

YuMi Deadly Maths

Year 7/8 Teacher Resource:

NA – Between the waterholes (add and subtract integers)

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT



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ACKNOWLEDGEMENT

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Year 7/8 Number and Algebra

Between the waterholes (add and subtract integers)

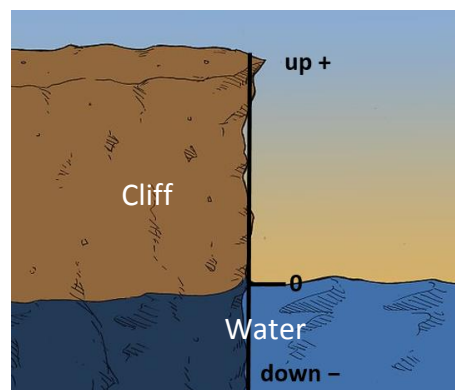
Learning goal	Students will represent, add and subtract integers.
Content description	Number and Algebra – Number and place value <ul style="list-style-type: none">Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)Compare, order, add and subtract integers (ACMNA280)
Big idea	Number – continuous vs discrete, number line
Resources	Rope; masking tape for three pictures – campfire: 0; waterfall pool: arrow cards 1 to 6; crocodile pool: arrow cards -1 to -6 ; two large foam dice marked as follows – one die marked F1, F2, F3, B1, B2, B3 (F is forward and B is back), and the other marked W (waterfall) on three sides and C (crocodile) on the other three sides

Reality

Local knowledge As with the earlier Year 7 YDM-CCP resource, *Into the waterhole – compare and order integers*, establish a context suitable for a vertical number line.

Prior experience Compare and order integers using both a vertical and horizontal number line.

Remind students of the context used for a vertical number line and again change it to a horizontal line:



- Move the number line rope from the vertical position to the horizontal position. *Where will we place the positive end of the number line?* [Right] *Where will we place the negative end?* [Left] (Why? Explain that this is a worldwide convention or agreement.)
- You can have both a vertical and a horizontal number line in place so students can see the relationship between the two, or you may wish to stay with a vertical number line. With a vertical number line you may need to adjust the following reality scenario.

Kinaesthetic Introduce the reality of between the waterholes.

Place the rope on the floor in a straight line and set a campsite (campfire) midway. At the positive end of the number line is a waterfall (W) with a large swimming hole. At the negative end of the number line is a muddy waterhole with crocodiles (C).



Create the number line with the help of the students. The cards may already be attached to the rope from the vertical number line, or maybe you are starting afresh; either way take the time to step out the number line. (*Note:* Start with the numbers -4 to $+4$ for the course between the waterfall and the crocodile; later this can be extended to -6 to $+6$.)

Put the zero card beside the campfire. What does that tell us? [It is a starting point.] Put the zero card on opposite side of the rope and a strip of narrow masking tape to mark the point for accuracy.

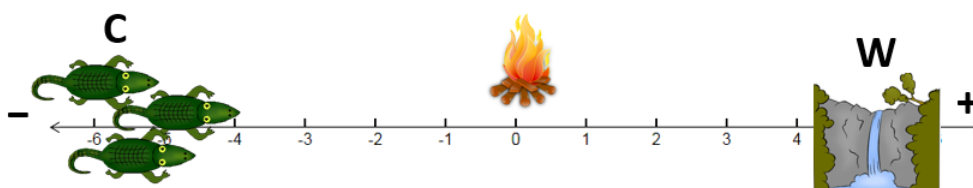
Create the positive section of the number line:

- Have a student walk four even steps forwards (from zero) and place the arrow cards 1 to 4 at each step. *These are the positive integers.*
- Another student places the picture of the swimming waterfall pool at the numeral 4 leaving a little bit of the rope continuing on. *Why have we left the rope going on?* [Numbers don't finish at 4, they go on forever (continuous).]
- *What direction are these steps going?* [These steps are going in a forwards or positive direction to the waterfall.]

If we stand at base (the campfire) and take one step back in the opposite direction towards the crocodile waterhole, how can we represent this with a number? [-1]

Now create the negative section of the number line:

- Repeat the above process for -1 to -4, with the student stepping backwards while facing zero (the campfire). *These are the negative integers.*
- Place the picture of the crocodile-infested pool at -4, leaving the rope continuing on. *Why have we left the rope going on?* [Numbers don't finish at -4, they go on forever (continuous).]
- *What direction are these steps going?* [They are going in a backwards or negative direction to the crocodile pool.]



Abstraction

Body

Develop familiarity with the number line without adding or subtracting.

This section develops the concept of opposites as a preliminary concept, then you need to establish the four movements as a preamble to the game that follows:

- facing waterfall (W)
- facing crocodiles (C)
- walking forwards, e.g. 2 steps (F2)
- walking backwards, e.g. 2 steps (B2).

1. Establish the concept of opposites

Example: -4 opposite to +4

Have two students stand at zero next to the picture of the campfire. Have both face W (positive). Ask one student to take four steps forward (F4).

- *What direction is this student going?* [a forward or positive direction]
- *Where will this take the student?* [to the waterfall]
- *How many steps backwards will the second student need to take to end up at the opposite point?* [four]
- *What number is at this point?* [-4]
- *What can you tell me about +4 and -4?* [They are opposites.]

Give similar examples to create understanding that opposite numbers or integers (positive and negative) are the same distance or same number of steps in each direction from zero.

2. Further establish the concept of opposites by using reversing

- *Start at 2. Walk forwards to its opposite, counting your steps as you go. What integer is opposite to 2? [-2] How many steps will be needed to get there? [4 steps: 2 steps back to zero and 2 steps below zero]*
- *Start at -4. Walk forwards to its opposite, counting your steps as you go. What integer is opposite to -4? [4] How many steps will be needed to get there? [8 steps: 4 steps up to zero and 4 steps on or forwards to 4].*
- Give other examples as needed to reinforce the notion of opposites in directed number.

3. Establish the concepts of facing the waterfall (W) or facing the crocodiles (C)

- Have the student start at base camp (0) and face W; take 2 steps forward (F2). *Where are you now? Are you closer to the waterfall or the crocodiles? [waterfall] How many steps are you away from the waterfall? [2 steps]*
- Have the student start at base camp (0) and face C; take 2 steps forward (F2). *Where are you now? Are you closer to the waterfall or the crocodiles? [crocodiles] How many steps are you away from the crocodiles? [2 steps]*

4. Establish the concepts of stepping forwards (F) or stepping backwards (B)

- Have the student start at base camp (0) and face W; take 2 steps backward (B2). *Where are you now? Are you closer to the waterfall or the crocodiles? [crocodiles] What was the effect of stepping backwards? [moved in a negative direction]*
- Have the student start at base camp (0) and face C; take 2 steps backward (B2). *Where are you now? Are you closer to the waterfall or the crocodiles? [waterfall] What was the effect of stepping backwards? [moved in a positive direction]. This time stepping backwards took us closer to the waterfall.*

Body

Game: "Swim or dinner?"

Two students play at a time. The dice are given to two students. The students, in turn, roll both dice:

- (a) **the direction dice**, which tells which way to face: W towards the waterfall and the safe swimming waterhole (this is positive) or C towards the crocodile waterhole to be eaten for the crocodile's dinner (this is negative); and
- (b) **the steps dice**, which tells which way to move and how much: F1, F2 or F3 (forward-positive) or B1, B2 or B3 (backward-negative) with the number telling how many steps to be taken.

Each time, ask the students: *What direction are you facing?* [positive or negative] *Which way will you be moving?* [forwards or backwards] *Where will this take you – closer to safety (positive) or danger (negative)?* Students in turn keep rolling both dice until one of them reaches the waterfall or the crocodiles. Two new students then get to play.

Repeat the game many times so that students gain the concept of forwards going in a positive direction and backwards going in a negative direction, with facing towards the waterfall being positive and facing towards the crocodiles being negative. Ensure they experience and understand the following four concepts:

1. Facing towards the waterfall (positive) and walking forwards (positive) means that they move closer to the waterfall (positive).

2. Facing towards the crocodiles (negative) and walking forwards (positive) means that they move closer to the crocodiles (negative).
3. Facing towards the waterfall (positive) and walking backwards (negative) means that they move closer to the crocodiles (negative).
4. Facing towards the crocodiles (negative) and walking backwards (negative) means that they move closer to the waterfall (positive).

Note: To ensure more students can participate, students can work in small groups outside using number lines drawn on the asphalt. Peer tutoring can improve understanding, along with teacher guidance.

**Body/Hand/
Mind**

Develop number sentences that start from base camp (zero).

Now transition from the waterfall/crocodile scenarios to number sentences by making the connection between the four terms *waterfall*, *crocodiles*, *forwards* and *backwards* to these four mathematical terms:

- two operations, either **addition (add)** or **subtraction (subtract)**
- the sign of the integer is either **positive** or **negative**.

Teaching notes: terminology and notation

- Avoid the use of the words *plus* and *minus* as they can be used for both the operation and the sign of the integer.
- Note the option to use superscripts for the + and – when used for the sign of the integer as opposed to when used as an operation, e.g. $^{-}3 + ^{-}2 = ^{-}5$.
- An alternative notation is $(-3) + (-2) = (-5)$. However, students can find the brackets confusing and difficult to read and write.
- Allow the students to discover these in the following activities; however, for your information the underlying connection is:
 - facing the waterfall (W) represents addition
 - facing the crocodiles (C) represents subtraction
 - stepping forwards is positive (e.g. F2 is +2)
 - stepping backwards is negative (e.g. B2 is -2).
- Use the following examples as a guide. Add more examples as needed for your students; allow students to create their own examples.
- As an introduction you could ask students to informally give you addition or subtraction stories: addition is joining together to make more so it goes in a forward direction; subtraction is taking away so quantity is reduced to a smaller amount if taking positive amounts away or a larger amount if taking negative amounts away.

Use scenarios that start from base camp (zero) to establish the mathematical terms.

Teacher (or students) write the accompanying number sentences on the whiteboard; students act out the scenarios. Use examples to demonstrate progressive understanding, or continue to use the dice.

Teaching notes:

- Notice the four types of number sentence listed in the table below. The aim is not to teach the four types but to provide experience with these different types of number sentence, leading through eventually to subtracting a negative.
- You may like to spend some time establishing the first three scenarios before moving to subtracting a negative.

Adding a positive number W F3	
<p>Say: Start at base camp – where are we? [0] Face the waterfall (looks like we are going to add; we are expecting to get an answer greater than zero). F3 is 3 steps forwards towards the waterfall (add positive 3). Where did we end up? Is it greater than zero? [Yes] Write: $0 + +3 = +3$ We know this as: $0 + 3 = 3$</p>	<p>Facing the waterfall means we add; stepping forwards is a positive number.</p>
Subtracting a positive number C F2	
<p>Say: Start at base camp – where are we? [0] Face the crocodiles (looks like we are going to subtract; we are expecting to get an answer less than zero). F2 is 2 steps forwards towards the crocodiles (subtract positive 2). Where did we end up? Is it less than zero? [Yes] Write: $0 - +2 = -2$ We know this as: $0 - 2 = -2$</p>	<p>Facing the crocodiles means we subtract; stepping forwards is a positive number.</p>
Adding a negative number W B3	
<p>Say: Start at base camp – where are we? [0] Face the waterfall (we are going to add; will we get an answer greater than our starting number?). B3 is 3 steps backwards towards the crocodiles (add negative 3). Where did we end up? Is it greater than zero? [No] Why not? [Because we added a negative number] Write: $0 + -3 = -3$ When we have negative numbers we need to write all the signs: $0 + (-3) = -3$</p>	<p>Facing the waterfall means we add; stepping backwards is a negative number.</p>
Subtracting a negative number C B2	
<p>Say: Start at base camp – where are we? [0] Face the crocodiles (we are going to subtract). B2 is 2 steps backwards towards waterfall (subtract negative 2). Where did we end up? Is it less than zero? [No] Why not? [Because we subtracted a negative number] Write: $0 - -2 = +2$ Say: What happened when we subtracted a negative number? [We got an answer greater than our starting number. It had the same effect as adding.]</p>	<p>Facing the crocodiles means we subtract; stepping backwards is a negative number.</p>

**Body/Hand/
Mind**

Move to scenarios that start from any position.

- Detailed examples have been provided in the appendix to support your own understanding.
- Students can work in small groups on number lines drawn on the playground with chalk. They can be given a set of number sentences to solve by walking the number line.

Hand/Mind

Students work with a number line and figurine: Students use number lines on their desks with small figurines to walk the number line. It is best to use a figure that has a front and a back rather than an object such as a cube of wood. This allows students to internalise the four movements they experienced on the larger number line.

$+3 + +1 = +4$ (swim at the waterfall)	$+1 - +3 = -2$ (on way to crocodile's dinner)
$+4 + -2 = +2$ (heading back to base)	$+1 - -3 = +4$ (let's swim!)

$+2 - +3 = -1$ (on way to crocodile's dinner)	$-5 - -3 = -2$ (on way back to base camp)
$-1 + +2 = +1$ (on way to swim again)	$-2 - -2 = 0$ (back to base)

Creativity

Students create their own scenarios/number sentences and relate the accompanying story to the number sentence and vice versa.

Return to the diving scenario (see Year 7 YDM-CCP resource, *Into the waterhole – compare and order integers*):

- *Imagine you are diving from the rocks on the hill into the waterhole below. You start at a rock 3 metres above the waterhole and dive to 2 metres down below the surface of the water in the waterhole. Then after you surface you climb to a ledge 4 metres up the hill.*
- *Draw a diagram of this scenario.*
- *How much distance did you cover in the dive? [3 m + 2 m = 5 m]*
- *How much distance did you cover in the swim back to the surface and the climb back up the hill? [2 m + 4 m = 6 m].*

Students choose their own context to create a story using words, symbols and pictures to depict going forwards and backwards from safe to unsafe places and vice versa; favourable to unfavourable conditions and vice versa.

Mathematics

Language/ symbols

number line, continuous, integer, positive, negative, zero, operations, addition, subtraction, sum, difference

Practice

1. Students use a thinkboard to identify context, pictures, word stories, number line, symbols for addition and subtraction of positive and negative integers. Set a context for their partner to solve.
2. Use the W and C method to answer problems like $1 - (-2)$; 1 is starting point, - is facing negative, -2 is walking backwards, so same as +2; answer is 3.
3. Practice these meanings and computations. Use digital technologies to compare, order, add and subtract integers.

Connections

Number line: Positive fractions and decimal numbers are placed between zero and one as well as negative fractions and decimal numbers placed between zero and negative one.

Reflection

Validation

Students validate their partner's thinkboard; explore and answer partner's example. Discuss other contexts – e.g. having and owing money.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently; e.g. problems relating to changes in temperature involving above and below zero temperatures, or problems relating to money (income and expenditure or assets and liabilities) that involve positive and negative integers in addition and subtraction.

Extension

Flexibility. Students are able to add and subtract integers starting from a positive integer, zero, or a negative integer, and progress in either a positive or negative direction.

Reversing. Students are able to create a context \leftrightarrow write a story \leftrightarrow draw pictures \leftrightarrow use symbols and number lines to demonstrate addition and subtraction of integers, starting from and moving between any given representation. This means going from symbols such as $3 - (-2)$ to a drawing or a story to get answer 5 and reversing this.

Generalising. Students come to see the patterns that are inherent in these additions and subtractions: $5 - +3 = 5 + -3 = 2$; $5 - -2 = 5 + 2 = 7$; and so on.

Changing parameters. Explore negative decimal numbers and fractions in contexts of moving forwards and backwards from zero.

Teacher's notes

- Further examples of addition and subtraction with integers have been provided in the Appendix for teachers who are less confident with this content.
- To develop full understanding at this stage it is important to include all the signs:
e.g. $+3 - +2 = +1$ OR $-5 - +2 = -7$

At some point the students will be ready to leave out the + sign when it is applied to an integer:

e.g. $3 - 2 = 1$ OR $-5 - 2 = -7$

This needs to be left until a later stage. If you do this too early the students do not have the chance to internalise the four movements they experience when walking the number line.

- This teaching resource needs to be adapted to suit the local environment, e.g. desert – safe and unsafe: rocks and desert taipan.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a waterhole, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a waterfall.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; <https://www.qcaa.qld.edu.au/3035.html>
- Teaching that **aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Appendix: Detailed examples with explanations

Teaching notes:	
<ul style="list-style-type: none"> • These examples are provided to support you and to confirm your own understanding, if you are less experienced in this topic. Create your own or continue to use the dice as appropriate. • They are structured from adding and subtracting positive numbers first then progress to adding and subtracting negative numbers. • Notice the gradual increase in complexity from operations on the positive side of the number line only, which is a familiar concept, through to working on both sides of the number line, through to working on the negative side only. • We are building familiarity with the concepts, building to subtracting a negative. Take the time with your students to build familiarity before introducing this. 	
Body/Hand/Mind Adding or subtracting a positive number Starting number may be positive or negative	Adding a positive number to a positive number (resulting in a positive number)
	Say: Start at +1; face waterfall (we are going to add; looks like the number will get bigger). F3 is 3 steps forwards towards the waterfall (add positive 3). Where did we end up? We added; did the number get bigger? [Yes] Write: $+1 + +3 = +4$ This is a number sentence we already know how to answer ($1 + 3 = 4$).
	Subtracting a positive number from a positive number (resulting in a positive number)
	Say: Start at +3; face crocodiles (we will subtract; looks like the number will get smaller, let's see). F1 is 1 step forwards towards the crocodiles (subtract positive 1). Where did we end up? We subtracted; did the number get smaller? [Yes] (Is $+2 < +3$?) Write: $+3 - +1 = +2$ This is a number sentence we already know how to answer ($3 - 1 = 2$).
	Adding a positive number to a negative number (resulting in zero)
	Say: Start at -2; face waterfall (we are going to add; looks like the number will get bigger). F2 is 2 steps forwards towards the waterfall (add positive 2). Where did we end up? We added; did the number get bigger? [Yes] (Is $0 > -2$?) Write: $-2 + +2 = 0$ This is a number sentence we already know ($-2 + 2 = 0$ or $2 - 2 = 0$).
	Subtracting a positive number from a positive number (resulting in a negative number)
	Say: Start at +1; face crocodiles (we will subtract; looks like the number will get smaller, let's see). F3 is 3 steps forwards towards crocodiles (subtract positive 3). Where did we end up? We subtracted; did the number get smaller? [Yes] (Is $-2 < +1$?) Write: $+1 - +3 = -2$
	Adding a positive number to a negative number (resulting in a positive number)
	Say: Start at -3; face waterfall. How many forward steps are needed to get to the waterfall? [7 steps forwards] Write: $-3 + +7 = +4$
	Subtracting a positive number from a negative number (resulting in a negative number)
	Say: Start at -2; face crocodiles (we will subtract; looks like the number will get smaller, let's see). F4 is 4 steps forwards towards the crocodiles (subtract positive 4). Where did we end up? We subtracted; did the number get smaller? [Yes] (Is $-6 < -2$?) Write: $-2 - +4 = -6$

Body/Hand/Mind Adding or subtracting a negative number Starting number may be positive or negative	Adding a negative number to a positive number (resulting in a negative number)
	Say: <i>Start at +1; face waterfall (we are going to add; looks like the number will get bigger). B3 is 3 steps backwards towards the crocodiles (add negative 3).</i> <i>Where did we end up? We added; did the number get bigger? [No] (Is -2 > +1?)</i> <i>Why not? [Because we added a negative number]</i> Write: $+1 + -3 = -2$
	Subtracting a negative number from a positive number (resulting in a positive number)
	Say: <i>Start at +3; face crocodiles (we will subtract; looks like the number will get smaller, let's see).</i> <i>B1 is 1 step backwards towards the waterfall (subtract negative 1).</i> <i>Where did we end up? We subtracted; did the number get smaller? [No] (Is +4 < +3?)</i> <i>Why not? [Because we subtracted a negative number]</i> <i>What is the effect of subtracting a negative number? [It has the same effect as adding.]</i> Write: $+3 - -1 = +4$
	Adding a negative number to a negative number (resulting in a negative number)
	Say: <i>Start at -2; face waterfall (we are going to add; looks like the number will get bigger).</i> <i>B2 is 2 steps backwards towards the crocodiles (add negative 2).</i> <i>Where did we end up? We added; did the number get bigger? [No] (Is -4 > -2?)</i> <i>Why not? [Because we added a negative number]</i> Write: $-2 + -2 = -4$
	Subtracting a negative number from a positive number (resulting in a positive number)
	Say: <i>Start at +2; face crocodiles (we will subtract; looks like the number will get smaller, let's see).</i> <i>B3 is 3 steps backwards towards the waterfall (subtract negative 3).</i> <i>Where did we end up? We subtracted; did the number get smaller? [No] (Is +5 < +2?)</i> <i>Why not? [Because we subtracted a negative number.]</i> <i>What is the effect of subtracting a negative number? [It has the same effect as adding.]</i> Write: $+2 - -3 = +5$
	Adding a negative number to a negative number (resulting in a negative number)
	Say: <i>Start at -3; face waterfall. How many backwards steps are needed to get to the crocodiles? [1 step backwards] How do we write this? [negative 1]</i> Write: $-3 + -1 = -4$
Subtracting a negative number from a negative number (resulting in a positive number)	
Say: <i>Start at -2; face crocodiles (we will subtract; looks like the number will get smaller, let's see).</i> <i>B4 is 4 steps backwards towards the waterfall (subtract negative 4).</i> <i>Where did we end up? We subtracted; did the number get smaller? [No] (Is +2 < -2?)</i> <i>Why not? [Because we subtracted a negative number]</i> <i>What is the effect of subtracting a negative number? [It has the same effect as adding.]</i> Write: $-2 - -4 = +2$	

Reversing	These examples use reversing: <i>How many steps do I need to get to ...?</i>
	<p>Say: Start at -2; face the waterfall (this means we will add). Do I need forward steps or backward steps to avoid the crocodiles? [forward steps] How many forward steps do I need to get to the waterfall? [F6] Write this as a number sentence. Write: $-2 + +6 = +4$</p>
	<p>Say: Start at -2; face the crocodiles (this means we will subtract). Do I need forward steps or backward steps to avoid the crocodiles? [backward steps] How many backward steps do I need to get back to the waterfall? [B6] Write this as a number sentence. Write: $-2 - -6 = +4$ Notice these two different number sentences have the same answer.</p>
	<p>Say: Start at -2; face the waterfall (this means we will add). Do I need forward steps or backward steps to get back to base camp? [forward steps] How many forward steps do I need to get back to base? [F2] Write this as a number sentence. Write: $-2 + +2 = 0$</p>
	<p>Say: Start at -2; face the crocodiles (this means we will subtract). Do I need forward steps or backward steps to get back to base camp? [backward steps] How many backward steps do I need to get back to base? [B2] Write this as a number sentence Write: $-2 - -2 = 0$ Notice these two different number sentences have the same answer.</p>