YuMi Deadly Maths

Year 8 Teacher Resource:

NA – Maths magic

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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QUT

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Growing community through education

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

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Year 8 Number and Algebra

Maths magic

Learning goal Students will create linear mathematical models to solve a problem.

Content description Number and Algebra – Linear and non-linear relationships

- Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)
- Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)

Number and Algebra – Patterns and algebra

- Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)
- Factorise algebraic expressions by identifying numerical factors (ACMNA191)
- Simplify algebraic expressions involving the four operations (ACMNA192)

Big idea Algebra – patterns, variable/pronumeral

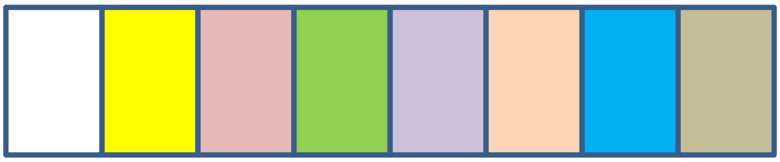
Resources Coloured sheets of A4 paper, Maths Mat, elastics

Reality

Local knowledge Discuss puzzles that students have encountered in their local world/environment where clues are given to find the missing part, e.g. "Think of a number". *How does this apply to problems in the real world?* [e.g. How much paint is needed to paint the house, how much area is needed for a dog kennel or a cat pen?]

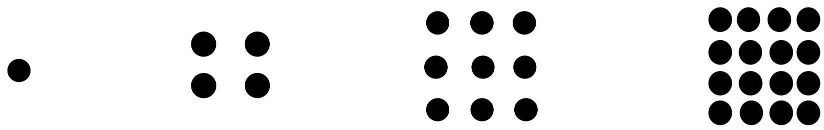
Prior experience *If you wanted to construct a set of boarding kennels to house dogs and cats while their owners were on holidays, what would you need to consider?* [the area to be allowed for each dog, the perimeter of each kennel/pen and the length required to place the kennels side by side and the pens together in a square]. *How do you find the area? How do you find the perimeter? In determining how many kennels can be placed side by side and pens together in a square, what measurements do you need to know?* [the available length/area respectively].

Kinaesthetic **Dogs**: The average size of a dog kennel is in the ratio of 2:3. Using a sheet of A4 paper to represent one kennel, have students lay out the model of the boarding kennels starting with one sheet of paper, followed by other sheets of differently coloured paper placed so that the lengths are adjacent. *The manager wants to fence around all the kennels. What will determine the amount of fencing she needs?* [the number of kennels].



As each new "kennel" is added, what is happening to the measurements? [The original length of the kennel remains constant and becomes the width of the total number of kennels placed side by side. The length required for the total boarding kennels depends on the number of kennels (widths) that are equal to the available length.] Have a student walk around one kennel. *What is the perimeter?* [2(3 + 1k's width) = 2(3 + 2) = 10]. Another student walks around two adjacent kennels. *What is the perimeter?* [2(3 + 2k's width) = 2(3 + 4) = 14]. Another student walks around 3 adjacent kennels. *What is the perimeter?* [2(3 + 3k's width) = 2(3 + 6) = 18]. Repeat this procedure until students understand the process. *What measurement is staying the same?* [the original length of 3]. *What measurement is extended by being multiplied?* [the original width of 2]. *What is the constant growth in the* *length each time another kennel is added?* [4]. *If "F" stands for the fencing required and "k" is the number of kennels, what is the algebraic expression for finding the length of fencing required?* [$F = 2(3 + 2k)$]. *Which of the terms, F and k, changes according to the value given to the other?* [F or fencing term as its value depends on the value that is substituted for k, the number of kennels.] *If a person can stand alone and not depend on others, how is that person described?* [an independent person]. *The words independent and dependent are also used to describe the variables in algebraic equations. What variable stands alone and what variable changes according to the value substituted for the other?* [k stands alone, independently, and F changes corresponding to k's value. F is dependent on k for its value but k is the independent variable.] *Would the manager be happy to have the help of maths magic to assist in planning the dog kennels?*

Cats: The manager wants to maximise the available area for her cat pens but have the least cost in fencing. *What shape gives the most area for the same perimeter?* [square; e.g. perimeter = 12; sides are 5 and 1, 4 and 2, 3 and 3 but area is respectively 5, 8, 9 square units]. The manager, therefore, decides to invest in square pens for the cats. The pattern below is used to calculate the number of cat pens that can be constructed and the amount of grass squares to turf the area.



Using the Maths Mat, ask: *Are there any cat pens already built?* [No.] *So where is the manager starting from?* [zero]. Have a student stand in one square to represent one cat pen that will be added to the zero. Have three more students join with the first student to create the next cat pen. *What shape has been made?* [a square]. *What is the side of this square?* [2]. *How has the side of this group changed from the original where the manager started from zero and then built one cat pen?* [This group has one pen plus one pen or 2 cat pens on each side]. *What is the algebraic expression for this?* [$p + 1$]. *How many cat pens are in this square with the side of 2?* [4]. *What operation has been used?* [2 has been multiplied by 2]. *What is another way of saying this?* [2 squared]. Repeat procedure for the next two diagrams. *How are the pens increasing each time?* [One is being added each time to each side.] *What is the algebraic expression for this?* [$s + 1$]. *Once a new side has been made, what is then created from each side?* [a square]. *So if each new side is $p + 1$ (where p stands for the original number of pens with one more added), what is the algebraic expression that shows each new side has been squared?* [$(p + 1)^2$]. *If T is used to indicate the total number of pens that can be made using this algebraic expression, what is the equation or function?* [$T = (p + 1)^2$]. *Which is the dependent variable and which is the independent variable?* [p is the independent variable and T is the dependent variable]. *Explain why this is so. How does the manager feel because maths magic is providing the options for the best plan to turf the optimal number of cat pens?*

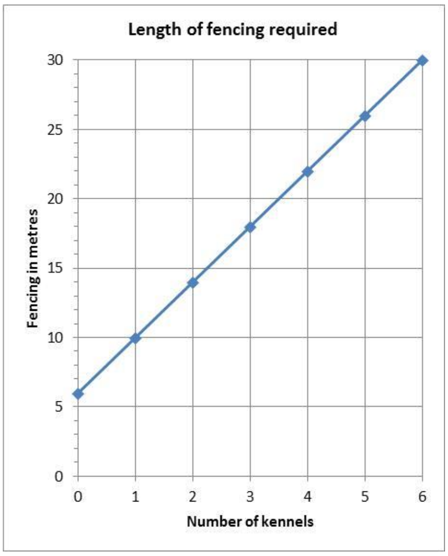
Abstraction

Body

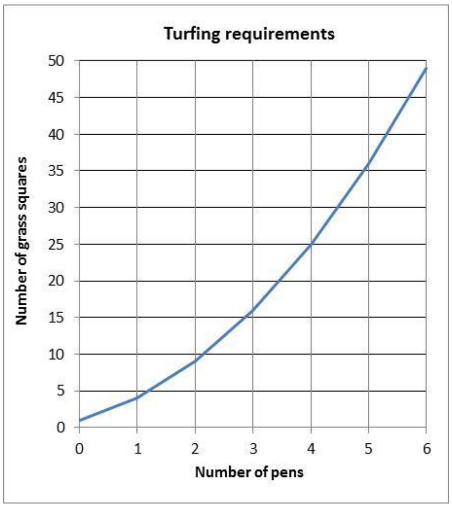
Dogs: Use the Maths Mat so that the long axis is F (fencing) and short axis is k (number of kennels). Have students plot and stand at the coordinates that demonstrate the algebraic expression above: $F = 2(3 + 2k)$. *What are the coordinates for this graph?*

k	0	1	2	3	4	5	6
F	6	10	14	18	22	26	30

Students then use an elastic or cord for each to hold and join all the coordinates to show that a linear graph has been made. Reinforce that all graphs must be named, axes are named, the scale remains constant and a legend is given where necessary to help interpret the graph.



Cats: Maths Mat as above where the long axis is T (total number of pens/grass squares for turfing the area) and short axis is p (initial number of pens in each side of the square before one more pen was added). Have students plot and stand at the coordinates that demonstrate the algebraic equation $T = (p + 1)^2$. Explain what this algebraic equation means. [Start with a number of pens (0–6), add one and then square the result to find the total in that group.] *What are the coordinates for this graph? Look at the mat. How many pens are there at present?* [0, the mat is empty or blank]. *So we start with 0 and add 1 and square it* [$(0 + 1)^2 = 1^2 = 1$]. *The first coordinate is 0, 1. How many do we start with in the next group?* [1]. *What steps follow?* [Add 1 and square the result: $(1 + 1)^2 = 2^2 = 4$]. *What are the coordinates for this group?* [1, 4]. *Explain the next group.* [Start with 2, add 1 and square the result: $(2 + 1)^2 = 3^2 = 9$]. *What are the coordinates here?* [2, 9]. Repeat procedure having a student stand at the point for each coordinate as it is made. *What type of graph has been made?* [a non-linear graph]. Students then use an elastic or cord for each to hold and join all the coordinates to show that a non-linear graph has been made. Reinforce all graphs must be named, axes are named, the scale remains constant and a legend is given where necessary to help interpret the graph.



p	0	1	2	3	4	5	6
T	1	4	9	16	25	36	49

Hand Students tabulate the coordinates from the above functions and draw the graphs on grid paper, making sure the graph and axes are named, an appropriate scale is chosen and a legend is included. They can also use geoboards and rubber bands to demonstrate the graphs.

Mind *Close your eyes and see a straight line graph that starts at 0, 0 and travels through 1, 1; 2, 2; 3, 3. What algebraic function does this demonstrate?* [$y = x$]. *What type of graph would be made if the points were plotted for $y = x^2$?* [non-linear graph].

Creativity Students create their own puzzles, algebraic expressions and graphs that represent the puzzles.

Mathematics

Language/ symbols variable, independent variable, dependent variable, linear, non-linear, algebraic expression, equation, coordinates, exponent, power, function

Practice

1. *The manager also needs to know the area that the number of dog kennels occupy and how many squares of grass will be needed to be ordered to turf the kennels. What two major factors will determine how many kennels she can construct?* [the area available and the cost of fencing and turfing]. *Develop an algebraic equation that demonstrates the turfing required that corresponds to the number of kennels. Use the length ratio as above: 3:2.*
2. Calendar puzzles. *Write an algebraic equation that shows how these puzzles may be solved*:
   (a) Draw a square around any four days in a calendar month. Using algebra, show that if the sum of the days is given, the first date can be found and then the other dates as well. (Hint: Let the first date be "n".)
   (b) If the sum of four dates that form a square on a calendar is 68, what are the four dates?
   (c) Select any three dates on the calendar that are on a diagonal line. Using algebra, show that if the sum of the days is given, the dates may be found. (Hint: Let the middle date be "n".)
3. Using algebra, show that if you think of any number, double it, add 10, halve it, take away the original number, your answer or final number will always be 5.
4. Using algebra, show that if any 3-digit number is chosen that has all digits the same, when that number is divided by the sum of its digits, the answer will always be 37.

Connections

Relate to function machines, tables, graphs, problem-solving, quadratic equations.

Reflection

Validation

Students explore any interesting pattern on the calendar, name one of the days x that could be the initial or middle day, and then name the other days in terms of x, total the days and work out a puzzle that can be solved algebraically. Test it out with a partner.

Validate the algebraic equations in the Practice examples above, share with the class and justify the solution.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently; e.g. a calendar puzzle: *Select any nine dates on the calendar. Using algebra, show that if the dates are added and then divided by the date in the middle square, the answer will always be 9.* (Hint: Let the middle date be "n".)

Extension

Flexibility. Students use variables to formulate algebraic equations that will assist them to solve mathematical problems in many different contexts.

Reversing. Students are able to move between writing an algebraic expression/equation ↔ acting it out ↔ writing and representing it in linear and non-linear relationships ↔ interpreting patterns and making generalisations in algebraic terms, starting from and moving between any given point.

Generalising. *A letter or symbol may be used to represent an unknown value. Variables, or pronumerals, stand for something that is not immediately known. The relationship in equations between the dependent and independent variables dictates whether the graph will be linear or non-linear. The use of algebra is a valuable tool in problem-solving. Equations are graphed to give a quick understanding of data*.

Changing parameters. Use the sentence starter, "How would this be changed if …?" to deviate from original scenarios that then propose new questions to be investigated.

Teacher's notes

- Ensure that students have a sound understanding of the features that make one variable dependent and the other the independent variable.
- Students must always name the graph, the axes, select an appropriate scale that remains constant, and include a legend where necessary.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a linear graph, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a non-linear graph.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that **aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.