YuMi Deadly Maths

Year 8 Teacher Resource:

NA – How many jerseys?

(distributive law)

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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ACKNOWLEDGEMENT

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Year 8 Number and Algebra

How many jerseys? (distributive law)

Learning goal
Students will apply the distributive law to expand and factorise algebraic expressions with numerical factors.

Content description
Number and Algebra – Patterns and algebra

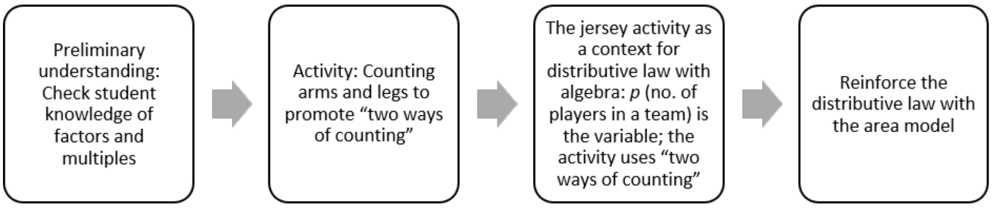
- Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)
- Factorise algebraic expressions by identifying numerical factors (ACMNA191)

Big idea
Number and Algebra – distributive law

Resources
Pencil and paper, 1 cm grid paper

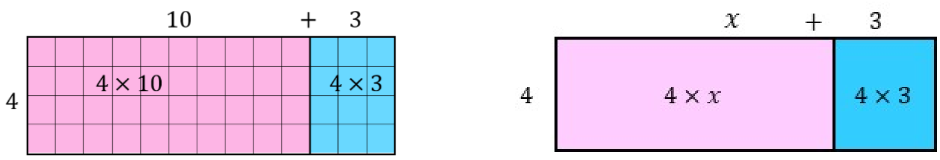
Reality

Local knowledge
This lesson plan introduces the distributive law applied to algebra with a story related to the students' context. It uses the idea that the distributive law is based on two ways of counting how many items you have: e.g. either 4 × 13 **or** 4 × 10 *and* 4 × 3. The lesson gives students another opportunity to see algebra in a context and to appreciate the power of using variables.



Of course, the distributive law applied to algebra can be understood in exactly the same way as it is for mental computation with number, i.e. using the area model.

e.g. $4 \times (10 + 3) = 4 \times 10 + 4 \times 3$ becomes $4 \times (x + 3) = 4 \times x + 4 \times 3$



Note: For this lesson context of jerseys for sports teams, you may wish to start with a real scenario related to the students' context: e.g. *How many rugby league jerseys are issued by the school?* or *How many jerseys are needed for the Cowboys/Broncos/Firebirds teams?* (use your own state sporting teams). Then move to the scenario in this lesson plan which will lead us to a simple algebraic expression.

Prior experience
Revise factors and multiples (see Appendix A)

Check student understanding of factors; students need to have capacity with factors and multiples to use the distributive law.

- The first activity in Appendix A uses counters and helps students establish the factors for whole numbers up to 16 (or 10 or 24). This is suitable when students need substantial revision of the concept.
- The second activity in Appendix A is a game from the Nrich website assuming a knowledge of factors and multiples.

Kinaesthetic *Note*: Use this activity as a warm up or go straight to the jersey question below.

How many arms and legs in the room?

Say: *Assume there are 23 people in the room.*

- *How many arms in the room?* (wave your arms) *Write your answer down*.
- *How many legs in the room?* (wave your legs) *Write your answer down*.
- *How many arms and legs in the room?* [92]

How did you count it?

Most students will answer:

23 × 2 arms and 23 × 2 legs = 46 arms and 46 legs = 92 arms and legs.

Write this number sentence on the whiteboard: $23 \times 2 + 23 \times 2 = 92$

How else could we have counted it?

Approach one student. *How many arms, how many legs?* [2 + 2]

Write: $(2 + 2)$ for 1 person.

But there are 23 people. So write this number sentence: $23 \times (2 + 2) = 92$

So there are two ways of counting arms and legs:

$$23 \times 2 + 23 \times 2 \;\; or \;\; 23 \times (2 + 2)$$

They both give 92 arms and legs altogether.

Mind **How many jerseys?**

A new sport is being offered at school and we need to provide new jerseys. We will need a jersey for each player plus one for each coach and there are two coaches on every team. We plan to have three teams. How many jerseys will we need? (I'm not telling you what sport it is.)

Students have to realise that we need to know the number of players in a team to be able to answer the question.

Say we have to buy 21 jerseys. How many players in each team? [$21 \div 3 = 7$. The 7 people includes the 2 coaches therefore 5 players in each team.]

How can we write this mathematically?

$$3 \times (\text{number of players} + 2 \text{ coaches}) = 21$$

We need a variable for the number of players.

What letter will we use for the number of players in each team? [n or p]

$$3 \times (p + 2) = 21$$

- *What does the 2 stand for?* [The number of coaches for each team.]
- *Why don't we need a variable for the number of coaches?* [Here it is a constant; it doesn't vary.]
- *What does the 3 stand for?* [The number of teams.]

We can see that $p = 5$ gives the answer of 21.

$LHS = 3 \times (5 + 2)$
$= 3 \times 7$
$= 21$
$= RHS$ *So there are 5 players in each team – what sport might it be?* [e.g. basketball]

Note: There aren't many sports with 5 in a team, but this number was used to keep the numbers in the example small.

Abstraction

Mind

How many jerseys? (Using diagrams – with number)

Our equation for the number of jerseys is

$$3 \times (p + 2) = 21$$

when $p = 5$

this becomes

$$3 \times (5 + 2)$$
$$3 \times 7 = 21$$

This can be counted as shown.

$$No. of\ jerseys = 3 \times (5 + 2)$$

But we all know that the coach's jersey should have some extra embroidery on it saying "Coach". So maybe we should count those jerseys separately like this:

$$No. of\ jerseys = (3 \times 5) \quad + \quad (3 \times 2)$$

15 players + 6 coaches

$$3 \times 5 + 3 \times 2 = 21$$
$$15 \quad + \quad 6 \quad = 21$$

Either way it is still 21 jerseys.

From the numbers we can see that:

$$\mathbf{3 \times (5 + 2)}$$
$$= \mathbf{3 \times 5 + 3 \times 2}$$

This pattern is the **distributive law**.

$3 \times (5 + 2)$ or applying "brackets first": $3 \times 7 = 21$

$= 3 \times 5 + 3 \times 2$

$= 15 + 6$

$= 21$

What if there were 7 in the team?

What sport might this be? [e.g. netball]

Still with 2 coaches for each team. (Keep the maths simple – maybe one coach is a trainee.)

$$No. of\ jerseys = 3 \times (7 + 2)$$

Or count the player jerseys separately to the coach jerseys.

$$No. of\ jerseys = \quad (3 \times 7) \quad\quad + \quad (3 \times 2)$$

It's just a matter of how we count it:

$$3 \times (7 + 2)$$
$$= 3 \times 7 + 3 \times 2$$

Also it works in reverse:

$$3 \times 7 + 3 \times 2$$
$$= \ 3 \times (7 + 2)$$

This is the pattern we are working with today.

Mind

How many jerseys? (Now with algebra)

Imagine we are doing this for every sport in the school, so p will change depending on the sport in the school, so p could be any number, but we still have 3 teams in every sport and 2 coaches with each team. (We can change them later, but let's keep it simple first.)

$$\begin{aligned} & 3 \times (p+2) \\ &= 3 \times p + 3 \times 2 \\ &= 3p + 6 \end{aligned}$$

Creativity

Change the number of coaches and the number of teams.

What if we have 10 teams and 1 coach for each team? Say the sport is Rugby League and there are 15 players per team (13 and 2 reserves).

$$\begin{aligned} & \text{No. of jerseys} \\ &= 10 \times (p+1) \\ &= 10 \times p + 10 \times 1 \\ &= 10 \times (15+1) \\ &= 10 \times 15 + 10 \ \times 1 \end{aligned}$$

You can develop variables for the number of teams and the number of coaches; realise that this is becoming more complex mathematically. Consider what would happen if there was only 1 coach for 2 teams. Mathematically this is ½ a coach per team! Discuss the difficulty of having different numbers of coaches.

Working individually or in pairs/small groups, students create an algebraic expression for the number of jerseys for different sports, using the sports in the school, and vary the number of teams and the number of coaches. Work with number or with algebra as appropriate.

Mathematics

Language/ symbols

factor, multiple, common factors, highest common factor, factorise, expand, distributive law, use of $\times$ and whether it is written or left out

Practice

At some point we need to leave the context and go to the mathematical construct, i.e. the expansion and factorisation of the distributive law.

This is the pattern we have established.

$$\begin{aligned} & 3 \times (p+2) \\ &= 3 \times p + 3 \times 2 \\ &= 3p + 6 \end{aligned}$$

Expand: Distribute the multiply-by-3 to every term in the brackets.

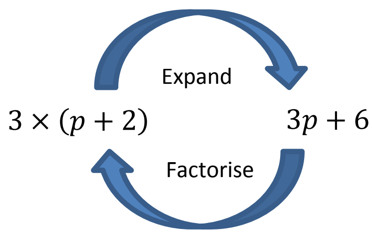
To reverse this process, start with $3p + 6$. We need to determine the common factor of $3p$ and 6.

- The factors of $3p$ are 3 and p.
- The factors of 6 are 3 and 2.
- ∴ The common factor is 3.

$$\begin{aligned} & 3p + 6 \\ &= 3 \times p + 3 \times 2 \\ &= 3 \times (p+2) \end{aligned}$$

Factorise: Find the highest common factor and place it as a factor in front of the brackets.

Emphasise that expanding and factorising are **inverse operations**.



Students now practice the skill of expanding and factorising algebraic expressions.

Explain that algebra is used in many complex stories and we are learning the building blocks to get us started. Practice can be provided from textbooks, worksheets, games, websites, and so on. Show students how to factorise expressions with more than two terms:

$4a + 6b + 2c$

$6k + 9m + 3r + 6t$

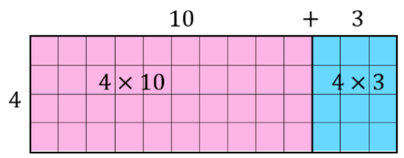
The challenge in factorising is to decide on the highest common factor. If students are weak with a knowledge of factors this becomes difficult. If your group is very weak try the activity in **Appendix B** to factorise as a group activity (see also Teacher's Notes).

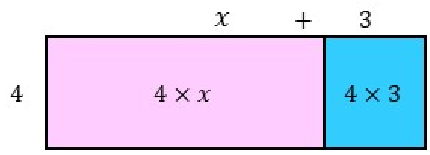
Connections

It is important at some stage to demonstrate the **area model** as was used for the distributive law for mental computation (as mentioned in the beginning of this lesson plan; see also the Year 7/8 *Distributive law* lesson plan).

This will be used again for binomial expansion, so it is important to reinforce it here.

e.g. $4 \times (10 + 3) = 4 \times 10 + 4 \times 3$ becomes $4 \times (x + 3) = 4 \times x + 4 \times 3$





Reflection

Validation

For algebra: students create scenarios and algebraic expressions for different sports and validate each other's answers.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently. For example, look at other items that can be counted in two ways, such as money: $20 \times \$1.20 = 20 \times \$1 + 20 \times 20\text{c}$.

Extension

Flexibility. Students demonstrate the different methods of representing the expression using the area models. This is especially meaningful with strategies to aid mental computation.

Reversing. Students are able to move back and forth between factorising an algebraic expression ↔ expanding the factorised expression.

Generalising. *The distributive law allows an expression to be expanded by removing the brackets and multiplying each term inside the brackets by the factor outside the brackets, which gives the same answer as doing the brackets first and then the multiplication. The highest common factor is used in factorising an algebraic expression*.

Changing parameters. Students expand and factorise algebraic expressions that include algebraic common factors, e.g. $4a(a - 5)$. (*Note*: This is not required for Year 8.)

Teacher's notes

- Ensure that students have a sound understanding of factors and finding the highest common factor before proceeding to factorise an algebraic expression. Students need to understand the distributive law before proceeding to expand algebraic expressions.
- **A body activity to assist with factorisation**. Only use this if it is suitable for your class. For students who are weak with the understanding of factors and factorisation in general it may help to complete a class activity where students act out the factorisation of an expression. The activity involves students completing the factorisation process for numbers by standing in front of the class and holding mini whiteboards in front of them, with numbers or symbols written on them. You could do this with number or algebra. This activity is in **Appendix B**.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Where possible students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- **Explicit teaching that aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Appendices

Appendix A: Revise factors and multiples

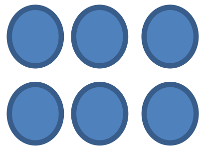
Hand

Check on student understanding of factors; students need to have capacity with factors and multiples to use the distributive law. This first activity is suitable if students need substantial revision of the concept; the second activity is a game assuming knowledge of factors and multiples.

Find the factors and multiples of all whole numbers up to 16

In this first activity choose an upper limit appropriate for your students; 8 or 10 may be sufficient or you may wish to go higher than 16.

Teacher arranges 6 counters as shown:

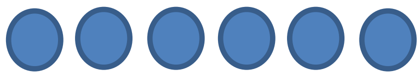


2 rows of 3 means $2 \times 3 = 6$

$\therefore 2\ is\ a\ factor\ of\ 6$
$and\ 3\ is\ a\ factor\ of\ 6$

because 2 divides evenly into 6 with no remainder, and 3 divides evenly into 6 with no remainder.

Now arrange counters as shown:



What does this represent? $1 \times 6 = 6$

$\therefore 1\ and\ 6\ are\ both\ factors\ of\ 6$

Factors of 6 are 1, 2, 3, 6 or, if it is easier, $1 \times 6,\ 2 \times 3$

What are the multiples of 6?

6, 12, 18 … and so on

Students continue working in pairs for all numbers 2 to 16. Realise that for 12 the counters can be arranged as 6×2 or 3×4 and 12×1.

Record the results in a table for future reference.

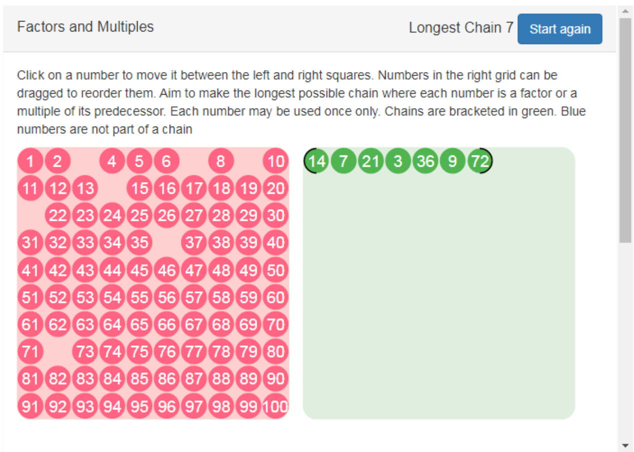
Note: You could use a rectangle drawn on grid paper (rather than counters) to demonstrate factors. This would reinforce the area model.

Mind

Factors and Multiples game from NRICH

Play the game Factors and Multiples from the NRICH website https://nrich.maths.org/5468

The challenge is to see how long you can keep a chain of factors and multiples going. If you want to use only 1 to 50, project a grid onto a board and cross them out with a pen as you go. A grid of 1 to 50 is provided.



Alternatively, you can print out some 1-100 square grids.

Appendix B: A body activity to reinforce the process of factorisation

Resources

Mini whiteboards or pieces of paper and a marker pen. Approximately 10 boards.
Note: Mini whiteboards can be made by laminating white paper.

Body/Mind

Use this activity if it is suitable for your class.

The activity involves students completing the factorisation by standing in front of the whiteboard and holding mini whiteboards in front of them, with numbers or symbols written on them.

We want to factorise the sum of two numbers: $12k + 20$.

(Note: Your first example could be an easier one, e.g. $2k + 6$.)

Write the sum $12k + 20$ on the board or a piece of paper. In front of this, the students will create the expression using people, with each student holding a number or a variable or an operation.

Form the initial expression:

- Student stands with the number 12.
- Student stands with the variable k and holds a $\times$ sign between the k and the 12 (or links arms with the student holding 12 to signify multiply if that is easier).
- Have a student stand with the $+$ sign.
- Have a student stand with the number 20.
- Read the expression: $12k + 20$.

Brainstorm factors together as a group:

- *What are the factors of 12? Think of factors in pairs.*
- *What are the factors of 20? Think of factors in pairs.*
- *What is the highest common factor?* [4] *So we choose the factor pairs with 4 in them*.

Replace the numbers with their factor pairs:

- Replace the 12 with the factor pair 3×4 by replacing the student holding 12 with two students holding 3 and 4 and a multiply sign between them. They are still connected to the k with a multiply sign (or by linking arms).
- The k stays in place ($12k$ should still be on the whiteboard).
- Replace the 20 with the factor pair 4×5 by replacing the student holding 20 with two students holding 4 and 5 and a multiply sign between them (20 should still be on the whiteboard).
- Read the expression: $3 \times 4k + 4 \times 5$.

Factorise the expression:

- *What factor is common?* [4]
- Both the students holding number 4 come to the left (one stands in front of the other with only one 4 sign visible), leaving the other pair of their factor behind.
- Students holding bracket signs move into place (or use something else to signify brackets). *What is the purpose of the brackets?*
- Read the new expression: $4 \times (3 \times k + 5)$
- Write both expressions: $12k + 20$ and $4 \times (3 \times k + 5)$ behind the students.

Repeat with another factorisation to allow all students to have a turn.