10 × 10 laminated board. Ask them to fill in the numbers. Do they complete the board accurately? Place a counter on a number and ask them to place a counter on the number that is one more; one less; 10 more; 10 less. Ask: *What patterns can you see?* Do they see patterns in the digits in the ones place? The tens place?
- Prepare 0–99 board laminated jigsaw pieces and ask the student to predict the numbers surrounding the piece chosen. Can they complete the puzzles?
- Repeat with 0–129 laminated puzzle pieces. Have student complete a blank 0–129 laminated 10 × 10 board. Can the student bridge tens and hundreds?

Counting patterns/Odometer principle

- Ask student to read out loud the numbers on their calculator as they use the constant function to count. Stop student at 9, then ask: *What number will be next? Check to see if you are correct. What is different about 9 and 10?* Do they identify the change in the number of digits? And the difference the place makes?
- Ask student to get 116 Unifix cubes. Did they group them? If not, ask them to group them in tens. Ask: *How many tens? How many ones left over?* Ask them to write the (three-digit) number (total). Add one more to the group. Ask: *How many now?* Have them record the number. Continue increasing by one Unifix cube in this way. When at 119 ask: *What will happen now?* Do it and ask them to record. Ask: *What has changed?* Continue on until they see the repeating 0–9 pattern in the tens and ones places. Ask: *What would happen if we had 198 cubes and we added 3 more? What changes? What is the pattern in the changes?*

Reading, writing and representing

- Play "Win/lose 300" (see "Trading games", p. 30). Can student construct PV chart HTO structure? Can they record two- and three-digit numbers correctly? Can they group and ungroup hundreds, tens and ones? Can they bridge hundreds and tens forwards and backwards? Do they record numbers with zero in them correctly? Do they name the zero in 204 as zero tens? The zero in 130 as zero ones? Ask: *What does zero do for us in these numbers?*

Renaming in additive place value

- Have student construct 328 with straws/Unifix/MABs/bead strings/paddle-pop sticks. Ask them to split the number into parts. Do they use standard place-value partitions? Ask them to explain their partitioned groups. Through careful questioning have the student make the standard partition of 3 hundreds, 2 tens and 8 ones. Ask them to record what they have represented. Say: *If I group them like this – 3 hundreds and 28 ones – are there still the same number? Is there another way we could group this number?* Say we can split numbers into different parts to help us think about them.
- Say: *The position of the digit tells us the quantity it represents*. *In 23, what does the 2 represent? In 238 what does the 2 represent?*
- Say: *The quantity can be represented in different ways*. Give the example of 37 as 3 tens and 7 ones or as 37 ones. Ask student to construct a number for you, then you make it a different way. Swap roles. Repeat with a three-digit number and so on. How well does the student do with flexibility and renaming?

3. Place-value multiplicative thinking

Multiplicative structure (how do adjacent places relate to each other multiplicatively?)

- Play "Next Number" and record. Ask student to count on from 81. Stop the count at 89 and ask: *How do you know what number should come next?* Have student continue to 99 and ask: *How do you know what number comes next?* Ask: *What changes after each 9 in a sequence?*

Demonstrating multiplicative structure (can the student explain with HTO and × 10?)

- Use Place-Value Calculator activity. Ask student to enter 1, then 0 into their calculator and say which place the 1 is in (tens). Have them enter another 0 and say where the one is now (hundreds). Continue recording the place names as they appear. Ask: *How many places across does the 1 need to move in order to say 1000? What is the pattern in the names of the places? Can you construct a picture/diagram to record what is happening?*

Odometer (0–9 in each place)

- Have students use a cup odometer to read numbers. Ask: *What number is one more than 99, 109, 189? Which numbers change when 1 is added to each 9? What is the pattern in these changes?*

Renaming and flexibility

- Ask student to make 74 on a PV chart HTO using materials and writing the number symbols. Roll a die to determine how many tens to increase the number by. (Say 4.) Do they adjust materials as either 11 tens and 4 ones or 1 hundred and 1 ten and 4 ones? Ask: *Can you model this another way? Can you write down the two different ways you have made? Is there another way to represent this number?*

Role of zero

- Repeat the above activity but start with no blocks on the PV chart and set a goal to "make 500". How well does your student deal with numbers with zeros? For example, 110 is 11 tens 0 ones; 207 is 20 tens 7 ones; 500 is 5 hundreds, 50 tens and 500 ones.

N2 Place Value: Observation Checklist				
Unit	**Concept**	**Knows**	**Can construct/do/tell/solve**	**Tripping points**
1. Early place value	Notion of a unit/one	Both can be ones	Identify groups and things in the group as different and related	Trusting the count
	Early grouping	How to put objects into equal groups	Groups and ones. Show both can be counted together. Skip count forwards/backwards from any number. Count up/down in 10s from any number	Group counting 2, 5, 10 Ability to count large collections efficiently
	Reading/ writing/ representing	How to read two-digit numbers; represent two-digit numbers with materials, words and digits	Hear the 4 to 9 part of the sequence in 14 to 19 (skip counting) Can explain the difference between 13 and 31	Teen numbers Bridging tens Forwards and backwards Reading zeros
	Seriation	How to order numbers to 99	Bridge decades up to 100 (skip counting) Order and compare	1 more 1 less, 10 more 10 less
	Renaming flexibility	23 is two tens and three ones and also 23 ones	Different representations of the same number with materials. Place-value table for tens and ones	The same number can look different, be represented in different ways
2. Place-value additive thinking	Additive structure	There is a pattern in the way we put digits together	Count in each place value Count on and back in ones, tens, hundreds	Grouping and ungrouping; reading place values
	Seriations hundreds/tens/ ones	The position of the digit tells us the quantity it represents	Bridge decades through hundreds; can read and write numbers in tens and ones	Bridging decades, hundreds
	Counting patterns/ Odometer principle	In any place-value position numbers count the same as in the ones place	Groupings based on tens making it easy to see how many there are; rounding	Place naming Repeated 0–9 cycle in each place; the effect on next place
	Reading, writing, representing	Read 3-digit numbers (ones/tens/hundreds) Order of the digits makes a difference 118 and 181	The position of the digit tells us the quantity it represents. In 526, 2 indicates 2 tens, in 247, 2 indicates 2 hundreds; zero is a place holder	Reading numbers with zero/s in them Writing numbers with a zero in them
	Renaming in additive place value	Splitting numbers; standard and non-standard partitions	Additively partition in a standard and a non-standard way	236 as 2 hundreds, 3 tens, 6 ones; 23 tens, 6 ones; 236 ones, etc.
3. Place-value multiplicative thinking	Multiplicative structure	Adjacent place values relate to each other multiplicatively	Construct HTO table using materials/symbols; relocate numbers on HTO table ×10 ÷10 and use calculator to verify	HTO HTO repeating structure in ones then thousands
	Demonstrating multiplicative structure	×10 relationship	Using chairs and cards; mat sequence and folding cards; bibs and digits; calculator wipe-out	Place naming Repeated 0–9 cycle in each place; the effect on next place
	Odometer	Standard place-value partitions	Using odometer ring cards/cups	Which numbers change when 1 is added to 9?
	Renaming and flexibility	Standard and non-standard place-value partitions	Multiplicatively regrouping tens and hundreds; representing 50 as 5 × 10 and 0 × 1	130 as 1 × 100 and 3 × ten and 0 × one; 1 × 10 × 10 and 3 × 10 and 0 × 1
	Role of zero	Zero as a place holder	What does zero tell us?	Reading and writing zero numbers

N2 Place Value: Observation Checklist Teacher Recording Instrument					
Unit	**Concept**	**Knows**	**Can construct/do/tell/solve**	**Tripping points**	**Comments/Observations**
1. Early place value	Notion of a unit/one	Both can be ones	Identify groups and things in the group as different and related	Trusting the count	
	Early grouping	How to put objects into equal groups	Groups and ones. Show both can be counted together. Skip count forwards/backwards from any number. Count up/down in 10s from any number	Group counting 2, 5, 10 Ability to count large collections efficiently	
	Reading/ writing/ representing	How to read two-digit numbers; represent two-digit numbers with materials, words and digits	Hear the 4 to 9 part of the sequence in 14 to 19 (skip counting) Can explain the difference between 13 and 31	Teen numbers Bridging tens Forwards and backwards Reading zeros	
	Seriation	How to order numbers to 99	Bridge decades up to 100 (skip counting) Order and compare	1 more 1 less, 10 more 10 less	
	Renaming flexibility	23 is two tens and three ones and also 23 ones	Different representations of the same number with materials. Place-value table for tens and ones	The same number can look different, be represented in different ways	
2. Place-value additive thinking	Additive structure	There is a pattern in the way we put digits together	Count in each place value Count on and back in ones, tens, hundreds	Grouping and ungrouping; reading place values	
	Seriations hundreds/tens/ ones	The position of the digit tells us the quantity it represents	Bridge decades through hundreds; can read and write numbers in tens and ones	Bridging decades, hundreds	
	Counting patterns/ Odometer principle	In any place-value position numbers count the same as in the ones place	Groupings based on tens making it easy to see how many there are; rounding	Place naming Repeated 0–9 cycle in each place; the effect on next place	

Unit	Concept	Knows	Can construct/do/tell/solve	Tripping points	Comments/Observations
2. Place-value additive thinking	Reading, writing, representing	Read 3-digit numbers (ones/tens/hundreds) Order of the digits makes a difference 118 and 181	The position of the digit tells us the quantity it represents. In 526, 2 indicates 2 tens, in 247, 2 indicates 2 hundreds; zero is a place holder	Reading numbers with zero/s in them Writing numbers with a zero in them	
	Renaming in additive place value	Splitting numbers; standard and non-standard partitions	Additively partition in a standard and a non-standard way	236 as 2 hundreds, 3 tens, 6 ones; 23 tens, 6 ones; 236 ones, etc.	
	Multiplicative structure	Adjacent place values relate to each other multiplicatively	Construct HTO table using materials/symbols; relocate numbers on HTO table ×10 ÷10 and use calculator to verify	HTO HTO repeating structure in ones then thousands	
3. Place-value multiplicative thinking	Demonstrating multiplicative structure	×10 relationship	Using chairs and cards; mat sequence and folding cards; bibs and digits; calculator wipe-out	Place naming Repeated 0–9 cycle in each place; the effect on next place	
	Odometer	Standard place-value partitions	Using odometer ring cards/cups	Which numbers change when 1 is added to 9?	
	Renaming and flexibility	Standard and non-standard place-value partitions	Multiplicatively regrouping tens and hundreds; representing 50 as 5 × 10 and 0 × 1	130 as 1 × 100 and 3 × ten and 0 × one; 1 × 10 × 10 and 3 × 10 and 0 × 1	
	Role of zero	Zero as a place holder	What does zero tell us?	Reading and writing zero numbers	
	Notion of a unit/one	Both can be ones	Identify groups and things in the group as different and related	Trusting the count	

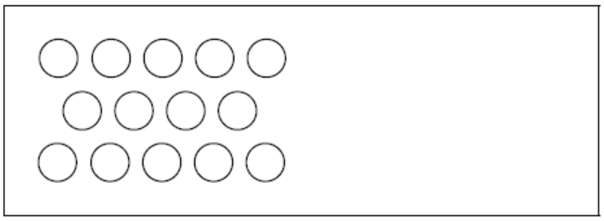
Subtest item types

Subtest 1 items (Unit 1: Early Place Value)

Early grouping

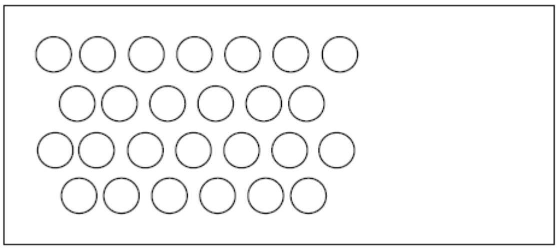
1. Complete the following.

(a) How many groups of four and how many ones can you make from these counters?



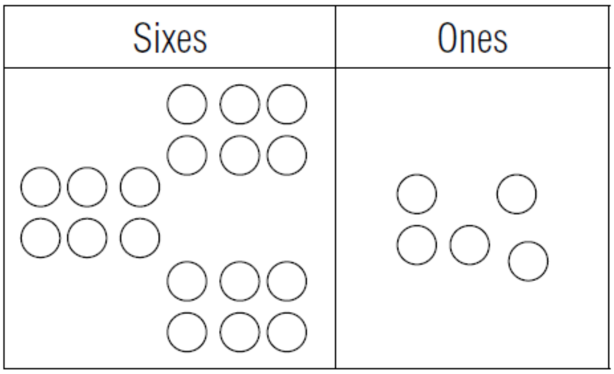
Write here: ______ fours ______ ones

(b) How many groups of seven and how many ones can you make from these counters?



Write here: ______ sevens ______ ones

(c) How many sixes and how many ones?



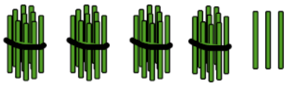
Write here: ______ sixes ______ ones

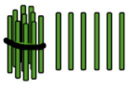
(d) Draw four sixes and two ones on the chart:

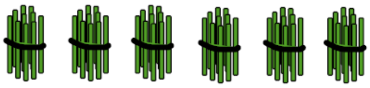
Sixes	Ones

Reading/writing/representing

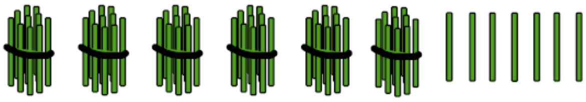
1. Write the ***number name*** to show how many sticks:

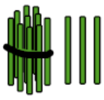
a. 

b. 

c. 

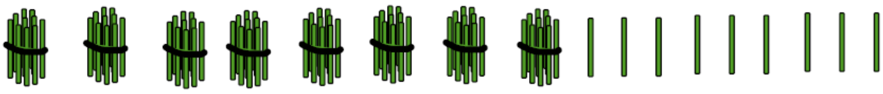
2. Write the ***number*** to show how many:

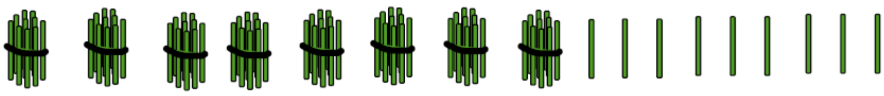
a. 

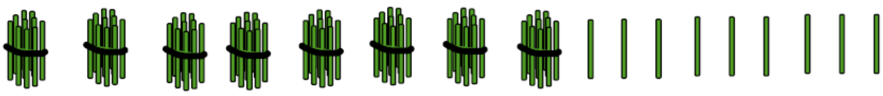
b. 

c. 

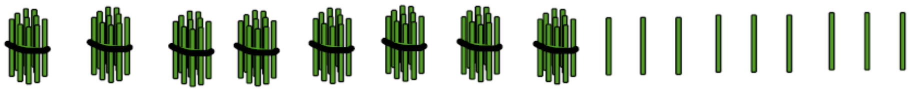
3. Circle the sticks you need to make the given number:

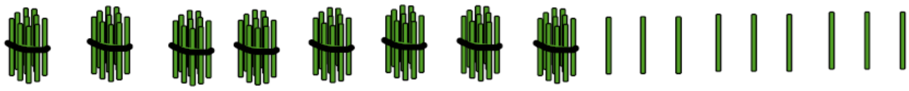
a. 65 

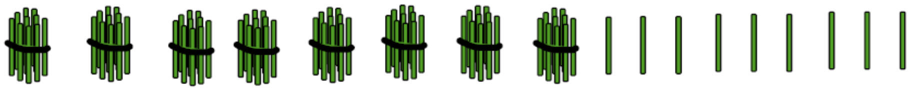
b. 12 

c. 70 

4. Circle the sticks you need to make the given number name:

a. forty-one 

b. eighteen 

c. sixty 

5. Write the number for each number name:

(a) sixty-seven ______________________________

(b) fifty ______________________________

(c) thirteen ______________________________

6. Write the number name for each number:

____ 8

____ 30

____ 17

7. Write the number that has:

3 tens 8 ones ____________________________

7 ones 6 tens ____________________________

0 tens 8 ones ____________________________

0 tens 4 ones ____________________________

8. Write the missing numbers:

(a) 66, 67, 68, _____, _____, _____

(b) 95, 96, 97, 98, 99, _____, _____, _____

(c) 57, 58, 59, _____, _____, 62, 63

(d) 28, 38, 48, 58, _____, _____, _____, _____

(e) 74, 64, 54, _____, _____, _____, 14

Seriation

1. What number is

   (a) 10 more than 42? ___________

   (b) 10 less than 26? ___________

   (c) 1 more than 58? ___________

   (d) 1 less than 70? ___________

2. In each example, circle the number that has the larger value:

   (a) 67 82

   (b) 47 45

   (c) 17 7

   (d) 59 6

   (e) 31 13

3. Put in order from the smallest to the largest value:

   40, 14, 41, 39, 47, ______, ______, ______, ______, ______

4. Write the number name for each number shown:

Tens	Ones
7	3

____

Tens	Ones
1	1

____

Tens	Ones
8	0

____

5. Fill in the Tens-Ones chart to show the number name:

(a) Fifty-eight

Tens	Ones

(b) Fifteen

Tens	Ones

(c) Fifty

Tens	Ones

6. For the number:

Write the number that is 10 more. ___________________________________

Write the number that is 10 less. ___________________________________

Write the number that is 1 more. ___________________________________

Write the number that is 1 less. ___________________________________

Renaming/flexibility

1. Write the missing numbers:

(a) 54 = 4 tens ______ ones

(b) 38 = _____ tens 18 ones

(c) _____ = 6 tens 17 ones

(d) _____ = 1 ten 10 ones

(e) 63 = 3 tens ______ ones

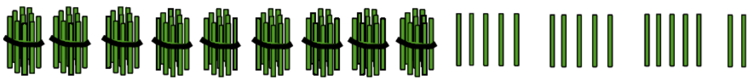
(f) _____ = 0 tens 37 ones

2. Complete the following:

(a) Write the number shown by the material.

____

(b) Circle the bundles of ten you would need to put with all the ones to show the number:



57 = __________

(c) Circle the ones you would need to put with all the bundles of ten to show the number:

63 = __________

Subtest 2 items (Unit 2: Place-Value Additive Thinking)

Seriation

1. What number is:

   (a) 10 more than738? ____________________

   (b) 100 less than 685? ____________________

   (c) 100 more than 310? ____________________

   (d) 10 less than 500? ____________________

   (e) 1 less than 40? ____________________

2. Complete the counting sequences:

   a. 286, 287, 288, ________, ________, ________

   b. 596, 696, 796, ________, ________, ________

   c. 433, 432, 431, ________, ________, ________

   d. 830, 820, 810, ________, ________, ________

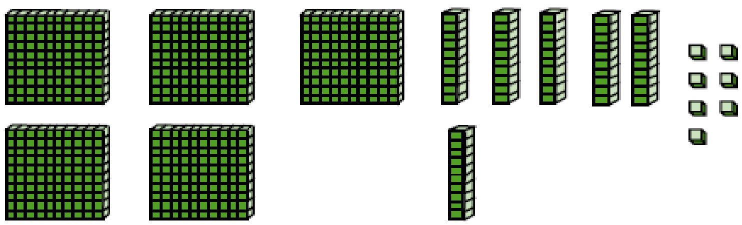
3. In each box, circle the number with the larger value:

a. 387, 404 b. 556, 552 c. 674, 92 d. 804, 840

4. Put these numbers in order from largest to smallest in value: 652, 625, 650, 605, 615

________, ________, ________, ________, ________

5. For this number:



   (a) Write the number that is 10 more. ____________________

   (b) Write the number that is 100 more. ____________________

   (c) Write the number that is 1 less. ____________________

   (d) Write the number that is 10 less. ____________________

Odometer

1. Circle the number in which 7 is worth the most:

    726 172 267

2. Round to the nearest 10:

    (a) 186 ____________________

    (b) 238 ____________________

    (c) 303 ____________________

3. Round to the nearest 100:

    (a) 256 ____________________

    (b) 438 ____________________

    (c) 850 ____________________

4. Circle the numbers that could be rounded to 300:

    (a) 713 (b) 267 (c) 325 (d) 380 (e) 250

Reading, writing, representing numbers

1. Write the number for each number name:

    a. three hundred and forty-nine ____________________

    b. five hundred and forty ____________________

    c. two hundred and twelve ____________________

    d. eight hundred and six ____________________

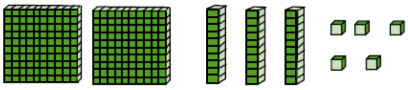
2. Write in words:

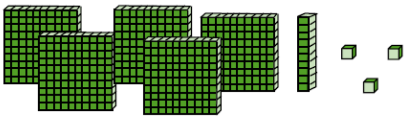
    a. 268 ____

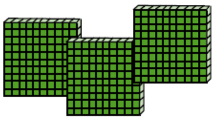
    b. 303 ____

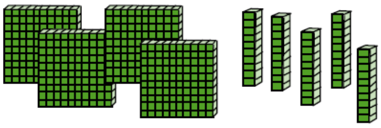
    c. 814 ____

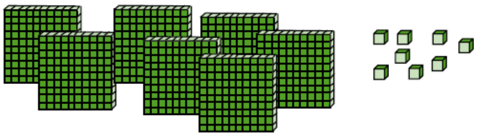
3. Write the number name for the number represented by the blocks:

a. 

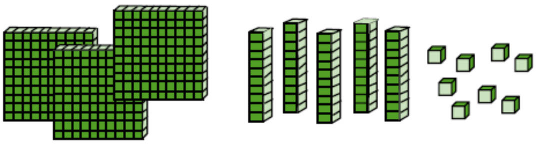
b. 

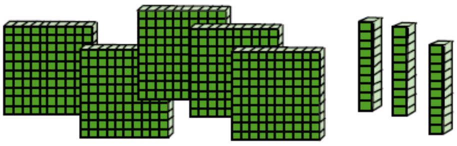
c. 

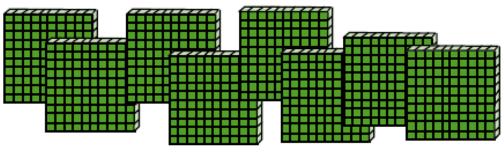
d. 

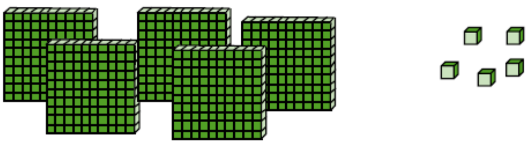
e. 

4. Write the numbers represented by the blocks:

a. 

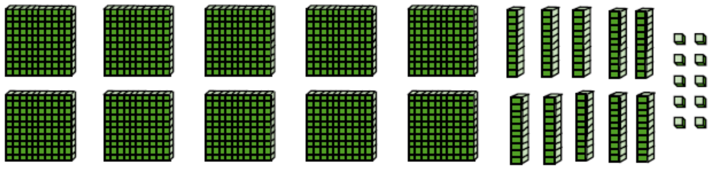
b. 

c. 

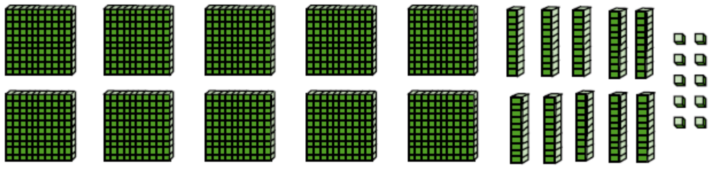
d. 

5. Circle the blocks needed to represent the number names:

a. two hundred and sixty-one

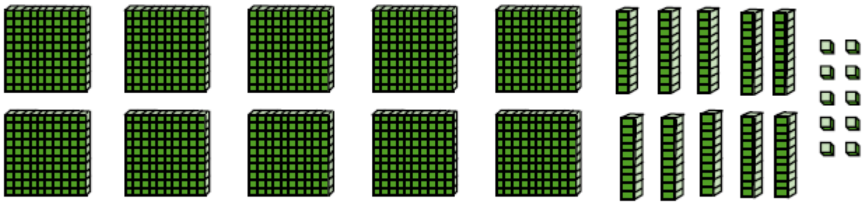


b. nine hundred and five

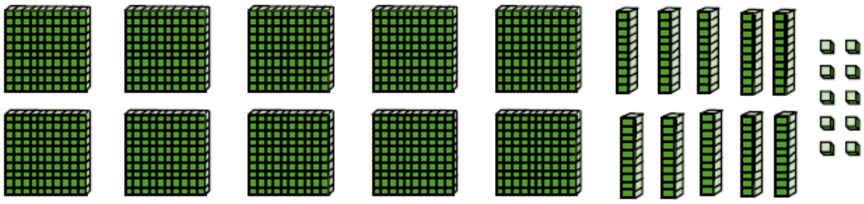


6. Circle the blocks needed to represent the following numbers:

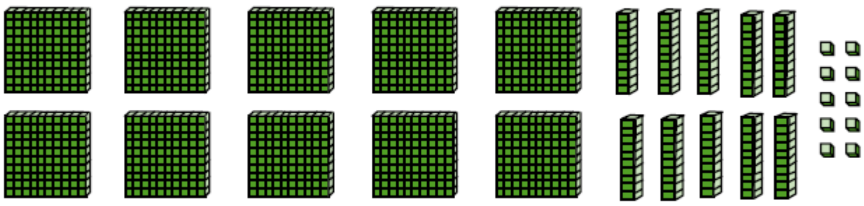
a. 704



b. 740



c. 714



7. Write the name for each number:

a.

Hundreds	Tens	Ones
4	1	5

____

b.

Hundreds	Tens	Ones
3	0	6

____

c.

Hundreds	Tens	Ones
8	3	0

____

8. Write the matching number on the Place Value Chart:

a. two hundred and forty

Hundreds	Tens	Ones

b. five hundred and eleven

Hundreds	Tens	Ones

c. six hundred and nine

Hundreds	Tens	Ones

Renaming flexibility

1. Write the number that has:

a. 3 hundreds 9 tens 7 ones ____________________

b. 8 tens 4 ones 4 hundreds ____________________

c. 6 ones 5 hundreds ____________________

2. Write the missing numbers:

a. 928 = 9 hundreds ____ tens ____ ones

b. 684 = 6 hundreds 7 tens ____ ones

c. ________ = 4 hundreds 19 tens 6 ones

d. 547 = ____ hundreds 14 tens 7 ones

e. 723 = 6 hundreds ____ tens 13 ones

f. ________ = 3 hundreds 26 tens 31 ones

Subtest 3 items (Unit 3: Place-Value Multiplicative Thinking)

Multiplicative structure

1. Write the missing place-value names:

   (a) 3 ones × 10 = 3 ________________

   (b) 4 tens ÷ 10 = 4 ________________

2. Write the missing numbers:

   (a) 6 ones × ___________ = 6 tens

   (b) 7 tens ÷ ___________ = 7 ones

3. Circle the numbers that could be rounded to 50:

   (a) 24 (b) 45 (c) 5 (d) 48 (e) 59

4. A calculator will shift:

   (a) 3 ones to 3 tens, if you:

   + 10; – 10; × 10; or ÷ 10
   (Circle which one.)

Tens	Ones
	3
3	0

   (b) 5 tens to 5 ones, if you:

   + 10; – 10; × 10; or ÷ 10
   (Circle which one.)

Tens	Ones
5	0
	5

5. Write the number on the Place Value Chart:

   (a) When 7 ones is × 10.

   (b) When 40 is ÷ 10.

Tens	Ones
	7

Tens	Ones

Tens	Ones
4	0

Tens	Ones

Odometer

1. Round to the nearest 10:

   (a) 686 ______________________

   (b) 725 ______________________

   (c) 303 ______________________

2. Round to the nearest 100:

   (a) 456 ______________________

   (b) 238 ______________________

   (c) 850 ______________________

3. Circle the numbers that could be rounded to 300:

   (a) 713 (b) 267 (c) 325 (d) 380 (e) 250

4. Complete the following:

a. Is this x 10, x 100, ÷ 10 or ÷ 100?

____

Hundreds	Tens	Ones
	4	7

→

Hundreds	Tens	Ones
4	7	0

b. Is this x 10, x 100, ÷ 10 or ÷ 100?

____

Hundreds	Tens	Ones
6	0	0

→

Hundreds	Tens	Ones
		6

c. Is this x 10, x 100, ÷ 10 or ÷ 100?

____

Hundreds	Tens	Ones
6	0	0

→

Hundreds	Tens	Ones
	6	0

d. What happens when × 10?

____

Hundreds	Tens	Ones
	7	4

→

Hundreds	Tens	Ones

e. What happens when ÷ 10?

____

Hundreds	Tens	Ones
	5	0

→

Hundreds	Tens	Ones

f. What happens when × 100?

____

Hundreds	Tens	Ones
		9

→

Hundreds	Tens	Ones

Renaming flexibility

1. Write the missing place-value names:

a. 8 ones $\times$ 10 $=$ 8 ______________________________

b. 4 ones $\times$ 100 $=$ 4 ______________________________

c. 62 ones $\times$ 10 $=$ 62 ______________________________

d. 15 tens $\times$ 10 $=$ 15 ______________________________

e. 9 hundreds $\div$ 10 $=$ 9 ______________________________

2. Write the missing operations (+, − , × or ÷) and numbers:

a. 7 ones ________________ $=$ 7 tens

b. 8 tens ________________ $=$ 8 hundreds

c. 9 tens ________________ $=$ 9 ones

d. 4 hundreds ________________ $=$ 4 ones

e. 5 ones ________________ $=$ 5 hundreds

f. 7 hundreds ________________ $=$ 7 tens

Role of zero

1. Write a zero in 368:

   (a) without changing its value. ____________________

   (b) so that its value changes. ____________________

2. What does zero represent:

   (a) in 10? ____________________

   (b) in 104? ____________________

Appendices

Appendix A: AIM Early Understandings Modules

Module content

1st module **Number N1: Counting** *Sorting/correspondence *Subitising *Rote *Rational *Symbol recognition *Models *Counting competencies	**2nd module** **Algebra A1: Patterning** *Repeating *Growing *Visuals/tables *Number patterns	**3rd module** **Algebra A2: Functions and Equations** *Functions* *Change *Function machine *Inverse/backtracking *Equations* *Equals *Balance
4th module **Number N2: Place Value** *Concepts* *Place value *Additive structure, odometer *Multiplicative structure *Equivalence *Processes* *Role of zero *Reading/writing *Counting sequences *Seriation *Renaming	**5th module** **Number N3: Quantity** *Concepts* *Number line *Rank *Processes* *Comparing/ordering *Rounding/estimating *Relationship to place value*	**6th module** **Operations O1: Thinking and Solving** *Early thinking skills *Planning *Strategies *Problem types *Metacognition
7th module **Operations O2: Meaning and Operating** *Addition and subtraction; multiplication and division *Word problems *Models	**8th module** **Operations O3: Calculating** *Computation/calculating *Recording *Estimating	**9th module** **Number N4: Fractions** *Concepts* *Fractions as part of a whole *Fractions as part of a group/set *Fractions as a number or quantity *Fraction as a continuous quantity/number line *Processes* *Representing *Reading and writing *Comparing and ordering *Renaming

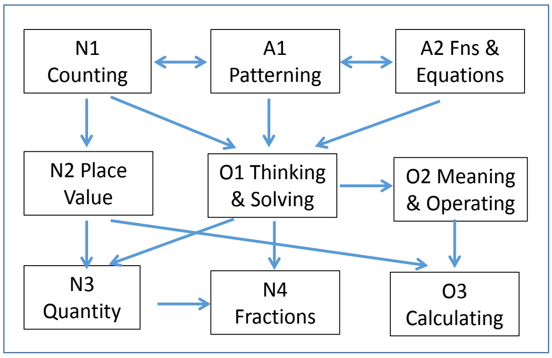
Module background, components and sequence

Background. In many schools, there are students who come to Prep/Foundation with intelligence and local knowledge but little cultural capital to be successful in schooling. In particular, they are missing basic knowledge to do with number that is normally acquired in the years before coming to school. This includes counting and numerals to 10 but also consists of such ideas as attribute recognition, sorting by attributes, making patterns and 1-1 correspondence between objects. Even more difficult, it includes behaviours such as paying attention, listening, completing tasks, not interfering with activity of other students, and so on.

Teachers can sometimes assume this knowledge and teach as if it is known and thus exacerbate this lack of cultural capital. Even when the lack is identified, building this knowledge can be time consuming in classrooms where students are at different levels. It can lead to situations where Prep/Foundation teachers say at the end of the year that some of their students are now just ready to start school and they wish they could have another year with them. These situations can lead to a gap between some students and the rest that is already at least one year by the beginning of Year 1. For many students, this gap becomes at least two years by Year 3 and is not closed and sometimes widens across the primary years unless schools can provide major intervention programs. It also leads to problems with truancy, behaviour and low expectations.

Components. The AIM EU project was developed to provide Years F–2 teachers with a program that can accelerate early understandings and enable children with low cultural capital to be ready for Year 3 at the end of Year 2. It is based on nine modules which are built around three components. The mathematics ideas are designed to be in sequence but also to be connected and related to a common development. The modules are based on the AIM Years 7–9 program where modules are designed to teach six years of mathematics (start of Year 4 to end of Year 9) in three years (start of Year 7 to end of Year 9). The three components are: (a) Basics – A1 *Patterning* and A2 *Functions and Equations*; (b) Number – N1 *Counting* (also a basic), N2 *Place Value*, N3 *Quantity* (number line), and N4 *Fractions*; and (c) Operations – O1 *Thinking and Solving*, O2 *Meaning and Operating*, and O3 *Calculating*. These nine modules cover early Number and Algebra understandings from before school (pre-foundational) to Year 2.

Sequence. Each module is a sequence of ideas from F–2. For some ideas, this means that the module covers activities in Prep/Foundation, Year 1 and Year 2. Other modules are more constrained and may only have activities for one or two year levels. For example, Counting would predominantly be the Prep/Foundation year and Fractions would be Year 2. Thus, the modules overlap across the three years F to 2. For example, Place Value shares ideas with Counting and with Quantity for two-digit numbers in Year1 and three-digit numbers in Year 2. It is therefore difficult, and inexact, to sequence the modules. However, it is worth attempting a sequence because, although inexact, the attempt provides insight into the modules and their teaching. One such attempt is on the right. It shows the following:



1. The foundation ideas are within *Counting*, *Patterning* and *Functions and Equations* – these deal with the manipulation of material for the basis of mathematics, seeing patterns, the start of number, and the idea of inverse (undoing) and the meaning of equals (same and different).

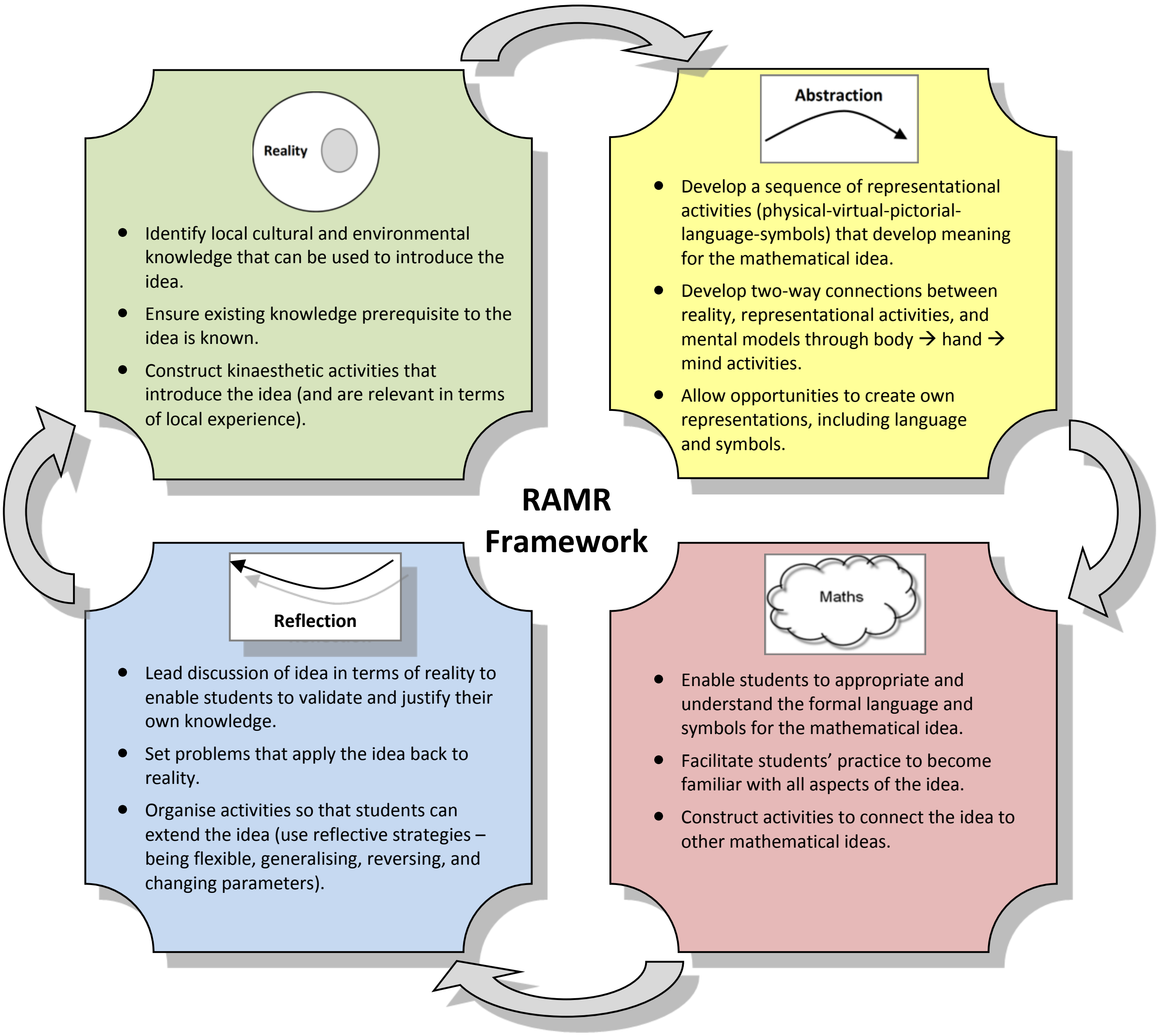
2. The central components of the sequence are *Thinking and Solving* along with *Place Value* and M*eaning and Operating* – these lead into the less important *Calculating* and prepare for *Quantity*, *Fractions* and later general problem-solving and algebra.

3. The *Quantity*, *Fractions* and *Calculating* modules are the end product of the sequence and rely on the earlier ideas, except that *Quantity* restructures the idea of number from discrete to continuous to prepare for measures.

Appendix B: RAMR Cycle

AIM advocates using the four components in the figure below, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem-solving, flexibility, reversing and generalising (see figure on right). The innovative aspect of RAMR is that Reality to Abstraction to Mathematics develops the mathematics idea while Mathematics to Reflection to Reality reconnects it to the world and extends it.

Planning the teaching of mathematics is based around the four components of the RAMR cycle. They are applied to the mathematical idea to be taught. By breaking instruction down into the four parts, the cycle can lead to a structured instructional sequence for teaching the idea. The figure below shows how this can be done.



The YuMi Deadly Maths RAMR Framework

Appendix C: Teaching Framework

Teaching scope and sequence for place value

TOPIC	SUB-TOPICS	DESCRIPTIONS AND CONCEPTS/STRATEGIES/WAYS
Place Value	Early place value	*Notion of a unit/one* – Trusting the count *Early grouping* – Group counting 2, 5, 10; ability to count large collections efficiently *Reading/writing/representing* – Teen numbers; bridging tens; forwards and backwards; reading zeros *Seriation* – 1 more 1 less, 10 more 10 less *Renaming flexibility* – The same number can look different and be represented in different ways; tens and ones
	Place-value additive thinking	*Additive structure* – There is a pattern in the way we put digits together *Seriations hundreds/tens/ones* – The position of the digit tells us the quantity it represents; HTO structure *Counting patterns/Odometer principle* – In any place-value position numbers count the same as in the ones place *Reading, writing representing* – Read three-digit numbers (ones tens hundreds); order of the digits makes a difference *Renaming in additive place value* – Splitting numbers; standard and non-standard partitions
	Place-value multiplicative thinking	*Multiplicative structure* – Adjacent place values relate to each other multiplicatively *Demonstrating multiplicative structure* – ×10 relationship actions *Odometer* – Standard PV partitions ×10 left and right *Renaming and flexibility* – Standard and non-standard place-value partitions *Role of zero* – Zero as a place holder

Proposed year-level framework

<table>
<tr><th rowspan="2">YEAR LEVEL</th><th colspan="2">NUMBER – PLACE VALUE</th></tr>
<tr><th>Semester 1</th><th>Semester 2</th></tr>
<tr><td>Prep</td><td>Concepts
Place value – experience early group activities (put objects into groups, say how many groups and ones left over).
Processes
Reading/writing – experience drawing numbers of personal significance.
Comparing/ordering – compare groups of objects to determine more or less.</td><td>Concepts
Place value – undertake early grouping activities with materials and on a groups/ones chart, stating groups and leftover ones; relate to tens and ones in teens.
Rank – experience number on a number track or ladder to 20.
Processes
Reading/writing – numbers to 20 (using tens and ones).
Comparing/ordering – numbers to 20 (comparing collections or using a number track); experience terms first and second to indicate order.</td></tr>
<tr><td>1</td><td>Concepts
Place value – introduce, model numbers with real-world objects up to two digits, language, set/line models; symbols. Experience number names into hundreds; investigate parts and wholes of quantities; standard partition of two-digit numbers [part-whole big idea].
Odometer – numbers to 50.
Processes
Reading/writing – numbers to 50.
Seriation – 1 more/less, 10 more/less to 50.
Comparing/ordering – up to 50.</td><td>Concepts
Place value – model numbers to 100 [look at notion of unit for one and tens]. Flexibly partition two-digit numbers; partition numbers into more than two parts; identify digit values.
Odometer – to 100 [discuss odometer big idea].
Multiplicative structure – experience to 100.
Processes
Reading/writing – numbers to 100.
Seriation – 1 more/less, 10 more/less to 100.
Comparing/ordering – up to 100.</td></tr>
<tr><td>2</td><td>Concepts
Place value – reinforce teens and zeros to 100, introduce symbol, language (number names) with materials and Place Value Charts to 130.
Multiplicative structure – experience to 100.
Processes
Reading/writing – to 130.
Seriation – to 130.
Comparing/ordering – to 130.
Renaming – to 100.
Rounding/estimating – to 100.</td><td>Concepts
Place value – connect symbol (teens and 10s), language (number names to 100), models to 1000.
Multiplicative structure – experience to 1000.
Processes
Reading/writing – to 1000.
Seriation – to 1000 (1, 10 and 100 more, 1, 10 and 100 less).
Comparing/ordering – to 1000.
Renaming – to 1000.
Rounding/estimating – to 1000.</td></tr>
<tr><td>Focus</td><td>Body activities → Hand activities → Mind activities (visualise then record).</td><td>Body activities → Hand activities → Mind activities (visualise then record).</td></tr>
</table>

The numbers in here will need to be decided by teachers.

Accelerated Inclusive Mathematics Early Understandings Project

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