YuMi Deadly Maths

Year 8 Teacher Resource:

MG – Transfigure the shape

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 8 Teacher Resource: **MG – Transfigure the shape**

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
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Year 8 Measurement and Geometry

Transfigure the shape

Learning goal Students will calculate perimeter and area of kites, rhombuses and trapeziums.

Content description Measurement and Geometry – Using units of measurement

- Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)

Big idea Measurement – Notion of unit, interpretation vs construction, equivalence, formulae

Resources Cards (measurement units), place-value strip; Maths Mat; 3.6 m velour strip, cord or thin rope (knotted at 30 cm intervals); elastics; grid paper, coloured pencils, pattern blocks

Reality

Local knowledge Discuss places in the local environment where four-sided 2D shapes are seen, e.g. faces of buildings, sporting fields, tops of tables.

Prior experience *What is the general name given to four-sided 2D shapes?* [quadrilaterals] *Tell me some names of specific four-sided 2D shapes and describe the properties of that shape. How are rhombuses different from parallelograms? How are squares similar to rectangles/rhombuses? How are squares different from kites? How are kites similar to rhombuses?*

Refer to the table of quadrilaterals below.

Properties	Square	Rhombus	Rectangle	Parallelogram	Kite	Trapezium
Two pairs adjacent sides equal	✓	✓			✓	
Opposite sides equal	✓	✓	✓	✓		
All sides equal	✓	✓				
All angles at the vertices equal 90°	✓		✓			
One or more pairs opposite angles equal	✓	✓	✓	✓	✓	
One or more pairs opposite sides parallel	✓	✓	✓	✓		✓
Diagonals are the same length	✓		✓			
Diagonals intersect at 90°	✓	✓			✓	
Diagonals bisect	✓	✓	✓	✓		

If you wanted to measure the perimeter, what would you be measuring? [the distance around all the edges]

What measurement units are used? [linear/length, mm, cm, m, km]

Have students place these cards appropriately on the place-value strip: millimetres, centimetres, metres, kilometres. Check students' knowledge of measurement facts and the relationships between metric measurements.

- *What metric measure is the unit in this place-value strip?* [metre]

One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
kilo metre			metre		centi metre	milli metre

- *If you are measuring the area, what is being measured?* [the amount of coverage or space within a given shape]
- *What measurement units are used?* [square units, mm^2, cm^2, m^2, h, km^2]

Have students place these cards appropriately on the place-value strip: square millimetres, square centimetres, square metres, hectares, square kilometres. Check students' knowledge of measurement facts and the relationships between metric measurements.

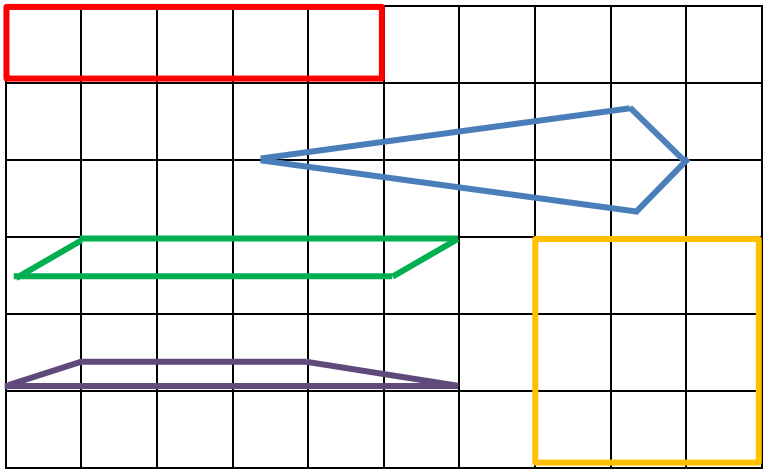
- *What metric measure is the unit in this place-value strip?* [square metres]

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
Trillions			Billions			Millions			Thousands			Ones		
	km^2			ha			m^2				cm^2			mm^2

Kinaesthetic

Maths Mat and 3.6 m velour strip/cord/rope: Give students the 3.6 m length of cord and have them make as many shapes on the Maths Mat as possible with that 3.6 m as the perimeter. Each square on the Maths Mat is 30 cm x 30 cm. There are 12 lots of 30 cm on the 3.6 m length of cord so the perimeter is 12 units. To verify lengths, a unit length mark every 30 cm will help. Alternatively, ensure students have a way of verifying the unit lengths of diagonals.

Possible shapes are: rectangles – 5 × 1, 4 × 2; parallelograms – same side lengths as rectangles, but angles are not 90°; square – 3 × 3; rhombus – same side lengths as square but angles are not 90°; kite – 5 × 1, 4 × 2; trapezium – 1 pair parallel sides of 6 and 3.



Remove these shapes and make others.

- *We know all these shapes have a perimeter of 12 units. What is the general process for finding perimeter?* [Add all the sides]
- *Can we establish a formula for each type of quadrilateral that would apply to any size?* Have the students explore the rules for perimeter of these shapes.

With elastics or longer cords, make given shapes and calculate the perimeter, e.g. *Make a kite that has adjacent sides of 3 units and 5 units. What is its perimeter?* [16 units]

Abstraction

Body

Maths Mat and elastics: Explore the area formulae for the following quadrilaterals by using elastics as shown in the diagrams. The aim is to "cut and paste" so that the development of the formulae is visually and kinaesthetically demonstrated.

Parallelogram/rhombus:

- *What are the properties of a parallelogram and a rhombus?* [A parallelogram is a quadrilateral that has opposite sides equal and parallel, opposite angles equal and diagonals bisect each other. A rhombus is a quadrilateral that has 4 sides equal, opposite sides parallel, opposite angles equal and diagonals bisect each other.]

(Full notes can be viewed in *Year 7 MG Allied shapes.* This resource also includes notes on area of triangle.)

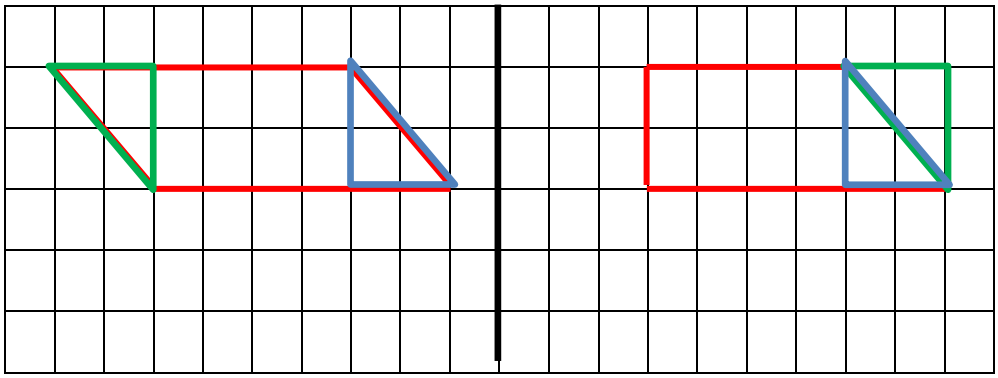


Diagram 1 Diagram 2

Area of parallelogram = Base × Perpendicular Height

Make new parallelograms on the mat and find the area. Give the area of a parallelogram and one linear dimension and students calculate the other dimension.

Trapezium:

- *What are the properties of a trapezium?* [A trapezium is a quadrilateral that has one pair of opposite sides parallel but not equal.]

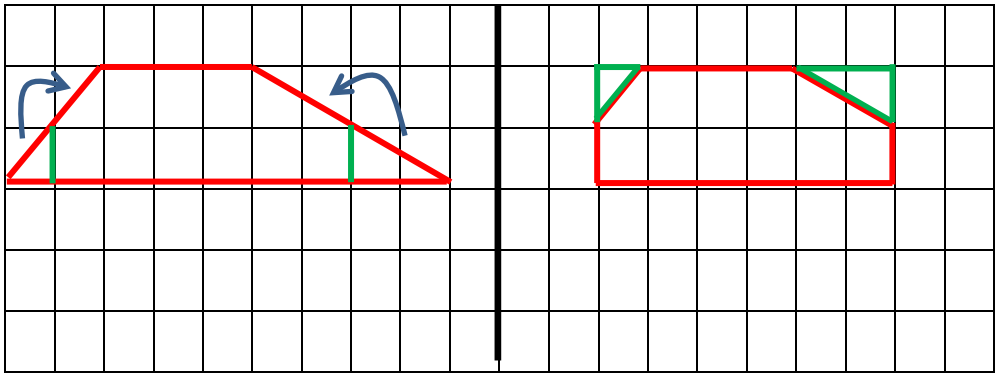


Diagram 1 Diagram 2

Have students make a trapezium on the Maths Mat with an elastic:

- *What could we do to make this trapezium into a regular rectangle?* [Cut off bottom end triangles, invert and paste them onto the top of the trapezium.]

Have students take elastics and show where the cuts need to be made (Diagram 1).

Using another elastic, have students make the new shape that remains when the end triangles are removed (Diagram 2 red). Ask two students to stand in Diagram 1 in the two triangles that are to be cut and pasted onto the top of the trapezium. Using two green elastics, have students rotate those triangles from Diagram 1 to the top of the new red shape (Diagram 2 green).

- *Describe the rotation* [small 180° clockwise, large 180° anticlockwise].

Ask the following questions to develop the area formulae with students:

- *What shape has been created now?* [rectangle]
- *How has the length of this rectangle been made?* [The lengths of both parallel lines of the trapezium have been made equal or the same.]
- *What is the mathematical term for adding lengths or numbers of different values and making them the same?* [mean or average]
- *How will the length of the rectangle be written?* [$\frac{L_1+L_2}{2}$, where L_1 and L_2 are the lengths of the two parallel sides]
- *How does the height of the trapezium compare with the height of the rectangle?* [They are the same.]
- *What is the formula for area of a rectangle?* [$L \times H$]
- *In comparing the height of both figures, what can be said about the measure of their height?* [The height is the same or equal in both figures.]
- *What formula could be written for the area of a trapezium?* [$A = \frac{L_1+L_2}{2} H$]

Area of trapezium $= \frac{L_1+L_2}{2} H$

Make new trapeziums on the mat and find the area. Give students the area and height of a trapezium and ask them to calculate possible dimensions of the two lengths.

Kite:

- *What are the properties of a kite?* [A kite is a quadrilateral that has two pairs of adjacent sides equal, one pair of opposite angles equal and diagonals that always intersect at 90°.]

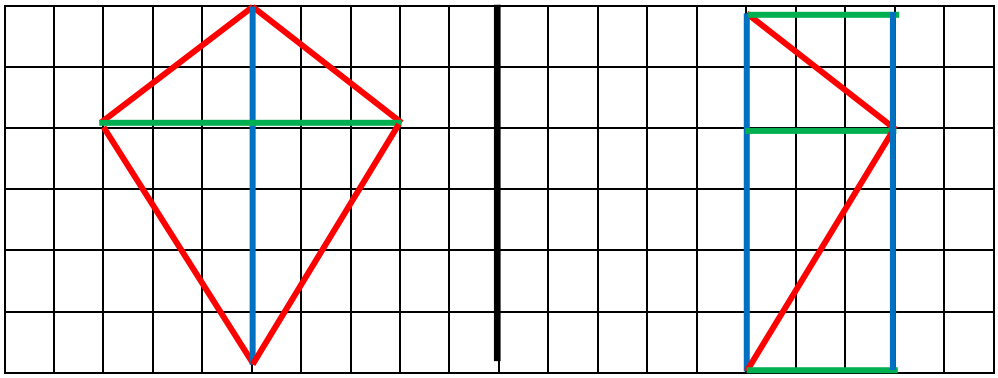


Diagram 1 Diagram 2

Have students make a kite on the Maths Mat.

- *What two shapes together make a kite?* [two triangles].

Have students use two elastics to show how the kite can be divided into two triangles (Diagram 1). Ask the following questions:

- *What are these lines called?* [diagonals]
- *What is the property of a kite's diagonals?* [The diagonals bisect each other at 90°.]

- *How can we use elastics to make the kite into a rectangle?* [Transform the two triangles on one side of the vertical diagonal to paste them next to the mirror image triangles on the other side (Diagram 2).]
- *How is this done?* [The top triangle is rotated 270° clockwise and the bottom triangle is reflected and translated to meet the hypotenuse of its image.]
- *In Diagram 2, what is the base/length of the rectangle and what is the height of the rectangle?* [Base/length is the measure of half the kite's green diagonal, height is the measure of the kite's blue diagonal.]
- *What is the formula for area of a rectangle?* [$L \times H$]
- *Substitute for area of a kite.* [Area of kite = $\frac{1}{2} D_1 \times D_2$, where D_1 is one diagonal and D_2 is the other diagonal.]

$$\textbf{Area of kite} = \frac{\boldsymbol{D_1 \times D_2}}{\mathbf{2}}$$

Given the area and measure of one diagonal, ask students to find the measure of the other diagonal.

Summarise the information:

- *To what shape can parallelograms, trapeziums and kites be converted?* [rectangle]
- *What is the base formula for the area of these shapes?* [Area of rectangle = $L \times H$]
- *What other mathematics knowledge is need for calculating area of trapezium?* [Finding the mean of the two lengths: $\frac{L1+L2}{2}$].
- *For area of kite?* [Diagonals bisect each other at 90°, so half one diagonal becomes the base or length and the other diagonal is the height.]

Hand

Distribute sheets of grid paper and ask students to draw parallelograms, rhombuses, trapeziums and kites to given dimensions. Using coloured pencils, demonstrate (as above) how these figures can be changed to a rectangle of equal proportions. Include all dimensions on the shapes.

Mind

Close your eyes and visualise, e.g. a parallelogram, rhombus, trapezium, kite. Walk around the perimeter of a rhombus; cover the area of the kite with newspaper; make the shape that has one pair of parallel lines only.

Creativity

Students create patterns with the above shapes using their knowledge of the relationship of all the shapes to that of a rectangle. These may be drawn or created using pattern blocks.

Mathematics

Language/ symbols

millimetres, centimetres, metres, kilometres, square millimetres, square centimetres, square metres, hectares, square kilometres, area, perimeter, quadrilateral, square, rectangle, parallelogram, kite, rhombus, trapezium, diagonal, triangle, mirror image, transformation, rotation, reflection, translation, clockwise, anticlockwise

Practice

1. Calculate the perimeter and area of all the shapes drawn in the Hand activity above.
2. Change the unit measure used in (1) above so that the unit is 100 times smaller and recalculate the perimeter and area.
3. Demonstrate and explain how the area of a kite can be determined by dividing the kite into two triangles.
4. Create thinkboards for their partner to solve regarding perimeter and area of a parallelogram, trapezium and kite. Swap boards and make calculations.
5. Write algebraic expressions for the formulae of perimeter and area of parallelogram, rhombus, trapezium and kite.

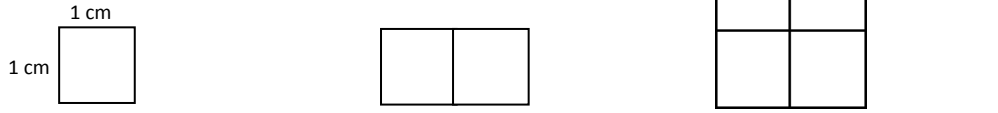
Connections	Relate to perimeter and area of triangles, pentagons, hexagons and so on, volume of quadrilaterals, metric system in the powers of 10.

Reflection

Validation	Students share their thinkboard with a partner and validate the calculations.
Application/ problems	Solve problems in finding perimeters and areas of parallelograms, trapeziums, rhombuses and kites that may be seen in the local environment. Problems may involve: • providing the shapes with all the sides labelled in the same units and requiring the students to find the perimeter and/or area; • providing shapes to the students but labelling the sides with different units and requiring the students to convert the units to solve the problem; • providing some of the side lengths of the shapes and the area and then requiring the student to rearrange the area equations to find an unknown side and then find the perimeter.
Extension	**Flexibility**. *S*tudents are able to work in any of the metric measures, change parallelograms, rhombuses, trapeziums and kites into equivalent rectangles and find the perimeter/area and the dimensions of any regular quadrilateral. **Reversing**. *S*tudents are able to move between shape ↔ properties, metric measure ↔ conversion to larger/smaller metric measure, calculate perimeter/area from given sides ↔ calculate missing side, given perimeter/area and one side. **Generalising**. *Regular quadrilaterals have specific properties that are always the same. Perimeter is a linear measure describing the length to go around all sides of a shape. It is the outside edge of the shape. Area is the amount of wrapping it takes to cover the space enclosed by the perimeter. The area formula for all regular quadrilaterals is based on the rectangle ($L \times H$). The measure taken as the unit affects conversions. Area is in square measure, made by combining square units.* **Changing parameters**. Students solve perimeter and area problems involving other regular polygons with more than four sides. They explore volume of prisms and pyramids with regular quadrilaterals as the base.

Teacher's notes

- The content in this resource will require more than one mathematics lesson to complete.
- Ensure that students have a sound understanding of metric measures, their relationships and conversions, and also the formula for area of rectangle as the rectangle is used as the base to derive the area formulae of other regular quadrilaterals.
- Ensure students understand the notation for square measure and how this is stated, e.g. cm × cm = square cm or cm².



1 square centimetre (1 cm²) 2 square centimetres (2 cm²) 2 centimetres squared (4 cm²)

Square centimetres vs centimetres squared

- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a shape, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different shape.

- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.