Developing Mathematics Understanding Through Cognitive Diagnostic Assessment Tasks

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I wish you good teaching and would welcome any feedback from you.

Dr Annette Baturo
Senior Lecturer in Mathematics Education, QUT (a.baturo@qut.edu.au)
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OVERVIEW and THEORY

1. INTRODUCTION

Assessment is undertaken for a variety of reasons but none so important as when it is undertaken to inform the teaching-learning process with respect to determining the extent of individual student knowledge and the effectiveness of teaching.

This book contains Cognitive Diagnostic Assessment Tasks (CDAT) to elicit students' understanding of the important mathematical concepts and processes that are required for processing whole numbers, fractions and probability effectively.

These CDAT are grounded in the results and theoretical frameworks of modern scientific research on the cognitions that underlie students' mathematics learning which, according to many researchers (e.g., Battista, 1999; Goldin, 2000; Lesh and Kelly, 2000; RAND, 2003), are generally missing from traditional assessment procedures. Cognition is the core substance of understanding and sense-making. As such, CDAT are absolutely critical for monitoring students' development of powerful mathematical thinking; they are not concerned with determining who are the ‘rememberers’ or ‘forgetters’ of drilled procedures.

CDAT are designed for use by teachers in formative and summative classroom assessment, namely, finding out what mathematical concepts and processes students understand before, during, or at the conclusion of teaching.

CDAT can provide a vehicle for deepening teachers' understanding of core ideas in elementary mathematics and consequently to modify or extend their instruction. As one teacher said when given a CDAT to trial with her class: Oh, that’s what I haven’t been doing! CDAT can also provide teachers with insights into how students learn those ideas, particularly when students' solution strategies are discussed. Thus, CDAT provide a springboard for intervention or prevention.

1.1 ASSESSMENT FRAMEWORK

Mathematics provides the utilitarian skills for normal participation in society, promotes problem-solving skills for employment, and produces the abstract, as well as the creative, knowledge required for innovation (Baturo and Cooper, 1998).

These different outcomes of a mathematics education reflect an increasing level of mathematical difficulty in the transition from real-world to abstraction (which prepare students for the highest level of mathematical thought).

CDAT focus on the abstract (decontextualised) mathematics that is based on system, pattern and structure. Because they are decontextualised, the mathematical ideas inherent in one domain (e.g., fractions) can be transferred to other domains such as decimal fractions, measurement, proportion and probability. Transfer can be very difficult for students whose thinking is grounded in real-world applications.
1.2 STAGES OF ASSESSMENT

There are four main stages of assessment, namely:

1. **Sweep** — across domains (to determine specific domains). This type of test instrument includes items from all mathematics domains (e.g., whole number, fractions, measurement); it is used when a student has several mathematics learning difficulties but the source is not known. The test is scored and analysed in terms of the mathematics domains being assessed.

2. **Scan** — within a domain (to determine specific concepts or processes). This type of instrument includes all concepts and processes within a specific domain (e.g., whole number). The results are scored and analysed in terms of the concepts and processes related to the domain (e.g., identifying numbers in word and digit form, comparing numbers, and so on).

3. **Probe** — within specific concepts and related processes (to determine the order of remediation). This type of test ideally is a teacher-made diagnostic instrument which is administered in individual interviews. The student’s responses are analysed for misconceptions and error patterns.

4. **Intervene** — for specific concepts and related processes. Ideally, these should be teacher-made tasks designed to remediate the particular problems.

The assessment tasks in this book are **scans** because they include all concepts and processes related to a specific mathematics domain (e.g., common fractions, decimal fractions). Teachers can then pinpoint, for each of these domains, at which stage in the learning cycle the student has experienced problems in understanding. Furthermore, scans provide the basis of remediation because the tasks can be replicated by the teacher (using different numbers or representations) to use as **intervention** tasks.

1.3 CDAT IN THIS BOOK

There are 31 tasks in this book for use by teachers. The tasks have been categorised as Levels 1–5 (see Table 1). These levels do not represent Year/Grade levels; rather, they represent concept development levels. For example, Level 1 indicates the Beginning stage of development and so Level 1 for Number would correspond with Year/Grade 1 while Level 1 for Decimal Fractions would correspond roughly with Years/Grades 3, 4 or 5 (depending on your curriculum). (See Table 1.)

There are two tasks for each level (with the exception of Probability): Type A tasks focus on the **structural knowledge** (see Section 2.1) that students have abstracted after extensive teaching/learning; Type B tasks focus on the **representational knowledge** (see Section 2.1) that students should have developed in acquiring structural knowledge. It is expected that **A will be done before B** to offset ‘on-the-spot’ (and therefore not likely to be robust) learning that can occur from representations of the concepts.
Table 1: Categorisation of CDAT in terms of levels and knowledge types

<table>
<thead>
<tr>
<th>Concept development stage</th>
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<th>Decimal Fractions</th>
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1.4 CDAT ADMINISTRATION

1.4.1 How to administer the instruments

The purpose of CDAT is to elicit students’ mathematics understanding. All students need to be given the best opportunity to demonstrate their understanding (in written and oral modes). Each task should be read to the class but no advice should be offered on how to answer the items. CDAT can be administered as:

- **Whole-class pencil-and-paper instrument.** The task is administered, collected, scored and analysed for common errors which would form the basis for intervention. Whole-class discussions of cognitive strategies used by the students will provide insight into common appropriate and inappropriate strategies.

- **Individual interview.** Administer the CDAT with individual students (both higher-performing and lower-performing students) to elicit how they process the tasks. The higher-performing students may supply new thinking strategies that could help the lower-performing students.

1.4.2 When to administer the CDAT

Assessment is most beneficial for informing teachers’ planning when it is administered before new concepts and/or processes are introduced (entry knowledge) and then re-administered when teaching of the concepts/processes is complete (exit knowledge).

1.4.3 What can you do with the CDAT data

Teachers should mark the test, enter the results into a spreadsheet (see Figure 1) and create a graph (see Figure 2, p. 4) to provide a visual record of the results.

From Figures 1 and 2 (showing a Year 6 class’s common fraction entry knowledge), the teacher can see which concepts/processes are known and which need to be focused on. For example, Task 1 (see Figure 3, p. 4) required the students to identify a half when the whole was represented by a variety of area models. As can be seen from Figures 1 and 2, this item elicited that only 42.9% of the students could recognise a half in a variety of representations. Without this basic conceptual understanding, further fraction concept development will be severely limited.
Figure 1: Year 6 students’ entry knowledge (pre-test) scores per common fraction item and sub-item.

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<td>66.2</td>
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</table>

Figure 2: Graph of results from Figure 1.

Figure 3: Item 1, CDAT Common Fractions 1B.

I. Tick the shapes that show halves.
2. THEORETICAL UNDERPINNINGS OF CDAT

Mathematics is best understood as a structure of concepts and principles rather than as a list of rules and definitions (Skemp, 1978). As a structure, it is sequenced and connected, and has recurring themes that provide a macrostructure across the topics. Mathematics teaching, learning and assessment should focus on helping students construct structural knowledge, which is the most transportable form of mathematical knowledge.

The CDAT are based on these views of mathematics and mathematics teaching, learning and assessment. Individual tasks have been developed to reflect the following:

- knowledge types (see Section 2.1)
- structural knowledge in terms of macrostructures, connections (see Section 2.2) and sequences (see Section 2.2)
- prototypic (standard) and nonprototypic (nonstandard) activities (see Section 2.3)
- unitising and re-unitising activities (see Section 2.4).

2.1 KNOWLEDGE TYPES

Current research on mathematics teaching and learning believes that:

(a) there are different types of knowledge to be constructed during the learning cycle
(b) these knowledge types represent a sequence of increasing abstraction
(c) the knowledge types must be connected.

Baturo’s teaching/learning framework (1998; see also Baturo, Warren, and Cooper 2004) synthesises earlier teaching/learning frameworks from research:

- **Entry knowledge** — knowledge constructed before formal instruction of a new concept or process. Entry knowledge can occur from out-of-school experiences and from prior related instruction (e.g. teaching hundredths after tenths have been taught). Thus entry knowledge needs to be examined before teaching a new concept or developing a concept.

- **Representational knowledge** — knowledge constructed during instruction. This type of knowledge is derived from increasingly abstract external (physical) representations of a concept or process. It provides the student with mental pictures of a concept or process. (See Section 2.1.2 for an elaboration of this knowledge which underlies all Level B tasks.)

- **Procedural knowledge** — knowledge constructed during instruction with respect to ‘working out’ how to solve a given problem or task.

- **Structural knowledge** — knowledge abstracted after instruction. This is usually the ‘aha!’ type of knowledge and is encoded for structure (like a family tree) (Ohlsson, 1993). (See Section 2.1.1 for an elaboration of this knowledge which underlies all Level A tasks.)

2.1.1 Structural knowledge

Structural knowledge is the macroknowledge constructed by experts (Baturo, 1998; Sfard, 1991) and should be the goal of all mathematics learning. It is the succinct yet global integrative cognitive knowledge that results from the refinement, abstraction and integration of concepts and processes (Sfard, 1991). It facilitates access to solutions in a variety of
problems; its construction requires teacher and learner to share an active role, both cognitively and kinaesthetically, with concrete and virtual materials and pictures, in making sense of learning through discussion, reflection and validation.

Macroknowledges provide the infrastructure (structural knowledge) of mathematics. The ability to perceive this infrastructure with its recurring themes separates the expert from the novice mathematician. The notions of inverse, identity and part/part/whole are examples of mathematics macroknowledges.

- **Inverse** — undoing actions that leave an object unchanged (e.g. adding/subtracting 1; turning left/turning right; partitioning [÷]/grouping [×]).
- **Identity** — operators that leave an object unchanged (e.g. adding/subtracting 0; multiplying/dividing by 1; rotating through 360°). For example:

  \[
  48 + 25 = 50 + 23 \quad 2 - 2 = 0
  \]

  Adding 0 does not change the value so \(48 + 25\) gives the same answer as \(50 + 23\).

  \[
  \frac{3}{4} \times 1 = \frac{3}{4}
  \]

  Multiplying by 1 does not change the value.

  The identity element, 1, can have many names (e.g., \(1 = 2\) halves, \(3\) thirds) so the value of \(\frac{3}{4}\) is unchanged.

- **Part/part/whole** — occurs in fractions and can help students differentiate between fractions and ratios …

  … and occurs in the basic operations. This may be a more effective strategy for solving word problems than the ‘looking for key words’ strategy.

  \[
  3 (\text{part}) + 4 (\text{part}) = 7 (\text{whole})
  \]

  \[
  3 (\text{part}) \times \frac{4}{7} (\text{whole}) = \frac{12}{3} (\text{whole})
  \]

  +, \times \; \text{know the parts; find the whole}

  \[
  7 (\text{whole}) - 4 (\text{part}) = \frac{3}{4} (\text{part})
  \]

  –, ÷ \; \text{know the whole & one part; find the other part}

Structural knowledge is best promoted in a social-constructivist learning environment where teachers and aides provide cognitive scaffolding (guidance) for students so that they can move from dependency upon experts or peers to being able to solve particular problems independently. In such an environment, both language and materials have a central role in articulating oral and written mathematical knowledge.
2.1.2 Representational knowledge

Instruction designed to enhance the acquisition of structural knowledge should be based on knowledge representations and connections (see Section 2.2 for Connections).

Representations may be external (concrete, pictorial, diagrammatic, written symbols, spoken words) or internal (mental images of external representations). Learning occurs when structural connections are made between internal and external representations (Halford, 1993).

External representations facilitate mental representations (cognitive) of the concept and/or process. Representations and actions accompanying representational materials should also become increasingly abstract. (See diagram below.)

**Representations**

CONCRETE
- Real-world (e.g., biscuits) and replicas (e.g., cars)
- Manipulatives (e.g., counters, Unifix cubes, MAB)
- Virtual (computer replications of the materials above)
- Pictures (of the above, diagrams, maps, number lines)
- Symbolpatterners (e.g., calculators, spreadsheets)

ABSTRACT

**Actions**

CONCRETE
- Whole body ("act it out")
- Hands
- Mind (image)

2.2 CONNECTIONS

The CDAT are based on connections across and within mathematics domains. Knowing how these domains are connected enables teachers to draw on similar representations and to help students connect their new learning to prior learning. For example, knowing that fractions and probability are structurally related through the part/whole concept helps teachers make better choices about sequencing probability materials and relating language.

2.2.1 Connecting mathematics domains

The mathematics domains to be taught from Years 1–10 are:

- Whole numbers
- Fractions
  - common fractions
  - decimal fractions
  - per cents
- Ratio, Proportion, Rate, Scale
- Money & Measurement
- Probability (chance)
- Statistics (data)
- Geometry (space)
- Algebra
- Integers
- Functions

Connected domains (samples only)
For example:

Measurement and Number are connected because of their decimal relationships. Measurement units are related by 1000 as are the Number periods (e.g. 7 ones \( \times 1000 = 7 \) thousands)

<table>
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<tr>
<th>Millions</th>
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<td>kilometre</td>
<td>metre</td>
<td>centimetre</td>
<td>millimetre</td>
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<td>megalitre</td>
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<td>litre</td>
<td>[centilitre]</td>
<td>millilitre</td>
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<tr>
<td>tonne**</td>
<td>kilogram</td>
<td>gram*</td>
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<td>kiloJoule</td>
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</table>

* Technically, the kilogram is the unit of mass measure.
** Tonne could be thought of as megagram

Fractions, Per cents and Probability are connected because they are all derived from ‘parts of a whole’.

2.2.2 Connecting representations

Payne and Rathmell’s (1977) model of concept/process abstraction incorporates four major two-way connections, namely:

- Problem Representation
- Representation Language
- Language Symbol
- Representation Symbol

Teaching would proceed from Problem→Representation→Language→Symbol.

However, for assessment, we ‘peel back’ by starting from the most abstract representation (i.e. symbol).

Remediation would continue to ‘peel back’ (i.e. Symbol→Language→Representation). For this reason, CDAT provide two tasks for each mathematics domain (except Probability) per year level. For example, for Year 4 Number, there will be a structural task (A) followed by a representational task (B).
2.3 PROTOTYPIC AND NONPROTOTYPIC

The CDAT contain prototypic (standard) and nonprototypic (nonstandard) tasks.

Tasks may be nonprototypic because of:

- Different orientations e.g.
  - Different representations e.g. tenths
  - Different directions (reversing) e.g.

Fractions: (a) Whole → part. This is the whole \( \square \); show \( \frac{2}{3} \).
(b) Part → whole. This is \( \frac{2}{3} \); show the whole.

Number: (a) Round 0.83, 0.39 to the nearest whole number.
(b) Write two numbers that can be rounded to 1, 0 (etc.).

2.4 UNITISING AND RE-UNITISING (NAMING AND RENAMING)

These are the mental processes required for naming and renaming numbers and fractions. Re-unitising can be very difficult for some children because it requires an ability to perceive a number from a different perspective, a facility that is required for whole numbers and fractions.

Unitising/re-unitising place values

Students need to be able to unitise 3 tens and then re-unitise as 30 ones with material and on the place value chart and in a written number. For example:
Students also need to be able to unitise (name) a given fraction (common or decimal) and then re-unitise (rename) the given fractions. Re-unitising may require physically or mentally repartitioning the given fraction to generate a new fraction name. For example:

**Unitising/re-unitising common fractions**

- Inserting (physically or mentally) partitions to make more equal parts without changing the value
- Deleting (physically or mentally) partitions to make fewer equal parts without changing the value

**Unitising/re-unitising decimal fractions**

Similar thinking needs to be applied to decimal fractions.

- 6 tenths = 60 hundredths
- 60 hundredths = 6 tenths
- 64 hundredths = 6 tenths 4 hundredths
- 6 tenths 4 hundredths = 64 hundredths
3. UNDERSTANDING WHOLE NUMBERS

3.1 THE DECIMAL NUMBER SYSTEM

There have been many number systems invented by humans since 3000 BC. All of these systems used names and symbols to indicate numbers and all of them had an adding property but no grouping/place value property.

Most countries today use the Decimal Number System, called ‘decimal’ because it has 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The word, decimal, came from the Roman word, decem, which means 10. (December used to be the 10th month of the year in early Roman times.)

The most important number in the system is 1 because it is the unit from which we make numbers larger or smaller. We can make numbers larger in two ways — by adding 1 more or by making groups (which is the basis of multiplying).

In the Decimal Number System, we are mainly interested in making groups of 10 (× 10) because each adjacent place is related by 10 (i.e. 10 ones = 1 ten; 10 tens = 1 hundred; 10 hundreds = 1 thousand; and so on). So, unlike all other number systems, the Decimal Number System has place value which makes it simple but powerful (and longlasting).

Place value is based on a multiplicative feature (i.e. 675 = 6 × 100, 7 × 10, 5 × 1) which utilises only 10 digits (Baturo, 1997; 2000). The unit (one) may be grouped to form larger units (e.g. tens, hundreds) or it may be partitioned to form smaller units (e.g. tenths, hundredths). Thus, a value is represented by its position/place on a continuum of places.

The decimal number system incorporates the following two powerful patterns that students need to internalise over the course of instruction.

3.2 MULTIPLICATIVE RELATIONSHIPS BETWEEN ADJACENT PLACES

This multiplicative structure is continuous across whole-number and decimal-number places, is bi-directional (× ‘shifts’ numbers to the left; ÷ ‘shifts’ numbers to the right), and is exponential (10^0 — ones, 10^1 — tens, 10^2 — hundreds, and so on).

Students need to be able to abstract this relationship, that is, to realise that the multiplicative relationships between any pair of places is the same.
3.3 OTHER MULTIPLICATIVE RELATIONSHIPS

Seeing large numbers as groupings of three digits is imperative for reading large numbers. Three-digit whole-number numeration (hundreds, tens, ones) encapsulates both patterns and is crucial to the development of understanding of the number system and the mathematics topics associated with this.

Understanding that each period is 1000 times larger/smaller (× 1000, ÷ 1000) than the adjacent period is essential for understanding our metric measurement system.

3.4 NUMBER PROCESSES

3.4.1 Overview

The number system structure is manifested in the numeration processes, namely:

- **number identification**: representing, reading and recording numbers when represented by material, word or symbol
- **place value**: knowing that the value of a digit is dependent on its position in a number (e.g. 33 has two ‘3s’ — the 3 on the left is worth more than the 3 on the right)
- **counting**: knowing the order of the number names and knowing that 9 is always followed by 0 when counting forwards by 1s/10s/100s (the ‘odometer’ principle)
- **seriating**: knowing 1/10/100… more than a given number (e.g. what is 100/10/1 more than 358?)
- **comparing**: determining which is larger or smaller in value (e.g. which is larger in value — 387 or 378?)
- **ordering**: arranging a set of numbers in order of value/size — requires comparing (e.g. arrange 482, 804, 842 in order from largest to smallest value)
- **renaming** (see Section 2.4): knowing that numbers can have many names without changing the value of the number (e.g. 265 can be thought of as 265 ones, 26 tens 5 ones, 2 hundreds 65 ones; 1 half can be thought of as 2 quarters, 3 sixths, 5 tenths, etc)
- **regrouping**: knowing that numbers can be rearranged in terms of place value without changing the value of the number (e.g. 265 can be thought of as 25 tens 15 ones; 7 eighths can be thought of as 1 half 3 eighths)
- **approximating**: ‘rounding’ to a given place (e.g. 372 to the nearest hundred would be 400; 2 fifths to the nearest whole would be 0)
- **benchmarking**: this is similar to approximating but, at times, it makes sense to benchmark to 0, $\frac{1}{2}$, or 1.
Prerequisite to these processes are the early number processes of sorting, classifying, patterning, rote and rational counting, digit recognition and early grouping.

The following concepts/processes are assessed in the CDAT.

<table>
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<tr>
<th>Concept</th>
<th>Numeration</th>
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<tr>
<td>— a whole thing (unit/one)</td>
<td>— seriation (1/10/100 … more/less)</td>
</tr>
<tr>
<td>— place value (position, names and order)</td>
<td>— counting (rote and rational)</td>
</tr>
<tr>
<td>— grouping, regrouping/re-naming, partitioning</td>
<td>— identifying numbers (place value)</td>
</tr>
<tr>
<td>— identifying numbers (place value)</td>
<td>— representing numbers (no zeros → external zero/s → internal zero/s)</td>
</tr>
<tr>
<td>— comparing and ordering</td>
<td>— approximating and estimating</td>
</tr>
<tr>
<td>— patterning</td>
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</tbody>
</table>

3.4.2 Renaming

Embedded in the renaming process are the processes of unitising and re-unitising. These are the mental processes required for naming and renaming numbers and fractions. (See Section 2.4.)
4. UNDERSTANDING FRACTIONS

4.1 SEQUENCE FOR TEACHING DECIMAL AND COMMON FRACTIONS

Because Australia is a metricated society, teaching emphasis should be on recording fractions in decimal, rather than common fraction form. However, in order to process decimal numbers (including measurements) with understanding, students need a solid understanding of the concept of fraction. In the early stages of concept development, teachers should use only informal recording (e.g. 3 fifths, 7 tenths) rather than the formal recording (\(\frac{3}{5}, \frac{7}{10}\)). The informal recording leads more directly to recording 7 tenths (for example) as 0.7 than does \(\frac{7}{10}\) (which has a ‘7’ and a ‘10’ to contend with).

4.2 FRACTION CONCEPT

A fraction is part of a whole (continuous or discrete)…

… and is generated by partitioning a whole into equal parts. The number of equal parts generates the fraction name.

This fraction process of partitioning into equal parts is similar in structure to the process of measuring where a length (for example) is partitioned into a number of equal units of measure such as millimetre, centimetre, metre.
Young children often confuse the part/whole notion of fractions with a part/part notion (ratio) and are likely to select the diagram A to represent \( \frac{1}{2} \).

### 4.3 MODELS TO DEVELOP THE FRACTION CONCEPT

There are three continuous models (area, linear, volume) used to develop the part/whole concept of fraction and one discrete model (set).

The **area model** should be used initially because it is easier for students to identify the whole (unitise) and the parts related to the whole. However, the main disadvantage of area models (except for circles) is that, once partitioned and then cut, students are likely to lose sight of the ‘whole’. Therefore, it is important that students always have a copy of the whole available to them in order to compare the part to the whole. However, when partitioning without cutting into equal parts is the main purpose of an activity, squares and rectangles are often easier to partition than circles. (See Pothier and Sawada, 1983, and Lamon, 1996, for the development of partitioning abilities.)

With the **linear model**, students tend to see the marks as discrete points on a line instead of as parts of a whole unit/segment and, again, the problem is related to unitising. However, it is worth persevering with this model as it can become a very powerful tool, particularly for identifying and representing mixed numbers. With the **volume model**, partitioning is generally unavailable for students and this notion of fraction is probably best developed through measurement activities.

With the **set model**, students find it very difficult to unitise a group of discrete objects (Behr et al., 1992; Nik Pa, 1989). This model should be delayed until the upper primary school.

Although the set, linear and volume models are not used in the initial development of the part/whole notion of fractions, they should not be avoided as full understanding of any notion requires an ability to abstract the salient features from a variety of materials (Dienes, 1969).

A fraction can also be the result of division: \( 3 \div 4 \rightarrow \frac{3}{4} \)

**Example:** 3 chocolate wafers shared amongst 4 people. How much chocolate per person?

\[
\begin{align*}
3 \text{ wholes} & \rightarrow 12 \text{ quarters} \left(\frac{12}{4}\right) \\
= & \quad 3 \text{ quarters} \left(\frac{3}{4}\right) \text{ each.}
\end{align*}
\]

Students also need to be able to interpret a fraction quantity multiplicatively and additively in terms of the unit fraction.

**Example:**

\[
\begin{align*}
\frac{3}{4} & = 3 \times \frac{1}{4} \\
\frac{3}{4} & = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}
\end{align*}
\]

**multiplicative** \hspace{1cm} **additive**
Thus, $\frac{3}{4}$ can be the result of four processes, namely:

- partitioning a whole into 4 equal parts and then considering 3 of the equal parts in relation to the whole
- dividing 3 by 4
- multiplying the unit fraction, $\frac{1}{4}$, by 3
- adding the unit fraction (i.e. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$).

4.4 FRACTION CLASSIFICATION

A fraction can be recorded formally as a decimal fraction, a common fraction, and as a per cent. Fractions that lie between 0 and 1 are referred to as proper fractions; fractions that are equal to 1 or more are referred to as either mixed numbers or improper fractions, depending on how they are recorded.

4.5 TEACHING FRACTIONS

4.5.1 Mathematical structure

The major process involved in generating fractions is that of partitioning a unit (whole). Prior to the introduction of fraction, the teaching focus has usually been on developing the notion of place value which is founded on the notion of grouping a number of units to form a new larger group. Both of these processes of partitioning and grouping are based on multiplication and research continues to show that young students have great difficulty in understanding the multiplication notion. Therefore it is not surprising that students have difficulty in understanding place value, fractions (including common, decimal and per cent forms), ratio, proportion, probability, and area.

Prior experiences with whole numbers may unwittingly have led students to connect addition, rather than multiplication, with grouping. These experiences would have included activities related to trading 10 ones for 1 ten and 10 tens for 1 hundred. To reinforce this notion, students are involved in trading games such as rolling a die and counting out that number of ones to place on a place value chart. Whilst these trading games are a fun way to reinforce the grouping notion underlying place value, they can also lead students to connect addition, rather than multiplication, with grouping. For example, the teacher may ask a student: How many ones do you have? (6) How many more do you need to make a ten? This latter question requires the student to think: $6 + ? = 10$. This is one of the major causes of students’ linking decimal fractions with subtraction, rather than division.
4.5.2 Connecting fraction representation, name, and symbol

When helping students focus on making these connections, teaching should focus on unit fractions (i.e. 1 part of the whole such as 1 half, 1 third, 1 tenth, etc.) before non-unit fractions such as 2 thirds, 4 fifths, 8 tenths, etc.

Payne and Rathmell’s model (see Section 2.2.2) is always a good basis for helping students develop a sound understanding of any new concept. For example, a real-world area model such as a chocolate bar could be used initially, then paper area models of chocolate bars before moving to the more abstract pictures of area models.

Once the concept of fractions has been established, students need to connect their understanding of whole-number processes (counting, comparing, ordering, approximating, estimating) to the corresponding fraction processes (see Section 4.5.6). This should be done with informal recording and then, after decimal numbers have been established (Years 4 and 5), all processes need to be repeated with formal recording (Year 6 or 7).

4.5.3 Connecting denominator with fraction name

When students are working with formal fraction recording, they need to establish a connection between the denominator number and ordinal number, not cardinal number. See the following diagram.

4.5.4 Teaching sequence for developing the concept of a fraction

To construct a sound understanding of fractions, the following three main processes need to be developed:

- identifying the whole (more difficult when considering linear or set models)
- ensuring that the whole has been partitioned/divided into equal parts
- ensuring that the fraction name is related to the total number of equal parts.
Fractions cannot be named until we know how many equal parts the whole has been divided into:

Is this a fraction? Yes, but it can’t be named until the whole has been partitioned into equal parts.

The size of a fraction is determined by two things:

- how many equal parts altogether
- the number of equal parts under consideration.

Fractions can be recorded as:

- common fractions \( \frac{1}{4} \)
- decimal fractions 0.7
- per cents 25%

4.5.5 Processes required for naming and then recording a fraction

IDENTIFYING the whole

PARTITIONING the whole into equal parts

NAMING/UNITISING

Determining – how many equal parts to be considered

Associating – the number of equal parts under consideration with the total number of equal parts that comprise the whole

Language – 1 half; 5 tenths

RECORDING

Informal – 5 tenths, 1 half; Formal, - 0.5, \( \frac{1}{2} \)

4.5.6 Fraction processes

Fraction processes are the same as those developed with whole numbers. These processes are: representing; identifying/naming numbers (common fractions, decimal fractions, per cents, etc.); counting, sequencing; re-unitising; regrouping; comparing, ordering; approximating, estimating; and calculating. As for whole numbers, the processes are first established through connecting concrete materials, language and symbols.

Being able to rote count (saying the number names in order) is fundamental to learning mathematics. However, counting should not be limited to counting whole numbers only. For example, fractions can (and should) be counted:

- 1 fifth, 2 fifths, 3 fifths, 4 fifths, 5 fifths, 6 fifths … or
- 1 fifth, 2 fifths, 3 fifths, 4 fifths, one, one and 1 fifth …
- 8 tenths, 9 tenths, 10 tenths, 11 tenths, 12 tenths …
4.5.7 Equivalent fractions

Equivalent fractions occur when a given fraction has been re-partitioned. This gives a fraction that has the same value but a different number of equal parts (and therefore a different name).

For example, start with a whole (see A in the diagram); partition (without cutting) the whole into two equal parts (B) and then repartition (without cutting) the whole (in the opposite direction) into three equal parts (C). The result is six equal parts.

Equivalence can be shown very well by having a paper template of a whole partitioned into halves and an overhead transparency (OHT) template of the same whole partitioned into thirds and placing the OHT on top of the paper half to show sixths.

If students are not given representational material to manipulate, they often take an additive view of the relationship between equivalent fractions. For example, Baturo (1998) found that, when asked to select the OHT fraction that would change halves to sixths, more than 80% of Year 5 students selected the OHT template showing quarters (C) (because 2 + 4 = 6) rather than the OHT template showing thirds (B) (2 × 3 = 6).

4.6 DECIMAL FRACTIONS

Decimal fractions share properties of whole number and fraction. Hence, the difficulties students have stem from a lack of understanding of what can be transferred from whole numbers and fractions and what needs to be adjusted. For example, the diagram below (repeated from Section 3.2) shows that adjacent places, irrespective of whether they are wholes or parts, are related multiplicatively by 10 (as expected from the base-10 nature of our decimal number system).

Therefore, 10 thousandths = 1 hundredth, 10 hundredths = 1 tenth, 10 tenths = 1 one (and so on). This means that each place immediately to the left of another place is 10 times larger in value and, conversely, each place immediately to the right of another place is 10 times smaller in value.

However, individual places can also be thought of as being either larger in value than 1 (e.g. ten, hundred, thousand, …) or smaller in value than 1 (e.g. tenth, hundredth, thousandth, …).
4.6.1 Role of the decimal point and the ‘symmetry’ model of place names

Many students know that the ‘dot’ in 6.25 is called a decimal, a point, or a decimal point; but many do not know why it is there – to separate the whole number from the fractional part. They often see the decimal point as being the point of symmetry when, in fact, it is the ones place that is central (numbers are either larger or smaller in value than 1).

Baturo (1998) found that, when asked to write the number that has 6 tens and 6 tenths, only 25% of 45 students were correct (60.6); the remaining 75% wrote ‘6.6’, explaining that they thought of the decimal point as the point of symmetry for the place names (see the following diagram). Those who were correct explained that they used a variety of strategies:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - Ones - Ignore the DP</td>
<td>H T O t h</td>
</tr>
<tr>
<td>B - Ones and DP</td>
<td>H T O t h</td>
</tr>
<tr>
<td>C - DP - Ignore the ones</td>
<td>H T O t h</td>
</tr>
<tr>
<td>D - DP - Insert oneth</td>
<td>H T O t h</td>
</tr>
</tbody>
</table>

These strategies clearly show that students try to make sense of mathematical concepts and processes and they were intuitively looking for the symmetry in the place names. The model below enabled students to understand this symmetry.

4.6.2 Equivalence in decimal fractions

It is important that students learn to think flexibly about place value. For example, sometimes a problem is solved more easily if one of the numbers can be renamed as the other (e.g. 1.4 m + 2.36 m) or in ordering numbers, for example:

Write the following numbers in order from smallest to largest in value: 7.16, 1.76, 1.67

These numbers can be thought of as 716 hundredths, 176 hundredths, 167 hundredths.

Similarly, when continuing a counting sequence such as 2.37, 2.38, 2.39, ___ ___ , students need to know that it is legitimate to think of these numbers as …

2 and 37 hundredths, 2 and 38 hundredths, 2 and 39 hundredths,
2 and 40 hundredths, 2 and 41 hundredths

... and write 2.40 (more legitimate in this example than 2.4) and 2.41

Students need to have concrete experiences where they can transform tenths into hundredths. (See Section 4.5.7 for activities involving paper and OHT representations of fractions.)
5. UNDERSTANDING PROBABILITY

Probability is the branch of mathematics that describes randomness, an attribute that should not be associated with ‘haphazardness’ but rather with a kind of order that is different from the deterministic one that is normally attributed to mathematics. (Steen, 1990).

Chance is a subjective informal estimate of an event whereas probability is the objective formal measurement of an event (Fischbein, 1975; 1997). In his research, Fischbein noticed that the subjects across all year levels tended to estimate the probability of an event by using ratios (number of favourable outcomes as opposed to the number of unfavourable outcomes) rather than fractions (number of favourable outcomes out of the total number of outcomes in the entire sample space).

The implication for teachers is that, until students have an understanding of the part/whole notion of fractions (irrespective of whether that fraction is recorded as a common fraction, decimal fraction or per cent), students will be incapable of true probabilistic reasoning. The example below indicates the types of probabilistic activities that students are exposed to. Note that only the second activity will elicit true probabilistic thinking because it requires students to compare fractions with unlike denominators.

Do you have the same chance of getting (without looking) a red marble from each container?

The student may give the correct answer (‘yes’) for the wrong reason, namely, s/he sees 3 red marbles in each container and makes the judgment on comparison/ratio thinking (i.e. 3 red in each container) without considering the total number of marbles in the sample space.

The student who answers ‘yes’ is definitely using the inappropriate comparison/ratio thinking (i.e. sees 3 red in each container) but does not consider this in relation to the total number of marbles in the sample space. This example requires fraction reasoning (i.e. parts in relation to the whole). Until students are able to compare unlike fractions, they will be unable to employ true probabilistic reasoning.

5.1 PROBABILITY TYPES

There are two main branches of probabilistic reasoning:

1. Theoretical — derived from making assumptions of equal likelihood within and between the sample spaces* (e.g. determining the likelihood of a colour being spun on a spinner)
2. Frequentist — calculated from observed frequencies of different outcomes in repeated trials.
A sample space refers to the materials used in a task. Single sample spaces (e.g. 1 bag of marbles, 1 set of tickets, 1 die, 1 coin, and so on) should be used before multiple sample spaces (e.g. 2 or more spinners, bags of marbles, coins, dice, etc.). A single sample space is much easier for students to determine and compare the probability of an event’s occurring as comparisons re likelihood of an event occurring involve fractions with the same name. Multiple sample spaces may have different numbers of outcomes and therefore comparing across sample spaces involves comparing unlike fractions (See Section 5.2).

Researchers and practitioners are aware that probabilistic reasoning, particularly in young students, is often generated from personal belief and perceptions which often result from overusing strategies such as ‘representativeness’ and ‘availability’. This type of probabilistic reasoning has been classified as subjective or intuitive probability.

5.2 ELEMENTARY PROBABILITY NOTIONS AND LANGUAGE

- The notions of an event and a chance in a single sample space — some events are impossible (have no chance of occurring), some events are certain (have every chance of occurring), and some events are possible (have some chance of occurring). For example, on the spinner below, it would be impossible to spin purple, possible to spin red, but cannot be certain of spinning red.

- The probability of an event’s occurring in one sample space (e.g. the probability of spinning red on the spinner below — the sample space in this experiment).

- The comparison of events in the same sample space (e.g. using the spinner on the left as the sample space, you could ask if yellow is just as likely to be spun as green or whether one colour has more chance of being spun than any other).

- The comparison of two or more sample spaces which have the same number of outcomes — \( \frac{a}{x}, \frac{b}{y} \) (e.g. two spinners each having the same number of equal sections, x, but different numbers of equal parts being considered, a and b). See the example below.

The probability of drawing red from each container from left to right is:

\[
\begin{align*}
\frac{2}{6} & \quad \frac{3}{6} & \quad \frac{2}{6} \\
\frac{3}{6} & \quad \frac{3}{7} & \quad \frac{3}{9}
\end{align*}
\]
5.3 TYPES OF ACTIVITIES/EXPERIMENTS

Probability activities are often referred to as experiments. Some experiments are more difficult than others and are referred to as one-stage experiments or two-stage experiments as the following will clarify.

One-stage: means one event only (e.g. finding the probability of spinning blue in one sample space)
- with replacement; that is, the sample space remains the same; spinners, dice, coins (area models) always maintain the same sample space.
- without replacement; that is, after one trial, the outcome is not replaced so the sample space becomes smaller; for example, when you flip a card from a deck of cards, the probability of it being a particular card is \( \frac{1}{52} \); if you do not replace the card before flipping again, the sample space has been reduced to 51. Experiments in which the outcome of each trial is not replaced in the sample space give increasingly better chances of the particular event's occurring for each successive trial. Experiments without replacement are based on set models of fractions (e.g. cards, tickets, marbles). This is not taught until upper middle school.

Two-stage: considering the probability of two events in more than one sample space; the sample space may be the same (e.g. two coins) or different (e.g. a coin and a die). Because the calculation of these probabilities requires an understanding of the multiplication of fractions, they are not taught until upper middle school.

5.4 BASIC PROBABILITY PROCESSES

Probability has many processes which are different from number processes. (Refer to Section 3.4 for number processes.) These are:
- Classifying an event as impossible, possible or certain
- Listing all possible outcomes of a sample space
- Stating which outcome is most likely in a single trial
- Choosing the situation in which an event is most likely/least likely to occur
- Listing all the possible outcomes in consecutive trials within one sample space:
  - (i) with replacement
  - (ii) without replacement
- Correctly assigning a numerical probability to an event (i.e. a fraction)
- Estimating the probability of an event using simulation methods (experimental probability).

5.5 MATHEMATICAL STRUCTURE

Probability is an application of fractions (i.e. part/whole). Therefore the same models (area, set, linear, volume) can be used to represent the sample space. The same sequence of models should be kept in mind (area before set). (See Sections 4.2 and 4.3)

5.6 TEACHING MODEL

In establishing the representation, language and symbolism of probability, Payne and Rathmell’s (1977) model should be followed (see Section 2.2).
5.6.1 Representations

A sample space can have its components related to the area model or the set model. As stated in Section 4, students find area models easier to interpret than set models; therefore, spinners should be used before discrete materials such as marbles in developing probabilistic reasoning. However, spinners can be partitioned into equal (units) or non-equal (gross) segments.

Moreover, like parts can be arranged together (contiguous) or split (noncontiguous). Unit measurement is easier for children to interpret than gross measurement and contiguous is easier to interpret than noncontiguous.

5.6.2 Language

Because probability is connected to the notion of fraction, it is recommended that students be encouraged to link language to the fraction part–whole concept through a number line showing 0–1. (See the number line in Section 2.2.1, which shows a continuum of formal and informal language ranging in meaning from impossible to certain.)

Young children find the “likelihood” language quite difficult as there is a “vagueness” to such terms as “highly likely”, “unlikely”, etc. However, there must always be an assumption that “likely” indicates there is some chance, no matter how small, of a particular event occurring within a given sample space. Therefore, if a given sample space has 3 green, 2 red and 4 blue marbles, then blue is more likely to be selected than either green or red; green is more likely to be selected than red. It does not make sense to say that green is more likely to be selected than purple when there is no purple in the sample space.

5.6.3 Symbol

Because probability is based on fraction understanding, it can be recorded as a common fraction (informally — 3 quarters; formally — \( \frac{1}{3} \)), a decimal fraction (0.75) or as a per cent (75%).
However, only the formal common fraction form shows the composition of the sample space. For example, with \( \frac{3}{4} \), there can be no doubt that the sample space had 4 equal parts and 3 of these were favourable. For example, a spinner could have been divided into 4 equal parts, 3 of which were red and 1 blue so that the probability of spinning red would be \( \frac{3}{4} \). It is recommended that teachers do not reduce common fractions to lowest terms (e.g. changing \( \frac{6}{8} \) to \( \frac{3}{4} \)) in the early stages of probability development because \( \frac{1}{2} \) does not represent the given sample space accurately.

5.6.4 Relationship of probability and fractions: Implications for teaching

Because probability and fractions share similar concepts and because probability answers are recorded as fractions, teaching probability requires the teacher to help students make the connection between fractions and probability. The following example shows how these two mathematical domains can be connected pedagogically as well as mathematically.

Example:
Aim: To lead the child to discover the probability of an event occurring (informal language and recording) using an area model and a set model.

Questions that can be asked:

Identify the whole (i.e. the sample space)
- What colours could you spin on this spinner? What colours could you get from this bag of marbles?

Partition the whole into equal parts or examine the parts for equality
- Has the spinner been divided into equal parts? Would the pointer be just as likely to stop on one part as on any other part? (OR: Would you have the same chance of stopping on any of the parts?)
- Are all the marbles equal, that is, the same size and shape? Would you be just as likely to get one marble as any other marble? (OR: Would you have the same chance of getting any of the marbles?)

Name the parts (establish the total number of chances, that is, the denominator)
- How many equal parts does this spinner have? So how many chances do you have altogether of spinning a colour?
- How many marbles does this bag have? So how many chances do you have altogether of getting a colour?

Determine the parts to be considered (the outcome preferred, that is, the numerator)
- How many blue parts are there? So how many chances do you have of spinning blue?
- How many blue marbles are there? So how many chances do you have of getting a blue marble?
Associate the two parts with the fraction name (the probability)

- What chance do you have of spinning blue? (2 chances out of 5 equal chances)
- What chance do you have of getting a blue marble? (2 chances out of 5 equal chances)

Record the probability

- 2 fifths (informal); $\frac{2}{5}$ (formal)

5.7 TEACHING SEQUENCE

The following provides a sequence for teaching probability.

- Follow the sequence of skills described in Section 5.6.4
  - use area and set models
  - look at fair and unfair situations
  - vary the position of the outcomes (contiguous/noncontiguous)
  - vary the number of outcomes
- One sample space
  - list the outcomes
  - discuss the likelihood of an event occurring (certain, possible, impossible)
  - compare two events to determine which is more/less likely to occur
- Two similar sample spaces
  - as for one sample space but:
    (a) with the same number of outcomes in each sample space (e.g. 2 spinners)
    (b) with different numbers of outcomes in each sample space (e.g. 2 bags of marbles with a different total number in each bag)
- No replacement
- Consecutive trials
  - from one sample space (e.g. 1 die)
  - from two similar sample spaces (e.g. 2 coins)
  - from two different sample spaces (e.g. 1 die, 1 coin)

Throughout the development of probabilistic notions and processes, it is recommended that teachers play games and undertake activities that focus on higher-level thinking. Go beyond the activities and discuss the situations and reflect on the results.

For example, using a spinner with 6 equal parts (2 red, 2 blue, 2 green sectors), ask: What is the probability of spinning blue? If you were to colour one of the red parts blue, would you have the same probability of spinning blue? red? green? This activity requires students to develop flexible thinking in a probabilistic context.
6. CONCLUSION

These cognitive diagnostic assessment tasks (CDAT) provide an invaluable insight into how students have structured their prior learning and how a teacher can build on this knowledge when introducing new concepts and processes or extending previously-learnt concepts and processes. Knowing how mathematics domains are connected is beneficial for both teachers and students in that ‘old ideas’ (those learned in one domain) may be transferred to other domains. As a teacher, the most useful questions to ask students to help them make connections, are: How are these the same? How are they different? How can you tell?

While not all of the mathematics domains have been included in this set of CDAT, they do have application outside of the included domains of whole number, fraction (common and decimal), and probability. For example, the relationship between metric measurements and place value means that the place value tasks can provide insights into the understandings that students need to develop in order to process metric measurements with understanding (particularly that 800 mm can be recorded as 0.8 m, 0.80 m, or 0.800 m). As well, numbers within the place value and decimal-fraction tests can generally be altered to metric measurements.

While the need for students to have real-world mathematical knowledge is recognised, these CDAT focus exclusively on the concepts, processes and connections that underpin the mathematics knowledge required to progress into higher mathematics courses in secondary school. For this reason, they have been dissociated from real-world contexts.

These CDAT have been developed over many years as a result of various research projects. The following readings are included for your interest.
RELATED READINGS


7. REFERENCES


CDA TASKS

NUMBER 1A

Name: ____________________________ Year Level: ________ Date: __________

1. a. Say: Start at 1 and show me how far you can count.
   b. In the box, write the number the student can get to without making an error.

2. Give the student a pencil and paper.
   a. Say: Maree has three books. Write the number that tells how many books.
   b. Say: Jack has seven books. Write the number that tells how many books.
   c. Say: Sue has no books. Write the number that tells how many books.

3. Place the set of number cards in front of the student.
   a. Say: Lizzie has this many lollies. [Point to the “2”.] Tell me how many lollies she has.
   b. Say: Frank has this many lollies. [Point to the “6”.] Tell me how many lollies he has.
   c. Say: Brian has this many lollies. [Point to the “0”.] Tell me how many lollies he has.

4. Place the set of number-name cards in front of the student.
   a. Say: Maree has this number of books. [Point to the “four”.
      Write the number that tells how many books.
   b. Say: Jack has this number of books. [Point to the “eight”.
      Write the number that tells how many books.
   c. Say: Sue has this number of books. [Point to the “zero”.
      Write the number that tells how many books.

5. Use the number cards and number-name cards.
   a. Say: Lizzie now has this many lollies. [Point to number “5”.
      Point to the number-name card that matches this number.
   b. Say: Frank now has this many lollies. [Point to number “9”.
      Point to the number-name card that matches this number.
   c. Say: Brian now has this many lollies. [Point to number “0”.
      Point to the number-name card that matches this number.

6. a. Point to the “4” number card and say: Write what number comes next after.
   b. Point to the “6” number card and say: Write what number comes next after.
   c. Point to the “9” number card and say: Write what number comes next after.
7. a. Point to the “8” number card and say: Write what number comes just before.
   b. Point to the “1” number card and say: Write what number comes just before.
   c. Point to the “10” number card and say: Write what number comes just before.

8. Put out two number cards as below. Say: Point to the larger number in each pair:
   a.  
      | 7 | 4 |
      |    |
   b.  
      | 3 | 5 |
      |    |
   c.  
      | 8 | 6 |
      |    |

9. Put out the number cards as below. Say: Put these in order from smallest to largest:
   
   | 6 | 2 | 0 | 9 | 7 | 4 |
   
10. a. Say: Count on one more from this number. [Point to the “6” number card.]
    Write the number that you finish on.
   b. Say: Count on two more from this number. [Point to the “4” number card.]
    Write the number that you finish on.
   c. Say: Count on three more from this number. [Point to the “7” number card.]
    Write the number that you finish on.

11. a. Say: Count back one from this number. [Point to the “5” number card.]
     Write the number that you finish on.
   b. Say: Count back two from this number. [Point to the “10” number card.]
     Write the number that you finish on.
   c. Say: Count back three from this number. [Point to the “7” number card.]
     Write the number that you finish on.
NUMBER 1B

Name: ________________________________ Year Level: _________ Date: __________

1. a. Put out 10 counters in a row (all the same colour).

Say: How many counters are there altogether? Touch the counters as you count.

Hint: Write the numbers inside the counters to show how they counted e.g.

8 9 10 1 2 3 4 5 6 7

b. If the student is correct, take 7 counters and arrange them like this:

Say: How many counters are there altogether? Count aloud and touch as you count.

2. a. If the student is correct again, take the same counters and arrange them like this:

Say: How many counters are there altogether? Touch as you count aloud.

b. If the student is correct again, take the same counters and arrange them like this:

Point to the counter shown. Say: Count them from this one.

c. If the student is correct, arrange five counters in a row and point to the middle counter.

Say: Count this set but finish here.
3. Put out 13 counters as shown below (all the same colour).

![Counters](image)

a. Say: Count me five counters.
b. Say: Count me eight counters

4. a. Put out 4 counters. Say: Jill has this many books: Tell me how many books she has.
b. Put out 7 counters. Say: Bob has this many books: Tell me how many books he has.
c. Say: Bill has eight crayons. Use counters to show how many crayons he has.
d. Say: Jan has zero crayons. Use counters to show how many crayons she has.

5. a. Put out 3 counters. Say: Gina has this many stickers.
   Write the number that tells how many stickers. _________
b. Put out 7 counters. Say: Gary has this many stickers.
   Write the number that tells how many stickers. _________
b. Say: Tania has this number of ribbons. [Write the number ‘5’.]
   Say: Use counters to show how many ribbons she has.
c. Say: Tom has this number of ribbons. [Write the number ‘9’.]
   Say: Use counters to show how many ribbons he has.

6. Put the set of number-name cards in front of the student.
a. Put out 4 counters. Say: Jill has this many toy dinosaurs:
   Show me the number-name card for this number of dinosaurs.
b. Put out 10 counters. Say: John has this many toy dinosaurs:
   Show me the number-name card for this number of dinosaurs.
c. Say: Jan has this many chocolates. [Point to the ‘three’ number-name card.]
   Use counters to show how many chocolates she has.
d. Say: John has this many chocolates. [Point to the ‘six’ number-name card.]
   Use counters to show how many chocolates he has.

7. Put counters on two plates. Say: Point to the plate with the larger number of counters.
a. 
b.
8. a. Put 6 counters in your hand; show the student. Close your hand so the counters aren’t visible.
   Say: How many counters in my hand?
   Drop one more counter into your hand.
   Say: How many now?

   b. Put 4 counters in your hand; show the student. Close your hand.
   Say: How many counters in my hand?
   Drop one more counter into your hand.
   Say: How many now?
   Drop another counter into your hand.
   Say: How many now?

   c. Put 7 counters in your hand; show the student. Close your hand.
   Say: How many counters in my hand?
   Drop one more counter into your hand.
   Say: How many now?
   Drop another counter into your hand.
   Say: How many now?
   Drop another counter into your hand.
   Say: How many now?

9. a. Put 5 counters in your hand; show the student. Close your hand.
   Say: How many counters in my hand?
   Drop one counter out of your hand.
   Say: How many now?

   b. Put 10 counters in your hand; show the student. Close your hand.
   Say: How many counters in my hand?
   Drop one counter from your hand.
   Say: How many now?
   Drop another counter from your hand.
   Say: How many now?

   c. Put 7 counters in your hand; show the student. Close your hand.
   Say: How many counters in my hand?
   Drop one counter from your hand.
   Say: How many now?
   Drop another counter from your hand.
   Say: How many now?
   Drop another counter from your hand.
   Say: How many now?
NUMBER 2A

1. Write the number for each number name:
   a. sixty-seven
   b. fifty
   c. thirteen

2. Write the number name for each number:
   a. ___________________________  83
   b. ___________________________  30
   c. ___________________________  17

3. Write the number that has:
   a. 3 tens 8 ones
   b. 7 ones 6 tens
   c. 0 ones 8 tens
   d. 0 tens 4 ones

4. a. Write a zero in 54 without changing its value. __________
    b. Write a zero in 54 so that its value changes. __________

5. Write the missing numbers:
   a. 66, 67, 68, _____, _____, _____
   b. 95, 96, 97, 98, 99, _____, _____, _____
   c. 57, _____, 59, _____, _____, 62, 63
   d. 28, 38, 48, 58, _____, _____, _____
   e. 74, 64, _____, 44, _____, _____, _____

6. a. What number is 10 more than 42? __________
    b. What number is 10 less than 26? __________
    c. What number is 1 more than 58? __________
    d. What number is 1 less than 70? __________

7. In each box, circle the number that has the larger value:
   a. 67  82
   b. 47  45
   c. 17  70
   d. 59  60
   e. 31  13
8. Put in order from smallest to largest value:
   40, 14, 41, 39, 47

9. Write the missing numbers:
   a. 54 = 4 tens ___ ones
   b. 38 = ___ tens 18 ones
   c. ___ = 6 tens 17 ones
   d. ___ = 1 ten 10 ones
   e. 63 = 3 tens ___ ones
   f. ___ = 0 tens 37 ones

10. Write the missing place value names:
    a. 3 ones × 10 = 3 _________
    b. 4 tens ÷ 10 = 4 _________

11. Write the missing numbers:
    a. 6 ones × _____ = 6 tens
    b. 7 tens ÷ _____ = 7 ones

12. Round to the nearest 10:
    a. 37 _______
    b. 22 _______
    c. 35 _______

13. Circle the numbers that could be rounded to 50:
    a. 24  b. 45  c. 55  d. 48  e. 59
NUMBER 2B

1. Complete the following:
   a. How many groups of four and how many ones can you make from these counters?

      Write here: _____ fours _____ ones

   b. How many groups of seven and how many ones can you make from these counters?

      Write here: _____ sevens _____ ones

   c. How many sixes and how many ones:

      | Sixes | Ones |
      |-----------|------|
      | ● ● ● ● ● | ● ● |
      | ● ● ● ●   | ● ● |
      | ● ● ●     | ● ● |
      | ● ●       | ● ● |

      Write here: _____ sixes _____ ones

   d. Draw four sixes and two ones on the chart:

      | Sixes | Ones |
      |-----------|------|
      |           | ● ● |
      |           | ● ● |
      |           | ● ● |
      |           | ● ● |
      |           | ● ● |
2. Write the **number name** to show how many sticks:
   a. 
   b. 
   c. 

3. Write the **number** to show how many:
   a. 
   b. 
   c. 

4. Circle the sticks you need to make the given number:
   a. 65 
   b. 12 
   c. 70 

5. Circle the sticks you need to make the given number name:
   a. forty-one 
   b. eighteen 
   c. sixty
6. Write the number name for each number shown:

<table>
<thead>
<tr>
<th></th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>b.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>c.</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

7. Fill in the tens-ones chart to show the number name:

<table>
<thead>
<tr>
<th></th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. fifty-eight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. fifteen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. fifty</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. For the number: 

a. Write the number that is 10 more. 

b. Write the number that is 10 less 

c. Write the number that is 1 more. 

d. Write the number that is 1 less. 

9. Complete the following:

<table>
<thead>
<tr>
<th></th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Write the number shown by the material.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| b. Circle the bundles of ten you would need to put with all the ones to show the number: |      | 57 =  
| c. Circle the ones you would need to put with all the bundles of ten to show the number: |      | 63 =  

```cda_tasks
```
10. a. A calculator will shift 3 ones to 3 tens, if you:

\[ +10; \quad -10; \quad \times 10; \quad \div 10 \]

(Circle which one.)

\[
\begin{array}{|c|c|}
\hline
\text{Tens} & \text{Ones} \\
\hline
3 & 0 \\
\hline
\end{array}
\]

b. A calculator will shift 5 tens to 5 ones, if you:

\[ +10; \quad -10; \quad \times 10; \quad \div 10 \]

(Circle which one.)

\[
\begin{array}{|c|c|}
\hline
\text{Tens} & \text{Ones} \\
\hline
5 & 0 \\
\hline
\end{array}
\]

11. a. Write the number when 7 ones is \( \times 10 \).

\[
\begin{array}{|c|c|}
\hline
\text{Tens} & \text{Ones} \\
\hline
0 & 7 \\
\hline
\end{array}
\]

b. Write the number when 40 is \( \div 10 \).

\[
\begin{array}{|c|c|}
\hline
\text{Tens} & \text{Ones} \\
\hline
4 & 0 \\
\hline
\end{array}
\]

12. Use the number line to help you answer the following:

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
16 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
63 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
85 \\
\hline
\end{array}
\]

a. Round 16 to the nearest 10.

b. Round 63 to the nearest 10.

c. Round 85 to the nearest 10.

13. Mark the following on the number line:

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\
\hline
\end{array}
\]

a. 17  

b. 71  

c. 38  

d. 89  

14. What number could be at:

\[
\begin{array}{|c|c|c|c|}
\hline
0 & 10 & 20 & 30 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
A & B & C \\
\hline
\end{array}
\]

a. A:  

b. B:  

c. C:  

NUMBER 3A

Name: ____________________________ Year Level: _________ Date: __________

1. Write the number for each number name:
   a. three hundred and forty-nine ______________________
   b. five hundred and forty ______________________
   c. two hundred and twelve ______________________
   d. eight hundred and six ______________________

2. Write in words:
   a. 268 ______________________
   b. 303 ______________________
   c. 814 ______________________

3. Write the number that has:
   a. 3 hundreds 9 tens 7 ones ______________________
   b. 8 tens 4 ones 4 hundreds ______________________
   c. 6 ones 5 hundreds ______________________

4. a. Write a zero in 368 without changing its value. ______________________
    b. Write a zero in 368 so that its value changes. ______________________

5. What number is:
   a. 10 more than 728? ______________________
   b. 100 less than 685? ______________________
   c. 100 more than 310? ______________________
   d. 10 less than 500? ______________________
   e. 1 less than 140? ______________________

6. Complete the counting sequences:
   a. 286, 287, 288, ______, ______, ______
   b. 596, 696, 796, ______, ______, ______
   c. 433, 432, 431, ______, ______, ______
   d. 830, 820, 810, ______, ______, ______

7. In each box, circle the number with the larger value:
   a. 387, 404
   b. 556, 552
   c. 674, 92
   d. 804, 840
8. Put these numbers in order from largest to smallest in value: 652, 625, 650, 605, 615

   ___________   ___________   ___________   ___________   ___________

9. Circle the number in which the 7 is worth the most:
   726       172       267

10. Write the missing numbers:
    a. 928 = 9 hundreds ___ tens ___ ones
    b. 684 = 6 hundreds 7 tens ___ ones
    c. ___ = 4 hundreds 19 tens 6 ones
    d. 547 = ___ hundreds 14 tens 7 ones
    e. 723 = 6 hundreds ___ tens 13 ones
    f. ____ = 3 hundreds 26 tens 31 ones

11. Write the missing place value names:
    a. 8 ones × 10 = 8 _________________
    b. 4 ones × 100 = 4 _________________
    c. 62 ones × 10 = 62 _________________
    d. 15 tens × 10 = 15 _________________
    e. 9 hundreds ÷ 10 = 9 _________________

12. Write the missing operations (+, −, × or ÷) and numbers:
    a. 7 ones = 7 tens
    b. 8 tens = 8 hundreds
    c. 9 tens = 9 ones
    d. 4 hundreds = 4 ones
    e. 5 ones = 5 hundreds
    f. 7 hundreds = 7 tens

13. Round to the nearest 10:
    a. 686
    b. 725
    c. 303

14. Round to the nearest 100:
    a. 456
    b. 238
    c. 850

15. Circle the numbers that could be rounded to 300:
    a. 713   b. 267   c. 325   d. 380   e. 250
1. Write the **number name** for the number represented by the blocks:

a. 

b. 

c. 

d. 

e. 

2. Write the **numbers** represented by the blocks:

a. 

b. 

c. 

d. 

3. Circle the blocks needed to represent the number names:
   a. two hundred and sixty-one
   b. nine hundred and five

4. Circle the blocks needed to represent the following numbers:
   a. 704
   b. 740
   c. 714

5. Write the number name for each number:
   a. Hundreds  Tens  Ones
      4        1      5
   b. Hundreds  Tens  Ones
      3        0      6
   c. Hundreds  Tens  Ones
      8        3      0

6. Write the matching number on the place value chart:
   a. two hundred and forty
   b. five hundred and eleven
   c. six hundred and nine
7. For this number: 

a. Write the number that is 10 more. 

b. Write the number that is 100 more. 

c. Write the number that is 1 less. 

d. Write the number that is 10 less. 

8. Write the following numbers on the number line in order from smallest to largest in value: 714, 782, 728, 741.

9. a. Write the number that is shown by the blocks:

b. Some blocks are missing. Circle the hundreds needed:

\[445 = ???\]

c. Circle the tens needed:

\[363 = ???\]

10. Complete the following:

a. Is this \( \times 10, \times 100, \div 10 \) or \( \div 100 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{4} & \text{7} & \\
\end{array}
\]

b. Is this \( \times 10, \times 100, \div 10 \) or \( \div 100 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{6} & \text{0} & \text{0} \\
\end{array}
\]

c. Is this \( \times 10, \times 100, \div 10 \) or \( \div 100 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{6} & \text{0} & \text{0} \\
\end{array}
\]

d. What happens when \( \times 10 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{7} & \text{4} & \\
\end{array}
\]

ey. What happens when \( \div 10 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{5} & \text{0} & \\
\end{array}
\]
f. What happens when \( \times 100 \)?

\[
\begin{array}{c|c|c}
\text{Hundreds} & \text{Tens} & \text{Ones} \\
\hline
\text{9} & & \\
\end{array}
\]
11. Look at the numbers on the number line

![Number Line Diagram]

a. Round 92 to the nearest 100  
   _________
b. Round 441 to the nearest 100  
   _________
c. Round 750 to the nearest 100  
   _________

12. What number could be at:

![Number Line Diagram]

a. A:  
   _________  
b. B:  
   _________  
c. C:  
   _________
NUMBER 4A

Name: __________________________ Year Level: __________ Date: __________

1. Write the number for each number name:
   a. three thousand, seven hundred and forty-nine
   b. five thousand, two hundred and forty
   c. two thousand and ninety-three
   d. eight thousand, three hundred and six
   e. four thousand and fourteen

2. Write in words:
   a. 5268
   b. 7007
   c. 6016

3. Write the number that has:
   a. 6 thousands 3 hundreds 9 tens 7 ones
   b. 8 tens 4 ones 2 thousands 4 hundreds

4. a. Write a zero in 542 without changing its value
   b. Write a zero in 542 so that its value changes

5. What number is:
   a. 1000 more than 7282?
   b. 100 less than 6285?
   c. 100 more than 310?
   d. 10 less than 5000?

6. Complete the counting sequences:
   a. 4286, 4287, 4288, __________, __________, __________
   b. 2596, 2696, 2796, __________, __________, __________
   c. 5344, 5334, 5324, __________, __________, __________

7. In each box, circle the number with the larger value:
   a. 2387, 2404 b. 1556, 1552 c. 8674, 892 d. 5804, 5840

8. Put in order from smallest to largest in value:
   a. 3652, 3625, 3650, 3605

9. Circle the number in which the 7 is worth the most:
   1726 3172 7104 9267
10. Write the missing numbers:
   a. 4928 = 4 thousands 9 hundreds ___ tens ___ ones
   b. 7684 = 7 thousands 6 hundreds 7 tens ___ ones
   c. 1723 = ___ hundreds 2 tens 3 ones
   d. 8547 = 8 thousands 4 hundreds ___ tens 17 ones

11. Write the missing place value names:
   a. 8 ones × 1000 = 8 _______________
   b. 6 thousands ÷ 1000 = 6 _______________
   c. 4 tens × 100 = 4 _______________
   d. 62 ones × 10 = 62 _______________
   e. 8 thousands ÷ 10 = 8 _______________
   f. 9 thousands ÷ 100 = 9 _______________

12. Write the missing operations (+, -, x or ÷) and numbers:
   a. 7 ones _____ = 7 thousands
   b. 8 hundreds _____ = 8 thousands
   c. 4 tens _____ = 4 thousands
   d. 9 thousands _____ = 9 hundreds
   e. 4 thousands _____ = 4 ones
   f. 3 thousands _____ = 3 tens

13. Round to nearest 1000:
   a. 2686 _____
   b. 4725 _____
   c. 9303 _____

14. Round to nearest 100:
   a. 4256 _____
   b. 9238 _____
   c. 6850 _____

15. Circle the numbers that could be rounded to 3000.
   a. 3713   b. 2267   c. 2325   d. 3380   e. 3250
Number 4B

Name: __________________________ Year Level: ______ Date: ______

1. Write the number and the number name shown by the blocks.

   a. 
      Number: __________________________
      Number name: __________________________

   b. 
      Number: __________________________
      Number name: __________________________

   c. 
      Number: __________________________
      Number name: __________________________

   d. 
      Number: __________________________
      Number name: __________________________

2. Circle the blocks needed to make the following number names:

   a. Two thousand, three hundred and sixty-four

   b. Five thousand and fifteen
1. Number

c. Six thousand, three hundred and seventy

d. Four thousand and forty

3. Circle the blocks needed to make each of the following numbers:

a. 5674

b. 1616

c. 7070

d. 4004
4. Write the numbers as number names:
   a. TH  H  T  O
      4  7  6  5
   b. TH  H  T  O
      3  5  0  1
   c. TH  H  T  O
      6  0  6  0
   d. TH  H  T  O
      2  0  1  2

5. Write the number on the Place Value Chart:
   a. Six thousand, three hundred and fifty-seven
      TH  H  T  O
   b. Three hundred and nineteen
      TH  H  T  O
   c. Four thousand, five hundred and eight
      TH  H  T  O
   d. Six thousand and six
      TH  H  T  O

6. For the number:
   a. Write the number that is 10 more.
   b. Write the number that is 100 more.
   c. Write the number that is 1000 less.
   d. Write the number that is 1 less.
   e. Write the number that is 10 less.
   f. Write the number that is 1000 more.

7. Write the following numbers on the number line in order from smallest to largest in value:
   5072  8619  5720  8961

8. a. Write the number represented:
b. Circle the thousands needed to make 5164:

![Diagram of thousands]

CDA TASKS

b. Circle the thousands needed to make 5164:

![Diagram of thousands]

c. Circle the tens needed to make 2626:

![Diagram of tens]

9. What operation will change the first number to the second number?

a. Is this x 10, x 100, x 1000, ÷ 10, ÷ 100, or ÷ 1000?

<table>
<thead>
<tr>
<th>TH</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

b. Is this x 10, x 100, x 1000, ÷ 10, ÷ 100, or ÷ 1000?

<table>
<thead>
<tr>
<th>TH</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

What happens when ÷ 1000?

<table>
<thead>
<tr>
<th>TH</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

What happens when x 100?

<table>
<thead>
<tr>
<th>TH</th>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

10. Use the number line to help you answer the following.

![Number line]

a. Round 2367 to the nearest 1000

b. Round 4651 to the nearest 1000

c. Round 8500 to the nearest 1000

11. Write the numbers that could be at:

<table>
<thead>
<tr>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
</tr>
<tr>
<td>A</td>
</tr>
</tbody>
</table>

a. A: 

b. B: 

c. C: 

1. Number
NUMBER 5A

Name: ___________________________  Year Level: __________  Date: __________

1. Write the number for each number name:
   a. four hundred and sixty-three million, one hundred and sixty-one thousand, seven hundred and forty-nine
      ____________________________
   b. sixteen million, four hundred and forty-two thousand and ninety-three
      ____________________________
   c. eighty-seven thousand, three hundred and six
      ____________________________
   d. thirty-six million, two hundred thousand, 4 hundred and eighty-eight
      ____________________________
   e. five hundred thousand, two hundred and forty
      ____________________________
   f. one hundred and seventeen million, four hundred and eleven
      ____________________________

2. Write in words:
   a. 5 675 268
      ____________________________
   b. 30 023 006
      ____________________________
   c. 400 040
      ____________________________
   d. 15 017 012
      ____________________________
   e. 560 670 200
      ____________________________

3. Write the number that has:
   a. 8 tens, 2 ten-thousands, 4 ones, 8 hundred-thousands, 2 thousands, 4 hundreds
      ____________________________
   b. 6 thousands, 9 ten-millions, 3 hundreds, 5 ten-thousands, 9 tens, 4 millions, 7 ones, 2 hundred-thousands
      ____________________________
   c. 7 hundreds, 4 thousands, 6 ten-thousands, 3 millions, 2 ones
      ____________________________
   d. 7 hundred-thousands, 4 tens, 5 hundred-millions, 6 hundreds
      ____________________________
4. a. Write a zero in 5 435 421 without changing its value. ________________
b. Write a zero in 5 435 421 so that its value changes. ________________

5. What number is:
a. 10 000 more than 724 682? ________________
b. 1 000 000 less than 627 851 446? ________________
c. 100 000 more than 31 967 086? ________________
d. 1 000 000 less than 560 450 081? ________________
e. 10 000 less than 30 006 429? ________________

6. Complete the counting sequences:
a. 34 286, 34 287, 34 288, ________________, ________________
b. 5 434 218, 5 424 218, 5 414 218, ________________, ________________
c. 679 596, 679 696, 679 796, ________________, ________________
d. 239 772 135, 239 872 135, 239 972 135, ________________, ________________
e. 40 036 761, 40 026 761, 40 016 761, ________________, ________________

7. In each box, circle the number with the larger value:
a. 2 867 903, 2 941 679
b. 24 487, 235 065
c. 2 867 426, 291 679
d. 584 067 002, 580 451 094

8. Write the following numbers in order from smallest to largest in value:
36 521 568, 3 621 568, 36 519 568, 36 059 568
________________________

9. Circle the number in which the 7 is worth the most:
1 726 459, 31 729 459, 7 104 298, 9 267 843
________________________

10. Write the missing numbers:
a. 543 768 = _____ thousands _____ ones
b. 45 063 501 = _____ millions _____ thousands _____ ones
c. 64 928 = 64 thousands 8 hundreds _____ tens _____ ones
d. 437 684 = 2 hundred-thousands _____ ten-thousands 7 thousands 6 hundreds 7 tens _____ ones
e. 37 234 067 = _____ ten-millions 17 millions 234 thousands _____ tens 27 ones
11. Write the missing place value names:
   a. $8 \times 1000 = 8\underline{\hspace{1cm}}$
   b. $4 \times 100 = 4\underline{\hspace{1cm}}$
   c. $5 \times 1000 = 5\underline{\hspace{1cm}}$
   d. $6 \div 1000 = 6\underline{\hspace{1cm}}$
   e. $9 \div 1000 = 9\underline{\hspace{1cm}}$
   f. $8 \div 1000000 = 8\underline{\hspace{1cm}}$

12. Write the missing numbers:
   a. $7 \times \underline{\hspace{1cm}} = 7\underline{\hspace{1cm}}$
   b. $8 \times \underline{\hspace{1cm}} = 8\underline{\hspace{1cm}}$
   c. $9 \div \underline{\hspace{1cm}} = 9\underline{\hspace{1cm}}$
   d. $4 \div \underline{\hspace{1cm}} = 4\underline{\hspace{1cm}}$
   e. $672 \div \underline{\hspace{1cm}} = 672\underline{\hspace{1cm}}$
   f. $56 \times \underline{\hspace{1cm}} = 56\underline{\hspace{1cm}}$

13. Round to nearest 10 000:
   a. $367 686\underline{\hspace{1cm}}$
   b. $4 722 577\underline{\hspace{1cm}}$
   c. $9 345 000\underline{\hspace{1cm}}$

14. Round to nearest 1 000 000:
   a. $46 256 896\underline{\hspace{1cm}}$
   b. $92 731 131\underline{\hspace{1cm}}$
   c. $687 500 000\underline{\hspace{1cm}}$

15. Circle the numbers that could be rounded to 30 000 000.
   a. $37 138 564$; b. $29 675 324$; c. $24 674 989$; d. $30 445 372$
1. Write the number name for each given number:

   a. Millions  Thousands  Ones
      H T O  H T O  H T O
      4 5 1  3 2 8  4 7 7

   b. Millions  Thousands  Ones
      H T O  H T O  H T O
      4 0 9 1 7 0 2 0

   c. Millions  Thousands  Ones
      H T O  H T O  H T O
      2 1 1 0 2 8 0 1 2

   d. Millions  Thousands  Ones
      H T O  H T O  H T O
      6 0 0 0 0 0 6 0

2. Write the matching number on the place value chart:

   a. Forty-five million, two hundred and sixty-one thousand, seven hundred and eighty-four

   b. Two hundred and seventy million, three hundred and five thousand, and twenty-nine

   c. Five million and five hundred

3. For the number shown in the place value chart:

   Millions  Thousands  Ones
   H T O  H T O  H T O
   3 9 0  8 6 1  2 7

   a. Write the number that is 10 000 more.

   b. Write the number that is 100 less.

   c. Write the number that is 1 000 000 more.

   d. Write the number that is 100 000 less.

   e. Write the number that is 10 000 000 less.

   f. Write the number that is 10 more.
4. Circle the number that is 10,000 less than the number in the place value chart:

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>H T O</td>
<td>H T O</td>
<td>H T O</td>
</tr>
<tr>
<td>5 6</td>
<td>7 9 2</td>
<td>4 36</td>
</tr>
</tbody>
</table>

56 791 436  56 892 436  56 782 436  55 792 436  56 802 436

5. Draw lines from each number to the number line to show where each could be on the number line:

61 897 434  5 869 488  24 643 292  99 120 541

6. Regroup the number in the place value chart and then write the new number:

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>H T O</td>
<td>H T O</td>
<td>H T O</td>
</tr>
<tr>
<td>5 6 2</td>
<td>3 1 7</td>
<td>5 3 7</td>
</tr>
<tr>
<td>6 2 4</td>
<td>4 6 7 1</td>
<td>8 4 7</td>
</tr>
<tr>
<td>5 1 4 7</td>
<td>9 1 6 0</td>
<td>1 5 0</td>
</tr>
</tbody>
</table>

7. Write the change shown in the place value charts as $\times$ or $\div$ by: 10, 100, 1000, 10,000, 100,000, 1,000,000.

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
<th>$\times$ or $\div$</th>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 6 7</td>
<td>8 9 0</td>
<td>1 2 3</td>
<td>$\times$ 100</td>
<td>3 6 0</td>
<td>8 9 0</td>
<td>1 2 3</td>
</tr>
<tr>
<td>4 0 0</td>
<td>0 0 0</td>
<td>2 3 4</td>
<td>$\times$ 1000</td>
<td>4 0 0 0</td>
<td>2 3 4</td>
<td>0 0 0</td>
</tr>
<tr>
<td>2 3 0 0</td>
<td>4 0 0</td>
<td>2 3 4</td>
<td>$\div$ 10</td>
<td>2 3 4</td>
<td>0 0 0</td>
<td>2 3 4</td>
</tr>
</tbody>
</table>

8. In each place value chart, write the change that comes from $\times$ or $\div$ the given number by:

<table>
<thead>
<tr>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
<th>$\times$ or $\div$</th>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 4 7</td>
<td>8 9 0</td>
<td>1 2 3</td>
<td>$\times$ 1000000</td>
<td>3 4 7</td>
<td>8 9 0</td>
<td>1 2 3</td>
</tr>
<tr>
<td>4 6 7</td>
<td>2 3 4</td>
<td>0 0 0</td>
<td>$\times$ 1000</td>
<td>4 6 7 0</td>
<td>2 3 4</td>
<td>0 0 0</td>
</tr>
<tr>
<td>3 8 8</td>
<td>2 3 4</td>
<td>0 0 0</td>
<td>$\div$ 10</td>
<td>3 8 8</td>
<td>2 3 4</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>
9. Use the number line to help you answer the following:

a. Round 27 511 245 to the nearest 10 000 000.

b. Round 55 000 000 to the nearest 10 000 000.

c. Round 83 837 600 to the nearest 10 000 000.

10. What number could be at:

a. A ____________________

b. B ____________________

c. C ____________________
COMMON FRACTIONS 1A

Name: ________________________________ Year Level: ________ Date: __________

1. a. Write the number that is 1 quarter more than 2 quarters. ________________
   b. Write the number that is 1 half more than 1 half. ________________

2. Complete the counting sequences:
   a. 1 quarter, 2 quarters, ________________, ________________, ________________
   b. 3 fifths, 4 fifths, ________________, ________________, ________________

3. a. If you cut one whole cake into sixths, how many pieces would you have? ________________
   b. Would the pieces be equal? YES NO DOESN'T MATTER (Circle your answer.)

4. Write the missing number: 1 whole = ___ halves.

5. In each box below, tick the number that has the larger value:
   a. 2 tenths
   b. 1 half
   c. 1 whole

   7 tenths 1 quarter 3 quarters

6. Circle the fraction that has a different value:
   1 half, 2 quarters, 1 tenth

7. Write the fractions, in order, from smallest to largest in value:
   1 half, 1 quarter, 1 tenth, ________________________________

8. Circle the fraction that is closest to 1 whole:
   1 half, 3 quarters, 3 tenths
COMMON FRACTIONS 1B

Name: ___________________________ Year Level: __________ Date: __________

1. Tick the shapes below that have been divided into halves:

   A
   B
   C
   D
   E
   F
   G
   H
   I

2. Match the fraction names with the picture:

   3 quarters
   2 halves
   1 third
   1 half

   a.
   b.
   c.
   d.

3. Colour each shape and set to match the given number:

   a. 1 quarter
   b. 1 third
   c. 1 half
4. This is 1 half of a ribbon. Draw the whole ribbon.

5. Partition (divide) the squares below to show halves in two different ways:

6. Partition the circle to show quarters:
COMMON FRACTIONS 2A

Name: ___________________________________ Year Level: _________ Date: ____________

1. Write these numbers in words:
   a. \( \frac{4}{5} \) __________________________ b. \( \frac{7}{8} \) __________________________

2. Write these numbers in digits:
   a. two sixths ________________ b. one quarter ________________ c. eight thirds ________________

3. Write the number that is 1 quarter more than:
   a. \( \frac{1}{4} \) __________________________ b. \( \frac{3}{4} \) __________________________

4. Complete the counting sequences:
   a. \( \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \) __________, __________, __________
   b. 3 fifths, 4 fifths, __________, __________, __________

5. Write any number that comes between: \( \frac{1}{2} \) and 1 ______________

6. Write the missing number: \( 1 = \quad \) fifths.

7. In each box below, tick the number that has the larger value:
   a. b. c. d.

8. Circle the fraction that has a different value: \( \frac{1}{2}, \frac{4}{5}, \frac{3}{6}, \frac{7}{10} \)

9. Write the fractions, in order, from smallest to largest in value: \( \frac{1}{2}, \frac{1}{6}, \frac{1}{4} \) ______________

10. Is \( \frac{1}{4} \) closer to 0 or to 1? ______________
COMMON FRACTIONS 2B

Name: ___________________________ Year Level: _________ Date: __________

1. Tick the shapes below that have been divided into halves:

   A  B  C  D  E  F  G  H  I

2. Write the missing fraction names:
   a. ________  b. ________  c. ________

   a. ________  {Fraction symbol}  b. ________  c. ________  d. ________

3. Write the fraction that shows how much of each shape is shaded:
   a. ________  b. ________  c. ________  d. ________
4. Colour each shape and set to match the given number:
   a. 3 fifths  
   b. 3 eighths  
   c. 1 third  
   d. 1 and 3 quarter pies

![Shapes](image)

5. This is 1 quarter of a ribbon. Draw the whole ribbon.

6. a. Write the missing fraction names:
   ![Fractions](image)

   2 4

   b. Do the two fractions have the same value?  YES  NO  (Circle your answer)

7. Partition (divide) the squares below to show quarters in three different ways:

![Squares](image)

8. Show 6 eighths on the circle.

![Circle](image)
COMMON FRACTIONS 3A

Name: _____________________________ Year Level: __________ Date: __________

1. Write these numbers in words:
   a. \( \frac{4}{5} \) ________________  b. \( \frac{7}{8} \) ________________

2. Write these numbers in digits:
   a. one quarter ________________  b. two fifths ________________  c. eight thirds ________________

3. Write the number that is 1 quarter more than:
   a. \( \frac{1}{4} \) ________________  b. \( \frac{3}{4} \) ________________

4. Complete the counting sequences:
   a. \( \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \) ________________  ________________  ________________
   b. 3 fifths, 4 fifths, ________________  ________________  ________________

5. Write any number that comes between:
   a. 1 sixth and 4 sixths ________________  b. \( \frac{1}{2} \) and 1 ________________

6. In each box below, tick the number that has the larger value:
   a. \( \frac{1}{6} \)  \( \frac{5}{6} \)  \( \frac{3}{4} \)  \( \frac{1}{2} \)  \( \frac{1}{3} \)
   b. \( \frac{1}{2} \)  \( \frac{3}{6} \)  \( \frac{3}{10} \)  \( \frac{7}{9} \)  \( \frac{12}{16} \)

7. In each box, circle the fraction that has a different value from the others:
   a. \( \frac{1}{2}, \frac{4}{8}, \frac{3}{6}, \frac{7}{10} \)
   b. \( \frac{3}{5}, \frac{4}{6}, \frac{5}{8}, \frac{9}{15} \)

8. Write these numbers from smallest to largest in value: \( \frac{7}{9}, \frac{1}{4}, \frac{3}{7} \) ________________

9. Round each number to the nearest whole number:
   a. 5 \( \frac{3}{8} \) ________________  b. \( \frac{3}{4} \) ________________
COMMON FRACTIONS 3B

Name: ___________________________ Year Level: _______ Date: __________

1. Tick the shapes below that have been divided into halves:

   A  B  C  D  E  F  G  H  I

2. Write the number that shows how much of each shape or set is shaded:

   a.  b.  c.  d.

3. Colour each shape and set to match the given number:

   a.  b.  c.  d.

4. a. Show $\frac{1}{3}$ on the number line below:

   0 1 2 3

2. Common Fractions
b. Show $1\frac{1}{2}$ on the number line below:

\[ \begin{array}{cccc} & & & \\ 0 & 1 & 2 & 3 \end{array} \]

5. a. What number could be shown at A on the number line below? _________

\[ \begin{array}{cccc} & & & \\ 0 & 1 & 2 & 3 \end{array} \]

b. What number could be shown at B on the number line below? _________

\[ \begin{array}{cccc} & & & \\ 0 & 1 & 2 & 3 \end{array} \]

6. This is $\frac{1}{4}$ of a ribbon. Draw the whole ribbon.

7. This is $\frac{3}{4}$ of a Mars Bar. Draw the whole chocolate bar.

8. What fraction of the shape is shaded?

9. Tick any of these fractions that are equivalent in value to the fraction in the box:
10. a. Partition and colour this whole to show $\frac{1}{3}$

b. Partition and colour this whole to show $\frac{3}{9}$

c. Are the fractions equal in value? YES NO (Circle your answer)

11. There are 2 chocolate bars to share among 3 people. Partition the chocolate bars below so that each person gets an equal amount. (Use different colours to show each share.)
COMMON FRACTIONS 4A

Name: ____________________________ Year Level: _________ Date: __________

1. Write these numbers in words:
   a. \( \frac{4}{5} \) ____________________________
   b. \( 3 \frac{7}{8} \) ____________________________

2. Write these numbers in digit form:
   a. seven and two fifths
   b. three quarters
   c. eight thirds

3. Write the number that has 5 sixths 2 ones.

4. Write the missing numbers:
   a. \( \frac{1}{2} \times 4 = \) __________
   b. \( \frac{5}{6} \times \) ___ = 5
   c. \( 2 \div \) ___ = \( \frac{2}{3} \)
   d. \( 5 \div 6 = \) __________

5. Write ‘Yes’ or ‘No’ to each of the following:
   a. Does 3 \( \frac{1}{2} \) have the same value as \( \frac{5}{2} \)?
   b. Does 1 \( \frac{1}{2} \) have the same value as \( \frac{5}{2} \)?
   c. Does 4 \( \frac{1}{3} \) have the same value as \( \frac{8}{3} \)?

6. Write the missing mixed numbers:
   a. 12 fifths = __________
   b. \( \frac{2}{3} \) = __________
   c. __________ = 15 halves
   d. __________ = \( \frac{15}{8} \)

7. Write the missing numbers:
   a. \( 6 \frac{1}{2} = 5 \) ones ___ halves
   b. 3 ones 5 thirds = __________

8. Write the number that is 1 quarter more than:
   a. \( 2 \frac{1}{4} \) __________________________
   b. \( 5 \frac{3}{4} \) __________________________
   c. 9 __________________________

9. Complete the counting sequences:
   a. \( \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \) _______: _______: _______: 
   b. \( 6 \frac{1}{5}, 6 \frac{2}{5}, 6 \frac{3}{5}, \) _______: _______: _______: 

10. Write any number that comes between:
    a. \( \frac{2}{5} \) and \( \frac{4}{5} \) __________________________
    b. \( 1 \frac{1}{2} \) and \( 1 \frac{1}{3} \) __________________________
11. In each box below, tick the number that has the smaller value:
   a. $\frac{5}{8}$  
   b. $\frac{7}{2}$  
   c. $\frac{4}{9}$  
   d. $\frac{8}{3}$  
   e. $\frac{5}{3}$

12. In each box, circle the fraction that has a different value:
   a. $\frac{1}{2}$  
   b. $\frac{5}{8}$  
   c. $\frac{3}{10}$  
   d. $\frac{7}{10}$  
   e. $\frac{12}{15}$

13. Write each set of numbers, in order, from smallest to largest in value:
   a. $\frac{1}{2}$, $\frac{1}{4}$  
   b. $\frac{5}{8}$, $\frac{7}{10}$, $\frac{2}{3}$

14. Round each number to the nearest whole number:
   a. $5\frac{1}{4}$
   b. $\frac{4}{5}$

15. Draw lines to show the matching numbers in Columns A and B:
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>$\frac{70}{100}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{5}{100}$</td>
</tr>
<tr>
<td>0.05</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$\frac{10}{10}$</td>
</tr>
<tr>
<td>0.25</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

16. To find the sum of 6 and 9 on your calculator, you would have to code the calculator like this:

   $6 + 9 = $

   Write the calculator code for changing $\frac{1}{7}$ to a decimal number.

   $\text{[ ] [ ] [ ] [ ] [ ]}$
COMMON FRACTIONS 4B

Name: ____________________________ Year Level: _________ Date: __________

1. Tick the shapes below that have been partitioned into quarters:

A  B  C  D  E  J

F  G  H  I

2. Write the number that shows how much of each shape or set is shaded:

a.  b.  c.  d.

3. Colour each shape to match the given number:

a. $\frac{3}{8}$  b. $\frac{3}{4}$  c. $\frac{1}{3}$
4. Colour each set to match the given number:
   a. \( \frac{1}{3} \) of the set of triangles
   b. 2 \( \frac{3}{4} \) pies
   c. 1 \( \frac{1}{2} \) sets of marbles

5. a. Show 2\( \frac{1}{2} \) on the number line below:

5. b. Show 1\( \frac{3}{4} \) on the number line below:

6. a. Write A to show where \( \frac{5}{6} \) is on the number line below:

6. b. Write B to show where \( \frac{11}{6} \) is on the number line below:

7. a. This is \( \frac{1}{2} \) of a ribbon. Draw the whole ribbon.

7. b. This is \( \frac{1}{3} \) of a set of marbles. Draw the set of marbles.

8. What fraction of the shape is shaded?
9. Tick any of these fractions that are equivalent in value to the fraction in the box:
   a. 
   ![Fraction Image]
   b. 
   ![Fraction Image]
   c. 
   ![Fraction Image]

10. a. Partition and colour this whole to show \( \frac{1}{3} \)
   ![Diagram]
   b. Partition and colour this whole to show \( \frac{2}{3} \)
   ![Diagram]
   c. Are the fractions equal in value? YES  NO  (Circle your answer)

11. There are 2 bags of chocolates to share among Jim, Anne and Bill.
   a. Colour the chocolates so that each person gets an equal amount from each bag.
      [Use different colours to show each share.]
   What fraction of the chocolates does Jim get from:
   b. Bag A?
   c. Bag B?
   d. Both bags?
COMMON FRACTIONS 5A

Name: ____________________________ Year Level: _________ Date: __________

1. Write these numbers in words:
   a. \(\frac{1}{5}\) ____________________________
   b. \(3\frac{1}{8}\) ____________________________

2. Write these numbers in digit form:
   a. seven and two fifths ________
   b. three quarters ________
   c. eight thirds ________

3. Write the number that has 5 sixths 2 ones: ________

4. Write the missing numbers:
   a. \(\frac{3}{4} \times 4 = \) ________
   b. \(\frac{5}{6} \times \) ___ = 5
   c. 2 ÷ ___ = \(\frac{2}{3}\)
   d. ___ \times 5 = 2
   e. 5 ÷ 6 = ________
   f. ___ ÷ 5 = \(\frac{2}{5}\)

5. Write ‘Yes’ or ‘No’ to each of the following:
   a. Does \(3\frac{1}{2}\) have the same value as \(\frac{7}{2}\) ________
   b. Does \(1\frac{1}{3}\) have the same value as \(\frac{5}{3}\) ________
   c. Does \(4\frac{1}{3}\) have the same value as \(\frac{5}{3}\) ________

6. Write the missing mixed numbers:
   a. 12 fifths = ________
   b. _____ = 15 sixths
   c. \(\frac{3}{4}\) = ________
   d. _____ = \(\frac{15}{6}\)

7. Write the missing numbers:
   a. \(\frac{6}{7} = 5\) ones ____ halves
   b. 3 ones 5 thirds = _______

8. Write the number that is 1 quarter more than:
   a. \(2\frac{1}{4}\) ____________________________
   b. \(5\frac{3}{4}\) ____________________________
   c. 9 ____________________________

9. Complete the counting sequences:
   a. \(\frac{1}{4}, \frac{2}{4}, \frac{3}{4}\) __________. __________. __________
   b. \(6\frac{1}{5}, 6\frac{2}{5}, 6\frac{3}{5}\) __________. __________. __________

10. Write any number that comes between:
    a. \(\frac{2}{7}\) and \(\frac{4}{7}\) ____________________________
    b. \(1\frac{1}{2}\) and \(1\frac{1}{3}\) ____________________________
11. In each box below, tick the number that has the larger value:
   a. $\frac{5}{6}$
   b. $\frac{7}{6}$
   c. $\frac{4}{3}$
   d. $\frac{8}{5}$
   e. $\frac{5}{3}$

12. In each box, circle the fraction that has a different value:
   a. $\frac{1}{2}$, $\frac{4}{5}$, $\frac{1}{5}$, $\frac{7}{10}$
   b. $\frac{12}{15}$, $\frac{4}{5}$, $\frac{5}{8}$, 0.75

13. Write each set of numbers, in order, from smallest to largest in value:
   a. $\frac{1}{2}$, $\frac{3}{6}$, $\frac{1}{4}$
   b. $7\frac{3}{6}$, $7\frac{3}{2}$, $7\frac{1}{4}$

14. Round each number to the nearest whole number:
   a. $5\frac{3}{6}$
   b. $\frac{4}{5}$
   c. $\frac{1}{2}$

15. Draw lines to show the matching numbers in Columns A, B and C:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>$\frac{40}{100}$</td>
<td>100%</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{5}{10}$</td>
<td>5%</td>
</tr>
<tr>
<td>0.05</td>
<td>$\frac{1}{2}$</td>
<td>40%</td>
</tr>
<tr>
<td>0.4</td>
<td>$\frac{10}{18}$</td>
<td>25%</td>
</tr>
<tr>
<td>0.25</td>
<td>$\frac{1}{4}$</td>
<td>75%</td>
</tr>
<tr>
<td>0.5</td>
<td>$\frac{3}{2}$</td>
<td>50%</td>
</tr>
</tbody>
</table>

16. To find the sum of 6 and 9 on your calculator, you would have to code the calculator like this:

$$ 6 + 9 = $$

a. Write the calculator code for changing $\frac{5}{7}$ to a decimal number.

b. Write the calculator code for changing $\frac{6}{3}$ to a per cent.
COMMON FRACTIONS 5B

Name: ___________________________ Year Level: _________ Date: __________

1. Tick the shapes below that have been partitioned into quarters:

A  B  C  D  E  F  G  H  I  J

2. Tick the shapes that show \( \frac{2}{3} \).

a.  b.  c.  d.  e.

3. Colour each shape to match the given number:

a.  b.  c.

4. Colour each set to match the given number:

a. \( \frac{1}{3} \) of the set of triangles  
   b. \( 2 \frac{1}{4} \) pies  
   c. \( 1 \frac{3}{4} \) sets of marbles
5. a. Write A to show $2\frac{2}{3}$ on the number line below:

```
0  1  2  3
```

b. Write B to show $1\frac{5}{9}$ on the number line below:

```
0  1  2  3
```

6. a. Write A to show $\frac{6}{5}$ on the number line below:

```
0  1  2  3  4
```

b. Write B to show $\frac{11}{5}$ on the number line below:

```
0  1  2  3  4
```

7. a. This is $\frac{3}{5}$ of a ribbon. Draw the whole ribbon.

```
[Green rectangle]
```

b. This is $\frac{3}{5}$ of a set of marbles. Draw the set of marbles.

```
[Set of marbles]
```

8. a. This is $1\frac{1}{3}$ of a ribbon. Draw the whole ribbon.

```
[Green rectangle]
```

b. This is $2\frac{1}{3}$ of a set of marbles. Draw the set of marbles.

```
[Set of marbles]
```

9. What fraction of the shape is shaded?

```
[Diagram with shaded square]
```
10. Tick any of these fractions that are equivalent in value to the fraction in the box:

![Fraction Images]

11. a. Partition and colour this whole to show $\frac{1}{3}$

![Fraction Image]

b. Partition and colour this whole to show $\frac{1}{2}$

![Fraction Image]

c. Are the fractions equal in value? YES NO (Circle your answer)

12. There are 2 bags of chocolates to share among Jim, Anne and Bill.
   a. Colour the chocolates so that each person gets an equal amount from each bag.
      [Use different colours to show each share.]

   What fraction of the chocolates does Jim get from:
   b. Bag A?
      _____________________
   c. Bag B?
      _____________________
   d. Both bags?
      _____________________

13. a. What fraction of the marbles in Bag A are coloured?
      _____________________
   b. What fraction of the marbles altogether (both bags) are coloured?
      _____________________

c. Are the fractions equal in value? YES NO (Circle your answer)
DECIMAL FRACTIONS 1A

Name: ___________________________ Year Level: _________ Date: ____________

1. Write these numbers in words:
   a. 3.8 ___________________________________________________________________
   b. 0.2 ___________________________________________________________________

2. Write these numbers in digits:
   a. five and three tenths __________
   b. seven tenths ________________

3. Write the number that has:
   a. 9 ones, 2 tenths ______________
   b. 7 tens, 7 tenths ______________
   c. 0 ones, 8 tenths _____________
   d. 5 tenths, 3 ones ______________

4. Circle the number in which the 7 is worth the most:
   9437 7.5 176 1.7

5. Write ‘Yes’ or ‘No’ to each of the following:
   a. Does 7.2 have the same value as 7.02? __________
   b. Does 7.2 have the same value as 07.2? __________
   c. Does 7.2 have the same value as 0.72 __________
   d. Does 7.2 have the same value as 70.2? __________

6. Write the number that is 1 tenth more than:
   a. 9.4 ______________
   b. 2.9 ______________

7. Write the number that is 1 tenth less than:
   a. 4.7 ______________
   b. 5 ______________

8. Complete the counting sequences:
   a. 1.6, 1.7, 1.8, __________, ________, ________
   b. 4.6, 5.6, 6.6, __________, ________, ________

9. In each box below, circle the number that has the larger value:
   a. 8.6  2.8
   b. 0.9  0.2
   c. 3.4  6
10. Write any number that comes between:
   a. 18.2 and 18.5 ____________________  
   b. 2 and 3 ____________________

11. Write each set of numbers in ascending order (smallest to largest):  
   a. 4.7, 4.2, 4.6 ____________________  
   b. 9, 3.8, 0.9 ____________________

12. Write the missing number or place name in each of the following:  
   a. 2.6  
   b. 70.3  
   c. ____________________  
   d. 12.7  

<table>
<thead>
<tr>
<th>Ones</th>
<th>6</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>3</td>
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<th>Ones</th>
<th>6</th>
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<tr>
<td>71</td>
<td>17</td>
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</table>

13. Write the missing numbers:
   a. 5.7 = ______ tenths altogether  
   b. 2 = ______ tenths altogether  
   c. 74 tenths = __________ [decimal number]  
   d. 5 ones 16 tenths = __________ [decimal number]

14. In 625.8, circle the part of the number that is worth:
   a. 2 tens  
   b. 8 tenths  
   c. 58 tenths  

   |   |   |   |
   | 6 | 2 | 5.8 |

   |   |   |   |
   | 6 | 2 | 5.8 |

   |   |   |   |
   | 6 | 2 | 5.8 |

15. Write the missing numbers:
   a. 0.3 × 10 = ________________  
   b. 6.2 × 10 = ________________  
   c. 4 ÷ 10 = ________________  
   d. 37 ÷ 10 = ________________

16. Write the operation (× 10 or ÷ 10) that changes the given number to the new number:
   a. 0.5 ________________ = 5  
   b. 73 ________________ = 7.3  
   c. 8 ________________ = 0.8  
   d. 2.9 ________________ = 29

17. Round each number to the nearest whole number:
   a. 8.3 ________________  
   b. 0.3 ________________  
   c. 0.7 ________________
18. Do the following:

a. Colour 0.9 of this shape.

b. Colour 0.4 of this shape.

c. This is 0.5 of a shape. Draw the whole shape.

d. Circle the number below that shows about how much of the shape has been shaded.

\[
\begin{array}{ccc}
0.2 & 0.5 & 0.9
\end{array}
\]
DECIMAL FRACTIONS 1B

Name: ___________________________ Year Level: _______ Date: ________

1. Partition (divide) the shapes below to show tenths in two different ways:

2. Tick the numbers that show how many chocolate bars Sally ate:
   a. 2 and 4 tenths
   b. 2 and four-sixths
   c. 2.4

3. Write the decimal number that shows how much of the shape has been shaded:

4. Colour 0.7 of this shape:

5. Answer the following in relation to the given shape.
   This is 0.1: _______ Draw the whole.

6. a. What decimal number is at A? ________________

   b. Show, with an arrow, where 2.6 is on the number line above.
DECIMAL FRACTIONS 2A

Name: ____________________________ Year Level: _________ Date: __________

1. Write these numbers in words:
   a. 3.82
   b. 607.02

2. Write these numbers in digits:
   a. Five, and thirty-six hundredths
   b. Eleven, and seven hundredths

3. Write the number that has:
   a. 5 tenths, 2 hundredths, 9 ones
   b. 7 tens, 7 tenths
   c. 2 hundredths, 8 tenths

4. Circle the number in which the 7 is worth the most:
   943.67  7.52  176  1.76

5. In 625.08:
   a. the 2 has a value of 2
   b. the 8 has a value of 8
   c. the 50 has a value of 50

6. Write ‘Yes’ or ‘No’ to each of the following:
   a. Does 7.2 have the same value as 7.02?
   b. Does 7.2 have the same value as 07.2?
   c. Does 7.2 have the same value as 7.20?
   d. Does 7.2 have the same value as 0.72?

7. Write the number that is 1 hundredth more than:
   a. 2.76
   b. 3.9
   c. 0.09
   d. 6
   e. 4.91

   Write the number that is 1 hundredth less than:
   f. 3.58
   g. 3.8
   h. 0.4
   i. 7
   j. 5.01
8. Complete the counting sequences:
   a. 1.67, 1.68, 1.69, ______, ______, ______
   b. 3.06, 3.07, 3.08, ______, ______, ______
   c. 4.65, 4.75, 4.85, ______, ______, ______

9. In each box below, tick the number that has the larger value:
   a. 8.62  b. 0.95  c. 0.7  d. 3.42  e. 7.35

10. Write any number that comes between:
    a. 18.28 and 18.5  b. 1.5 and 1.6

11. Write each set of numbers in ascending order (smallest to largest):
    a. 4.73, 4.28, 4.65
    b. 9, 3.81, 3.09
    c. 6.93, 6.08, 6

12. Write the missing number or place name in each of the following:
    a. 2.64  
       Ones  5  Tenths  Hundredths
    b. 70.35
       703  5
    c. ______
       71  Tenths  4  Hundredths
    d. 2.87
       27  17

13. Write the missing numbers:
    a. 5.07 = ______ hundredths
    b. 6.2 = ______ hundredths
    c. 3 tenths 16 hundredths = ______
    d. 74 tenths 10 hundredths = ______

14. Write the missing numbers:
    a. 0.7 × 100 = _____________
    b. 72.5 ÷ 10 = _____________
    c. 0.04 × 100 = _____________
    d. 0.9 ÷ 10 = _____________
    e. 6.23 ÷ 10 = _____________
15. Write the missing operations (e.g. $\times 100$, or $\div 10$ and so on):
   a. $8 \ \underline{\hspace{2cm}} \ = 0.08$
   b. $2.16 \ \underline{\hspace{2cm}} \ = 216$
   c. $0.47 \ \underline{\hspace{2cm}} \ = 4.7$
   d. $0.02 \ \underline{\hspace{2cm}} \ = 0.2$
   e. $356 \ \underline{\hspace{2cm}} \ = 3.56$
   f. $8.5 \ \underline{\hspace{2cm}} \ = 0.85$

16. Round each decimal number to the nearest whole number.
   a. $8.39 \ \underline{\hspace{2cm}}$
   b. $0.347 \ \underline{\hspace{2cm}}$
   c. $1.09 \ \underline{\hspace{2cm}}$

17. Do the following.
   a. Colour 0.9 of this shape.
   b. Colour 0.4 of this shape.
   c. This is 0.5 of a shape. Draw the whole shape.
   d. Circle the number below that shows about how much of the shape has been shaded.
      \[0.2 \quad 0.5 \quad 0.9\]
DECIMAL FRACTIONS 2B

Name: __________________________ Year Level: ________ Date: __________

1. a. What decimal number is at A? ________________

2 3 4 5

b. Show, with an arrow, where 0.16 could be on the number line below:

0 0.1 0.2 0.3

0.16

c. Show, with an arrow, where 6.47 could be on the number line below:

4 5 6 7

6.47

d. What decimal number could be at Z on the line above? ________________

2. Tick the numbers that show how many chocolate bars Sally ate:

a. 2 and 4 tenths
b. 2 and four-sixths
c. 2.4
d. 2.04

3. Write the decimal number that shows how much of each shape has been shaded:

a. ________________
b. ________________
c. ________________
d. ________________
4. a. Shade 0.57 of the shape below.

b. Shade 0.4 of the shape below.

c. Shade 0.28 of the shape below.

d. Shade 0.6 of the shape below.

5. Colour 0.3 of the shape below.

6. Answer the following about the shapes below:

a. What decimal fraction of the shape is shaded?

b. Colour 0.23 of this shape.
7. Answer the following in relation to the given shape.
   a. This is 0.1:  
      [Blank]
      Draw 0.01
   b. This is 0.1:  
      [Blank]
      Draw 0.10
   c. This is 0.1:  
      [Blank]
      Draw the whole.

8. Partition (divide) the shape below to show hundredths.

   [Diagram of a shape divided into hundredths]
1. Write these numbers in words:
   a. 3.826  
      ____________________________  
   b. 607.012  
      ____________________________

2. Write these numbers in digits:
   a. Five, and three hundred and six thousandths  
      ____________________________  
   b. Eleven, and seven thousandths  
      ____________________________

3. Write the number that has:
   a. 5 tenths, 2 hundredths, 9 ones, 3 thousandths  
      ____________________________  
   b. 7 tens, 7 tenths  
      ____________________________  
   c. 2 thousandths, 8 tenths  
      ____________________________

4. Circle the number in which the 7 has the least value:
   a. 94  
   b. 376  
   c. 70.523  
   d. 1  
   e. 762  
   f. 1.762

5. In 625.078:
   a. the 2 has a value of  
      ____________________________ [place name]  
   b. the 8 has a value of  
      ____________________________ [place name]  
   c. the 50 has a value of  
      ____________________________ [place name]

6. Write ‘Yes’ or ‘No’ to each of the following:
   a. Does 7.2 have the same value as 7.020?  
      __________________  
   b. Does 7.002 have the same value as 07.002?  
      __________________  
   c. Does 7.02 have the same value as 7.020?  
      __________________  
   d. Does 7.2 have the same value as 0.72?  
      __________________

7. Write the number that is 1 thousandth more than:
   a. 3.563  
      ____________________________  
   b. 5.269  
      ____________________________  
   c. 0.09  
      ____________________________  
   d. 4.591  
      ____________________________  
   e. 6  
      ____________________________

Write the number that is 1 thousandth less than:
   a. 6.723  
      ____________________________  
   b. 4.26  
      ____________________________  
   c. 9.2  
      ____________________________  
   d. 3.701  
      ____________________________  
   e. 8  
      ____________________________
8. Complete the counting sequences:
   a. 8.527, 8.528, 8.529, _______, _______, _______
   b. 2.197, 2.198, 2.199, _______, _______, _______
   c. 2.803, 2.802, 2.801, _______, _______, _______

9. In each box below, tick the number that is worth more:
   a. 8.623   b. 0.956   c. 0.07   d. 3.421   e. 7.357
   2.898    0.279    0.600    6    7.62

10. Write any number that comes between:
    a. 18.283 and 18.5  _____________  b. 1.36 and 1.37  _____________

11. Write each set of numbers in ascending order (smallest to largest):
    a. 7.329, 6.201, 3.914  _____________
    b. 2.919, 0.399, 3.019, 3.09  _____________
    c. 4.73, 4.285, 4.6  _____________

12. Write the missing value or place name in each of the following:
    a. 2.614  
       |  Ones |  Tenths | Thousandths |
       |  703  |    65   |            |
    b. 70.365  
       |  Ones |  Tenths | Thousandths |
       |  62   |    3    |     7      |
    c. 2.007  
       |  Ones |  Tenths | Thousandths |
       |  200  |     7   |        |

13. Write the missing numbers:
    a. 5.017 = __________ thousandths
    b. 0.235 = 22 hundredths __________ thousandths
    c. 3 tenths 6 hundredths 17 thousandths = __________

14. Write the missing numbers:
    a. 0.02 × 1000 = __________
    b. 0.075 × 10 = __________
    c. 62 ÷ 1000 = __________
    d. 7.03 ÷ 10 = __________
    e. 3.256 × 100 = __________
    f. 0.481 × 1000 = __________
15. Write the missing operations (e.g., × 100 or ÷ 1000, and so on).
   a. \(0.04 \underline{\text{[operation]} } = 4\)
   b. \(6.3 \underline{\text{[operation]} } = 0.063\)
   c. \(62 \underline{\text{[operation]} } = 0.062\)
   d. \(7.0 \underline{\text{[operation]} } = 0.07\)
   e. \(6.75 \underline{\text{[operation]} } = 6750\)
   f. \(0.081 \underline{\text{[operation]} } = 81\)

16. Round each number to the nearest whole number:
   a. \(8.623 \underline{\text{[operation]} }\)
   b. \(0.347 \underline{\text{[operation]} }\)
   c. \(1.096 \underline{\text{[operation]} }\)

17. Do the following.
   a. Colour 0.9 of this shape.
   b. Colour 0.4 of this shape.
   c. This is 0.5 of a shape. Draw the whole shape.
   d. Circle the number below that shows about how much of the shape has been shaded.
   
   \[0.2 \ 0.5 \ 0.9\]
DECIMAL FRACTIONS 3B

Name: ______________________________ Year Level: __________ Date: __________

1. a. Show, with an arrow, where 7.285 metres could be on the number line below:
   
   6 m  C  7 m

   b. What measurement could be at C on the line above? ________________

2. Tick all the diagrams that show 2.4:
   a. 
   b. 
   c. 

3. a. Shade 0.6 of the shape below.
   
   b. Shade 0.28 of the shape below.

   

4. Colour 0.3 of the shape below.

   

3. Decimal Fractions
5. Answer the following about the shapes below:
   a. What decimal fraction of the shape is shaded?
   b. Colour 0.230 of this shape.

6. Answer the following in relation to the given shape:
   a. This is 0.1: __________
   b. This is 0.1: __________
   c. This is 0.1: __________

   Draw 0.01
   Draw 0.10
   Draw the whole.

7. Partition (divide) the shape below to show hundredths.
PROBABILITY 1

Name: ___________________________ Year Level: __________ Date: __________

1. Mummy says: It will be a nice sunny day for our picnic next Sunday.
   Tick the correct box to tell whether:
   - Sundays are always sunny.
   - Sundays are sometimes sunny.
   - Sundays are never sunny.

2. a. If you were playing a game with this spinner, what colours could you spin?
   Circle the colours: Green, Yellow, Pink, Blue, Red

   b. Would you always spin pink? Yes / No
   c. Is this a fair spinner to use in a game? Yes / No

3. a. What numbers could you get if you were playing a game with this normal die?

   b. Is it harder to get a 1 than a 6? Yes / No
   c. Could you sometimes get a 7 on a normal die? Yes / No

4. a. Colour the spinner below so that you will be certain to spin red.

   b. Colour the spinner below so that it is possible but not certain that you will spin red.

   c. Colour the spinner below so that it will be impossible to spin red.

5. a. Colour the marbles so that your friend would have no chance of picking a blue marble without looking.

   b. Colour the marbles so that your friend would have every chance of picking a blue marble without looking.

   c. Colour the marbles so that your friend would have some chance of picking a blue marble without looking.
If you were to close your eyes and select a toy animal from the bag, circle the animal you think you’d most likely get. Tell your teacher why.

Because
Because
Because
PROBABILITY 2

Name: ___________________________ Year Level: _________ Date: ____________

1. Your teacher says: It will be a nice sunny day for our sports day next Friday.
   Tick the correct box to tell whether:
   - Fridays are always sunny. ☐
   - Fridays are sometimes sunny. ☐
   - Fridays are never sunny. ☐

2. a. If you were playing a game with this spinner, what colours could you spin?
   Circle the colours: Green, Yellow, Pink, Blue, Red, Orange

   b. Would you always spin green? Yes / No
   c. Is this a fair spinner to use in a game? Yes / No

3. a. If you were playing a game with this spinner, what colours could you spin?
   Circle the colours: Green, Purple, Pink, Blue, Red, Orange

   b. Would you always spin pink? Yes / No
   c. Is this a fair spinner to use in a game Yes / No
   Why or Why not? ______________________________________________________

4. a. What numbers could you get if you were playing a game with this normal die?
   ______________________________________________________

   b. Is it harder to get a 1 than a 6? Yes / No
   c. Could you sometimes get a 7 on a normal die? Yes / No

5. a. Colour the spinner below so that you will be certain to spin red.
   b. Colour the spinner below so that it is possible but not certain that you will spin red.
   c. Colour the spinner below so that it will be impossible to spin red.
6. a. Colour the marbles so that your friend would have **no chance** of picking a blue marble without looking.

   ![Marbles with no blue ones]

   ![Marbles with no blue ones]

   ![Marbles with no blue ones]

   b. Colour the marbles so that your friend would have **every chance** of picking a blue marble without looking.

   ![Marbles with all blue ones]

   ![Marbles with all blue ones]

   ![Marbles with all blue ones]

   c. Colour the marbles so that your friend would have **some chance** of picking a blue marble without looking.

   ![Marbles with some blue ones]

   ![Marbles with some blue ones]

   ![Marbles with some blue ones]

7. If you were to close your eyes and select a toy animal from the bag, circle the animal you think you’d **most likely** get. Tell your teacher why.

   ![Bag of toy animals]

   ![Lion]

   ![Lion]

   ![Lion]

   **Because**

   ![Polar Bear]

   ![Polar Bear]

   ![Polar Bear]

   **Because**

   ![Elephant]

   ![Elephant]

   ![Elephant]

   **Because**
8. This is a game for 2 players. The player who colours the most bows on the kite’s tail is the winner. Decide who will be Yellow and who will be Blue.

Each player takes turns in spinning the pointer (10 spins per player). Each player must spin his or her own colour to colour a bow on his or her kite. For example, if the Blue player spins yellow then he or she cannot colour a bow on his/her kite’s tail.

Do you think the spinner is fair? Explain your thinking.

When you’ve finished the test, play the game with a friend!
9. This is a game for 2 players. The player who colours the most bows on the kite’s tail is the winner. Decide who will be Yellow and who will be Blue. Each player takes turns in spinning the pointer (10 spins per player). Each player must spin his or her own colour to colour a bow on his or her kite. For example, if the Blue player spins yellow then he or she cannot colour a bow on his/her kite’s tail.

Do you think the spinner is fair? Explain your thinking.

When you’ve finished the test, play the game with a friend!
1. Your teacher says: It will be a nice sunny day for our sports day next Friday. Tick the correct box to tell whether:

- it is possible that Friday will be a nice sunny day
- it is impossible that Friday will be a nice sunny day
- it is certain that Friday will be a nice sunny day

2. a. If you were playing a game with this spinner, circle the numbers you could spin.
   
   0, 1, 2, 3, 4, 5, 6, 7, 8, 9

   b. Is this a fair spinner to use in a game? Yes / No

3. a. If you were playing a game with this spinner, what numbers could you spin? Circle the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

   b. Would you always spin 4? Yes / No

   c. Is this a fair spinner to use in a game? Yes / No

   Why or Why not?

4. Using the spinner shown here, circle TRUE or FALSE for each question:
   a. You have an equal chance of getting a 3 or a 4. TRUE FALSE
   b. Getting a 3 is less likely than getting a 4. TRUE FALSE
   c. Getting a 3 is more likely than getting a 4. TRUE FALSE
   d. You will always get a 4 after you get a 3. TRUE FALSE

5. a. What numbers could you get if you were playing a game with this normal die?

   b. Is it harder to get a 1 than a 6? Yes / No

   c. If you rolled a die 600 times, circle how often you would expect to get a 6?

   10 times 50 times 100 times 300 times
6. a. Colour the spinner below so that you will be certain to spin red.

![Spinner A](image1)

b. Colour the spinner below so that it is possible but not certain that you will spin red.

![Spinner B](image2)

c. Colour the spinner below so that it will be impossible to spin red.

![Spinner C](image3)

7. a. Colour the marbles so that your friend would have no chance of picking a blue marble without looking.

![Bag A](image4)

b. Colour the marbles so that your friend would have every chance of picking a blue marble without looking.

![Bag B](image5)

c. Colour the marbles so that your friend would have some chance of picking a blue marble without looking.

![Bag C](image6)

8. Spinner A  

![Spinner A](image7)

Spinner B  

![Spinner B](image8)

Spinner C  

![Spinner C](image9)

Spinner D  

![Spinner D](image10)

a. On which spinner(s) is it impossible to spin purple (P)?

b. On which spinner(s) would you be:

1. More likely to spin blue than yellow?

2. Less likely to spin green than blue?

3. Just as likely to spin yellow as blue?

9. Draw lines to join the picture and the correct statement.

a. On this spinner, you will be just as likely to spin red as blue.

![Spinner 1](image11)

b. On this spinner, you will be more likely to spin red than blue.

![Spinner 2](image12)

c. On this spinner, you will be less likely to spin red than blue.

![Spinner 3](image13)
10. Using purple, pink and orange, colour the spinner below so that you would be:

Most likely to get pink and just as likely to get orange as purple.

11. Tick the spinner that will give you a better chance of getting blue.

Explain your choice.

12. If you were to flip a coin 100 times, write how many heads and how many tails you would be likely to get.
   a. Heads?  
   b. Tails?

Explain your thinking.
PROBABILITY 4

Name: ____________________________ Year Level: __________ Date: ___________

1. a. If you were playing a game with this spinner, what numbers **could** you spin?  
Circle the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

   ![Spinner Diagram]

b. Is this a **fair** spinner to use in a game? **Yes** / **No**  
   Why or Why not? ___________________________________________________________________

2. a. If you were playing a game with this spinner, what numbers **could** you spin?  
Circle the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

   ![Second Spinner Diagram]

b. Would you **always** spin 4? **Yes** / **No**

c. Is this a **fair** spinner to use in a game? **Yes** / **No**  
   Why or Why not? ___________________________________________________________________

3. Using the spinner shown here, circle **TRUE** or **FALSE** for each question:
   a. You have an **equal chance** of getting a 3 or a 4. **TRUE** **FALSE**
   b. Getting a 3 is **less likely** than getting a 4. **TRUE** **FALSE**
   c. Getting a 3 is **more likely** than getting a 4. **TRUE** **FALSE**
   d. You will **always** get a 4 after you get a 3. **TRUE** **FALSE**

4. a. What numbers **could** you get if you were playing a game with this normal die?

   ![Dice Diagram]

b. Is it harder to get a 1 than a 6? **Yes** / **No**

c. If you rolled a die 600 times, circle how often you would expect to get a 6?  
   10 times  50 times  100 times  300 times

5. a. Is there a bag of marbles (shown below) from which you could be certain to pick (without looking) a yellow marble? **YES** **NO**

b. If “**YES**”, which one/s (A, B, C)?

   ![Bag Diagrams]
6. **Spinner A**  
   ![Spinner A Image]

   **Spinner B**  
   ![Spinner B Image]

   **Spinner C**  
   ![Spinner C Image]

   **Spinner D**  
   ![Spinner D Image]

   a. On which spinner(s) is it impossible to spin purple?  
   b. On which spinner(s) would you be:
      - More likely to spin blue than yellow?  
      - Less likely to spin green than blue?  
      - Just as likely to spin yellow as blue?  

7. On the spinner, write the numbers 1, 3, 6 and 9 so that you would be:
   
   Most likely to get a 9, just as likely to get a 3 as a 6 but least likely to get a 1.

8. Draw lines to join the picture and the correct statement.
   a. On this spinner, you will be just as likely to spin red as blue.  
   b. On this spinner, you will be more likely to spin red than blue.  
   c. On this spinner, you will be less likely to spin red than blue.

9. Tick the spinner that will give you a better chance of getting blue.
   
   Explain your choice.
10. Does Spinner A give you the same chance of getting red as Spinner B?  
   Spinner B?  YES  NO  
   Explain your choice.  
   [Diagram of Spinner A and Spinner B]  
   [Diagram of Spinner A and Spinner B]

11. If you were to flip a coin 100 times, write how many heads and how many tails you would be likely to get.  
   a. Heads?  
   b. Tails?  
   Explain your thinking.  

12. If you are playing a game with two dice, would you be more likely to roll a 7 or 11?  
   Explain your thinking.  

[Diagram of two dice]
PROBABILITY 5

Name: ___________________________ Year Level: ______ Date: __________

1. a. If you were playing a game with this spinner, what numbers could you spin?
   Circle the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   b. Is this a fair spinner to use in a game? Yes / No
      Why or Why not? _____________________________________________

2. a. If you were playing a game with this spinner, what numbers could you spin?
   Circle the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
   b. Would you always spin 4? Yes / No
   c. Is this a fair spinner to use in a game? Yes / No
      Why or Why not? _____________________________________________

3. Using the spinner shown here, circle TRUE or FALSE for each question:
   a. You have an equal chance of getting a 3 or a 4. TRUE FALSE
   b. Getting a 3 is less likely than getting a 4. TRUE FALSE
   c. Getting a 3 is more likely than getting a 4. TRUE FALSE
   d. You will always get a 4 after you get a 3. TRUE FALSE

4. a. What numbers could you get if you were playing a game with this normal die?
   b. Is it harder to get a 1 than a 6? Yes / No
   c. If you rolled a die 600 times, ring how often you would expect to get a 6?
      10 times 50 times 100 times 300 times

5. a. Is there a bag of marbles (shown below) from which you could be certain to pick (without looking) a yellow marble? YES NO
   b. If “YES”, which one/s (A, B, C)? _____________________________
6. **Spinner A**  
   ![Spinner A Diagram]

   **Spinner B**  
   ![Spinner B Diagram]

   **Spinner C**  
   ![Spinner C Diagram]

   **Spinner D**  
   ![Spinner D Diagram]

   a. On which spinner(s) is it impossible to spin purple(P)?

   b. On which spinner(s) would you be:
      - More likely to spin blue than yellow?
      - Less likely to spin green than blue?
      - Just as likely to spin yellow as blue?

7. On the spinner, write the numbers 1, 3, 6 and 9 so that you would be:

   Most likely to get a 9, just as likely to get a 3 as a 6 but least likely to get a 1.

8. Draw lines to join the picture and the correct statement.

   a. On this spinner, you will be just as likely to spin red as blue.

   b. On this spinner, you will be more likely to spin red than blue.

   c. On this spinner, you will be less likely to spin red than blue.

9. Does Spinner A give you the same chance of getting blue as Spinner B?  
   YES   NO

   Explain your choice.
10. Does Spinner A give you the same chance of getting red as Spinner B?  
YES  NO

Explain your choice.

11. If you were to flip a coin 100 times, write how many heads and how many tails you would be likely to get.
   a. Heads?  
   b. Tails?

Explain your thinking.

12. If you are playing a game with two dice, would you be more likely to roll a 7 or 11? 

Explain your thinking.

13. Draw marbles in the bag so that the probability of getting a red marble is:
   a. \( \frac{2}{5} \)
   b. 1
   c. 0
   d. more than \( \frac{1}{2} \)
14. Draw lines to connect these words to an appropriate place on the number line below.

impossible  certain  unlikely  likely  even chance  more chance  50-50

0  1

15. Box A below has 1 Mars bar and 4 Snickers; Box B has 2 Mars bars and 6 Snickers. (You cannot see through the boxes)

Box A

Box B

If you want to get a Mars bar when you take one out, which box is best to choose from, or do they both give you the same chance of getting a Mars bar?

Tick your answer. Either box  Box A  Box B

Explain your thinking.

16. Rhonda is thinking of a number between 1 and 10 and challenges Tom to guess it. She said, ‘I’ll give you a clue. The number is greater than 6.’

Has this clue given Tom a better chance of guessing the number or would it not make any difference?

Tick your answer. Has helped  Hasn’t helped

Explain your thinking.
## ANSWERS and OBJECTIVES

### 1. NUMBER (Refer To Overview and Theory, Section 3)

**Number 1A**

**Teacher notes:** Number 1A is designed to be administered by the teacher or teacher assistant one item at a time to one child at a time.

**Materials required:** Pencil and paper; number cards (0–10); number-name cards (zero–ten).

**Answers/Objectives:** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open</td>
<td>Rote counting — checking whether students can count correctly</td>
</tr>
<tr>
<td>2</td>
<td>(a) 3</td>
<td>(b) 7</td>
</tr>
<tr>
<td>3</td>
<td>(a) two</td>
<td>(b) six</td>
</tr>
<tr>
<td>4</td>
<td>(a) 4</td>
<td>(b) 8</td>
</tr>
<tr>
<td>5</td>
<td>(a) five</td>
<td>(b) nine</td>
</tr>
<tr>
<td>6</td>
<td>(a) 5</td>
<td>(b) 7</td>
</tr>
<tr>
<td>7</td>
<td>(a) 7</td>
<td>(b) 0</td>
</tr>
<tr>
<td>8</td>
<td>(a) 7</td>
<td>(b) 5</td>
</tr>
<tr>
<td>9</td>
<td>0, 2, 4, 6, 7, 9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(a) 7</td>
<td>(b) 6</td>
</tr>
<tr>
<td>11</td>
<td>(a) 4</td>
<td>(b) 8</td>
</tr>
</tbody>
</table>
Number 1B

Teacher notes: Number 1B is designed to be administered by the teacher or teacher assistant one item at a time to one child at a time.

Materials required—Counters (same colour); pencil and paper; number-name cards.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 10</td>
<td>(b) 7</td>
</tr>
<tr>
<td>2</td>
<td>(a) 7</td>
<td>(b) 7</td>
</tr>
<tr>
<td>3</td>
<td>(a) 5</td>
<td>(b) 8</td>
</tr>
<tr>
<td>4</td>
<td>(a) four</td>
<td>(b) seven</td>
</tr>
<tr>
<td>5</td>
<td>(a) 3</td>
<td>(b) 7</td>
</tr>
<tr>
<td>6</td>
<td>(a) four</td>
<td>(b) ten</td>
</tr>
<tr>
<td>7</td>
<td>(a) Plate 2: 5</td>
<td>(b) Plate 1: 8</td>
</tr>
<tr>
<td>8</td>
<td>(a) six, seven</td>
<td>(b) four, five, six</td>
</tr>
<tr>
<td>9</td>
<td>(a) five, four</td>
<td>(b) ten, nine, eight</td>
</tr>
</tbody>
</table>
# Number 2A

**Teacher notes:** Read each item to the class.

**Materials required** — Pencil and paper.

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 67</td>
<td>(b) 50</td>
</tr>
<tr>
<td>2</td>
<td>(a) eighty-three</td>
<td>(b) thirty</td>
</tr>
<tr>
<td>3</td>
<td>(a) 38</td>
<td>(b) 67</td>
</tr>
<tr>
<td>4</td>
<td>(a) 054</td>
<td>(b) 540; 504</td>
</tr>
<tr>
<td>5</td>
<td>(a) 69, 70, 71</td>
<td>(b) 100, 101, 102</td>
</tr>
<tr>
<td>6</td>
<td>(a) 52</td>
<td>(b) 16</td>
</tr>
<tr>
<td>7</td>
<td>(a) 82</td>
<td>(b) 47</td>
</tr>
<tr>
<td>8</td>
<td>14, 39, 40, 41, 47</td>
<td><strong>Ordering</strong> — 2-digit numbers from smallest to largest (numbers having characteristics of Item 7)</td>
</tr>
<tr>
<td>9</td>
<td>(a) 14</td>
<td>(b) 2</td>
</tr>
<tr>
<td>10</td>
<td>(a) tens</td>
<td>(b) ones</td>
</tr>
<tr>
<td>11</td>
<td>(a) 10</td>
<td>(b) 10</td>
</tr>
<tr>
<td>12</td>
<td>(a) 40</td>
<td>(b) 20</td>
</tr>
<tr>
<td>13</td>
<td>45, 55, 48</td>
<td><strong>Rounding</strong> — 2-digit numbers to a given ten (rounded number ➔ number; reverse of Item 12)</td>
</tr>
</tbody>
</table>
Number 2B

Teacher notes: Read each item to the class.

Materials required — Pencil and paper.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 3; 2</td>
<td>Early grouping — forming and reading groups and ones (group size less than 10)</td>
</tr>
<tr>
<td></td>
<td>(b) 3; 5; (c) 3; 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) 4 groups of 6 in Ones place; 2 single ones in Ones place</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(a) forty-three</td>
<td>Place value — pictorial → language (number name) for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(b) sixteen</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) sixty</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) 67</td>
<td>Place value — pictorial → number for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(b) 13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 30</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Circle: (a) 6 tens, 5 ones</td>
<td>Place value — number → pictorial for 2-digit numbers (reverse of Item 3)</td>
</tr>
<tr>
<td></td>
<td>(c) 7 tens 0 ones</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) 1 ten, 2 ones</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Circle: (a) 4 tens, 1 one</td>
<td>Place value — number name → pictorial for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(c) 6 tens, 0 ones</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) 1 ten, 8 ones</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a) seventy-three</td>
<td>Place value — number → number name for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(b) eleven</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) eighty</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a) 5 tens 8 ones</td>
<td>Place value — number name → number for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(b) 1 ten 5 ones</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 5 tens 0 ones</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(a) 57</td>
<td>Seriality (10 more/10 less, 1 more /1 less) — pictorial → number for 2-digit numbers</td>
</tr>
<tr>
<td></td>
<td>(b) 37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) 46</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(a) 34</td>
<td>Regrouping tens and ones — pictorial → number (a); number → pictorial (b, c)</td>
</tr>
<tr>
<td></td>
<td>(c) 23 ones circled</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(a) × 10</td>
<td>Multiplicative structure — knowing the relationship between adjacent places (one shift to the left makes the digit 10 times larger in value; one shift to the right makes the digit 10 times smaller in value)</td>
</tr>
<tr>
<td></td>
<td>(b) ÷ 10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(a) 70</td>
<td>Multiplicative structure — knowing the relationship between adjacent places (reverse of Item 10)</td>
</tr>
<tr>
<td></td>
<td>(b) 4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(a) 20</td>
<td>Rounding — 2-digit numbers to the nearest 10</td>
</tr>
<tr>
<td></td>
<td>(b) 60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 80 or 90</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>17 34 71 85</td>
<td>Estimating — positions on a number line using benchmarks (half of 10)</td>
</tr>
<tr>
<td></td>
<td>0 10 20 30 40 50 60 70 80 90 100</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0 10 20 30 40 50 60 70 80 90 100</td>
<td>Estimating — reverse of Item 13</td>
</tr>
<tr>
<td></td>
<td>16/17 62/63 95</td>
<td></td>
</tr>
</tbody>
</table>

CDAT Answers and Objectives
### Number 3A

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil and paper.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 349 (b) 540 (c) 212</td>
<td>Identification (number name → number) — reading 3-digit numbers (‘normal’, zero and teen numbers)</td>
</tr>
<tr>
<td></td>
<td>(d) 806</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(a) two hundred and sixty-eight (b) three hundred and three (c) eight hundred and fourteen</td>
<td>Identification (number → number name; reverse of Item 1) — writing 3-digit numbers (normal, zero, teen)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) 397 (b) 484 (c) 506</td>
<td>Place value (place value → number) — identifying a 3-digit number from place-value positions given out of sequence</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(a) 0368 (b) 3680; 3608; 3068</td>
<td>Place value — understanding the role of zero in whole numbers (when a zero changes/not changes a number’s value)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(a) 738 (b) 585 (c) 410 (d) 490 (e) 139</td>
<td>Seriation — determining 1/10/100 more or less than a given 3-digit number</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a) 289, 290, 291 (b) 896, 996, 1096 (c) 430, 429, 428 (d) 800, 790, 780</td>
<td>Seriation — counting by 1s, 10s, 100s forwards and backwards for 3-digit numbers</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a) 404 (b) 556 (c) 674 (d) 840</td>
<td>Comparing/ordering — comparing two 3-digit numbers for a variety of situations (e.g. internal/end zeros; ‘different length’ numbers)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>652, 650, 625, 615, 605</td>
<td>Comparing/ordering — ordering 3-digit numbers from largest to smallest value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>726</td>
<td>Comparing/ordering — linking to place value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>(a) 2; 8 (b) 14 (c) 596 (d) 4 (e) 11 (f) 591</td>
<td>Regrouping — 3-digit numbers in both directions (identifying individual places and constructing the number)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(a) tens (b) hundreds (c) tens (d) hundreds (e) tens</td>
<td>Multiplicative structure — determining how place value positions change as a result of multiplying and dividing numbers by 10 (× 10, ÷ 10 → place-value change)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(a) × 10 (b) × 10 (c) × 10 (d) ÷ 100 (e) × 100 (f) ÷ 10</td>
<td>Multiplicative structure — determining the operation that makes a place-value position change (place-value change → × 10, ÷ 10; reverse of Item 11)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(a) 690 (b) 720 or 730 (c) 300</td>
<td>Rounding — 3-digit numbers to the nearest 10 (number → rounded number)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(a) 500 (b) 200 (c) 800 or 900</td>
<td>Rounding — 3-digit numbers to the nearest 100 (number → rounded number)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>267, 325, 250</td>
<td>Rounding — rounded number → number; reverse of Item 14</td>
</tr>
</tbody>
</table>
### Number 3B

**Teacher notes:** Read each item to the class.

*Item 8 is an ordering activity, not an estimation activity, therefore exact/approximate positions of the individual numbers are not essential.

**Materials required —** Pencil and paper.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives/</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) two hundred and thirty-five  (b) five hundred and thirteen  (c) three hundred  (d) four hundred and fifty  (e) seven hundred and eight</td>
<td>Place value — pictorial → number name for 3-digit numbers</td>
</tr>
<tr>
<td>2</td>
<td>(a) 358  (b) 530  (c) 800  (d) 505</td>
<td>Place value — pictorial → number for 3-digit numbers</td>
</tr>
<tr>
<td>3</td>
<td>(a) 2 hundreds, 6 tens, 1 one  (b) 9 hundreds, 0 tens, 5 ones</td>
<td>Place value — number name → pictorial for 3-digit number names (reverse of Item 1)</td>
</tr>
<tr>
<td>4</td>
<td>(a) 7 hundreds, 0 tens, 4 ones  (b) 7 hundreds, 4 tens, 0 ones  (c) 7 hundreds, 1 ten, 4 ones</td>
<td>Place value — number pictorial for 3-digit numbers (reverse of Item 1)</td>
</tr>
<tr>
<td>5</td>
<td>(a) four hundred and fifteen  (b) three hundred and six  (c) eight hundred and thirty</td>
<td>Place value — number name for 3-digit numbers represented on a PV Chart</td>
</tr>
<tr>
<td>6</td>
<td>(a) 2 hundreds 4 tens 0 ones  (b) 5 hundreds 1 ten 1 one  (c) 6 hundreds 0 tens 9 ones</td>
<td>Place value — number name for 3-digit numbers names (reverse of Item 5)</td>
</tr>
<tr>
<td>7</td>
<td>(a) 577  (b) 667  (c) 566  (d) 557</td>
<td>Seriation — pictorial → number (100 more/100 less; 10 more/10 less; 1 more/1 less)</td>
</tr>
<tr>
<td>8*</td>
<td><img src="image" alt="Number Line" /></td>
<td>Comparing/Ordering — numbers on a number line</td>
</tr>
<tr>
<td>9</td>
<td>(a) 436  (b) Need 3 H  (c) Need 15 T</td>
<td>Regrouping — ones as tens/ones, tens as hundreds/tens and reverse; pictorial → number (a); number pictorial (b, c)</td>
</tr>
<tr>
<td>10</td>
<td>(a) × 10  (b) ÷ 100  (c) ÷ 10  (d) 7 hundreds 4 tens 0 ones  (e) 5 ones  (f) 9 hundreds 0 tens 0 ones</td>
<td>Multiplicative structure — knowing the relationship between adjacent and non-adjacent places (one/two shift/s to the left makes the digit 10/100 times larger in value; one/two shift/s to the right makes the number 10/100 times smaller in value)</td>
</tr>
<tr>
<td>11</td>
<td>(a) 100  (b) 400  (c) 700 or 800</td>
<td>Rounding — 3-digit numbers to the nearest 100 on a 0 to 1000 number line with benchmarks of 50 and 100</td>
</tr>
<tr>
<td>12</td>
<td>(a) 100 to 120  (b) 450 to 480  (c) 890 to 915</td>
<td>Estimating — numbers on a 0 to 1000 number line (reverse of Item 11)</td>
</tr>
</tbody>
</table>
### Number 4A

**Teacher notes:** Read each item to the class.

**Materials required** — Pencil and paper.

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) 3749  
(d) 8306  
(e) 4014 | **Identification** (number name → number) — for 4-digit numbers (‘normal’, zero and teen numbers) |
|      | (b) 5240  
(c) 2093  
(e) 4014 | |
|      | (c) six thousand and seven | |
|      | (d) five thousand, two hundred and sixty-eight | **Identification** (number → number name; reverse of Item 1) — for normal, zero and teen numbers. |
| 2    | (a) 2896  
(b) 2896  
(c) 5314  
(d) 5294 | **Serialisation** — counting by 1s, 10s, 100s forwards/backwards by ones, tens and hundreds |
|      | (b) 2996  
(b) 2996  
(c) 5304  
(d) 5294 | |
| 3    | (a) 6397  
(b) 2484 | **Place value** — identifying a 4-digit number from place-value positions out of sequence. |
|      | (c) 410  
(d) 4990 | |
| 4    | (a) 0542, 542.0  
(b) 5420, 5402, 5042 | **Place value** — understanding the role of zero in 4-digit numbers (when a zero changes/not changes a number’s value) |
|      | (b) 6185  
(c) 410  
(d) 4990 | |
| 5    | (a) 8282  
(b) 6185  
(c) 410  
(d) 4990 | **Serialisation** — determining 10/100 more or less than a given 4-digit number |
| 6    | (a) 4289, 4290, 4291  
(b) 2896, 2996, 3096  
(c) 5314, 5304, 5294 | **Serialisation** — determining 10/100 more or less than a given 4-digit number |
|      | (b) 2896, 2996, 3096  
(c) 5314, 5304, 5294 | |
| 7    | (a) 2404  
(b) 1556  
(c) 8674  
(d) 5640 | **Comparing** — 4-digit whole numbers (internal/end zeros; ‘different length’ numbers) |
| 8    | 3605, 3625, 3650, 3652 | **Comparing/ordering** — 4-digit numbers from smallest to largest value |
| 9    | 7104 | **Comparing/ordering** — linking to place value |
| 10   | (a) 2 : 8  
(b) 14  
(c) 17  
(d) 13 | **Regrouping** — ones as tens/ones, tens as hundreds/tens and hundreds as thousand/hundreds |
| 11   | (a) thousands  
(b) ones  
(c) thousands  
(d) tens  
(e) hundreds  
(f) tens | **Multiplicative structure** — determining how place value positions change as a result of × and ÷ numbers by 10/100 /1000 (×/÷ → place-value change) |
| 12   | (a) × 1000  
(b) × 10  
(c) × 100  
(d) ÷ 10  
(e) × 1000  
(f) ÷ 100 | **Multiplicative structure** — determining how place value positions change as a result of multiplying and dividing numbers by 10/100 /1000 (place-value change → ×/÷ ; reverse of Item 11) |
| 13   | (a) 3000  
(b) 5000  
(c) 9000 | **Rounding** — to the nearest 1000 |
| 14   | (a) 4300  
(b) 5000  
(c) 9000 | **Rounding** — to the nearest 100 |
| 15   | 3380, 3250 | **Rounding** — to the nearest 1000 |
Number 4B

Teacher notes: Read each item to the class.

*Item 7 is an ordering activity, not an estimation activity, therefore exact/approximate positions of the individual numbers are not essential.

Materials required — Pencil and paper.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) 4347; four thousand, three hundred and forty-seven  
(b) 8013; eight thousand and thirteen  
(c) 3809; three thousand, eight hundred and nine  
(d) 5470; five thousand, four hundred and seventy | Identification — pictorial \( \rightarrow \) number \( \rightarrow \) number name |
| 2    | (a) 2 Th, 3 H, 6 T, 4 Ones  
(b) 5 Th, 1 T, 5 Ones  
(c) 6 Th, 3 H, 7 T  
(d) 4 Th, 4 T | Place value — number name \( \rightarrow \) pictorial; (reverse of Item 1) |
| 3    | (a) 5 Th, 6 H, 7 T, 4 Ones  
(b) 1 Th, 6 H, 1 T, 6 Ones  
(c) 7 Th, 7 T  
(d) 4 Th, 4 Ones | Place value — number \( \rightarrow \) pictorial; (reverse of Item 1) |
| 4    | (a) four thousand, seven hundred and sixty-five  
(b) three thousand, five hundred and one  
(c) six thousand and sixty  
(d) two thousand and twelve | Place value — number \( \rightarrow \) number name represented on a PV chart |
| 5    | (a) 6 Th, 3 H, 5 T, 7 Ones  
(b) 3 H, 1 T, 9 Ones  
(c) 4 Th, 5 H, 0 T, 8 Ones  
(d) 6 Th, 0 H, 0 T, 6 Ones | Place value — number name \( \rightarrow \) number represented on PV Chart (reverse of item 4) |
| 6    | (a) 5694  
(b) 5784  
(c) 4684  
(d) 5683  
(e) 5674  
(f) 6684 | Seriation — pictorial \( \rightarrow \) number (1/10/100/1000 more and/or less) than a given number |
| 7*   | 5072  
5720  
8619  
8961 | Comparing/ordering — numbers on a number line |
| 8    | (a) 4276  
(b) 3 thousands  
(c) 22 tens | Regrouping — tens as hundreds/tens and hundreds as thousands/hundreds |
| 9    | (a) \( \times \) 1000  
(b) \( \div \) 100  
(c) 9 Ones  
(d) 6 H, 0 T, 0 Ones | Multiplicative structure — knowing that the relationship between adjacent and non-adjacent places (one/two/three shifts to the left makes the number 10/100/1000 times larger in value; one/two/three shifts to the right makes the number 10/100/1000 times smaller in value |
| 10   | (a) 2000  
(b) 5000  
(c) 8000 or 9000 | Rounding — to the nearest 1000 on a 1000—10 000 number line benchmarked in 1000s |
| 11   | (a) 700 to 1000  
(b) 4200 to 4600  
(c) 7800 to 8200 | Estimating — numbers on a 0—10 000 number line (reverse of Item 10) |
**Teacher notes:** Read each item to the class.

**Materials required —** Pencil and paper.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 463 161 749 (b) 16 442 093 (c) 87 306 (d) 36 200 488 (e) 500 240 (f) 117 000 411</td>
<td>Identification (number name → number) — writing large numbers (no zeros, zero, teen numbers)</td>
</tr>
<tr>
<td>2</td>
<td>(a) five million, six hundred and seventy-five thousand, two hundred and sixty-eight (b) thirty million, twenty-three thousand and six (c) four hundred thousand and forty (d) fifteen million, seventeen thousand and twelve (e) five hundred and sixty million, six hundred and seventy thousand, two hundred</td>
<td>Identification (number → number name; reverse of item 1) — reading large numbers (no zeros, zero, teen)</td>
</tr>
<tr>
<td>3</td>
<td>(a) 822 484 (b) 94 256 397 (c) 3 064 702 (d) 500 700 640</td>
<td>Place value — identifying large numbers from place-value positions given out of sequence</td>
</tr>
<tr>
<td>4</td>
<td>(a) 05 435 421 or a decimal number 5 435 421.0, 5 435 421.00, etc (b) any internal zero (e.g. 50 435 421), external right (54 354 210)</td>
<td>Place value — understanding the role of zero (when a zero changes/not changes a number's value)</td>
</tr>
<tr>
<td>5</td>
<td>(a) 734 682 (b) 626 851 446 (c) 32 067 086 (d) 559 450 081 (e) 29 996 429</td>
<td>Seriation — determining the number that is 1 more and 1 less in any place value up to a hundred million</td>
</tr>
<tr>
<td>6</td>
<td>(a) 34 289, 34 290 (b) 5 404 218, 5 394 218 (c) 679 896, 679 996 (d) 240 072 135, 240 172 135 (e) 40 006 761, 39 996 761</td>
<td>Seriation — counting large numbers forwards and backwards [See Teaching Notes.]</td>
</tr>
<tr>
<td>7</td>
<td>(a) 2 941 679 (b) 235 065 (c) 2 867 426 (d) 584 067 002</td>
<td>Comparing/ordering — comparing two large numbers for a variety of number types</td>
</tr>
<tr>
<td>8</td>
<td>3 621 568, 36 059 568, 36 519 568, 36 521 568</td>
<td>Comparing/ordering — large numbers from smallest to largest</td>
</tr>
<tr>
<td>9</td>
<td>7 104 298</td>
<td>Comparing/ordering — linking to place value</td>
</tr>
<tr>
<td>10</td>
<td>(a) 543 thousands, 768 ones (b) 45 millions, 63 063 thousands, 501 ones (c) 12 tens, 8 ones (or 11 tens, 18 ones, etc.) (d) 23 ten-thousands, 14 ones (e) 2 ten-millions, 4 tens</td>
<td>Regrouping — places [See teaching Notes]</td>
</tr>
<tr>
<td>11</td>
<td>(a) thousands (b) hundred-thousands (c) millions (d) ones (e) ten-thousands (f) tens</td>
<td>Multiplicative structure — determining how place value positions change as a result of multiplying and dividing numbers by 1 000 and 1 000 000 (×/÷; reverse of Item 11).</td>
</tr>
<tr>
<td>12</td>
<td>(a) 1000 (b) 1000 (c) 1000 (d) 1 000 000 (e) 1000 (f) 1 000 000</td>
<td>Multiplicative structure — determining the operation that makes a place-value position change (place-value change → ×/÷; reverse of Item 11).</td>
</tr>
<tr>
<td>13</td>
<td>(a) 370 000 (b) 4 720 000 (c) 9 340 000 or 9 350 000</td>
<td>Rounding — to the nearest 10 000 (number → rounded number)</td>
</tr>
<tr>
<td>14</td>
<td>(a) 46 000 000 (b) 93 000 000 (c) 687 000 000 or 688 000 000</td>
<td>Rounding — to the nearest 1 000 000 (number → rounded number).</td>
</tr>
<tr>
<td>15</td>
<td>(b) 29 675 324 (d) 30 445 372</td>
<td>Rounding — to a given 10 million (rounded number → number; reverse of Items 13 and 14)</td>
</tr>
</tbody>
</table>
Number 5B

Teacher notes: Read each item to the class.

* Item 5 is an ordering activity, not an estimation activity, therefore exact/approximate positions of the individual numbers are not essential.

Materials required — Pencil and paper.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) four hundred and fifty-one million, three hundred and twenty-eight thousand, four hundred and seventy-seven</td>
<td>Identification — (number (\rightarrow) number name) writing large numbers in words</td>
</tr>
<tr>
<td></td>
<td>(b) forty million, nine hundred and seventeen thousand and twenty</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) two hundred and eleven million, twenty-eight thousand and twelve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) sixty million and sixty</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(a) 45 261 784</td>
<td>Identification — (number name (\rightarrow) number) writing large numbers in digit form (reverse of Item 1)</td>
</tr>
<tr>
<td></td>
<td>(b) 270 305 029</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 5 000 500</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) 39 096 127</td>
<td>Seriation — determining the number that is more/less by a power of 10 than a given number; recognising which place/s change and which don’t</td>
</tr>
<tr>
<td></td>
<td>(b) 39 086 027</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 40 086 127</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) 38 986 127</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(e) 29 086 127</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(f) 39 086 137</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>56 782 436</td>
<td>Seriation — determining 10 000 less than a given number; knowing that places to the right of the TTh place don’t change</td>
</tr>
<tr>
<td>5*</td>
<td>5 869 488</td>
<td>Comparing/ordering — numbers on a number line</td>
</tr>
<tr>
<td></td>
<td>24 643 282</td>
<td></td>
</tr>
<tr>
<td></td>
<td>61 897 434</td>
<td></td>
</tr>
<tr>
<td></td>
<td>99 120 541</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a) 5 462 372</td>
<td>Regrouping — determining which places change/don’t change when a place contains a number larger than 9</td>
</tr>
<tr>
<td></td>
<td>(b) 64 688 473</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 794 750 016</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a) (\times) 1000</td>
<td>Multiplicative structure — knowing the relationship between adjacent and non-adjacent places (one/two/three/… shifts to the left/right makes the number 10, 100, 1000… times large/smaller in value)</td>
</tr>
<tr>
<td></td>
<td>(b) (\div) 1 000 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) (\div) 100</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(a) 347 000 000</td>
<td>Multiplicative structure — writing the number that results from (\times/\div) by 10, 100, 1000 etc (reverse of Item 7)</td>
</tr>
<tr>
<td></td>
<td>(b) 23 467 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 38 800</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(a) 30 000 000</td>
<td>Rounding — to the nearest 10 million on a 0—100 million number line benchmarked in 10 millions</td>
</tr>
<tr>
<td></td>
<td>(b) 50 000 000 or 60 000 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 80 000 000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(a) 5 to 8 million</td>
<td>Estimating — numbers on a 0—100 million number line (reverse of Item 9)</td>
</tr>
<tr>
<td></td>
<td>(b) 42 to 47 million</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 84 to 88 million</td>
<td></td>
</tr>
</tbody>
</table>
### 2. COMMON FRACTIONS (Refer to Overview and Theory, Section 4)

#### Common Fractions 1A

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) 3 quarters  
(b) 1 (or 2 halves) | Seriation — 1 part more                        |
| 2    | (a) 3 quarters, 4 quarters, 5 quarters  
(b) 5 fifths, 6 fifths, 7 fifths | Counting                                       |
| 3    | (a) 6  
(b) Yes | Relating — fraction name to number of parts and that parts must be equal |
| 4    | 2 | Relating — the whole to a number of equal parts |
| 5    | (a) 7 tenths  
(b) 1 half  
(c) 1 whole | Comparing — different number/same name; same number/different name; different number/different name |
| 6    | 1 tenth | Comparing — same number/different name cannot be equal; renaming halves as quarters |
| 7    | 1 tenth, 1 quarter, 1 half | Ordering — unit fractions (1 part) with different names (more parts in the whole, the smaller the fraction) |
| 8    | 3 quarters | Estimating to the whole                        |

#### Common Fractions 1B

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B, C, D, H, I</td>
<td>Identifying fractions — number name pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
</tbody>
</table>
| 2    | (a) 1 third  
(b) 1 half  
(c) 3 quarters  
(d) 2 halves | Identifying fractions — pictorial number name (fraction); reverse of Item 1 |
| 3    | (a) 1 of the 4 equal parts  
(b) 1 of the 3 cherries  
(c) 3 of the 6 equal parts | Identifying fractions |
| 4    | A ribbon twice the size of the one shown | Constructing — the whole from a part (reverse of Item 3) |
| 5    | Any two of the following ways: | Relating — the fraction name to the number of equal parts in the whole; developing flexible thinking |
| 6    | | Relating — the fraction name to the number of equal parts in the whole |
Common Fractions 2A

Teacher notes: Read each item to the class.

Materials required — Pencil.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) four fifths or 4 fifths  &lt;br&gt;(b) seven eighths or 7 eighths</td>
<td>Identification — number (fraction) to number name (fraction name)</td>
</tr>
<tr>
<td>2</td>
<td>(a) $\frac{2}{6}$  &lt;br&gt;(b) $\frac{1}{4}$  &lt;br&gt;(c) 8/3</td>
<td>Identification — number name to number; reverse of item 1</td>
</tr>
<tr>
<td>3</td>
<td>(a) $\frac{2}{4}$ (preferred as it maintains the name) or $\frac{1}{2}$  &lt;br&gt;(b) $\frac{1}{4}$ (preferred as it maintains the name) or 1</td>
<td>Seriation — adding 1 part (quarter) more</td>
</tr>
<tr>
<td>4</td>
<td>(a) $\frac{4}{4}$, $\frac{6}{6}$, $\frac{8}{8}$  &lt;br&gt;(b) 5 fifths, 6 sixths, 7 sevenths</td>
<td>Counting — fractions in digit and word form</td>
</tr>
<tr>
<td>5</td>
<td>Any number between $\frac{1}{2}$ and 1, most common would be $\frac{3}{4}$, $\frac{2}{3}$ but could also be for example, $\frac{3}{5}$, $\frac{5}{6}$, etc.</td>
<td>Ordering — identifying fractions that lie between a half and 1</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>Relating — the number of equal parts in a whole to the fraction name</td>
</tr>
<tr>
<td>7</td>
<td>(a) $\frac{5}{6}$  &lt;br&gt;(b) $\frac{1}{2}$  &lt;br&gt;(c) $\frac{1}{2}$  &lt;br&gt;(d) 1</td>
<td>Comparing — unit and non-unit fractions with the same name, different number of parts; different names, same number of parts; fraction to a whole</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{7}{10}$</td>
<td>Comparing — fractions with different denominators with $\frac{1}{2}$</td>
</tr>
<tr>
<td>9</td>
<td>$\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$</td>
<td>Ordering unit fractions — knowing that the more parts in a whole (denominator), the smaller the part (reverse of whole numbers)</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>Estimating — knowing that proper fractions lie between 0 and 1 and benchmarking to $\frac{1}{2}$</td>
</tr>
</tbody>
</table>
### Common Fractions 2B

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B, C, D, H, I</td>
<td>Identifying fractions — number name (fraction) → pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
<tr>
<td>2</td>
<td>(a) half (b) quarters (c) halves</td>
<td>Identifying fractions — pictorial → number name (fraction); reverse of Item 1</td>
</tr>
<tr>
<td>3</td>
<td>(a) ( \frac{1}{3} ) (b) ( \frac{2}{4} ) (c) ( \frac{3}{2} ) (d) ( \frac{10}{10} ) [Informal recording (e.g. 1 third) is very acceptable.]</td>
<td>Identifying fractions — pictorial → number/fraction; items include: contingent and non-contiguous shading; unit and non-unit fractions; whole</td>
</tr>
<tr>
<td>4</td>
<td>(a) Colour 3 of the 5 equal parts (b) Colour 3 of the 8 equal parts (c) Colour 1 of the 3 cherries (d) Colour 1 whole and 3 out of 4 equal parts of the second whole (internal partitioning must be evident — colouring such as that in the diagram is not acceptable)</td>
<td>Identifying fractions — number → pictorial</td>
</tr>
<tr>
<td>5</td>
<td>A ribbon four times the length of the one shown. The purpose is to identify that there must be 4 equal parts so it doesn’t matter whether the students place the 4 parts differently (e.g. on top of each other).</td>
<td>Constructing — the whole from the part.</td>
</tr>
<tr>
<td>6</td>
<td>(a) thirds, sixths (b) YES</td>
<td>Identifying equivalent fractions — pictorial → fraction name</td>
</tr>
<tr>
<td>7</td>
<td>Any three of the following ways:</td>
<td>Partitioning — whole → equal parts in different ways (flexible thinking); reverse of Item 5</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>Repartitioning — the whole (re-unitising quarters as eighths)</td>
</tr>
</tbody>
</table>
# Common Fractions 3A

**Teacher notes:** Read each item to the class.  
**Materials required —** Pencil.  
**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) four fifths or 4 fifths  
       (b) seven eighths or 7 eighths | Identification — number (fraction) to number name (fraction name) |
| 2    | (a) $\frac{1}{4}$  
       (b) $\frac{2}{5}$  
       (c) $\frac{3}{5}$ (preferred to $\frac{2}{5}$ in this activity) | Identification — number name to number; reverse of item 1 |
| 3    | (a) $\frac{2}{5}$ (preferred as it maintains the name) or $\frac{1}{2}$  
       (b) $\frac{3}{4}$ (preferred as it maintains the name) or 1 | Seriation — adding 1 part (quarter) more |
| 4    | (a) $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{6}$, $\frac{4}{7}$  
       (b) 5 fifths, 6 sixths, 7 sevenths | Counting — fractions in digit and word form |
| 5    | (a) 2 sixths, 3 sixths or $\frac{2}{6}$, $\frac{3}{6}$ (Others could be: $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{6}{12}$ …)  
       (b) $\frac{1}{6}$, $\frac{1}{5}$, etc. | Ordering — identifying fractions that lie between zero and half, and half and 1 |
| 6    | (a) $\frac{5}{6}$  
       (b) $\frac{3}{4}$  
       (c) 1 | Comparing — unit and non-unit fractions; requires equivalence to $\frac{1}{2}$ and $\frac{3}{4}$ |
| 7    | (a) $\frac{7}{10}$  
       (b) $\frac{4}{5}$ | Comparing — fractions with different names with $\frac{1}{2}$ and $\frac{3}{4}$ |
| 8    | $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$ | Ordering unit fractions — knowing that the more parts in a whole (denominator), the smaller the part (reverse of whole numbers) |
| 9    | (a) 5  
       (b) 1 | Estimating — knowing that proper fractions lie between 0 and 1 and benchmarking to $\frac{1}{2}$ |
## 2. Common Fractions

**Common Fractions 3B**

**Teacher notes:** Read each item to the class. This is the first time the set model is used to represent the whole.

**Materials required** — Pencil, colour pencils.

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B, C, D, H, I</td>
<td>Identifying fractions — number name → pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
<tr>
<td>2</td>
<td>(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{10}{10}$ (d) $1\frac{3}{8}$ [These numbers can be written informally (e.g. 3 eighths)]</td>
<td>Identifying fractions — pictorial → number name (fraction); reverse of Item 1. The whole is represented pictorially by area and set models.</td>
</tr>
<tr>
<td>3</td>
<td>(a) Colour 8 of the 8 equal parts (b) Colour 3 of the 5 equal parts (c) Colour 1 of the 3 cherries (d) Colour 1 whole and 3 out of 4 equal parts of the second whole (internal partitioning must be evident — colouring such as that in the diagram is not acceptable)</td>
<td>Identifying fractions — number → pictorial</td>
</tr>
<tr>
<td>4</td>
<td>(a) [Diagram] (b) [Diagram]</td>
<td>Identifying fractions and mixed numbers — number → pictorial (number line)</td>
</tr>
<tr>
<td>5</td>
<td>(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (Acceptable) (c) $1\frac{1}{3}$</td>
<td>Estimating — fractions and mixed numbers on a number line</td>
</tr>
<tr>
<td>6</td>
<td>Three times the size of the given shape.</td>
<td>Constructing — the whole from the unit part.</td>
</tr>
<tr>
<td>7</td>
<td>[Diagram]</td>
<td>Constructing the whole from a non-unit part — requires partitioning the given part (3 quarters) into 3 equal parts to ascertain the size of 1 quarter and then constructing the whole that is four times the size of the unit part (1 quarter)</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{1}{6}$</td>
<td>Constructing — the whole from the part and naming the fraction</td>
</tr>
<tr>
<td>9</td>
<td>(a) $\frac{5}{8}$ is equivalent in value to $\frac{2}{3}$ (fraction in the box)</td>
<td>Equivalence — pictorial representations</td>
</tr>
<tr>
<td>Item</td>
<td>Answers</td>
<td>Objectives</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>10</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td><strong>Partitioning and repartitioning</strong></td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>(c) Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>11</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><strong>Partitioning 2 wholes — the division model of fractions</strong></td>
</tr>
<tr>
<td></td>
<td>or</td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>
### Common Fractions 4A

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) four fifths or 4 fifths (b) three and 7 eighths</td>
<td>Identification — number (fraction) → number name (fraction name)</td>
</tr>
<tr>
<td>2</td>
<td>(a) (rac{2}{5}) (b) (rac{3}{4}) (c) (rac{8}{3}) (or (2\frac{2}{3}))</td>
<td>Identification — number name → number; reverse of item 1</td>
</tr>
<tr>
<td>3</td>
<td>(rac{5}{6})</td>
<td>Identification — number name → number</td>
</tr>
<tr>
<td>4</td>
<td>(a) 3 (b) 8 (c) 3 (d) (rac{5}{6})</td>
<td>Multiplicative relationship — between the number of parts and the whole</td>
</tr>
<tr>
<td>5</td>
<td>(a) No (b) Yes (c) No</td>
<td>Understanding the relationship — between mixed numbers and improper fractions</td>
</tr>
<tr>
<td>6</td>
<td>(a) (rac{2}{5}) (b) (rac{1}{4}) (c) (rac{1}{7}) (d) (rac{7}{18})</td>
<td>Renaming — improper fractions as mixed numbers</td>
</tr>
<tr>
<td>7</td>
<td>(a) 3 halves (b) (4\frac{2}{3})</td>
<td>Regrouping — parts and wholes</td>
</tr>
<tr>
<td>8</td>
<td>(a) (2\frac{1}{2}) (or (2\frac{1}{3})) (b) (6) (c) (9\frac{1}{4})</td>
<td>Seriation — adding 1 part (quarter) more</td>
</tr>
<tr>
<td>9</td>
<td>(a) (rac{4}{2}), (rac{5}{4}), (rac{5}{6}), (\frac{1}{17}), (\frac{1}{17}), (\frac{2}{17}), (\frac{1}{17}) (d) (rac{5}{112})</td>
<td>Seriation — counting forwards by quarters; counting backwards by fifths</td>
</tr>
<tr>
<td>10</td>
<td>(a) (rac{9}{15}) or (\frac{3}{5}), (\frac{1}{15}), (\frac{1}{2}) (b) (\frac{5}{112})</td>
<td>Seriation — knowing that there are many fractions between any two given fractions (requires renaming/ equivalence)</td>
</tr>
<tr>
<td>11</td>
<td>(a) (rac{5}{6}) (b) (\frac{3}{6}) (c) (\frac{3}{4}) (d) (\frac{8}{3}) (e) (\frac{2}{3})</td>
<td>Comparing — fractions with the same/different names</td>
</tr>
<tr>
<td>12</td>
<td>(a) (rac{7}{10}) (b) (\frac{4}{5})</td>
<td>Comparing — fractions with different names with (\frac{1}{2}) and (\frac{3}{4})</td>
</tr>
<tr>
<td>13</td>
<td>(a) (rac{1}{8}), (\frac{1}{2}) (b) (\frac{1}{2}), (\frac{2}{3}), (\frac{5}{6})</td>
<td>Ordering — unit fractions and mixed numbers; knowing that the more parts in a whole (denominator), the smaller the part</td>
</tr>
<tr>
<td>14</td>
<td>(a) 5 (b) 1</td>
<td>Rounding — mixed numbers and fractions to the nearest whole number</td>
</tr>
<tr>
<td>15</td>
<td>0.75 = (rac{3}{4}); 1 = (\frac{10}{10}); 0.05 = (\frac{5}{100}); 0.4 = (\frac{4}{10}); 0.25 = (\frac{1}{4}); 0.5 = (\frac{1}{2})</td>
<td>Equivalence — between decimal and common fractions</td>
</tr>
<tr>
<td>16</td>
<td>1 ÷ 2 =</td>
<td>Multiplicative structure — connecting the action of partitioning a whole into a number of equal parts with ↓</td>
</tr>
</tbody>
</table>
Common Fractions 4B

Teacher notes: Read each item to the class.

Materials required — Pencil, colour pencils.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D, F</td>
<td>Identifying fractions — number name → pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
<tr>
<td>2</td>
<td>(a) 3/8  (b) 3/5  (c) 10/10 (d) 3/4</td>
<td>Identifying fractions — pictorial → number name (fraction); reverse of Item 1. The whole is represented pictorially by area and set models.</td>
</tr>
<tr>
<td>3</td>
<td>(a) Colour 8 of the 8 equal parts (b) Colour 6 of the 8 equal parts (c) Colour 1 diamond or 2 triangles</td>
<td>Identifying fractions — number → pictorial; Items (b) and (c) require reunitising/ repartitioning the whole</td>
</tr>
<tr>
<td>4</td>
<td>(a) Colour 3 triangles (b) Colour 2 wholes and 3 out of 4 equal parts of the third whole (internal partitioning must be evident – colouring such as that in the diagram is not acceptable) (c) Colour 1 whole set of marbles and 6 out of 8 marbles in the 2nd set</td>
<td>Identifying fractions — number → pictorial; re-unitising</td>
</tr>
<tr>
<td>5</td>
<td>(a)</td>
<td>Identifying fractions and mixed numbers — number → pictorial (number line)</td>
</tr>
<tr>
<td>6</td>
<td>(a) and (b)</td>
<td>Estimating — estimating improper fractions and mixed numbers on a number line (requires reunitising/ repartitioning)</td>
</tr>
<tr>
<td>7</td>
<td>(a) four times the size of the part shown. (b)</td>
<td>Constructing — the whole from a non-unit part.</td>
</tr>
<tr>
<td>8</td>
<td>1/6</td>
<td>Constructing — the whole from the part and naming the fraction</td>
</tr>
<tr>
<td>9</td>
<td>(a) 6/1 is equivalent in value to 1/2 (fraction in the box)</td>
<td>Equivalence — pictorial representations</td>
</tr>
<tr>
<td>Item</td>
<td>Answers</td>
<td>Objectives</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>10</td>
<td><img src="image1.png" alt="Diagram" /> <img src="image2.png" alt="Diagram" /> or <img src="image3.png" alt="Diagram" /> <img src="image4.png" alt="Diagram" /></td>
<td>Partitioning — and repartitioning</td>
</tr>
<tr>
<td>11</td>
<td>Bag A Bag B</td>
<td>Partitioning 2 wholes — the division model of fractions</td>
</tr>
<tr>
<td></td>
<td>(a) <img src="image5.png" alt="Diagram" /> <img src="image6.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) $\frac{2}{6}$ or $\frac{1}{3}$ (c) $\frac{2}{6}$ or $\frac{1}{3}$ (d) $\frac{4}{12}$ or $\frac{1}{3}$</td>
<td></td>
</tr>
</tbody>
</table>
# Common Fractions 5A

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) four fifths or 4 fifths  
(b) \( \frac{3}{8} \) (three and 7 eighths) | Identification — number (fraction) \( \rightarrow \) number name (fraction name) |
| 2    | (a) \( \frac{7}{8} \)  
(b) \( \frac{1}{4} \)  
(c) \( \frac{8}{3} \) (or \( \frac{2}{7} \)) | Identification — number name \( \rightarrow \) number; reverse of item 1 |
| 3    | \( \frac{5}{6} \) | Identification — number name \( \rightarrow \) number |
| 4    | (a) 3  
(b) 8  
(c) 3  
(d) \( \frac{7}{3} \)  
(e) \( \frac{5}{6} \)  
(f) 2 | Multiplicative relationship — between the number of parts and the whole |
| 5    | (a) No  
(b) Yes  
(c) No | Understanding — the relationship between mixed numbers and improper fractions |
| 6    | (a) \( \frac{2}{3} \)  
(b) \( \frac{1}{2} \) or \( \frac{1}{2} \)  
(c) \( \frac{7}{2} \)  
(d) \( \frac{7}{8} \) | Renaming — improper fractions as mixed numbers |
| 7    | (a) 3 halves  
(b) \( \frac{7}{1} \) | Regrouping — parts and wholes |
| 8    | (a) \( \frac{1}{3} \) (or \( \frac{2}{3} \))  
(b) 6  
(c) \( \frac{1}{4} \) | Seriation — adding 1 part (quarter) more |
| 9    | (a) \( \frac{4}{4} \), \( \frac{5}{4} \), \( \frac{6}{4} \) (or \( \frac{1}{1} \), \( \frac{1}{4} \) or \( \frac{1}{1} \))  
(b) 6, \( \frac{4}{3} \), \( \frac{3}{2} \) | Seriation — counting forward by quarters; counting backwards by fifths |
| 10   | (a) \( \frac{9}{15} \) or \( \frac{3}{5} \)  
(b) \( \frac{8}{13} \)  
(c) \( \frac{1}{2} \)  
(d) \( \frac{5}{12} \) | Seriation — knowing that there are many fractions between any two given fractions (requires renaming/ equivalence) |
| 11   | (a) \( \frac{5}{6} \)  
(b) \( \frac{7}{4} \)  
(c) 6  
(d) \( \frac{1}{2} \)  
(e) \( \frac{5}{2} \) | Comparing — fractions with the same/different names |
| 12   | (a) \( \frac{7}{10} \)  
(b) \( \frac{4}{5} \) | Comparing — fractions with different names and representations (common, decimal) with \( \frac{1}{2} \) and \( \frac{3}{4} \) |
| 13   | (a) \( \frac{1}{6} \), \( \frac{1}{4} \), \( \frac{1}{2} \)  
(b) \( \frac{7}{4} \), \( \frac{7}{8} \), \( \frac{7}{6} \) | Ordering — unit fractions, mixed numbers |
| 14   | (a) 5  
(b) 1  
(c) 0 | Rounding — mixed numbers and fractions to the nearest whole number |
| 15   | 0.75, \( \frac{3}{4} \), 75%;  
1, \( \frac{10}{1} \), 100%;  
0.05, \( \frac{5}{100} \), 5%;  
0.4, \( \frac{4}{10} \), 40%;  
0.25, \( \frac{1}{4} \), 25%;  
0.5, \( \frac{5}{10} \), 50% | Equivalence — between decimal and common fractions |
| 16   | (a) 5 \( \div \) 8 =  
(b) 5 \( \div \) 8 % | Multiplicative structure — connecting the action of partitioning a whole into a number of equal parts with \( \div \) |
# Common Fractions 5B

**Teacher notes:** Read each item to the class.  
**Materials required —** Pencil, coloured pencils.  
**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D, F</td>
<td>Identifying fractions — number name → pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
<tr>
<td>2</td>
<td>(b) (c) (e)</td>
<td>Identifying fractions — number name → pictorial; relating fractions to equal parts of a whole and the number of equal parts to be considered</td>
</tr>
<tr>
<td>3</td>
<td>(a) Colour 8 of the equal parts  (b) Colour 6 of the 8 equal parts  (c) Colour 1 diamond or 2 triangles</td>
<td>Identifying fractions — number → pictorial; Items (b) and (c) require re-unitising/ repartitioning the whole</td>
</tr>
<tr>
<td>4</td>
<td>(a) Colour 3 triangles  (b) Colour 2 wholes and 3 out of 4 equal parts of the third whole (internal partitioning must be evident – colouring such as that in the diagram is not acceptable)  (c) Colour 1 whole set of marbles and 6 out of 8 marbles in the 2nd set</td>
<td>Identifying fractions — number → pictorial; re-unitising</td>
</tr>
<tr>
<td>5</td>
<td>(a)</td>
<td>Identifying fractions and mixed numbers — number → pictorial (number line)  Re-unitising</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a) and (b)</td>
<td>Estimating — estimating improper fractions and mixed numbers on a number line (requires reunitising/ repartitioning)</td>
</tr>
<tr>
<td>7</td>
<td>(a)</td>
<td>Constructing — the whole from a non-unit part.</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(a)</td>
<td>Constructing — the whole from the part and naming the fraction</td>
</tr>
</tbody>
</table>
|      | (b) | Colour /Draw 4 marbles.  
$2T=5$ halves;  
$5$ halves $=10$ marbles  
$1$ half $=2$ marbles  
$1$ whole $=4$ marbles |
<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td><img src="image" alt="Fraction" /></td>
<td>Constructing — the whole from the part and naming the fraction</td>
</tr>
<tr>
<td>10</td>
<td>(a) 1/8 is equivalent in value to 1/4 (fraction in the box)</td>
<td>Equivalence — pictorial representations</td>
</tr>
<tr>
<td>11</td>
<td><img src="image" alt="Partitioning" /></td>
<td>Partitioning — and repartitioning</td>
</tr>
</tbody>
</table>
| 12   | (a) Bag A Bag B  
(b) 2/8 or 1/4  
(c) 4/8 or 1/2  
(d) 4/12 or 1/3 | Partitioning 2 wholes — the division model of fractions |
| 13   | (a) 4/8 or 2/4  
(b) 4/12 or 1/3  
(c) No | Unitising — when the part is constant but the whole changes |
### 3. DECIMAL FRACTIONS (Refer to Overview and Theory, Section 4)

#### Decimal Fractions 1A

**Teacher notes:** Read each item to the class.

**Materials required** — Pencil.

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) three and eight tenths  
(b) two tenths (or zero and two tenths) | **Identification** — number ➔ number name |
| 2    | (a) 5.3  
(b) 0.7 | **Identification** — number name ➔ number (reverse of Item 1) |
| 3    | (a) 9.2  
(b) 70.7  
(c) 0.8  
(d) 3.5 | **Place value** — tens, ones, tenths |
| 4    | 176 | **Place value** — knowing that the position of the digit gives its value |
| 5    | (a) No  
(b) Yes  
(c) No  
(d) No | **Place value** — understanding the role of zero in numbers (when zero changes/not changes a number’s value) |
| 6    | (a) 9.5  
(b) 3.0 (or 3) | **Seriation** — determining 1 tenth more (including odometer principle — 1>9) |
| 7    | (a) 4.6  
(b) 4.9 | **Seriation** — determining 1 tenth less (including odometer principle — 1<10) |
| 8    | (a) 1.9, 2.0, 2.1  
(b) 7.6, 8.6, 9.6 | **Seriation** — counting forward by tenths, ones (recognising which place should change first) |
| 9    | (a) 8.6  
(b) 0.9  
(c) 6 | **Comparing/Ordering** — comparing decimal numbers with other decimal numbers and whole numbers |
| 10   | (a) 18.21 to 18.49  
(b) 2.1 to 2.9 | **Seriation** — knowing that there are many fractions between any two given fractions |
| 11   | (a) 4.2, 4.6, 4.7  
(b) 0.9, 3.8, 9 | **Comparing/Ordering** — writing numbers in order of value |
| 12   | (a) 2  
(b) ones, tenths  
(c) 7.1  
(d) ones, tenths | **Place value** — number ➔ place value names (a, b, d); place value ➔ number (c) |
| 13   | (a) 57  
(b) 20  
(c) 7.4  
(d) 6.6 | **Regrouping** — ones/tenths as tenths; tenths as ones/ tenths |
| 14   | (a) 2 is circled  
(b) 8 is circled  
(c) 5.8 is circled | **Place value** |
| 15   | (a) 3  
(b) 62  
(c) 0.4  
(d) 3.7 | **Multiplicative structure** — writing the number that results from ×/÷ by 10 |
| 16   | (a) ×10  
(b) ÷10  
(c) ×10  
(d) ×10 | **Multiplicative structure** — one shift to the left/right makes the number 10 times larger/smaller in value |
| 17   | (a) 8  
(b) 0  
(c) 1 | **Rounding** — to the nearest whole number |
| 18   | (a) Almost the whole star  
(b) less than half the triangle (c)  
(d) 0.2 (must be less than a half – 0.5) | **Estimation** — benchmarking to a half (0.5) and 1. |
Decimal Fractions 1B

Teacher notes: Read each item to the class.
Materials required — Pencil.
Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 4, 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2, 3, 4</td>
<td>10 equal parts</td>
</tr>
<tr>
<td>2</td>
<td>2 and 4 tenths (a) and 2.4 (c)</td>
<td>Identification — pictorial → number name and number</td>
</tr>
<tr>
<td>3</td>
<td>0.3 (3 tenths is acceptable)</td>
<td>Identification — pictorial → number</td>
</tr>
<tr>
<td>4</td>
<td>Any 7 of the 10 equal parts</td>
<td>Identification — number → pictorial (reverse of Item 3)</td>
</tr>
<tr>
<td>5</td>
<td>Many different answers are acceptable but the 10 equal parts must be visible.</td>
<td>Constructing — part → whole</td>
</tr>
<tr>
<td>6</td>
<td>(a) 4.2</td>
<td>Identification — pictorial → number and reverse (number → pictorial)</td>
</tr>
<tr>
<td></td>
<td>(b) 2.6</td>
<td>10 equal parts</td>
</tr>
</tbody>
</table>

**Teacher notes:**
- Read each item to the class.
- Materials required — Pencil.
- Answers/Objectives — See the table below.
Decimal Fractions 2A

Teacher notes: Read each item to the class.

Materials required — Pencil.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) three and eighty-two hundredths (b) six hundred and seven, and two hundredths</td>
<td>Identification — number (\rightarrow) number name</td>
</tr>
<tr>
<td>2</td>
<td>(a) 5.36 (b) 11.07</td>
<td>Identification — number name (\rightarrow) number (reverse of Item 1)</td>
</tr>
<tr>
<td>3</td>
<td>(a) 9.52 (b) 70.7 (c) 0.82</td>
<td>Place value — tens, ones, tenths, hundredths</td>
</tr>
<tr>
<td>4</td>
<td>176</td>
<td>Place value — knowing that the position of the digit gives its value</td>
</tr>
<tr>
<td>5</td>
<td>(a) tens (b) hundredths (c) tenths</td>
<td>Place value — (a, b) and regrouping (c)</td>
</tr>
<tr>
<td>6</td>
<td>(a) No (b) Yes (c) Yes (d) No</td>
<td>Place value — understanding the role of zero in decimal numbers (when zero changes/ not changes a number’s value)</td>
</tr>
<tr>
<td>7</td>
<td>(a) 2.77 (d) 6.01 (h) 0.39 (b) 3.91 (e) 4.92 (i) 6.99 (c) 0.10 (or 0.1) (f) 3.57 (g) 3.79 (j) 5 (or 5.00)</td>
<td>Seriation — knowing the effect of adding/subtracting 1 hundredth</td>
</tr>
<tr>
<td>8</td>
<td>(a) 1.70 (or 1.7), 1.71, 1.72 (b) 3.09, 3.10 (or 3.1), 3.11 (c) 4.95, 5.05, 5.15</td>
<td>Seriation — counting by hundredths and tenths</td>
</tr>
<tr>
<td>9</td>
<td>(a) 8.62 (c) 0.7 (d) 6</td>
<td>Comparing — decimal numbers (hundredths/hundredths; tenths/ hundredths; whole number/hundredths)</td>
</tr>
<tr>
<td>10</td>
<td>(a) 18.29 to 18.49 (b) 1.51 to 1.59</td>
<td>Seriation — knowing that there are many fractions between two given fractions</td>
</tr>
<tr>
<td>11</td>
<td>(a) 4.28, 4.65, 4.73 (c) 3.09, 3.81, 9 (b) 3.11, 3.12 (d) 6</td>
<td>Comparing/Ordering — writing numbers in order of value</td>
</tr>
<tr>
<td>12</td>
<td>(a) 2 ones, 14 hundredths (c) 7.14 (b) tenths, hundredths (d) tenths, hundredths</td>
<td>Place value — number (\rightarrow) place value names (a, b, d); place value (\rightarrow) number</td>
</tr>
<tr>
<td>13</td>
<td>(a) 507 (b) 620 (c) 0.46 (d) 7.5</td>
<td>Regrouping — ones/tenths/hundredths</td>
</tr>
<tr>
<td>14</td>
<td>(a) 70 (b) 7.25 (c) 4 (d) 0.09 (e) 62.3</td>
<td>Multiplicative structure — writing the number that results from (\times/\div) by 10, 100</td>
</tr>
<tr>
<td>15</td>
<td>(a) (\div) 100 (b) (\times) 100 (c) (\times) 10 (d) (\div) 10 (e) (\times) 100 (f) (\div) 10</td>
<td>Multiplicative structure — knowing the relationship between adjacent and non-adjacent places (one/two/three/ (\ldots) shifts to the left/right makes the number 10/100, times large/smaller in value (reverse of item 4)</td>
</tr>
<tr>
<td>16</td>
<td>(a) 8 (b) 0 (c) 1</td>
<td>Rounding — decimal numbers to the nearest whole number</td>
</tr>
<tr>
<td>17</td>
<td>(a) Almost the whole star (e.g.) (\star) (b) less than half the triangle (e.g.) (\triangle) (c) (\square) or (\triangle) or (\star) (d) 0.2 (must be less than a half (-) 0.5)</td>
<td>Estimation — benchmarking to a half (0.5) and 1.</td>
</tr>
</tbody>
</table>
Decimal Fractions 2B

Teacher notes: Read each item to the class.

Materials required — Pencil.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Identification — pictorial → number and reverse (number → pictorial)</td>
</tr>
<tr>
<td></td>
<td>(a) 4.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) 0.1, 0.16, 0.2, 0.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 4, 5, 6, 6.47, 7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) 4.06 to 4.09</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 and 4 tenths (a) and 2.4 (c)</td>
<td>Identification — pictorial → number name and number</td>
</tr>
<tr>
<td>3</td>
<td>(a) 0.47</td>
<td>Identification — pictorial → number</td>
</tr>
<tr>
<td></td>
<td>(b) 0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 0.6 (or 0.60)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) 0.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(a) Any 57 square shaded (preferably contiguous)</td>
<td>Identification — number → pictorial (reverse of Item 3)</td>
</tr>
<tr>
<td></td>
<td>(b) Any 4 columns shaded (preferably contiguous)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Any 2 columns and eight-tenths of a third column shaded (preferably contiguous)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) Any 60 squares shaded (preferably contiguous)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30 of the 100 squares shaded (preferably contiguous)</td>
<td>Renaming — tenths as hundredths on a non-prototypic pictorial representation of hundredths</td>
</tr>
<tr>
<td>6</td>
<td>(a) 0.41</td>
<td>Re-unitising and repartitioning — nonprototypic representations of tenths and hundredths</td>
</tr>
<tr>
<td></td>
<td>(b) 2 stripes and less than half of a third stripe</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a) one-tenth of the shape drawn</td>
<td>Regrouping — part → whole</td>
</tr>
<tr>
<td></td>
<td>(b) same as the given shape</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) ten times the shape drawn. The 10 equal parts must be visible. For example: 1 × 10 (1 row of ten) or 10 × 1 (1 column of ten), or 2 × 5 or 5 × 2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td><img src="image" alt="Diagram" /></td>
<td>Partitioning — a nonprototypic shape to represent hundredths</td>
</tr>
</tbody>
</table>
## Decimal Fractions 3A

**Teacher notes:** Read each item to the class.

**Materials required** — Pencil.

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | (a) three, and eight hundred and twenty-six thousandths  
(b) six hundred and seven, and twelve thousandths | Identification — number → number name |
| 2    | (a) 5.306  
(b) 11.007 | Identification — number name → number (reverse of Item 1) |
| 3    | (a) 9.523  
(b) 70.7  
(c) 0.802 | Place value — ones, tenths, hundredths, thousandths |
| 4    | (a) 1.762 | Place value — knowing that the position of the digit gives its value |
| 5    | (a) tens  
(b) thousands  
(c) tenths | Place value — (c) requires regrouping |
| 6    | (a) No  
(b) Yes  
(c) Yes  
(d) No | Place value — understanding the role of zero in decimal numbers (when zero changes/not changes a number’s value) |
| 7    | (a) 3.564  
(b) 5.270 (or 5.27)  
(c) 0.091  
(d) 4.592  
(e) 6.001  
(f) 6.722  
(g) 4.259  
(h) 9.199  
(i) 3.700 (or 3.70 or 3.7)  
(j) 7.999 | Seriation — determining 1 thousandth more/less (including odometer principle — 1>9; 1<10) |
| 8    | (a) 8.530 (or 8.53)  
(b) 8.531  
(b) 2.200 (or 2.2)  
(b) 2.201, 2.202  
(b) 2.800 (or 2.8)  
(b) 2.799, 2.798 | Seriation — counting forwards/backwards by thousandths (recognising which place should change first) |
| 9    | (a) 8.623  
(b) 0.956  
(c) 0.600  
(d) 6  
(e) 7.62 | Comparing/Ordering — comparing decimal numbers with other decimal numbers and whole numbers |
| 10   | (a) 18.284 to 18.499  
(b) 1.361 to 1.369 (other answers with ten-thousandths could also be correct) | Seriation — knowing that there are many fractions between any two given fractions |
| 11   | (a) 3.914, 6.201, 7.329  
(b) 0.399, 2.919, 3.019, 3.09  
(c) 4.285, 4.6, 4.73 | Comparing/Ordering — writing numbers in order of value |
| 12   | (a) 2, 6, 14  
(b) tenths, thousandths  
(c) 6.203  
(d) hundredths, thousandths | Place value — number → place value names (a, b), (with regrouping); place value → number (c) (with regrouping) |
| 13   | (a) 5017  
(b) 15  
(c) 0.377 | Regrouping |
| 14   | (a) 20  
(b) 0.75  
(c) 0.062  
(d) 0.703  
(e) 325.6  
(f) 481 | Multiplicative structure — knowing one/two/three shifts to the left/right makes the number 10, 100, 1000 times larger/smaller in value |
| 15   | (a) × 100  
(b) ÷ 100  
(c) × 1000  
(d) ÷ 100  
(e) × 1000  
(f) × 1000 | Multiplicative structure — writing the number that results from ×/÷ by 10, 100, 1000 etc. (reverse of Item 14) |
| 16   | (a) 9  
(b) 0  
(c) 1 | Rounding — decimal numbers to the nearest whole number |
| 17   | (a) Almost the whole star  
(b) less than half the triangle  
(c) 0.2 or 0.2  
(d) 0.2 (must be less than a half – 0.5) | Estimation — benchmarking to a half (0.5) and 1. |
Decimal Fractions 3B

Teacher notes: Read each item to the class.

Materials required — Pencil.

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) 6 m</td>
<td>Identification — pictorial → number and reverse (number → pictorial)</td>
</tr>
<tr>
<td></td>
<td>(b) 6.02 m to 6.05 m</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(b)</td>
<td>Identification — pictorial → number</td>
</tr>
<tr>
<td>3</td>
<td>(a)</td>
<td>Identification — number → pictorial (reverse of Item 3)</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>or</td>
<td>Renaming — tenths as hundredths on a non-prototypic pictorial representation of hundredths</td>
</tr>
<tr>
<td></td>
<td>or 3 rows of 10 (3 tenths); correct but inappropriate would be 30 individual squares (as it’s difficult to see the part as one chunk of the whole).</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(a) 0.41</td>
<td>Reunilising and repartitioning — nonprototypic representations of tenths and hundredths</td>
</tr>
<tr>
<td></td>
<td>(b) (2 stripes and less than half of a third stripe)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a)</td>
<td>Regrouping — part → whole</td>
</tr>
<tr>
<td></td>
<td>(b) (same as the given shape)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Many different answers are acceptable but the 10 equal parts must be visible.</td>
<td>2 × 5 or 5 × 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 × 10 or 10 × 1</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Partitioning — a nonprototypic shape to represent hundredths</td>
</tr>
</tbody>
</table>
4. PROBABILITY (Refer to Overview and Theory, Section 5)

Probability 1

Teacher notes: Read each item to the class.

Materials required — Pencil; coloured pencils (including red, blue).

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sometimes sunny (middle box)</td>
<td>Language (informal) — of probability in a real-world context</td>
</tr>
<tr>
<td>2</td>
<td>(a) Circle: Pink, Red (b) No (c) Yes (fair)</td>
<td>Possible outcomes — on a spinner (area model); estimating chances (equal, unequal)</td>
</tr>
<tr>
<td>3</td>
<td>(a) 1, 2, 3, 4, 5, 6 (b) No (c) No</td>
<td>Listing all possible outcomes — on a die (area/set model); estimating chances; identifying impossible events</td>
</tr>
<tr>
<td>4</td>
<td>Colour: (a) the whole spinner red (c) no part red (b) 1 of the 2 parts red</td>
<td>Language (formal) — impossible, possible, certain (area model)</td>
</tr>
<tr>
<td>5</td>
<td>Colour: (a) 0 marbles blue (c) 1, 2, 3, or 4 marbles blue</td>
<td>Language (formal) of probability — no chance, some chance, every chance (set model)</td>
</tr>
<tr>
<td>6</td>
<td>Elephant: Because there are more elephants than either lions or polar bears</td>
<td>Comparing events — in a single sample space (set model)</td>
</tr>
</tbody>
</table>
Probability 2

Teacher notes: Read each item to the class.

Materials required — Pencil; coloured pencils (including red, blue, yellow).

Answers/Objectives — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sometimes sunny (middle box)</td>
<td>Language (informal) — of probability in a real-world context</td>
</tr>
</tbody>
</table>
| 2    | (a) Circle: Green, Blue, Orange (fair)  
(b) No  
(c) Yes | Possible outcomes — on a spinner (area model);  
estimating chances (equal, unequal) |
| 3    | (a) Purple, Pink, Blue  
(b) No  
(c) No; Because there is more pink on the spinner than either purple or blue (i.e. 2 chances of spinning pink and only 1 chance each of spinning either purple or blue) | Comparing — the probability of an event in a single sample space (area) |
| 4    | (a) 1, 2, 3, 4, 5, 6  
(b) No  
(c) No | Listing all possible outcomes — on a die (area/set model);  
estimating chances;  
identifying impossible events |
| 5    | Colour: (a) the whole spinner red (6 of the six parts)  
(b) 1, 2, 3, 4 or 5 of the six parts red  
(c) no parts red | Language (formal) — impossible, possible, certain (area model) |
| 6    | Colour: (a) 0 marbles blue  
(b) all marbles blue  
(c) 1, 2, 3, or 4 marbles blue | Language (formal) of probability — no chance, some chance, every chance (set model) |
| 7    | Elephant; Because there are more elephants than either lions or polar bears | Comparing — the probability of an event in a single sample space (set model) |
| 8    | Yes, the spinner is fair because all the spinner parts are equal and three of these are yellow, and three are blue | Comparing — the probability of an event in a single sample space (area) |
| 9    | No; although all the spinner parts are equal in size, there are 5 chances of spinning blue and only 3 chances of spinning yellow | Comparing — the probability of an event in a single sample space (area) |
## Probability 3

**Teacher notes:** Read each item to the class.

**Materials required** — Pencil; coloured pencils (including red, blue, purple, pink, orange).

**Answers/Objectives** — See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Possible (top box)</td>
<td>Language (formal) — impossible, possible, certain in a real-world context</td>
</tr>
<tr>
<td>2</td>
<td>Circle: (a) 0, 2, 3, 4, 5, 6, 8, 9 (b) Yes (fair) — all parts are equal and a different number in each part</td>
<td>Possible outcomes — on a spinner (area model); estimating chances (equal, unequal)</td>
</tr>
<tr>
<td>3</td>
<td>(a) 1, 2, 3, 4 (b) No (c) No; Because, although the parts are equal, there are more 4s and 1s than 2s or 3s</td>
<td>Comparing outcomes — in a single sample space with equal parts but unequal outcomes</td>
</tr>
<tr>
<td>4</td>
<td>(a) True (b) False (c) False</td>
<td>Comparing outcomes — in a single sample space with equal parts using informal language; (d) Knowing that the order in which the chances are shown does not affect the outcome</td>
</tr>
<tr>
<td>5</td>
<td>(a) 1, 2, 3, 4, 5, 6 (b) No (c) 100 times</td>
<td>Possible outcomes — on a die (area/set model); estimating chances (equal, unequal); relating probability of an event to number of outcomes of a given number of trials</td>
</tr>
<tr>
<td>6</td>
<td>Colour: (a) the whole spinner red (6 of the six parts) (b) 1, 2, 3, 4 or 5 of the six parts red (c) no parts red</td>
<td>Language (formal) of probability — impossible, possible, certain (area model)</td>
</tr>
<tr>
<td>7</td>
<td>Colour: (a) 0 marbles blue (b) all marbles blue (c) some, but not all, marbles blue</td>
<td>Language (formal) of probability — no chance, some chance, every chance (set model)</td>
</tr>
<tr>
<td>8</td>
<td>(a) A, B, D (b) D, C, A, and B* (See Overview and Theory, 5.6.2.)</td>
<td>Comparing — the probability of two or more events within a single sample space (area model)</td>
</tr>
<tr>
<td>9</td>
<td>(a) Spinner 2 (b) Spinner 1 (c) Spinner 3</td>
<td>Comparing — the probability of two or more events within a single sample space and across sample spaces (area model)</td>
</tr>
<tr>
<td>10</td>
<td>Colour: 6 pink, 1 orange, 1 purple or 4 pink, 2 orange, 2 purple</td>
<td>Matching — a sample space (area model) with the informal ‘likelihood’ language of probability; looking for all potential solutions</td>
</tr>
<tr>
<td>11</td>
<td>Spinner 2; because there are 3 out of 6 equal parts blue (Spinner 1 has only 2 out of 6 equal parts blue)</td>
<td>Comparing — probability across two sample spaces (area models); articulating appropriate explanation (e.g. both spinners have the same number of equal parts but the first spinner gives only 2 out of 6 chances to get blue while the second spinner gives 3 out of 6 chances). Inappropriate response would be: there are more blue parts on the second spinner than on the first spinner. The second response shows that only the number of blue sectors was compared (2, 3) whereas probability requires comparison of fractions. [See Overview and Theory.]</td>
</tr>
<tr>
<td>12</td>
<td>(a) 50 (b) 50 There is 1 out of 2 chances (1/2) of getting either a head or a tail and half of 100 is 50. In reality, 100 trials is too few to hope for a 50/50 outcome so answers ranging from 30 to 70 could be accepted if students can explain their thinking.</td>
<td>Relating — probability of an event to number of outcomes of a given number of trials.</td>
</tr>
</tbody>
</table>
## Probability 4

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil.

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| 1    | Circle: (a) 0, 2, 3, 4, 5, 6, 8, 9  
      (b) Yes (fair) — all parts are equal and a different number in each part | Possible outcomes — on a spinner (area model); estimating chances (equal, unequal) |
| 2    | (a) 1, 2, 3, 4  
      (b) No  
      (c) No; Because, although the parts are equal, there are more 4s and 1s than 2s or 3s | Comparing — outcomes in a single sample space with equal parts but unequal outcomes |
| 3    | (a) True  
      (b) False  
      (c) False  
      (d) False | Comparing — outcomes in one sample space with equal parts using informal language; (d) Knowing that the order in which the chances are shown does not affect the outcome |
| 4    | (a) 1, 2, 3, 4, 5, 6  
      (b) No  
      (c) 100 times | Possible outcomes — on a die (area/set model); estimating chances (equal, unequal); relating probability of an event to number of outcomes of a given number of trials |
| 5    | (a) Yes  
      (b) Bag A | Language (formal) of probability — certain (set model) |
| 6    | (a) A, B, D  
      (b) D; C; A and B* (See Overview and Theory, 5.6.2.) | Comparing — the probability of two or more events within a single sample space (area model) |
| 7    | Write: 9 in three sectors; 3 in two sectors; 6 in two sectors; 1 in one sector | Matching — a sample space (area model) with the informal ‘likelihood’ language of probability |
| 8    | (a) Spinner 2  
      (b) Spinner 1  
      (c) Spinner 3 | Comparing — the probability of two or more events within a single sample space and across sample spaces (area model) |
| 9    | Spinner 2; because there are 3 out of 6 equal parts blue (Spinner 1 has only 2 out of 6 equal parts blue) | Comparing — probability across two sample spaces (area models); articulating appropriate explanation (e.g. both spinners have the same number of equal parts but the first spinner gives only 2 out of 6 chances to get blue while the second spinner gives 3 out of 6 chances). Inappropriate response would be: there are more blue parts on the second spinner than on the first spinner. The second response shows that only the number of blue sectors was compared (2, 3) whereas probability requires comparison of fractions. [See Overview and Theory.] |
| 10   | Yes; Spinner B can be partitioned to show 6 equal parts with 2 blue, 2 red and 2 green (as for Spinner A) | Knowing — that the order in which outcomes are placed does not affect their probabilities |
| 11   | (a) 50  
      (b) 50 (There is 1 out of 2 chances (1/2) of getting either a head or a tail and half of 100 is 50. In reality, 100 trials is too few to hope for a 50/50 outcome so answers ranging from 30 to 70 could be accepted if students can explain their thinking.) | Relating — probability of an event to the number of outcomes of a given number of trials |
| 12   | 7; there are more dice combinations that make 7 (1,6; 6,1; 2,5; 5,2; 3,4; 4,3) than 11 (5,6; 6,5) | Relating — outcomes to probabilistic thinking |
# Probability 5

**Teacher notes:** Read each item to the class.

**Materials required —** Pencil; coloured pencils (including red).

**Answers/Objectives —** See the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circle: (a) 0, 2, 3, 4, 5, 6, 8, 9</td>
<td>Possible outcomes — on a spinner (area model); estimating chances (equal, unequal)</td>
</tr>
<tr>
<td></td>
<td>(b) Yes (fair) — all parts are equal and a different number in each part</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(a) 1, 2, 3, 4</td>
<td>Comparing — outcomes in one sample space with equal parts but unequal outcomes</td>
</tr>
<tr>
<td></td>
<td>(b) No</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) No; although the parts are equal, there are more 4s and 1s than 2s or 3s</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) True</td>
<td>Comparing — outcomes in one sample space with equal parts using informal language; (d) Knowing that the order in which the chances are shown does not affect the outcome</td>
</tr>
<tr>
<td></td>
<td>(b) False</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) False</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) False</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(a) 1, 2, 3, 4, 5, 6</td>
<td>Possible outcomes — on a die (area/set model); estimating chances (equal, unequal); relating probability of an event to number of outcomes of a given number of trials</td>
</tr>
<tr>
<td></td>
<td>(b) No</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) 100 times</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(a) YES</td>
<td>Language (formal) of probability — certain (set model)</td>
</tr>
<tr>
<td></td>
<td>(b) Bag A</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(a) A, B, D</td>
<td>Comparing — the probability of two or more events within a single sample space (area model)</td>
</tr>
<tr>
<td></td>
<td>(b) D; C; A and B</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Write: 9 in three sectors; 3 in two sectors; 6 in two sectors; 1 in one sector</td>
<td>Matching — a sample space (area model) with the informal ‘likelihood’ language of probability</td>
</tr>
<tr>
<td>8</td>
<td>(a) Spinner 2</td>
<td>Comparing — the probability of two or more events within a single sample space and across sample spaces (area model)</td>
</tr>
<tr>
<td></td>
<td>(b) Spinner 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Spinner 3</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>No; there are 3 out of 6 equal parts blue on Spinner 2 and only 2 out of 6 equal parts blue on Spinner 1</td>
<td>Comparing — probability across two sample spaces (area models); articulating appropriate explanation (e.g. both spinners have the same number of equal parts but the first spinner gives only 2 out of 6 chances to get blue while the second spinner gives 3 out of 6 chances). Inappropriate response would be: there are more blue parts on the second spinner than on the first spinner. The second response shows that only the number of blue sectors was compared (2, 3) whereas probability requires comparison of fractions. [See Overview and Theory.]</td>
</tr>
<tr>
<td>10</td>
<td>YES; Spinner B can be partitioned to show 6 equal parts with 2 blue, 2 red and 2 green (as for Spinner A)</td>
<td>Knowing — that the order in which outcomes are placed does not affect their probabilities</td>
</tr>
<tr>
<td>11</td>
<td>(a) 50</td>
<td>Relating — probability of an event to the number of outcomes of a given number of trials</td>
</tr>
<tr>
<td></td>
<td>(b) 50</td>
<td></td>
</tr>
</tbody>
</table>

(4) Probability
<table>
<thead>
<tr>
<th>Item</th>
<th>Answers</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>7; there are more dice combinations that make 7</td>
<td>Relating — outcomes to probabilistic thinking</td>
</tr>
<tr>
<td></td>
<td>(1,6; 6,1; 2,5; 5,2; 3,4; 4,3) than 11 (5,6; 6,6)</td>
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</tr>
<tr>
<td>13</td>
<td>(a) 5 marbles, 2 red</td>
<td>Relating — formal recording of probability to a pictorial representation of the sample space</td>
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<tr>
<td></td>
<td>(b) any number, all red</td>
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<tr>
<td></td>
<td>(c) any number, no red</td>
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<td></td>
<td>(d) any number, more than half red</td>
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<tr>
<td></td>
<td>(e.g. 8 marbles, 5 or more red)</td>
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<tr>
<td>14</td>
<td>unlikely; more chance; likely; certain</td>
<td>Relating — probability to fractions (part/whole)</td>
</tr>
<tr>
<td></td>
<td>impossible; equal chance; 50-50</td>
<td></td>
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<tr>
<td>15</td>
<td>Box B: (Box A gives 1 chance out of 5 (1/5) of getting a Mars bar while Box B gives 2 chances out of 8 (1/4) and 1/4 is larger than 1/5</td>
<td>Abstracting — the mathematics of probability (i.e. its relation to the part/whole notion of fraction)</td>
</tr>
<tr>
<td>16</td>
<td>Has helped: There are eight numbers between 1 and 10</td>
<td>Linking — probability to fraction</td>
</tr>
<tr>
<td></td>
<td>(2, 3, 4, 5, 6, 7, 8, 9). Without the clue, Tom had $\frac{1}{5}$ chance of being correct; with the clue, he has $\frac{3}{8}$ chance</td>
<td></td>
</tr>
</tbody>
</table>