YUMI DEADLY CENTRE
School of Curriculum

Enquiries: +61 7 3138 0035
Email: ydc@qut.edu.au

http://ydc.qut.edu.au

Numeracy for Transition to Work

Year 11 Prevocational Mathematics

Booklet 11.2: "Big Day Out"

Number, tables, budgeting, algebra and problem solving

Bundamba State Secondary College
Queensland University of Technology
DEADLY MATHS CONSORTIUM

TEACHER RESOURCE BOOKS
NUMERACY FOR TRANSITION TO WORK
YEAR 11: SEMESTER 2

11.2

PREVOCATIONAL MATHEMATICS

Booklet 11.2: "Big Day Out"

VERSION 1, 2009

Bevan Penrose
Bundamba State Secondary College
Tom J Cooper
Annette R Baturo
with
Kaitlin Moore
Elizabeth Duus
Deadly Maths Consortium
Faculty of Education, QUT

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

School of Curriculum
QUT Faculty of Education
S Block, Room S404, Victoria Park Road
Kelvin Grove Qld 4059
Phone: +61 7 3138 0035
Fax: + 61 7 3138 3985
Email: ydc@qut.edu.au
Website: http://ydc.qut.edu.au

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Bundamba State Secondary College &
Queensland University of Technology

TEACHER RESOURCE BOOKLETS

NUMERACY FOR TRANSITION TO WORK

YEAR 11 PREVOCATIONAL MATHEMATICS

BOOKLET 11.2: "BIG DAY OUT"

VERSION 1: 2009

Research Team:
Bevan Penrose
Tom J Cooper
Annette R Baturo
with
Kaitlin Moore
Elizabeth Duus

Deadly Maths VET
School of Mathematics, Science and Technology Education,
Faculty of Education, QUT

DEADLY MATHEMATICS VET

Deadly Maths VET was the name given to the materials produced to support the teaching of numeracy to vocational education and training students, particularly those from Indigenous backgrounds. These booklets were produced by the *Deadly Maths Consortium* at Queensland University of Technology (QUT) but also involving a researcher from Nathan Campus of Griffith University.

At the time of the production of this booklet, Deadly Maths VET was producing materials as part of an ARC-funded Linkage grant LP0455667 (12 booklets on Years 11 and 12 Prevocational Mathematics course and 2 booklets on pesticide training) and ASISTM-funded 2008 grant (3 booklets on construction; 3 booklets on engineering; 3 booklets on marine; 1 booklet on retail; and 2 booklets and a series of virtual materials on basic mathematics).

CONTENTS

THE PREVOCATIONAL MATHEMATICS BOOKLETS

In 2005 to 2008 researchers Annette Baturo and Tom Cooper from the Deadly Maths Consortium received an ARC Linkage grant (LP0455667) to study mathematics learning of Year 11 and 12 and adult students undertaking vocational education and training (VET) courses who had low achievement in mathematics. The title of this ARC Linkage project was *Numeracy for Access to traineeships, apprenticeships, and vocational studies and empowerment and post year ten students.* The project studied better ways to teach mathematics to VET students at Bundamba State Secondary College, Metropolitan Institute of TAFE (Moreton Campus), Gold Coast Institute of TAFE (Ridgeway Campus), Tagai College (Thursday Island Campus), and the Open Learning Institute of TAFE.

As part of the study, the Deadly Maths research team developed booklets and other resources to be trialled by VET students and teachers. The project's activity with Bundamba State Secondary College focussed on the Year 11 and 12 Prevocational Mathematics subjects taught at that college to less able mathematics students. The project used a series of intervention case studies to research learning. As part of this, the following 12 Prevocational Mathematics resource booklets were produced (6 for Year 11 and 6 for Year 12). The booklets are numbered 11.1 to 11.6 and 12.1 to 12.6.

11.1 – "Using Numbers of Numbers" – Yr 11 prevocational maths booklet: Number, decimals, fractions and problem solving.

11.2 – "The Big Day Out" – Yr 11 prevocational maths booklet: Number, tables, budgeting, algebra and problem solving.

11.3 – "Rating our World" – Yr 11 prevocational maths booklet: Rate, area and volume activities and problems.

11.4 – "Exchange Student" – Yr 11 prevocational maths booklet: Operations, discounts, tables, metric conversion and best buys.

11.5 – "Planning a Roster" – Yr 11 prevocational maths booklet: Tables, 24 hour time, percentages and computation strategies.

11.6 – "The Man from Hungary" – Yr 11 prevocational maths booklet: Time relationships, time calculations, timetables and efficient scheduling.

12.1 – "Beating the Drought" – Yr 12 prevocational maths booklet: Fractions, probability, graphing and data.

12.2 – "Monopoly" – Yr 12 prevocational maths booklet: Fractions, probability, game strategies, property finance, graphs and tables.

12.3 – "How tall is the Criminal?" – Yr 12 prevocational maths booklet: Multiplicative structure, ratio and proportion, problem solving.

12.4 – "Design a Kitchen" – Yr 12 prevocational maths booklet: Visual imagery, percent, rate, ratio, perimeter, area and volume.

12.5 – "Healthy Eating" – Yr 12 prevocational maths booklet: Data collection and analysis, tables and graphing (line and histograms).

12.6 – "Rocking around the World" – Yr 12 prevocational maths booklet: Time and angle, time operations and problem solving strategies.

OVERVIEW

1. Theoretical position

The Bundamba Prevocational Mathematics booklets are based on the notion of Renzulli (1977) that mathematics ideas should be developed through three stages.

Stage 1: Motivate the students – pick an idea that will interest the students and will assist them to engage with mathematics.

Stage 2: Provide prerequisite skills – list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.

Stage 3: Culminating task – end the teaching sequence by setting students an open-ended investigation to explore.

These booklets use Stage 3 as an assessment item and so we have added an assessment rubric whereby the culminating task can be used to check the knowledge held by the student.

The booklets combine two approaches to teaching:

(1) structural activities that lead to the discovery and abstraction of mathematical concepts, processes, strategies and procedures; and

(2) rich-style tasks which allow students an opportunity to solve problems and build their own personal solution.

2. Mathematics for this booklet

The motivating idea behind this booklet is that of setting up and budgeting for a trip to the Big Day Out, an Australian and New Zealand wide music festival. This task should have resonance with the students in that it is something they themselves may have to do for themselves in their future. It is not a simple task, having many things to consider, so it is not only a relevant thing to do, it has many options. The culminating task is thus motivating and open. The prerequisite skills for budgets are not so obvious. However, for this booklet, they will be deemed to consist of the following:

(1) *Identification and computation* – calculators or spreadsheets can be used to calculate totals; therefore, all that is required is to be able to identify which operators and which numbers are needed to determine the total cost; however, we also provide some methods for calculating (if there is no access to calculator or mobile phone),

(2) *Using tables* – the best way to set up the budget is with a table (as the culminating task directs); therefore, how to use tables is part of the prerequisite skills,

(3) *Budgeting and simple algebra* – any form of budgeting has unknowns and variables (spending more on drink means spending less on food); this section looks at how simple algebraic thinking operates with money in budgets,

(4) *Multi-step problems* – this section looks at the strategies that will help in multi-step problems, plus how to determine what has to be taken into account for a Big Day Out type problem.

2.1. Identification and computation

Many students believe that you determine operation by the language that is used in describing the task, for example, "take-away" means subtraction. However, consider this problem: *John had some bottles. Fred came and took away 6 bottles. This left 18 bottles. How many bottles were there to start with?* This has words and actions of "take-away" yet to get the answer, 24, one has to add the 6 and the 18.

Thus, there is a need for new ways to determine which operation is required. These are:

(1) All descriptions of operations have three numbers; two numbers are given and the third is requested. If these numbers all refer to the same thing (e.g., bottles), then the operation is either addition or subtraction. If two of these numbers refer to the same thing and the third refers to "groups" of these things, then the operation is either multiplication or division.

(2) When the three numbers refer to the same thing: (a) addition is when you know the parts and want the total; and (b) subtraction is when you know the total and one part and you want the other part.

(3) When two of the numbers refer to the same thing and the third number refers to "groups" of that thing: (a) multiplication is when you know the factors and want the product; and (b) division is when you know the product and one factor and want the other factor.

The crucial thing in modern mathematics is: (a) to have a repertoire of strategies for computation that also can be used in other mathematics topics (if pen and paper computation is not required – calculator can be used); and (b) to enable students to find the strategy that works for them to get answers and to relate this to a recording system that works for them (if pen and paper computation is required).

Thus, teachers need to know the three major strategies for computation as follows:

(1) Separation: In this, numbers are separated into place values, the operations are done separately, and then results combined. For example: 46 + 28 is 40 + 20 and 6 + 8 is 60 + 14 is 78. Similarly: 38 × 7 is (30 + 8) × 7 is 30 × 7 and 8 × 7 is 210 and 56 is 266.

(2) Sequencing: In this, one number is left unchanged, the other is broken into parts and the operation acts on the unchanged numbers and parts in sequence. For example: 46 + 28 is 46 + 20 + 8 is 74. Similarly: 38 × 7 is 38 × 1 plus 38 × 2 plus 38 × 4 is 38 + 76 + 152 is 266, or: 38 × 7 is 38 × 5 + 38 × 2 is 190 (½ 380) + 76 is 266.

(3) Compensation: In this, both numbers are left unchanged but nearby numbers for which the operation is easy are used and then compensated for. For example: 46 + 28 is 46 + 30 subtract 2 is 76 – 2 = 74. Similarly: 38 × 7 is 40 × 7 minus 2 × 7 is 280 – 14 is 266.

2.2. Using tables

Tables are a way of representing information which shows relationships of the information to two different classifications of this information. For example: times can be organised in relation to trains and stations (2 classifications) in cells.

Stations

	Chelm	Hod	Pend	Fedt			
Train 1	9.20 am	9.40 am	10.20 am	10.50 am			
Train 2							
Train 3							

Cells

Trains

Times are put here

Tables are highly structured – information is given horizontally and vertically (in rows and columns). However, information is also given in a table in the relationships between cells in rows and columns. For example: people and weights can be organised in a table (2 classifications of people and attributes).

	Name	Mass
Person 1	Fred	115 kg
Person 2	Jack	102 kg
Person 3	Joe	96 kg

Attributes

(Note: The people column is often left out)

Information is also found within these relationships

People

Tables can also be used for calculations; see figure right:

Item	Restaurant	Number of Meals	Number of Items	Cost per Item	Item Reduction	Totals
Sausage & Egg muffin	Mc Edible	4	8	$3.20	0	$25.60
Orange Juice	Mc Edible	4	4	$2.90	0	$11.60
					OVERALL TOTAL	$168.40

2.3. Budgeting and simple algebra

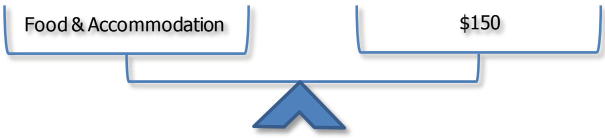
Budget situations are actually algebra situations in the general sense of algebra. Algebra in the generalisation of arithmetic is when you do or think about operations in terms of "any number" or in general. It is where you can't immediately find an answer. For example: *I bought a hat for $20 and a coat for $60, how much did I spend?* is arithmetic, but *I bought a hat and then I bought a coat for $60, how much did I spend?* is algebra. In equations, the first is 20 + 60 = 80, while the second is Hat + 60 = total.

What you have to do in these algebra situations is to think about operations in terms of general numbers. For example, for Hat + 60 = total, you think of things like: (a) the total depends on the hat cost; and (b) the hat is $60 less than the total (Hat = total – 60).

Budgeting is a particularly strong algebra situation because there is a fixed total and if something costs more you have less to spend on something else. So budgeting requires thinking of numbers generally and relating increases in one amount to decreases in another number. For example: if your budget is $150 for food and accommodation, you do not have much to spend on food if your accommodation is expensive (and neither food nor accommodation can be over $150 in total). This budget equation is:

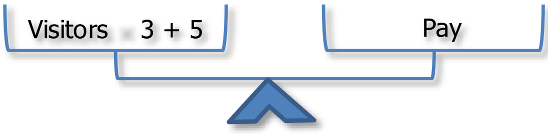
Food + accommodation = 150

To assist thinking in budget and other algebra situations, there are some models that can help. One of the best is to think of equations as balance. See example right:



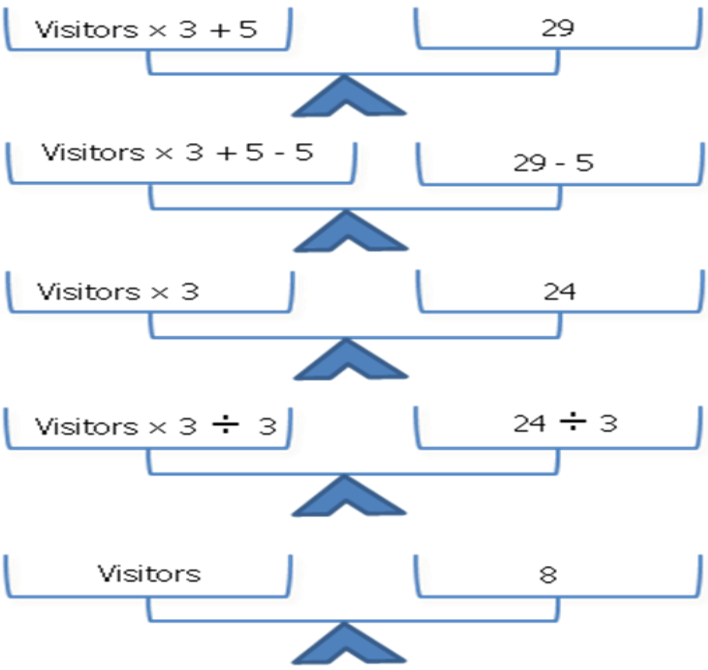
Once this is done, then it is easy to see that, to keep the equation correct, we need to do the same thing to both sides (the balance principle).

This enables complicated algebra to be solved. For example: Fred buys pies for $3 for his visitors and a sandwich for $5 for himself. How much does he pay? The equation and balance are shown right:



Equation: $3V + 5 = P$

If he paid $29, then we can find the number of visitors as follows:



$3V + 5 = 29$

$3V + 5 - 5 = 29 - 5$

$3V = 24$

$3V \div 3 = 24 \div 3$

$V = 8$

Note: The basis of this section, and of section 2.1 earlier, is that equations tell stories (stories can be represented by equations and one can make up a story for an equation).

So, spend time relating stories and equations (particularly the reverse, equation to story).

$3 + 4 = 7$ There were 3 children playing and 4 more joined to make 7 children.

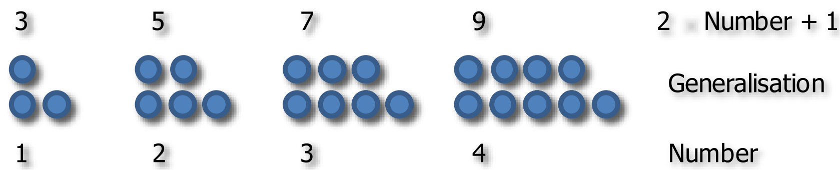
$3 + 4 = ?$ There were 3 children playing and 4 more joined. How many children?

$x + 4 = 7$ There were some children playing and 4 more joined to make 7 children. How many were playing at the start?

$x + 4 = y$ There were some children playing and 4 more joined to make a larger group of children.

Note: There are three ways to introduce the idea of a variable (which can be any number):

(1) generalise a pattern or a change, for example,



(2) formulae, for example, Area of Rectangle = length × width

(3) from problems where we solve for unknown(s), e.g., the selling shirts problem when we sell for $45 (a profit of $25): Number × $25 = $500; Number = 20

In this booklet, we will be using way (3) to look at variables. Of course, what we are really doing with unknowns is pre-algebra (not full algebra). This is because, when there is an unknown, the calculations only use normal arithmetic (there are no variables in calculations).

2.4. Multi-step problems

The problem with most VET situations and rich tasks is that they involve more than one operation. To work out what operations to use with what numbers, requires ways to represent the situations and tasks. Some ways include:

(1) Break into parts: Break the problem into sections and tackle each section one at a time,

(2) Identify the given, the needed and the wanted: Try to break the information into what you start with (the given), where you have to go (the wanted) and then work out the steps to get there (the needed).

(3) Restate the problem: This can also include acting the situation out. A good drawing can help work out what to do. Good drawings are based upon length or sets, and are often diagrammatic. Take this problem, for example, "Each day Fred bought a pie for lunch. How much change from $20?" The following is an example of a good drawing:

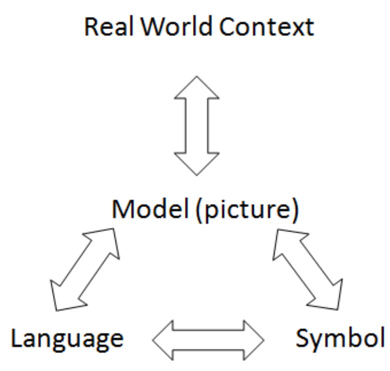
Monday	Tuesday	Wednesday	Thursday	Friday
$ 3.40	$ 3.40	$ 3.40	$ 3.40	$ 3.40

$20

3. Pedagogy

The pedagogy for the culminating task is to: (a) interest the students in the situations so they are engaged in the task, (b) provide them with all they need mathematically to gather information and computer totals, and (c) let them develop their solution as they see fit.

The pedagogy for the prerequisite skills is to develop mental models (pictures in the mind) and connect all representations for the mathematics concepts, processes, strategies and procedures that are needed to tackle the culminating task. Thus instruction is based on the so-called Rathmell Triangle (Payne & Rathmell, 1978):



The prerequisite pedagogy also involves three major generic strategies:

(1) Flexibility – trying to ensure students understand things in a variety of ways (e.g., discount, reduction, etc.; %, part per 100, decimal hundredths, fraction out of 100, and so on).

(2) Reversing – trying to teach in all directions (e.g., real-world situation to best buy; best buy to real-world situation).

(3) Generalising – trying to teach things in the most general way (going beyond the needs of a task).

Particular strategies for tackling some of the preliminary activities will be provided at the start of the activities.

4. How to use this booklet

The major focus of the unit is the culminating task. The preliminary activities are only suggestions for prior work if you think your students require this work before they begin the culminating task. Therefore:

(1) use the culminating task as the focus of the unit – to motivate engagement;

(2) look through the preliminary activities and pick and choose things that you believe will be useful for your students – it is not necessary to do everything and to do it in the order that it appears in this booklet (although there is a logic to the order);

(3) do these activities as a lead in to the culminating activity; and

(4) try to organise things so that the students can do the culminating activity as an assessment of their abilities to do mathematics.

The preliminary activities are in sections and there are real activities at the end of each section. The earlier parts of each section simply explain the ideas/models/pictures in the mind that are being attempted. Although how they are presented gives some hints on pedagogy, you will have to determine your own way to teach these.

PRELIMINARY ACTIVITIES

1. Identification and computation

In many situations, you can use a calculator to work out the answer to a computation, but you need to know the numbers and the operations – the calculator will not tell you what those are. Therefore, this section looks at identifying operations in real-world situations. However, because the real world often happens without calculators we also include the strategies for mental calculation and estimation.

1.1. Identifying operations

Addition and subtraction were invented by mathematicians as ways to describe, and solve problems with respect to, joining and separating situations. However, it is not as simple as joining being addition and separation (e.g., take-away being subtraction). Consider the following problem: *I went to the bank and took out $55. This left $212 in the bank. How much did I have to start with?* Although the action and the language in the problem are take-away, the question requires the two numbers to be added.

Thus, the part-part-total approach is the best way to think about addition and subtraction:

(1) Addition – knowing the parts and wanting the total, and

(2) Subtraction – knowing the total and one part and wanting the other part.

Then, when the problem above is analysed, it can be seen that the $55 and the$212 are parts and the unknown is the total. This makes the problem addition regardless of the words and actions in the description.

Addition and subtraction can also describe comparison situations. Once again, determining the operation is not straightforward. For example, the problem, *John has $36, Frank has $15 more than John, how much does Frank have?*, is solved by addition; while the problem, *Frank has $15 more than John, Frank has $62, how much does John have?*, is solved by subtraction. However, the part-part-total approach still applies here if a little creativity is used.

Multiplication and division were invented by mathematicians as ways to describe, and solve problems with respect to, combining equal groups and partitioning into equal groups situations. Once again, things are not as simple as combining being multiplication and partitioning being division. For example, the following problem is solved by multiplication but is a partitioning action: *The money was shared among 11 people, each person got $156, how much was there to be shared?*

As before, there is an approach that enables the problems to be correctly interpreted – it is called factor-factor-product:

(1) Multiplication – the two factors are known (the number of groups and the number in each group) and the product is wanted, and

(2) Division – one factor and the product are known and the other factor is wanted.

Multiplication and division can also describe comparison situations. Once again, determining the operation is not straightforward. For example: the problem, *Mary has 4 times the money Jane has, Jane has $26, how much does Mary have?*, is solved by multiplication; while the problem, *Mary has 4 times the money Jane has, Mary has $184, how much does Jane have?*, is solved by division. However, similar to addition-subtraction, the factor-factor-product approach still applies if used with a little creativity.

Thus, we have a two-step process for determining operators:

Step 1 Work out whether problem is addition/subtraction or multiplication/division.

Step 2 Use part-part-total or factor-factor-product to determine actual operation.

Identifying whether a problem is addition-subtraction or multiplication-division is based on what the numbers mean in the problem. For addition-subtraction, we have a number for the first part, a number for the second part and a number for the total. Since there is joining and separation, all 3 numbers are in relation to the same thing (e.g., cats, money, bells, children). For example, *12 children joined 3 adults, how many people were there?*

The numbers all refer to people. 12 people + 3 people = 15 people.

However, for multiplication-division, the first factor is the number of groups, the second factor is things and the product is things. Thus only two of the three numbers refer to the same thing and the third number refers to groups (e.g., number of cans at $3, or number of groups of children, number of boxes of balls). For example: *12 children got into 3 cars evenly, how many children were there in each car?*

Only two of the numbers refer to children. 12 children ÷ 3 cars = 4 children.

Teaching how to determine an operation is made difficult by the many ways language is used to say things. The answer to this is to first give problems where the language is very simple and the order of the solution procedure is the order of the language.

For example: (a) *I had $56, spent $37, how much change?* (b) *I bought 3 bags of Easter eggs for $14 each. How much change do I have from $50?*

Then start to make the language more complex and break the nexus between the order of mathematics and the order of the language.

For example: (a) *I went to the shop and spent $37. I had $56. What was my change?*; (b) *The Easter eggs were $14 a bag. I paid with $50. What change did I get back if I bought 3 bags of eggs?*

Finally, move on to unfamiliar contexts for language:

For example: (a) *I bought 1000 Farmel shares for $37 each and sold them when they were $56. How much did I make/share?* (b) *The three companies combined. I bought shares in each for $14. The new combined shares were $50. If I sold them, how much profit per share would I make?*

1.2. Computation strategies for addition and subtraction

Without a calculator there are three methods of adding and subtracting numbers. These methods are also the basis of estimation

(1) Separation – separate both numbers into parts, add or subtract the parts separately and then combine the answers:

Addition

Add tens first	Add ones first	Add highest place value first	
	1		
56	56	84.47	
+ 28	+ 28	+ 47.55	
70	84	120.00	
14		11.00	
84		1.02	
		132.02	130.00 will do as an estimate

Subtraction

Subtract tens first	Subtract ones first	Subtract whole nos first	
	7 1	6 1	
82	$\cancel{8}2$	$\cancel{7}4.08$	
− 27	− 27	− 45.69	
60	55	29.00	
down 5		down 0.61	
55		28.39	29.00 will do as estimate

(2) <u>Sequencing</u> – keep the first number whole and add parts of the second number:

56	84.47	82	74.08
+ 28	+ 47.55	− 27	− 45.69
76	124.47	62	34.08
+ 8	+ 7.55	− 7	− 5.69
$84	131.47	$55	29.08
	+ 0.55		− 0.69
	131.97		28.48
	+ 0.05		− 0.09
	132.02		28.39

Subtraction by sequencing can be done additively (building from smaller to larger instead of taking smaller from larger):

$82	→	27	3
− 27		30	50
		80	2
		82	
			$55

$74.08	→	45.69	0.31
− 45.69		46.00	4.00
		50.00	24.00
		74.00	0.08
		74.08	
			$28.39

(3) <u>Compensation</u> – both numbers are kept whole, numbers are changed to make operations easy, then change is compensated for:

56	→	56		82	→	82	
+ 28		+ 30	change	− 27		− 30	change
		86				52	
		− 2	compensation			+ 3	compensation
		84				55	

84.47	→	84.47		74.08	→	74.08	
+ 47.55		+ 50.00	change	− 45.69		− 44.00	change
		134.47				30.08	
		− 2.45	compensation			− 1.69	compensation
		132.02				↓	
						30.08	
						− 2.00	change
						28.08	
						+ 0.31	compensation
						28.39	

(4) <u>Estimation/Approximation</u> – to estimate, ignore the smaller values and just try to get to the nearest 10 or 5. Later you can try to get closer if needed:

368.57	→	370.00
+459.68		+460.00
		830.00

917.47	→	920.00
−348.69		−350.00
		570.00

1.3. Computation strategies for multiplication and division

These are the same as for addition and subtraction.

(1) Separation – separate numbers into parts and multiply and divide separately:

Multiplication

	24
	x 37
tens × tens	600
tens × ones	140
ones × tens	120
ones × ones	28
	888

	30	7
20	tens × tens = 600	tens × ones = 140
4	ones × tens = 120	ones × ones = 28

Division

$$\begin{array}{r|l l}
 & 256 & \\
\hline
3 & 768 & \\
 & 6 & \text{Sharing 100s} \\
\hline
 & 16 & \\
 & 15 & \text{Sharing 10s} \\
\hline
 & 18 & \\
 & 18 & \text{Sharing 1s} \\
\hline
 & 0 &
\end{array}$$

	200	50	6
3	600	150	18

(2) Sequencing – keep one number whole and multiply and divide by parts of the other:

Multiplication

$$\begin{array}{r} 37 \\ \times\ \underline{24} \end{array} \longrightarrow \begin{array}{r} 37 \\ \times\ \underline{\ \ 4} \\ 148 \\ \times\ \underline{\ \ 6} \\ 888 \end{array} \quad 24 = 4 \times 6$$

$$\begin{array}{rl} 37 & \\ \times\ \underline{24} & \\ 370 & \times 10 \\ 370 & \times 10 \\ 74 & \times\ 2 \\ \underline{74} & \times\ 2 \\ 888 & \end{array}$$

Division

$$\begin{array}{r|l}
24 & 696 \\
 & \downarrow \\
 & 174 \\
4 & 696 \\
 & \downarrow \\
 & 29 \\
6 & 174
\end{array}
\qquad\qquad
\begin{array}{r|r|l}
24 & 696 & \text{10 lots of 24} \\
 & -240 & \\ \cline{2-2}
 & 456 & \text{10 lots of 24} \\
 & -240 & \\ \cline{2-2}
 & 216 & \text{5 lots of 24} \\
 & -120 & \\ \cline{2-2}
 & 96 & \text{2 lots of 24} \\
 & -\ 48 & \\ \cline{2-2}
 & 48 & \text{2 lots of 24} \\
 & -\ 48 & \\ \cline{2-2}
 & 0 & 29
\end{array}$$

(3) Compensation – keep both numbers whole just change to something that is easy and then compensate for the change:

Multiplication

$$\begin{array}{r}
29 \\
\times\ 36
\end{array}
\longrightarrow
\begin{array}{rl}
30 & \text{Change} \\
\times\ 36 & \\ \hline
1080 & \\
-\quad 36 & \text{Compensate} \\ \hline
1044 &
\end{array}
\qquad\qquad
\begin{array}{r}
83 \\
\times\ 24
\end{array}
\longrightarrow
\begin{array}{rl}
83 & \\
\times\ 25 & \text{Change} \\ \hline
2075 & \\
-\quad 83 & \text{Compensate} \\ \hline
1992 &
\end{array}$$

Note:
× 25 is × 100 ÷ 4
83 ÷ 4 = 20 and ¾ hundreds

Division

$$7\,\overline{)\,1862} \qquad
\begin{array}{rl}
7\,\overline{)\,2100}^{\;300} & \text{Change} \\
-\ 7\,\overline{)\,210}^{\;30} & \text{Compensate} \\
-\ 7\,\overline{)\,28}^{\;4} & \text{Compensate} \\
=\ 266 &
\end{array}$$

(4) Estimation/approximation – the strategy of rounding still works but the straddling strategy where larger and smaller numbers are used to find an estimate above and below:

Multiplication

$$\begin{array}{r}
34 \\
\underline{\times\ 48}
\end{array}
\qquad
\begin{array}{r}
30 \\
\underline{\times\ 40} \\
1200
\end{array}
\quad \text{to} \quad
\begin{array}{r}
40 \\
\underline{\times\ 50} \\
2000
\end{array}$$

Somewhere in the middle, say, a bit over 1600

Division

$$73\overline{)2745} \longrightarrow 70\overline{)2800} \longrightarrow 40$$

1.4. Identifying/Computing activities

(1) Circle which operation has to be used:

(a) There are 8 times as many apples as oranges, there are 56 apples, how many oranges? **+ − × ÷**

(b) There were 24 more cattle sent to market from Jensen Station than there were from Tropica Station. Jensen sent 328 cattle, how many cattle did Tropica send to market? **+ − × ÷**

(c) There were 88 crates of cans, there were 11 cans in each crate, how many cans? **+ − × ÷**

(d) There were 16 less people after lunch, this made 84 people, how many started the day? **+ − × ÷**

(e) The men and women were loading sheep into 8 trucks, there were 124 sheep in each truck, how many sheep? **+ − × ÷**

(2) Circle the operation needed to complete the table and fill in the ?

	Item	Number of Items	Cost/item	Total Cost	Operation?
(a)	Can of beans	6	?	$12.54	+ - x ÷
(b)	Bottle of juice	3	$3.45	?	+ - x ÷
(c)	Packet of Pasta	?	$2.56	$12.80	+ - x ÷
(d)			SUB TOTAL	?	+ - x ÷
(e)			Reduction	?	+ - x ÷
			TOTAL	$31.85	

(3) Construct an everyday money/shopping problem for each equation and situation:

	EQUATION	SITUATION
(a)	$28 ÷ 7	Jack buys 7 times as much as Jill.
(b)	$48.27 + $63.21	Sue loses some money.
(c)	$128 – $56.54	Bill buys two shirts, one of which costs $56.54.
(d)	$34.68 × 8	Fred packs the windcheaters into bags of 8.

(4) Calculate the following using some pen and paper or mental method:

(a) $\begin{array}{r} 68.27 \\ \underline{+\ 49.56} \end{array}$

(b) $\begin{array}{r} 487.85 \\ \underline{+\ 234.19} \end{array}$

(c) $\begin{array}{r} 44 \\ \underline{\times\ \ 57} \end{array}$

(d) $\begin{array}{r} 128 \\ \underline{\times\ \ 49} \end{array}$

(e) $\begin{array}{r} 147.21 \\ \underline{-\ \ 86.48} \end{array}$

(f) $\begin{array}{r} 643.87 \\ \underline{-\ 256.49} \end{array}$

(g) $6\overline{)858}$

(h) $23\overline{)4531}$

(5) Estimate using one of the methods

(a) $\begin{array}{r} 356.87 \\ +\ \underline{298.37} \end{array}$ (b) $\begin{array}{r} 861.28 \\ -\ \underline{282.43} \end{array}$

(c) $\begin{array}{r} 56 \\ \times\ 74 \\ \hline \end{array}$ (d) $82\overline{)1722}$

2. Using tables

In tasks like a Big Day Out, the important thing is to ensure that everything that is needed (for the trip) has been taken into account. An effective way to do this is to use tables based on lists of requirements.

2.1. Lists of requirements

To ensure that a budget is accurate, complete and exhaustive lists have to be developed of what is needed to make a trip and then turned into a form that will facilitate computation. The best form for this is to turn the lists into tables. It is also good if the table can contain all the information required in a form that allows it to use technology (e.g., Excel spreadsheets). This section looks at how to turn lists into tables; the next section will look at how to set up the table for computation.

To show how lists can become tables, we look at a simpler example than budgeting for Big Day Out, for example, *What would be the cost of one person on a 4-day trip?* There are three steps:

Step 1 – Develop a list: One way to do this is to break the trip, or whatever is being budgeted, into parts, to form a list; for example, a list for a trip could be

Travel

Accommodation

Tickets

Food

Drink

Step 2 – Consider all needed for table: One way to do this is to think of one item and what is needed to work out cost (e.g., the number of items, where to buy it, the cost and so on).

Item (food/drinks)

Place to buy it (restaurant/shop)

Number of meals (how many meals)

Number of items (how many of this)

Cost of each item (different amounts for different meals)

Any reductions

Totals

Step 3 – Translate to a table: An effective way to do this is to put the needs (Step 2) across the top and the list of all items (Step 1) down the left hand side of the table.

Item	Restaurant	Number of Meals	Number of Items	Cost per Item	Item Reduction	Totals
Sausage and Egg muffin						
Orange Juice						
Coffee						
Coca Cola						
Sandwiches						
Noodles/Pasta						
Ice Creams						
Salad						
					OVERALL TOTAL	

Step 4 – Determine numbers/costs: The next step is to complete the table by filling in the gaps – determining the number of items, the shop, and the cost/item (plus any reductions).

Item	Restaurant	Number of Meals	Number of Items	Cost per Item	Item Reduction	Totals
Egg muffin	Mc Edible	4	8			
Orange Juice	Mc Edible	4	4			
Coffee	Coffee Stall	6	12			
Coca Cola	Corner Deli	6	12			
Sandwiches	Sandwich Bar	4	8			
Noodles/Pasta	Rok a Wok	3	9			
Ice Creams	Ice Cream Stall	3	3			
Salad	Salad Bar	3	6			
					OVERALL TOTAL	

2 Serves.
E.g: 2 muffins per meal means 8 muffins overall

Could add on an extra column for item/meal – then number of items is a calculation. From the resulting table, totals can be calculated. How this is done is left to the next section.

2.2. Calculating with tables

Once the table has been set up, then operations can be used to determine totals (budgets) as follows.

Step 1 – Select appropriate operations: Selecting the right operations within a table is easier than in real-life situations, but still needs to be accurate. In general, the rules are: (a) addition for totals and overall totals; (b) subtraction for any reductions (discounts); and (c) multiplication of number × cost or number × reduced cost for total in each row.

Of course, if we have to work backwards (e.g., find the cost per item when know cost of 4), the operations can invert (e.g., from multiplication to division).

Step 2 – Determine form of totalling: In a table, you can find the overall total at the end or you can total cumulatively – keep a running total that advances at each row.

Step 3 – Complete the operations: Here are examples for overall total and cumulative total based on the cost of a 4-day trip for one person.

Item	Restaurant	Number of Meals	Number of Items	Cost per Item	Item Reduction	Totals
Sausage & Egg muffin	Mc Edible	4	8	$3.20	0	$25.60
Orange Juice	Mc Edible	4	4	$2.90	0	$11.60
Coffee	Coffee Stall	6	12	$2.50	$0.25	$27.00
					OVERALL TOTAL	$168.40

Multiplication
4 x $2.90 = $11.60

Addition of the column.

4 meals, 2 items per meal.
4 x 2 = 8 muffins

Subtraction
$2.50 - $0.25 = $2.25

Multiplication
12 x $2.25 = $27.00

Cumulative total:

Item	Restaurant	No. of Meals	No. of Items	Cost per Item	Item Reduction	Cumulative Totals
Sausage & Egg muffin	Mc Edible	4	8	$3.20	0	$25.60
Orange Juice	Mc Edible	4	4	$2.90	0	$37.20
Coffee	Coffee Stall	6	12	$2.50	$0.25	$64.20

Adding $25.60 to total of row ($11.60)

Adding $37.20 (the amt above) to total of the row ($27.00)

2.3. Table activities

(1) Develop a list of items for the following situations: (a) things other than clothing that you need to take on a trip to the Big Day Out; (b) food that you would have to buy over 3 days of a Big Day Out; and (c) forms of travel that you would have to take to get to the Big Day Out.

(2) Translate the lists from (1) into tables as follows: (a) a table that determines the cost for these items that you take to the Big Day Out; (b) a table that determines the cost of the food for one day; and (c) a table that would give the cost of travel for you and 4 mates from different places.

3. Budgeting money and simple algebra

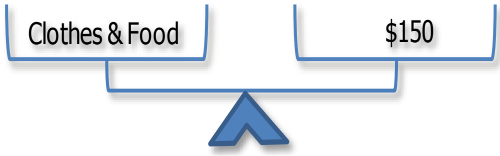
Budgeting means looking at what happens if more is spent in one area: its impact on what can be spent on another area. Thus it requires algebraic thinking, the ability to think of number in a general sense.

3.1. Thinking of budgeting as balance

Consider the example, *I had $150 to spend on clothes and food, how much do I spend on food?* This initial answer is "I cannot work it – I don't have enough information" or "it depends – on what I spend on clothes". We need a way of thinking about the problem. One way to do this is to think of the budgeting problem as balance.

Step 1: Put the problem in language in a straight forward manner, e.g., *cost of clothes and food is $150*.

Step 2: Put this in the language of operations (there are only 4 operations – it can't be subtraction, multiplication or division, so it must be addition:
cost of clothes plus cost of food is $150



Step 3: Make this an equation: clothes + food = $150

Step 4: Think of an equation as a balance (see right)

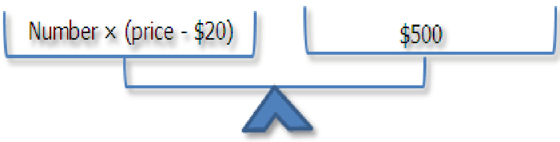
Now we can discuss the problems, for example: (a) If food = $50, clothes is $100 to keep the balance; and (b) If food increases by $20 to $70, clothes decreases to $80 to keep the balance.

Try another example: *I buy shirts wholesale at $20 to sell at a market. It costs $300 for a stall at the market. How many shirts do I need to sell to make $200 for myself in a day?* Once again, "it depends" on how many shirts are sold and how much I charge per shirt. So we go through the steps:

Step 1: The number of shirts I sell times the profit I make has to be $200 more than the cut of $300.

Step 2: numbers sold × sale price minus $20 is $500

Step 3: numbers sold × (price-$20) = $500



Step 4: Write the balance as we have done at right:

So again we can discuss:

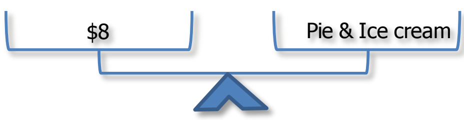
(a) If I sell at $30 then make $10 per shirt and I need to sell 50 shirts.

(b) If I sell 40 shirts I need to make $500/40 shirts = $12.50 on each so I need to sell at $32.50

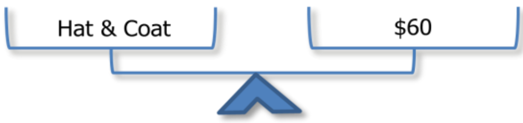
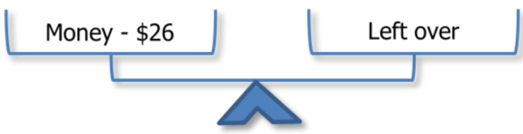
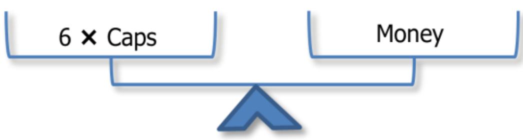
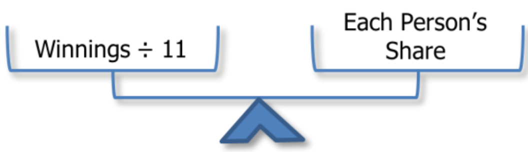
This leads to seeing relationships such as, "the less profit I make per shirt, the more shirts I have to sell."

3.2. Relating problem, balance and equation

Consider a problem where there is an unknown: *I spent $ on a pie and ice cream. How much was the ice cream?* Think about it as a balance. Write it as an equation: $8 = pie & ice cream. Turn the balance around in the mind, the equation is also a turnaround: pie & ice cream = $8

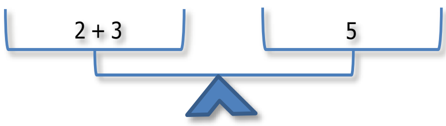


Consider all operations:

	Problem	Balance	Equations
(a)	I bought a hat and coat for \$60 How much was the coat?		hat & coat = \$60 \$60 = coat & hat
(b)	I saved some money. I bought a \$26 lunch. How much do I have left to spend?		money – \$26 = left left = money – \$26
(c)	I bought 6 caps with my money. How much did each cap cost?		6 × caps = money money = 6 × caps
(d)	I shared the winnings amongst 11 people. How much did each get?		winnings ÷ 11 = share share=winnings ÷ 11

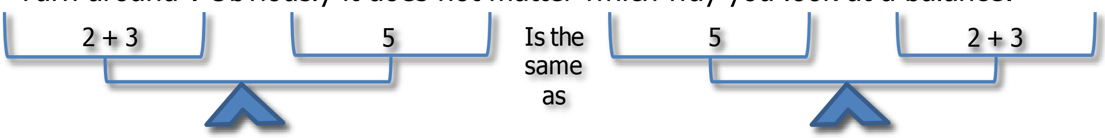
The idea of balance in this section is not a physical balance – it is a mathematical balance in that numbers on each side can be added, subtracted, multiplied and divided – something which is very difficult on a physical balance.

We have seen that by making "=" balance, the equation 2+3=5 become the balance at right:



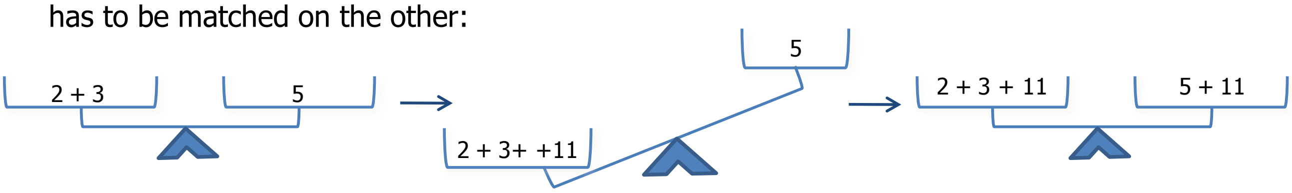
What makes the other way? What ideas of balance help us to understand equations?

(1) "Turn around". Obviously it does not matter which way you look at a balance.



So 2+3=5 and 5=2+3 are both allowable and say the same thing.

(2) "Balance Principle". If one is to keep a balance balanced, then a change in one side has to be matched on the other:



So 2+3=5 changed to 2+3+11≠5 means that the same change has to be made to the other side so that 2+3+11=5+11. So, to keep an equation equal, do the same things to both sides. In this way, the Balance Principle is what enables us to solve problems for unknowns.

3.3. Solving equations with the balance principle

The objective of the sub-section is to comprehend how to represent and solve the unknown in various situations. Materials used are picture of money balance, pens and paper. The problem is thought of as a balance and the Balance Principle is used.

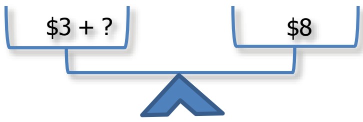
<u>Consider the problem</u>: *I spend $8 on a pie and ice-cream. How much was the ice cream?* Represent this with an equation: pie + ice-cream = $8

Now consider the pie was $3 – this changes the equation to: $3 + ice-cream = $8

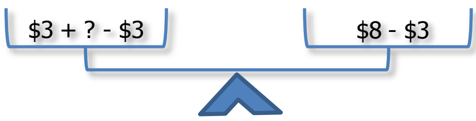
Ask the students: *What is not known or the unknown?* (cost of ice-cream). *How could we represent this?*

Note: Adopt your class choice for unknown – but we'll use "?" (or the name of the object) for unknown in this booklet. Rewrite the equation: $3+? = $8

Now solve the problem by thinking of the equation as a balance, right:



$3 + ? = 8

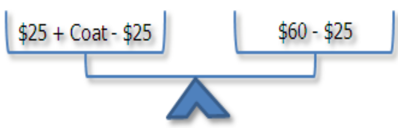


$3 + ? – $3 = $8 - $3
? = $5
(So, ice cream = $5)

Ask students: (a) *How can we change the sides so that it still remains balanced?*; (b) *What do we need to do to make sure the balance is maintained?*; (c) *How can we make the "?" alone on the left hand side?* [subtract 3]; (d) *What do we then do to the right hand side?* [also subtract $3].

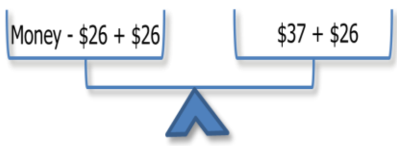
This approach can work for all operations. Let us consider the four problems from section 3.2.

(1) *The hat was $25. I bought a hat and coat for $60. How much was the coat?* To get the cost of the coat alone subtract $25.



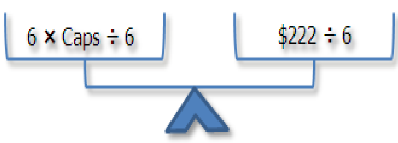
$25 + Coat = $60
$25 + Coat - $25 = $60 - $25
Coat = $35

(2) *I saved some money. I bought a $26 lunch. I was left with $37. How much had I saved?* To get the amount of the money alone add $26.



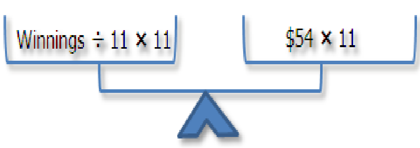
Money - $26 = $37
Money - $26 + $26 = $37 + $26
Money = $63

(3) *I bought 6 caps. The total amount was $222. How much for each cap?* To get the cost of the caps alone ÷ 6



6 × Caps = $222
6 × Caps ÷ 6 = $222 ÷ 6
Cap = $37

(4) *The winnings were shared amongst 11 people. Each person got $54. How much were the winnings?* To get the amount of the winnings alone, × 11.

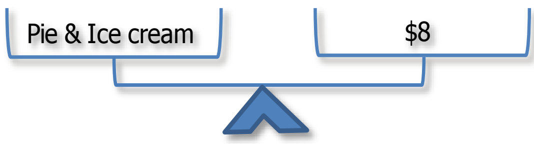


Winnings ÷ 11 = $54
Winnings ÷ 11 × 11 = $54 × 11
Winnings = $594

3.4. Approaching budget problems with more than one unknown

The objective of this sub-section is to comprehend how to approach variable money problems. Materials used are pictures of money, pens, paper. The situation is again considered in terms of balance.

Consider the open problem, *I bought a pie and ice cream for $8.* Look at it in terms of balance and equation.



Pie + Ice cream = $8

Discuss possibilities, likelihoods, for example, if the cost of both items was in whole dollars? What would be reasonable options? List them and describe them. Use a table.

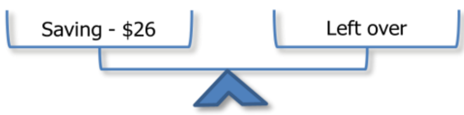
Pie	Ice cream	Explanation
$2	$5	Upmarket ice-cream with a cheap pie
$3	$5	Normal pie & upmarket ice-cream
$4	$4	Boutique pie with a normal double ice-cream
$5	$3	Large pie with a simple ice-cream
$6	$2	Family pie with a cheap ice-cream cone

This technique could be used with other problems from 3.2 (with one number given).

Problem	Questions
(a) "I saved some money. I bought a $26 lunch. How much do I have left?"	In what range could the left over amount be? Could I have $22 left over? Is there a limit on how much is left?
(b) "I bought some caps for $222. How much for each cap?"	What would be the range of prices for the caps? What is the highest amount? Why?
(c) "The winnings are shared amongst some people. Each person got $54. How much was won?	How big could the winnings be? How small?

The way to think about things is to use the balance idea:

(a)

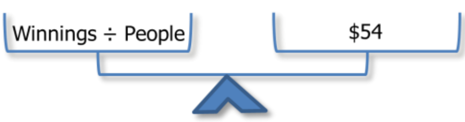


Savings could be $26 or above but not less than $26. The money left over could be $0 or anything larger.

(b)

The lowest number of caps is 2, so each cap could be $111. If there were 100 caps, then each would cost $2.22.

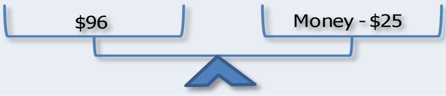
(c)



2 people means winnings of $108. This is the smallest number. 100 people is $5,400. The winnings can be anything above $108.

3.5. Budgeting money and simple algebra activities

(1) Complete the table by filling in the blank cells:

Problem	Balance	Equation
(a) John bought jeans for $68 and a shirt. He spent $100		
(b)		
(c)		Number of T-shirts 7 = $98
(d) Frank sold 7 shirts and paid $50 rent. How much money did he make?		
(e)		($250 – cost of entry) 4 = spectators money

(2) Use a balance diagram to solve the following:

(a) I bought a cake and a pie for $26. The cake cost $12. How much for the pie?

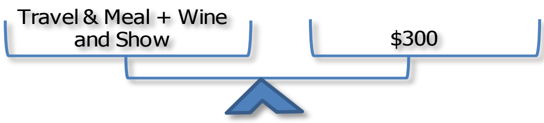
(b) Jack paid $220 rent. This left $157. How much did he have to start with?

(c) Fred bought a $3 pie for each of his visitors and a chocolate for $5. He spent $26. How many visitors did he have?

(d) Sue shares some of her money amongst her 6 nieces and then used the rest to buy a dress for $65. If she had $275, how much did she give each niece?

(3) Explore the problem 3 equally priced CDs and a DVD cost $109. How much for each CD and now much for the DVD? What are the possibilities? What are the restrictions?

(4) You and a friend have money for a night out, a special celebration! This will include travel (can't take own car), restaurant, meal, wine, and a show, movie or play. They all have to be within your money limit which is $300. Use the internet to check prices. What could you do for the best night out on this money? Remember, this situation means that it is algebra: Hint: The balance is shown above.



Try a table of values and describe how each option would be reached.

Travel	Meal	Wine	Show	Explanation
(a) $0	$200	$100	$0	Walked to restaurant, had a great meal and wine and did not go to the show.
(b) $20	$30	$20	$230	Tickets cost $115 each, so grabbed a burger, and a drink at a pub on the way to the show.
(c)				
(d)				
(e)				

4. Multi-step problems

When working with money, it is common for problems to have more than one step. Consider an example, *I bought 4 lunches for $17 each and drinks for $22. How much did I pay all together?* This example requires the multiplication of 4 and $17 to get the total amount paid for lunches the addition of the extra $22 for the drinks to get the answer. Thus this is a scenario where there are two steps required – multiplication and addition – 4 × $17 + $22 = $65 + $22 = $90.

Thus, in multi-step problems there are two components: (a) determining the steps; and (b) determining the operations in each step. Component (a) requires students coming to understand (a) strategies or rules of thumb which point towards a solution (e.g. make a drawing, break the problem into parts); and (b) a general plan of attack (Polya's 4 Step approach is best (SEE, PLAN, DO, CHECK – see the next section). Component (b) uses the knowledge from Section 1, that is part-part-total, and factor-factor-product to identify where addition, subtraction, multiplication and division has to be used.

4.1. Polya's stages

In this section we will focus on Polya's 4 stages of problem solving:

(1) See – work out what you have to do

(2) Plan – make a plan to do it

(3) Do – do the plan

(4) Check – check your answer and see what you can learn from the problem.

The best way to do this is to make a POSTER of the 4 stage and always use the 4 steps when discussing, adding, problem solving. For example: Problem: I sell discs for $30 which I buy for $15. I pay $200 rent for the stall. I sell 25 discs. How much did I make for the day?

(1) See: What is going on here? Stall, discs for sale, bought for $15 sell for $30. To sell 25 discs means the profit is increased for each disc. Have to pay $200 rent.

(2) Plan: What strategy is best? I'll choose to break the problem into parts: (a) I buy discs at $15 and sell them at $30 each means profit on each disc, (b) 25 discs sold means 25 lots of this profit, (c) Have to reduce this profit by $200 for rent to be paid.

(3) Do the steps: (a) $30 – $15 = $15 profit, (b) 25 × $15 = $375 made from 25 discs, (c) $375 – $200 rent means I make $175 profit.

(4) Check your working: $175 + $200 rent = $375 – my answer means that this is the profit that I make in a day; 25 discs is $375 ÷ 25 = $15 – my answer means that this is the profit on each disc; $30 – $15 is $15 – my answer means that this is the amount I pay for each disc I sell for $30. So checking/working backwards gives me the correct starting point – so my answer is correct.

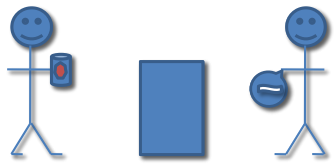
Emphasise the following: (a) Do each step in turn, (b) Finish each step before moving on to next, (c) Make sure you take everything into account (e.g., must subtract the $200 at the end). It is necessary to reinforce Polya's 4-stage method with many other examples.

4.2. Strategies for simpler problems

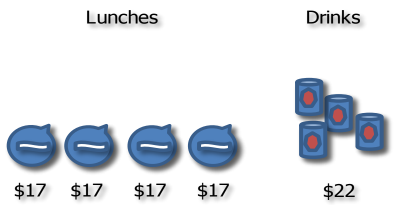
For simpler 2 and 3 step problems, there are 3 major strategies. These are: (a) Make a drawing – draw something useful that will help solve the problem, (b) Given, needed or wanted – determine what is given, what is needed to get you to where you want to go and what is wanted (where you want to go) and (c) Restate the problem – rethink the problem in your mind so it becomes easier. The first step is to introduce the three strategies. To do this, take a problem and try them out. For example, *I bought 4 lunches for $17 each and drinks for $22. How much did I pay?*

(1) Drawing: Get students to make drawings and discuss with them which is the most useful picture and why? For example, which is the most useful drawing below to solve the problem?

(a)



(b)



(2) Given, needed and wanted: Get students to identify these 3 things and record:

(a) Given: 4 lunches – $17 for each lunch, $22 for drinks

Needed: 4 × $17 to work out lunches.

Wanted: the total amount (food and drinks)

(3) Restate the problem: Think of it in an easier way and write it down. Think: How could I make the problem easier for a child without giving them the answer? For example, *Work out what 4 lunches at $17 is and add that answer to $22 for drinks to get how much you spent.*

Once the strategies have been introduced reinforce the strategies with similar problems before changing problem type too much. After reinforcement, the strategies can be applied to many types of problem. One good way is to use the three strategies together. The framework at the end of this section is an example of this.

In summary, the method advocated for teaching strategies is called the Match-Mismatch method. To do this method, organise students to initially do a set of problems that all use the one strategy (matching or similar to each other). This enables the students to focus on getting to know the strategy and to understand the process that they will follow in problem solution. They should do this using Polya's four stages. In this way, they become familiar with both plan and strategy.

When students are able to do these matching problems, move on to problems that are mismatched with respect that only some problems use the strategy (the others use different strategies or dissimilar/not matching). In these situations, students have to think about when to use a strategy. This method is based on the sequence: (a) Teach students about a strategy; (b) Teach students how to use a strategy, (c) Teach students when to use a strategy.

4.3. Strategies for solving complex problems or rich tasks

Solving the "Big Day Out" type problems means ensuring all requirements of travel, accommodation and food are met. This means covering all necessities needed for the occasion. To do this, three strategies become important. They are: (a) <u>breaking the problem into parts</u> (checking that you have not missed any parts), (b) <u>making a table or chart</u> (this helps ensure nothing is missed and keeps things systematic, and (c) <u>exhaust all possibilities</u> (go through and do everything on a list).

It should be noted that the advent of computers has increased the mathematical importance of these strategies. The speed of computers means that checking all possibilities (exhaustion) is a useful and effective strategy. This, in turn, has meant that problems now can handle much larger data sets, which has increased the need to be able to break things into parts to make them manageable. Finally, the computer enables access to powerful table and chart techniques such as spreadsheets.

The application of the three strategies of "parts," "table" and "exhaustion" requires the student to make sure that everything is covered. To do this, it is parts first, and then tables and finally exhaustion. An example will illustrate: Suppose we were to prepare for a trip say for 2 weeks, what would we take?

(1) <u>Break the problem into parts and make a list/table of types of things to be taken</u>:

- (a) clothing to wear during the day;
- (b) things for night time;
- (c) bathroom/toiletries/personal hygiene goods;
- (d) entertainment goods; and
- (e) other necessities (e.g. money).

(2) <u>Continue building the table by making sub-lists for each of these</u>:

- (a) clothing – shoes, socks, underwear, shirts, pants, dresses, jumpers and coats, hats, belts;
- (b) night – pyjamas, alarm clock, book;
- (c) bathroom – soap, shampoo, toothbrush and toothpaste, deodorant, perfume, makeup, brush, comb, etc.;
- (d) entertainment – iPod, camera, charger, mobile phone, computer, cards, board games; and

(e) other – scissors, tape, working material, medicines, wallet, passport, maps, information booklets.

(3) Try to exhaust all possibilities: imagine what you will be doing and what you need to have as you are doing it, and ask others to check your list.

As well as requiring parts, tables and spreadsheets, and exhaustion strategies, complex tasks also require more than one operation. For example, preparing for a party requires multiple amounts of each item. If there is a budget, it requires continually looking to see how much money is left and what this money should be spent on.

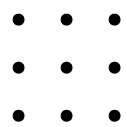
An example may suffice. Suppose we are buying a drink at $5.50 a bottle with 10% off if a crate of 12 is bought. We have $200 but we must return $70 for the pizzas. How many bottles can we buy?

First there is subtraction: $200 – $70 = $130 for the drinks; (2) Then there is division: $130 ÷ $5.50 = 23.64 – so 23 bottles at $5.50 each; (3) Then we must deal with the dozens and the 10% discount that occurs, meaning multiplication and subtraction are needed: The first 12 bottles cost $66; 10% discount is $6.60 (multiplication by 0.1). So they really cost $59.40 (subtraction $66 minus $6.60); (4) Then there is further subtraction: There is $70.60 still to spend (subtraction $59.40 from the $130 that we had left to spend); and (5) finally more division and subtraction to the answer: More can be bought - we can buy another dozen for $59.40 from the $70.60 leaving $11.20 (subtraction) which enables two more bottles to be bought (adding $5.50 to $5.50 to give $11). Thus, the total number of bottles is 26.

4.4. Multi-step problem activities

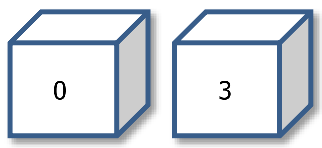
(1) Apply Polya's 4 stages to the following problems:

(a) Six couples meet. They all shake hands with each other. How many hands shake?



(b) Draw 4 connected straight lines so that a line goes through each dot of a 3 × 3 array of dots

(a) Place numbers on 2 dice so that when placed side by side they can give all numbers in any month from 01 to 31.



(2) Use the Strategy Template to solve the following problems. Complete all the 3 strategies.

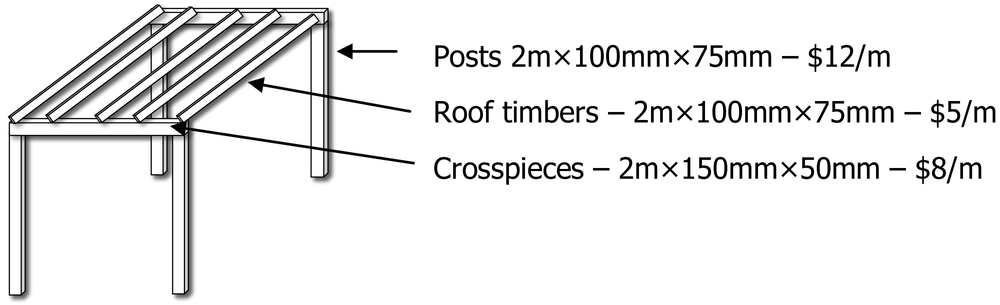
(a) A koala was climbing a 10m tree. He climbed 3m and back 2m each hour. How many hours to the top of the three.

(b) Fred walked 90km in 4 days. On the first day he walked 16km, the second 32km and the third 23km. When he finished the 90km he walked another 11km to the motel on the next day. How far did he walk on the 4^{th} day?

(c) There were 37 players for a knockout tennis competition. How many games if there is one winner?

(3) Build a pergola:

Consider the problem, How much change would you receive from \$200 when buying wood for a pergola as below?



(a) In order, list the operations needed to solve the following problem (it is not necessary to solve the problem – just list the operations you would use).

(b) Construct a table from the list and set up a spreadsheet to do the calculations.

(c) Do the calculations and work out the cost.

(d) Calculate how much each month for three months to pay this cost interest free.

(4) Plan a party

(a) Design a party for 20 people – what will you do; where will you have it; what will you have?

(b) Use parts and exhaustion to list everything needed for the party.

(c) Place your list on a table or spreadsheet and use the internet to find the costs of each item and then calculate to find the total cost of the party.

Strategy Framework	
Problem:	
Drawing:	**Given:** **Needed:** **Wanted:**
Restated Problem:	
Working: **Answer:** ______________________	

CULMINATING TASK

THE BIG DAY OUT

Directions: This year, you and two of your friends are going to the Big Day Out (BDO) in Perth. Your friends will be paying for all of their own costs. In order to get there many different arrangements have to be made. You are to prepare a report detailing the trip and will need to read the following information.

(1) Ensure that the trip is cost effective. Therefore certain luxuries should not be used, for example using a taxi as a form of transportation.

(2) For your trip you will have a budget of $1250 dollars (which will not include the transportation to Perth or the tickets for the BDO).

(3) The transportation to Perth and the tickets to the BDO do not need to be purchased with your $1250, but will be paid using a brand new credit card.

(4) BDO tickets usually sell out within 30 minutes of sale and go on sale at 9:00 am (Perth Time), 8 weeks before the event.

(5) Your parents are paying for 15% of the trip.

The report should include information under the following headings.

Accommodation

K	• Where in Perth will you stay? Will you share a room with friends or pay for separate rooms?	
K	• How much will it cost per night? (State the amount of GST included)	
K	• What are your check-in and check-out times?	
A	• How many days will you rent accommodation? What will be the total cost of your accommodation?	
A/E	• How far is accommodation from the BDO? Explain your accommodation choice.	

Tickets to the Big Day Out

A	• At what time and on what day will you need to purchase the tickets in Qld?	
K	• How much do the tickets cost? (State the amount of GST included)	
K	• Are there any possible ways of getting refunds on the tickets?	
K	• Find the list of bands and the times they will be performing.	

Transportation

K	• How long will the flight take from Queensland to Perth?	
A	• What is the most efficient transport to get from your accommodation to BDO?	
A	• Estimate how long it would take to walk from your accommodation to the BDO.	

E	•	Explain why you chose the particular transportation throughout your trip.	
K	•	Collect a group of maps for the trip.	
A	•	Mark and state the grid reference of your accommodation and the BDO.	
A	•	What is the total cost of transportation for the trip? (State the amount of GST)	

Budget

A	•	What is the total cost of the trip for your friends?	
A	•	How much money has been allowed for food?	
A	•	How many meals will be eaten?	
K	•	How much money are your parents giving you for the trip?	
K	•	How much will your trip cost?	
A	•	How much spending money will you have? (Money after expenses)	
A	•	What fees will you incur if you pay your credit card off in 3 months time?	
A	•	Calculate the total cost of the trip.	

Itinerary

K	•	What time will you be leaving Queensland?	
K	•	What time will you arrive in Perth?	
K	•	Detail the proposal for transportation in Perth. (Time of trips, arrival/departure)	
E	•	Why does Queensland have a different time to Perth?	
K	•	Detail any other plans you have while in Perth.	
A	•	Detail what bands you will be seeing. (Times and stages)	

Additional Notes:

- All results involving time should be in both 12-hour and 24-hour formats. (A)
- All results involving an amount of time should be in both fractional and decimal form. (A)
- The budget should be made in a spreadsheet program. (K)
- All costs in the budget should be rounded to the nearest five dollars. (K)
- Add any additional expenses to your budget. (K)
- When deciding which bands you wish to see, make sure your day is as full as possible. (A)
- Ensure you read the website when deciding your arrival time at the BDO; you don't want to be late entering the venue. www.bigdayout.com will give you all details you need for the day. (K)

Useful Websites

Ticket/Event Information

- www.bigdayout.com
- www.ticketmaster.com.au

Transportation

- http://www.virgin.com/home.aspx
- www.qantas.com.au
- www.jetstar.com.au
- www.transperth.wa.gov.au

Accommodation

- www.hotel2k.com
- www.lastminute.com
- www.wotif.com.au

Itinerary

- www.flightcentre.com.au

ASSESSMENT RUBRIC

Assessment Form

QUESTION	KNOWLEDGE (K)	APPLICATION (A)	EXPLAINING (E)
Accommodation			
Tickets to the Big Day Out			
Transportation			
Budget			
Itinerary			
Overall Grade			

Assessment Criteria

GRADE	KNOWLEDGE (K)	APPLICATION (A)	EXPLAINING (E)
A	Effectively uses given rules to carry out tasks.	Applies rules across all contexts effectively.	Presents detailed solutions logically and clearly.
B	Uses given rules to carry out tasks.	Applies rules across most contexts effectively.	Presents solutions logically and clearly.
C	Uses given rules adequately to carry out tasks.	Applies rules adequately across most contexts.	Presents readable solutions.
D	Uses given rules to carry out parts of tasks.	Applies rules in some contexts adequately.	Presents partial solutions.
E	Did not meet standard D.	Did not meet standard D.	Did not meet standard D.