YuMi Deadly Maths

Year 7/8 Teacher Resource:
NA – Distributive law

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

YuMi Deadly Maths Year 7/8 Teacher Resource: **NA – Distributive law**

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Growing community through education

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
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Year 7/8 Number and Algebra

Distributive law

Learning goal Students will apply the distributive law to help with mental computation.

Content description Number and Algebra – Patterns and algebra

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)

Big idea Number and Algebra – distributive law

Resources Pencil and paper, 1 cm grid paper

Reality

Prior experience Revise factors and multiples (**Appendix A**).

Check student understanding of factors; students need to have capacity with factors and multiples to use the distributive law.

- The first activity uses counters and helps students establish the factors for whole numbers up to 16 (or 10 or 24). This is suitable when students need substantial revision of the concept.
- The second activity is a game from the Nrich website assuming a knowledge of factors and multiples.

Kinaesthetic The reality we are using to lead into the distributive law with algebra is the **distributive law with number**. The principles of algebra are all based on the principles of number – it is important to emphasise this connection.

Mentals warm up

Start with easier mental arithmetic (adjust the level as appropriate for your students). Finish with two or three problems that can be solved by applying the distributive principle.

- Double 7; half of 64; 4×15; $18 + 4$; $98 + 4$; 13×4; 16×7; 98×3

As you go through the answers ask students how they solved each problem (especially the last three). You are looking for an answer that

- splits the 13×4 into 10×4 and 3×4
- splits the 98×3 into $100 \times 3 - 2 \times 3$

If students give you this, affirm that they are using a number law that we will learn about today, otherwise demonstrate the principle with these numbers.

Today we will investigate this a little further: $13 \times 4 = 52$

We know the answer is 52; we aren't looking for the answer, we are looking at how this shortcut works.

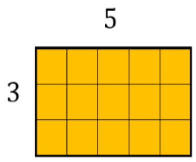
Abstraction

Hand/Mind **The area model for multiplication**

Remember the area model for arithmetic?

Demonstrate with an easy example, such as $3 \times 5 = 15$.

We can use the area of a rectangle to represent a simple multiplication.



$3 \times 5 = 15$

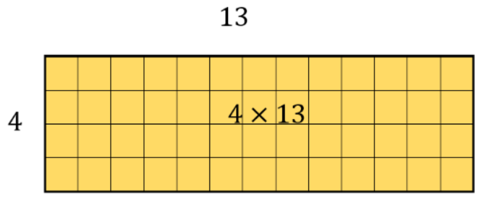
Establish $3 \times 5 = 5 \times 3$.

If your students are used to this concept move on, otherwise use sufficient examples to establish the area model (this simple diagram) as a valid method for modelling multiplication.
15 equals 3 rows of 5 or 5 columns of 3.

If appropriate remind students that this is the **commutative law** or **turnaround principle** and it also applies for addition but not subtraction or division.

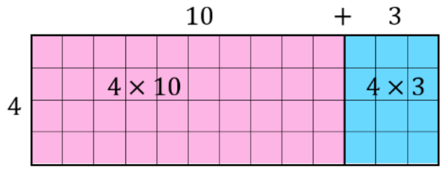
Now back to our example: $13 \times 4 = ?$

Now we have a larger number – it's hard to count all the squares!



So let's split up the rectangles to make it easier for us. We can split the 13 into 10 + 3. Why 10? [Because multiplying by 10 is easy!]

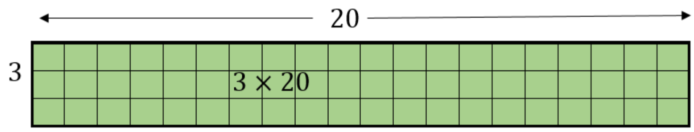
$$\begin{aligned} 4 \times 13 &= 4 \times (10 + 3) \\ &= (4 \times 10) + (4 \times 3) \\ &= 40 + 12 \\ &= 52 \end{aligned}$$

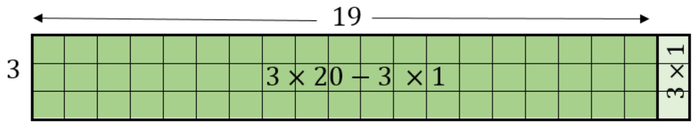


Clearly link the number sentences with the diagrams

Also demonstrate the same principle with subtraction:

$3 \times 19 = ?$





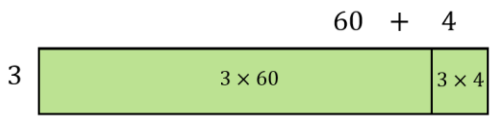
$$\begin{aligned} 3 \times 19 &= 3 \times (20 - 1) \\ &= (3 \times 20) - (3 \times 1) \\ &= 60 - 3 \\ &= 57 \end{aligned}$$

Establish this with sufficient examples of suitable difficulty, drawing the area model and showing the connection between the number sentences and the diagram each time.

At some point you need to stop drawing the actual divisions for the numbers and use the model for the concept. This will later lead into algebra.

e.g. 64×3

$$\begin{aligned} 3 \times 64 &= 3 \times (60 + 4) \\ &= (3 \times 60) + (3 \times 4) \\ &= 180 + 12 \\ &= 192 \end{aligned}$$



Hand/Mind

Activity: Students draw their own area models

To this point, you as the teacher have drawn all the models (diagrams). Students will more fully understand the model when they draw it themselves. The diagrams may need to be scaffolded.

- Now ask students to draw a small number of their own area diagrams. They are to draw the diagram even if they know the answer.
- Instruct students to use either 1 cm grid paper or plain paper. The 1 cm grid is more suitable for students who want to count the squares. Once you have stopped counting squares the plain paper is easier.
- The beginning of a task sheet is in **Appendices B and C**. Use examples that are suitable for your students. Realise that these problems can be quite difficult, e.g. $98 \times 12;\ \ 997 \times 11$
- Allow students to work on mini whiteboards and share with other students.
- Allow students to create their own number problems and swap with other students

Creativity

After practising drawing their own models, students create their own number problems to be solved using the distributive law.

Mathematics

Language/ symbols

factor, multiple, common factors, highest common factor, factorise, expand, distributive law

Connections

The distributive law: factorisation and expansion with numbers

It is called the distributive law because multiplication "distributes over addition" (and subtraction).

Where else do we use the word distribute?

- If I **distribute** 5 counters to each person, I am handing the 5 counters to each person.
- With $5 \times (4 + 2) = 5 \times 4 + 5 \times 2$ we are "handing out" the multiply-by-5 to every number in the brackets.
- To further illustrate this point: $5 \times (4 + 2 + 3) = (5 \times 4) + (5 \times 2) + (5 \times 3)$
- The **distributor** in a car (in spark ignition not electronic ignition cars) distributes the spark to the leads for the cylinders.

Students have learnt two other number laws:

- **Commutative** law: $3 + 4 = 4 + 3;\ 5 \times 2 = 2 \times 5;\ 5 - 3 \neq 3 - 5;\ 8 \div 4 \neq 4 \div 8$. This is informally called the *turnaround principle*. Note that *commute* means to travel to work – travel forward, travel back.
- **Associative** law: $3 + 4 + 5 = (3 + 4) + 5 = 3 + (4 + 5)$. Note here that the order doesn't change but the way we associate the numbers with the brackets changes.

Practice

Establish that we are using the distributive law:

5×12
$= 5 \times (10 + 2)$
$= 5 \times 10 + 5 \times 2$
$= 50 + 10$
$= 60$

Expansion

The following is using the distributive law for expansion:

$5 \times (10 + 2)$
$= 5 \times 10 + 5 \times 2$

Expand *because it is written out in full – applying (or distributing) the multiply 5 to every number in the brackets.*

Reflection

Validation

For number: Students use the area model to demonstrate evaluating the numeric expression; they can also use the calculator to validate the answer.

Extension into Year 8

Reversing. Ensure that students see expanding and factorising as inverse processes.

The Australian Curriculum content descriptor relates only to using the distributive law for mental computation, i.e. using expansion. A section on factorisation with number is included here for the extension concept of reversing. The Australian Curriculum places factorisation in Year 8 with algebra.

Factorisation

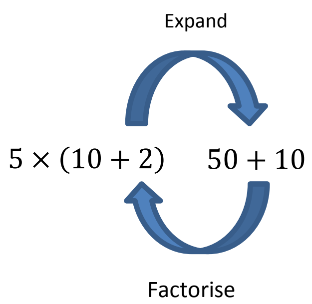
Using the 5×12 example from the Mathematics (Practice) section above, establish that the reverse process is factorising – or finding a common factor.

$$\begin{aligned} &5 \times 10 + 5 \times 2 \\ =\ &5 \times (10 + 2) \end{aligned}$$

or

$$\begin{aligned} &50 + 10 \\ = &5 \times 10 + 5 \times 2 \\ = &5 \times (10 + 2) \end{aligned}$$

Expanding and factorising are inverse operations.



Expand	Factorise
$5 \times (10 + 2)$ $= 5 \times 10 + 5 \times 2$ $= 50 + 10$	$50 + 10$ $= 5 \times (10 + 2)$

Note that in this example 5 is not the highest factor. See the following example

An example with several common factors

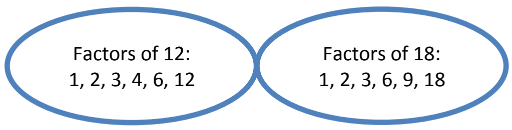
$12 + 18$

We know $12 + 18 = 30$ *but we will explore further*.

What are the common factors of 12 and 18?

2, 3 and 6

The highest common factor is 6.



Taking 6 as the highest common factor:

$$\begin{aligned} &12 + 18 \\ = &6 \times 2 + 6 \times 3 \\ = &6 \times (2 + 3) \end{aligned}$$

Notice you could also take 3 as a common factor:

$$\begin{aligned} &12 + 18 \\ = &3 \times 4 + 3 \times 6 \\ = &3 \times (4 + 6) \end{aligned}$$

You could also take 2 as a common factor:

$$\begin{aligned} &12 + 18 \\ = &2 \times 6 + 2 \times 9 \\ = &2 \times (6 + 9) \end{aligned}$$

All answers are equal to 30.

Teacher's notes

- Ensure that students have a sound understanding of factors and finding the highest common factor before proceeding to factorise an algebraic expression. Students need to understand the distributive law before proceeding to expand algebraic expressions.
- It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Where possible students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- **Explicit teaching that aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Appendices

Appendix A: Revise factors and multiples

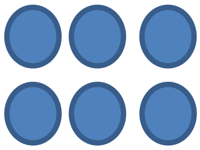
Hand

Check on student understanding of factors; students need to have capacity with factors and multiples to use the distributive law. This first activity is suitable if students need substantial revision of the concept; the second activity is a game assuming knowledge of factors and multiples.

Find the factors and multiples of all whole numbers up to 16

In this first activity choose an upper limit appropriate for your students; 8 or 10 may be sufficient or you may wish to go higher than 16.

Teacher arranges 6 counters as shown:

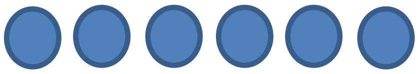


2 rows of 3 means $2 \times 3 = 6$

$\therefore 2\ is\ a\ factor\ of\ 6$
$and\ 3\ is\ a\ factor\ of\ 6$

because 2 divides evenly into 6 with no remainder, and 3 divides evenly into 6 with no remainder.

Now arrange counters as shown:



What does this represent? $1 \times 6 = 6$

$\therefore 1\ and\ 6\ are\ both\ factors\ of\ 6$

Factors of 6 are 1, 2, 3, 6 or, if it is easier, $1 \times 6,\ 2 \times 3$

What are the multiples of 6?

6, 12, 18 … and so on

Students continue working in pairs for all numbers 2 to 16. Realise that for 12 the counters can be arranged as 6×2 or 3×4 and 12×1.

Record the results in a table for future reference.

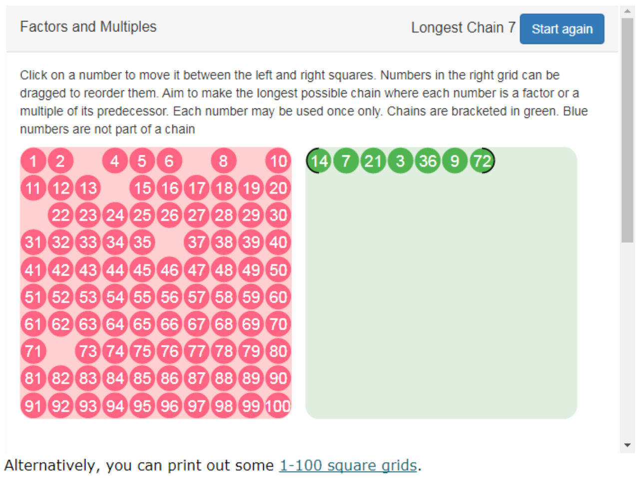
Note: You could use a rectangle drawn on grid paper (rather than counters) to demonstrate factors. This would reinforce the area model.

Mind

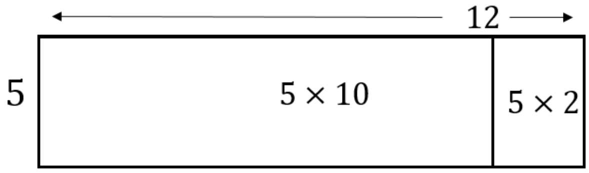
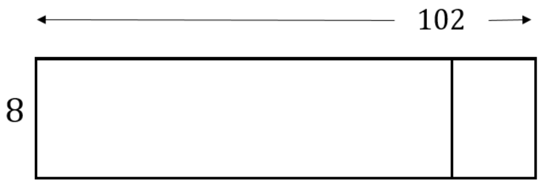
Factors and Multiples game from NRICH

Play the game Factors and Multiples from the NRICH website https://nrich.maths.org/5468

The challenge is to see how long you can keep a chain of factors and multiples going. If you want to use only 1 to 50, project a grid onto a board and cross them out with a pen as you go. A grid of 1 to 50 is provided.



Appendix B: Using area diagrams to help with calculations – addition

<table>
<tr><td colspan="2">Make each of the calculations easier by splitting up one of the numbers.
Draw an area diagram for each and write the calculation.
• Write the calculations on the diagrams.
• The first ones have been started for you.
• You can use the calculator to check your answers.</td></tr>
<tr><td>$5 \times 12 = ?$
</td><td>5×12
$= 5 \times (10 + 2)$
$= 5\ \times 10 + 5 \times 2$
$=$</td></tr>
<tr><td>$8 \times 102 = ?$
</td><td>8×102
$= 8 \times ($</td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
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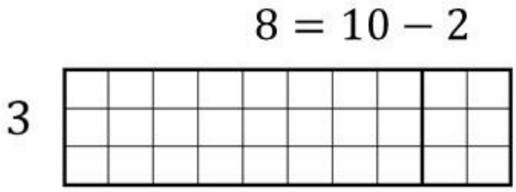
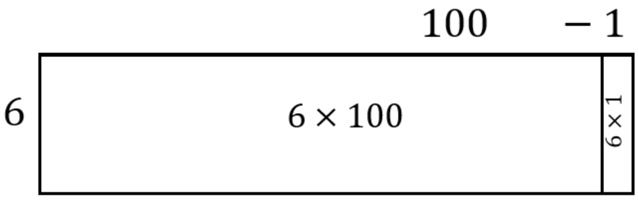
</table>

Appendix C: Using area diagrams to help with calculations – subtraction

Make each of the calculations easier by splitting up one of the numbers.
Draw an area diagram for each and write the calculation.

- Write the calculations on the diagrams.
- The first ones have been started for you.
- You can use the calculator to check your answers.

$3 \times 8 = ?$  	3×8 $= 3 \times (10 - 2)$ $= 3 \times 10 - 3 \times 2$ $=$
$6 \times 99 = ?$  	6×99 $= 6 \times (100 - 1)$ $= 6 \times 100 - 6 \times 1$ $=$

