YuMi Deadly Maths

Year 9 Teacher Resource:

NA – Time exploration

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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Growing community through education

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
refined and presented in professional development sessions.

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research.qut.edu.au/ydc/

Year 9 Number and Algebra

Time exploration

Learning goal

Students will

- convert between metric unit representations of time
- solve problems involving very small and very large time scales and intervals.

Content description

Number and Algebra – Real numbers

- Apply index laws to numerical expressions with integer indices (ACMNA209)
- Express numbers in scientific notation (ACMNA210)

Measurement and Geometry – Using units of measurement

- Investigate very small and very large time scales and intervals (ACMMG219)

Big ideas

Number – multiplicative structure (time and metric); pattern of threes

Resources

Ten strip mat, time units and blank cards

Reality

Local knowledge

Discuss where very small and very large numbers are found in the local knowledge and environment; e.g. speed of light, distances in solar system, time from BC to AD, number of seconds in a century.

Prior experience

Check that students understand the progression of the pattern of threes in the real number system up to trillions/trillionths. Students need to have automaticity in knowing the sequence in powers of ten and related number of places; for example, they need to know that a positive integer with twelve places is in the trillions, that $n \times 10^9$ is billions, $n \times 10^{14}$ is hundred trillions and so on, also applying this to the negative exponents to trillionths.

Revise index notation and the laws for multiplying and dividing by powers of ten:

- *Expand 10^4. What is the result?*
- *When you multiply 10^4 by 10^2, what is the result? What did you do?*
- *When you divide 10^5 by 10^3, what is the result. What did you do?*

Revise conversion from standard decimal form to scientific notation and vice versa. Check that students know units of time measurement and the speed formula:

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \text{ or } \text{Distance} = \text{speed} \times \text{time}$$

Kinaesthetic

Use the ten strip mat to plot units of time. Distribute cards: seconds to century and milliseconds to picoseconds for students to place in the appropriate place. Identify the seconds' place on the strip mat.

century	years	days	hours	minutes	seconds

petasec	terasec	gigasec	megasec	kilosec	seconds	millisec	microsec	nanosec	picosec	femtosec
10^{15}	10^{12}	10^9	10^6	10^3	10^0	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}

Have students take a blank card and write a time measurement relationship fact from seconds to century (60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 365 days = 1 year, 100 years = 1 century) and place the card under the appropriate place on the mat.

Repeat the process going from milliseconds in a second to picoseconds in a second using standard decimal form and scientific notation: one thousand milliseconds in one second (1 000 or 10^3), one million microseconds in one second (1 000 000 or 10^6), one billion nanoseconds in one second (1 000 000 000 or 10^9), one trillion picoseconds in one second

(1 000 000 000 000 or 10^{12}). Reverse: One millisecond is one thousandth of a second (10^{-3}), and so on, to one femtosecond is one quadrillionth of a second (10^{-15}).

Abstraction

Body

Have students make various calculations using their scientific calculators while stepping from one place to the required place, e.g. *How many seconds are there in a century?*

- Start at 1 second: calculator displays 1.
- Move to minutes: $1 \times 60 = 60$ sec/min.
- Move to hours: $60 \times 60 = 3600$, and convert to scientific notation: 3.6×10^3 sec/hr.
- Move to days: $3.6 \times 24 \times 10^3 = 86.4 \times 10^3$ sec/day.
- Move to years: $86.4 \times 365 \times 10^3 = 31\,536 \times 10^3$ sec/yr and convert to scientific notation: 31.536×10^6 sec/yr.
- Move to century: $31.536 \times 10^6 \times 10^2 = (31.536 \times 10^8) + (25 \times 86.4 \times 10^3$ to include leap years$) = (31.536 \times 10^8) + (2160 \times 10^3) = (31.536 \times 10^8) + (0.0216 \times 10^8) = (31.536 + 0.0216) \times 10^8$ (distributive law) $= 31.5576 \times 10^8$ sec/century.
- Standard decimal form = 3 155 760 000 sec/century.

Make other calculations in a similar manner, e.g. *How many nanoseconds are there in 15 hours?*

Hand

Use calculators to complete the following operations using the appropriate index law. Express the answer in both scientific notation and standard decimal form. Check first that students understand that brackets may be removed across multiplication and factors re-grouped to facilitate the operation, e.g. *Looking at the common factor, 10, what law applies to the indices? How will you now proceed in performing the operation?* The first example is demonstrated.

1. $(5.7 \times 10^6) \times (3.9 \times 10^9)$
   - Undo brackets: $5.7 \times 10^6 \times 3.9 \times 10^9$
   - Re-group factors: $5.7 \times 3.9 \times 10^6 \times 10^9$
   - Re-bracket: $(5.7 \times 3.9) \times (10^6 \times 10^9)$
   - Calculate: 22.23×10^{15} (add the indices)
   - Decimal form: 22 230 000 000 000 000 or 22.23 quadrillions
2. $(3.8 \times 10^2) \times (6.4 \times 10^7)$
3. $(7.2 \times 10^4) \times 9.4 \times 10^{-11}$

Mind

Shut your eyes and imagine a line, say from 0 to a billion (which is 1000 million) – think of this as similar to the 0 to 1000 line – now place numbers like 435 678 121 on it. Extend to trillions and trillionths. What place do you associate with 10^{12}, 10^{-9}? In your mind, progress along the number line from the ones to the given place.

It is important that students understand and know the progression of the pattern of threes and their names: ones → thousands → millions → billions → trillions → quadrillions; and parts of ones: ones → thousandths → millionths → billionths → trillionths → quadrillionths. They also need to associate how many places / what exponent is represented in each, e.g. billions have ten places, 10^9, and so on.

Creativity

Students create a poster showing very large and very small numbers expressed in both scientific notation and standard decimal form.

Mathematics

Language/ symbols

scientific notation, standard decimal form, billions, trillions, quadrillions, billionths, trillionths, quadrillionths; prefixes: milli, micro (micron), nano, pico, femto; kilo, mega, giga, tera, peta

Practice

1. Use the laws of index notation to find how many:
   (a) picoseconds in a millisecond?
   (b) microseconds in one hour?
   (c) milliseconds in a terasecond?

2. (a) What is your age today:
   - in years and days?
   - in days?
   - in megaseconds?

   (b) Is one megasecond longer or shorter than a year? Justify your response.
   (c) How many birthdays will you have celebrated when you reach one gigasecond of age?
   (d) How many nanoseconds have elapsed in an 80-minute maths lesson?
   (e) The universe is estimated to be 13.8 billion years old. What is its age in gigaseconds?
   (f) Light travels at a speed of approximately 300 000 000 metres per second. How long does it take for light to travel one metre?

3. Convert these standard decimal forms to scientific notation:
   (a) Distance between the Earth and the sun: 149 598 000 km
   (b) Distance that light travels in one year: 9 460 730 472 580.8 km

4. In 1977, *Voyager 1* was launched as an unmanned probe travelling at a speed of 17 km/sec. It has now left our solar system and is in interstellar space. Consider how fast the speed of 17 km/sec is by answering the following:
   (a) Name a place that is approximately 17 km away from the school.
   (b) How long would it take to get to Noosa from Brisbane (approx. 148 km) travelling at 17 km/sec?
   (c) How long would it take to travel the 1 700 km from Brisbane to Cairns travelling at 17 km/sec?
   (d) The Earth's circumference at the equator is 40 075.16 km. How long would it take to circumvent the Earth at the equator travelling at 17 km/sec?

   Remember the speed formula: Distance = Rate × Time

Creativity

Relate to the metric system and computer bytes.

Reflection

Validation

Students validate where very large and very small numbers are found in the real world; e.g. statistics, hits on websites, Gross National Products, census data, chances of winning Lotto, timeline of the Big Bang. Students validate the creative work of their partners.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently; e.g. *Find how much time it takes to travel one light year (1 ly) of distance in Voyager 1 and then find the time to travel to neighbouring stars within the Milky Way galaxy, such as Gliese 581 (20 ly from the sun), Epsilon Eridani (10.5 ly from the sun), Proxima Centauri (4.24 ly from the sun).*

Extension

Flexibility. Ensure flexibility in working with very large and very small numbers by offering many examples in the metric system and computer bytes.

Reversing. Give a number, e.g. 64 gigaseconds, and students have to find where to put this on a diagram of the system. Then reverse, give a position and students have to make up a situation. That is: number situation → system AND system → number situation.

Generalising. Can the students see the two forms of the system in their mind? Can they make up their own system? What happens when we move two places in multiplicative structure? If we are at 10^6, and we multiply by 10 000, we are now at 10^{10}, ten billions. Can students generalise – 2 places is 10^2, 3 places is 10^3 and so on?

Changing parameters. What if we changed base (e.g. seconds, minutes, hours)? What would the Mayan base 20 system look like? What would the Babylonian base 60 system look like?

Teacher's notes

- It is important to make sure students understand multiplicative structure – that moving 5 places left is × 100 000 and moving 5 places to the right is ÷ 100 000. This is the basis of multiplication/division by powers of ten and most metric conversions. Once students understand thousands/millions, extend the hundreds-tens-ones understanding to large numbers by breaking the numbers and writing the numbers in groups of 3 numerals. Focus understanding of the multiplicative structure on the pattern of threes by moving 3-digit numbers left and right to show what happens as the places move left and right across the patterns of three (i.e. moving numbers between patterns of three is left ×1000 and right ÷1000).
- Students need to develop automaticity in knowing the sequence in powers of ten and related number of places, e.g. know that a positive integer with ten places is in the ten billions and $n \times 10^6$ is millions, $n \times 10^{12}$ is trillions and so on, also applying this to the negative exponents to trillionths. This can be extended to the relevant Greek prefixes. Google: Orders of magnitude (time).
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a quadrilateral, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different quadrilateral.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- **Explicit teaching that aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.