Accelerated Inclusive Mathematics Early Understandings Project

YuMi Deadly Maths

AIM EU

Module A1

Algebra: Patterning

DRAFT 3, January 2017

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, written and refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning. The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life. YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

DEVELOPMENT OF THE AIM EARLY UNDERSTANDINGS MODULES

In 2009, the YuMi Deadly Centre (YDC) was funded by the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This resulted in a Years 7 to 9 program of 24 half-term mathematics modules designed to accelerate the learning of very underperforming Indigenous students to enable access to mathematics subjects in the senior secondary years and therefore enhance employment and life chances. This program was called Accelerated Indigenous Mathematics or AIM and was based on YDC's pedagogy for teaching mathematics titled YuMi Deadly Maths (YDM). As low-income schools became interested in using the program, it was modified to be suitable for all students and its title was changed to Accelerated Inclusive Mathematics (leaving the acronym unchanged as AIM).

In response to a request for AIM-type materials for early childhood years, YDC decided to develop an Early Understandings version of AIM for underperforming Years F to 2 students titled Accelerated Inclusive Mathematics Early Understandings or AIM EU. This module is part of this new program. It uses the original AIM acceleration pedagogy developed for Years 7 to 9 students and focuses on developing teaching and learning modules which show the vertical sequence for developing key Years F to 2 mathematics ideas in a manner that enables students to accelerate learning from their ability level to their age level if they fall behind in mathematics.

YDC acknowledges the role of the Federal Department of Education in the development of the original AIM modules and sees AIM EU as a continuation of, and a statement of respect for, the *Closing the Gap* funding.

CONDITIONS OF USE AND RESTRICTED WAIVER OF COPYRIGHT

Contents

Module Overview

This module, A1 *Patterning*, is the second of the nine Accelerated Inclusive Mathematics Early Understandings (AIM EU) modules. These modules are designed to provide support in Years F to 2 to improve Year 3 mathematics performance. The nine modules covering Number and Algebra Years F to 2 are shown in **Appendix A**.

AIM EU uses the YuMi Deadly Mathematics (YDM) pedagogy, which is based around the structure of mathematics (sequencing, connections and big ideas) and a Reality–Abstraction–Mathematics–Reflection (RAMR) teaching cycle (see **Appendix B**). The YDM pedagogy endeavours to achieve three goals:

(a) to reveal the structure of mathematics;

(b) to show how the symbols of mathematics tell stories about our everyday world; and

(c) to provide students with knowledge they can access in real-world situations to help solve problems.

YDM argues that the power of mathematics is based on how the structure of connections, big ideas and sequences relates descriptively (with language) and logically (through problem-solving) to the world we live in.

This module (*Patterning*) and the third module (*Functions and Equations*) of AIM EU focus on algebra. However, it should be noted that we are talking here about the algebra of relationships and properties **not** the algebra of manipulation of letters, that is, **early algebra not algebra early**. In the early years, algebra is not about x's and y's; it is about doing and understanding arithmetic in a deeper way that builds arithmetic structure and prepares students for algebra. Thus, this module focuses on the act of generalisation (seeing patterns) while the third module focuses on the relationship and change processes at the basis of arithmetic. They are designed to follow on from counting, precede place value and operations, and lay the foundations for arithmetic and algebra.

Patterning in early childhood and primary

Patterning is an important mathematical idea across primary (and secondary) for many reasons; we will highlight three.

1. **Patterning develops the ability to generalise**. Algebra generalises arithmetic – it focuses on relationships that always hold with numbers and operations, for example, first number plus second number always equals second number plus first number; that is, $a + b = b + a$ for any numbers a and b (what is called the *commutative law*). The basic component for understanding algebra, therefore, is **how to generalise**, that is, understanding the act of generalisation that transforms particular arithmetic results into general rules. The best method for building students' ability to generalise is through patterning – seeing how a pattern can be formed from particular examples.

2. **Patterning develops the concept of variable**. The generalisations from patterning have to hold for all numbers, so they need to be represented in a way that indicates this, by a variable designated as a letter. This development of variable has four steps:

   (a) the student can show the pattern for numbers similar to those given, often with gestures;

   (b) the student can show the pattern/generalisation for any number given (this is called a "quasi-generalisation");

   (c) the student can describe the generalisation in language; and

   (d) the student can show the generalisation by using a letter to stand for "any number".

These four steps to describe a generalisation are shown below:

Development of variable

PATTERN:	Position	1	2	3	4	5	6	7	
	Number	3	5	7	9	11	13	15	and so on

4 STEPS:

Give numbers close to positions shown, e.g. for position 10 the pattern is 21
Give any position, say 257; the pattern is 2 × 257 + 1 = 515 (quasi-generalisation)
Double the position number and add one (language)
$2n + 1$ where n is any number (variable)

3. **Patterning is fundamental to all mathematics**. Many mathematics educators believe that patterning is fundamental to all mathematics and success in mathematics is success in seeing patterns.

Pattern types

There are two pattern types to explore in algebra to promote generalisation and early algebraic thinking and introduce the notion of variable, namely **repeating** and **growing** patterns as follows.

1. **Repeating patterns**. These are linear sequences of objects, pictures or numbers that form a pattern because a section of them repeats; for example, pattern is o I I o I I o I I o I I o I I . . .; repeating part is o I I.
2. **Growing patterns**. These are series of terms where there is a fixed part and a growing part as on right. In the pattern on the right, 0 is fixed and X is growing by one each time.

			X
		X	X
	X	X	X
0X	0X	0X	0X

Repeating patterns are part of the early childhood curriculum and are simplest for introducing activities that engage students in noticing and identifying patterns, whereas growing patterns introduce more complex relationships between terms. We recommend beginning with repeating and then showing how these can be extended to growing patterns.

There are two types of growing patterns, **linear** and **nonlinear**. Linear patterns change by the same amount each time and are simpler than the more complex nonlinear patterns. Linear patterns are represented by straight-line graphs, while nonlinear patterns lead to quadratic, triadic and exponential graphs. In AIM EU, we restrict our focus to the beginning of linear growing patterns, covered in Unit 3 of this module. However, in the *Module Review* section under the heading "Later patterning", we provide a broader view of the sequence across all primary and junior secondary years, so teachers know where the work in Years F to 2 leads.

Activities

The generalising activities in repeating patterns are based on **identifying the repeating part**. Thus, early patterning is copying, continuing, completing and constructing patterns in relation to a repeating part. These activities can be extended to being able to **identify where elements appear** in the repeating sequence, and growing patterns can be introduced by changing the representation of repeating patterns to repeats and then growing a component of the repeat as in the following example:

Repeating pattern		Repeating pattern as repeats		Growing pattern
0 x 0 x 0 x 0 x	➔	0x 0x 0x 0x	➔	0x 0xx 0xxx 0xxxx
		1 2 3 4		1 2 3 4

The generalising activities in growing patterns are to be able to determine the pattern rules. There are two rules as follows below. Only the sequential could be considered as something that could be given to Years F to 2 students for the small amount of growing patterns in this module.

1. **Sequential rule**. This is the amount the growing pattern grows each step. The growing pattern on the right grows by 1 so its sequential rule is + 1.
2. **Position rule**. This is the relationship between position and number of objects. The growing pattern on the right has circle + 1 for position 1, circle + 2 for position 2 and so on. So its position rule is $n + 1$ where n is the position.

			X
		X	X
	X	X	X
OX	OX	OX	OX
1	2	3	4

As will be seen in the *Module Review* section under "Later patterning", both growing and repeating patterns can introduce the concept of a variable. This introduction is stronger in growing patterns where there is more flexibility. The variables are used in determining the position rule. If the pattern grows the same amount at each step, then growing patterns lead to linear relationships (e.g. $y = 2x + 3$) and to line graphs. These ideas are not part of this module.

Finally, as the RAMR model stresses, all teaching activities need to be **reversed**. Thus, pattern activities need to include:

(a) repeating part → repeating pattern AND repeating pattern → repeating part;

(b) position in a repeating pattern → type of object AND type of object → position; and

(c) growing pattern → sequential rule AND sequential rule → pattern.

A full list of activities for primary and junior secondary is given under "Later patterning" in the *Module Review* section.

Patterns in other mathematics strands

Patterns in other strands can be used to obtain understanding of ideas and recall of facts in these strands. For example, the order of place-value positions, the relationship between adjacent place-value positions, counting patterns and the odometer principle, multiplication basic facts, higher decade facts ($3 + 4 = 7$ → $30 + 40 = 70$) and repeated addition are all examples of understandings and facts that can be obtained by seeing patterns. Unit 4 of this module will focus on patterns in number and arithmetic.

Consequently, the ideas in this patterning module are a part of the growth of number through counting and place value (and on to decimal numbers, common fractions and percent, rate, ratio) and a part of the operations of addition, subtraction, multiplication and division. This patterning module has therefore been designed to follow on from counting and to precede the place-value and operations modules, and to lay the foundations for the arithmetic and algebra that follows. In particular, patterning focuses on building the skill of generalisation (seeing the general from a collection of particulars) which is the basis of all mathematics and, especially, the relationship between arithmetic and algebra (which is best understood as the "generalisation of arithmetic").

Connections and big ideas

The starting point for all YDC AIM modules is the connections between mathematics topics and using these connections to accelerate learning, in particular in the formation of big ideas whose learning will provide understanding across mathematics topics and across year levels.

Mathematics is best understood and applied in a schematic structured form which contains knowledge of when and why as well as how. Schema has knowledge as connected nodes, which facilitates recall and problem-solving. It is the basis of YDM that knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning. It enables teachers to:

(a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;

(b) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem-solving;

(c) *choose effective instructional materials, models and strategies* – mathematics that is connected to other ideas or based around a big idea can be taught with similar materials, models and strategies; and

(d) *teach mathematics in a manner that enables later teachers to teach more advanced mathematics* – by preparing linkages to other ideas and foundations for big ideas the later teachers will use.

Patterning connections

Patterning has connections with all mathematics because of its role in generalising or abstracting new ideas from more particular examples. Even 2 + 3 = 5 is a generalisation of examples such as 2 dogs + 3 dogs = 5 dogs, and 2 rulers + 3 rulers = 5 rulers, and so on. The **important connection** for patterning is how it is used in other topics to teach the act of generalisation in those topics, that is, grasping a pattern from particular examples.

It is important to note that there are **two parts** to generalisation – <u>finding/determining</u> the generalisation, and <u>expressing</u> the generalisation. These two must not be confused. Students may determine patterns but express the pattern in a difficult way. In particular, incorrect expressions (particularly if they are only syntactically incorrect) should not result in correct generalisations being marked as incorrect.

Major connections of patterning with other mathematics in early childhood are as follows.

1. **Patterns ←→ counting**. Counting is the ability to follow a pattern (e.g. 56, 57, 58, 59, 60, 61, 62, and so on) both in numerals and most of the language used. (Note: the teens and numbers with zeroes tend not to follow the pattern).
2. **Patterns ←→ place value**. Place value is based on many patterns (e.g. the 100s-10s-1s sub-pattern and the … billions-millions-thousands-ones-thousandths, millionths, billionths … pattern).
3. **Patterns ←→ operations**. Operations are also based on patterns. There are the basic fact patterns such as 8 + 5 = 10 + 3 in that 2 is added and subtracted to maintain the equivalence. There are the multiplication patterns such as ×5 always ends on 5 or 0. There are the overall basic patterns or laws that first number + second number always equals second number + first number. As well, repeating patterns provide a method for determining multiples. For example, the pattern based on repeat A A B is A A B A A B A A B A A B … , and in this pattern, the B marks out the multiples of three.
4. **Patterns ←→ fractions**. Fractions have many patterns, such as being named by the number of equal parts that the whole is divided into, or that the fractions with larger denominator are smaller in quantity.

Other connections are evident when we go up the years of schooling. For example:

(a) *patterns are connected to variables* – the position rule in growing patterns determines the number of elements at each position, and as this is determined by the position number which can be any number, then a variable has to be used; and

(b) *patterns are connected to and connect arithmetic and algebra* – the laws of arithmetic can be translated to algebra by seeing the connections between the patterns in arithmetic computation and the way this relates to algebraic computation.

Patterning big ideas

The major patterning big ideas that have some application to early childhood are as follows.

1. **Change and relationship**. There are two perspectives from which mathematics can be viewed and approached: *transformation* (e.g. 2 → 5 by +3) and *relationship* (e.g. 2 + 3 = 5). In patterning, we tend to take a predominantly transformation/change approach as we focus on patterns. However, when we give position, we start to move into a relationship mode.

2. **Relating symbols to reality (symbols tell stories)**. This is not strong in patterning but relationships can be seen between position and the reality of the pattern.

3. **Pre-empting and prerequisites**. The step from particular to general must be well built because the step from pattern to variable is built upon it.

4. **Generalising**. Patterning is built around seeing the general in the specific. This is a big, big idea for patterning.

5. **Unnumbered before numbered instruction**. Students appear more easily able to look for patterns in unnumbered activities than in numbered situations, where they tend to look for answers. Thus, it is recommended that instruction start with unnumbered activities, then move to numbered activities and then to variable activities as in the diagram below.

Unnumbered activities ↓ Numbered activities ↓ Variable activities	**GENERALISATION**	↓ Small to large numbers Everyday language Variables

It should be noted that there are more patterning big ideas if we go beyond the early childhood years. Some of these are as follows.

1. **Concept of variable**. There are three ways of introducing the concept of variable as an unknown number: (a) using letters to express a generalisation for a pattern, (b) solving changes and relationships in terms of unknowns, and (c) relating numbers through formulae (e.g. volume of a cylinder = $\pi r^2 \times h$). Obviously patterning is closely related to variable.

2. **Sequencing arithmetic to algebra**. The difference between arithmetic and algebra is that algebra holds for any number. Therefore it is a pattern and so algebra expresses the patterns seen in arithmetic.

3. **Generalisations themselves**. Because algebra is the generalisation of arithmetic, it is important to know and understand common generalisations or common patterns (often in number) that cover arithmetic and algebra. The most important are the arithmetic principles (or the Field properties):

   (a) *identity* – addition and multiplication have identities (0 and 1 respectively) that do not change anything;

   (b) *inverse* – numbers/variables like a have inverses for $+, \times$ ($-a$ and $\frac{1}{a}$);

   (c) *commutative law* – $a + b = b + a$ and $p \times q = q \times p$;

   (d) *associative law* – $a + (b + c) = (a + b) + c$ and $p \times (q \times r) = (p \times q) \times r$; and

   (e) *distributive law* – $a \times (b + c) = (a \times b) + (a \times c)$.

Sequencing

To teach for rich schema, it is essential for teachers to know the mathematics that precedes, relates to and follows what they are teaching, because they are then able to build on the past, relate to the present, and prepare for the future. Thus, YDM presents information in this module as sequences of ideas that relate to and connect with each other. To build this sequence, we listed the important components of patterning in the early childhood years and then sequentially related them into the units within the module, taking into account how ideas interact as they develop. The list was as follows.

(a) copying and continuing repeating patterns (learning to differentiate between designs and patterns);

(b) completing and creating repeating patterns (without direction and when given a repeating part);

(c) discerning relationships between repeats and repeating patterns;

(d) finding position in repeating patterns, from position to object and object to position;

(e) breaking repeating patterns into repeats and changing into growing patterns, for example,
0 x 0 x 0 x 0 x … → 0x 0x 0x 0x … → 0x 0xx 0xxx 0xxxx … ;

(f) copying, continuing, completing and creating growing patterns;

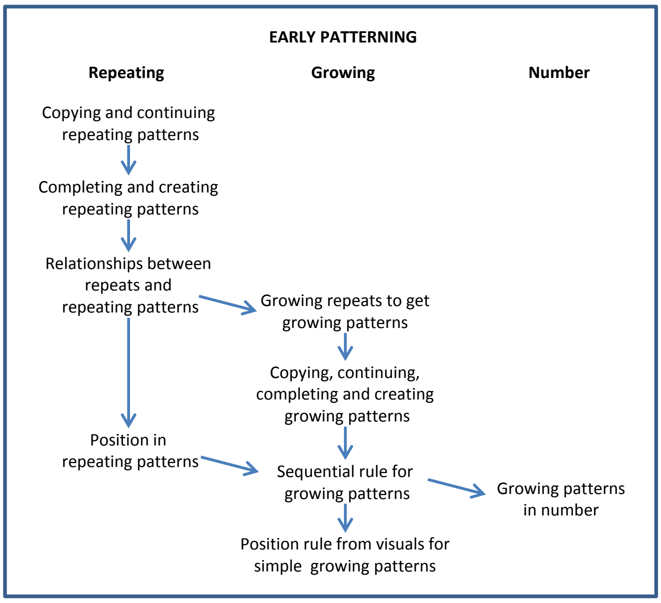
(g) identifying fixed and growing parts and sequential pattern rule;

(h) identifying position rule for simple examples, using visuals and then tables of numbers; and

(i) identifying patterns in number, that is, patterns from counting, seriation, odometer, and addition and multiplication relationships.

The resulting sequence for the module is shown on the right. It has three columns – repeating patterns on left, patterns using number on the right, and the sequence repeating to growing repeats to growing patterns in the centre. Crucial components of the sequence are where we study the role of repeats and look at elements and their positions for repeats. The diagram starts with repeating patterns and moves on to growing patterns and patterns in number. It represents a focus on integrating repeating and growing patterns.

Appendix C shows the teaching scope and sequence as a table, and provides a proposed year-level teaching framework.



Teaching and culture

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework and the impact of Western number teaching on Indigenous and low-SES students.

Teaching implications

Teaching implications are as follows.

1. **RAMR cycle**. Use the RAMR cycle in pattern lessons. Start with something students know such as music or dance, move on to representing the patterns with blocks and letters, with the focus on repeating patterns, and finally on to generalisation. The RAMR cycle is in **Appendix B**.

2. **Inquiry approach**. Patterning is an opportunity for students to be creative and to learn by inquiry – to construct their own patterns and to analyse other students' patterns.

3. **Pattern/design distinction**. Designs are creations that appeal to our artistic senses. They are often symmetrical. A pattern, though also artistic, must convince the observer that it will go on/repeat forever. For example, ABBCDDDCBBA is a design that, as it stands, does not convince us that it repeats, but ABBCDABBCDABBCD does convince us that it goes on forever – with ABBCD as the ongoing repeating part.

4. **Repeating/growing distinction**. Traditionally, repeating and growing patterns were studied separately because they look and act so differently, especially syntactically. The teaching position advocated here is to see growing patterns as repeating patterns with a growing part and to build syntactical similarity by

breaking repeating patterns into repeating parts. From the other direction, repeating patterns broken into repeats can be seen as growing patterns that grow by zero.

5. **Sequence for repeating patterns**. First use sound (e.g. drumming) and movement (e.g. dance) before using objects and then numbers. Over time, the use of numbers will become more important.
6. **Bringing in numbers**. Repeating patterns can be represented by numbers but the numbers are simply replacing a sound, a movement, or a block – they do not relate with each other in terms of operations (e.g. 1223 1223 1223 and so on). This starting point with numbers can then be extended to counting, skip counting and multiples. Then there is a relationship that is based on operations. However, for all situations, teaching and learning should focus on **unnumbered situations before numbered situations**.
7. **Where to start – 0 or 1**. Growing patterns have each element or pattern position numbered. When making the pattern with materials, it is more sensible to call the first element 1. However, when in a more abstract situation, it is better to start from zero, particularly as this better relates to the function and graph of the pattern.

Finally, because algebra is the generalisation of arithmetic, it will be necessary to focus on the development of the new concept of variable as standing for any number and on the big ideas from arithmetic that carry through into algebra (e.g. concepts of operations and equals, principles associated with operations and equals).

Cultural implications

There are two implications for algebra from the discussion above: (a) what is the best way to teach it? and (b) what is the best way to teach it to Aboriginal and Torres Strait Islander and low-SES students?

Teaching patterning. The power of mathematics lies in the structured way it relates to everyday life. Knowledge of these structures gives learners the ability to apply mathematics to a wide range of issues and problems. This is best achieved if the knowledge is in its most generalised form, which is algebraic form. Thus, the most effective way to present mathematical knowledge is through algebra, with patterning an essential part of learning how to build generalisation. Any topic of mathematics can be presented instrumentally (as a set of rules). Although algebra is the direction for power in mathematics, it has to be presented structurally, showing the generalisations that can be used in many examples. Powerful algebra teaching focuses on extending arithmetic to generalisations that can apply across all arithmetic, which is achieved by seeing pattern; that is, teaching through patterning that builds holistic understandings of structure that can then be applied to particular instances (from the whole to the part). If students are fortunate enough to gain this structured understanding of mathematics, the subject becomes easy. This is because it is no longer seen as a never-ending collection of rules and procedures but rather as the reapplication of a few big ideas.

Teaching Indigenous students. Aboriginal and Torres Strait Islander students tend to be high context – their mathematics has always been built around pattern and relationships. Their learning style is best met by teaching patterning that presents mathematics structurally as relationships, without the trappings of Western culture. Powerful Indigenous teaching is therefore holistic, from the whole to the part. As Ezeife (2002) and Grant (1998) argued, Indigenous students should flourish in situations where teaching is holistic (from the whole to the parts). Thus, patterning as holistic algebra teaching has two positive outcomes for Indigenous students: (a) it teaches a powerful form of mathematics, and (b) it teaches it in a way that is in harmony with Indigenous learning styles. Algebra taught structurally through patterns, then, is something in which Indigenous students should excel. However, this is just a general finding. What does this mean in practice for the teaching of algebra? It means that we will not be teaching rules for manipulating letters. Letters and algebraic expressions and equations will be understood in terms of everyday life and algebraic ideas will be generalised from arithmetic through seeing patterns. This will mean a lesser focus on algorithms and rules, and a greater focus on patterning, generalisations and applications to everyday life.

Teaching low-SES students. Interestingly, holistic teaching is also positive for low-SES students. Three reasons are worth noting. First, low-SES students tend to have strengths with intuitive-holistic and visual-spatial teaching approaches rather than verbal-logical approaches. Thus, an algebraic and patterning focus on teaching mathematics should also be positive for low-SES students. Second, many low-SES students in Australia are immigrants and refugees from cultures not dissimilar to Aboriginal or Torres Strait Islander cultures. They are also advantaged by holistic algebraic and patterning approaches to teaching mathematics. Third, many low-SES students have themselves experienced failure in traditional mathematics teaching, and so have members of their families. This results in learned helplessness with regard to mathematics and what is called *mathaphobia*, where students believe that no effort on their part will enable them to learn mathematics. Holistic-based algebraically oriented teaching of mathematics through patterns is sufficiently different that students may not apply their phobia to it – particularly if taught actively and from reality as in the RAMR model.

Thus, for the Indigenous and low-SES students for whom YDM was developed, algebra patterning is the key for mathematics success – not x's and y's but the generalised holistic thinking that is the basis of it.

Structure of module

Components

Based on the ideas above, this module is divided into this overview section, four units, a review section, test item types, and appendices, as follows.

Overview: This section covers a description of patterning in early childhood, connections and big ideas, sequencing, teaching and culture, and summary of the module structure.

Units: Each unit includes examples of teaching ideas that could be provided to the students, some in the form of RAMR lessons, and all as complete and well sequenced as is possible within this structure.

Unit 1: Early repeating patterns. This unit covers the initial teaching of repeating patterns (e.g. sounds, actions and materials) and the role of the repeating part.

Unit 2: Position in repeating patterns. This unit extends the role of repeating parts and explores the relationship between repeating parts, objects and position.

Unit 3: Repeating to linear growing patterns. This unit covers how repeating patterns can be extended to growing patterns, and looks at growing and fixed parts and visuals for determining sequential and position pattern rules for simple patterns.

Unit 4: Patterns in number. This unit covers number patterns in terms of counting, place value, and additive and multiplicative structure.

Module review: This section reviews the module, looking at important components across units. This includes the major ways of modelling patterns, the important competencies associated with patterning, and a brief description of where patterning leads to in Years 3 to 9.

Test item types: This section provides examples of items that could be used in pre- and post-tests for each unit.

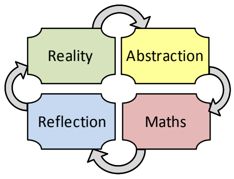
Appendices: This comprises three appendices covering the AIM EU modules, the RAMR pedagogy, and proposed teaching frameworks for patterning.

Further information

Sequencing the teaching of the units. The four units are in sequence and could be completed one at a time. However, each of the units is divided into sub-ideas (concepts and processes) that are also in sequence within the unit. Therefore, schools may find it advantageous to: (a) teach earlier sub-ideas in a later unit before completing all later sub-ideas in an earlier unit; (b) teach sub-ideas across units, teaching a sub-idea in a way that covers that sub-idea in all the units together; or (c) a combination of the above.

The AIM EU modules are designed to show sequences within and across units. However, it is always YDC's policy that schools should be free to adapt the modules to suit the needs of the school and the students. This should also be true of the materials for teaching provided in the units in the modules. These are exemplars of lessons and test items and schools should feel free to use them as they are or to modify them. The RAMR framework itself (see Appendix B) is also flexible and should be used that way.

Together, the units and the RAMR framework are designed to ensure that all important information is covered in teaching. Therefore, if modifying the order, try to ensure the modification does not miss something important (see Appendix C for detailed teaching frameworks).



RAMR lessons. We have included RAMR lessons or part lessons as exemplars wherever possible in the units of the module. Activities that are given in RAMR framework form are identified with the symbol on the right.

Suggestions for improvement. We are always open to suggestions for improvement and modification of our resources. If you have any suggestions for this module, please contact YDC.

Special note: For ease of writing on a computer with Word, this module has simple descriptions of patterns mainly represented by noughts and crosses (O and X). This does not mean that these are important patterns. Repeating patterns could be done using sounds, actions, blocks, pasta stuck on string, dancing hippopotamuses with and without colourful parasols – anything the imagination can come up with. Similarly, growing patterns can be built by blocks, straws, real-world instances (trees and leaves), actions like climbing stairs – anything that enables a pattern to emerge. Do not let the narrow ability to type with variety affect the variety in your patterns.

Unit 1: Early Repeating Patterns

This unit introduces the idea of repeating patterns, that is, a linear representation of different attributes that repeat a basic pattern. This pattern can be in a variety of attributes. It could be a clapping or drumming pattern, or a movement or action pattern (as in a repetitious dance), or a sequence of shapes, sizes and colours. What sets the repeating pattern apart from other arrangements of actions, materials and attributes is the repeat – the pattern is this repeat over and over again. Therefore, it has to be seen as different from a design. A design is a bounded set of actions, materials and attributes that does not repeat a part of the design continually. Of course, a pattern can show the action of its repeats in more than one direction. For example, in geometry, we have tessellations or tiling patterns. In these, shapes cover 2D space in repeating patterns that have two dimensions. However, we will restrict ourselves to linear or 1D repeating patterns that follow a line.

This unit begins with copying and continuing patterns, looks at difference from non-repeating designs, uses repeats to complete repeating patterns, and finally looks at creating repeating patterns. Throughout each sub-unit, it is important to go from **simple to complex** when examples of repeating patterns are being considered. Repeating patterns become more complicated by *increasing the number of elements* in the repeat (e.g. number of actions or colours), by *increasing the number of different attributes* (e.g. colour and action together), or by *making the repeat more difficult to sequence* with other repeats (e.g. repeating O X O – does this give O X O X O X O X … or O X O O X O O X O O X …). These ways of going from simple to complex are shown diagrammatically on the right.

INCREASING COMPLEXITY

Number of elements in repeat	**Number of attributes in repeat**	**Structure of repeat**
Two elements	One attribute	Repeat starts/finishes with different element
↓	↓	↓
Many elements	Many attributes	Difficult to discern start/finish of repeat

Background information

1. Ask the students to copy simple two-attribute patterns (e.g. O X X X O X X X O X X X ...) where O and X are actions or materials. Act out or place out the pattern with materials and let the students copy it. It seems to be a better teaching sequence and easier for students if they copy patterns before continuing them. With an eye to the RAMR model, YDC recommends action patterns (e.g. clapping and stomping, or hand out and feet up) before material actions (e.g. Unifix in a row with colours showing pattern).

2. Once the students have shown they are able to copy the pattern, ask them to continue the pattern. Teacher starts the pattern (e.g. O X X X O X X X O) and students continue it (in this example, continuing gives … X X X O X X X O X ...).

3. When students are experienced in this, allow them to have more than two attributes and more complex repeats (e.g. X O O Y repeated).

4. Also, if you feel the students are ready, you can see if the students can continue the patterns in both directions, that is:

   O X X O X X O X X → O X X O X X O X X O X X O X X O and so on to the right

   → O X X O X X O X X O X X O X X O X X O X X O X X and so on in both directions.

   The bi-directional continuing is more difficult with actions, as you have to go in the reverse direction which can be difficult.

Note: Students should not simply watch the teacher give examples. They should join in the actions and use materials to copy the patterns. Repeats are visual images and best learnt by body/hand activities. This means that we often need a way of talking about the pattern and its repeats when students are using different materials. Capital letters of the alphabet are commonly used. For example, one clap and two stamps repeated is called an ABB pattern, as is a triangle-square-square pattern.

1.1 Copying and continuing repeating patterns

1. **Copying**. Let students copy what you do – keep it to simple repeating patterns.
2. **Continuing**. Do this with a variety of activities and when you feel students are ready, let them continue the pattern themselves.
3. **Action ←→ Action**. Use actions, sounds, and attributes of materials to act out patterns and ask students to copy the patterns – simple movements with hands, steps left and right, dance moves, clapping, stomping and drumming patterns, putting out Unifix in colour patterns, putting out blocks in size patterns, and so on.
4. **Language ←→ Action**. State the moves and colours you want and let students copy these in actions and materials, and then continue.
5. **Other set ups**. Do not always be linear – get students to sit in a circle – then make the repeat, say, first person leg up and next person two hands in air and continue around the circle. Get an odd number of students so when the circle is complete and they are starting the second circle students are not doing the same thing. Reverse the direction of the pattern. Repeat for three actions with the number of students that allows repeats and a different number that does not.

1.2 Relating patterns to repeats and differentiate from non-repeating designs

1. Get students to **identify the repeating part** when they create repeating patterns. Students often identify only part of the repeating part (e.g. X X X not O X X X), so language must direct students to find **all** of the repeating part.
2. Next, develop **language of repeats** so that it is straightforward to record and talk about patterns in terms of repeats. One way to do this is to use capital letters to describe the repeat (e.g. an AABBB pattern would be clap, clap, stomp, stomp, stomp, clap, clap, stomp, stomp, stomp, and so on).
3. Get students to **continue patterns from using language**. For example, what is an AAB patterns if A is a clap and B a stomp? Start it off and see if students can continue. (*Note:* Some students may be at 1.4 – do not hold them back at this point.)
4. Highlight the **difference between a repeating pattern and a linear design** by focusing on the presence or non-presence of a repeat. For example,

   O X X O X X O X X O X X

   is a pattern with an ABB repeat, while

   O X X O O O X X O O O X X O

   is a design. Keep the patterns to simple two-attribute designs initially and then allow all proposals to be attempted and discussed. (*Note:* Many students see patterns in terms of design. So use what you are given by the students to stress the difference.)
5. Show students repeating patterns and designs and ask them to pick the difference. Ask students to make designs and **convert into repeating patterns** and vice versa.

1.3 Completing repeating patterns

1. Begin by getting students to **experience and continue** repeating patterns that are more complex. Note that most early repeating patterns may have only two objects (X and O). Begin to use three or more objects and actions to make patterning more difficult.

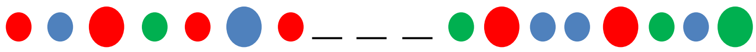
2. Next, focus on the activity that is to **complete** a pattern – to fill in the empty spaces. For example, what shapes will fill the gap in X X O O O X X O O O X X O _ _ _ X O O O ...? Show patterns with gaps and ask students to fill in the spaces. Do this for all types of repeating patterns – actions, sounds, blocks, and so on.

3. Make the pattern completions **more difficult**, for example:

   (a) this is easy: X X O X X O _ _ _ X X O X X O ... ;

   (b) this is harder: X X O X X O X _ _ X ... ;

   (c) this is even harder because it has two attributes (size and colour), with the repeating pattern based only on the size attribute (two small, one large), but colour tends to dominate the pattern:

4. Get students to **make repeating patterns**, remove a few places and hand on to another student to fill in the places.

Be careful with sequencing here – younger students find it easier to work with repeating patterns if the objects' attributes are very different and the difference obvious. This often means two attributes is much harder and that colour tends to predominate, as shown in 3(c) above.

1.4 Creating repeating patterns

1. Finally, ask students to **create** their own patterns. Depending on their development with patterns, this could be closed, with suggestions on what to start with, or very open, allowing any actions, sounds and objects. Although limited, Unifix or multilink cubes are effective as they can be connected into a line. This activity, because of its openness, often results in designs rather than repeating patterns. This is a further opportunity to discuss the ideas in 1.2, and is also the reason for the importance of 1.2 – teachers should ensure students see the necessity to show that pattern can go on forever (with continuous repeats).

2. Continue asking students to create their own pattern but ask the students, when showing the pattern, to **identify the repeating part**. This means that, from now on, a description of a repeating pattern includes describing the repeating part.

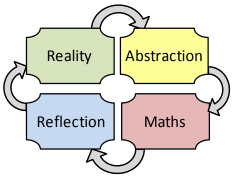
3. Now reverse the activities and ask the students to **create a repeating pattern when given the repeating part**. For example, when given the following repeating part, X X O Y O, students can create the pattern X X O Y O X X O Y O X X O Y O ... and so on. This means that pattern activities go **both ways**: pattern → repeat <u>and</u> repeat → pattern.

4. Make sure that you **provide activities of some difficulty**. For example, an activity can be made much more difficult if the repeat given is O X X X O, because the repeating part starts and finishes with the same object and many students will put O X X X O X X X O ... as the pattern, instead of O X X X O O X X X O O .... So start with repeats that are simpler, with few attributes and little complexity. Then see how far the students can go.

5. Ensure you encourage students to make a **wide variety of repeating patterns** using a variety of forms. For example, a Year 1 teacher had students make patterns out of types of pasta (all one colour) and stick them onto string so they had vertical patterns hanging from the ceiling (with no colour).

Note: (a) Begin with examples with only two objects. Three or more can be used to make more difficult repeating patterns; and (b) younger students find it easier to work with repeating patterns if the objects are very different. This often means two attributes different – both colour and shape (e.g. red O, blue X).

1.5 RAMR lesson for early repeating patterns



Learning goal: Identify, describe, copy, continue, construct and record patterns.

Big ideas: Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity.

Resources: General classroom items, manipulatives, toys, things from nature that range in size, colour, texture, purpose, mass, sound, and so on.

Reality

Local knowledge: Items that are familiar to students, play items, construction items, games, music. Go outside and find patterns.

Prior experience: Playing, sorting, collections and materials, threading, building.

Kinaesthetic: Large rope for playing jump rope using pattern chants, drumming, music, clapping. Have students copy, continue and accompany patterns chants. Have one student create a chant, another copy and/or extend.

Abstraction

Body: Have students skip over footpath divisions and/or paving patterns; dancing patterns and foot movements.

Hand: Have students explore and make patterns with kinaesthetic activity: toe tapping, clapping, feely materials; threading macaroni/beads; playdough patterns; percussion; reciting nursery rhymes and poetry; reading rhyming stories, object cards.

Pattern in the round: Use china plates with patterns around the outside edge as models. Give students paper plates and allow them to create patterns around the edges. The same can be done with cups. Play copy your partner's pattern. Have students place markers between repeats.

Create Unifix trains working together; allow student time to describe and read pattern. Use string for student to identify repeating part.

Mind: Paint imaginary pattern pictures. Visualise pattern. Tell/ describe for partner to draw with chalk on cement.

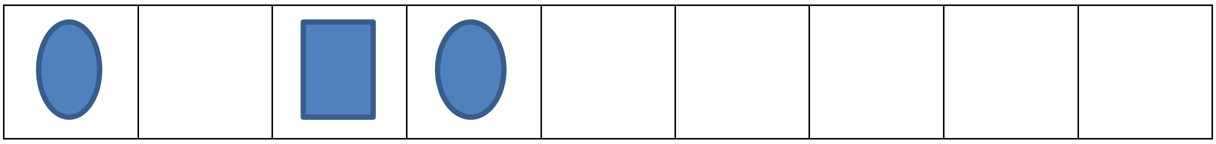
Creativity: Students make up their own patterns with paint/printing/stamping/swirling e.g. jogger treads. Make multiple crayon rubbings of jogger treads. Use these to create two-part patterns. Discuss the similarities and differences between the treads to ensure the students can determine the two elements. **Junk Box:** Students create patterns with little or no direction from 'maths junk box'.

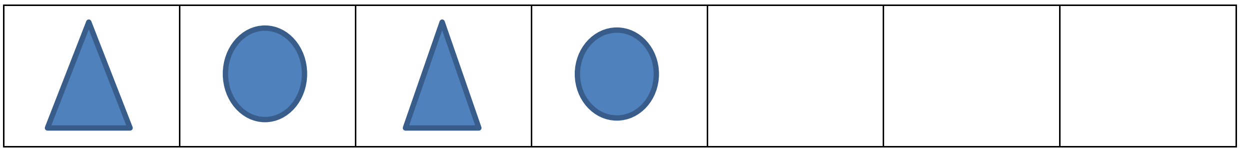
Mathematics

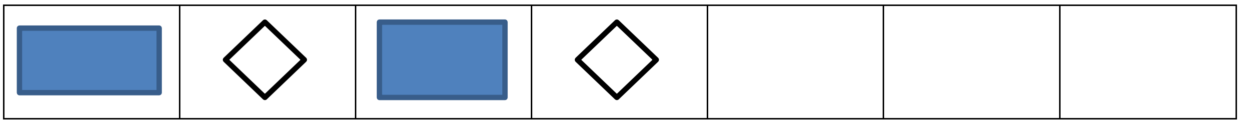
Language/symbols: Same, different, element, part, repeat, pattern, non-pattern words to describe patterns.

Practice: Using beads have students create a pattern, read it to their partner then have their partner copy and continue their pattern. *What happens?* Have students discuss this. Make a list of the pattern words students come up with. Have them sort the words. Ask students to record their pattern on a whiteboard. *Where does your pattern repeat?* Ask student to draw a line between each repeat. **Repeat** with **more complex** patterns.

Keep Going is a game to name the elements of the pattern and continue this pattern. Use strip grid cards with shapes, one for each child. Place a grab bag with the shapes in the middle of the group. Student selects a shape from the bag. If the block is appropriate for their strip grid pattern they verbalise the reason for it being appropriate and place it in the next space. If not, they replace it in the bag. Next student has a go. The game is completed when the first strip card is complete.







Connections: Encourage students to describe attributes of their pattern. *How many repeats?* Ask: *What might come next? Where might you see a pattern like this? Where do we see/use patterns in other maths?*

Reflection

Validation: Roll a dice to choose an element to build a pattern. Record the patterns created. Compare the patterns. *Where do they repeat? Can you continue this pattern? How many repeats have you made?*

Build a snake pattern with toilet rolls threaded on rope. Decorate into a two-part pattern.

Can students continue a simple two-part pattern when given at least three repeats?

Can students generate a simple two-part pattern when given two types of materials?

Can students identify the repeated element in a pattern?

Can students create a simple two-part pattern when given no specific attributes to that pattern, e.g. “Make a pattern with these things?”

Have students construct a non-pattern. Ask: *Why is this not a pattern?*

Reversing: Give students a repeat and ask them to construct the pattern.

Extension: Take 24 counters. Use 12 of them to make a pattern and the other 12 to make a non-pattern. Ask students to tell you why one is a pattern and the other is not.

Unit 2: Position in Repeating Patterns

One of the major outcomes of upper primary and junior secondary patterning involves students finding the growing patterns so that they can say how many elements are at any position of the pattern. Since any position is a variable, it can be designated as n and the number at that position can be given as an expression (sentence with variable, numbers and operations). For example, the growing pattern on right has an O object in each term and grows by one X object for each term, giving 3 X's for position 3 and 10 X's for position 10, so one O at every position and as many X's as the position number, altogether giving $n + 1$ objects in the n^{th} position. Thus, growing patterns not only teach how to find a pattern from particular examples, they also introduce variable and expression.

			X
		X	X
	X	X	X
0X	0X	0X	0X
1	2	3	4

Position can also be part of the repeating pattern repertoire of activities. In the long run, it also introduces variable and, in a restricted manner, expression. In this unit, we look at the very early positioning activity in repeating pattern activities. The idea is to put out the repeat as a starting point, number each object from 1 in the pattern (or two patterns as we recommend) and see if students can work out what objects will be in what positions.

Background information

1. If we are relating object to position in a repeating pattern, we have to coordinate two things as we move along the patterns. This is not hard if we can put out the pattern as we go, for example, finding position 17:

X	O	O	X	O	O	X	O	O	X	O	O	X	O	O	X	O	…
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**17**	

2. However, if we have to do this in our mind, determining what object is in a position requires students to:

   (a) **coordinate two things in their minds**, the pattern and the position number; or, more difficult, **synchronise these two things** as they move, in their mind, along the pattern of objects and along the numbers for the position of the objects; and

   (b) identify the whole repeat and recognise its components (e.g. one X and two O's).

   The latter is easier to do if the students can put out the objects as they go, the number for the position past the last object placed is within the students' subitisation range (normally less than or equal to 5), or the students have familiarised themselves with the pattern by placing it out themselves.

3. Sometimes, the students use a wrapping technique as below – this enables larger numbers to be set, e.g. find the 27th position. The students often use multiples of six not three and see 27 as three more than counting the 6 objects four times, that is, counting to 24 and counting halfway along the six objects (making the 27th position an O as below).

X	O	O	X	O	O
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	**27**			

   The students just continually count along the six objects until they reach the position – then whatever they are at will be the object. (*Note:* Of course, students could wrap around the pattern of three components to the same effect – just double the wrap-arounds.)

4. However, there is an even more efficient way. As students get older and gain skip counting proficiency or improved understanding of multiplication, they can use the length of the repeat to find the position of the object by looking at multiples (in the case above, multiples of three). This means that even larger numbers can be set and generalities can be found as to object and position. The students can determine that, for example, the 27th term is O because they see that X O O is a repeat of length three and 27 as a multiple of three must be the last object in the pattern of three (the O). It relies on students making jumps of three. As a beginning to this, students start to manipulate the objects emphasising features in the repeat – they start to think of the repeat and the pattern in terms of its last object.

5. Because skip counting by five and the five times tables are more familiar to students, five-object patterns are easier for skip counting than three- or four-object patterns; for example, students find that a pattern like O O X X X ... is easier than a pattern like O X X X ... . Finally, students seem to find patterns like O O X ... easier with respect to finding objects in positions than patterns like X O O ... because the students can tag the third or repeat ending object.

6. In summary, determining what object is in what position is difficult and should also be part of patterning when growing patterns are being developed. It requires students to: (a) **coordinate** (synchronise) counting and pattern, (b) identify the **whole** repeat and the number of objects in it and relate this to the position of the object to be found, and (c) **tag** the last term and use this for skip counting towards the required term.

2.1 Discerning repeats and relating to repeating pattern

1. Continue on from Unit 1 by creating patterns and repeats and relating pattern → repeat and repeat → pattern. However, this time ask the students to show you where the repeats were in the pattern and why they know that the pattern is based on that repeat. For example:

   REPEAT: X O X X PATTERN: X O X X <u>X O X X</u> X O X X X O X X X O X X …

2. You could ask students to do the following:
   (a) Show how they can see the repeat, X O X X, in the pattern (underlined) – perhaps use a highlighter pen to show the repeats.
   (b) Show how certain aspects of the pattern, such as only ever having one O in sequence (or never two or more O's together), are because there is only one O in the repeat.
   (c) Show how the three X's in the pattern are because the first X joins with the last two X's.

3. Try to get discussion going in both directions – from the pattern to the repeat and the repeat to the pattern.

4. Try more complex repeats in number and attributes (such as O P X X Y) and repeats that join together in complex ways because they have the same start and finish (such as O X O X O). Look at simple characteristics of repeats such as the length of repeat and organisation of things within the repeat and the effect they have on the pattern.

2.2 Position to object in repeating patterns

1. Provide two repeats of a simple repeating pattern (e.g. X O O X O O ...) and number each object in the two repeats as below.

X	O	O	X	O	O	__	__	__	__
1	2	3	4	5	6				10

   The teacher then asks the students to identify the object (X or O) that is in a particular position, e.g. position 10 as in the example above. Students can simply guess and be lucky, so ask the students to show why they know what object will be in the position.

2. To set up the problem, the sequence involved is as follows:

   (a) Kinaesthetic – state aloud the pattern X O O X O O in the repeats as you put out repeats, then put out again counting to six.

   (b) Hand/Kinaesthetic – Get students to move finger along the repeat as they say the pattern and then repeat this finger movement as they count to six. (*Note:* Could walk along the repeats if you have large copies.)

   (c) Fingers – Get students to chant the pattern as they move fingers past the six objects and run-on count as they move fingers past the six.

   (d) Discuss what would be at various positions past 6 – position 7, 8, 10, 15 – chant with students to find it.

3. To begin to work on the problem, the sequence involved is as follows:

   (a) Initially allow the students to put out extra objects until the position is reached before asking them to work it out in their head.

   (b) Initially allow students to do one of the set up activities, or at least act out or copy the pattern, when they are asked to determine the object in positions before asking them to find a position in a pattern that the teacher has put out.

   (c) Initially give the students positions to find that are five or fewer ahead of the last item placed before going to positions further out (e.g. finding the 10th position is easier than the 13th position).

   (d) Let groups of students work together to find the object at a position and discuss their methods – see if you can get them to find a mental strategy better than writing down the pattern beside numbers up to the set position.

2.3 Encouraging more powerful strategies

1. Let students explore how to find objects that are at positions 15–30. Let them use synchronisation by saying the object and term together. For example, in repeating pattern X O O X O O, they could continue from 6, saying "7 X, 8 O, 9 O" in order to work towards "23 ?". Where possible, these activities should move through kinaesthetic patterns (e.g. jump 7, kick 8, and so on), sound patterns, object patterns to picture patterns.

2. When the pattern is made out of objects or pictures, ask students if they can find a way to use the one or two repeats shown as a starting point for determining what object is in what position. See if by questioning and discussion you can get some students to use the wrap-around strategy.

3. If students are still using laborious methods to find what object is in what position (e.g. writing out all the positions and objects), try to encourage them to use better strategies by questioning. For example, ask them to look at the form of the repeat: *How many objects? How are the different components arranged? Can any of this be used?* Try to get them to see that the length of the repeat is important – it sets up a cycle in the pattern which is that many units long. For example, repeat X O O Y has four objects; this means that every four positions in the pattern you get a Y, just after this Y you get an X, and the two O's are just before Y and just after the X. If looking at the 27th position, this is just before 28, the multiple of 4, which is a Y. Thus, the 27th position is an O. This reasoning would just req**u**ire the students to mark every 4th position – a lot less than writing out all the possibilities.

4. Even if students' skip counting and multiplication tables are poor, it is possible to get students to see what they could do if they had more knowledge. They could use a calculator for the skip counting if they could see that, for instance in the XOO pattern, they needed to find where the position was in the multiple of threes.

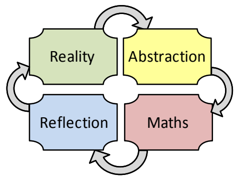
5. Provide the patterns for which the students have to find positions in sequence from easy to hard. Note that five objects in the repeats is easier for students than three or four objects as students are better at multiples of five. Also note that a repeat with a single last object at the end (e.g. X X O O Y) is easier than having two or more objects the same at the end of the repeat (e.g. X Y O O). This is because the single object can be tagged to the multiples of the repeat's length.

2.4 Object to position in repeating patterns

1. Reverse the activities in 2.2 and 2.3. Change from position→object to object→position. For the XOO pattern, for instance, no longer ask *What object is at position 53?*; ask *What positions are X's in?* Explore this with the pattern created or drawn with objects or pictures respectively. *Circle the X's. Is there a pattern in where they are?*
2. Put out a large copy of the pattern with position numbers beside each component of the repeat, for example, made with blue and yellow discs (blue, yellow, yellow, blue, yellow, yellow, and so on). Walk this, saying X O O X O O X (or whatever attributes used in the pattern) and so on, placing arrows beside each of the positions. *What do you notice? Can we use skip counting?*
3. If students are starting to see something, switch to the X X X X O repeat and pattern. The students, with help, can usually see that the O is at 5, 10, 15, and so on. So the 127th position is easy – it is 125, where the O is, plus two more which takes us to the second X in the XXXXO repeat.
4. Try to see, in easier patterns, whether students can describe where all of one object is, even roughly. (*Note:* The basis of this is the relation between components of the repeat and the pattern. This is why this is the focus of section 2.2 and a lot of Unit 1.)

Note: Determining what object is in what position is difficult. It requires students to: (a) **coordinate** (synchronise) counting and pattern, (b) identify the **whole** repeat and the number of objects in it and relate this to the position of the objects to be found, and (c) **tag** the last term and use this for skip counting towards the required term.

2.5 RAMR lesson for position in repeating patterns



Learning goal: Discern repeats; identify position to object and reading object to position; recognise that patterns enable us to predict and plan.

Big ideas: Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity.

Resources: General classroom items, manipulatives, toys, picture cards, whiteboard, pens, paper, things from nature that range in size, colour, texture, purpose, mass, sound, and so on.

Reality

Local knowledge: Items and events that are familiar to students, e.g. sun setting every day; stories, songs and verses that follow patterns; day night day night…; things with segments that continuously occur. The segment can vary in size and level of complexity, but begin with the simplest that includes just two items.

Prior experience: Playing, sorting, collections and materials, threading, building, blocks.

Kinaesthetic: Have students arrange themselves in a pattern using their characteristics, e.g. short, tall, short, tall, … or girl, boy, girl, boy, … or jumper on, jumper off, jumper on, jumper off, … or standing up, sitting down, standing up, sitting down. Ask a student to read the pattern. Ask: *What comes next?* Have students create a more complex pattern (say, three parts). Ask students to read pattern; *What comes next? What else can you see?* Provide divider strips to place between students where the pattern repeats. Ask: *How many repeats?* Introduce concept of position. Have students number and label the positions. Ask: *Who is at position 3?* etc.

Abstraction

Body: Have students work in groups to construct repeating body patterns, use dividers to identify each repeat, and cards to label the positions. Ask: *What object is at position 3?*

Hand: Have students create patterns with materials and cards. Feely Box: Place a large number of two elements into a box (hands-on materials). The first student pulls out one to start the pattern. The second student pulls out a different element to develop the pattern. Other students take turns to pull out the appropriate element to continue the pattern. You can have students record the pattern as you go (on little whiteboards).

Play copy your partner's pattern. Have students place markers between repeats, label repeats with number cards. Ask: *What object is at position 4? If you were to continue this pattern what object would be at position 7?*

Mind: Paint imaginary pattern pictures. Visualise pattern. Tell/describe for partner to draw with chalk on cement. Have students mark repeats with dividers and number the object positions. Ask: *What object is at position 5?*

Creativity: Students make up their own patterns with painting/printing/stamping/swirling materials of choice; place dividers to mark repeats; and label the positions with number cards.

Mathematics

Language/symbols: Same, different, element, part, repeat, pattern, non-pattern, position, object, number, label, divider, marker, and words to describe patterns. Build a word list and illustrate it.

Practice: Using beads have students create a pattern, read it to their partner then have their partner copy and continue their pattern. *What happens?* Have students discuss this. Ask students to record their pattern on a whiteboard. *Where does your pattern repeat?* Ask student to draw a line between each repeat. Have students label each object position with a number card. **Repeat** with **more complex** patterns. Ask: *What object is at position 6? If you were to continue this pattern what object would be at position 10?* Have students create their own pattern with nine pattern repeats. Ask them to remove an object from within the pattern. Ask: *At what position did you remove an object? Who can tell what object is missing? How do you know which object it is?*

Have students create and record their own pattern in their work book, mark the repeats and label the object positions.

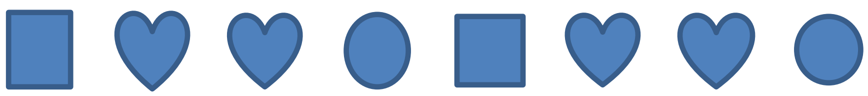
Connections: Encourage students to describe attributes of their pattern. *How many repeats?* Ask: *What might come next? Where might you see a pattern like this? Where do we see/use patterns in other maths?*

Reflection

Validation: Play create a pattern. Use a bag with pattern-making items. Students take turns to pull out three items to create a pattern repeat. Have students continue the pattern to three repeats and record the patterns created. Compare the patterns. Have the students label the object positions Ask: *What is at position 5? Can you continue this pattern? How? What object would be at position 10?*

Reversing: Give students a repeat and label the positions. Ask them to construct the pattern. Ask object position questions. Give students a picture of a pattern with some objects missing. Ask them to identify what objects these might be.

Generalise: Have students make a pattern they think is like this and say why the patterns are alike:



Unit 3: Repeating to Linear Growing Patterns

Repeating patterns are commonly undertaken before growing patterns. Both types of patterns build students' ability to generalise (to see the pattern), but the growing patterns build understanding of variable when developing position rules. There is a tendency to simply stop the repeating patterns and move on to the growing patterns. This is because the repeating patterns, when dealing with objects and numbers, are just a line of materials and numbers, while the growing patterns are divided into positions which hold a collection of materials or a total number. For example, a repeating pattern is X O O X O O X O O and so on – a line of things all separated, while the growing pattern has collections of things as on right. They are syntactically so different. However, as we will now see, growing patterns can come from repeating patterns and this provides a sequential relationship between the two types of patterns.

XOO	X XOO	X X XOO	**or**	O XO	O O XO	O O XO
0	1	2		0	1	2

This unit looks at how repeating patterns can grow into linear growing patterns and then explores growing patterns, looking at copying, continuing, creating and completing these patterns. The unit also explores growing patterns in terms of constant and growing parts and sequential and position pattern rules.

Background information

The following sequence represents an effective way to move from repeating to growing patterns.

1. Set up a repeating pattern, say X O O X O O X O O ... . Ask the students to identify the repeating part and then to break the pattern into repeats and to **separate** the repeats as shown on right below. Note that when separating the repeats it can be useful to discuss and trial other ways of representing the repeat, as also shown on right.

XOO	XOO	XOO	**or**	O XO	O XO	O XO

2. Have the students construct a set of **number cards** and place these under the repeats as on right. Repeating patterns have the same term structure as growing patterns but they do not grow. Here, having the zero makes sense as it is the original and the 1 is the first repeat.

XOO	XOO	XOO .... and so on
0	1	2

3. Ask students to pick one of the objects and **grow** it, as on right. Provide a variety of activities – grow one object, grow both objects, grow both at different rates, change the way the objects are presented, and so on. This presents repeating patterns as a precursor to growing patterns and links the two together.

XOO	X XOO	X X XOO	**or**	O XO	O O XO	O O XO
0	1	2		0	1	2

4. Similar to how Unit 1 explored repeating patterns, move on to students **copying and continuing** growing patterns. Set up a growing pattern up to the 3rd term, as on the right. [This example has X increasing by two and O increasing by one in each new term.] Have the students make their own copy of this pattern and continue this pattern for the next few terms; for example, this could involve making the 3rd and 4th terms as in the diagram on left. It is useful for students to have number cards and place these under the terms.

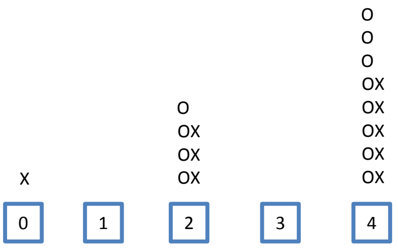
O XO	XO XO XO	X XO XO XO XO
0	1	2

O XO	XO XO XO	X XO XO XO XO	X X XO XO XO XO XO	X X X XO XO XO XO XO XO
0	1	2	3	4

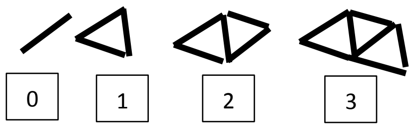
5. Ask the students to make some further terms (or to say what is involved in these terms) such as the 7th, 10th or 20th term. This requires some understanding of the **pattern rule** but can be completed by considering what happens term by term.

6. Ask the students to **create** their own growing pattern and to make/draw the first five terms and explain what is involved in the patterns and how the terms are growing.

7. Have students **complete** a pattern – this is where there are gaps in the example given, as on the right. It should be noted that it is more difficult to complete than continue a pattern. It is also harder if the terms given are not regular, e.g. when you are given the 1st, 2nd and 5th terms. [The example on the right has X increasing by one and O increasing by two in each new term.]



8. The final activity is to provide students with growing patterns as in the example on the right. This pattern is making triangles out of bundling sticks. Its purpose is to determine the number of bundling sticks to make 1, 2, 3 and so on triangles. The idea is to find the pattern that relates number of triangles to sticks – to answer questions like: *How many sticks to make 15 triangles?* and *How many triangles can 37 sticks make?*



   There are two pattern rules. The first and simplest is the **sequential rule** – how much does the pattern change as we move along positions? In the triangle example, this is +2 as we add two sticks each time. The second is the **position rule** – how many sticks for each position? To see this, we look for **what changes and what remains constant**. There are two ways to look at the pattern – **visually** and **numerically** using a table.

   (a) *Visually* – the pattern has one starting stick and increases by two each time, so at position 1 it is 1 stick plus 2 more , position 2 it is 1 stick plus 4 more, position 3 it is 1 stick plus 6 more. One can then see that, at position 20, it is 1 stick plus 40 sticks (and so the pattern is 1 stick plus double the position number). Visual activity should precede numerical.

   (b) *Numerically* – the patterns is shown in the table below:

POSITION NUMBER	0	1	2	3	4	5
NUMBER OF STICKS	1	3	5	7	9	11

   Once again we can see that the number of sticks in any given position is 1 + position × 2, that is, $2n + 1$ in upper primary/junior secondary.

9. Early childhood activity should be restricted to **simple growing patterns**. The activity is to provide the students with the beginning of the growing pattern (the first three steps above) and ask the students to continue the pattern to the fifth step. From this point, the lesson could discuss: (a) the constant and growing part, (b) the sequential rule, (c) the 10th and 20th positions, and (d) first ideas of the position rule using examples. Do not focus on right and wrong answers; simply encourage students to go as far as they can. Teach and use visual methods.

3.1 Moving from repeating to linear growing patterns

1. Create a simple repeating pattern, say, X O O X O O X O O … and so on. Divide the pattern into repeats, that is, XOO XOO XOO. Discuss other ways of presenting the repeats when they are in clusters – for example, you could have the following representations:

```
  O    O    O    O                      X    X    X    X
X O  X O  X O  X O   and so on    OR   O O  O O  O O  O O   and so on
```

2. Make small cards with numbers 0, 1, 2, 3, 4 and 5 on them. Place them under the repeats, so we have small collections of objects now numbered 0, 1, 2, and so on.
3. Pick one of the objects in the repeats and grow it one object at a time – this means that the 0 position stays as it is, the 1 position gets one extra object, the 2 position gets 2 extra objects (the same attribute), and so on, like below, where the left-hand side grows X and the right-hand side grows O:

O	O	O	O		X	X	X	X
X O	X X O	X X X O	X X X X O	AND	O O	O O O	O O O O	O O O O O
0	1	2	3		0	1	2	3

4. Give the students other repeating patterns to start from, making them more complex with more objects and attributes in the repeats. Allow/encourage the students to have their own cluster setting out and to grow a part of the repeat. Then, allow different growings, that is, different numbers of each object being grown at each grow, or the number of objects being grown in different/same ways at each grow. This means, for the example above, growing two X's at a time, or growing both X's and O's at the same time and in different amounts (e.g. one X and two O's at the same time).

3.2 Relating growing and repeating patterns

1. Continue with activities similar to activity 4 from 3.1, encouraging students to grow a variety of growing patterns from repeating patterns. However, when this is done, ask students to compare the growing with the repeating pattern from which it was grown. Ask questions: *What are the similarities and differences? What is growing? What are all the growing parts together? What is staying the same? Are there growings that do not start from one component of the repeat? Is this allowed?* Consider an example like the one below:

   REPEATING: X O O X O O X O O X … and so on
   REPEATS: XOO XOO XOO XOO … and so on
   GROWING: XOO XOOY XOOYY XOOYYY … and so on

   Are there other ways to do this?

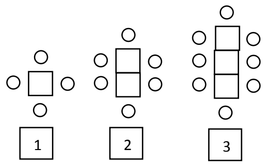
2. Continue with repeating → growing activities, but this time ask the students to show:
   (a) the repeating pattern with which they started;
   (b) the growing pattern with which they finished;
   (c) the growing part of the pattern and its relation to the repeat; and
   (d) how many objects are in that growing part (the number by which the pattern increases each position – call the **sequential rule**).
3. Repeat activity 2 above, but this time identify the constant part as well. This means that students show original repeating pattern, final growing pattern, growing part and number of objects, and constant part and number of objects. You may wish to highlight that a repeating pattern has no growing part and the repeat is the constant part – it is a special growing pattern.
4. In this activity, reverse what we have done before. That is, go from growing to repeating. Ask students to share their growing patterns with each other (or provide examples as the teacher). Encourage students to look at the growing pattern and determine what the original repeating pattern was. Use experience from the activities above where you related repeating to growing. Discuss examples with the whole class – get students to tell how they found the repeats. Ask questions: *Is it always possible to find a repeating pattern on which the growing pattern is based? What part, constant or growing, is the repeating pattern based on? Does this help identify growing and constant parts?*

3.3 Copying, continuing, creating and completing growing patterns

1. Find examples of growing patterns and represent them for the students. Get students to use materials or drawings or both to copy the first three positions. Ask students for different ways to represent the growing patterns. Ask students to state how they see the growing patterns. For example, in the pattern below, some people see two rows with the bottom row one more than the top, some see a double row with an extra on the bottom and some see a double row with one lost on the top:

	O	OO	OOO	OOOO
O	OO	OOO	OOOO	OOOOO
0	1	2	3	4

   Try to relate patterns to real-world situations, for example, chairs around a restaurant table (starts with four around one table, two tables together seats six, three tables together seats eight, and so on – see diagram on right) or as a small tree which started with one leaf, grows two leaves a day, and so on.



   Discuss the growing patterns with the students. Look at a variety of examples. How do the students see each position growing? Will this help the students to continue the growing pattern? How?

2. Provide the students with the growing patterns to positions 3 or 4 (as in the example above). Ask the students to copy them and then continue for a few extra positions. Discuss with them what would be position 4 and 5? What would be position 10? Let them work out all the positions up to 10 as this will help in the long run. However, in class discussions and talks with students and groups of students, try to get them to see some relationship between position number and objects that will help work out position 10.

3. Move on to the creation of linear growing patterns. Ask the students to create their own growing pattern. First discuss what will be the properties – it cannot be a random thing. Ask students for "rules" they have seen in what they have done up to now. Write these on the board – encourage them to get the important one which is that a growing pattern has to grow in the same way and for the same amount each time you change position. Relate this to the new term **sequential rule** that we first used in 3.2 if this is appropriate. When students create their own growing pattern, they should design positions 0 to 5 and show the sequential rule as the same change for each new position.

4. Finally, we have to complete linear growing patterns. To do this, provide the students with positions 0, 1 and 2 and position 5 and the students have to fill in positions 3 and 4. This is difficult and it is an activity that early childhood students should try but may not achieve. Get students who do work it out to share their methods with other students. The critical thing is to get the change from position 0 to position 1; this should be the same as the change from position 1 to 2. Continue this change from 2 to 3 and 3 to 4 and 4 to 5 to see if you match the position 5. If so, you have the basis of the pattern. *Note:* A simpler completion is giving positions 0, 1, 2 and 4 and asking for position 3.

3.4 Identifying position rule using growing and constant parts

Finding the position rule that relates objects to positions would not be expected in Years F to 2, certainly not in a generalised way using variables. However, it could be worth challenging students, for easier linear growing patterns, to visually see relationships through identifying the constant and growing part of a pattern.

1. Provide students with simple growing patterns. For each pattern, ask the students to provide:
   (a) the pattern up to position 5;
   (b) the constant and growing part;
   (c) a visual way to differentiate between and see the constant and growing part separately; and
   (d) the pattern up to position 7.

It is important that students make and draw the positions given and use materials and drawings as part of the discussion for this activity and for all the activities in this unit. Students can also work in groups.

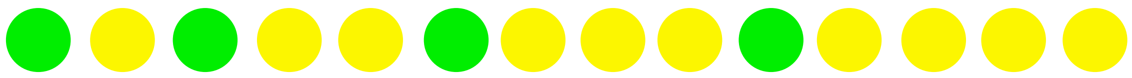
2. Repeat the activity in 1 but, this time, discuss visuals as a way to differentiate and see the constant and the growing parts. Use the results of this discussion to draw the pattern for positions 6 to 10. Look at different visuals and discuss which is most effective.

3. Repeat activity 2 but, this time, ask students to draw, in some way that makes sense to them, the 10th, 20th, and 100th positions. Discuss what different students do. Work out the most effective. Show students some options for the growing patterns being considered. That is, explore **with** the students.

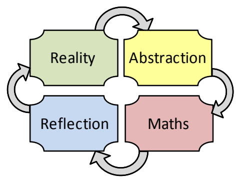
4. Encourage students to state what they see as the pattern that enables the teacher to say the position and them to give how many objects; in other words, the pattern that enables the students to say, for example, how many objects in the 38th position. Use all we have done so far.

   You can also reverse the position rule – that is, give the number of objects and find the position. For example, work towards getting students to be able to answer: *Find the position that has 38 objects*.

Note: As mentioned in section 2.4, determining what object is in what position is difficult and should also be part of patterning when growing patterns are being developed. It requires students to: (a) **coordinate** (synchronise) counting and pattern; (b) identify the **whole** repeat and the number of objects in it and relate this to the position of the objects to be found; and (c) **tag** the last term and use this for counting towards the required term. In the growing pattern example below: (a) the number of parts (terms) is four; (b) the first part has two objects, the second part three objects, and so on; and (c) the growing part is growing by one each time.



3.5 RAMR lesson for repeating to linear growing patterns



Learning goal: Discern and represent repeats in different ways, number repeats, add different "growings", predict, copy, continue, complete, and create growing patterns.

Big ideas: Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity.

Resources: General classroom items, manipulatives, toys, counters, blocks, picture cards, whiteboard, pens, paper, things from nature that range in size, colour, texture, purpose, sound, and so on.

Reality

Local knowledge: Items and events that are familiar to students that demonstrate pattern.

Prior experience: Playing, sorting, collections and materials, threading, building, blocks, etc.

Kinaesthetic: Have half the students create a two-part repeating pattern using themselves (a people pattern). Demonstrate the creation of a growing pattern using the other students. Have the students read/tell the two patterns. Ask: *What is different about the two patterns? What is the same?*

Abstraction

Body: Have students work in groups to construct a growing pattern; use dividers to identify each repeat.

Hand: Have students make a two-part repeating pattern. Have student identify the parts. Create a second pattern the same as the first. Pick one object in the second pattern example and ask them to grow one object at a time. Have the students to compare the two patterns. Ask: *What is similar? What is different? What is growing? What is staying the same? Is there another way to do this?*

Mind: Paint imaginary pattern pictures. Visualise the pattern. Tell/ describe for partner to draw with chalk on cement. Have students mark repeats with dividers and number the repeats.

Creativity: Students make up their own patterns with painting/printing/stamping/swirling materials of choice, place dividers to mark repeats, and label the repeats with number cards.

Mathematics

Language/symbols: Same, different, element, part, repeat, growing, pattern, non-pattern, position, object, number, label, divider, marker, and words to describe patterns. Build a word list and illustrate it.

Practice: Have each student make a two-part repeating pattern. Have student identify the parts. Create a second pattern the same as the first. Pick one object in the second pattern example and ask them to grow one object at a time. Ask the student to compare the two patterns. Have them ask these questions of a partner. What is similar? What is different? What is growing? What is staying the same? Is there another way to do this? Have students create and record their own pattern in their work book. Have them consider further patterns you have already made and ask the same questions of each other.

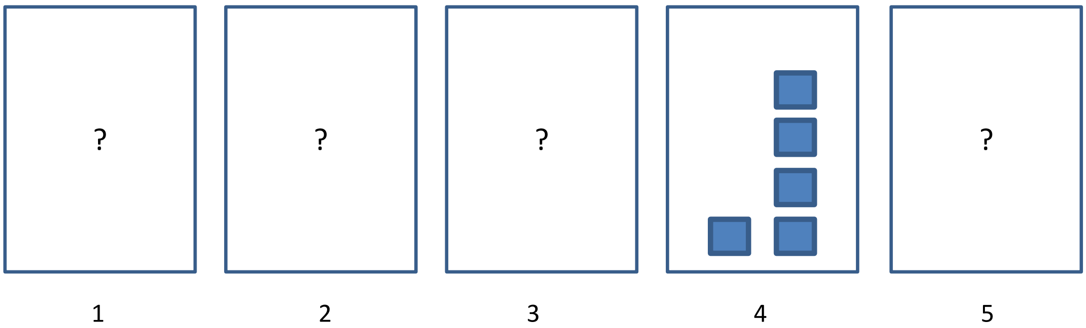
Connections: Encourage students to describe attributes of their pattern. How many repeats? Ask: *What might come next? Where might you see a pattern like this? Where do we see/use patterns in other maths?*

Reflection

Validation: Have students create their own growing pattern. Pass to a partner to describe their pattern, read their pattern and answer: What is similar? What is different? What is growing? What is staying the same? Is there another way to do this?

Extend this activity to copy, continue, complete and create growing patterns. Or provide a simple growing pattern. Ask the student to complete to position 5; what is growing? What is staying the same? Can they do the next position in the pattern?

Generalise: The fourth picture in a pattern consists of five small squares as shown. What could the first, second, third and fifth pictures look like?



Unit 4: Patterns in Number

In general counting, adding and subtracting by the same amounts (skip counting), and multiplying by the same amounts (99 board patterns), numbers follow growing patterns. These patterns assist with understanding number and with basic facts.

We have been looking at patterns starting from repeating patterns. Now, we look at patterns that come from counting and operations. In many ways, this is similar. It requires students to copy, continue, create and complete, and to understand what the pattern is in the long run.

Thus, in this unit, we look at the patterns in counting and place value, seriation and odometer, addition and subtraction, and multiplication and division.

Background information

The work to be covered with respect to patterns in numbers is what YDC normally covers in additive and multiplicative structure, two of the major big ideas in whole number and place value. It is a major focus in the Australian Mathematics Curriculum under algebra.

According to the Australian Mathematics Curriculum, the patterning in relation to number includes:

(a) identifying and describing patterns in number sequences;

(b) identifying missing elements and errors in familiar sequences; and

(c) reasoning and suggesting solutions for problems that involve missing elements in a pattern.

Examples of activities include the ideas below.

1. Follow a rule to generate a number sequence based on an addition or subtraction strategy; e.g. *Follow the rule: start with one, two, then each time add the two previous numbers.*
2. Fill in number sequences involving addition or subtraction by a constant amount; e.g. *Fill in 6, 11, …, 21, 26, …, to make a pattern.*
3. Use patterns in sequences of related additions or subtractions to generate new equations; e.g. *Fill in the missing numbers in 19 – 14 = 5, 29 – 14 = 15, 39 – 14 = 25, ? – 14 = 35, 59 − 14 = ?*
4. Describe a sequence sufficiently for a peer to reproduce it; e.g. for 3, 7, 11, …, it is insufficient to say a rule that works is "add four". It is also necessary to indicate the starting point, e.g. *A rule that works is begin with three and add four each time.*
5. Describe and continue number sequences based on addition or subtraction but involving more than adding or subtracting a constant amount; e.g. say: *For the sequence 100, 99, 97, 94 …, a rule that fits is start with 100 and take one, then two, then three, so the next would be four.*
6. Identify the starting number and the constant multiplier needed to generate a number sequence; e.g. 6, 12, 24, 48 … could have the rule: *Start with six and keep doubling.*
7. Fill in number sequences involving constant multiplication or division; e.g. doubling and halving, or 2, 6, __, 54, 162.

In particular, ideas that are built around investigating and describing number patterns formed by skip counting and patterns with objects are a focus and include:

- using materials to represent number patterns, involving skip counting, e.g. twos, fives, tens;
- identifying even numbers as collections where the objects can be paired, with no left overs;

- representing patterns on a number line or number chart (number board); and
- copying, continuing, completing and creating skip counting patterns that increase and decrease.

The last activity above indicates the activities possible: (a) **continuing patterns** that have started, and (b) **completing (filling in spaces, looking for missing numbers) patterns**. In what follows, it will be assumed that the ideas will be used in the above two types of activities.

4.1 Counting and place-value patterns

1. Look at counting itself. There is a pattern behind counting numbers that relates to nine and zero – for example, …, 47, 48, 49, 50, 51, 52, … for counting forwards and ..., 62, 61, 60, 59, 58, … for counting backwards. This is the odometer pattern which says that you count up to nine then revert to zero and the position on the left-hand side goes up one, and you count back to zero then jump up to nine and the position on the right-hand side decreases by one.
2. Bridging tens and hundreds also relies on the above pattern and can be particularly difficult when doing both together. For example, complete the following patterns: 296, 297, 298, 299, __, __, __ ; 503, 502, 501, __, __, __ ; and 667, 677, 687, ___, ___, ___ .
3. Once we have checked that the students can count in ones (forwards and backwards), we move on to counting in twos, fives and tens. For example, complete 25, 30, 35, __, __, __, 55, 60; 32, 34, 36, 38, 40, 42, __, __, __, __, __ ; 770, 780, 790, ___, ___, ___ ; and 980, 990, ____, ____, ____ .
4. Teens and zeros do not follow the counting pattern in terms of language. Special attention needs to be given to these, for example, 816, 817, 818, ___, ___, ___, ___ ; and 414, 413, 412, ___, ___, ___, ___ .
5. Finally, get students to create their own patterns and share with the rest of the class.

4.2 Seriation and odometer patterns

1. Seriation means being one different (larger and smaller) in terms of a place value. For three-digit numbers, this means knowing +1, −1, +10, −10, +100 and −100. The 99 board assists with this skill. On the board on the right, 46 has been circled and we can see that +1/−1 is on the right/left, of the 46, while +10/−10 is below/above the 46. The activities are for students to:
   (a) identify the seriation numbers for a given number, e.g. for 582, what is +10, −100, and so on;
   (b) identify a number from the seriation numbers, e.g. 287 is the number +10, what is the number?; and
   (c) identify the number that −10 gives 397.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

2. You can make things more difficult if you do not give all the information. For example, *adding 100 makes the number in the 700s, subtracting 10 makes the number have teens, and adding 1 means that the number ends in 5, what is the number?* Play number heads – a number stuck on a student's forehead and the student can ask seriation questions.
3. Construct patterns of the type 356, 366, 376, and so on for different place values (also have decreasing patterns). After constructing, then get students to decipher the patterns – they have to find the changing place value. Make sure students know what happens as the place-value position goes up to 9 or down to 0. It is easier when changing by 100s (258, 358, 458, …) or 1s (272, 273, 274, …). Perhaps bring in thousands, e.g. 4383, 4283, 4183, ____, ____, ____, ____, … ?

4. You can decide that seriation patterns can jump more than one place-value position – this leads to, for example, patterns such as 385, 387, 389, ___, ___, ___ ; 161, 361, 561, ___, ___ ; and 658, 638, 618, ___, ___, ___. You can also do two things at once, e.g. 457, 438, 419, ___, ___, ___. These are particularly difficult.

5. Finally, similar to 4.1, get students to create their own patterns and share with the rest of the class.

4.3 Additive patterns

1. Organise students to copy, continue, complete and construct patterns based on addition and subtraction. For example, 3, 7, 11, 15, ___, ___; or 345, 341, 337, 333, ___, ___. The students should be able to describe the pattern in terms of (a) starting point, and (b) what is being added or subtracted each time. Note again that completing the pattern is important, e.g. 745, 775, 805, ___, ___, 895, …

2. It is possible to use the above to develop patterns that are not linear (because they are not adding or subtracting a constant amount), but involve more complex ideas, such as 3, 3, 6, 9, 15, 24, 39, ___, ___, ___? This is starting at 3 and 3 and adding the previous two numbers. Also, the following pattern, 2, 2, 4, 8, 16, 32, ___, ___, which involves adding all numbers before the position being considered.

3. Patterns also emerge from how things are added and subtracted. For example, 8 + 3 = 11, 18 + 3 = 21, 28 + 3 = 31, 38 + 3 = ___, 48 + 3 = ___, and so on. Can we extend the pattern to 280 + 30 and 580 + 30? What about 48 + 13 = 61, 58 + 13 = ___, and so on? Do the same for 13 – 6 = 7; extend to 43 − 6, 230 − 60, and so on. There are also multiple-of-ten facts. If 3 + 4 = 7, then, as well as 23 + 4 = 27, 30 + 40 = 70, 300 + 400 = 700, and so on. This is more difficult when carrying: 5 + 8 = 13, 50 + 80 = 130, ___, ___ and so on. We also have examples for subtraction, such as 1200 – 400 = 800 because 12 – 4 = 8. These patterns are more complex but may be a suitable challenge for more advanced students.

4. Simple patterns such as 3, 7, 11, 15, … are easy to copy, continue, complete and create, but they can be considered as linear growing patterns. In this example, the constant part is 3 and growing part 4 (the sequential rule is +4). Thus, we can use them to start the process of finding the position rule, see below:

PATTERN	3	7	11	15	19
POSITION	0	1	2	3	4

We can see that position 1 is 3 plus one 4, position 2 is 3 plus two 4s, and position 3 is 3 plus three 4s. Thus, position 10 is 3 plus 10 4s and position 100 is 3 plus 100 4s. Overall, the number at each position is:

position × 4 + 3

5. Similar to 4.1 and 4.2, get students to create their own patterns and share with the rest of the class.

4.4 Multiplicative patterns

1. Similar to the additive patterns in 4.3, we can have patterns based on multiplication, for example 2, 6, 18, 54, ___, ___, and so on. This is much more difficult because the multiplications rapidly get to the point where they cannot be completed in the mind. YDC recommends using calculators. The students need to know that these patterns have a starting point and a constant multiplication, similar to addition patterns having a starting point and a constant addition. However, constant multiplication patterns are not linear and not usually part of algebra until late junior secondary school.

2. Patterns can also emerge from how things are multiplied or divided. For example, 2 × 3 = 6, so 20 × 3 = 60, 2 × 30 = 60, 20 × 30 = 600, and so on. Seeing the patterns in multiplication and division can be done by getting, for example, 7 × 4 = ___, 70 × 4 = ___, 70 × 40 = ___, and so on completed by calculator, then asking students to complete the following without a calculator: 8 × 6 = 48, 80 × 6 = ___, 8 × 60 = ___, 80 × 60 = ___.They have to see the pattern relating to zeros. *Note*: 3000 ÷ 60 = 50 not 500 because here an extra zero has been found as a result of 6 × 5 = 30. The zero law for multiplication is not straightforward.

3. One activity very useful in Years 1 and 2 is to look at multiplication tables. Give students calculators and a 99 board. Put 0 + 3 in calculator. Each time students press =, they put a counter on the number in the board that is shown on the calculator. The result is the three times tables with a 99 board visual – we are adding, but adding 3 at a time is skip counting threes which is multiplication by three. The visual pattern for ×3 is diagonals to left – this means 3, 6, 9 in first row, 12, 15, 18 (one back in terms of ones) in second row, and 21, 24, 27 (one more back in terms of ones) in third row. The visual of the first three rows can assist the 3 times tables.

   Continue this development of 99 board visuals for other numbers – this assists with multiplication tables.

4. Finally, the 99 board activity above leads to interesting patterns with regard to what numbers are divisible by a certain number. For example, the fives pattern is in two columns – one at 5 and one for tens (ending in 0). It is possible for Years 1and 2 students to determine the rule for multiples of 5 or what numbers divide by 5. They can see that numbers ending with 5 or 0 are divisible by 5.

5. As in 4.1, 4.2 and 4.3, give students opportunities to construct their own number patterns. Give them a calculator to do the computation.

Module Review

This section reviews the units in this module. It looks across the units and identifies outcomes that go beyond the particularities of the units. The first of these is **models and representations**, that is, ways of teaching ideas common within the units and across most of mathematics. The second is **competencies**, that is, abilities that are important across the units and into the future. The third is **later patterning**, information on the mathematics that grows out of this module and provides the reason for its importance.

Models and representations

The major models for this unit are some way to represent patterns and position numbers. YDC has found that magnetic counters on magnetic whiteboards are a good way to represent patterns. Of course, virtual pictures that can be manipulated are just as good. And a lot can be done with drawings on whiteboards.

Coloured Unifix cubes are also good for students to use for copying and continuing patterns.

The other models that are useful are tables if you wish to go that far, and 99 boards.

Competencies and critical teaching points

Students demonstrate their understanding of the principles of patterning by:

- sorting and classifying items;
- developing the concept of patterns;
- recognising and using patterns;
- identifying items that are repeated;
- constructing patterns as often and where ever possible;
- extending linear patterns in both directions; circular patterns outwards; 3D patterns upwards;
- reading the pattern in as many ways as possible;
- recognising non-patterns;
- predicting the next item in the pattern and where it will be placed;
- describing patterns as relationships; and
- validating predictions.

Act of generalisation

The act of generalisation, where particular numbers and images are turned into a pattern of generalisation that applies to many numbers and images, is one of the most powerful things in mathematics.

It has four steps:

(a) giving the pattern for numbers near where they were presented;

(b) giving the pattern for any number (quasi-generalisation);

(c) giving the pattern in language; and

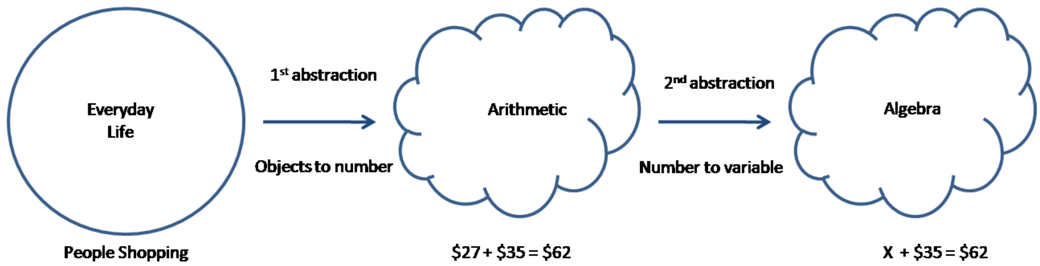
(d) representing the pattern with an expression that includes a variable.

Later patterning

Generalising and patterning

In the upper primary and early secondary years, algebra is about **understanding the world algebraically**, manipulating equations and expressions, solving equations, and expressing and representing functions. (An *expression* is a number sentence without an equals sign, and an *equation* is two expressions with an equals sign between them.) However, the move from the real world to algebra is a two-step process that goes through arithmetic (see below). This means the following:

1. The act of generalising is at the core of algebra and proficiency must be built in both the act of generalising (how to generalise) and the products of generalisation (the mathematical ideas/principles that result from generalising).
2. The symbols of algebra, notably the letters, are far removed from everyday life and their meaning must be built with care through:
   (a) continuous connections being made between symbols and real-world stories; and
   (b) using sequences of materials and activities that become progressively more abstract.



Repeating to growing patterns

The crucial skill is to be able to go from pattern to repeating part **and** repeating part to pattern. The normal sequence of activities is as follows:

(a) copying, continuing and completing repeating patterns and identifying repeating part;

(b) constructing repeating patterns (without direction and when given a repeating part);

(c) finding what object is at a position or finding positions for objects (e.g. what object is the 13th term?; what terms are square red counters?);

(d) breaking pattern into repeats and connecting to growing patterns, for example:

O x O x O x O x … → Ox Ox Ox Ox … → Ox Oxx Oxxx Oxxxx

(e) representing repeats on tables and generalising tabled numbers to variables; and

(f) using tables of repeats to introduce fractions, equivalent fractions, ratio and equivalent ratio (proportion).

The common materials to be used consist of objects of different colour, size, shape, and so on, with any one or more attributes determining the basis of the pattern, plus small numbered cards (1 to 5), tables, and calculators. It is useful to have magnetic copies of these objects/cards that can be placed on whiteboards.

The major ideas to be developed from repeating patterns are:

(a) generalisation;

(b) representing generalisations with variables (introduction to algebra);

(c) fractions and ratio; and

(d) equivalent fraction and ratio.

Linear growing patterns, pattern rules and graphing

Growing patterns are series of terms where there is a fixed part and a growing part as on right. If the growing amount is always the same, then it is linear. In the pattern on the right, 0 is fixed and X is growing by one each time.

			X
		X	X
	X	X	X
0X	0X	0X	0X

When the growing part does not grow (grows by zero), you have a repeating pattern as on right.

0X 0X 0X 0X

It is possible for the fixed part to not exist (to be zero) as on right.

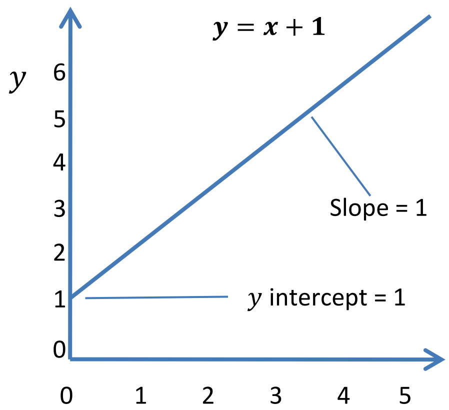
			x
		x	x
	x	x	x
x	x	x	x

The focus on growing patterns is to identify what is called the *pattern rule* which describes the growth. For patterns like that on the right, there are two types of rules:

(a) *Sequential*: the nth term is the previous term + 1.

(b) *Position*: the nth term is 1 + n, since:

1st term is	1 O and 1 X	(1 + 1)
2nd term is	1 O and 2 X's	(1 + 2)
3rd term is	1 O and 3 X's	(1 + 3)
and so on		



Position rules enable linear growing patterns to be used to introduce the notion of variable. They also can be used to plot graphs as straight lines, as shown on right.

Reversing

For all activities, it is crucial to go from pattern to pattern rule **and** pattern rule to pattern. The normal sequence of activities is as follows:

(a) copying, continuing and completing growing patterns;

(b) constructing growing patterns (without direction and when given a pattern rule);

(c) finding what objects are at a position and what position has certain objects (e.g. what term has the 20th red circle?);

(d) identifying growing and fixed parts of visual patterns;

(e) identifying pattern rules (sequential and position) from visual patterns with and without use of number tables;

(f) identifying growing and fixed parts and pattern rule from number tables;

(g) identifying different versions of pattern rules (leads to equivalence of expressions – number sentences with no equals), and justifying why it works for all terms;

(h) using pattern rules to introduce variable and algebraic expression; and

(i) representing patterns with graphs and relating growing and fixed parts to slope and y intercept of graph respectively.

Test Item Types

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

Instructions

Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different from pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "don't know" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going**. Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

Information on the Module A1: Patterning test item types

This section includes:

1. Pre-test instructions;
2. Diagnostic Mapping Points;
3. Observation Checklist and Teacher Recording Instrument; and
4. Test item types.

Pre-test instructions

When preparing for assessment ensure the following:

- Students have a strong sense of identity; feel safe, secure and supported; develop their emerging autonomy, interdependence, resilience and sense of agency; and develop knowledgeable and confident identities.
- Students are confident and involved learners, and develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity.

When conducting assessment, take the following into consideration:

- Student interview for diagnostic assessment in the early learning stages is of paramount importance.
- Use materials and graphics familiar to students’ context in and out of school.
- Use manipulatives rather than pictures wherever possible.
- Acknowledge the role of using stories in this early number learning, enabling students to tell stories and act out understandings to illustrate what they know.
- Playdough and sand trays are useful for early interview assessment situations.

Ways to prepare students for assessment processes include the following:

- In **individual teaching times**, challenge students’ thinking. “Challenging my thinking helps me to learn by encouraging me to ask questions about what I do and learn. I learn and am encouraged to take risks, try new things and explore my ideas.”
- In **group time**, model and scaffold question-and-answer skills by using sentence stems to clarify understandings and think about actions. Encourage students to think of answers to questions where there is no one correct answer, and to understand that there can be more than one correct answer (e.g. *How can we sort the objects?*).
- In **active learning centres**, use activities such as imaginative play, sand play, playdough, painting, ICTs and construction to think and talk about different ways of using materials, technologies or toys. Ask questions and take risks with new ideas.

Other considerations:

- Preferred/most productive assessment techniques for early understandings are observations, interviews, checklists, diary entries, and folios of student work.
- Diagnostic assessment items can be used as both pre-test and post-test instruments.

Remember:

Testing the knowledge can imply memory of stuff; asking the students **what they can do with knowledge** requires construction and demonstration of their understanding at this early understandings level.

A1 Patterning: Diagnostic Mapping Points

1. Early repeating patterns

Copy and continue

- Have students reproduce a string of beads that shows a repeating pattern such as green, green, yellow, green, green, yellow, … Draw out pattern by asking students to "read"/chant aloud the colour sequence. Ask students to make their string longer than the original. Ask: *How do you know what comes next?*

Relate pattern to repeats, also non-repeating patterns

- Repeat the above activity, using a variety of materials/objects where the only repetition is in the colour. Draw out that this pattern is in the colour, but the actual objects vary.
- Repeat the above activity but create a non-pattern. Ask: *What have we made here?*

Complete patterns

- Repeat the above activity but have a set number of beads. Ask the students to complete the pattern using all the beads.

Create repeating pattern

- Provide a range of materials and ask students to create a repeating pattern and describe it to you.
- Can they extend linear patterns in both directions; circular patterns outwards; 3D patterns upwards?

2. Position in repeating patterns

Discerning repeats

- Can the students generate a two-part pattern when given two types of materials? Ask: *What is the repeated element/part?*

Position to object

- Ask the students to label the parts of their pattern with numbers. Ask: *What is at position 2?* etc.

Object to position

- Lay out number cards. Ask students to place pattern objects at varying positions. Ask: *What would be at position 8?*

3. Repeating to linear growing patterns

Moving from repeating to linear growing patterns

- Ask students to make a two-part repeating pattern. Have students identify the parts. Create a second pattern same as the first. Pick one object in the second pattern example and ask them to grow one object at a time. Ask the students to compare the two patterns. *What is similar? What is different? What is growing? What is staying the same? Is there another way to do this?*

Copying, continuing, creating and completing growing pattern

- Use above activity to ask students to copy, continue, complete and create growing patterns.

Identifying position growing and repeating parts

- Provide a simple growing pattern. Ask the students to complete to position 5. Ask: *What is growing? What is staying the same? Can you do the next position in the pattern?*

4. Patterns in number

Counting and place value

- Count in rhythm with students as they go up, over and through the decade numbers. Use variation in pitch and volume of voice to emphasise one to nine repeating within each decade. Stop occasionally and ask: *How do you know what comes next? What is the pattern? How does it help?*

Seriation and odometer

- Repeat the above activity but include the decades following this pattern. Ask the questions as above.

Additive

- Represent simple adding and subtracting patterns visually, choosing materials to help make the pattern obvious, e.g. represent 21, 31, 41, 51, … by grouping materials to show the tens in some way. Ask: *What can you see? What is the pattern? How does it help?*

Multiplicative

- Have students represent the decades pattern 10, 20, 30, etc. with materials. Ask: *What do you see? Can you read the pattern? What groups do you have? What pattern do you see with these groups?*

A1 Patterning: Observation Checklist			
Unit	**Concept**	**Knows**	**Can construct/do/tell**
1. Early repeating patterns	Copy	To repeat what you do	With a range of materials and settings
	Continue	You start they continue	With a range of materials/actions
	Relate pattern to repeats	Can they identify the repeating part?	Use the language of repeats
	Non-repeating patterns (non-pattern)	What's missing?	What's different?
	Complete patterns	More complex patterns	Others' patterns
	Create repeating pattern	Given repeating part	In a range of forms
	Does student recognise underlying regularity or ongoing repetition in a situation?	Uses language of pattern, over and over, repeat, again	Can student describe/read/ tell their pattern?
2. Position in repeating patterns	Discerning repeats	Where are the repeats?	Why they know the pattern is based on this repeat
	Position to object	What is at position 3?	Identifying
	Object to position	What position is … in?	Reading
	Does student recognise that patterns enable us to predict and plan?	Knows what to do and to say what is likely to happen; predicts	Predicting
3. Repeating to linear growing patterns	Moving from repeating to linear growing patterns	Represents repeats in different ways; numbers repeats	Add different "growings"
	Copying growing pattern	To repeat what you do	Use range of materials
	Continuing	You start they continue	Use range of materials
	Completing	More complex patterns	Other patterns
	Creating/constructing	Given repeating part	In a range of forms
	Identifying position growing and repeating parts	Can they see visual relationships?	Can they find ways to represent and describe these?
4. Patterns in number	Counting and place value	Pattern that relates to 9 and zero	Forwards and backwards
	Seriation and odometer	Identify seriation numbers: +1 +10 −1 −10	Identify from the seriation number
	Additive	3, 7, 11, 15, __, __	Knows "starting with 3 each number is 4 more than the one before'
	Multiplicative	Twos and fives multiplication tables patterns	With counters or similar
	Represent number patterns with materials in ways to help expose the pattern	Represent a variety of patterns using numbers	Can they use the numbers to help them continue the pattern?

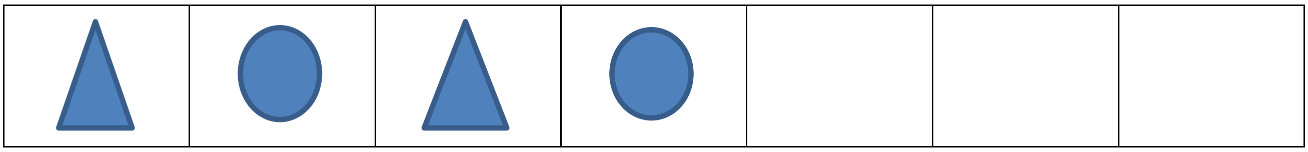
A1 Patterning: Observation Checklist Teacher Recording Instrument				
Unit	**Concept**	**Knows**	**Can construct/do/tell**	**Comments/Observations**
1. Early repeating patterns	Copy	To repeat what you do	With a range of materials and settings	
	Continue	You start they continue	With a range of materials/actions	
	Relate pattern to repeats	Can they identify the repeating part?	Use the language of repeats	*(Vary repeating part: 1–4 elements and repeated elements)*
	Non-repeating patterns (non- pattern)	What's missing?	What's different?	
	Complete patterns	More complex patterns	Others' patterns	
	Create repeating pattern	Given repeating part	In a range of forms	
	Does student recognise underlying regularity or ongoing repetition in a situation?	Uses language of pattern, over and over, repeat, again	Can student describe/read/ tell their pattern?	
2. Position in repeating patterns	Discerning repeats	Where are the repeats?	Why they know the pattern is based on this repeat	
	Position to object	What is at position 3?	Identifying	
	Object to position	What position is … in?	Reading	
	Does student recognise that patterns enable us to predict and plan?	Knows what to do and to say what is likely to happen; predicts	Predicting	

Unit	Concept	Knows	Can construct/do/tell	Comments/Observations
3. Repeating to linear growing patterns	Moving from repeating to linear growing patterns	Represents repeats in different ways; numbers repeats	Add different "growings"	
	Copying growing pattern	To repeat what you do	Use range of materials	
	Continuing	You start they continue		
	Completing	More complex patterns	Other patterns	
	Creating/constructing	Given repeating part	In a range of forms	
	Identifying position growing and repeating parts	Can they see visual relationships?	Can they find ways to represent and describe these?	
4. Patterns in number	Counting and place value	Pattern that relates to 9 and zero	Forwards and backwards	
	Seriation and odometer	Identify seriation numbers: +1 +10 −1 −10	Identify from the seriation number	
	Additive	3, 7, 11, 15, __, __		
	Multiplicative	Twos and fives multiplication tables patterns	With counters or similar	
	Represent number patterns with materials in ways to help expose the pattern	Represent a variety of patterns using numbers	Can they use the numbers to help them continue the pattern?	

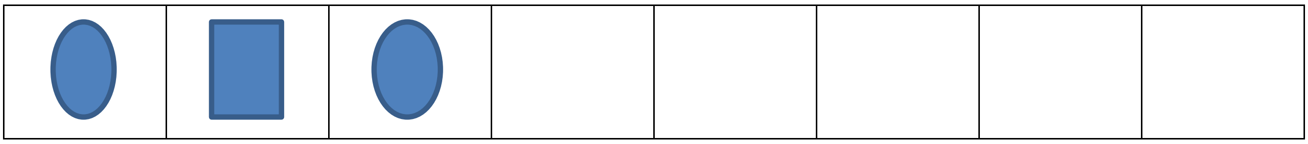
Subtest item types

Subtest 1 items (Unit 1: Early Repeating Patterns)

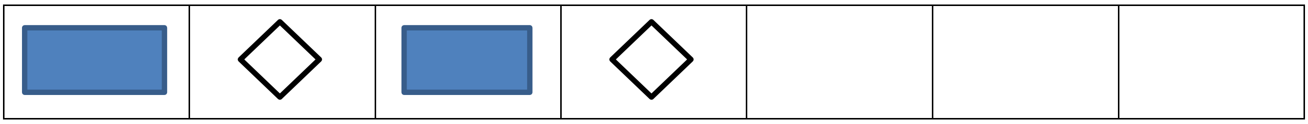
1. Copy this pattern



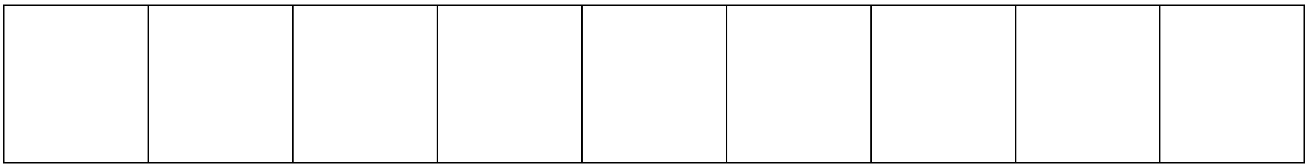
2. Continue this pattern



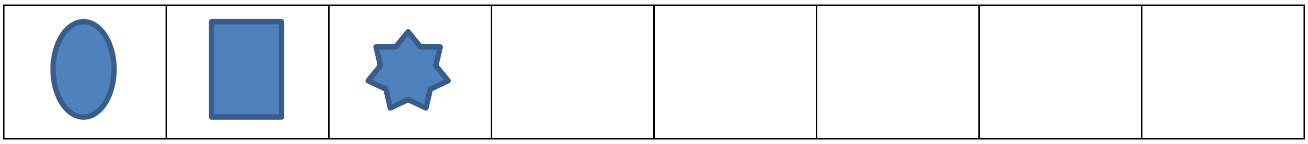
3. Circle the repeating part of this pattern



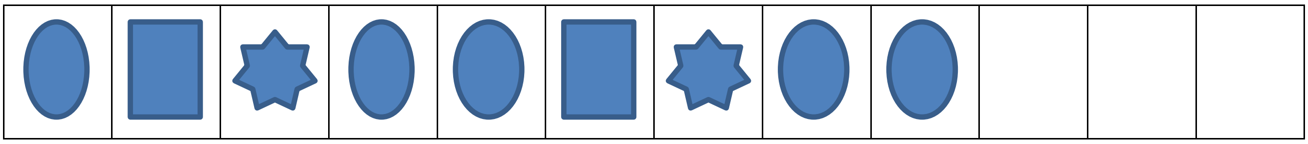
4. Draw a repeating pattern



5. Make this pattern



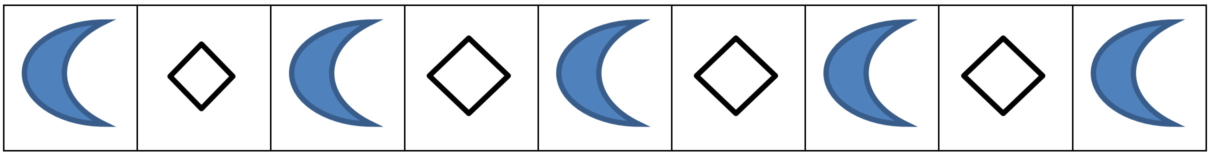
6. Complete this pattern (The repeating part has 4 elements, first and fourth the same)



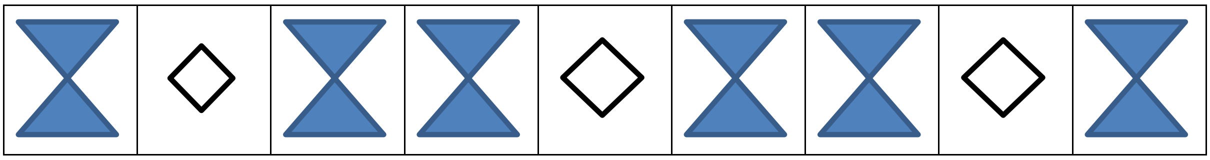
Subtest 2 items (Unit 2: Position in Repeating Patterns)

1. Circle the repeating part of the pattern (second example has more complex repeating part)

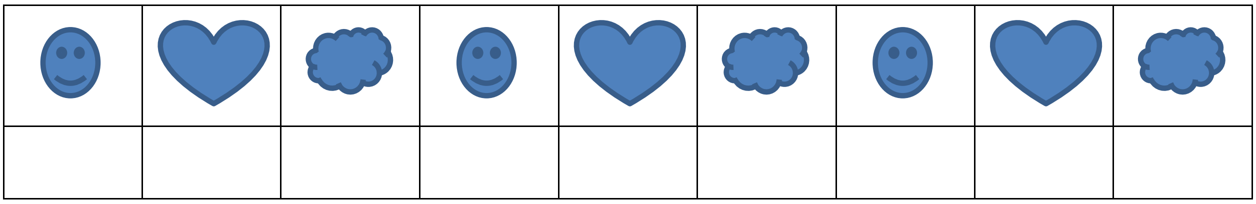
(a)



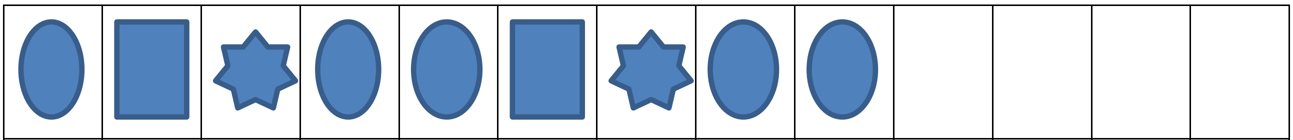
(b)



2. Number the position parts



3. Look at this pattern:



1	2	3	4	5	6	7	8	9	10	11	12	13

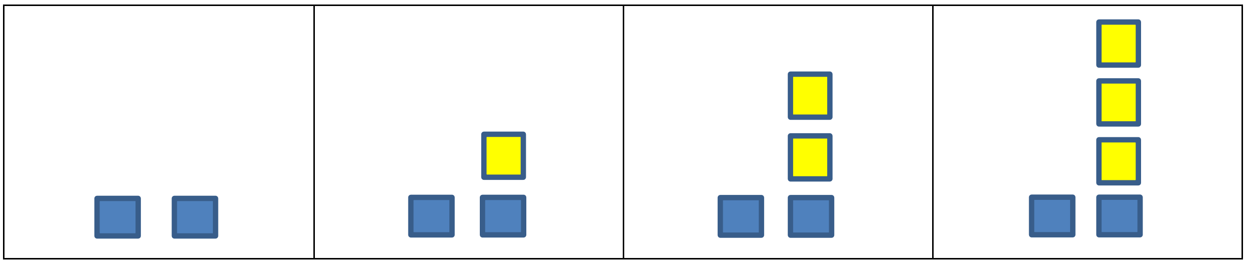
(a) What object is at position 3?

(b) What position is the rectangle?

(c) What will be at position 11?

Subtest 3 items (Unit 3: Repeating to Linear Growing Patterns)

1. Look at this pattern:

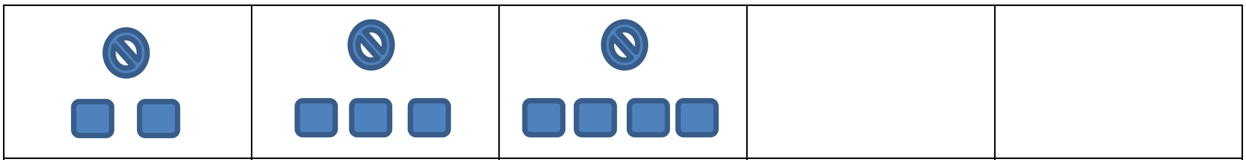


   (a) Which part is staying the same? ______________________________

   (b) Which part is growing? ______________________________

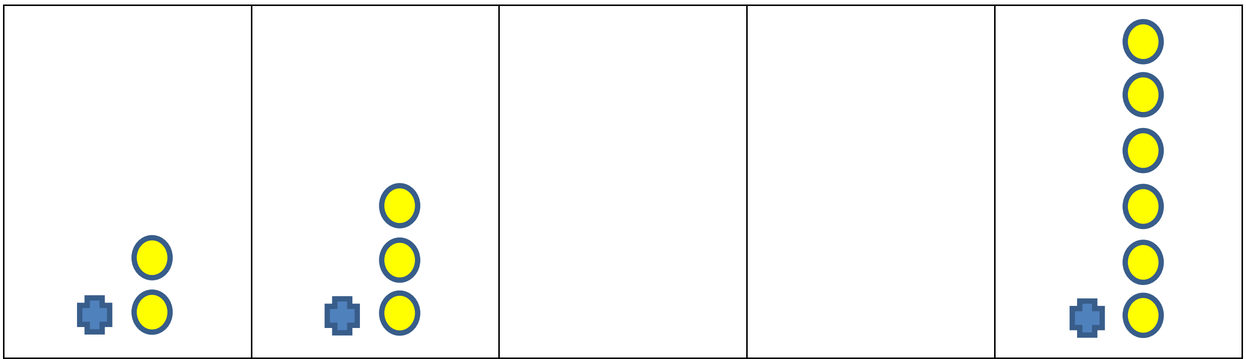
2. Copy this pattern to make your own growing pattern.

3. Continue this pattern:



1	2	3	4	5

4. Complete this pattern:



1	2	3	4	5

5. Create your own growing pattern.

1	2	3	4	5

(a) Which is the repeating part?

(b) Which is the growing part?

(c) Identify what is in position 3 of your pattern.

Subtest 4 items (Unit 4: Patterns in Number)

(Many examples of these item types can be found in the Year 2 Diagnostic Net materials.) Ensure students can: (a) identify and describe patterns in number sequences, (b) identify missing elements and errors in familiar sequences, and (c) reason and suggest solutions for problems that involve missing elements in a pattern.

1. Counting and place value

   (a) Continue this pattern: 15, 16, 17, _____, _____, _____

   (b) Complete this pattern: 33, 32, 31, _____, _____, _____, 27, 26

   (c) Here is a pattern: 1, 2, 3, 4, 5, 6, …

   What is the rule? ____

   (Make examples for backwards patterns as well.)

2. Seriation and odometer

   (a) Continue this pattern: 36, 46, 56, _____, _____, _____

   (b) What is the rule? ____

   (c) Complete this pattern: 141, 131, 121, ______, ______, _______

   (d) What is the rule? ____

3. Additive

   (a) Continue this pattern: 3, 7, 11, 15, _____, _____, _____

   (b) What is the rule? ____

   (c) Complete this pattern: 45, 41, 37, 33, _____, _____, _____

   (d) What is the rule? ____

4. Multiplicative

   (a) Continue this pattern: 1, 2, 4, 8, 16, 32, _____, _____

   (b) What is the rule? ____

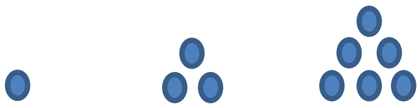
5. Represent with materials to expose the pattern.

(a) Use counters to make the pattern for the 3 times table.

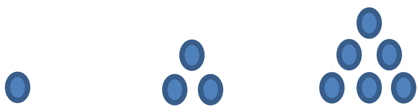
(b) Draw your pattern.

(c) Describe the rule.

(d) Use counters to make this pattern:



(e) Continue the pattern for two more positions.



Appendices

Appendix A: AIM Early Understandings Modules

Module content

1st module **Number N1: Counting** *Sorting/correspondence *Subitising *Rote *Rational *Symbol recognition *Models *Counting competencies	**2nd module** **Algebra A1: Patterning** *Repeating *Growing *Visuals/tables *Number patterns	**3rd module** **Algebra A2: Functions and Equations** _Functions_ *Change *Function machine *Inverse/backtracking _Equations_ *Equals *Balance
4th module **Number N2: Place Value** _Concepts_ *Place value *Additive structure, odometer *Multiplicative structure *Equivalence _Processes_ *Role of zero *Reading/writing *Counting sequences *Seriation *Renaming	**5th module** **Number N3: Quantity** _Concepts_ *Number line *Rank _Processes_ *Comparing/ordering *Rounding/estimating _Relationship to place value_	**6th module** **Operations O1: Thinking and Solving** *Early thinking skills *Planning *Strategies *Problem types *Metacognition
7th module **Operations O2: Meaning and Operating** *Addition and subtraction; multiplication and division *Word problems *Models	**8th module** **Operations O3: Calculating** *Computation/calculating *Recording *Estimating	**9th module** **Number N4: Fractions** _Concepts_ *Fractions as part of a whole *Fractions as part of a group/set *Fractions as a number or quantity *Fraction as a continuous quantity/number line _Processes_ *Representing *Reading and writing *Comparing and ordering *Renaming

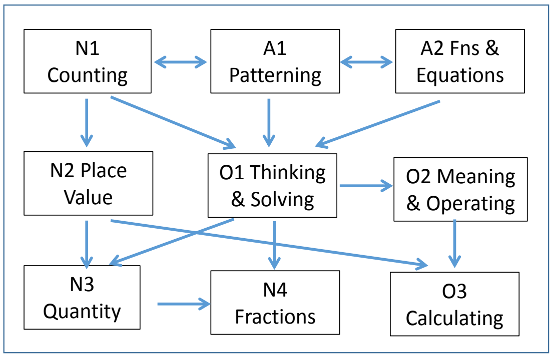
Module background, components and sequence

Background. In many schools, there are students who come to Prep/Foundation with intelligence and local knowledge but little cultural capital to be successful in schooling. In particular, they are missing basic knowledge to do with number that is normally acquired in the years before coming to school. This includes counting and numerals to 10 but also consists of such ideas as attribute recognition, sorting by attributes, making patterns and 1-1 correspondence between objects. Even more difficult, it includes behaviours such as paying attention, listening, completing tasks, not interfering with activity of other students, and so on.

Teachers can sometimes assume this knowledge and teach as if it is known and thus exacerbate this lack of cultural capital. Even when the lack is identified, building this knowledge can be time consuming in classrooms where students are at different levels. It can lead to situations where Prep/Foundation teachers say at the end of the year that some of their students are now just ready to start school and they wish they could have another year with them. These situations can lead to a gap between some students and the rest that is already at least one year by the beginning of Year 1. For many students, this gap becomes at least two years by Year 3 and is not closed and sometimes widens across the primary years unless schools can provide major intervention programs. It also leads to problems with truancy, behaviour and low expectations.

Components. The AIM EU project was developed to provide Years F–2 teachers with a program that can accelerate early understandings and enable children with low cultural capital to be ready for Year 3 at the end of Year 2. It is based on nine modules which are built around three components. The mathematics ideas are designed to be in sequence but also to be connected and related to a common development. The modules are based on the AIM Years 7–9 program where modules are designed to teach six years of mathematics (start of Year 4 to end of Year 9) in three years (start of Year 7 to end of Year 9). The three components are: (a) Basics – A1 *Patterning* and A2 *Functions and Equations*; (b) Number – N1 *Counting* (also a basic), N2 *Place Value*, N3 *Quantity* (number line), and N4 *Fractions*; and (c) Operations – O1 *Thinking and Solving*, O2 *Meaning and Operating*, and O3 *Calculating*. These nine modules cover early Number and Algebra understandings from before school (pre-foundational) to Year 2.

Sequence. Each module is a sequence of ideas from F–2. For some ideas, this means that the module covers activities in Prep/Foundation, Year 1 and Year 2. Other modules are more constrained and may only have activities for one or two year levels. For example, Counting would predominantly be the Prep/Foundation year and Fractions would be Year 2. Thus, the modules overlap across the three years F to 2. For example, Place Value shares ideas with Counting and with Quantity for two-digit numbers in Year1 and three-digit numbers in Year 2. It is therefore difficult, and inexact, to sequence the modules. However, it is worth attempting a sequence because, although inexact, the attempt provides insight into the modules and their teaching. One such attempt is on the right. It shows the following:

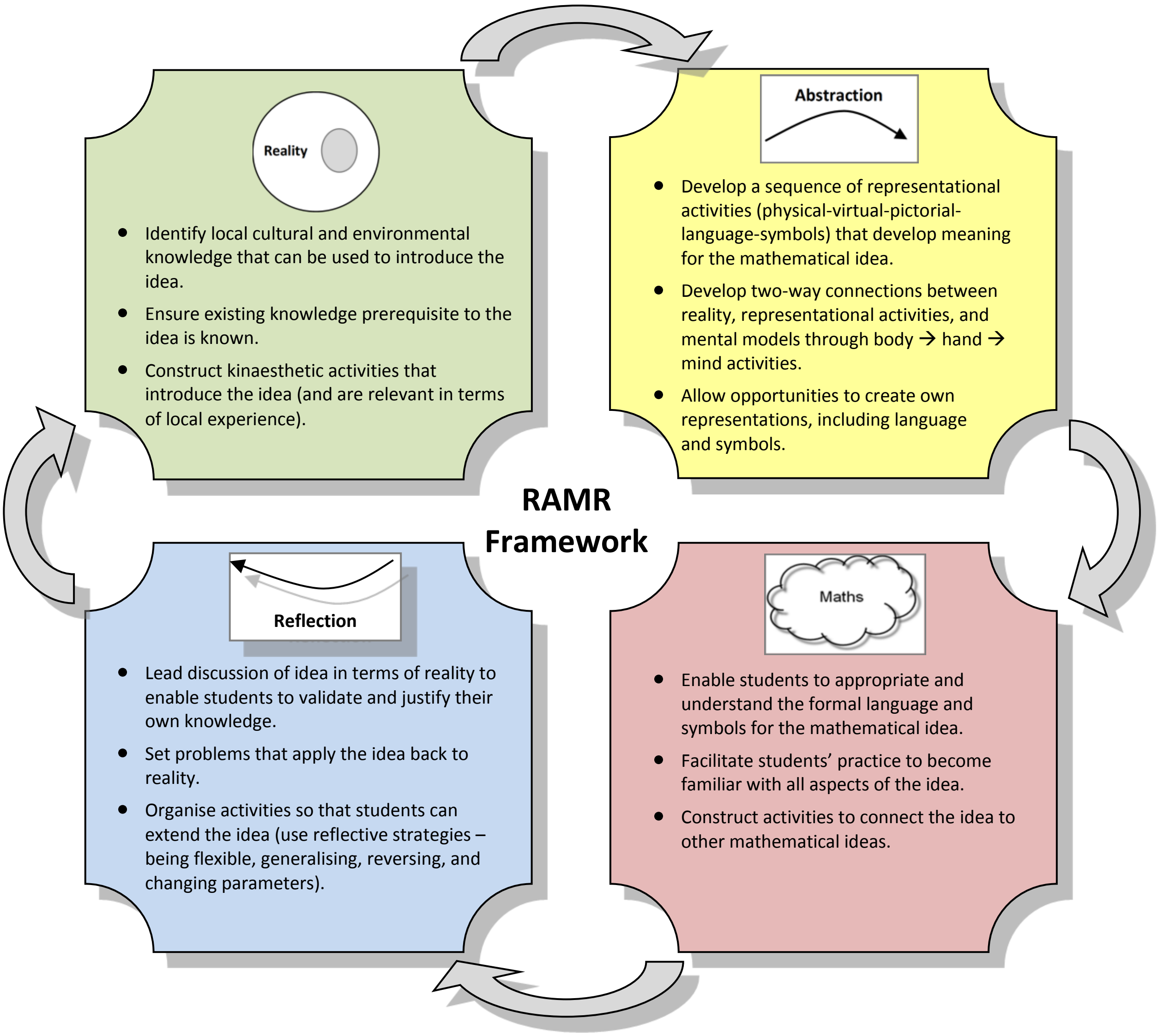


1. The foundation ideas are within *Counting*, *Patterning* and *Functions and Equations* – these deal with the manipulation of material for the basis of mathematics, seeing patterns, the start of number, and the idea of inverse (undoing) and the meaning of equals (same and different).
2. The central components of the sequence are *Thinking and Solving* along with *Place Value* and M*eaning and Operating* – these lead into the less important *Calculating* and prepare for *Quantity*, *Fractions* and later general problem-solving and algebra.
3. The *Quantity*, *Fractions* and *Calculating* modules are the end product of the sequence and rely on the earlier ideas, except that *Quantity* restructures the idea of number from discrete to continuous to prepare for measures.

Appendix B: RAMR Cycle

AIM advocates using the four components in the figure below, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem-solving, flexibility, reversing and generalising. The innovative aspect of RAMR is that Reality to Abstraction to Mathematics develops the mathematics idea while Mathematics to Reflection to Reality reconnects it to the world and extends it.

Planning the teaching of mathematics is based around the four components of the RAMR cycle. They are applied to the mathematical idea to be taught. By breaking instruction down into the four parts, the cycle can lead to a structured instructional sequence for teaching the idea. The figure below shows how this can be done.



The YuMi Deadly Maths RAMR Framework

Appendix C: Teaching Frameworks

Teaching scope and sequence for patterning

<table>
<tr><th>TOPICS</th><th>SUB-TOPICS</th><th>DESCRIPTIONS AND CONCEPTS/STRATEGIES/WAYS</th></tr>
<tr><td rowspan="4">Repeating and growing patterns</td><td>Repeating patterns</td><td>Following/copying patterns
Continuing patterns
Completing patterns
Creating patterns
Identifying repeats
Non-patterns</td></tr>
<tr><td>Position in repeating patterns</td><td>Discerning repeats
Relating position to item
Relating item to position
Finding rule for this relationship</td></tr>
<tr><td>Repeating to linear growing patterns</td><td>Copying, continuing, completing and creating patterns using objects
Determining growing and fixed parts
Visuals versus tables of numbers</td></tr>
<tr><td>Patterns in number</td><td>Using patterns to find number and operation ideas and relationships</td></tr>
</table>

Proposed year-level framework

YEAR LEVEL	ALGEBRA – PATTERNS	
	Semester 1	**Semester 2**
Pre-Prep	*Repeating patterns* – sort and classify and explain reasons; follow and create sound and movement patterns; copy and create repeating patterns – one attribute repeats.	*Growing patterns* – follow patterns where one component grows in a counting manner.
Prep	*Repeating patterns* – copy, continue, create and describe repeating patterns with objects using more than one attribute.	*Growing patterns* – explore extending repeating patterns put in form of repeats to simple growing patterns. *Patterns in number* – use numbers in repeating patterns; look at patterns in even/odd numbers.
1	*Repeating patterns* – copy, continue, create and describe patterns with objects and numbers to 100 (e.g. RBGGRBGGRBGG ...; 20 30 60 20 30 60 ...). *Patterns in number* – explore counting patterns (e.g. odometer property for ones and tens, patterns in counting by 2s, 5s, 3s, etc.).	*Growing patterns* – extend repeating patterns to simple growing patterns with one attribute (e.g. RB RBB RBBB ...). Look at growing patterns with objects and numbers to 100 (e.g. 2, 4, 6, ...; 1, 4, 7, ...). *Patterns in number* – explore place-value patterns (e.g. all numbers in forties have a 4 followed by a number; counting in tens from 6 always has a 6 in the ones position).
2	*Repeating patterns* – complete repeating patterns with objects and numbers using more than one attribute and increasing complexity. *Patterns in number* – describe and predict place-value and counting patterns (e.g. skip counting, multiplication/constant addition patterns from 99 board, odometer rule).	*Growing patterns* – copy, continue, complete, create and describe simple growing patterns based on visuals and numbers; explore sequential and position rules.
3	*Repeating patterns* – copy, continue, create and describe patterns with objects and numbers to 100. *Patterns in number* – continue place-value and counting patterns for whole numbers and simple fractions, describing patterns (i.e. identifying next elements).	*Growing patterns* – explore extending repeating patterns put in form of repeats to simple growing patterns.
Focus	Kinaesthetic: Body → Hand → Mind. Building early algebraic thinking: ideas; logic; techniques; habits of mind. Practising language.	Kinaesthetic: Body → Hand → Mind. Building early algebraic thinking: ideas; logic; techniques; habits of mind. Practising language and representation.

Accelerated Inclusive Mathematics Early Understandings Project

YuMiDeadly
Growing community through education

Faculty of Education
School of Curriculum
S Block, Room 404
Victoria Park Road
KELVIN GROVE QLD 4059

CRICOS No. 00213J

Phone: +61 7 3138 0035
Fax: +61 7 3138 3985
Email: ydc@qut.edu.au
Website: ydc.qut.edu.au