TEACHING MEASUREMENT
Processes in the Primary School

by

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PREFACE

It is the intention of the mathematics education lecturers at Calseldine Campus of QUT to produce a series of books on teaching primary mathematics under the series title: *Mathematics Learning Series*.

The topics to be covered in the series are:

- Numeration
- Operations
- Basic Facts of Arithmetic
- Decimal and Common Fractions
- Rational Numbers (Common or Decimal Fractions)
- Ratio, Proportion and Percentage
- Geometry (Space and Shape)
- Measurement
- Statistics, Graphs and Probability
- Problem Solving
- Calculators

This book is the draft materials for the measurement book which will be rewritten under the title: *Teaching Measurement Processes in the Primary School*.

Tom Cooper

January 1988
OVERVIEW OF BASIC MEASUREMENT

BASIC MEASUREMENT IN PRIMARY MATHEMATICS

This book focusses on the basics of Measurement. The purpose of the book is, therefore, on how to develop in children

(i) understanding of the concepts of length, area, volume, mass, time, money, temperature and angle, the units used and the formulae applied and

(ii) competence in the process of measuring, in using instruments and in choosing appropriate units.

Measurement is an important human activity. It is an everyday skill. It is an essential tool of science, and it provides a useful link between the real world and mathematics. Measurements are as diverse as the length of a straight line, the I.Q. of a human being, and the speed of light. Measurement skills include simple dexterity, the techniques of calculus, and the ability to construct models of human thought and behaviour. This book concentrates on those aspects of measurement which are basic in the elementary school.

In simple terms, measurement involves the comparison of an amount of an attribute with a unit amount of the attribute in order to determine a number. For example, to measure the amount of the attribute length possessed by a pencil one might compare the pencil with a unit of length such as the centimeter to determine a number, say 15.

Within the unique and important place mathematics has had in Western Civilization, measurement plays an important part. The two well springs of mathematical thought have been number and geometry and these are also the starting points for primary mathematics. But the impetus for the development of number and geometry has often been problems of measurement (e.g. astronomy, surveying, taxes) and measurement still plays a crucial role in the opportunities it offers for consolidation, application and development of number and geometric concepts and processes. It is convenient to consider the primary mathematics syllabus as three major topic areas as is diagramatically described in figure 1 below.

<table>
<thead>
<tr>
<th><strong>NUMBER</strong></th>
<th><strong>GEOMETRY</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Numeration</td>
<td>Shape</td>
</tr>
<tr>
<td>Operations</td>
<td>e.g. 2-D, 3-D, Symmetry</td>
</tr>
<tr>
<td>Decimal and common</td>
<td>tessellations etc.</td>
</tr>
<tr>
<td>fractions</td>
<td>Position</td>
</tr>
<tr>
<td>Percentage, rate and</td>
<td>e.g. coordinates, etc.</td>
</tr>
<tr>
<td>ratio</td>
<td>Size</td>
</tr>
<tr>
<td>Early Algebra</td>
<td>e.g. congruence, similarity, etc.</td>
</tr>
</tbody>
</table>

**MEASUREMENT**

Length, Area, Volume, Mass,
Time, Money, Temperature, Angle
Probability, Statistics, Graphs, and Charts.

*figure 1.*
It is also convenient, although somewhat simplistic, to think of measurement as the application of number to geometry (to shape and position). This book focusses only on the basic applications—length, area, volume and capacity, mass, time, money, temperature and angle. Probability, statistics, graphs and charts, is the focus of another book. Measurements such as IQ, velocity, force, etc., not being in primary school mathematics, are not covered.

SEQUENCING MEASUREMENT ACTIVITIES

Measurement raises some problems for children. It requires the recognition of attributes which may not yet have become conscious parts of a child's experience. This frequently involves the introduction of new words in the child's vocabulary. The comparisons involved in measurement depend on a clear understanding of what can be done to an object without changing the quantity of an attribute that is present. For example, is the volume of a quantity of liquid changed when its container is changed? Dexterity is also required to manipulate measurement instruments. Consequently, children seem to require both experience and maturity to grasp measurement skills and concepts. This need suggests a spiral approach to measurement instruction, in which difficult concepts are visited and revisited throughout a child's schooling.

Children need to come to terms with units, how they are used, the comparisons that can be made with them and how they interrelate with the number that results. Children must become proficient with metrics and in applying commonly used measurement formulae. For these reasons, it is recommended that all topic areas in measurement be developed with children through the five stages below.

(1) IDENTIFYING THE ATTRIBUTE TO BE MEASURED

Before they can compare or measure an attribute such as mass, children must be aware of what mass is. This has to come from experiencing instances of the attribute. It requires careful development of language.

Much of this is completed by the time children reach school age but it is essential for teachers to check that this has happened. For instance, many children mistake mass for size or volume, believing always that the largest object has more mass.

One method of exhibiting a new attribute to a child lacking experience with it is to show the child:

(a) instances where everything varies except the attribute (e.g. ribbons, sticks, cylinders, pens all with the same length); and

(b) instances where the only thing that varies is the attribute (e.g. pink ribbons of different length but of identical colour, width and material).

(2) COMPARING AND ORDERING THE ATTRIBUTE (no numbers)

Before children use numbers in measuring situations, it is useful to experience the concepts of length, area, etc. by comparing and ordering different examples. Children should undertake comparing activities between two examples, before ordering three or more examples. The change from comparing two to ordering three or more is difficult. It requires a focus on betweenness, in identifying the example which is between the other two.
Comparing activities should include all the types below and should be introduced to children in the sequence below:

(a) direct comparison where only the attribute being compared varies (e.g. comparing 2 pencils of different lengths);

(b) direct comparison where more than the compared attribute varies (e.g. comparing the lengths of a pencil and a pair of scissors);

(c) indirect comparison via an intermediary (e.g. comparing the distance around a can to the length of a pencil); and

(d) comparing different representations of the attribute (e.g. comparing the diameter of a bicycle wheel with the width of a door).

Ordering activities should include all the types below and should be introduced to children in the sequence below:

(a) copying an ordered sequence of examples;

(b) placing an example correctly in an ordered sequence of other examples when the example to be placed is such that, in terms of the attribute, it is greater or lesser than all the other examples (i.e. it has to be placed at an end of the already existing sequence);

(c) placing an example correctly in an ordered sequence of other examples when the example is such that it could be placed anywhere; and

(d) ordering three or more examples without any assistance.

(3) NON STANDARD (OR CHILD CHOSEN) UNITS

This is the first point at which number is introduced in the measurement activities. It is introduced with units from the everyday world of the child not with metrics. For example:

(i) handspans, paces, blackboard dusters can be used to measure length;

(ii) cups full, glasses full to measure volume; and

(iii) paper clips, books to measure area.

The reasons for using non-standard units are threefold:

(i) they are more natural, personal and familiar (and more fun), are commonly used in real life and do not have the added problem of notation and conversion factors;

(ii) they show that standard units are not absolute (but were chosen for reasons of history and convenience); and

(iii) they provide a vehicle for teaching the role of units in the process of measuring.
The objectives with regard to measuring that can be achieved by using non-standard units are:

(a) when measuring, the unit being used must not change;

(b) when comparing or ordering examples by number of units, the same units must be used in all measurements;

(c) when using common units, the example with the largest number has the greatest amount of attribute and vice versa;

(d) in arriving at a number, the larger the size of the unit the smaller will be the number and vice versa (the inverse relation property);

(e) in assessing measurements, the numbers should not be seen as correct in the sense that 2 + 3 gives 5 but should be seen as accurate to the nearest unit (accuracy vs correctness) - there should be a tolerance for error;

(f) in measurement situations and with a variety of units, being able to accurately estimate the amount of attribute in terms of those units;

(g) in measurement situations, being able to choose appropriate units (matching unit size to the object being measured and to the need for accuracy); and

(h) in measurement situations, there is a need to standardize units.

(4) STANDARD UNITS.

With the successful teaching of the need for standardized units comes the opportunity for instruction in our society's standard units - metrics. Primary schools use only some metric units as laid down in the System Internationale conventions and these are:

(i) **Length**

\[ \begin{align*}
1 \text{km} &= 1000 \text{m} && \text{(kilometre)} \\
1 \text{m} &= 100 \text{cm} && \text{(metre)} \\
1 \text{cm} &= 10 \text{mm} && \text{(centimetre and millimetre)}
\end{align*} \]

(ii) **Area**

\[ \begin{align*}
1 \text{km}^2 &= 100 \text{ha} && \text{(square kilometre)} \\
1 \text{ha} &= 10,000 \text{m}^2 && \text{(hectare)} \\
1 \text{m}^2 &= 100 \text{cm} \\n\end{align*} \]

(iii) **Volume and capacity**

\[ \begin{align*}
1 \text{m}^3 &= 1,000,000 \text{cm}^3 && \text{(cubic metre and cubic centimetre)} \\
1 \text{kL} &= 1000 \text{L} && \text{(kilo litre)} \\
1 \text{L} &= 1000 \text{mL} && \text{(litre and millilitre)} \\
1 \text{mL} &= \text{cm}^3
\end{align*} \]

(iv) **Mass**

\[ \begin{align*}
1 \text{t} &= 1000 \text{kg} && \text{(tonne)} \\
1 \text{kg} &= 1000 \text{g} && \text{(kilogram and gram)} \\
1 \text{g} &= \text{mass of one cm}^3 \text{ of water at } 4^\circ \text{C}; 1 \text{kg} = \text{mass of one L of water at } 4^\circ \text{C}
\end{align*} \]

(v) **Time**

\[ \begin{align*}
1 \text{century} &= 100 \text{ years} \\
1 \text{decade} &= 10 \text{ years} \\
1 \text{year} &= 52 \text{ weeks } = 12 \text{ months } = 365 \text{ days} \\
1 \text{fortnight} &= 2 \text{ weeks } = 14 \text{ days} \\
1 \text{week} &= 7 \text{ days} \\
1 \text{day} &= 24 \text{ hours (h)} \\
1 \text{h} &= 60 \text{ minutes (min)} \\
1 \text{min} &= 60 \text{ seconds (s)}
\end{align*} \]
(vi) Temperature

freezing point of water = 0°C (zero degrees celsius)
boiling point of water = 100°C

(vii) Angle

360° = full turn

Metrics (or any standard unit) should be introduced in three stages:

(a) identification (constructing actual units out of materials, or experiencing examples of them);

(b) internalization (finding examples in students’ own bodies, or in common everyday items, that equal units in size); and

(c) estimation (being able to “think” in the units and visualize them sufficiently to be able to make good “educated” guesses at the measure of examples).

The introduction of metrics is also the opportunity for continuing to develop the seven objectives discussed in non standard units.

(5) FORMULAE

The final stage in the development of skill and understanding in measurement is the application of formulae. It is a sad indictment of much of modern teaching of measurement that formulae is often the first instance with which many children are introduced to measurement concepts and processes. For example, many children have their first real experiences with area with the formula for the area of a rectangle. This means that they have great difficulty understanding area for irregular shapes.

Formulae enable us to calculate measures of regular shapes from their dimensions - a much faster process than counting units. The formulae most appropriate for primary schools are -

Perimeter: Rectangle

\[ W \]
\[ L \]
\[ 2 (L + W) \]

Circle

\[ R \]
\[ 2 \pi R \]

Area:

Rectangle

\[ W \]
\[ L \]
\[ L \times W \]

Triangle

\[ W \]
\[ L \]
\[ \frac{1}{2} (L \times W) \]

Parallelogram

\[ W \]
\[ L \]
\[ L \times W \]

Circle

\[ R \]
\[ \pi R^2 \]
Volume: Rectangular Prism \[ L \times W \times H \]
Prism \[ \text{Area of base} \times H \]
Cylinder \[ \pi R^2 H \]

Formulae can be directly applied to examples of the shape or may require analysis and synthesis as is shown below:

(1) What is the area of the concrete path?
Area of large rectangle = 6000m²
Area of small rectangle = 80 \times 40 = 3200m²
Area of path = 2800m²

(2) What is the area of the shape on the right
Area = \((30 \times 60) + \frac{1}{2} (30 \times 30) + \pi \times 15^2 \) m²

MEASUREMENT TOPICS

There is a famous saying of Thorndike:

"Anything that exists, exists to some degree and can be measured."

The attributes to which we could apply the sequence of measurement development activities described above are therefore almost limitless. Hence there is a need to develop a general understanding of the measurement process that could be applied to any attribute. But, within primary school, there are also particular attributes which have to be specifically covered. These are discussed below.

(1) **Length/Perimeter.**

This is a one-dimensional concept related to the notions, from geometry, of direction and line. It is a measure of the separation, along a line, of two points. (We should note that, at its basis, distance is related to straight line. It is applied to curved line by dividing that line into small straight segments.)

[A particular measurement of length, the distance around a plane shape, is of specific interest and is called perimeter]
(2) **Area.**

This is a planar or two-dimensional concept. It is related to the geometric notion of region. It is a measure of the amount of coverage of that region. It is related to two lengths in two directions.

(3) **Volume/Capacity.**

This is a three dimensional concept. It is related to the geometric notion of a solid - it is a measure of the interior size of a solid. It is the amount of space enclosed by such a solid shape. It is related to three lengths in three non co-planar directions.

(4) **Mass.**

This is not directly related to solid shape - although solid shapes have it as a property. It is the measure of the inertia of an object - how much force it takes to move the object, and how much force it takes to stop the object from moving. This may not be proportional to volume.

For school children, it is related to the force exerted by an object as a result of gravity (the weight of the object) - and can be intuitively understood as the amount of pressure the objects exert down on the hands holding it up (the "heft" of the object). It should be remembered that weight varies depending on the planet the person is standing on. (On the moon our weight is approximately 1/6th of what it is on earth.) Mass does not vary for this reason, although the convention is to give as Mass the objects earth weight, invariant mass has replaced varying weight as the "inertial" attribute of an object which is studied by school children.

(5) **Time.**

Time is often called the fourth dimension. It is the duration we spend undertaking an activity. We are fixed within the flow of time. We are unable to move through time at our own volition or at our own speed and direction.

Children are continually experiencing time. We do not have to set up special "changes" for them to experience. They only need to be directed to reflect on their own experience. But, of course, we can not set up any contrasting experience (such as no change in time or twice as fast a change in time) with which to compare. Time is also a very subjective experience.

(6) **Money/Value.**

Money is the units for cost or value that we put on objects. As a measure it is not as absolute as length but varies according to demand. It may seem strange to place money within the measurement context but measurement teaching approaches can be useful in instructing children about money.

(7) **Temperature.**

Temperature is a measure of how hot or cold objects are - a very subjective experience without measuring instruments. The most important reference points are the freezing point and boiling point of water.
(8) Angle

Angle is the amount of turn - a measure of change in direction. As a concept it is contained both within geometry and measurement.

APPROACHES TO TEACHING MEASUREMENT

The topic of measurement contains both concepts and processes to be learnt. The processes consist of general ones which pertain to measuring with units and particular ones which are related to rules and formulae.

ACTIVITY TYPES:

It is the recommendation of this book that the concepts and processes of Measurement are best developed by the use of exemplars (materials and pictures) such as the following measuring equipment

- rulers, tape-measures, trundle wheels
- grid paper
- measuring cylinders, blocks, containers
- beam balances, spring balances, weights
- clock faces, timers, eggtimers
- play money, money stamps
- thermometers
- protractors, rotagrams

It is further recommended that, once materials are chosen, behaviour that can indicate that the content to be covered has been learned must be identified, kept in mind during teaching and used for evaluation. Then teaching activities that keep a balance between developing the ideas, consolidating, the ideas and applying the ideas should be prepared and undertaken as figure 2 below diagrammatically shows.

The approach of teaching should be to ensure that the children integrate the new ideas into their already existing knowledge to form conceptual schemas - that the ideas are not retained as isolated pigeonholed pieces of information but interconnected with previous ideas. To this end, schematizing activities are important - there should be a focus on, e.g., connecting the formulae for area of a triangle with area of a rectangle so that both can be recalled from one memorization.
ACTIVE TEACHING:

Measurement concepts and processes (particularly formulae) should be discovered by children. Being basically a procedural knowledge (a knowledge of how to do something), measurement is best learnt from the children's own experiences - by doing. Teaching should be active not imitative as figure 3 below diagrammatically shows.

<table>
<thead>
<tr>
<th>Teach actively</th>
<th>not imitatively</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Children explore</td>
<td>(1) Teacher describes (and &quot;explains&quot;) the idea</td>
</tr>
<tr>
<td></td>
<td>(2) Teacher demonstrates the ideas or the procedures that use the idea</td>
</tr>
<tr>
<td>(2) Children record and analyse results looking for pattern</td>
<td>(3) Children memorize the idea/procedures</td>
</tr>
<tr>
<td>(3) Children &quot;discover&quot; ideas/procedures</td>
<td>(4) Children work similar examples using the idea/procedures</td>
</tr>
<tr>
<td>(4) Children practice these ideas and the procedures that depend on them</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3**

It is important that in the development of ideas and procedures, exemplars (models), language and symbols be interrelated. This means that during the development and consolidation of an idea that teaching activities vary to include all six interactions.

- teacher asks questions using exemplars/models → language
- language → exemplars/models
- exemplars/models → symbols
- symbols → exemplars/models
- language → symbols
- symbols → language

It is important that children experience:

(i) many embodiments (examples using different materials or showing different types) of concepts/processes; and
(ii) both examples and non examples of concepts/processes.

GENERAL OBJECTIVES:

There are many specific objectives within the area of mathematics that teaching has to meet - specific activities in length, area, volume, etc. But it is important to remember that while these are being achieved, there are higher level more general objectives that should also be the aim of instruction. Some of these are described over.
(1) **Estimation.**

Thinking in terms of measurement requires the ability to estimate in terms of units. Children should be given many opportunities to estimate lengths, areas, masses, etc. In fact, it is a good idea to require an estimate before any measure is made.

(2) **Reasoning.**

It is important that thinking skills be developed in measurement situations. Many practical measurement activities and applications of formulae require children to solve problems. Such activities should be included to ensure that children can reason with measures.

(3) **Applications.**

Measurement offers great opportunities for real world applications (e.g. the cost of carpeting the school). This should be a major focus of measurement teaching.

(4) **Appreciation of the use of measurement.**

Measurements make up a large part of the information we use to make judgements. Are these measurements appropriate and valid for the questions we face? Are measurements an appropriate source of data for the decisions? Children need to grow to appreciate when measurement is useful (and when it is not) and the extent that its use is valid.

(5) **Interrelating attributes.**

When objects are made of the same material, mass is proportional to volume. The volume of a prism relates to height and the area of its base. In a similar way other attributes are related to each other. Children should know when and how the various attributes are related.

(6) **Developing measures.**

In their study of length, area, etc., children should acquire the general ability to work with measures in investigations. This includes

- finding and defining a problem to be investigated,
- designing the investigation and determining the measures to be undertaken,
- designing the instruments to do the measurement with (where necessary),
- determining the criteria for calibrating the instruments (where necessary).

Examples of questions to be investigated which are sufficiently open ended to allow wide scope to the children are determining the best seat to sit in in the classroom, determining the happiest person in the school, etc.

**DEVELOPMENTAL LEVEL:**

Measurement is one of the areas of mathematics for which there is some evidence that age and maturation place limits on what children can do. For this reason, teachers need to watch the following:-
(1) **Motor skills.**

Children must coordinate reasonably complicated motor skills to undertake some measurements accurately. For example, rulers must be placed in a straight line and meet end to end for length and water must be poured accurately into cylinders for volume. Some children find it hard, for example, to align the end of an object with the zero of a ruler when measuring length or not to be clumsy when measuring capacity by immersion and overflow. Teachers should expect only the accuracy children are capable of.

(2) **Conservation.**

Young children may find it difficult, in some situations, to realize that lengths stay the same when one object is placed above another, or that volume remains the same for an amount of water regardless of whether the glass is short and wide or tall and narrow or that mass is the same for a ball of plasticine as it is when the plasticine is rolled out into a "snake" etc. Such activities should be given to all children but correct answers only expected from children whose development is such that they are capable of answering correctly.

Mass is a particular problem. Children commonly confuse mass with volume/size or with quantity. They think larger size means greater mass and, e.g. five foam balls are heavier than one steel ball because there are more of them.

**HISTORY:**

Measurement has a colourful history. This can be used to enliven its teaching. Unfortunately the metric system is much younger than the old British System of units and has a much more clinical history. Still, time and angle go back to Babylon and the history of the Napoleon inspired "scientific" development of metrics by the French can be contrasted to the more "natural" growth of the British system out of body measurements. Children can then see that, although metrics are much more logical in their conversion rates, the British units were a more understandable and manageable size. For example, inches are related to finger-size, feet to foot size and yard to arm length. Metres and centimetres are not so easily related and not such a nice manageable size.

**CONNECTIONS TO OTHER TOPIC AREAS:**

Possibly the most important teaching approach is to connect the teaching of measurement to number and geometry.

(1) Length, area, volume and capacity, mass and angle can easily be related to shape (both line, plane shape and solid shape) and measures such as length and angle can be used to classify shapes and are the basis of concepts such as similarity and congruence.

(2) Metrics and money can easily be related to decimals (in actuality, this is a relation which is essential for the development of all three areas of metrics, money and decimals).

(3) Measurement topics provide excellent opportunities to apply (and consolidate) number knowledge.

In terms of actual activities, it is an excellent idea to:
(i) Teach metrics along with decimals - dollars and cents and metres and centimetres with 2 decimal places and litres and millilitres and kilograms and grams with three decimal places; and

(ii) continue work on solids and plane shapes into length (perimeter), area, volume and mass, and then use these measurements to classify the shapes (e.g. isosceles triangles).

**SUMMARY**

The basis of this book lies in the sequence of measurement activities described above and diagrammatically represented in figure 4 below.

```
Identify the attribute
  ↓
Compare and order
  ↓
Non standard units
  ↓
Standard units (e.g. metrics)
  ↓
Formulae
```

**Figure 4**

To achieve this, the chapters in this book are designed around this sequence. Chapter One will look at identifying the different attributes and recognising differences and similarities (or relationships) between different attributes. Chapter Two will focus on comparing and ordering different amounts of attributes without using units and number. Chapter Three will then introduce units and number into measurement via non standard or child chosen units. Chapter Four will focus on standard units - metrics, hours, minutes, etc. dollars and cents and degrees (for both angle and temperature). Chapter Five will look at formulae and their application in measurement situations. Finally Chapter six will discuss the teaching of measurement looking at approaches to teaching, limitations to learning and diagnosis and remediation, and Chapter Seven provides a variety of measurement activities, classified by topic, for the various stages of Measurement instruction.

**SPECIAL NOTE**

The sequence in figure 4 above is a cornerstone of this book, but it should not be seen as a hard and fast rule, particularly in terms of finishing one stage before starting the next. It is very possible, and even commendable, to start, say, standard units while still completing non-standard work.
CHAPTER ONE: IDENTIFYING ATTRIBUTES FOR MEASURE

Being able to identify the attribute that has to be measured is obviously the first step in developing skill and understanding in measuring that attribute. This chapter focusses on how this is to be done.

Children come to school with some ideas of the major attributes. Length in usually reasonably well known, as is volume, but mass is often misunderstood and area may not be known at all. There is some understanding of money, time and temperature but angle usually needs formal development. The early teachers job is to check what is known, build what is not known and flesh out what is partially known.

But identifying attributes is not something that should be left only to Grade 1 teachers. Many children take some years and many experiences to gain complete schematic understanding of notions such as area, volume, mass, money or value, time and temperature. Witness the difficulties most people have with the change to metrics - many people rely on a symbolic rote understanding of attributes and do not have a meaningful understanding that transcends unit names and sizes.

Furthermore, there is a need to develop in children a general knowledge of how to identify attributes whose measurement would help them to solve problems. This requires the ability to define and categorise, to construct measuring instruments and to calibrate them. And much of the later formulae work relies on relationships between different attributes - an area that builds on individual attribute understandings. Hence Unit 1.1 will focus on identifying attributes, Unit 1.2 will look at relationships between attributes and Unit 1.3 will look at instrumentation - how to develop, calibrate and use instruments appropriately for the attribute under investigation.
UNIT 1: IDENTIFYING ATTRIBUTES

FOCUS:

In this unit we focus on enabling children to identify the attributes of length, area, volume, mass, time, money, temperature and angle.

BACKGROUND:

If you do not know what it is, you cannot measure it. The first task of measurement instruction is to ensure children are aware of what it is we want them to measure (Read carefully the discussions of the attributes in the overview of this book.)

This awareness must extend to all the words that are associated with that attribute. This can be quite dramatic a list as the words for length below give some indication:

- long
- longer
- longest
- short
- shorter
- shortest
- wide
- wider
- widest
- narrow
- narrower
- narrowest
- high
- low
- up
- down
- above
- below
- near
- far
- thick
- thicker
- thickest
- thin
- thinner
- thinnest
- close
- away
- next
- to
- distant
- tall
- taller
- tallest
- height
- width
- shortness
- narrowness
- thickness
- thinness
- distance

ACTIVITIES:

Materials Required: Scissors, tape, paper, cardboard, pens, pencils

1. Acting in groups, look amongst the pens, pencils scissors and other materials in the room and find as many different articles as you can that are the same length. Cut pieces of cardboard and paper (of different colours, widths and thicknesses), ribbon, string, tape, etc. to equal this length. Call this collection set A.

2. Acting in groups, cut strips of paper of equal width (and colour) but differing lengths. Call this collection set B.

3. Spread set A and set B separately on your table. Consider that you have a young child who is looking at the two sets. The following questions could direct children to the attribute length.

"What is the same about the objects in each set?"
"What is different about the objects in each set?"

For which set (A or B) would you expect the questions to cause the most difficulty?

4. What could we do with the items in set B to make the attribute of length more obvious to children?
5. In the above activity, set A is composed of items of equal length where other attributes vary and set B is composed of items of differing length which are otherwise the same. What collections of items could you make to achieve the same purpose for:

Area?
Volume?
Mass?

6. Time, Money (Value), Temperature and Angle require more thought in introducing to children.

Angle can be introduced by looking at things that turn (door knobs, wheels, drills etc.). What could we look at to introduce:

Time?
Money (or Value)?
Temperature?

7. To introduce children to temperature, a teacher visited a cold storage warehouse and the children were allowed to stay in the refrigerated room for a short time. How could we use this experience to highlight temperature? What questions would you ask?

8. What similar excursions/activities/experiences would assist children to focus correctly on:

Time?
Money (or Value)
Mass?

9. Pasting paper on objects to cover them is an excellent introduction to area. What other activities could we do to introduce area? What about:

Temperature?
Volume?
Mass?

10. In the background to this unit we listed a collection of words which are used in reference to length. List a similar collection for:

Volume?
Temperature?
Money or Value?

11. Acting in groups, secretly look at the members of your group to identify human attributes that all members have to a lesser or greater degree. Choose one of these attributes (it may be colour of hair, length of fingernails or the amount of blue being worn) and decide which member has most/least of it. As directed by your lecturer, present yourselves to the rest of the class lined up so that the ordering from most to least (left to right) is evident. The rest of the class must guess your attribute. Try to choose something that is fun, unusual and "clever" but not impossible to guess.
12. For the attributes below, determine how you would describe each attribute to people who were not familiar with it, describe some experiences you might give to these people to make them familiar it and describe procedures which could be used to compare and order:

   Colour
   Intelligence
   Pain

TEACHING HINTS:

1. Use the everyday experiences of the children. This is particularly valuable for attributes such as length, time, money (value) and temperature.

2. Exploration and questioning should be the major tools of teaching. An understanding of an attribute can not be told to a child.

3. Materials must be experienced by children. Children should be given every opportunity to heft different objects, build with bricks, pour water and sand, etc.

4. The teachers role should be to encourage language (appropriate to the attribute) and manipulative skills. The teacher should also prepare the child for the next stage by drawing out ideas of relative size.

5. Use a variety of activities. For example, for length,

   - name tall short narrow ....objects
   - tie string or ribbon together
   - unroll balls of string
   - draw short or long lines
   - sort objects by length
   - cut streamers as long as your pen
   - direct students to follow instructions (e.g. walk to the low tree)
   - make a tall tower of lego

6. Teachers should remember, that in life, they continually make comparisons on the basis of many attributes, e.g. beauty, temperatures, velocity and size. Some of these attributes (e.g. humidity, intelligence or warmth) may be as nebulous to them as mass, e.g., is to children. Keeping this in mind may result in more tolerance and understanding of children's difficulties.
UNIT 1.2 RELATIONSHIPS BETWEEN ATTRIBUTES

FOCUS:

In this unit we explore the extent to which differing attributes, e.g. mass and volume, are related to each other.

BACKGROUND:

There are definite relationships between some attributes that show up in formulae. Other attributes are not generally related but form particular relationships in some special instances. Other attributes are independent and form no relationships.

Some of the relationships that are worth exploring with children are below.

(1) Volume, area and length - these are directly related as is shown in the formulae for area and volume.

(2) Mass and volume - these are not generally related but are proportional when density remains the same.

(3) Time and temperature - these are independent of the other attributes and each other.

(4) Money - the value of a particular substance is proportional to its size and mass but value (money) is not generally related to mass or volume.

(5) Angle and length - these are directly related as long as the length is along the arc a fixed distance from the centre of the turn.

ACTIVITIES:

Materials required: Pen paper.

1. Young John believes the pillow is heavier than the brick. What is John focussing on? Mass or Volume? How can we get him to see his error?

2. John believes that the large vase will be heavier than the small vase. Is this always true? When is it true? In what situations could it be false?

3. A group of children were throwing objects of different masses and measuring how far they went. For a particular child, how would you expect distance thrown to relate to mass? Would this always be so?

4. What would be the mass/length relationship if the objects were being hung from large rubber bands?
5. The children were playing a game - objects of different masses were being placed in a large ice cream container and the lid put on and the children had to guess which object by hefting the container.

Why is this game so suitable for introducing mass?

6. Investigations
(a) What would happen to the height of a litre milk container if we doubled the dimensions of its base?
(b) Debate this position - "large things are more expensive than small"!
(c) Why are cheap houses square? (Hint - look at relation of perimeter to area!)
(d) Why do we have to be careful about babies getting too hot or too cold? (Hint - look at relation of volume/mass to surface area!)
(e) Why do large 1kg parcels heft lighter than small 1kg parcels?

TEACHING HINTS:

1. The world of merchandising and packaging is a constant source for interrelating attributes.
   . Why does a litre cold drink bottle seem so much larger than a MAB block?
   . How can a package be made to look expensive? Or Cool?
   . Why does the 33\% segment of the turnip look bigger than that of the circle?

   . Which contains the most?

2. Even as adults, we are constantly fooled by packaging. We do not have good concepts of, e.g. the attributes of mass and volume.

Work on appropriately and correctly perceiving attributes should continue all through primary school.
UNIT 1.3 INSTRUMENTATION

FOCUS:

In this unit, we shall focus on instrumentation - how to develop, calibrate and use instruments appropriately for the attribute under investigation.

BACKGROUND:

In many cases, the measurement of an attribute (e.g. intelligence) requires the development, calibration and use of a special instrument.

Refinements of measure (e.g. accuracy to thousandths of a milimetre) have also required the development through technology, of special instruments.

It is important that children be made aware of the role of instrumentation and its relationship to the attribute under investigation. It is particularly important that they be made aware of how an instrument that gives good range of measures can come to change perception of an attribute and seduce the measurer into replacing the attribute by the instrument. For example, I.Q. measurements on Stanford Binet tests replaced intelligence (for many years) as the subject under study - intelligence became synonymous with this test. In the end, it could be argued that researchers were not studying the effect of intelligence but the effect of test score results from this instrument.

ACTIVITIES:

Materials required: Pen and paper
Materials for the investigation.

1. Before we can develop instruments, it is necessary to understand what is involved in measurement in terms of attribute, comparison and unit (so that an instrument can be developed which will be able to compare with a unit to get a measure in the given attribute).

Example 1: Angle

(1) Describe in your own words the attribute being measured when you measure an angle. What would be appropriate units for measurement? What would be an appropriate measuring instrument for angle?

(2) Why would a child consider that the two angles represented below have different measures? What "attribute" are children considering when they make such a mistake? What experiences could you give to remediate such children?

-----

(3) Look at a rotagram and a protractor. Why are these both good instruments for measuring angle? Why is the degree, the radian and the "whole turn" appropriate units?
Example 2: Temperature

(4) How does a thermometer actually measure temperature? (What physical principles are involved?) Why can this instrument use height to measure temperature? Draw and label the parts of a thermometer. How could a "home-made" version of this instrument be made?

Example 3: Probability

(5) What does probability measure? What is the relation between statistics and probability? If seven heads result from ten flips of a coin, what are the implications for probability (which would have predicted 5 out of 10 heads)? What kind of instrument could be used to measure probability?

2. There are strong differences of opinion between educators on the importance of measuring human ability. Teachers, of course, do a lot of such measurement (e.g. tests, assignments, projects) and use the results to make crucial decisions about children.

For the instruments below, discuss

(1) the attribute being measured,

(2) the conditions under which an individual or a group can possesses more of this attribute,

(3) the errors inherent in the instrument,

(4) procedures likely to give rise to errors, and

(5) the potential advantages and dangers in the instrument.

I.Q. tests
School or College grades
Attitude questionnaires
Vocational preference scales
Standardised Arithmetic tests
Gallup polls

What measures of human ability seem to be the least prone to misunderstanding and error? What human characteristics are the easiest to measure?

3. Investigation

(1) Choose a question that requires the development of a measuring instrument

   e.g. What's the happiest time of day?
       What are the best type of TV programs?
       What is the most comfortable chair in the institution?
       How much is a degree worth?

(2) Design and administer the instrument.

(3) Write up a report of your question, your instrument design and your findings.

(4) Discuss the validity of your instrument for the task it was used for.
TEACHING HINTS:

1. It is important to allow children freedom for these investigations. They should see it as something they are controlling.

2. The role of the teacher is facilitator - to react the children's difficulties/impasses. The teacher has to encourage, suggest ideas or new directions and supply resources. It is not necessary to have the answers - in many cases there are no correct answers but just a lot of options.

3. The reporting back is an important part of the investigation and is the point which may require much teacher intervention. To steal an idea from the process approach to writing, teachers could conference each group with rough drafts of their presentations before the final one is completed.

4. Investigations can be an open-ended form of instruction. Once the questions to be investigated are raised, how the lesson proceeds can be determined by the interest of the children. The questions can be the basis for whole-class or small-group discussion and activity. Such activity need only proceed for a short time, or the investigation can be expanded into a "full-blown" series of activities. The children can be encouraged to pursue anything that is of interest to them - as far as they wish to go.
CHAPTER TWO: COMPARING AND ORDERING.

Many of the real world situations involving measurement do not use units. For example, working out whether a table will fit through a door, sawing a plank to make a shelf, or cutting paper to go round a bucket. In these situations we can use either direct or indirect comparison to determine what to do. For example, we can use our bodies to check the width of the table against the door, we can place the plank beside the cupboard and mark its length direct and we can use a piece of string to transfer the bucket's circumference to the paper.

There are many other situations where we order objects without using units. For example, in buying stepping stones we may select the widest in a collection, in choosing cucumbers we may select the longest displayed, in choosing what work to do on a family camp, we may select what will take the shortest time, etc.

For these everyday occurrences and because the activities extend their understanding of the various attributes, children should experience activities in which they compare and order attributes. To this end, the Units in this chapter have been designed so that Unit 2.1 focusses on direct comparisons, Unit 2.2 extends this to indirect comparisons and Unit 2.3 focusses on ordering amounts of attribute.

A good ability to compare and order depends on rich experiences and the ability to visualize amounts. It also depends on an imaginative approach to the use of everyday items as intermediaries in the comparison process. For example, you would know that a board was not long enough to go right along the side of your garage if it was shorter than your car.
UNIT 2.1 DIRECT COMPARISON

FOCUS:

This unit looks at methods for directly comparing attributes and how these can be related to appropriate teaching experiences for children.

BACKGROUND:

Each of the various attributes that make up the Measurement area of primary mathematics have different comparison techniques. For example:

(1) length - requires two objects to be placed beside each other with one end aligned;
(2) area - requires the objects to be placed on top of each other for overlap;
(3) volume - sand or water can be poured from one object to the other if it is hollow, solid objects would have to be immersed to see differences in water level rise;
(4) mass - these can be directly compared on a beam balance but each arm length must be the same;
(5) time - requires the two activities to be run concurrently (as a "race"), starting together;
(6) money or value - this depends on personal preference unless units are used and this could only be determined by what children were willing to do for the items;
(7) temperature - by feel; and
(8) angle - by placing a copy of one angle over top of the other.

ACTIVITIES:

Materials required: wire coathanger, string, margarine containers, large icecream containers, assorted materials for measuring.

1. Construct a beam balance out of a wire coat hanger, string and two margarine containers. Use it to compare:

   (1) a duster and ten pieces of chalk; 
   (2) a stapler and 3 pairs of scissors and
   (3) other items as directed by your instructor.

2. Half fill a large icecream container with water. Mark the water height with a pen. Immersing objects will raise the water level. Which of the following objects raises it the most.

   (1) an ashtray or ten unifix cubes;
   (2) three MAB flats or a coke can; and
   (3) other items as directed by your instructor.
3. Here are some examples of comparison activities.

(a) The children held "races" between themselves comparing, for example, taking off and putting on their shoes with running around the school

(b) The children were discussing what clothes would be comfortable outside versus the airconditioned room.

Can you think up some activities to compare:

(1) area;
(2) value;
(3) angle; and
(4) length.

4. John placed the two sticks as below:

and said the "fat" one was longer. What is the problem here? What can we do about it? (Children do not seem to have a problem in comparing heights - could this be used to help John?)

5. Frank placed the red table on top of the blue as below:

blue table

but it did not help him to see which covered the most area? What could he do?

6. The teacher stood on one end of the board as below and none of the children could lift him off the ground when they stood on the other end.

Then Jenny said she knew how to do it. She shifted the brick and lifted the teacher. Where did she shift the brick to? What can this activity do to help children's understanding of beam balances?

7. Jack and Bill were seeing if writing their name 5 times was quicker than 20 hops. "Go" said Anne. But Bill was still searching for his pencil when Jack started hopping. How can we lead them to see the problem here?
TEACHING HINTS

1. These activities will give some insight into whether children are fully familiar with the attributes being compared. Teachers need to keep "one eye" open for this and use the activities to help consolidate these attribute understandings.

2. Make the activities fun. Rely on discussion afterwards, where children describe what they have done and found out, to draw out problems of aligning ends or sides or starting points.

3. Some attributes, e.g. temperature, value, are very subjective in comparison - this should be kept in mind and discussion should be the focus of the activity not "correct answers".

4. We should never forget to connect activity to the real world. For example, when a farmer was asked which property he would like (he was shown a picture of a large area of land and a smaller area of land), he could not give an answer because he did not know the quality of the soil, the availability of water, etc.

Many decisions are not made on one attribute alone but on the interaction of many.
UNIT 2.2 INDIRECT COMPARISON

FOCUS:

In this unit we will extend comparing activities to those many situations where it can not be directly achieved.

BACKGROUND:

In the real world, you often need to know whether, for example, the piano will fit before you move it to that position.

In these situations where you can not, or do not want to, move one item to beside the other, an intermediary has to be used. For the various attributes that make up primary measurement, examples of intermediaries are:

1. length - string, parts of our bodies;
2. area - paper (cutting and pasting);
3. volume - amounts of water or sand;
4. mass - plasticine or sand or rice or etc.;
5. time - shading a long rectangle on a blackboard;
6. money or value - gold;
7. temperature - amount of clothes; and
8. angle - rotation or paper cut-out of the angle.

ACTIVITIES:

Materials required: String, scissors, glue, tape, paper, tangrams, rotagrams.

1. Use a piece of string to compare

   (1) the width of the door to the height of the blackboard;
   (2) the height of the table to its width; and
   (3) any other items as directed by your instruction.

2. In Unit 2.1, activity 5, Frank had a problem comparing the top of the red table with the top of the blue. Frank could solve his problem by using paper as an intermediary. How? And what would scissors be used for?

3. Cutting and refitting are important techniques for developing area understanding. Tangram activities can be used to develop this skill and the understanding behind it. Complete the TANGRAM ACTIVITIES below.
Tangram puzzles have been around for over 3000 years. The tangram comes from a square that has been cut into seven specific pieces!

Cut out pieces....
C. How many of these

Just cover these?

D. The triangle

The area of

is 1

The area of

is
The triangle is 1

The area of square is __________

The area of triangle is __________

The square is 1

What is the area of the parallelogram?
6. Triangle is 1

What is the area of this shape?

4. Square is 1

What is the area of this shape?
4. Use a protractor to complete the ANGLE ACTIVITIES below.

Join the angles of the same size to each other with a line.
5. There are many real world situations where we have to use an intermediary, i.e. in comparing the circumference of a barrel with a length of 'hoop iron' to cut a length to go around it.

Think up some real world situations where intermediaries are used for comparison for the following attributes.

(1) length; 
(2) area; 
(3) volume; 
(4) mass; 
(5) time; 
(6) money or value; 
(7) temperature; and
(8) angle

TEACHING HINTS:

1. In developing comparison techniques in young children, we should move through four stages as below.

   *(1) Direct comparison where only the attribute being compared varies (e.g. comparing 2 pencils of different length). *

   *(2) Direct comparison where more than the compared attribute varies (e.g. comparing the lengths of a pencil and a pair of scissors). *

   *(3) Indirect comparison via an intermediary (e.g. comparing the distance around a can to the length of a pencil). *

   *(4) Comparing different representations of an attribute (e.g. comparing the diameter of a bicycle wheel with the width of a door). *

2. Teachers should make every endeavour to relate these comparison activities to real world situations - to make them real life.
UNIT 2.3 ORDERING

FOCUS:

In this unit, we look at the change from comparing two examples in terms of amount of attribute to order more than two examples.

BACKGROUND:

Understanding the process of ordering items by, for example, length or area is not a simple extension of comparing. It relies, at its basis, on an understanding of transitivity - that, for example, if A is longer than B and B is longer than C, then A is longer than C. Transitivity is a concept that is associated with maturation and development and may not be available to some children.

For this reason, the following sequence of activities is recommended.

(1) Copying an ordered sequence of examples.
(2) Placing an example correctly at the end of an ordered sequence to complete it.
(3) Placing an example correctly in an ordered sequence to complete it.
(4) Ordering three or more examples without any assistance.

ACTIVITIES:

Materials required: Cardboard for the Tall Men's Task and materials for chosen attribute in activity 2.

1. Read the Tall Men's Task (prepared by mathematics education lecturer at Carseldine, Rod Mason) below. Construct the required lengths of cardboard and trial the activities with another member of your group. Note that the activities follow the sequence given above.
Material: Six pieces of cardboard each of (noticeably) different length and colour. (In order of length these are: pink, white, yellow, blue, red, green.)

<table>
<thead>
<tr>
<th>TEACHER ACTION</th>
<th>CHILD'S RESPONSE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Subtask A</strong></td>
<td></td>
</tr>
<tr>
<td>Place cards randomly on table</td>
<td></td>
</tr>
<tr>
<td>Say: SHOW ME THE RED CARD</td>
<td></td>
</tr>
<tr>
<td>Say: SHOW ME THE GREEN CARD</td>
<td></td>
</tr>
<tr>
<td>* For children unable to complete this task, the teacher must select all future colour cards for the child.</td>
<td></td>
</tr>
<tr>
<td>Place green and red cards next to each other with bases aligned in this manner:</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Cardboard arrangement" /></td>
<td></td>
</tr>
<tr>
<td>Say: WHICH CARD IS LONGER?</td>
<td></td>
</tr>
<tr>
<td>Say: WHICH CARD IS SHORTER?</td>
<td></td>
</tr>
<tr>
<td><strong>Subtask B</strong></td>
<td></td>
</tr>
<tr>
<td>Say: SHOW ME THE BLUE CARD</td>
<td></td>
</tr>
<tr>
<td>Say: IF YOU ARE GOING FROM THE SHORTEST TO THE LONGEST (teacher points and moves hand from right to left), WHERE SHOULD YOU PLACE THE BLUE CARD?</td>
<td></td>
</tr>
<tr>
<td>(Have child predict and then place card).</td>
<td></td>
</tr>
<tr>
<td>If the child places the card this way</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Cardboard arrangement" /></td>
<td></td>
</tr>
<tr>
<td>Say: IS THE BLUE SHORTER THAN THE GREEN AND RED?</td>
<td></td>
</tr>
<tr>
<td>TEACHER ACTION</td>
<td>CHILD'S RESPONSE</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>If the child says yes, move blue card to correct position and say: <strong>IS THE BLUE NOW SHORTER THAN THE GREEN AND RED?</strong></td>
<td><em>This is the end of task 1 for children unable to complete this subtask.</em></td>
</tr>
</tbody>
</table>

**Subtask C**

Say: **SHOW ME THE YELLOW CARD**

Say: **IF I AM GOING FROM THE SHORTEST TO THE LONGEST (Teacher points) WHERE SHOULD YOU PLACE THE PINK CARD?**

(Have child predict and then place the pink card).

*This is the end of task 1 for children unable to complete this subtask*

**Subtask D**

Say: **SHOW ME THE WHITE CARD**

Say: **IF YOU ARE GOING FROM THE SHORTEST TO THE LONGEST (Teacher points), WHERE SHOULD YOU PLACE THE WHITE CARD?**

(Have child predict and then place the white card).

If incorrect, give back white card, remove blue, red and green and try again to get child to correctly place white card (when only pink and yellow are present).

If still incorrect, say: **IS THE WHITE LONGER THAN THE PINK?**

Say: **IS THE WHITE SHORTER THAN THE YELLOW?**
Say:  ARE THEY GOING FROM SHORTEST TO LONGEST?

(IF child sees error, ask child to place correctly).

"This is the end of task 1 for children unable to complete this subtask.

<table>
<thead>
<tr>
<th>TEACHER ACTION</th>
<th>CHILD'S RESPONSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtask 2</td>
<td></td>
</tr>
</tbody>
</table>

Shuffle cards and say:  **PUT THESE CARDS IN ORDER FROM SHORTEST TO LONGEST** (If child has difficulty, ask to locate smallest card, next smallest and so on).

If order completed, say:  **POINT TO THE LONGEST CARD**

Say:  **POINT TO THE SHORTEST CARD**

Say:  **POINT TO THE SECOND LONGEST CARD**

Say:  **POINT TO THE SECOND SHORTEST CARD**

Say:  **POINT TO THE THIRD LONGEST CARD.**
2. Choose one of the attributes below. For this attribute, prepare materials, a series of activities and appropriate questions to lead children through the four steps in the sequence on page 34 for introducing ordering.

(1) Area  
(2) Volume  
(3) Mass  
(4) Time  
(5) Temperature  
(6) Angle

TEACHING HINTS:

1. The teaching of the ability to order items by attributes must continue on into non-standard and standard units. The work here should be to prepare for this as well as to achieve the immediate objectives.

2. Young children should not be pushed to achieve correct answers in these activities if they are not ready.
CHAPTER THREE: NON-STANDARD UNITS

As is stated in OVERVIEW OF BASIC MEASUREMENT, a unit is a quantity of an attribute which has to be used as a basis for comparison in measurement. Once a unit has been chosen, comparisons can be made between this unit and the object to arrive at a number. The number reflects the amount of the attribute possessed by the object as compared with the unit.

When the unit chosen varies from one measurer to another, the unit is called non-standard. Such non-standard units are paces, arm lengths, cupsfull, etc. When the unit has been defined so that society in general uses the same one, it is called standard. Such standard units are litres, metres, seconds, etc.

Units are the basis of measurement in the real world - it enables numbers to be used in relation to space and time. We have to develop facility with these units - to overcome the mystique of their special names. Non standard units are an excellent way to introduce units - they are personal and natural and allow children to see our standard units as products of history and convenience.

The work on non-standard units is the most important section of primary mathematics as it enables teachers to develop the processes that lie behind measurement:

- the same units should be used when measuring and comparing;
- the largest number means the most attribute when units are common;
- the larger the unit, the smaller the number (and vice versa);
- measurements are accurate not correct (tolerance for error);
- estimation;
- choosing appropriate units; and
- the need for a standard.

This will ultimately lead to understandings that can control the measurement process (not just comprehend it) - to children being able to identify and define appropriate units for attributes under investigation, to developing instruments calibrated to measure these attributes, and to knowing the limitations of these measurements.

This chapter, therefore, focusses firstly on the non standard units suitable for each attribute in primary measurement (in Unit 3.1) and then on the development of the processes described above (in Unit 3.2). Finally, special attention is given (in Unit 3.3) to a process which can cause particular difficulty, the process 'tolerance for error'.
UNIT 5: NON-STANDARD UNITS FOR PRIMARY MEASUREMENT

FOCUS:

This unit looks at the variety of non-standard units that can be used for length, area, volume, mass, time, money or value, temperature and angle.

BACKGROUND:

Non-standard units can be anything that the measurer wishes - body parts and common objects. Some examples for the various attributes in primary measurement are:

1. length - digit (finger width, or length from end of finger to first knuckle), palm (length across the palm), hand span, cubit (length from fingers to elbow), fathom (finger tip to finger tip), pace, pencil length, blackboard duster length;

2. area - any tessellating figure (triangle, quadrilateral, regular hexagon, etc.), stamps, hands;

3. volume - cups, glasses, any tessellating solid (cubes, pyramid, prism, etc.);

4. mass - paper clips, ballbearings, marbles, pens, blackboard dusters, rubber band and margarine containers;

5. time - clapping, hopping, counting, pulse, pendulum, candles, sandtimers;

6. money or value - play money, shells;

7. temperature - column of coloured water in a thin tube; and

8. angle - small paper cut out of a 'thin' angle, e.g.

ACTIVITIES:

Materials required: glass coffee jar, large rubber band, paper, string, margarine container, felt pen, assorted junk materials for measuring (jars, weights, cups, solid objects, etc.), water, scissors, cardboard, thin glass pipe, red colour, containers.

1. Measure the length of the room in

   (a) cubits
   (b) foot lengths
   (c) fathoms
   (d) paces

   Estimate first.
Questions

1. The length of the room has not changed—why do different people get different numbers for its length with the same method?

2. What does this exercise say about using cubits or paces to order a product by length over the phone?

3. Despite these problems are there advantages in using cubits, paces, etc. over, say, a metre stick for measuring the length of a room? For introducing length measurement to children?

4. Can you think of real world situations where cubits, paces, etc. would be more useful than metres and centimetres, say?

5. What are the commonly used standard units for length? How will their use enhance communication in length?

2. Measure the length of your desk in

(a) palms  
(b) digits  
(c) hand spans

Estimate first.

Repeat the questions from 1 above.

3. Measure the area of your desk in

(a) hands  
(b) dusters  
(c) A4 pages

Repeat the questions from 1 above.

4. Construct a volume measurer and a mass measurer as follows.

Volume measurer

- Materials - glass coffee jar, felt pen.

- Use little finger to divide the height of the jar into finger widths as on right. Number these lines.

Mass Measurer

- Materials - large rubber band, hook, paper, felt pen, string, margarine container.

- Attach margarine container to end of rubber band with string and attach both to the hook as on right.

- Stick the paper behind the rubber band and, using two fingers, divide the height of the paper into two finger widths as on right. Number these.
Use your volume and mass measures to find the volume and mass of objects as directed by your instructor. Estimate first.

5. (1) How would you explain to a child what a clock measures?

(2) Could we have metric or base 10 units for time? What limitations would there be on any unit? Are there any "natural" time units?

(3) Brainstorm a number of interesting ways to measure time that children could use.

(4) There is a collection of measurers for time given below. Gather suitable materials and construct one that interests you.

(5) With each of the timers below, activities are given for using the timers for standard units. For each timer, develop one activity (and write it beside the timer) for using the timer to teach non-standard units.
TIMERS

(i) Candle Clock

Materials: Candles or tapers or birthday candles, pins or marker pen, tin lids, matches, timer.

Construction: Light one candle (or taper) and use timer to help mark, with pins or marking pen, the other candle (or taper) at suitable time intervals. Use shorter time intervals for birthday candles and shortest time intervals for tapers.

(ii) Sand Timer

Materials: Containers, card, very dry sand (or salt), timer, scissors, sticky tape.

Construction: Cut a circle of card and make a cone. Fill cone with sand (or salt) and let run into container for 1 minute or another suitable time interval (use timer). This amount of sand will then enable you to time a minute or the other suitable time interval. (Note: can also mark the cone to indicate level giving various time intervals.)
(iii) **Water Timer**

**Materials:** Plastic containers, bricks, timer, marking pen.

**Construction:** Pierce small hole in upper container. Fill this container with water. Cover hole with finger. Let water run into bottom container and mark bottom container every 30 seconds or other suitable time intervals (1 minute or 2 minutes), or mark top container.

![Diagram of Water Timer](image)

(iv) **Sinking Timer**

**Materials:** Large bowl, containers, plasticine, timer

**Construction:** Pierce hole in base of container and weigh it with ring of plasticine so it sinks evenly. Use timer to find the level the container sinks to in 1 minute (or other suitable time interval).

![Diagram of Sinking Timer](image)
(v) **Obstructed Slope Timer**

**Materials:** Plank of soft wood (1m x 15m x 1cm), cardboard, marbles, timer.

**Construction:** Attach cardboard barriers and edges to plank. Experiment with position of barriers and angle or slope of the board so that it takes 10 seconds (or some other suitable time interval) for the marble to roll down the course.

---

(vi) **Pendulum Timer**

**Materials:** Hook, ruler or stick (or line and weight), timer, string, weight (washer)

**Construction:** Make a pendulum. Count how many swings (forward and back) it makes in 30 seconds (or other suitable time interval). Change the length of the pendulum to change the timing.
(vii) **Rolling Timer**

**Materials:** Ruler or thin long board with groove in middle, marble, timer, plasticine, marker pen

**Construction:** Set up ruler in doorway.

(a) Use timer to find out how many full rolls (forward and back) the marble makes in 20 seconds (or some other suitable time interval).

(b) Place plasticine in groove at centre and roll it firmly with marble. Mark the starting position from which the marble takes 20 seconds (and other suitable time intervals) to come to rest on the plasticine.

6. Use scissors, cardboard, a glass container, a thin glass pipe, red colour and a felt pen to make a measurer for

(1) temperature

(2) angle

7. What can we do to develop non-standard units for money (or value)?
B. Choose which of the following reasons for using non-standard units you feel are most valid. Justify your choices.

A. Non-standard units are easier to use than standard.
B. Historically, non-standard units were used before standard units were established.
C. Children are more involved when they choose a unit than when a unit is imposed.
D. Children do not have to use non-standard units with the same accuracy as standard units.
E. It is good practice to give children experience with a variety of instruments.
F. By using non-standard units, children are shown that anything can be measured with some object that one possesses.
G. Materials are easier to obtain for non-standard units than for standard.

TEACHING HINTS:

1. Teachers should focus on ensuring that children use the non-standard units "correctly"
   - that they use the same units throughout
   - that they "align" units appropriately

(e.g. leave no large gaps when finding area, keep units in a straight line when measuring length, do not spill water when measuring volume).

2. Teachers should also encourage imagination in the use of units - non standard units can be a great fallback when measuring instruments are left behind and people should become aware of a wide range of possibilities in these situations.

3. History can be a tremendous fallback to give interest to non standard units. For example
   - As the Roman army marched, it counted each time its left foot came down - this was called a passus.
   - 125 passus (or passi) was a stadia and 1000 passus was a mile passus (which became the mile).
   - In old England, fields were made a stadia long - their width was determined by how much room was needed to turn the oxen around (and this was determined by the chain from the lead oxen to the plough).
   - The stadia became the furrow long (and so the furlong) - and the stadia by chain area became the acre.
UNIT 3.2 DEVELOPING MEASUREMENT PROCESSES

FOCUS:

This unit focuses on how to use now standard units to develop the processes that lie behind the overall measuring process.

BACKGROUND:

Measurement is comparison with units. To understand the measurement process is to understand how units relate to the numbers that are given to the objects to measure their attributes as a result of comparison with them. The basis of this understanding is 8 processes:

(1) The same units must be used throughout a measurement;
(2) Common units must be used for comparing;
(3) Object with the most attribute has the largest number (and vice versa);
(4) The larger the unit, the smaller the number (and vice versa);
(5) Measurements are accurate not correct (tolerance for error);
(6) Skill in estimation is essential;
(7) Appropriate units must be chosen; and
(8) There is a need for a standard.

One of the major foci of primary measurement should be developing these processes, firstly with non-standard units and then with metrics. But the main thrust of the development should be in non-standard units where names, notation and conversion rates are not necessary.

ACTIVITIES:

Materials required: Pen, paper, cuisenaire rods, angle pieces, pendulums, materials for investigation.

1. Cuisenaire rods can be used as follows for the measurement processes.

   Process (1): Give each child a pile of different rods and ask them to place the rods end to end along their book. Discuss the reasons for the different number of rods each child will have along their book. Discuss which rod could be used to name the number.

   Process (2): Give each child different rods and ask them to measure how many of their rods end to end equal the length of their book. Discuss the different answers.

   Process (3): Organise the children to have two objects, one longer than the other. Direct them to measure the shorter object with rods and then add rods until this length is extended to the length of the longer object.
Process (4): Direct the children to measure their books with two types of rod (preferably where one rod is double the length of the other). Discuss the resulting numbers.

Process (5): Discuss situations where the length is, say, 4 and a bit rods. These situations emerge when long rods are used.

Process (6): Require the children to estimate the number of rods before measuring.

Process (7): Give a variety of measurement tasks and require the children to select the most appropriate rod for each task.

Process (8): Discuss the problems people have in deciphering the numbers when they do not know the rod you are using.

Try these activities with your group. Can you think of more imaginative ways of achieving the same purpose?

2. Working in groups, brainstorm activities to introduce the measurement processes for the attributes and for the materials below:

   Angle - a variety of different sized cardboard sectors of a circle; and

   Time - pendulums with a variety of different string lengths.

3. Investigation

At the end of this unit there are two sequences of classroom activities to teach non-standard units and the measurement processes. (They were developed with the assistance of Rod Nason and the author by students at Riverina C.A.E.)

The first of these sequences (pages 51 to 61) is on length. The measurement processes being developed are listed at the beginning of each activity and include the following:

. the larger the object, the more units needed
. the bigger the unit, the fewer units needed (and vice versa)
. units can not be mixed in measurement
. the same units must be used in comparison
. choosing appropriate units
. the need for a standard.

The second of these (pages 62 to 73) is on Time and focusses on the following processes:

. the need for same and constant units for comparison
. the longer the interval, the more units needed (and vice versa)
. the smaller the unit, the larger the number (and vice versa)
. choosing appropriate units
. the need for a standard.

Read these activities. Note their purpose, sequencing and use of materials. Prepare a similar sequence of activities, for non standard units and for the basic measurement processes, for anyone of the following:

. Area
. Volume
. Mass
. Money or Value
. Temperature
. Angle
TEACHING HINTS:

1. The focus of the activities herein should be on the objectives described not solely on proficiency with units. Sequences should be determined so that the process ends with the need for the introduction of metrics.

2. Yet proficiency with units is one of the necessary products of the non-standard unit stage and should not be forgotten while the concepts are being tackled.

3. The introduction of units, the proficiency in their use and the development of the concepts, necessitates a large amount of time (many years) of the primary syllabus being devoted to non-standard units.
LENGTH ACTIVITY ONE - Measuring Height

Concepts to be developed: a) the larger the object the more units needed,
b) ordering,
c) betweenness
d) graphing.

Materials: - streamers, bundling sticks, large sheet paper.

Class Organisation: Normal seating, as long as it allows for each child to be able to see front of room.

PROCEDURE: 1. As an introductory lesson, this activity should begin with a class discussion on the concept of betweenness - i.e. tall, taller, tallest; short, shorter, shortest (etc.)
2. Children will be chosen to demonstrate these concepts. Beginning with 1 child (and gradually bringing more and more in - up to about 6), assemble a line increasing from shortest to tallest. Randomly choose children, and ask class to place child in correct position. This is developing the notions of: order betweenness and height. This section is based entirely on questioning e.g. 'Where should John stand in this line?' Are you sure - why?
3. Using these 6 children the teacher quickly takes a streamer measurement (height) of each child - from head to foot. Next, divide class into 6 small groups - each group containing 1 measured child. Distribute sticks and each group to measure streamer using these - length to be written onto streamer.
4. Class is re-united and the 6 streamers placed in order from longest to shortest by class-extensive questioning by teacher to get children to justify placements. These streamers will be pasted onto paper to form graph, the respective lengths (number of stick units) will be written on graph.
5. Once the graph has been constructed, the remainder of the lesson consists of questioning, examples of these questions would be:-

i) How many units long is the longest streamer?
ii) How many units long is the shortest streamer?
iii) Do the streamers in the middle have more, or less or the same number of bundling stick units as the (a) longest, (b) shortest streamer?
iv) What do you notice about the streamers - in particular the lengths in relation to the number of units required? Hopefully the longer the streamer the more units required.
v) What is the relationship between the length of the streamers and the height of the respective people?
LENGTH ACTIVITY TWO - Arm Lengths

Concepts to be developed: the larger the object, the more units needed, graphing, ordering.

Materials: streamers, string, pegs, large sheet paper, unifix units.

Class Organisations: normal classroom setting.

Procedure: This lesson is a reinforcement lesson, and the concepts to be developed are the same as in the previous lesson. However, in this activity, the children are able to work on their own streamer, rather than having to share one.

1. Take 2 streamer measurements of each child's arm length. One is to be pegged onto string, (fastened by drawing pins from one end of the blackboard to the other), with 1 peg securing streamer to string and another peg on bottom of streamer to keep it straight. Other streamer is to be kept by children. (Names to be written on)

2. Streamers or string are to be ordered from longest to shortest by class: - (involves estimating and testing).

3. Distribute unifix and ask children to measure 2nd streamer using these. When this is done write number of units onto small piece of cardboard and pin this onto appropriate streamer hanging on the string. (Collect second streamer for use in a later lesson).

4. The piece of cardboard is added to the streamers to show the correspondence between number and length.

Key questions

i) How many units long is the longest arm length?

ii) How many units long is the shortest arm length?

iii) Do the streamers in the middle have more, less or the same number of unifix units as the (a) longest, (b) shortest streamer?

iv) Does the longest streamer have the most number of units? Do the middle streamers always have more units than the shortest streamer?

v) What is a rule we could make up for this?

N.B. These are only examples of questions that could be asked - the teacher must respond to all questions asked, and also must ask many questions to gain an understanding of the children's depth of understanding.

At the end of this activity, the teacher and the class should remove the streamers from the string, and paste them to paper to form a graph, which will be used later.
LENGTH ACTIVITY THREE - Measuring our Book.

Concepts to be developed: - The bigger the unit the fewer units are needed
The smaller the unit the more units are needed. (Inverse proportion)

Materials:- approximately 6 musk sticks per child (lollies) (child
2 large handfuls of smarties per child motivation)
Childrens number books.

Class organisation: Children seated at desks and to work individually.
Class discussion.

Procedure: 1. Teacher instructs and demonstrates to children the
procedures to follow.
2. Children to open books and place flat on table.
3. Children to measure book from end to end with musk sticks (making
sure that musk sticks are also placed end to end, and in a relatively
straight line). Estimate before actually doing.

<table>
<thead>
<tr>
<th>Number</th>
<th>Book</th>
</tr>
</thead>
</table>
|        | musk sticks

Children will count how many musk sticks were needed.
4. Step 3 is repeated using the smarties. Careful instruction by the
teacher is required in order that children place the smarties side by
side i.e. O O O and not O O O

Children will count how many smarties were needed.
5. A number of children are asked to give their results. Were your
estimations correct?
Quest:
Did everybody have the same results? Why/Why not?
Were more musk sticks or smarties needed?
Why did we need so many smarties to measure our book?
6. Children to eat lollies (motivation and reward.)
LENGTH ACTIVITY FOUR - Magnetic board.

Concepts to be developed: - The smaller the unit the more units needed to measure the same object (i.e. the notion of inverse proportion) - Develop the notion that measurements cannot be compared unless there is a uniformity of units.

Materials: - streamers made in a previous lesson.
  - yellow cuisenaire rods
  - cardboard cards - 2 per child
  - orange cuisenaire rods
  - graph previously made
  - magnetic board and magnetic orange and yellow rods.

Class organisation: Children to work individually. Seating allows for class discussion.

Procedure: - 1. Teacher clips a streamer to the magnetic board and demonstrates to children how to measure it with orange rods.
  2. Children are given a streamer each and proceed to measure it with orange rods, counting the number of units (orange rods) long it is and writing it on one side of their card.
  3. The card is then pinned under each child's streamer on the graph.
  \textbf{Quest:} Has the longest streamer got the most orange units?
  Has the smallest streamer got more or less units? Why?
  4. Repeat steps 1 - 3 using yellow rods.
  \textbf{Quest:} - How many yellow rods has the longest streamer got?
  How many yellow units has the smallest streamer got?
  Why has the longest streamer got more yellow units than the other ones? Has the longest streamer got the most number of orange units? Has the same streamer got the most number of yellow units?
  Were more orange or yellow units needed? Why?
  Did everyone use more yellow units than orange ones?
  If child says "Shouldn't we have twice as many yellow rods as orange ones"? Point out that this may not be the case as measuring is approximate not always accurate.
LENGTH ACTIVITY FIVE - Cannot mix units.

Concepts to be developed: Cannot mix units when measuring the lengths of objects. Must use the same units when comparing the length of two objects. Estimation skills.

Materials: Part 1. 1 long streamer (30 units)   Part 2. two streamers of equal length. 1 short streamer (50 units) 1 set cuisenaire rods.

Class organisation: Children seated in a position so as they can see the teacher demonstrating with the streamers.

Procedure: Part 1. 1. Children asked to estimate (guess) and mark a point on the blackboard where they think 30 units would be. (Brief class discussion and consensus taken). 2. Children repeat step 1 with 50 units. Would it be bigger or smaller than 30 units? Why?

3. Teacher shows the streamer 30 units long (long streamer). Does it match the point we marked? 4. Show streamer 50 units long (short streamer). Does this one match our point? Is it longer or shorter than the other streamer? Why does the shorter streamer have more units? What sort of units might we have used for each one? 5. Teacher shows children the units (i.e. a small and a large unit). Notice different sizes of units and why the shorter streamer has more units.

Part 2. 1. Three streamers are displayed on board. Each streamer has a mixture of cuisenaire rods measuring it (also displayed) N.B. could be displayed along base of blackboard. 2. Children count the number of units in each streamer. Quest - Why do we get different numbers of units if all 3 streamers are of equal length? What did we measure each one with? Is it fair to say that one has more units than another? Why? What or how should we have measured them to make it fair.
LENGTH ACTIVITY SIX - Measuring our Licorice

Concepts to be developed: - You must use the same units when comparing the length of two objects (i.e. you cannot mix units when measuring the lengths of two objects). Develop estimation skills.

Materials: Long and short licorice sticks (one of each per child)
motivation: White and pink cuisenaire rods.

Class organisation: Pupils to work individually but seating must allow for sharing of rods and class discussion.

Procedure: 1. Teacher tells children of a piece of licorice she has that was 5 units long. Class asked to estimate length by demonstrating with hands (N.B. Class not told the units that were used i.e. pink rods).
2. Teacher shows licorice and 3 different sized rods. Children are asked to predict which unit was used to measure 5 units.
3. Licorice (long piece) and rods are distributed. Children test prediction. Gain consensus on which unit was used by questioning children.
4. Key questions - Using the same unit, get class to predict how long a 10 unit, 2 unit and 40 unit piece of licorice would be.
5. Tell class of another piece of licorice that is 20 units long. Would it be longer or shorter than the first piece? Get several ideas. Show the licorice. Why is it shorter than the first piece? How many units were in each one? What units did we use for the first piece? Did we use the same units in the second piece? What size units might we have used? Why do you think this? Is it fair to say that the shorter piece has more units than the longer piece? If we used the same sized unit which one would be longer?
Children to eat licorice.
LENGTH ACTIVITY SEVEN - measuring our foot.

Concepts to be developed: a) Appropriate units.

Materials: 1 sheet blank paper/child, scissors, square stickers. (Square rather than round stickers are used because Year 2 pupils find it easier to put together items that have straight edges rather than curved edges, and it is important that the stickers are as close as possible to each other, e.g. ___________)

Class Organisation: No particular organisation required.

Procedure: 1) Ask students to trace around own foot onto piece of paper (with shoes on). Carefully cut out shape.
2) Ask children to see how many sticker units long their foot is, next ask pupils to measure their foot using their pencil.
3) Ask a number of children what they have found, children will be interested to see how their measurements compare with others, so ask quite a few.

Questions: (i) Was one unit better to measure than another? If so, why, or why not? (ii) Did you have problems with 'left-over' pieces? Yes? Which unit caused the most problem in this area? (iii) Why do you think this problem occurred? (iv) If there is class concensus about the sticker being the best unit to measure feet with: - ask the following type of question. (1) Why was this unit (stickers) better to use? (2) Would it necessarily be the best unit to use? (3) Would it be good for measuring everything? Why or why not?
LENGTH ACTIVITY EIGHT - Measuring various objects

Concepts to be developed: - a) appropriate units for particular objects,  
(b) comparing.

Materials: Broom handle, dowel, pencils, rods, toothpicks, drinking straws, paper clips. (the use of these types of units of measurement require less of the teachers time in preparation, and are easier for the children to relate to as they form part of their 'everyday' lives.)

Class organisation: Tables should be in clusters to allow several children to be working there at the one time.

Procedure: - Materials are to be distributed around room, several on each cluster of tables. About five children to a table should be adequate. A list of objects to be measured is to be written onto board, this list could include: length of blackboard, height of chair, width of room, length of book, length of sheet of newspaper etc.

Children are to choose an object to measure, and select from the available non-standard units, the most appropriate unit to measure the nominated object with. Pupils are to move around the room, experimenting with as many different units as possible, and are to measure as many of the objects listed as possible.

Questions (i) Ask several children what unit they found to be most appropriate for specific objects, e.g. length of blackboard etc.  
(ii) How did you come to this decision? (iii) How long did you have to experiment before you decided? (iv) Did you have to use every unit before you decided? (v) Could you look at a unit and decide whether or not it would be appropriate without having to test it? (vi) In general could you say that the larger the object, the larger the unit should be? Why or why not?
LENGTH ACTIVITY NINE - Measuring with our feet.

Concepts to be developed: Use of informal units to measure, use of body parts as units.

Materials: streamers, stencil (next page)

Class organisation: Students are to work on their own initially, and then are to discuss results in groups.

Procedure: 1) Students are to have a streamer measurement taken of their foot. This is the students individual standard informal unit - based on the size of the child's foot. Write name on streamer.
2) Distribute stencil.
3) Students are to measure the nominated objects and then to choose several objects of their own to measure with their unit. The measurements are to be written onto the stencil.
4) Children are to discuss own results and then to compare their measurements with others.
5) Questions that the students need to consider may be: - Is everybody's measurement stencil the same? How many different measurements for the door, the table, the blackboard, and the teacher are there? Why should these differences occur?
6) To conclude this activity, a discussion and explanation of differences should occur, e.g. differences in size of foot.

Other concepts:

- By doing this activity, students may gain a slightly better understanding of the appropriateness of the unit.
- The concept of ... the larger/longer the object the more units needed, is consolidated.
- The concept of ... the smaller the unit, the more are needed, is also emphasized.
ACTIVITY NINE STENCIL - Measuring without Foot.

Name: _____________   Date: ___________

<table>
<thead>
<tr>
<th>Object to be measured</th>
<th>Number of units needed to measure object.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Door</td>
<td></td>
</tr>
<tr>
<td>Table</td>
<td></td>
</tr>
<tr>
<td>Blackboard</td>
<td></td>
</tr>
<tr>
<td>Chair</td>
<td></td>
</tr>
<tr>
<td>Bookcase</td>
<td></td>
</tr>
<tr>
<td>Dictionary</td>
<td></td>
</tr>
<tr>
<td>Teacher</td>
<td></td>
</tr>
<tr>
<td>Cupboard</td>
<td></td>
</tr>
<tr>
<td>Window</td>
<td></td>
</tr>
<tr>
<td>Heater</td>
<td></td>
</tr>
</tbody>
</table>
LENGTH ACTIVITY TEN - comparison of units

Concepts to be developed: - important to use a standard unit in comparisons estimating choice of appropriate unit.

Materials: One large fish (made from paper) per child, 3 different sized blocks for measuring (enough for all children), small fish for teacher's use.

Class organisation: pupils to work individually but seating must allow for sharing.

Procedure: 1) Teacher tells class of a fish that was 9 units long (9 pink cuisenaire rods long). Class asked to estimate length by demonstrating with hands (note - class not told yet what units were used). - take several examples.
2) Teacher shows larger fish, and whilst showing 3 different measuring units (pink cuisenaire, orange cuisenaire and unifix blocks), asks children to predict which unit was used to measure 9 unit fish. Fish and measuring units are distributed, children to test prediction of unit used, by testing with the 3 units. Gain consensus on which unit was used by questioning children.
3) Key questions - Using this same unit, get class to predict how long is, 3 unit, 6 unit, 20 unit, 35 unit and 18 unit fish would be - children are to justify their estimations.
4) Next, tell class there is also a 65 unit fish "would this fish be longer or shorter?" - get several ideas. Show smaller fish. Ask children to predict why 65 unit fish is shorter than 9 unit fish - hopefully they will state that a smaller measurement unit must have been used. Ask children whether it would be just to compare the 2 fish without giving the unit used. Teacher then to stress the importance of the use of a standard unit when comparing, by asking children pertinent questions, such as:

1. Why is the 65 unit fish smaller than the 9 unit fish?
2. Why is it important the same sized unit is used when comparing?
3. If we used the same sized unit which fish would be longer?
TIME ACTIVITY ONE - Introduction to non-standard units.

Concepts Developed:

Non-standard unit
realise the need to have the same (and constant units) for comparison.

Materials/Aids/equipment:

Each student will need:-
(1) a copy of workcard No. 1.
(2) a pencil
Each pair of students will need -
(1) a piece of paper

Class organisation:

Work in pairs

Activity

Step 1. Questioning.
Refer to the comparisons made in workcard No. 1:

e.g. set 1. a. Do 10 knee bends
b. Touch your toes 15 times.
(a) How can we tell which activity is the fastest (or slowest) without racing each other in the activities? Elicit the need to time the activities.

(b) What's a way we can time the activities to see which is the fastest without using a clock? Elicit: counting, tapping clapping etc.

Step 2. Use counting, tapping and clapping to measure the three intervals of the activities on workcard.

Step 3. Fill in workcard No. 1.

Step 4. Questioning
(a) How good are these (counting, tapping, clapping) for measuring time?

(b) What if I counted like 1, 2, 3, . . . . . . . . 4 . . . . . . . . . . . .5, 6, 7, 8 . . . . . . . . . . . .9 . . . . . . . . . . . .10 etc.
(i.e. an irregular count) to measure one time interval and like 1, 2, 3, 4 etc. (very quickly for the second time interval? Would the counting be good for a comparison? Elicit: the need to have constant units for comparison.

(c) What if we used tapping to measure the time interval for one activity and clapping for another activity? Would we be able to make a fair comparison? Why not? Elicit: the need to have the same units for comparison.
WORKCARD NO. 1: Introduction to Non-standard Units

Instruction: Complete the following sets of activities and fill in the table.

<table>
<thead>
<tr>
<th>Sets of Activities</th>
<th>The number of counts</th>
<th>The number of taps</th>
<th>The number of claps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a. 10 knee bends</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Touch your toes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 times</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. a. Writing the</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alphabet twice</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Saying your name</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and address aloud</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. a. Hopping 20 times</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Blinking 20 times</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TIME ACTIVITY TWO - Pendulum and sand timer

Concept Developed

The longer the interval, the more units needed.

Materials/aids/equipment:

Each child will need -
(1) a copy of workcard No. 2.
(2) a pencil

Each group will need the following to make a pendulum
(1) a ruler
(2) a piece of wool (about 30 cm long)
(3) a small ball of plasticine
(4) 2 books to rest on the ruler
(5) a table to set the pendulum from

Each group will also need the following to make a sand timer -
(1) 2 orange juice bottles
(2) some glue
(3) sand to fill one bottle
(4) a piece of heavy cardboard to fit between the bottles with a small hole in it.

Each group will also need - (1) a tennis ball (2) a skipping rope.

Class organisation:

Work in groups of 3.

Activity

Step 1. Questioning.
Problem: Sally wants to find out which activity would take the longest to complete
(1) copy out the word berries 30 times or
(2) standing up and laying down 50 times?

(a) How could we find out which is the longest activity for Sally without having a race of the activities or using a clock?
Elicit: time the intervals by clapping, tapping etc.
make some kind of device to measure the time intervals e.g.
a pendulum.

(b) How will we be able to tell which activity is longer by using the pendulum.

Step 2. Teacher demonstrates how to set up a pendulum -
(a) Place a ruler on the table, making sure half the ruler is balancing over the edge of the table.

(b) Place 2 books on the ruler to keep it in place.

(c) Roll a ball of plasticine about the size of a 10c piece and stick one end of the length of wool on the ball of plasticine.

(d) Place the other end of the wool on the top of the ruler and place a small ball of plasticine on top of the wool to keep it in place.
The final display should then look like this:

---

**Step 3.** In groups of 3 set up a pendulum and complete the activities on workcard no. 2.

**Step 4.** Fill in workcard No. 2.

**Step 5.** Questioning.
Refer to the results of Sally's activities on Workcard No. 2
(a) Which activity was the longest?

(b) How could you tell?
Elicit: the longer activity had more swings of the pendulum.

Refer to the results of the other activities on Workcard No. 2.

(a) Which activity was the longest?

(b) How could you tell?
Elicit: more swings of the pendulum were counted for the longer activity.

**Step 6.** Let's test this idea - the longer the interval the more units needed - with another measuring device, the sandtimer.

Teacher demonstrates how to set up a sandtimer.

(a) fill one orange juice bottle with sand.

(b) place glue around the rim of the bottle

(c) stick the piece of cardboard on top, making sure the hole is over the middle of the bottle.

(d) place glue around the rim of the other bottle.

(e) stick the bottle on the opposite side of the cardboard.
The final display should look like this -

This equals one
sand timer full

Step 7. In groups of three set up a sandtimer and complete the activities on Wordcard No. 2.

Step 8. Questioning.
Refer to the results of the sandtimer activities on Wordcard No. 2.

(a) Which activity was the longest?

(b) How could you tell?
Elicit: the longer the activity the more units needed.

(c) Tell a story: On the planet Uropia a new unit has been worked at to measure time. This unit has been called zillfills. If we wanted to measure the time intervals of the following activities in zillfills which activity do you think would require the most zillfills?
(a) blinking 5 times
(b) drinking a glass of water
or
(c) reading a 200 page story.
WORKCARD NO. 2: Pendulum and Sandtimer.

A. Make a pendulum like this one.

B. Complete the following activities and fill in the table.

<table>
<thead>
<tr>
<th>Sally's Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
</tr>
<tr>
<td>1. Copying out the word berries 30 times.</td>
</tr>
<tr>
<td>2. Standing up and laying down 50 times.</td>
</tr>
</tbody>
</table>

C. Make a sandtimer like this one -

   This is one sandtimer full

D. Complete the following activities and fill in the table.

<table>
<thead>
<tr>
<th>Activity</th>
<th>How many times the sandtimes fills?</th>
<th>Which activity is the longest? (✓)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bounce a ball 20 times</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Do 30 skips with a rope</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Draw 30 circles in the air</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Read your spelling list words aloud, once.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TIME ACTIVITY THREE - Pulse and candle

Concept Developed:
The smaller the unit- the larger the number of units needed.

Materials/aids/equipment.
Each student will need -
(1) a pencil
(2) a copy of Wordcard No. 3
Each pair of students will need -
(1) 2 or 3 small birthday candles (about 6cm long)
(2) some plasticine
(3) a saucer
(4) matches
(5) a small bouncing ball
(6) a ruler measured in centimetres.

Class organisation:
Work in pairs.

Activity.

Step 1. Questioning
How can we find out which unit of time
(a) a pulse beat or
(b) a small birthday candle is the smallest?
Elicit: have a race, use a clock.

Step 2. Let's have a race to find out which (a or b) is the smallest.

(a) Teacher demonstrates how to set up a candle timer -
   (i) place a small ball of plasticine in the middle of the
       saucer. Press firmly on the plasticine so that it sticks.
   (ii) stick the small birthday candle in the middle of the
       plasticine.
       The candle timer should look like this:-

       * mark the candle with a
texta colour at 2cm intervals.

       plasticine
       saucer.

(b) Teacher demonstrates how to check your pulse:-
   (1) hold your left hand, palm upwards
   (2) place the fingers of your right hand and close together along
       your wrist to feel for pulse beat.

       Put your fingers
       of your right hand here.
(c) In pairs, have a race to see which is the smallest (a or b).

(d) Which unit is the smallest?
Elicit: the pulse beat.
How did you know it has the smallest time unit?
Elicit: when one pulse beat was counted the candle was still burning.

Step 3. Here is a problem - Janis selects to use the pulse beat and a small birthday candle to measure the time intervals for some activities - She is trying to find out whether the pulse beat or the birthday candle has the largest number of units for each activity.

Which unit of time (a) the pulse beat or (b) the small birthday candle, do you think will have the largest number of units each time?
Note: This question is asked to determine the children's prior knowledge of the concept to be developed.

Step 4. Let's test our ideas about which unit of time (a) or (b) has the largest number of units by doing several activities.

Step 5. In pairs, children complete the activities on wordcard No. 3 and fill in the workcard.

Step 6. Questioning.

(a) Which unit did we find out was the smallest?
Elicit: the pulse beat.

(b) Which unit had the largest number of units for the first activity?
Elicit: the pulse beat?

(c) Was this the same for every activity?
Elicit: Yes

(d) What generalisation can we make about the number of units needed as the time unit becomes smaller?
Elicit: The smaller the unit, the larger the number of units needed.

WORKCARD NO. 3: Pulse and Candle

Complete the following activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of pulse beats</th>
<th>Amount of candle used (in centimetres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Counting up to 30 on a centimetre ruler.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Roll a ball along a table 10 times.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Hop on the spot 50 times.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Breathe in and out 100 times.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TIME ACTIVITY FOUR - which unit should I use?

Concept Developed:

The ability to choose appropriate units.

Materials/aids/equipment:

Each student will need -

(1) a pencil
(2) a copy of workcard No. 4

Each group will need the following to make a pendulum -

(1) a table
(2) 2 books
(3) a ruler
(4) some plasticine
(5) a piece of wool 30 cm long

Each group will need the following to make a candle timer -

(1) a small birthday candle (about 6 cm long)
(2) some matches
(3) some plasticine
(4) a saucer

Each group will also need -

(1) a piece of paper
(2) a ruler

Class organisation.

Work in groups of 2 or 3

Activity

Step 1. Questioning

(a) If you wanted to time how long it takes you to run around the oval which unit would you choose to time the activity? -
   (a) a pendulum swing
   (b) a candle timer

(b) Why would you choose that unit?
   This questioning is used to find out the children's prior knowledge of the concept intended to be developed.

Step 2.

(a) Consider the following problem -

Robert is given (1) a pendulum and (2) a candle timer. He needs to select from these the best unit to use to measure the time intervals of some of his football exercises.
   Note: Robert's exercises are on Workcard No. 4.

How can we help Robert?
Elicit: do the activities - select the most appropriate unit to time the activities.

(b) The teacher demonstrates how to set up a pendulum - as in Time Activity Two.

(c) The teacher demonstrates how to set up a candle timer - as in Time Activity Three.
(d) Complete the activities on Wordcard No. 4 and fill in the table.

Step 3. Questioning

(a) Which unit did you use to time activity 1?

(b) Why did you choose that unit?
Elicit: It is the most appropriate unit.
Note: Ask questions a and b for all the activities.

Step 4. In order to determine whether the children have grasped the idea of choosing appropriate units allow them to write down on their workcards any other appropriate units for each activity.

For this, the children may need to refer back to previous work done on time to help recall the various types of units used.

WORKCARD NO. 4: Which unit should I use.

A. Set up a pendulum and a candle timer.
   (see workcards No. 2 and 3)

B. Complete the following activities and fill in the table.
Robert's football exercises -

<table>
<thead>
<tr>
<th>Activity</th>
<th>Which unit should I use a pendulum swing or candle?</th>
<th>The number of units used.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. do 20 situps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. jump up and down</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 times</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. fold and unfold arms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 times</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. jog on the spot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 times.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. Complete the following table.

<table>
<thead>
<tr>
<th>Activity</th>
<th>What are some other appropriate units to use?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. do 20 situps</td>
<td></td>
</tr>
<tr>
<td>2. jump up and down</td>
<td></td>
</tr>
<tr>
<td>60 times</td>
<td></td>
</tr>
<tr>
<td>3. fold and unfold arms</td>
<td></td>
</tr>
<tr>
<td>5 times</td>
<td></td>
</tr>
<tr>
<td>4. Jog on the spot</td>
<td></td>
</tr>
<tr>
<td>200 times.</td>
<td></td>
</tr>
</tbody>
</table>
TIME ACTIVITY FIVE - WHY DO WE NEED A STANDARD UNIT

Concept Developed:

The need for a standard unit.

Materials/aids/equipment:

Each child will need -

(1) a copy of workcard No. 5
(2) a pencil

Each group will need -

(1) a piece of paper
(2) a clock.

Everyone will need to view a clock with a second hand.

Class organisation:

Work in groups of 2 or 3.

Activity

Step 1. Consider the following problem -

Jane and Graeme wanted to measure how long it takes to do several activities and then compare their results. They decided that they would both use pendulums to measure the time intervals and ask David to do the activities for them.

When Jane and Graeme compared their measurements they found out that they were quite different.

(a) What do you think caused the different results?
Elicit: the pendulums may have been different -

(1) different lengths of string
(2) different weights of plasticine

(b) Is there anything wrong with having different results?
Elicit: a fair comparison cannot be made, reliable patterns cannot be established.

(c) What are some other ways we can find out how long the activities take?
Elicit: use a clock, count, clap, tap, etc.

(d) Which of these units would allow for the best comparisons?
Elicit: the clock - seconds.

(e) Why do you think it is the clock?
Elicit: Timing on a clock is very regular so results are reliable and comparisons are fair.

Step 2. Show the class a clock. Allow the children to count the seconds aloud up to 2 minutes to get a feel of the regularity of timing.

Step 3. Complete the activities on Workcard No. 5 and fill in the table.

Step 4. Compare your group's results with other groups.
Step 5. Questioning.

(a) When you compared your group’s results with other groups which unit had the closest number of units for each activity
   - (a) clapping,
   - (b) counting
   OR (c) seconds?
Elicit: the seconds, by the clock

(b) What do you think caused the results of the other units
   (i) clapping
   (ii) counting to be so different between the various groups.
Elicit: the people in the different groups clapped differently. Some may have clapped slowly, others may have clapped quickly. This applies to the counting also.

(c) Why then, do we need a clock to measure time?
Elicit: (i) reliable results
   (ii) fair comparisons

WORKCARD NO. 5. Why do we need a standard unit?
Complete the following activities and fill in the table -

<table>
<thead>
<tr>
<th>Activity</th>
<th>The number of claps</th>
<th>The number of counts</th>
<th>The number of seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write the alphabet twice.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Stretch up and then touch the floor 10 times.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Take off your shoes and put them back on.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Crawl under the table.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
UNIT 3.3 TOLERANCE FOR ERROR

FOCUS:

This Unit focusses on one of the measurement processes: tolerance for error. In arithmetic, answers to exercises are exact. This is not so for measurement where the practical difficulties with instruments allow answers only to be accurate to the needs of the task.

BACKGROUND:

It is essential that children grasp the fact that exactness is not possible in measurement and that they learn to tolerate error. This can be difficult to acquire and much of what we cover in this unit may not be easily teachable to children.

When we measure something, say, the area of a tennis court, we go through four steps as follows:

(1) we assume that the tennis court is a rectangle and that it is flat [we create a model];
(2) we choose a tape measure [we choose a measuring instrument];
(3) we measure the length and breadth with the tape measure (in the same units) [we apply the instrument]; and
(4) we multiply the length by the breadth to determine the area of the tennis court [we perform the computations].

Each of these four steps has the potential for error as follows:

(1) the tennis court may not be perfectly flat or a perfect rectangle;
(2) the tape measure may not be perfectly constructed and/or its markings may only be to the nearest cm so estimates will have to be made;
(3) the tape could be imperfectly applied to the sides of the court, the markings could be wrongly read (there is a lot of error in measuring something like a court); and
(4) round off error will result from the computations (there are also arithmetic errors).

Measurement is a complex human activity which is doomed to inaccuracy. But in real situations, exactness is not needed. Only measurements accurate to certain tolerances are required. For example, races are timed only to hundredths of a second, windows are measured to the nearest mm and mass to the nearest kg.
ACTIVITIES:

Materials required: ruler marked in cm only, ruler marked in mm, measuring cylinders, beam balances and masses, clock marked in minutes.

1. Measure the line below with a ruler:
   (1) marked only in cm; and
   (2) marked in mm.

What differences exist between the two measures? What is the innacuracy of each instrument?

2. For each of the examples below, do the following:
   (1) measure each with appropriate units and instruments;
   (2) compare your answers with others in your group;
   (3) discuss in your group the four steps you used (model, instrument, application, computation); and
   (4) discuss why your answers differ (if they do) and which of the four steps was the major source of error (in your opinion).

   * length of classroom wall
   * volume of water in a glass
   * mass of a bag of marbles
   * as directed by your lecturer

3. What is the instrument error in

   * volume of a jug with measuring cylinder marked in 10 mL steps,
   * one hour interval with a clock marked in minutes, and
   * area of a table with a ruler marked in cm (remember step 4).

4. Can you think of a measurement that is exact? How can we reduce error? How can we reduce error in particular for measures of children's ability?

5. When would you introduce the notion of innacuracy to children? How?
TEACHING HINTS:

1. The best way to introduce tolerance for error appears to be in discussing the children's own measurement experiences. All children will make errors when measuring. Allow them to discuss the differences that occur. Ensure that discussion covers errors due to measuring skill and errors due to instrument limitations.

2. In the long run, instrument errors, particularly those related to unit markings, should be the major focus of instruction.

3. A tolerance for error is essential if children are to use investigations involving measurement as a vehicle for discovering formulae and relationships.
CHAPTER FOUR: STANDARD UNITS.

The units that our society has adopted at present for measurement come from the metric system. This is a base 10 system and can be related to numeration. Parts of it are also highly related and this should be used in its teaching.

\[ \text{e.g. } 1 \text{g is the mass of } 1 \text{cm}^3 \text{ or } 1 \text{mL of water at 4°C} \]

\[ 1 \text{kg is the mass of } 1 \text{ L of water at 4°C} \]

\[ 1 \text{ tonne is the mass of } 1 \text{ kL of water at 4°C}. \]

Standard units should not just be told to children. They need to be introduced through stages so that children can become familiar with them. To this end, Unit 4.1 focusses on the four stages of introducing standard units

- common unit,
- identification,
- internalization, and
- estimation;

and Unit 4.2 focusses on the metrics themselves and how they relate to the base 10 decimal numeration system.

To add detail to the chapter, unit 4.3 discusses how standard units can be developed for area and volume and Unit 4.4 how time is developed.
UNIT 4.1 INTRODUCING STANDARD UNITS

FOCUS:

In this unit we look at the stages involved in introducing standard units in measurement.

BACKGROUND:

Standard units are the formal basis of measurement. They are the units used in everyday life in our society - in industry, commerce and science. If one has a poor concept of a particular unit, or worse still, no concept at all, it is extremely difficult to use the unit in everyday life. To be able to use the unit is to be able to estimate with it. To be able to estimate, one must be conversant with the unit. Memorizing conversion rates and symbols and names is not enough, one must become actively involved in measuring activities.

To learn a standard unit, one must learn to think in terms of it - the unit must be used constantly without referring back to or translating from better known units. One has to think of mass in kilograms and height in metres and centimetres.

The development of standard units is therefore to get across the idea of standard and then to build the ability to estimate in those units. The following four stage process is recommended.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Common unit</td>
<td>to gain experience in working with the same unit as others and the benefits therein.</td>
</tr>
<tr>
<td>2. Identifying standard</td>
<td>to develop a 'feel' for the relative size of the standard unit.</td>
</tr>
<tr>
<td>3. Internalizing standard</td>
<td>to conceptualize (internalize) the standard unit with respect to common objects (this forms a reference base for estimating).</td>
</tr>
<tr>
<td>4. Estimating with standard</td>
<td>to estimate using the standard unit and to improve accuracy of estimates.</td>
</tr>
</tbody>
</table>

ACTIVITIES:

Materials required: Coloured straws, measuring tapes (large/small), stop watches, calculators, cm grid paper, tape, scissors, glue, pens, rulers, 1 L soft drink bottle, collection of glasses/jars/bottles/jugs, collection of small boxes, measuring cylinders, overflow trays, plastic bags, collection of plastic containers, string, collection of materials for weighing (sand, pasta, rice, marbles, sawdust, etc.), bathroom scales, beam and spring balances, masses, rubber bands, plasticine.
1. A piece of dowelling (about the length of an arm) is an excellent common measure for length that can be adopted by a class. Make up a suitable common measure for a primary classroom for the following attributes.

(1) Area  (2) Volume  
(3) Mass  (4) Time  
(5) Money or value  (6) Angle  

Temperature is a little more difficult. What can we do for it?

2. Complete the following activities for introducing identification, internalization and estimation for standard units for length.

Identification

(1) Cut 1 cm pieces from different coloured drinking straws. Thread these pieces along a string in groups of 10 of one colour followed by 10 of another colour.

(2) Using lcm grid paper, cut ten strips that are 10 cm in length. Tape these together to form a folding 1 m measuring strip. This should be placed on cardboard to make it more durable.

Internalization

(3) Use a measuring tape to measure and record your personal body measures:

<table>
<thead>
<tr>
<th>Height</th>
<th>Head</th>
<th>Length of hand</th>
<th>Index finger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm span</td>
<td>Neck</td>
<td>Ankle to knee</td>
<td>Middle finger</td>
</tr>
<tr>
<td>Waist</td>
<td>Leg</td>
<td>Wrist to elbow</td>
<td>Ring finger</td>
</tr>
<tr>
<td>Chest</td>
<td>Arm</td>
<td>Left hand</td>
<td>Little finger</td>
</tr>
<tr>
<td>Hip</td>
<td>Foot</td>
<td>Thumb</td>
<td></td>
</tr>
</tbody>
</table>

(4) Find a reference length in your body which is approximately:

1 cm ............ 10 cm ............ 1 m ............

(5) Mark out a 10 m distance using a large measuring tape. Determine how many of your paces equal this 10 m. Pace the following distances, as directed by your lecturer, and use this value to convert your paces to metres:
DISTANCE | PACES | METRES
Length of room


(6) Mark out 25 m. Use a stop watch to time how long it takes you to walk this distance. Use this time to determine how long it would take you to walk a kilometre.

Estimation

(7) First estimate, and then measure the length of the objects given to you by your lecturer to complete the table below. Estimate and measure each distance before starting the next.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of lecturer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of blackboard</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of top shelf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of another student</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


3. Complete the following activities for introducing identification, internalization and estimation for standard units for volume/capacity.

Identification

(1) Using 1 cm grid paper, draw and cut out a net for a cube of side 1 cm. Fold and tape to make the cube. Connect 1 m dowels to form a cubic metre.

![Diagram of a cube](image)

Fill the 1 cm cube with water (or sand). Pour this into another container. This represents 1 mL.

(2) Use cardboard to make a net for a cube of side 10 cm. Tape and fold this cardboard to make the cube. This cube is 1 L. Check this by pouring 1 L of water or sand into it. Check that this cube holds the same as a 1 L soft drink bottle.

![Diagram of a 10 cm cube](image)
(3) Calibrate a container into 100 mL levels in either of the following ways:

- Way 1 - take a glass jar and pour 100 mL amounts into it, marking the levels with tape as you go;
- Way 2 - take a 1 L milk carton, cut off the top and use a ruler to divide the height into 10 equal intervals.

**Internalization**

(4) Obtain a collection of small rectangular prisms (boxes - e.g. matchbox, cigarette box, cassette case) and pack these with MAB units to find their volume. Check by using the formula \(V = L \times B \times H\). Try to estimate first.

(5) Obtain a collection of jars and jugs and pour 250 mL and 500 mL into them and note levels. Try to estimate where the levels will be before pouring.

**Estimation**

(6) First estimate and then measure the volumes of objects as given by your lecturer. Estimate and measure each object before moving onto the next. ESTIMATE THE VOLUME - DO NOT ESTIMATE LENGTH, BREADTH, HEIGHT. Use a tape for volume and measuring cylinders for capacity.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
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<tr>
<td>Chalk box</td>
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<tr>
<td>Shoe box</td>
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<tr>
<td>Cupboard</td>
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<td>Under the table</td>
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<tr>
<td><strong>Capacity</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cup</td>
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<td></td>
<td></td>
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<tr>
<td>Glass</td>
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<td></td>
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<tr>
<td>Bottle</td>
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<td></td>
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<tr>
<td>Plastic container</td>
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</tbody>
</table>

(7) Use an overflow tray and a measuring cylinder, to find the volume of objects by immersion and overflow. Estimate first.
OBJECT
Lump of plasticine
A rock
Your fist

ESTIMATE

MEASURE

DIFFERENCE

4. Complete the following activities for introducing identification, internalization and estimation for standard units for mass.

**Identification**

(1) Construct mass measurers as follows:

   way 1 - a wire coathanger, string and 2 margarine containers (plus masses);

   way 2 - a long piece of paper, 3 rubber bands, string and a margarine container (calibrate the 'spring balance' with masses - mark lengths on the paper).

Use these measurers to make up plastic bags, or other containers, containing 100 g, 250 g, 500 g and 1 kg of various materials (pasta, sand, marbles, rice, etc.).

**Internalization**

(2) Use a bathroom scale to measure your own mass in kg.

(3) Measure 1 L of water

(4) Find objects in the environment that measure approximately 1 kg, 500 g, 250 g, 100 g, 50 g and 1 g. Make up lumps of plasticine to these measures.

**Estimation**

(5) Estimate first and then measure the masses of the following objects as
given by your lecturer. Complete estimates and measures of each object before moving onto the next.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ESTIMATE</th>
<th>MEASURE</th>
<th>DIFFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duster</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case or port</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Another student</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lecturer</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Make up a series of activities to take children through identification, internalization and estimation for standard units for one of the following:

(1) Area
(2) Time
(3) Money
TEACHING HINTS:

1. These activities need a laboratory set up - materials, tables, water. This requires the same attention to safety that is required in science experiments.

2. It also requires the development in children of the expectation that learning will occur from what they do and from other children as well as the teacher. Investigation style activities do not work where children have to show everything to the teacher and be shown how to do everything by the teacher.
UNIT 4.2 METRIC AND DECIMAL NUMERATION

FOCUS:

This Unit focusses on metrics, the standard units for length, area, volume, mass, temperature and angle. It shows how these can be interrelated with decimal numeration.

BACKGROUND:

Schools use a simplified version of the metric system, the SI system of measurement. SI is an abbreviation of Systeme Internationale (International System). It is a base 10 system of 6 base units (metre, kilogram, second, ampere [electric current], kelvin [temperature] and candela [luminous intensity]) agreed to at an international conference in 1960.

Primary schools use only the SI base units of metre, kilogram and second. All other SI units are derived, e.g.

- a centimetre is 1/100 of a metre,
- a cubic centimetre is a cube of side 1 centimetre,
- a litre is 1000 cubic centimetres.

Once a name has been fixed for a SI unit (e.g. metre for length), larger and smaller units of that measurement are made by putting a code word, or prefix, before that name (e.g. centimetre, kilometre) — like a given name and a family name. The prefixes mean the same for every "family". SI has two prefixes for all units

- "kilo" — meaning 'a thousand times', and
- "milli" — meaning 'a thousandth of';

and for length only

- "centi" — meaning 'a hundredth of'.

SI units in the primary school

The following SI units are used in the primary school:

<table>
<thead>
<tr>
<th>ATTRIBUTE</th>
<th>NAME OF UNIT</th>
<th>USE</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>metre (not &quot;meter&quot;)</td>
<td>base unit</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>centimetre</td>
<td>convenient unit</td>
<td>cm</td>
</tr>
<tr>
<td></td>
<td>millimetre</td>
<td>fine measurement</td>
<td>mm</td>
</tr>
<tr>
<td></td>
<td>kilometre</td>
<td>distance</td>
<td>km</td>
</tr>
</tbody>
</table>
Mass
kilogram base unit kg
gram fine measurement g
tonne ("o" as in "tot") large masses t

Time
second base unit s
minute
hour
day

Area
square metre derived unit (floor area) m²
square centimetre graphs, models cm²
hectare (10 000 m²) land area ha

Volume/capacity
cubic metre derived unit (sand, cement) m³

cubic centimetre volume of boxes cm³

litre fluid measure (note capital) L

millilitre fine measurement mL

Speed
metres per second derived unit m/s

kilometres per hour speed of cars km/h

Angle
degrees direction °

Temperature degrees celsius °C

When writing the symbols, there is a space and no 's' or full stop, e.g.

- correct - 40 kg; incorrect - 40 kg., 40 kgs or 40kg,

It should also be noted that a comma is no longer used as a thousands separator, a space is used instead, e.g.

- correct - 49 216; incorrect - 49,216;

and a zero should always be placed before the decimal point, e.g.

- correct - 0.53; incorrect -.53.

ACTIVITIES:

Materials required: Metre rules, measuring tapes, measuring cylinders, cm grid paper, metric masses, beam balances, thermometers, protractors, stop watches, timers, callipers, cm cubes.

1. Obtain examples of measuring instruments for metrics and measure the objects below:

   . Length of room - metre stick,
   . length of blackboard - measuring tape,
2. Metric expanders

(a) Copy the four number expanders (or construct larger copies) at the end of this unit and cut them out. Use them to construct metric expanders as follows:

- expander A - metres, centimetres and millimetres,
- expander B - kilometres, metres and millimetres,
- expander C - tonnes, kilograms and grams, and
- expander D - litres and millilitres.

Fold the metric expanders like number expanders. Use them, as directed by your lecturer, to relate metres, kilograms and litres to the other units.

Note: The decimal point has been placed so that expanders A and B read as metres, expander C as kilograms and expander D as litres when folded.

(b) Construct an expander for hectares, square metres and square centimetres (which reads as square metres when folded).

3. Metric slide rule

Copy the metric slide rule at the end of this unit (metric slide rule is an idea from Baturu A and English L, Sunshine Mathematics, Melbourne, Longman/Cheshire, 1985/86). Using scissors, cut out the slides and the scale and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide from the back up through the slit on the left of the scale and across the front and out the slit on the right of the scale.

Use the slide rule to relate metrics and decimal numeration.

4. Time is often represented as

2.43 hours.

What is the numeration difficulty here? What can we do to diminish this difficulty?
TEACHING HINTS:

1. Give children experience with specialist measuring instruments such as callipers, verniers, etc.

2. Undertake outdoor activities such as height measurement, orienteering and scale drawings/surveying (see Geometry book).

3. Metric conversion rates need to be consolidated through drill - some examples

   (a) Dominos

   1 Kg : 100 cm

   1 m

   1000 m

   (b) Bingo

   1000 mm

   1 kg

   100°C

   500 g

   Free

   1 tonne

   2000 ml

   90°

   1 h

   (c) Mix and match cards

   2 m

   200 cm

   (d) Card desks (for concentration, gin rummy, snap, etc.)

   5 Kg

   5000 g

   0.005 tonne

   5 kilograms

4. Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance:

   (1) 2 decimal places are related to money (dollars and cents) and length (m and cm); and

   (2) 3 decimal places are related to length (m and mm), mass (Kg and g and t and Kg) and volume (l and ml).

The only problem is time which is base 60.
METRIC EXPANDERS

MILLILITRES

LITRES

TONNES

KILOMETRES

KILOMETRES

CENTIMETRES

METRES

MILLIMETRES

GRAMS
<table>
<thead>
<tr>
<th>Whole Numbers</th>
<th>Decimal Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>H</td>
</tr>
<tr>
<td>O</td>
<td>T</td>
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<tr>
<td>E</td>
<td>H</td>
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<table>
<thead>
<tr>
<th>km</th>
<th>m</th>
<th>cm</th>
<th>mm</th>
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<tr>
<th>L</th>
<th>mL</th>
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<tr>
<th>t</th>
<th>kg</th>
<th>g</th>
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</tbody>
</table>
UNIT 4.3 AREA AND VOLUME

FOCUS:

In the development of standard units, there are techniques and materials that enable children to experience measurement without the need to apply formulae. This Unit looks at particular techniques and approaches useful in Area and Volume. It also offers activities that can be part of the development of the process of measuring Area and the development of the process of measuring Volume.

BACKGROUND:

The sequence of activities to standard units involves

* identifying the attribute,
* comparing and ordering, and
* non-standard units.

Hence, when you begin to teach children about area, for example, you should give preliminary activities which focus on covering and comparing the different amounts of cover needed for different objects. It is also useful to do some activities with non-standard units. It is only at the end of such activities that the experiences included in this Unit are meant to be presented.

It is also important to remember that measures are not exact. This is particularly important for area and volume work where squares and cubes are counted. The numbers that result are only approximations for the areas or volumes.

Volume is a particular difficulty in that there are two units of measure - cubic cm for ordinary volume and mL for capacity (liquid volume). This dichotomy is exacerbated by the differences that exist in the difficulty of each unit. Capacity is simple and quick to measure with the special measuring cylinders and cups. Children can pour from one container to another to do this at an early age although conservation does not exist to late. The computations involved in the rest of volume make it a topic for the upper years.

Capacity through the use of measuring cylinders is much easier for irregular shapes that can contain sand or a fluid. Computations that give rise to ordinary cubic measures are easier for regular shapes for which a formula is known.

The two models of volume are related - a cubic cm is a mL and 1 000 L is a cubic m.
ACTIVITIES:

Materials required: Square grids (at end of Unit), tracing paper, pens, cm cubes, measuring cylinders, collection of glasses and jars, cm cubes, cuisenaire rods, water or sand, geoboards and rubber bands (dot paper).

1. Beginnings

(1) What is area? How would you describe it to a child? How would you decide which of two objects has more area?

(2) Which of the three shapes below would be best for finding the area of this page? Which would make the best unit?

(3) Which of these three shapes covers best? How well do copies fit together? Which is common and easy to draw? Which is easiest to count when copies of it have covered a shape? Experiment with a few copies of each shape!

2. The adoption of the square

For historical and practical reasons, the square is our society's unit for area. Hence, to find area, we need to determine how many squares of a certain size cover a shape. At the end of this Unit are three grids of different sized squares (2 cm, 1 cm and 2 mm). Use these squares to do the activities below.

(1) Copy the following shapes onto tracing paper and use the grids to find their areas. [Note: this can also be done by copying the grids onto plastic and placing the grids over the shapes.] ESTIMATE first!
<table>
<thead>
<tr>
<th>SHAPE</th>
<th>ESTIMATE</th>
<th>NUMBER OF SQUARES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 mm</td>
</tr>
</tbody>
</table>

A
B
C
D

(2) Do the same for the following shapes, but in each case determine the number of squares that are completely inside [I], the number of squares that are completely and partially inside [F] and then the average between these two numbers [A]. ESTIMATE first!
<table>
<thead>
<tr>
<th>SHAPE</th>
<th>ESTIMATE</th>
<th>NUMBER OF SQUARES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 cm</td>
<td>I P A I P A 2 mm</td>
</tr>
<tr>
<td>E</td>
<td>I P A I P A</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>I P A I P A</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>I P A I P A</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>I P A I P A</td>
<td></td>
</tr>
</tbody>
</table>

(3) For each of the three grids for G above, what is the error in taking the average? In terms of 1 cm squares, which grid gives the least error? (Remember that each 2 cm square is four 1 cm squares and each 2 mm square is one twenty-fifth of a 1 cm square.) Hence, which square is the most accurate? How does this compare with which square is the easiest to count?

3. Removing tedious counting

To become increasingly accurate in area, we have to use finer and finer grids. But then the counting becomes tedious. In some cases, the accuracy required demands too fine a grid. Hence, the invention of formulae to give relief to the calculation of area for regular shapes. Irregular shapes still require the counting of squares, but the invention of calculus enables this also to be achieved without tedious counting.

How fine a grid would you need to be within 1% of the correct area of an irregular shape by counting squares and averaging (as we did in 2 above)?

4. Using geoboards

Geoboards are a useful aid for area measurement. The nails enclose squares and rubber bands can be used to outline the shapes. There are two particularly useful techniques for polygons as are described below.

(a) Breaking into parts

This relies on techniques as below:

- is half and therefore is \( \frac{1}{2} \) cm²
- is half and therefore is \( 1 \) cm²
- is half and therefore is \( 1\frac{1}{2} \) cm²
(b) Enclosing with a rectangle

Area is \(16 - 4 - 4 - 1 - 1 = 6 \text{cm}^2\).

(1) Use the most appropriate of the above two methods to determine the area of the following shapes.

(2) Which of the following shapes is it possible to make on a geoboard?

- **A** - Rectangle: area of 1, perimeter of 4
- **B** - Rectangle: area of 3, perimeter of 8
- **C** - Rectangle: area of 4, perimeter of 8
D - Rectangle: area of 4, perimeter of 10
E - Triangle: area of 3
F - Parallelogram: area of 4
G - Trapezium: area of 5
H - Pentagon: area of 8
I - Hexagon: area of 8
J - Octagon: area of 8

5. Variation on the geoboard

Other shapes can be used for the unit of area, e.g. triangle units - the small right triangle made by placing a rubber band around three adjacent nails:

(1) what is the area of the figures below in triangle units.

(2) Is it possible to construct a pentagon with an area of 11 triangle units.

6. Counting cubes

(1) Resisting the use of any formulae you may know, count the number of cubes that could be packed into the following 3D shapes. Try to be systematic in your counting!

(2) Use cm cubes (or cuisenaire rods) to estimate the volume of a can or jar.

(3) A box is 2m by 3m by 40 cm. What pitfalls might children fall into in calculating its volume? What would you suggest be done with a child who comes up with the answer of 240 cubic units?
7. Capacity

(1) Obtain a clear cylindrical jar or glass as your unit of capacity. Use a ruler to divide it into tenths of units. Use this unit to measure the capacity of three containers. Estimate first!

(2) Discuss in your groups strategies to determine the volume of an irregular container of any size. Also discuss strategies for finding the volume of an irregular solid object of any size.

TEACHING HINTS:

1. The geoboard is a flat (usually square board) from which nails or pegs protrude. The nails are in a regular array (rows and columns). Coloured rubber bands are used to outline shapes. Geoboards are useful teaching aids for multiplication, geometry and measurement. They are particularly good for children to use in investigations because shapes can be quickly made, tested and remade. They do have problems if discipline is a difficulty as their rubber bands can be dangerous if used other than on the boards. The book (part of this series) Geometry: Space and shape in the primary school has many activities for the geoboard.

A geoboard can be replaced with dot paper, ruler and coloured pens - it is not so flexible but a permanent record is available and there is less danger of discipline problems.

2. There is a formula (Pick's theorem) for relating the area inside a polygon on a geoboard with the number of nails inside \([I]\) and the number of nails on the boundary \([B]\):

\[
\text{AREA} = I + \frac{B}{2} - 1.
\]

This formula is an excellent topic for an investigation. Direct the children to attempt to find a formula when there are no nails inside the shapes, then when there is always one nail inside the shapes and then when there are two nails inside and so on \(\ldots\). Then combine the different formulae for all these cases into one formula for all cases.

3. Children require experience counting squares and cubes and measuring capacities if they are to understand what lies behind formulae. Even though it is slow and messy, experiments of this type should be an important part of measurement.

4. Measurement is a great opportunity to get outside and into everyday activities. Try to find local situations where measures are taken and take your children to them.
UNIT 4.4  TIME AND TIME-TELLING

FOCUS:

This Unit focuses on a sequence for teaching time. Because of its central place within everyday life, time requires instruction on the concept of time and the reading of common instruments for measuring time, e.g. clock faces. In fact, this Unit will separate the topic of time into two areas: the concept of time and time-telling skill.

BACKGROUND:

The development of standard units for time (hours, minutes, days) requires the acquisition of knowledge of time-telling, i.e. how to ‘tell the time’. A suggested sequence for teaching time is as follows:

1. sequencing events;
2. cycles of events;
3. associating events with times of day;
4. telling time on the hour;
5. movement of hands on clock
   - 5 minute intervals,
   - minutes after the hour,
   - digital notation, and
   - other conventions (e.g. ‘half-past’, ‘quarter-to’, etc.);
6. passage of time (calculating time intervals); and
7. 24-hour time.

Because of the need to develop the ability to read a clock face, standard units and time-telling has to be introduced early to children. In fact, some educators recommend that time-telling (the ability to read a clock and to set a clock) be taught separately to the concept of time (the understanding of various units of time, their duration and their relationships to events). The identification of the attribute of time and comparing times has to occur before the introduction of these standard units and time-telling, but the main non-standard unit work on the measurement processes can be undertaken after this standard unit or time-telling instruction.

People make continual references to time in everyday life. The day’s program is determined by time and the measured with the use of clocks and watches. Some of the language associated with time is confusing. As stated by an article central
to this Unit I Nelson, Glen, "Teaching time-telling", The Arithmetic Teacher, May 1982), the following all describe the same time:

Forty-five after six
Forty-five past six
Fifteen till seven
Fifteen to seven
Quarter till seven
Quarter to seven
Quarter before seven
Six forty-five

The understanding of time, therefore requires the simplification of this language and, because of its analog form, the use of the tactile and kinaesthetic senses as well as the visual and auditory senses.

ACTIVITIES:

Materials required: Materials for construction of clock faces, geared clocks, pen, paper.

1. The non-geared clock face.

Construct a clock face - a circle, twelve markings, the numbers 1 through to 12, two hands.

(1) What knowledge can the construction of such a clock face assist children to acquire? What questioning will assist this acquisition?

(2) Move the hands of the constructed clock so that the little hand is directly at the 4 and the big hand is directly at the 6. Is this half-past 3 or half-past 4? Can a real clock face ever show these numbers? What then is the problem of non-geared clock faces as a teaching aid?

(3) What teaching activities is a non-geared clock face useful for? (Remember the materials --> language --> symbols model for determining appropriate instructional activities.)

(4) Use your non-geared clock face to set a clock for

. a quarter to 4
. 4 O'clock
. 6:55
. 25 minutes past 8
. 17 minutes to 11

2. The geared clock face

(1) How can you use the gearing to help introduce the fact that at the O'clock, the big or 'minute' hand points to the 12?
(2) How can you use the gearing to introduce 'clockwise' and 'anti-clockwise'?

(3) How can you use the gearing to introduce the role of the 'minute' or big hand?

(4) How can you emphasise the use of tactile and kinaesthetic senses in this teaching? [Think of ways that the children can focus on feeling where the hands are and on 'acting out' the action of a clock face!]

(5) What activities are geared clock faces most useful for?

3. Digital clocks/ 24 h clocks.

Most clocks are digital. The numbers on a digital clock can be read as soon as numbers can be read, but there may be little understanding of what the numbers mean in terms of time. For the purposes of teaching, digital representations of time can be considered as the symbols in the materials — language — symbols model of instruction. Hence digital representations of time can be introduced in relation to clock faces (the materials) and then later related to language (statements like "half-past 4", "25 minutes to 6", "7 o'clock", etc.).

(1) The triangular model on right shows that six activity types should be included in a teaching session. Complete the table below by continuing the labelling of activity types and briefly describing an activity for each type.

<table>
<thead>
<tr>
<th>ACTIVITY TYPE</th>
<th>TEACHING ACTIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials ——&gt; Language</td>
<td></td>
</tr>
<tr>
<td>Language ——&gt; Materials</td>
<td></td>
</tr>
<tr>
<td>Materials ——&gt; Symbols</td>
<td></td>
</tr>
</tbody>
</table>

..............................

..............................

..............................
(2) 24 h clocks are becoming common in digital form. How do we amend the activities in (1) above to make them applicable to the 24 h clock? What particular difficulties does the 24 h clock have for children? (Remember that children are used to working in base 10!)

4. In your groups, prepare an activity for steps (1), (2), (3) and (6) in the suggested sequence for teaching time given in the background to this unit.

TEACHING HINTS:

1. Let the children's experiences, especially with geared clocks, do a lot of the instructional work for you. Geared clocks always have the 'minute' hand pointing at the 12, for instance, whenever the 'hour' hand points directly at a number. Children can be lead to discover this.

2. Collect old unworking clocks for your classroom. The gearing should still work.

3. Have the children act out the actions of a clock. For instance, their hands can become the hands of a clock. Have the children shut their eyes and feel where the clock hands are at various times. Remove the glass from the front of the clocks to allow this to be done. The kinaesthetic and tactile senses are essential in the teaching of time.

4. Do not forget the concept of time. Children need experiences with time intervals, comparing lengths of time, timers of all sorts that use non-standard units and particularly the relating of times with parts of the day - start of school, end of school, dinner time, bed-time, etc.
CHAPTER FIVE: FORMULAE AND APPLICATIONS

Once standard units have been acquired, measures can be calculated by directly comparing the unit with the object - in most cases by seeing how many copies of the unit fit into the object. This can be a slow and tedious process, therefore, to make it simpler, ways have been found to speed up the process. One such way in formulae.

For most attributes, technology has been the means by which measurement has been made easier. For example:

1. mass can be quickly measured by weighing machines;
2. length can be easily (though not so quickly) measured by tapes and surveying devices;
3. time is quickly and very accurately measured by clocks;
4. temperature is measured by thermometers and other more sophisticated devices; and
5. angle is measured by protractors and surveying equipment.

Value (or how much money something is worth) is not so technologically based and area can be very tedious to measure. Volume of liquids (capacity) can be easily measured by instruments (e.g. petrol pumps) although volume in other situations is difficult.

But volume and area are related to length. And for regular shapes this relation can be enshrined in formulae. Hence area and volume can be calculated from more easily measured length. Similarly, formulae for perimeter has also been developed. The first unit in this Chapter (Unit 5.1) focusses on these formulae, looking at how they can be developed in children. It looks particularly at two teaching approaches or strategies that can be used with children to enhance understanding and memorization of the formulae: discovery and relating to known formulae.

Measurement, being the application of number to the world, is the main area of applications in mathematics. Most routine classroom problems arise in money or measurement as do many of the real problems that children can be directed to. The second Unit in this Chapter (Unit 5.2), therefore, focusses briefly on applications and problem solving (problem solving is the total focus of another book in this series).
UNIT 5.1 DEVELOPING FORMULAE WITH UNDERSTANDING

FOCUS:

The focus on this unit is on the development of formulae understandings in children. This unit describes two strategies for effectively introducing formulae: discovery and relating to known formulae. It also stresses the importance of checking discoveries.

BACKGROUND:

The formulae which is the focus of primary mathematics is within length, (perimeter) area and volume. The most common formulae that have to be acquired are as below.

(1) Perimeter
   - Perimeter of a square = 4L
   - Perimeter of a rectangle = 2(L + W)
   - Circumference of a circle = 2πR

(2) Area
   - Area of a square = L²
   - Area of a rectangle = L x W
   - Area of a parallelogram = L x W
   - Area of a triangle = ½ (L x W)
   - Area of a circle = πR²

(3) Volume
   - Volume of a cube = L³
   - Volume of a rectangular prism = L x W x H
   - Volume of a prism = A x H
     (A is area of base)
   - Volume of a cylinder = πR²H

ACTIVITIES:

Materials required: Geoboards, rubber bands, pen, paper, scissors, tape, glue.

1. The first strategy for children acquiring formulae is discovery. For example
(a) Construct 6 different rectangles on your geoboard.

(b) Draw up and complete the following table for your rectangles.

<table>
<thead>
<tr>
<th>RECTANGLE</th>
<th>LENGTH</th>
<th>WIDTH</th>
<th>NUMBER OF SQUARES (AREA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Looking at your table, can you see a pattern for relating area to length and width? How would you use your relationship to find the area of a rectangle of length 30 and width 40?

What questions would you ask to ensure children saw the pattern above?

2. The formulae for circumference of a circle can be found the same way. For example -

(a) Measure radius and circumference of several circles.

(b) Record this information on a chart

<table>
<thead>
<tr>
<th>RADIUS (R)</th>
<th>CIRCUMFERENCE (C)</th>
<th>C/R (use calculator)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Can you see a pattern?

What difficulties would children have with measurement that may make their C/R computations too inaccurate. How could we overcome this?

(For example - one group measured cylinders. Ten cylinders were placed side by side as below, measured and this divided by 20 for the radius.

String was then wound around them ten times, measured and this divided by 10 for the circumference).

3. Which is the best way to discover the formula for volume of a rectangular prism?

Way A - fill various prisms with blocks and record this on a chart looking for a pattern.
Way B - construct with blocks various prisms of

(i) height one (look for a pattern - relate resulting volume to area of base),
(ii) height two (again relate to area of base), and
(iii) height 3, then height 4, and so on (attemping to relate volumes to the product of the height with the area of base).

Note: The area of a rectangle can also be discovered using way B above. It can be seen as a natural consequence of counting one row, then a second row, and a third, ... up to the height.

4. Describe a way you could discover the formula for the perimeter of a rectangle.

5. Sometimes discovery is assisted by simple activities. For example, a cardboard circle could be rolled along a line for one complete revolution, as below.

\[ \text{start} \rightarrow \text{circle} \]

It could then be shown that the length of this line is just a little more than 3 diameters or 6 radii.

Would this help activity 2 above? Would it suffice for primary knowledge of circumference of a circle?

6. The second strategy for children acquiring formulae is relating to known formulae (schematizing).

As an example, complete the MAKING CONNECTIONS activity below.

(1) Cut a rectangle out of paper. By cutting and taping, form two triangles as below:
Are these two triangles the same? How does their area relate to that of the beginning rectangle? Is the base length and height of the triangle equal to the length and width of the rectangle?

(2) Cut a triangle out of paper. Cut a copy of it. By further cutting and taping, form a rectangle from the two triangles as below:

\[ \begin{array}{c}
\text{triangle} \rightarrow \\
\text{rectangle} \rightarrow \\
\text{rectangle} \rightarrow \\
\text{rectangle} \\
\end{array} \]

How does the area of the rectangle relate to the area of the triangle? What about length and width?

(3) Cut an obtuse triangle out of paper (make it different to the triangle in (2) above). Cut a copy of it. By further cutting and taping, form a rectangle from these two triangles as below:

\[ \begin{array}{c}
\text{triangle} + \\
\text{triangle} \\
\rightarrow \\
\text{rectangle} \rightarrow \\
\text{rectangle} \\
\end{array} \]

What is the relation between the triangles and the rectangle in this instance? Is it the same as for (1) and (2) above?

(4) If the area of the rectangle on the right is \( P \times Q \), what is the area formula for the related triangle? Where are \( P \) and \( Q \) on this triangle (do a drawing)? What about the area of the associated parallelogram?

How does this activity help children to remember formulae? What insight does it give into shapes and their properties?

7. The formula for area of a circle can be got from the formula for circumference of a circle. The formulae for volumes of cones and pyramids can be get from the formulae for the volumes of cylinders and prisms. Do some research and find out how these are done.
8. The results from area formula must be related to the number of squares that cover the shape. Hence, formula must be checked.

(a) Find the area of the following triangles by

(i) counting squares, and

(ii) the formula for the area of a triangle.

(b) Repeat the above for the following parallelograms.
9. Why would children think the two shapes below have the same area? Which is more useful for showing they do not - formulae or counting squares?

10. In these activities, we have used paper and geoboards to embody area. What are the relative advantages and disadvantages of each material?

11. In 2 above, an activity to discover the formula for the circumference of a circle was described. A similar method could be used for the formula for the area of a circle. Squared paper could be used and the results for different circles recorded under the headings below:

```
CIRCLE  RADIUS (R)  R SQARED  AREA (A)  A/R  SQUARED
```

What would be the difficulties with this?

TEACHING HINTS:

1. Discovery and schematizing activities are the basis of good teaching because they enable children to structure their own knowledge in a form it is easy to recall from and reconstruct particular knowledge from.

2. Formulae can be used to practice decimal operations. For example

   (1) Addition of decimals - perimeter of a rectangle, and

   (2) Multiplication of decimals - area of a rectangle.

3. Seeing pattern and generalization is an activity that should be left to the upper primary years. Even children at this age may have to be "led by the hand" through discovery. Do not expect abstract generalization.

4. There is a tendency for children to launch into formulae and to ignore the meaning of numbers generated by the formulae. For example, it is important that children first visualise area in terms of coverage with unit squares. Only when this covering concept is established, should the counting process be organised into the L x W formula for rectangles. Once this has been achieved, the formulae for the parallelogram and the triangle can be established. Then the coverage visualisation must be used to ensure that children realise that the numbers that come from the formulae are related to the number of cm squares that cover the shape. Hence, the importance of the checking procedure, where numbers from formulae are related to the actual coverage in squares.
UNIT 5.2 APPLICATIONS AND PROBLEM SOLVING

FOCUS:

This unit looks at measurement in relation to the other topics in primary mathematics and focuses on its role in providing applications and problems for knowledge of number and shape.

BACKGROUND:

In the companion book to this on problem solving [Mathematics Education, Carseldine Campus, Problem Solving (Draft Materials), Brisbane, Brisbane C.A.E., 1986], problems are divided into a continuum routine (based on mathematics content) to creative (requiring mathematical thinking and little content base). Measurement problems are routine in that they use mathematics content but can be sufficiently complex in their setting that they require creative thought as well. As such, they seem to lie mid-way in the routine-creative continuum.

Therefore, measurement problems are applications of number and shape that require problem-solving skills for both routine and creative problems. Hence, they require:

* good mathematics content knowledge (number, shape and measurement);
* a positive attitude, self concept and attribution;
* a good plan for attacking problems;
* a wide repertoire of problem-solving strategies;
* good thinking skills; and
* good executive processes (for monitoring, planning and evaluating problem-solving progress)

Of particular importance to measurement are the following problem-solving skills.

(1) Polya's four stage plan of attack

SEE - understand the problem,
PLAN - develop a plan to go about solving the problem,
DO - solve the problem, and
CHECK - check the solution and look back to see what can be learnt from how the problem was solved.

It is crucial that a measurement application be understood before any attempt is made to solve it. Likewise, the complexity of measurement applications requires that they often be solved in steps. This necessitates that an overall plan is developed before the solution is attempted.

(2) Problem solving strategies that help with understanding and enable problems with more than one step to be tackled. Such strategies include (see Unit 4
of the Problem Solving book and Heiring, S. Problem solving ... A basic mathematics goal, Palo Alto, Cal., Dale Seymour, 1980):

'Identify given, needed and wanted information,
- Restate the problem in your own words,
- Make a drawing, diagram or graph,
- Identify a subgoal (break the problem into parts), and
- Work backwards.

The use of pen and paper to make drawings and to list the different parts that have to be completed is crucial to these problems.

(3) Visual, flexible and creative thinking and the ability to both monitor what is happening at present and to keep in mind what has to happen next (and where what is happening now fits into the total solution).

As always, it has to be kept in mind that problem solving is a two edged teaching tool. It can be used to apply knowledge. It can also be used to teach that knowledge in the first place. The same problem in one instance could be given for the processes that have to be experienced to complete it - as a vehicle to apply old knowledge and improve thinking. In another instance, that same problem may be given for its answer. It may, for example, be the approach used to get the children to discover a formula.

ACTIVITIES:

Materials required: Pen, paper, any material that you think may help with the problems, your imagination.

1. Measurement problems should be related to the everyday world of the child. For instance, children at a school beside a road and next to a river could be set the following measurement problems:

   - How many trees worth of paper does our school use in a year?
   - How many tonnes of metal pass our school on the road in a week?
   - What volume of water flows down the river in a fortnight?

   (1) How would you go about determining the amount of ink (ball point pen ink, felt pen ink, typewriter ribbon ink, etc.) that is used in your institution in a week? Don't work the problem out. Just list the things you would have to do to solve this problem.

   (2) Are there any investigations that you would have to do in order to solve the above problem? [Remember you may have to work out what the average ink use is in a student's pen!] Do these investigations add to the problem? What extra difficulties do they add to a teacher in a classroom? Are they worth it?
(3) Consider that you are to take a class of children in measurement problem solving at your institution. In your groups, develop 5 measurement problems that could be set for these children that use the environment of your institution.

2. Measurement problems can be extensions of measurement instruction. For example, try these problems.

(1) What is the largest possible square that can be constructed completely inside the octagon below? (The square may touch the boundary of the octagon.)

(2) What is the area of the square region below?

(3) What is the volume of the stairwell on the right? The two doors are 3 m high and 1 m wide. Each step is 0.3 m high and 0.3 m deep.

(4) What can the floor space of the following addition to a home be? The ceiling has to match the rest of the house and be 3.1 m high. The airconditioner is only adequate to cool and extra 200 cubic metres of space.
3. Measurement problems can be used to introduce, or to give a reasonable appearance to, formulae. For example, the volume of a prism/cylinder is a product of the base area and the height. Can you devise a measurement problem for investigation that might make this formula reasonable to a child for the shapes below. (For example, stacking coins may help for the volume of the cylinder.)

4. The MMP Measurement book alluded to in the acknowledgements of this book contained a collection of measurement experiments. They provide examples of good measurement applications and problems. Try some of the examples below.

(1) What is the relationship between the height a ball is dropped from and the height it bounces to? Does this relationship vary with the ball? [Experiment with different balls.]

(2) How could you find the mass of a suit made from 3 m by 1.5 m of material from the scraps left over?

(3) What is the effect on the volume of a cube when you double the length of it sides? What is the effect on the volume of a sphere when you double its diameter? [Experiment with cubes made of blocks and plasticine spheres. Find the volume by counting blocks or by immersion in water (or by mass?).]

(4) What measures best answer the question "how big are you?"? Do any of these measures correlate? [Plot results for a number of people on graph paper.]

(5) What is the ratio between the volume of a cylinder and the volume of a cone? [Make a cylinder and cone of the same height and diameter out of cardboard and check.]

(6) What is the volume of water wasted in 24 h by a dripping tap?

(7) What is the surface area of a tennis ball? [Do not use the formula.]

(8) What rectangle of perimeter 36 cm has the largest area? What 2D shape of perimeter 36 cm has the largest area? [Use a geoboard. Tie a loop of string of perimeter 36 cm and make the different shapes with this.]
TEACHING HINTS:

1. Measurement problems are an excellent opportunity for out of doors work, for investigations in the library, for experimentation and for open ended activity (where each child does as much as they are able).

2. Measurement problems allow different abilities to be catered for. Use them particularly when catering for talented children. Their open ended nature allows different children to perform at different levels.

3. Measurement problems do require flexibility of the teacher. Fixed times and timetables are difficult to work within. There is also a need to organise for many messy materials to be used. Safety is also a problem. Cleaning up, a time consuming activity, must also be planned for.

4. Use the ideas of the children. Children can often think up excellent investigations. Do not be afraid of leading children to a situation, say a boat on the river, and requiring them to make up their own problems for answering.

5. Do not help children having difficulties by answering the question. Rather direct them to an experiment they can perform that may throw light on the question.
CHAPTER SIX: TEACHING MEASUREMENT

The teaching of measurement requires the active investigation of measurement attributes in situations that allow the discovery of concepts and formulae and the practice of skills, techniques and standard unit notation and conversion rates. It also requires taking into account development and maturation (conservation) and ensuring that students acquire all the understandings in the five stages of measurement instruction.

- identifying the attribute
- comparing and ordering
- non standard units
- standard units
- formulae

To this end, therefore, this Chapter has been organised to cover development, the five stages, and how to use these stages to prepare teaching sequences and to diagnose difficulties. Unit 6.1 looks at conservation and developmental levels. Unit 6.2 focusses on the five stages and on ensuring that the type of activity suitable for each stage is clearly delineated. Finally, Unit 6.3 looks at sequencing in measurement (writing measurement activities which move through stages) and Unit 6.4 looks at diagnosis and remediation (using anecdotes to describe common error patterns).
UNIT 6.1 CONSERVATION AND DEVELOPMENTAL LEVELS

FOCUS:

Young children's observed performance of many measurement tasks appears to be different to adults' performance, particularly in how they seem to understand the tasks. This observed difference has lead psychologists and educators from Piaget onwards to hypothesise the existence of qualitatively different levels through which children develop. For measurement, the important problem is that of conservation, realising when the quantity of an attribute does or does not change during a measurement process. This Unit focusses on conservation and developmental levels and how they may effect the teaching of measurement.

BACKGROUND:

The Swiss psychologist Jean Piaget, in his study of how children develop their thinking and grow in their ability to learn and handle new information has placed warning flags over certain aspects of the measurement process. When performing measurement tasks, there often comes a time when something has to be done to the object being measured (e.g. the object is moved or containers are changed). The change does not effect the quantity of the attribute being measured. We say that the attribute is conserved. In many basic measurement situations, children have been observed to behave as if they do not believe the quantity has been conserved.

The development of conservation is, in this theory, a characteristic of the attainment of certain developmental levels that children must pass through as they move to adult thinking. Young children, not yet to these levels will not be able to conserve, nor, in some theories, be capable of learning how to conserve. This can lead to measurement difficulties as the following examples of non conservation show.

(1) Length

A child can be shown two rods of the same length lined up so that it is easily seen that this is so, e.g.

-----------------------------
-----------------------------

The child will agree they are the same length. But if one of the rods is moved as below

-----------------------------
-----------------------------

the child will now say that the lower rod is longer even though the rods have not changed in length. This lack of conservation of length is
particularly noticeable when a child is given the task of building a tower of blocks on a lower table to the same height as an already built tower on another table, e.g.

```
\[ \ldots \]
```

The child will attempt to build the second tower so that its top matches the top of the existing tower, not so that its length matches, e.g.

```
\[ \ldots \]
```

(2) Area

A child can be shown a picture of a paddock with houses (X) in it as below

```
\[ \ldots \]
```

The houses can be moved as below

```
\[ \ldots \]
```
The child will now say there is less grass in the paddock even though there has been no change in the size of the field or the number of houses.

(3) Volume

A child can be asked to pour water into a short wide container as on the right. The child can then be directed to pour that water into a tall thin container as on the right. The child will say that the tall container has more water regardless of the fact that he/she has just poured the water from the short to the long container.

(4) Mass

A child can make a lump of plasticine to balance a 500 g mass as below

\[ \text{\includegraphics[width=0.5\textwidth]{lump_of_plasticine.png}} \]

The child can then be directed to roll the lump of plasticine out into a long “sausage”. The child will now believe this plasticine has more mass (and also more volume) regardless of the fact that no plasticine has been added or taken from the lump.

Children are said to attain conservation when they are capable of reversing in their minds the change made on the object being measured. For example, the child mentally pours back the water from the tall thin container into the short wide container and, in doing so, comprehends that the volume is unchanged.

Modern theories of developmental levels are based on working memory capacity. It is felt that as children mature they gain the ability to hold more items of information in working memory. The reversals necessary for conservation require a capacity, so the theories go, that is not available in young children.

Developmental levels are a contentious issue. Some researchers argue that they are an artifact of the tasks given to children and the language used in the questions asked of the children. Others argue that, rather than representing fixed biological stages that children move through, they are only crude indicators of the sequence through which knowledge is acquired. Whether levels are biologically based or instructionally based is an open debate. Some educators argue that a child must be ready in terms of biological maturation before something can be taught, while others believe that with appropriate instruction any child is ready to learn anything.
It seems that much of the evidence for levels has assumed a global view of concepts, that acquisition of, say, number is a single, all or nothing, ability. Modern research is showing that many of these so-called global concepts are made up of sequences of subconcepts that children can be shown to slowly acquire as a result of instruction. It also seems that much of the evidence for levels has underestimated the power of knowledge of context in solving tasks. Modern research is focussing on domain-specific knowledge as having a strong effect on thinking. Children given tasks in a familiar context show remarkable abilities to confound the levels they have been classified into. Still the Piagetian notion of levels has inspired a plethora of powerful modern theories of cognitive development by such researchers as Pascual-Leone, Case, Fischer, Collis and Biggs and Halford.

Instructionally, there has been a turning away from levels as a basis for curriculum. The three main reasons for this are the difficulties in evaluating and determining children’s levels, the existence of many situations where children operate at different levels in different topics and the ability of children to use task specific thinking to solve problems amenable, it seemed, only to thinking from a higher developmental level.

With the literature inconclusive, the best approach for a teacher is to be sensitive to the children and to realise that

(a) there is prerequisite understanding before a child is ready to learn some things, and

(b) there may be good reasons for so-called wrong answers and for slowness in mastering some activities.

Reading:


ACTIVITIES:

Materials required: Pen, paper.

1. Read the background to this Unit, particularly the four examples of non-conservation. Answer the following questions.

   (1) For example (a), what difficulties would children who do not conserve length have with a ruler?

   (2) For example (c), what sense would children who do not conserve volume make of a measuring cup?

   (3) For example (c), a child chose the taller container when asked the question "which holds more?". What could be the problem with the word 'more' here? How could this question be rephrased to ensure the child's answer reflected the intended idea of 'more'?

   (4) What difficulties could children have who fail to conserve as in examples (b) and (d)?

   (5) Do you think that a child would perform differently in these examples under normal conditions than under the test conditions that most of these tasks are given to children?

2. There are many situations where adults have difficulty with conservation. For example:

   * different volume bottles having different shapes;
   * taller cans having smaller diameters;
   * 1 L softdrink bottles having the same volume as a MAB block; and
   * a special of 100 sheets for 50c when 500 sheets is normally $2.

(1) Determine which of the following conserves the stated attribute.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>ATTRIBUTE</th>
<th>CHANGE</th>
<th>CONSERVED?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity of liquid in a glass</td>
<td>Volume</td>
<td>Four into a measuring cup</td>
<td>Yes</td>
</tr>
<tr>
<td>Quantity of liquid in a glass</td>
<td>Volume</td>
<td>Freeze the water in the cup</td>
<td>No</td>
</tr>
<tr>
<td>Two sticks</td>
<td>Relative length</td>
<td>Change positions</td>
<td></td>
</tr>
<tr>
<td>Quantity of liquid</td>
<td>Temperature</td>
<td>Turn on an intense light</td>
<td></td>
</tr>
<tr>
<td>Rectangle 10 cm by 50 cm</td>
<td>Area</td>
<td>Change to a parallelogram of side length 10 by 50</td>
<td></td>
</tr>
</tbody>
</table>
(2) How important was experience in answering these questions? Where did you learn the answers to them? At school? Would a young child have been able to answer them? Why?

(3) What effect would it have on one's ability to measure the attribute in question if you did not know whether the change was conserved or not? In particular, discuss any questions you got wrong!

(4) What does your attempts at these questions tell you about children who may fail to correctly answer conservation tasks?

TEACHING HINTS:

1. Take opportunities to trial some of the conservation activities with young children. Try hard not to lead the children.

2. If you are in doubt about readiness, then run measurement activities which are exploratory in nature, i.e. activities that are open-ended and that do not have a single end point or correct answer which has to be found. The children simply undertake measurement experiences and describe what they found and what they think it means.

3. Do not mark children right/wrong (or demand the "correct answer") when undertaking measurement activities where conservation may be a problem. Simply accept whatever children find important and think is correct.
UNIT 6.2 THE FIVE MEASUREMENT STAGES

FOCUS:

The OVERVIEW of this book recommended five stages in the development of measurement: identifying the attribute; comparison and order; non-standard units; standard units; and formulae and applications. The focus of this unit is to clearly delineate the type of classroom activities that are suitable for each stage.

BACKGROUND:

There is difficulty in delineating between the first two stages: identifying an attribute and comparison and order. Otherwise the various stages are straightforward. Identifying an attribute and comparison and order activities have no numbers. Non-standard unit activities have numbers but the units are not the formal metric ones. Standard unit activities deal with metrics and formal units and measuring instruments, while formulae activity is just that – activity which uses formulae.

ACTIVITIES:

Materials required: People, Cuisenaire rods, string, scissors, trundle wheels, measuring tapes, geoboards, rotagrams, angle pieces (sectors), paper, paper circles, rulers, glue and tape.

1. Complete the following length activities which have been classified by stage with your lecturer.

   **Stage 1** (identifying the attribute)
   
   Use body movements to do the following
   
   - acorn growing to a tree, uncurling and growing higher
   - hands moving wider apart, telling a story of a growing fish
   - same and different in heights, a tall person bending to be the same height as a short person and vice versa

   **Stage 2** (comparing and ordering)

   Use Cuisenaire rods to do the following
   
   - taking two rods and discussing longer and shorter
   - taking a collection of rods and ordering (discussing shortest, longest and in between)
Use string to do the following

- comparing length of black-board to length of wooden plank and then
to distance around small table top

**Stage 3** (non-standard units)

Use body measurements (i.e. digit, palm or hand, hand-span, cubit, fathom) to do the following

- find the length of table in palms and hand-spans (look at the
  inverse relation process)
- discuss how stadia, mille-passus, furrow-long, chain and acre were
  historically developed
- discuss why the first yard was made a yard and 2 thumbwidths

**Stage 4** (standard units)

Use the trundle wheel to measure the length of room.
Use a measuring tape to find your own height in cm.

**Stage 5** (formulae and applications)

Use geoboards to discover the formula for the perimeter of a rectangle

2. Complete the following **angle activities** which have been classified by stage in your groups.

   **(1) Activity One** *(Identifying the attribute)*

   Use your bodies to act out

   - direction, turn, turn to right, turn to left, clockwise turn and
     anti-clockwise turn;
   - straight and curved line; and
   - angle (amount of turn).

   Extra ideas: Take turns to give instructions to each other on how to get
   from one point to another (e.g. "turn left, forward two paces,
   etc.").

   **(2) Activity two** *(Comparing and ordering)*

   Complete Unit 2.2 Activity 4 [pages 27 and 33].
   Order the angles in the left hand column of the worksheet on page 33 from
   smallest to largest.

   **(3) Activity three** *(Non-standard units)*

   Obtain twelve sets of angle pieces (sectors), each set containing 12 sectors
of approximately ten degrees, B sectors of approximately twenty degrees and 5 sectors of approximately thirty degrees.

Use the sectors to measure the angles below as follows (estimate first in all cases).

- use a mixture of sectors to measure angle A - what problems arise when units are not the same size?; 
- measure angle B with the largest sectors and angle C with the smallest sectors - what difficulties arise in making comparisons when different size units are used for different measures?; and 
- measure angle D three times with different units - what is the relation between size of unit and number of units? what are the appropriate units for this angle? why do we need a standard unit for measuring angles?

(4) Activity four (Standard units)

Use the protractor to cut a sector of angle one degree out of the paper circle. (If this is too difficult, try an angle of two degrees.)

Place the rotagram on top of the protractor. Open the rotagram to make angles of 75, 23, 154, 265 and 298 degrees.

(5) Activity five (Formulae)

Use the ruler to mark out a scalene triangle. Use the scissors to cut out the triangle and to cut the three corners off. Glue these corners together to show that the interior angle sum of a triangle is 180 degrees.

3. Develop your own set of activities for each stage for the attributes of

- Area
- Time
TEACHING HINTS:

1. The various stages would, in practice, be taught over many years, but it is not recommended that any stage be missed. This means that it is possible, in the higher grades, to have to complete activities from earlier stages before, e.g., standard units or formulae are introduced.

2. Observe children in measurement activities for evidence of lack of experience in any stage. The five stages are particularly useful for diagnosis (as will be seen in Unit 6.4).
UNIT 6.3 SEQUENCING MEASUREMENT ACTIVITIES

FOCUS:

This unit focusses on the use of the five stages (identifying the attribute, comparing and ordering, non-standard units, standard units, and formulae) to develop teaching sequences of measurement activities for the primary classroom.

BACKGROUND:

Planning of instruction in schools is both short term and long term. Individual lessons and sequences of lessons across weeks have to be prepared. Plans have also to be developed for the seven years the children will be in the primary school.

It is important that this planning is done within a framework of how instruction will fit into children's long term experience with measurement. It is also important that each lesson is part of a short term sequence to accomplish a certain objective. The material in the Overview of this book has focussed on the important aspects of planning instruction. It should be read again at this point.

It is crucial to note that for whatever planning occurs, implementation of this planning in the classroom must be tempered by the reactions of the children. If children have difficulty, then lessons/activities must be repeated with variation until these difficulties disappear. If children find the work easy/boring, then new work can be moved to at a faster rate that planned for. Planning must prepare the teacher for flexibility in instruction.

ACTIVITIES:

Materials required: Pen, paper, materials as listed for the length cards, scissors.

1. Read STEPS IN DEVELOPING LENGTH IN THE PRIMARY SCHOOL below.

   (1) Copy out each step onto a strip of paper. In your groups sequence the steps in the order across the years of primary school they should be given to children. Give reasons for your sequence.

   (2) Is the sequence exhaustive? Does it follow the five stages recommended in the Overview to this book? Brainstorm any extra steps that should be added to the list!

   (3) Do the steps follow a sequence of greater sophistication? Of progressively greater dexterity?
(4) Briefly describe a classroom activity that might be undertaken with children for each step.

**STEPS IN DEVELOPING LENGTH IN THE PRIMARY SCHOOL**

A. Introducing the metre.

B. Discussing lengths measured in paces and determining the need for a standard.

C. Investigating relationships between perimeter and area.

D. Comparing two lengths of the same material.

E. Experiences with small units (e.g. mm).

F. Working with cuisenaire rods to determine that small units give large numbers and vice versa.

G. Connecting diameter and the circumference of a circle.

H. Distances on a globe of the earth/distances in space.

I. Measuring the length around the table and the tennis court.

J. Discovering the formula for the perimeter of a rectangle.

K. Recording measurements in decimal notation.

L. Developing accuracy in length measurement (to nearest mm).

M. Determining what length means.

N. Classroom experiences with a common length unit determined by the class.
G. Finding the perimeter of a collection of objects.

P. Determining which of a collection of objects would be the most appropriate unit for a length measurement task.

Q. Measuring the school with handspans, cubits, paces, etc..

R. Estimating distances in m and cm.

S. Ordering a collection of objects by length.

T. Experiences with larger units (km).

U. Comparing the circumference of a can with the length of a feather.

V. Understanding all the different words for length (e.g. wide, short, etc.).

W. Metric units, conversion rates and decimal numeration.

X. Introducing the centimetre.

2. Prepare a sequence of steps of a similar form for the measurement topic Area!

3. Gather the required material and complete the length workcards in the SAMPLE WORKCARDS ON LENGTH at the end of this Unit. Answer the following questions concerning these cards.

(1) List the objectives of each card! Do the cards follow the sequence of stages recommended in the Overview to this book? Are there any missing steps?

(2) Why not start at Card 4?

(3) Are there advantages in writing tasks on separate cards as these are? Here are two:


* separate cards are more flexible - extra cards can be inserted for struggling students and cards can be omitted for talented students; and

* separate cards give the teacher more freedom - they can be given to a small group of children, to one child, or to the whole class; the teacher does not have to give continuous directions and therefore is free to help those in need.

Can you think of more?

(4) Was there enough activity in each card? Would you do any cards differently and why? Would children have difficulty with any cards?

(5) Can you find a card that exemplifies conservation of length?

4. Choose a measurement topic other than length. Write a series of cards similar to those in 3 above which would develop the central activities of that topic through the seven years of primary.

5. Take one step in the sequence in 4 above and write six activities, two to develop the idea in the step, two to consolidate this idea and two to apply it. Use the ACTIVITY TYPES from the Overview of this book.

TEACHING HINTS:

1. Be a reactive teacher. It is necessary to plan. Find the knowledge level your children have at present in the topic. Determine where you would like them to be. Prepare a plan that takes the children directly to your end-point as below.

```
START
/    /
/    /
/    /
/    /
/    /
/    /
END-POINT
```

But implement your plan (i.e. teach) so that you react to the children's reactions to your teaching by turning away from the end-point wherever necessary to help the children, using the end-point only as a beacon to turn
SAMPLE WORKCARDS ON LENGTH
----------------------------------------

These workcards come from the MMP Measurement book referred to in the acknowledgements of this book. Copyright ended on this material in December 1981.

The cards show a sequence of activities from comparing and ordering to standard units.

Children ready for the later cards (the introduction of the metre) should be allowed to do the early cards even though they will be easy. The concern of the teacher should be on the methods they use in completing the cards. If children have difficulty with the early cards, they should be provided with more activities at this level before proceeding.

Above each card, there is a list of materials for that card. The teacher would supply these materials. They are listed for the convenience of the reader. Children would not be given the list of materials.

3 pieces of ribbon, red, blue, yellow, in that order of size.

**Card 1**

Is the red ribbon longer than the blue ribbon?
Is the blue ribbon longer than the yellow ribbon?
Is the red ribbon longer than the yellow ribbon?
Which ribbon is shorter than the blue ribbon?
How many ribbons are shorter than the yellow ribbon?
Which ribbons are they?
3 pieces of ribbon, blue, red, green, in that order in size.

Card 2

Is the blue ribbon longer than the red ribbon?
Is the red ribbon longer than the green ribbon?
Is the blue ribbon longer than the green ribbon?
Which ribbon is the longest?
Which ribbon is the shortest?

A feather which is shorter but fatter than a pencil, a piece of string longer than the pencil, a can which has a perimeter longer than the string.

Card 3

Is the feather longer than or shorter than the pencil?
Is the piece of string longer than or shorter than the pencil?
Is the distance around the can longer than the length of the feather?
A can, 3 pieces of string, one shorter than the perimeter of the can, one longer and one the same size. A line drawn on a piece of card so that it is equal in length to a multiple of "can" perimeters. A piece of blue string which is twice the length of the line.

Card 4

Which piece of string is equal in length to the distance around the can?
How many "can" distances are there in the length of the line?
How many "can" distances are there in the length of the blue string?
The distance around the can is called the PERIMETER.

Table, chair, and book.

Card 5  GUESS FIRST—THEN MEASURE

How many handspans in width is the table?
How many handspans in width is the chair?
How many handspans in width is the book?
How many thumbwidths is the book?
How many thumbwidths is the chair?
How many thumbwidths in a handspan?
How many thumbwidths in the length of the table?
A pencil, a Cuisenaire rod or similar wood or plastic "brick" (about 5 cm. long), collection of objects with straight edges; e.g., books of different sizes, desk, pieces of card, ribbon, drinking straw.

**Card 6**

Measure the length of each of the objects, using the pencil and also the brick. (Make a guess first)

Record your results like this.

<table>
<thead>
<tr>
<th>Object</th>
<th>Guess</th>
<th>Length</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book</td>
<td>3 pencils</td>
<td>2 pencils</td>
<td>1 pencil</td>
</tr>
<tr>
<td></td>
<td>5 bricks</td>
<td>3 bricks</td>
<td>2 bricks.</td>
</tr>
</tbody>
</table>

Why do you have different results when you measure?

**Card 7**

How many paces in length is this room?
How many of your friend's paces in length is this room?
Why do you have different answers?
How many paces in length is the corridor?
Would you expect your friend's answer to be more or less than yours?
A stick, unmarked and about 80 cm. long.

**Card 8** Take the stick. In each question, guess what your answer should be before you measure.

How many stick lengths are there in the length of the room?
How many stick lengths are there in the length of the corridor?
How much longer is the corridor than the room?
Measure four things with the stick. Was your guess correct?

Meter rule, not marked in centimeters.

**Card 9** Take the meter rule

(Make a guess first.)

How many meters long is the classroom?
How many meters long is the corridor?
What is the difference in length of the room and the corridor?
What do you buy in meters?
Measure 5 things in meters. Make a guess first. What was your error?
Meter trundle wheel.

Card 10  Take the trundle wheel

What is the distance all around the trundle wheel?
Place the "click" on the wheel so that it is just about to click. Now turn the pointer until it reaches the wooden block again. How far has the pointer travelled?
How many "clicks" do you hear when you wheel along the line on the floor? (Start with the pointer and wooden block together.) How long is the line?
Measure the room with the wheel.
UNIT 6.4 DIAGNOSIS AND REMEDIATION

FOCUS:

In the previous Unit, we stressed the need to base instruction on the behaviour of the children. In this Unit, we will focus on how to evaluate children's performance, to diagnose the underlying cause of any difficulty and to prepare appropriate remedial activities.

BACKGROUND:

When children have difficulty with a mathematics problem or procedure, teachers should not limit their assistance to the actual point of difficulty. Giving a rule to get the answer and directing the children to memorize and practice this rule will not help the children to understand the concept or process behind the procedure and can have a deleterious effect on attitude.

Teachers should treat a child having difficulty as a sign that understanding may be lacking. The child must be probed to discover what he/she knows and, hence, what requires reteaching.

The steps are as follows:

1. observe the children and evaluate their work for any difficulties;
2. when a difficulty is found, analyse it for any pattern that may indicate the cause of the error / discuss the difficulty with the child (listen to the child);
3. list the possible causes of the error and check these with the child, endeavouring to find what the child knows more than what is not known; and
4. reteach the missing knowledge starting from what the child knows using the same procedures as if teaching for the first time.

The basis for determining cause has to be the particular knowledge of concepts, processes and measurement techniques in each topic area, but also the existence of usable knowledge in each of the five stages of the measurement process: attribute, ordering, non-standard units, standard units and formulae.

We should also note that evaluating performance in measurement is in some ways more difficult than in number. Correct and incorrect answers may not be the best indicator of satisfactory performance and difficulties. In measurement tasks it is possible for more than one solution to be appropriate. It is also possible to arrive at a satisfactory conclusion to a task and yet exhibit unsatisfactory techniques and understandings. Likewise an incorrect answer may be the result of one careless measure in an otherwise flawless performance.
ACTIVITIES:

Materials required: Pen, paper.

1. Read the MEASUREMENT ANECDOTES below. For each anecdote, briefly answer the questions accompanying it. (Note: The anecdotes are based on those from the MMP Measurement book referred to in the acknowledgements of this book.)

MEASUREMENT ANECDOTES

A. Jane said "I can't tell whether the table is longer than the cupboard; I don't have a ruler!".

(1) What is the misconception that Jane has? What is the cause of this misconception? (i.e. what experiences are lacking from her background?)

(2) What would you do if Jane said this to you when you were teaching?

(3) What sequence of experiences would ensure that this didn't happen to children in your class?

B. Fred could not work out what to do. Frank had told him that the place they had visited was 35 paces long, but what length of string was equal to that?

(1) How could Fred work out how much string to cut? (Can you think of more than one way?)

(2) What experiences seem to be lacking in Fred's background? What would you do if you were teaching Fred?

(3) How could you use this to teach the need for a standard?
C. Anne was dismayed. When comparing the two pieces of ribbon, she found that they were not the same length, yet they were both marked 25 cm.

(1) Is it possible for both Anne and the markings to be correct?

(2) What experiences does Anne lack that would account for her dismay here? (What is the cause of the dismay?)

(3) What might you do with Anne to give her insight into the situation?

(4) How would you use this situation to teach tolerance for error?

D. Henry did not know what to do! His teacher had asked him to measure the school's fence and had only given him a 15 cm ruler.

(1) How could you help Henry?

(2) What lack of experiences could be the cause of Henry's lack of creativeness and flexibility here?

(3) What would you do to promote measurement flexibility in your class so that they did not have Henry's limitations?
E. Janine was measuring the table as on the right.

(1) What is wrong?

(2) What can be done to help Janine?

(3) What other situations like this can occur in measurement where difficulties other than conceptual ones cause problems? List some cases! [Remember discipline, psych-motor skills, etc.]

F. Joe could find the area of the left figure because he could use the formula \[ A = l \times w \], but he could not find the area of the right figure. As he said: "I don't have a formula for that shape!".

(1) What is missing from Joe's concept of area?

(2) What experiences would remediate Joe?
G. Sue was puzzled. Yesterday she had poured water from the jar to the measuring cup without changing its volume. She could not understand why today she could not find the area of her geostrip parallelogram by simply straightening it up into a rectangle and using the formula \( A = L \times B \).

1. Is there any justification for Sue's puzzlement?

2. What would you say to Sue? What would you do with her to show her that she could not do as she wanted?

\[ \text{\includegraphics[width=0.5\textwidth]{parallelogram}} \]

H. John looked at the illustration on the right and said that the container on the left had the greater mass.

\[ \text{\includegraphics[width=0.2\textwidth]{mass}} \]

1. Is John's answer as you expected?

2. Is John's answer necessarily correct?

3. How could you make John aware of the problem with the question as it is posed? How could the question be posed to avoid ambiguity?

4. If, even when all ambiguity is removed, John still believes that larger objects are heavier, what experiences would you give him to overcome this misconception?
1. Tom sorted the coin, book, chair, scissors, pencil and table into two groups: large objects — table, chair and book; small objects — scissors, pencil and coin. Mary sorted the same objects into: large — table and chair; small — scissors, book, pencil and coin.

(1) Which of these two results is correct?

(2) Why does this situation emerge?

(3) What does this say about correct answers in some measurement situations?

(4) What important measurement results can emerge from this beginning?

3. Bill was not interested in the sundial project. "Why did they make sundials?" he said, "Why didn't they just look at their watches?"

(1) What is lacking in Bill's measurement experiences?

(2) Is an historical perspective important? Why?

(3) How would you weave history into measurement teaching?
TEACHING HINTS:

1. Base your evaluation of children's measurement performance on observations of them in measurement tasks. In this way, you will be able to determine any difficulties in measurement techniques, use of materials, problem solving skills, group co-operation and number and space skills as well as the concepts and processes of measurement.

2. Look for difficulties as you teach. The best teaching opportunities arise from the statements and questions of children, particularly those that seem to show that the children is on a different track to you. Do not categorise the statements as wrong and move on, but probe the children to see the reasons for the statements. Use it as an opportunity to draw out the concepts or procedures that are the basis of the statements. Do not be afraid to abandon the planned lesson to follow the questions of your children.

3. As we have said in 2 above, the most effective teaching action is what you do in immediate response to a child's question or statement. To make this response appropriately, you need a good model in your mind of what complete measurement knowledge is and a belief that all children act from what they think is correct. The five stages, plus the general objectives, in the Overview at the beginning of this book provide a recommended model of complete measurement knowledge for the primary school.
CHAPTER SEVEN: MEASUREMENT ACTIVITIES

This Chapter describes activities that can be used in primary classrooms to teach the measurement topics of length, perimeter, area, volume, mass, time, angle, money/value and temperature. The activities in each topic have been organised under the 5 stages of measurement. As such, they supplement the earlier writing in this book.

The Chapter is divided into four Units. Unit 7.1 looks at length and area activities, Unit 7.2 looks at volume and mass activities, and Unit 7.3 looks at the remaining attributes: time, money, temperature and angle. A separate unit, Unit 7.4, has been provided at the end for activities that relate attributes and that can be the basis of workstations.

It is the contention of this book that effective teaching of measurement requires knowledge of the stages through which measurement develops and a rich variety of motivating and practical activities. This Chapter exists to increase the richness of activity available through this book and to provide a view of the stages within separate attributes.

DELINEATING ACTIVITIES FOR THE FIVE STAGES

Stage 1 - Identify the attribute:

It is difficult to differentiate 'identify the attribute' activities from 'comparing and ordering' activities. In practice, this need not be done and both types of activities can be taught together. It should also be noted that activities to identify particular measurement attributes should follow rich experiences with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, etc.

There are two general ways to introduce any attribute. They should work for every measurement topic. They are

1. providing examples where the only thing that is the same is the attribute, and

2. providing examples where the only thing that varies is the attribute.

Stage 2 - Comparing and ordering:

Comparing means two instances. Ordering involves 3 or more instances. The process of ordering is based on comparisons. In this handout, ordering activities are interrelated with comparison activities.

The classroom activities under this section can be divided into three types: direct comparison of two instances of an attribute; indirect comparison of two instances of an attribute; and ordering, directly or indirectly, more than two instances of an attribute. The activities involve no units and no
numbers. The total amount of the attribute present is compared or ordered in one action.

Stage 3 - Non-standard units:

Non-standard units have two central uses: introducing the notion of the unit; and introducing the eight measurement processes (see page 48 of this book). They also are useful in introducing children to the appropriate techniques for using units and measuring instruments. The information in this handout will focus on materials that can be used for non-standard units. It will not focus on the teaching needed to draw out the uses to the same extent.

Stage 4 - Standard units:

Similar to the non-standard units, this section will focus on instruments and techniques. It should be remembered that these units should only be introduced after the need for a standard has been determined and it is recommended that they be preceded by the use of a class chosen common unit (if appropriate). It is also recommended (page 78 of this book) that the first experiences with the unit involve identifying the unit through experiencing it or constructing it. Then the unit can be used, firstly on everyday things to facilitate internalization and, secondly on everything to develop estimation skills.

Stage 5 - Formulae and applications:

The focus here will be on activities and ideas to enhance discovery and problem solving. Situations involving two or more attributes are the most fruitful.

SPECIAL NOTE: Both time and money/value differ from other attributes in that basic standard unit skills have to be developed early. For time, time-telling skill for analogue timers (i.e. the clock face), and digital timers, should be taught early and may well precede work on the concept of time at the comparing and ordering and non-standard units stages. For money/value, money-handling skill (recognition of coins and notes and computation with money) may well precede work on the concept of value, on bartering (the comparing and ordering stage) and on non-standard units.

Therefore, for the time and money/value sections of this Chapter, time-telling skill and money-handling skill will be the first section covered.
UNIT 7.1 LENGTH AND AREA ACTIVITIES

FOCUS:

This unit focusses on classroom activities for length, perimeter and area.

BACKGROUND:

Length and perimeter are the measure of one dimension and area is the measure of two dimensions. As such, length and perimeter are associated with strips of various types (e.g. rulers) and area is associated with covering and planes (e.g. graph paper). Of course, there are other ways than the obvious to measure each, e.g. trundle wheels to measure length, and it is hoped that a rich and varied mixture is given below.

ACTIVITIES:

Materials required: Pen, paper, a library or collection of resources, the materials that are mentioned in the activities.

1. Read the following activities for length and perimeter. In the space that has been left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

LENGTH AND PERIMETER ACTIVITIES

CLASSROOM ACTIVITIES EXTRA CLASSROOM ACTIVITIES

Stage 1 - Identify the attribute:

Body actions, movements and dances that represent trees and plants growing higher or fish growing longer etc. Discussing heights, cutting strips of ribbon to children's heights and sticking on walls, marking heights on walls, drawing around a child lying down, drawing around a child's foot.

Cutting ribbon, string, paper strips to the length of various things (e.g. width of door, height of blackboard). Constructing "lines" of blocks or other material (representing walls or roads), constructing other "lines" equal in length to the first one. Rebuilding the "lines" with the same material but in a different order.

Playing "Mr. Here" - hiding a doll and then giving directions to find it - "it is near, it is far, too high, too low, ...", etc.
"Getting close"
An interesting idea for a length activity from Burns, H. The I hate mathematics book.

Find a group of birds (or animals) - say pigeons. Pick a pigeon. Walk slowly towards it. Try not to make sudden movements or noises. Try to look inconspicuous. When it flies away drop something. Keep your eye on the spot where the pigeon was. Mark the length between where pigeon was and the dropped object on a piece of string. Do this 10 times. Is there a pattern? What is the average fright distance for pigeons?

Try the same experiment with people. Talk about something as you approach another person. When do they begin to look uncomfortable and back off? Mark this distance. Compare the pigeons' fright distance with that of people. Try the same experiment to find the distance at which, speaking normally, you can be heard and understood by another person. Walk towards this person talking quietly until they motion you to stop. The person could have his/her back turned.

CLASSROOM ACTIVITIES

Perimeter: Cutting strips of material to fit around head and waist, running around the oval. Walking around shapes on the ground. Tying string around objects (how much do we need?), putting tape around objects. Constructing "fences" or "walls" to enclose objects.

Stage 2 - Comparing and ordering

Directly comparing lengths where both objects can be moved for the comparison: e.g., heights of children, ribbons, strips of paper, string, pencils, sticks, etc.. Also should compare thicknesses and widths of objects. Introduce language such as shallow and deep, etc.. Comparing lengths where only one of the objects can be moved: e.g. stick with side of a table, string with height of shelves, child with height of a cupboard, roll of toilet paper with the distance around the oval, span of a child with width of doorway, etc..
Comparing lengths through the use of an intermediary: e.g. string, paper strips, body lengths - width of table and width of door, length of blackboard and height of cupboard, distance around a can and length of pencil, etc.
Comparing different things: e.g. width and thickness, nearness of chair and height of ceiling, perimeter and length, etc. Ordering objects from shortest to longest/tallest.
Categorising objects by near and far, nearby and away, close and distant, etc.

Skeletons

Each member of the group (three pairs to a group) chooses a different coloured ribbon. Working in pairs and using the ribbon, cut off the lengths of ribbon to correspond to perimeter of head (H), perimeter of face (F), perimeter of left foot (L), perimeter of neck (H), waist (W), total height (T) and arm span (A). Compare the lengths of ribbon. Can any interesting relationships be found between any of these measures?

Make comparisons between H & F, H & W, T & A, H & L, F & L. Record which is larger. Are you a square (T=A), a tall rectangle (T>A) or a wide rectangle (T<A)?

Use one set of ribbons to prepare a graph showing the variations of H, F, W, T, A and L within your group.

Use the second set of ribbons to make 6 skeletons for your group: 2 full size, 2 half size and 2 quarter size. What other measures do you need to do this? What happens to the cross section area and volume of your head when you halve and quarter the perimeters? What age children would correspond to your half size and quarter size children? Check this with actual children of these ages. What do you notice?

CLASSROOM ACTIVITIES

Perimeter: Perimeter to length: rolling bicycle wheel one revolution along a wooden plank, wrapping a paper strip around a cylinder/can, opening out a wire rectangle to compare it with a stick, etc.

Perimeter to perimeter: using string or paper as an intermediary, opening out wire or geostrip shapes and comparing the total lengths of their sides, rolling two cans one revolution each to see which can rolls further, etc.

EXTRA CLASSROOM ACTIVITIES
Drying glasses (an idea from the I hate mathematics book)

When you dry a glass with a towel, you dry up the side and around the top rim. Which is longer, the distance up the side (the height) or the distance around the top (the circumference of the circle)? Use your towel. Mark off the height of the glass on the towel. See if this distance will wrap around the top?

Get the rest of the drying up so that a glass can’t be found for which the height is longer than the circumference. You’re sure to win!

CLASSROOM ACTIVITIES

Stage 3 - Non-standard units

Using various body parts to measure length

finger → digit ← palm ← hand → cubit ← foot → pace → fathom

Using various objects to measure length, e.g. pencils, pencil cases, sheets of paper, blackboard dusters, smarties, cuisenaire rods, blocks, straws, cardboard strips, pegs, paper clips, lengths of dowel, length of string, etc.. Good to have things that are multiples of lengths of other things. Cut out a copy of your foot. Use this as a non-standard measure.

Using anything that rolls and counting the revolutions turned when the object is "wheeled" along the length, e.g bicycle wheel, cotton reel, can, dish, etc..

Parameter: All the above but going "around the outside" of a shape. The rolling method is particularly good when the shape is irregular. So is putting string around the shape and then measuring the string.

Stage 4 - Standard units

Introducing the kilometre, metre, centimetre and millimetre, e.g. constructing the metre out of graph paper strips and the centimetre out of straws, cutting paper strips to the length of 1 or 2 millimetres, walking a kilometre. Measuring parts of their bodies, measuring common objects,
e.g. cars, door heights, blackboards, books, sheets of paper, ports, distance from school to home, distance around the oval, etc. Finding things shorter than, and longer than, 1 m.

Measuring in a variety of situations, e.g. diameters (internal and external), thickness of gaps, depth, etc. Using a variety of measuring instruments, e.g. rulers, tapes,

Using rolling measurers, e.g. metre trundle wheels, or other wheels calibrated to standard units (can get a small trundle wheel for cm).

**Perimeter:** As above, but going around the outside of a shape. Again it is often useful to use an intermediary (like string).

**Stage 5 - Formulae and applications**

Most length activity in this section is in relation to other attributes and appears in Unit 7.4. An important area of focus is special lengths, e.g. diameter, radius, base, height, etc., and especially perpendicular distance (shortest distance).

**Perimeter:** Discovering the formulae for perimeter of a square and rectangle. Draw shapes on graph paper and record lengths, widths, perimeters on a table for discussion. Cut out rectangle shape from paper and fold along diagonal as below to show that the total perimeter is two lengths and two widths.

\[ \text{Perimeter of a rectangle} = 2L + 2W \]

Discovering the formula for circumference of a circle using cylinders, string and calculators, or by rolling the circle out along a page and then seeing how many times the circle by diameter fits into this line (see pages 107-108 of this book).
2. Read the following activities for area. In the space that has been left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

**AREA ACTIVITIES**

<table>
<thead>
<tr>
<th>CLASSROOM ACTIVITIES</th>
<th>EXTRA CLASSROOM ACTIVITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stage 1 - Identifying the attribute</strong></td>
<td></td>
</tr>
<tr>
<td>Dressing dolls, wrapping packages, wrapping strangely shaped (or hard to wrap) parcels, painting and colouring in.</td>
<td></td>
</tr>
<tr>
<td>Covering desktops with paper, covering play areas with paper, cutting and pasting one shape to cover another (i.e. a square piece of paper to cover a long thin rectangle of same area).</td>
<td></td>
</tr>
<tr>
<td>Using stamps or paint on hands to cover a space with colour. Using wet hands to cover part of the blackboard.</td>
<td></td>
</tr>
<tr>
<td><strong>Stage 2 - Comparing and ordering</strong></td>
<td></td>
</tr>
<tr>
<td>Directly comparing areas by placing on top of each other, e.g. table on top of table, book on top of chair, pencil case on top of sheet of paper, etc. Drawing around body (or part of body) and comparing this with another child's body (or part).</td>
<td></td>
</tr>
<tr>
<td>Indirectly comparing area by covering one instance with paper and then transferring this paper to another instance. This is very good for instances of differing shape where the paper has to be cut and rejoined to fit on the other instance. Using other material to do the same thing, e.g cloth, plastic, etc.</td>
<td></td>
</tr>
<tr>
<td>Using dissections, e.g. tangrams (and other shape puzzles), to cover space, and then breaking the dissection apart and rejoining differently to cover other spaces.</td>
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<tr>
<td><strong>Stage 3 - Non-standard units</strong></td>
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<tr>
<td>Using a variety of materials as units, e.g. dusters, blocks, cubes, tiles, sheets of paper, hands, etc. Particularly interesting are chalk-filled dusters on paper, stamps on paper,</td>
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wet hands on blackboard, etc. Use geoboards, both square and isometric (triangles).

Using tessellations as non-standard units, e.g. triangles, quadrilaterals, hexagons, escher type drawings, etc. Make plastic overlays of these tessellations (e.g. O/H plastic) so can place easily over shapes. Otherwise cut out shapes and place on top of tessellating grids. Note: do not be afraid to use non-tessellating shapes to cover and count.

Using tangrams to cover shapes and assigning unit value to one of the tangram pieces (see page 29 of this book).

Stage 4 - Standard units

Construct a square cm and a square m. Attempt to experience a hectare and a square km. by walking around such a space.

Use cm graph paper (preferably in the form of plastic overlays) to measure area of hand, A4 paper, bag, etc. (anything that is an everyday item for the child). Use centicubes or MA8 units and pack into spaces. Use geoboards (1 cm squares) and rubber bands.

Learn the two techniques on pages 94-95 of this book ('breaking into parts' and 'enclosing with a rectangle'), plus the technique of finding the area of a triangle by halving the appropriate rectangle. Use these techniques to calculate the areas of polygons (any type). Calculate the area inside curved shapes by counting the squares that are all or more than half way inside the shape (or by counting squares all in and squares partly in and averaging).

Stage 5 - Formulae and applications

Discover the formulae for the area of a rectangle (or a square) by constructing a variety of rectangles on graph paper (or geoboards) and recording length, width and area (by counting squares) - see pages 106-107 of this book. The area formulae for triangles and parallelograms can also be discovered in a similar manner to this.
Relate the area of a parallelogram to the rectangle by cutting and moving one end of the parallelogram (see page 111 of this book). Relate the area of two triangles to a rectangle using the techniques of page 111 of this book. In this way the area formulae for triangles and parallelograms can be related to the area formula for rectangle.

By cutting a circle into sectors and rearranging the sectors to form a rectangle (see below) of dimensions half the circumference by the radius, the formulae of the area of a circle can be discovered.

Calculate the areas of standard figures by using the formulae. Measure examples of these figures in the environment (e.g. tennis courts, rooms, etc.) and calculate their area. Relate this to multiplication of decimal fractions. Find the area of composite figures by breaking them into parts and calculating the area of each part (see below). Find the area of more complicated figures by making them the subtraction of two areas (see below).

Discover Pick's theorem for polygons on a geoboard (or dot paper). Pick's theorem is that the Area of a polygon is equal to the number of nails inside the polygon plus half the number of nails on the boundary of the polygon minus 1.

**Feet and hands**

**Draw around your foot.** Measure it's area with plastic squares, blocks, triangles, etc. Compare your answers with those of a partner. Are the comparisons the same ratio? Cut out this foot. Calculate its area on squared paper. Calculate the area of the rectangle that it just fits inside. Is there a relation between these two measures? Check with others' results.

**Work out your mass.** Determine the mass per square cm supported by the sole of your foot. Compare with others. Compare with children and animals.

**Draw around your hand and calculate its area with fingers together and open.**
Compare these areas. Compare these areas with your foot area.

Draw around your body to get your 'half skin' area. Determine the ratio of your hand area, and foot area, to double this 'half skin' area, i.e. your total skin area. How many 'hand areas' are needed to cover your legs, arms, head, etc.? What percentage are these areas of your total skin area? For what medical emergency is this measure used?

TEACHING HINTS:

Be prepared for movement, mess, noise and students discovering their own knowledge when doing many of these activities. The approach should be the same as for Science experiments.
UNIT 7.2 VOLUME AND MASS ACTIVITIES

FOCUS:

This Unit focusses on classroom activities for Volume, capacity and mass.

BACKGROUND:

Volume is the measure of three dimensional space. Mass is a measure of the inertia of an object - the amount of effort to move and stop it moving.

Volume activities are usually associated with filling and packing and mass activities with pressure downwards from gravity.

ACTIVITIES:

Materials required: Pen, paper, a library or a collection of mathematics education resources, materials for the activities as described below.

1. Read the following activities for volume and capacity. In the space that has been left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

VOLUME AND CAPACITY ACTIVITIES

CLASSROOM ACTIVITIES

Stage 1 - Identifying the attribute

Sand and water play, pouring from one container to another, filling and emptying, building sandcastles, building with blocks.

Packing things away, filling a box or carton with material, filling a bucket with junk material. Enclosing space, building a house or a fort.

Stacking materials.

Blowing up a balloon, making cakes (or any type of cooking), mixing ingredients. Acting out, or miming, big and small with children's bodies, enclosing space with children's bodies. Acting out stories like 'three Billygoats Gruff', 'Goldilocks and the three bears', etc.

Immersing objects in water and watching the level rise or the water spill over.
Stage 2 - Comparing and ordering

In simple cases, directly comparing by placing one object inside the other. Use 3D shape puzzles (e.g. soma cubes).

Indirectly comparing and ordering by pouring water, sand, rice, macaroni, etc. from one container to another and seeing which container holds more.

Doing the same thing by immersing objects in water - determining the larger object either by the height the water rises or the amount that is spilled over (in both cases the water level must be the same at the start). Two interesting ways to do this are to mark the increase for one object and to see if the other makes the water go higher or lower, or to fill a bucket to the brim with the first object in it and then to remove this object and to place in a second to see if there is any spill.

Discuss empty and full, large and small, etc.
Order pictures of objects or pictures of partly filled glasses.

Stage 3 - Non-standard units

Packing the container with small objects and counting them, e.g. cubes, lollies, marbles, etc.
Using any tessellating three dimensional object in a similar manner.

Pour water, etc. into containers from smaller containers and count how many small containers full. Use spoons, thimbles, lids, small glasses, egg cups, buckets, etc.

Make a home made 'measuring cylinder', calibrated by regularly spaced lines, from a glass jar which has constant diameter. Use this to measure volume of containers, by pouring water etc. into the 'cylinder' from the containers, and to measure volume of objects, either by immersion and rise in water level (get the level right to start with) or by measuring the overflow.
Stage 4 - Standard units

Construct a cubic cm, a cubic m and a L (see page 81-82 of this book). Look at L bottles and packages (and other regular sizes). Calibrate a 1 L milk carton into 11 mL intervals. Calibrate a glass jar similarly. Classify containers as larger and smaller than 1 L.

Relate mL to cubic cm, L to 10 cm cube and kL to cubic metre.

Use a measuring cylinder. Measure the volume of everyday things such as a cup, a glass, a child’s fist, etc. Measure directly or by immersion. Look at the rise in level or collect the overflow (there are commercial examples of overflow measurers).

Pack containers with cm cubes. Construct rectangular prisms (and cubes) from cm cubes.

Stage 5 - Formulae and applications

Discover the formula for the volume of a rectangular prism directly by constructing prisms and recording lengths, widths, heights and volumes or from the area of a rectangle by constructing prisms of height 1, 2, 3 (and so on) units (see ways A and B on pages 107-108 of the text). Calculate volumes after measuring lengths of sides.

Relate cylinders to prisms to justify the formula for the volume of a cylinder.

Construct a cylinder and a cone of the same height and diameter and pour cones full of sand into the cylinder to show that the volume of a cylinder is three times the volume of a cone.

HINT: Make the cone 1st - cut the cylinder to the cone’s height
2. Read the following activities for mass. In the space left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

MASS ACTIVITIES

CLASSROOM ACTIVITIES

Stage 1 - Identifying the attribute

Hefting objects, trying to lift different objects - "how much can I lift?" (be careful here).
Looking at how hard things press down on the hand.
Placing things on the end (or middle) of a stick and seeing how much the stick bends (or bows).

The balance beam - trying different objects on each side, using a see-saw and the children themselves. Developing the notion of balance and how we can balance and unbalance. Look at questions such as "how can we get this side to go up?".

Using a spring (or a rubber band) to see how long different things stretch out the spring when lifted with it or how much they compress the spring when placed on top of it.

Stage 2 - Comparing and ordering

Comparing two objects by hefting them. Using a beam balance to compare two objects. A homemade beam balance can be constructed from a wire coathanger, string and two margarine containers (see below).

A homemade spring balance or rubber band balance can be constructed from string, a wire hook, a margarine container and rubber bands (see above). This balance can be used to compare mass by seeing which objects stretches out the rubber band the furthest. Mass can also be compared with a flotation measurer (see above). This is a tall container floating in water with sufficient
plasticine in the bottom to prevent it tipping. Objects placed in the container cause it to sink deeper into the water. The object causing the furtherest sinking is the one with the most mass.

Other homemade mass measurers can be made from pieces of wood or plastics or metal as below (rubber bands can also be used in the RHS measurer). How much the material bends or stretches determines the mass of the object.

Stage 3 - Non-standard units

Use beam balance and determine the number of small objects required to balance the object being measured. Small objects that can be used include dusters, ball bearings, marbles, books, KAB units, stones, pencils, etc.

The homemade spring balance can also be placed against a blackboard or a sheet of paper. The container can be weighted with plasticine to make the rubber bands taught. This position can be marked and called 0. Then regular intervals can be marked on the blackboard/sheet going down. When objects are placed in the container, their mass can be read off the position that the rubber band stretching allows the container to move down to.

A similar mass measurer can be made by marking regular intervals on the side of the flotation measurer.

Both the spring balance and the flotation measurer can also be used similar to the beam balance. An object is placed in the container and the position where the container ends up is marked. The object is removed and smaller objects added until the same position is reached.

Stage 4 - Standard units

Construct and heft a g and a kg. Measure out on a balance material to equal 500g, 250g, etc. Children measure their own mass and the mass of
the teacher.

Use a variety of balances and standard masses to measure a variety of everyday objects. Use beam, spring, bathroom and kitchen balances.

Calibrate a homemade spring balance and a homemade flotation measurer by adding 100g masses and marking. Stress that 1 mL of water is extremely close to 1 g in mass and 1 L of water is extremely close to 1 kg.

Measure the water that overflows when an object is added to a flotation measurer floating on top of a full container. The mass of the water overflowing is the same as the mass of the object added. In fact, the object's mass is the same as the volume of the overflowing water.

Stage 5 - Formulae and applications

Mass really requires comparison with other attributes to give rise to many investigations. There are no formula.

Nett and gross mass are worth investigating (e.g. in contents of a jam jar, etc.).

TEACHING: HINTS:

1. The relation between mass and volume is at the basis of much of the above activity. There will be more on this in Unit 7.4. Children should be free to explore these relationships.

2. Obviously, volume and capacity involve water and sand play and will need to be especially planned for. It is a good opportunity to get outside.
UNIT 7.3 TIME, ANGLE, MONEY AND TEMPERATURE ACTIVITIES

FOCUS:
This Unit focusses on classroom activities for time, angle, money/value and temperature.

BACKGROUND:
These four attributes are not as straightforward as length, area, volume and mass in the application of the 5 stages. As stated in the overview to the Chapter (page 147), time-telling and money-handling skills have to be taught separately. There are not the range of activities in each stage.
The stages are applicable to the four attributes and will enrich their teaching if used in planning.

ACTIVITIES:
Materials required: Pen, paper, a library or a collection of mathematics education resources, materials as described in the activities.

1. Read the following activities for time. In the space left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

TIME ACTIVITIES

CLASSROOM ACTIVITIES EXTRA CLASSROOM ACTIVITIES

Time-telling skill

Turns, part turns, quarter and half turns, full turns. Studying the clock face and the two hands, working with non-gear and geared clocks (see pages 102-103 of this book). Clockwise and anti-clockwise turns, relating full turn of large hand to movement of small hand. Relating a full turn to an hour. O'clock, half-past, quarter past and quarter to. Geared clock activities - discovering relation between small hand and large hand movements (see the Nelson article, Arithmetic Teacher, May 1982).

Focussing on fraction of a full turn - angle wheel, rotagrams, rotascan clock (see below).
Counting by fives, telling time in five minute intervals (5 past, 10 past, etc.). Introducing sixtieths in relation to clock face. Introducing notion of minute. Reading time on digital clocks. 24 hour clock. Relating, e.g., 5:43 to 17 minutes to 6.

Worksheets with clock faces, relating drawings of time on face with language and digital time. Relating different ways to use language to tell time (see page 102 of this book).

Introducing the second, using stopwatches. Applications to sport.

Stage 1 - Identifying the attribute

Arranging pictures of events in the order in which they happened, hanging pictures with pegs on a clothes line in this order (with the size of the pictures related to the length of the event and the closeness of the pictures related to how closely they occur next to each other in time).

Associating events with the time of the day, e.g. when do we come to school? Looking at how cycles repeat themselves, e.g. out of bed, dress, eat, clean teeth, etc..

Experiencing activities that take a short or a long time. Waiting while others do things (e.g. hop ten times, run around oval, etc.). Noting when things start and finish. Experiencing a variety of activities which all take the same time. Relating the passing of time to another activity, e.g. shading in a column or row on a blackboard

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Stage 2 - Comparing and ordering

Directly comparing which activity takes longer by "having a race", e.g. time for a ball to stop
bouncing and walking across a room, running around building and writing name and address, etc..
Ensure children start both activities at the same time.

Indirectly comparing activities by timing both events with the same technique, i.e. the amount of sand to run out of a small hole, the length of a taper candle that is burnt down, the length of the column on a blackboard that is shaded, the distance a ball rolls down an incline, the depth a holed container sinks to, etc..

Relating events to a common event, e.g. eating breakfast is longer than a short cartoon on TV but dressing takes less time, etc..

Labelling strips of paper with events, the length of the strip of paper is determined by the length of time the event took.

Stage 3 - Non-standard units

Using a regular action to time events, e.g. counting, pulse, a pendulum's swing, hopping, the roll of a ball bearing in a curved groove, etc. (see pages 44-47 of this book).

Calibrating the regular change in something to time events, e.g. a candle burning down is marked with pins at regular distances (thin taper candles are best), water or sand pouring out a small hole into another container marked into height intervals, a container marked at intervals down its side sinking in water, a ball rolling slowly down an incline marked into distance intervals, etc. (see pages 44-47 of this book).

See activities on pages 62-73 of this book.

Relating time to the movement of the sun.
Constructing a sun dial. Discuss historical time measurers. Discussing terms such as midday, morning, afternoon, evening (use worksheets and relate times to drawings).

Stage 4 - Standard units

Introducing minutes, hours, seconds, clock faces, digital time, 24 hour clock. Determining what children can do in a minute. Working out the
length of a favourite TV program.

Experiencing a minute and other time intervals by doing something for this time. Relating these times to everyday events like brushing teeth or eating, or school lessons, etc.. Turning back and calling out when 20 seconds is up. Doing this while doing something else, e.g. hopping, holding breath, etc.. Choose actions that may change perception of time.

Using stop watches and clocks, etc. to time events. Calibrating a sun dial in hour, e.g. putting a stick in the ground and marking where shadow points when each hour is up.

Relating daily events to the times they commonly occur (use worksheets and join pictures to times). Working out a roster (daily or for a special activity). Relating digital time to clock faces.

Stage 5 - Formulae and applications

There is no formulae. Applications are in relation to other attributes, mainly in the context of rate, e.g. speed, water wasted by a dripping tap, etc..

Many good mathematics process problems are related to clock faces and to timing events.
2. Read the following activities for money/value. In the space left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

ANGLE ACTIVITIES

CLASSROOM ACTIVITIES

Stage 1 - Identifying the attribute

Discussing things that turn, experiencing turning with the children's bodies, following the turns of another ("follow the leader", "simon says", etc.). Relating the turns to the directions to which the body points at the start and end of the turns. Rotagram and angle wheel activities.

Turning the pages of a book, turning geostrips in relation to each other. Giving directions to a blindfolded partner (e.g. "turn right, turn left, turn further, ..", etc.).

Stage 2 - Comparing and ordering

Comparing angles by taking a copy of one angle as a sector of a circle and placing this copy over the other angle. The copy can be made on tracing paper and cut out or by using an angle wheel or rotagram (see below).

An angle can also be copied with two hinged strips joined by string a set distance from the hinge. In this angle copier, the length of the string determines the size of the angle (see above). A compass and a ruler can also copy an angle by measuring the distance between the two rays (or directions) with the ruler along an arc at a fixed radius from the vertex (arc is drawn using compass held at a fixed radius).

Constructing a right angle measurer by folding paper and comparing with angles to classify them as acute or obtuse (see below for fold).
Stage 3 - Non-standard units

Measuring extent of turn by how many small sectors will fit in the angle. Can also get approximate measures by how many of an object can be fitted into the angle at a fixed distance from the vertex, e.g. fingers, pencils, etc. A rotagram, angle wheel or a home-made protractor can be calibrated with regular intervals along an arc and the angle read from these intervals (see below).

Stage 4 - Standard units

Constructing a sector equal to 1 or 2 degrees.
Experiencing the protractor. Measuring common angles.

Using an inclinometer to measure incline (see below for how to make a home-made one from ruler, protractor, string and mass).

Stage 5 - Formulae and applications

Tearing off the corners of polygons to determine the interior angle sum rules for these polygons. Triangulating polygons into triangles for the same purpose (see Geometry text). "Walking" around regular polygons to find relation of number of sides to the exterior angle (the amount that has to be turned in, e.g., $180^\circ$).

Relating angles in regular polygons, relating interior and exterior angles in triangles, relating angles in circles (particularly those from diameters), relating angles in parallel line situations, relating angles between diagonals to the various quadrilaterals, relating angles to tessellation, relating angles in line and rotational symmetry, etc. (all this is in the Geometry text).
3. Read the following activities for temperature. In the space left on the RHS
of the descriptions, write in extra activities. Look through mathematics
education resources in your library for good ideas.

MONEY/VALUE ACTIVITIES

CLASSROOM ACTIVITIES EXTRA CLASSROOM ACTIVITIES

Money-handling skill

Recognition of coins - coin rubbings, drawing
coins, coin posters, etc.

Value of coins - relating coins to each other,
recognising what can be bought for certain coins,
adding and subtracting coins and making change.

Setting up a "shop" - things to buy and sell, play
money, etc. Computation and word problems in
money situations.

Stage 1 - Identifying the attribute

Discussing what things are the most valuable for
them (e.g. "what one thing would you take to be
marooned with, ...", etc.). Discussing why they
value it. How they would show it had value.
Choosing amongst alternatives.

Discussing things that have little value.
Pictures of expensive and cheap things.

Stage 2 - Comparing and ordering

Subjectively comparing objects to see which one is
valued more highly. In directly comparing by
relating to a common object, e.g. I like my toy
more than this pen but the trip to get hamburgers
is better than the toy, etc.

Bartering - discussing how many of one object is a
fair trade for another. Organising the class into
groups all with a different product and discussing
how the tribes would barter one thing for another
(introduces supply and demand).

Stage 3 - Non-standard units

Discussing the use of a common or valued object as
a medium of exchange, e.g. Cowrie shells, oxen or
pigs or horses or other animals, marbles, etc. Discuss historical development of money.

Discuss the talent, an iron ring that had the mass of the gold needed to buy and oxen - became a basis for money and mass.

Stage 4 - Standard units

See money handling skills above.

Play "shop", determine everyday items that can be bought for the coin/note under discussion or multiples of that coin/note.

If possible, involve children in actual buying and selling, e.g. tuckshop activities - organising lunches and lunch money, buying, cooking and making and selling food on days the tuckshop is closed, organising an outing and determining the money needed, etc.

Stage 5 - Formulae and applications

Other than early work on interest and percentage there is little formulae here. There is a huge supply of money problems both routine and process.
4. Read the following activities for angle. In the space left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

TEMPERATURE ACTIVITIES

CLASSROOM ACTIVITIES

Stage 1 - Identifying the attribute

Experiencing a variety of temperatures.
Describing different types of days (e.g. a summer scorcher or a winter frost, etc.). Heating ice until it boils. Pictures of hot and cold.
Reading stories involving temperature (e.g. Goldilocks and the three bears).

Feeling your the temperature of your head after your hand has been in hot water and then after your hand has been in cold water.

Stage 2 - Comparing and ordering

Comparing temperatures directly with the hand, moving hand from one instance to another, the hand will feel the change. Relating temperatures by comparison with to known things, e.g. that one is hotter than my head feels yet this one is not, etc.. Fill cups with water at different temperatures and let children order the cups from warmest to coldest.

Can indirectly relate temperatures by what they do to other things, e.g. the paraffin is runny in that situation but remains solid here, etc..

Constructing a simple temperature measurer with a thin tube and coloured water in a small container. Discovering how the coloured water goes up the tube as the temperature rises.

Discuss hot days and cold days, order pictures that show hot and cold. Discuss summer and winter, what it is common to wear, what is commonly eaten and where is the common vacation area. Keep a weather chart, drawing and describing the weather on each day. Relate this work to other subjects, e.g. Science (effect of heat and cold on humans and animals, also on food and other materials), Language (talk about
feelings and observations concerning different types of weather and its effects on them), Environmental studies (how changes of temperature affect things such as butter, care of pets in hot and cold conditions) and Health (survival in hot and cold situations).

"How does it feel?"

1. Have students place their hands on their cheeks to feel how warm or cool their hands are, then have the children rub their hands together briskly for about 30 seconds, and put them against their cheeks again. Ask the children if their hands are warmer or cooler, after they rubbed them together?

2. Arbitrarily label several spots in the room which have different temperatures for the children to touch; for example, window glass, metal shelf, spot in the sunlight, spot in the shade, and so on. Let the pupils try to determine which spot has more heat and which spot has less heat. Let the students make statements such as, "The metal shelf has more heat than the window glass." Let the children attempt to order the spots by the amount of heat they feel. Point out to the children that the things that have more heat than their hands feel warm or hot to them and vice versa.

3. Obtain two glasses of water from the tap, and let the children feel them to see that they are both cool. Set one in the shade and one in the sun. Thirty minutes later, let the children feel them again. Discuss why one glass now feels warm, whereas the other is still cool.

4. Let two pupil "judges" stand in front of the room. Have 6 or so pupils file past and lay their hands on the "judges" cheeks. The "judges" try to decide who has the coldest/warmest hands.

CLASSROOM ACTIVITIES

Stage 3 - Non-standard units.

The only possibility is to calibrate the measurer above by regular distance intervals along the length of the tube. This could be used to order the cups of different temperature water.

Stage 4 - Standard units

Learn to use a thermometer. Familiarise the children with how temperature change causes changes to the height of mercury in the thermometer. Let children discover this movement of the mercury.

Get a feel for the degree by feeling water that
has been heated 5 degrees, say, in comparison with water from the tap.

Measure the temperature of common things, e.g. ice, boiling water (be careful!), body temperature, etc.. Use a thermometer to find the warmest and coldest spots in the classroom.

Stage 5 - Formulae and applications

There is no formulae, but some interesting investigations can be held around boiling and freezing points, e.g. why does salty water not freeze as readily as ordinary water?, etc..

TEACHING HINTS:

Many excellent suggestions for measurement activities are to be found in other primary subjects other than science. For example, a social studies module on eskimos would have many opportunities for relevant work on temperature.
UNIT 7.4 ACTIVITIES FOR RELATING ATTRIBUTES AND WORKSTATIONS

FOCUS:

This Unit completes the survey of classroom activities for measurement by focussing on activities that relate attributes. It also gives examples of investigatory activities that can be the basis of rotating workstations and examples of measurement experiences for the early childhood years.

BACKGROUND:

Many of the most interesting measurement activities with the greatest problem-solving capacity involve more than one attribute. In fact, they are based on the interrelation between two attributes. For example, mass and volume, two attributes that are commonly confused, interrelate as density with many interesting conclusions. One particularly enticing investigation can be studied with a displacement experiment. If an object is immersed in a full bucket of water, the resulting overflow will equal the volume of the object. If, on the other hand, the object is placed in a container floating on top of a full bucket of water, the resulting overflow will equal the mass of the object (if it does not sink).

These types of activities often involve a longer time than the normal class and an intricate equipment set-up. It is difficult to have multiple copies of these set-ups. One way to overcome this is to organise different experiments and materials in different positions and rotate the children, in groups, through each experiment. This teaching approach is called 'rotating workstations'. Of course, such workstations can be organised for individual attributes as well as situations where more than one attribute is involved.

ACTIVITIES:

Materials required: Pen, paper, a library or other collection of mathematics education resources, materials as required for the activities below.

1. Read the following activities for relating attributes. In the space that has been left on the RHS of the descriptions, write in extra activities. Look through mathematics education resources in your library for good ideas.

ACTIVITIES FOR RELATING ATTRIBUTES

CLASSROOM ACTIVITIES EXTRA CLASSROOM ACTIVITIES

Length/Perimeter —> Area:

Cutting up "fat" rectangles of paper to cover long thin rectangles. Drawing shapes with a fixed
perimeter and determining which shape has the largest area for a given perimeter (it's the circle). (Restricting this to rectangles gives the square as the largest area for the smallest perimeter - this is why cheap houses are square.)

Starting with a fixed number of blocks and making a variety of shapes of the same area and comparing their perimeters.

**Length --> Volume:**

Pouring from long thin containers into short fat containers. Studying packaging in supermarkets. Comparing litre soft drink bottles with MAB blocks (both 1 L).

**Length --> time:**

Looking at speed in stories such as the tortoise and the hare. Measuring how far can walk or run in 1 minute and using this to calculate how long to walk or run 1 km.

**Area --> Volume:**

Comparing surface area with volume and also area of base with volume, particularly in relation to height, e.g. halve the length and breadth of the base, what happens to the height if the volume wishes to stay the same?

*Roll ups (an idea from the The I hate mathematics book)*

Obtain 2 pieces of A4 paper, tape, extra paper and some beans. Roll the two pieces of paper into cylinders, one "longways" and one "shortways" (see below). Add a bottom to each container and stick down. Use the beans to determine which container holds the most?

What is the better container, the short fat one or the tall skinny one?

Make a shape from 5 squares counters. Note the area and perimeter. Make the shape again but double the original scale, and again, but triple. Note the areas and perimeters for all three. Record this information on a table. What is the pattern in the relationship between area and perimeter as the size increases?
Repeat the above for a 9 counter square, doubling and tripling the length and width. What pattern is emerging in the perimeter area relationship?

Start with 3 cubes and make any 3D shape. Double and triple this in scale. Calculate volume and surface area. What is the effect on surface area of doubling and tripling lengths of sides? What is the effect on volume?

Make cubes of side 1, 2, 3, 4, 5 and 6. Calculate area of base, volume and surface area for these cubes. Put these on a table. What pattern emerges? What happens to the surface area to volume ratio? What important consequence does this have for babies?

CLASSROOM ACTIVITIES

Mass $\longrightarrow$ Volume:

Hefting large amounts of low density material (e.g. pillows) and small amounts of high density material (e.g. lead weights). Balancing dusters and bricks. Looking at pictures and discussing big and heavy, small and light, small and heavy, etc.

Investigating density. Using the flotation mass measurer (see Unit 7.2) to determine that the water displaced by a floating object equals the mass of the object and using a displacement volume measurer to determine that the water displaced by an immersed object represents the volume of the object.

Money $\longrightarrow$ Volume:

Discussing what happens when we add more to the pile being valued. Valuing such things as large and small blocks of chocolate, diamonds versus pillows, etc.

2. (1) Complete the following workstation activities as directed by your lecturer.

A. COMPARING AND ORDERING WORKSTATIONS

B. NON-STANDARD UNIT WORKSTATIONS

C. MEASUREMENT INVESTIGATIONS

(2) Collect examples of such activities. Place them on card with a list of materials required and instructions and questions for the activity.
A. COMPARING AND ORDERING WORKSTATIONS

STATION 1: LENGTH

Materials - string, ball, pen and paper.

Use string to determine which is the larger

. the distance around the ball OR
  the length of the blackboard compass (closed)?

. the height of glass cupboard OR the length of blackboard?

. The distance up and down the six flights of stairs outside OR
  the distance from the maths building to the stairs in front
  of the refectory?

STATION 2: AREA

Materials - cardboard carton, paper, scissors, tape, pen and paper.

What has the larger area

. the bottom of the carton OR the base of the chair?

. area covered by the hat OR two A4 sheets?

STATION 3: VOLUME

Materials - sand, containers, pen and paper.

Place the containers in order from largest to smallest volume.

STATION 4: MASS

Materials - coathanger, string, margarine containers, scissors, objects,
  pen and paper.

Construct a balance with the coathanger, string and margarine containers. Use
this balance to place the objects in order from largest to smallest mass.
STATION 5: TIME

Materials - ball, pen and paper.

(1) Which takes longer
   . walking across the room OR tying your shoe laces?
   . time taken for a bouncing ball to stop OR
     writing the heading of this handout?

(2) Make up 5 activities to time (be creative) and place in order.

STATION 6: ANGLE

Materials - paper, scissors, cars, pen and paper.

By cutting paper copies of the angles,
   . determine which can form the sharper angle, your elbow OR your knee.
   . place cars in order from largest to smallest by the angle their front
     door opens.

STATION 7: MONEY/VALUE

Materials - pen and paper.

(1) What box of materials in this room would you most value in your classroom
    next year and why?

(2) Choose six important necessities of a student’s life other than money (e.g.
    shelter, food, etc.) and order them in terms of importance to you. Is
    this the same order that you would spend money on them?

STATION 8: TEMPERATURE

Materials - building, pictures of hot and cold, pen and paper.

(1) Order the pictures from coldest to hottest.

(2) Find five objects of different temperatures in this building and order from
    hottest and coldest.
B. NON-STANDARD UNIT WORKSTATIONS

COMPLETE THE FOLLOWING ACTIVITIES:

1. **Body-length**
   (Unit 3.1, activities 1 and 2)

   Materials - blackboard, body lengths, pen and paper.

   Directions - use the following body-lengths to measure the length of the blackboard, estimate first and complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>ESTIMATE</th>
<th>MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAND SPANS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUBITS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FOOT LENGTHS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Cardboard-area**
   (Unit 3.1, activity 3)

   Materials - large piece of cardboard, blackboard duster, A4 pages, pen and paper.

   Directions - use the following materials to measure the area of the cardboard, estimate first and complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>ESTIMATE</th>
<th>MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>HANDS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BLACKBOARD DUSTER</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4 PAGES</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. **Volume measurer**  
(Unit 3.1, activity 4)

Materials - coffee jar marked with felt pen divisions, water, three containers, pen and paper.

Directions - use the marked coffee jar to measure the volume of the three containers, estimate first and complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>ESTIMATE</th>
<th>MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONTAINER 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONTAINER 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONTAINER 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. **Mass measurer**  
(Unit 3.1, activity 4)

Materials - margarine container connected to a rubber band hung against a page with divisions, three objects, pen and paper.

Directions - use the mass measurer to measure the mass of the three objects, estimate first and complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>ESTIMATE</th>
<th>MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJECT 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBJECT 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBJECT 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. **Tangrams and area**  
(Unit 2.2 - Tangram activities)

Materials - tangram sets, worksheets pages 30 - 32, pen and paper.

Directions - use the tangrams to complete cards C, D, E, F, G and H, estimate first and complete the following table.
6. Pendulums and time
(Unit 3.2, activity 2)

Materials - two pendulums of different string length, pen and paper.

Directions - use the two pendulums to time three activities, estimate first and complete the following table.
C. MEASUREMENT INVESTIGATIONS

Most of these investigations are taken from LeBlanc, J.F. et al (1973) Measurement (A MMP book)
Bloomington: Indiana University,
acknowledged at the beginning of this book.

A. How many of your hands to cover the surface of your body? Is there a relationship between hand area and body area? What is this measure used for? (Hint - its medical).

B. Drop a ball from different heights and measure the height of the bounce. What is the relationship, if any, between these two quantities? Does the relationship vary with the ball? Repeat the task with a ball of different size or elasticity.

C. The bag of scraps is left over from making a garment from material which is 150cm wide. Given the length of material (written on the bag), how much does the garment weigh? (Not including zips etc.)

D. What is the effect on the volume of a cube if you double the length of the sides? How about if you triple the length of each side? What is the effect on the volume of a sphere if you double or triple its radius?

E. What relationships, if any, exist between volume and surface area as size increases. Record results for cubes of sides 1, 2, 3, 4, 5, and 6. What implication does this have for babies?

F. Find 10 measures which might answer the question, "How big are you?" Collect the data for each measure from the members of your group. Do any two measures seem to correlate (i.e., a person who has a large measurement in one measure tends to have a large measurement in the other)? Plot the data for two correlated measures on graph paper. (You might want to get these measures from other members of the class in order to have more data.) Does your graph have an interesting shape?
G. Make a cylinder and a cone which have the same vertical height and the same circular base. What is the relationship between the volume of the cylinder and the volume of the cone? Check your answer by constructing another cylinder-cone pair. Do you know a formula for the volume of a cylinder? If so, what would be the formula for the volume of a cone?

H. Find the surface area of the tennis ball.

I. Find the volume and weight of water wasted in 24 hours by a dripping tap.

J. Determine the dimensions of the rectangle of perimeter 36 cm. that has the greatest area. Use your string and graph paper to experiment. If you are not restricted to rectangles, what shape of perimeter 36 cm. would you say has the greatest area?

K. Make two cylinders from A4 paper, one rolled lengthwise, the other oppositely. Which holds the most volume? Why? What does this indicate for the surface area/volume relationship?

L. Obtain a bucket of water and a deep container. Place plasticine in container until it floats without tipping. Use this flotation measure and a variety of objects to investigate how mass and volume vary. Check that the amount of water that an object displaces on its own is its volume, while the amount it displaces when in the flotation measure is its mass.
3. Workstations where children read instructions are only useful for the middle and upper primary years. In the lower grades, instructions must be verbally given. The simplicity of the equipment does allow some replication of the situations so that all may experience them.

Below are some activities for capacity and mass given as examples of measurement activities for the early grades prepared by Rosemary Irons of Carseldine Campus of Brisbane C.A.E..

**CAPACITY EXPERIENCES FOR EARLY CHILDHOOD**

<table>
<thead>
<tr>
<th>1. Development of Manipulative skills</th>
<th>Allow time for exploring a variety of materials and how to use them.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Awareness of the attribute of capacity</td>
<td>&quot;Packing Up&quot; activities enable the children to further their ideas of capacity - unifix cubes into buckets (shaking them down so they'll fit in).</td>
</tr>
<tr>
<td>3. Ordering containers according to capacity</td>
<td>Fill your bucket with sand. Find one that will hold less sand. Check to see if you are correct. Arrange in order from least to most.</td>
</tr>
<tr>
<td>4. Awareness that containers of different shape may have the same capacity.</td>
<td>Find all the containers which hold just enough water to fill the red tin. Which containers need to be filled three times, with water, in order to fill this bottle?</td>
</tr>
<tr>
<td>5. Measurement of capacity with informal units</td>
<td>Use the biggest spoon to fill the mug. How many spoons of water did you use? Use the yellow vase to fill the bucket. How many vases of water did you use? Find something else that holds the same number of yellow vases.</td>
</tr>
<tr>
<td>6. Choosing appropriate units.</td>
<td>Discussion may be initiated by the teacher posing nonsense questions. &quot;How long do you think it would take to fill the school pool with a spoon?&quot; &quot;What could we use to fill the fish tank?&quot;</td>
</tr>
<tr>
<td>7. Awareness of the need for standard capacity units.</td>
<td>Have the children measure how many jugs fill a bucket then how many cups fill a large bowl. Discuss both answers then ask which one holds the most. Why don't we know? What should we do? Cooking experiences.</td>
</tr>
<tr>
<td>Skill</td>
<td>Objectives</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Free and directed play.</td>
<td>To learn by discovery.</td>
</tr>
<tr>
<td></td>
<td>To develop problem solving skills.</td>
</tr>
<tr>
<td>Awareness of the attribute of mass.</td>
<td>To become aware of experiences which require them to balance equipment, or push, pull, lift, in order to define</td>
</tr>
<tr>
<td>Comparison of two masses.</td>
<td>To develop the understanding of heavy &amp; heavier, light &amp; lighter.</td>
</tr>
<tr>
<td>Understanding of an equal arm balance.</td>
<td>To develop an understanding of balance, does not balance.</td>
</tr>
<tr>
<td>Use of an equal arm balance beam</td>
<td>To realize when two items have the same mass they balance.</td>
</tr>
<tr>
<td></td>
<td>To develop an understanding of unequal masses.</td>
</tr>
<tr>
<td>Skill</td>
<td>Objectives</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Measurement with informal mass units</td>
<td>To use a simple pan balance with multiple informal units to measure the mass of an object.</td>
</tr>
<tr>
<td>Conservation of mass</td>
<td>To develop an awareness that mass remains constant regardless of changes in form.</td>
</tr>
<tr>
<td>Ordering three masses.</td>
<td>To develop the ability to estimate and then measure which of the three objects has the greatest mass, the least mass, and to</td>
</tr>
<tr>
<td>Awareness of the need for a standard mass unit.</td>
<td>To become aware of the need for a more accurate unit of measure.</td>
</tr>
<tr>
<td>The kilogram as a formal unit.</td>
<td>To find various objects that balance against a kilogram mass.</td>
</tr>
</tbody>
</table>
TEACHING HINTS:

Workstations are an excellent way to teach measurement, but they require careful planning. Work with other teachers to develop a collection of tasks to be shared across classes.

These activities, of course, are also an important part of problem solving and should be integrated into the problem solving program.