YuMi Deadly Maths

Year 9 Teacher Resource:

MG – Look alikes

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 9 Teacher Resource: **MG – Look alikes**

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

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Year 9 Measurement and Geometry

Look alikes

Learning goal

Students will:

- identify similar figures
- generalise the conditions of similarity
- determine scale factors.

Content description

Measurement and Geometry – Geometric reasoning

- Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)
- Solve problems using ratio and scale factors in similar figures (ACMMG221)

Big idea

Geometry – similarity as enlargement

Resources

Torch, screen, projector, cardboard triangles, squares, rectangles, circles, tracing paper, rulers, protractors, string, hook, paper, Maths Mat, elastics

Reality

Local knowledge

Discuss where similar figures occur in the local environment through shapes being enlarged, e.g. projection of movies. *How does this work? How does it leave the picture undistorted?* Discuss enlarging a picture, diagram or tag. Discuss the enlarging of pictures and diagrams on a computer. Discuss the enlargement of lettering on sign posts, banners, shop fronts. *How does a builder construct a house from a plan?*

Prior experience

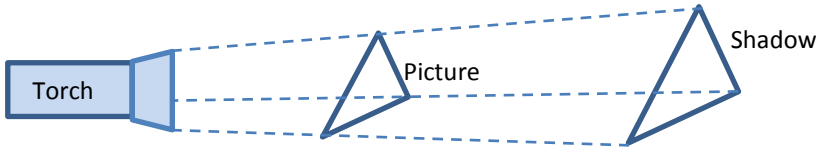
Discuss the students' experiences in producing similar shapes, e.g. photography:

- *If you want a close-up shot of a person or object, where do you stand?*
- *How does the camera have the ability to produce a standard or enlarged picture?*
- *How do you enlarge shapes/diagrams on the computer?*

Kinaesthetic

Torch shadows

Similarity means the same shape. Use a torch in a darkened room so that shadows of various shapes (held parallel to the screen) are cast onto the screen that is at right angles to the light from the torch.

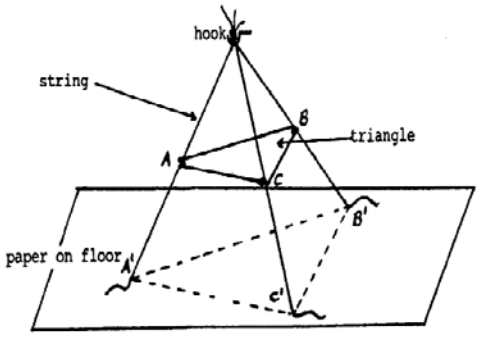


Trace the image of the shadow onto tracing paper and compare the objects and their shadows. Measure the angles and ratios of lengths of sides. *What conclusions can be made?* [Similarity projections are enlargements (or reductions); the shape is not changed.]

Children's coloured torches with a variety of pictures will also give the same experience. A projector can also be used to give students as many experiences as possible in comparing objects and their shadows.

String projections

Hold a triangle parallel to a large paper sheet (with notches at corner of shape), put a hook in the roof and pull down a string through each corner – this marks the corners of the enlarged triangle. Draw the enlarged triangle on the paper and



compare to original. Note the following:

- The strings must be straight.
- The triangle must be held horizontal.
- The strings must pass through where the corners of the triangle would be, so one person should hold the triangle flat while others hold the strings on the paper on the floor.
- When the strings are straight and through the notches, mark on the paper on the floor where each string ends. Connect these points to form a triangle.

This triangle is an **enlargement** of the notched triangle and shows **similarity**. Use a ruler and protractor to compare side lengths and angles to discover the properties of similarity. Complete the table below:

Triangle ABC	**Triangle $A^1B^1C^1$**	**Ratio**
$AB =$	$A^1B^1 =$	$\frac{AB}{A^1B^1} =$
$AC =$	$A^1C^1 =$	$\frac{AC}{A^1C^1} =$
$BC =$	$B^1C^1 =$	$\frac{BC}{B^1C^1} =$
$BAC =$	$B^1A^1C^1 =$	
$ABC =$	$A^1B^1C^1 =$	
$ACB =$	$A^1C^1B^1 =$	

Abstraction

Body **Maths Mat and elastics**

Have students make a right triangle where one student stands on the bottom left corner of the mat holding two elastics. A second student takes one end of one elastic and walks three squares along the horizontal base stretching the elastic to form the base of the triangle. A third student takes one end of the other elastic and walks two squares up the vertical boundary stretching the elastic to form the perpendicular height of the triangle. A fourth student takes a third elastic, gives one end to the student on the base and walks to the student at the top end of the vertical elastic stretching the elastic and gives the other end to the student to form the hypotenuse. Measure and record the two angles thus made with a protractor.

If this triangle were enlarged by a scale factor of 2, what length would the new base be?

Make the new base with elastics. Repeat for other sides. Measure the angles. Calculate and record the ratios of the sides. Then reduce the original triangle by a scale factor of 0.5. Repeat the process and record the data on a table as below:

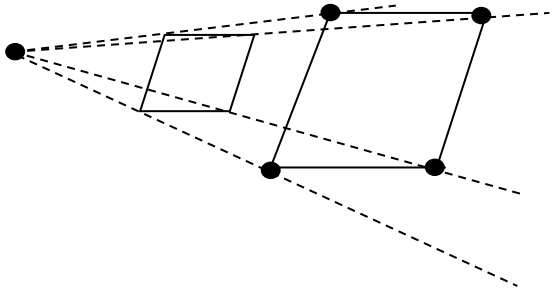
Shape	**Angle 1**	**Angle 2**	**Angle 3**	**Side 1**	**Side 2**	**Side 3**	**Ratio of sides 1**	**Ratio of sides 2**	**Ratio of sides 3**
1.									
2.									
3.									

Ask students:

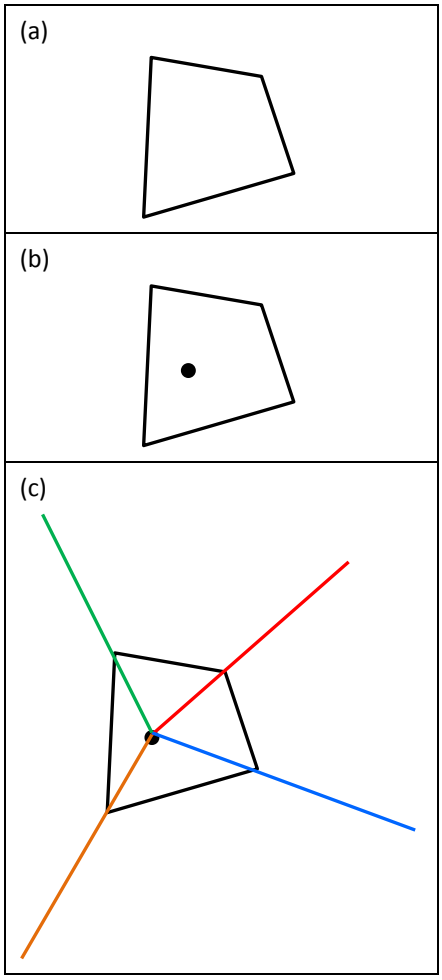
- *What do you notice about the ratios of the corresponding sides?*
- *Are the three triangles similar? Why?*
- *Is triangle 2 an enlargement of triangle 3? True or False? Justify your reasoning.*
- *Describe the shapes that have been made. What are the determining criteria of similar shapes?*

Hand

1. Draw a shape and a dot outside it (or can be inside). Draw dotted lines from the dot through the corners. Measure the distance from the dot to the corner, then measure on the same length and put a dot. In this way, the shape can be doubled to a similar shape, as shown on the right (a triple-size similar shape requires measuring double distance past the corner, and so on for larger similar shapes). This "animation" method allows students to experience similarity as an enlargement. Measure the similar shapes to investigate the properties of similar shapes.



2. Create a film strip or slide show to illustrate enlargement in making similar shapes to a factor or scale of 2.

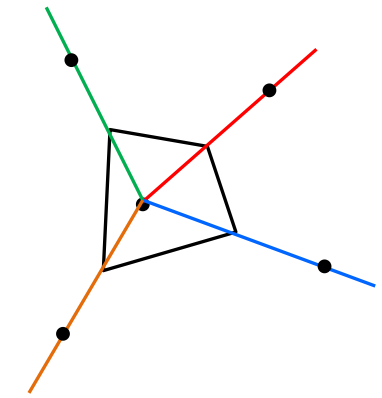


(a) The film strip starts with an irregular quadrilateral on a plain background.

(b) Place a dot inside the quadrilateral.

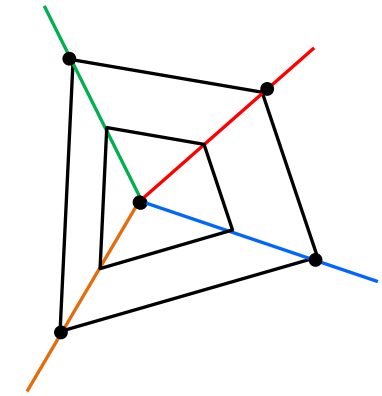
(c) Draw coloured lines from the dot through each of the four vertices so that the lines disappear off the screen.

(d)



(d) Measure the distance from the centre dot along the four lines to each of the vertices. Mark an equal distance along each line outside the quadrilateral.

(e)



(e) Join the new dots to create an enlargement of the original.

Note: Enlargements with factors of 3 or 4 are made by having a distance 2 or 3 times the distance from each vertex to the centre dot.

Mind *Visualise a clock face that would be suitable for your kitchen. Now see an enlargement that would be the size of Big Ben. What scale factor may have been used? See a traffic Stop sign in your mind. What would a similar sign look like enlarged 3 times the size? Put an octagon in the middle of the Stop sign that has been reduced by 0.5.*

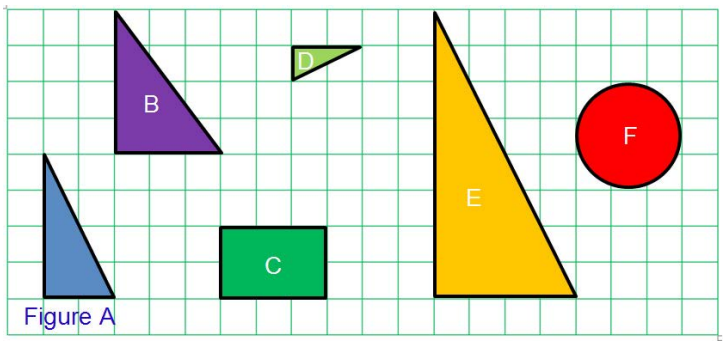
Creativity Use grid paper to plot a number of shapes, their enlargements and reductions.

Mathematics

Language/ symbols transformation, similar, scale, enlargement, ratio, reduction

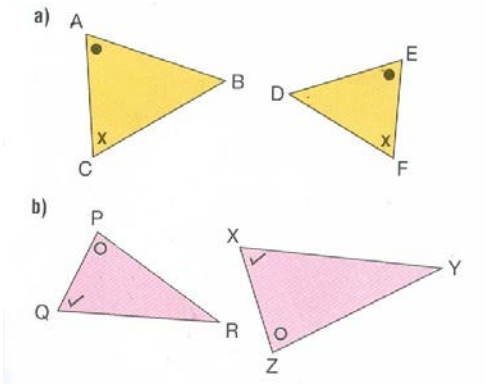
Practice

1. Which of the shapes below is similar to Figure A? Why?

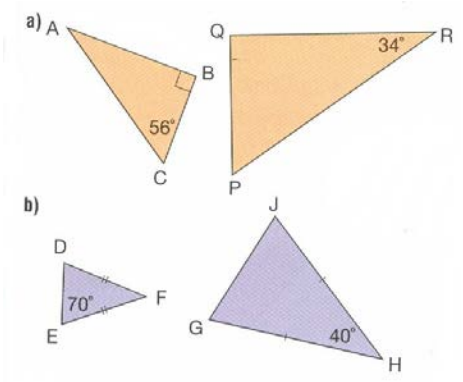


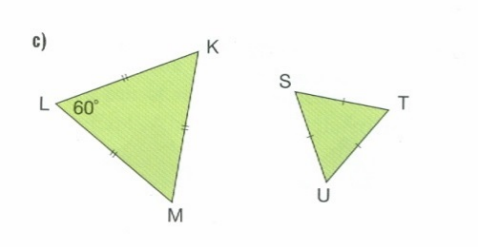
2. Draw similar figures using a coordinate grid. Identify the scale factor for each of the enlarged or reduced figures.

3. For each pair of similar triangles, list the corresponding sides and angles.

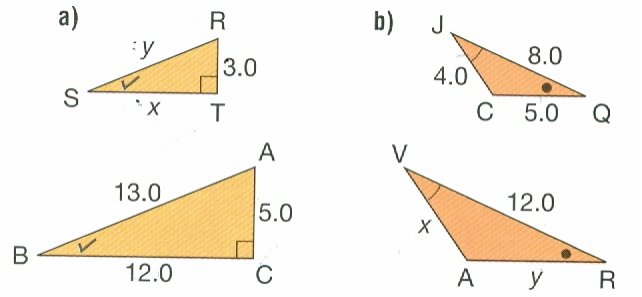


4. Each pair of triangles is similar. In each case, state the measures of the angles that are not marked.



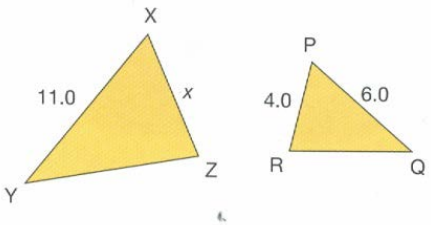


5. Explain why each pair of triangles is similar. Find the values of *x* and *y*.

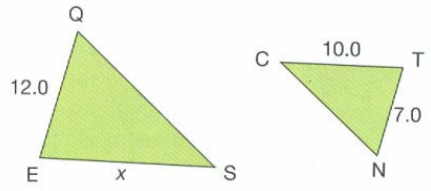


6. State the ratios of the corresponding sides of each pair of similar triangles. Find each value of x.

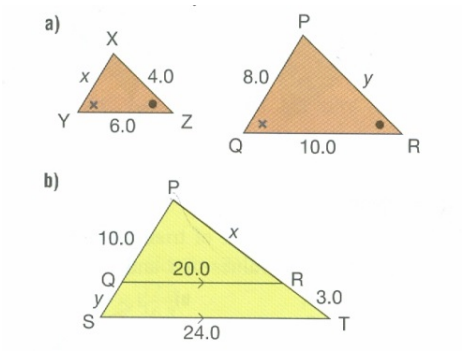
a) △XYZ is similar to △PQR.



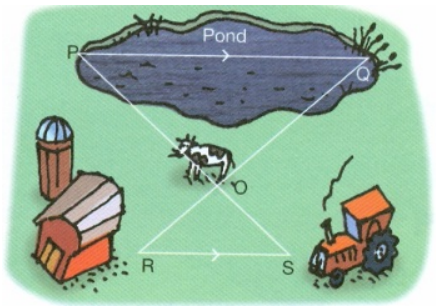
b) △TNC is similar to △EQS.



7. State which triangles are similar. Find the values of *x* and *y*.



8. To find the distance PQ across a farm pond, Marty marks out points R and S so that RS is parallel to PQ. By measuring, she finds that RS = 5.7 m, OP = 19.5 m, and OS = 4.2 m. What is the distance PQ?



Connections

Ensure the relationship between similarity as blow up or enlargement and similarity as properties of angles and ratio of sides are related. Make connections between similarity and other projections. One connection is to relate general similarity to specific rules for shapes. An example of such an activity is as follows.

Similar triangles

Consider you have drawn a small map of a triangular garden and you wish to provide the minimum information that would enable another person to make an exact copy of your triangle or to make an enlargement. Materials are paper, tracing paper, rulers, protractors.

1. Make a triangle that is scalene (no equalities).
2. Find a partner. Provide the partner with three pieces of information (made up from side lengths and angles) and partner has to make an exact copy of your map. They use your data to make this copy on tracing paper so it can be directly compared to the original.
3. Determine whether data given is enough for an accurate copy. Data should state how angles and side lengths relate.
4. There are three sets of data that will do this and one set that is close. See if you can find them.

Reflection

Validation

Investigate similarity in the real world, e.g. computers, chips, USBs, maps, building plans, construction and engineering. Validate a partner's creativity.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently; e.g. Mirror method for measuring height. Obtain mirror and paper/pen to record and draw.

1. Obtain a mirror. Place mirror on floor between you and the object whose height is to be measured (e.g. the wall).
2. Stand up straight. Move mirror so that you can see the top of wall in it. Measure height to your eyes, distance from you to mirror and distance of mirror to wall.

3. Divide distance of mirror to wall by distance of mirror from you and multiply this by the height to your eyes. This is the height of the wall. Calculate it and check.
4. *Why does this work?* (Hint: Draw a diagram of you, the wall, the mirror and lines from your eyes to mirror to top of wall.)

Extension

Flexibility. So that students obtain a comprehensive understanding of similarity, provide many applications, enlargement and reduction, using a variety of resources and materials for students to manipulate.

Reversing. Go from properties to similarity and similarity to properties. For example:

- *The corresponding angles in two triangles are equal and their corresponding sides are in the ratio of 2:5. How would you describe the triangles?*
- *Triangle ABC is similar to triangle DEF, being a reduction in the ratio of 3:4. Draw possible triangles and list their properties.*

Generalising. Two shapes are similar if one is an enlargement of the other. It is sufficient for their corresponding angles to be the same or the lengths of the corresponding sides to be in ratio to give similar figures. For triangles, two are similar if:

- all lengths of corresponding sides are in the same ratio;
- all corresponding angles are the same;
- two angles are the same (hence the third is – add to 180^{o}); or
- two sides are in ratio and the enclosed angle is equal.

Similar figures have the same shape but are different sizes. The scale factor in similar triangles is the ratio of the lengths of two corresponding sides:

$$\text{Scale factor} = \frac{\text{image length}}{\text{actual length}}$$

Changing parameters. Investigate further the affine projection and the divergent or perspective projection.

Teacher's notes

- Scaffold tasks so that the first enlargement/reduction coordinate is given. Extend to drawing complex shape enlargements/reductions on a coordinate grid.
- Some students may have difficulties in plotting the coordinates of enlarged/reduced images using a given scale factor.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a pattern block of a right triangle, students look at it, remove the pattern block, students then close their eyes and see the right triangle in their mind, then make a mental picture of a different type of right triangle.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.