YuMi Deadly Maths

Year 8 Teacher Resource:

MG – How many layers?

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 8 Teacher Resource: **MG – How many layers?**

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
refined and presented in professional development sessions.

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Year 8 Measurement and Geometry

How many layers?

Learning goal Students will develop the general formula for the volume of a prism.

Content description

Measurement and Geometry – Using units of measurement

- Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)
- Develop formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)

Big idea Measurement and Geometry – Interpretation vs construction; formulae

Resources 3D shapes (see Prior experience), clear plastic open square-based container, measuring jug of water, A4 sheet of paper, wrapped ream of A4 paper, wedge of cheese cut in triangular prism shape, Maths Mat, coloured elastics, geoboards, rubber bands, grid paper

Reality

Local knowledge Name some 3D shapes and discuss places where these are found in the local environment, e.g. rectangular prism – brick or classroom; cube – dice; triangular-based prism – Toblerone chocolate box; square-based prism – carton of milk.

Prior experience Procure some of the 3D shapes mentioned by students and put them on display. Ask:

- *What determines the name given to a prism?* [the shape of its base]
- *How many bases does a prism have?* [two only]

Have students come out and show the bases of the prisms on display. *What shape is the face of any prism?* [rectangle].

Have students show faces of the prisms on display. *How many faces will a prism have?* [It depends on the base: a triangular-based prism will have 3 rectangular faces and 2 triangular bases; a prism with any of the quadrilaterals as its base will have 4 faces and 2 bases that are quadrilaterals.]

Check that students know the different types of triangles and quadrilaterals:

- triangles: acute and obtuse scalene, right-angled, isosceles, equilateral
- quadrilaterals: trapezium, kite, parallelogram, rectangle, rhombus, square.

Ask:

- *How do you find the area of any triangle?* [$\frac{1}{2}B \times H$]
- *What shape gives the general formula for finding the area of a quadrilateral?* [rectangle: $L \times H$]
- *How is this general formula changed for a trapezium and a kite?* [Area of trapezium = $\frac{1}{2}(L_1 + L_2)H$, Area of kite = $\frac{1}{2}D_1 \times D_2$]

Check that students know linear, square and cubic metric measurement facts and can convert between them, e.g. m, m × m (m²), m × m × m (m³).

Kinaesthetic Display the clear plastic open square-based container, measuring jug of water, A4 sheet of paper, wrapped ream of A4 paper and wedge of cheese on a table in front of the class.

Show the open square-based container and ask:

- *What is the shape of the base?* [square]
- *How many bases does it have?* [One solid and one open = 2]
- *What faces does it have and how many?* [4 rectangles]
- *How could I find the volume of water it holds?* [Pour water from the jug into the container]

Pour a little water into the container so that it just covers the base. *How much of the container is filled?* [sufficient to cover the base]. Pour more water into the container, stopping at intervals to ask: *What shape is the cross-section or bird's eye view that is seen every time I stop pouring the water into the container?* [a square]. *What part of the prism is the square?* [base].

Stop pouring at the half-way mark and ask:

- *What fraction of the height of the container has been filled?* [half]
- *What will I need to do to find the total volume of water the container will hold?* [Add water so that all the layers in the height of the container are filled]
- *What dimensions have been used in filling the container with water?* [side × side in the base and height of the container]
- *What does side multiplied by side give?* [area of the square base]
- *How is the volume calculated?* [multiplying the area of the base by the height of the container]

Show the sheet of A4 paper. *What 2D shape is this?* [rectangle]. Show the wrapped ream of A4 paper. *What 3D shape is this?* [rectangular prism]. Ask:

- *What shape are the faces of the prism?* [all rectangles]
- *If I unwrap the ream of paper, what shape are all the cross-section pieces of paper?* [rectangles]
- *What is being repeated throughout all the layers of paper in the ream?* [the A4 rectangular base]
- *What is the multiplier of the base to get the volume?* [the height]
- *How is the volume of the ream of paper calculated?* [multiplying the area of the base by the height].

Show the wedge of cheese:

- *What 3D shape is this?* [triangular-based prism]
- *What is the shape of the 2 bases?* [triangles]
- *How many faces are there and what shape are the faces?* [3 rectangular faces]
- *If I sliced the cheese across the triangular base, how many 2 mm slices would I get?* [height of the cheese divided by 2 mm]
- *How would I calculate the volume that is in this piece of cheese?* [number of 2 mm slices multiplied by the area of the triangular base, or area of base multiplied by the height]

Develop the general formula for volume of a prism:

- *In each of these examples, what has remained constant in determining the volume of the prism?* [area of base times height]
- *What then are the facts we need to be given to find the volume of a prism?* [the dimension and shape of the base and the height of the prism]

Volume of prism = Area of base × height

Abstraction

Body

Maths Mat and coloured elastics

Have four students make a square base on the mat with students holding down the elastic at each of the corners (see red square on diagram below). (*Note*: Later you can have them make other shapes such as a rhombus, rectangle, parallelogram, trapezium, kite, triangle.)

Have another four students, two on each opposite side of the square, hold additional elastics (see blue and green below) above the original elastic to form two opposite and congruent faces so that the bottom of the new elastics are also being held by the two students holding the original elastic. A square base with two vertical faces has now been formed (Diagram 1).

A fourth elastic (see orange below) is passed through the top corners held by the second four students parallel to the elastic on the mat (Diagram 2). The shape of a square-based prism has now been made. The second four students can demonstrate that the volume of the square-based prism can be altered by adjusting the height of the elastics (blue and green) either down or up (Diagram 3: height halved).

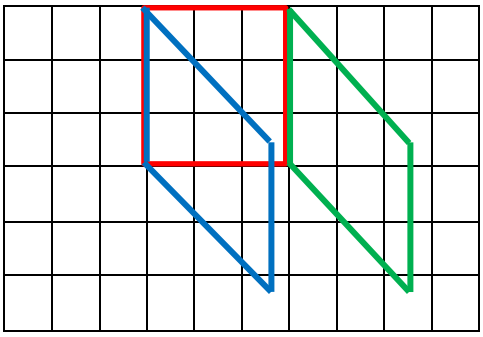


Diagram 1

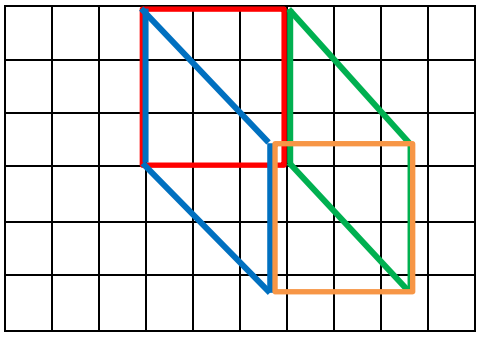


Diagram 2

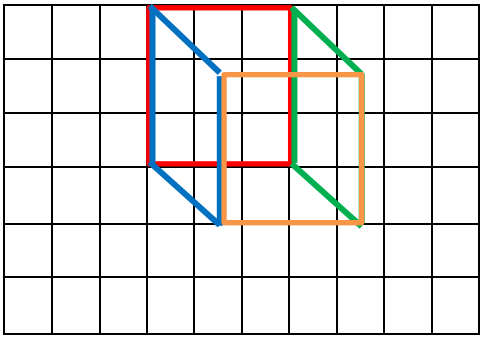


Diagram 3

What happens to the volume of the prism that maintains the same base but the height is lowered or raised? [The volume is adjusted according to the height factor – volume decreases when the height is lowered, increases when the height is raised.]

Hand

In pairs using geoboards and rubber bands, repeat the above process making prisms with different bases and raising/lowering the height of the prism to reinforce the notion that the base of a prism remains the same but the base layer (or cross-section of the prism) is repeated as a constant throughout the height. The height of the layers where the base of the prism is extended affects the volume.

Mind

Visualise a rectangular prism in the shape of a brick. See it in your mind twice its size by doubling the height. Visualise a triangular prism of Toblerone. See it in your mind as half its size. What was done to its height?

Creativity

Students are given grid paper to create their own set of prisms, exploring the effect a change in one dimension has on the volume.

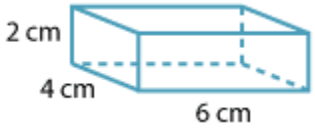
Mathematics

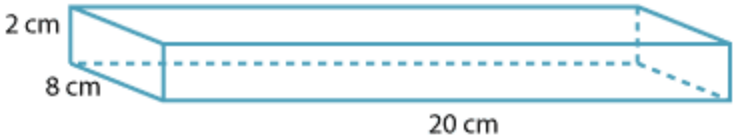
Language/ symbols

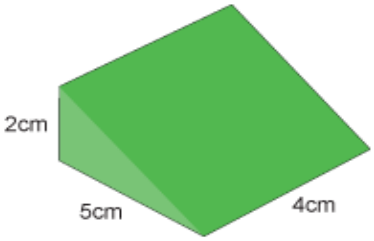
Names of 2D and 3D shapes, cross-section, bird's eye view, prism (e.g. square-based prism), cuboid, trapezoid

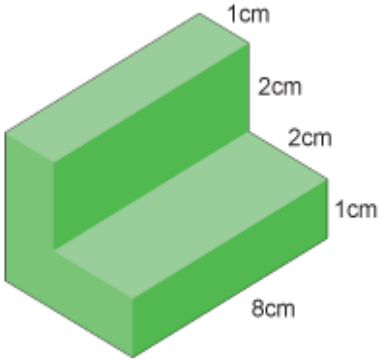
Practice

1. Find the volume of the following prisms:









2. A tank in the form of a rectangular prism has base dimensions 2.3 m by 1.4 m and holds water to a depth of 2 m. If the depth of the water is increased to 6 m, by how much has the volume of water increased?
3. A swimming pool has a volume of 1024 m^3. If its length is 32 m and the depth of water is 4 m, how wide is the pool?
4. Students create their own thinkboards with examples of finding dimensions and volume of prisms that have regular quadrilaterals as the base.

Connections

Relate to mass and capacity, place value, exponents.

Reflection

Validation

Students discuss where volume is found in their world, e.g. volume of cereal in a packet, comparison of volume and prices. They share their thinkboard with a partner and check the calculations.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently, e.g. *A Rubik's cube has a side length of 8 cm. By what fraction has its volume been reduced in creating a smaller version with a side length of 4 cm?*

Extension

Flexibility. *S*tudents are able to apply the general formula to calculate the volume of any prism that has the base of a regular quadrilateral or triangle. From the volume and two other dimensions, they can find the third dimension.

Reversing. *S*tudents are able to move between calculating the volume ↔ finding a missing dimension ↔ calculating the increase or decrease in volume given an increase or decrease in one dimension, starting from and moving between any given points.

Generalising. *The volume of a regular prism is found by multiplying the area of the base by the height. The base layer is repeated throughout the height of the prism. A cross-section parallel to the base will always be congruent with the base.*

Changing parameters. Students explore finding the volume of a cylinder; they investigate how increasing/decreasing more than one dimension of a prism has a compounding effect on the volume, e.g. decreasing each of the dimensions of a rectangular prism by half reduces the volume by one-eighth.

Teacher's notes

- Ensure that students have a sound understanding of area of regular quadrilaterals and triangles before proceeding to volume. Metric measurements and conversions need to be related to place value and powers of ten.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a rectangular prism, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a triangular prism.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.