## Numeracy for Transition to Work Year 11 Prevocational Mathematics Booklet 11.6: "The Man from Hungary"

 Time relationships and calculations, timetables and efficient scheduling

## Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".
"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: Growing community through education.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

## Restricted waiver of copyright

This work is subject to a restricted waiver of copyright to allow copies to be made for educational purposes only, subject to the following conditions:

1. All copies shall be made without alteration or abridgement and must retain acknowledgement of the copyright.
2. The work must not be copied for the purposes of sale or hire or otherwise be used to derive revenue.
3. The restricted waiver of copyright is not transferable and may be withdrawn if any of these conditions are breached.

# © QUT YuMi Deadly Centre 2008 

Electronic edition 2013

School of Curriculum
QUT Faculty of Education
S Block, Room S404, Victoria Park Road
Kelvin Grove Qld 4059
Phone: +61 731380035
Fax: + 61731383985
Email: ydc@qut.edu.au
Website: http://ydc.qut.edu.au
CRICOS No. 00213J

This booklet was developed as part of a project which ran from 2005-2008 and was funded by an Australian Research Council Linkage grant, LP0455667: Numeracy for access to traineeships, apprenticeships, and vocational studies and empowerment and post Year 10 students.

# TEACHER RESOURCE BOOKLETS NUMERACY FOR TRANSITION TO WORK 

## YEAR 11 PREVOCATIONAL MATHEMATICS BOOKLET 11.6: "THE MAN FROM HUNGARY" VERSION 1: 2008

Research Team:
Bevan Penrose
Tom J Cooper
Annette R Baturo
with
Kaitlin Moore
Elizabeth Duus

Deadly Maths VET
School of Mathematics, Science and Technology Education, Faculty of Education, QUT

## DEADLY MATHEMATICS VET

Deadly Maths VET was the name given to the materials produced to support the teaching of numeracy to vocational education and training students, particularly those from Indigenous backgrounds. These booklets were produced by the Deadly Maths Consortium at Queensland University of Technology (QUT) but also involving a researcher from Nathan Campus of Griffith University.

At the time of the production of this booklet, Deadly Maths VET was producing materials as part of an ARC-funded Linkage grant LP0455667 (12 booklets on Years 11 and 12 Prevocational Mathematics course and 2 booklets on pesticide training) and ASISTM-funded 2008 grant ( 3 booklets on construction; 3 booklets on engineering; 3 booklets on marine; 1 booklet on retail; and 2 booklets and a series of virtual materials on basic mathematics).

## CONTENTS

Page
THE PREVOCATIONAL MATHEMATICS BOOKLETS ..... iv
OVERVIEW ..... 1

1. Theoretical position ..... 1
2. Mathematics for this booklet ..... 1
2.1. Structures of time ..... 1
3. Pedagogy ..... 3
4. How to use this booklet ..... 4
PRELIMINARY ACTIVITIES ..... 5
5. Time relationships ..... 5
1.1. Read and analyse clock ..... 5
1.2. Relating analogue and digital time ..... 6
1.3. 24-hour time ..... 7
1.4. Time relationship activities ..... 7
6. Time representations ..... 9
2.1. Time as a fraction ..... 9
2.2. Time as "place-value" ..... 11
2.3. Time representation activities ..... 12
7. Time calculations ..... 15
3.1. Traditional algorithms ..... 15
3.2. Non-traditional algorithms ..... 17
3.3. Time calculation activities ..... 18
8. Timetables ..... 19
4.1. Interpreting tables ..... 19
4.2. Constructing a timetable ..... 20
4.3. Timetable activities ..... 21
9. Efficient scheduling ..... 21
5.1. Other travel-time factors ..... 21
5.2. Fast and efficient time allocation ..... 22
5.3. Efficient scheduling activities ..... 22
CULMINATING TASK ..... 23
ASSESSMENT RUBRIC ..... 25

## THE PREVOCATIONAL MATHEMATICS BOOKLETS

In 2005 to 2008 researchers Annette Baturo and Tom Cooper from the Deadly Maths Consortium received an ARC Linkage grant (LP0455667) to study mathematics learning of Year 11 and 12 and adult students undertaking vocational education and training (VET) courses who had low achievement in mathematics. The title of this ARC Linkage project was Numeracy for access to traineeships and apprenticeships, vocational studies, empowerment and low-achieving post Year 10 students. The project studied better ways to teach mathematics to VET students at Bundamba State Secondary College, Metropolitan Institute of TAFE (Moreton Campus), Gold Coast Institute of TAFE (Ridgeway Campus), Tagai College (Thursday Island Campus), and the Open Learning Institute of TAFE.

As part of the study, the Deadly Maths research team developed booklets and other resources to be trialled by VET students and teachers. The project's activity with Bundamba State Secondary College focused on the Year 11 and 12 Prevocational Mathematics subjects taught at that college to less able mathematics students. The project used a series of intervention case studies to research learning. As part of this, the following 12 Prevocational Mathematics resource booklets were produced (6 for Year 11 and 6 for Year 12). The booklets are numbered 11.1 to 11.6 and 12.1 to 12.6 .
11.1 - "Using Numbers of Numbers" - Yr 11 prevocational maths booklet: Number, decimals, fractions and problem solving.
11.2 - "The Big Day Out" - Yr 11 prevocational maths booklet: Number, tables, budgeting, algebra and problem solving.
11.3 - "Rating our World" - Yr 11 prevocational maths booklet: Rate, area and volume activities and problems.
11.4 - "Exchange Student" - Yr 11 prevocational maths booklet: Operations, discounts, tables, metric conversion and best buys.
11.5 - "Planning a Roster" - Yr 11 prevocational maths booklet: Tables, 24-hour time, percentages and computation strategies.
11.6 - "The Man from Hungary" - Yr 11 prevocational maths booklet: Time relationships, time calculations, timetables and efficient scheduling.
12.1 - "Beating the Drought" - Yr 12 prevocational maths booklet: Fractions, probability, graphing and data.
12.2 - "Monopoly" - Yr 12 prevocational maths booklet: Fractions, probability, game strategies, property finance, graphs and tables.
12.3 - "How tall is the Criminal?" - Yr 12 prevocational maths booklet: Multiplicative structure, ratio and proportion, problem solving.
12.4 - "Design a Kitchen" - Yr 12 prevocational maths booklet: Visual imagery, percent, rate, ratio, perimeter, area and volume.
12.5 - "Healthy Eating" - Yr 12 prevocational maths booklet: Data collection and analysis, tables and graphing (line and histograms).
12.6 - "Rocking around the World" - Yr 12 prevocational maths booklet: Time and angle, time operations and problem-solving strategies.

## OVERVIEW

## 1. Theoretical position

The Bundamba Prevocational Mathematics booklets are based on the notion of Renzulli (1977) that mathematics ideas should be developed through three stages.

Stage 1: Motivate the students - pick an idea that will interest the students and will assist them to engage with mathematics.

Stage 2: Provide prerequisite skills - list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.

Stage 3: Culminating task - end the teaching sequence by setting students an openended investigation to explore.

These booklets use Stage 3 as an assessment item and so we have added an assessment rubric whereby the culminating task can be used to check the knowledge held by the student.

The booklets combine two approaches to teaching:
(1) structural activities that lead to the discovery and abstraction of mathematical concepts, processes, strategies and procedures; and
(2) rich-style tasks which allow students an opportunity to solve problems and build their own personal solution.

## 2. Mathematics for this booklet

The culminating task for this unit is to use train timetables to determine an appropriate schedule for a visitor, Mr Schien, to get to his appointments by train.

The task requires the following prerequisites:
(1) Time relationships - an understanding of how to tell time in terms of hours and minutes.
(2) Time representations - an understanding of how to record and read time in fractions and decimal terms and in 12 and 24 hour terms (especially am and pm).
(3) Time calculations - ability to add and subtract times to work out duration of journeys and total travel times.
(4) Timetables - ability to read railway timetables and represent schedules in a simple tabular form.
(5) Other travel-time factors - appreciation that train travel involves adding to and from stations and taking account of the train being early or late.
(6) Fast and efficient time allocation - understanding of what is involved in and how to determine the fastest time to destination and the most efficient way to achieve this.

### 2.1. Structures of time

Time differs from other measures in that it is not decimal (based on 10). It is a base 60 system (e.g. 60 minutes to one hour). This means that counting in time follows 1 hr 58 mins, $1 \mathrm{hr} 59 \mathrm{mins}, 2 \mathrm{hrs} 0 \mathrm{mins}, \ldots$ not $1 \mathrm{~m} 58 \mathrm{~cm}, 1 \mathrm{~m} 59 \mathrm{~cm}, 1 \mathrm{~m} 60 \mathrm{~cm}, 1 \mathrm{~m} 61 \mathrm{~cm} .$. as in length. It
also means that addition and subtraction of time involves renaming/regrouping around 60 . For example:

|  |  | 5 | 71 |
| ---: | ---: | ---: | :--- |
| 3 hr | 34 min | 6hr | X1min |
| +4 hr | 47 min | $-\underline{2 h r}$ | 28 min |
| 8 hr | 21 min | 3 hr | 43 min |

In the addition computation, $34 \mathrm{~min}+47 \mathrm{~min}=81 \mathrm{~min}$ which is 1 hr and 21 min . Similarly, in the subtraction computation, to enable the subtraction, 1 hour is changed to 60 min which gives 5 hr and 71 min from which to subtract 2 hr 28 min .

Secondly, historically, time has developed both an analogue and digital form of representation. The first is based on fractions (and is circle based) while the second is pseudo-decimal (and is symbol based). For example:

Half past 1


Analogue representation

## 1:30

Digital representation

Further, time can be given in two 12 -hour sections a day, am and pm, or 24 -hour times where pm times continue to 24 . For these times, the 24 -hour pm times (e.g. 15:46) are 12 hours more than pm time (here 3:46pm).

Timetables are tables where activities (e.g. attending school, catching a bus) are related to times. Train timetables give the starting and ending times of a train and also the time that the train stops at each station in between. Catching a train somewhere, however, is not just calculated from the timetable. It involves walking time to the starting station and from the end station plus any time for delays and to ensure the train is not missed.

Overall hours and minutes must be seen as one component of a time place-value system that is not consistent with the decimal number system. For example, the decimal number system is as follows with 10 giving the multiplicative change


However, time is much more complex (and this is without putting in weeks and fortnights):


For the train timetables, possibly all we need is the relationship:


## 3. Pedagogy

To teach time requires a pedagogy based around 60 for hours-minutes and an hours-minutes-seconds place-value situation, again based around 60. However, it is a little more complex because the hours-minutes relationship has a substructure of 12 on analogue systems, and the hours-day relationship can have a substructure of 2 (am and pm). For example:

Hour:


Day:

| 12 | 12 |
| :--- | :--- |

am pm

And, further, the hour-minute relationship recognises $3 / 12=1 / 4,6 / 12=1 / 2$ and $9 / 12=3 / 4$ or $1 / 4$ to.

The answer to this dilemma is to treat each relationship separately. That is, teach hoursminutes separately to days-hours and teach analogue separately to 12 -hour digital and 24hour digital. Then convert different representations; for example, a quarter to $5=45$ minutes past $4=4: 45$.
Thus, teaching should focus on the following:
(1) Relationships between model, language and symbols:

Real World Context ( 15 minutes before 5 pm )


Model

( $1 / 4$ to 5 in afternoon) (4:45pm / 16:45)
(2) Relationships between models - analogue, 12-hour digital, 24hour digital:


4:45pm or $16: 45$

Finally, in the development of time ideas, we should ensure teaching covers:
(3) Flexibility and reversibility - e.g. "how many different ways to represent 15 minutes before 5pm" and "draw a clock face to show 4:45 and write this clock face as digital".
(4) The full sequence of time telling as below:
(a) o'clock (full hours);
(b) easy fractions, e.g. " $1 / 4$ past", " $1 / 2$ past", " $1 / 4$ to";
(c) multiples of 5, e.g. "10 past", "20 to";
(d) using all 60 numbers, e.g. " 27 minutes to 8 ";
(e) digital time (am and pm); and
(f) 24-hour time.

The prerequisite pedagogy also involves three major generic strategies:
(1) Flexibility - trying to ensure students understand things in a variety of ways (e.g. discount, reduction, etc.; \%, part per 100, decimal hundredths, fraction out of 100, and so on).
(2) Reversing - trying to teach in all directions (e.g. real-world situation to best buy; best buy to real-world situation).
(3) Generalising - trying to teach things in the most general way (going beyond the needs of a task).

Particular strategies for tackling some of the preliminary activities will be provided at the start of the activities.

## 4. How to use this booklet

The major focus of the unit is the culminating task. The preliminary activities are only suggestions for prior work if you think your students require this work before they begin the culminating task. Therefore:
(1) use the culminating task as the focus of the unit - to motivate engagement;
(2) look through the preliminary activities and pick and choose things that you believe will be useful for your students - it is not necessary to do everything and to do it in the order that it appears in this booklet (although there is a logic to the order);
(3) do these activities as a lead in to the culminating activity; and
(4) try to organise things so that the students can do the culminating activity as an assessment of their abilities to do mathematics.

The preliminary activities are in five sections and there are real activities at the end of each section. The earlier parts of each section simply explain the ideas/models/pictures in the mind that are being attempted. Although how they are presented gives some hints on pedagogy, you will have to determine your own way to teach these.

## PRELIMINARY ACTIVITIES

## 1. Time relationships

The number 6 is the first perfect number, that is, the factors of 6 , which are 1,2 and 3 , add to 6 . It is also half of 12 which is an abundant number (factors of 12 , which are $1,2,3,4$ and 6 add to 16 which is more than 12) and which uses the number of moons in a year. Finally, there are 6 equilateral triangles fitting around a point.


Therefore the Babylonians took 60 as the base of their number system. This had three outcomes:
(1) their numbers read like $326=3 \times 60+26=206$;
(2) the degrees in a full turn were 6 triangles $\times 60=360$ degrees;
(3) time was two 12 's ( am and pm ) for hours and 1 hour $=60$ minutes and 1 minute $=60$ seconds.

### 1.1. Read and analyse clock

To ensure people can read a clock face we need to go through the following stages:
(1) Kinaesthetic: Place materials to make a large clock on the ground. Students stand in the centre and use hands (one stretched out and one held in so it's shorter) to show the time.
(a) For hours (use short hand)

Students stand as before and turn, stopping 12 times and saying as they go: $1,2,3,4,5$, $6,7,8,9,10,11,12$.

(b) For fractions of hours (use long hand)

Students stand with two hands clasped pointing forward and turn $360^{\circ}$ stopping four times (every $90^{\circ}$ ) and saying as they stop: "quarter past, half past, quarter to, o'clock".

(c) For minutes (use long hand)

Students stand as before and turn, stopping 12 times and saying as they go: 5, 10, 15, $20,25,30,35,40,45,50,55,60$ followed by 5 past, 10 past, 15 past, etc. and then followed by 25 to, 20 to, 15 to, 10 to, 5 to, o'clock.
(2) Materials: Use clock faces, virtual clock faces on computers, pictures of clock faces to do activities that connect representations in both directions.
(a) Teacher gives time, e.g. " 26 minutes to 4 o'clock" and students model
 time.
(b) Teacher models time using body clock, and students give time to teacher "17 minutes past one o'clock".


These activities are done for quarters ("1/4 past 6" etc.), 5 minute intervals ("10 to 7" etc.) and whole minutes ("11 past 4" etc.).
(3) Imagining: Get students to imagine a clock face and to think of where hands will be for various times (show clock face in the air).

### 1.2. Relating analogue and digital time

Decimal numbers have place values as follows:

| Hundreds | Tens | Ones | tenths | hundredths |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

Time is based on 60, therefore it is better considered as:


It is common to show this distinction by a colon or double dot (":") instead of the single dot of decimals ("."). For example, 3.48 is three and forty-eight hundredths (or 3 ones and 4 tenths and 8 hundredths), while 3:48 is 3 hours and 48 minutes. Note: This also means that numbers like 4:71 do not exist; the highest number after ":" is 59.

The steps involved are:
(1) Understand digital time - 3:23 is 3 hours and 23 minutes or $323 / 60$ of an hour or 3 and 23 sixtieths.
(2) Relate "to the hour" to "after the hour" - stress that digital is always after the hour so 16 to 8 is really 7:44.
(a) Look at numbers adding to 60, for example (i) $32+?=60$; (ii) $?+51=60$; (iii) $13+?=60$. Show this on a picture of a clock face - show how the "to 7 " really means between 6 and 7 so it's "something" past 6 or 6 :"something".
(b) Get students to generalise that the "something" is 60 subtract the amount "to". For example: 18 to 7 is $6:(60-18)=6: 42$.
(3) Practise digital to analogue time - reverse this process by:

Teacher shows analogue:
AND
Teacher writes digital: 12:17

(4) Add in language - again reversing the process by:

Teacher says time $\longrightarrow$ Student draws $\longrightarrow$ Student writes digital

Teacher writes digital $\longrightarrow$ Student draws $\longrightarrow$ Student writes analogue language 2:24

Student writes digital: 12:17

Student draws analogue:
 "24 past two"


### 1.3. 24-hour time

The division of the day into am and pm can be overcome by simply continuing to count hours up to 24. Get the students to consider two representations of a day:


12 Hour am pm

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ | $\mid$ |

## 24 Hour

$\begin{array}{llllllllllllllllllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24\end{array}$
| | | | | | | | | | | | | | | | | | | | | | | |
(1) Ask if they can see a pattern between 12-hour time and 24-hour time: (a) am remains the same, (b) pm is 12 plus the pm time, and (c) pm time is 24-hour time subtract 12.
(2) Then practise adding and subtracting 12 hours for pm time.

### 1.4. Time relationship activities

Fill in the missing information: The first one is done for you.

|  | Analogue language |  | 12-hour digital <br> symbols <br> afternoon | 24-hour digital <br> symbols |
| :--- | :--- | :--- | :---: | :---: |
| (1) | 6 past 8 in the <br> morning |  | $6: 19 \mathrm{pm}$ | $18: 19$ |
| (2) |  |  |  |  |
| (3) |  |  |  |  |


| (4) |  |  |  | 06:04 |
| :---: | :---: | :---: | :---: | :---: |
| (5) | 13 to 7 in the morning |  |  |  |
| (6) |  |  |  |  |
| (7) |  |  | 8:41 am |  |
| (8) |  |  |  | 12:56 |
| (9) | 21 past 2 in the afternoon |  |  |  |
| (10) |  |  |  |  |
| (11) |  |  | 3:14 pm |  |
| (12) |  |  |  | 21:26 |


| (13)12 to 10 in the <br> afternoon |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| (14) |  |  |  |  |
| (15) |  |  |  |  |

## 2. Time representations

As an analogue, time reflects a fraction understanding of a circle, while as a digital, time reflects a "place-value" structure (even though it is sixtieths and not tenths or hundredths).

### 2.1. Time as a fraction

To understand this aspect of time students require an understanding of fractions and equivalent fractions, for circles particularly. A fraction is part of a whole. It has 5 components:
(1) a whole (identified, known and retained across the representations);
(2) divided into equal parts;
(3) the number of equal parts is the number of the fraction (e.g. 3 parts = "thirds"; 6 parts = "sixths");
(4) the number of equal parts considered (shaded, removed, etc.) gives the rest of the value (e.g. whole partitioned into 6 equal parts and 5 shaded is "five-sixths"); and
(5) the common fraction notation is derived from the same numbers (e.g. 5 out of 6 equal parts of one whole shaded is $5 / 6$ ).
Examples:


One whole, 5 equal parts, 2 shaded - "two-fifths" or 2/5


One whole but not equal parts - not the fraction half

One whole, 4 equal parts, 3 shaded - "three-quarters" or "three-fourths" or 3/4.

Common misconceptions are not to ensure equal parts and to think the fraction is the shaded part to the unshaded part (e.g. to see the figure
 on right as $2 / 3$ ).

## Equivalent fractions

Two fractions are equivalent if they represent the same amount (e.g. $2 / 3=4 / 6$ ). This means that the change from one fraction to another is equivalent to multiplying by 1 (e.g. $2 / 3=2 / 3$ $\times 1=2 / 3 \times 2 / 2=4 / 6$ and so on). To see this in practice, take a common whole and find both fractions, for example:


It can be easily seen that $2 / 3=4 / 6$.
Mixed numbers
Mixed numbers are wholes and fractions together. They are based on "place-value" the same as ones and tenths but with different groups than 10. For example:

| "Place-value charts" | Ones | Tenths | Ones | Fifths | Ones | Eighths |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 6 | 3 | 2 | 7 | 7 |
| "Decimal symbols" | 4.6 |  | N/A |  |  | /A |
| Fractions (mixed num | rs) $4 \frac{6}{10}$ |  | $3 \frac{2}{5}$ |  | $7 \frac{7}{8}$ |  |

To teach mixed numbers (wholes and fractions), use materials, diagrams and charts, for example:


Then do many activities that relate language to diagrams to charts to symbols (forwards and backwards).

## Teaching fractions

To teach fractions and equivalent fractions, there is a need to take wholes (use food items such as pizzas, oranges, cakes, liquorice, chocolate bars first, then cut up and fold paper, then use computer shapes and drawings) and partition them into equal amounts, removing or shading the parts needed. Teach the fraction names (thirds, sixths, etc.) before teaching the full names (five-sixths, etc.). To teach equivalent fractions, fold one way for original fraction and the other way for equivalent fractions, for example:


Or use overlays (works really well with computers):

| $\square$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

## Clock-face fractions

The clock face is divided into 60 intervals. So the basic fraction is 60ths. However, to also show hours, each 5 minute sequence is given a second number. The hours divide the clock face into 12ths. Finally, the 12ths are further changed to make fourths, or quarters, for common analogue language ("a quarter to"). For example:


Sixtieths


Twelfths


Quarters/Fourths

As the clock face operates, the "big" or long hand moves around each minute (showing sixtieths) while the short or "little" hand moves between hours (showing twelfths).

### 2.2. Time as "place-value"

As digital, time is similar to ones and tenths but with 60 as the group size, for example:

|  | Ones | Tenths | Hours | Minutes |
| :---: | :---: | :---: | :---: | :---: |
| "Place-value" charts: | 6 | 3 | Ones | Sixtieths |
|  |  |  | 4 | 27 |
| Symbols: | 6.3 |  | 4:27 |  |
| As a fraction: | $6^{3} / 10$ |  | $4^{27} / 60$ |  |

To teach this, you need to do the following:
(1) Relate the various representations; e.g.

| LANGUAGE | CHART |  | SYMBOL |
| :--- | ---: | :--- | :---: |
|  | Hrs | Mins |  |
| Three hours and <br> forty-eight minutes |  | 48 | $3: 48$ |

(2) Change hours to minutes and vice versa; e.g.
$3: 48=3 \mathrm{hrs} 48 \mathrm{~min} \quad=\quad 3 \times 60+48=228 \mathrm{mins}$
(3) Restart the count in minutes at the hour; e.g.

3 hrs 58 mins, 3 hrs 59 mins, 4 hrs 0 mins, 4 hrs 1 min, ...

### 2.3. Time representation activities

(1) Fill in the following spaces for the short or "little" hand (the first has been done for you):

|  | Clock Face | Fraction Language | Fraction Symbol |
| :---: | :---: | :---: | :---: |
|  | $\left[\begin{array}{c}-1 \\ -\infty\end{array}\right.$ | Two twelfths | 2/12 |
| (1) | Coss |  |  |
| (2) |  | Nine twelfths |  |
| (3) |  |  |  |
| (4) |  |  | 11/12 |
| (5) |  |  | $3 / 4$ |
| (6) |  | One half |  |

(2) Fill in the following spaces for the long or "big" hand (the first has been done for you):

|  | Clock Face | Fraction Language | Fraction Symbol |
| :---: | :---: | :---: | :---: |
|  | $(\mathrm{Cl}$ | Eight sixtieths | 8/60 |
| (1) | $(-2)$ |  |  |
| (2) |  | Forty-seven sixtieths |  |
| (3) |  |  |  |
| (4) | $\left[\begin{array}{ll} 1 & \\ -6 & \\ -1 \end{array}\right)$ |  |  |
| (5) |  |  | 20/60 |
| (6) |  | Thirty-eight sixtieths |  |

(3) Fill in the following spaces for both hands (the first one has been done for you):

|  | Clock Face | Fraction Language | Fraction Symbol |
| :---: | :---: | :---: | :---: |
|  | $\left\langle\theta^{\sim}\right\rangle$ | 2 twelfths and 28 sixtieths | $2^{28} / 60$ |
| (1) | - |  |  |
| (2) |  | 6 twelfths and 4 sixtieths |  |
| (3) |  |  | $11^{36 / 60}$ |
| (4) |  |  |  |
| (5) |  |  | $4^{16} / 60$ |
| (6) |  | 7 twelfths and 38 sixtieths |  |

(4) Fill in the following for both hands:


## 3. Time calculations

The culminating task requires that time be added and subtracted. We shall look at traditional and non-traditional (but often more useful) algorithms.

### 3.1. Traditional algorithms

In traditional algorithms, two numbers are added or subtracted by separating them into "place-values", adding or subtracting each position separately, and recombining at the end.

## Addition

Step 1: The two numbers are identified and placed values are aligned. For example:

| PROBLEM |  | CHART |  |  | SYMBOLS |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Hrs | Mins |  |  |
| 3 hrs \& 52 mins |  | 3 | 52 |  | 3: 52 |
| + 4 hrs \& 29 mins | + | 4 | 29 | + | 4:29 |

Step 2: The hours and minutes are added separately:

| Hrs | Mins |
| ---: | :--- |
| 3 | 52 |
| + | 29 |
| 7 | 81 |

Step 3: If there are enough minutes to make an hour, this change is completed (i.e. carrying is undertaken) and the final amount calculated.

| Hrs | Mins |
| ---: | :--- |
| 3 | 52 |
| 4 | 29 |
| 7 | 81 |
| 8 | 21 |$\quad$| 1 |
| :---: |
| $3: 52$ |$\quad$| $8: 29$ |
| :--- |
| $8: 21$ |

Working out that $52+29=81$
and this is 1 hour and 21
minutes so 1 hour is carried

Note: there are other possible ways to set out the addition algorithm:

| Hrs | Mins | Doing the addition as two steps so there is no need to carry | Hrs | Mins | Adding first and then carrying/renaming second |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 52 |  | 3 | 52 |  |
| + 4 | 29 |  | + 4 | 29 |  |
| 1 | 21 |  | 7 | 81 |  |
| 7 | 00 |  | 8 | 21 |  |
| 8 | 21 |  |  |  |  |

## Subtraction

Step 1: The two numbers are identified and placed so positions are aligned. For example, PROBLEM: "The train left at 2:58 and arrived at 6:21. How much time did it take?"

| CHART |  |  |
| ---: | :--- | :--- |
| Hrs | Mins |  |
| 6 | 21 |  |
| 2 | 58 | $-\quad 6: 21$ |

Step 2: The minutes in the first number are checked to see if sufficient to subtract the minutes of the second number (if not, an hour is changed for 60 minutes):

| Hrs | Mins |
| ---: | :--- |
| 5 | 81 |
| 6 | 21 |
| -2 | 58 |

$$
\begin{array}{r}
5: 81 \\
6: \not 21 \\
-\quad 2: 58 \\
\hline
\end{array}
$$

Step 3: The hours and minutes are subtracted and answers found:

Note: There is one other possible way of setting out:

$$
\begin{array}{r}
6: 21 \\
-\quad \begin{array}{c}
2: 58 \\
\hline 4: 00 \\
\text { down } 37 \text { mins }
\end{array} \\
\hline 3: 23
\end{array}
$$

### 3.2. Non-traditional algorithms

Time is something for which it is sometimes easier to use non-traditional or "mental" algorithms.

## Addition

First possibility - Sequencing: Keep the first number and add parts of the second:

| $3: 52$ |  |
| ---: | :--- |
| $+4: 29$ | First add the hours |
| $7: 52$ |  |
| $+0: 29$ | Then add the minutes to make the next hour |
| $8: 00$ |  |
| $+0: 21$ | Then add remaining minutes |

Second possibility - Compensation:

$$
3: 52
$$

$+4: 29$
8:29 $\quad$ Add 4 hours to $4: 29$ as $3: 52$ is close to 4 hours

- 0:08 This is 8 minutes too much so subtract the 8 minutes

8:21

## Subtraction

First possibility - Sequencing: Keep large numbers and subtract parts of the smaller:

$$
6: 21
$$

- 2:58 Subtract the 2 hours
- 0:58 Subtract the minutes to reduce to hours
- 0:37 Subtract the rest

Second Possibility - Compensation:

$$
6: 21
$$

- 2:58 $\quad$ Subtract 3 hours as 2:58 is close to 3 hours
+ 0:02 Add 2 minutes because subtracted 2 minutes too much

Third possibility - Additive Sequencing:
Instead of subtracting smaller amounts from large, add from smaller to larger and then calculate what was added. (Note: This is similar to the shopkeeper method of giving change.) It has a special algorithm setting out:

| $2: 58$ |  |  |
| :--- | :--- | :--- |
|  | $>0: 02$ | This is the method most people |
| $3: 00$ |  | use to work out time intervals in their head. |
| $6: 00$ | $>3: 00$ |  |
| $6: 21$ | $>0: 21$ |  |
|  | $3: 23$ |  |

This is particularly useful when time change is across 12 o'clock. For example, the time difference between 10:48 am and 3:16 pm is worked out as follows:

| $10: 48$ |  |
| ---: | :--- |
|  | $>0: 12$ |
| $11: 00$ |  |
|  | $>1: 00$ |
| $12: 00$ |  |
|  | $>3: 00$ |
| $3: 00$ | $>0: 16$ |
| $3: 16$ | $4: 28$ |

### 3.3. Time calculation activities

(1) Complete the following calculations using the traditional algorithms:
a) $6: 44$
b) $2: 47$
c) $4: 37$
d) $4: 12$
e) $7: 28$
f) $8: 36$
$+3: 34$
$+1: 53$
$+6: 25$

- 1:47
- 3:35
$-5: 28$
(2) Complete the following calculations using the stated algorithms:
a) $7: 48$
b) $5: 55$
$+2: 36$ Sequencing
$+2: 46$ Compensation
c) 7:27
-4:42 Sequencing
d) $8: 12$
-3:52 Compensation

| e) $2: 28 \mathrm{pm}$ | Additive | f) $4: 38 \mathrm{pm}$ | Additive |
| :--- | :--- | :--- | :--- |
| $-11: 18 \mathrm{am}$ | sequencing | $-10: 52 \mathrm{am}$ | sequencing |

## 4. Timetables

The culminating task requires interpretation of train timetables and the development of a schedule which would be best developed as a table.

### 4.1. Interpreting tables

Consider the following timetable for trains - columns are stations, rows are trains, and cells are times. There are four ways to read the timetable as described below.

| Train | Start | Jackin | Karlin | Mont | Nanty | Ooptan |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $9: 16 \mathrm{am}$ | $9: 47 \mathrm{am}$ | $10: 23 \mathrm{am}$ | $11: 26 \mathrm{am}$ | $11: 58 \mathrm{am}$ | $12: 22 \mathrm{pm}$ |
| 2 | $9: 45 \mathrm{am}$ |  | $10: 43 \mathrm{am}$ |  | $12: 06 \mathrm{pm}$ | $12: 42 \mathrm{pm}$ |
| 3 | $10: 07 \mathrm{am}$ | $10: 39 \mathrm{am}$ |  | $12: 09 \mathrm{pm}$ |  | $1: 35 \mathrm{pm}$ |
| 4 | $10: 54 \mathrm{am}$ | $11: 28 \mathrm{am}$ | $12: 03 \mathrm{pm}$ | $1: 01 \mathrm{pm}$ | $1: 42 \mathrm{pm}$ | $2: 06 \mathrm{pm}$ |

(1) Way 1 - Finding information in a cell (here, finding a time): To do this, look across and down. For example: to find when Train 2 gets to Karlin, we start at Train 2 and move across to Karlin's column. Then circle what time appears in that box - 10:43 am.

| Train | Start | Jackin | Karlin | Mont | Nanty | Ooptan |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  | $\searrow$ |  |  |  |
| 2 |  |  | $10: 43 \mathrm{am}$ |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

(2) Way 2 - Finding which row to use (here, finding a train): To do this, look for the station and then go down for the time and across for the train. For example: if you need to get to Nanty just before 12:30 pm, you start at Nanty, and look down until you get to $12: 06 \mathrm{pm}$, then across to the left for the train. Then you can circle which train number is needed - Train 2.

| Train | Start | Jackin | Karlin | Mont | Nanty | Ooptan |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  | $12: 06 \mathrm{pm}$ |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

(3) Way 3 - Finding which column to use (here, finding a station): To do this, find the train required, go across to the time and look up for the station. For example, to find which station you would have to get off from Train 3 that was just after 12.00 pm , we start at Train 3 and move across to a train near 12:00 pm, then move up to the station. Then circle the station you need to get off at.

| Train | Start | Jackin | Karlin | Mont | Nanty | Ooptan |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  | $12: 09 \mathrm{pm}$ |  |  |
| 4 |  |  |  |  |  |  |

(4) Way 4 - Finding things that have no answers (here, some cells are blank): To do this, you need to understand that sometimes tables have blank spaces because, here, the train does not travel to certain stations. For example, Train 2 does not stop at Jackin. To find these things out, look for gaps and missing spaces.

### 4.2. Constructing a timetable

The best way to understand timetables is to construct one. For this, you have to consider:
(1) "Activities" - this is the focus of the timetable, for example, trains, buses, classes in a school, appointments. These are usually down the side of the table.
(2) "Places" - this is where the activities occur or may stop, for example, buses stop at stations; appointments occur in numbered surgery rooms. These are usually across the top of the table (but may not be, sometimes timetables reverse things).
(3) "Times" - these are what are in the squares or cells of the table - the times at which the activities occur at a certain place.

Thus a timetable is as follows:


To teach this, think of a timetable situation. Try to choose something different. For example, when bands are playing at different venues, get students to create their own personal weekly or monthly "gig guide".


### 4.3. Timetable activities

(1) Get students to interpret timetables:
(a) Obtain a collection of timetables for buses, trains (from bus/train operators, or off the internet) or something different (e.g. planes).
(b) Set question from the timetables for students to answer. For example, for the train timetable in section 4.1, questions could go from simple (e.g. "What time does Train 3 get to Mont?") to complex (e.g. "How long for Train 3 is it from Mont to Ooptan? Is this the same for all trains? Why would these times be different?")
(2) Get students to construct a timetable:
(a) Get students to think of a situation (e.g. rock bands and venues).
(b) Ask them to research their area of interest.
(c) Ask to create names etc., and develop a timetable for their situation.

## 5. Efficient scheduling

The culminating task requires a schedule of appointments to be developed based on train travel. This means that more has to be taken into account than duration of the train trip as the appointment may be away from the station requiring walking. As well, the appointment may be close to two train lines and require working out the best line to take.

### 5.1. Other travel-time factors

To get to an appointment from home by train requires Mr Schien to:
(a) walk from home to the train station;
(b) travel on train to the destination station; and
(c) walk from this station to the appointment.

When planning the trip, additional factors need to be taken into account for each of these stages and extra time allowed. For example:
(a) error or hold up during the walk from home to the station or early/late arrival of the train;
(b) delay or slowness in the train travel to the destination;
(c) error or hold up in walking from the destination station to the appointment; and
(d) arrival at appointment with enough time to find the office, talk to receptionists, etc., and prepare self for the appointment.

For instance, in the timetable from section 4.1:

| Train | Start | Jackin | Karlin | Mont | Nanty | Ooptan |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $9: 16 \mathrm{am}$ | $9: 47 \mathrm{am}$ | $10: 23 \mathrm{am}$ | $11: 26 \mathrm{am}$ | $11: 58 \mathrm{am}$ | $12: 22 \mathrm{pm}$ |
| 2 | $9: 45 \mathrm{am}$ |  | $10: 43 \mathrm{am}$ |  | $12: 06 \mathrm{pm}$ | $12: 42 \mathrm{pm}$ |
| 3 | $10: 07 \mathrm{am}$ | $10: 39 \mathrm{am}$ |  | $12: 09 \mathrm{pm}$ |  | $1: 35 \mathrm{pm}$ |
| 4 | $10: 54 \mathrm{am}$ | $11: 28 \mathrm{am}$ | $12: 03 \mathrm{pm}$ | $1: 01 \mathrm{pm}$ | $1: 42 \mathrm{pm}$ | $2: 06 \mathrm{pm}$ |

What time would you allow to get to an appointment at Nanty at $12: 30$ pm if you lived 3 blocks from Karlin station and the location where the appointment is being held was 3 blocks walk from Nanty? In other words, what time would you leave home and which train would you catch, and why?

The following activities will help teach this:
(1) Have students work out for themselves what they have to take into account in the travel (set it out).
(2) Give them an example like the one for the timetable in 4.1 where there is more than one option and ask them to come up with a solution and an argument for that solution.
(3) Get them to go outside and work out how long to walk a block (do hills matter?) and how often trains are late or early and whether their trip will be affected by where they live in relation to the train line.

### 5.2. Fast and efficient time allocation

Sometimes, an appointment can be reached more than one way. For instance, say you rushed to go from Ipswich to QUT at Gardens Point. You could catch the train that goes through Indooroopilly and get off at Brisbane Central and walk to Gardens Point through the city. Or, you could catch the train that goes through South Bank, get off there and walk across the Goodwill Bridge.

Which is best and why?
To do this requires taking into account:
(a) the schedule for each train;
(b) the time needed for each of the walks; and
(c) the time of the appointment.

To teach students about this, we need to set up examples like the Ipswich to QUT one above. However, it can also be useful to get students to turn the problem around and not only give accounts of when one station/route is better than the other, but also when they are equally efficient and time saving.

Letting students share their experiences and ideas can be an effective method of getting all factors important to solving the problem accounted for and listed. Ask students to give their answers and why, and then list those points that seem to have general application. In this way a list of factors can be built up.

### 5.3. Efficient scheduling activities

Obtain the timetables from section 4.3 but set problems that require more than train times to solve. Set problems of the type in 5.1 and 5.2.

Get students to make up problems for schedules and timetables they have or can get that they think will fool other students.

## CULMINATING TASK

## THE MAN FROM HUNGARY

(Note: For this assignment the following website will be useful http://www.citytrain.com.au/)
Directions: An international visitor is visiting Australia from Hungary. He is arriving on an international flight which arrives at Brisbane airport at 6:20 am, on Monday morning. His name is Mr Schien.

Mr Schien is going to stay at your place for the time he is in Queensland. You can meet him at your local train station but he will need, in advance, a timetable for the week.

Mr Schien is an expert on suburban train infrastructure. He wants to experience as much time on the trains in south-east Queensland during his stay. He has a number of engagements scheduled already. This information is in the table below.
He needs a comprehensive timetable organised for him so that he doesn't miss any appointments. He will need to know where and when he has to be during the week.

## Task One:

Construct a timetable for Mr Schien which includes all the train times, arrivals and departures, in an easy-to-follow format. Remember that Mr Schien will be on his own and in a strange country. Write a paragraph or two explaining any decisions you have made with regard to the timetable. Below is a table of places Mr Schien needs to be, during his stay in Queensland.

| Day of stay | Appointment places | Appointment times |
| :--- | :--- | :--- |
| Monday | Airport to your place | Plane arrives 6:20 am |
|  | Afternoon at Botanic Gardens | Approximately 1:00 pm until 3:00 <br> pm |
|  | Evening at Queensland University of <br> Technology, Gardens Point Campus | Lecture from 7:00 pm until 9:00 pm |
| Tuesday | Trip to Rosewood | Rosewood from 8:30 am until <br> $11: 30$ am |
|  | Lunch in Ipswich | Lunch date 12:30 pm until 2:00 pm |
| Wednesday | Morning at Bond University | Lecture from 3:00 pm until 5:00 pm <br> Robina railway station at 8:30 am <br> Will be dropped back at Robina at <br> $1: 30$ pm |
| Thursday | Evening lecture at the University of <br> Queensland, Ipswich Campus | Lecture from 5:00 pm until 7:00 pm <br> Mr Schien has no commitments <br> today. He desires to spend the day <br> sightseeing in Brisbane City. |
| Needs to be back in Ipswich by <br> $6: 00 ~ p m$ |  |  |
| Friday | Morning visit to Bundamba State <br> Secondary College | BSSC from 8:30 am until 11:00 am |


|  | Lunch at Redbank Plaza | Lunch date from 12 noon until <br> 2:00 pm |
| :--- | :--- | :--- |
|  | Meeting at Sherwood RSL | Meeting from 3:30 pm until 5:30 <br> pm |
| Evening at South Bank | Dinner and theatre from 6:30 pm <br> until 11:00 pm |  |
| Saturday | Free day with you and your family | Organise the day as you see fit but <br> train travel is expected. |
| Sunday | Flight to Budapest via Frankfurt | Flight leaves at 10:30 am and all <br> travellers are expected to be at the <br> airport 75 minutes before their <br> scheduled flight. |

## ASSESSMENT RUBRIC

## Assessment Form:

| General <br> Objectives | Specifics | Assignment <br> Grade (A-E) |
| :--- | :--- | :--- |
| Knowing | Relationship between minutes and hours. <br> Fractional and decimal representation of time. <br> Read, record and calculate with 12 and 24 hour <br> time, especially am and pm. |  |
| Applying | Select the appropriate train times to plan the week <br> for Mr Schien. <br> Interpret and solve problems related to time <br> management. |  |
| Explaining | Present the timetable in a coherent and easy-to- <br> follow format. <br> Explain choices such as fastest journey, distance, <br> direction and speed of travel. |  |

## Assessment Criteria:

| GRADE | KNOWLEDGE | APPLICATION | EXPLAINING |
| :--- | :--- | :--- | :--- |
| A | Effectively uses given <br> rules to carry out tasks. | Applies rules across all <br> contexts effectively. | Presents detailed <br> solutions logically and <br> clearly. |
| B | Uses given rules to carry <br> out tasks. | Applies rules across most <br> contexts effectively. | Presents solutions <br> logically and clearly. |
| C | Uses given rules <br> adequately to carry out <br> tasks. | Applies rules adequately <br> across most contexts. | Presents readable <br> solutions. |
| D | Uses given rules to carry <br> out parts of tasks. | Applies rules in some <br> contexts adequately. | Presents partial solutions. |
| E | Did not meet standard D. | Did not meet standard D. | Did not meet standard D. |

