YuMi Deadly Maths

Year 8 Teacher Resource:

MG – The triangle detective (congruent triangles)

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 8 Teacher Resource: **MG – The triangle detective**

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
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refined and presented in professional development sessions.

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Year 8 Measurement and Geometry

The triangle detective (congruent triangles)

Learning goal Students will develop the conditions for congruence of triangles.

Content description Measurement and Geometry – Geometric reasoning

- Develop the conditions for congruence of triangles (ACMMG201)

Big idea Geometry equivalence – equal angles, equal lengths, equal shapes; congruence

Resources Protractors, rulers, 1 cm grid paper

Reality

Local knowledge Use the scenario of a triangle detective. The triangle detective is a junior detective who is helping the boss find the criminal shapes in town.

"There's a couple of bad triangles on the loose. These two are WANTED for some serious tri-crimes and I need to bring them in. They are new in town, but we have word that they are a double-trouble team – they are two identical triangles.

I have to get a good look at them; then I have to decide if they are ***identical or not****. If they are identical we need to bring them in.*

But they've got to be ***exactly the same****: the sides and the angles of one triangle have got to equal the sides and angles of the other triangle. Boss calls it* ***CON-GRU-ENT****. He says that's the technical term.*

So I've got to collect evidence and get this right. Don't want to accuse an innocent triangle."

Prior experience Students need to understand:

- congruence as related to any shape (not just triangles)
- comparing congruence to similarity for any shape (copying or enlarging images).

How will we know if the shapes are congruent/similar? We need to be able to prove it.

Kinaesthetic **Equal shapes activity**

Prepare a variety of large cardboard triangles of different colours. Have pairs (or even three) that are congruent. Place them on the floor and ask students to find shapes that are the same.

This activity gives students the opportunity to pick the shapes up and place them on top of one another to see that they are equal. Make sure some shapes that are congruent are of different colours to indicate that colour is not a required property. You could also include equal shapes that are not triangles.

Leave the use of the word *congruent* as a final definition.

Abstraction

Hand/Mind Follow the story of the triangle detective outlined in the **Appendix**.

The context of a detective collecting evidence gives an understanding that we are trying to prove a claim, and that we need evidence to support this claim. Many students don't understand the need for proof in the case of congruency because it is quite an abstract concept to deal with ("why would anyone be bothered proving this anyway?") and many students are quite happy to just look at the triangles and say "the sides look the same to me". In the context of the detective, the need for evidence is obvious and the fact that the detective does not initially collect enough evidence helps students understand the need for minimum proof.

The triangle detective activity in the appendix relates to the congruency test SSS (side, side, side). To understand it, students are taken through the process of asking the following:

- *Is having one side of each triangle equal in length enough for the triangles to be congruent?* [no] *Is having two sides of each triangle equal in length enough for the triangles to be congruent?* [no] *Is having three sides of each triangle equal in length enough for the triangles to be congruent?* [yes]
- At each stage, the triangle detective makes mistakes and the students are asked to draw a pair of triangles to show that the detective's conclusion doesn't hold. It is only in the third stage that it is impossible to disprove.
- Thus we find that having three corresponding sides equal in length is sufficient to conclude that two triangles are congruent. Of course, **this exercise does not mathematically prove the congruency test**. It gives an informal understanding or experience of the test.
- Drawing the triangles by hand can be difficult, but students can persevere to get the "hands-on" experience. The triangles pairs with three sides equal is best drawn with a compass. It is unlikely that hand-drawn triangles will be exactly congruent, so the angles when measured with a protractor may not be equal. These errors need to be explained.
- You could use software to draw the triangles, if your students are familiar with software. This is not without its challenges and exactly congruent triangles can be hard to create unless you copy them (which actually defeats the purpose of the exercise, i.e. to see if two different triangles are congruent).

Hand/Mind

Repeat this investigative process for another congruency test, for example, SAS (side, angle, side):

- *Is having two corresponding sides equal and any angle equal sufficient to prove that two triangles are congruent?* [no]
- *Is having two corresponding sides equal and the included angle equal sufficient to prove that two triangles are congruent?* [yes]

If appropriate, complete for all four congruency tests; for each test, ensure students try it first with a condition that is not sufficient for congruency, allowing students to draw triangles that are not congruent, then they try it with all conditions required for congruency.

Mathematics

Language/ symbols

congruent shapes, similar shapes, congruent triangles, similar triangles, equal in length, side, angle, vertex, vertices, corresponding sides and corresponding angles (distinguish between the use of the word *corresponding* here and in corresponding angles in parallel lines), included angle, prove, congruency test, minimum conditions for proof, rotate, orientation, SSS (side, side, side), SAS (side, angle, side), ASA (angle, side, angle), AAS (angle, angle, side), $\Delta ABC \equiv \Delta DEF$

Practice

Use practice to develop the concept of congruency of any shape. For example:

1. Use squared grid paper to copy a shape congruently.
2. Use Mira mirrors – two shapes are congruent if you can transform one to the other by flips, slides or turns only (i.e. reflection, translation or rotation). Use this to check whether shapes are congruent.

Once students understand the concept, use further practice questions to enable them to see the patterns for the congruency tests for triangles, then move on to formal proofs (as needed).

Connections	Connect the concept of congruence in geometry with the concept of balance (equals) in number and algebra. Compare congruence and similarity.

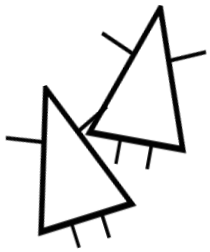
Reflection

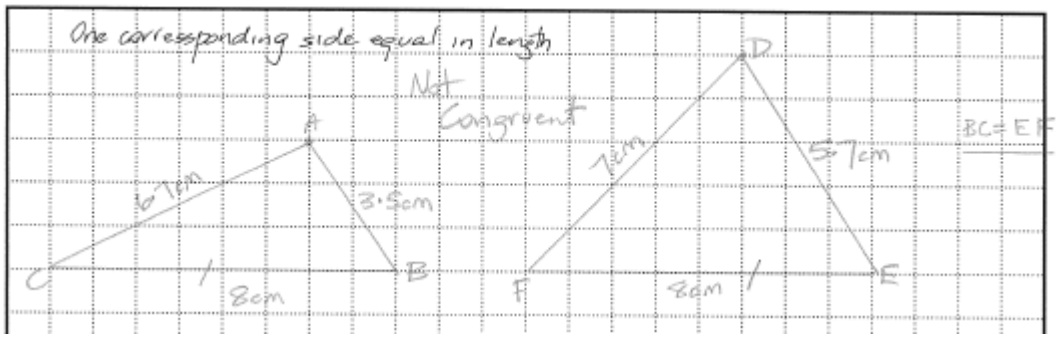
Validation	Students discuss where congruent shapes are found in the environment, e.g. bricks, tiles on floors or in a bathroom.
Application/ problems	Provide applications and problems for students to apply to different real-world contexts independently, e.g. explore congruent and non-congruent triangles in terms of tessellation. (Note: All triangles tessellate because their angle sum is 180°, so putting all three corners of three congruent triangles together at a point will give a straight line. Non-congruent triangles will not tessellate with each other.)
Extension	**Flexibility**. Students understand the four different congruency tests and can apply them to any triangles. **Reversing**. As well as giving triangles that students can test for congruency, ensure students can draw triangles that meet the conditions of congruency for each of the four congruency tests. **Generalising**. *Two triangles are congruent if their corresponding sides are equal in length, in which case their corresponding angles are equal in measure. Any two shapes are congruent if one shape can be transformed to the other by flips (reflection), slides (translation) or turns (rotation) only.* **Changing parameters**. Use the congruency proofs and tests to underpin the following: • Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202) The activity promotes an understanding of the need for proof in geometry.

Teacher's notes

- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a triangle, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different triangle.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

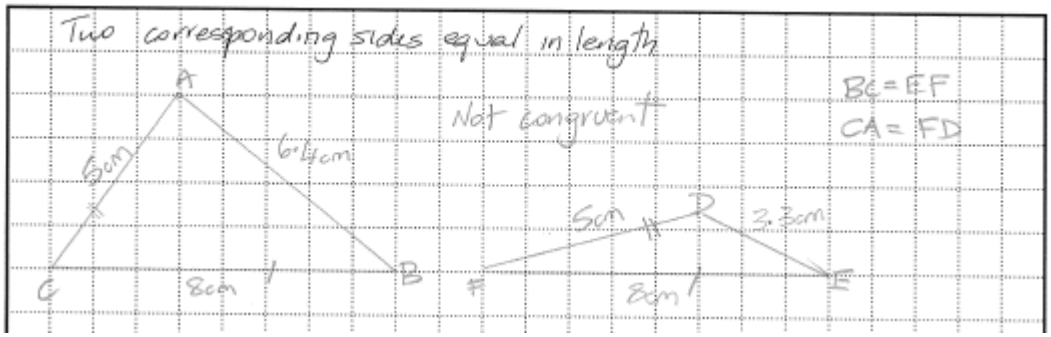
Appendix: The triangle detective activity

<table>
<tr><td colspan="2">Can you help the triangle detective?
Are the triangles CONGRUENT?</td></tr>
<tr><td rowspan="2"> 
  </td><td>The triangle detective is a junior detective, who is helping the boss find the criminal shapes in town.

“There’s a couple of bad triangles on the loose. These two are WANTED for some serious tri-crimes and I need to bring them in. They are new in town, but we have word that they are a double-trouble team – the only thing we know about them is that they are two identical triangles.
I have to get a good look at them; then I have to decide if they are identical or not. If they are identical we need to bring them in.
But they’ve got to be exactly the same: the sides and the angles of one triangle have got to equal the sides and angles of the other triangle. Boss calls it CON-GRU-ENT. He says that’s the technical term.
So I’ve got to collect evidence and get this right. Don’t want to bring in an innocent triangle.”</td></tr>
<tr><td>I just can’t get a good look at them. They’re hiding out.
I got a quick sight of one suspect triangle – has a side of 8 cm.
Then I saw another suspect triangle, also got a side of 8 cm.
I figure that’s enough … they must be congruent! Boss can we bring them in?</td></tr>
<tr><td> 
BOSS</td><td>Are you crazy!
Any secondary school maths student can tell me that’s not enough detail!
Just because they have ONE SIDE the same – that’s not enough to say they are the equal triangles. We want proof!
Let’s ask our MATHS TEAM for help!</td></tr>
<tr><td colspan="2">Give an example of two triangles that each have a side of 8 cm but are not equal triangles, i.e. not CONGRUENT.</td></tr>
<tr><td colspan="2"> 
</td></tr>
<tr><td colspan="2"></td></tr>
<tr><td rowspan="2">  </td><td>Mmm … think I need to get a better look at those triangles.
I’ve gotta play smart.
I’ll wait around until I see TWO SIDES of each triangle.</td></tr>
<tr><td>Ok Boss.
I’ve been staking out both of our suspect triangles. One triangle has one side 8 cm long and another side 5 cm long.
And guess what? The other triangle also has one side 8 cm long and another side 5 cm long.
We’ve got them this time.
They have got to be CON-GRU-ENT and I’m sure these are the two we are after. Can we bring them in?</td></tr>
</table>

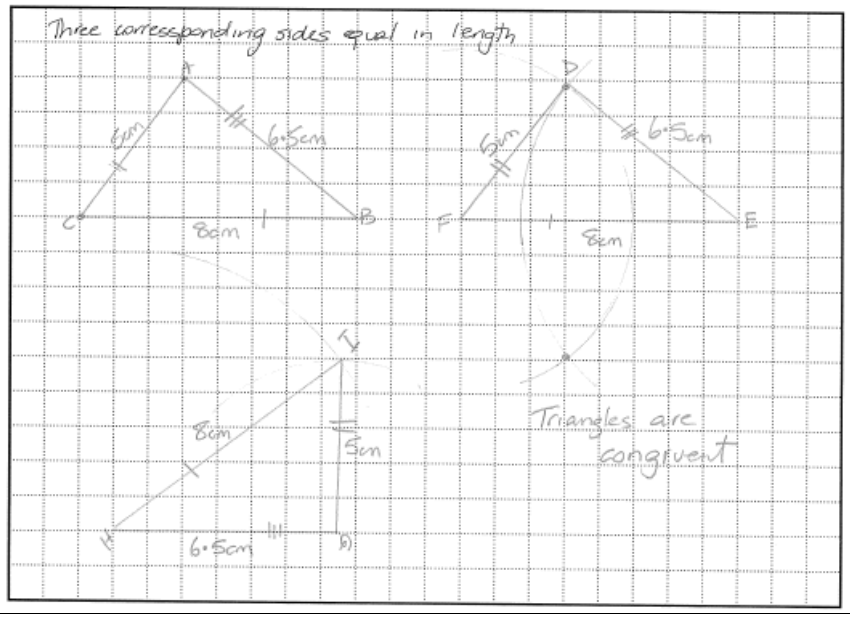
 **BOSS**	Good work Detective. Much better than last time. I think you are onto something. But … we need to be really sure. Let's ask our **MATHS TEAM** if they can help us.

Can you draw two triangles that each have sides of 8 cm and 5 cm but are different triangles?
Is having two corresponding sides equal enough to be sure that the triangles are exactly the same?



	WOW, I can't believe it! Two equal sides was not enough to be sure that the triangles are equal. I've got to look really closely now. I have to look at **THREE SIDES OF EACH TRIANGLE.** This is going to take a lot of patience. I need to do some serious surveillance!
	Now I've got my details: • First triangle … 3 sides: 8 cm, 5 cm and 6.5 cm • Second triangle … 3 sides: 8 cm, 5 cm and 6.5 cm I've got them now – surely they can't be different!
 **BOSS**	OK, let's double-check that data: • First triangle … 3 sides: 8 cm, 5 cm and 6.5 cm • Second triangle … 3 sides: 8 cm, 5 cm and 6.5 cm You've got the length of ALL 3 sides – good work! **Will the triangles really be identical?** Let's check with our **MATHS TEAM**.

Is this enough information to decide the triangles are **CONGRUENT**?
Can you draw two triangles that each have sides of 8 cm, 5 cm and 6.5 cm but are different?
Try to do this yourself first but the teacher can show you how to use a compass to help you.

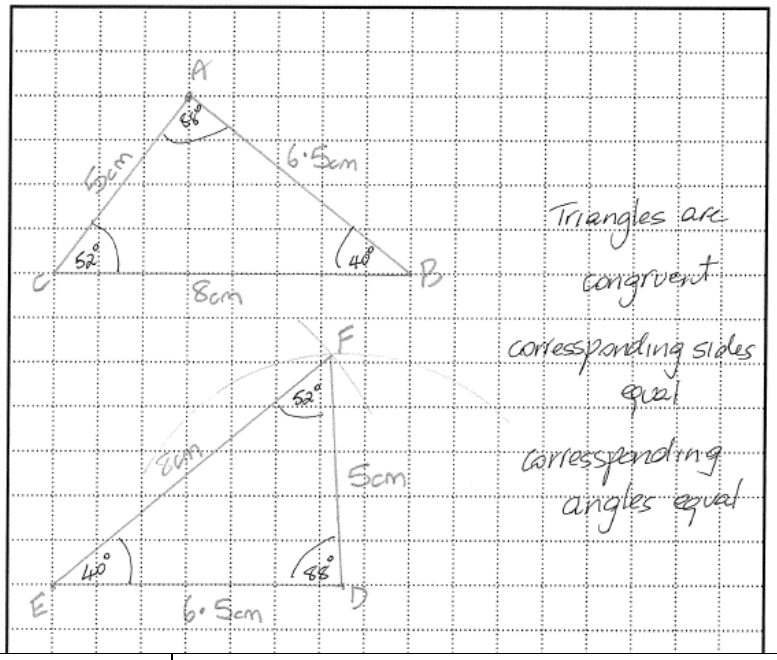


Looks like we've got them now …
Surely that's enough information – three sides of one triangle equal in length to the three sides of the other triangle!
But wait, what about the angles?
Are the corresponding angles equal too?

Good question … I think you are learning the tricks of the trade.
Look at the diagrams below and see for yourself.
Check your triangles and see.

I don't need to – I remember from school.
When the three corresponding pairs of sides in each triangle are equal this means the triangles are congruent and that the three corresponding pairs of angles for each triangle will be equal as well.
So these triangles are congruent …
Bring 'em in! We need to question these two.

This test is called Side–Side–Side (SSS). It is one of **four tests for congruent triangles**.
SSS: If the three sides of one triangle are equal in length to the three corresponding sides of the other triangle, then the two triangles are congruent. This means that all pairs of corresponding sides and angles are equal.



WAIT BOSS!!
I was trying to bring them in and one of the triangles just turned upside down and flipped over! Trying to pretend he's not congruent.

Are they still congruent?

What do you think?

