YuMi Deadly Maths

Year 9 Teacher Resource:

NA – Exponential change

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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QUT

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ACKNOWLEDGEMENT

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research.qut.edu.au/ydc/

Year 9 Number and Algebra

Exponential change

Learning goal Students will:

- investigate the patterns of numbers arising for the powers of ten for positive and negative exponents
- write numbers in scientific notation
- convert scientific notation to standard decimal form and vice versa.

Content description Number and Algebra – Real numbers

- Express numbers in scientific notation (ACMNA210)

Big idea Multiplicative structure, pattern of threes, exponents

Resources Three ten-strip mats or masking tape; orange; place value cards, place value chart; exponent, numeral, digit, factor and word cards; chairs; scientific calculators; expanders

Reality

Local knowledge Discuss where very large and very small numbers are found in the local environment or real world, e.g. speed of sound, distance to planets, the weight of a human skin cell, time margins in motor racing. Find out if students themselves have situations where very large and very small numbers are used.

Prior experience Check that students understand the progression of the pattern of threes in the real number system up to millions/millionths:

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
Millions			Thousands			Ones			Thousandths			Millionths		
	4	6	3	1	9	2	5	7 ●	8	3	7	4	0	5

The above number would be read as:

forty-six millions, three hundred and nineteen thousands, two hundred and fifty-seven, eight hundred and thirty-seven thousandths, four hundred and five millionths

Check that students understand that $\frac{30}{1000} = \frac{3}{100}$ (i.e. thirty thousandths is three hundredths); similarly, $\frac{800}{1000} = \frac{8}{10}$ (i.e. eight hundred thousandths is eight tenths).

- *What does the same HTO pattern in the millionths mean?* [four hundred millionths is four ten thousandths, etc.]

Ensure students understand that 3.4673 could be read as: *three and four thousand, six hundred and seventy-three* ***ten-thousandths***.

- *Why are we calling them ten-thousandths?* [4 decimal places so 10^{-4}]

It can also be read as: *three and four hundred and sixty-seven* ***thousandths*** *and three hundred* ***millionths***.

Revise index notation and the laws for multiplying and dividing by powers of ten:

- *Expand 10^4. What is the result?* [10 × 10 × 10 × 10 = 10 000]
- *When you multiply 10^4 by 10^2, what is the result?* [10^6 or 1 000 000] *What did you do?* [added the indices or powers: 4 + 2 = 6]
- *When you divide 10^5 by 10^3, what is the result?* [10^2 or 100] *What did you do?* [subtracted the indices or powers: 5 – 3 = 2]

Kinaesthetic

Using the pattern of threes, extend to billions, trillions and then in the opposite direction showing that the pattern of threes also continues into the decimal numbers. It is important that students know the names in the progression of the pattern of threes:

trillions ← billions ← millions ← thousands ← **ones** → thousandths → millionths→ billionths → trillionths

Set up three ten-strip mats. Mark the ones of the Ones place with, for example, an orange as the decimal point. See **Appendix A** for an example.

- *What place does the decimal determine?* [The ones place; the decimal point always comes after the ones.]

Distribute cards of four types to be placed on the strip mats:

- words – trillions to trillionths
- sets of H T O cards
- numerals – sets of 0–9
- exponents – 10^{14} to 10^{-12} (on cards underneath each place; intersperse the negative index with the index as a fraction).

Check that students understand the link between the exponent and the number of times 10 is repeated as a factor.

Have students lay these cards in the appropriate place on the place-value strips so that each of the four types correspond along the strip mats (see **Appendix A**).

- *What do you notice about 10^{10} compared with 10 000 000 000 and 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10?*
  [The number of places after the digit in the largest place value in the number, and before the decimal point, is the same as the exponent and also the same as the number of repeated factors of ten.]
- **Note**: In the above example, the places happen to be filled with zeros, so the number of zeros is the same as the exponent, but stating the rule this way causes later problems when there are digits other than zero in the lower places.

Discuss the pattern in terms of indices or powers of 10 as above. Give more examples with both whole and decimal numbers so that students link the exponent to the number of places below the largest place and before the decimal point, and the number of times ten is repeated as a factor in any power of ten.

Read the number shown on the strip mat (see **Appendix A**):

Five hundred and forty-one trillions, eight hundred and forty billions, six hundred and thirty-four millions, five hundred and sixty-seven thousands, nine hundred and eighty-two [and] five hundred and forty-one thousandths, eight hundred and forty millionths, six hundred and thirty-four billionths, five hundred and sixty-seven trillionths

Note how the number system pivots around the ones (not around the decimal point):

541 840 634 567 **982 .** 541 840 634 567

trillions billions millions thousands **ones** thousandths millionths billionths trillionths

Abstraction

Body

Place 14 chairs in a straight line with students standing behind the chairs holding PV cards going from tens of millions to millionths. Have students sit on the appropriate chairs with digit cards to make the number, e.g. 34.567. Place the orange after the 4 in the ones place.

- *What happens when 34.567 is multiplied by one million?*
- *Firstly, how many zeros are there in one million?* [6]
- *So how is a million expressed exponentially?* [10^6]

- *If a number is being multiplied by a whole number does it get smaller or bigger?* [bigger]
- *How many places will the students need to move around the decimal point if the number is being multiplied by a million?* [6 places]

All the students stand and move up 6 chairs and sit down in the chair 6 places up.

- *What needs to be done with the empty places on the whole-number side of the decimal point?* [Zeros need to be added as place holders.]

Have three students take three zeros and sit on the chairs to the left of the decimal point (from the point of view of the rest of the class).

- *Read the number that has been made.* [thirty-four millions, five hundred and sixty-seven thousands]

Give similar examples with multiplication by powers of 10, e.g. 4.71×10^9 (Think: 4.71 is ones, multiplying by 10^9 will give billions, so the answer is 4.71 billion or 4 710 000 000.)

Reverse: Use division by a power of ten where the given number (whole or decimal) becomes smaller by moving the students back by the same number of places as the exponent, e.g. $34.567 \div 10^3 = 0.034\ 567$ = thirty-four thousand five hundred and sixty-seven millionths = $34\ 567 \times 10^{-6}$.

- *What is the easiest way to say this very small number, in words or in exponential form?* [exponential form]

Give similar examples.

- *So, very large or very small numbers are usually written and said in exponential form.*

Give some examples for students to move along the chairs, using both multiplication (moving up to make the number bigger) and division or multiplication by a negative exponent (moving back to make the number smaller), the same number of places (chairs) as the exponent, e.g. $2.64 \times 10^9 = 2\ 640\ 000\ 000$; $5.92 \times 10^{-7} = 0.000\ 000\ 592$.

Emphasise that multiplying by the positive exponent moves the digits around the decimal point so that the number increases exponentially by the same number of places as the exponent; multiplying by a negative exponent moves the digits around the decimal point so that the number decreases exponentially by the same number of places as the exponent.

By what power of ten has 74.9 been multiplied to give:

(a) 7 490 000 000 and (b) 0.000 007 49?

This gives the basis for scientific notation.

Sit students in chairs on the strip mat to represent 74.9. Have students move up 8 places and fill 7 places with placeholders:

- *What is used for placeholders?* [zeros]
- *What were we multiplying by each time we moved up one place?* (× 10)
- *How many times or places did we shift/move all the digits?* [8]
- *So what power of ten was 74.9 multiplied by?* [10^8]
- *When we write the number we made in decimal notation as 7 490 000 000, we see it can also be written in scientific notation as* 74.9×10^8.

Use scientific calculators as in 'Hand' below to verify answers. Repeat process for (b) above. Note that multiplying by a negative power of 10 is the same as division where the number moves back (to the right) to make it smaller. Have students act out: 3.6×10^{-4} moving students with these digits back 4 places to give 0.000 36. Emphasise the role of zeros as placeholders. Provide other examples of these types.

Hand Using scientific calculators have students find the value of 7×10^6. The process is:

- enter the number being multiplied
- press the $\times 10^x$ button
- enter the required power of ten
- press equals.

The display will show the answer. For example, enter 7, then press the $\times 10^x$ button, enter 6, press equals:

7	$\times 10^x$	6	=	The display should read **7 000 000**.

Find the value of 5.78×10^4

5	.	7	8	$\times 10^x$	4	=

The display should read **57 800**.

Describe the exponent of 10 when you are multiplying by thousandths. [The exponent or index of 10 is negative or written as a fraction, e.g. 10^{-3} or $10^{1/1000}$]

Find the value of 64.9×10^{-3}

6	4	.	9	$\times 10^x$	(–)	3	=

The display should read **0.0649**. Notice that the (–) key needs to be pressed to indicate that the exponent or index is negative.

Give other examples using scientific notation to multiply whole and decimal numbers using both positive and negative exponents.

For students having trouble reading large numbers, use expanders (see **Appendix B**).

Mind Encourage the students to find and write down patterns in movements and their relation to ×10, ÷10, ×100, ÷100, ×1000, ÷1000 and so on. Ask students for a pattern (i.e. move left one place is ×10 and move right one place is ÷10).

Spend time on seeing in their mind the three PV position movements (×1000 and ÷1000), that is, movements across the macrostructure from ones to thousands to millions to billions to trillions and also the reverse, ones to thousandths to millionths to billionths.

Creativity Find examples where very large and very small numbers are used and express these in standard decimal form and in scientific notation, for example:

- distance from Earth to the sun is 92 935 700 miles or 9.3×10^7
- width of the average human hair is 75 micrometres or microns (a micrometre is $\frac{1}{1\,000\,000}$ of a metre), so the width = 0.000 075 or 7.5×10^{-5}

Mathematics

Language/ symbols scientific notation, decimal notation, scientific calculators, powers of ten, exponent, index, indices, millionths or micrometres or microns

Practice 1. Complete the following tables using a scientific calculator and describe the pattern you see.

Power of ten	Expanded form	Number	Name
10^0	$\frac{1}{1}$	1	one
10^1	$\frac{10}{1}$	10	ten
10^2	$\frac{10\times10}{1}$		

10^3	$\frac{10\times10\times10}{1}$		
10^4	$\frac{10\times10\times10\times10}{1}$	10 000	ten thousand
and so on to			
10^{12}	$\frac{10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10}{1}$		one trillion

Power of ten	Expanded form	Number	Name
10^0	$\frac{1}{1}$	1	one
10^{-1}	$\frac{1}{10}$		one tenth or one hundred thousandths
10^{-2}	$\frac{1}{10\times10}$	0.01	one hundredth or ten thousandths
10^{-3}	$\frac{1}{10\times10\times10}$		
10^{-4}	$\frac{1}{10\times10\times10\times10}$	0.0001	one ten thousandth or one hundred millionths
and so on to			
10^{-12}	$\frac{1}{10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10}$		one trillionth

2. Convert these numbers from standard decimal form to scientific notation or vice versa. Check these websites:
   http://www.universetoday.com/14824/distance-from-earth-to-mars/
   http://en.wikipedia.org/wiki/10_micrometres

Item	Standard decimal form	Scientific notation
Age of the Earth	4 500 000 000 years	
Closest distance Earth to Mars	56 000 000 km	
Furthest distance Earth to Mars	401 000 000 km	
Average distance Earth to Mars	225 000 000 km	
Height of Mount Everest	8 840 m	
Length of red blood cell	0.000 01 mm	
Twenty microseconds	0.000 02 sec	
Length of DNA base pair	0.000 000 000 34 m	
Earth's population (2018)		7.633×10^9
Sun's temperature		8.5×10^{17} °C
Estimated number of stars in the universe		6×10^{22}
Jupiter's surface area		3.79×10^{10} km²
Wavelength of green light		5.6×10^{-7} m
Length of bacterium		2.7×10^{-4} cm
A nanosecond		1×10^{-9} sec
Width of a silk fibre		1.5×10^{-7} m

Connections Relate to the metric system:

H	T	O	H	T	O	H	T	O	H	T	O
millions			thousands			ones			thousandths		
megalitres			kilolitres			litres			millilitres		
ML			KL			L			mL		
10^6L			10^3L			10^0L			10^{-3}L		

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
ones			thousandths			millionths			billionths			trillionths		
metres			millimetres			micrometres			nanometres			picometres		
m			mm			μm			nm			pm		
10^0			10^{-3}			10^{-6}			10^{-9}			10^{-12}		

The cognitive load is decreased if students have automaticity in knowing the sequence in the decimal system for the pattern of threes from ones moving left to trillions (multiplying by a power of 10) and from ones moving right to trillionths (dividing by a positive power of ten or multiplying by a negative power of 10).

Reflection

Validation Validate where very small and very large numbers are found and used in the world. Share personal creativity and validate another student's creativity, checking that standard decimal form is expressed correctly in scientific notation.

Application/ problems Provide applications and problems for students to apply to different real-world contexts independently; e.g. bytes to terabytes. *If the memory on the hard drive is one terabyte, how much memory is left when 12 963 000 000 bytes have been used?* Hint: Construct a table as above showing bytes to terabytes.

Extension **Flexibility**. Construct a poster showing many examples of systems of numbers: kilo, mega, giga, tera, milli, micro, nano, pico, and so on. Relate these measures to as many different examples as you can.

Reversing. Give a number, say 64 gigabytes, and students have to find where to put this on a diagram of the system. Then reverse, give a position on the system and students have to make up a number situation. That is:

number situation → system AND system → number situation

Generalising. Can the students see the two forms of the system in their mind? Can they make up their own system? What happens when we move two places in the multiplicative structure? We are at 10^6, we multiply by 100, we are now at 10^8. Can students generalise – 2 places is 10^2, 3 places is 10^3?

Changing parameters.

- *Does the same procedure follow for exponents and factors of numbers in other bases?* [yes, $6^4 = 6 \times 6 \times 6 \times 6$]
- *How do you write 8 × 8 × 8 × 8 × 8 × 8 × 8 in exponential form?* [8^7]
- *Expand 7^5* [7 × 7 × 7 × 7 × 7]
- *What if we changed base (e.g. seconds, minutes, hours)? What would the Mayan base 20 system look like? What would the Babylonian base 60 system look like?*
- *Because the decimal system follows the pattern of threes, can you discover a trick that some mathematicians use whereby they easily calculate multiplication by powers of ten?*

Teacher's notes

- It is important to make sure students understand multiplicative structure – that moving three places to the left is ×1000 and moving three places to the right is ÷1000. This is the basis of multiplication/division by powers of ten and most metric conversions. Once students understand thousands, extend the hundreds-tens-ones understanding to large numbers by breaking the numbers and writing the numbers in groups of three numerals. Focus understanding of the multiplicative structure on the pattern of threes by moving 3-digit numbers left and right to show what happens as we move left and right across the patterns of three (i.e. moving numbers between patterns of three is left ×1000 and right ÷1000).
- Students need to develop automaticity in knowing the sequence in powers of ten and related number of places, e.g. know that a positive integer with ten places is in the ten billions and $n \times 10^6$ is millions, $n \times 10^{12}$ is trillions and so on, also applying this to the negative exponents to trillionths.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Appendices

Appendix A: Strip mat

Trillions			Billions			Millions			Thousands			**Ones**			thousandths			millionths			billionths			trillionths		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
5	4	1	8	4	0	6	3	4	5	6	7	9	8	2	5	4	1	8	4	0	6	3	4	5	6	7
10^{14}	10^{13}	10^{12}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}
															$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	etc.								

Appendix B: Expanders

4	6	2	Billions	3	8	1	Millions	7	5	9	Thousands	8	0	6	Ones

H	T	O	H	T	O	H	T	O	H	T	O
Billions			Millions			Thousands			Ones		
10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	10^{0}

3	7	4	Ones	1	0	9	thousandths	2	4	6	millionths	0	8	5	billionths

H	T	O	H	T	O	H	T	O	H	T	O
Ones			thousandths			millionths			billionths		
10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}