Accelerated Inclusive Mathematics Early Understandings Project

YuMi Deadly Maths

# AIM EU
# Module N1
# Number: Counting

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## ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at the Queensland University of Technology which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning. The YuMi Deadly Centre (YDC) can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life. YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

## DEVELOPMENT OF THE AIM EARLY UNDERSTANDINGS MODULES

In 2009, the YuMi Deadly Centre (YDC) was funded by the Commonwealth Government's *Closing the Gap: Expansion of Intensive Literacy and Numeracy* program for Indigenous students. This resulted in a Year 7 to 9 program of 24 half-term mathematics modules designed to accelerate learning of very underperforming Indigenous students to enable access to mathematics subjects in the senior secondary years and therefore enhance employment and life chances. This program was called Accelerated Indigenous Mathematics or AIM and was based on YDC's pedagogy for teaching mathematics titled YuMi Deadly Maths (YDM). As low income schools became interested in using the program, it was modified to be suitable for all students and its title was changed to Accelerated Inclusive Mathematics (leaving the acronym unchanged as AIM).

In response to a request for AIM-type materials for Early Childhood years, YDC is developing an Early Understandings version of AIM for underperforming Years F to 2 students titled Accelerated Inclusive Mathematics Early Understandings or AIM EU. This module is part of this new program. It uses the original AIM acceleration pedagogy developed for Years 7 to 9 students and focuses on developing teaching and learning modules which show the vertical sequence for developing key Years F to 2 mathematics ideas in a manner that enables students to accelerate learning from their ability level to their age level if they fall behind in mathematics.

YDC acknowledges the role of the Federal Department of Education in the development of the original AIM modules and sees AIM EU as a continuation of, and a statement of respect for, the *Closing the Gap* funding.

# Contents

# *Module Overview*

This module on early counting is the first of the Accelerated Inclusive Mathematics Early Understandings (AIM EU) modules. These modules are designed to provide support in Years F to 2 to improve Year 3 mathematics performance. It is the intention of the YuMi Deadly Centre to prepare nine of these modules to cover Number and Algebra Years F to 2 (see **Appendix A**).

The original Years 7–9 AIM module on Number focused on the connections between whole numbers, decimal numbers, fractions, and so on. This first module in AIM EU covers early number ideas such as matching, sorting and 1:1 correspondence, rote and rational counting, subitisation, and early work on number language and symbols. It does this to lay the foundation for number and to cover all the teaching needed if these foundations have not been covered.

Similar to the Years 7–9 AIM modules, this Year F–2 module provides teaching and learning information in terms of a vertical sequence of units. Each unit is a step in the sequence and provides information and teaching ideas for that step. This section, Module Overview, precedes the units and looks at the content and pedagogy covered by the module. It does this in subsections on counting in early childhood, connections and big ideas, sequencing, teaching and culture, and a summary of the module structure.

## Counting in early childhood

This section briefly overviews what is involved in counting in the early years of schooling. To teach effectively, it is essential for teachers to know the mathematics that precedes, relates to and follows what they are teaching, so they can build on the past, relate to the present, and prepare for the future. The information in this module is presented as sequences of ideas that relate to and connect with each other. This module on counting covers:

(a) *pre-counting* – identifying attributes, matching, sorting, comparing, ordering, number as an attribute and subitisation, ideas for counting before number is introduced;

(b) *rote counting* – stating the numbers in order; this is separate from pre-counting but sets the scene for the next section;

(c) *rational counting* – using rote counting to count a set and count out a subset, based on 1:1 correspondence between object and number name, separating those that have been counted from those that are yet to be counted, and the last number says how many (includes starting and finishing at preordained objects for robustness and 1:many counting based on 1:many correspondence for flexibility);

(d) *extension to large numbers* – ensuring all this can be done for numbers up to 99 and then up to 999, based on flexible counting of groups and ones, and groups of groups, groups and ones, knowing seriation and odometer principles;

(e) *introducing numerals and relating these to number names and representations of objects* – based on being able to move between any of the three ways of representing number and relying on symbol recognition; and

(f) *using a variety of models* – set, number track, number ladder, number line, odometer cards, and so on.

## Connections and big ideas

The unit or the one is the starting point for number. It is the basis for all numbers. Units can be grouped to produce whole numbers (ones, tens, hundreds, and so on), or they can be partitioned to produce decimal numbers (tenths, hundredths, and so on) or common fractions (halves, quarters, sixths, and so on). However, before this can be attempted, there is pre-number work on attributes and early counting, and the relationship between number names, numerals, and models.

## Connections

The starting point for all YuMi Deadly Maths (YDM) AIM modules is the connections between mathematics topics and using these connections to accelerate learning, in particular in the formation of big ideas whose learning will provide understanding across mathematics topics and across year levels.

Mathematics is best understood and applied in a schematic structured form which contains knowledge of **when and why as well as how**. Schema has knowledge as **connected nodes**, which facilitates recall and problem-solving. It is the basis of YDM that knowledge of the structure of mathematics, particularly of connections and big ideas, can assist teachers to be effective and efficient in teaching mathematics, and enable students to accelerate their learning. It enables teachers to:

(a) *determine what mathematics is important to teach* – mathematics with many connections or based on big ideas is more important than mathematics with few connections or little use beyond the present;

(b) *link new mathematics ideas to existing known mathematics* – mathematics that is connected to other mathematics or based on the one big idea is easier to recall and provides options in problem-solving;

(c) *choose effective instructional materials, models and strategies* – mathematics that is connected to other ideas or based around a big idea can be taught with similar materials, models and strategies; and

(d) *teach mathematics in a manner that enables later teachers to teach more advanced mathematics* – by preparing linkages to other ideas and foundations for big ideas the later teachers will use.

Counting is the basis for all number work; it is connected to whole numbers, decimal numbers, common fractions, and percent, rate and ratio. It is also connected to operations, measurement, statistics and probability and, in general form, to algebra. Counting is therefore important in all mathematics and is not only an early childhood activity; pre-counting ideas such as attribute recognition and classification remain important throughout all year levels.

## Big ideas

There are five major big ideas that apply in a similar way to all numbers (whole and decimal numbers, common fractions, and percent, rate and ratio): notion of unit/part-whole, additive structure, multiplicative structure, continuous vs discrete/number line, and equivalence. Some of these apply to early number:

(a) *notion of unit/part-whole* – absolutely basic in terms of 1:1 and 1:many correspondence – have to recognise that a unit may be a group as well as a single and, when it is a group, we are counting 1:many;

(b) *additive structure* – understanding seriation, that is, adding and subtracting a 1, a 10 or a 100, and counting patterns and odometer to 999; and

(c) *continuous vs discrete/number line* – what happens when number is applied to continuous entities is important and leads to the number-line model where notion of zero changes.

Other ideas are also big:

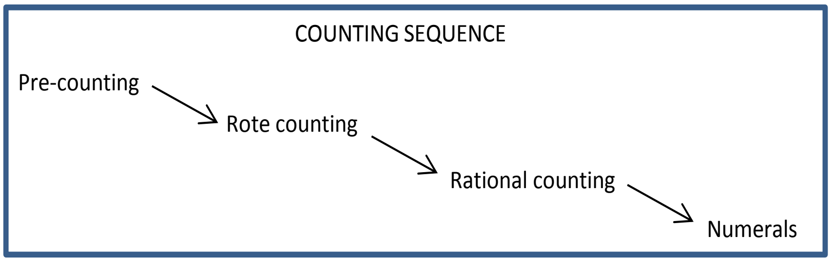
(a) *flexibility* – the unit in groups and individuals can change as can the unit for hundreds, tens and ones;

(b) *trust the count* – anything discrete can be counted, and counting a different way does not change the count;

(c) *bridging tens and hundreds* – counting quantities to 999 through counting patterns that move across tens and hundreds; and

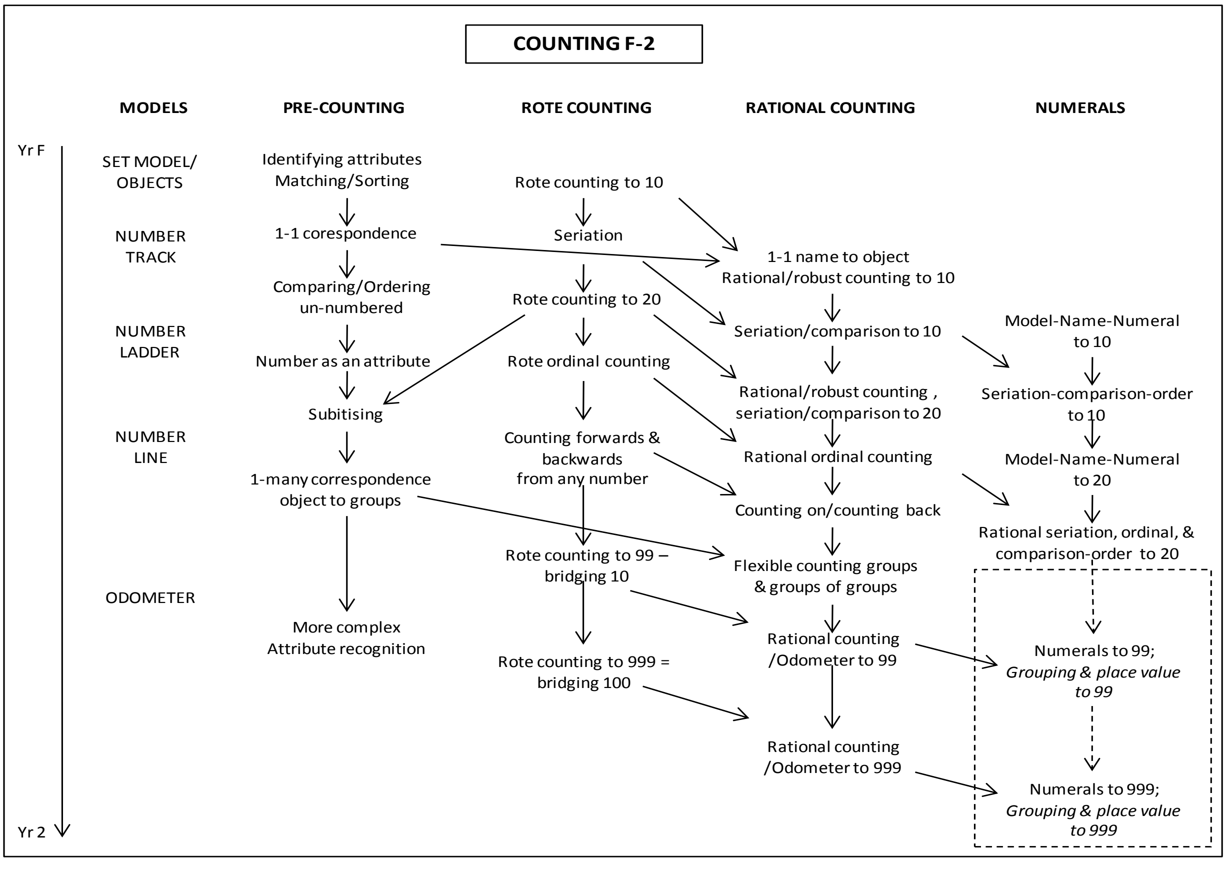
(d) *odometer* – this is the pattern followed in counting forward and bridging ten (e.g. 38, 39, 40, 41) and hundred (e.g. 197, 198, 199, 200, 201), that is, the count goes to 9 or 99, then reduces to 0 or 00 with the position to the left increasing by one 10 or one 100; and counting back and bridging ten (e.g. 62, 61, 60, 59, 58) and hundred (e.g. 223, 213, 203, 193, 183), that is, the count goes backward to 0 or 00, increases to 9 or 99 with the position on the right decreasing by one.

## Sequencing




Ignoring the size of the numbers, the counting sequence has four components, as in the figure on right. It follows the sequence: (a) **pre-counting** (e.g. attributes, same and different, sorting and classifying, matching, 1:1 correspondence, and so on); (b) **rote counting** (e.g. stating number names in correct order); (c) **rational counting** (e.g. counting a set and counting out a subset by 1:1 correspondence between number names and objects and where the last number name gives how many); and (d) **numerals** (e.g. learning the numeral for each number and relating to models and names).

This is a simple sequence which is made more complex by the size of the numbers for each section. For example, a student may have rote counting to 20 but not be able to write the numerals for 1 to 20, meaning that students could be at different size numbers for different concepts. As well, language and symbols are difficult to match as the language for certain numbers does not follow the symbol structure (e.g. the teens do not follow the language counting pattern, and the zeros are silent). Therefore, we have in the figure below, an attempt to show this complexity.




The sequence for AIM Early Understandings Module N1 on Counting

The figure above represents the sequences (and connections) that need to be followed in counting in early number Years F–2. The figure has vertical arrows to show the sequence from Year F at the top to Year 2 at the bottom, and diagonal arrows showing the connections across the different parts of the sequence. Some further points with regard to the figure are as follows:

(a) the dotted lines around numerals to 99 and 999, and the dotted arrows, indicate that place value is part of the development of this section and that this will be done in Module N2, leaving this module's involvement in this section to counting with numerals;

(b) 1:1 correspondence and 1:many correspondence are important early activities – 1:many correspondence is the basis of place value plus operations such as multiplication;

(c) 1:many correspondence can be effectively developed by subitisation which is an ability to determine number as a whole (hence the connection backwards from rote counting);

(d) the final topic in the pre-counting is there to show that this area can be taken further and has an important role in reasoning;

(e) rote and rational go alongside each other with the rational using the rote language sequence from rote counting, with this part also involving the development of ordinal numbers;

(f) seriation, comparison and order can start from rote counting if they are based on number names not numerals;

(g) model ←→ language teaching precedes model ←→ language ←→ symbol teaching;

(h) the bridging 10 and 100 in rote counting precedes the odometer principle in rational counting; and

(i) rote and rational counting are also important across all Years F to 6 and counting should be used in decimal numbers, common fractions, money, measures, and other areas.

## Teaching and culture

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework and the impact of Western number teaching on Indigenous and low-socio-economic status (SES) students.

### Teaching implications

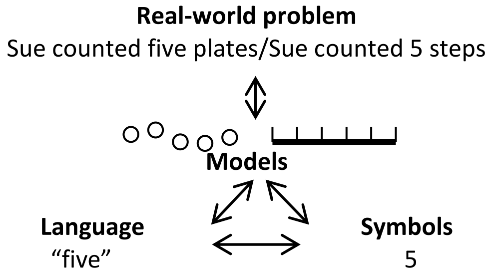
The first teaching implication is that mathematical ideas that are structurally similar can be taught by similar methods. For example, fractions and division are similar and both are taught by partitioning sets into equal parts – except that the set is seen as one whole for fractions and a collection of objects for division. Similarly, much of pre-counting has to do with attributes, correspondences and sorting, and set and number track/line representations. These commonalities will be highlighted in the Module Review section.

The second teaching implication is that number instruction is both diverse and generic. There are particular ways to teach the varied topics in this module and they will be discussed as we meet them. However, there are also generic methods that hold for most topics. Two are given below.

1. **RAMR cycle**. This is the basis of lesson planning in YDM because of the generic teaching ideas contained in the framework, and is applicable to number teaching. For a start, it grounds all mathematics in reality and provides many opportunities for connections, flexibility, reversing, generalisations and changing parameters, as well as body → hand → mind. The idea is to use the framework and all its components throughout the years of schooling and this will help prevent learning from collapsing back into symbol manipulation and the quest for answers by following procedures. The framework is shown in **Appendix B** and is the basis of all YDM materials.

2. **Payne and Rathmell triangle**. This is one of the underpinnings of the Abstraction and Mathematics stages of the RAMR framework. It is important in teaching number because of its focus on the relationship

between models, language and symbols. Activities and questions should be constructed that encourage students to connect and move flexibly between models, language and symbols in all directions.




The Payne and Rathmell (1977) triangle for early number

### Cultural implications

Aboriginal and Torres Strait Islander students may find the teaching of number confronting because of the differences of the number-oriented culture of the mathematics classroom and their culture, and because many students are from low-SES backgrounds.

Aboriginal and Torres Strait Islander cultures followed a different path from number-oriented cultures (European, Indian-Arabic, and Chinese-Japanese) in the development of mathematics; for Indigenous cultures, people were seen as more important than number so their mathematics specialised in areas other than number. This different focus could be seen as emanating from their cultural beliefs with regard to group rather than individual ownership. Thus, the teaching of number, operations and measurement can bring Australian mainstream Eurocentric school teaching into conflict with Indigenous students; it can be a topic that can, in the terms of Indigenous mathematics and mathematics-education researcher Dr Chris Matthews, designate these cultures as primitive. It must be taught with care as part of a European culture that Indigenous people need to understand. It should not be celebrated as something that raises some people above others.

For low-SES Aboriginal and Torres Strait Islander students in Australia, the outcome is exacerbated. As low-income people, these students are sometimes considered as unsuccessful. The number systems created as part of Eurocentric mathematics have always benefited high-SES people at the expense of low-SES people, and promulgated the idea that bigger numbers (e.g. money, house cost, cars) are better, and mean that the person with the bigger numbers is more successful. The way the numbers function within Eurocentric societies achieves two outcomes simultaneously: (a) it benefits one class of people at the expense of the other, and (b) it puts the blame for their lack of benefit on the actions of the class that is not benefited. The mathematics of number, operations and measurement must be taught with care to low-SES students because its teaching can designate these students as failures. And if the students are both Indigenous and low SES, even greater care must be taken.

## Structure of module

### Components

Based on ideas above, the module is divided into this overview, four units, a review, test item types, and appendices as follows.

**Overview**: This section covers pre-counting ideas, connections, big ideas, sequencing, teaching and culture, and summary of units.

**Units**: Each unit includes examples of teaching ideas that could be provided to the students, some in the form of RAMR lessons, and all as complete and well sequenced as is possible within this structure

*Unit 1: Pre-counting*. This unit covers concepts and processes that underlie counting such as sorting and classifying and one-to-many correspondence.

*Unit 2: Introducing rote and rational counting*. This unit covers the two types of counting without numerals and for small numbers – rote, which is memorising order of names, and rational, which is being able to count how many.

*Unit 3: Bridging and counting patterns*. This unit covers rote and rational counting for larger numbers, to 99 and then to 999, covering bridging tens and hundreds, but without numerals.

*Unit 4: Numerals*. This unit covers the introduction of numerals and pedagogy for teaching and learning number through relating models, language and symbols.

**Module review**: This section reviews the module, looking at important components across units. This includes the major ways of modelling number concepts and processes and the important competencies associated with effective and efficient counting.

**Test item types**: This section provides examples of items that could be used in pre- and post-tests for each unit.

**Appendices**: This comprises three appendices covering the AIM EU modules, the RAMR pedagogy, and proposed teaching frameworks for counting.

## Further information

**Sequencing in modules and in figures above**. Units 2 and 3 do not match the headings in the figures under sequencing. Rote and rational counting are both in Unit 2. Rote is covered before rational to allow a sequence from simply stating the number names to using them to count. This sequence is the basis of Unit 2. The numbers are not large. The focus is on the types of counting. Rote and rational are also both in Unit 3. However, here, the numbers are larger (up to 999) and so we look at the development of the counting patterns, focusing on bridging across the tens and the hundreds. This means looking at the pattern in counting that enables the count to go across tens and hundreds:

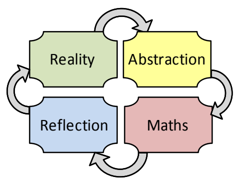
twenty-seven, twenty-eight, twenty-nine, thirty, thirty-one and so on; and

three hundred & ninety-eight, three hundred & ninety-nine, four hundred, four hundred & one, and so on.

Unit 4 covers numerals and the important model–language–symbol relationship. However, the numbers are kept small and place value is just touched on because it is the focus of AIM EU Module N2.

**Sequencing the teaching of this module**. The four units are in sequence and could be completed one at a time. However, each of the units is divided into sub-ideas (concepts and processes) that are also in sequence within the unit. Therefore, schools may find it advantageous to: (a) teach earlier sub-ideas in a later unit before completing all later sub-ideas in an earlier unit; (b) teach sub-ideas across units, teaching a sub-idea in a way that covers that sub-idea in all the units together; or (c) a combination of the above.

The AIM EU modules are designed to show sequences within and across units. However, it is always YDC's policy that schools should be free to adapt the modules to suit the needs of the school and the students. This should also be true of the materials for teaching provided in the units in the modules. These are exemplars of lessons and test items and schools should feel free to use them as they are or to modify them. The RAMR framework itself (see Appendix B) is also flexible and should be used that way. Together, the units and the RAMR framework are designed to ensure that all important information is covered in teaching. Therefore, if changing and modifying the order, try to ensure the modification does not miss something important (see Appendix C for detailed teaching frameworks).




**RAMR lessons**. We have included RAMR lessons as exemplars wherever possible in the units of the module. Activities that are given in RAMR framework form are identified with the symbol on the right.

**Suggestions for improvement**. We are always open to suggestions for improvement and modification of our resources. If you have any suggestions for this module, please contact YDC.

# Unit 1: Pre-Counting

This unit covers and sequences a variety of concepts and processes that underlie counting and number, namely, identifying attributes, matching, sorting and classifying, unnumbered comparing and ordering, subitising, and one-to-one and one-to-many correspondence. It involves recognition of different attributes and using these attributes to match, sort, compare and order. It puts different sets in one-to-one or one-to-many correspondence.

This unit's focus is on the activities that come before counting although some intersect with rote counting, for example, the first form of counting (e.g. subitising). The unit identifies, defines and provides examples of the concepts and processes that lead into counting, sequencing them in an order. It also provides an exemplar RAMR cycle lesson for each of the concepts/processes.

The unit covers the sequence of ideas on the right.

**PRE-COUNTING**

Identifying attributes
Matching/Sorting
↓
1-1 corespondence
↓
Comparing/Ordering
un-numbered
↓
Number as an attribute
↓
Subitising
↓
1-many correspondence
object to groups

## 1.1 Identifying and describing attributes

An attribute is a property, characteristic or feature of an object. Attributes young children notice are colour, size, mass, shape, sound, taste, texture and an object's function or use.

Young children need to engage in activities that require the use of the senses when identifying and describing attributes. This exposes them to new vocabulary and they learn words that are relevant to each attribute. The process of identifying and describing attributes is the ability to recognise likenesses and differences between objects and pictures, e.g. recognising everything that has the colour red, or can be sat on, or rattles, and so on. Children should be encouraged to notice detail and to be observant.

### 1.1.1 Attribute ideas and activities

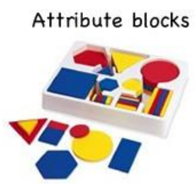
Activities should foster a range of language so children will talk about the likenesses and differences between objects. Teachers need to listen to the children as they play with the toys and encourage them to use their senses to identify the likenesses and differences and then describe these attributes in their own language. This also is promoting reasoning at a very young age.

The ability to recognise likenesses and differences is important in the development of young children's beginning mathematics. Number cannot be understood unless students can recognise when things are the same and when they are different. This is the first step. Difference is the basis of number because number tells how many different things there are. Same is the opposite of different and is essential for understanding difference. It leads to recognising place values, fractions, percent, ratios and rates.

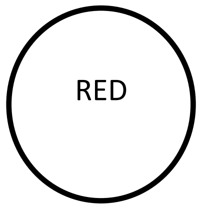
It is worth noting that the objective is to introduce vocabulary to young students as they explore maths ideas with concrete objects, not to make them memorise vocabulary. Teach maths vocabulary as naturally as you would introduce the names of different foods, animals or toys. The more children hear math vocabulary used in everyday activities the more they will begin to use it correctly. Children learn the language of mathematics just like they learn to talk about other topics. Language and communication are essential elements of the process of learning mathematics. We model appropriate mathematical terms and encourage children to share their ideas. In this way, language bridges the gap between the abstractedness of mathematics and the child's real world.

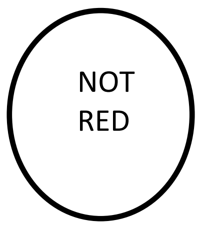
The following activities encourage children to talk about their experiences and connect new ideas to existing knowledge.

1. Feely box – invite students to reach in and feel and describe an object for others to guess. After some discussion, child then removes object to show everyone and describe its colour. This can also be done with a board of matching outlines for items to be placed on. Extend as a pairs activity. Are students showing an interest in the attributes and noting likeness? Can they describe the attributes of the object?
2. Texture cards – have pairs of students find and describe squares of fabric that feel the same and different according to colour and texture. Extend to matching bingo boards, or a "quilt" board. Ask children: *Why are they the same? Why are they different?*
3. Paint imaginary pictures – have students close their eyes and put certain things in their minds e.g. cats, boxes, blocks, balls, etc. *Can you make a picture? Tell me about your picture. What can you see?*
4. Opposites – use children's books about opposites. Find opposites in the room and have children create situations of the opposite relationship.
5. Constructions – construct things with boxes, blocks, materials and observe attributes. Can students describe why things go together?
6. Outside play – making collections, nature trails, sorting balls. Are students beginning to make purposeful collections?
7. Attribute blocks – use attribute blocks to play attribute games. Attribute blocks are mathematics manipulatives that have four different features. These are shape, colour, size and thickness. The shapes are circles, hexagons, squares, rectangles and triangles. The three colours are red, blue, and yellow. The two sizes are big and small. The two thicknesses are thick and thin. Activities include sorting the blocks, stating attributes, identifying blocks from attributes (reverse of previous activity), playing "what's my block" games, and so on.



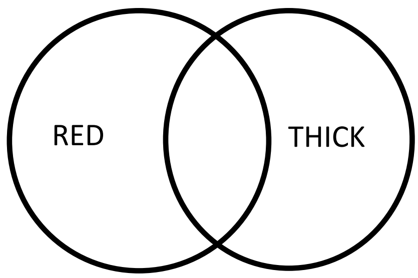

8. Hoop and block activities – the blocks with hoops are great for the logical connectives: "not", "and", "or", and "if…then". For example, to experience not, place hoop and label it with an attribute (say, RED). Get students to place blocks inside and outside the hoop following the label. The ones outside are NOT the attribute. This can be reinforced by placing the hoop with the label NOT RED inside of it, and asking the students to place blocks again.



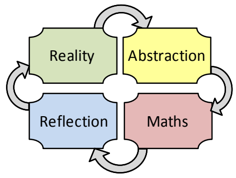

   For "AND" and "OR", place two hoops so they intersect (overlap) and label them with different attributes (e.g. THICK and RED). Get students to place blocks inside the hoops, the intersection of the hoops, and outside the hoops, following the labels. The materials in the intersection show "AND". The materials in both hoops and the intersection show "OR". This means AND signifies both attributes are present, while OR means one or both are present.

   - Play "What belongs inside the hoop?"
   - Build an attribute bridge/wall/path/trail. Each student takes a turn to add a block that relates to previous block by one attribute. Then play using two attributes; three attributes.
   - Play "What's missing?" Spread blocks on floor, remove one while students are not watching and ask.




   *Note*: More complex attribute recognition activities are in the YDM Number book pp. 24–30.

### 1.1.2 RAMR lesson for attributes




Learning goal: Identify and describe attributes.

Big ideas: Language ←→ picture ←→materials ←→ action; visualising through kinaesthetic activity.

Resources: General classroom items, manipulatives, toys, things from nature that range in size, colour, texture, purpose, mass, sound, and so on.

## Reality

*Local knowledge*: Items that are familiar to students, play items, construction items, games.

*Prior experience*: Collections each child has at home or school.

*Kinaesthetic*: Place different collections in hoops around the room. Have students work in small groups to identify and describe collections. Have groups share back their work processes.

## Abstraction

*Body*: Have students group and regroup by same and different attributes.

*Hand*: Have students repeat kinaesthetic activity above but with own boxes of objects; play Feely boxes.

*Mind*: Have students paint imaginary pictures and "tell" their picture. Visualise objects.

*Creativity*: Students make up their own objects and descriptions.

## Mathematics

*Language/symbols*: Same, different, words to describe attributes.

*Practice*: Facilitate becoming familiar with idea. Create funny faces with paper plates. Describe results.

*Connections*: Encourage students to describe colour, shape, size, length. Extend/connect to taste and sound.

## Reflection

*Validation*: Play "create a creature". Roll the dice to choose an attribute to put on creature.

*Extension*: Have students feel object in feely bag and draw it on the white board for other students to name.

- *Reversing*: Make a collection, ask students to work out and describe how they have made it. Describe attributes; have students identify object from a collection.

## 1.2 Matching and sorting

Matching activities focus on the "sameness" of properties of objects. Children need to look at the characteristics of different items and find characteristics that are the same. Young children learn to select objects that have at least one attribute the same, then describe why the objects match using language associated with the attribute they have chosen. Extend children's reasoning through asking questions that encourage them to think at an early age, e.g. *Why do these match?*

Sorting involves matching but with a larger number of objects. The process of sorting involves the three "Ds" – Decide, Do, and Describe. Many experiences of sorting objects and in a variety of ways is necessary, for children to feel confident sorting, describing and inventing their own systems.

### 1.2.1 Matching and sorting ideas and activities

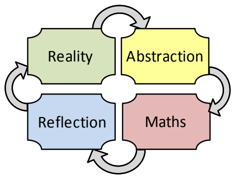
Matching activities can be sequenced as follows: matching objects visually focussing on colour, size and shape; then matching objects using other senses – touch, smell, hearing and taste; follow with matching objects to outlines; then matching pictures to pictures; and finally matching pictures to outlines. Matching leads to understanding the concept of one-to-one correspondence. When a child passes out biscuits, each child in the room gets one biscuit. Maybe there is just the right amount of biscuits or maybe there are extra biscuits. Matching forms the basis for our number system. When a child can create "the same", it then becomes possible to match two sets. This becomes a prerequisite skill for the more difficult tasks associated eith conservation.

The sorting sequence of activities should be to start with sorting by one attribute, going on to sorting by two attributes, and then to three or more. Reversing activities should be included – asking students to sort objects, and asking students to identify the attributes used with objects that have already been sorted. Remember also that sorting can be done in terms of no attribute (e.g. not round or not large). This is the basis of equivalence – identifying no change.

Some ideas for activities are as follows.

1. Use a wide range of memory, matching and pairs games.
2. Make picture outline boards and matching picture cards of items that are in children's life or are of interest to them.
3. Match puzzle shapes to puzzle back boards. Many wooden jigsaws are available for this.
4. Create a felt board for children to match felt shapes to shape pictures.
5. Use road mats to match cars and trucks and vans.
6. Balance containers by filling with various materials to match the mass.
7. Play domino games to match the patterns.
8. Sort objects from the environment: boxes, leaves, stones, flowers, cups, balls, bottle tops, sticks, straws, and so on.
9. Use sorting circles to sort by more than one attribute; do overlaps.
10. Cut and sort pictures from magazines of objects, people and so on to sort by chosen characteristics.
11. Use organising trays to sort pictures into categories such as clothes, hats, food, toys.
12. Match pairs of objects, e.g. socks and socks, shoes and shoes; do open and closed sorting; put things out in separate piles.

### 1.2.2 RAMR lesson for matching and sorting




Learning goal: Sorting size, colour, shape and thickness.

Resources: Attribute blocks.

## Reality

*Local knowledge*: Find and discuss with students shapes in their environment.

*Prior experience*: Identifying, describing and matching games with shape.

*Kinaesthetic*: Provide each group of four with a box of attribute blocks. Allow students a few minutes free play to construct and create pictures using the attribute blocks. Have students describe their pictures.

## Abstraction

*Body*: Give each student an attribute block. Have them sort themselves by colour; regroup for size; regroup for shape; regroup for thickness.

*Hand*: Write these words on the board: large/small; red/blue/yellow; square/circle/triangle/hexagon; thick/thin. As you read and identify each word, direct students to hold up a block displaying the particular attribute.

*Mind*: Have students close their eyes and imagine the blocks by attribute. Ask: *What picture can you make?*

*Creativity*: How else could we invent to sort these blocks?

## Mathematics

*Language/symbols*: Match, sort, describe, identify, attribute, display, collections, thick, thin, large, small, shape names, colours.

*Practice*: Facilitate becoming familiar with idea. Instruct one student in each group to sort four blocks by one attribute (e.g. colour). The rest of the students in the group try to identify the attribute by which the example blocks are sorted. Have students alternate roles and continue sorting and classifying by one attribute. As you circulate among the groups encourage the students to verbalise the sorting strategies they use.

*Connections*: Identifying, describing, and measuring.

## Reflection

*Validation*: Look for things around school, home, in community, etc. where things are sorted (e.g. shops, building sites, and so on).

*Application/problems*: Encourage the students to sort and classify by more than one attribute, e.g. all blocks are blue and large; or all blocks are blue, large and thin, and so on.

*Reversing*: Put out sorted objects and students have to identify the attributes used in the sorting, that is, attributes to sort and sort to attributes.

## 1.3 Comparing and ordering (unnumbered)

Comparing is looking at the amounts of an attribute that two objects have. Students look at items and compare by understanding difference. Comparing is looking at two objects and seeing which has more of a particular attribute (big/little, hot/cold, smooth/rough, tall/short, heavy/light, and so on).

Ordering is looking at more than two objects and putting them in sequence from the one that has the least of the attribute to that which has the most. Ordering is foundational to our number system. Students have to be able to put items in an order so they are counted once and only once. Putting items in order is a prerequisite to ordering numbers. The word seriation is often used in place of ordering in maths writing.

This section looks at comparison and order without number – this is difficult to do sometimes but unnumbered activities build more powerful foundations for mathematics ideas. For example the following are not unnumbered, they are numbered: (a) order costs (value) using toy catalogues; and (b) roll dice and compare quantities.

### 1.3.1 Ideas and activities on comparing and ordering (unnumbered)

At the preparatory level students should make comparisons of more, less and same by making visual comparisons. The comparing process can be very difficult for students to comprehend. They need to understand that comparison is relative and that a thing can change its description and have more than one comparison relationship. At first, students want the description to relate to one and only one relationship.

Students' vocabulary is developed greatly through the process of comparing. Teachers should model the "er" words associated with comparisons. Children need lots of opportunities to use these comparison words. Some words are: tall, taller; big, bigger; short, shorter; late, later; early, earlier; narrow, narrower; dark, darker; thick, thicker; heavy, heavier; light, lighter; wide, wider; thin, thinner.

Ordering with three objects is an important understanding. For three objects, the first activities should involve comparing two objects and then adding in a third object which is the smallest or the largest. Then, activities should move to comparing two objects and then adding in a third which is in between the first two.

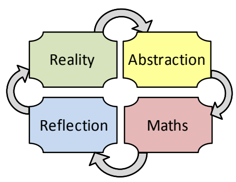
The important feature in the move from two objects (comparing) to three (ordering) is the development of the notion of "betweenness" – that objects can have attributes which means that one is between the other two in terms of these attributes. Comparing means a lot of new language. In terms of length we have long/short, thick/thin, narrow/wide, short/tall, etc. When giving a child directions use ordinal words (first, next, last).

Some activities are as follows.

1. Children learn best from real-world experiences. They enjoy putting items, such as boxes, tubs, bowls, cups in order of size; posting letters in order of size; making towers in order of size; building pathways in order of lengths with play-dough. Doing activities with three objects to order leads to extending vocabulary to the "est" words, e.g. big, bigger, biggest. Order tubes and other sporting play equipment.
2. Compare and order building blocks in terms of size of building to be built.
3. Make a range of pictures of creatures. Have each creature represented in three or four sizes. Cut them in half and jumble them up. Can students sort, compare and order them?
4. Go outside and make shadows. Compare and order them.
5. Have students in small groups and ask them to put themselves in order. Ask: *What order are you in? How did you decide that?* Ask other students: *What did they compare?*
6. Vehicles: Have students order and compare a collection of vehicles. Ask: *What order have you placed them in? What did you compare?* Have students record by drawing a picture.
7. Provide a basket of socks. Ask students to sort them. Ask: *How have you sorted them? What order have you put them in? What did you compare?*
8. Provide balance scales and a range of items. Ask students to sort the items using the balance scale. Ask: *What groups have you made? How did you decide which group to place items in? What attribute did you compare?*
9. Packing activity: Provide students with a box each and an assortment of items. Ask them to pack the box with items. Discuss how they compare and order to sort items.

Any collection of objects can be ordered in some unnumbered way – colour, temperature, size, mass, and so on. It is important to do this as we need this unnumbered approach in length, mass, area, volume, capacity and so on, in the measurement area.

### 1.3.2 RAMR lesson for comparing and ordering (unnumbered)




Learning goal: Arrange shapes in order of size.

Resources: Attribute blocks or similar.

## Reality

*Local knowledge*: Ask students: *Where do we see things arranged? Why might they be arranged like this?*

*Prior experience*: Identifying attributes; matching; sorting.

*Kinaesthetic*: Have students select three similar things from in the room and bring them to the learning circle and arrange them in order. Have students discuss and share the ways they did this.

## Abstraction

*Body*: Play "team grab bag". Students form two teams and take turns in pairs to "grab" three items out of the bag. Students then order their items. First done wins the point for their team.

*Hand*: Place and describe a range of three different-sized shapes on the table. Ask students to describe a shape and put in the bag. When all shapes are in the bag, have students take turns to feel in the bag and describe what shape they have. Then pull it out to confirm. Encourage each child to find a different shape.

*Mind*: Have the students visualise different shapes then place them in the bag.

*Creativity*: Have students invent a shape and place in the bag. Repeat above games.

## Mathematics

*Language/symbols*: Big, bigger, biggest; small, smaller, smallest.

*Practice*: Have students draw and label the different shapes as they pull them from the bag.

*Connections*: Mass; height; area; weather.

## Reflection

*Validation:* Where in the world do we take account of size (e.g. when lifting things, fitting them into backpack, etc).

*Extension*:

- *Flexibility*: What other things do we compare and order in size?
- *Reversing*: Arrange shapes and ask students to explain the order (to determine the attribute).
- *Generalising*: Where else do we make comparisons to find a solution? Does this work with other objects? Demonstrate how.
- *Changing parameters*: Extend to thick, thicker, thickest; thin, thinner, thinnest, and so on. We can also see if students can extend size to number – e.g. put out seven objects and five objects of same size and see if students can pick the larger – ask why? Put out seven small things and five large things – ask which is larger – ask where it would be possible for the seven things to be larger. (*Note*: may have to have the seven and five things being in a bag.)

# 1.4 The attribute of number

Number is a special attribute in its own right – it identifies something (usually a collection or group) by the amount of objects (in terms of counting discrete objects). For this attribute, other attributes do not matter. For example, the number of objects is two regardless of any differences between the two objects such as colour, size, difference in types, and so on. It just has to be two identifiable discrete objects; that is, discerning when objects are separate and can be counted.

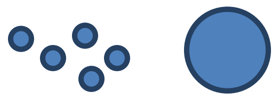
## 1.4.1 Ideas and activities on attribute of number

We can only show number by examples which have other attributes and this requires abstraction by the learner. Number is complex because it is not natural like wood, red, rabbits, and so on. It is something we invented to apply to these things. So, we can always show two hands, two marbles, two pencils, and so on, but the attribute of number as the twoness requires the learner to abstract a commonality from these examples. Therefore, it is easier to understand number if, initially, we are comparing sets of same things where the only attribute that changes is number (e.g. sets of red blocks where the blocks are all the same but the sets change in number); or comparing sets where the only thing that does not change is number (e.g. set of six red balls, set of six blue blocks, set of six pencils, and so on). Start in situations where other attributes do not change across the objects making up the number and then move on to differentiating other attributes (e.g. counting red balls of the same size before balls of many colours and sizes before objects of any size and type).

Number can be applied to other attributes (e.g. length) and so is part of the descriptions of that object even though the central attribute is not number (e.g. I took the 6 cm lengths of wood away from the steel rods). Of course, sometimes the attribute is number even though it is also lengths of wood (e.g. sort the wood by length). The important think here is to note that **the number line model applies to number as well as the set model**. So walk distances that have different lengths, look at commonality of six steps, six hops, six double feet jumps and so on. Start number-track activities.

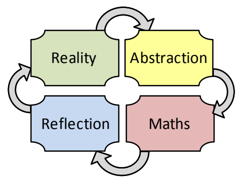
Some activities are as follows.

1. Interesting to compare collections that have one object and collections that have more than one – for example which of A and B is larger? (*Note*: A and B could be one big cake and many little cakes – which has more?) Would some children start to look at things in terms of number? – A has more because it has more than one rather than B is bigger because it is bigger? How many little cakes until they believe little cakes give more food?




2. Differentiating in 1 above is better initially if you keep all other attributes out of the situation as is possible.
3. Look at groups where the only difference is number; look at groups where the only thing not different is number, e.g. different coloured pens in sets of four to different everything in sets of four; and identical objects in different set sizes.
4. Collections: Take students for a walk to collect items outdoors, e.g. stones, or leaves, or seeds, etc. Notice amounts increasing as more is collected. What is changing?
5. Matching bags: Have zip bags with a number of stick-on stars on them. Students have to place the matching number of items in the plastic zip bag.
6. Walk side by side and stop at different points – what changed between the walkers?

### 1.4.2 RAMR lesson for attribute of number




<u>Learning goal</u>: Number is not a natural thing, it is something we have invented to apply to other things.

<u>Resources</u>: Buckets of counters, counting things, blocks, and boxes, pack of cards, large die, and dice.

## Reality

*Local knowledge*: Find things in the local environment to which we can apply numbers.

*Prior experience*: Sorting, classifying, making collections.

*Kinaesthetic*: All students have a small pile of counting things in front of them. Cover with their hands. Split group without looking under their hands. Ask: *Without lifting your hands what can you feel about the groups under your hands?* Have some students share. *Lift your hands and see. What do you see?* Have some other students share. *What do you see about your neighbours' groups?* Discuss the ones, threes, twos etc. Repeat process. In second discussion ask: *How is, say, G's two trucks the same as A's two buttons? How is this the same if the trucks are bigger than the buttons?* etc. with varying attributes.

## Abstraction

*Body*: Play "dice dual" with large die; students in pairs with little dice. Roll die, make group. Ask: *Who has larger group? Why?*

*Hand*: Matching bags – have zip bags with a number of stick-on stars on them. Students have to place the matching number of items in the plastic zip bag.

*Mind*: Visualise groups of large things and groups of small things, groups of tall things, and groups of short things of the same number.

*Creativity*: Have students construct a picture telling everything they know about, say 2.

## Mathematics

*Language/symbols*: Attribute, collection, group.

*Practice*: Provide activities to facilitate becoming familiar with the idea that the number is, for example, two regardless of whether the objects are short, large, containers and so on. It just has to be two identifiable discrete objects.

*Connections*: Numbers can be applied to measurement.

## Reflection

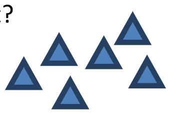
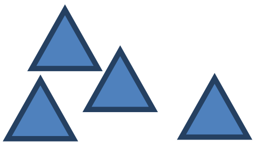
*Validation*: Set up a poster of things in everyday life that are based on number (e.g. the number attending the football game) and not on number (e.g. the tastiness of a cake).

*Application/problems*: Arrange groups of objects so some groups appear small but have a greater number of objects and some appear large but have fewer items. Ask students to describe the group then find and place the matching number symbol next to each group of objects.

*Extension*:

- *Reversing*: In a container place cards with pictures of multiple things. Students take a card and find as many situations as they can that this card might apply to. In another container place cards with number symbols. Students take a number then find as many ways they can to use that number to apply to things.
- *Changing parameters*: Interesting to see if students can extend this understanding to seeing "who has the most" without using number (counting). For example – who has more?

Which has most?

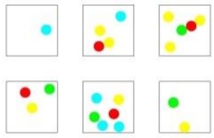
**A**  OR **B**?  WHY?

## 1.5 Subitising

Most very young children can recognise collections of one, two or three things without counting, simply by looking at a collection as a whole. This seeing a pattern at a glance without counting is called *subitising*. The pattern can be reconstructed without knowing the amount. Subitising helps the children see small collections as one unit. It develops before counting and underpins it.

Later, children "see" that two looks different from and less than three and come to connect this with the counting sequence. Children who do not readily distinguish "oneness", "twoness" and "threeness" just by looking, are unlikely to benefit from counting experiences. They need explicit help to develop the capacity to subitise small numbers.

Students also should learn to think of a collection in component parts, coming to see that: it is easier to see how many there are when collections are in special arrangements (e.g. see diagram on right); any collection can be separated into parts and each part can be represented by a number (thinking part-part-whole can help us to see "how many" there are); the same number can be thought of in parts in different ways; a number can also be thought of in more than two parts.



### 1.5.1 Ideas and activities for subitising

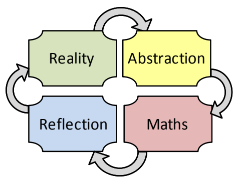
Activities to ensure students develop images of what quantities look like include: "see at a glance" with tray of objects covered by a tea towel; dots on a ten frame card sets; recognising sets of objects in different arrangements of the same number; visualising a collection in parts (certain arrangements make it easier to subitise); and being able to visualise parts removed.

Have two students face each other, then clap their hands three times before holding up between 5 and 10 fingers. Have them show all the fingers on one hand and some extra fingers on the second hand. Together, students say how many fingers are held up altogether.

Teaching suggestions are as follows.

1. Students' informal recordings of numbers and photos of students' constructions of numbers.
2. Drawings of actual objects randomly placed to represent numbers.
3. Photos of number models constructed by students and the teacher, taken from different perspectives.
4. Drawings of actual objects arranged in a structured way to represent quantities.
5. Plane shapes arranged randomly and then in a structured way to represent quantities.
6. Dot patterns – from dominoes, dice, ten frame, odd/even and Caldwell patterns (selecting one as a focus).
7. Terminology to develop an understanding of mathematical concepts in terms of the child's everyday language but continuing to be quantitative.
8. Develop an understanding of the mathematical significance of relational positional, quantitative and operational terms.
9. Use the language associated with the materials being used that reflects the action being performed (e.g. join parts to make a whole).

### 1.5.2 RAMR lesson for subitisation




<u>Learning goal</u>: Students will recognise amounts to five without counting, that is, subitise small collections of objects.

<u>Big idea</u>: Number – arrangement does not affect quantity; i.e. rearrangement without change in value.

<u>Resources</u>: Cards with pictures of numbers arranged in different ways (e.g. straight line, array, circle), pack of playing cards, small toys, counters, blocks.

## Reality

*Local knowledge*: Identify groups of things in the environment. How many toes on each foot? How many children sit at the green table?

*Prior experience*: Stories – How many? e.g. "A Wangkatha Counting book" and/or recent/relevant stories to the community.

*Kinaesthetic*: Inside, students can point to one nose; put up two arms; show me five fingers on one hand; now show me four fingers on the other hand. Outside, students find their own space where they are just on their own, that is, they can turn around in a circle with arms outstretched. Ask initially how many are in their group, how many are in – just your circle? Then make groups of 2, 3, 4, 5.

## Abstraction

*Body*: Play game "Hands up" – teacher calls a number and flashes picture cards of numbers that are arranged in different ways in a jumbled pack. As soon as the identified picture number is shown, students raise their hands. The pictures are displayed for only a couple of seconds so that students must be able to recognise (e.g. "4") without counting the pictures on the card. This can also be done with fingers. Students can also make the group shown with materials.

Other activities are "How many fingers?" What number is shown on the jumbled picture cards – teacher displays a picture card and turns it over quickly. Students show the number of fingers that correspond to the picture.

*Hand*: Students are given small toys/counters/blocks – teacher nominates a number in range 1 to 5 – students asked to arrange in materials in groups that are that number in as many different ways as possible. Different students are asked to describe their arrangements. Teacher records arrangements on whiteboard beside the appropriate quantity.

*Mind*: Students are asked to visualise: 1 flower; 2 trees; 3 apples; 4 balls; 5 stars, and so on. Teacher calls numbers randomly with any attribute. Encourage students to see as many different arrangements of the number as possible in their mind (with eyes shut where appropriate). Draw dots in the air with their eyes closed.

*Creativity*: Students choose the arrangement/s they like best of each number and record each numeral and its arrangement/s in picture form.

## Mathematics

*Language/symbols*: Count, visualise, how many, quantity, total, arrange. Use dice and dominoes to do activities to develop talking and writing.

*Practice*: Facilitate becoming familiar with idea. Use packs of dot cards, object cards, playing cards (using only Ace to 5 shuffled and dealt into two or three hands. In turn, each player turns over a card from his/her hand and places it in the middle of the group. When two cards of the same number are turned over consecutively, the first player to call "Snap" wins the pile. Players drop out as they have no more cards to play. Player who gets all the cards is the winner.)

Play the game "Snap" with cards that have pairs (or more) of numbers 1–5 but with different arrangements of dots for each number, all repeated many times. In pairs, students draw cards, turn the cards over, one by one, and the first to see a match, swats the pile and keeps the cards. Person with most cards wins.

## Reflection

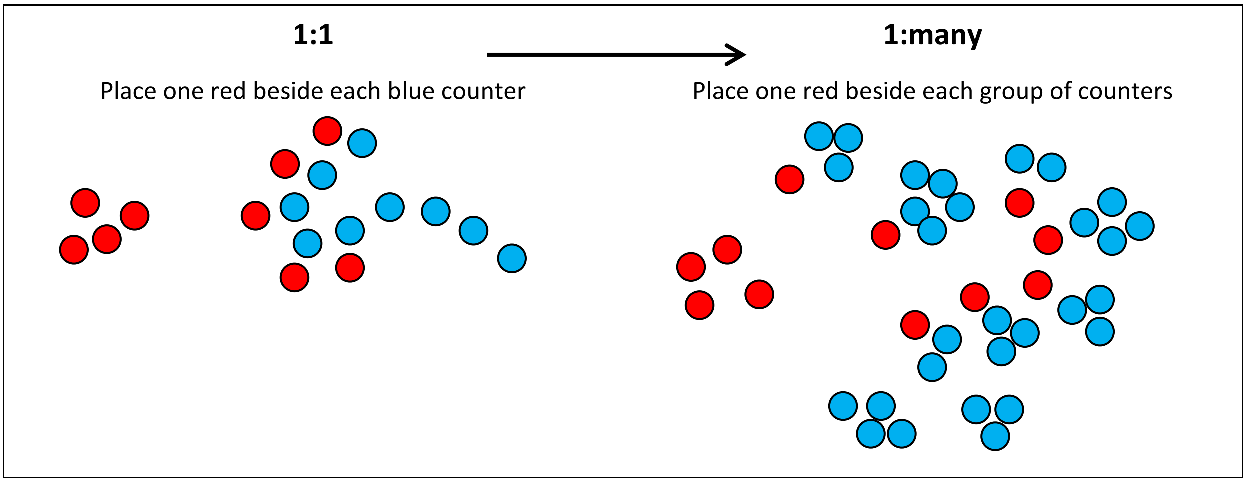
*Validation*: Students check to see whether objects in their world, e.g. books, toys, have the same number as the first time they are counted when they are arranged differently.

*Extension*:

- *Flexibility*: Identify amounts to five within larger collections and show how the numbers may be arranged in different ways.
- *Generalising*: The number will still be the same if we change how the blocks/counters look.

## 1.6 One-to-one and one-to-many correspondence

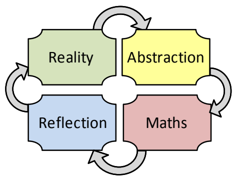
This is to move students away from dependence on 1:1 correspondence (and, later, rational counting) towards the 1:many counting that will have to be done as numbers get large (and then small). This is the process of moving students from 1:1 to 1:many, as in diagram below. This enables us to extend from counting in ones to counting in twos, fives, tens, hundreds, and so on. It begins by showing that one can correspond to a group of things (as below).




### 1.6.1 Ideas and activities for 1:1 and 1:many

1. Initially put one thing with one thing, then put one thing with a group (all groups the same), then put one thing with a group (groups different in number, size, colour and so on).
2. Go past object-to-object correspondence to name-to-object correspondence (naming students) to name-to-group correspondence (naming a class).
3. Set the table, e.g. one plate, one fork, one knife; cups and saucers; pairs of shoes and socks then one plate for spoon, knives and forks. Use birthday claps to relate numbers to age.
4. Make age groups with students' photos/names. Compare the groups. As students have birthdays ask student to move their name to appropriate group. Invite all students to describe what has happened. Ask: *Which group must get smaller? Why? Which group got larger? Why?*
5. Make a poster of students who have lost teeth/who haven't lost teeth. Compare 1:1.
6. Each student makes a Unifix cube train to match the number of students in the group. Ask: *How can we check this?*
7. Play totals. Each student needs a paintbrush and a pencil. Ask a student to get the brushes and another to get the pencils. Compare how many each has. Ask: *Is it possible to have two different amounts? Why? Why not?*

### 1.6.2 RAMR lesson for 1:1 and 1:many




Learning goal: Understanding one-to-one matching "Same for all"

Resources: Paper, small balls, array of sporting equipment, or art equipment, or construction equipment (depending what you have at hand).

## Reality

*Local knowledge*: What situations require that everyone has one of something?

*Prior experience*: Identifying, describing, and matching. Sorting, comparing, and ordering.

*Kinaesthetic*: Whistle ball – Teacher blows whistle, say, peep, peep. Students run to ball baskets and take the number of balls to match the whistle. In this case, two peeps so two balls. Repeat with varying number of whistles. Ask: *Have you the right number of balls? What if I blow whistle more times? Will there be enough balls? How could you check?*

## Abstraction

*Body*: Organise the class into different-sized groups. Select a student in each group to get enough paper for each member of their group. Ask: *Have you got the right amount of paper? Will you have to come back for more or to return some? How will you know? How could you check? Will counting help?*

*Hand*: Play delivery truck. Have a student deliver "fruit" from a basket to each student group. However each student group has to put in an order (phone, post-it note, etc.) Ask: *How will the delivery person know they will have enough fruit?*

*Mind*: Have students visualise how many flowers they would need to give one to each person in their group. Have them visualise their family. Now give them each a present. *Visualise you and three friends going to the dodgem cars. Can you see each of you getting in a dodgem car?*

*Creativity*: Choose two friends. Now choose a collection for each friend. Draw the collection for each of them and one for you.

## Mathematics

*Language/symbols*: Ensure each student has the opportunity to discuss their reasoning and portioning.

*Practice*: Play "Enough for all". Invite students to suggest ways they can check if there will be enough equipment for different numbers of people in different situations. For example, collect enough plastic cups for group at the table; collect enough bean bags for whole group activity, etc. Give a reason for bringing just enough beanbags for the group; for example, say: *Another class wants the leftover bean bags, so we can't take the whole box. How will we know when we have enough bean bags for our group? How could you check to see if there will be enough bean bags?*

*Connections*: Making groups.

## Reflection

*Validation*: Make and place orders to tuckshop.

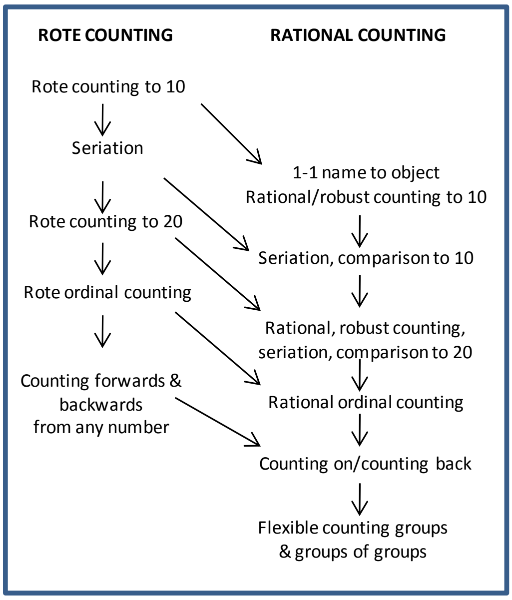
*Application/problems*: What different groups can you individually make holding hands?

*Extension*:

- *Reversing*: Use a basket of shoes. *I have 12 shoes. How many people can have a pair of shoes?*

# Unit 2: Introducing Rote and Rational Counting

This unit is based around the principles students need to internalise for counting a collection if they are to fully accept that counting "works" and must always give the same answer each time. This requires a focus on rote, rational, robust and flexible counting as in diagram on the right.




**Rote counting** is reciting the whole number names in their right order beginning at one. **Rational counting** checks a collection one by one in order to say how many are in it or to count out a subset (using rote counting in 1:1 correspondence with what is being counted). **Robust and flexible** counting is when students can count in a variety of different ways involving different orders of objects. This needs experience with different sequences and types of learning activities.

As the first unit on rote and rational counting, we restrict activity to numbers up to 20. We also restrict activity to models and language. Symbols or numerals are the focus of Unit 4.

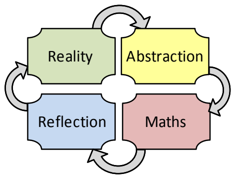
## 2.1 Pre-rote and rational counting

To prepare for rote and rational counting, we need to build on the pre-counting concepts and processes of Unit 1. There are also some particular activities from pre-number that particularly prepare for counting. We shall call this sub-section pre-rote and rational counting and begin with it.

### 2.1.1 Ideas and activities for pre-rote and rational counting

1. When children begin to count, they often use their own words and their own repeated sequence of words before learning the conventional sequence. This is important and has a role to play to establish 1:1 correspondence and make comparisons with objects they count (rational counting).
2. As well, rhythm, music, movement and clapping activities are influential in developing children's early awareness of the pattern of how we say, read, and write numbers. This section looks at this; in particular, the role of invented words and sequences.
3. To lead into rote counting, use the following activities:
   (a) say number rhymes, make up number rhymes, involve parents and the community; and
   (b) allow students to make up number names and orders as long as they keep working towards the correct one. Let them use their incorrect sequence for one-to-one correspondence and to tag objects.
4. To lead into rational counting, use the following activities:
   (a) making groups/collections of objects and finding/making paths through the collections;
   (b) separating objects into those that have been "marked" and those that have not been marked;
   (c) moving collections (changing how they are spread out) and seeing this as no change;
   (d) pulling out an object for each name as they state names in order; and
   (e) repeating the subitising activities with small numbers.

### 2.1.2 RAMR lesson for pre-rote and rational counting




Learning goal: Subitise small collections of objects.

Resources: Cards with pictures of numbers arranged in different ways (e.g. straight line, array, circle), Unifix cubes, flash cards, small toys, counters, blocks, pocket dice.

#### Reality

*Local knowledge*: Where do we find small collections of things?

*Prior experience*: Sorting, ordering.

*Kinaesthetic*: Sing number songs with appropriate objects collected into whole group circle or tell/read story.

#### Abstraction

*Body*: Play "Body number game" – this makes numbers with students' bodies. Play "Clumps game" – roll special pocket dice and move into groups of 1, 2 or 3.

*Hand*: Unifix cubes – students in circle – each child has three cubes – arrange the cubes into a pattern (e.g. line of three, group of three [2 and 1; 1 and 2]); 3 ones – add another cube – show different patterns; add 5$^{th}$ cube (*How many different patterns can we make for 5?*). Repeat for 6$^{th}$ cube.

*Mind*: Animal flash cards – students in circle with Unifix cubes – they respond to each card held up by putting the same number of cubes in front of them on the floor.

#### Mathematics

*Language/symbols*: Groups, collections, how many, same/different, more/less, one more/one less.

*Practice*: Facilitate becoming familiar with ideas as follows:

(a) Clown – counters on pants by die roll till all 10 are covered;

(b) Gingerbread men with dots – match to plate with numeral;

(c) Teddy bear race – roll die and move the corresponding teddy one space; and

(d) Humpty matching game – roll die and match dots on pieces and place on Humpty.

*Connections*: Comparing, ordering, and sorting.

#### Reflection

*Validation*: Hold up cards with dots quickly. *How many did you see? How do you know? What did you see?* Play snap with subitising cards. Play "subitise footprints game" with objects, coins, and dots.

## 2.2 Rote and ordinal counting

To count can mean to recite numbers in the right order. It can also mean to check a collection one by one to say how many are in it. Rote counting is the first of these. Thus rote counting is remembering to put the right names in the right order, that is, remembering the number names in the right order. In contrast, rational counting has the added meaning of putting names and objects in 1:1 correspondence and being able to use the last number name to decide how many.

Rote ordinal counting means that students can remember the ordinal number names in order, that is, first, second, third, fourth, fifth, sixth, and so on.

### 2.2.1 Ideas and activities for rote and rote ordinal counting

1. Rote counting is important for rational counting, in fact for all counting. It requires stating the counting numbers always in the same order. Students need to learn to do this in different ways and in different orders. The two ideas they are consolidating here are:

   (a) each object to be counted must be touched exactly once as the numbers are said; and

   (b) the numbers must be said once and always in the conventional order.

2. Rote counting is achieved when students can say the number names in the right order without worrying about whether they touch one object as they say each name. For example:

   *one, two, three, four, five* (good enough for starting Prep student)

   *six, seven, eight, nine, ten* (even better for starting Prep student).

3. Rote counting includes counting for whole numbers and in later years, decimal numbers, common fractions, mixed numbers, measures and percent. Counting begins with rote counting (i.e. saying the number names in order), moves to using the names to count out a set, goes on to applications such as comparison, conservation, ordination and labelling, and then on to advanced counting skills such as counting on, counting back, flexibility, and counting to and through 100.

4. If students cannot say the number names in order, then they need to sing a lot of nursery rhymes, like "One, two, buckle my shoe …" (traditional number rhyme) or "One, two, wallaby stew …" (a number rhyme made up by Indigenous teacher aides), and so on.

5. The aim for Prep students is to: (a) count forwards up to 20 and beyond from a given number, and (b) count backwards from a given number (to 10).

6. Other activities include the following:

   (a) Counting as students do something active (e.g. walking along a line – forwards and backwards);

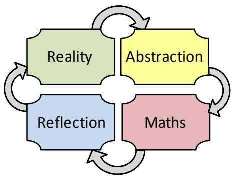
   (b) Saying number rhymes/songs, e.g. Ten in a Bed; Five Little Ducks;

   (c) Setting up calculator to count on (+1) and backwards (−1) and saying numbers as pressing equals;

   (d) Playing number games such as Snakes and Ladders, counting circles, Hopscotch, number ladder, number strip, and so on; and

   (e) Recording oral counting to play back.

### 2.2.2 RAMR lesson for rote counting




Learning goal: Recognising numbers 1 to 3.

Resources: Items from classroom (small toys) and from nature (leaves, stones/pebbles, seeds, shells).

#### Reality

*Prior experience:* Rote counting – collections at home, in day-care/kindy, candles on birthday cake matching age.

*Local knowledge:* Noticing numbers on road signs, shops, clocks, car number plates, TV, phones.

*Kinaesthetic*: Writing numbers in sand, feeling numbers on card – numbers made from fabric glued on card or drawn on card with glue and sand scattered over number shape (and allowed to dry).

## Abstraction

*Body:* For each number 1, 2 and 3 in turn – items counted to get the numbers; the numerals are drawn and the students copy the number with their bodies. Roll over any students who are inverting the numbers.

Students write numbers on the back of a friend. Friend guesses what is being written on their back.

*Hand:* Match a number card with the amount of objects; write the number on a piece of paper. Make number 1, 2 and 3 out of play dough.

*Mind:* Visualise the number – teacher gives directions for movement of fingers, students move fingers and teacher checks that students are doing it correctly.

*Creativity:* Students make up their own symbols for the numbers.

## Mathematics

*Language/symbols:* Say name ←→ students draw symbol. Extend to pictures and materials.

*Practice*: Facilitate becoming familiar with idea: Hide objects behind a piece of fabric. Reveal the objects briefly. Students say how many they think are there or hold up a card with the number on it. Add or take an object away. Repeat, hiding a different number of objects behind the fabric.

*Connections:* Spend time on going from symbol to language to actions to objects to pictures and then reversing.

## Reflection

*Application/Validation:* Do a poster of ones, twos and threes in the world. Do a walk around the school to find ones, twos and threes in the various forms (three objects, symbol 3).

*Extension:* Game: Clown with circles drawn on the pants. Child rolls die. Child counts out that number of counters. The counters are put onto the circles on the clown's pants. The next child rolls the die and does the same.

## 2.3 Rational counting

Rational counting means students understand and use the following five counting principles:

1. Each object to be counted must be touched or included exactly once as the numbers are said (**one-to-one correspondence)**. This usually also needs the ability to plot a path through the objects to achieve this. It also requires being able to keep track of the objects that have been counted from those that have not yet been counted.
2. The numbers must said once and always in the conventional order (**rote counting)**.
3. The objects can be touched in any order, and the starting point and the order in which the objects are counted does not affect how many there are (**robust counting**).
4. The arrangement of the objects does not affect how many there are (**robust counting**).
5. The last number said tells how many in the whole collection. It does not describe the last object touched (**cardinal principle**).

As well as this, students move to understanding flexibility in counting. Groups and groups of groups can be counted as well as individual items and we have to move easily between different interpretations (e.g. both of these are correct – John had 36 eggs; John had 3 cartons of eggs) (**flexible counting**).

### 2.3.1 Ideas and activities for rational counting

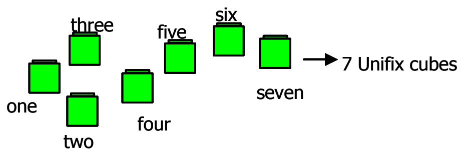
1. One crucial skill in number is putting number names in one-to-one correspondence with objects and one-to-many with groups of objects – one number name for each object or group of objects. One idea here is that this skill can be assisted by putting one set of objects in one-to-one correspondence with another set (e.g. one egg for each eggcup, one knife for each fork, one fish for each fishing line, etc.). The evidence for this particular approach is not strong. There appears to be evidence that putting two sets of objects in one-to-one correspondence with each other is **more difficult** than putting number names (words) in one-to-one correspondence with objects.

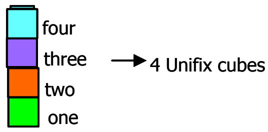
2. The one-to-one correspondence of number names and objects also requires students to be able to:

   (a) choose a path through the objects so that objects are counted once;

   (b) separate in the mind those that have been counted from those that need to be counted; and

   (c) know that the last number said tells how many in the whole collection (it does not describe the last object touched), as below.



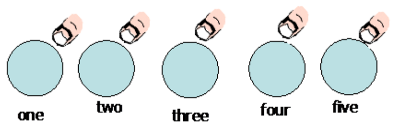

3. Rational counting also requires that students can:

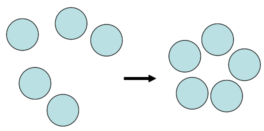
   (a) *count a set* – point to (or touch) each object in a set and say one number name for each object, as shown in the figure below; and

   (b) *count out a subset* – identify a given number of items from a larger collection – requiring knowing that the last number name tells how many.

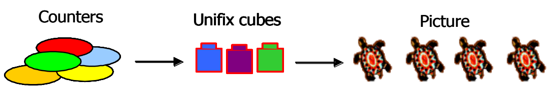
4. Rational counting should be applied to continuous (e.g. number line) as well as discrete situations (e.g. sets). It is important that students, from the start, count in a number-line sense as well as a set sense, i.e. count steps, count along a number track, and play games like Snakes and Ladders.

5. When rationally counting, the student has to keep track of objects – remember which objects s/he has counted, and which objects s/he still has to count. This is easy to do in a line, but less so in a circle, as below.



6. Discrete materials should be used for younger students when they begin learning, starting from *real-world* materials and then replicas of the real-world materials, such as toys. When students are able to count rationally, *concrete* ("hands on") materials can be used, such as counters and then Unifix cubes. Following this, students should be asked to count things in *pictures* (see figure below). This is more difficult as students cannot pick up or move the objects in a picture. Finally, students have to know that they can apply number to even imaginary discrete things and to number lines.



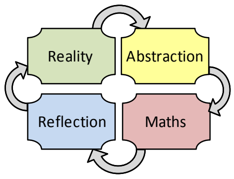

7. It is important that while students can count out a subset as well as a set, they can always go from object to number name <u>and</u> number name to object (**reversing**).

8. Further activities:

   (a) How many? Count themselves/cakes/chocolates and so on. Form students into groups – check by counting, go both directions: group → number, and number → group (**reversing**). Label one name per item, e.g. stickers.

   (b) Scatter students, walk through touching each one; do same with objects; draw lines touching points so none missed (begin with easy scatterings – nearly a line, nearly a circle, close to an array, etc.). Stop about halfway through and state the number counted and yet to be counted.

### 2.3.2 RAMR lesson for rational counting




Learning goal: Understanding rational counting.

Resources: Numerous classroom counting materials, small containers with 20 counters, one for each student.

## Reality

*Local knowledge*: Identify when and where students use counting going about their day-to-day business.

*Prior experience*: Pre-counting concepts and activities.

*Kinaesthetic*: Play "Beat the Teacher". Set a number target, say 16. Roll a die, the teacher takes that many items; students roll the die, they each take that many items and so on; first to get to the target wins. Repeat with different targets. As you go, ask: *How many do you have now? How do you know? How many do I have? How do you know? What if I grouped them like this (in twos), do I still have the same number?* Count the total as a whole group together. Count the total of yours and the students together.

## Abstraction

*Body*: Have students establish how many jumps/hops/skips they can do. Record daily and compare. Do it in pairs so other student verifies.

Do student number trains – count down who is present or are in line. Ask: *Could we find out how many are here if we count by twos? Will we get the same number?*

*Hand*: Have students count and label large collections of objects. Have other students verify the count. Ask: *What did you do to count all the objects? Does it matter where you begin?*

Play bead string counting games.

*Mind*: Have students visualise numbers of items and then count them.

*Creativity*: Create displays of items of the biggest number students can make. Build on it daily. Ask: *How can we check this?*

## Mathematics

*Language/symbols*: Number names, forwards, backwards, group counting, numerals, conventions for recording counting.

*Practice*: With collections of counting items on hand, play "which number is more?". Students have to say which of two numbers is more. Then invite them to explain how they could convince someone this has to be.

*Connections*: Counting on counting back; more/less.

## Reflection

*Validation*: Have each student make a thinkboard to associate five different ideas about a given number.

*Extension*:

- *Flexibility*: How many different counting patterns can you make about the number 3?
- *Reversing*: I am thinking of a number of beads that is more than 21. What might it be?
- *Generalising*: Create a number counting pattern starting at 17.

## 2.4 Robust counting

Robust counting is the understanding that objects can be touched in any order, and the starting point, ending point and the order in which the objects are counted does not affect how many there are. It is also a skill in being able to count in situations where it is difficult to determine order and to separate those that have been counted from those that have not yet been counted.

In rational counting students first combine rote and sorting/correspondence skills to be able to put number names one-to-one or one-to-many with objects or groups and are able to chart a path through the objects or groups so that none are missed or counted twice. To ensure counting is a robust knowledge for the students, give counting activities where the start and/or finish are identified beforehand and are not at the start or finish of any line or identified grouping. These counting activities (shown below) are more difficult than the scattered arrangement or the circle because students are given a starting or a finishing point that is not at the end of a line.

### 2.4.1 Ideas and activities for robust counting

1. Robust counting can be affected if students do not know:

   (a) any discrete objects can be counted;

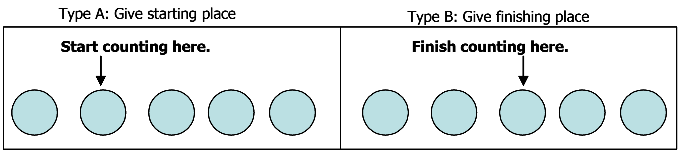
   (b) the conventional number-name order is always required (stable order principle);

   (c) how to count small collections with one-to-one correspondence; and

   (d) the arrangement of the objects does not affect how many there are.

   Check these understandings before starting. Robust counting should follow success with rational counting.

2. A starting way to check and teach robust counting is to provide a collection of objects and stipulate the starting point, ending point or both. Start with objects in a line (as below) and then move to circles and irregular patterns. These activities help students to understand that counting is "robust" – the quantity in a collection remains the same when the count starts with different objects and when the start and finish or both are given to you.




3. It is much more difficult to count when the objects to be counted are scattered and most difficult when the objects are arranged in a circle. This often requires the student to be able to determine a **visual path or pattern** through the objects which includes all items, does not repeat any, and knows when to stop. Students need experiences counting along lines (e.g. counting steps) as well as objects.

4. Some further ideas and activities are as follows.

   (a) scatter counters with one blue and one red. Ask students to start to count at red, to finish counting at blue, or do both;

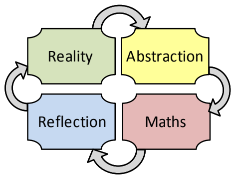
   (b) count different arrangements – in a line, in a circle, having a pattern to the arrangement, totally irregular – relate to choosing a path through objects and keeping track of those that have been counted;

(c) count forward and back on the 99 board, particularly what happens when bridging tens, that is, when the second numeral is a 9;

(d) point to, touch or move aside one object with each number (enables the understanding to be bridged or scaffolded from rational counting); and

(e) count a variety of objects – real objects, drawings, imagined images, jumps, skips, turns, indoors and out of doors.

### 2.4.2 RAMR lesson for robust counting




Learning goal: Learning to trust the count.

Resources: Countless counting materials.

## Reality

*Local knowledge*: What needs to be counted in this community? (e.g. trees, trucks, cows, birds, etc.)

*Prior experience*: Rote counting.

*Kinaesthetic*: Go outdoors and find things to count that are in position, e.g. trees, garden beds, etc. Discuss that these are arranged in different ways but have the same number. Ask: *How is this so?*

## Abstraction

*Body*: Count all the fingers/toes/feet/hats and so on in the room. Rearrange the people/things and count again. *Is the number the same? Is it different? How can this be so?*

*Hand*: Have students count collections. If students arrive at different totals ask: *Could we all be right? Why? Why not?* Have students rearrange collections and count again. Ask: *Have you the same number? Why/Why not?*

*Mind*: Have students visualise a collection and count. Have them rearrange the collection in their mind's eye; count again, etc.

*Creativity*: Look at counting when it is difficult – like counting students on an oval at lunchtime when they are all running around. How do farmers count their sheep when the sheep are always milling around? Can students think up a way to estimate counts – not accurate but near enough?

## Mathematics

*Language/symbols*: More, less, the same number, one by one, forwards, backwards, beginning, end, group, collection, how many, total.

*Practice*: Invite students to rearrange collections of things to make them easier to count e.g. *In our class, how many students are at school today?* Ask: *Can we arrange ourselves so it is easy to count? Is there another way?* Have them record what they do and their totals. Ask: *What do you notice about how many we get every time we count? Why don't we get a different number if we start with a different person?*

*Connections*: One-to-one correspondence, group counting, one more than and one less than, patterns.

## Reflection

*Validation*: Have students make a collection for their partner to count. Check totals. *How did you do this?*

*Application/problems*: Have students choose and use materials to show why nine is less than ten when counting collections. Have students find the same total when counting different collections. Ask: *How can you be sure?*

*Extension*:

- *Reversing*: Here is the count/number; make/show me the number of items.
- *Generalising*: Have students find everyday counting opportunities and explore how counting is used every day.
- *Changing parameters*: Have students use the constant function on a calculator to count things.

## 2.5 Flexible counting

Flexible counting has three important outcomes with respect to ways that counts can be made other than counted 1:1.

1. **Any collection has only one count**. Robust counting shows students that a collection cannot have two counts. This is important for later number work. For example, trusting the count means that we can tell which collection is bigger by the count alone. For example, if we have two groups with counts 21 and 22 respectively, it must be obvious to students from the numbers alone that the collection of 22 will always have more than a collection of 21. Until they realise this, they cannot fully understand numbers as abstractions with properties of their own such as 22 is greater than 21.

2. **It is possible to expand from 1:1 to 1:many without changing the count**. Group counting is crucial for students' development of place value with its tens and ones. Thus, it is important that students learn to:

   (a) use equal groupings or parts to help count large collections;

   (b) skip count by reciting every second number or jumping along a number line saying 2, 4, 6, and so on;

   (c) subitise and add, that is, see six objects as, for example, two objects and another three objects and a final single object joined together; and

   (d) realise that subitising and adding and skip counting tell how many, the same as 1:1 counting, with the same trust of the count as in 1:1 counting.

   It should be noted that students need lots of practical experience in order to see that pulling out two at a time and counting by twos, subitising and adding, or counting in groups give the same answer as if they had counted by ones. Trusting that all the different ways of counting give the same number is key to saying that a student is able to count. Thus, groups and groups of groups have to be counted as well as individual items and we have to move easily between different interpretations (e.g. both of these are correct – John had 36 eggs; John had 3 cartons of eggs).

3. **Counting on and counting back**. It is important that advanced counting techniques such as counting forward, or counting on, and counting backward, or counting back, can be undertaken, again with trust of the count. Group counting forwards and backwards from any number is an essential skill for the later development of place-value understandings. Counting backwards is often overlooked and needs to be revisited with all new numbers learned.

### 2.5.1 Flexible counting ideas and activities

Ideas and activities for flexible counting are around using other counts than 1:1. This means advanced counting in its different forms (e.g. counting subgroups, skip counting, subitising and adding, and counting on/back) all have to be experienced. Group counting when objects are not presented as a single collection is the basis of the other advance counting such as 1:many correspondence and 1:many counting as in place values. Some examples that should be experienced by students are as follows:

(a) *abstract counting* – counting imagined objects;

(b) *composite counting* – counting when there are groups or groups of groups, for example, counting the number of lollies when they are in bags (with same or different numbers of lollies in each bag); and

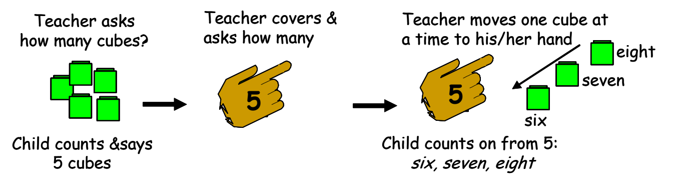
(c) *measure counting* – counting the number of groups of objects and ignoring how many in each group (e.g. forming bundling sticks into groups of 10 and then counting only the number of tens).

Some further ideas and activities for group counting are as below.

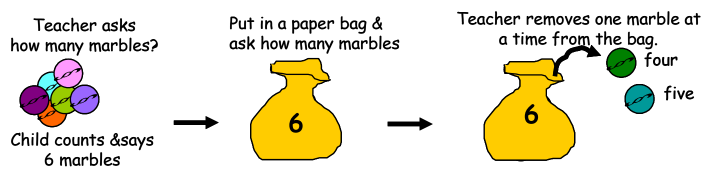
1. Count objects placed in groups in different boxes by counting each box and joining, or counting number trains of carriages with objects in them. The group/carriages of objects can be the same or different.
2. Partition large group into small groups and count by groups, matching numerals to the right number of items in the groups and in the total. Do this to divide objects between people so there is "enough for all". See if changes in numbers affect the division into groups. Look at all the ways a large group can be divided into small groups. Ask: *Which is easier/easiest to count?*
3. Use constant function on your calculator to practise skip counting and use this skill to skip count large collections including money.
4. Play "which number is bigger" when composed of groups, and play "running totals" with dice. Act the dice game out on a racetrack – first to finish wins.
5. Place orders by having students use plastic food or other props for role play. One child can be the shop owner to whom the other students have to phone their orders. Students have to think about whether they have enough of each thing for their group. Ask: *How will the delivery person know that there is enough food for everyone? How will counting help the delivery person to bring enough food?*
6. Start skip counting for twos and threes, and continue the pattern – use the pattern to make a staircase out of blocks/paper with levels labelled, or threaded beads of own choice of colours.

**Counting on and back** are advanced counting activities that are part of flexible counting. They involve counting forward and backward. They can be taught using the methods below:

**Counting on** (example 5 counting on 3)




**Counting back** (example 6 counting back 2)

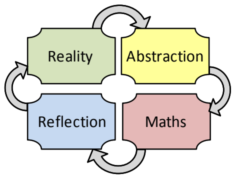



Some further activities for counting on and back area as follows:

1. Get students to imagine a ladder or the YDM strip mat and walk it in their minds – walking up and back (e.g. start at 6 and walk up 3 more or at 5 and walk down 2). Get students to make up their own names and symbols for the ladder and walk these.
2. Build a large racetrack and run forward and backward races, where students count on and back numbers from dice – have positions marked with a star where students have to take a card when they land on the star – cards require counting on and back.

*Note*: It is important, particularly for flexible learning, that students both count discrete objects (sets), and count along continuous lines (number lines). The number-line model can be experienced by counting steps and using a number track, and with games like Snakes and Ladders and Ludo and any other board games where they have to count along lines, or games where they have to take the number of strides that they are told or work out (such as the number of times they have the letter "N" or "T" in their name). Body and hand activities with both discrete and continuous are used to encourage students to imagine and gain mental models of numbers **as sets of objects or steps along lines**.

## 2.5.2 RAMR lesson for flexible counting




Learning goal: Students can count in different ways.

Resources: Counting materials, counting boxes.

### Reality

*Local knowledge*: When do we need to count large amounts?

*Prior experience*: Pre-counting activities, rote and rational counting.

*Kinaesthetic*: Whistle groups. Play music, students move freely around space. When you blow the whistle (music stops) hold up a number, students make groups of assigned number, e.g. 3. Students check their groups then ask: *How many groups of three?* Count the total students in threes.

### Abstraction

*Body*: As above.

*Hand*: Sorting and counting into containers for partners.

*Mind*: Imagining other groupings.

*Creativity*: Students create their counting activities/items/ arrangements.

### Mathematics

*Language/symbols*: More, less, the same number, one by one, forwards, backwards, beginning, end, group, collection, how many, total, skip counting, group counting.

*Practice*: Have a collection of takeaway food container boxes with a variety of items. Each box has similar items within and is different from other boxes in both item and number. Have students predict how many are in the box. Have them count the contents in groups of three or four students. Ask: *How did you count it? How did you decide how to count it? What is your total? How do you know?* Compare how different groups are counting their items – Ones? Twos? Threes? Fives? Tens? and so on.

Have them count their items a different way. Ask: *Will you get the same total? Why? Why not?*

*Connections*: One-to-one correspondence; one more than, one less than; patterns; number multiples.

### Reflection

*Validation*: Have students count collections of different items that all have the same total. Compare their counts.

*Application/problems*: Fifteen students are at school this morning; seven more arrive late. How many are here now? How many different ways can you count that?

*Extension*:

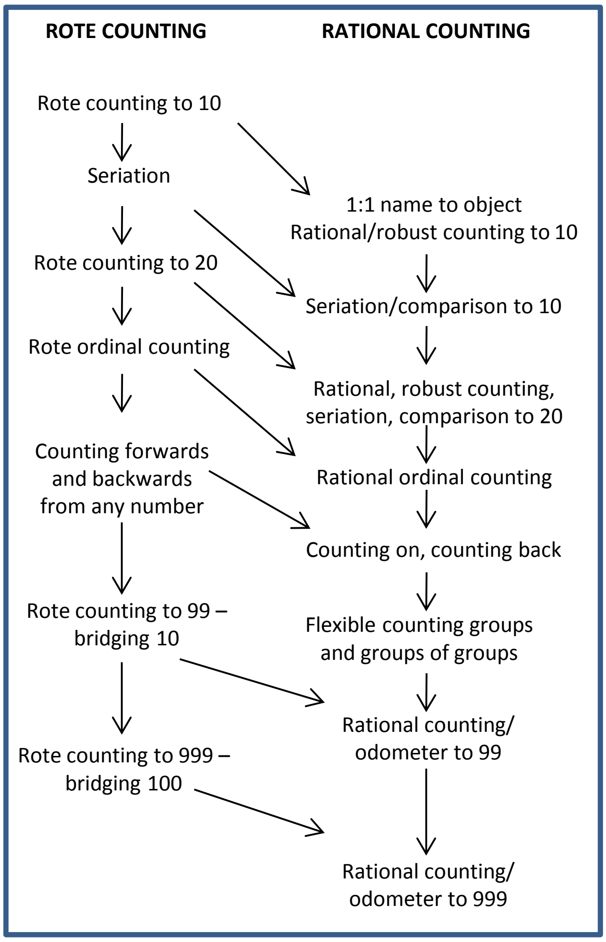
- *Flexibility*: Count on in threes from your total for four counts. How many would you have now? Count back in threes from your total for five counts. What number have you now?
- *Reversing*: Here is the number, make the collection.
- *Generalising*: How far could you count with your chosen strategy of counting?
- *Changing parameters*: Count your collection in fours (if they have counted in twos or ones etc.)

# Unit 3: Bridging and Counting Patterns

This unit extends the rote and rational counting of Unit 2 to two-digit and then three-digit numbers, as in the diagram on right. It does this by focusing on the pattern in the numbers as they are said (counted out aloud), forward and back. Obviously, the pattern in the numbers is easier to see when there are numerals, as numerals do not have the zero and teen problems of spoken numbers. However, **this unit focuses on oral patterns**. The numerals are presented in Unit 4. They can then reinforce the number pattern.

Two of the learning challenges for young children are realising that the numbers have a particular order and remembering the order. We recommend that this **largely occurs orally**. This means that we have to assist students from an early age to notice the patterns in the way we say the numbers. Students need to understand that they do not have to remember every number name **because the patterns in the numeration system enable us to predict a number** even if we have never heard it before.

In this unit, we look at the counting pattern before numerals. This includes time on oral versions of seriation and odometer principle and comparison and order.




## 3.1 Oral counting pattern

When students know the number names to ten, they have all they need to count to trillions or thousandths, and to count fractions. This counting should be reinforced across all the years of schooling. At the beginning, once they know the number names to nine, students should be encouraged to count further. Initially, they need to do this often until they can say all the number names in the right order to twenty and hundred.

The basis of the pattern is counting to nine. Regardless of what else is said before the counting numbers, the counting numbers roll on in sequence, for example,

**one two three** ... **seven eight nine**
fifty **one** fifty **two** fifty **three** ... fifty **seven** fifty **eight** fifty **nine**
two hundred sixty **one** two hundred sixty **two** ... two hundred sixty **eight** two hundred sixty **nine**

Once we hit "nine", we start again but with a zero and different words before each count.

### 3.1.1 Ideas and activities for oral counting pattern

In order to develop the capacity to generate any numbers in sequence students need to work through the following understandings.

1. **Word sequence**. Students need to learn to say the number names in order into the teens since there is no inherent pattern in the sounds. They need to memorise the 1 to 13 words in sequence. Ways to do this can include having students play games that involve chanting numbers; displaying collections and their matching numbers; and having students counting aloud matching the count to the rhythm of actions.

2. **Hear the words**. Students need to be able to hear the 4 to 9 part of the sequence in 14 to 19. Display collections of 14 to 19 objects and their matching numbers; making, displaying and using number lines and chunks of number lines. Asking: *What sounds the same about the new numbers? How does each number sound different from the others? What comes after 13? What parts of the twenties sounds the same as the thirties?* are some useful strategies.

3. **Bridging the decades**. Students need practice to predict and name the decades by following the 1 to 9 sequence, then repeating the 1 to 9 sequence within each decade. One way they can do this is to generate number sequences using the constant function on their calculators over the decades while reading, saying, predicting and verifying the numbers from the calculator display. However, this is using the numerals – but in reality Units 3 and 4 will overlap and numerals will work with language to build the pattern of counting. Later place value will be added to the mix to strengthen the pattern.

4. **Bridging the hundreds**. Next, students need to be able to predict and name the hundreds by following the 1 to 9 sequence; then repeating the decade sequence and 1 to 9 sequences within each of the hundreds. One way to do this is play "Biggest Number" to build this understanding. Select students to write the biggest number they know at the top of a display board. Ask each student: *What is one more*? Write the new number beneath the first. Then have students add to the sequence each day and say the new number. Ask: *Can this number sequence come to an end*? Select a number close to a hundreds to begin activity on other occasions. Once students know numerals, extend the number/hundreds charts and use flip cards to assist as well.

5. **Bridging the thousands**. Once students know the pattern, it may be useful to extend it to thousands. To do this, the next step is to ensure students can predict and name the thousands by following the 1 to 9 sequence and repeat the hundreds, tens and 1 to 9 sequences within each of the thousands. Extend the range of the Next Number, Biggest Number and Hundreds charts activities above to include numbers in the thousands. Ask: *What number should be next*? *How do you know what number should be next*?

6. **Extra practice for teens and zeros**. Because these do not follow the oral pattern, special attention needs to be given to these cases.

There are many activities to assist bridging tens (e.g. thirty-eight, thirty-nine, forty, forty-one, and so on) and bridging hundreds (e.g. three hundred and ninety-eight, three hundred and ninety-nine, four hundred, four hundred and one, four hundred and two, and so on). Some are below.

1. Have students decide each day (week) how many jumps/hops etc. to include in their fitness routine and record number. Ask students to decide whether they need more or less of each action and to record this new number.

2. Each player (or group of players) throws two dice and counts on, from where they were last time, the number of dots shown. First past or to a given number (say one hundred and eleven) dots wins. Also count back.

3. Climbing ladders: Students have toys progress up and down ladder frames. Roll die to determine how many rungs to progress. They have to determine how to keep track of their progress.

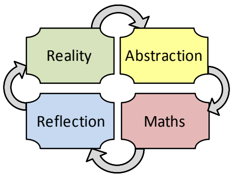
4. Magnetic game boards and dice games.

5. Jack in the Box: Students chant various number sequences e.g. through the teens, or 29–49 etc. Have students choose a number, say 7, nominate a "Jack", and every time the students chant a number with a seven in it the "Jack" has to jump up. Ask: *On what numbers did Jack jump?*

6. Play number actions: ball bouncing and count; skipping and count; hoop hopping and count; ball catching and count, etc.

7. Make a number line and change the parameters of stepping and counting. Ask: *What comes before and what comes after the "ty" numbers?*
8. Have students in line count off in sequence beginning at one. Change the starting number to increase the numbers experienced.
9. Build number charts with cash register tape. Begin with varying numbers. Predict: *What will come next?*
10. Have students write the biggest number they know. (They have to be able to read it back.) Ask: *What is one more? Can this number sequence come to an end?*

### 3.1.2 RAMR lesson for oral counting pattern




***Ten On Three Off***

Learning goal: Practising oral counting patterns.

Resources: Beads, long shoelaces, bead strings, chairs, ten frames, counters.

## Reality

*Local knowledge*: Find what needs to be kept track of by counting in the student's world – e.g. numbers of seats when travelling maybe on a weekend away.

*Prior experience*: Pre-counting, rote counting, counting personally significant collections.

*Kinaesthetic*: Students on chairs play/model getting on and off a bus/train/plane.

## Abstraction

*Body*: Act out the getting on and off and keeping count of passengers with chair arrangements. Use toys/stuffed animals to increase the numbers past the 25 students in the class.

*Hand*: Use ten frames and counters to model; use bead strings to model.

*Mind*: Imagine and visualise the events, describe what happens; sing coming/leaving songs.

*Creativity*: Enable students to represent the count with any materials/props/actions they like.

## Mathematics

*Language/symbols*: Number names, especially "ty" and "teen" names; patterns; hearing the pattern; bridging; repeating; forwards; backwards; group counting.

*Practice*: Starting at number 16 say, then ten on, two off; three on, four on, one off, etc. Read the Dr Seuss book and change the numbers to suit.

Put several buses/planes/train carriages together to make larger numbers. Model it all again with bead strings, then with ten frames.

*Connections*: Collections, partitioning, groups.

## Reflection

*Validation*: What did we do? Tell the person next to you. Who needs to know this? When might you use this?

*Application/problems*: Give problems and applications which require the student to follow counting (on and back) when it is most difficult – bridging tens, particularly when near bridging hundreds.

*Extension*:

- *Flexibility*: other ways to represent counting numbers.

- *Reversing:* Students act out story without speaking; the other students have to identify the counting numbers and the count direction (forwards or backwards, more or less).
- *Generalising*: Bridge over 100 and bridge over 10 and 100 together.
- *Changing parameters*: Look at bridging over 1000; use patterns of twelves or eights instead of ten frames.

## 3.2 Seriation and odometer

**Seriation** is the ability to add or subtract 1 from any place-value position. For numbers 1 to 999 this means adding and subtracting 1, 10 or 100 from a number that has these values, for example, 5 + 1 = 6, 34 – 1 = 33, 56 + 10 = 66, 79 – 10 = 69, 349 + 1 = 350, 257 – 1 = 256, 638 + 10 = 648, 607 – 10 = 597, 532 + 100 = 632, 164 – 100 = 64, and so on. In this unit this activity must be done **orally** without using numerals (although, of course, later, we can integrate the two). Thus, we have to remember/understand the pattern in oral names, for example, "thirty-nine plus one is forty", "ninety-six plus ten is one hundred and six", and so on.

**Odometer** is the name for the principle that occurs when counting bridges tens and hundreds. When counting forward by one in ones or tens, the count goes seven, eight, nine in the ones and the ones goes back to zero and the tens or hundreds increase by one. For example, counting forward by ones is

> "fifty-eight", "fifty-nine", "sixty", "sixty-one", and so on.

It does the same when counting forward by ones and tens in hundreds and tens and ones. For example, see below for counting that bridges tens, bridges hundreds and bridges both:

> "three hundred and forty-eight", "three hundred and forty-nine", "three hundred and fifty", "three hundred and fifty-one";
>
> "three hundred and eighty-one", "three hundred and ninety-one", "four hundred and one", "four hundred and eleven";
>
> "two hundred and ninety-eight", "two hundred and ninety-nine", "three hundred", "three hundred and one".

Similarly for counting backwards, the odometer principle is three, two, one, zero and then nine, eight, seven, and so on in the place value in which the counting occurs, while the place on the left decreases by one as the following show.

> "three hundred and forty-two", "three hundred and forty-one", "three hundred and forty", "three hundred and thirty-nine";
>
> "five hundred and twenty-one", "five hundred and eleven", "five hundred and one", four hundred and ninety-one".

It should be noted that seriation and odometer is difficult with oral names because of the teens and not stating zeros; for example, 202, 201, 200, 199 is two hundred and two, two hundred and one, two hundred, one hundred and ninety-nine.

### 3.2.1 Ideas and activities for seriation and odometer

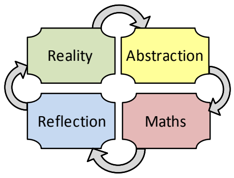
1. Build an oral version of a 99 board, that is, a 99 board with language (oral names) for each number. Use the board to count in ones forward and back, and to bridge tens forward and back.
2. Build an oral or number name version of the 99 board that starts from two hundred and forty and finishes at three hundred and thirty-nine. Use the board to count in tens forward and back starting from any position and to bridge hundreds.

3. Build an oral or number name version of the 99 board that starts from one hundred and thirty-five. Again count in ones forward and back and bridge tens and hundreds.

4. Build an oral or number name version of the three-digit flip numbers – see on right. Each section has ten cards from “zero” to “nine” that can be flipped over so you can practise the odometer patterns as you count.

| 0 | 0 | 0 |
|---|---|---|
| one | four | six |

### 3.2.2 RAMR lesson for seriation and odometer




Learning goal: Experiencing and identifying numbers 1 more, 1 less, 10 more, 10 less.

Resources: Bead strings, ten frames, ten strips, counters, counting materials, collections, skipping rope.

## Reality

*Local knowledge*: Counting out materials to share – where and when in your local community is this required?

*Prior experience*: Say the 0 to 19 numbers, connect them with their names and put them together to represent numbers after 19; say the “ty” numbers.

*Kinaesthetic*: Play skipping in – students run in to skip counting to stay as long as they can. Order each other in terms of how many skips they stayed in for. Clapping rhymes.

## Abstraction

*Body*: Partner ball tosses until a drop. Order pairs – *Who had one more? One less? 10 more? 10 less?* etc. *How many altogether?* Walk the mat.

*Hand*: Make counting window strips and count along; use counting tracks; number boards.

*Mind*: Visualise counters, numbers changing as you count.

*Creativity*: Invite students to represent the increasing numbers any way they can think of.

## Mathematics

*Language/symbols*: More, less, digits, tens place, ones place.

*Practice*: Recognise that the order of the digits makes a difference to the number so 18 is different from 81; the position of the digit tells us the quantity it represents. *What happens to the digits when there is 10 more? Ten less? One more? One less?* Act out and move for each count on the mat or a number ladder.

*Connections*: Relate adding one and subtracting one to counting on and counting back. Look at adding and subtracting tens and relation to the counting patterns of skip counting in tens (e.g. forty-six add 10 is fifty-six).

## Reflection

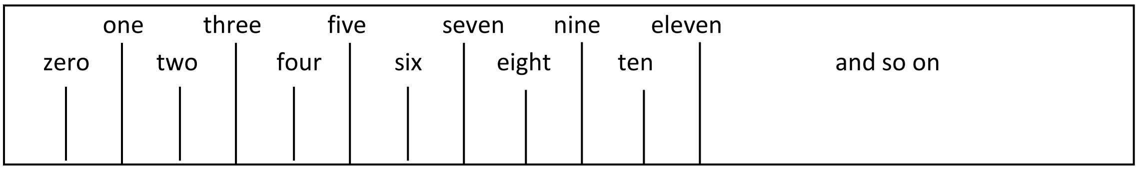
*Extension*:

- *Flexibility*: “Unwrap” a mat. “Wrap” a number track.
- *Reversing*: Go backwards; act out number action, students tell what’s happening.
- *Generalising*: What does the digit zero do?
- *Changing parameters:* Go to larger numbers.

## 3.3 Comparison and order

Numbers can be ordered orally as well as numerically. The simple rule is that if we have two groups and we count them, the larger is further along the sequence of counting numbers, for example, thirty-five, thirty-six, thirty-seven, thirty-eight, thirty-nine, forty, forty-one, forty-two means that forty-one is larger than thirty-six and thirty-six is smaller than forty-one. In this way two numbers can be compared. This comparison can then be used to order, from smallest to largest or vice versa, more than two numbers (by focusing on "betweenness").

The number-line model can also help here. Versions with numbers placed as number names not numerals can show order easily because the number names are in order along the line in terms of distance (and thus counting) from zero, for example:

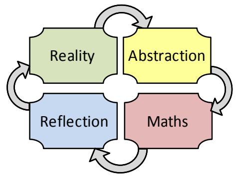



Obviously, nine is larger than six but smaller than eleven.

### 3.3.1 Ideas and activities for comparison and order

1. Pick two numbers, work out where they are in the counting sequence (e.g. *Which is further away from zero?*). Use this information to determine the larger.
2. Construct a number-name ruler and use it to explore why one number is larger than another.
3. Shuffle a card deck for Ace (1) to 9. Each player selects two cards and makes a numeral. Players compare numbers and use counting to determine the larger. Player with larger (further on in the counting sequence) number gains a point. First to five points wins.
4. Use same deck as above. Select four cards, place cards as selected to make two numbers; if number on left is smaller than that on right, score one point; again first to five wins.

### 3.3.2 RAMR lesson for comparison and order




Learning goal: Comparing and ordering – before, after, next to, between.

Resources: Maths Mat, cards with numbers on them, ladder mat, laminated ladders and grids, counters.

#### Reality

*Local knowledge*: What can you find that is between things? Where is placement important?

*Prior experience*: Pre-counting, rote and rational counting, number tracks.

*Kinaesthetic*: Use a rope ladder for students to order and compare objects and number sets. Give out items/numbers to students. Choose one to come and stand on ladder. Choose others to place themselves on the ladder. Ask: *Why did you choose there?* Ask other students where else might they have chosen. Have students rearrange their order. Discuss. Try different set of objects, then sets of numbers.

#### Abstraction

*Body*: Use ladder mat for students to act out before, after, between; do it again with different sets of numbers; extend to grid mat and repeat. Ask: *What is between each item/number placed on the ladder/grid?*

*Hand*: Peg items on a piece of rope, label the order; use laminated ladders/grids and coloured counters for students to order and label and compare.

*Mind*: Visualise order of items. Change the order; describe what you are seeing in your mind's eye.

*Creativity*: Encourage students to find and create their own objects/items display, explaining order and comparing.

## Mathematics

*Language/symbols*: next to, before, after, between, item, order, compare, more, less, reasoning, questioning.

*Practice*: Have students record orders of items they create on grids/ladders and label them; repeat with sets of numbers they are confident to work with.

*Connections*: Sorting, comparing, ordering sets, number lines.

## Reflection

*Application/problems*: Order your family by age. Compare with your partner.

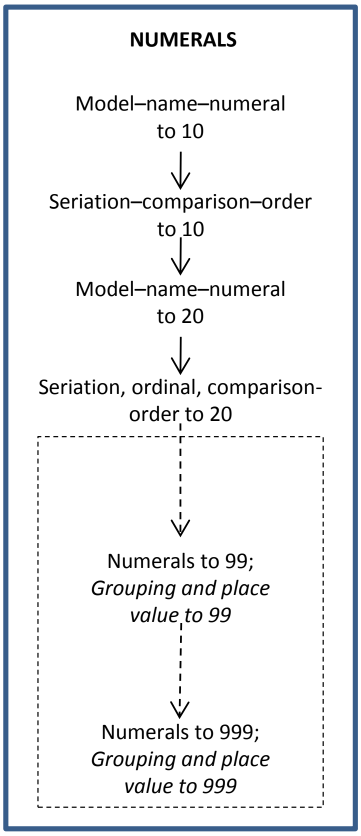
What is the number of your house? What are the numbers of the houses next door? Compare with another student in the group. Are they the same? Are they different? Why might this be?

Order these numbers: 34, 42, 43, 37, 40, 33, 44, 35, 39, 36, 38, 41. How did you do this?

*Extension*:

- *Flexibility*: Use different sets of numbers.
- *Reversing*: Create grids/ladders with missing number. Find the number.
- *Generalising*: Ascending and descending patterns.

# Unit 4: Numerals




This unit looks at introducing numerals for numbers. It follows the diagram on right. The dotted rectangle contains ideas that can be considered in terms of place value, which is being left to the AIM EU Module N2 *Place Value*. In this module, we will consider numerals as counting numbers.

It is important here to remember that when introducing new ideas, we need to consider there are four languages, from everyday child's language to the language of mathematics symbols as follows:

(a) *child's language* – the everyday language that involves mathematical ideas;

(b) *materials language* – the language that comes from using concrete, virtual and pictorial materials;

(c) *mathematical language* – the language specific to mathematics concepts; and

(d) *symbol language* – the symbol for the concept word or abbreviated word.

Children will enjoy communicating what they understand of number in mathematics. Communicating about early number means discussing ideas, manipulating objects, drawing pictures, explaining by way of writing, using diagrams and charts, and expressing ideas in informal and formal symbols. As children mature, communication helps them to organise their thinking, create strategies, and use correct mathematical language. The teacher's role is to model correct mathematical language, provide opportunities for verbal interaction, listen attentively, and to ask questions to clarify students' reasoning. The communicating and reasoning activities here encourage students to discuss and reflect on their experiences and help them connect new number ideas to existing knowledge.

## 4.1 Numerals to 9

Reading and writing numbers involves the use of language, representations of number such as pictures, materials and models, and symbols (numerals). This means that it requires **three different things to be related**. Although this is an extra difficulty, it also offers opportunities for diagnosing weaknesses. Reading and writing numerals to represent numbers can assist teachers to recognise student understandings about numbers.

Writing numbers includes students' ability to recognise symbols as representing a specific quantity of items and to draw the related symbols. When number names are known and can be used to count a set and count out a subset, the next step is to attach the words and symbols for the number names and the numerals up to nine, namely, "zero" 0, "one" 1, "two" 2, "three" 3, "four" 4, "five" 5, "six" 6, "seven" 7, "eight" 8, and "nine" 9, to the oral names and to any representation of that number of objects either with materials and pictures, or imagined in the mind. It is based on relating symbols (e.g. 7) to real-world situations (e.g. seven pencils), models (e.g. set of seven sticks representing pencils) and language (e.g. "seven"). This learning of written names and numerals is often restricted to repetition of the associations. However, as we will see below, there should be **a major use of kinaesthetic teaching** (body and hand) with regard to acting out the shape of the numeral.

Finally, we are looking at numerals 0 to 9 first because these are the only numeral symbols needed to be known. All numbers can be made up from the 10 numerals 0 to 9. This section therefore focuses on how to learn these numerals, that is, relate them to examples and language.

### 4.1.1 Ideas and activities for numerals to 9

1. Symbol language is the mathematics in which numerals are introduced. Along with oral and written number names, the **written numerals are introduced** in the Abstraction stage of the **RAMR cycle** and consolidated in the Mathematics stage.
   (a) Start with a situation from the real world, then model this with sets and number lines. For example, count objects and count along number tracks and number lines – note that the number line, or length, model requires counting *jumps* not starts and ends of jumps.
   (b) Sequence the numbers/numerals: do 1–4 first, 0 second, 5–9 third, and all 0–9 last.
   (c) Move on to using numerals in activities focusing on one more/one less, comparing and ordering, and ordinal numbers (first, second, third, etc.).
2. Introduce numerals through **stories** (ordinary language). Posing stories that match mathematical concepts can be an enriching activity at all levels from stories about addition to stories about rate and percentage. The level of abstraction (language, numerals) in the language of the story can develop over time and be varied to suit particular student or conceptual needs.
   (a) Pose stories as a teacher; for example, "you are five", "I have four bears sitting at the table".
   (b) Move on to using a numeral instead of language; for example, "pick up the numeral that gives your age", "put out the numeral that shows how many bears are at the table".
   (c) Move on to the student posing the story and illustrating with numerals.
3. Introduce symbols with **kinaesthetic activities**. The symbols for the **numerals** can be difficult for students in terms of visual distinction. For example, 2 and 5 have aspects that are reflections of each other, 7 and 1 can look similar and 9 and 6 are rotations of each other. As visual images, they need kinaesthetic experience. Examples are:
   (a) walk numerals (e.g. draw large numerals on the ground and have students walk them);
   (b) make numerals with body (e.g. lie on the floor and contort body to make a 2);
   (c) trace numerals out with hands (e.g. trace in the air with hand many times, and have numerals on felt stuck to cardboard for students to trace out); and
   (d) detect numerals by shape with eyes blindfolded, and so on.
4. Use numerals in the following examples (numbers → numerals AND numerals → numbers):
   (a) groups of students and parts of their bodies, e.g. arms, limbs, facial features, fingers;
   (b) environmental materials, e.g. fruit, shoes, hats, cases, books and pencils;
   (c) represent actual objects with unstructured materials (e.g. counters and blocks) and repeat but in a structured way (e.g. counters placed in dot patterns);
   (d) practise writing numerals – trace them, use them in rhymes; and
   (e) use materials suggested above without reference to the actual objects that these materials represent.
5. Start with natural and everyday materials, then use materials specifically constructed for the study of number, for example:
   (a) counters, Unifix cubes, array mats, number dominoes, bingo, number jigsaws, mind reader, and so on;
   (b) money amounts (twice as many, five times as many, ten times as many); and
   (c) thinkboards – divided into four sections, two sections for models/materials, one for numeral and the fourth for number name.

*Note*: This can include drawings as well as materials, and number-line models as well as set models, if students are ready. Photograph students' number boards to keep a record. May need more than four sections.

6. Sequence in Prep/Foundation as follows:

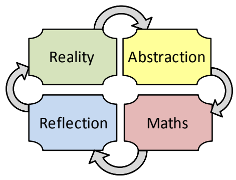
   (a) connect number names to representations (set and line) and represent numbers to nine using objects and pictures;

   (b) introduce numerals, match number names and numerals, and represent numbers using objects, words and numerals;

   (c) attach number names and numerals to collections, and use numerals in counting on/back, considering composite numbers (separate subgroups) and other advanced counting ideas; and

   (d) move on to numerals to 19 and then to larger numbers as students become proficient.

### 4.1.2 RAMR lesson for numerals to 9




Learning goal: Students will match numerals to quantities, rational counting, connecting number names, numerals and quantities, including zero, initially up to 10.

Big idea: Part-whole, notion of the unit.

Resources: Ladder mat (large/small); cards: numerals, familiar objects (e.g. animals/flowers/cars/balls), dots; counters.

## Reality

*Local knowledge*: Counting, sorting, matching, familiar objects from home and from outside, e.g. *Show me five pebbles, four leaves, three fingers* – to 10 fingers.

*Prior experience*: Counting number of people in family: *How many brothers/sisters do you have?*

*Kinaesthetic*: Draw large numerals on the ground. Teacher shows each numeral on a card and students walk the shape of the numerals 0–10. Teacher asks for a story about each number, e.g. *I had 5 candles on my cake*.

Teacher tells a story about a number, e.g. *My hen laid 2 eggs*; students make the shape of that numeral by lying on the ground/floor and contorting their bodies to form the shape of the given numeral. A student is then asked to find the corresponding numeral card and place it on the correct square of the large ladder mat.

## Abstraction

*Body*: Teacher shows a numeral. Students form groups of the given number and all the class clap their hands the number of times represented by that numeral. Students then trace that numeral in the sandpit. Repeat for all numerals 0–10. Reverse.

Ladder mat: Students are given counters or picture quantities or dots cards, one to 10, to arrange in order on the squares of the ladder mat. After each has been placed, students find and place its corresponding numeral card beside it. Students then count, e.g. one ball, numeral 1 and so on to 10.

*Hand*: Show numeral: With one finger, students draw the shape of each numeral on the desk or floor, in pairs on each other's back, in the sand.

Show numeral, students clap their hands to match the numeral. Reverse: Teacher claps and students draw the numeral in the air.

On individual small ladder mats, students arrange counters on the squares in ascending order (one to 10) and match with a small numeral card that they place beside each of the corresponding counters in the squares.

*Mind*: Students shut their eyes and visualise either the numeral or picture quantity of a number that is called out. With eyes shut, they hold up that number of fingers, then write the numeral in the air or on the desk.

*Creativity*: Students choose a numeral, draw a picture and tell the class a story about that numeral, e.g. *I've chosen 5* (show numeral and tell the story about the drawing).

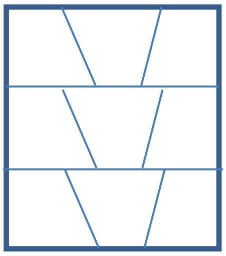
## Mathematics

*Language/symbols*: Count, visualise, how many, quantity, total, arrange, number, numeral, match.

*Practice:* Practice focuses on students identifying names, numerals and models as the same number. Activities are language based relating language to numerals and models. Final practice can focus on language ←→ numeral relationships. The following activities/games can assist.

1. **Find your partners.** Distribute cards (0–10) of numerals, picture quantities/dots to students; students find other cards that match theirs. Then put the groups of students with the same number in order zero to 10.

2. **Mix-and-match**. Photocopy onto A4 cards (all same colour) three rows as on right with two lines cutting the rows identically for the three rows. Then using same colour pen, or computer, write numeral on the left (cover all numerals 0–9), the word for that numeral in the middle and a drawing of that many objects on the right of each row. Then cut up the card along lines. This gives a pile of cards to be sorted into same number. This can be used by group or individual student – you give them a pile of different-shaped card and they sort into number groups.



3. **Card games**. Photocopy 3 or more sheets of A4 paper into 10 equal rectangles. On one A4 paper write numerals 0–9, on the second write number names 0–9, and on the third give drawings of objects for all numbers 0–9 (can extend this to more A4 sheets by having different drawings, e.g. number lines). Copy the three or four different forms onto coloured card, one colour per form (e.g. all numeral cards are blue). Cut up cards into rectangles. Make sets of cards with all 0–9 numerals, names and pictures like suits. Use the cards to play the following:

   (a) *Roll the dice.* In groups, each student has a set of cardboard cards ($\frac{1}{4}$ or $\frac{1}{8}$ of AV card are good sizes). On these cards are drawings of numerals 0–9, objects or picture quantities, or dots. Roll one die and people display the card and number of objects that match. Extend the game by using two dice. Students tell stories about the various numbers that are rolled.

   (b) *Snap.* For Snap, simply play with the card deck. Shuffle and deal cards between students/groups (two students is the normal way to play but can have more). Students play one card at a time face-up in front of them. When they see a common card, i.e. a match, first to say snap gets cards that have been played. Continue until a player has no cards. One who gets most cards or all the cards wins.

   (c) *Cover the board.* For cover the board, leave one of the forms as an A4 sheet and use other cards in turn to cover it (cards dealt randomly to players or groups; take turns playing one card). Cards can only be put on top of same number. If wrong, miss turn or ask for help. At end, the player with most of his/her cards on top wins.

   (d) *Bingo*. Use same cards as for games above but choose one suit for the caller. Other suits are shuffled and dealt to players who put cards face up in front of them. Caller shows first card, people look at their cards – any cards of same number turned over; first to have all cards turned over wins.

4. **Worksheets**. The final type of practice can be a worksheet. Set up a sheet with columns: pictures (set), pictures (number line), language and numeral. On each row of the table, complete one column; students complete the other three columns so that the same number is represented on each row. The focus of the worksheet is to practise which numeral goes with which number name and with which drawings.

*Connections*: Relate to number names, subitising and telling how many. Discuss how the 0–9 numerals relate to counting. Count using the numerals. Do activities from Units 1–3 with numerals.

## Reflection

*Application/problems*: Provide applications and problems for students to apply to different contexts independently, e.g. identify numbers in different media and patterns and write the corresponding numeral.

*Extension*:

- *Flexibility*. Think of all the ways you can use to show how many there are in a group. Think of situations where a big number is needed and where a small number applies.
- *Reversing*. Provide opportunities for reversal, for example, (a) teacher shows the numeral and students make the number and say what it is (language); (b) teacher calls out a number and students show the numeral card and make the number with counters; and (c) teacher gathers a number of objects and students call the number and show the corresponding numeral card.
- *Changing parameters*. How do we show numbers that are bigger than 10? Look at the clock to see the number that comes after 10.

## 4.2 Numerals to 19

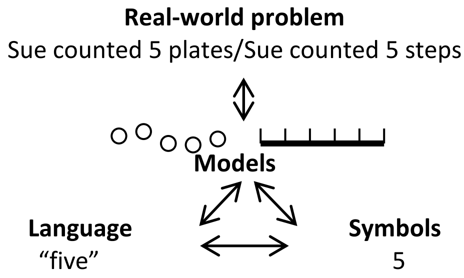
Mathematics contains many **symbols** which students need to interpret to be able to communicate mathematically and to understand mathematical concepts. Often the symbols used in mathematics are particular just to mathematics and sometimes they are part of everyday life, for example, the numerals for numbers are one of the first mathematical symbols introduced and widely used in the world. Understanding mathematics symbols is best left until after students understand the concept the symbols represent. This is why this unit on numerals is the last in the module of counting.

In Unit 3, we covered that patterns in number names are not useful for learning number names 10–19 and that the names for 10 to 19 contradict the place-value structure for these numbers as numerals. The reason for this is that numerals are Hindu-Arabic and were base 10 while the English language at the time of the adoption of the Hindu-Arabic number system was base 20. This meant that the English language had 20 names for numbers (i.e. “zero”, “one”, “two”, … , “eighteen”, “nineteen”), where the base-10 Hindu-Arabic number system only required 10 (i.e. “zero”, “one”, … , “nine”). The English language kept the now unnecessary “ten” to “nineteen”. It could have changed these to “onety”, “onety-one”, “onety-two”, … “onety-eight”, onety-nine”. Thus, the English language for teens breaks the pattern between number names and numerals (as does the decision not to say “zero” – e.g. 30 is really “thirty-zero”).

Thus special care must be spent on the numbers 10 to 19 – teaching them as a special part of number that does not follow the pattern of other numbers in its **number names**.

### 4.2.1 Ideas and activities for numerals to 19

1. Learn numerals for 10 to 19, particularly the relationship between number names and numerals, through **repetition** in rhymes and stories. After 13, things become a little easier in that students are able to hear the repeating pattern from four, five, six, seven, eight, nine, within the number. From 20 onwards, number names take a recognisable pattern which allows students to learn the name for the tens and then rely on the known pattern for the remaining numbers to the next 10.
2. **Effectively connect** reality, models, language and numerals for numbers 10–19 so that students can move back and forth flexibly between them by using the **Payne and Rathmell (1977) triangle**. The RAMR cycle is based on this triangle (in part) and this triangle has particular application to learning number because it is an excellent vehicle for connecting real-world problems, representations, language and symbols. Activities and questions should be constructed that encourage students to connect and move flexibly between model, language and symbols in all directions.




The Payne and Rathmell (1977) triangle for early number

**Spend time practising the six connections** in as many ways/contexts (stories, materials, drawings) as you can, that is:

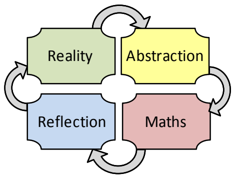
- number name → objects/lines AND objects/lines → number name;
- objects/lines → numeral AND numeral → objects/lines; and
- number name → numeral AND numeral → number name.

This also means connections between real-world situations and above need to be added to this.

3. Nonsense counting – use patterned nonsense language to recognise and help the understanding of the structure of teen numbers and decade numbers (10, 20, 30, 40) (e.g. onety-one, onety-two, onety-three, …; twoty-one, twoty-two, … , and so on).
4. Have students count a line of 19 objects, e.g. bears/fish, trucks etc. Ask: *Will there be the same number if we start counting from the other end? Why? Why not?* Count the objects again. This time start in the middle. Ask: *Does it matter where you begin?* Have students label the objects with numbers 1–19 and/or words.
5. Use two YDM 1×10 line mats placed end on end to represent teen numbers. Count and then place numerals.
6. Have students investigate the ages of students in the school. *Where are the children whose ages are 0–5? 6–9? 10–13? 14–19?*
7. Students can do fitness investigations. The aim is to be able to do up to 20 of every skill, say, catches of a large ball; bounces of a small ball; bean bags in hoop; hops; skips; jumps on spot; jumps forward; jumps backwards. Students have to count as they go and do each action consecutively without stopping/dropping ball/falling over, etc. Label each activity station with the number required for each day's practice, e.g. 11, 16, 17, 13, 19 – different numbers for each activity station.
8. Provide 30 takeaway food boxes containing up to 20 similar items. Students estimate how many, open and count, write the numeral for the quantity/total/amount and pop in box and close. Exchange boxes to check.
9. Play Jack-in-the-box. Have students play games that involve chanting numbers and counting teens. Students choose a number between 10 and 20. In unison, the class counts up to the chosen number and one student, playing the role of "Jack", jumps up in the air with the numeral.
10. Three other activities are:
    (a) *Daily grouping numbers* – pick a number for the day and wherever possible students attempt to stack, put away or create groups using that number.
    (b) *Play totals* – create a total of 0–19 with whatever you do today (students record their totals and sets).
    (c) *Counting coins* – activities such as, *If I have 18 10c coins how much money do I have?* Label number of coins and write value on a table. Try other numbers of same coins.

*Note*: Counting teaching activities should be part of every year level. RAMR activities for sorting and correspondence, rote and rational counting, and symbol recognition are included here as exemplars for sequencing instruction and ideas to start from.

### 4.2.2 RAMR lesson for numerals to 19




Materials: Everyday objects (e.g. toys, fabrics, pegs, cardboard cylinders, eggs, eggcups, spoons); other objects (e.g. blocks, counters, Unifix cubes, sticks, plastic animals); pattern blocks, logic attribute blocks, number tracks, number lines, racetrack games, Snakes and Ladders games; pictures of collections of things.

## Reality

*Local knowledge*. Teach numerals 10–19 by extending your teaching of counting and numerals 0–9. Base the teaching on reality by using 10–19 materials from the students' local environment or culture. For example, 10–19 sets of different things from home and environment (e.g. toys, fabric, pegs), counting and labelling with numeral objects from outside (e.g. seeds, nuts, sticks or rocks – and then plastic counters).

Get objects from supermarkets that have numerals on them. Put out collections that students have to label with numerals 10–19. If over 19, remove some objects and ask to label how many left. For example, 24 pegs of three colours, how many blue, how many left after the blue pegs are removed, and so on. Label paper plates with numbers 10–19 and students have to put that number of objects on the plate, and so on with examples numeral ←→ number name/objects.

*Prior experience*. Check the existing knowledge students have of 10–19. Get them to experience what the numerals they know mean. Do they have older siblings – *what are their ages? How old do you have to be to be in Grade 7?* and so on. Check things in their environment, find things that are 10–19 – when they find them, get students to trace them with hands or on paper. Connect this to counting 0–19.

*Kinaesthetic*. Get students to show numerals 0–9 with bodies or trace with fingers on large posters or walk the shapes. Then move on to 10–19 by adding in a 1 on the left-hand side of numeral. Get two people to act out the numbers 10–19 with their bodies – one acts the 1 the other the 7, say. Put 1 beside numerals they know in 0–9. Make sure that the 1 is on the left.

## Abstraction

*Body/Hand:* Rote count 10–19 and show numerals. As you do this rote counting, get students to act out the numerals with their bodies, by lying down and making the numbers, by "tracing" the numbers out with their hands, or by walking large copies of each of the numbers. Focus on the numeral similarity between the numbers 0–9 and 10–19 as on right. Add the number names to the list to make a poster – first row "zero 0 10 ten", second row "one 1 11 eleven" down to last row "nine 1 9 nineteen". Continuously relate number name, numeral and models (objects, hops along a line). Note that set model should precede hops along a line, and that doing things with the body should precede doing them with the hand/mind. Use activities that teach 0–9 and adapt them to extend to 19 (e.g. plates with numerals on them, and so on).

| | |
|---|---|
| 0 | 10 |
| 1 | 11 |
| 2 | 12 |
| 3 | 13 |
| 4 | 14 |
| 5 | 15 |
| 6 | 16 |
| 7 | 17 |
| 8 | 18 |
| 9 | 19 |

Play games with numbers 10–19 so that students can see that 13 is 3 further along than 10 or 3 more than 10. For example, make a racetrack game 0 to 100 by rolling a 0 to 9 die and then counting along the track 10 plus the 0–9 number thrown. To stress body activity, this could be played on an NRL field with students walking 10 metres plus the metres thrown on the 0–9 die. Use the 0–9 and 10–19 similarity to relate numerals and objects with number names.

*Mind.* Look at the relationship between 0–9 and 10–19 and try to find the patterns that will help numeral ←→ number name ←→ pictures/objects relationships. They include (a) special name for 10 objects "ten" which is never repeated for more than one ten; (b) special names for 11 and 12, "eleven" and "twelve"; (c) special names for 13–19 which relate to how many more than ten there are (e.g. four more than ten is "fourteen", six more is "sixteen" and so on); (d) the "twenty", "thirty" and "fifty" are the same at the start as the teen changes "twelve", "thirteen" and "fifteen" (i.e. "twe", "thir" and "fif"). Thinking of thirteen as ten plus three or seventeen as ten plus seven helps in counting out the set (number counted ←→ numeral) and later place-value teaching.

*Creativity*. Give students opportunities to share the number names from their home culture and language if different from normal mathematics. Let students create their own number names and number numerals, even if using their own made-up groupings.

## Mathematics

*Language/symbols*: The end of the Payne and Rathmell triangle is the beginning of formal mathematics – the appropriation by students of formal number names and numerals. Ensure there are activities that relate language/number names and symbols/numerals and real-world situations (materials, models, pictures, etc.) in all six ways described in the Payne and Rathmell triangle. Of course, the real-world situation needs also to interact with these components. This means having lessons where:

- teacher gives symbols/numerals and students give language/names, models/materials and situations;
- teacher gives language/names and students give symbols/numerals, models/materials and situations;
- teacher gives models and students give language, symbols and situations; and
- teacher gives situation and students give language/names, symbols/numerals and models/materials.

*Practice*. This is an essential part of learning. New mathematical ideas need prerequisite knowledge to be familiar, not just known, so there must be practice of previous mathematical ideas before going on to new ideas. This means games, oral question and answer sessions, and simple worksheets. However, games and worksheets need not just be with symbols; excellent practice activities involve relating symbols to language, models and reality (in fact, this is the definition of learning for many mathematics educators – that students can connect different representations of the same idea). The activities for teaching 0–9 apply to 0–19 just with larger numbers of objects and so on. Ensure to use number line as well as set activities. Some good ideas are:

1. **Act it out**. Use kinaesthetic activities to act out numerals and even words with body and hand; make a set of numerals on cards with sandpaper or silk fabric for students to trace.
2. **Number rhymes**. This is also the time to get out the students' number rhymes and counting books. YDC has a collection of counting rhymes developed by Aboriginal and Torres Strait Islander teacher aides – create your own using local language and environment.
3. **Make the amount**. Place numerals on plates; students have to place counters on plates to match numerals (can also place language cards). Reverse the activity: put counters on the plates and students have to match with numeral and language.
4. **Mix-and-match cards**. Draw up cards (all the same colour) with some or all of the four options for different examples, cut cards up the same way, mix together, and students match to reconstruct cards.
5. **Cover-the-board**. Make decks of cards (different colours) for some or all of the options, make a base board from some of the options; students take turns placing card on board or over another person's card that represents the same thing; the person with the most of their colour on the top at the end wins (can also play Snap with these cards).
6. **Bingo**. Make up boards with different representations of different numbers, construct call cards; one player shows card, others cover with a counter the same representation on their board; the first player with three in a row, column or diagonal wins.
7. **Racetrack games**. Make racetracks, throw numeral dice (or select numeral cards) and move along track the amount shown – counting out the step shown by the numerals.

8. **Calculators**. These can be a great teaching aid at this point – they allow students to show numerals by pressing buttons. If students enter 4 + 1 and keep pressing equals the calculator counts showing the numerals that go with the language of the number names as they are being said.
9. **Tabular worksheets**. Have columns for some or all of reality, models, language and symbols; fill in one column and require the students to fill in the rest.

*Connections*. This is the time also for building connections between the knowledge being learnt and other knowledges. One important example of this is linking counting forward with adding one and counting backward with subtracting one. Calculators can be very useful: (a) to reinforce counting forward and back, and (b) for recording through adding and subtracting the actions of counting forward and back.

## Reflection

*Validation*. Once students have developed proficiency with numbers in terms of relating numerals, language and models, get the students to check their understanding of early number with their experience of the world (e.g. Does what they know about numbers make sense? Does it appear reasonable?). In particular the question of whether the knowledge would be useful in the students' everyday world is worth discussing. A poster "numbers up to 19 in my world" may also be useful.

*Application*. After validating students' knowledge of number, this knowledge can be applied back into their reality. For example, students can use numbers to help them live in their world, that is, recognise the number of things, relate amount to counting (e.g. Who has more/less?), use numbers in sport, and so on.

*Note*: Some of these application activities would make good reality activities.

*Extensions*. There are many ways in which reflection on mathematics knowledge can lead to extensions.

- The first of these is **being flexible**. For example, discuss all the ways in which the number 15 can be used (e.g. three groups of five or a 10 and a 5).
- The second way is **reversing.** This is a major component of the Payne and Rathmell triangle – encouraging students to think both ways. For example: (a) looking at how many different ways we can set out counters so that the total is 12, going from numeral → objects as well as objects → numeral; and (b) acting out breaking into parts and getting students to state the parts (e.g. 10 and 7 is 17, and 18 is 10 and 8).
- The third way is **generalising.** This is a major part of mind in abstraction in this RAMR lesson. Get students to generalise the pattern between 0–9 and 10–19. For example, ask *How do we know the numeral for a group of objects over 9?* Answer would be if it is 10 objects it is "ten", 1 or 2 more than 10 is "eleven" and "twelve", from then on, the number of objects over 10 gives us the numeral (3 – "thirteen", 4 – "fourteen", and so on up to 9 – "nineteen").
- The last way is **changing parameters** – this may be one step too far but you can ask students what they think 356 could be as a number name or what is the numeral for "three hundred and seventy-eight"?

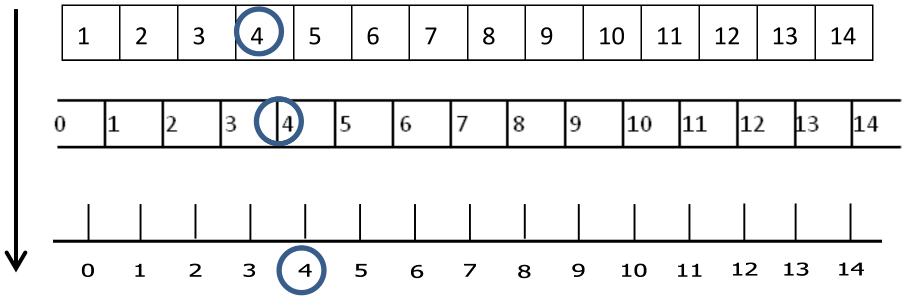
## 4.3 Number tracks, ladders, lines and boards

This section is not really in sequence with 4.1, 4.2 and 4.4, but we have put it here to stress the need for number line as well as set activities in lessons about counting and numerals.

The big idea behind this is **continuous vs discrete** which focuses on a fundamental difference in the world around us. Things are either: (a) discrete, that is, can be identified as separate entities and counted (e.g. fingers, people, chairs, lollies); or (b) continuous, that is, flowing forever without being naturally broken into pieces that can be counted (e.g. line or length, coverage or area, heft or mass). Because of the importance placed on determining number and quantity, Western culture found a way of "discretifying" the continuous by inventing **units**. A unit is a small amount of the continuous attribute used to break up the attribute into equal pieces that can be counted so the attribute can be quantified (e.g. metres for length, hectares for area, and kilograms for mass). Thus, number is applied to both discrete and continuous attributes in our world, and this is essential for all number types. Whole numbers, decimal numbers, common fractions, rates and ratios must be understood in terms of discrete and continuous – that is, seen in relation to separate objects and seen in relation to continuous entities. In practice this means that number should be represented in terms of **sets** and **number lines**. This leads to number tracks and number ladders, and their extensions, the 99 board.

*Note*: This application of number to the continuous world changed numbers, 0 now became something, the starting point, and the other numbers became the ends of things, such as steps, not the step itself.

Below are examples of the three continuous → discrete models in the early years – (a) the **number track** that counts spaces, (b) the **number ladder** that labels the end of spaces, and (c) the **number line or ruler** that abstracts the ladder (with 4 labelled).




On the right is the final model for discrete vs continuous and that is the **99 board** with example 46 showing.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

*Note*: There is also a 100 board which starts at 1 and finishes at 100. We tend to use the 99 board because it has all the twenties, thirties, etc. on the same line.

The 99 board can be considered as a wrap-around number line or a number line broken into strips. It counts left to right and then curves back to start at the left of the next line. Tens are added/subtracted vertically and ones horizontally. It is excellent for patterns and can assist counting to 100.

It can also begin at numbers other than 0 and so it can be used from any number (e.g. 37 or 278) and can, in these cases, show bridging across tens and hundreds.

*Note*: The number board is similar to number tracks in that it counts spaces. It should not be used in relation to line graphs, which like number lines have the numbers on the lines and intersections of lines, not the squares.

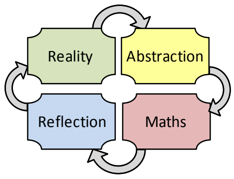
### 4.3.1 Ideas and activities for number tracks, ladders, lines and boards

The idea here is to count jumps as well as objects – using number lines and number boards. One way to begin this is to count steps and this leads to games and activities that use number tracks. However, the number line places numbers at the ends of steps while the number track has numbers in the spaces. Therefore, a number ladder (like a track but numbering the lines) can be a useful intermediary.

A number line is a continuous line. To enable numbers to be applied to the number line, an interval of length one is used as a unit to divide the line into discrete parts. The placement of the number is based on this partitioning of the line. Since many measures are associated with lines, it is important that all numbers are related to positions on lines as well as sets of objects. In general, activities with sets and numerals can be replaced with number-line models and numerals but there are kinaesthetic differences.

1. Give experience to students of using tracks to represent numbers, then experience with number boards (note that games such as Snakes and Ladders use a number board as the base), and then move on to ladders and lines. Then relate all these models to number names and numerals. Discuss similarities and differences for tracks, ladders and lines.
2. Use ovals/playing fields and outside areas to act out number-line ways of counting and identifying numerals. Use rope to construct a line with a material tied every 10 steps – walk this line given numerals and place numerals at points given. For example, students can be asked to walk 63 or to label a point with a numeral to which they have walked.
3. Add number name → line model, line model → number name, numeral → line model, and line model → numeral activities to what was done in numerals work with sets. This means using number tracks/ladders/lines models (activities and pictures) and number boards everywhere and in every activity that uses set models, activities and pictures. Better still, use a variety of models and try to have both track and set models shown in numeral work.
4. Although the 99 board is like a number track cut into 10 interval parts and the parts placed in order beneath each other, it has the capacity to show relationships that are not so readily available in number tracks – ones are counted across and tens are counted down. Begin activity by allowing students to explore the board – counting aloud across and down to see the patterns. Break the board into pieces and get students to reassemble and provide parts of the board with only one number showing and students have to complete the board. See subsection 4.4.2 for more on the 99 board.

### 4.3.2 RAMR lesson for number tracks, ladders, lines and boards – number ladder




Materials: Large number ladder on floor, small number ladders with counters (with and without numerals). (This activity focuses on ladders.)

## Reality

*Local knowledge:* Look for things that count jumps – walking, games, kangaroos, etc.

*Prior experience*: Check that students have counting and an understanding of counting and numerals for objects.

*Kinaesthetic*: Have students walk the large number ladder – focus on new role for zero and counting movements (e.g. jumps) not stopping points.

## Abstraction

*Body/Hand*: Repeat counting activities using small number ladders and counters, without numerals and then with numerals; relate real world to model to language and finally to symbols using Payne and Rathmell triangle.

*Mind*: Students close eyes and imagine the number ladder and walk it in their minds.

*Creativity*: Students make their own number ladder with own symbols and walk it.

## Mathematics

*Language/symbols*: Ensure students understand formal language and symbols.

*Practice*: Relate numerals to positions on the ladder and vice versa. Count out to get positions. Play race games.

*Connections*: Connect to set model – relate numeral, number name, drawing of a position on a number ladder and set of objects using physical and pictorial materials and games such as Bingo and Mix-and-Match.

## Reflection

*Validation*: Discuss where the action of a number ladder is used (e.g. ruler).

*Application*: Apply to number-ladder problems in world (e.g. house numbers).

*Extension*:

- *Flexibility*: Brainstorm number ladder or length applications (e.g. speedometers).
- *Reversing:* Ensure students go from position on number ladder to language and symbols.
- *Generalising:* Have students state what zero is for number ladder (start point); students discuss what the numbers do (tell jumps); show examples where it is necessary to count ends instead of jumps (e.g. 5 + 3 is 5, 6, 7, 8 which is four numbers but only three jumps).
- *Changing parameters*. Set up ladder so that every two or five steps is bolded or coloured red – counts twos and fives. Extend ladder to hundreds and count in tens or fives. Make each step more than one (e.g. to count as two or five), discuss what this means.

# 4.4 Counting numerals to 99 and 999

For this module, unit and sub-unit, building numbers to 99 and 999 will have to be done without the place-value structure but relying on relationship between oral number names and numerals. Place value is left to AIM EU Module 4 (N2) which is on Place Value. However, we would expect the first activities for the Place Value units to be done with these last units of Counting and that there will be integration. Everything should be done to build understanding of tens and ones, and hundreds, tens and ones – one of the most important building blocks of number. Artificial barriers due to form of materials should never get in the way of: (a) using what students know, and (b) integrating place value with what is being done here.

This section looks at numerals for 0 to 99 and 0 to 999. These are built from counting and repetition with regard to the digits that will be used. Initial counting will show that two digits are required for 10 to 99 and three for 100 to 999. Obviously, for 0 to 99, we will rely on the 99 board to give us the pattern in numerals to go with the counting. For 0 to 999, we can extend the 99 board to cover hundreds as well.

## 4.4.1 Ideas and activities for counting numerals to 99 and 999

1. Place value provides the structured theory that determines numerals. We have a sub-pattern of three that are hundreds (H), tens (T) and ones (O). Each use of these HTOs is with a name for three positions with the number of digits for whole numbers as follows:

| HTO millions | HTO Thousands | <u>HTO ones</u> | HTO thousandths | HTO millionths |
|---|---|---|---|---|
| 9 digits | 6 digits | 3 digits | | |

   Without this place-value theory, we have to rely on repetition and the rules that this brings:

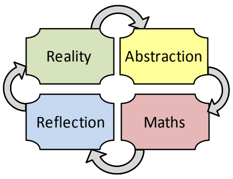
   (a) counting from 0 gives two digits when we pass 9, and three digits when we pass 99; and

   (b) the pattern in counting is to get to "nine" at the end of a name and then repeat the pattern for the next count in line – that is, "fifty-eight", "fifty-nine", "sixty", "sixty-one", … "sixty-nine", and so on.

2. The 99 boards can assist with 0 to 99 numerals as can be seen in 4.4.2.

3. Extending 99 boards to where they have three-digit numerals can also assist with numerals to 999. The activities in 4.4.2 can be extended to three digits and assist with this numeral pattern. Take the ideas in the 99 board activities in 4.4.2 and extend these ideas to 99 boards that start with 100, 200, 47, 643, etc. and thus have three-digit numbers. Look to see the patterns in these boards (e.g. 356, 357, 358, and so on, and 356, 366, 376 and so on; also have decreasing patterns). After constructing, then get students to decipher the patterns – they have to find the changing place value. Make sure students know what happens as place-value positions go up to 9 or down to 0.

### 4.4.2 RAMR lesson for numerals to 99 and to 999 – 99 board




Learning goal: To introduce and become competent in numerals to 99.

Big ideas: Trust the count. Tens and one, odometer pattern.

Resources: 99 board large, 99 boards small, cards 0–99, counters, 99 board windows and worksheets.

## Reality

*Local knowledge*: Something in the world of students that relates to two digits and numbers to 99, e.g. house numbers, sport scores in NRL, cents in a dollar.

*Prior experience*: Check students understand numerals to 19 and rational counting to 99 (plus knowledge of 99 board).

*Kinaesthetic*: Get a 10×10 or 6×10 mat or equivalent. Place in numerals 0–19 starting from top LHS square. Walk these numbers, counting as you go. Then continue walking and counting – discuss what the numerals could be. Try to fill in the whole board – discuss the patterns in the board.

## Abstraction

*Body*: Go back to large 99 board and place cards 0–99 on it. Encourage students to see that the 99 board represents number in terms of rows down for tens and columns across for ones. Have students read aloud columns and rows, for example, 4, 14, 24, ...; and 60, 61, 62, ...., as is shown on right. Encourage students to notice the patterns by thinking what word are we repeating – down from 4 it is "four" at the end of the name; across from 60, it is the "sixty" at the start of the name. So downs keeps the one (in this case the "four") and across keeps the 10 (in this case the "sixty").

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

Get students to find numbers by walking from the zero. For example, a student walks four down from zero and seven across, where is the student? Student is at 47! What happens when student steps to left, right, up and down? Repeat for other students with different examples. Do not forget difficult examples – three down and zero across, five down and nine across, nine down and four across. See diagram on right. Remove all numbers and repeat, say seven down and two across, where is the student? Get student to stay at this number – what happens if they were to move left, right, up and down? Repeat this for different movements.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

*Hand*: Students have a small A4 version of 99 board with all numbers. Above actions/questions can be repeated. Students can be given another A4 version with no numbers and things are repeated again. Always start at zero. For position to number – get students to start at zero and move down and across, encouraging students to see the pattern, e.g. that three down and seven across is 37. For number to position – move the tens down and ones across, e.g. 54 is five down and four across (starting at zero). For extra activities, cut 99 boards into jigsaw puzzles and get students to re-form them, get students to make puzzles for each other, hand out 99 boards with parts missing and students have to complete the numbers. Hand out pieces of boards with only a few numbers on squares –complete all squares.

*Mind*: Ask students to close their eyes and imagine the 99 board. Get them, with eyes shut, to find numbers by pointing, to state numbers that are one more and less or 10 more and less, to point with eyes shut to where they think other numbers are. Ensure they have a picture of the board they can use for seriation and patterns.

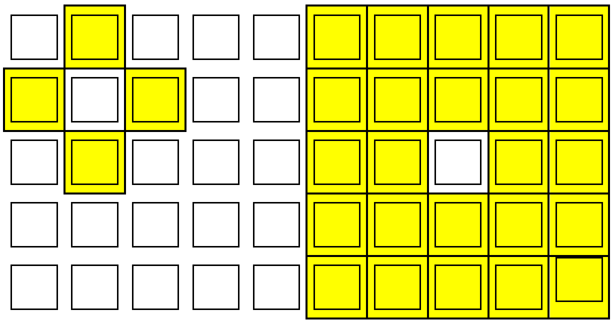
*Creativity*: Ask students to draw up a 99 board starting at 37 – discuss what it would have – particularly focus on how it goes past 100.

## Mathematics

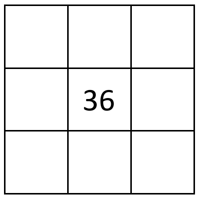
*Language/symbols*: Ensure students all understand important language – down, up, left, right, column, row, add 1, subtract 1, add 10, subtract 10, and so on. Ensure students can place numerals on board correctly.

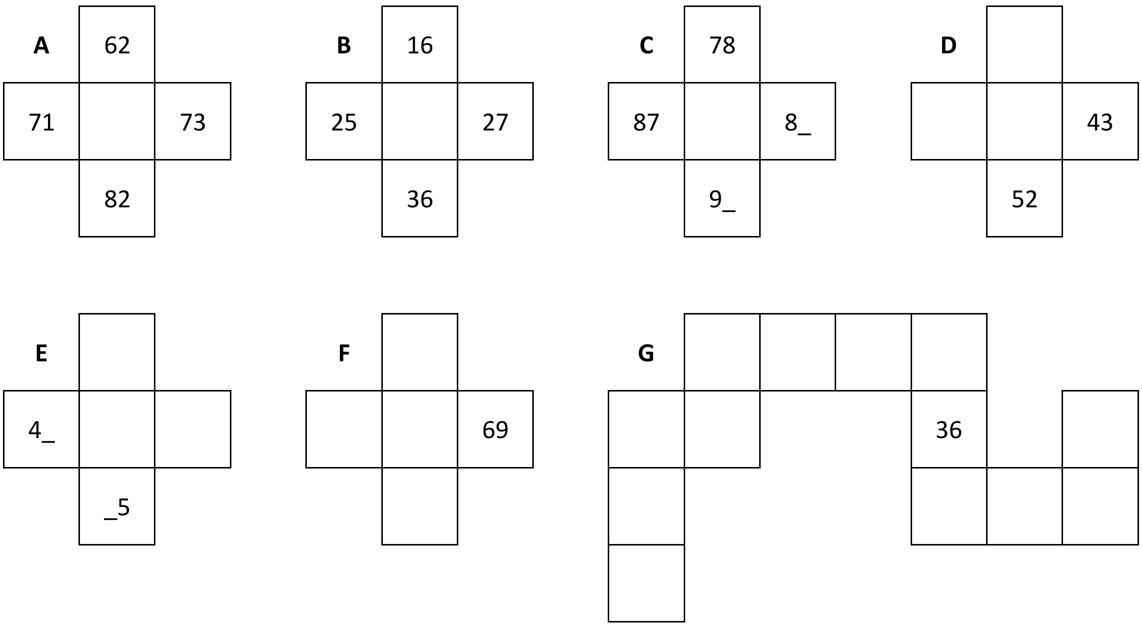
*Practice*:

1. Play "Three in a row". Players in turn take two cards from a pack with 1 to 9 in it (A is 1; K, Q, J and 10 removed) and cover any number they can make (e.g. 4 and 3 could be 43 or 34) with a counter (students can remove opponent's counter to place theirs). The first player to get three in a row (column or diagonal) wins.
2. Construct a 99 board window with a hole in the middle; place over a number so only that number is visible and ask students to write numbers one less and one more, above and below. Examples of windows are:



3. Give 3×3 squares with number in middle and ask for other numbers as on right.
4. Give 3×3 squares with numbers on outside and ask for number in middle (reversal of 3 above). Give a section of a number board and provide some numbers; vary the difficulty or number of squares; extend to use larger numbers. See examples A to G below:




*Connections*: Relate 99 board activities to seriation; what is one more or less, 10 more or less?

## Reflection

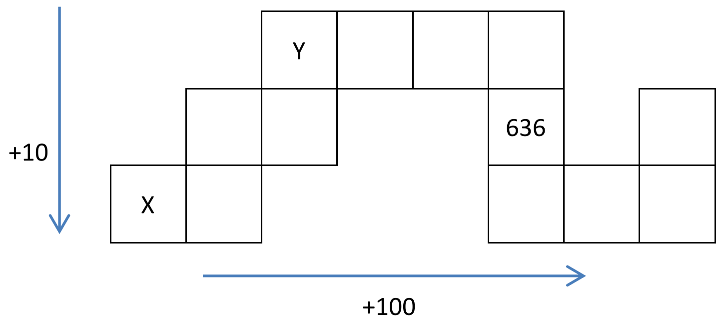
*Validation*: Make a poster on “0–99 numbers in my area” and fill it in. Remember that numbers may not be static on a display – they can move, such as in kilometres travelled, or hours passed (how many hours in a day-and-a-half?).

*Applications/Problem-solving*. Provide students with applications and problems that use two-digit numbers and any of the skills that have been covered (e.g. *John was told to wait 25 minutes. He counted 7 extra minutes. How long did he wait?*)

*Extension*:

- *Flexibility.* Think of different ways that a number could be achieved – for example 46 is 6 more than 40, 10 more than 36, 10 less than 56, and so on. More difficult, it is also 1 more than $^3/_4$ of an hour and 1 more than $^1/_2$ a right angle, and so on.
- *Reversing*: Reverse everything, for example, go from what is 10 more than 78 (number to seriation) to 56 is 10 more than what number (seriation to number). So think!: what am I asking and what are students doing – next time I’ll do what the students are doing and they have to find what I am asking.
- *Generalising/Changing parameters.* What are the patterns for numerals 0–99 (asking students to generalise)? How can we extend these patterns to patterns for 0–999 (asking students to consider a parameter change from two to three digits)?

  Extend the worksheets so that changes are greater – for example, across changes by 100 and down/up changes by 10 in the example below, making the square X to be 146 and the square Y to be 326.



# *Module Review*

This review looks across the units and identifies outcomes that go beyond the particularities of the units. The first of these is **models and representations**, that is, ways of teaching ideas common within the units and across most of mathematics. The second is **competencies**, that is, abilities that are important across the units and into the future. The third is **later counting**, information on the mathematics that grows out of this module and provides the reason for its importance.

## Models and representations

In this subsection, we look at set models, the continuous vs discrete big idea, number tracks, ladders and lines, and number boards.

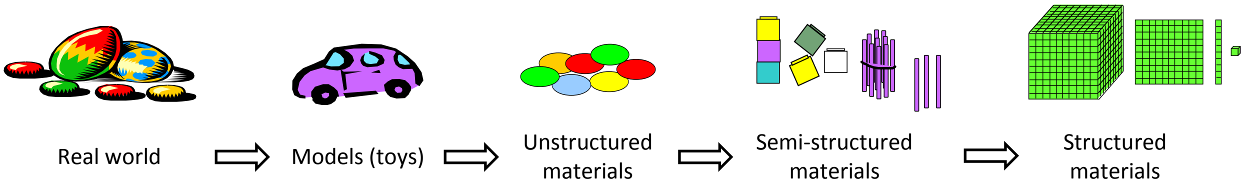
Young students should begin learning with discrete materials, starting from real-world materials (e.g. biscuits, marbles) and then replicas of the real-world materials. These can include counting steps when walking. When they are able to rationally count, the teacher starts to use concrete ("hands on") materials such as counters and then Unifix cubes which allow lengths to be counted, before asking students to count things in pictures, which is more difficult as students cannot pick up or move the objects in a picture. Finally, students have to know that they can apply number to even imaginary discrete things and to number lines.

The abstracting component of RAMR is where oral and written **number names** and written **numerals** are introduced. Start with a situation from the world, model this with sets and number lines (e.g. count objects and count along number tracks and number lines, noting that the number line, or length, model requires counting jumps not starts/ends of jumps). Sequence the numbers – do 1–4 first, 0 second, 5–9 third and all 0–9 finally. Move on to using numerals in activities focusing on one more/one less, comparing and ordering, and ordinal numbers (first, second, third, etc.). Then use numerals for 10, counting on/back, composite numbers (groups of groups) and other advanced counting ideas.

### Set model (discrete)

New mathematical concepts are introduced to students through the use of **language**. First, the ordinary language of the students can convey mathematical concepts; for example, "I am three", "my bear is next to the table". As students progress in their understanding and experiences with mathematics, the language they use also progresses to become more mathematical and symbolic. For example, the concept of subtraction can be described using *lost*, *covered up*, *take away*, *subtract*, and the symbol "–". The level of abstraction in the language used can develop over time and be varied to suit particular student or conceptual needs. However, having students pose stories that match mathematical concepts can be an enriching activity at all levels, from stories about addition to stories about rate and percentage.

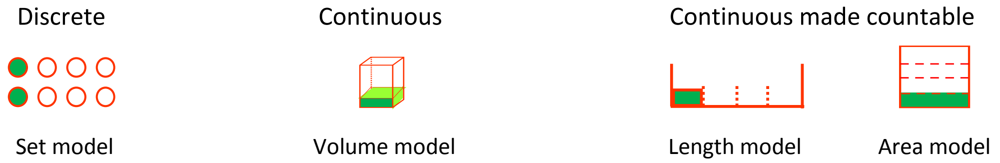
**Physical materials** are commonly used to support student learning of mathematics. As students develop their mathematical understanding the materials used to represent real-world contexts need to gradually decrease in the amount of contextual information they provide for students, as shown below.




Sequence of physical material representations

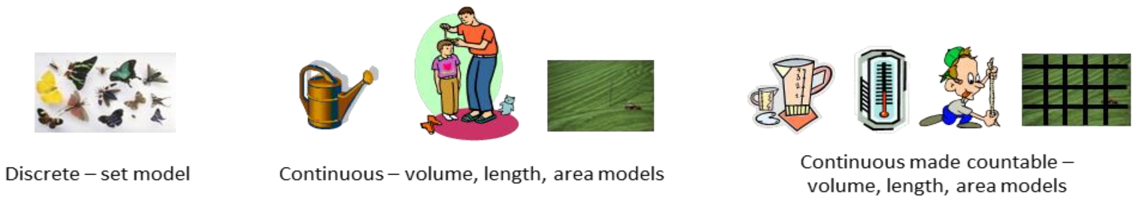
## Continuous–discrete relationship

When using **models for number**, it is important to include both discrete and continuous models to assist students with understanding the *continuous vs discrete* big idea, as well as providing enough examples of items in the real world to ensure students build flexible and robust understandings. Note that continuous models from the real world become discrete and countable when arbitrary measures are applied. For example, water in a jug with no markings is continuous; marking increments up the side breaks the continuous into discrete countable units. Similarly a streamer is continuous but its length becomes discrete and countable when placed alongside a ruler or measuring tape.




Examples of discrete and continuous models to support maths learning

Some real-world models for discrete, continuous, and continuous made countable (or discrete) are pictured below, showing set model, volume model, length model and area model.




Real-world models for discrete, continuous, and continuous made countable

Examples of discrete models for whole number are: actual objects – buttons, toy cars; models of actual objects – counters, blocks; and structured models – ten frames, bundling sticks, MAB.

Examples of continuous models for whole number are: number tracks, number ladders and lines; and number boards – hundred boards and 99 boards.

Lines are continuous; to enable numbers to be applied to get a number line, an interval of length one is used as a unit to divide the line into discrete parts. The placement of the number is based on this partitioning of the line. Since many measures are associated with lines, it is important that all numbers are **related to positions on lines** as well as sets of objects.

## Number track and number ladder

It is important that students both count discrete objects (sets) and count along continuous-made-countable lines (number lines). The number-line model can be experienced by counting steps and using a number track or number ladder. Body and hand activities with both discrete and continuous are used to encourage students to imagine and gain mental models of numbers as sets or steps on lines. Thus, it is important that early number work includes things that lead to the number line as well as to groups and leftovers.

There are three steps to the number line:

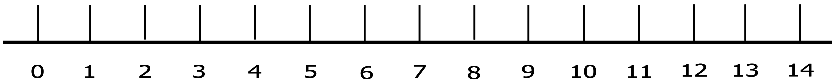
1. **Number track**. The number track is partially set model in that it is the steps between the lines that are counted and there is no zero. Tracks are used in most of the games, like snakes and ladders.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

2. **Number ladder**. The number ladder has the numerals on the steps and not between them. Now zero appears and is the starting point. Ladders are used to transition from track to line.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

3. **Number line**. The number line is the most abstract line – the numbers are at the points, not between them, and 0 is the start.




Counting ladder and line situations differ from counting of objects in two ways: (a) counting focuses on jumps (or steps) between numbers/lines, not counting the numbers or lines; and (b) the zero represents the start not nothing. Because of this difference, and because of the role measurement has in the modern world, students need to understand number lines. They lead to ideas of rank, comparison, rounding and estimation.

As students move through the year levels, the mathematics they study moves to the number line away from the number track. Thus, the sequence to be followed in this length model is number track → number ladder → number line. The number ladder assists students to move from track to line. It is important in graphs – simple bar graphs come from vertical number tracks but later graphs (e.g. line graphs, graphs of quadratic functions, trigonomic graphs) require students to use where lines cross as the points.

### Ninety-nine board

The last model/representation to look at is the number board. We tend to use the 99 board on right. It is a number of tracks from 0–9, 10–19, 20–29, and so on, that are placed above each other. Numbers are found by starting at zero and going down tens and across ones. It is useful for numerals to 99.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

There is also a board that starts from 1 and goes to 100 – this is called the hundred board. We use the 99 board because all of a ten (e.g. all the seventies) are in one row. The board can start from any number and so boards starting above 0 and in the hundreds are useful with numerals to 999.

## Counting competencies and critical teaching points

In this subsection we look across the units and pick out major skills and teaching ideas.

### Counting competencies

Essentially, counting competencies are what students can do and need to be able to do with all this counting knowledge, how and why it is useful, and where it will lead to help with later maths ideas. These competencies include the following.

1. Students can do 1:1 correspondence, particularly number name to object.
2. Students can rote count accurately even with large numbers.
3. Students can rationally count, robustly and flexibly, and know that the last number named tells how many.
4. Students trust the count regardless of whether skip counting, counting on and back or other forms of group counting are used.
5. Students can model with both set models (e.g. objects real and imagined) and line models (e.g. number tracks, number ladders, number lines and number boards).
6. Students can understand how to bridge across tens and hundreds and both together.

## Critical teaching points

Critical teaching points represent important steps in the teaching sequence. They are as below.

***Prep years***

1. Ask open-ended questions to elicit descriptive responses, e.g. *Tell me what you've done?*
2. Encourage oral language that is descriptive and specific, in relation to attributes, e.g. colour, size, position, etc. Expand on language.
3. Focus terminology on being descriptive (e.g. red, large, soft) and relational (e.g. before/after, same/different, between, next).
4. Form equal and unequal groups – this relates to one-to-one correspondence, one-to-many correspondence, multiplication/division, fractions, and so on.
5. Realise that the above activities extend to one-to-one correspondence between names and objects (and one to many).
6. When moving to name–object correspondence, build on known numbers to introduce unknown numbers into the student's preferred sequence; choose numbers next to or near the known numbers; and use the benchmark numbers of 3, 5 and 10 to build up known numbers.
7. Relate models, language and symbols until students translate between the three in all directions using all possible models.
8. Recognise common error patterns in reading, writing and saying larger numbers. For example, 630, or six hundred and thirty written, can be wrongly symbolised as: 60030, 600,30; 6,00,30; 6H30; and so on.

***Later counting in primary and junior secondary***

It is crucial to continue counting across the years of mathematics and to count on and back, as in the following situations.

1. **Counting in fractions**. For example, count in sixths: $\frac{5}{6}$, 1, $1\frac{1}{6}$, $1\frac{2}{6}$, $1\frac{3}{6}$, $1\frac{4}{6}$, $1\frac{5}{6}$, 2, $2\frac{1}{6}$ and so on. Fractions count like whole numbers but the new whole number appears when the numerator equals the denominator, not when it passes 9.
2. **Counting in decimal numbers**. All place values count and follow the odometer pattern as follows:
   - 345.2, 355.2, 365.2, 375.2, 385.2, 395.2, 405.2, 415.2, …, and so on.
   - 2.463, 2.473, 2.483, 2.493, 2.503, 2.513, …, and so on.
3. **Counting in measures**. All measures (e.g. length, area, mass, time, and so on) can be counted with the point at which the new whole appears, which depends on unit conversion, for example:
   - 3 m 997 mm, 3 m 998 mm, 3 m 999 mm, 4 m, 4 m 1 mm, 4 m 2 mm, …, and so on.

# *Test Item Types*

This section presents instructions and the test item types for the subtests associated with the units. These will form the bases of the pre-test and post-test for this module.

## Instructions

### Selecting the items and administering the pre-post tests

This section provides an item bank of test item types, constructed around the units in the module. From this bank, items should be selected for the pre-test and post-test; these selected items need to suit the students and may need to be modified, particularly to make post-test items different from pre-test items. The purpose of the tests is to measure students' performance before and after the module is taught. The questions should be selected so that the level of difficulty progresses from easier items to more difficult items. In some modules this will follow the order of the units and subtests, and in other modules it will not, depending on the sequencing across the module and within units. The pre-test items need to allow for students' existing knowledge to be shown but without continual failure, and the post-test items need to cover all the sections in a manner that maximises students' effort to show what they can do.

In administering the pre-test, the students should be told that the test is not related to grades, but is to find out what they know before the topic is taught. They should be told that they are not expected to know the work as they have not been taught it. They should show what they know and, if they cannot do a question, they should skip it, or put "don't know" beside questions. They will be taught the work in the next few weeks and will then be able to show what they know. Stress to students that **any pre-test is a series of questions to find out what they know** before the knowledge is taught. They should do their best but the important questions come at the end of the module. For the **post-test**, the students should be told that **this is their opportunity to show how they have improved**.

For all tests, **teachers should continually check to see how the students are going.** Items in later subtests, or more difficult items within a particular subtest, should not be attempted if previous similar items in earlier subtests show strong weaknesses. Students should be allowed to skip that part of the test, or the test should be finished. Students can be marked zero for these parts.

### Information on the Module N1: Counting test item types

This section includes:

1. Pre-test instructions;
2. Observation Checklist and Teacher Recording instrument; and
3. Test item types and Diagnostic Mapping Points.

## Pre-test instructions

When preparing for assessment ensure the following:

- Students have a strong sense of identity; feel safe, secure and supported; develop their emerging autonomy, interdependence, resilience and sense of agency; and develop knowledgeable and confident identities.
- Students are confident and involved learners, and develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity.

When conducting assessment, take the following into consideration:

- Student interview for diagnostic assessment in the early learning stages is of paramount importance.
- Use materials and graphics familiar to students' context in and out of school.
- Use manipulatives rather than pictures wherever possible.
- Acknowledge the role of using stories in this early number learning, enabling students to tell stories and act out understandings to illustrate what they know.
- Playdough and sand trays are useful for early interview assessment situations.

Ways to prepare students for assessment processes include the following:

- In **individual teaching times**, challenge students' thinking. "Challenging my thinking helps me to learn by encouraging me to ask questions about what I do and learn. I learn and am encouraged to take risks, try new things and explore my ideas."
- In **group time**, model and scaffold question-and-answer skills by using sentence stems to clarify understandings and think about actions. Encourage students to think of answers to questions where there is no one correct answer, and to understand that there can be more than one correct answer (e.g. *How can we sort the objects?*).
- In **active learning centres**, use activities such as imaginative play, sand play, playdough, painting, ICTs and construction to think and talk about different ways of using materials, technologies or toys. Ask questions and take risks with new ideas.

Other considerations:

- Preferred/most productive assessment techniques for early understandings are observations, interviews, checklists, diary entries, and folios of student work.
- Diagnostic assessment items can be used as both pre-test and post-test instruments.

Remember:

**Testing the knowledge** can imply memory of stuff; asking the students **what they can do with knowledge** requires construction and demonstration of their understanding at this early understandings level.

| N1 Counting: Observation Checklist | | | |
|---|---|---|---|
| **Unit** | **Concept** | **Knows** | **Can construct/do/tell** |
| **1. Pre-counting** | Identifying and describing attributes | To observe details and recognise likeness and difference between<br>Objects and pictures | Notice colour, size, shape, sound, taste, texture and an object's function or use<br>Discuss and use appropriate language/vocabulary to talk about likeness and differences |
| | Matching and sorting | To focus on "sameness" of properties of objects | Sort by one attribute the same; Why do these match?<br>Sort involving matching; "Decide Do and Describe" |
| | Comparing and ordering (unnumbered) | Can look at items and compare by understanding difference<br>Order items from least to most | Identify big/little, hot/cold, smooth/rough, tall/short, heavy/light, and so on<br>Put two or more objects in sequence from the least of an attribute to that which has the most |
| | The attribute of number | Number is a special attribute in its own right | Identify something by the amount of objects; for this attribute other attributes do not matter |
| | Subitising | See a pattern at a glance without counting | Recognise collections of one, two, and three things without counting |
| | One-to-one and one-to-many correspondence | 1:1 and can extend to 1:many | Count in ones; count in groups (twos, fives, tens, hundreds and so on) |
| **2. Introducing rote and rational counting** | Pre-rote and rational counting | Rhyme, rhythm, movement, music and clapping | Invent words and sequences for rhyme, rhythm, movement, music and clapping |
| | Rote and ordinal counting | Number names in the right order | Recite number names in the right order<br>Remember ordinal number names in order |
| | Rational counting | The 5 counting principles | 1. Count objects by touching each object just once<br>2. Say numbers just once in conventional order<br>3. Objects can be touched in any order and the starting point and the order in which they are counted does not affect how many there are<br>4. The arrangement of the objects does not affect how many there are<br>5. The last number said tells how many in the whole collection |
| | Robust counting | That objects can be touched in any order, and the starting point, ending point and the order in which the objects are counted does not affect how many there are | Can separate those that have been counted from those that have not yet been counted<br>Always uses the conventional number name order<br>Can count collections in varying arrangements<br>The arrangement does not affect how many there are |
| | Flexible counting | Any collection has only one count<br>You can expand from 1:1 to 1:many without changing the count<br>Counting on and counting back | Collection of 22 will always be more than 21; use equal groups to count large collections<br>Skip count by reciting every second number or jumping along a number line<br>Subitise and skip count to arrive at the count<br>Count on / count backwards from any number |

| Unit | Concept | Knows | Can construct/do/tell |
|---|---|---|---|
| **3. Bridging and counting patterns** | Oral counting pattern | Number names to ten, then twenty, then one hundred | Memorise word sequence 1–13<br>Hear the 4 to 9 part of the sequence 14–19<br>Bridge the decades<br>Bridge the hundreds then the thousands |
| | Seriation and odometer | What number comes before and after each number<br>Odometer is the principle that occurs when bridging tens, hundreds, thousands | Orally tell:<br>39 plus one becomes 40; 20 take one is 19<br>The difficulty of the teen numbers that don't fit the pattern<br>Zero numbers: ten, twenty, thirty, forty, etc. |
| | Comparison and order | Numbers can be ordered orally as well as numerically | We have two groups and we count them, the larger is further along the sequence of counting numbers<br>Smallest, largest, betweenness |
| **4. Numerals** | Numerals to 9 | Child's language; materials language; mathematical language; symbol language | Language, reading, writing and symbols for numbers 0–9<br>All numbers can be made up from these symbols |
| | Numerals to 19 | Child's language; materials language; mathematical language; symbol language | Language, reading, writing and symbols for numbers 0–19<br>Teen numbers do not follow the pattern of other numbers in their number names |
| | Number tracks, ladders, lines and boards | Different models exist (Will be explored fully in N2 and N3) | Set, number board and number line can be used to represent numbers |
| | Counting numerals to 999 | Relationship between oral number names and numerals to 999 | Hundreds tens and ones using number board, number line and sets 0–99 first, then 0–999 |

| N1 Counting: Observation Checklist Teacher Recording Instrument | | | | |
|---|---|---|---|---|
| **Unit** | **Concept** | **Knows** | **Can construct/do/tell** | **Comments/Observations** |
| **1. Pre-counting** | Identifying and describing attributes | To observe details and recognise likeness and difference between<br>Objects and pictures | Notice colour, size, shape, sound, taste, texture and an object's function or use<br>Discuss and use appropriate language/vocabulary to talk about likeness and differences | |
| | Matching and sorting | To focus on "sameness" of properties of objects | Sort by one attribute the same; Why do these match?<br>Sort involving matching; "Decide Do and Describe" | |
| | Comparing and ordering (unnumbered) | Can look at items and compare by understanding difference<br>Order items from least to most | Identify big/little, hot/cold, smooth/rough, tall/short, heavy/light, and so on<br>Put two or more objects in sequence from the least of an attribute to that which has the most | |
| | The attribute of number | Number is a special attribute in its own right | Identify something by the amount of objects; for this attribute other attributes do not matter | |
| | Subitising | See a pattern at a glance without counting | Recognise collections of one, two, and three things without counting | |
| | One-to-one and one-to-many correspondence | 1:1 and can extend to 1:many | Count in ones; count in groups (twos, fives, tens, hundreds and so on) | |
| **2. Introducing rote and rational counting** | Pre-rote and rational counting | Rhyme, rhythm, movement, music and clapping | Invent words and sequences for rhyme, rhythm, movement, music and clapping | |
| | Rote and ordinal counting | Number names in the right order | Recite number names in the right order<br>Remember ordinal number names in order | |

| | | | | |
|---|---|---|---|---|
| | Rational counting | The 5 counting principles | 1. Count objects by touching each object just once<br>2. Say numbers just once in conventional order<br>3. Objects can be touched in any order and the starting point and the order in which they are counted does not affect how many there are<br>4. The arrangement of the objects does not affect how many there are<br>5. The last number said tells how many in the whole collection | |
| | Robust counting | That objects can be touched in any order, and the starting point, ending point and the order in which the objects are counted does not affect how many there are | Can separate those that have been counted from those that have not yet been counted<br>Always uses the conventional number name order<br>Can count collections in varying arrangements<br>The arrangement does not affect how many there are | |
| | Flexible counting | Any collection has only one count<br>You can expand from 1:1 to 1:many without changing the count<br>Counting on and counting back | Collection of 22 will always be more than 21; use equal groups to count large collections<br>Skip count by reciting every second number or jumping along a number line<br>Subitise and skip count to arrive at the count<br>Count on / count backwards from any number | |

| Unit | Concept | Knows | Can construct/ do/tell | Comments/Observations |
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| | Comparison and order | Numbers can be ordered orally as well as numerically | We have two groups and we count them, the larger is further along the sequence of counting numbers<br>Smallest, largest, betweenness | |
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| | Numerals to 19 | Child's language; materials language; mathematical language; symbol language | Language, reading, writing and symbols for numbers 0–19<br>Teen numbers do not follow the pattern of other numbers in their number names | |
| | Number tracks, ladders, lines and boards | Different models exist (Will be explored fully in N2 and N3) | Set, number board and number line can be used to represent numbers | |
| | Counting numerals to 999 | Relationship between oral number names and numerals to 999 | Hundreds tens and ones using number board, number line and sets 0–99 first, then 0–999 | |

## Subtest item types and Diagnostic Mapping Points

### Subtest 1 items (Unit 1: Pre-Counting)

**Identifying and describing attributes**

1. Have you provided students with frequent experiences manipulating a range of different materials? e.g. toys, counters, animals, bears, balls, blocks, shapes, buttons, bottle tops etc. Ask students to tell you what they can about these collections of buttons/blocks/bears etc.

   (a) Place a range of objects in a box. Have student put hand in sleeve hole into box and pick up one item. Ask them to describe what they can feel. Can they guess what they have? Have them pull it out to identify. Re-examine the object. Repeat with other objects.

   (b) Place a range of objects on a tray. Cover with a tea towel. Have student put hand under tea towel and pick up, describe and identify an object. Repeat until they have identified all objects. Then uncover tray and check.

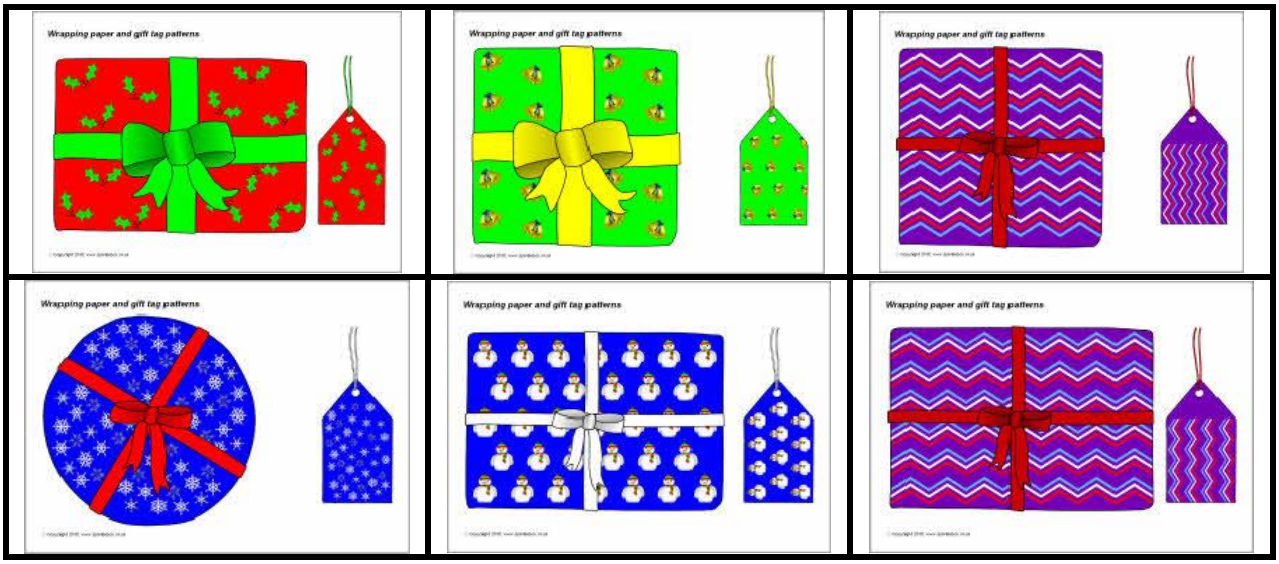
   (c) Place collections of mixed objects on tray. Have student tell you what they can about the collection. Which attributes do they identify? Can student talk about attributes of size, colour, shape, texture, movement, smell, etc.?

**Matching and sorting**

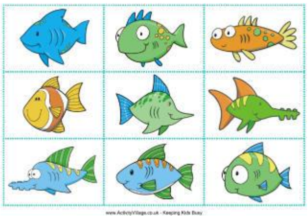
2. Have you provided students with frequent opportunities to sort collections? Use sorting trays, grids, egg cartons, cups, containers, plastic component sorting boxes, sorting circles, etc.

   - identifying similarities
   - identifying differences
   - sets, sorting circles

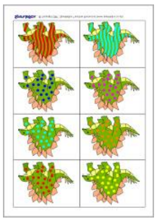
   (a) Can students match tags to presents? Use cards.



(b) Play the fish matching game. Ask: *How did you do this?*



(c) Using sorting games, can the student identify what's the **same**? What's **different**? Ask: *How have you sorted the cards?* Can they sort according to similarities? Can they sort by identifying differences?



(d) Using sorting circles, give student sets of materials to sort by one attribute, say, all red; encourage them to tell you what they are doing; then two attributes, say, red and small; once again encourage them to tell you what they are doing; and so on. Can they nominate attributes and sort appropriately?

Ask: *What is your sorting rule?*

(e) Give the student a container containing 5 red, 5 blue, 5 yellow, and 5 green teddies/counters. Ask the student to tip out the counters and sort them. Once they have sorted the counters, ask them to explain their sorting.

**Observe**: Did they sort according to attributes? Can they explain similarities and differences of the group?

(f) Using bear counters or similar, place the following in front of student (large bears – 1 red, 2 blue, 1 yellow; medium bears – 1 red, 1 blue, 1 yellow, 1 green; small bears – 2 red, 1 blue, 2 green.

- Can the student sort the bears?
- Can the student sort the bears according to size?
- Can the student sort the bears another way?

**Comparing and Ordering (Unnumbered)**

3. Have students order a collection of vehicles (matchbox toys or similar). Ask: *What order have you placed them in? What did you compare?*

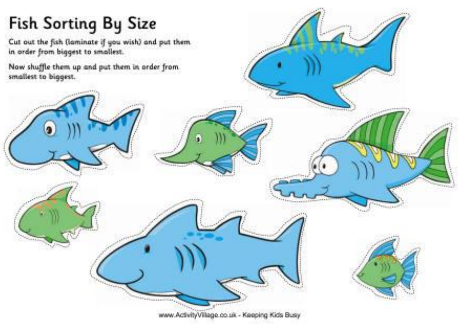
(a) Ask student to compare two objects you have placed, e.g. large/small; tall/short; big/little; long/short; narrow/wide; light/heavy; thick/thin, etc.

(b) Have student create their own pairs. Ask: *How did you compare that?*

(c) Introduce wide/wider; thin/thinner; large/larger, etc. Can student create these?

(d) Ask student to order with three objects. Can they identify the object "between"? What language do they use? Is the student using "-est" words, e.g. small, smaller, smallest?

(e) Have student put fish cards in order. Ask: *What order have you put them in? How did you decide that? What did you compare?* Note what language they use.



**The attribute of number**

4.

(a) Put out three groups of identical objects. In one group have two objects, in one group have three objects, and in third group have four objects. Ask: *What do you see?* Does the student identify what attribute discerns the groups?

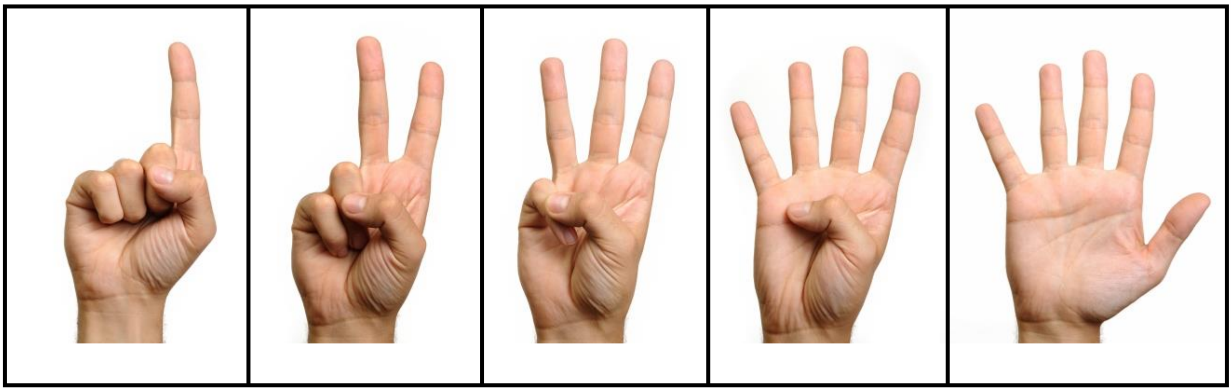
(b) Put four different coloured pencils in one group. Put four different objects in second group. Ask: *What do you see?* Does the student identify the attribute that is constant?

**Subitising**

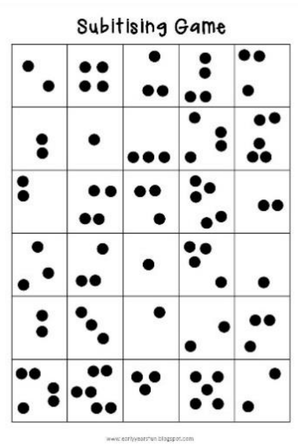
5.

(a) Flash small groups of things to student. Ask: *How many at a glance do you see?*

(b) Use a set of five finger cards. Hold a card up quickly, ask: *How many did you see?*



(c) Use subitising dot cards. Repeat above activity.



**One-to-one and one-to-many correspondence**

6. Using the one-to-one card on the following page:

   (a) Ask student to gather enough straws to "give" one to each child.

   (b) Then ask them to put an apple in each shopping basket.

   (c) Then a ball to each sports team.

   Display a card showing five circles. Ask the student to pick up enough counters to put one in each circle and then to distribute them. Did the student collect all five counters before placing one in each circle?

## One-to-one and one-to-many work card



## Subtest 2 items (Unit 2: Introducing Rote and Rational Counting)

1.

(a) Say: Start at 1 and show me how far you can count.

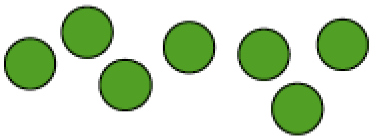
In the box, write the number the student can get to without making an error.

(b) Put out 10 counters in a row (all the same colour).

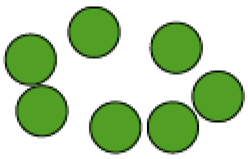
Say: How many counters are there altogether? Touch the counters as you count.
Hint: Write the numbers inside the counters to show how they counted e.g.


(c)

If the student is correct, take 7 counters and arrange them like this:

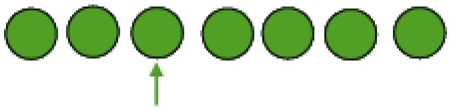


2. a. If the student is correct again, take the same counters and arrange them like this:



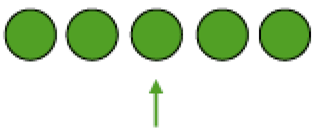
Say: How many counters are there altogether? Touch as you count aloud.

b. If the student is correct again, take the same counters and arrange them like this:



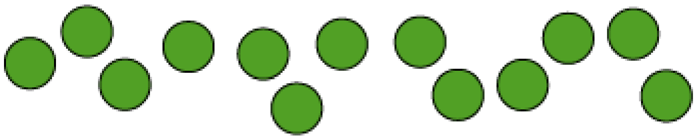
Point to the counter shown. Say: Count them from this one.

c. If the student is correct, arrange five counters in a row and point to the middle counter.



Say: Count this set but finish here.

3. Put out 13 counters as shown below (all the same colour).



a. Say: Count me five counters.

b. Say: Count me eight counters

4. a. Put out 4 counters. Say: Jill has this many books: Tell me how many books she has.

b. Put out 7 counters. Say: Bob has this many books: Tell me how many books he has.

c. Say: Bill has eight crayons. Use counters to show how many crayons he has.

d. Say: Jan has zero crayons. Use counters to show how many crayons she has.

5.

Using counters or farm animals or toys, count 10 to be used for this question. (student to do) and ask the student to place them in a row. Repeat this process for the following numbers. Place a tick in box if done correctly.

| | | | | |
|---|---|---|---|---|
| a) 10 | ☐ | selected amount | ☐ | counted orally |
| b) 2 | ☐ | selected amount | ☐ | counted orally |
| c) 7 | ☐ | selected amount | ☐ | counted orally |
| d) 6 | ☐ | selected amount | ☐ | counted orally |
| e) 8 | ☐ | selected amount | ☐ | counted orally |

6. Give the student a container with six blocks, six counters and six pencils (18 objects). Ask the student to tip the objects out and count them. After counting, ask the student how many items are in the group.

Did the student: tag each item with one number name?
identify "how many" in the group as the last number?
predict that the number of objects in the group remained the same?

7. Say: *I have some paddle pop sticks in this container. Tip them out, how many do you see?*

Did the student: count them individually?
count them in groups?
skip count them?

If the student counts them in ones, ask: *Can you count them a different way?*

When they have done so, ask: *Are there still the same number?*

Use another container of objects, repeat. When student has completed first count, ask: *If I count them in groups of two will I still get the same number? Why or why not?*

## Subtest 3 items (Unit 3: Bridging and Counting Patterns)

### Counting orally

1. Can the student recite the numbers orally especially when bridging ten, twenty, thirty, etc. then 100, 200, 300 etc. then 1000, 2000, 300 etc.?

2. Does student discern these counting patterns?

3. Does student discern fourteen and forty? eighteen and eighty? Nineteen and ninety etc.?

4. Can the student count forwards to 20? (or given number)

5. Can the student count backwards from 10? (or given number)

6. Can the student predict and name the decades by following the 1 to 9 sequence?

7. Can the student count forwards from and backwards from a given number? Do they discern the pattern, say, seventy-nine comes before eighty, sixty-nine before seventy and so on?

8. Can they predict and name the hundreds by following the 1 to 9 sequence?

9. Do they repeat the decade sequence and 1 to 9 sequence within each of the hundreds?

10 Do they predict and name the thousands by following the 1 to 9 sequence?

11. Do they repeat the hundreds, decades and 1 to 9 sequence within each of the thousands?

### Seriation and odometer

1. Ask student to generate a number sequence using the constant function on their calculator. Have student read, say, predict and verify the numbers from the display. Take note especially as they approach the decades then the hundreds.

2. Begin a counting sequence, have the student continue it. Begin at one, then at other numbers.

3. Have student tell you the biggest number they know. Ask: *What is one more? How do you know?*

4. Have student manipulate odometer rings. Ask student what changes after each 9 in a sequence? Have them use the odometer to read a number sequence. Ask: *What number is 1 more than 99? (109, 189, 1099 ...) What numbers change when 1 is added to each 9? What is the pattern in these changes?*

**Comparison and order**

Using a collection of numbers on cards 1–10 and some other numbers from 11–20 do the following:

1. Pick two numbers. Ask: *Where are they in the counting sequence? Which is further away from zero?*

2. Give student varying number cards. Ask them to place them in order.

3. Can student explain why a number is larger than another?

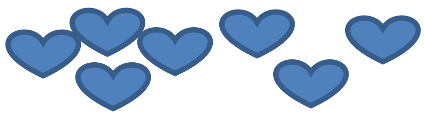
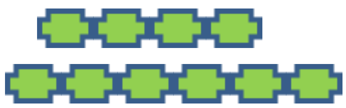
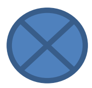
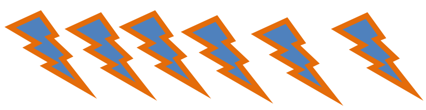
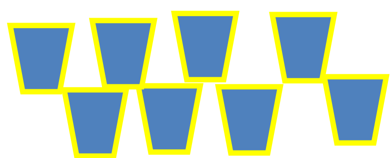
4. Can student order number cards in ascending and descending order?

## Subtest 4 items (Unit 4: Numerals)

Can student:

- Interchange: numerals, materials, language, symbol language
- Label collections
- Represent numbers using materials, digit numbers, words
- Write numbers in sequence
- Identify missing numbers in spoken and written sequence
- Write numbers in ascending and descending order
- Identify numbers between other numbers

1. Match the picture, numeral and number name. (This can be done effectively with manipulables, numerals and number names on cards for many students.)

| Picture | Numeral | Number name |
|---|---|---|
|  | 8 | two |
|  | 1 | seven |
|  | 3 | ten |
|  | 10 | three |
|  | 2 | five |
|  | 9 | nine |
|  | 4 | one |
|  | 6 | four |
|  | 7 | eight |
|  | 5 | six |

**CDAT Items**

2. Give the student a pencil and paper.
   a. Say: Maree has three books. Write the number that tells how many books.
   b. Say: Jack has seven books. Write the number that tells how many books.
   c. Say: Sue has no books. Write the number that tells how many books.

3. Place the set of number cards in front of the student.
   a. Say: Lizzie has this many lollies. [Point to the "2".] Tell me how many lollies she has.
   b. Say: Frank has this many lollies. [Point to the "6".] Tell me how many lollies he has.
   c. Say: Brian has this many lollies. [Point to the "0".] Tell me how many lollies he has.

4. Place the set of number-name cards in front of the student.
   a. Say: Maree has this number of books. [Point to the "four".]
      Write the number that tells how many books.
   b. Say: Jack has this number of books. [Point to the "eight".]
      Write the number that tells how many books.
   c. Say: Sue has this number of books. [Point to the "zero".]
      Write the number that tells how many books.

5. Use the number cards and number-name cards.
   a. Say: Lizzie now has this many lollies. [Point to number "5".]
      Point to the number-name card that matches this number.
   b. Say: Frank now has this many lollies. [Point to number "9".]
      Point to the number-name card that matches this number.
   c. Say: Brian now has this many lollies. [Point to number "0".]
      Point to the number-name card that matches this number.

6. a. Point to the "4" number card and say: Write what number comes next after.
   b. Point to the "6" number card and say: Write what number comes next after.
   c. Point to the "9" number card and say: Write what number comes next after.

7. a. Point to the "8" number card and say: Write what number comes just before.
   b. Point to the "1" number card and say: Write what number comes just before.
   c. Point to the "10" number card and say: Write what number comes just before.

8. Put out two number cards as below. Say: Point to the larger number in each pair:
   a. | 7 | | 4 |
   b. | 3 | | 5 |
   c. | 8 | | 6 |

9. Put out the number cards as below. Say: Put these in order from smallest to largest:

| 6 | 2 | 0 | 9 | 7 | 4 |

10. a. Say: Count on one more from this number. [Point to the "6" number card.] Write the number that you finish on.
    b. Say: Count on two more from this number. [Point to the "4" number card.] Write the number that you finish on.
    c. Say: Count on three more from this number. [Point to the "7" number card.] Write the number that you finish on.

11. a. Say: Count back one from this number. [Point to the "5" number card.] Write the number that you finish on.
    b. Say: Count back two from this number. [Point to the "10" number card.] Write the number that you finish on.
    c. Say: Count back three from this number. [Point to the "7" number card.] Write the number that you finish on.

12.

Write the number for each number name:

a. sixty-seven ____________________

b. fifty ____________________

c. thirteen ____________________

Write the number name for each number:

a. ______________________________________ 83

b. ______________________________________ 30

c. ______________________________________ 17

13.

(a) Here is 19.

- What number comes next?
- What number comes before?

(b) Here is 100.

- What number comes next?
- What number comes before?

(c) Here is 109.

- What number comes next?
- What number comes before?

(d) Here is 119.

- What are the next three numbers?
- What number comes before?

14.

(a) Here is a counting pattern: 10, 20, 30, ____, ____, ____

What are the next three numbers in this pattern?

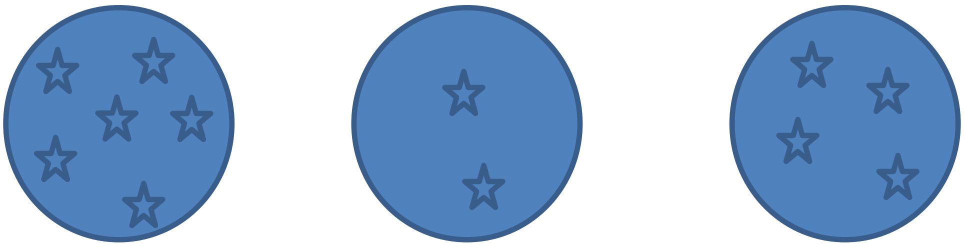
(b) In this counting pattern, what comes before these three numbers:

____, ____, ____, 50, 60, 70?

(c) Use numbers relevant to your students: ones, then two, tens, fives. Remember to always ask to count backwards as well as forwards.

(d) Then ask for numbers between other numbers.

(e) Use visual representations to order number sets, e.g.



# Appendices

## Appendix A: AIM Early Understandings Modules

### Module content

<table>
<tr>
<td>1st module<br>Number N1: Counting<br>*Sorting/correspondence<br>*Subitising<br>*Rote<br>*Rational<br>*Symbol recognition<br>*Models<br>*Counting competencies</td>
<td>2nd module<br>Algebra A1: Patterning<br>*Repeating<br>*Growing<br>*Visuals/tables<br>*Number patterns</td>
<td>3rd module<br>Algebra A2: Functions and Equations<br>Functions<br>*Change<br>*Function machine<br>*Inverse/backtracking<br>Equations<br>*Equals<br>*Balance</td>
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<tr>
<td>4th module<br>Number N2: Place Value<br>Concepts<br>*Place value<br>*Additive structure, odometer<br>*Multiplicative structure<br>*Equivalence<br>Processes<br>*Role of zero<br>*Reading/writing<br>*Counting sequences<br>*Seriation<br>*Renaming</td>
<td>5th module<br>Number N3: Quantity<br>Concepts<br>*Number line<br>*Rank<br>Processes<br>*Comparing/ordering<br>*Rounding/estimating<br>Relationship to place value</td>
<td>6th module<br>Operations O1: Thinking and Solving<br>*Early thinking skills<br>*Planning<br>*Strategies<br>*Problem types<br>*Metacognition</td>
</tr>
<tr>
<td>7th module<br>Operations O2: Meaning and Operating<br>*Addition and subtraction; multiplication and division<br>*Word problems<br>*Models</td>
<td>8th module<br>Operations O3: Calculating<br>*Computation/calculating<br>*Recording<br>*Estimating</td>
<td>9th module<br>Number N4: Fractions<br>Concepts<br>*Fractions as part of a whole<br>*Fractions as part of a group/set<br>*Fractions as a number or quantity<br>*Fraction as a continuous quantity/number line<br>Processes<br>*Representing<br>*Reading and writing<br>*Comparing and ordering<br>*Renaming</td>
</tr>
</table>

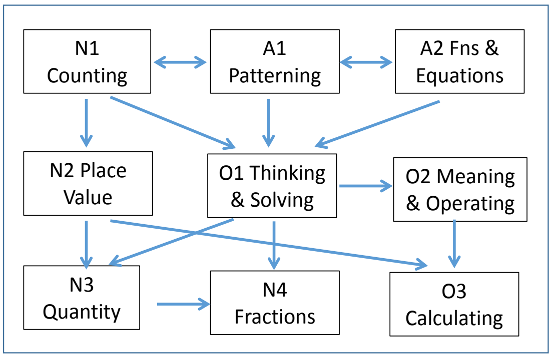
## Module background, components and sequence

**Background**. In many schools, there are students who come to Prep/Foundation with intelligence and local knowledge but little cultural capital to be successful in schooling. In particular, they are missing basic knowledge to do with number that is normally acquired in the years before coming to school. This includes counting and numerals to 10 but also consists of such ideas as attribute recognition, sorting by attributes, making patterns and 1-1 correspondence between objects. Even more difficult, it includes behaviours such as paying attention, listening, completing tasks, not interfering with activity of other students, and so on.

Teachers can sometimes assume this knowledge and teach as if it is known and thus exacerbate this lack of cultural capital. Even when the lack is identified, building this knowledge can be time consuming in classrooms where students are at different levels. It can lead to situations where Prep/Foundation teachers say at the end of the year that some of their students are now just ready to start school and they wish they could have another year with them. These situations can lead to a gap between some students and the rest that is already at least one year by the beginning of Year 1. For many students, this gap becomes at least two years by Year 3 and is not closed and sometimes widens across the primary years unless schools can provide major intervention programs. It also leads to problems with truancy, behaviour and low expectations.

**Components**. The AIM EU project was developed to provide Years F–2 teachers with a program that can accelerate early understandings and enable children with low cultural capital to be ready for Year 3 at the end of Year 2. It is based on nine modules which are built around three components. The mathematics ideas are designed to be in sequence but also to be connected and related to a common development. The modules are based on the AIM Years 7–9 program where modules are designed to teach six years of mathematics (start of Year 4 to end of Year 9) in three years (start of Year 7 to end of Year 9). The three components are: (a) Basics – A1 *Patterning* and A2 *Functions and Equations*; (b) Number – N1 *Counting* (also a basic), N2 *Place Value*, N3 *Quantity* (number line), and N4 *Fractions*; and (c) Operations – O1 *Thinking and Solving*, O2 *Meaning and Operating*, and O3 *Calculating*. These nine modules cover early Number and Algebra understandings from before school (pre-foundational) to Year 2.

**Sequence**. Each module is a sequence of ideas from F–2. For some ideas, this means that the module covers activities in Prep/Foundation, Year 1 and Year 2. Other modules are more constrained and may only have activities for one or two year levels. For example, Counting would predominantly be the Prep/Foundation year and Fractions would be Year 2. Thus, the modules overlap across the three years F to 2. For example, Place Value shares ideas with Counting and with Quantity for two-digit numbers in Year1 and three-digit numbers in Year 2. It is therefore difficult, and inexact, to sequence the modules. However, it is worth attempting a sequence because, although inexact, the attempt provides insight into the modules and their teaching. One such attempt is on the right. It shows the following:




1. The foundation ideas are within *Counting*, *Patterning* and *Functions and Equations* – these deal with the manipulation of material for the basis of mathematics, seeing patterns, the start of number, and the idea of inverse (undoing) and the meaning of equals (same and different).

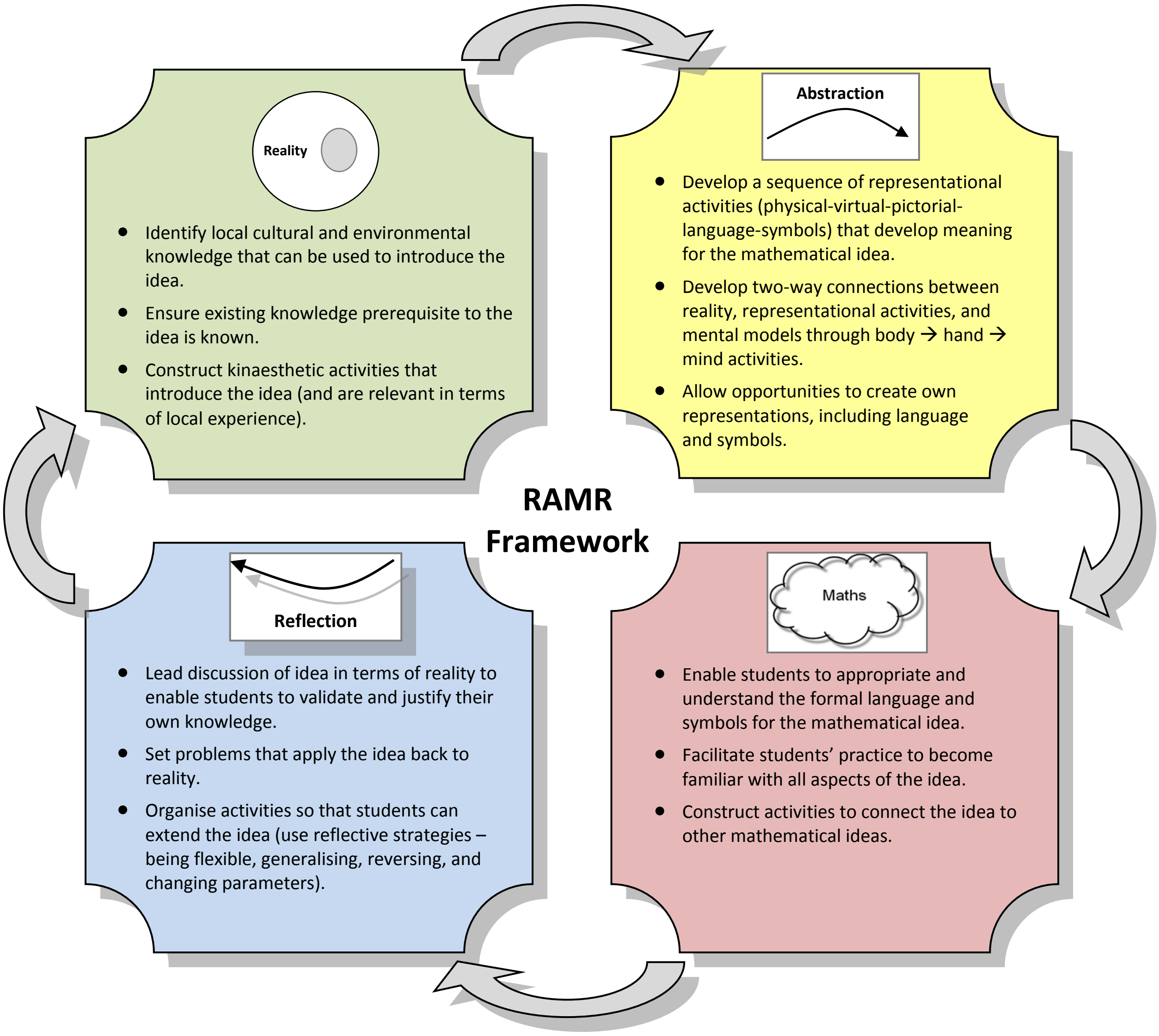
2. The central components of the sequence are *Thinking and Solving* along with *Place Value* and M*eaning and Operating* – these lead into the less important *Calculating* and prepare for *Quantity*, *Fractions* and later general problem-solving and algebra.

The *Quantity*, *Fractions* and *Calculating* modules are the end product of the sequence and rely on the earlier ideas, except that *Quantity* restructures the idea of number from discrete to continuous to prepare for measures.

## Appendix B: RAMR Cycle

AIM advocates using the four components in the figure below, reality–abstraction–mathematics–reflection (RAMR), as a cycle for planning and teaching mathematics. RAMR proposes: (a) working from reality and local culture (prior experience and everyday kinaesthetic activities); (b) abstracting mathematics ideas from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language and symbolic representations, i.e. body → hand → mind); (c) consolidating the new ideas as mathematics through practice and connections; and (d) reflecting these ideas back to reality through a focus on applications, problem-solving, flexibility, reversing and generalising. The innovative aspect of RAMR is that Reality to Abstraction to Mathematics develops the mathematics idea while Mathematics to Reflection to Reality reconnects it to the world and extends it.

Planning the teaching of mathematics is based around the four components of the RAMR cycle. They are applied to the mathematical idea to be taught. By breaking instruction down into the four parts, the cycle can lead to a structured instructional sequence for teaching the idea. The figure below shows how this can be done.




The YuMi Deadly Maths RAMR Framework

# Appendix C: Teaching Frameworks

## Teaching scope and sequence for counting

| TOPIC | SUB-TOPICS | DESCRIPTIONS AND CONCEPTS/STRATEGIES/WAYS |
|---|---|---|
| Counting | Pre-counting | Identifying and describing attributes<br>Matching and sorting<br>Comparing and ordering (unnumbered)<br>The attribute of number<br>Subitising<br>One-to-one and one-to-many correspondence |
| | Introducing rote and rational counting | Pre-rote and rational counting<br>Rote and ordinal counting<br>Rational counting<br>Robust counting<br>Flexible counting |
| | Bridging and counting patterns | Oral counting pattern<br>Seriation and odometer<br>Comparison and order |
| | Numerals | Numerals to 9<br>Numerals to 19<br>Number tracks, ladders, lines and boards<br>Counting numerals to 99 and 999 |

## Proposed year-level framework

| YEAR LEVEL | NUMBER – COUNTING | |
|---|---|---|
| | **Semester 1** | **Semester 2** |
| **Prep** | *Sorting/correspondence* – experience sorting objects by attribute, putting sets into 1:2 correspondence; experience 1:1 and 1:many correspondence.<br>*Rote* – forwards to 10.<br>*Rational* – experience making and counting collections to 10; match 1:1 to compare collections through Body activities → Hand activities → Mind activities.<br>*Symbol recognition* – experience identifying numerals of personal significance (e.g. age); experience real-world, language, set/line models, up to 10 (e.g. storytelling, forming sets of objects, acting out story on an unnumbered number track). | *Sorting/correspondence* – reinforce sorting objects by attribute, experience; experience 1:1 and 1:many correspondence with number names.<br>*Rote* – rote counting forwards and backwards to 20; experience ordinal numbers to 20.<br>*Rational* – rationally counting to 20, identifying 1:1 with objects to 20, with and without counting (subitising); count robustly (recognising can start and end count at any number and can count groups and objects in groups).<br>*Symbol recognition* – introduce identifying digits and relate to objects to 20; introduce number names as words and relate to symbols and collections of objects to 20. |
| **1** | *Sorting/correspondence* – sort objects of different attribute (e.g. same colour but different shape); experience 1:1 and 1:many (notion of unit big idea).<br>*Rote* – experience forwards/backwards to 20 and 40 by 1s and 2s; strategies to count collections (song and rhyme); introduce ordinal numbers to 40.<br>*Rational* – make and count collections to 50; count out a subset from a set up to 50.<br>*Symbol recognition* – reinforce digit recognition to 40 (real world, models, language, symbol). | *Sorting/correspondence* – sort objects of different attributes; reinforce 1:many correspondence (particularly 1:10).<br>*Rote* – forward and backwards to 100 by 1s, 2s, 5s and 10s; reinforce ordinal numbers to 1000; discuss different things that could be counted (e.g. objects, groups, steps, position along number track and number line).<br>*Rational* – to 100, counting out subset (last number name tells how many).<br>*Symbol recognition* – introduce symbols to 100. |
| **2** | *Rote* – forwards and backwards to 130 in 1s, 2s, 5s, 10s.<br>*Rational* – to 130.<br>*Symbol recognition* - reinforce digit recognition to 1020 (real world, set/ line/ grid, language, symbol).<br>*Odometer* – experience to 130 (particularly at bridging the 100). | *Sorting/correspondence* – reinforce 1:100 as well as 1:10 and 1:1 correspondence.<br>*Rote* – forwards and backwards to 1000 in 2s and 5s; counting halves, thirds and quarters.<br>*Rational* – reinforce to 130; experience to 1000 (including collections of 10 and 50).<br>*Odometer* – experience counting forwards and backwards past multiples of 10 and 100 (10, 20, 30, and so on); discuss outcomes (patterns of ones, teens and decades); understand to 130, experience to 1000; look at odometer process as count halves, thirds, quarters. |
| **3** | *Rote* – reinforce forwards and backwards to 100 in 1s, 2s, 5s, 10s and 100<br>*Rational* – reinforce to 1000.<br>*Odometer* – reinforce to 1000. | *Rational* – extend beyond 1000 to 10,000; count whole and parts for fractions.<br>*Odometer* – extend beyond 1000 to 10,000; experience with wholes and fractions. |
| **Focus** | Body activities → Hand activities → Mind activities (visualise) then record. | Body activities → Hand activities → Mind activities (visualise) then record. |

The numbers in here will need to be decided by teachers.

Accelerated Inclusive Mathematics Early understandings Project

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